Can Inflation Expectations Be Deduced From the Development in Danish Consumption?

Michael Pedersen, Economics Department

INFLATION EXPECTATIONS

The real rate of interest indicates borrowing costs (or the return on an investment in financial assets), disregarding price increases. The expected real interest rate is important to investments, consumption and savings, and is therefore significant in the evaluation of economic prospects. The problem is that since the inflation expectations of investors and borrowers are not known, it cannot be observed directly.

Inflation expectations can be estimated in several ways. The actual rate of inflation can be used as an approximation of the expected inflation rate, but since a period of high inflation is often followed by a period of low inflation, the actual inflation rate may be a poor measure of expected inflation.

In countries whose central bank has an inflation target, the latter may be the basis for inflation expectations provided, however, that the central bank enjoys absolute credibility.

Another possibility is to employ various prognoses as the basis for expectations. These can be prognoses by national authorities or international organisations such as the OECD or the IMF. In addition, a large number of private banks prepare prognoses and an average of these can be taken to express expectations. This method assumes that, on average, borrowers and investors concur with the prognoses' future expectations.

The yield on index-linked bonds can be used as an approximation of the real interest rate since this yield is adjusted for inflation. A simple expression of inflation expectations can be derived by subtracting the yield on index-linked bonds from a nominal yield. However, this method entails some elements of uncertainty, due to the special characteristics and taxation terms of index-linked bonds.

Questionnaire surveys are another method of charting inflation expectations. A representative selection of households or business enter-

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1 See Topp (1996) concerning the method applied using Danish data. See Andersen & Gyntelberg (1999) for a description of the special characteristics of Danish index-linked bonds.
prises are asked about their expectations of inflation, whereby an overall picture is achieved. Statistics Denmark has carried out monthly questionnaire surveys for several years. An article in a previous Monetary Review\(^1\) presents a method of translating the households' responses into a measure of inflation expectations.

This article presents a measure of inflation expectations in Denmark based on a theoretical economic model, the Consumption-based Capital Asset Pricing Model (CCAPM). According to this model, at the equilibrium point the real interest rate is determined on the basis of growth in private consumption. The inflation expectations found deviate considerably from other results and the conclusion is that the underlying assumptions are so restrictive that the method cannot be applied to the Danish economy without difficulty.

Similar surveys for the USA, Germany and the UK appear to yield more plausible results. An obvious conclusion is therefore that the model is more suitable for larger, more closed economies in which domestic factors play a greater role in determining interest rates than is the case in Denmark\(^2\).

**THE CCAPM MODEL**

CCAPM is a general equilibrium model, i.e. all markets are assumed to be in equilibrium: supply equals demand via price formation and interest rates. The model's starting point is a representative consumer who is also an investor and who is assumed to act rationally. Within that consumer's budget he seeks to achieve the best possible situation for himself, i.e. to maximise utility. Subject to these assumptions, the representative consumer will distribute his consumption over time, so that the growth in consumption corresponds to the real interest rate, taking into account time preferences, risk aversion and other consumer-specific factors, cf. Box 1. Technically, this model makes it possible to determine a real interest rate as well as inflation expectations on the basis of the development in consumption per capita and the nominal interest rate. The technical details of the model are described in Appendix A\(^3\).

The expected real return on a nominal bond depends on the expected inflation during the period of ownership of the bond. The actual inflation rate will be uncertain, so the investor will require a premium on the yield – an inflation-risk premium.

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\(^1\) See Christensen (1996).

\(^2\) See Ireland (1996) and an internal working paper of 1997 from the European Monetary Institute. In Ayuso & López-Salido (1996, 1997) the model is e.g. applied to Spanish data.

\(^3\) It should be noted that the model considered here does not take taxation aspects into account. Introduction of taxation may therefore alter the results. Ayuso & López-Salido (1996) discusses the introduction of a proportional income tax in the model.
The size of the inflation-risk premium depends on e.g. the maturity of the bond. The longer the maturity, the greater the uncertainty concerning inflation. Moreover, the investor's risk aversion, i.e. the importance the investor attaches to avoiding risk, also plays a role. The size of the risk premium also depends on the stability of the inflation rates. The risk premium can be viewed as a premium on the expected real interest rate and inflation rate, and thus increases the nominal interest rate. This gives the following relation:

\[ R = r + \pi^e + RP \]

R is the nominal interest rate, r is the real interest rate, \( \pi^e \) is the expected inflation and RP is the risk premium. In CCAPM the real interest rate is determined on the basis of the growth in private consumption:

\[ r = f (\Delta c) \]

\( \Delta c \) is the change in consumption. Since the nominal interest rate can be observed, an expression of inflation expectations and risk premium can be calculated.
EMPIRICAL APPLICATION OF THE THEORETICAL MODEL

Application of the model requires consumption data and a nominal interest rate. The nominal interest rate applied is the yield on a 5-year government bond, which means that a 5-year real interest rate and inflation expectations over 5 years are derived. The consumption data is the data for non-durable goods and services from the national accounts. This consumption concept is often used in relevant literature because the important aspect of the model is the underlying consumption growth. Consumption per capita is found by dividing the value by the population aged between 15 and 74.

To illustrate how the model functions, a number of parameters to describe e.g. the consumer's risk aversion must be selected. The parameters chosen here give the greatest possible correlation between the generated real interest rate and a 5-year realised real interest rate calculated as the nominal 5-year interest rate less actual inflation for the duration of the bond. It is also taken into account that the averages of the two interest rates are at the same level. The choice of parameters indicates an almost risk-neutral consumer, i.e. the consumer is not particularly hesitant to assume risks.

Charts 1 and 2 show the generated inflation expectations and real interest rates.

In Chart 1 the actual inflation is also drawn. The trend for the generated inflation expectation generally matches the actual inflation. The averages for the period considered are by and large at the same level, but the expectation's volatility is considerably greater than for the actual inflation. It is also noteworthy that inflation expectations and actual inflation are only at the same level at the beginning of the period, while there are considerable deviations for by far the greatest part of the period. The deviations from actual inflation, as well as from other measures of inflation expectations, appear too large to be plausible. In particular, the low level in recent years is difficult to interpret.

Chart 2 shows that the nominal interest rate followed a generally falling trend during the period under consideration. On the other hand, there were no major fluctuations in the generated real interest rate. Overall, the model implies that there is no close relation between the nominal and the real interest rates.

Furthermore, the calibration of the model results in a relatively low inflation-risk premium. For the period under consideration it is generated at between -0.2 and 0.4 percentage points.

1 The calibration of the model is described in Appendix B.
Fluctuations in the calculated real interest rate and risk premium were relatively limited. On the other hand, according to the model changes in the nominal interest rate must be attributed first and foremost to changes in inflation expectations.
COMMENTS ON THE RESULTS

The results concerning inflation expectations, etc. derived from the model naturally depend on the parameters and calibration method chosen. The purpose here was not to state estimates for all the parameters in the model, but to illustrate a specific example of the derivation of inflation expectation and real interest rate\(^1\).

Without becoming too absorbed in the academic discussion it can be stated that major fluctuations in inflation expectations are a characteristic feature, irrespective of the choice of utility function, although the level varies considerably. It can also be discussed whether the relevant expression of the nominal interest rate is to be compiled as the amount after tax at the consumer's disposal. In the first instance this will not affect the generated real interest rate, but will solely reduce the generated inflation expectations. It gives no immediate improvement in the applicability of the model to a description of the Danish economy.

As stated, CCAPM is a very simple representation of reality and the question must be asked of whether the model provides a satisfactory description of an economy such as Denmark's. At first sight the calculated inflation expectation seems far removed from the inflation estimates of e.g. the OECD, the IMF and other prognosis brokers. Their estimates were relatively stable in the period considered. The same picture is painted by other methods to derive inflation expectations, such as questionnaire surveys.

Use of the aforementioned model gives a positive relation between the increase in consumption and the real interest rate, cf. Chart 3. This can be attributed to the type of consumer chosen, but also to the model's correlation between real interest rate and development in consumption. Since strong consumption growth indicates a low savings ratio, a negative correlation is also established between the real interest rate and the propensity to save, cf. Chart 4. This seems to be a contra-intuitive effect. In principle, it could be defended as an adjustment of the interest rate to reduced savings. This effect from consumption and savings on the interest rate is conceivable in a large economy or

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\(^1\) A sensitivity analysis considers three different types of consumer. One with a standard utility function, i.e. the utility achieved depends only on the consumer's own consumption in the current period. The second is a habitual consumer where there is a connection between yesterday's and today's consumption. The third is a consumer for whom the utility also depends on the general level of consumption in society as a whole. One can be envious or pleased about the success of others. The sensitivity of the model to the choice of utility function and parameters is shown in Tables 1 and 2 of Appendix C. It can be said that on its own merits the model has the expected characteristics. The more impatient and averse to risk the consumer, the higher the real interest rate he will demand. The more stability consumers want in their consumption, the higher the real interest rate, and the more the consumers are pleased about the success of others, the lower the real interest rate they will demand. Reference is made to Ayuso & López-Salido (1996, 1997) for further discussion of the model's sensitivity to choice of utility function.
currency area. However, this interpretation does not match Danish data, where the interest rate to a high degree is determined by external factors. It is conceivable that lower savings increase inflation and reduce the real interest rate at an externally-determined nominal interest rate. However, this is not incorporated in the model, which considers equilibria for consumption and capital market at an exogenous inflation rate.
As stated, in an open economy such as Denmark's with a fixed-exchange-rate policy the nominal interest rate is to a great degree determined by external factors. In this situation the propensity to save can barely affect the nominal interest rate. This means that there is no adjustment of the interest rate to consumption behaviour, but solely an adjustment of consumption behaviour to the interest rate. In the short term, this adjustment to a higher interest rate reflects the consumers' desire to increase their savings ratio. Once the savings ratio has been established at a higher level an equilibrium has been reached with less consumption today and more consumption in the future.

This corresponds to an increase in the planned consumption growth. A high real interest rate and strong consumption growth will thus be related in the long term, even with an exogenous interest rate. Against this background the CCAPM relation between interest rate and consumption growth can probably survive since CCAPM is an equilibrium model.

Unfortunately, the necessary adjustment takes up both time and significance. The economy is characterised by cyclical fluctuations which often seem to reflect an adjustment to changes in interest rates, cf. the drop in interest rates prior to the upswing in the 1980s and 1990s. It follows that the dominating pattern is the negative relation between consumption growth and the real interest rate as perceived by consumers. This impedes or perhaps even rules out, an empirical application since account must be taken of persistent, dominant cyclical fluctuations in consumption growth and savings.

Besides the influence on interest-rate formation, another effect of the open economies in reality is that consumption can be financed abroad. This leads to a current-account deficit, which the model does not take into account. This problem is probably also relevant on calibration of the model for other countries such as the USA, whose current-account deficit is currently approximately 3 per cent of GDP.
APPENDIX A

We consider a CCAPM which describes an exclusive barter economy with no explicit formulation of the production technology. It is assumed that the output cannot be stored – certain authors call it a fruit-tree model. In addition, the model is based on the following assumptions:

- All agents are identical with an infinite lifetime. This means that only one representative agent (consumer or household) is considered.
- The agents have rational expectations and wish to maximise the expected utility within their budget constraints.
- Business enterprises in the model are identical, operating in full competition. This means that they can be represented by one single enterprise. It manufactures one product with a very short lifetime, so it cannot be stored. The entire output is consumed within the same period. Output is given exogenously.
- There are two types of bond in this economy: a nominal bond and a real bond. The nominal bond has a price of kr. 1 at the time t and yields \((1+R_t)\) kroner in the following period. The real bond costs 1 consumption unit at the time t and yields \((1+r_t)\) units at the time \(t+1\). \(R_t\) and \(r_t\) are the nominal and real interest rates respectively.
- The bonds are perfectly divisible.
- The markets are perfect and thus without friction – no transaction costs.
- The model is formulated in discreet time \(t=0,1,...,\infty\). The timing is stated in subscript.

The remaining notation in the model is as follows: \(y_t\) is the consumer’s income, \(P_t\) is the price of the product, \(B_t\) is the consumer’s holdings of nominal bonds, \(b_t\) is the consumer’s holdings of real bonds, and \(U(c_t)\) is the consumer’s utility function, which is dependent on consumption \(c_t\). \(\beta\) is the consumer’s subjective discounting factor, which is assumed to be constant over time.

Since the output cannot be stored, it will all go to consumption: in other words: consumption is determined by output. In each period the consumer must decide how many (nominal and real) bonds he will buy. At the time t the consumer therefore faces the following problem:

\[
\max_{B_t,b_t} \quad E_t \left[ \sum_{j=0}^{\infty} \beta^j U(c_{t+j}) \right] \\
\text{u,b. } \quad y_t + (1 + r_{t-1})b_{t-1} + \frac{(1 + R_{t-1})B_{t-1}}{P_t} \geq c_t + b_t + \frac{B_t}{P_t} \tag{A1}
\]
\( E_t[\bullet] \) is the mathematical expectation. (A1) is a dynamic programming problem which can be solved by means of Bellmann’s principle of optimality. The first-order conditions give the following relations:

\[
E_t[MRS_{t+1}](1 + r_t) = 1 \\
E_t[MRS_{t+1}\pi_{t+1}^{-1}](1 + R_t) = 1
\]  

(A2) (A3)

\( MRS_{t+1} \) is the marginal rate of substitution between consumption in the period t and t+1, while \( \pi_{t+1} \) is the relationship between the price level in the period t and t+1:

\[
MRS_{t+1} = \beta \frac{U'(c_{t+1})}{U'(c_t)}, \quad \pi_{t+1} = \frac{P_{t+1}}{P_t}
\]  

(A4)

(A2) is the real interest rate generated by the model. At the equilibrium point it equals the reciprocal of the marginal rate of substitution. The following relation is achieved by using (A2) and (A3):

\[
\frac{1}{1 + R_t} = \frac{1}{1 + r_t} E_t[\pi_{t+1}^{-1}] + Cov_t[MRS_{t+1}, \pi_{t+1}^{-1}]
\]  

(A5)

\( Cov[\bullet, \bullet] \) is the covariance. By using the statistical definition of the correlation coefficient, which is between -1 and 1, and using the approximation \( 1/E_t[1/\pi_{t+1}] = E_t[\pi_{t+1}] \), it is now possible to delimit inflation expectations:

\[
(1 + R_t)(E_t[MRS_{t+1}] - \sigma_t[MRS_{t+1}]) \leq E_t[\pi_{t+1}]
\]

\[
\leq (1 + R_t)(E_t[MRS_{t+1}] + \sigma_t[MRS_{t+1}])
\]  

(A6)

\( \sigma_t(\bullet) \) is the standard deviation. The conversion to (A6) includes the assumption that the volatility of the (reciprocal) inflation rate is limited:

\[
\frac{\sigma_t[\pi_{t+1}^{-1}]}{E_t[\pi_{t+1}^{-1}]} \leq 1 \iff \sigma_t[\pi_{t+1}^{-1}] \leq E_t[\pi_{t+1}^{-1}]
\]  

(A7)

In empirical terms this assumption is rather conservative. The fraction is far below 1 for the entire period under consideration.

Knowing the marginal rate of substitution for consumption the model can now be used to generate a real interest rate, cf. (A2), and to estab-

\footnote{The conversion takes into account that \( E[ab] = \text{cov}[a,b] + E[a]E[b] \), where a and b are stochastic variables.}
lish limits for inflation expectations, cf. (A6). The variables to be used in
the model are consumption per capita and a nominal interest rate. Data
is available for both of these variables.

**Consumer preferences**

Empirical tests of CCAPM yield mixed results, depending on the specifi-
cation of preferences and on the country on which the model is tested.
Relevant literature often uses the concept of an isoelastic utility function
with a constant relative risk aversion. Besides this type of preference
other types are considered here, i.e. a consumer forming habits and a
consumer whose individual consumption is determined in consideration
of total consumption in the economy.

**Constant relative risk aversion**

The most frequently used utility function in the literature is an isoelastic
function with a constant relative risk aversion:

\[ U(c_t) = \frac{c_t^{1-\alpha} - 1}{1-\alpha}, \quad \alpha \neq 1 \]  

(A8)

For \( \alpha = 1 \) the utility is logarithmic in consumption. \( \alpha \) is the coefficient of
the relative risk aversion, which is assumed to be constant over time.
One of the properties of this utility function is that the coefficient of the
relative risk aversion equals the reciprocal inter-temporal elasticity of
substitution for consumption.

With this type of utility function the marginal rate of substitution is as
follows:

\[ MRS_{t+1} = \beta (g_{t+1})^{-\alpha}, \quad g_{t+1} = \frac{c_{t+1}}{c_t} \]  

(A9)

**Habit formation**

In the isoelastic utility function consumption is independent of time.
There is often a certain relation between today's and yesterday's con-
sumption, i.e. a certain inertia in the development of consumption. This
can be captured in a utility function in various ways. A simple method,
inspired by e.g. Heaton (1995), is to subtract a weighted measure of
yesterday's consumption from today's consumption:

\[ U(c_t) = \frac{(c_t - \lambda c_{t-1})^{1-\alpha}}{1-\alpha}, \quad \alpha \neq 1 \]  

(A10)

\( \lambda \) is the degree of habit formation where the requirement is \( \lambda < \min \{ \bar{c}_i \} \)
for the function to be well-defined. Furthermore, it is assumed that \( \lambda > 0 \),
which introduces habit formation into the utility function. It is noteworthy that the relative risk aversion is not constant in (A10), as it is a function of the absolute risk aversion $\alpha$, the degree of habit formation $\lambda$ and the consumption growth rate $g_t$. Given this utility function the marginal substitution rate can be calculated as follows:

$$MRS_{t+1} = \beta \frac{(g_{t+1} g_t - \lambda g_t)^{-\alpha} - \lambda \beta (g_{t+1} g_t)^{-\alpha} (g_{t+1} - \lambda)^{-\alpha}}{(g_t - \lambda)^{-\alpha} - \lambda \beta g_t^{-\alpha} (g_{t+1} - \lambda)^{-\alpha}}$$

(A11)

Relative consumption

The last type of preference to be considered here is the assumption that the consumer considers not only his own consumption, but also that of his neighbour. This can be captured in a utility function in a simple way by following Galí (1994), where the aggregate consumption is part of the individual utility function:

$$U(c_t, c_{t-1}) = \frac{c_t^\alpha}{1-\phi} c_{t-1}^\phi, \quad \alpha \neq 1$$

(A12)

$c_{t-1}$ is the total consumption in the economy in the preceding period, while $c_t$ is the individual consumption. $\phi$ is the extent to which the utility to the consumer depends on total consumption in the most recent period. If $\phi$ is positive, the consumer is pleased at the success of others (altruism). If $\phi$ is negative, on the other hand, the consumer is envious of the consumption of others. In this model using one single representative consumer, at equilibrium individual consumption and consumption in society will coincide. The marginal rate of substitution can thus be derived as follows:

$$MRS_{t+1} = \beta g_{t+1} g_t^\phi$$

(A13)
APPENDIX B

Expected values and standard deviation for the marginal rate of substitution must be used in the calibration of the model. Ireland (1996) is adhered to here, and an auto-regressive process is assumed for the rate of substitution:

\[ MRS_{i+1} = \gamma + \Gamma(L)MRS_i + \varepsilon_{i+1} \]  

(\text{B1})

\( \gamma \) is a constant, \( \Gamma(L)=\gamma_0+\gamma_1L+\gamma_2L^2+...+\gamma_kL^k \) is a polynomial in the lag operator \( L \), and \( \varepsilon_{i+1} \) is a noise element which meets the conditions \( E[\varepsilon_{i+1}]=0 \), \( \sigma[\varepsilon_{i+1}]=\sigma \), \( E[\varepsilon_{i+1}\varepsilon_{i-j}]=0 \) and \( E[\varepsilon_{i+1}MRS_j]=0 \) for all \( i,j=0,1,2,... \). By estimating (\text{B1}) it is possible to establish expected values and standard deviation for MRS. On the basis of the relatively limited amount of data a lag length of one is applied to the estimation of (\text{B1}).

Quarterly data is used for the period 1st quarter 1977 to 4th quarter 1998. Since a 5-year nominal interest rate is used, each period in the model is 5 years. A 5-year real interest rate and inflation expectations over 5 years are thus generated.

Initial choice of parameters

The first parameter chosen in the model is the subjective discounting factor \( \beta \). Here two values of \( \beta \) are considered: 0.9 and 0.7. The first value represents a patient consumer with an annual discounting rate of 1.9 per cent, while the other represents an impatient consumer with an annual discounting rate of 5.4 per cent.

Four values of the relative risk aversion coefficient are considered: \( \alpha: 0.1; 0.5; 2 \) and 5. They represent consumers ranging from almost risk-neutral to a relatively high degree of aversion.

Furthermore, two types of habit formation are considered: weak \( (\lambda=0.2) \) and strong \( (\lambda=0.8) \), and four values for the degree of altruism (envy) shown by the consumer \( \phi=[0.75, 0.25, -0.25, -0.75] \).

Choice of utility function and parameters

The highest correlation between the real interest rate generated by the model and a realised real interest rate appears where the consumer is very altruistic and almost risk-neutral. The discounting factor is determined on the basis of simple linear interpolation between the average generated real interest rates and the realised real interest rate. This gives a factor equivalent to an annual discounting rate of 4.09 per cent. The model is calibrated using the parameters \( (\beta, \alpha, \phi)=(0.778; 0.1; 0.75) \).
## APPENDIX C

GENERATED REAL INTEREST RATES ($\beta = 0.7$), AVERAGES  

<table>
<thead>
<tr>
<th>Risk aversion</th>
<th>$\alpha = 0.1$</th>
<th>$\alpha = 0.5$</th>
<th>$\alpha = 2$</th>
<th>$\alpha = 5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Isoelastic utility function</td>
<td>7.53</td>
<td>8.07</td>
<td>10.14</td>
<td>14.39</td>
</tr>
<tr>
<td>Habit formation</td>
<td>7.55</td>
<td>8.16</td>
<td>10.47</td>
<td>15.23</td>
</tr>
<tr>
<td>$\lambda = 0.2$</td>
<td>7.95</td>
<td>9.96</td>
<td>*</td>
<td>*</td>
</tr>
<tr>
<td>$\phi = 0.75$</td>
<td>8.11</td>
<td>8.66</td>
<td>10.74</td>
<td>15.03</td>
</tr>
<tr>
<td>$\phi = 0.25$</td>
<td>7.34</td>
<td>7.88</td>
<td>9.95</td>
<td>14.21</td>
</tr>
<tr>
<td>$\phi = 0.75$</td>
<td>6.95</td>
<td>7.49</td>
<td>9.56</td>
<td>13.80</td>
</tr>
</tbody>
</table>

Note: * means that the selected parameter gives unrealistic real-interest-rate values.

GENERATED REAL INTEREST RATES ($\beta = 0.9$), AVERAGES  

<table>
<thead>
<tr>
<th>Risk aversion</th>
<th>$\alpha = 0.1$</th>
<th>$\alpha = 0.5$</th>
<th>$\alpha = 2$</th>
<th>$\alpha = 5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Isoelastic utility function</td>
<td>2.26</td>
<td>2.78</td>
<td>4.74</td>
<td>8.78</td>
</tr>
<tr>
<td>Habit formation</td>
<td>2.27</td>
<td>2.86</td>
<td>5.06</td>
<td>9.60</td>
</tr>
<tr>
<td>$\lambda = 0.2$</td>
<td>2.87</td>
<td>5.20</td>
<td>*</td>
<td>*</td>
</tr>
<tr>
<td>$\phi = 0.75$</td>
<td>2.81</td>
<td>3.33</td>
<td>5.31</td>
<td>9.39</td>
</tr>
<tr>
<td>$\phi = 0.25$</td>
<td>2.44</td>
<td>2.96</td>
<td>4.94</td>
<td>9.00</td>
</tr>
<tr>
<td>$\phi = 0.75$</td>
<td>2.08</td>
<td>2.59</td>
<td>4.56</td>
<td>8.61</td>
</tr>
<tr>
<td>$\phi = 0.75$</td>
<td>1.71</td>
<td>2.22</td>
<td>4.19</td>
<td>8.22</td>
</tr>
</tbody>
</table>

Note: See Table 1.
REFERENCES


Topp, J., Indicators of the Market’s Interest-Rate and Inflation Expectations in Denmark, Danmarks Nationalbank, Monetary Review, May 1996.