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Optimal Contract Currencies and Exchange Rate Policy

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Optimal Contract Currencies and Exchange Rate Policy*

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Abstract

The paper develops a simple stochastic new open macroeconomic model in which price-setting firms' choice between producer currency pricing and local currency pricing is endogenous. We show that, in equilibrium, firms will denominate their export price contracts in the currency of the country with the lowest level of monetary variability. A welfare maximising government's choice of exchange rate regime is also analysed, and we find that a fixed exchange rate is preferable if the domestic monetary variability is higher than the foreign one.

Resume

Papiret udvikler en simpel, stokastisk, ny-åben-makro model, hvor prisfastsættende virksomheders valg mellem at fastsætte eksportpriser i egen eller i forbrugerens valuta er endogent. Vi viser, at virksomhederne i ligevægt vil denominere deres eksportpriskontrakter i den valuta, der er forbundet med mindst monetær usikkerhed. En velfærdsmaksimerende regerings valg af valutakursregime analyseres også, og vi finder, at en fast valutakurs er at foretrække, hvis den indenlandske monetære variabilitet er højere end den udenlandske.

Keywords: New open-economy macroeconomics; producer currency pricing; local currency pricing; endogenous contract currencies; optimal exchange rate regime.
JEL classification: F3, F4.

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1 Introduction

The optimal choice of exchange rate regime is a long-standing theme in international monetary economics that has recently received renewed interest from academic economists. This development has been closely related to the introduction of a new framework suitable for welfare analysis of different exchange rate regimes. The new generation of models fall within the class of explicitly stochastic general equilibrium models and they furthermore include both imperfect competition and nominal rigidities.

This paper offers a more thorough analysis of firms' price-setting problem within the new framework. While it is clear that in the presence of international trade, firms must not only decide what price to set but also in which currency to set it, previous work has simplified the analysis by assuming either that prices are set in the producer's currency or that producers set prices in their consumers' currency. To remedy this, we present an explicit analysis of firms' choice of price-setting currency which reveals the trade-offs faced by individual firms and also shows which (aggregate) outcomes are consistent with equilibrium.

The new framework for analysing exchange rate regimes was proposed by Obstfeld and Rogoff (2001). In that paper, Obstfeld and Rogoff present a simple stochastic general equilibrium model that can be solved without assuming certainty-equivalence for price setting and which allows for a proper consideration of how risk affects agents in the economy. The nominal rigidity is introduced by assuming that one-period nominal contracts specify the prices of consumption goods. Firms are assumed to fix prices in their own currency implying that there is complete pass-through of exchange rate changes to consumer prices and, accordingly, a strong expenditure-switching effect. Under these assumptions, Obstfeld and Rogoff (2001) show that exchange rate volatility entails a welfare cost that can potentially be quite large and that there will never be a conflict between the two governments over the choice of exchange rate regime.

Devereux and Engel (1998) consider the implications of assuming that firms set prices in their consumers' currency instead. The absence of exchange-rate pass-through eliminates the expenditure-switching effect and shifts the adjustment after an exchange rate change to the profit margins of exporters. They show that the optimal exchange rate regime depends on whether firms use producer or consumer currency pricing: with the traditional assumption of producer currency pricing used by Obstfeld and Rogoff (2001), a fixed exchange rate regime will be optimal if the consumers are sufficiently risk averse, while with consumer currency pricing (the preferred pricing assumption of Devereux and Engel (1998)), floating exchange rates always dominate fixed exchange rates.[1]

A number of papers have considered extensions of this basic framework. Obstfeld and Rogoff (2000, 2002) consider a model with sticky nominal wages and non-traded goods in which they study optimal monetary policy rules and exchange rate regimes in the presence of productivity shocks. Devereux and Engel (2000) analyse similar issues but focus on the differences between local and producer currency pricing. Betts and Devereux (2000) present a model in which a fraction of firms use producer currency pricing and the rest use consumer currency pricing. While this allows the model to better fit the empirical evidence on the degree of exchange rate pass-through, the analysis still suffers from the fact that contract currency choices are exogenous. Finally, Engel (2001) considers the importance of asset markets for the optimal choice of exchange rate regime.

The main difference between the approach taken in this paper and that employed in previous studies is that we endogenise firms' choice of contract currency. Our motivation for considering this extension is based on the findings of Devereux and Engel (1998, 2000), who show that the welfare results associated with the optimal choice of exchange rate regime are sensitive to firms' choice of currency for export price contracts. It is, therefore, critical for the models to be based on the right specification of contract currencies in order for their policy recommendations to be useful. A natural way to determine the correct specification is to look at the empirical evidence and this is indeed the approach that has been taken previously: Devereux and Engel (1998) argue that the observation of volatile real exchange rates due to the observed broad failure of the law of one price points to local currency pricing as being the best way to describe price setting. Obstfeld and Rogoff (2000), on the other hand, present data which rejects an important implication of local currency pricing (that unexpected exchange rate depreciations are associated with improvements of the terms of trade) and conclude that the traditional framework with producer currency pricing is more consistent with empirical evidence. The present state of uncertainty about which contract currency assumption is the more empirically relevant one leads us to suggest that the practice of exogenously imposing the contract currency choice should be abandoned.[2] In this paper, we show how to replace this assumption by an

[1] These results are derived under the assumption that there is only monetary variability in the foreign country. When domestic monetary variability is also present, a fixed exchange rate may be preferred also in the case of consumer currency pricing, but the condition will be stricter than with producer currency pricing.

[2] Obstfeld (2001) suggests that new pricing assumptions are necessary to improve the match between the models and the data. He argues that a distinction between retail and wholesale price may be useful since while there is virtually no pass-through of exchange rate changes to retail prices, there is some (but not full)

explicit model of how contract currencies are determined.

Our model of the contract currency choice is conceptually very simple. Previous models with one-period nominal price contracts have assumed that firms operate in monopolistically competitive goods markets and that equilibrium is given by the non-cooperative Nash solution to a price-setting game. We extend this game by considering firms' choice of currency for export price contracts to be part of their strategy. An equilibrium will then specify not only the prices set by firms but also whether they use producer or local currency pricing. It turns out that in the simple model of this paper in which the only uncertainty stems from monetary variability, equilibrium existence depends on the relative level of monetary variability in the two countries. In fact, even though a firm's choice of currency for fixing export prices depends on both average levels of and covariances between several aggregate variables, the decision rule is surprisingly simple: use producer currency pricing if monetary variability is lower domestically than abroad and use local currency pricing if domestic monetary variability is higher than it is in the foreign country. That is, firms (both domestic and foreign) fix export prices in the currency that is associated with the lowest monetary variability. We use this result to determine the equilibrium of the contract currency game for all possible combinations of domestic and foreign monetary variability.

We also analyse the choice of exchange rate regime by governments in the presence of endogenous contract currencies. This analysis yields welfare results that are more generally applicable than those derived in previous papers, because the conclusions are robust to firms' contract currency choices. We show that a government that finds domestic monetary variability to be greater than that abroad will choose to give up its monetary independence and fix the exchange rate in order to reduce domestic monetary variability. In case the two countries face the same level of monetary variability, we show that despite the fact that there are multiple equilibria of the contract currency game, governments will indeed be able to ensure the highest possible level of welfare by choosing a fixed exchange rate regime.

The rest of the paper is structured as follows. Section 2 contains a presentation of the model. In section 3, we provide a detailed analysis of firms' contract currency choice and state three propositions which contain our results on equilibrium contract currencies. In section 4, we consider the governments' problem of choosing the optimal exchange rate regime when contract currencies are endogenous. Finally, we offer some concluding remarks in section 5.

pass-through to wholesale prices.

2 The Model

We consider a simple two-country stochastic general equilibrium model with one-period nominal price contracts. The basic setup parallels that of Engel (2001) and Devereux and Engel (1998), which is in turn similar to the model in Obstfeld and Rogoff (2001). However, these papers take contract currencies as exogenously given, while we allow firms to choose their contract currency optimally.

The representative consumer in the Home country maximises expected lifetime utility

$$U_t = E_t \left(\sum_{s=t}^{\infty} \beta^{s-t} u_s \right), \quad 0 < \beta < 1, \tag{1}$$

where the period utility function is given by

$$u_s = \frac{1}{1-\rho} C_s^{1-\rho} + \chi \ln \left(\frac{M_s}{P_s} \right) - \eta L_s, \quad \rho > 1, \quad \chi > 0, \tag{2}$$

where C is a consumption index to be defined below, M/P is real balances and L is labour supply. Consumers are assumed to be risk averse, and the degree of risk aversion is measured by the parameter ρ. In line with most empirical estimates, we assume $\rho > 1$, that is, consumers are more risk averse than implied by a logarithmic utility function. The consumption index is a geometric average of Home and Foreign goods

$$C = \frac{C_h^n C_f^{1-n}}{n^n (1-n)^{1-n}}, \quad 0 < n < 1. \tag{3}$$

The parameter n is also taken to be a measure of the relative size of the Home population. The subindexes are given by (Home goods are indexed by numbers in the interval $[0, n)$ while Foreign goods belong to the interval $[n, 1]$)

$$C_h = \left[n^{-1/\lambda} \int_0^n C_h(i)^{\frac{\lambda-1}{\lambda}} di \right]^{\frac{\lambda}{\lambda-1}}; \quad C_f = \left[(1-n)^{-1/\lambda} \int_n^1 C_f(i)^{\frac{\lambda-1}{\lambda}} di \right]^{\frac{\lambda}{\lambda-1}}, \quad \lambda > 1. \tag{4}$$

The elasticity of substitution between different varieties of each of the national composite goods is assumed to be greater than one and, therefore, greater than the elasticity of substitution between the two national composite goods, which equals one. The price index in the Home country is given by

$$P = P_h^n P_f^{1-n}, \tag{5}$$

with subindexes

$$P_h = \left[\frac{1}{n} \int_0^n P_h(i)^{1-\lambda} di \right]^{\frac{1}{1-\lambda}}; \quad P_f = \left[\frac{1}{1-n} \int_n^1 P_f(i)^{1-\lambda} di \right]^{\frac{1}{1-\lambda}}. \tag{6}$$

The description of the Foreign consumers is similar; importantly, the consumption indexes are identical to those of the Home consumers.

From consumer optimisation, one derives the Home representative consumer's demands for individual goods and expenditure shares for the two composite goods

$$C_h(i) = \frac{1}{n}\left(\frac{P_h(i)}{P_h}\right)^{-\lambda} C_h; \quad C_f(i) = \frac{1}{1-n}\left(\frac{P_f(i)}{P_f}\right)^{-\lambda} C_f; \tag{7}$$

$$P_h C_h = nPC; \quad P_f C_f = (1-n)\,PC. \tag{8}$$

We assume that asset markets are complete.[3] The flow budget constraint of the Home consumer may be written as

$$P_t C_t + M_t + Q_t B_t = W_t L_t + \pi_t + M_{t-1} + B_{t-1} + T_t, \tag{9}$$

where $Q_t B_t$ denotes expenditure on internationally traded state-contingent nominal bonds denominated in Home currency purchased in period t and carried into period $t+1$, and B_{t-1} is proceeds from bonds purchased in period $t-1$. W_t is the wage rate, π_t is profits from Home firms and T_t is transfers from the government.

Consumer optimisation also implies the following money demand equation

$$\frac{M_t}{P_t} = \frac{\chi C_t^{\rho}}{1 - E_t d_{t+1}}, \tag{10}$$

where $E_t d_{t+1}$ is the inverse of the gross nominal interest rate when we define the stochastic discount factor (or asset pricing kernel) as

$$d_{t+1} = \beta \frac{C_{t+1}^{-\rho} P_t}{C_t^{-\rho} P_{t+1}}. \tag{11}$$

The optimal trade-off between consumption and leisure is governed by

$$\frac{W_t}{P_t C_t^{\rho}} = \eta. \tag{12}$$

[3] Obstfeld and Rogoff (2001) only allow for international trade in a riskless real bond. But, as noted by the authors, allowing for international trade in equity would not affect the equilibrium of their model because of the assumed utility function and the assumption that the law of one price always holds. In our model, the law of one price does not hold in general and international trade in equity will, therefore, not be redundant as in Obstfeld and Rogoff (2001). We follow Devereux and Engel (1998) in assuming complete asset markets to focus on the effects of monetary and exchange rate uncertainty while abstracting from imperfections in international asset markets. For an interesting analysis of the effects of asset market incompleteness, see Engel (2001).

Finally, optimal risk sharing implies that in equilibrium

$$\frac{S_t P_t^*}{P_t} = \left(\frac{C_t}{C_t^*}\right)^{\rho}, \tag{13}$$

where S_t is the nominal exchange rate giving the price of Foreign currency in terms of Home currency. To interpret this condition, note that the left-hand side is the relative price of a unit of the Foreign consumption index in terms of the Home consumption index, while the right-hand side is the ratio of Foreign to Home marginal utility of consumption.

The government increases the money supply using lump sum transfers and keeps the budget balanced each period

$$M_t = M_{t-1} + T_t. \tag{14}$$

Each firm (indexed by i) produces a single variety of its country's differentiated good. Labour is the only input to production and transformation takes place according to the production function

$$Y(i) = L(i). \tag{15}$$

The uncertainty in the model is assumed to stem entirely from monetary shocks in the two countries. We follow Engel (2001) in taking the shocks to be outside the control of the monetary authorities. Possible interpretations include control errors and shocks to the money multiplier (or more generally to the relationship between the control variable of the central bank and the relevant monetary aggregate). In particular, we assume that the shock to the money supply is multiplicative and lognormally distributed

$$M_{t+1} = \frac{M_t}{\mu \exp(-\sigma_m^2/2)} V_{t+1}, \quad V_{t+1} \sim \Lambda\left(0, \sigma_m^2\right), \tag{16}$$

where μ is a drift parameter and it can easily be shown that $E_t\left(M_t/M_{t+1}\right) = \mu$. The log of the money supply follows a random walk with drift $-\ln(\widetilde{\mu})$

$$m_{t+1} = m_t - \ln(\widetilde{\mu}) + v_{t+1}, \quad v_{t+1} \sim \mathrm{N}\left(0, \sigma_m^2\right), \tag{17}$$

where we have defined $\widetilde{\mu} = \mu \exp\left(-\sigma_m^2/2\right)$ and lowercase letters denote logs of uppercase letters. The process for the foreign money supply is similar and we allow for the foreign money supply to have a different drift parameter and for its shock to have a different variance. We assume that the two money supply processes are independent.

Given the assumed money supply process, we can derive the following money market equilibrium condition

$$C_t^{\rho} = \left(\frac{1-\mu\beta}{\chi}\right) \frac{M_t}{P_t}, \tag{18}$$

and a similar condition holds for the foreign country. Plugging these into the risk sharing condition (13), we get the following solution for the exchange rate

$$S_t = \frac{(1-\mu\beta)\,M_t}{(1-\mu^*\beta)\,M_t^*}, \tag{19}$$

which shows that the (change in the) exchange rate is proportional to (the change in) relative money supplies.

Taking logs in equations (18) and (19), we get

$$\rho c_t = m_t - p_t + \ln\left(\frac{1-\mu\beta}{\chi}\right); \tag{20}$$

$$s_t = m_t - m_t^* + \ln\left(\frac{1-\mu\beta}{1-\mu^*\beta}\right). \tag{21}$$

An equation similar to (20) holds for the foreign country as well.

3 Choice of Contract Currency

As mentioned above, we assume that firms write one-period price contracts that fix prices for the following period. Recent literature on the optimal choice of exchange rate regime differ in the assumptions made about the currency used for writing price contracts: Obstfeld and Rogoff (2001) assume that contracts are written in terms of the producer's currency (producer currency pricing), Devereux and Engel (1998) assume that contracts are written in terms of the consumer's currency (local currency pricing), while Engel (2001) considers the possibility of asymmetric contract currencies in the sense that firms in one country write contracts in their own currency while firms in the other country write contracts in their consumers' currency. But instead of assuming that firms write export contracts in their own or in their consumers' currency, we will consider a model in which firms' choice of contract currency is endogenous.

For the purpose of including the choice of contract currency in the model, it is useful to note that our assumption that each firm is a monopoly producer of a specific variety of the national composite good implies that firms have the power to set the price of their product. Correspondingly, we will assume that firms also have the power to choose which currency to set their price in. Furthermore, the contract currency decision and the price-setting decision take place at the same point in time and we maintain the assumption (of the monopolistic competition model) that firms' choices are simultaneous as well as non-cooperative.

Finally, we assume that firms take the exchange rate regime as given when they choose their contract currency and fix prices. That is, governments choose the exchange rate regime before the firms choose their contract currency. We find this to be the most plausible sequential structure both because the governments are likely to have better opportunities for commitment than the individual firms and because the costs involved in switching exchange rate regime are large compared to those of choosing a contract currency implying that the horizon involved in the choice of exchange rate regime is likely to be significantly longer than the contract period.

We consider two exchange rate regimes in our analysis, a flexible exchange rate regime in which neither government intervenes to affect the exchange rate and a fixed exchange rate regime in which one of the governments choose to control its money supply so as to keep the exchange rate fixed no matter which state of the world is realised.[4]

It is important to note that while the equilibrium in a flexible exchange rate regime depends on firms' choice of contract currency, the equilibrium under a fixed exchange rate will actually be independent of whether firms use producer or local currency pricing. The reason for this difference is simple: when the exchange rate is completely fixed, it does not matter whether a firm fixes its export price in its own or in foreign currency. The implication for our analysis is that we only need to consider firms' choice of contract currency explicitly in the case of a flexible exchange rate—a task to which we now turn.

3.1 Potential Equilibria in the Flexible Exchange Rate Regime

The model we have set up has the property that firms are identical within each country. In considering the game in which firms simultaneously and non-cooperatively choose their contract currency when the exchange rate is flexible, we will, therefore, focus on symmetric equilibria in which each country's firms behave identically. Furthermore, we will restrict attention to pure strategy Nash equilibria. This leaves us with four possible equilibrium configurations: 1) worldwide producer currency pricing, 2) worldwide local currency pricing, 3) producer currency pricing in Home, local currency pricing in Foreign and 4) local currency pricing in Home, producer currency pricing in Foreign. To evaluate whether these configurations can in fact be supported as equilibria of the game, we solve for each potential equilibrium and consider whether Home and/or Foreign firms have incentives to deviate.

The objective of firms is to maximise the utility of their owners. Given the complete set of state-contingent claims, this is achieved by maximising the expected value of profits discounted by the stochastic discount factor derived in the consumer's problem. Firms are monopolistic competitors so they set prices to maximise their objective subject to the demand for their

[4]The commitment to a fixed exchange rate is assumed to be fully credible.

product variety. We do not impose the restriction that the law of one price holds as in Obstfeld and Rogoff (2001) and other models with producer currency pricing.[5] Instead, we assume that markets are sufficiently segmented that firms are able to charge different prices in different markets.[6] As an example of firms' price-setting problems, we will consider the decision problem of a Home firm in some detail.

When firm i in Home uses producer currency pricing, it sets $P_{ht}(i)$ and $S_t P^*_{ht}(i)$ (both in Home currency) to maximise

$$E_{t-1}\left(d_{t-1}\left[P_{ht}(i)\,X_{ht}(i)+S_t P^*_{ht}(i)\,X^*_{ht}(i)-W_t\left(X_{ht}(i)+X^*_{ht}(i)\right)\right]\right), \tag{22}$$

where $X_{ht}(i)=nC_{ht}(i)$ is the quantity sold to Home consumers and $X^*_{ht}(i)=(1-n)\,C^*_{ht}(i)$ is the quantity sold to Foreign consumers (recall that the world population is normalised to one). The first order conditions read

$$E_{t-1}\left(d_{t-1}\left[X_{ht}(i)+P_{ht}(i)\frac{\partial X_{ht}(i)}{\partial P_{ht}(i)}-W_t\frac{\partial X_{ht}(i)}{\partial P_{ht}(i)}\right]\right)=0; \tag{23}$$

$$E_{t-1}\left(d_{t-1}\left[X^*_{ht}(i)+S_t P^*_{ht}(i)\frac{\partial X^*_{ht}(i)}{\partial\left(S_t P^*_{ht}(i)\right)}-W_t\frac{\partial X^*_{ht}(i)}{\partial\left(S_t P^*_{ht}(i)\right)}\right]\right)=0. \tag{24}$$

From (7) and its foreign equivalent, we get

$$\frac{\partial X_{ht}(i)}{\partial P_{ht}(i)}=-\lambda\left(\frac{P_{ht}(i)}{P_{ht}}\right)^{-\lambda}\frac{C_{ht}}{P_{ht}(i)}; \tag{25}$$

$$\frac{\partial X^*_{ht}(i)}{\partial\left(S_t P^*_{ht}(i)\right)}=\frac{1-n}{n}(-\lambda)\left(\frac{P^*_{ht}(i)}{P^*_{ht}}\right)^{-\lambda}\frac{C^*_{ht}}{S_t P^*_{ht}(i)}. \tag{26}$$

Substituting (7) and (25) into (23) and rearranging, we have

$$E_{t-1}\left(d_{t-1}\left[(1-\lambda)\left(\frac{P_{ht}(i)}{P_{ht}}\right)^{-\lambda}C_{ht}+\lambda W_t\left(\frac{P_{ht}(i)}{P_{ht}}\right)^{-\lambda}\frac{C_{ht}}{P_{ht}(i)}\right]\right)=0;$$

$$P_{ht}(i)=\frac{\lambda}{\lambda-1}\frac{E_{t-1}\left(d_{t-1}W_t C_{ht}\right)}{E_{t-1}\left(d_{t-1}C_{ht}\right)}, \tag{27}$$

[5] Indeed, the empirical evidence against the law of one price is substantial. See Froot and Rogoff (1995) for a survey.

[6] Note that this is really not an independent assumption as it is implied by the assumption that firms can write export price contracts in foreign currency which specify prices that may differ from the prices charged domestically.

where we have used that Home firm i takes the Home domestic goods' price index P_{ht} as given. Using the definition of d_{t-1} given in (11) and the consumption shares in (8), we get

$$P_{ht}(i) = \frac{\lambda}{\lambda - 1} \frac{E_{t-1}\left(W_t C_t^{1-\rho}\right)}{E_{t-1}\left(C_t^{1-\rho}\right)} = P_{ht}, \tag{28}$$

where the last equality holds in a symmetric equilibrium. To interpret the expression for the price charged of Home consumers, note first that in the absence of uncertainty, the optimal price reduces to the well-known formula in which the price is a markup on marginal cost: $P_{ht}(i) = (\lambda / (\lambda - 1)) W_t$. In the presence of uncertainty, however, the price also includes a risk premium term since optimal prices are set to lead firm profit to serve as a hedge against the owners' consumption risk.

To derive the optimal price for the Home good in the Foreign country, we substitute the Foreign version of (7) and (26) into (24) and rearrange to get

$$S_t P_{ht}^*(i) = \frac{\lambda}{\lambda - 1} \frac{E_{t-1}\left(d_{t-1} W_t C_{ht}^*\right)}{E_{t-1}\left(d_{t-1} C_{ht}^*\right)}; \tag{29}$$

$$S_t P_{ht}^*(i) = S_t P_{ht}^* = \frac{\lambda}{\lambda - 1} \frac{E_{t-1}\left(W_t C_t^{*1-\rho}\right)}{E_{t-1}\left(C_t^{*1-\rho}\right)}, \tag{30}$$

where we have used the risk sharing condition (13) and that Home firms take the Home currency value of the Foreign import price index $S_t P_{ht}^*$ as given.[7] Note that while the price set for Foreign consumers is predetermined in Home currency, its Foreign currency value will vary with the exchange rate, implying that Foreign consumers will observe changes in the price of imported goods when the exchange rate changes.

A comparison of equations (27) and (29) shows that Foreign consumers are not necessarily charged the same (Home currency) price as Home consumers. To see why this is so, we take advantage of the fact that all variables will be lognormally distributed in equilibrium, which allows us to write the prices as the product of three terms: the certainty equivalent price and two risk premium terms

$$\begin{aligned} P_{ht}(i) &= \frac{\lambda}{\lambda - 1} E_{t-1}(W_t) \exp\left(Cov_{t-1}(\ln d_{t-1}, \ln W_t)\right) \exp\left(Cov_{t-1}(\ln W_t, \ln C_{ht})\right); \\ S_t P_{ht}^*(i) &= \frac{\lambda}{\lambda - 1} E_{t-1}(W_t) \exp\left(Cov_{t-1}(\ln d_{t-1}, \ln W_t)\right) \exp\left(Cov_{t-1}(\ln W_t, \ln C_{ht}^*)\right). \end{aligned}$$

[7] The assumption that Home firms take the Home currency value of the Foreign import price index as given may not seem innocuous, but it amounts to no more than the standard assumption that a monopolistically competitive firm takes the prices of its competitors as given.

The first risk premium term is common to the two prices and depends on the (endogenous) covariance between firm owners' marginal value of income d_{t-1} and the unit cost of production W_t. If the covariance is negative (reflecting that the unit cost is low when the marginal value of income is high and vice versa), the risk premium will be small since the fact that costs are low precisely when high profits are desirable provides consumption insurance for the firm owners which leads the firm to command a relatively low price. The second risk premium term is market specific and involves the covariance between the unit cost and the market specific demand for the firm's product. If demand in a market is positively correlated with the unit cost, the firm will set a high price for that market to compensate for the added risk associated with the firm having to produce a lot when the unit cost is high. Finally, this decomposition of the Home firm's prices allows us to conclude that it will charge different prices in the two markets unless the demands are identically correlated with the unit cost of production.

If the Home firm writes price contracts in the local currency instead as suggested by e.g. Devereux and Engel (1998), its optimisation problem is to choose local currency prices $P_{ht}(i)$ and $P_{ht}^*(i)$ to maximise (22). The optimal price to charge domestic consumers is again given by (27), while the optimal export price takes the following form

$$P_{ht}^*(i) = \frac{\lambda}{\lambda - 1} \frac{E_{t-1}(d_{t-1} W_t C_{ht}^*)}{E_{t-1}\left(d_{t-1} S_t C_{ht}^*\right)}. \tag{31}$$

Using that the variables are lognormally distributed in equilibrium, we can write this as

$$E_{t-1}(S_t) P_{ht}^*(i) = \frac{\lambda}{\lambda - 1} E_{t-1}(W_t) \frac{\exp\left(Cov_{t-1}(\ln d_{t-1}, \ln W_t)\right) \exp\left(Cov_{t-1}(\ln W_t, \ln C_{ht}^*)\right)}{\exp\left(Cov_{t-1}(\ln d_{t-1}, \ln S_t)\right) \exp\left(Cov_{t-1}\left(\ln S_t, \ln C_{ht}^*\right)\right)}.$$

This ex-ante Home currency value of the export price includes two additional risk premium terms compared to the export price of a Home firm that uses producer currency pricing. Both the new terms involve the exchange rate, which affects profits because the value of export revenues now depends on the exchange rate since the export price is fixed in Foreign currency. A negative covariance between the exchange rate and the firm owners' marginal value of profit income leads to a higher export price because a low Home currency value of the export revenues (following a low realisation of the exchange rate S_t equivalent to a depreciation of the Foreign currency) will tend to be associated with firm owners' having a high marginal value of income. In a similar vein, a negative covariance between the exchange rate and Foreign consumption of the Home firm's good will imply a high price to compensate Home firm owners for the added risk associated with high Foreign sales usually being coupled with a depreciated Foreign currency.

This discussion of the risk premia in price setting has shown how changes in the covariances between a number of aggregate variables affect the prices set by a Home firm. However, as we have already noted above, while these covariances are taken as given by individual firms, they are indeed endogenous variables of the model. Therefore, our comments may not provide a proper account of general equilibrium effects.

An important thing to note is that in the analysis of a Home firm's price-setting problems above, we have not assumed anything about the behaviour of Foreign firms. Consequently, the optimal prices we have derived are valid independently of whether Foreign firms use producer or local currency pricing.

The optimal prices set by Foreign firms are symmetrical to those of the Home firms and we report the results in Table 1.

Firm	Consumer	Producer currency pricing	Local currency pricing
Home	Home	$P_{ht} = \frac{\lambda}{\lambda-1}\frac{E_{t-1}\left(W_t C_t^{1-\rho}\right)}{E_{t-1}\left(C_t^{1-\rho}\right)}$	
	Foreign	$P_{ht}^{*} = \frac{\lambda}{\lambda-1}\frac{S_t^{-1}E_{t-1}\left(W_t C_t^{*1-\rho}\right)}{E_{t-1}\left(C_t^{*1-\rho}\right)}$	$P_{ht}^{*} = \frac{\lambda}{\lambda-1}\frac{E_{t-1}\left(S_t^{-1}W_t C_t^{*1-\rho}\right)}{E_{t-1}\left(C_t^{*1-\rho}\right)}$
Foreign	Home	$P_{ft} = \frac{\lambda}{\lambda-1}\frac{S_t E_{t-1}\left(W_t^{*} C_t^{1-\rho}\right)}{E_{t-1}\left(C_t^{1-\rho}\right)}$	$P_{ft} = \frac{\lambda}{\lambda-1}\frac{E_{t-1}\left(S_t W_t^{*} C_t^{1-\rho}\right)}{E_{t-1}\left(C_t^{1-\rho}\right)}$
	Foreign	$P_{ft}^{*} = \frac{\lambda}{\lambda-1}\frac{E_{t-1}\left(W_t^{*} C_t^{*1-\rho}\right)}{E_{t-1}\left(C_t^{*1-\rho}\right)}$	

Table 1: Optimal Prices

Having solved for all individual prices, we can now solve for the aggregate quantities in the two countries. Price levels are immediate from Table 1 and equation (5). Using the optimality conditions (12), (13) and (18), we can solve for consumption levels and variances. Finally, labour supply can be found using the production function and the goods market equilibrium conditions

$$nY_t = nC_{ht} + (1-n)\,C_{ht}^{*} \quad \Rightarrow \quad L_t = n\frac{P_t C_t}{P_{ht}} + (1-n)\,\frac{P_t^{*} C_t^{*}}{P_{ht}^{*}}; \tag{32}$$

$$(1-n)\,Y_t^{*} = nC_{ft} + (1-n)\,C_{ft}^{*} \quad \Rightarrow \quad L_t^{*} = n\frac{P_t C_t}{P_{ft}} + (1-n)\,\frac{P_t^{*} C_t^{*}}{P_{ft}^{*}}. \tag{33}$$

3.2 Existence of Equilibria in the Flexible Exchange Rate Regime

In the last subsection, we have solved fully for each of the four proposed symmetric equilibria of the contract currency game. We now consider whether and under what parameter restrictions these potential equilibria are indeed equilibria of the game. To this end, we compute whether any individual firm has an incentive to deviate from its equilibrium strategy. By solving for the parameter restrictions under which no firm finds deviation profitable, we determine the Nash equilibria of the contract currency game.

As an example of this analysis and to get some insight into firms' contract currency choice, we consider whether firms have an incentive to deviate from the (potential) equilibrium with worldwide producer currency pricing by choosing to write price contracts for exports in their consumers' currency. Since a potential deviator expects the other firms to stick to their equilibrium strategies and since a deviator does not take the aggregate effects of a change in its strategy into account, all aggregate quantities, prices, variances and covariances will be taken as constants in this analysis.

When Home firm j deviates from the equilibrium by using local currency pricing, it will maximise profits (22) with respect to $P_{ht}(j)$ and $P_{ht}^{*}(j)$. The optimal price to charge domestic consumers turns out to be unchanged, while the optimal export price is easily shown to be given by

$$P_{ht}^{*}(j) = \frac{\lambda}{\lambda-1}\frac{E_{t-1}\left(d_{t-1}S_t^{-\lambda}W_tC_{ht}^{*}\right)}{E_{t-1}\left(d_{t-1}S_t^{1-\lambda}C_{ht}^{*}\right)} = \frac{\lambda}{\lambda-1}\frac{E_{t-1}\left(S_t^{-\lambda}W_tC_t^{*1-\rho}\right)}{E_{t-1}\left(S_t^{1-\lambda}C_t^{*1-\rho}\right)}. \tag{34}$$

In the absence of uncertainty, this price is equal to that charged in the proposed (worldwide producer currency pricing) equilibrium (cf. (30)). With uncertainty, we see that deviation to local currency pricing leads to a different risk premium in the export price than does the equilibrium strategy of producer currency pricing. The distinct risk premia stem from the fact that the hedge properties of export profits depend on whether the Home firm writes export price contracts in Home or Foreign currency.

The payoff to deviation for Home firm j is (index i denotes non-deviation)

$$D_h(j) = E_{t-1}\left[d_{t-1}\left(S_tP_{ht}^{*}(j)X_{ht}^{*}(j) - S_tP_{ht}^{*}(i)X_{ht}^{*}(i) - W_t\left(X_{ht}^{*}(j) - X_{ht}^{*}(i)\right)\right)\right], \tag{35}$$

since $P_{ht}(j) = P_{ht}(i)$ and $X_{ht}(j) = X_{ht}(i)$. Substituting the prices (30) and (34) into this expression along with the equilibrium solutions for aggregate variables, we show in the Appendix that it pays for the Home firm to deviate from the worldwide producer currency pricing equilibrium if monetary variability is higher in Home than in Foreign

$$D_h(j) > 0 \quad \Leftrightarrow \quad \sigma_m^2 > \sigma_{m^*}^2. \tag{36}$$

This implies that the proposed equilibrium breaks down if the Home monetary variability is higher than the Foreign one because then it will be profitable for Home firms to switch to writing export price contracts in Foreign currency.

A number of mechanisms underlie this strikingly simple result and we will provide a brief overview in the following. Because firms take into account that their profits serve as consumption insurance for their owners, their optimal choice of contract currency will depend on the variation in profits across states as well as on the level of profit. The level of profits is affected by deviation since a change in contract currency leads to a different risk premium in the export price (and, of course, an associated change in demand).

The way profits vary across states also depends on the choice of contract currency. One channel through which this occurs is the difference in the state-dependence of the domestic currency value of export prices. A firm that uses producer currency pricing will be passing on exchange rate changes to its consumers abroad so that the domestic currency value of its price will be independent of the current state. With local currency pricing, however, exchange rate changes will be borne entirely by the producer. And in this case, the effect of exchange rate changes is straightforward: depreciations (appreciations) lead to a higher (lower) domestic currency value of the export price and, therefore, a higher (lower) level of export revenue for a given level of export demand.

Another channel leading to state-dependence in profits is the variation in demand for individual goods. This stems from two sources: First, in a world of producer currency pricing, there is a strong expenditure-switching effect which shifts world demand towards the country with a depreciating currency (here, the country with the largest money supply increase). Second, changes in demand for an individual good result from changes in the relative price of the good on the export market. Since the expenditure-switching effect does not depend on the pricing policy of any individual firm, this is irrelevant for the deviation decision. The same is, however, not true for the (export-market) relative price effect. To see why, note that when the Home firm uses producer currency pricing like its competitors, its export-market relative price will remain constant in all states of the world, while the Home firm, if it deviates to local currency pricing, will experience changes in its relative price on the export market. Specifically, suppose that there is a positive money supply shock in Home. This leads to an exchange rate depreciation and the deviating Home firm j will experience an increase in its relative price in Foreign and, therefore, a fall in the Foreign demand for its product. At the same time, Home firm owners have a relatively low marginal value of money income as consumption and the Home price level are both relatively high. In case of a positive Foreign monetary shock, the export-market relative price effect will lead to an increase in the Foreign demand for the product of Home firm j. The marginal value of money income for Home firm owners remains unchanged after an expansionary Foreign shock because the effects of higher consumption and a lower Home price level outweigh each other.

A change in contract currency thus has implications for both the expected level of profit and the covariance between profits and firm owners' marginal value of income, and a firm that considers deviating from the equilibrium will use its objective function to weigh these effects. As shown in (36) above, a Home firm that considers deviating from the worldwide producer currency pricing equilibrium will find it optimal to do so if the monetary variability in Home exceeds that in Foreign.

We now turn to Foreign firms. They also use producer currency pricing in the proposed equilibrium, so the only differences between a Foreign and a Home firm are that they belong to countries that may be of different sizes and that may have different money supply drifts and variances. However, we have seen that only money supply variances affect a Home firm's deviation decision so that, by symmetry, a Foreign firm's decision to deviate will also only depend on these variances. The following result is then immediate: Foreign firm j has an incentive to deviate if monetary variability is highest in Foreign

$$D_f(j) > 0 \quad \Leftrightarrow \quad \sigma_{m^*}^2 > \sigma_m^2. \tag{37}$$

The proposed equilibrium with worldwide producer currency pricing can only be supported if no firm has an incentive to deviate. From (36) and (37) we can conclude that Home firms will stick to their equilibrium strategy if (and only if) Home monetary variability is less than or equal to Foreign monetary variability while Foreign firms similarly will resist deviation if and only if the opposite (weak) inequality holds. Therefore, we have the following result on the existence of an equilibrium with only producer currency pricing (in a flexible exchange rate regime):

Proposition 1 (a) *Producer currency pricing by both Home and Foreign firms is an equilibrium if and only if the variances of the Home and Foreign money supplies are equal.*

In considering the existence of an equilibrium with worldwide local currency pricing, we apply the same reasoning as above. The effects to be considered by potential deviators are equivalent to those just given for the producer currency pricing equilibrium and we will, therefore, proceed directly to the result on equilibrium existence. We find that the equilibrium with worldwide local currency pricing exists if and only if there is the same level of monetary variability in both countries. The equilibrium fails when money supply variances are asymmetric because Home firms have an incentive to use producer currency pricing when the monetary variability at Home is less than in Foreign, while Foreign firms will deviate if monetary conditions are most stable in Foreign.

Proposition 1 (b) *Local currency pricing by both Home and Foreign firms is an equilibrium if and only if the variances of the Home and Foreign money supplies are equal.*

Taking Propositions 1 (a) and (b) together, we conclude that globally symmetric equilibria only exist when money supply variances are equal across the two countries. Thus, in a world in which firms choose their contract currencies optimally, firms in the two countries will not choose identical contract currency policies unless money supplies have identical variances. The next proposition considers the existence of globally asymmetric equilibria.

Proposition 1 (c) *Local currency pricing by Home firms and producer currency pricing by Foreign firms is an equilibrium if and only if the variance of the Home money supply is greater than or equal to the variance of the Foreign money supply.*
Producer currency pricing by Home firms and local currency pricing by Foreign firms is an equilibrium if and only if the variance of the Home money supply is less than or equal to the variance of the Foreign money supply.

One and/or the other type of globally asymmetric equilibrium always exist—if money supply variances are different, one asymmetric equilibrium can be supported and if the money supply variances are equal, both asymmetric equilibrium types can be supported. The particular type of globally asymmetric equilibrium which prevails in case monetary conditions are different depends on the relative sizes of the variances. Specifically, firms in the country with the lowest (highest) money supply variability choose to write export contracts in their own (the buyers') currency.

Finally, we note that the results given in Propositions 1 (a) through (c) can be summarised as follows.

Proposition 1 *Producer currency pricing by firms in the country with the lowest money supply variance and local currency pricing by firms in the country with the highest money supply variance is an equilibrium of the contract currency game.*

4 Choosing Exchange Rate Regime

Now we take one step back and consider optimal choice of exchange rate regime. In particular, we analyse the problem of the Home government which chooses between having a flexible exchange rate vis-à-vis Foreign and controlling their money supply so as to keep the exchange rate fixed.[8] In addition, we consider whether the Foreign government agrees or disagrees with the Home government's decision.

In their analysis of optimal exchange rate regime in a model with exogenous contract currencies, Devereux and Engel (1998) identify a trade-off between fixed and flexible exchange rate regimes when producers use producer currency pricing. The advantage of a flexible exchange rate is its ability to alleviate the impact of monetary shocks, while its disadvantage is that the associated exchange rate uncertainty leads firms to increase their mark-ups implying a lower average level of consumption. If the domestic monetary variability exceeds that in the other country, a fixed exchange rate is always preferable. A fixed exchange rate regime can also be superior even with lower domestic than foreign monetary variability if the domestic economy is small or the level of risk aversion is high. In case firms write export price contracts in their consumers' currency, there is an additional difference between the two regimes in that expected leisure is higher under a floating exchange rate. This implies that the condition for a fixed exchange rate to be optimal is stricter in a model with local currency pricing than with producer currency pricing. To be precise, a fixed exchange rate regime is preferred if and only if the domestic monetary variability is greater than that abroad.

We assume that the Home government chooses the exchange rate regime which maximises the welfare of its citizens. Our framework provides a consistent measure of the representative consumer's (expected) welfare in the form of the (expected value of the) utility function (1).[9] Since variables are lognormally distributed in equilibrium, the expected welfare is given by

$$\begin{aligned} E\omega &= \frac{1}{1-\rho}E\left(C^{1-\rho}\right) - \eta E\left(L\right) \\ &= \frac{1}{1-\rho}\left(\exp E\left(C\right)\right)^{1-\rho}\exp\left(\frac{\rho\left(\rho-1\right)}{2}\sigma_c^2\right) - \eta E\left(L\right), \end{aligned} \tag{38}$$

and it is increasing in the expected level of consumption, decreasing in the variance of log

[8] The derivation of the (unique) equilibrium under fixed exchange rates is omitted as it is straightforward and not central to our analysis. As noted above, the choice of contract currency is immaterial in the fixed exchange rate regime.

[9] To maintain comparability of our results with those of the existing literature, we disregard the term involving real balances in the utility function.

consumption and decreasing in the expected level of labour supply.[10]

The analysis in the previous section showed that the relative size of the money supply variances determines which of the proposed equilibrium types may be supported. It is, therefore, convenient to split the analysis into three cases, which we consider in turn in the following subsections.

4.1 Case I: Home Money Supply Variance Lowest

In case the money supply is less variable in Home than in Foreign, we have from Proposition 1 that the unique (symmetric) equilibrium of the contract currency game is for Home firms to use producer currency pricing and for Foreign firms to use local currency pricing. The Home government compares Home welfare in this equilibrium to Home welfare in the fixed exchange rate equilibrium to determine which exchange rate regime is preferable. Equilibrium Home welfare under flexible exchange rates, PCP in Home and LCP in Foreign is given by

$$\omega\left(PCP, LCP\right) = \left(\frac{1}{1-\rho} - \frac{n\left(\lambda-1\right)}{\lambda}\right)\left(\frac{\lambda-1}{\lambda\eta}\right)^{\frac{1-\rho}{\rho}} \exp\left(\frac{\rho-1}{2\rho^2}\sigma_m^2\right) - \eta\left(1-n\right)$$
$$\times\left(\frac{\lambda-1}{\lambda\eta}\right)^{\frac{1}{\rho}} \exp\left(\frac{\rho-1}{2\rho^2}\left[n\left(\rho\left(1-n\right)+n\right)\sigma_m^2 + \left(1-n\right)\left(1+n\left(\rho-1\right)\right)\sigma_{m^*}^2\right]\right), \quad (39)$$

while Home welfare under a fixed exchange rate equals

$$\omega\left(FIX,\cdot\right) = \left(\frac{1}{1-\rho} - \frac{\lambda-1}{\lambda}\right)\left(\frac{\lambda-1}{\lambda\eta}\right)^{\frac{1-\rho}{\rho}} \exp\left(\frac{\rho-1}{2\rho^2}\sigma_{m^*}^2\right). \quad (40)$$

One can show that under the assumption $\sigma_m^2 < \sigma_{m^*}^2$,

$$\omega\left(PCP, LCP\right) > \omega\left(FIX,\cdot\right), \quad (41)$$

that is, the Home government prefers a flexible exchange rate regime.

How does the Home government's decision to let the exchange rate float affect Foreign welfare? To evaluate this, we use that Foreign welfare levels in the two scenarios are given by

$$\omega^*\left(PCP, LCP\right) = \left(\frac{1}{1-\rho} - \frac{\left(1-n\right)\left(\lambda-1\right)}{\lambda}\right)\left(\frac{\lambda-1}{\lambda\eta}\right)^{\frac{1-\rho}{\rho}}$$
$$\times\exp\left(\frac{n\left(\rho-1\right)\left(\rho\left(1-n\right)+n\right)}{2\rho^2}\sigma_m^2 + \frac{\left(1-n\right)\left(\rho-1\right)\left(1+n\left(\rho-1\right)\right)}{2\rho^2}\sigma_{m^*}^2\right)$$
$$-\eta n\left(\frac{\lambda-1}{\lambda\eta}\right)^{\frac{1}{\rho}} \exp\left(\frac{\rho-1}{2\rho^2}\sigma_m^2\right) \quad (42)$$

[10] One can easily show that welfare also depends negatively on the variance of the level of consumption.

and (40). In the Appendix, we show that the Foreign government would have preferred that the Home government fixed the exchange rate if risk aversion is high

$$\omega^* (PCP, LCP) < \omega^* (FIX, \cdot) \quad \text{if} \quad \rho \geq \rho^* (\lambda, n), \tag{43}$$

and there will also be disagreement if risk aversion is somewhat lower and the Foreign money supply variance is above a critical value

$$\omega^* (PCP, LCP) < \omega^* (FIX, \cdot) \quad \text{if} \quad \frac{2-n}{1-n} < \rho < \rho^* (\lambda, n) \quad \text{and} \quad \sigma_{m^*}^2 \geq \widehat{\sigma_{m^*}^2 (\rho)}. \tag{44}$$

If the Foreign money supply variance is below this critical value, then there exists a critical value of the Home money supply variance such that the Foreign government supports (opposes) the Home decision to let the exchange rate float for values of the Home variance less (greater) than the critical value

$$\begin{aligned} \omega^* (PCP, LCP) \quad &\gtreqless \quad \omega^* (FIX, \cdot) \quad \text{for} \quad \sigma_m^2 \lesseqgtr \widehat{\sigma_m^2 \left(\sigma_{m^*}^2\right)} \\ \text{if} \quad \frac{2-n}{1-n} \quad &< \quad \rho < \rho^* (\lambda, n) \quad \text{and} \quad \sigma_{m^*}^2 < \widehat{\sigma_{m^*}^2 (\rho)}. \end{aligned} \tag{45}$$

If risk aversion is low, we have

$$\omega^* (PCP, LCP) \gtreqless \omega^* (FIX, \cdot) \quad \text{for} \quad \sigma_m^2 \lesseqgtr \widehat{\sigma_m^2 \left(\sigma_{m^*}^2\right)} \quad \text{if} \quad \rho \leq \frac{2-n}{1-n}. \tag{46}$$

Thus there seems to be a potential policy conflict between the two governments if risk aversion is high: the Home government chooses to let the exchange rate float while the Foreign government prefers that Home fixes the exchange rate. But when the Foreign government also has the option of fixing the exchange rate, a conflict may not materialise. In fact, one can conclude from the analysis in the next section that if the Foreign government has the option of fixing the exchange rate, it will choose to do so (in this case of higher monetary variability in Foreign than in Home) and, furthermore, the Home government would approve of Foreign's move to a fixed exchange rate.

4.2 Case II: Home Money Supply Variance Highest

In the other asymmetric case, the Home money supply is more variable than the Foreign money supply, and we have from Proposition 1 that the unique (symmetric) equilibrium of the contract currency game is for Home firms to use local currency pricing and for Foreign firms to use producer currency pricing. Equilibrium Home welfare under flexible exchange rates, LCP in Home and PCP in Foreign is given by

$$\omega\left(LCP,PCP\right)=\left(\frac{1}{1-\rho}-\frac{n\left(\lambda-1\right)}{\lambda}\right)\left(\frac{\lambda-1}{\lambda\eta}\right)^{\frac{1-\rho}{\rho}}\exp\left(\frac{\rho-1}{2\rho^{2}}\left[n\left(\rho\left(1-n\right)+n\right)\sigma_{m}^{2}\right.\right.$$
$$\left.\left.+\left(1-n\right)\left(1+n\left(\rho-1\right)\right)\sigma_{m^{*}}^{2}\right]\right)-\eta\left(1-n\right)\left(\frac{\lambda-1}{\lambda\eta}\right)^{\frac{1}{\rho}}\exp\left(\frac{\rho-1}{2\rho^{2}}\sigma_{m^{*}}^{2}\right). \tag{47}$$

Comparing this to Home welfare under a fixed exchange rate regime (40), we can show that under the assumption $\sigma_m^2>\sigma_{m^*}^2$,

$$\omega\left(LCP,PCP\right)<\omega\left(FIX,\cdot\right), \tag{48}$$

implying that the Home government prefers to give up monetary independence to keep the exchange rate fixed.

Foreign welfare levels with a flexible and a fixed exchange rate are given by, respectively,

$$\omega^{*}\left(LCP,PCP\right)=\left(\frac{1}{1-\rho}-\frac{\left(1-n\right)\left(\lambda-1\right)}{\lambda}\right)\left(\frac{\lambda-1}{\lambda\eta}\right)^{\frac{1-\rho}{\rho}}\exp\left(\frac{\rho-1}{2\rho^{2}}\sigma_{m^{*}}^{2}\right)$$
$$-n\eta\left(\frac{\lambda-1}{\lambda\eta}\right)^{\frac{1}{\rho}}\exp\left(\frac{\rho-1}{2\rho^{2}}\left[n\left(\rho\left(1-n\right)+n\right)\sigma_{m}^{2}+\left(1-n\right)\left(1+n\left(\rho-1\right)\right)\sigma_{m^{*}}^{2}\right]\right). \tag{49}$$

and (40). Comparing these, we find that under the assumption $\sigma_m^2>\sigma_{m^*}^2$,

$$\omega^{*}\left(LCP,PCP\right)<\omega^{*}\left(FIX,\cdot\right), \tag{50}$$

which shows that Foreign welfare is higher under the fixed exchange rate regime than it is if the Home government chooses to leave the exchange rate floating. Consequently, the Foreign government will approve of a decision by the Home government to fix the exchange rate.

In contrast to the result derived in the previous case, there is no possibility of disagreement in this case where the government with the highest monetary variability chooses to fix the exchange rate. The difference in results shows that it is important to distinguish between whether the government which considers the optimal choice of exchange rate regime is the government with low monetary variability or the government with high monetary variability.

4.3 Case III: Equal Money Supply Variances

If the two countries have equally variable money supplies, we have from Proposition 1 that there exist multiple (symmetric) equilibria of the contract currency game. In fact, all the equilibrium types we have considered can be supported when money supply variances are equal. Home welfare is given by (39), (40), (47),

$$\omega\left(PCP,PCP\right)=-\frac{1+\rho\left(\lambda-1\right)}{\left(\rho-1\right)\lambda}\left(\frac{\lambda-1}{\lambda\eta}\right)^{\frac{1-\rho}{\rho}}\exp\left(\frac{\left(\rho-1\right)\left(1+2n\left(1-n\right)\left(\rho-1\right)\right)}{2\rho^{2}}\sigma_{m}^{2}\right) \quad (51)$$

and

$$\omega\left(LCP,LCP\right)=-\frac{1+\rho\left(\lambda-1\right)}{\left(\rho-1\right)\lambda}\left(\frac{\lambda-1}{\lambda\eta}\right)^{\frac{1-\rho}{\rho}}\exp\left(\frac{\rho-1}{2\rho^{2}}\sigma_{m}^{2}\right). \quad (52)$$

The presence of multiple equilibria under flexible exchange rates complicates the analysis of the optimal choice of exchange rate regime by the Home government. In our specification of the model, the firms decide on their contract currency in a non-cooperative game and the governments have no obvious way of ensuring that the best of the multiple equilibria will be the outcome.

Sidestepping the issue of choosing the exchange rate regime for a moment, it is useful to note that we can derive the following results on the relative welfare levels under flexible exchange rates (when $\sigma_m^2=\sigma_{m^*}^2$): Welfare in the equilibrium with local currency pricing in both countries is strictly greater than welfare in the other three equilibria. In the Appendix, we show that relative welfare levels of the asymmetric equilibria depend on the parameters in the following way

$$\omega\left(PCP,LCP\right)\lesseqqgtr\omega\left(LCP,PCP\right)\quad\text{as}\quad\rho\gtreqqless\frac{\left(1-2n\right)\left(\lambda-1\right)+\lambda}{\left(1-2n\right)\left(\lambda-1\right)}=\widehat{\rho}\quad\text{and}\quad n<1/2;$$

$$\omega\left(PCP,LCP\right)=\omega\left(LCP,PCP\right)\quad\text{for}\quad n=1/2;$$

$$\omega\left(PCP,LCP\right)>\omega\left(LCP,PCP\right)\quad\text{for}\quad n>1/2.$$

If the Home country is the largest, the (PCP, LCP) equilibrium yields the highest Home welfare; if the two countries are equal in size, the two asymmetric equilibria will yield identical Home welfare levels; and if the Home country is the smallest, the (PCP, LCP) equilibrium will yield the highest Home welfare if the risk aversion parameter is smaller than the critical value $\widehat{\rho}$, while the (LCP, PCP) equilibrium will be best for the Home country if $\rho>\widehat{\rho}$. Finally, the equilibrium in which both firms in the Home and the Foreign country use producer currency pricing will be associated with lower Home welfare than any of the other equilibria.

From these results, we see that if the Home government is able to affect which of the equilibria ends up being the outcome under flexible exchange rates, it will wish to ensure that firms in both countries use local currency pricing. That is, in order to obtain the highest possible level of Home welfare, the Home government must not only convince Home firms that they should use local currency pricing, it must also persuade Foreign firms to do the same.

In our model, however, the Home government has no way of influencing the players so that the equilibrium which maximises Home welfare is reached. In fact, the only action available to the government is that of fixing the exchange rate. Comparing Home welfare under fixed exchange rates and flexible exchange rates, we find that

$$\omega\left(FIX, \cdot\right) = \omega\left(LCP, LCP\right),$$

that is, the (unique) fixed exchange rate equilibrium yields the same Home welfare level as the equilibrium which yields the highest Home welfare in the set of equilibria under flexible exchange rates. So, in spite of the problems with multiple equilibria and limited influence on the outcome, the Home government does indeed have the ability to ensure that Home welfare is maximised: by choosing to fix the exchange rate, it can achieve—with probability one—the optimal outcome for the Home representative consumer.

The Foreign government will support the decision of the Home government to fix the exchange rate since this ensures the highest possible level of Foreign welfare, as well. As a consequence, there is no need for international policy coordination when money supply variances are equal—the optimal regime will result from the Home (or Foreign) government's unilateral move to a fixed exchange rate.

5 Concluding Remarks

In this paper, we have analysed governments' optimal choice of exchange rate regime and firms' optimal choice of contract currency in a stochastic general equilibrium model with nominal rigidities and monetary uncertainty. In contrast to earlier studies in this field, we have given explicit consideration to firms' choice of currency for quoting export prices. We have shown that the contract currency assumptions of the models considered by Obstfeld and Rogoff (2001) and Devereux and Engel (1998) do in fact correspond to equilibria of our model, but that these equilibria only exist in the special case of identical money supply variances. In the absence of equal variances, we have shown that a world-wide asymmetric (but country-wide symmetric) equilibrium exists in which firms in the country with the most unstable money supply use local currency pricing while firms in the other country use producer currency pricing. These results document the importance of incorporating a consideration of firms' choice of contract currency into the analysis to ensure that welfare results do not rest on questionable assumptions about how firms fix export prices.

Turning next to our results on the optimal choice of exchange rate regime when contract currencies are endogenous, we have shown that a government that tries to maximise the welfare of its consumers will opt for a fixed exchange rate regime if the monetary uncertainty in its own country is greater than or equal to the monetary uncertainty in the other country. Conversely, a government faced with more stable monetary conditions domestically than abroad will choose a flexible exchange rate regime. We also considered the possibility of a policy conflict between the two governments and found that a government will always support a move to a fixed exchange rate regime by the other government. Hence, our model predicts that there should be no conflicts between governments.

The model specification we have chosen has the advantage that we can derive closed-form solutions and it allows a clear demonstration of our main point: research in this field needs to take firms' choice of contract currency explicitly into account and leave the current practice of making ad hoc assumptions. Obvious extensions to be considered in future research include a more general utility function, other types of shocks, non-tradable goods and asset market imperfections.

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Appendix

Proof of Proposition 1 (a)

To see whether (and under what conditions) an equilibrium with producer currency pricing in both countries exists, we now consider the payoff to a (Home) firm that deviates from the proposed equilibrium strategy profile by choosing to use local currency pricing. The deviating Home firm j sets its export price in Foreign currency $P^*_{ht}(j)$ to maximise profits and we derive the following expressions for optimal prices

$$P^*_{ht}(j) = \frac{\lambda}{\lambda-1}\frac{E_{t-1}\left(d_{t-1}W_t S_t^{-\lambda} C^*_{ht}\right)}{E_{t-1}\left(d_{t-1} S_t^{1-\lambda} C^*_{ht}\right)};$$

$$P^*_{ht}(j) = \frac{\lambda}{\lambda-1}\frac{E_{t-1}\left(W_t S_t^{-\lambda} C_t^{*1-\rho}\right)}{E_{t-1}\left(S_t^{1-\lambda} C_t^{*1-\rho}\right)}.$$

The increase in expected profit associated with deviation is given by (since firm j deviates and firm i does not)

$$\begin{aligned}
\Delta_h^{PCP,PCP} &= E_{t-1}\left(d_{t-1}\left[P_{ht}(j)X_{ht}(j) + S_t P^*_{ht}(j) X^*_{ht}(j) - W_t\left(X_{ht}(j) + X^*_{ht}(j)\right)\right]\right) \\
&\quad -E_{t-1}\left(d_{t-1}\left[P_{ht}(i)X_{ht}(i) + S_t P^*_{ht}(i) X^*_{ht}(i) - W_t\left(X_{ht}(i) + X^*_{ht}(i)\right)\right]\right) \\
&= E_{t-1}\left(d_{t-1}\left[S_t P^*_{ht}(j) X^*_{ht}(j) - S_t P^*_{ht}(i) X^*_{ht}(i) - W_t\left(X^*_{ht}(j) - X^*_{ht}(i)\right)\right]\right),
\end{aligned}$$

since $P_{ht}(i) = P_{ht}(j)$ and $X_{ht}(i) = X_{ht}(j)$. Substitution of optimal prices, demand levels and equilibrium aggregate variables leads to

$$\begin{aligned}
&\Delta_h^{PCP,PCP} = E_{t-1}\left(d_{t-1}\left[S_t P^*_{ht}(j) X^*_{ht}(j) - S_t P^*_{ht}(i) X^*_{ht}(i) - W_t\left(X^*_{ht}(j) - X^*_{ht}(i)\right)\right]\right) \\
= \;& \beta(1-n)\frac{P_{t-1}\left(S_t P^*_{ht}\right)^{\lambda-1}}{C_{t-1}^{-\rho}} E_{t-1}\left(C_t^{*1-\rho}\left[S_t^{1-\lambda}\left(\frac{\lambda\eta}{\lambda-1}\frac{E_{t-1}\left(S_t^{1-\lambda}P_t^* C_t^*\right)}{E_{t-1}\left(S_t^{1-\lambda}C_t^{*1-\rho}\right)}\right)^{1-\lambda}\right.\right. \\
&\left.\left. - \left(S_t P^*_{ht}\right)^{1-\lambda} - \eta S_t P_t^* C_t^{*\rho}\left(S_t^{-\lambda}\left(\frac{\lambda\eta}{\lambda-1}\frac{E_{t-1}\left(S_t^{1-\lambda}P_t^* C_t^*\right)}{E_{t-1}\left(S_t^{1-\lambda}C_t^{*1-\rho}\right)}\right)^{-\lambda} - \left(S_t P^*_{ht}\right)^{-\lambda}\right)\right]\right).
\end{aligned}$$

This is positive if

$$E_{t-1}\left(C_t^{*1-\rho}\left[S_t^{1-\lambda}\left(\frac{\lambda\eta}{\lambda-1}\frac{E_{t-1}\left(S_t^{1-\lambda}P_t^*C_t^*\right)}{E_{t-1}\left(S_t^{1-\lambda}C_t^{*1-\rho}\right)}\right)^{1-\lambda}-\left(S_tP_{ht}^*\right)^{1-\lambda}\right.\right.$$

$$\left.\left.-\eta S_tP_t^*C_t^{*\rho}\left(S_t^{-\lambda}\left(\frac{\lambda\eta}{\lambda-1}\frac{E_{t-1}\left(S_t^{1-\lambda}P_t^*C_t^*\right)}{E_{t-1}\left(S_t^{1-\lambda}C_t^{*1-\rho}\right)}\right)^{-\lambda}-\left(S_tP_{ht}^*\right)^{-\lambda}\right)\right]\right)>0,$$

which simplifies to

$$\left(E_{t-1}\left(S_t^{1-\lambda}P_t^*C_t^*\right)\right)^{1-\lambda}\left(E_{t-1}\left(S_t^{1-\lambda}C_t^{*1-\rho}\right)\right)^{\lambda}>\left(E_{t-1}\left(S_tP_t^*C_t^*\right)\right)^{1-\lambda}\left(E_{t-1}\left(C_t^{*1-\rho}\right)\right)^{\lambda}.$$

We use (17), (21) and equilibrium foreign consumption and price levels to compute

$$\sigma_s^2=\sigma_m^2+\sigma_{m^*}^2;\quad \sigma_{p^*}^2=n^2\sigma_m^2+n^2\sigma_{m^*}^2;$$

$$\sigma_{sp^*}=-n\sigma_m^2-n\sigma_{m^*}^2;\quad \sigma_{sc^*}=\frac{n}{\rho}\sigma_m^2-\frac{1-n}{\rho}\sigma_{m^*}^2;\quad \sigma_{p^*c^*}=-\frac{n^2}{\rho}\sigma_m^2+\frac{n(1-n)}{\rho}\sigma_{m^*}^2.$$

The conditional expectation terms in the preceeding inequality are now easily evaluated

$$\begin{aligned}E_{t-1}\left(S_t^{1-\lambda}P_t^*C_t^*\right) &= \exp\Big((1-\lambda)E_{t-1}s_t+E_{t-1}p_t^*+E_{t-1}c_t^*\\ &\quad+\frac{1}{2}\left((1-\lambda)^2\sigma_s^2+\sigma_{p^*}^2+\sigma_{c^*}^2+2(1-\lambda)\sigma_{sp^*}+2(1-\lambda)\sigma_{sc^*}+2\sigma_{p^*c^*}\right)\Big)\\ &= \left(\frac{\mu^*(1-\mu\beta)}{\mu(1-\mu^*\beta)}\right)^{1-\lambda}\left(\frac{1-\mu^*\beta}{\mu^*\chi}\right)\left(\frac{\lambda\eta}{\lambda-1}\right)^{\frac{\rho-1}{\rho}}\exp\big((1-\lambda)m_{t-1}+\lambda m_{t-1}^*\\ &\quad+\left(\rho\lambda(\rho(\lambda-3)+2n(\rho-1))+n(\rho-1)(n(\rho-1)-2\rho)+2\rho^2\right)\frac{\sigma_m^2}{2\rho^2}\\ &\quad+\left(\rho\lambda(\rho(\lambda-1)+2(1+n(\rho-1)))+(\rho-1)^2(1-n)^2\right)\frac{\sigma_{m^*}^2}{2\rho^2};\end{aligned}$$

$$\begin{aligned}&E_{t-1}\left(S_t^{1-\lambda}C_t^{*1-\rho}\right)=\left(\frac{\mu^*(1-\mu\beta)}{\mu(1-\mu^*\beta)}\right)^{1-\lambda}\left(\frac{\lambda\eta}{\lambda-1}\right)^{\frac{\rho-1}{\rho}}\exp\Big((1-\lambda)\left(m_{t-1}-m_{t-1}^*\right)\\ &+\left(\rho\lambda(\rho(\lambda-3)+2n(\rho-1))-n(\rho-1)(\rho(1+n)-n)+2\rho^2\right)\frac{\sigma_m^2}{2\rho^2}\\ &+\left(\rho\lambda(\rho(\lambda-3)+2(1+n(\rho-1)))-(\rho(1+n)+1-n)(\rho n+1-n)+2\rho^2\right)\frac{\sigma_{m^*}^2}{2\rho^2};\end{aligned}$$

$$\begin{aligned}E_{t-1}\left(S_tP_t^*C_t^*\right) &= \left(\frac{1-\mu\beta}{\mu\chi}\right)\left(\frac{\lambda\eta}{\lambda-1}\right)^{\frac{\rho-1}{\rho}}\exp\Big(m_{t-1}+\left(2\rho^2-n(\rho-1)(\rho(2-n)+n)\right)\frac{\sigma_m^2}{2\rho^2}\\ &\quad+(\rho-1)^2(1-n)^2\frac{\sigma_{m^*}^2}{2\rho^2}\Big);\end{aligned}$$

$$E_{t-1}\left(C_t^{*1-\rho}\right) = \left(\frac{\lambda\eta}{\lambda-1}\right)^{\frac{\rho-1}{\rho}} \exp\left((\rho-1)\,n\,(\rho\,(1-n)+n)\,\frac{\sigma_m^2}{2\rho^2}\right.$$
$$\left. +\,(1-n)\,(\rho-1)\,(n\,(\rho-1)+1)\,\frac{\sigma_{m^*}^2}{2\rho^2}\right).$$

Substitution into the inequality then yields (after simplifying)

$$\sigma_m^2 > \sigma_{m^*}^2.$$

We conclude that a Home firm has an incentive to deviate from an equilibrium in which all firms fix their export prices in their own currency if (and only if) the variance of the domestic monetary shock is strictly greater than that of the Foreign shock. By symmetry, a Foreign firm will have an incentive to deviate if (and only if) the opposite inequality holds. This implies that an equilibrium in which firms in both countries fix their export prices in terms of their own currency exists if (and only if) the two monetary shocks have equal variance.

Proof of Proposition 1 (b)

An equilibrium in which all firms use local currency pricing exists if no firm has an incentive to deviate and set export prices in their own currency. To check this, we consider the payoff to a Home firm that fixes its export price in the Home currency. This deviating firm maximises profits with respect to $P_{ht}(j)$ and $S_t P_{ht}^*(j)$. The first order conditions are given by (23) (with index j instead of i) and

$$E_{t-1}\left(d_{t-1}\left[X_{ht}^*(j) + S_t P_{ht}^*(j)\,\frac{\partial X_{ht}^*(j)}{\partial\left(S_t P_{ht}^*(j)\right)} - W_t \frac{\partial X_{ht}^*(j)}{\partial\left(S_t P_{ht}^*(j)\right)}\right]\right) = 0.$$

The former condition implies that the optimal price $P_{ht}(j)$ is given by (28). The latter first order condition implies that

$$S_t P_{ht}^*(j) = \frac{\lambda}{\lambda-1}\,\frac{E_{t-1}\left(W_t S_t^{\lambda-1} C_t^{*1-\rho}\right)}{E_{t-1}\left(S_t^{\lambda-1} C_t^{*1-\rho}\right)};$$

where we have used that the deviating firm's decision variable is $S_t P_{ht}^*(j)$ and that it takes the Foreign currency value of the Foreign import price index P_{ht}^* as given since all other Home firms set their export prices in Foreign currency.

The payoff to deviation is now given by

$$\Delta_h^{LCP,LCP} = E_{t-1}\left(d_{t-1}\left[S_t P_{ht}^*(j) X_{ht}^*(j) - S_t P_{ht}^*(i) X_{ht}^*(i) - W_t\left(X_{ht}^*(j) - X_{ht}^*(i)\right)\right]\right)$$

$$= \beta(1-n)\frac{P_{t-1}P_{ht}^{*\lambda-1}}{C_{t-1}^{-\rho}}E_{t-1}\left(S_t^{\lambda-1}C_t^{*1-\rho}\left(\frac{\lambda}{\lambda-1}\frac{E_{t-1}\left(W_t S_t^{\lambda-1}C_t^{*1-\rho}\right)}{E_{t-1}\left(S_t^{\lambda-1}C_t^{*1-\rho}\right)}\right)^{1-\lambda}\right.$$

$$\left.-C_t^{*1-\rho}P_{ht}^{*1-\lambda} - W_t\left(S_t^{\lambda-1}C_t^{*1-\rho}\left(\frac{\lambda}{\lambda-1}\frac{E_{t-1}\left(W_t S_t^{\lambda-1}C_t^{*1-\rho}\right)}{E_{t-1}\left(S_t^{\lambda-1}C_t^{*1-\rho}\right)}\right)^{-\lambda} - S_t^{-1}C_t^{*1-\rho}P_{ht}^{*-\lambda}\right)\right).$$

This is positive if

$$\left(\frac{\lambda}{\lambda-1}\frac{E_{t-1}\left(W_t S_t^{\lambda-1}C_t^{*1-\rho}\right)}{E_{t-1}\left(S_t^{\lambda-1}C_t^{*1-\rho}\right)}\right)^{1-\lambda}E_{t-1}\left(S_t^{\lambda-1}C_t^{*1-\rho}\right)$$

$$-\left(\frac{\lambda}{\lambda-1}\frac{E_{t-1}\left(W_t S_t^{\lambda-1}C_t^{*1-\rho}\right)}{E_{t-1}\left(S_t^{\lambda-1}C_t^{*1-\rho}\right)}\right)^{-\lambda}E_{t-1}\left(W_t S_t^{\lambda-1}C_t^{*1-\rho}\right)$$

$$> P_{ht}^{*1-\lambda}E_{t-1}C_t^{*1-\rho} - P_{ht}^{*-\lambda}E_{t-1}\left(W_t S_t^{-1}C_t^{*1-\rho}\right);$$

$$\left(E_{t-1}\left(S_t^{\lambda}C_t^*\right)\right)^{1-\lambda}\left(E_{t-1}\left(S_t^{\lambda-1}C_t^{*1-\rho}\right)\right)^{\lambda} > \left(E_{t-1}\left(C_t^*\right)\right)^{1-\lambda}\left(E_{t-1}\left(C_t^{*1-\rho}\right)\right)^{\lambda}.$$

Computing the conditional expectations and substituting these into the inequality, we get after simplifying

$$\sigma_m^2 < \sigma_{m^*}^2.$$

We conclude that a Home firm has an incentive to deviate from its equilibrium strategy if (and only if) the Home monetary variability is strictly lower than the Foreign monetary variability. By symmetry, a Foreign firm will deviate if (and only if) the Foreign monetary variability is the lowest. It follows that an equilibrium with worldwide local currency pricing exists if and only if there is no difference in Home and Foreign monetary variability $\sigma_m^2 = \sigma_{m^*}^2$.

Proof of Proposition 1 (c)

Consider first the profitability of deviating from the proposed equilibrium (LCP in Home, PCP in Foreign) for a Home firm—that is, a Home firm's change in profits when it fixes export prices in Home currency instead of in Foreign currency. The deviating firm j maximises profits with respect to prices in Home currency $P_{ht}(j)$ and $S_tP^*_{ht}(j)$. The first order conditions may be rewritten to give optimal prices (28) and

$$S_tP^*_{ht}(j) = \frac{\lambda}{\lambda-1}\frac{E_{t-1}\left(W_tS_t^{\lambda-1}C_t^{*1-\rho}\right)}{E_{t-1}\left(S_t^{\lambda-1}C_t^{*1-\rho}\right)}.$$

The payoff to deviation is

$$\begin{aligned}
\Delta_h^{LCP,PCP} &= E_{t-1}\left(d_{t-1}\left[S_tP^*_{ht}(j)\,X^*_{ht}(j) - S_tP^*_{ht}(i)\,X^*_{ht}(i) - W_t\left(X^*_{ht}(j) - X^*_{ht}(i)\right)\right]\right) \\
&= \beta(1-n)\frac{P_{t-1}P_{ht}^{*\lambda-1}}{C_{t-1}^{-\rho}}E_{t-1}\left(C_t^{*1-\rho}\left[S_t^{\lambda-1}P_t^{*1-\lambda}\left(\frac{\lambda\eta}{\lambda-1}\frac{E_{t-1}\left(S_t^{\lambda}C_t^*\right)}{E_{t-1}\left(S_t^{\lambda-1}C_t^{*1-\rho}\right)}\right)^{1-\lambda}\right.\right. \\
&\quad \left.\left. -P_{ht}^{*1-\lambda} - \eta P_tC_t^{\rho}\left(S_t^{\lambda-1}P_t^{*-\lambda}\left(\frac{\lambda\eta}{\lambda-1}\frac{E_{t-1}\left(S_t^{\lambda}C_t^*\right)}{E_{t-1}\left(S_t^{\lambda-1}C_t^{*1-\rho}\right)}\right)^{-\lambda} - S_t^{-1}P_{ht}^{*-\lambda}\right)\right]\right),
\end{aligned}$$

which is positive if

$$\begin{aligned}
&\left(\frac{\lambda\eta}{\lambda-1}\frac{E_{t-1}\left(S_t^{\lambda}C_t^*\right)}{E_{t-1}\left(S_t^{\lambda-1}C_t^{*1-\rho}\right)}\right)^{1-\lambda} E_{t-1}S_t^{\lambda-1}C_t^{*1-\rho} \\
&-\left(\frac{\lambda\eta}{\lambda-1}\frac{E_{t-1}\left(S_t^{\lambda}C_t^*\right)}{E_{t-1}\left(S_t^{\lambda-1}C_t^{*1-\rho}\right)}\right)^{-\lambda} \eta E_{t-1}\left(S_t^{\lambda}C_t^*\right) \\
> \; & E_{t-1}\left(C_t^{*1-\rho}\right) - \eta E_{t-1}\left(C_t^*\right);
\end{aligned}$$

We compute the expected values, substitute these into the inequality and simplify to find

$$\sigma^2_{m^*} > \sigma^2_m, \tag{53}$$

that is, Home firms have an incentive to deviate from the equilibrium in which Home firms use local currency pricing and Foreign firms set prices in their own currency if (and only if) the variance of the Foreign monetary shock exceeds that of the domestic monetary shock.

Consider next the profitability of deviating from the proposed equilibrium for a Foreign firm—that is, a Foreign firm's change in profits when it fixes export prices in Home currency instead of in Foreign. The deviating firm j maximises profits with respect to prices $P_{ft}(j)$ and $P^*_{ft}(j)$. The first order conditions yield optimal prices

$$P_{ft}(j) = \frac{\lambda}{\lambda-1}\frac{E_{t-1}\left(W_t^*S_t^{\lambda}C_t^{1-\rho}\right)}{E_{t-1}\left(S_t^{\lambda-1}C_t^{1-\rho}\right)};$$

$$P_{ft}^{*}(j)=\frac{\lambda}{\lambda-1}\frac{E_{t-1}\left(d_{t-1}^{*}W_t^{*}C_{ft}^{*}\right)}{E_{t-1}\left(d_{t-1}^{*}C_{ft}^{*}\right)}.$$

The payoff to deviation reads

$$\begin{aligned}
\Delta_f^{LCP,PCP} &= E_{t-1}\left(d_{t-1}^{*}\left[\frac{P_{ft}(j)}{S_t}X_{ft}(j)-\frac{P_{ft}(i)}{S_t}X_{ft}(i)-W_t^{*}\left(X_{ft}(j)-X_{ft}(i)\right)\right]\right)\\
&= \beta n\frac{P_{t-1}^{*}\left(P_{ft}/S_t\right)^{\lambda-1}}{C_{t-1}^{*-\rho}}E_{t-1}\left(C_t^{1-\rho}\left[S_t^{\lambda-1}\left(\frac{\lambda\eta}{\lambda-1}\frac{P_t^{*}E_{t-1}\left(S_t^{\lambda}C_t^{*\rho}C_t^{1-\rho}\right)}{E_{t-1}\left(S_t^{\lambda-1}C_t^{1-\rho}\right)}\right)^{1-\lambda}\right.\right.\\
&\left.\left.-\left(P_{ft}/S_t\right)^{1-\lambda}-\eta P_t^{*}C_t^{*\rho}\left(S_t^{\lambda}\left(\frac{\lambda\eta}{\lambda-1}\frac{P_t^{*}E_{t-1}\left(S_t^{\lambda}C_t^{*\rho}C_t^{1-\rho}\right)}{E_{t-1}\left(S_t^{\lambda-1}C_t^{1-\rho}\right)}\right)^{-\lambda}-\left(P_{ft}/S_t\right)^{-\lambda}\right)\right]\right),
\end{aligned}$$

which is positive if

$$\begin{aligned}
& P_t^{*1-\lambda}\left(\frac{\lambda\eta}{\lambda-1}\right)^{1-\lambda}\frac{1}{\lambda}\left(E_{t-1}\left(S_t^{\lambda}C_t^{*\rho}C_t^{1-\rho}\right)\right)^{1-\lambda}\left(E_{t-1}\left(S_t^{\lambda-1}C_t^{1-\rho}\right)\right)^{\lambda}\\
> \; & \left(P_{ft}/S_t\right)^{1-\lambda}E_{t-1}\left(C_t^{1-\rho}\right)-\eta\left(P_{ft}/S_t\right)^{-\lambda}P_t^{*}E_{t-1}\left(C_t^{*\rho}C_t^{1-\rho}\right).
\end{aligned}$$

We compute the expected values and substituting these into the inequality, we ultimately arrive at

$$\sigma_{m^*}^2>\sigma_m^2, \tag{54}$$

that is, Foreign firms have an incentive to deviate from the equilibrium in which Home firms use local currency pricing and Foreign firms set prices in their own currency if (and only if) the variance of the Foreign monetary shock exceeds that of the Home monetary shock.

LCP in Home and PCP in Foreign is an equilibrium of the price-setting game if no firm has an incentive to deviate. From (53) and (54), we conclude that no firm has an incentive to deviate if (and only if) the Home monetary shock has weakly higher variance than the Foreign monetary shock. This completes the proof of the first part of Proposition 1 (c).

To prove the second part, note that the only exceptions from symmetry in the model are: 1) country size, 2) money supply drift and 3) money supply shock variance. The results of the last section showed that the equilibrium in which there is LCP in the economy with relative size n, drift μ and variance σ_m^2 and PCP in the economy with parameters $\left(1-n,\mu^*,\sigma_{m^*}^2\right)$ can be supported when $\sigma_m^2\leq\sigma_{m^*}^2$. Given these observations, it is clear that the analysis of the existence of an equilibrium in which there is PCP in Home and LCP in Foreign can be carried out as a reinterpretation of the results of the last section. In the present case, there is PCP in the economy with parameters $\left(n, \mu, \sigma_m^2\right)$ and LCP in the economy with parameters $\left(1-n, \mu^*, \sigma_{m^*}^2\right)$. It follows immediately that this equilibrium can be supported when $\sigma_{m^*}^2 \leq \sigma_m^2$.

Proof of Proposition 1

This Proposition follows immediately from Propositions 1 (a) through (c).

Welfare Comparisons

Case I: Home Money Supply Variance Lowest

The Foreign government agrees with the Home government if

$$\omega^*\left(PCP, LCP\right) > \omega^*\left(FIX, \cdot\right);$$

$$\begin{aligned}
&\left[\frac{1+\rho\left(\lambda-1\right)}{\lambda+\left(\rho-1\right)\left(\lambda-1\right)\left(1-n\right)}-1\right]\exp\left(\frac{\rho-1}{2\rho^2}\sigma_m^2\right)\\
<\; &\exp\left(\frac{\rho-1}{2\rho^2}\sigma_{m^*}^2\right)\left[\frac{1+\rho\left(\lambda-1\right)}{\lambda+\left(\rho-1\right)\left(\lambda-1\right)\left(1-n\right)}-\exp\left(\frac{n\left(\rho-1\right)\left(\rho\left(1-n\right)+n\right)}{2\rho^2}\sigma_m^2\right.\right.\\
&\left.\left.+\frac{n\left(\rho-1\right)\left(n+\left(1-n\right)\rho-2\right)}{2\rho^2}\sigma_{m^*}^2\right)\right]. \qquad (55)
\end{aligned}$$

Suppose $\sigma_m^2 = \sigma_{m^*}^2$. Then the inequality reduces to

$$1 > \exp\left(\frac{n\left(1-n\right)\left(\rho-1\right)^2}{\rho^2}\right),$$

which is clearly false as the argument of the exponential function is strictly positive. The left hand side of the inequality (55) is strictly increasing in Home money supply variance since

$$\frac{1+\rho\left(\lambda-1\right)}{\lambda+\left(\rho-1\right)\left(\lambda-1\right)\left(1-n\right)}-1=\frac{n\left(\lambda-1\right)\left(\rho-1\right)}{\lambda+\left(\lambda-1\right)\left(\rho-1\right)\left(1-n\right)}>0,$$

while the right hand side is strictly decreasing in Home money supply variance

$$\begin{aligned}
&\frac{\partial\left\{\exp\left(\frac{\rho-1}{2\rho^2}\sigma_{m^*}^2\right)\left[\frac{1+\rho(\lambda-1)}{\lambda+(\rho-1)(\lambda-1)(1-n)}-\exp\left(\frac{n(\rho-1)(\rho(1-n)+n)}{2\rho^2}\sigma_m^2+\frac{n(\rho-1)(n+(1-n)\rho-2)}{2\rho^2}\sigma_{m^*}^2\right)\right]\right\}}{\partial\sigma_m^2}\\
= \;& -\frac{n\left(\rho-1\right)\left(\rho\left(1-n\right)+n\right)}{2\rho^2}\exp\left(\frac{\rho-1}{2\rho^2}\sigma_{m^*}^2\right)\\
&\exp\left(\frac{n\left(\rho-1\right)\left(\rho\left(1-n\right)+n\right)}{2\rho^2}\sigma_m^2+\frac{n\left(\rho-1\right)\left(n+\left(1-n\right)\rho-2\right)}{2\rho^2}\sigma_{m^*}^2\right)<0.
\end{aligned}$$

It follows that the inequality (55) may be true when the Home money supply variance is less than the Foreign one. To investigate whether this is the case, we compute the limits of the two sides of the inequality as the Home money supply variance goes to zero

$$\begin{aligned}&\lim_{\sigma_m^2\to 0}\left\{\left[\frac{1+\rho(\lambda-1)}{\lambda+(\rho-1)(\lambda-1)(1-n)}-1\right]\exp\left(\frac{\rho-1}{2\rho^2}\sigma_m^2\right)\right\}\\ =&\ \frac{n(\lambda-1)(\rho-1)}{\lambda+(\lambda-1)(\rho-1)(1-n)};\end{aligned}$$

$$\begin{aligned}&\lim_{\sigma_m^2\to 0}\left\{\exp\left(\frac{\rho-1}{2\rho^2}\sigma_{m^*}^2\right)\left[\frac{1+\rho(\lambda-1)}{\lambda+(\rho-1)(\lambda-1)(1-n)}\right.\right.\\ &\left.\left.-\exp\left(\frac{n(\rho-1)(\rho(1-n)+n)}{2\rho^2}\sigma_m^2+\frac{n(\rho-1)(n+(1-n)\rho-2)}{2\rho^2}\sigma_{m^*}^2\right)\right]\right\}\\ =&\ \exp\left(\frac{\rho-1}{2\rho^2}\sigma_{m^*}^2\right)\left[\frac{1+\rho(\lambda-1)}{\lambda+(\rho-1)(\lambda-1)(1-n)}-\exp\left(\frac{n(\rho-1)(n+(1-n)\rho-2)}{2\rho^2}\sigma_{m^*}^2\right)\right].\end{aligned}$$

The inequality (55) is true for some value of the Home money supply variance between zero and the Foreign money supply variance if and only if the limit of the left hand side of (55) is less than the limit of the right hand side. This is the case if

$$\begin{aligned}L=\frac{n(\lambda-1)(\rho-1)}{\lambda+(\lambda-1)(\rho-1)(1-n)}&<\exp\left(\frac{\rho-1}{2\rho^2}\sigma_{m^*}^2\right)\\ \left[\frac{1+\rho(\lambda-1)}{\lambda+(\rho-1)(\lambda-1)(1-n)}-\exp\left(\frac{n(\rho-1)(n+(1-n)\rho-2)}{2\rho^2}\sigma_{m^*}^2\right)\right]&=R\left(\sigma_{m^*}^2\right)\end{aligned}\tag{56}$$

where we have defined the constant L and the function R. As the Foreign money supply variance goes to zero, we have

$$\lim_{\sigma_{m^*}^2\to 0}R\left(\sigma_{m^*}^2\right)=\frac{1+\rho(\lambda-1)}{\lambda+(\rho-1)(\lambda-1)(1-n)}-1=L.$$

The derivative of R is

$$\begin{aligned}R'\left(\sigma_{m^*}^2\right)&=\left(\frac{\rho-1}{2\rho^2}\right)\left(\frac{1+\rho(\lambda-1)}{\lambda+(\rho-1)(\lambda-1)(1-n)}\right)\exp\left(\frac{\rho-1}{2\rho^2}\sigma_{m^*}^2\right)\\ &-\frac{(\rho-1)(1-n)(n\rho+1-n)}{2\rho^2}\exp\left(\frac{(\rho-1)(1-n)(n\rho+1-n)}{2\rho^2}\sigma_{m^*}^2\right),\end{aligned}$$

which is positive if

$$\begin{aligned}&\left(\frac{\rho-1}{2\rho^2}\right)\left(\frac{1+\rho(\lambda-1)}{\lambda+(\rho-1)(\lambda-1)(1-n)}\right)\exp\left(\frac{\rho-1}{2\rho^2}\sigma_{m^*}^2\right)\\ >&\ \frac{(\rho-1)(1-n)(n\rho+1-n)}{2\rho^2}\exp\left(\frac{(\rho-1)(1-n)(n\rho+1-n)}{2\rho^2}\sigma_{m^*}^2\right);\end{aligned}$$

$$\frac{1+\rho(\lambda-1)}{(\lambda+(\rho-1)(\lambda-1)(1-n))(1-n)(n\rho+1-n)} > \exp\left(\frac{n(\rho-1)(n+(1-n)\rho-2)}{2\rho^2}\sigma^2_{m^*}\right). \tag{57}$$

It can be shown that the left hand side depends on ρ in the following way

$$\frac{1+\rho(\lambda-1)}{(\lambda+(\rho-1)(\lambda-1)(1-n))(1-n)(n\rho+1-n)} \gtreqless 1 \quad \text{for}$$

$$\rho \lesseqgtr \rho^* = \frac{\lambda\left(3-4n+2n^2\right)-4+5n-2n^2+\sqrt{\lambda^2\left(5-8n+4n^2\right)+\lambda\left(-8+10n-4n^2\right)+(n-2)^2}}{2(1-n)^2(\lambda-1)}.$$

The right hand side of (57) depends on $\sigma^2_{m^*}$ in the following way

$$\frac{\partial\left\{\exp\left(\frac{n(\rho-1)(n+(1-n)\rho-2)}{2\rho^2}\sigma^2_{m^*}\right)\right\}}{\partial\sigma^2_{m^*}} \lesseqgtr 0 \quad \text{for} \quad \rho \lesseqgtr \frac{2-n}{1-n} < \rho^*.$$

From this, we can deduce that the inequality (57) is true if $\rho \leq (2-n)/(1-n)$ and that it is false if $\rho \geq \rho^*$. For intermediate values of ρ, the inequality is true (false) when the Foreign money supply variance is less (weakly greater) than a finite critical value $\widetilde{\sigma^2_{m^*}}(\rho)$.

We can use this to determine whether the inequality (56) is fulfilled or not. If $\rho \leq (2-n)/(1-n)$,

$$\begin{aligned} R(s)-L &= \lim_{\sigma^2_{m^*}\to 0} R\left(\sigma^2_{m^*}\right)+\int_0^s R'(x)\,dx - L \\ &= \int_0^s R'(x)\,dx > 0, \quad \forall s>0, \end{aligned}$$

where the inequality follows from the fact that $R'(x)$ is strictly positive for all positive x when $\rho \leq (2-n)/(1-n)$. This implies that the inequality (56) is fulfilled for low values of ρ.

If $\rho \geq \rho^*$,

$$\begin{aligned} R(s)-L &= \lim_{\sigma^2_{m^*}\to 0} R\left(\sigma^2_{m^*}\right)+\int_0^s R'(x)\,dx - L \\ &= \int_0^s R'(x)\,dx < 0, \quad \forall s>0, \end{aligned}$$

because $R'(x) < 0$ for all $x > 0$ when $\rho \geq \rho^*$. For large values of ρ, the inequality (56) is, therefore, false.

If $(2-n)/(1-n) < \rho < \rho^*$,

$$R(s) - L = \lim_{\sigma^2_{m^*}\to 0} R\left(\sigma^2_{m^*}\right) + \int_0^s R'(x)\,dx - L$$

$$= \begin{cases} \int_0^s R'(x)\,dx > 0 & \text{if } s \le \widetilde{\sigma^2_{m^*}(\rho)}; \\ \underbrace{\int_0^{\widetilde{\sigma^2_{m^*}(\rho)}} R'(x)\,dx}_{+} + \underbrace{\int_{\widetilde{\sigma^2_{m^*}(\rho)}}^s R'(x)\,dx}_{\div} & \text{if } s > \widetilde{\sigma^2_{m^*}(\rho)}. \end{cases}$$

The sign of the sum of integrals depends on the value of s: for sufficiently large values of s, the sum will be negative as the first integral is a finite positive number while the second integral becomes an arbitrarily large negative number as s goes to infinity. Thus the inequality (56) will be true (false) for values of the Foreign money supply variance less (weakly greater) than a critical level $\widehat{\sigma^2_{m^*}(\rho)}$.

Finally, we note that the existence of a critical level of Home money supply variance below which Foreign agrees with Home's decision to keep a flexible exchange exchange rate is equivalent to the inequality (56) being true. Thus we conclude that there exists a critical value $\widehat{\sigma^2_m\left(\sigma^2_{m^*}\right)}$ such that Foreign agrees with Home if and only if $\sigma^2_m < \widehat{\sigma^2_m\left(\sigma^2_{m^*}\right)}$ if either $\rho \le (2-n)/(1-n)$ or if $(2-n)/(1-n) < \rho < \rho^*$ and $\sigma^2_{m^*} < \widehat{\sigma^2_{m^*}(\rho)}$. If, on the other hand, $\rho \ge \rho^*$ or $(2-n)/(1-n) < \rho < \rho^*$ and $\sigma^2_{m^*} \ge \widehat{\sigma^2_{m^*}(\rho)}$, then Foreign prefers that Home fixes the exchange rate.

Case III: Equal Money Supply Variances

Home prefers the asymmetric equilibrium in which its firms use LCP to the other asymmetric equilibrium if

$$\omega(PCP, LCP) < \omega(LCP, PCP);$$

$$(\rho-1)(1-2n)(\lambda-1) - \lambda < ((\rho-1)(1-2n)(\lambda-1) - \lambda)\exp\left(\frac{2n(\rho-1)^2(1-n)}{2\rho^2}\sigma^2_m\right).$$

Note that the argument of the exponential function on the right hand side is strictly positive implying that the value of the function is a number strictly greater than one. It follows that the inequality is true if (and only if) the left hand side is positive

$$(\rho-1)(1-2n)(\lambda-1) - \lambda > 0,$$

which is equivalent to

$$\rho > 1 + \frac{\lambda}{(1-2n)(\lambda-1)} > 1 \quad \text{for} \quad n < 1/2,$$

and

$$\rho < 1 + \frac{\lambda}{(1-2n)(\lambda-1)} < 1 \quad \text{for} \quad n > 1/2.$$

The former inequality holds if ρ is sufficiently large, and the latter inequality is always false. We conclude that

$$\omega(PCP, LCP) \lesseqqgtr \omega(LCP, PCP) \quad \text{as} \quad \rho \gtreqqless \frac{(1-2n)(\lambda-1)+\lambda}{(1-2n)(\lambda-1)} \quad \text{and} \quad n < 1/2;$$

$$\omega(PCP, LCP) = \omega(LCP, PCP) \quad \text{for} \quad n = 1/2;$$

$$\omega(PCP, LCP) > \omega(LCP, PCP) \quad \text{for} \quad n > 1/2.$$