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Resumé

Estimerede DSGE-modeller er blevet et vigtigt redskab for empirisk funderet makroøkonomisk analyse gennem de seneste år. I denne artikel præsenteres en estimeret DSGE-model for Danmark. Modellen er estimeret ved hjælp af Bayesianske metoder baseret på et datasæt bestående af 23 makroøkonomiske variable. Modellen anvendes i artiklen til at identificere de vigtigste bidragsydere til konjunkturudsvingene i dansk økonomi. Resultaterne indikerer, at stød til udlandet forklarer mere end 50 pct. af udsvingene i dansk BNP. Eksempelvis var det økonomiske tilbageslag, som ramte Danmark i kølvandet på den finansielle krise i 2008, i høj grad drevet af udenlandske faktorer. Udviklingen i udlandet spillede også en vigtig rolle i årene op til krisen. Indenlandske faktorer bidrog dog også i stort omfang til det kraftige økonomiske opsving i Danmark i disse år, mens finanspolitikken ikke var tilstrækkeligt stram gennem opsvinget.

What Drives the Business Cycle in a Small Open Economy? Evidence from an estimated DSGE Model of the Danish Economy*

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Abstract

Estimated DSGE models have become the standard workhorse model for empirically based macroeconomic analysis in recent years. In this paper, we present an estimated DSGE model for Denmark. The model has been estimated using Bayesian methods and a dataset consisting of 23 macroeconomic variables. We use the model to identify the most important determinants of business cycle fluctuations in Denmark. Our results indicate that foreign shocks explain more than 50 pct. of the variation in Danish real GDP. As an example, the recession that hit Denmark in the wake of the recent financial crisis was to a large extent caused by foreign factors. Shocks originating abroad also played an important role in the build-up to the crisis. However, domestic factors also contributed substantially to the boom in Danish GDP in the years before the crisis, while fiscal policy was not sufficiently contractionary during the boom.

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Keywords: DSGE Models, Small Open Economies, Bayesian Estimation.

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1 Introduction

Over the last decade, a large number of policy institutions have adopted a class of structural models for forecasting and policy analysis. These so-called Dynamic, Stochastic General Equilibrium (DSGE) models mostly derive from the New-Keynesian tradition (see Woodford [2003] or Galí [2009] for textbook treatments), and are thus in accordance with state-of-the-art academic research in macroeconomics; see e.g. Blanchard [2009]. Following Smets and Wouters [2003] and Smets and Wouters [2007], the models are typically estimated with Bayesian techniques. In recent years, the DSGE models of many institutions have been improved through the addition of a number of empirically relevant features, most notably frictions in the labor market (e.g., Galí et al. [2011]) and in financial markets (e.g., Christiano et al. [2010]). As a result, a number of central banks now use estimated DSGE models as their primary tool for forecasting and policy analysis.[1]

In contrast, in Denmark there is a longstanding tradition of using large-scale macroeconometric models for these purposes. This includes the MONA model of Danmarks Nationalbank, the ADAM model developed by Statistics Denmark and used by the Ministry of Finance, and the SMEC model of the Danish Economic Councils.[2] While the use of similar models in different policy institutions has a number of advantages in terms of transparency, communication, and model development, traditional macroeconometric models come with a set of problems on their own. One is the lack of forward-looking behaviour by private agents, another is the exposure of these models to the Lucas [1976] critique. This critique is particularly relevant when using the models for policy analysis, and less so when it comes to forecasting. In addition, the vast majority of academic research in the field of monetary macroeconomics uses DSGE models as the theoretical workhorse, whereas macroeconometric models have largely disappeared from the research agenda over the last couple of decades.

In this paper, we present an estimated DSGE model of the Danish economy. The model builds on recent academic research as well as on models developed by other central banks. We model Denmark as a small open economy as in Gali and Monacelli [2005]. While the exchange rate is fixed towards the eurozone, reflecting Denmark's currency peg, we allow for fluctuations in the exchange rate towards the rest of the world. Moreover, the model features a fairly detailed description of fiscal policy, which is the central stabilization tool in the Danish economy. Finally, we model labor market frictions following Galí et al. [2011], introducing a role for involuntary unemployment.

The model is estimated using Bayesian techniques. The estimated parameter values are generally in line with estimates from similar studies. To validate the model, we compare the estimated impulse responses from the model to impulse responses estimated from structural VAR models of the Danish economy. We find that the impulse responses from the DSGE model to shocks to domestic

[1] A non-exhaustive list includes the Riksbank, Norges Bank, and the Bank of England.

[2] One notable exception is the DREAM model, which is, however, mainly a model for the long run.

government spending and to foreign output are roughly in line with the VAR-based evidence. For a shock to the policy rate of the European Central Bank, the DSGE model delivers a quantitative impact on Danish output in line with the data, although the shape of the impulse responses are somewhat different.

We use the estimated model to shed light on the driving forces behind business cycle fluctuations in Denmark. The main finding of this analysis is that foreign shocks are the most important source of movements in Danish GDP. Our variance decomposition shows that for the period 1995-2012 taken together, foreign shocks account for around 50 percent of output fluctuations in Denmark at all frequencies. This result is in contrast to other recent studies of the effects of foreign shocks in small open economies, e.g. Justiniano and Preston [2010] and Adolfson et al. [2007], who - somewhat surprisingly - find that foreign shocks explain less than 5 percent of output fluctuations in Canada and Sweden, respectively. It should be noted, however, that the results of Justiniano and Preston [2010] are a bit of a puzzle in the literature, where different ways of resolving this issue have been proposed. Their results are also in contrast to VAR-based evidence for the Canadian economy, see, e.g., Cushman and Zha [1997].

The main reason for the striking difference between these studies and our results is Denmark's fixed exchange rate towards the euro, which opens up an important channel through which foreign shocks can be transmitted *directly* to the Danish economy. For example, interest rate decisions of the European Central Bank have a direct effect on consumption and investment decisions of Danish households. Indeed, as discussed by Aastveit et al. [2013], part of the explanation for the surprisingly small results obtained by Justiniano and Preston [2010] is the lack of a direct effect of foreign shocks on domestic variables, and the lack of other transmission mechanisms than the international trade channel. A fixed exchange rate resolves these issues.[3] Furthermore, our finding of an important role for foreign shocks is in line with empirical evidence based on VAR-studies for Denmark (see Ravn and Spange [2013]) as well as for other small open economies (see, e.g., Cushman and Zha [1997], or Aastveit et al. [2013]).

We also perform a historical decomposition of deviations in Danish output from its trend for the period 2004-2012. This decomposition confirms the importance of foreign shocks. In particular, the recent economic crisis in Denmark was to a large extent driven by shocks originating abroad. Foreign shocks also contributed to the economic boom in Denmark in the years leading up to the crisis. However, domestic demand also increased substantially in the years preceding the crisis. Finally, our analysis suggests that shocks to the supply side may have played a more important role during the boom than previously thought, possibly due to an inflow of foreign workers, and in part because of the above-mentioned use of macroeconometric models which typically ascribe little importance to the supply side.

Due to Denmark's fixed exchange rate, it is mainly the job of fiscal policy-

[3] It should be noted, however, that while the choice of exchange rate regime is very important in this class of models, the empirical literature tends to find that this choice is less critical for the driving factors behind business cycles.

makers to stabilize the domestic economy. In the years before the crisis, the high levels of domestic and foreign demand called for a tight fiscal policy. Our analysis shows, however, that fiscal policy was not tightened sufficiently during these years so as to counteract the boom. Instead, fiscal policy shocks exerted a neutral or even stimulating effect on the Danish economy in these years.

The remainder of this paper is structured as follows: In section 2, we give a general introduction to fluctuations in the Danish economy over the period we consider. We then describe the model in section 3, and the estimation of the model in section 4. In section 5, we analyze some of the properties of the estimated model, including its ability to match impulse responses from VAR studies. We then perform a historical decomposition of the quarter-to-quarter movements in the output gap in Denmark in section 6. Finally, section 7 concludes. In the appendix, we provide additional details on the model as well as a number of tables and graphical illustrations.

2 Some stylized facts about the Danish economy

We start with a brief tour of the development of the main Danish macroeconomic variables to set the stage for the modeling task which lies ahead. The Danish economy experienced repeated devaluations of the krone during the 1970's and the beginning of the 1980's. This was the case until 1982 when a newly elected government introduced a currency peg vis-a-vis the German mark, which in 1999 was changed to a peg against the euro. The fixed exchange rate regime has succesfully been defended ever since. As can be seen in figure (1), top-left, in our data sample there has only been a noteworthy positive spread between the Danish monetary policy rate and the equivalent in the eurozone on two occasions: During the EMS crisis at the begining of the 1990s and during the recent financial crisis. This confirms the high credibility of the Danish exchange rate regime.

The top-right graph in figure (1) shows two among many challenges in estimating a DSGE model for Denmark. Firstly, the quarterly growth rate of Danish real GDP is quite volatile relative to as an example real GDP in the eurozone. This is not surprising: Denmark is a very small and very open economy. Secondly, we have included data for the financial crisis but it imposes another challenge when having a model with positive trend growth as it raises an end-point problem. That is, the trend in the data is heavily influenced by the financial crisis and we do not yet know whether Danish GDP growth will return to the pre-crisis trend. We address this problem by including forecasted data, as explained in section (4.2).

The bottom-left graph in figure (1) shows year-on-year inflation rates. A necessary condition for a successful fixed-exchange rate regime is that the inflation rate in pegging country, Danmark, on average equals the inflation rate in the, pegged country, the eurozone. Although closely related, there is some evidence that Danish prices have increased slightly more than prices in the eurozone in our sample. Moreover, inflation in Denmark is more volatile than inflation in the eurozone.

Turning to the components on the national accounts balance, private consumption has taken an increasing share of GDP in the Danish economy from the early 2000's until the outbreak of the financial crisis, after which it plummets. Private consumption as a share of GDP has since not recovered to pre-crisis levels. Investment as a share of GDP dropped significantly during the brief international recession in the late 1990's/early 2000's, but recovered strongly until the financial crisis, see figure (1), bottom-right. Like most other countries, Denmark is affected by globalisation, which is reflected in increasing export and import shares, as showed in figure (2), top-left. The trend in exports and imports exceeds the trend in output. In the estimation, this additional growth is removed following, for example, Adolfson et al. [2013].The collapse in world trade during financial crisis is also clear in Danish data. It is noteworthy that during the entire sample, Denmark has had positive net exports.

Even though the nominal Danish exchange rate is fixed towards the euro, that does not imply that the *effective*, trade-weighted Danish exchange rate is constant, which can be seen in figure (2), top-right. These movements clearly indicate that a two-country setup for Denmark and the euro-area is not sufficient. As an example, the depreciation of the effective Danish krone from the onset of the financial crisis is likely to have helped Danish exporters survive the meltdown of global trade. We include these effects in the model through a three-country setup: Denmark, the eurozone, and the rest of the world. The rest of the world consists of the weighted sum of Denmark's trading partners excluding of course the trading partners which reside within the eurozone.

The Danish labour market has since the early 1990's experienced profound structural reforms. This is reflected in a decrease in the natural rate of unemployment and a corresponding decrease in actual unemployment, as can be observed in figure (2), bottom-right. In the same period the real wage has increased more or less with output growth, while increased above trend during the boom before the financial crisis. We have not attempted to incorporate this downward trend in structural unemployment, which instead is left for future research.

Finally, turning to public debt and expenditures, Denmark's public finances displayed a fairly large primary surplus (relative to GDP) during the years before the crisis. This turned into a large deficit after the crisis, as seen in figure (2), bottom-left. One reason behind the worsening of public finances is the expansionary fiscal policy conducted in order to mitigate the effects of the financial crisis. This can easily be seen from the large increase in the public consumption to GDP ratio starting from 2009, although much of the increase in this ratio is due to the drop in output. We will come back to the role of fiscal policy during the boom-bust cycle around the financial crisis. First we need to set up a model which can explain the movements in Danish data as presented in this section.

3 The Model

This section sets up the model. The model is closely related to a number of existing medium-sized DSGE models like the Christoffel et al. [2008], Adolfson et al. [2007], Burriel et al. [2010]. The main building blocks of these models are, however, modified to reflect key aspects of the Danish economy as set out in the previous section. Denmark is a small open economy with a fixed exchange rate against the euro. In the model it is assumed that this regime is 100 percent credible. Effectively, this amounts to an assumption that Denmark is part of a currency union with the eurozone. However, that does not imply that exchange rate effects do not play a role in the determination of the Danish business cycle. As an example, the second and third most important trading partners are outside the eurozone (Sweden and U.K.). Consequently, Denmark has seen some fluctuations in the effective exchange rate and the model needs to reflect that.

The model therefore puts Denmark inside a currency union with the eurozone but also allows for trade with what will be denoted Rest-of-the-World, RoW, which consists of the Danish trading partners excluding countries within the eurozone. It is assumed that the exchange rate vis-a-vis the RoW is floating. The two foreign countries, the eurozone and RoW, are assumed to be exogenously given and independent of each other and especially of Denmark reflecting the small open economy assumption.

The problem of the household sector is setup in section (3.2), production is presented in section (3.3) and (3.4) while the consumers choice between home produced goods and foreign produced goods is presented in section (3.5). Fiscal and monetary policy are presented next in section (3.6), and the labour market in section (3.7). Finally, the foreign economies are described in section (3.8), while section (3.9) and (3.10) present the exports and imports sectors.

3.1 Trends

Fundamentally, there are two ways of dealing with the presence of non-stationary data in an estimated DSGE model. One is to write a stationary model, and detrend all non-stationary variables before mactching them to their model counterparts. The other option is to introduce growth in the relevant variables in the model, so as to be able to estimate the model using the non-filtered, nonstationary data series. In recent years, the latter approach has become best practice in the literature, not least because the process of de-trending variables that may have different trend growth rates is complicated and involves a loss of information. For this reason, we introduce growth in our model.

The first step is to identify the relevant trends in the data. Many recent studies based on US data include two trends in the model; a total factor productivity (TFP) trend to account for the growth rate in output, and an investment-specific trend to account for the continuous decline in the relative price of investment goods, such as computers, in terms of consumptions goods.[4] We follow this

[4]See among others the studies by Justiniano et al. [2011], Christiano et al. [2013] and Liu

practice after confirming that the same two trends are present in Danish data for our sample period. We can write the overall growth rate of the economy as:

$$d\Gamma_t = \left(dA_t dZ_t^{\alpha}\right)^{\frac{1}{1-\alpha}}, \tag{1}$$

where dA_t and dZ_t denote the growth rate of TFP and the (inverse) relative price of investment, respectively. Finally, while the share of imports and exports to GDP has shown an upward trend over our sample period we have decided to detrend these variables, so that the data for imports and exports used in the estimation follow the same trend as domestic GDP, see also section (2). This greatly simplifies the modeling task as concerns the import and export sectors.

3.2 Household Sector

The problem of the representative household is to choose consumption, C_t, holdings of domestic, B_t^{DK}, and international, B_t^I, real bonds, capital, K_t, capital utilization, u_t, and the level of investment, I_t, so as to maximize its stream of discounted future utility, which is given by:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{1}{con_t} \log\left(C_t - hC_{t-1}\right) - \eta_N \chi_t O_t \int_0^1 \frac{N_t\left(i\right)^{1+\phi}}{1+\phi} di \right), \tag{2}$$

where $0 < \beta < 1$ is the discount factor, $h > 0$ measures the degree of (external) habit formation in private consumption. The second term in the utility function denotes disutility of labor. We will define the variables and parameters associated with this in subsection (3.7). Finally, con_t is a shock to the household's preference for consumption today versus tomorrow, which is given by:

$$\frac{con_t}{\overline{con}} = \left(\frac{con_{t-1}}{\overline{con}}\right)^{\rho_{con}} \exp \varepsilon_t^{con}, \tag{3}$$

where $\overline{con} > 0$, $0 < \rho_{con} < 1$, and where ε_t^{con} is an i.i.d. stochastic process with mean zero and variance σ^{con}. Utility maximization is subject to the following budget constraint:

$$\begin{aligned}
& \left(1+\tau_t^{VAT}\right)\frac{P_t^C}{P_t}C_t + \frac{P_t^I}{P_t}I_t + B_t^{DK} + B_t^I + T_t \\
= \quad & \Pi_t + \left(\left(1-\tau_t^K\right)r_t^K u_t cap_t + \tau_t^K \delta^K - z^u\left(u_t cap_t\right)\right)K_{t-1} + \frac{R_{t-1}B_{t-1}^{DK}}{\pi_t^{DK}} + \\
& + \frac{R_{t-1}^{ECB}\exp(-\psi_d\left(\frac{B_{t-1}^I}{Y_t} - \frac{\overline{B}^I}{\overline{Y}}\right))\left(\frac{RPD_t}{\overline{RPD}}\right)B_{t-1}^I}{\pi_t^{DK}} - \frac{\tau_t^B B_{t-1}^{DK}\left(R_{t-1}-1\right)}{\pi_t^{DK}} + \\
& + \left(1-\tau_t^n\right)w_t N_t + \kappa_U w_t U_t^N,
\end{aligned} \tag{4}$$

where P_t is the overall price level to be defined below, T_t denotes real lump-sum taxes, and Π_t is the profits obtained from firms in the intermediate goods sector.

et al. [2013].

Moreover, r_t^K is the real rental rate on capital, and R_t and R_t^{ECB} denote the Danish and the foreign risk-free rate of interest. Y_t is output, while N_t denotes hours worked, with w_t representing the corresponding real wage rate, $w_t \equiv \frac{W_t}{P_t}$. U_t^N is the unemployment rate, to be defined later. We let τ_t^{VAT}, τ_t^K, τ_t^B and τ_t^n be the tax rates on consumption (i.e., a VAT), capital, bond returns and labor, while $0 < \kappa_U < 1$ is the compensation rate in unemployment benefits. $\delta^K > 0$ is the capital depreciation rate.

We assume that if the ratio of foreign debt to GDP exceeds its steady state level, Danish households will have to pay a risk premium on top of the interest rate set by the ECB. This reflects that foreign investors will be less willing to hold Danish debt. In turn, the higher interest rate will make it less attractive for Danish households to borrow abroad, so that eventually the debt-to-output ratio will return to its steady state level. In this respect, $\psi_d > 0$ measures the sensitivity of the risk premium with respect to the net level of holdings of foreign bonds, or equivalently, Denmark's net foreign asset position. The assumption of a risk premium on foreign bonds is only made to ensure a stationary model as in Schmitt-Grohé and Uribe [2003]. Without such an assumption it would be possible for the consumers to borrow indefinitely in the international bond market and consume the proceeds.

We assume that each household does not internalize the effects on Denmark's net foreign asset position, and thus on the risk premium, of changes in its individual international borrowing or lending. We let RPD_t denote a shock to the risk premium. This shock evolves as:

$$\frac{RPD_t}{\overline{RPD}} = \left(\frac{RPD_{t-1}}{\overline{RPD}}\right)^{\rho_{RPD}} \exp \varepsilon_t^{RPD}, \tag{5}$$

where $\overline{RPD} = 1$, $0 < \rho_{RPD} < 1$, and where ε_t^{RPD} is an i.i.d. stochastic process with mean zero and variance σ^{RPD}.

The degree of capital utilization is measured by the variable u_t, and is subject to the capital utilization shock cap_t, i.e. an exogenous shock that changes the degree of capital utilization. The function $z^u(u_t cap_t)$ measures the cost of changing the degree of capital utilization, which we assume takes on the following functional form:

$$z^u(u_t cap_t) = c_1(u_t cap_t - \overline{u}) + \frac{c_2}{2}(u_t cap_t - \overline{u})^2, \tag{6}$$

where $c_1, c_2 > 0$ are parameters, and $\overline{u}$ is the steady state level of capital utilization, which we set to 1.[5] The utilization shock cap_t evolves according to:

$$\frac{cap_t}{cap} = \left(\frac{cap_{t-1}}{cap}\right)^{\rho_{cap}} \varepsilon_t^{cap},$$

where $0 < \rho_{cap} < 1$, $cap = 1$ is the steady state value of the shock process, and ε_t^{cap} is an i.i.d. normal shock.

[5] When we solve the model, we then need to scale the tax deduction from capital depreciation and the utilization cost in the budget constraint with the trend growth of investment-specific technology so as to ensure that these are not eroded over time.

The stock of capital evolves as follows:

$$K_t = \left(1 - \delta^K\right) K_{t-1} + (1 - S_t) Z_t^T I_t, \tag{7}$$

where $S_t = \frac{\kappa_I}{2}\left(\frac{I_t}{I_{t-1}} - \gamma^I\right)^2$ is the investment adjustment cost function, with the parameter $\kappa_I > 0$ measuring the cost of changing the investment level, and where $\gamma^I > 0$ denotes the steady state growth rate of investment. Z_t^T is a transitory investment-specific technology shock, which evolves according to:

$$\frac{Z_t^T}{\overline{Z}^T} = \left(\frac{Z_{t-1}^T}{\overline{Z}^T}\right)^{\rho_Z} \exp \varepsilon_t^{Z_T}, \tag{8}$$

with $\overline{Z}^T > 0$, $0 < \rho_Z < 1$, and where ε_t^Z is an i.i.d. stochastic process with mean zero and variance σ^Z. Moreover, the model features a permanent investment-specific technology shock Z_t^P, so that $Z_t = Z_t^T Z_t^P$. [6] The permanent component follows the process:

$$\frac{Z_t^P}{Z_{t-1}^P} = \lambda_{zt}, \tag{9}$$

where, in turn,

$$\frac{\lambda_{zt}}{\overline{\lambda_z}} = \left(\frac{\lambda_{zt-1}}{\overline{\lambda_z}}\right)^{\rho_{\lambda_Z}} \exp \varepsilon_t^{Z_P}.$$

Thus, λ_{zt} denotes the time t growth rate of investment-specific technology, while $\overline{\lambda_z} > 0$ is the steady state growth rate. $\varepsilon_t^{\lambda_Z}$ is an i.i.d. stochastic process with mean zero and variance σ^{λ_Z}, while $0 < \rho_{\lambda_Z} < 1$.

The first-order conditions related to the utility maximization problem of the household are as follows:

$$\frac{P_t^C}{P_t}\frac{\lambda_t}{con_t} = \frac{1}{(C_t - hC_{t-1})\left(1 + \tau_t^{VAT}\right)}, \tag{10}$$

$$\lambda_t = \beta \frac{E_t \lambda_{t+1}}{E_t \pi_{t+1}^{DK}} R_t^{ECB} \exp(-\psi_d \left(\frac{B_t^I}{Y_t} - \frac{\overline{B}^I}{\overline{Y}}\right)) \left(\frac{RPD_t}{\overline{RPD}}\right), \tag{11}$$

$$\begin{aligned} Q_t = \beta E_t[\frac{\lambda_{t+1}}{\lambda_t}(r_{t+1}^K(1 - \tau_{t+1}^K)u_{t+1}cap_{t+1} \\ + \delta^K \tau_{t+1}^K - z^u(u_{t+1}cap_t) + (1 - \delta^K)Q_{t+1})], \end{aligned} \tag{12}$$

$$\left(1 - \tau_t^K\right) r_t^K = z^{u\prime}\left(u_t cap_t\right), \tag{13}$$

$$\frac{P_t^I}{P_t} = Q_t Z_t^T \left[1 - S_t - S_t' I_t\right] + \beta E_t \left[Q_{t+1} Z_{t+1}^T \frac{\lambda_{t+1}}{\lambda_t} S_{t+1}' I_t \left(\frac{I_{t+1}}{I_t}\right)^2\right]. \tag{14}$$

[6] We point out for clarity that the total investment shock, Z_t, affects the economy through equation (7). We have however written the model in detrended form and consequently only the part of Z_t which is related to the transitory part, Z_t^T, shows up in (7).

Here, we let $Q_t \equiv \frac{\mu_t}{\lambda_t}$ denote the price of installed capital, which differs from the price of new capital (i.e., the price of investment) due to the presence of investment adjustment costs. λ_t and μ_t denote the Lagrange multipliers associated with the budget constraint and the law of motion for capital, respectively, in the optimization problem. Moreover, given the functional form for $z^u(u_t cap_t)$, it follows that $z^{u\prime}(u_t cap_t) = c_1 + c_2(u_t cap_t - 1)$, while for S_t, we obtain that $S_t' = \frac{\kappa_I}{I_{t-1}}\left(\frac{I_t}{I_{t-1}} - \gamma^I\right)$.

3.3 Intermediate Goods Producers

There is a continuum (of unit length) of firms in the intermediate goods sector, each of which operates under monopolistic competition. These firms are owned by the household. Each firm j uses private and public capital as well as labor to produce a firm-specific output according to the following production function:

$$Y_t(j) D_t = A_t^T \left(\overline{K}_{t-1}(j)^{1-\eta}\left(K_{t-1}^G\right)^{\eta}\right)^{\alpha} (N_t(j))^{1-\alpha}, \tag{15}$$

where $\alpha, \eta > 0$ are parameters, $\overline{K}_t(j) = u_t cap_t K_t(j)$ is the effective capital stock being utilised in a given period, D_t is a measure of price dispersion as described below, and A_t measures aggregate total factor productivity (TFP). It is assumed that A_t consists of two terms; a transitory component A_t^T, and a permanent component A_t^P, so that $A_t = A_t^T A_t^P$. The transitory component evolves according to:

$$\frac{A_t^T}{\overline{A}^T} = \left(\frac{A_{t-1}^T}{\overline{A}^T}\right)^{\rho_A} \exp \varepsilon_t^{A_T}, \tag{16}$$

with $\overline{A}^T > 0$, $0 < \rho_A < 1$, and where $\varepsilon_t^{A_T}$ is an i.i.d. stochastic process with mean zero and variance σ^A. The permanent component follows the process:

$$\frac{A_t^P}{A_{t-1}^P} = \lambda_{At}, \tag{17}$$

where, in turn,

$$\frac{\lambda_{At}}{\overline{\lambda_A}} = \left(\frac{\lambda_{At-1}}{\overline{\lambda_A}}\right)^{\rho_{\lambda_A}} \exp \varepsilon_t^{A_P}, \tag{18}$$

with λ_{At} measuring the growth rate in aggregate technology or TFP, while λ_A is the steady state growth rate, $0 < \rho_{\lambda_A} < 1$, and where $\varepsilon_t^{A_P}$ is an i.i.d. stochastic process with mean zero and variance σ^{λ_A}. [7]

The problem of each firm is to maximize its profits subject to the production function. This problem gives rise to the following first-order conditions, where we have dropped the j's for simplicity:

$$r_t^K = \frac{\alpha Y_t mc_t}{u_t cap_t K_{t-1}}, \tag{19}$$

[7] As for permanent investment shock, Z_t, we point out for clarity that the total productivity shock, A_t, affects the economy through equation (15). We have however written the model in detrended form and consequently only the part of A_t which is related to the transitory part, A_t^T, shows up in (15).

$$\left(1+\tau_t^n\right) w_t = \frac{\left(1-\alpha\right) Y_t mc_t}{N_t}, \tag{20}$$

where mc_t is the marginal cost of production, which is identical to the Lagrange multiplier associated with the production function in the optimization problem.

We introduce sticky prices into the model by assuming that intermediate goods firms are subject to staggered price setting. In particular, following Calvo [1983] each firm is only allowed to change its price in any given period with probability $(1-\theta_P) < 1$. Since all firms are identical ex ante, this implies that only a fraction $(1-\theta_P)$ of firms will reset their price each period. Of the remaining θ_p firms, we allow a fraction Γ_P to index their price to the steady state rate of inflation, $\bar{\pi}$, while the remaining fraction of firms keep their price unchanged. When a given firm is allowed to re-optimize its price, it solves a dynamic optimization problem, taking into account that the price it sets is likely to prevail for $\frac{1}{1-\theta_p}$ periods. We can write the resulting first-order condition as:

$$\widetilde{P}_t\left(j\right) = \frac{\epsilon_t^P}{\epsilon_t^P - 1} \mathrm{E}_t \sum_{s=0}^{\infty} \left(\beta\theta_P\right)^s \frac{\lambda_{t+s}}{\lambda_t} \frac{Y_{t+s}\left(j\right) mc_{t+s} P_{t+k}}{Y_{t+s}\left(j\right)}, \tag{21}$$

where $\widetilde{P}_t\left(j\right)$ is the price set by intermediate firm j if it is allowed to change its price in period t. As all firms are identical, this price will be the same for all firms. Note also that we use the stochastic discount factor of households, as these are the owners of the firms. Finally, ϵ_t^P is the elasticity with which final goods producers substitute between different varieties of the intermediate good, and is given by:

$$\left(\frac{\epsilon_t^P}{\epsilon^P}\right) = \left(\frac{\epsilon_{t-1}^P}{\epsilon^P}\right)^{\rho_{\epsilon^P}} \exp \varepsilon_t^{e^P}, \tag{22}$$

where $\varepsilon_t^{e^P}$ is an i.i.d. stochastic process with mean zero and variance σ^{ϵ^P}, and where $0 < \rho_{\epsilon^P} < 1$. $\epsilon^P > 0$ measures the steady state elasticity of substitution. We can then write the evolution of the aggregate price index as

$$P_t = \left[\theta_P \left(\bar{\pi}^{\Gamma_P} P_{t-1}\right)^{1-\epsilon_t^P} + \left(1-\theta_P\right) \left(\widetilde{P}_t\right)^{1-\epsilon_t^P}\right]^{\frac{1}{1-\epsilon_t^P}} \tag{23}$$

highlighting that the share $(1-\theta_P)$ of prices are reset in each period. Finally, D_t measures the loss associated with price dispersion, and is given by

$$D_t = \left(1-\theta_P\right) \left(\widetilde{P}_t\right)^{-\epsilon_t^P} + \theta_P \left(\pi_t\right)^{\epsilon_t^P} D_{t-1}. \tag{24}$$

and where π_t is the domestic inflation rate of the Danish producer price index.

3.4 Final Goods Producers

Firms in the final goods sector operate under perfect competition. They collect a variety of intermediate goods and repackage these into a final good to be used for consumption or investment. In doing so, they solve a cost minimization problem by choosing intermediate input goods so as to produce the final output, Y_t, at the lowest possible price. Final goods producers aggregate intermediate goods according to:

$$Y_t = \left(\int_0^1 Y_t\left(j\right)^{\frac{\epsilon_t^P - 1}{\epsilon_t^P}} dj \right)^{\frac{\epsilon_t^P}{\epsilon_t^P - 1}} . \tag{25}$$

We can write the price index of domestically produced final goods as:

$$P_t = \left(\int_0^1 P_t\left(j\right)^{1-\epsilon_t^P} dj \right)^{\frac{1}{1-\epsilon_t^P}} , \tag{26}$$

where $P_t\left(j\right)$ is the price set by intermediate goods firm j.

3.5 Final Consumption and Investment Goods

We assume that households combine domestically, C_t^{DK}, and foreign, C_t^F, produced goods into the final composite consumption good, C_t, according to a constant elasticity of substitution (CES) technology:

$$C_t = \left(\vartheta_c^{\frac{1}{\upsilon_c}} \left(C_t^{DK}\right)^{1-\frac{1}{\upsilon_c}} + \left(1-\vartheta_c\right)^{\frac{1}{\upsilon_c}} \left(\left(1-\chi_t^C\right) C_t^F\right)^{1-\frac{1}{\upsilon_c}} \right)^{\frac{1}{1-\frac{1}{\upsilon_c}}} , \tag{27}$$

where $\upsilon_c > 1$ measures the elasticity of substitution between foreign and domestic goods, and $\vartheta_c > 0$ measures the steady state share of foreign and domestic goods in the consumption basket, and thus also the degree of home bias in consumption. Moreover, we follow Erceg et al. [2000] and Christoffel et al. [2008] and assume that there is a cost to adjusting the share of imported consumption goods, represented by the function χ_t^C, which is given by:

$$\chi_t^C = \frac{\chi_C}{2} \left(\frac{\frac{C_t^F}{C_t}\omega_t^I}{\frac{C_{t-1}^F}{C_{t-1}}} - 1 \right)^2 , \tag{28}$$

with $\chi_C > 0$ measuring the adjustment cost, and where ω_t^I is an import shock, which follows the process:

$$\left(\frac{\omega_t^I}{\overline{\omega}^I} \right) = \left(\frac{\omega_{t-1}^I}{\overline{\omega}^I} \right)^{\rho_{\text{Im}}} \exp\left(\varepsilon_t^{\text{Im}}\right) , \tag{29}$$

with $\overline{\omega}^I > 0$, $0 < \rho_{\text{Im}} < 1$, and where $\varepsilon_t^{\text{Im}}$ is an i.i.d. stochastic process with mean zero and variance σ^{Im}.

As in Erceg et al. [2000] , the optimal composition of final consumption is found by choosing the values of C_t^{DK} and C_t^F that solve a cost-minimization problem subject to (27). The two resulting first-order conditions are:

$$\frac{P_t^{DK}}{P_t^C} = \left(\frac{\vartheta_c}{C_t^{DK}} \right)^{\frac{1}{\upsilon_c}} \left(\vartheta_c^{\frac{1}{\upsilon_c}} \left(C_t^{DK}\right)^{1-\frac{1}{\upsilon_c}} + \left(1-\vartheta_c\right)^{\frac{1}{\upsilon_c}} \left(\left(1-\chi_t^C\right) C_t^F\right)^{1-\frac{1}{\upsilon_c}} \right)^{\frac{1}{\upsilon_c - 1}} ,$$

$$\frac{P_t^F}{P_t^C} = (\frac{(1-\vartheta_c)}{(1-\chi_t^C)C_t^F})^{\frac{1}{\nu_c}}(1-\chi_t^C-(\chi_t^C)' C_t^F)$$

$$(\vartheta_c^{\frac{1}{\nu_c}}(C_t^{DK})^{1-\frac{1}{\nu_c}}+(1-\vartheta_c)^{\frac{1}{\nu_c}}(C_t^F(1-\chi_t^C))^{1-\frac{1}{\nu_c}})^{\frac{1}{\nu_c-1}}, \quad (30)$$

which can be combined to yield:

$$\frac{C_t^{DK}}{C_t^F} = \frac{\vartheta_c}{1-\vartheta_c}\left(\frac{P_t^F}{P_t^{DK}}\right)^{\upsilon_c}\left(1-\chi_t^C\right)\left[1-\chi_t^C-\left(\chi_t^C\right)' C_t^F\right]^{-\upsilon_c}, \quad (31)$$

where P_t^{DK} and P_t^F denote the price of domestic and foreign goods, respectively. Note that in the absence of adjustment costs, the optimal composition would depend only on the relative price, the elasticity of substitution and the steady state consumption shares.

Likewise, firms combine foreign and domestic investment goods into a final investment good using a similar CES technology:

$$I_t = \left(\vartheta_I^{\frac{1}{\upsilon_I}}\left(I_t^{DK}\right)^{1-\frac{1}{\upsilon_I}}+(1-\vartheta_I)^{\frac{1}{\upsilon_I}}\left(\left(1-\chi_t^I\right)I_t^F\right)^{1-\frac{1}{\upsilon_I}}\right)^{\frac{1}{1-\frac{1}{\upsilon_I}}}, \quad (32)$$

where the parameters are defined as above. The adjustment cost function χ_t^I is defined similarly to that for consumption goods, while the import shock is the same. Cost minimization by firms therefore implies that:

$$\frac{I_t^{DK}}{I_t^F} = \frac{\vartheta_I}{1-\vartheta_I}\left(\frac{P_t^F}{P_t^{DK}}\right)^{\upsilon_I}\left(1-\chi_t^I\right)\left[1-\chi_t^I-\left(\chi_t^I\right)' I_t^F\right]^{-\upsilon_I}. \quad (33)$$

Finally, we can write the relative prices of consumption and investment goods as follows:

$$\frac{P_t^C}{P_t} = \left(\vartheta_c+(1-\vartheta_c)\left(\frac{\frac{P_t^F}{P_t^{DK}}}{1-\chi_t^C-\left(\chi_t^C\right)' C_t^F}\right)^{1-\upsilon_c}\right)^{\frac{1}{1-\upsilon_c}}, \quad (34)$$

$$\frac{P_t^I}{P_t} = \left(\vartheta_I+(1-\vartheta_I)\left(\frac{\frac{P_t^F}{P_t^{DK}}}{1-\chi_t^I-\left(\chi_t^I\right)' I_t^F}\right)^{1-\upsilon_I}\right)^{\frac{1}{1-\upsilon_I}}, \quad (35)$$

while the related relative inflation rates are defined as:

$$\pi_t^C = \frac{\frac{P_t^C}{P_t}\pi_t}{\frac{P_{t-1}^C}{P_{t-1}}}, \quad (36)$$

$$\pi_t^I = \frac{\frac{P_t^I}{P_t}\pi_t \frac{1}{Z_t}}{\frac{P_{t-1}^I}{P_{t-1}} \frac{1}{Z_{t-1}}}. \tag{37}$$

We point out that PPP does not hold in this model due to the presence of home bias. That is, for equal cost of buying a basket consisting of goods produced at home versus a basket consisting of goods produced abroad, the Danish household would prefer the home basket.

3.6 Fiscal and monetary policy

The role of the public sector in the model is to raise taxes to be used for public consumption, public investment, and transfers. Public consumption, C_t^G, evolves according to:

$$\frac{C_t^G}{\overline{C}^G} = \left(\frac{C_{t-1}^G}{\overline{C}^G}\right)^{\rho_G} \exp\left(\varepsilon_t^G\right), \tag{38}$$

where ε_t^G is an i.i.d. stochastic process with mean zero and variance σ^G, $0 < \rho_G < 1$, and where $\overline{C}^G$ is given by:

$$\overline{C}^G = G^Y Y \tag{39}$$

where Y denotes total steady state output, and G^Y is the steady state share of government spending of goods and services produced by the intermediate goods producers and public production. As for government investments, we assume that these are implemented with a lag. Specifically, we assume that an investment that is decided on in period t can only be initiated in period $t + M$ and is finalized in period $t+N$. In other words, we allow for *time to build* as well as *time to plan* as in Leeper et al. [2010]. To this end, we need to distinguish between planned public investment denoted by $I_t^{G,B}$ and implemented public investment denoted by I_t^G. Planned public investment evolves according to:

$$\frac{I_t^{G,B}}{\overline{I}^G} = \left(\frac{I_{t-1}^{G,B}}{\overline{I}^G}\right)^{\rho_{IG}} \exp\left(\varepsilon_t^{IG}\right), \tag{40}$$

where ε_t^{IG} is an i.i.d. stochastic process with mean zero and variance σ^{IG}, $\overline{I}^G$ is the steady state level of government investment, and $0 < \rho_{IG} < 1$. Due to our assumption of time to build, implemented investment only adds to the stock of public capital with a lag:

$$K_t^G = \left(1 - \delta^G\right) K_{t-1}^G + I_{t-N}^{G,B}, \tag{41}$$

where $\delta^G > 0$ is the depreciation rate of public capital, and N is the number of periods it takes from an investment project is decided upon and until the investment is finalized. Note that investment-specific technology shocks also affect the accumulation of the public capital stock. This ensures a stable long-run relationship between the size of the public and the private capital stock along the balanced growth path.[8] Moreover, to take into account that planned investments affect the actual investment level (and hence, economic activity) with a lag, we let actual public investment be given by:

$$I_t^G = \sum_{i=M}^{N-1} \phi_i^I I_{t-i}^{G,B}, \tag{42}$$

with $\phi_i^I > 0$, and where M is the number of periods that pass from a project is decided on until it is initiated. I_t^G is thus a measure of all ongoing government investment projects at time t.

On the revenue side, the government raises five different types of taxes: A labor income tax, τ_t^N, a capital income tax, τ_t^K, a value added tax, τ_t^{VAT}, a tax on domestic bond returns, τ_t^B, and a lump-sum tax T_t. By adjusting the tax rates, the government ensures that its intertemporal budget constraint, to be presented below, is always satisfied. This is done via the following type of tax rule:

$$\frac{X_t}{\overline{X}} = \left(\frac{X_{t-1}}{\overline{X}}\right)^{\rho_{tX}} \varepsilon_t^{tX} \left(\frac{B_{t-1}/Y_{t-1}}{\omega^D}\right)^{(1-\rho_X)e_X^{aux}\zeta_X},$$

for $X = \left\{\tau_t^N, \tau_t^K, \tau_t^{VAT}, \tau_t^B, T_t\right\}$. Here, $\overline{X}$ is the steady state value of X, while $0 < \rho_{tX} < 1$. ε_t^{tX} is a white noise shock associated with shocks to each tax rate X. Moreover, $\zeta_X > 0$ measures how strongly each fiscal instrument reacts to deviations of the debt-to-GDP ratio from its long-run target value, ω^D, reflecting that if the debt-to-GDP ratio overshoots its long-run target, one or more of the tax rates will eventually have to be raised. Finally, the dummy variable e_X^{aux} essentially switches the adjustment term on or off. We can set this to zero in order to undertake simulation experiments in which the government only starts raising taxes after a certain number of periods.

We are now ready to present the government's intertemporal budget constraint, which takes the following form:

$$B_t^{DK} + TR_t = \frac{R_{t-1}}{\pi_t^{DK}} B_{t-1}^{DK} + G_t + w_t U_t^N \kappa_U, \tag{43}$$

where we have defined tax revenues TR_t and government expenditure G_t as:

$$TR_t = T_t + \tau_t^{VAT} \frac{P_t^C}{P_t} C_t + \tau_t^K \left(r_t^K u_t cap_t - \delta^K\right) K_{t-1} + \tau_t^N w_t N_t + \tau_t^B \frac{R_{t-1} - 1}{\pi_t^{DK}} B_{t-1}^{DK}, \tag{44}$$

$$G_t = C_t^G + \frac{P_t^I}{P_t} I_t^G. \tag{45}$$

Moreover, recall that κ^B denotes unemployment benefits, while U_t is the unemployment rate in the model, to which we return in the following subsection.

[8] As we shall see, the growth in investment-specific technology is related to the negative trend in the relative price of investment goods such as high-tech products, IT, software etc. Since many public investments also comprise such products, it seems reasonable to assume that public investments are also affected by the negative trend in the relative price of these.

Finally, we assume that only 20 percent of the public debt needs to be refinanced in each period, so that changes in the interest rate set by the ECB only has a 20 percent impact on the interest rate on public debt. Moreover, we assume that the interest rate at which the government borrows will increase if the ratio of government debt to output exceeds its steady state level. This reflects that the household sector, which buys the bonds issued by the Danish government, will demand a risk premium if they are to hold the bonds. In technical terms, this gives rise to the following condition:

$$\frac{R_t}{\overline{R}} = \left(\frac{R_{t-1}}{\overline{R}}\right)^{\rho_{RDK}} \left[\frac{R_t^{ECB}}{\overline{R^{ECB}}} \exp(\psi_G \left(\frac{B_{t-1}}{Y_{t-1}} - \omega^D\right))\right]^{(1-\rho_{RDK})}, \tag{46}$$

where $\rho_{RDK} = 0.8$ and $\psi_G > 0$.

3.7 The Labor Market

We model the labor market following Galí et al. [2011] and we refer to that paper for the details. The model of Galí et al. [2011] may be seen as a somewhat simpler alternative to the well-known search-and-match approach in the tradition of Diamond, Mortensen and Pissarides. As discussed by Galí [2011], what really matters (at least quantitatively) for the dynamics of unemployment fluctuations is nominal wage rigidities and not search frictions. For our purposes, it therefore seems natural to stick to the formulation of Galí et al. [2011], as it involves fewer equations (as compared to the search-and-match approach) in an already large model, and brings in fewer additional parameters to be estimated using Bayesian methods.

The main building block of our labour market are wage-setting households and sticky wages. Just as for the pricing behaviour of the firms in our model, sticky wages are achieved by assumption using the theory of Calvo [1983]. We assume the existence of a representative household with a continuum of members indexed by

$$(i, j) \in [0, 1] \times [0, 1]$$

Here index i refers to differentiated labour services. Hence, we assume the existence of heterogeneous types of labour specialized in various fields. This implies that each labour supplier has some market power to set its wage. We assume the existence of a continuum of labour unions each representing the different labour types. Index j refers to the household member's disutility from work. Hence, the household consists of many labour types who each have a certain degree of disutility from work.[9]

We assume full consumption risk sharing across the household and we give that household a utility function. The full risk sharing implies that we do not need to take care of different consumption levels and hence marginal utilities, and that the individual members of the household have the household in

[9] The parameter η_N in the utility function will help us to determine the steady-state employment, but is left out in what follows.

mind when maximizing utility. When these members work, they work full time. Hence, labour supply movements are *inter*-marginal and not *intra*-marginal. The assumptions about households setting wages and working full time implies that the employment level is determined on the firm side - the household simply supplies the given number of workers at the going real wage.

When the household chooses its labor supply it equalizes the marginal rate of substitution between supplying more labour and consumption to the real wage

$$MRS_t \equiv \frac{\chi_t O_t \left(N_t\right)^{\phi}}{\lambda_t} = w_t \tag{47}$$

where $MRS_t \equiv -\frac{U_{N,t}}{U_{C,t}}$ is the household's marginal rate of substitution between consumption and leisure, and $\phi > 0$ is the inverse of the Frisch elasticity. That is, $\frac{1}{\phi}$ measures by how much the households' labor supply changes in percent when the real wage increases by one percent holding consumption constant. Intuitively, at the optimum the disutility of working more must be compensated by what the real wage can buy in utility terms. If not, the household would be able to reshuffle between consumption and labour and achieve a higher utility.

The variable O_t is defined as:

$$O_t = \frac{z_t}{\left(C_t - hC_{t-1}\right)\left(1+\tau_t^{VAT}\right)\left(P_t^C/P_t\right)\frac{1}{con_t}} = z_t \lambda_t, \tag{48}$$

with z_t evolving according to:

$$z_t = z_{t-1}^{1-\nu}\left[\left(C_t - hC_{t-1}\right)\left(1+\tau_t^{VAT}\right)\left(P_t^C/P_t\right)\frac{1}{con_t}\right]^{\nu}, \tag{49}$$

where $\nu \in [0,1]$. Following Galí et al. [2011], we may interpret z_t as a smooth trend for (habit-adjusted) aggregate consumption. In other words, O_t is smaller than one when consumption grows faster than this smooth trend, and vice versa. As seen from (47), this implies a drop in the marginal disutility of labor, so that each individual will be willing to work at a lower wage rate, *ceteris paribus*. The parameter ν determines the strength of the wealth effect on labor supply. That is, by how much labor supply is affected by changes in wealth: If ν is close to 1, the wealth effect is quite strong, while the wealth effect disappears when ν tends to 0.[10]

Finally, in (47), the term χ_t represents an exogenous shock to labor supply, which evolves according to:

$$\frac{\chi_t}{\overline{\chi}} = \left(\frac{\chi_{t-1}}{\overline{\chi}}\right)^{\rho_\chi} \exp\left(\varepsilon_t^{\chi}\right), \tag{50}$$

where ε_t^{χ} is an i.i.d. stochastic process with mean zero and variance σ^{χ}, while $0 < \rho_\chi < 1$.

[10] As discussed by Galí et al. [2011], a low value of ν is necessary to ensure that not only employment, but also the labor force moves in a procyclical fashion in response to shocks originating from the demand side.

In equilibrium, a given individual will participate in the labor market if and only if the net benefits from doing so exceed that individual's total disutility of labor. We can write this participation condition as:

$$\lambda_t \left(1 - \tau_t^n\right) w_t \geq \Upsilon_t \left(j\right), \tag{51}$$

where the left-hand side measures the after-tax real wage rate as measured in utility units, and where $\Upsilon_t\left(j\right) \equiv \chi_t O_t j^{\phi}$ represents total disutility from working. Here it is important that the individuals of each type of labour i are ordered by their disutility of labor and that the condition is related to the household's marginal disutility of work. Total disutility from working thus consists of the exogenous shock to labor supply χ_t, the endogenous process O_t as described above, and individual-specific labor disutility.

This implies that the labor force will consist of all individuals for which the above condition is satisfied. We can write the labor force L_t in a symmetric equilibrium as:

$$L_t = \left(\frac{\left(1 - \tau_t^n\right) w_t}{\chi_t z_t}\right)^{\frac{1}{\phi}}. \tag{52}$$

That is, the labour force consists of the j individuals for which condition (51) is satified. The sum of these participation rates across labour types gives the model's aggregate labour force. Notice that the labour force is time-varying. It may increase, for example, due to labour supply shocks which decrease the marginal disutility of working.

Next we define our notion of unemployment as $U_t \equiv \frac{L_t}{N_t}$, i.e. the ratio between the labor force and total employment. Notice that this definition differs slightly from official unemployment measures, which are typically given by $\frac{L_t - N_t}{L_t}$. However, around a log-linear approximations, these definitions of unemployment are equal for small levels of unemployment as is the case for Denmark. Define the (log) average wage markup in the economy as the difference between the real wage and the marginal rate of substitution between consumption and work as

$$\mu_t^w \equiv \log(w_t) - \phi\left(z_t + n_t + \log\left(\chi_t\right)\right).$$

The wage markup varies as long as wages are not fully flexible, and it is non-zero as long as the labour market is not fully competitive. We can use this expression together with the participation condition, (52), to write

$$\mu_t^w = \phi u_t,$$

where $u_t \equiv \log\left(U_t\right)$. Notice that the natural rate of unemployment is given by $\mu^w = \phi u_t^n$. Hence, unemployment in this model is due solely to a non-competitive labour market in which heterogenous types of labour can set a wage above the market clearing wage, and unemployment varies due to changes in the average wage markup in the economy. That is, due to wage rigidities. The natural rate of unemployment is higher the higher is the degree of monopolistic competition and the higher is the Frisch elasticity of labour supply. When this elasticity is high, the members of the household are more willing to substitute in and out of employment.

Finally, we describe the wage formation. Recall that households supply differentiated types of labor services, giving rise to monopolistic competition for labor. Furthermore, we assume that households face Calvo-style wage stickiness. The nature of the problem implies that all households who can reoptimize the wage rate in a given period choose the same wage $\widetilde{w}_t^P$ according to the following first-order condition:

$$\widetilde{w}_t^P(i) = \frac{\epsilon_t^W}{\epsilon_t^W - 1} \mathrm{E}_t \sum_{s=0}^{\infty} (\beta\theta_W)^s \frac{\lambda_{t+s}}{\lambda_t} \frac{N_{t+s}^P(j)\,\chi_{t+s} O_{t+s}}{N_{t+s}^P(j)}, \tag{53}$$

where $0 < \theta_W < 1$ is the wage stickiness parameter, and ϵ_t^W is the elasticity of substitution between labor types, which evolves according to:

$$\left(\frac{\epsilon_t^W}{\epsilon^W}\right) = \left(\frac{\epsilon_{t-1}^W}{\epsilon^W}\right)^{\rho^{\epsilon^W}} \exp \varepsilon_t^{e^W}, \tag{54}$$

where $\varepsilon_t^{\epsilon^W}$ is an i.i.d. stochastic process with mean zero and variance σ^{ϵ^W}, and where $0 < \rho_{\epsilon^W} < 1$. Of the remaining types of workers θ_W, we allow a fraction Γ^W to index their wage to the steady state rate of inflation, while the remaining fraction keep their wage unchanged. We can consequently write the evolution of the wage level in the private sector as:

$$w_t^P = \left[\theta_W \left(\bar{\pi}^{\Gamma^W} w_{t-1}^P\right)^{1-\epsilon_t^W} + (1-\theta_W)\left(\widetilde{w}_t^P\right)^{1-\epsilon_t^W}\right]^{\frac{1}{1-\epsilon_t^W}}. \tag{55}$$

3.8 Trade and the two foreign economies

As discussed in the introduction and in section (2), Denmark's fixed exchange rate towards the euro implies that we need to include two foreign economies in the model: One (the eurozone, EA for short) towards which Denmark has a fixed exchange rate, and one (the Rest of the World; RoW for short) towards which the exchange rate is fully flexible and exogenous for Denmark due to the small-economy assumption and with monetary policy given from the eurozone. The two foreign economies are otherwise completely identical, and are taken as completely exogenous, so that movements in the Danish economy does not affect the foreign economies. We also do not model trade or other interactions between the eurozone and the rest of the world. Each of foreign economies is described by a basic 3-equation New Keynesian model, so that for $j = (EA, RoW)$ we have:

$$\frac{Y_t^j}{\overline{Y}^j} = \left(\frac{Y_{t+1}^j}{\overline{Y}^j}\right)^{\rho_Y^j} \left(\frac{Y_{t-1}^j}{\overline{Y}^j}\right)^{1-\rho_Y^j} \left(\frac{\frac{R_t^j}{\overline{R}^j}}{\frac{\pi_{t+1}^j}{\overline{\pi}^j}}\right)^{-\phi_Y^j} \left(\frac{\epsilon_{Y,t}^j}{\overline{\epsilon}_Y^j}\right), \tag{56}$$

$$\frac{\pi_t^j}{\overline{\pi}^j} = \left(\frac{\pi_{t+1}^j}{\overline{\pi}^j}\right)^{\rho_\pi^j} \left(\frac{\pi_{t-1}^j}{\overline{\pi}^j}\right)^{1-\rho_\pi^j} \left(\frac{Y_t^j}{\overline{Y}^j}\right)^{\phi_\pi^j} \left(\frac{\epsilon_{\pi,t}^j}{\overline{\epsilon}_\pi^j}\right), \tag{57}$$

$$\frac{R_t^j}{\overline{R}^j} = \left(\frac{R_{t-1}^j}{\overline{R}^j}\right)^{\rho_R^j} \left[\left(\frac{\pi_t^j}{\overline{\pi}^j}\right)^{\Gamma_\pi^j} \left(\frac{Y_t^j}{\overline{Y}^j}\right)^{\Gamma_Y^j}\right]^{1-\rho_R^j} \left(\frac{\epsilon_{R,t}^j}{\overline{\epsilon}_R^j}\right). \tag{58}$$

Here, (56) is a hybrid dynamic IS-type relation that links output to the real interest rate, (57) is a version of a hybrid New Keynesian Phillips Curve linking the rate of inflation to real activity, and (58) is a Taylor rule that determines monetary policy in each of the two regions as a function of inflation and economic activity. See Galí [2009] for a detailed exposition of the 3-equation New Keynesian model. In turn, the shock processes in each of these equations are given as AR(1)-processes:

$$\frac{\epsilon_{k,t}^j}{\overline{\epsilon}_k^j} = \left(\frac{\epsilon_{k,t-1}^j}{\overline{\epsilon}_k^j}\right)^{\rho_{\epsilon^k}^j} \varepsilon_t^{k,j}, \tag{59}$$

for $j = (EA, RoW)$ and $k = (Y, \pi, R)$, and where the $\varepsilon_t^{k,j}$'s are i.i.d. normal processes. The parameters in the IS curve $\left(\rho_Y^j, \phi_Y^j\right)$ and the New Keynesian Phillips Curve $\left(\rho_\pi^j, \phi_\pi^j\right)$, as well as the reaction parameters in the Taylor rule $\left(\Gamma_\pi^j, \Gamma_Y^j\right)$ are chosen in line with the literature, as described in the appendix.[11] The six shocks are included in the estimation to account for the contribution to the Danish business cycle of foreign shocks.

Finally, we can write world output and inflation as:

$$\frac{Y_t^W}{\overline{Y}^W} = \left(\frac{Y_t^{EA}}{\overline{Y}^{EA}}\right)^{\omega_X} \left(\frac{Y_t^{RoW}}{\overline{Y}^{RoW}}\right)^{1-\omega_X}, \tag{60}$$

$$\frac{\pi_t^W}{\overline{\pi}^W} = \left(\frac{\pi_t^{EA}}{\overline{\pi}^{EA}}\right)^{\omega_X} \left(\frac{FX_t}{FX}\frac{\pi_t^{RoW}}{\overline{\pi}^{RoW}}\right)^{1-\omega_X}, \tag{61}$$

where the parameter $\omega_X > 0$ measures the relative size of the eurozone, and where FX denotes the change in the *effective* exchange rate of the Danish krone. The effective exchange rate is given by:

$$\frac{FX_t}{FX} = \frac{\frac{R_t^{ECB}}{\overline{R}^{ECB}}}{\frac{R_t^{RoW}}{\overline{R}^{RoW}}} \varepsilon_t^{UIP},$$

so that the effective exchange rate moves in response to interest rate differentials between the eurozone and the rest of the world. ε_t^{UIP} is an i.i.d. normal shock process.

[11] It may be difficult to distinguish interest rate smoothing from persistence in the shocks hitting the interest rate rule. We therefore decide to eliminate the latter by fixing the parameter $\rho_{\epsilon^R}^j = 0$ for $j = (EA, RoW)$.

3.9 Exports

The role of the export sector is to buy final domestic goods, differentiate them, set a price and sell them to import firms in the eurozone or the rest of the world. The motivation behind the introduction of the import and export sectors is to be able to model an imperfect pass-through from changes in prices and the exchange rate to the Danish economy through estimation of the parameters in the export- and import relations. Hence, we can let the data determine the degree of the pass-through.

We can write the world demand for Danish exports, Ex_t, as:

$$Ex_t = x_t^Z Y_t^W \left(\frac{P_t^X}{Px_t^W} \right)^{-\varepsilon^W}, \tag{62}$$

where the parameter ε^W denotes the elasticity with which world consumers substitute between Danish and foreign goods. The demand for Danish exports is thus increasing in world output and decreasing in the ratio between the relative price of Danish exports, P_t^X, and the relative world market price, Px_t^W. We define the latter as:

$$Px_t^W = Px_{t-1}^W \frac{\pi_t^W}{\pi_t^{DK}}, \tag{63}$$

where π_t^W is the world inflation rate, as described above. The relative price of Danish exports, P_t^X, is defined as:

$$P_t^X = P_{t-1}^X \frac{\pi_t^X}{\pi_t^{DK}}, \tag{64}$$

where π_t^X is the inflation rate in Danish exports price, as described below. Finally, the export demand shock x_t^Z evolves according to:

$$\frac{x_t^Z}{\overline{x}^Z} = \left(\frac{x_{t-1}^Z}{\overline{x}^Z} \right)^{\rho_{EX}} \exp\left(\varepsilon_t^{X^{Ex}} \right), \tag{65}$$

where $\varepsilon_t^{X^{Ex}}$ is an i.i.d. stochastic process with mean zero and variance $\sigma^{\varepsilon^{EX}}$, and where $0 < \rho_{EX} < 1$.

Firms in the export sector are faced with price rigidities of the same form as in the domestic production sector. We can therefore write the optimal export price $\widetilde{P}_t^X$ set by a given firm j in the export sector that is allowed to change its price in period t as:

$$\widetilde{P}_t^X(j) = \frac{\epsilon_t^X}{\epsilon_t^X - 1} \mathrm{E}_t \sum_{s=0}^{\infty} (\beta\theta_X)^s \frac{\lambda_{t+s}}{\lambda_t} \frac{Y_{t+s}^W(j)\, mc_{t+s}^X P_{t+k}^X}{Y_{t+s}^W(j)}, \tag{66}$$

where θ_X is the Calvo stickiness parameter in the export sector, and mc_t^X is the marginal cost for the export firms, which is simply given by the inverse of the export price; $mc_t^X = \frac{1}{P_t^X}$. Finally, ϵ_t^X is the elasticity of substitution between the goods produced by each individual firm in the export sector, which follows the process:

$$\left(\frac{\epsilon_t^X}{\epsilon^X}\right) = \left(\frac{\epsilon_{t-1}^X}{\epsilon^X}\right)^{\rho_{\epsilon^X}} \exp \varepsilon_t^{e^X}, \tag{67}$$

where $\varepsilon_t^{\epsilon^X}$ is an i.i.d. stochastic process with mean zero and variance σ^{ϵ^X}, and where $0 < \rho_{\epsilon^X} < 1$. Of the remaining θ_X firms, we allow a fraction Γ^X to index their price to the steady state rate of inflation, while the remaining fraction of firms keep their price unchanged. The inflation rate in Danish export prices will then satisfy:

$$1 = \theta_X \left(\bar{\pi}^{\Gamma^X} \pi_t^X\right)^{\epsilon_t^X - 1} + (1 - \theta_X) \left(\frac{\widetilde{P}_t^X}{P_t^X}\right)^{1-\epsilon_t^X}. \tag{68}$$

3.10 Imports

The structure of the importing sector can be described as follows: A continuum of import differentiators import a homogenous final good from foreign exporters, differentiate the good (say, by adding brand names), and sell the differentiated products to Danish households and firms, who, as described above, solve a cost minimization problem when they choose between imported and domestically produced goods. The world market prize of import goods, which in turn determines the marginal cost of Danish import differentiators, is computed as a weighted average of prices in the eurozone and the rest of the world.[12] We can write the marginal cost for an import differentiator as:

$$mc_t^M = \frac{Px_t^W}{P_t^M}, \tag{69}$$

where, as described in the previous subsection, Px_t^W is the relative world market price, and P_t^M is the price of imported goods relative to Danish goods;

$$P_t^M = P_{t-1}^M \frac{\pi_t^M}{\pi_t^{DK}}, \tag{70}$$

We define the inflation rate of import prices in Denmark, π_t^M, below. Just like domestic and exporting firms, the firms in the import sector face sticky prices as in Calvo [1983]. We can therefore write the optimal price $\widetilde{P}_t^M$ chosen by a given import differentiator j that is allowed to change its price in period t as:

$$\widetilde{P}_t^M(j) = \frac{\epsilon_t^M}{\epsilon_t^M - 1} \mathrm{E}_t \sum_{s=0}^{\infty} (\beta\theta_M)^s \frac{\lambda_{t+s}}{\lambda_t} \frac{\mathrm{Im}_{t+s}(j)\, mc_{t+s}^M P_{t+k}^M}{\mathrm{Im}_{t+s}(j)}, \tag{71}$$

[12] Our modeling of the import sector involves one important drawback: Consider for example a situation where the US dollar appreciates against the Danish krone. In turn, this drives up the aggregate import price faced by Danish households and firms, who in turn choose to buy fewer imported goods from the rest of the world AND from the Euro area, even though the exchange rate towards the Euro is unaffected.

where θ_M is the Calvo stickiness parameter in the import sector. Im_t denotes Danish demand for imported goods, which consists of two terms: Imports used for consumption by households, and imports used as investment goods by Danish firms. As shown in the appendix, we can write Danish import demand as:

$$\text{Im}_t = \frac{P_t^C \left(C_t - C_t^{DK}\right) + P_t^I \left(I_t - I_t^{DK}\right)}{P_t^M}. \tag{72}$$

In the expression for the optimal price, ϵ_t^M is the elasticity of substitution between the goods of each import differentiator, which follows the process:

$$\left(\frac{\epsilon_t^M}{\epsilon^M}\right) = \left(\frac{\epsilon_{t-1}^M}{\epsilon^M}\right)^{\rho_{\epsilon^M}} \exp \varepsilon_t^{e^M}, \tag{73}$$

where $\varepsilon_t^{\epsilon^X}$ is an i.i.d. stochastic process with mean zero and variance σ^{ϵ^X}, and where $0 < \rho_{\epsilon^M} < 1$. Of the remaining θ_M firms, we allow a fraction Γ^M to index their price to the steady state rate of inflation, while the remaining fraction of firms keep their price unchanged.Finally, analogous to the previous subsection, the inflation rate in Danish import prices satisfies:

$$1 = \theta_M \left(\bar{\pi}^{\Gamma^M} \pi_t^M\right)^{\epsilon_t^M - 1} + (1 - \theta_M) \left(\frac{\widetilde{P}_t^M}{P_t^M}\right)^{1-\epsilon_t^M}. \tag{74}$$

We finally point out that the presence of staggered import- and export prices implies that the model in the short run allows for deviation from the law of one price. That is, the same good can be sold at different exchange-rate adjusted prices in different countries.

3.11 Market Clearing

We can write the aggregate resource constraint of the Danish economy as:

$$Y_t = \frac{P_t^C}{P_t} C_t^{DK} + \frac{P_t^I}{P_t} I_t^{DK} + G_t + \frac{P_t^I}{P_t} I_t^G + z^u(u_t) K_{t-1} + \frac{P_t^X}{P_t} Ex_t. \tag{75}$$

Moreover, Denmark's net foreign asset position is given by:

$$B_t^I = \frac{R_{t-1}^{ECB} \exp(-\psi_d \left(\frac{B_{t-1}^I}{Y_t} - \frac{\overline{B}^I}{\overline{Y}}\right))}{\pi_t^{DK}} B_{t-1}^I + \frac{P_t^X}{P_t} Ex_t - \frac{P_t^M}{P_t} \text{Im}_t, \tag{76}$$

so that net holdings of foreign assets increase if Danish exports exceed imports in a given period.

3.12 Stationary equilibrium and steady state

As already described, the model features two deterministic trends: growth in total factor productivity, A_t, and in investment-specific technology, Z_t. This implies that aggregate macroeconomic variables, such as output and consumption, fluctuate around a balanced growth path. In order to solve the model, we therefore need to rewrite the equations in terms of detrended stationary variables and find the steady state of the stationary model. Observe that we can write the compounded trend growth of these two variables as $d\Gamma_t \equiv (dA_t dZ_t^{\alpha})^{\frac{1}{1-\alpha}}$, where we have taken into account that both public and private capital are affected by investment-specific technological progress.

To obtain a stationary equilibrium, we then make the following transformations of the endogenous variables: We define $\widetilde{Y}_t = \frac{Y_t}{\Gamma_t}$ as the stationary counterpart of Y_t. Similarly, we define $\widetilde{C}_t = \frac{C_t}{\Gamma_t}$, $\widetilde{G}_t = \frac{G_t}{\Gamma_t}$, $\widetilde{T}_t = \frac{T_t}{\Gamma_t}$, $\widetilde{B}_t^{DK} = \frac{B_t^{DK}}{\Gamma_t}$, $\widetilde{w}_t = \frac{w_t}{\Gamma_t}$, and so forth, and we define $\widetilde{K}_t = \frac{K_t}{Z_t\Gamma_t}$, $\widetilde{I}_t = \frac{I_t}{Z_t\Gamma_t}$, and $\widetilde{K}_t^G = \frac{K_t^G}{Z_t\Gamma_t}$, where we have taken into account that capital and investment grow at a faster rate than output in the non-stationary model. We also define $\widetilde{\lambda}_t = \lambda_t\Gamma_t$ so as to ensure that the shadow price of consumption remains stable as the level of consumption grows, and we let $\widetilde{Q}_t = Q_t Z_t$, so that the relative price of investment goods changes over time along with investment-specific technological progress.

3.12.1 Steady state

We normalize GDP and the price level in all three economies to 1 in steady-state. These normalisations give us the rest of the prices in the economy: Import-, export-, investment-, and consumer prices and their relative prices. We also assume that the exchange rate is constant against the euro and 1 against rest-of-world. Given the monetary policy regime in place in Denmark, the domestic nominal interest rate is equal to the ECB nominal interest rate. In steady state risk premia for holding foreign bonds are zero. Moreover, all adjustment costs are zero in steady state, as all the variables to which they apply grow at their steady state growth rate. Specifically, the cost of changing the import content of consumption and investments, steady state utilization costs, and investment costs are all zero in steady state.

Given steady state exports and the CES functions for private consumption and investment as well as steady state investment, and imposing the law of one price and normalizing foreign output we can derive steady state imports and the net foreign asset position in the steady state.

Following Galí et al. [2011], we set the elasticity of substitution among labour varieties, ε^{subW}, such that unemployment in steady states equals Nationalbanken's estimate of the natural rate of unemployment of around 4 pct., see Andersen and Rasmussen [2011].

The average quarterly growth rate of Danish real GDP in our sample is around 0.4 pct., while that of the relative investment price is -0.2 pct. We use these estimates to determine the steady states of the two processes which together determine the growth in the model, Λ_t^A, Λ_t^Z.

Regarding the fiscal policy side of the model, we can observe the steady state ratios of public consumption (public expenditures on goods and services as well as public employment), public investment, all tax rates as well as the debt ratio. Finally, steady state lump-sum taxes are determined such that public debt obligations can eventually be expected to be honored.

4 Estimating the model

Our goal is to estimate all the structural shocks in our model and a majority of the parameters. We outline the econometric approach in section 4.1, the data and description of the shocks used in the estimation in section 4.2, the calibrated parameters in the model in section 4.3, while our parameter estimates and prior distributions are discussed in section 4.4.

4.1 Econometric methodology

We confront the model with data using Bayesian methods. In this section we only outline the methodology. For a more thorough introduction to Bayesian estimation of DSGE-models, see among many Smets and Wouters [2003], Smets and Wouters [2007], An and Schorfheide [2007], or a series of papers by Jesus Fernandez-Villaverde and coauthors; (Fernández-Villaverde [2010], Fernández-Villaverde et al. [2010] or Fernández-Villaverde and Rubio-Ramírez [2007]).

We follow the Bayesian approach for a number of reasons. Firstly, as is well-known the use of priors allows us to introduce presample information and to reduce the dimensionality problem associated with the large parameters to data ratio. Secondly, Bayesian methods have well-known and important computational advantages over maximum likelihood in larger DSGE models. That is, the use of simulation provides us with a much easier method to derive the marginal distribution of the parameters in the model than the traditional frequentist maximum likelihood approach involving maximisation. The use of priors gives curvature to a highly dimensional likelihood-function, which is likely to be flat in many dimensions due to poorly identified parameters. Even the most sophisticated algorithms have a hard time finding the global maximum of such a function; it is much easier to simulate the posterior distribution of the parameters than to maximize the likelihood function.

Our goal is to report features of the posterior distribution. The Bayesian methodology provides a mapping from the prior distribution to the posterior through data. Let Θ denote all the parameters in the model we aim to estimate and let Y^T denote all the observed data we will use in the estimation. Let $p(\Theta)$ denote the prior distribution over these parameters. The model implies a likelihood $p\left(Y^T|\Theta\right)$ and we then have a posterior distribution of Θ given by:

$$p\left(\Theta|Y^T\right) = \frac{p\left(Y^T|\Theta\right)p\left(\Theta\right)}{p\left(Y^T\right)} \tag{77}$$

The posterior distribution equals the prior distribution times the likelihood function divided by a scaling factor. It can perhaps be helpful to think of Bayesian analysis as traditional maximum likelihood with a penalty function in form of the prior distribution, $p(\Theta)$. That is, the prior distribution assign low values to parameter values which the Bayesian econometrician finds implausible.

The posterior distribution, $p(\Theta|Y^T)$, summarizes the uncertainty regarding the parameter values. The posterior is difficult to characterize and we consequently generate draws from it using a Metropolis-Hastings algorithm. The resulting empirical distribution is used to obtain point estimates, uncertainty bands etc.

4.2 Data and shocks

We estimate the model on data running from 1990 to 2015. We use the first 5 years as 'training' sample for the Bayesian estimation which we afterwards discard in the analysis. This has the advantage that initial conditions in the historical shock decomposition are likely to have vanished in the sample period, see also section (6). We add the most recent forecast produced by Nationalbanken for the 3-year period after the sample to get better estimates of the long term trend in data. The latter must be seen as a consequence of the crisis period which might bias the long-run growth rate downwards leaving a worse fit at the end of our sample. Strictly speaking we should allow for a structural break but we leave that for future work. We use data from 1990 although the european currency union was not in place before 1999. Hence, we weight the costs of having less data less than the costs of using data from a group of countries within a currency union, which was not in place at that point in time. We believe, however, that the initial eurozone countries to some extent shared business cycles already in 1995, as also suggested by Dam [2008].

In estimating the model, we use times series for 23 macroeconomic variables. The following time series for Danish variables are taken from Statistics Denmark and Nationalbanken:[13]

- Real GDP
- Private consumption
- Government consumption including public production
- Government investment
- Exports
- Imports
- Total investment including inventories and construction
- Labour income tax
- Private employment
- Unemployment (net)
- Industry nominal wages deflated by CPI
- Investment deflator
- PPI index
- Import deflator
- Export deflator
- Danish nominal interest rate

[13] See the appendix for a detailed description of the source for each data series.

The 'Rest of World'-variables are defined as the weighted sum of GDP, inflation or the policy rate of the Danish trading partners excluding trading partners within the eurozone. The eurozone and the Rest of the World are approximately of equal size. Data is taken from the Ecowin data base and OECD. For the foreign economies we use the following variables:

- Inflation in the eurozone and in the rest of the world
- Real GDP in the eurozone and in the rest of the world
- ECB policy rate
- Implied rest of the world policy rate
- Effective exchange rate

Prior to estimation, we transform the time series into quarter-on-quarter growth rates, approximated by the first difference of their logarithm. As explained above, in the model, we include a trend in productivity and in the relative price of investments goods. The variables in the model therefore have trends and consequently, we do not demean data. Also, a number of additional transformations are made to ensure that variable measurement is consistent with the properties of the model's growth path. Firstly, we remove the sample growth rate differentials between the export and import variables and Danish GDP, as these variables in the sample have grown faster than GDP reflecting globalisation, see figure (2) and the discussion in section (2). Secondly, for the effective exchange rate we band-pass filter and demean the data. Lastly, we HP-filter data for the foreign economies as we do not model trends in the foreign economies.

Data for Denmark including the effective exchange rate against the rest-of-world is shown in figures (3), (4) and (5). The time-series used in the estimation for the foreign economies are shown in figure (6).

4.2.1 Shocks

The model includes all in all 27 structural shocks but we only use 22 including the shocks from the exogenous AR-models for public consumption, public investments and labour income tax in the estimation of the model. Finally, we use the 6 shocks from the DSGE-model for the two foreign economies:

- Public consumption shock, ε^{G}
- Perm. tech shock, ε^{A_P}
- Temp. tech shock, ε^{A_T}
- Temp. investment shock, ε^{Z_T}
- Cap. utilization shock, ε^{cap}
- Consumption shock, ε^{con}
- Price markup shock, ε^{e^P}
- Wage markup shock, ε^{e^W}

- Export shock, $\varepsilon^{X^{E}x}$
- Import shock, ε^{Im}
- Import price markup shock, $\varepsilon^{e^{M}}$
- Export price markup shock, $\varepsilon^{e^{X}}$
- Riskpremia shock, ε^{RPD}
- Public investment shock, ε^{IG}
- Tax on labour income shock, ε^{tN}
- EA price shock, $\varepsilon^{pi,EA}$
- EA output shock, $\varepsilon^{y,EA}$
- ECB interest rate shock, $\varepsilon^{r,EA}$
- RoW price shock, $\varepsilon^{pi,RW}$
- RoW output shock, $\varepsilon^{y,RW}$
- RoW interest rate shock, $\varepsilon^{r,RW}$
- UIP shock, ε^{UIP}

As already described, all shocks are assumed to feed into first-order autoregressive processes, except for the shock to the ECB policy rate, the shock to the policy rate for the rest of the world, and the UIP shock, which are all white noise, and the shocks to public investment and the labour income tax, which follow higher-order autoregressive processes.

The shocks which are not used in the estimation are shocks to three taxes, capital income tax, ε^{tK}, VAT, ε^{tVAT}, and tax on bond income, ε^{tB}. These tax rates have been kept almost fixed throughout the sample but we keep the shocks in the model for policy experiments. We also do not include shocks to the trend in the relative price of investment goods, ε^{Z_P}. Lastly, we do not use the labour supply shock, ε^{χ}, as the model is not able to distinguish between this shock and the wage markup shock in the estimation. In the estimation we assume that all data are measured with measurement error except for the ECB policy rate.[14]

4.3 Calibration

It is well known that some parameters in DSGE-models are hard to identify and we do not assume that our model is any different. Also, some parameters such as the depreciation rate on private capital, δ, are better estimated using micro data. The parameters we have calibrated in the estimation are shown in table (2). Also, we do not estimate the standard deviations of non-estimated shocks as described above, ε^{to}, ε^{VAR}, $\varepsilon^{tk}$and ε^{L}, nor their autoregressive parameters. These are only in use for policy analysis.

4.4 Parameter estimates and Prior Distributions

Tables (3), (4), and (5) show our assumptions regarding the priors and the results of the estimation. That is, the posterior mode estimates of the structural parameters and the shocks in the model.

[14] We do assume measurement errors for the interest rate in the rest-of-world, as this variable is an artificial variable constructed mainly by aggregating the monetary policy rates in Sweden, U.K. and the U.S.

4.4.1 Priors

The prior distribution for the estimated parameters are shown in the left column of the tables. Regarding the priors, we generally follow the literature and make broadly similar assumptions about our priors, see Christoffel et al. [2008], Adolfson et al. [2013], Burriel et al. [2010]. The prior distributions for the parameters are chosen in conformity with the constraints on the parameter space implied by theory. For those parameters that are bounded between 0 and 1, we choose a standardised beta distribution. For parameters that are bounded from below at zero, we have chosen either a gamma or an inverse gamma distribution to model the prior distributions. An easier way to evaluate the choices we have made regarding our priors is to plot them. First, we discuss the posterior distributions.

4.4.2 Posterior estimates

The right hand side of tables (3), (4), and (5) show the posterior distribution of the parameters for our preferred specification of the model. The entries in the posterior-mode column refer to the values of the estimated parameters that are obtained by maximizing the model's posterior distribution. The distributions are computed based on a Markov chain with 500.000 draws. We use Dynares build-in figures for convergence (not shown for brevity). We found that the 500.000 draws was enough to obtain convergence.

We highlight the following in our estimation results. On the labour market, we find a elasticity of labour supply of 2.9. The inverse of this parameter determines the size of the elasticity of employment with respect to the real wage holding consumption constant. We note that the elasticity is with respect to employment and not hours. Hence, the elasticity determines how many workers substitute in and out of employment in response to changes in wages, and the parameter is not related to the hours worked by a particular person.

We find a value for υ above $\frac{1}{2}$. This implies some degree of wealth effect on the labour supply. This has implications for, e.g., the size of the fiscal multiplier, see Monacelli and Perotti [2008].

On the nominal side of the economy, we find some differences across sectors of the economy. The estimate of the Calvo parameter is quite low for domestic goods, θ_p, with a value of around 1/2. We obtain a similar value for export prices, but a much higher value for import prices. According to the model estimates, wages are more sticky than prices. This might reflect that a large share of Danish wages are set according to 2-3 year wage agreements. The relative flexibility of export prices might reflect that Danish exporters can not to a great extent rely on the nominal exchange rate to adjust to changes in market conditions. Also, the indexation of export prices is almost zero, while quite large for wages with a value of around a half. Regarding the the shocks, we do not find that some shocks dominate in the sense of having a very big standard deviation.

Another way to check the quality of the estimation is by comparing the prior and posterior distributions of each parameter. As discussed above, this is also a method to evaluate the choices we have made regarding the priors. This is done in figures (8) to (14). In general the figures show that the data is informative about the posterior distribution. That is, the posterior distribution is not equal to the prior. However, some parameters do seem to be 'defined' by their prior; as an example the capital utilization cost parameter. This may reflect that our data sample is not informative about these parameters. Our parameter estimates can from this perspective be regarded as calibrated or can be viewed as being estimated with a high degree of outside information.

As revealed by the figures, some priors are set quite tight. That reflects to a large degree a necessity; without these tight priors the model would not work well in some important dimensions like impulse response functions. Finally, the point estimates for the autoregressive parameters of shock processes show that some shocks are very persistent, especially those related to temporary techonology shocks, consumption shocks, and export shocks. This may suggest that the model has some difficulties in generating the level of endogenous persistence present in the data and therefore the model opts for these exogenous shocks to be highly persistent.

5 Model properties

Having looked at the prior-posterior plots, point estimates and smoothed shocks, we now move on to study the model's empirical properties. We do this by reporting the model's impulse-response functions in section (5.1). Here for selected we also compare impulse-response functions with the impulse-response functions from a structural VAR. We move on to study forecast-error-variance decompositions for various horizons in section (5.2).

We first highlight some results from the model which may seem in contrast to comparable results from other medium-sized DSGE-models like the ones in Christoffel et al. [2008], Adolfson et al. [2013], or Burriel et al. [2010]. The key to the differences is straightforward: Denmark's fixed exchange rate regime.

Firstly, the variance decomposition shows that foreign shocks are the most important drivers of the Danish busines cycle. This stands in contrast to existing results, see, e.g., Justiniano and Preston [2010]. The fixed exchange rate regime implies that shocks originating in the eurozone have a direct effect on the Danish economy. Hence, eurozone shocks are transmitted directly to the Danish economy through the interest rate, and therefore have a large impact on the Danish business cycle on top of the effects via international trade.[15] Secondly, all shocks which affect inflation cause an over- or undershooting. This is because in the model all goods are tradable and because the Danish economy

[15] For example, while the effects on the Danish economy of a rise in the interest rate in each of the two foreign regions are qualitatively very similar, the impact is roughly twice as big after a rise in the euro-area interest rate as compared to a rise in the interest rate in the rest of the world.

can effect neither foreign prices nor exchange rates due to the fixed exchange rate regime and the small open economy assumption. This implies that the steady state level of Denmark's terms of trade needs to be achieved eventually. This provides the economy with a nominal anchor and ensures determinacy of the model. The nominal anchor imposes a lot of restrictions on the economy and give rise to differences between the effect of shocks in the model in this paper and the previously mentioned papers.

5.1 Impulse-response functions

We focus on a few of the most important shocks in our analysis of the impulse-response functions, namely a transitory techonology shock, a domestic price markup shock, a shock to foreign output, a shock to the ECB policy rate, and the most important policy instrument for Denmark, government spending. The interest rate shock provides a view on the monetary policy transmission mechanism from the ECB to the Danish economy, while the government spending shock both provides a fiscal multiplier and an example of a demand shock. The first three shocks constitute examples of supply, cost-push, and foreign demand shocks respectively. We further compare the monetary policy shock, the public consumption shock and the foreign consumption shock with impulse-response functions from a structural VAR. We focus on the effects on a subset of the endogenous variables in the model.

5.1.1 Consequences of a temporary techonology shock

The impulse-response to a positive shock to the temporary technology shock of size one standard deviation is shown in figure (19). A temporary technology shock decreases real marginal costs and hence domestic prices initially. However, the forward looking firms correctly anticipate higher supply and demand in the economy. As it turns out, the higher demand effect in the estimated economy rapidly mitigates the increase in inflation.

The increase in aggregate demand both stems from the initial lower prices but also from wealth effects on consumption: The forward looking households realize that they have become more productive and hence richer wich induces them to consume some of the wealth today. This is amplified by a subsequent drop in the real interest rate due to higher inflation, which is not combatted through nominal interest rate increases. The latter reflects the fixed-exchange rate regime in Denmark. The more productive capital leads to an investment boom further increasing demand. Exports fall due to the subsequent rise in inflation. Together with increasing imports, this counteracts the rise in domestic demand from investments and consumption.

Turning to the labour market, higher productivity increases supply of goods in the economy leading to an increase in the demand for labour. That is, the workers have become more productive which causes an outward shift in the labour demand curve leading to higher real wage and employment. This is the so-called classic effect, but not the only effect. Workers have become more productive and richer which makes them decrease their labour supply - a wealth effect on employment. This puts a further upward pressure on real wages but downward pressure on employment. If the model was model did not feature any 'Keynesian' effects, employment would fall in the absence of an increase in aggregate demand, as more goods can be produced with fewer workers after a technology shock. However, as explained above, aggregate demand does rise and initially pushes employment up.

5.1.2 Consequences of a domestic price markup shock

The impulse-response to a one standard deviation positive shock to the domestic price markup is shown in figure (20). A positive shock can be interpreted as a situation in which domestic firms set a higher markup over marginal costs. That is, firms set a higher price, all else equal. This causes inflation to rise, but only in the first few quarters, despite the fact that the shock is quite persistent with a half-life of around $2\frac{1}{2}$ quarters. This is because the forward-looking firms anticipate that the price increase leads to a decline in economic activity and thereby lower marginal costs. Ultimately, firms seek to reestablish the optimal relationship between prices and marginal costs and therefore decrease their prices over time.

The shock leads to lower output because households face higher prices. This effect is mitigated by lower real interest rates, as the policy rate does not react to the higher inflation. After a few quarters, the decline in prices leads to an improvement in competitiveness and higher exports, which helps in driving output back towards the steady state. The decline in aggregate demand causes employment to fall and exerts downward pressure on real wages.

5.1.3 Consequences of a shock to foreign demand

While less obvious for the technology and domestic markup shock, we have an observable variable for foreign demand, which in this case is measured by GDP in the rest of the world. This series can be included into a VAR and compared to the estimated impulse-response functions produced by the DSGE model. We do the same comparison for shocks to the ECB policy rate and government spending in the subsequent subsections. We use the same data as in the estimation of the DSGE model. All series are in logs except for inflation, which is in levels. The VAR is identified using a Choleski decomposition. We order the 'shock' variable, i.e. GDP in the rest of the world, last in the VAR. This implies that Danish variables in the VAR cannot react in the first quarter to shocks to foreign GDP. By including the 'shock' variable last in the causal structure we try to 'control' for exogenous movements in the other variables in the VAR. This can be considered the most cautious approach, as discussed by Abildgren [2010].

The results for a shock to GDP in the rest-of-world region are shown in figure (21). Notice that we do not provide SVAR results for all the endogenous variables in the figure. We look at a one percent increase in rest-of-world output.

In the following quarters GDP in Denmark rises both in the SVAR and in the DSGE-model though the effect is much larger in the DSGE-model. Both models predict a negative response from around 4 quarters. The recession is, however, more severe in the DSGE-model than in the SVAR. The initial larger response in the DSGE-model can be attributed primarily to the identification strategy in the SVAR.

In the DSGE model export booms. The boom pushes employment and the real wage upwards with higher marginal costs as a consequence, and thus higher inflation. This higher inflation is, as usual in our DSGE-model, followed by deflation such that the initial loss of competitiveness can be regained. The inflationary initial period implies a fall in the real rate of interest, while the deflation period implies an increase in the real rate of interest, which explains the small response of consumption. Finally, notice that the uncertainty bands in the DSGE-model are much smaller than in the SVAR. This can be explained by the estimation procedure in which the foreign block is estimated seperately of the Danish model. The uncertainty regarding parameter values in the foreign block, which obviously play a central role in this type of shock, consequently does not show up in the confidence bands.

5.1.4 Consequences of a shock to the ECB monetary policy rate

Next we consider the effects of a shock to the ECB policy rate, which we include in levels in the VAR. This is the de facto monetary policy rate for the Danish economy in normal times. In the VAR we order the ECB policy rate after the real variables in the causual structure, following Christiano et al. [1999]. We normalize the shock so that it is equivalent to a 25 basis points increase in the ECB policy rate. The impulse responses from the VAR and the DSGE-model are pictured in figure (22). Notice that again we do not provide SVAR results for all the endogenous variables in the figure.

GDP in the VAR stays unchanged in the first quarter, which is due to the identification of the VAR. GDP in the DSGE model falls around 0.15 percent on impact. The shock is less persistent in the DSGE-model than in the VAR-model and only statistically significant during the first year or so.

Notice that in the DSGE-model a shock to the interest rate in the eurozone does not imply that the interest rate is the only variable that moves. The eurozone model is estimated as a small scale DSGE-model along the line of Galí [2009] and Woodford [2003], see section (3.8) and appendix (9). Hence, a positive shock to the policy rate in the eurozone also causes a fall in GDP and inflation in the eurozone. The fall in eurozone GDP puts downward pressure on Danish exports, all else equal. However, the fall in inflation in the eurozone is smaller than the fall in Danish inflation causing an improvement in Danish competitiveness. This depresses imports and stimulates exports and therefore GDP in Denmark. This explains the strong rebound of Danish GDP.

We notice that the initial fall in GDP for Denmark as a consequence of contractionary monetary policy is of about the same size as the estimated effect in DSGE-models for the eurozone, see Christoffel et al. [2008]. We also notice that the effects of a monetary policy shock from the rest-of-world economy are qualitatively similar, but that quantitatively speaking, the effects are only little more than half as big as the effect from the monetary policy shock from the eurozone. This is due to the fact that the monetary policy rate enters directly in the Danish economy, which partly explains why the foreign shocks contribute so much to the variation in Danish GDP, as discussed in section (5.2) below.

5.1.5 Consequences of a government spending shock

Ravn and Spange [2013] perform a VAR-based analysis of the effects of a temporary increase in public consumption. We refer to that paper for a detailed description of the VAR-model. Here we use and compare the results in that paper with the estimated response to expansionay fiscal policy in the DSGE-model. That is, we compare the implied fiscal multipliers. In figure (23) we compare the impulse response functions from Ravn and Spange [2013] to the impulse response functions from the estimated DSGE-model. For a deeper analysis of the effects of fiscal shocks under fixed exchange rates in a DSGE-model we refer to Pedersen [2012].

The following observations from this analysis merit some comments. The fiscal multiplier in the DSGE-model resides within the confidence bands produced by the VAR-model although the multipliers are somewhat different. As can be seen from the figure, the DSGE-model predicts a fiscal multiplier of around 0.7, which is substantially smaller than the fiscal multiplier predicted by the VAR of around 1.3. The results are comparable to the findings of Pedersen [2012], who discusses fiscal multipliers in a DSGE-model calibrated to the Danish economy. Note also that once again, the confidence bands produced by the VAR are a lot wider than those from the DSGE-model. The reason is that the most important parameter in determining the impact of this shock; the persistence of the shock, has been calibrated so as to match the estimated persistence from Ravn and Spange [2013]. This means that there is no uncertainty surrounding this parameter, so that the uncertainty surrounding the impact of the shock becomes small as well.

The economic effects driving the multiplier are quite standard: Consumption falls due to a wealth effect, imports increase, while exports decrease due to loss of competitivenes as inflation rises. The increase in inflation is, however, shortlived: Danmark needs to regain competitivenes and therefore the economy experiences deflation.

In the labour market, the wealth effect gives rise to a rightwards shift in the labour supply curve as the households feel poorer and thus work more. This puts downward pressure on wages and drives up employment. At the same time, however, the firms wish to meet the extra demand as their prices are sticky and they do so by hiring more labour, pushing real wages up. As seen from figure (23), real wages increase and therefore the demand effect on the labour market wins. That is, the existence of nominal rigidities makes the wedges in the economy move countercyclically, driving up real wages. After some quarters real wages start to fall.

There is one common theme from the analysis of the impulse response functions in this section: Increasing prices, due to either cost-push, productivity shocks or demand shocks, leads to a subsequent deflationary period as the terms of trade need to revert to the initial value. If not, Denmark would be loosing competiveness indefinitely, and hence a share of world trade. This is because Denmark is a small open economy which cannot influence the macroeconomic state of its trading partners and due to the fixed echange rate: Deflation is the only means to regain the initial terms of trade.

5.2 Forecast-error-variance decompositions

In tables (6), (7), and (8) we show the contribution of the structural shocks in the model to the forecast error variances of a selected set of observed variables. This exercise tries to provide a quantitative insight as to which structural shocks on average in the estimated model during the sample period give rise to variability in the endogenous variables in the model. For simplicity we only look at the variance decomposition for the 1^{st}, 4^{th}, and 40^{th} quarter, and in what follows we concentrate on the variance decomposition for real GDP.

The tables illustrate that foreign shocks are very important drivers of the Danish business cycle. The group of structural shocks originating in the two foreign economies (inflation, output, interest rates, and the effective exchange rate) account for between 60 percent of the variations in real GDP at the very short term, 1 quarter, and 70 percent at longer horizons, 40 quarters. In particular, the shock to output in the euro-area is by far the largest contributor to movements in Danish GDP at all horizons. This should not be surprising, as Denmark is a very small and open economy with a fixed exchange rate towards the Euro.

Another important source of GDP variations is the temporary technology shock, which account for 10-15 percent of the variations in real GDP at all horizons. Domestic price markup shocks account for around 5 percent at all horizons.

While our finding that variations in Danish GDP are to a large extent driven by shocks from abroad may not seem very surprising, it does stand in contrast - and remarkably so - to the results of Justiniano and Preston [2010]. After estimating a small open economy model in the tradition of Gali and Monacelli [2005] for Canada, they study the contribution of shocks to the US economy for fluactuations in Canadian GDP. They find that US shocks account for less than 3 percent of the movements in a number of Canadian macroeconomic variables, including GDP, at all horizons. In line with this result, the estimated DSGE model of the Swedish Riksbank, see Adolfson et al. [2007] also ascribes less than 10 percent of GDP fluctuations in Sweden to foreign shocks. Obviously, the main difference between Denmark and these other small open economies is that Denmark has pegged its currency to the Euro, whereas Canada and Sweden both have a flexible exchange rate. Aastveit et al. [2013] suggest two main reasons for the small explanatory power typically assigned to foreign shocks in estimated small open economy models: The absence of other cross-country linkages than the trade channel (for example via financial markets or consumer and investor sentiments), and the lack of direct effects of foreign-economy shocks on domestic variables (for example through common shocks). A fixed exchange rate fills both of these gaps, as it makes way for another channel, the interest rate channel, through which shocks originating in the eurozone have a direct effect on the Danish economy. As the variance decomposition shows, the contribution of shocks in the eurozone is much larger than that of shocks in the rest of the world for output fluctuations at medium and long horizons, as well as for movements in inflation and private consumption at all horizons. This confirms that eurozone shocks are transmitted directly to the Danish economy through the interest rate, and therefore have a large impact on the Danish business cycle on top of the effects via international trade.

Finally, even though the transmission of shocks works through the interest rate, the variance decomposition shows that interest rate shocks in the eurozone are much less important than output shocks. The explanation is that movements in the eurozone interest rate, which is set according to a Taylor rule, are primarily driven by shocks to eurozone output and inflation, whereas monetary policy shocks are less important. These insights, which are confirmed by the variance decomposition of the eurozone interest rate, merely suggest that the rule-based component of monetary policy is much more important than monetary policy shocks, in line with a number of empirical studies, see e.g. Christiano et al. [1999].

6 Application - Historical Shock Decomposition

In this paper we focus on a historical shock decomposition of Danish real GDP for the period 2004 to the present and leave forecasting exercises and counterfactual analysis to future work. We focus on this period as it compromise the build up to the financial crisis, Lehman's collaps and the subsequent period. We decompose real GDP into the contributions of the model's 22 structural shocks. To facilitate the presentation, we group the structural shocks into five categories: markup, demand, productivity and capacity shocks, the foreign economies and fiscal policy.

- Markups: Domestic, export and import price markup and wage markup shocks
- Demand: Consumption, investment, import, export shocks
- Productivity and capacity: Temporary and permanent techonology shocks, utilization shock
- Foreign shocks: All shocks to production, inflation, interest rates and uip-shocks from euroarea and rest-of-world
- Fiscal policy: All shocks to taxes and public consumption and investment

We show in figur (24) the combination of structural shocks which according to the model have given rise to the historical development in real GDP for Denmark in period 2004-12. The thick line in figur (24) shows the actual development in GDP around an estimated growth rate and we will in what follows denote it as the output gap.[16] The sum of all the structural shocks add up to the quarter-to-quarter growth rate for GDP. A group of variables taken together has contributed positively to growth in GDP in a given quarter if their sum is positive.

In the beginning of our sample GDP is below trend but on a upward path after the low-growth period during the beginning of the 90's. This cycle ends at the beginning of the 00's. Around this period, the Danish economy experienced a shortlived and mild economic downturn. This downturn was succeeded by a large upturn culminating at the outbreak of the financial crisis. Denmark was, as many developed economies, hit hard by the financial crisis causing a large output gap, which stays negative for the remainder of the sample.

The next subsections dig deeper into the effects of these 5 subgroups of structural shocks on the historical development of real GDP in Denmark. We will look at the full sample but emphasize the build up to the financial crisis and the following downturn.

6.1 Foreign shocks

The first observation we make is that by comparing the 5 subgroups the group of foreign shocks provides the greatest impact on the historical development of Danish real GDP. This confirms the results in the variance decomposition in section (5.2).

In figure (25) we have decomposed the sum of the foreign shocks in the respective shocks within the group: Production, inflation, interest rates in the two economies as well as shocks to the exchange rate. The Danish economy booms along with the global economy from 2005 until the outbreak of the financial crisis. From 2008, foreign shocks exert a strong negative impact on the production gap in Denmark. As can be seen from figure (25), at the onset of the financial crisis the contraction in output is counteracted by low interest rates in both the eurozone and rest-of-world. Later on, the zero lower bound starts to bind and therefore the interest rates contribute negatively to GDP growth; the economic situation would prescribe even lower interest rates but central banks are restricted by the zero lower bound. At the end of our sample, the contribution of the foreign sectors is roughly zero. This is the result of two offsetting effects: While the rest-of-world block exerts a positive effect, likely reflecting the economic recovery and maintained expansionary monetary policy in the US, the euro-crisis still contributes negatively to the growth in Danish GDP.

[16] This is not equivalent to the output gap in DSGE terminology. We leave the 'DSGE output gap' - the difference between actual output and the level of production which would have prevailed in the absence of any nominal rigidities - to future work.

6.2 Demand shocks

Shocks to domestic demand contributed to the strong growth in Danish GDP before the outbreak of the financial crisis, as seen from figure (24). We show in figure (26) the contribution of each shock in this group to movements in the output gap. The subgroup consists, as an example, of investment shocks. A positive shock to investments means that a given investment in capital in the current period leads to a bigger increase in the stock of installed capital in the next period than what would normally be the case. In the model a shock to investment also consists of residential investments as we do not have a separate residential sector in the model, and because the observed data series for investments consists of all investments as we wish to cover the entire national account. We therefore assign an important part of the positive investment shocks during the period 2005-2008 to higher residential investments.

From figure (26) we also observe that shocks to Danish households' preferences for imported goods contributed positively to the output gap during the period 2005-2008. This may reflect that Danish households were more eager to buy Danish goods and services than usually. A possible explanation to this may be that Danish households consumed a lot of 'residential' goods and services.

6.3 Markup shocks

We now turn to the group which consists of shocks to the economy's markup shocks. This subgroup consists of shocks to wage and price markups (domestic prices, import prices and export prices). We decompose the contribution of the markup shocks to the growth rate in real GDP in figure (27).

In the build-up to the financial crisis, the markup shocks taken together affected the output gap negatively. This is primarily due to the domestic price markup shock which was relatively high before the crisis. The economic intuition behind this observation is that domestic producers in those years utilized the extraordinary high domestic and foreign demand to increase their margins more than usually. That is a sign of an overheated economy.

A bit more surprising is that figure (27) shows that the wage markup shock affected GDP positively during the build up to the financial crisis, 2005-2008. This implies that the wage markup was relatively low in these years. Looking back at figure (2) in section (2), a relatively smooth increase can be observed in the industrial wage deflated by the consumer price index, along with a fall in unemployment and a strong increase in employment (not shown). Only in the latter phase of the build-up to the financial crisis an equally strong increase in real wages can be observed. The limited wage increases may be attributed to inflows of foreign labour and increases in the labour force. To explain these movements the model identifies a decrease in the wage markup, which contributes positively to GDP growth.

The subsequent bust leads to a downward pressure on prices due to a drop in demand. As they anticipate falling domestic and foreign demand, domestic producers lower their margins, which affects GDP positively. On the contrary, the wage markup contributes negatively to the growth rate in GDP. This can be explained by a rapid increase in unemployment, while wages did not fall accordingly, see also figure (1).

6.4 Productivity and capacity shocks

The decomposition in figure (24) points to productivity and capacity (or supply) shocks as an important driver of the boom during the years 2005-2007 and, to a smaller extent, the subsequent bust. This may seem somewhat surprising at a first glance. In this subsection, we argue that this finding is in line with the data as well as with economic theory. Firstly, the data for detrended hourly productivity shows a similar pattern over the relevant period. Secondly, during this period, output was booming while inflation remained fairly low and stable, suggesting that shocks originating on the supply side played an important role for business cycle fluctuations.

We emphasize that productivity and capacity shocks should not be interpreted as a factor behind the overheating of the Danish economy and the associated loss of competitiveness against Denmark's trading partners. From a DSGE perspective the economy's response to productivity shocks is efficient and hence does not call for economic stabilization policy. Rather, our findings suggest that the actual overheating of the Danish economy during these years may have been smaller than previously thought as the economy seems to have been able to expand potential production. This points to the importance of identifying the fundamental drivers of the business cycle especielly if policy makers wish to react to output gaps.

Figure (7) shows the movements in measured hourly productivity in the Danish non-farm sector. The figure illustrates that from around 2003 and until the end of 2006, productivity growth was in fact above the estimated trend for our sample period. From late 2006, measured productivity then dropped massively, reaching its trough in the fourth quarter of 2008 at the peak of the financial crisis. This development fits fairly well with the productivity shocks observed by our estimated model during the same years. In particular, the large positive productivity shocks in 2005-2006 and the subsequent reversal coincides closely with the data, although the model-implied reversal seems to lag the data by a few quarters. In addition, we suspect that other related factors may have contributed to the model-implied productivity shocks during the boom. This includes inflow of foreign workers, which probably played a role in keeping marginal costs down during these years as well the role of China in keeping imported inflation on a relatively low level. Overall, figure (7) illustrates that supply-side factors are likely to have played an important role in the Danish boom-bust cycle during the mid-2000's.

In general, the years leading up to the recent crisis were characterized by a boom in output without an associated rise in inflation, as evidenced, for instance, by the quarterly growth rates in Danish producer prices displayed in figure (5). Danish inflation took off only during the final part of the boom, i.e. from the second half of 2007 onwards. Even then, part of the increase in inflation was driven by a global rise in food and energy prices. This sounds almost like a textbook description of the effects of a supply shock; enabling firms to increase production at an unchanged level of marginal costs. In light of this, it is not surprising that our model ascribes at least part of the boom in output to factors originating on the supply side.

6.5 Fiscal policy

In a fixed exchange rate regime such as Denmark's, monetary policy can not be used to dampen domestic demand and price pressures. This implies that fiscal policy plays the role as the main stabilization instrument in the Danish economy. Our analysis so far has indicated that the large positive output gap in Denmark before the financial crisis can be attributed to a combination of supply side factors and a strong growth in domestic as well as foreign demand. From a stabilization perspective, such a situation would call for a contractionary fiscal policy so as to dampen the growth in aggregate demand. However, figure (24) shows that during the boom years, discretionary fiscal policy, which in our setup is defined as the contribution from the group of fiscal policy shocks to the output gap, exerted a small but mostly positive effect on output. In other words, fiscal policy appears to have been too expansionary, or at the very least not sufficiently contractionary, during the boom years.

In 2009, Denmark adopted an expansionary fiscal policy in response to the financial crisis. From figure (24) this is reflected in positive contributions from discretionary fiscal policy. Due to the large automatic fiscal stabilizers in the Danish economy, Denmark's public finances suffered, and from 2010, the fiscal stimulus was withdrawn and replaced by fiscal austerity. By construction, the effects of automatic fiscal stabilizers do not show up in figure (24), as these are not associated with their own set of structural shocks to the economy, but instead work by attenuating other shocks.

As a robustness check we compare the contribution of discretionary fiscal policy shocks in the estimated model to other available measures of the effects of fiscal policy. In Denmark, the effect on GDP from discretionary fiscal policy are traditionally measured by the so-called (one-year) 'fiscal effects'. Fiscal effects measure the contribution from discretionary fiscal policy on the growth rate of GDP relative to a situation in which fiscal policy is *neutral*. As an example, a neutral stance of government spending may be thought of as a situation in which the growth rate of public spending follows the steady-state growth rate of GDP. A positive fiscal effect thus implies that the discretionary part of fiscal policy in a given year contributes positively to the growth in real GDP. The fiscal effects are calculated by the Ministry of Finance and the Economic Council, and are based on the change in expenditure/revenues for a given item on the primary government balance and the associated multiplier for that item in the traditional macroeconometric models used by these institutions. Differences in the fiscal effects reported by the two institutions may therefore arise due to differences in the models used and due to different assumptions as regards a neutral fiscal policy.

In the DSGE model the effect on GDP of discretionary fiscal policy is measured by the size of the green columns in figure (24). It may be useful to think of these columns as impulse responses where the size of the shock is computed in the estimation of the model. The baseline for the impulse responses is the model's steady state in which discretionary fiscal policy is neutral. Hence, there is a fairly close connection between the effects of fiscal policy in the DSGE model and the fiscal effects.

In figure (28) we compare the fiscal effects computed by the Ministry of Finance and the Economic Council with the fiscal policy shocks from the DSGE-model, i.e. the green columns in figure (24) transformed to an annual basis. The main impression is that the different ways of measuring fiscal policy yield roughly the same results. In most years, there is a fairly close link between the three measures. There are some differences between the fiscal effects and the effects from the DSGE-model; especially in 1999-2000 and in 2005. However, there are also notable differences between the fiscal effects as computed by the Ministry of Finance and the Economic Council. This suggests that while the exact contribution of fiscal policy in a given year may be hard to measure, the three different approaches give rise to similar conclusions for the sample period taken together.

7 Conclusion

In this paper, we have set up a newly developed DSGE-model for the Danish economy, and shown and explained how to estimate it using Bayesian techniques. We have presented estimation results and examined the empirical properies through impulse-response functions and variance decompositions. Our results indicate that the model has economically plausible properties. Finally, we have used the model to compute a historical shock decomposition of Danish real GDP growth.

The model is, however, not finished and we expect a continuing development in light of economic and academic developments. We can easily point to some improvements: We have not provided a DSGE-based output gap, the model does not have inputs of imported goods in the production function, and finally, and perhaps most importantly, the model does not feature house prices, financial frictions or banking. In the future, we plan to incorporate these elements in the model. Also, we think that the set of possible applications of this model is fairly large. As an example, we have not talked about forecasting or counterfactual analysis in this paper.

8 Appendix A: The Data

As described in the main text, we use times series for 23 macroeconomic variables. These include 16 Danish series, 6 foreign series, and the effective Danish exchange rate. The 'Rest of World'-variables are defined as the weighted sum of GDP, inflation or the policy rate of Denmark's trading partners excluding the trading partners within the eurozone. We transform the time series into quarter-on-quarter growth rates, approximated by the first difference of their logarithm. In the following, we describe the source of each of the variables used in the estimation.

- Real GDP: Danmarks Nationalbank, MONA (variable name: *fy*).
- Private consumption: Danmarks Nationalbank, MONA (*fcp*).
- Government spending: Danmarks Nationalbank, MONA (*fco*).
- Government investment: Danmarks Nationalbank, MONA (*fio*).
- Exports: Danmarks Nationalbank, MONA (*fe*).
- Imports: Danmarks Nationalbank, MONA (*fm*).
- Total private investment: Danmarks Nationalbank, MONA (*fip*; chained sum of all types of private investment, including inventories and construction).
- Labor income tax: Danmarks Nationalbank, MONA (*bsda*).
- Private Employment: Danmarks Nationalbank, MONA (*qp+qs*).
- Unemployment: Danmarks Nationalbank, MONA (*ul* / (*qp+qo+qs*)).
- Industry nominal wages deflated by CPI: Danmarks Nationalbank, MONA (*lna* / *pcp*).
- Investment deflator: Danmarks Nationalbank, MONA (*pip*; relative price of total private investment).
- Producer price index: Danmarks Nationalbank, MONA (*pyfbx*).
- Import price deflator: Danmarks Nationalbank, MONA (*pm*).
- Export price deflator: Danmarks Nationalbank, MONA (*pe*).
- Danish nominal interest rate: Danmarks Nationalbank, MONA (*idi*, quarterly rate).
- Effective Danish exchange rate: Danmarks Nationalbank, MONA (*efkrks*).
- eurozone inflation: OECD (obtained from Ecowin).

- Rest of World inflation: OECD (Ecowin).
- eurozone real GDP: OECD (Ecowin).
- Rest of World real GDP: OECD (Ecowin).
- ECB policy (nominal) interest rate: OECD (Ecowin).
- Rest of World implied nominal interest rate: OECD (Ecowin).

9 Appendix B: The eurozone and rest of world models

The main impacts of the foreign countries on the Danish economy work through trade and interest rates. We consequently aim for the most flexible model for these two economies and downplay the microfoundations Further, we do not aim to estimate a common trend for all the three economies as the data points to different steady state growth rates in output. Denmark's role as a small open economy implies that we can model the foreign economies as being exogenous to the Danish economy. Also, we do not aim to model the interrelations between the eurozone and rest of the world, and we consequently do not model trade between these two economies. We do, however, include a UIP-relation between the policy rate in the two countries so that we can pin down the effective exchange rate between Denmark and rest of world.

The model equations are shown in section (3.8) while the parameter estimates are shown in table (1). We use the same sample as for the Danish economy. Data are HP-filtered and shown in figure (6). We estimate the international linkages using a two-step procedure: In the first step, we estimate the two separate small-scale DSGE models of each of the foreign blocks. In the second step, where we estimate the main model for Denmark, we include these estimated relations, and then estimate the shocks in the foreign models by including again the data for the foreign economies.

10 Appendix C: Market Clearing

In this appendix, we demonstrate that all markets clear in the model. Start from the budget constraint of the household, which reads:

$$
\begin{aligned}
& \left(1+\tau_t^{VAT}\right) \frac{P_t^C}{P_t} C_t + \frac{P_t^I}{P_t} I_t + B_t^{DK} + B_t^I + T_t \\
= \; & \Pi_t + \left(\left(1-\tau_t^K\right) r_t^K u_t + \tau_t^K \delta^K - z^u\left(u_t\right)\right) K_{t-1} + \frac{R_{t-1} B_{t-1}^{DK}}{\pi_t^{DK}} + \\
& + \frac{R_{t-1}^{ECB} \exp(-\psi_d \left(\frac{B_{t-1}^I}{Y_t} - \frac{\overline{B}^I}{\overline{Y}}\right)) B_{t-1}^I}{\pi_t^{DK}} - \frac{\tau_t^B B_{t-1}^{DK}\left(R_{t-1}-1\right)}{\pi_t^{DK}} + \\
& + \left(1-\tau_t^n\right) w_t N_t + \kappa^B w_t U_t^N .
\end{aligned}
$$

Now use the government's budget constraint to insert for B_t^{DK} on the left-hand side, and rewrite to obtain:

$$
\begin{aligned}
& \left(1+\tau_t^{VAT}\right) \frac{P_t^C}{P_t} C_t + \frac{P_t^I}{P_t} I_t + \left(\frac{R_{t-1}}{\pi_t^{DK}} B_{t-1}^{DK} + G_t + w_t U_t^N \kappa^B - TR_t\right) + B_t^I + T_t \\
= \; & \Pi_t + \left(\left(1-\tau_t^K\right) r_t^K u_t + \tau_t^K \delta^K - z^u\left(u_t\right)\right) K_{t-1} + \frac{R_{t-1} B_{t-1}^{DK}}{\pi_t^{DK}} + \\
& + \frac{R_{t-1}^{ECB} \exp(-\psi_d \left(\frac{B_{t-1}^I}{Y_t} - \frac{\overline{B}^I}{\overline{Y}}\right)) B_{t-1}^I}{\pi_t^{DK}} - \frac{\tau_t^B B_{t-1}^{DK}\left(R_{t-1}-1\right)}{\pi_t^{DK}} + \\
& + \left(1-\tau_t^n\right) w_t N_t + \kappa^B w_t U_t^N .
\end{aligned}
$$

Next, insert the expressions for TR_t and G_t presented in the main text, as well as for profits, which we can write as $\Pi_t = Y_t - w_t N_t - r_t^K u_t K_{t-1}$:

$$
\begin{aligned}
& \left(1+\tau_t^{VAT}\right) \frac{P_t^C}{P_t} C_t + \frac{P_t^I}{P_t} I_t + B_t^I + T_t - \left(Y_t - w_t N_t - r_t^K u_t K_{t-1}\right) - \\
& \left(\left(1-\tau_t^K\right) r_t^K u_t + \tau_t^K \delta^K - z^u\left(u_t\right)\right) K_{t-1} - \frac{R_{t-1} B_{t-1}^{DK}}{\pi_t^{DK}} + \frac{\tau_t^B B_{t-1}^{DK}\left(R_{t-1}-1\right)}{\pi_t^{DK}} \\
& - \frac{R_{t-1}^{ECB} \exp(-\psi_d \left(\frac{B_{t-1}^I}{Y_t} - \frac{\overline{B}^I}{\overline{Y}}\right)) B_{t-1}^I}{\pi_t^{DK}} - \left(1-\tau_t^n\right) w_t N_t - \kappa^B w_t U_t^N \\
= \; & T_t + \tau_t^{VAT} \frac{P_t^C}{P_t} C_t + \tau_t^K \left(r_t^K u_t - \delta^K\right) K_{t-1} + \tau_t^N w_t N_t \\
& + \tau_t^B \frac{R_{t-1}-1}{\pi_t^{DK}} B_{t-1}^{DK} - \frac{R_{t-1}}{\pi_t^{DK}} B_{t-1}^{DK} - C_t^G - \frac{P_t^I}{P_t} I_t^G - w_t U_t^N \kappa^B .
\end{aligned}
$$

We can now begin to cancel out terms:

$$\begin{aligned}
& \frac{P_t^C}{P_t}C_t + \frac{P_t^I}{P_t}I_t + B_t^I - Y_t + z^u\left(u_t\right)K_{t-1} - \frac{R_{t-1}^{ECB}\exp(-\psi_d\left(\frac{B_{t-1}^I}{Y_t} - \frac{\overline{B}^I}{\overline{Y}}\right))B_{t-1}^I}{\pi_t^{DK}} \\
= \quad & -C_t^G - \frac{P_t^I}{P_t}I_t^G,
\end{aligned}$$

which we can rewrite as:

$$Y_t = \frac{P_t^C}{P_t}C_t + \frac{P_t^I}{P_t}I_t + C_t^G + \frac{P_t^I}{P_t}I_t^G + z^u\left(u_t\right)K_{t-1} + \left(B_t^I - \frac{R_{t-1}^{ECB}\exp(-\psi_d\left(\frac{B_{t-1}^I}{Y_t} - \frac{\overline{B}^I}{\overline{Y}}\right))}{\pi_t^{DK}}B_{t-1}^I\right).$$

Now apply the resource constraint as defined in the main text:

$$\begin{aligned}
& \frac{P_t^C}{P_t}C_t^{DK} + \frac{P_t^I}{P_t}I_t^{DK} + C_t^G + \frac{P_t^I}{P_t}I_t^G + z^u\left(u_t\right)K_{t-1} + \frac{P_t^X}{P_t}Ex_t \\
= \quad & \frac{P_t^C}{P_t}C_t + \frac{P_t^I}{P_t}I_t + C_t^G + \frac{P_t^I}{P_t}I_t^G + z^u\left(u_t\right)K_{t-1} + \left(B_t^I - \frac{R_{t-1}^{ECB}\exp(-\psi_d\left(\frac{B_{t-1}^I}{Y_t} - \frac{\overline{B}^I}{\overline{Y}}\right))}{\pi_t^{DK}}B_{t-1}^I\right) \Leftrightarrow
\end{aligned}$$

$$\frac{P_t^X}{P_t}Ex_t - \frac{P_t^C}{P_t}\left(C_t - C_t^{DK}\right) - \frac{P_t^I}{P_t}\left(I_t - I_t^{DK}\right) = B_t^I - \frac{R_{t-1}^{ECB}\exp(-\psi_d\left(\frac{B_{t-1}^I}{Y_t} - \frac{\overline{B}^I}{\overline{Y}}\right))}{\pi_t^{DK}}B_{t-1}^I.$$

The final step is to apply the equation determining Denmark's net foreign asset position, which reads as:

$$\begin{aligned}
B_t^I &= \frac{R_{t-1}^{ECB}\exp(-\psi_d\left(\frac{B_{t-1}^I}{Y_t} - \frac{\overline{B}^I}{\overline{Y}}\right))}{\pi_t^{DK}}B_{t-1}^I + \frac{P_t^X}{P_t}Ex_t - \frac{P_t^M}{P_t}\,\mathrm{Im}_t \Leftrightarrow \\
B_t^I - \frac{R_{t-1}^{ECB}\exp(-\psi_d\left(\frac{B_{t-1}^I}{Y_t} - \frac{\overline{B}^I}{\overline{Y}}\right))}{\pi_t^{DK}}B_{t-1}^I &= \frac{P_t^X}{P_t}Ex_t - \frac{P_t^M}{P_t}\,\mathrm{Im}_t\,.
\end{aligned}$$

Insert this to obtain:

$$\frac{P_t^X}{P_t}Ex_t - \frac{P_t^C}{P_t}\left(C_t - C_t^{DK}\right) - \frac{P_t^I}{P_t}\left(I_t - I_t^{DK}\right) = \frac{P_t^X}{P_t}Ex_t - \frac{P_t^M}{P_t}\,\mathrm{Im}_t,$$

which we can rewrite as:

$$\begin{aligned}
-\frac{P_t^C}{P_t}\left(C_t - C_t^{DK}\right) - \frac{P_t^I}{P_t}\left(I_t - I_t^{DK}\right) &= -\frac{P_t^M}{P_t}\,\mathrm{Im}_t \Leftrightarrow \\
\mathrm{Im}_t &= \frac{P_t^C\left(C_t - C_t^{DK}\right) + P_t^I\left(I_t - I_t^{DK}\right)}{P_t^M}, \qquad (78)
\end{aligned}$$

which is the expression for imports used in the main text.

11 Tables and figures

		Prior distribution				Posterior distribution					
		Type	Source	Mean	s.d.	Mean	Mode	s.d.	Median	5 pct.	95 pct.
Parameters for euroarea model											
Weight on expected output	ρ_Y^{EA}	Normal	-	0.75	0.1	0.64	0.63	0.061	0.64	0.54	0.74
Weight on real rate of interest	ϕ_Y^{EA}	Normal	-	0.4	0.05	0.35	0.35	0.052	0.35	0.26	0.44
Weight on expected inflation	ρ_π^{EA}	Normal	-	0.75	0.1	0.86	0.86	0.067	0.86	0.75	0.97
Weight on output	ϕ_π^{EA}	Normal	-	0.75	0.1	0.59	0.59	0.092	0.59	0.44	0.74
Policy interest rate smoothing parameter	ρ_R^{EA}	Normal	-	0.85	0.1	0.98	0.98	0.0027	0.98	0.98	0.99
Persistence of shock to output	$\rho_{\epsilon^Y}^{EA}$	Beta	-	0.85	0.1	0.9	0.91	0.02	0.9	0.86	0.93
Persistence of shock to inflation	$\rho_{\epsilon^\pi}^{EA}$	Beta	-	0.85	0.1	0.76	0.76	0.049	0.76	0.68	0.85
Shock to output	ϵ_Y^{EA}	Inv. gamma	-	0.01	2	0.003	0.003	0.001	0.003	0.002	0.004
Shock to inflation	ϵ_π^{EA}	Inv. gamma	-	0.01	2	0.026	0.026	0.004	0.026	0.019	0.032
Monetary policy shock	ϵ_R^{EA}	Inv. gamma	-	0.01	2	0.003	0.003	0.00	0.0025	0.002	0.0031
Policy interest rate, reaction to inflation	Γ_π^{EA}	Calibrated		1.5							
Policy interest rate, reaction to inflation	Γ_Y^{EA}	Calibrated		0.125							
Parameters for rest of world model											
Weight on expected output	ρ_Y^{RoW}	Normal	-	0.75	0.1	0.69	0.68	0.068	0.69	0.57	0.8
Weight on real rate of interest	ϕ_Y^{RoW}	Normal	-	0.5	0.05	0.52	0.51	0.051	0.52	0.43	0.6
Weight on expected inflation	ρ_π^{RoW}	Normal	-	0.75	0.1	0.88	0.88	0.084	0.88	0.73	1
Weight on output	ϕ_π^{RoW}	Normal	-	0.75	0.1	0.32	0.28	0.063	0.31	0.21	0.44
Policy interest rate smoothing parameter	ρ_R^{RoW}	Beta	-	0.85	0.1	0.84	0.85	0.021	0.85	0.81	0.88
Persistence of shock to output	$\rho_{\epsilon^Y}^{RoW}$	Beta	-	0.85	0.1	0.76	0.77	0.037	0.76	0.7	0.82
Persistence of shock to inflation	$\rho_{\epsilon^\pi}^{RoW}$	Beta	-	0.85	0.1	0.8	0.8	0.053	0.8	0.72	0.89
Shock to output	ϵ_Y^{RoW}	Inv. gamma	-	0.01	2	0.002	0.002	0.0003	0.002	0.002	0.003
Shock to inflation	ϵ_π^{RoW}	Inv. gamma	-	0.01	2	0.002	0.002	0.00032	0.002	0.002	0.003
Monetary policy shock	ϵ_R^{RoW}	Inv. gamma	-	0.01	2	0.002	0.002	0.00022	0.002	0.002	0.002
Policy interest rate, reaction to inflation	Γ_π^{RoW}	Calibrated		1.5							
Policy interest rate, reaction to inflation	Γ_Y^{RoW}	Calibrated		0.125							

Table 1: **Estimated and calibrated parameters for the foreign countries:** This table reports the prior distribution and the posterior mode estimates of the structural parameters for the model.

	Value	Parameter explanation	Explanation
β	0.99	Discount factor	To match annualized SS real rate
δ^K	0.0250	Private capital depreciation	Standard value
ϵ^P	6	Sub. between good	Markup of 20%, standard
α	0.30	Capital share	SS labour share in production
ψ_d	0.0100	Risk premia for inv. in foreign bonds	Small but keep model stationary
ϵ^W	13	Sub. between laboures	Natural rate of unemployment of 4%
ψ_G	0.020	Ela. of Danish interest rate risk premia wrt. debt	Corsetti et al.
ω_X	0.5	Relative size of foreign countries	Average through period
η	0.0150	Public capital influence in production	Microdata, Kamps 2004
δ^G	0.0250	Depreciering of public capital	Standard
ϵ^X	6	Sub. between export goods	Markup of 20%, standard
ϵ^M	6	Sub. between import goods	Markup of 20%, standard
ϕ_0^I	0	Immediate implementation of pub. inv.	Time-to-build, Leeper et al. [2010]
ϕ_i^Ii=1,2,3	$\frac{1}{3}$	1st-3rd q. implementation of pub. inv.	Time-to-build, Leeper et al. [2010]
ζ_T	0.5	Ela. of lump-sum taxes wrt. public debt	St. model is stationary
κ_U	0.5	Unemployment compensation	LR average for the sample
$\bar{u}$	1	SS capital utilization cost	Simplification
c_1	0.0349	Set st. $\bar{u}$=1	Simplification
ϑ_C	0.60	Degree of home-bias in con.	Import content in total private consumption
ϑ_I	0.5	Degree of home-bias in inv.	Import content in investments.

Table 2: **Calibrated parameters:** This table shows that parameters which are calibrated in the estimation.

		Prior distribution				Posterior distribution					
		Type	Source	Mean	s.d.	Mean	Mode	s.d.	Median	5 pct.	95 pct.
Preferences and production											
Habit formation	h	Beta	SW07	0.15	0.05	0.36	0.63	0.064	0.35	0.24	0.47
Wage indexation	Γ_W	Beta	Medea	0.5	0.2	0.5	0.45	0.27	0.5	0.17	0.83
Price indexation	Γ_P	Beta	Medea	0.4	0.005	0.4	0.4	0.005	0.4	0.39	0.41
Import price indexation	Γ_{Mt}	Beta	Medea	0.5	0.2	0.42	0.63	0.22	0.42	0.14	0.69
Export price indexation	Γ_{Xt}	Beta	Medea	0.35	0.2	0.074	0.24	0.22	0.058	0.00054	0.15
Calvo, wages	θ_w	Beta	SW03	0.7	0.05	0.69	0.73	0.041	0.69	0.63	0.75
Calvo, import price	θ_M	Beta	SW03	0.75	0.05	0.76	0.77	0.047	0.76	0.68	0.84
Cavlo, export price	θ_X	Beta	SW03	0.75	0.05	0.5	0.51	0.048	0.5	0.44	0.57
Calvo, price	θ_P	Normal	SW03	0.75	0.05	0.48	0.48	0.025	0.48	0.44	0.52
Elasticity	ϵ^W	Normal	ADLS07	20	1	19	22	1	19	17	20
Frisch labour supply	ϕ	Normal	Gali11	2.5	1	2.9	2.3	0.2	2.9	2.6	3.1
Pref. for degree of wealth-effect	ν	Beta	Gali11	0.5	0.2	0.57	0.46	0.14	0.56	0.32	0.83
Investment adj. cost	κ_I	Normal	SW03	0.25	0.05	0.34	0.4	0.044	0.34	0.27	0.41
Utilization adj. cost	C_2	Normal		1.5	0.1	1.5	1.5	0.1	1.5	1.3	1.7
Cost of adj import in C.	χC	Normal	Medea	2	0.01	2	2	0.01	2	2	2
Cost of adj import in I.	χI	Normal	Medea	2	0.01	2	2	0.01	2	2	2
CES ela. C.	υ_C	Gamma	SW03	7.5	1	3.1	3.8	0.45	3.1	2.6	3.7
CES ela. I.	υ_I	Gamma	SW03	7.5	1	4.8	5.3	0.67	4.7	4	5.5

Table 3: **Estimated parameters:** This table reports the prior distribution and the posterior mode estimates of the structural parameters for the model. ADLS07 refers to Adolfson (2007), SW03 refers to Smets and Wouters (2003), Medea refers to Fernández-Villaverde et al. (2009) and Gali refers to Galí et al. (2011)

		Prior distribution				Posterior distribution					
		Type	Source	Mean	s.d.	Mean	Mode	s.d.	Median	5 pct.	95 pct.
Persistence of shocks											
Persistence of perm. tech. shock	ρ_{A_P}	Beta	SW07	0.85	0.1	0.94	0.95	0.031	0.94	0.89	0.99
Persistence of temp. tech. shock	$\tilde{A}_T$	Beta	SW07	0.85	0.1	0.67	0.79	0.12	0.68	0.49	0.86
Persistence of C. shock	ρ_{con}	Beta	SW07	0.85	0.1	0.78	0.91	0.089	0.82	0.51	0.99
Persistence of price markup shock	ρ_{ϵ_P}	Beta	SW07	0.85	0.1	0.89	0.94	0.058	0.9	0.79	0.99
Persistence of inv. shock	ρ_{Z_T}	Beta	SW07	0.7	0.1	0.35	0.69	0.11	0.35	0.22	0.48
Persistence of export shock	$\rho_{X^{Ex}}$	Beta	SW07	0.85	0.1	0.85	0.94	0.071	0.87	0.7	1
Persistence of import shock	ρ_{Im}	Beta	SW07	0.85	0.1	0.79	0.92	0.089	0.83	0.57	0.99
Persistence of export price markup shock	ρ_{e^X}	Beta	SW07	0.85	0.1	0.88	0.91	0.063	0.89	0.78	0.98
Persistence of import price markup shock	ρ_{e^M}	Beta	SW07	0.85	0.1	0.84	0.92	0.079	0.86	0.7	0.99
Persistence of riskpremia shock	ρ^{RPD}	Beta	SW07	0.85	0.1	0.57	0.63	0.091	0.58	0.41	0.74
Persistence of wage markup shock	ρ_{ϵ_W}	Beta	SW07	0.9	0.1	0.91	0.98	0.03	0.94	0.8	1
Persistence of labour income tax shock	ρ_{tn}	Beta	SW07	0.5	0.1	0.24	0.22	0.056	0.23	0.14	0.32
Persistence of capacity utilization shock	ρ^{cap}	Beta	SW07	0.85	0.1	0.85	0.92	0.084	0.87	0.71	1
Std. of shocks											
Perm tech shock	ε^{A_P}	Inv. gamma		0.001	0.1	0.00046	0.00034	4.5e-05	0.00044	0.00028	0.00065
Wage markup shock	ε^{e^W}	Inv. gamma		0.01	2	0.0058	0.0041	0.0014	0.0054	0.0027	0.009
Temp. tech shock	ε^{A_T}	Inv. gamma		0.01	2	0.008	0.0072	0.0011	0.008	0.0066	0.0095
Consumption shock	ε^{con}	Inv. gamma		0.01	2	0.0093	0.0042	0.0014	0.0073	0.0023	0.019
Price markup shock	ε^{e^P}	Inv. gamma		0.01	2	0.0048	0.004	0.0011	0.0047	0.0026	0.0069
Investment shock	ε^{Z_T}	Inv. gamma		0.01	2	0.023	0.0042	0.0015	0.023	0.016	0.03
Export shock	$\varepsilon^{x^{E_x}}$	Inv. gamma		0.01	2	0.0067	0.0045	0.0018	0.006	0.0025	0.011
Import shock	ε^{Im}	Inv. gamma		0.01	2	0.0065	0.0043	0.0013	0.0059	0.0028	0.01
Import price markup shock	ε^{e^M}	Inv. gamma		0.01	2	0.0065	0.0044	0.0017	0.0058	0.0025	0.011
Export price markup shock	ε^{e^X}	Inv. gamma		0.01	2	0.0032	0.003	0.00064	0.0031	0.0022	0.0043
Riskpremia shock	ε^{RPD}	Inv. gamma		0.01	2	0.0035	0.0032	0.00086	0.0033	0.0021	0.0049
Public consumption shock	ε^{G}	Inv. gamma		0.01	2	0.0071	0.0071	0.00052	0.0071	0.0063	0.008
Public investment shock	ε^{IG}	Inv. gamma		0.05	2	0.063	0.062	0.0044	0.063	0.056	0.07
Tax on labour income shock	ε^{n}	Inv. gamma		0.03	2	0.011	0.011	0.00077	0.011	0.0097	0.012
EA price shock	$\varepsilon^{\Pi,EA}$	Inv. gamma		0.01	2	0.003	0.0027	0.0005	0.003	0.0021	0.004
EA output shock	$\varepsilon^{y,EA}$	Inv. gamma		0.01	2	0.0016	0.0015	0.00012	0.0016	0.0014	0.0018
ECB policy rate shock	$\varepsilon^{r,EA}$	Inv. gamma		0.01	2	0.0053	0.0054	0.00043	0.0053	0.0046	0.006
RoW output shock	$\varepsilon^{y,RW}$	Inv. gamma		0.01	2	0.0021	0.002	0.0002	0.002	0.0017	0.0024
RoW price shock	$\varepsilon^{\Pi,RW}$	Inv. gamma		0.01	2	0.0022	0.0021	0.00028	0.0021	0.0017	0.0027
RoW interest rate shock	$\varepsilon^{r,RW}$	Inv. gamma		0.01	2	0.0016	0.0015	0.00016	0.0016	0.0013	0.0019
UIP shock	ε^{UIP}	Inv. gamma		0.01	2	0.0029	0.0027	0.00053	0.0028	0.0021	0.0037
Utilization shock	ε^{cap}	Inv. gamma		0.001	0.1	0.00079	0.00046	0.00013	0.00065	0.00025	0.0014

Table 4: **Estimated parameters:** This table reports the prior distribution and the posterior mode estimates of the structural parameters for the model. ADLS07 refers to Adolfson (2007), SW03 refers to Smets and Wouters (2003), Medea refers to Fernández-Villaverde et al. (2009) and Gali refers to Galí et al. (2011)

		Prior distribution				Posterior distribution					
		Type	Source	Mean	s.d.	Mean	Mode	s.d.	Median	5 pct.	95 pct.
Std. of measurement errors											
Inflation, RoW	$pieRowt$	Inv. gamma		0.15	2	0.014	0.013	0.00095	0.014	0.012	0.015
Output, RoW	$YRoWt$	Inv. gamma		0.25	2	0.023	0.023	0.0017	0.023	0.02	0.026
Interest rate, RoW	$RRoWt$	Inv. gamma		0.015	2	0.0017	0.0017	0.00014	0.0017	0.0015	0.002
Inflation, EA	$pieEAt$	Inv. gamma		0.01	2	0.064	0.063	0.0044	0.064	0.057	0.072
Output, EA	$YEAt$	Inv. gamma		0.01	2	0.061	0.061	0.0044	0.061	0.054	0.068
Interest rate, EA	$RECBt$	Inv. gamma		0.01	2	0.0047	0.0046	0.00032	0.0047	0.0042	0.0052
Unemploytment	Ut	Inv. gamma		0.01	2	0.062	0.061	0.0044	0.062	0.055	0.07
Employment	Npt	Inv. gamma		0.01	2	0.0041	0.0051	0.0019	0.004	0.0024	0.0056
Wages	Wpt	Inv. gamma		0.01	2	0.0074	0.0071	0.00064	0.0074	0.0063	0.0086
Output	$YBNPt$	Inv. gamma		0.01	2	0.0043	0.0057	0.00085	0.0043	0.0034	0.0052
Consumption	Ct	Inv. gamma		0.01	2	0.0039	0.011	0.00089	0.0038	0.0024	0.0054
Investments	It	Inv. gamma		0.01	2	0.025	0.049	0.0046	0.025	0.017	0.032
Inflation	$pieDKt$	Inv. gamma		0.01	2	0.0075	0.0055	0.0011	0.0075	0.0059	0.0091
Import	Mt	Inv. gamma		0.01	2	0.016	0.016	0.0021	0.016	0.012	0.02
Export	Xt	Inv. gamma		0.01	2	0.008	0.011	0.01	0.0076	0.003	0.013
Import prices	pm	Inv. gamma		0.01	2	0.014	0.013	0.00099	0.014	0.012	0.015
Export prices	pe	Inv. gamma		0.01	2	0.014	0.013	0.00097	0.014	0.012	0.016
Exchange rate	fx	Inv. gamma		0.01	2	0.98	0.97	0.066	0.98	0.87	1.1
Investment deflator	pi	Inv. gamma		0.01	2	0.032	0.033	0.0024	0.032	0.028	0.036

Table 5: **Estimated parameters:** This table reports the prior distribution and the posterior mode estimates of the structural parameters for the model.

Forecast horizon: 1 quarter

	Real GDP	Consumption	Investment	Export	Import	Inflation	Real wages	Employment	Unemployment
Public consumption shock	0.90	0.01	0.00	0.46	0.01	0.08	0.06	1.05	0.42
Perm tech shock	0.00	1.17	0.74	4.04	0.89	0.33	0.37	0.05	0.57
Wage markup shock	0.10	0.30	0.16	0.18	0.15	0.01	0.00	0.11	0.27
Temp. tech shock	14.02	6.65	9.67	0.19	3.92	0.54	5.75	0.07	0.46
Consumption shock	0.10	0.84	0.07	0.25	0.16	0.02	0.00	0.11	0.01
Price markup shock	4.52	2.16	4.99	0.22	2.21	0.04	8.07	5.24	3.49
Investment shock	0.98	0.05	4.96	2.21	2.35	0.29	0.36	1.14	0.58
Export shock	0.33	0.03	0.02	0.87	0.05	0.06	0.07	0.38	0.23
Import shock	0.07	0.03	0.14	0.85	0.18	0.07	0.01	0.08	0.08
Import price markup shock	0.01	0.01	0.05	0.17	0.22	0.01	0.00	0.01	0.02
Export price markup shock	4.24	0.71	0.45	11.17	1.08	1.39	1.38	4.95	3.24
Riskpremia shock	1.17	1.50	2.68	1.46	1.93	0.25	0.48	1.36	1.75
Public investment shock	0.01	0.00	0.00	0.02	0.00	0.00	0.00	0.01	0.01
Tax on labour income shock	0.58	0.09	0.10	0.42	0.00	0.26	0.29	0.70	0.44
EA price shock	4.37	1.45	1.25	3.98	1.91	3.55	2.83	5.08	3.66
EA output shock	27.12	32.99	22.54	0.08	29.58	60.11	42.27	31.44	32.71
ECB policy rate shock	11.67	32.86	32.11	33.51	32.75	15.50	19.28	13.52	22.16
RoW output shock	4.66	0.00	0.12	28.67	0.02	0.85	0.62	5.45	2.43
RoW price shock	4.02	0.98	0.75	5.46	1.21	1.56	1.57	4.68	3.31
RoW interest rate shock	14.21	15.68	16.75	1.23	18.34	12.46	13.70	16.50	17.82
UIP shock	6.92	2.50	2.45	4.56	3.06	2.62	2.89	8.06	6.34
Capital utilization shock	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

Table 6: **Variance decompositions:** This table reports posterior mean estimates for the forecast-error-variance decomposition of selected variables at the 1 quarter horizon. The decomposition is conducted only for the structural shocks part of the forecast errors, while the shares of the forecast errors due to measurement errors and unobserved state variables are skipped.

Forecast horizon: 4 quarter

	Real GDP	Consumption	Investment	Export	Import	Inflation	Real wages	Employment	Unemployment
Public consumption shock	0.23	0.01	0.01	0.26	0.00	0.06	0.01	0.82	0.27
Perm tech shock	0.13	1.10	0.94	5.75	1.19	0.25	0.25	1.11	0.51
Wage markup shock	0.17	0.36	0.22	0.28	0.23	0.01	0.00	0.38	1.00
Temp. tech shock	13.97	6.54	10.12	2.60	6.34	1.16	7.84	4.50	6.01
Consumption shock	0.15	0.91	0.07	0.32	0.20	0.02	0.00	0.35	0.01
Price markup shock	4.34	2.09	5.56	2.44	3.39	0.31	9.20	13.35	6.46
Investment shock	0.71	0.04	3.24	1.66	1.59	0.20	0.26	1.09	0.45
Export shock	0.08	0.03	0.01	0.09	0.04	0.04	0.03	0.29	0.15
Import shock	0.03	0.02	0.08	1.40	0.98	0.05	0.00	0.08	0.06
Import price markup shock	0.01	0.00	0.02	0.32	1.01	0.01	0.00	0.01	0.02
Export price markup shock	1.62	0.67	0.30	1.22	0.94	0.93	0.69	3.97	2.45
Riskpremia shock	0.36	0.37	0.80	0.72	0.62	0.19	0.15	1.04	1.22
Public investment shock	0.02	0.00	0.01	0.03	0.00	0.00	0.00	0.06	0.01
Tax on labour income shock	0.21	0.04	0.05	0.10	0.02	0.43	0.13	1.64	0.76
EA price shock	3.28	1.82	1.82	0.92	2.08	2.96	2.38	4.13	3.12
EA output shock	50.82	37.69	29.14	5.01	28.32	68.49	49.00	26.21	31.76
ECB policy rate shock	9.04	28.98	27.57	45.19	30.79	11.97	16.38	11.97	20.84
RoW output shock	2.03	0.00	0.53	7.20	0.08	0.63	0.15	5.68	2.00
RoW price shock	1.67	0.94	0.58	0.97	0.93	1.05	0.84	3.82	2.60
RoW interest rate shock	8.97	15.81	16.48	19.46	18.33	9.41	10.94	13.20	15.84
UIP shock	2.16	2.56	2.47	4.08	2.90	1.83	1.77	6.29	4.46
Capital utilization shock	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

Table 7: **Variance decompositions:** This table reports posterior mean estimates for the forecast-error-variance decomposition of selected variables at the 1 quarter horizon. The decomposition is conducted only for the structural shocks part of the forecast errors, while the shares of the forecast errors due to measurement errors and unobserved state variables are skipped.

Forecast horizon: 40 quarter

	Real GDP	Consumption	Investment	Export	Import	Inflation	Real wages	Employment	Unemployment
Public consumption shock	0.11	0.01	0.02	0.06	0.00	0.05	0.01	0.22	0.25
Perm tech shock	0.70	0.53	1.55	4.07	1.69	0.22	0.13	1.35	0.48
Wage markup shock	1.23	0.57	0.33	0.14	0.30	0.01	0.05	0.72	6.50
Temp. tech shock	9.64	5.01	8.27	2.90	4.12	0.94	5.54	4.28	5.67
Consumption shock	0.37	0.43	0.09	0.17	0.14	0.02	0.02	0.71	0.01
Price markup shock	5.79	2.13	5.42	1.39	2.31	0.28	9.28	3.64	6.04
Investment shock	0.63	0.14	1.98	0.44	0.45	0.19	0.25	0.35	0.42
Export shock	0.05	0.03	0.02	0.05	0.05	0.03	0.02	0.10	0.14
Import shock	1.09	0.10	0.63	4.42	10.29	0.04	0.60	0.40	0.06
Import price markup shock	0.35	0.02	0.33	0.80	3.26	0.01	0.13	0.11	0.02
Export price markup shock	0.99	0.67	0.37	1.08	1.34	0.78	0.39	1.83	2.30
Riskpremia shock	0.19	0.04	0.45	0.18	0.15	0.16	0.03	0.27	1.15
Public investment shock	0.01	0.00	0.01	0.01	0.00	0.00	0.00	0.02	0.02
Tax on labour income shock	0.10	0.02	0.03	0.04	0.02	0.34	0.03	0.44	0.70
EA price shock	1.93	1.92	2.00	1.86	2.05	2.38	1.71	2.59	2.91
EA output shock	61.29	49.03	37.82	24.43	28.46	74.43	56.24	36.79	30.50
ECB policy rate shock	5.92	23.43	23.44	34.09	26.56	9.61	14.82	23.99	19.44
RoW output shock	2.21	0.02	0.82	1.98	0.29	0.69	0.15	1.79	2.04
RoW price shock	0.95	0.85	0.52	1.53	1.03	0.86	0.48	2.03	2.44
RoW interest rate shock	5.26	12.88	13.73	17.16	14.97	7.50	8.70	14.80	14.77
UIP shock	1.18	2.17	2.16	3.20	2.53	1.45	1.42	3.56	4.13
Capital utilization shock	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

Table 8: **Variance decompositions:** This table reports posterior mean estimates for the forecast-error-variance decomposition of selected variables at the 1 quarter horizon. The decomposition is conducted only for the structural shocks part of the forecast errors, while the shares of the forecast errors due to measurement errors and unobserved state variables are skipped.

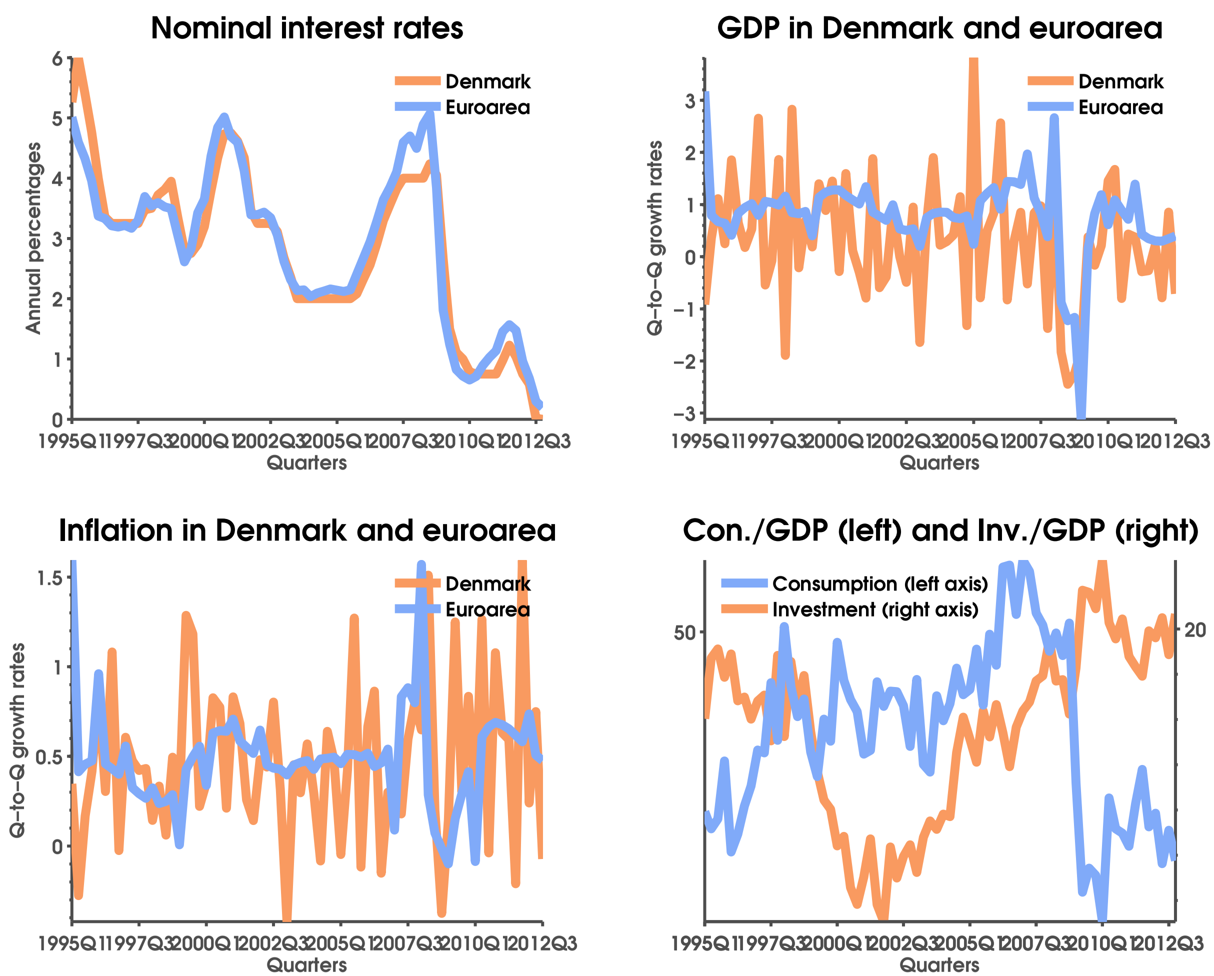

Figure 1: **Stylized facts about the Danish economy.**

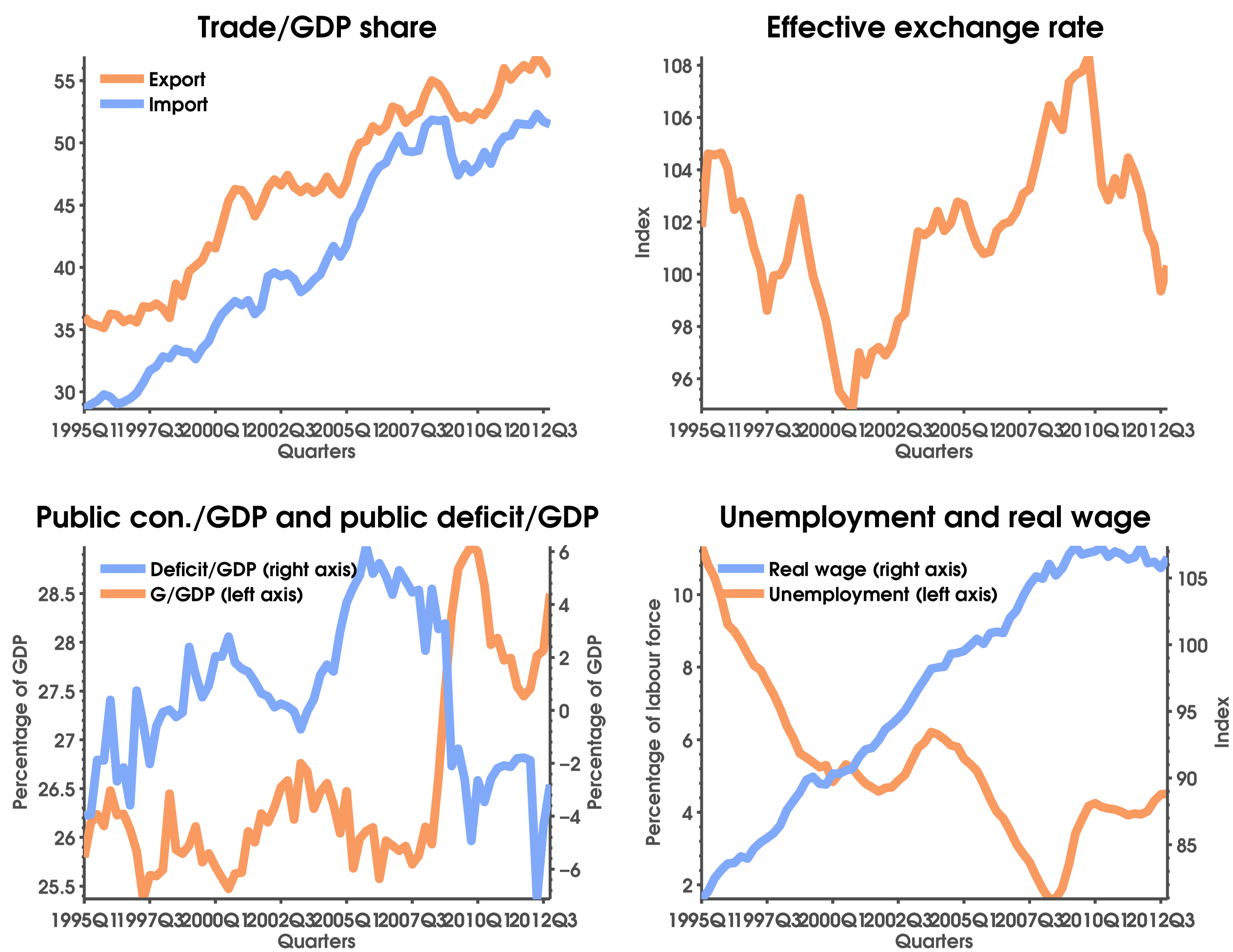

Figure 2: **Stylized facts about the Danish econom**

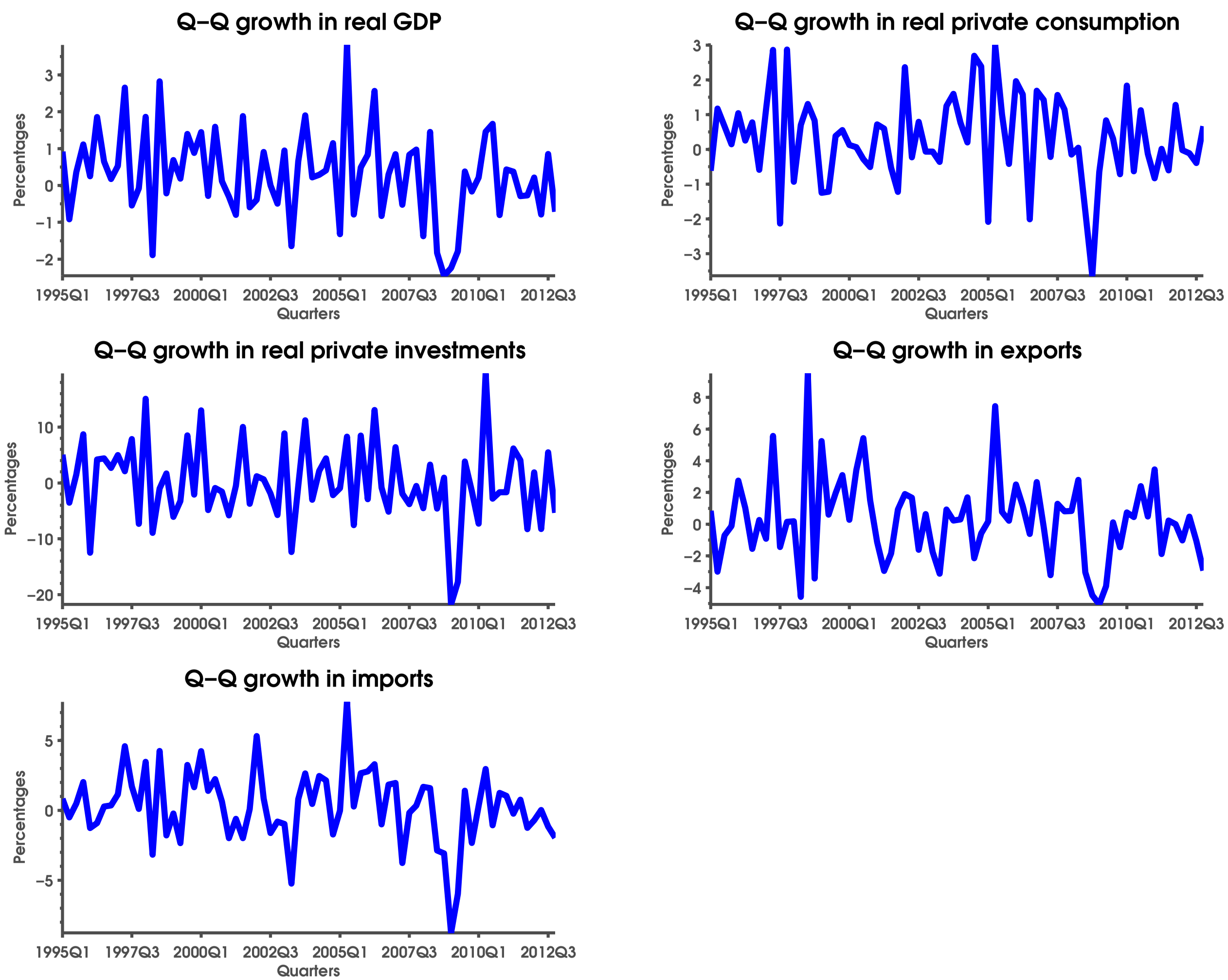

Figure 3: **Transformed observed time series**
This figure shows the time series of the observed variables used in the estimation of the model.

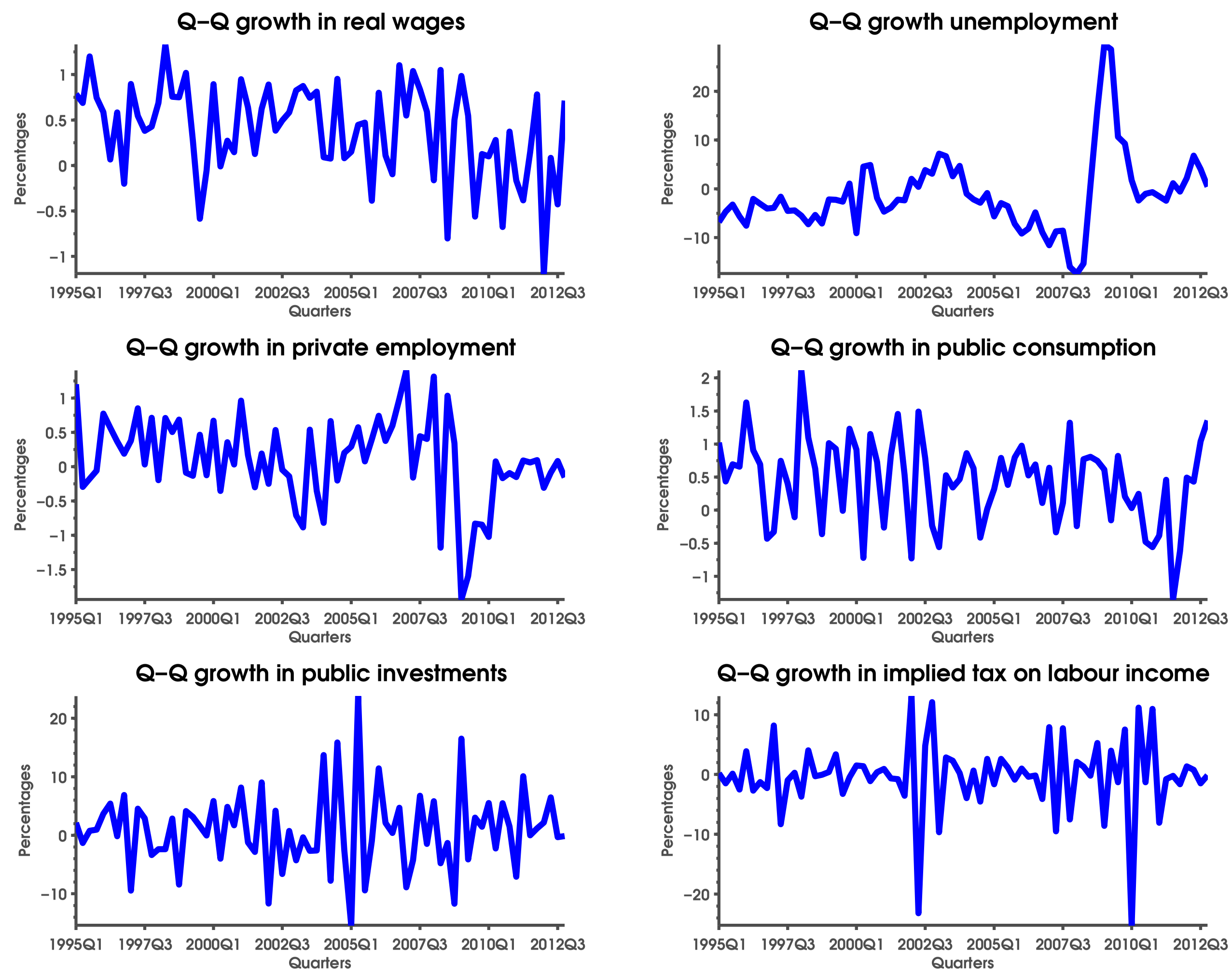

Figure 4: **Transformed observed time series**
This figure shows the time series of the observed variables used in the estimation of the model.

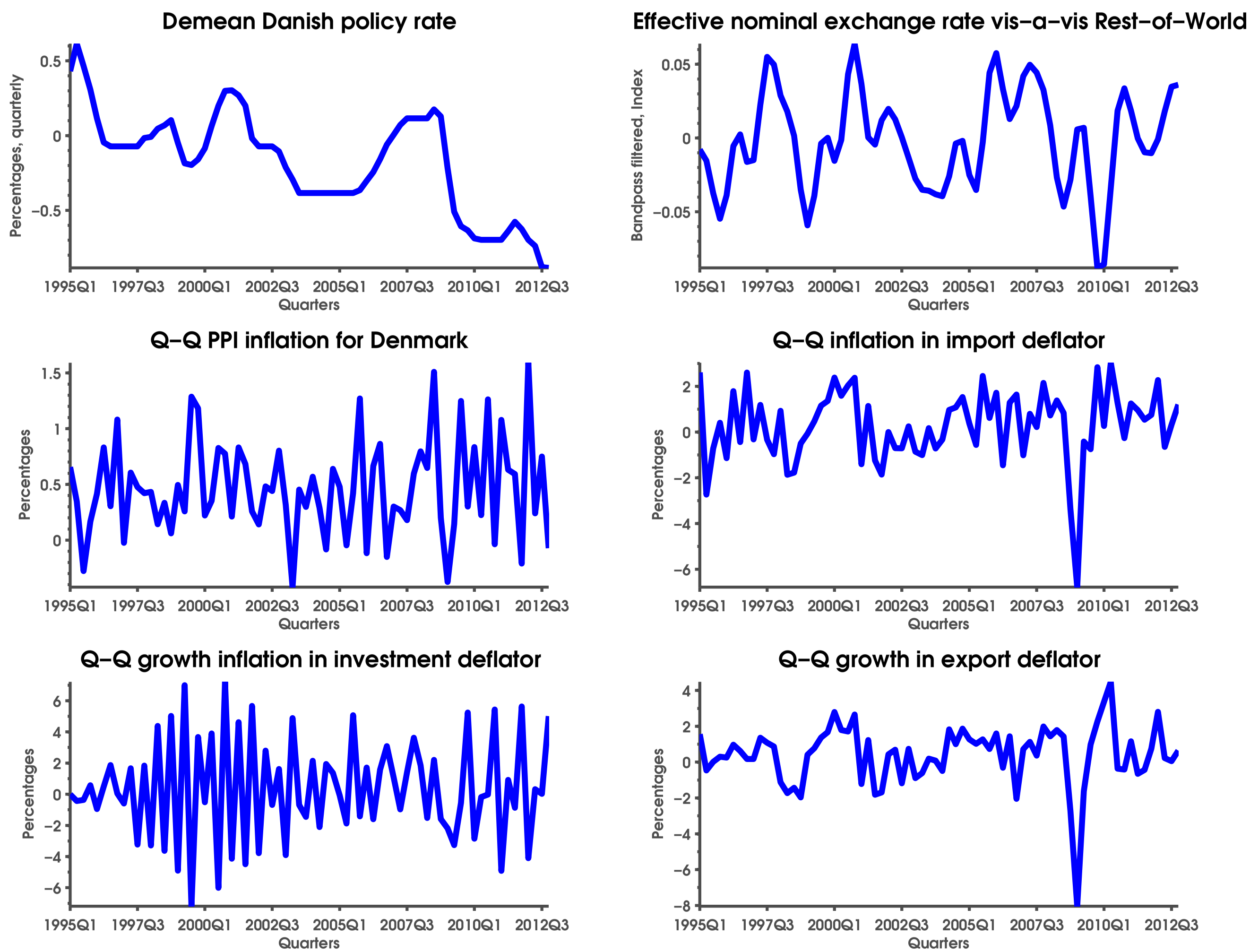

Figure 5: **Transformed observed time series**
This figure shows the time series of the observed variables used in the estimation of the model.

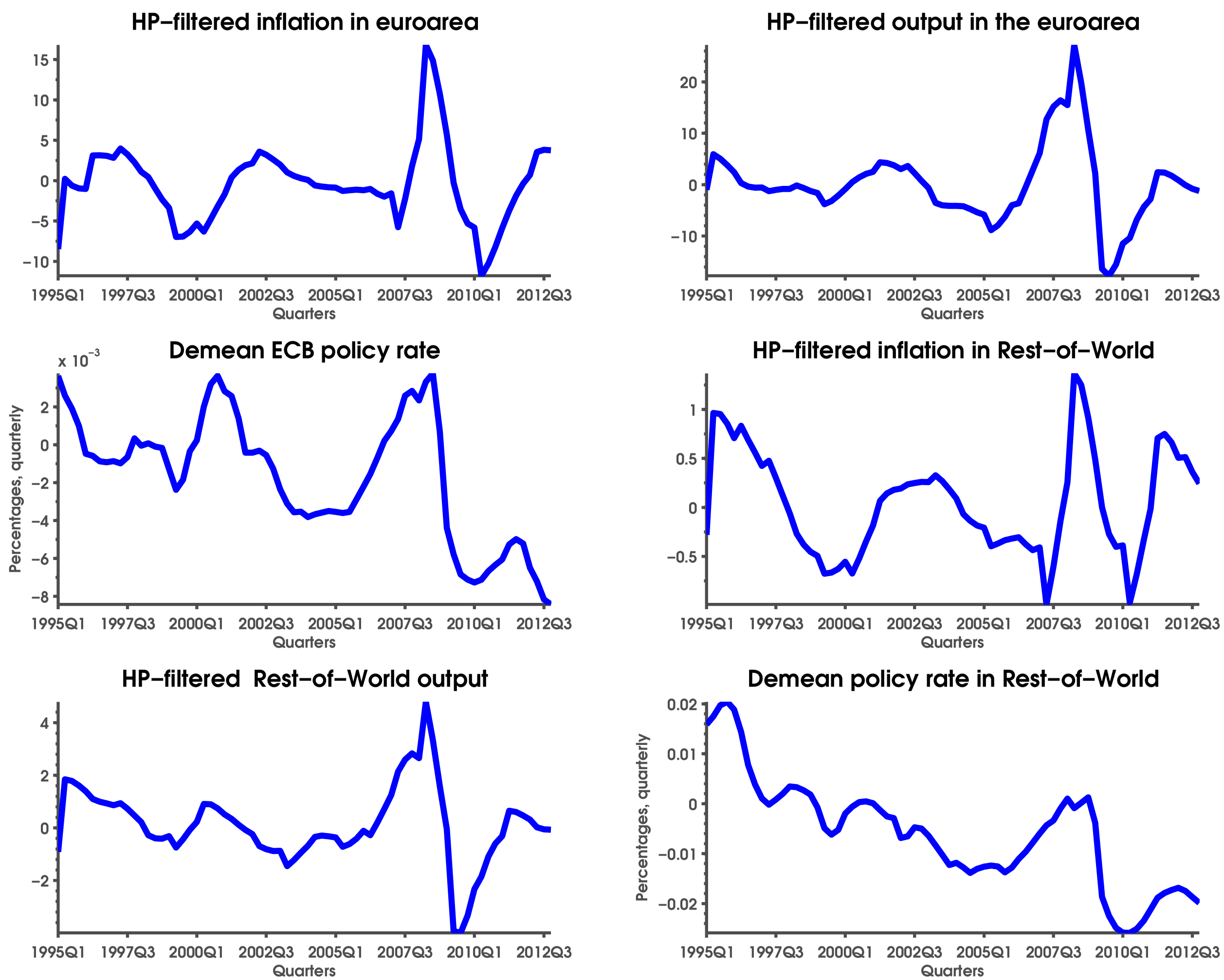

Figure 6: **Transformed observed time series**
This figure shows the time series of the observed variables used in the estimation of the model.

Figure 7: **Detrended Measured Hourly Productivity**
This figure shows the time series of measured hourly productivity in the non-farm sector.

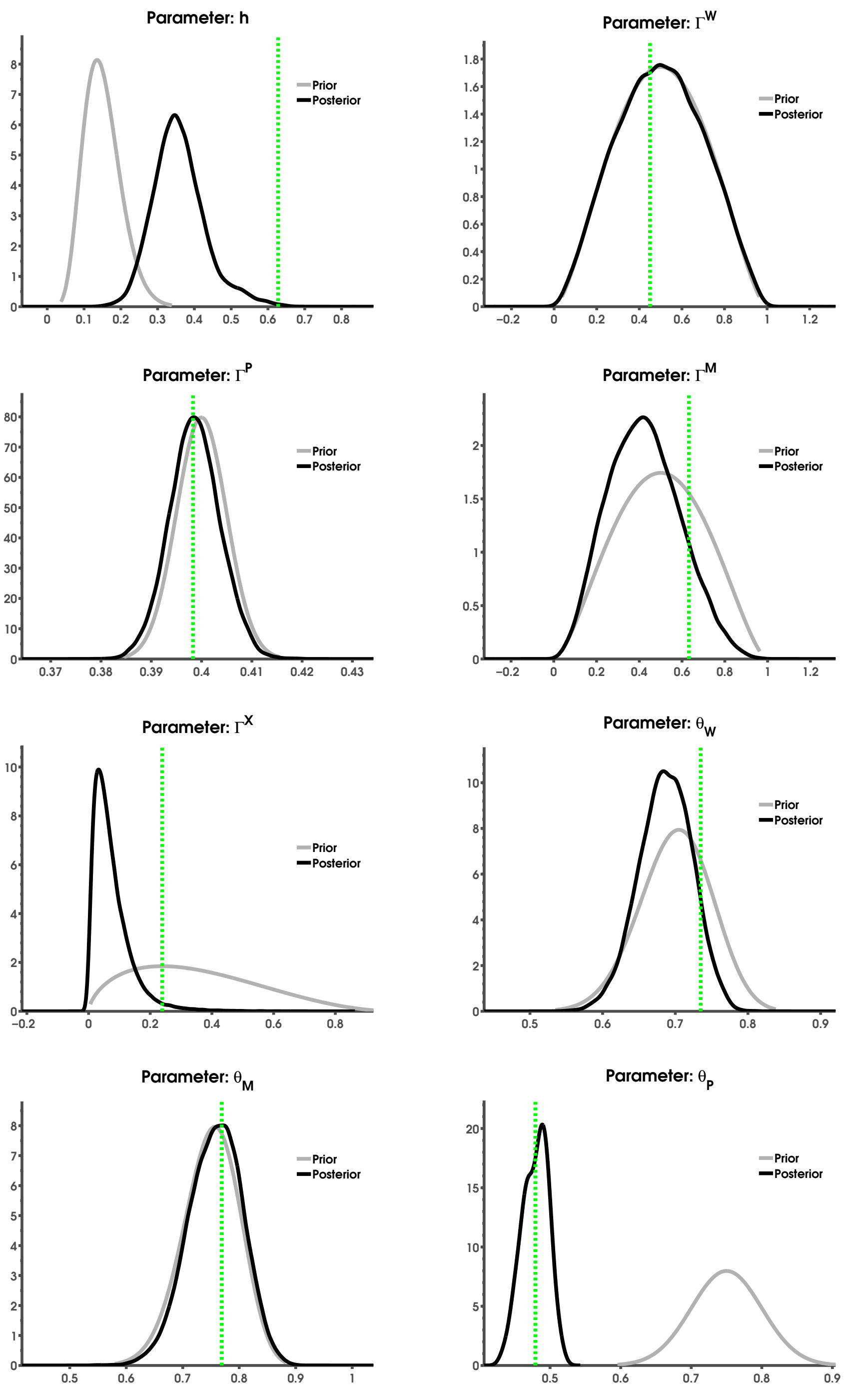

Figure 8: **Prior and posterior**
Prior and posterior distributions of the estimated parameters.

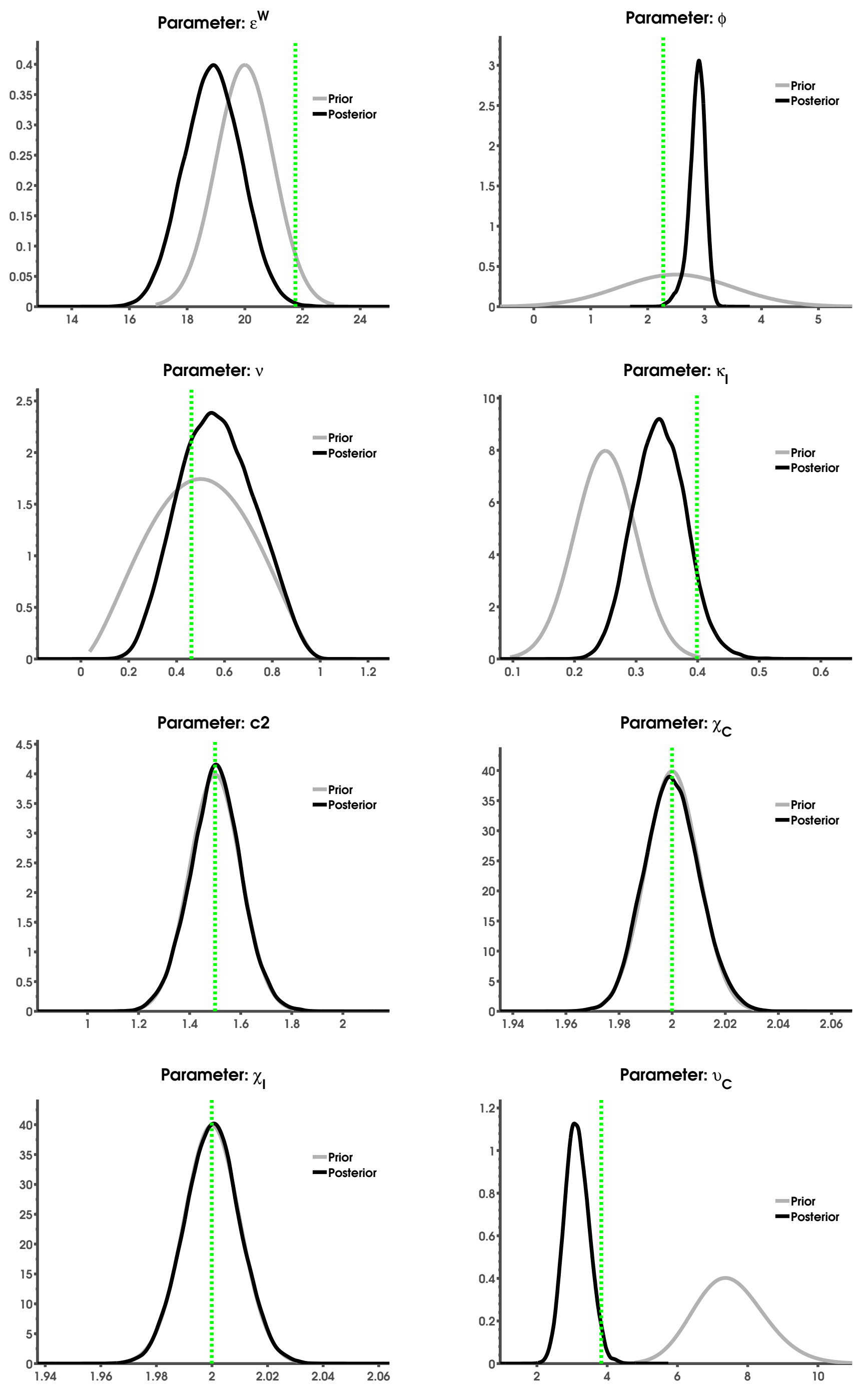

Figure 9: **Prior and posterior**
Prior and posterior distributions of the estimated parameters.

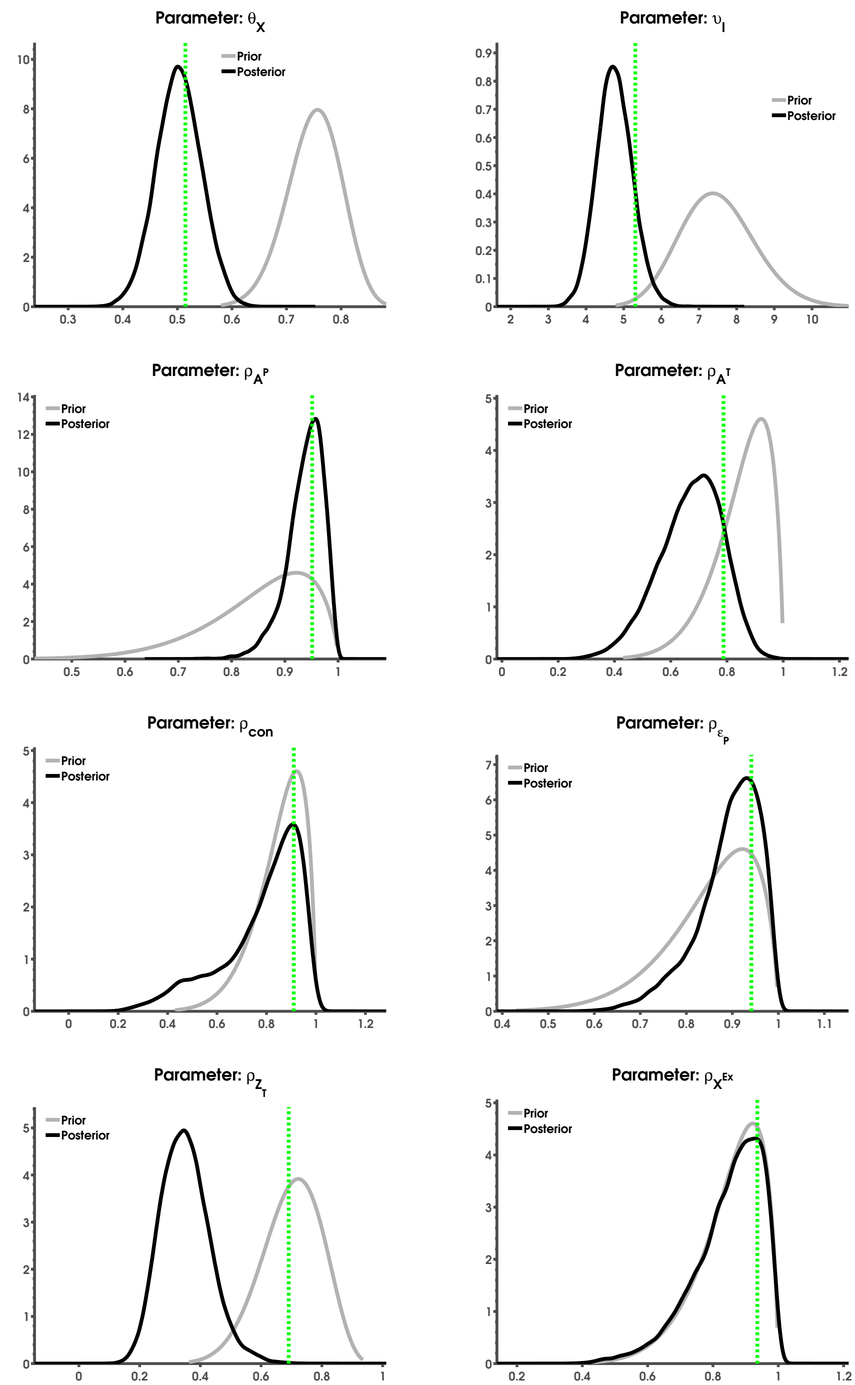

Figure 10: **Prior and posterior**
Prior and posterior distributions of the estimated parameters.

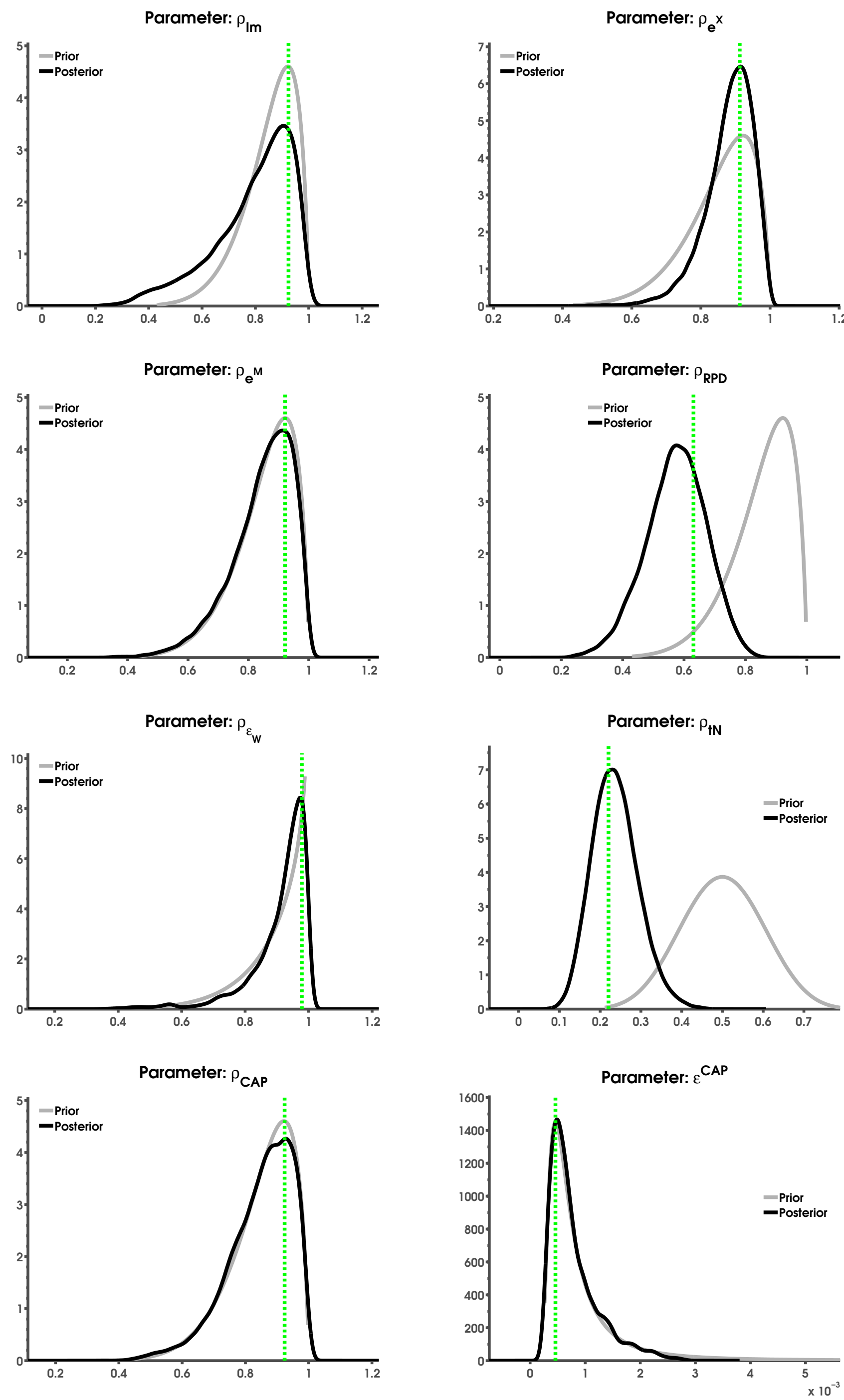

Figure 11: **Prior and posterior**
Prior and posterior distributions of the estimated parameters.

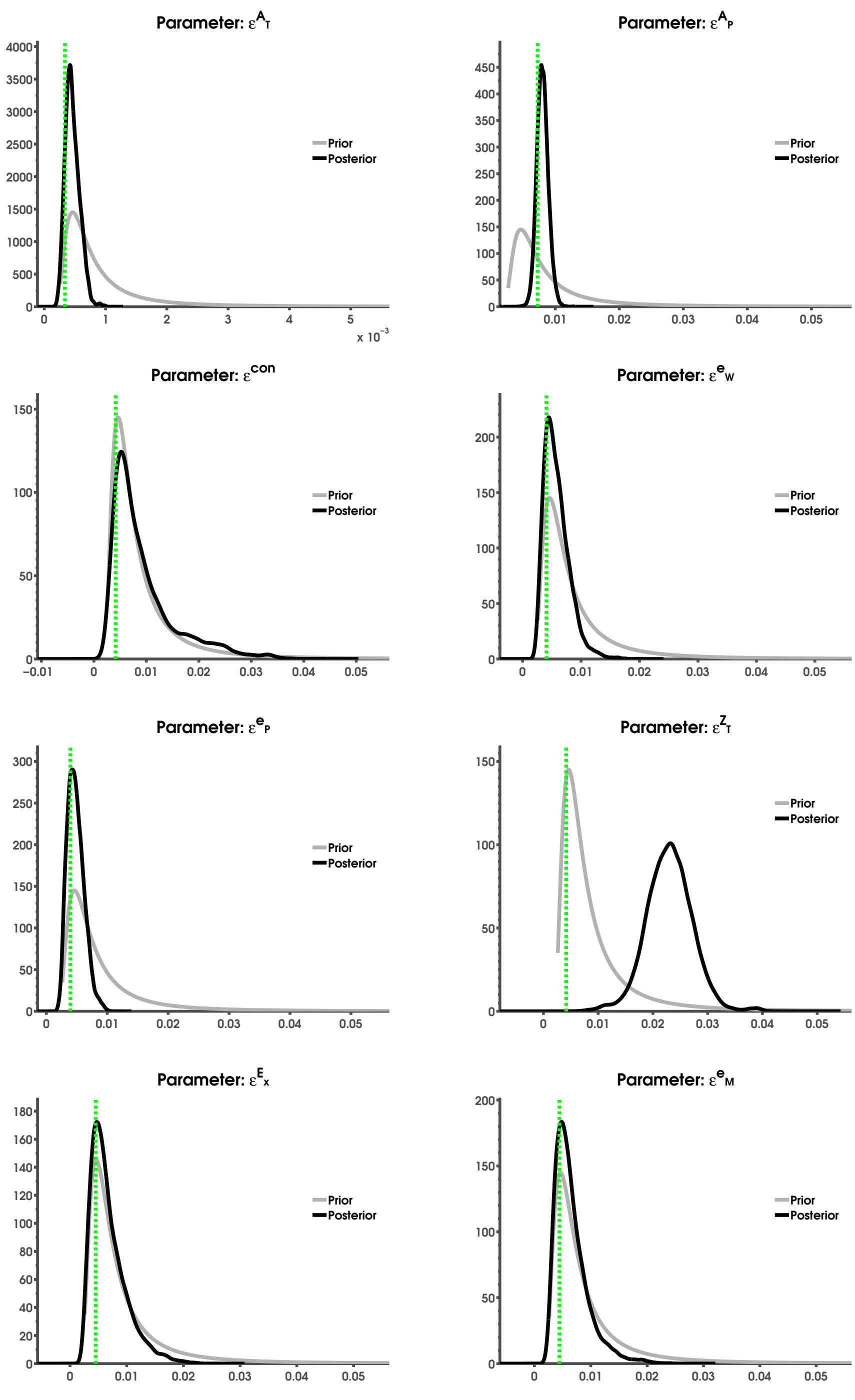

Figure 12: **Prior and posterior**
Prior and posterior distributions of the estimated parameters.

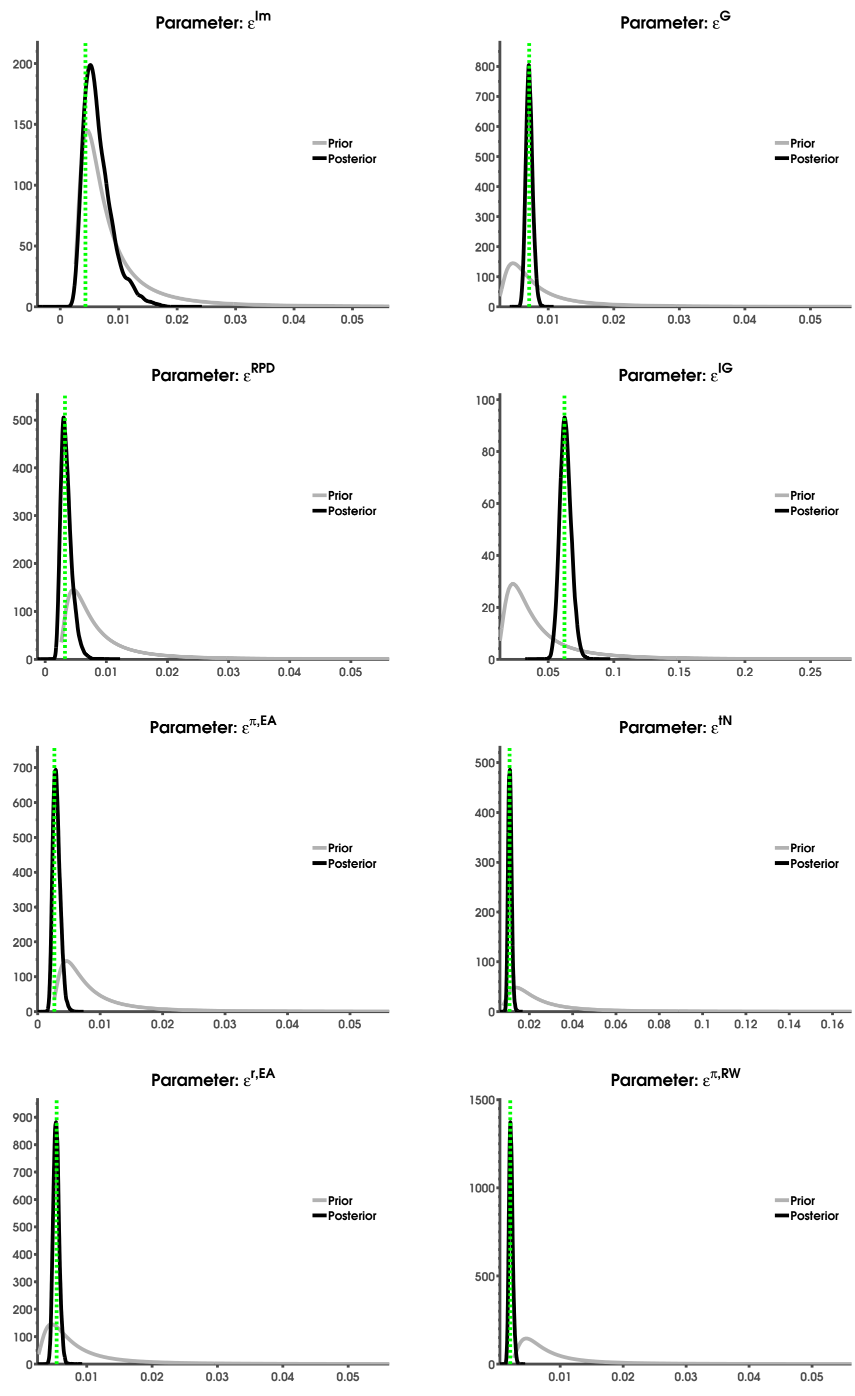

Figure 13: **Prior and posterior**
Prior and posterior distributions of the estimated parameters.

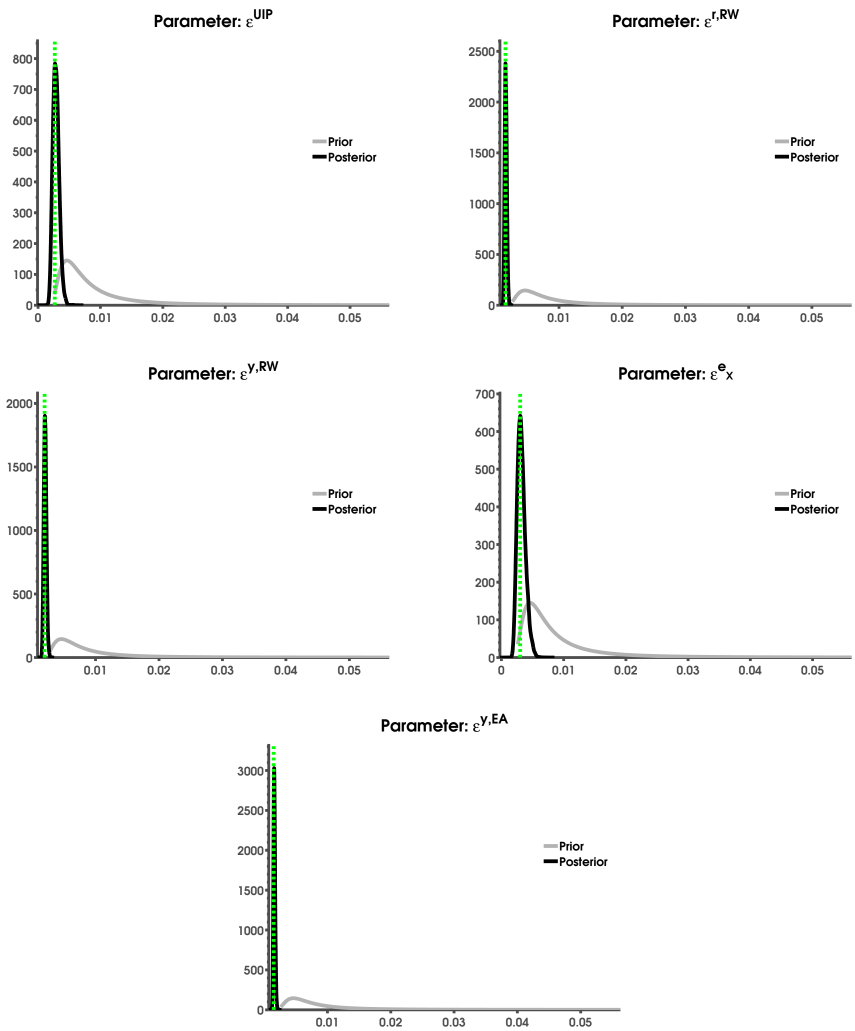

Figure 14: **Prior and posterior**
Prior and posterior distributions of the estimated parameters

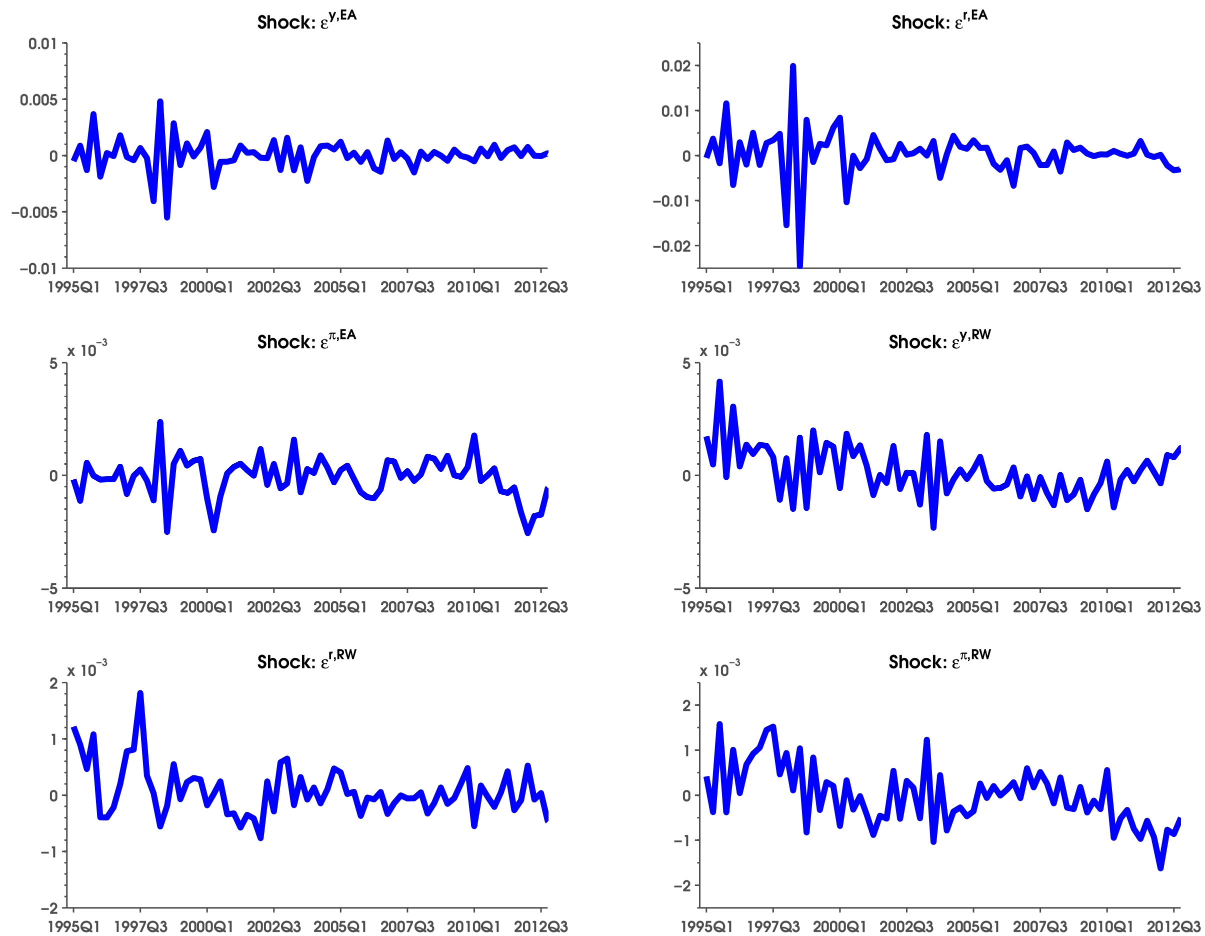

Figure 15: **Smoothed shocks**

This figure shows the smoothed estimates of the models structural shocks used in the estimation based on the posterior mode estimates of the model's structural parameters.

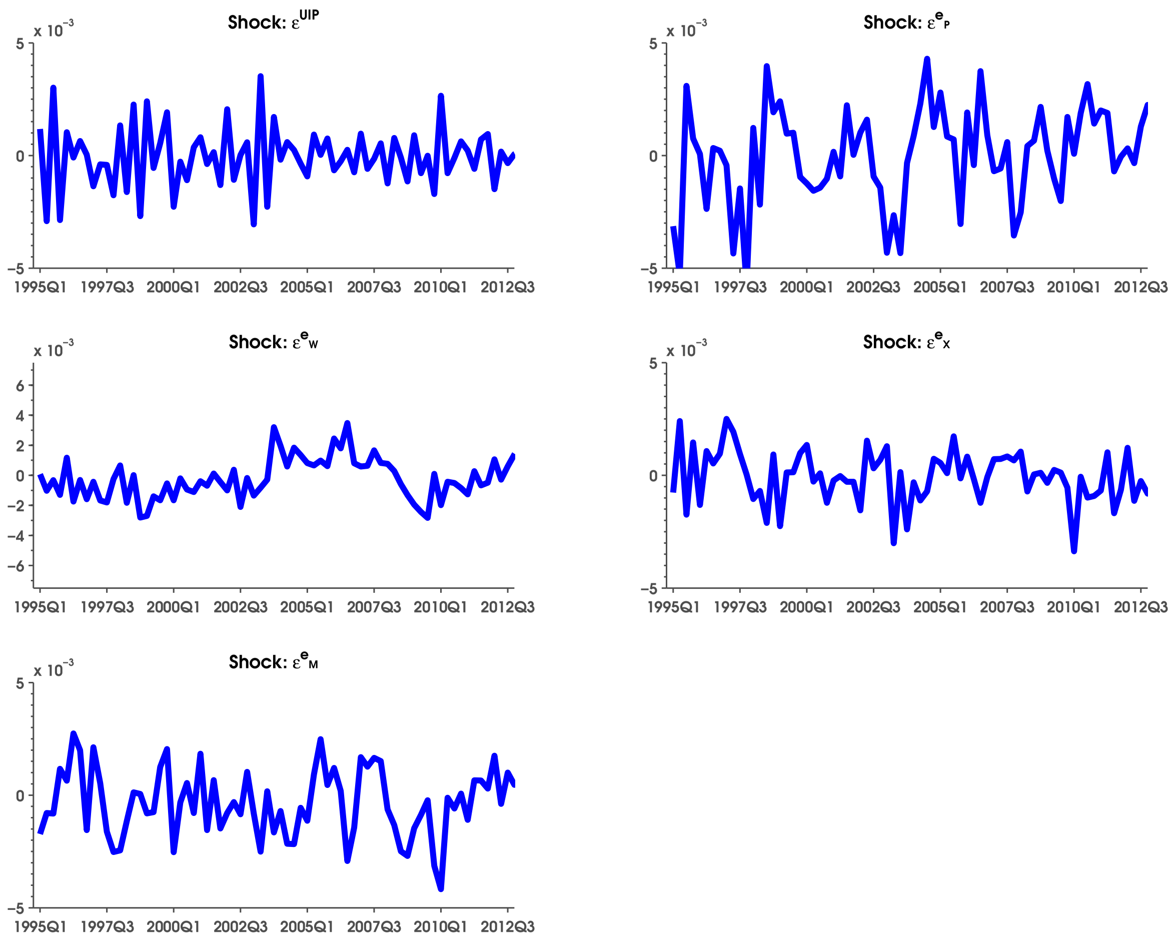

Figure 16: **Smoothed shocks**
This figure shows the smoothed estimates of the models structural shocks used in the estimation based on the posterior mode estimates of the model's structural parameters.

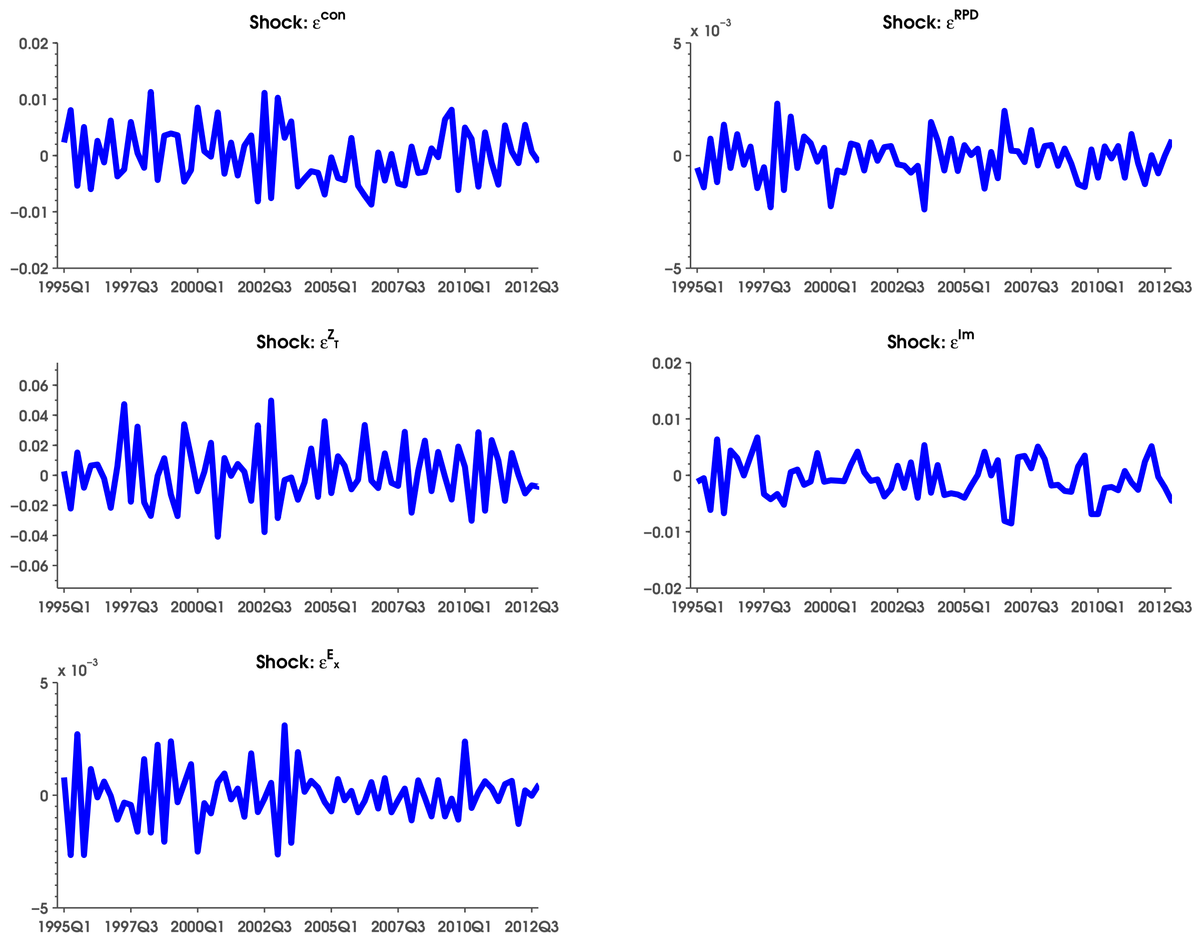

Figure 17: **Smoothed shocks**
This figure shows the smoothed estimates of the models structural shocks used in the estimation based on the posterior mode estimates of the model's structural parameters.

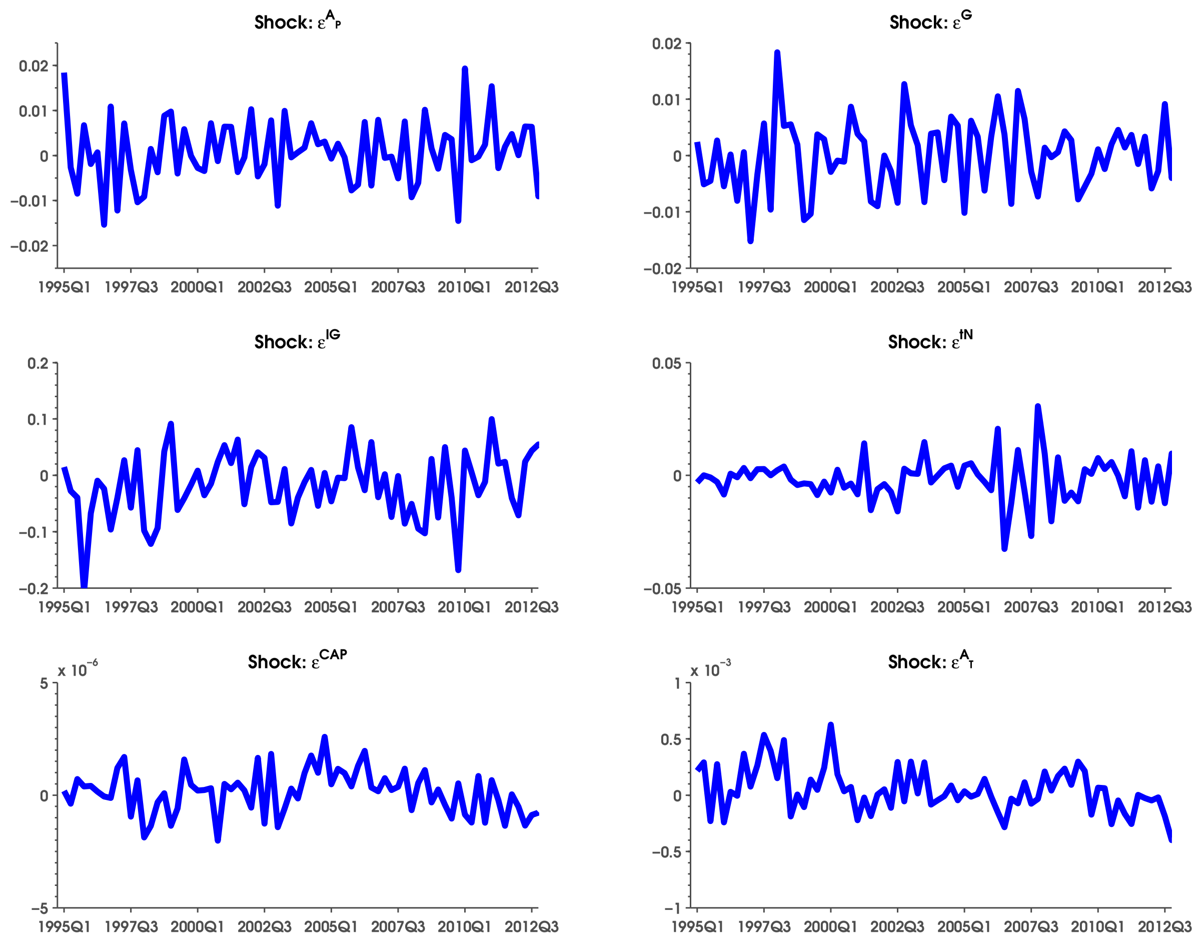

Figure 18: **Smoothed shocks**
This figure shows the smoothed estimates of the models structural shocks used in the estimation based on the posterior mode estimates of the model's structural parameters.

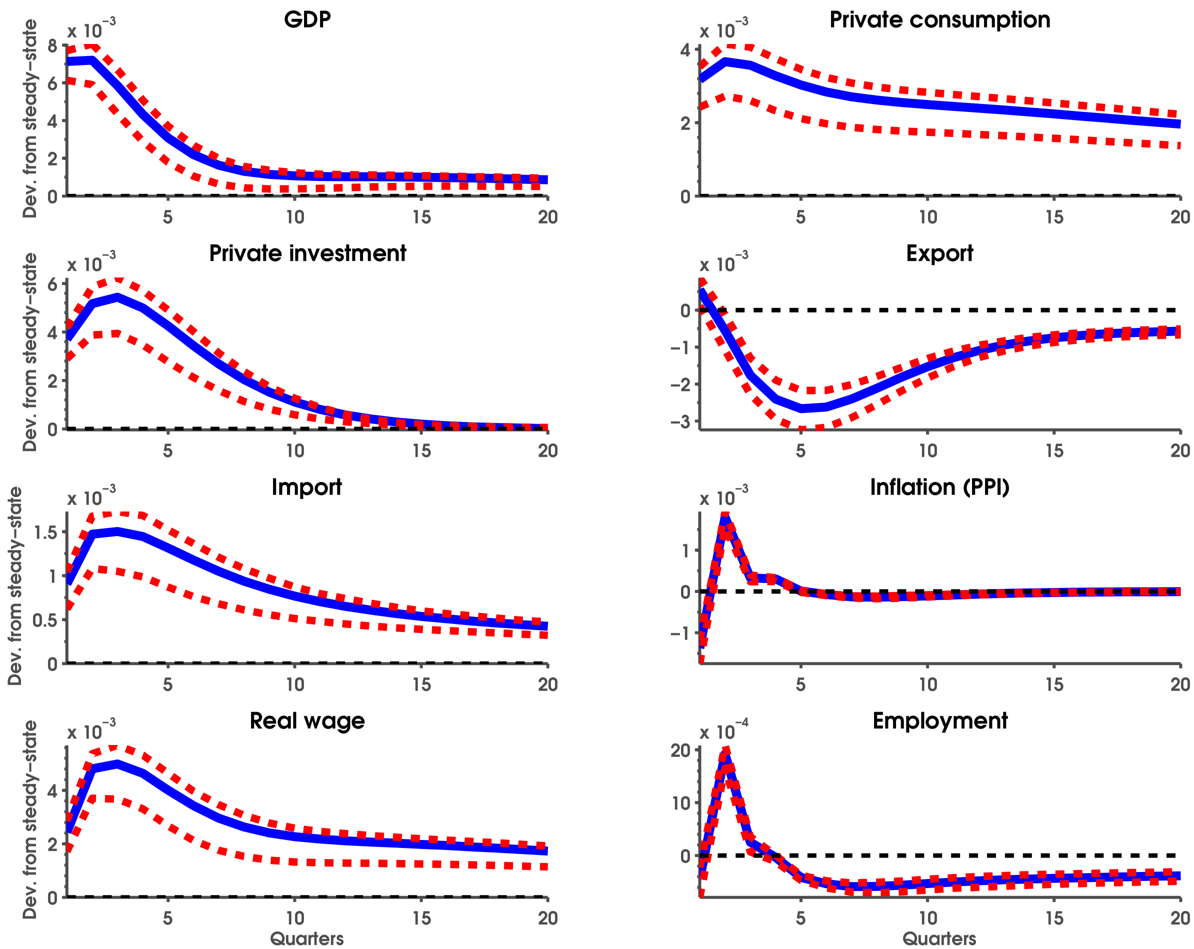

Figure 19: **Effects of a Temporary Technology shock**
This figure shows the mean (solid lines) and the 70 and 90 percent equal-tail uncertainty bands (dotted lines) of the impulse responses of selected variables to a temporary techonology shock equal to one standard deviation. The responses are reported as percentage deviations from steady state.

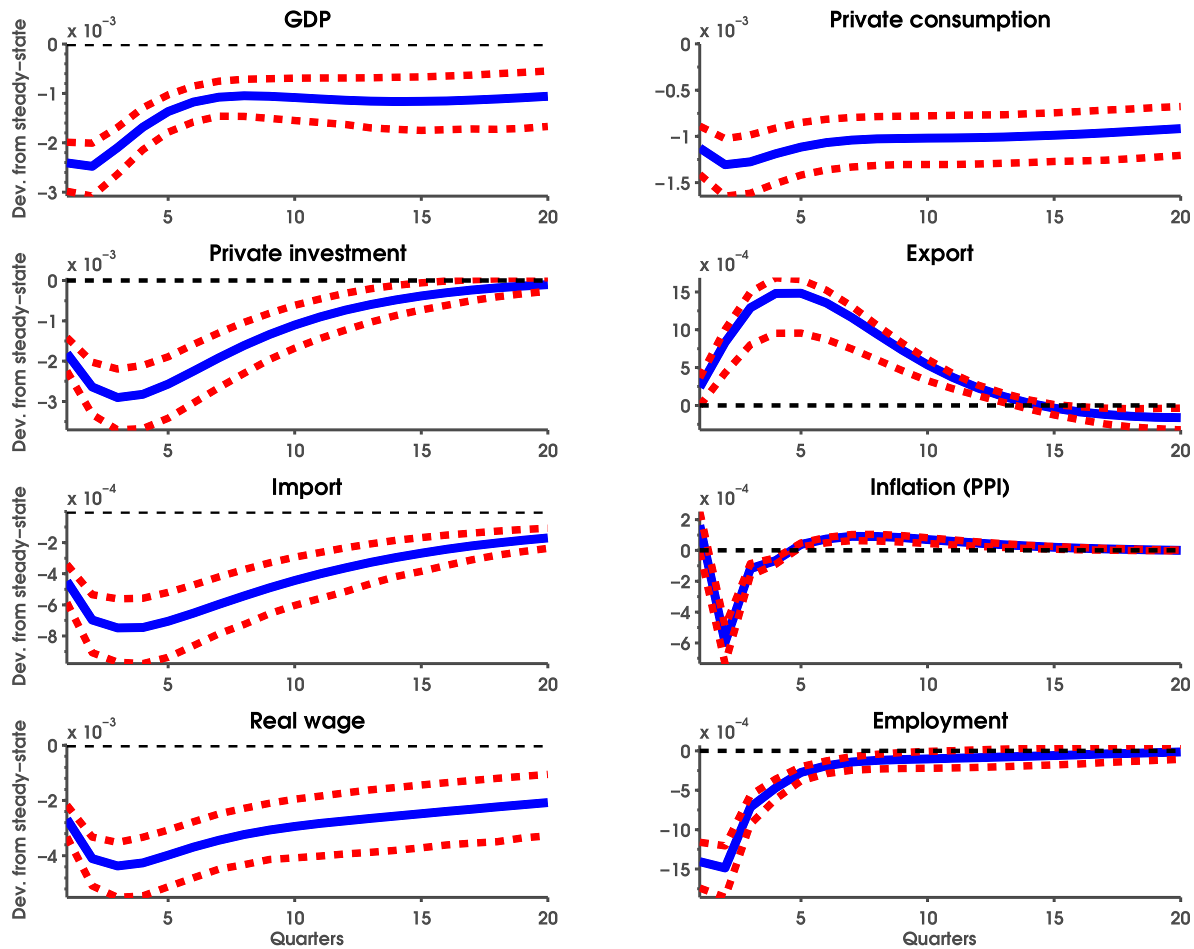

Figure 20: **Effects of a Domestic price markup shock**
This figure shows the mean (solid lines) and the 70 and 90 percent equal-tail uncertainty bands (dotted lines) of the impulse responses of selected variables to a domestic price markup shock equal to one standard deviation. The responses are reported as percentage deviations from steady state.

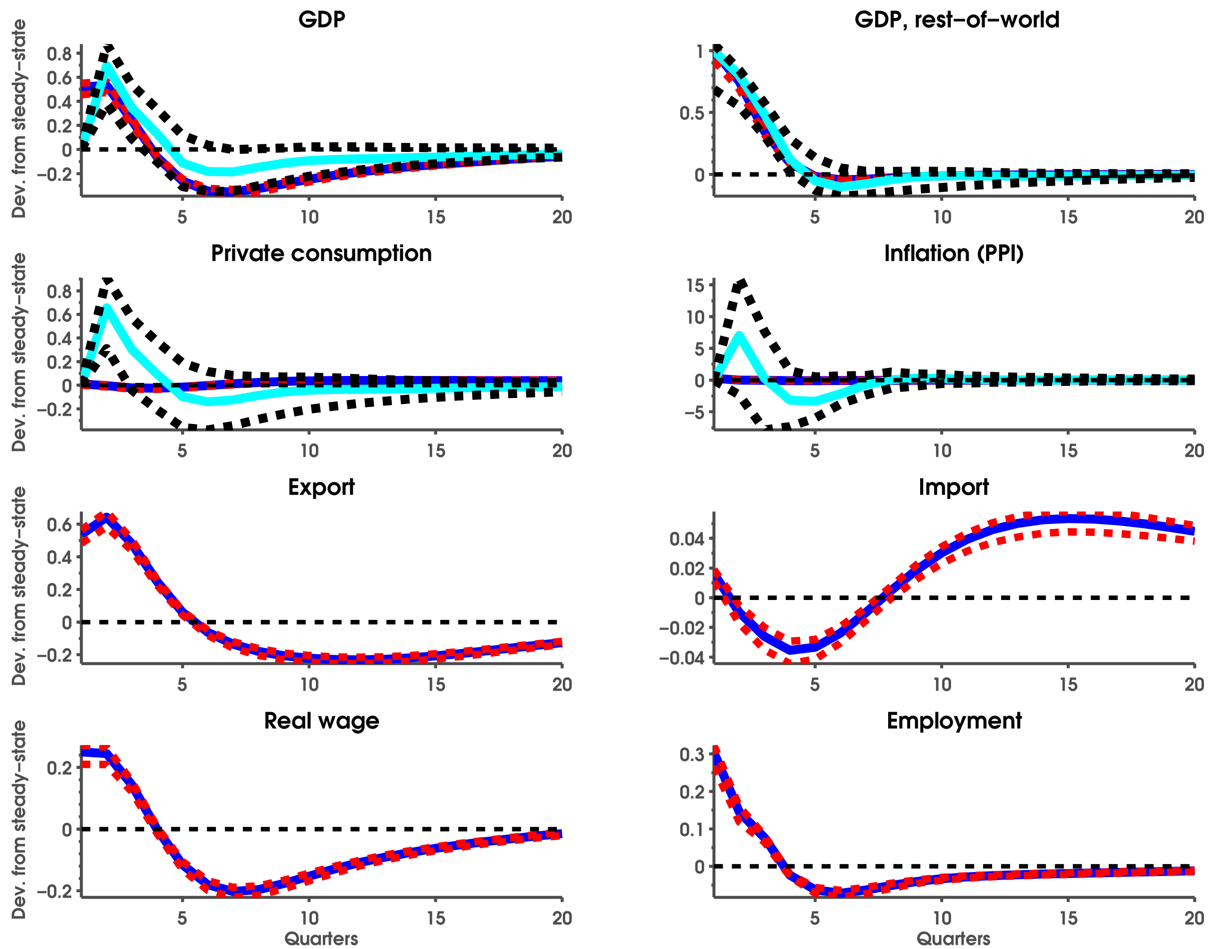

Figure 21: **Effects of a shock to GDP in rest-of-world**
This figure shows the mean (solid lines) and the 70 and 90 percent equal-tail uncertainty bands (dotted lines) of the impulse responses of selected variables to a temporary techonology shock equal to one standard deviation. The responses are reported as percentage deviations from steady state. The red and blue lines are from the DSGE-model. The cyan and thick dotted black lines are from a SVAR.

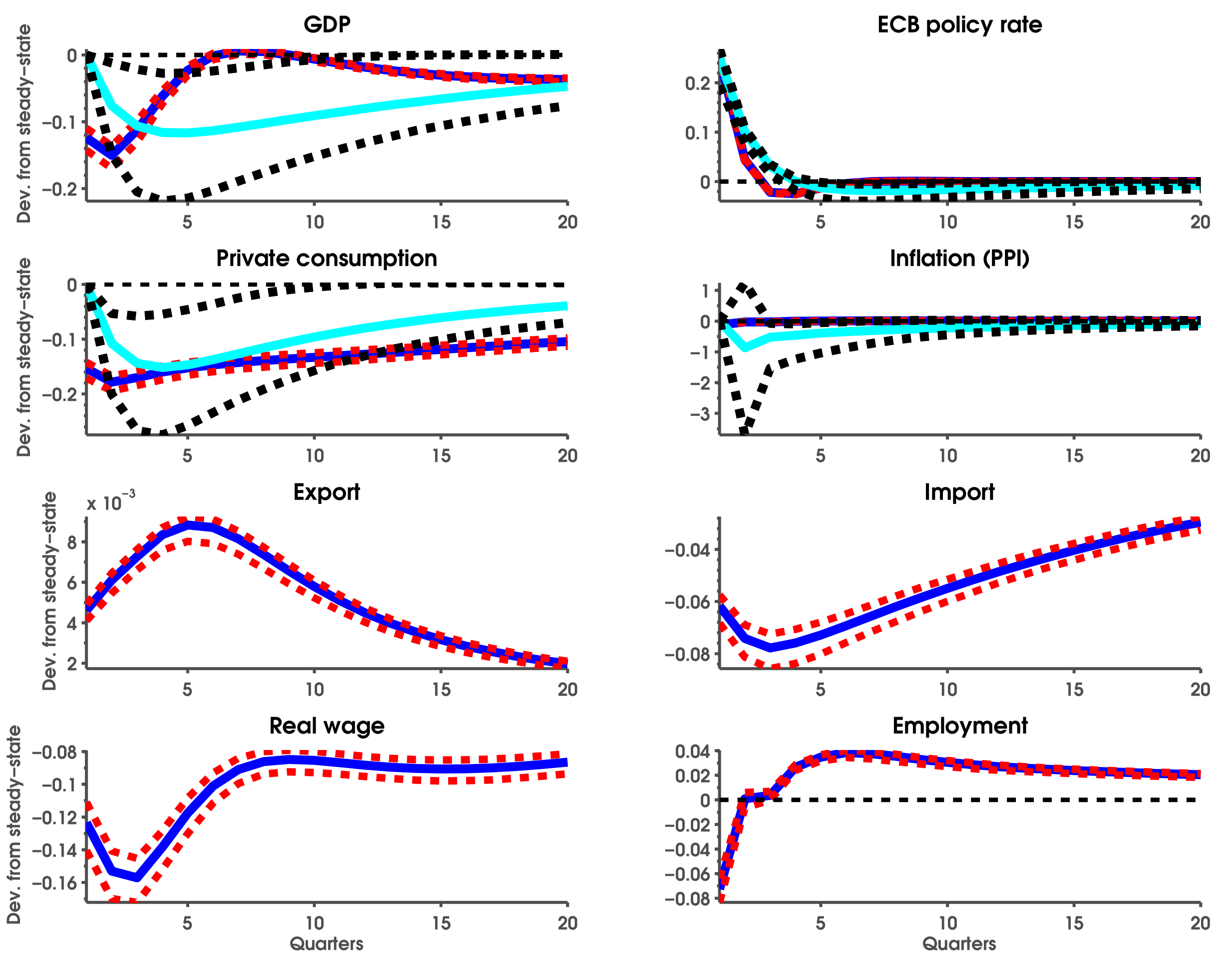

Figure 22: **Effects of a shock to the ECB policy rate**
This figure shows the mean (solid lines) and the 70 and 90 percent equal-tail uncertainty bands (dotted lines) of the impulse responses of selected variables to a temporary shock to the ECB policy rate equal to 0.25 basis point increase in the policy rate. The red and blue lines are from the DSGE-model. The cyan and thick dotted black lines are from a SVAR.

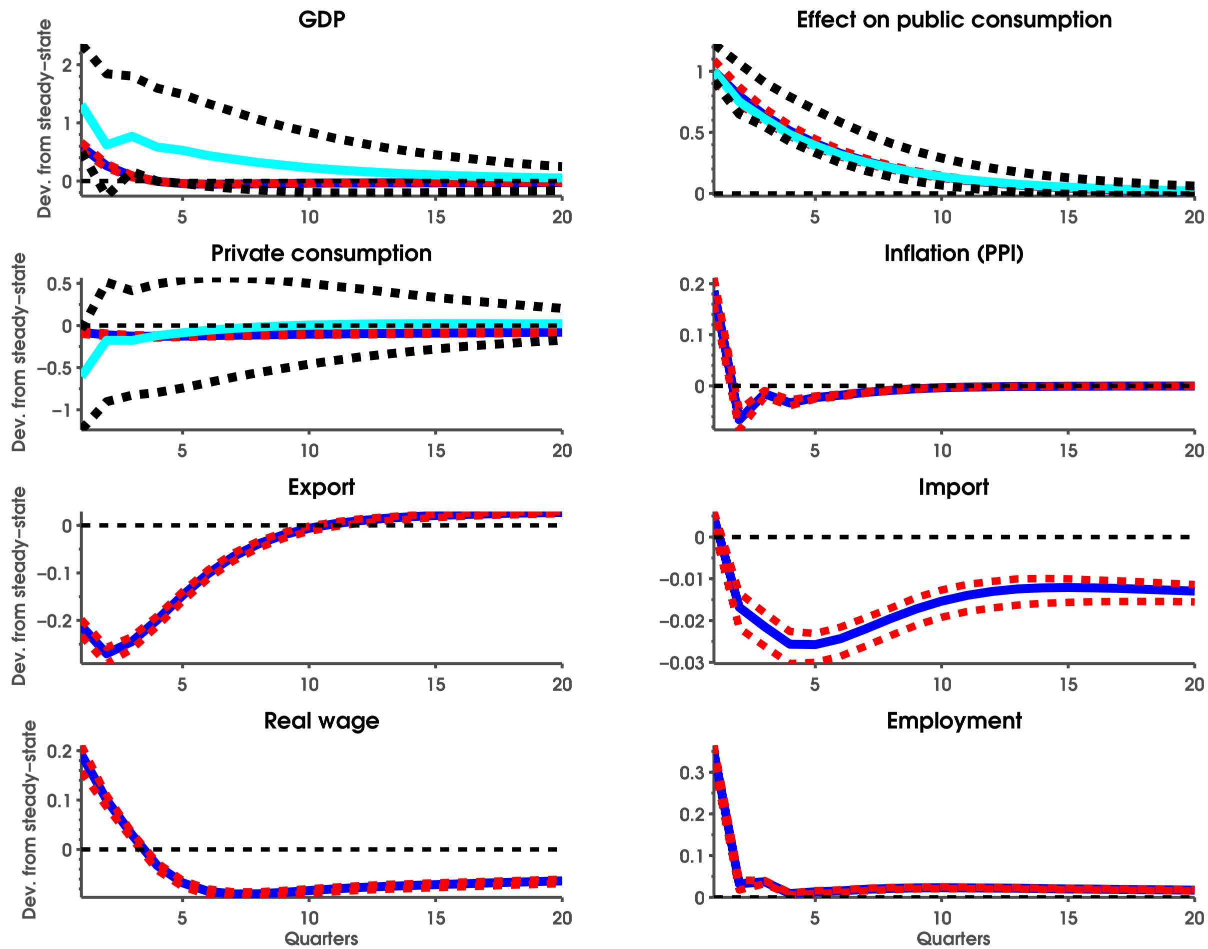

Figure 23: **Effects of a fiscal policy shock**
This figure shows the mean (solid lines) and the 70 and 90 percent equal-tail uncertainty bands (dotted lines) of the impulse responses of selected variables to a temporary fiscal policy shock equal to one percent of GDP. The responses can therefore be interpreted as multipliers. The red and blue lines are from the DSGE-model. The cyan and thick dotted black lines are from a SVAR.

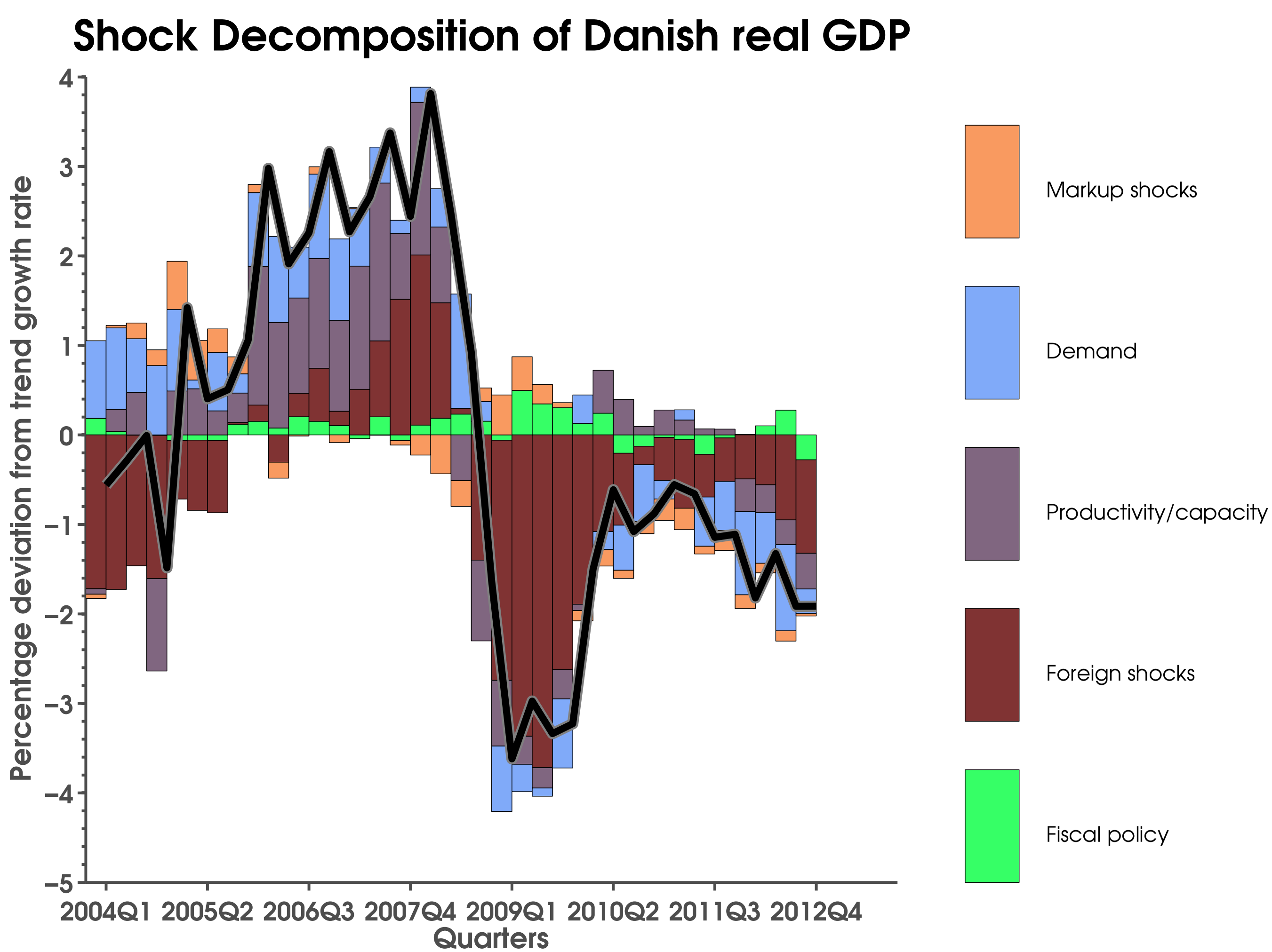

Figure 24: **Historical Shock Decomposition for real GDP**
This figure shows the historical quarter to quarter growth rates in Danish real GDP (solid lines) decomposed into the structural shocks in the model. The 22 shocks are grouped into 5 categories as explained in the text. Real GDP growth is reported in deviation of the steady-state mean growth rate of approximately 0.4 percent per quarter. Residual contributions, which capture the influence of the initial state of the economy and measurement errors, are not shown. The decomposition have been computed using the posterior mode estimates of the model's structural parameters.

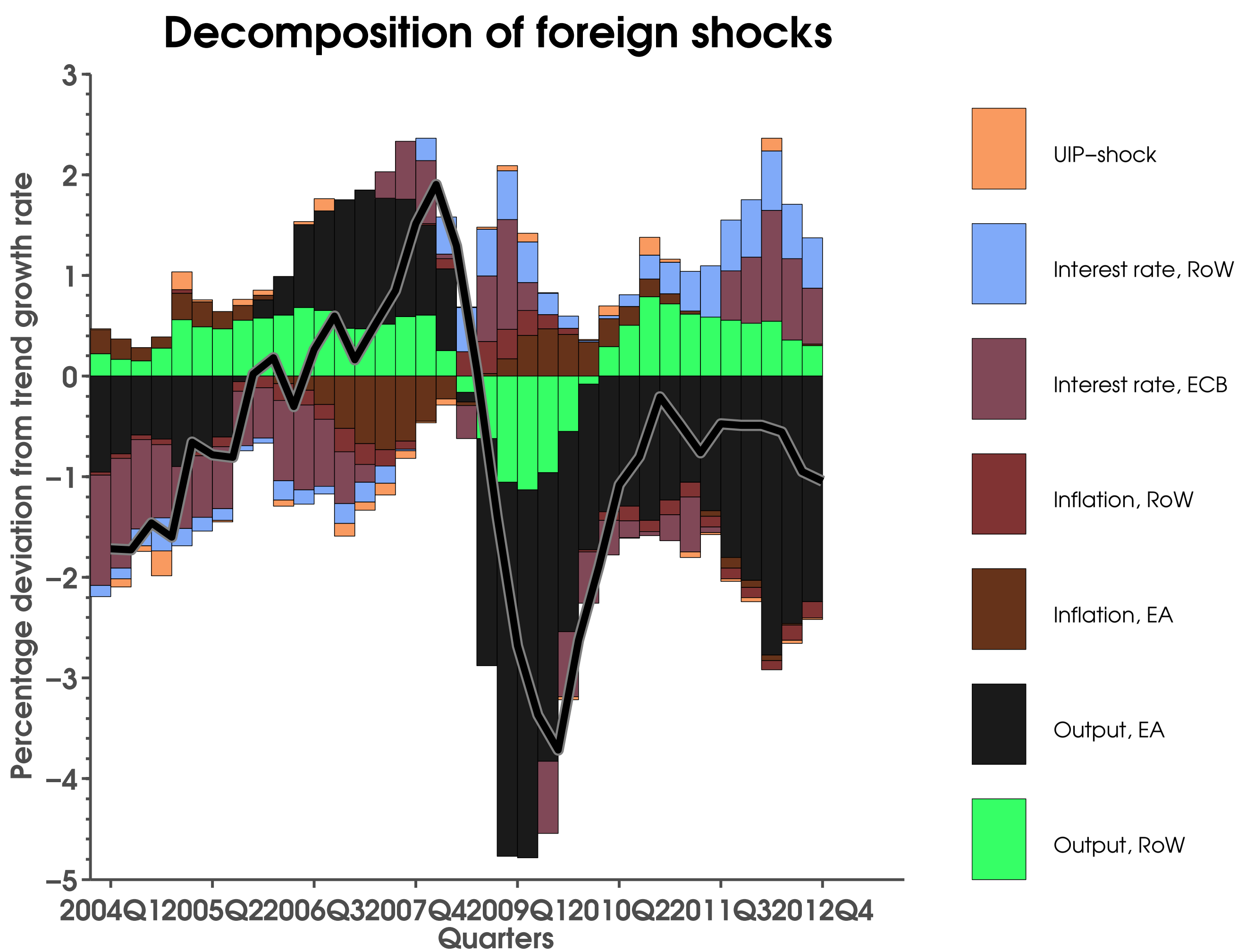

Figure 25: **Historical Shock Decomposition, shocks from foreign economies**
This figure shows the combined contribution of the category named foreign sector (solid lines), as explained in the text. The shocks in this category are shocks to GDP, inflation, interest rates and the uip shocks. Residual contributions, which capture the influence of the initial state of the economy and measurement errors, are not shown. The decomposition have been computed using the posterior mode estimates of the model's structural parameters.

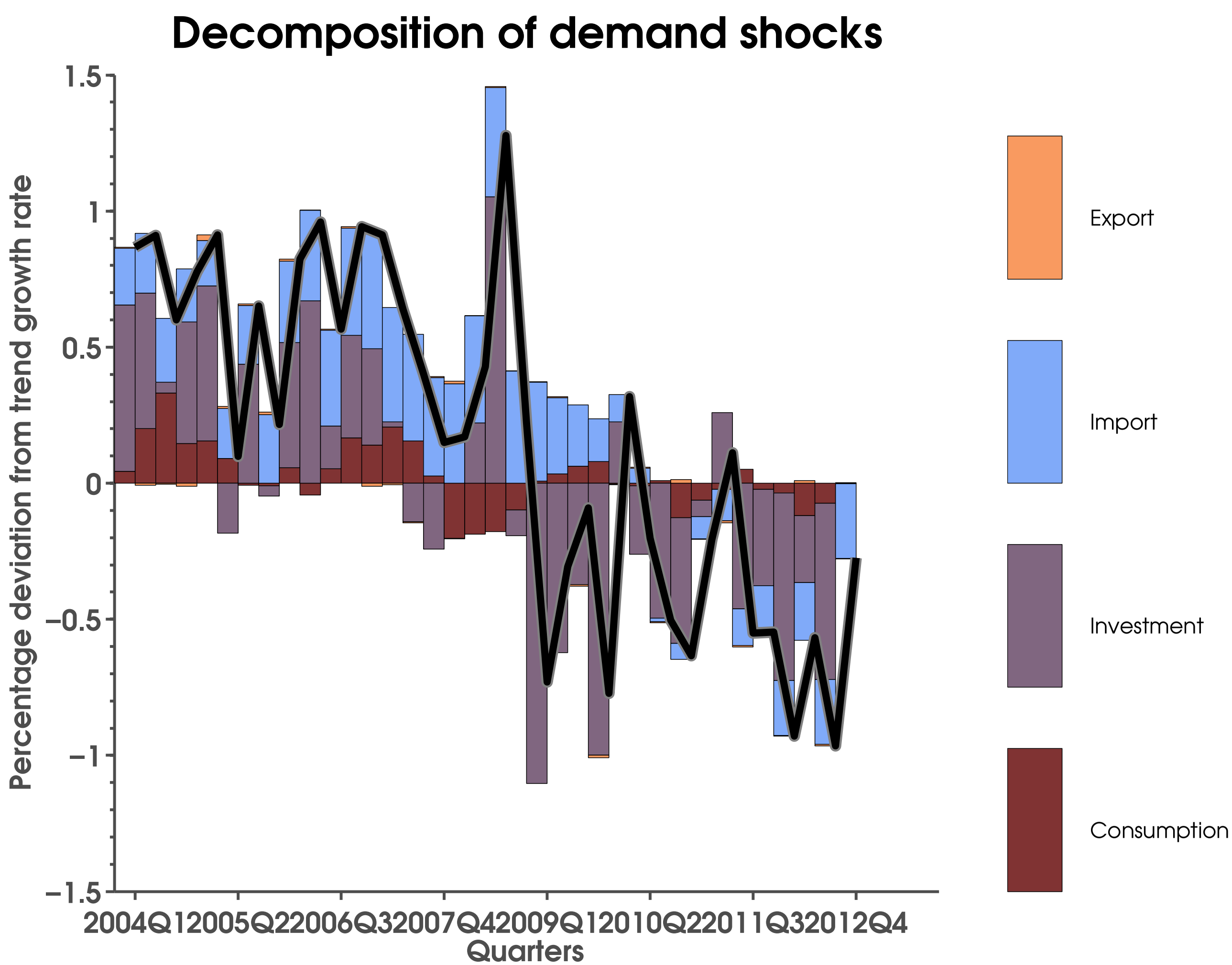

Figure 26: **Historical Shock Decomposition, demand shocks**
This figure shows the combined contribution of the category named demand (solid lines), as explained in the text. The shocks in this category are preference shocks affecting consumption, investment shocks, import and export shocks. Residual contributions, which capture the influence of the initial state of the economy and measurement errors, are not shown. The decomposition have been computed using the posterior mode estimates of the model's structural parameters.

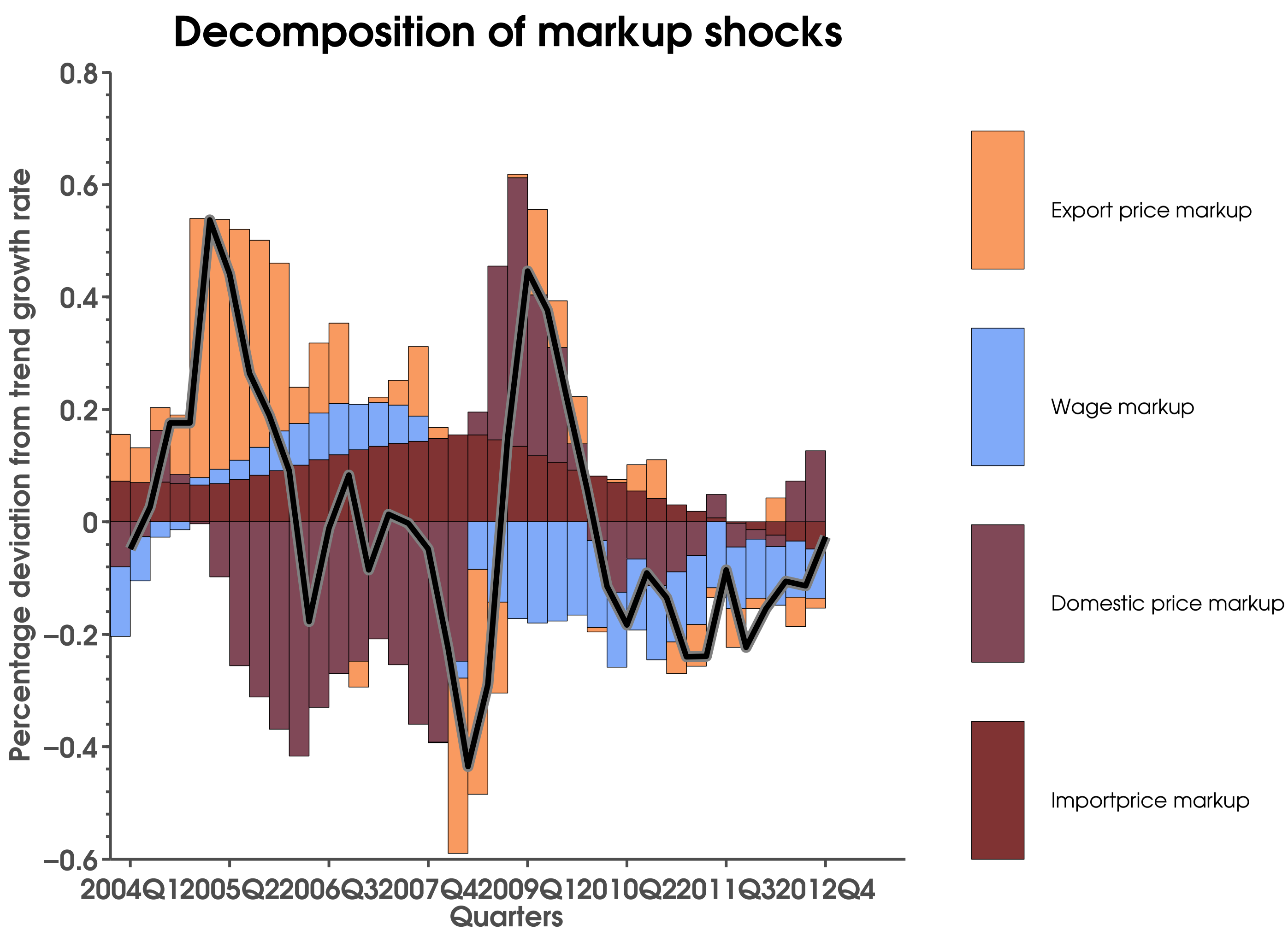

Figure 27: **Historical Shock Decomposition, markup shocks**
This figure shows the combined contribution of the category named markup shocks (solid lines), as explained in the text. The shocks in this category are markup shocks to domestic-, import-, and export prices as well as the wage markup shock. Residual contributions, which capture the influence of the initial state of the economy and measurement errors, are not shown. The decomposition have been computed using the posterior mode estimates of the model's structural parameters.

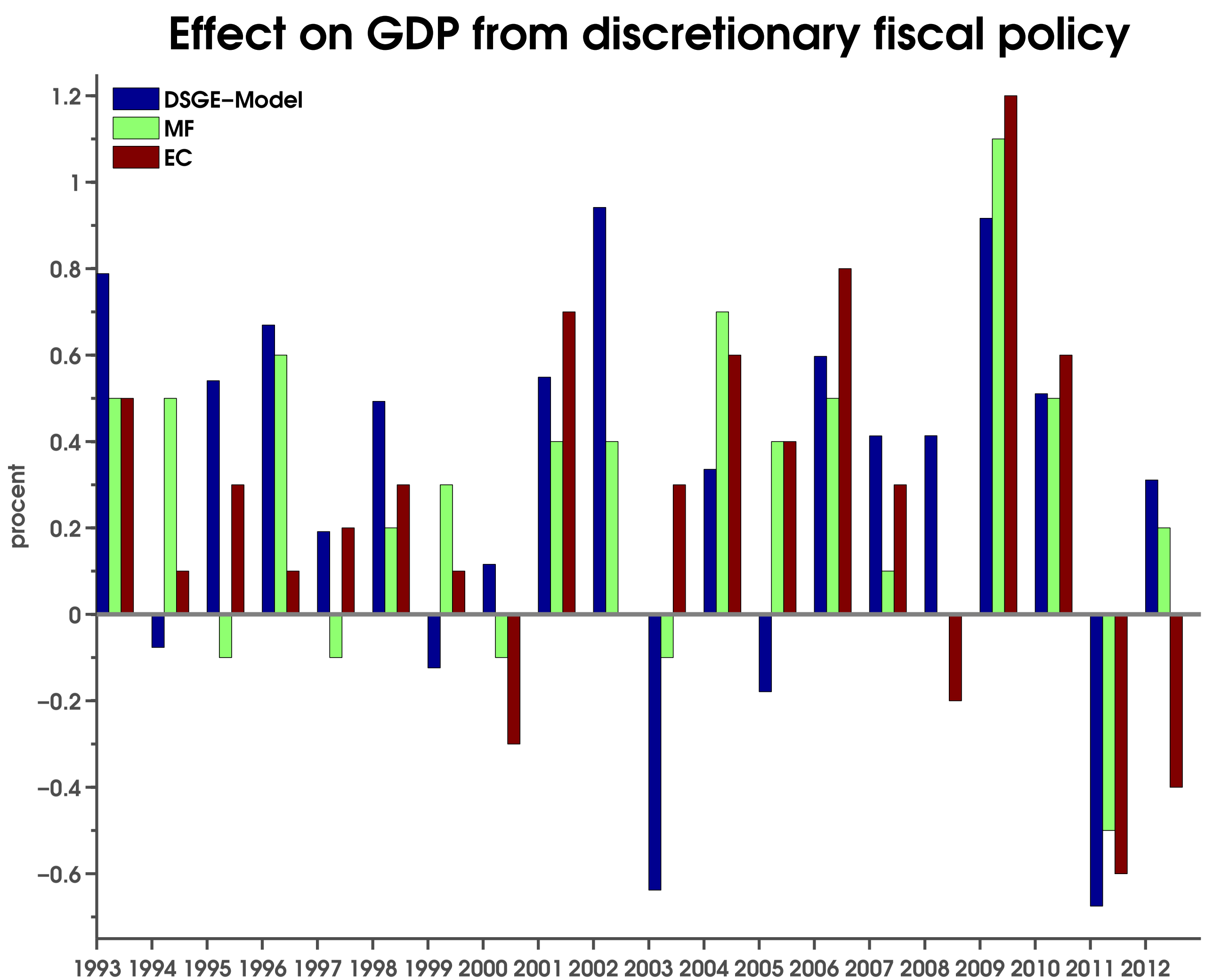

Figure 28: **Effects of fiscal policy in the DSGE model and 'fiscal effects'**
The DSGE effects consist of the contributions to the output gap from shocks to government spending, government investment and the implied labor income tax. That is, the green columns in figure 24. The fiscal effects are obtained from various publications of the Ministry of Finance (MF) and the Economic Council (EC).

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