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2002 • 5

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Propagation of nominal shocks in open economies

September 2002

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ISSN (trykt/print) 1602-1185

ISSN (online) 1602-1193

Abstrakt

Empiriske undersøgelser viser, at der er betydelig træghed (persistens) i tilpasningsprocessen til nominelle stød. Eksisterende modeller for åbne økonomier kan ikke generere forløb, der matcher tilpasningsprocessen til nominelle stød både kvalitativt og kvantitativt. Vi analyserer transmissionen af nominelle stød i en fuldt specificeret intertemporal model for en åben økonomi med ukomplette kapitalmarkeder og asynkrone nominelle lønkontrakter. Det bliver vist, at træghed i tilpasningsprocessen afhænger af løn-pris interdependens (løn-pris spiral), der – i en generel ligevægtssammenhæng – bestemmes af strukturelle parametre fra både efterspørgsels- og udbudssiden af økonomien. Det illustreres såvel analytisk som numerisk, at parametervalg, der styrker interdependensen imellem lønninger og priser, også leder til øget træghed i tilpasningen til nominelle stød. I papiret udvikles desuden en løsningsmetode til en stokastisk, intertemporal "Ny Åben Makro"-model.

Abstract

Empirical evidence documents substantial persistence in the adjustment process to nominal shocks. Existing open-economy models have failed either to generate interesting dynamics or found that the mechanisms are quantitatively weak. We consider the propagation of nominal shocks in a fully specified stochastic intertemporal open-economy model with incomplete capital markets and staggered nominal wage contracts. It is shown that persistence depends on wage-price interdependencies (spiral), which in turn in a general-equilibrium setting depends on structural parameters characterizing both the demand and the supply side of markets. Parameter choices strengthening wage-price interdependencies thus strengthen persistence as is demonstrated analytically and illustrated numerically. A further product of the paper is that it develops a method by which to solve explicitly for a stochastic intertemporal version of the "New Open-Economy Macroeconomics" model in which the expenditure switching effect is effective.

Propagation of nominal shocks in open economies

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Empirical evidence documents substantial persistence in the adjustment process to nominal shocks. Existing open-economy models have failed either to generate interesting dynamics or found that the mechanisms are quantitatively weak. We consider the propagation of nominal shocks in a fully specified stochastic intertemporal open-economy model with incomplete capital markets and staggered nominal wage contracts. It is shown that persistence depends on wage-price interdependencies (spiral), which in turn in a general equilibrium setting depends on structural parameters characterizing both the demand and the supply side of markets. Parameter choices strengthening wage-price interdependencies thus strengthen persistence as is demonstrated analytically and illustrated numerically. A further product of the paper is that it develops a method by which to solve explicitly for a stochastic intertemporal version of the "New Open-Economy Macroeconomics" model in which the expenditure switching effect is effective.

Keywords: current account; exchange rates; incomplete capital markets; nominal shocks; persistence; staggering

JEL classification: E32; F41

* We gratefully acknowledge comments and suggestions from anonymous referees, Frédérique Béc, Niels Haldrup, Bo W. Hansen, Phillip R. Lane, Lars M. Nielsen, Peter E. Storgaard, and Hans Timmer as well as by seminar participants at the University of Aarhus, Danish Doctoral Programme in Economics Annual Meeting (Aarhus), NHH (Bergen), University of Warwick, and at the conferences "General Equilibrium and Monetary Transmission" organized by the Dutch Central Bank, and "Instability, Inefficiency, and Public Policy: Lessons from Dynamic General Equilibrium Models" at THEMA, Cergy-Pontoise, France. The views expressed are our own and not necessarily those of Danmarks Nationalbank.

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1 Introduction

Important questions for open-economy macroeconomics are how nominal shocks are transmitted between countries, and whether the effects tend to be persistent. Ample empirical evidence documents that monetary shocks are important for open-economy variables, non-neutral, and that the real effects are long lasting.[1] A significant achievement of the new open-economy macroeconomics literature is the explicit formulation of dynamic general equilibrium models, which potentially allow for an explanation of both the impact effect of nominal shocks and their transmission over time.[2] However, research on the transmission over time in the open economy has not yet obtained *analytical* results and reached a level understanding comparable to the research in first-wave models with one-period dynamics, i.e. impact and long-run effects only (e.g., Obstfeld and Rogoff, 1995). This paper fills this gap by analyzing the propagation over time, or medium-term dynamics, of nominal shocks in an open-economy setting with staggered nominal contracts. Staggering is a candidate for generating persistence since it implies delayed adjustment of nominal variables, and the interesting question is whether the lack of coordinated nominal decisions is sufficient to generate dynamic paths as those observed in the data. We characterize analytically the conditions for generating persistent effects in international relative prices, and illustrate the mechanisms numerically.

A few papers have studied staggered contracts as a potential propagation mechanism in open economies. A prominent contribution is Chari, Kehoe, and McGrattan (1998, 2001). Based on numerical results they conclude that staggering can account for the empirically observed persistence in international relative prices only if nominal contracts have a very long duration and risk aversion is high. Specifically, they do not find significant effects of nominal shocks beyond the contract lengths, i.e. endogenous persistence is weak. Bergin and Feenstra (2001) show how persistence is increased when the demand elasticity is increasing in the price since this induces firms to smoothen their price responses over the cycle. Lastly, Betts and Devereux (1999) calibrate a model, where they compare the transmission of nominal shocks across pricing regimes and capital market structures. They find that empirical im-

[1] Recent empirical evidence on the role of nominal shocks for business cycle fluctuations in general, and movements of exchange rates and the trade balance in particular can be found in, e.g., Canova and De Nicoló (2000), Eichenbaum and Evans (1995), Prasad (1999), and Rogers (1999).

[2] The literature took off with the framework suggested in the redux-paper by Obstfeld and Rogoff (1995). For a recent survey of the new open-economy macroeconomics, see Lane (2001).

pulse responses to monetary shocks are replicated better by an economy with pricing to market than an economy, where exports are invoiced in the producers' currency (producer currency pricing), and that the capital market structure is inconsequential for the transmission of monetary shocks. Only numerical methods are used, and although impulse-responses are presented, no in-depth analysis of the underlying mechanisms is performed. Furthermore, the specific issue of persistence is not addressed.[3]

We take the Obstfeld and Rogoff (1995) model as a starting point, because it is well-known and the case of one-period nominal contracts is well understood in the literature. The model has the advantage of featuring the expenditure switching mechanism traditionally emphasized in the open-economy literature. However, it also implies PPP, which leaves the terms of trade as the key international relative price in the model. In the recent literature there has been much debate on the appropriateness of the producer currency pricing assumption accounting for the PPP-property. Since empirical evidence tends to fail to support this property there has been a growing interest in models with pricing to market and local currency pricing, i.e. where prices are sticky in consumers' currency (see, e.g., Chari, Kehoe and McGrattan, 1998, 2001; and Bergin and Feenstra, 2001). One important feature of these models is, however, that the terms of trade improve following a positive monetary shock, and this lacks empirical support (see, Obstfeld and Rogoff, 2000, for a lengthy discussion). Furthermore, and perhaps more importantly, a pricing-to-market structure leaves out an expenditure switching effect as a potentially important transmission mechanism across countries. We consider this effect to be instrumental in open economies (again, see Obstfeld and Rogoff, 2000), and while a combination of both pricing to market and producer currency pricing in different sectors might be a future research strategy, we pursue as a first step an in-depth analysis of the transmission mechanism, when expenditure switching is present.[4] This is crucial since international relative-price changes are important for both price and wage formation, which in turn may be propagated over time via, e.g., staggered contracts. Moreover, with incomplete capital markets wealth reallocations induced by international relative-price changes may have not only a direct effect on relative demand across countries, but also an effect on wage formation (via wealth/income effects) and therefore in turn prices.

[3]Two other open-economy contributions with (Calvo) staggered contracts and pricing to market are Kollmann (2001a, 2001b). Neither the former nor the latter focus on persistence, but primarily on volatility in a semi-small economy and cross-country correlations in a two-country model, respectively. More importantly, both papers pursue a quantitative approach only.

[4]See also Obstfeld (2001) for a discussion on the pricing issue.

Accordingly, the aim of this paper is to investigate the conditions needed for persistent effects of nominal shocks in open economies. In relation to the literature our contribution is four-fold: (i) we solve a stochastic intertemporal general equilibrium two-country model *analytically* to trace out the adjustment paths to shocks. Thereby we avoid the black box often involved in simulations of fairly complicated models to gain better insight into the key structural parameters driving the processes generated for the endogenous variables; (ii) we assume incomplete financial markets (bonds only) allowing for wealth reallocations as a propagation mechanism; (iii) under producer currency pricing and two-period overlapping wage contracts we analyze how wage and price interactions affect the propagation of shocks over time. In this context we adopt a more flexible specification of the technology where the usual constant returns to scale assumption made in the literature (Hau, 2000; Obstfeld and Rogoff, 2000) is a limiting case of our specification; (iv) numerical examples are presented to yield some quantitative information on how key parameters affect persistence.

The main contribution of the paper is the analytical characterization of what could be labeled an open economy wage-price spiral. The general formulation of the model allows an identification of key parameters affecting the wage and price interdependencies in open economies. We show that the link between wages and prices is crucial for the propagation process, and the stronger this link, the more persistent are the real effects of nominal shocks. The usual explanation for the lack of persistence (in price staggering models) is that wages are too sensitive to activity (see Lane, 2001) which actually implies that the link from prices to wages is weak. Our analysis clarifies the role of the wage and price interdependencies for persistence in an international context, and shows that both the price-to-wages and the wages-to-price links are determining the dynamic adjustment process.

This paper is organized as follows. Section 2 sets up a stochastic version of a by now fairly standard dynamic two-country model augmented with staggered wage contracts. The main analysis on the dynamic adjustment to monetary shocks is presented in section 3. In section 4 quantitative evidence is presented, and three-period staggering is briefly introduced in order to evaluate the role of the contract length and number of contracting groups. Section 5 summarizes and concludes the paper. The appendix provides the technical details.

2 A stochastic two-country model

We consider a new open-economy macroeconomics two-country model with a flexible exchange rate. Both countries, Home and Foreign, produce a separate tradable commodity (specialized production) which is demanded by consumers in both countries. Money is demanded for the transaction services it provides which is captured by including real balances, in the (semi-indirect) utility function of households (cf. Feenstra, 1986). There is a real asset (bond) which is traded in a perfect international capital market. To focus on the interdependencies between the two countries, the model is symmetric.

Product markets are assumed competitive, while there are nominal wage rigidities, in the form of staggered contracts. Although nominal rigidities may prevail both in product and labor markets, we find it natural to focus on nominal wage rigidities since empirical evidence indicates that they are more important than nominal price rigidities (see, e.g., Spencer, 1998; Obstfeld and Rogoff, 2000). Traditional open-macro models tend also to be based on an assumption of rigid nominal wages. Agents behave optimally given the exogenous contract structure and the strategy is to analyze the implications of staggered contracts for the dynamic adjustment process. In the concluding section we comment on the importance of nominal price versus wage staggering. All technical details of the model are given in the appendix.

2.1 Consumers

The representative consumer's preferences are given by

$$U_t = E_t \sum_{j=0}^{\infty} \delta^j \left[\frac{\sigma}{\sigma - 1} C_{t+j}^{\frac{\sigma-1}{\sigma}} + \frac{\lambda}{1-\varepsilon} \left(\frac{M_{t+j}}{P_{t+j}} \right)^{1-\varepsilon} - \frac{\kappa}{1+\mu} N_{t+j}^{1+\mu} \right], \qquad (1)$$

$$\sigma > 0, \quad \lambda > 0, \quad \varepsilon > 0, \quad \kappa > 0, \quad \mu > 0, \quad 0 < \delta \leq 1. \qquad (2)$$

E_t is the expectations operator conditional on period t information (see below), δ the subjective discount factor, N is the amount of labor worked, M denotes nominal balances, P is the consumer price index, and C is a real consumption index defined over consumption of the Home good and the Foreign good as

$$C_t = \left[\left(\frac{1}{2} \right)^{\frac{1}{\rho}} \left(C_t^h \right)^{\frac{\rho-1}{\rho}} + \left(\frac{1}{2} \right)^{\frac{1}{\rho}} \left(C_t^f \right)^{\frac{\rho-1}{\rho}} \right]^{\frac{\rho}{\rho-1}}, \ \rho > 0, \qquad (3)$$

where ρ is the elasticity of substitution between Home and Foreign goods. The minimum cost at which one unit of the consumption bundle can be acquired defines the corresponding price index

$$P_t = \left[\frac{1}{2}\left(P_t^h\right)^{1-\rho} + \frac{1}{2}\left(P_t^f\right)^{1-\rho}\right]^{\frac{1}{1-\rho}}, \tag{4}$$

where P_t^h (P_t^{*h}) is the price of the Home good in Home (Foreign) currency and P_t^f (P_t^{*f}) is the price of the Foreign good in Home (Foreign) currency. We assume that there are no impediments to trade so that the law of one price holds for both goods, i.e.,

$$P_t^h = S_t P_t^{*h}, \quad P_t^f = S_t P_t^{*f}. \tag{5}$$

An asterisk refers to Foreign variables. S is the nominal exchange rate defined as the Home price of Foreign currency. The assumption that the law of one price holds implies straightforwardly that purchasing power parity holds as well; that is, $P_t = S_t P_t^*$.

We assume that there is one internationally traded real bond denoted in the composite consumption good C, where r_t is the consumption based real interest rate between dates t and $t+1$. Note that we make only a small deviation from the case of complete capital markets, since we in a context of nominal shocks allow for a *real* bond, which is traded perfectly across countries.

The consumer's dynamic budget constraint is given by

$$P_t B_t + M_t + P_t C_t = (1 + r_{t-1}) P_t B_{t-1} + M_{t-1} + W_t N_t + \Pi_t + P_t \tau_t. \tag{6}$$

The right-hand side gives available resources as the sum of the gross return on bondholdings $(1+r_{t-1})P_t B_{t-1}$, initial money holdings M_{t-1}, labor income $W_t N_t$, nominal profit income Π_t and transfers from the government $P_t \tau_t$. Resources are allocated to consumption $P_t C_t$, nominal money holdings M_t and bondholdings $P_t B_t$.

Given the constant elasticity consumption index Home consumers' demands for the Home good and the Foreign good are

$$D_t^h = \frac{1}{2}\left(\frac{P_t^h}{P_t}\right)^{-\rho} C_t, \quad D_t^f = \frac{1}{2}\left(\frac{P_t^f}{P_t}\right)^{-\rho} C_t, \tag{7}$$

respectively, and similarly for the Foreign consumers' demands. Aggregating demands, we find demands for the Home and Foreign goods to be

$$D_t = \left(\frac{P_t^h}{P_t}\right)^{-\rho} C_t^W, \quad D_t^* = \left(\frac{P_t^{*f}}{P_t^*}\right)^{-\rho} C_t^W, \tag{8}$$

where world consumption $C_t^W \equiv \frac{1}{2}C_t + \frac{1}{2}C_t^*$.

The consumer maximizes expected utility subject to the budget constraint and the first-order conditions determining the optimal choice of B_t, and M_t are readily found. Wages are set by unions, and labor is demand determined; see section 2.4 below. In order to solve the model analytically, it is convenient to work with the model written in logs. Later it will be shown that the variables of the model are lognormally distributed under the assumed stochastic processes for the exogenous variables. The first-order conditions can be written

$$E_t c_{t+1} = c_t + \sigma \log(1 + r_t), \tag{9}$$

$$m_t - p_t = \eta_{mc} c_t - \eta_{mc}^1 E_t c_{t+1} + \eta_{mp}(p_t - E_t p_{t+1}). \tag{10}$$

The money-demand elasticities are given by: $\eta_{mp} = \frac{\delta}{(1-\delta)}$, $\eta_{mc} = \frac{1}{\sigma(1-\delta)\varepsilon}$, and $\eta_{mc}^1 = \delta\eta_{mc}$. It is assumed that the usual transversality condition holds. Lower-case letters denote the log-deviations from a symmetric steady state of the corresponding upper-case variables. All constants – including conditional variance terms which are time invariant – are suppressed since the focus of this paper is on the adjustment process to shocks.

2.2 Firms

There is perfect competition in the product markets. The representative firm is a price and wage taker and produces subject to a decreasing returns technology linking output Y^h and labor input N[5]

$$Y_t^h = N_t^\gamma, \qquad 0 < \gamma < 1. \tag{11}$$

This specification includes the case of constant returns as a special case ($\gamma \to 1$). This case has been mostly studied in the literature but implies that nominal wage contracts automatically makes nominal output prices rigid (Hau, 2000; Obstfeld and Rogoff, 2000). Since this has strong implications for persistence (see below) it is useful to work with this slightly more general set-up. Note, that there are no nominal price rigidities, i.e. prices are allowed to vary following changes in demand.

[5] Real capital is disregarded to simplify. Decreasing returns can be interpreted as arising from a second factor of production in fixed supply. The present formulation avoids the specific assumption concerning disutility of labor and the production technology underlying the often used yeoman-farmer specification.

Maximizing profits yields the following labor demand and output supply for the representative firm

$$N_t = \alpha_n \left(\frac{W_t}{P_t^h} \right)^{\eta_{nw}^d}, \qquad \alpha_n = \gamma^{\frac{1}{1-\gamma}}, \qquad \eta_{nw}^d = \frac{1}{\gamma - 1}, \tag{12}$$

$$Y_t^h = \alpha_y \left(\frac{W_t}{P_t^h} \right)^{\eta_{yw}}, \qquad \alpha_y = \gamma^{\frac{\gamma}{1-\gamma}}, \qquad \eta_{yw} = \frac{\gamma}{\gamma - 1}. \tag{13}$$

For later use, note that the elasticity of labor demand with respect to the product real wage is η_{nw}^d, and the elasticity of output supply with respect to the product real wage is η_{yw}. Both are determined by the structural productivity parameter γ. When γ is high, firms are very sensitive to product real-wage changes both with respect to labor demand and output supply. Profits are distributed to households.

2.3 Government

We assume the only role for the government is to issue money. Thus the government's budget constraint is:

$$M_t - M_{t-1} = P_t \tau_t. \tag{14}$$

Money is transferred to consumers in a lump-sum fashion. Home money is only held by Home residents. We introduce shocks into the model by assuming a specific stochastic process for the (relative) money supply.

$$m_t - m_t^* = m_{t-1} - m_{t-1}^* + u_t, \tag{15}$$

where $u_t \sim nid(0, \sigma_u^2)$. This specification implies that all (unanticipated) nominal shocks are fully permanent. We assume full current information; u_t is commonly observed in period t.

2.4 Wage setting under risk and imperfect competition

To endogenize nominal wage setting we build on a vast literature introducing imperfect competition into the labor market.[6] Workers are organized in (monopoly) unions, and each union represents a (small) subset of workers

[6] See, for example, Blanchard and Fisher (1989, chapter 9), and, Moene and Wallerstein (1993).

supplying labor to a given group of firms.[7] We assume an exogenous contract structure, where half the unions sign contracts in even periods, and the other half in odd periods. Our staggering framework can be interpreted as introducing the least restrictive assumptions regarding nominal staggering to capture the dynamic implications of asynchronized nominal decision making in a decentralized market economy. The (utilitarian) union presets the wage for periods t and $t+1$ given all available information in period $t-1$ to maximize the expected utility of its members, which in turn depends on the wage income received and the disutility of work. We assume a right-to-manage structure in which employment is determined by firms given the wage.[8] Given this contract structure unions behave optimally and the union setting a wage at the end of period $t-1$ applying for periods t and $t+1$ chooses it so as to maximize

$$E_{t-1}\left[\zeta_t \frac{W}{P_t} N_t - \frac{\kappa}{1+\mu} N_t^{1+\mu} + \delta \left(\zeta_{t+1} \frac{W}{P_{t+1}} N_{t+1} - \frac{\kappa}{1+\mu} N_{t+1}^{1+\mu}\right)\right], \quad (16)$$

where ζ_t measures the shadow value of wage income to the household ($\zeta_t = C_t^{-\frac{1}{\sigma}}$, cf. the consumer's optimization problem). This way of presenting the wage decision problem mimics the standard trade-union literature. The union takes into account that employment is determined according to labor demand and the optimal nominal wage to be quoted for period t can now be written

$$W_t = \kappa \gamma^{-1} \frac{E_{t-1}\left(N_t^{1+\mu} + \delta N_{t+1}^{1+\mu}\right)}{E_{t-1}\left(C_t^{-\frac{1}{\sigma}} \frac{N_t}{P_t} + \delta C_{t+1}^{-\frac{1}{\sigma}} \frac{N_{t+1}}{P_{t+1}}\right)}. \quad (17)$$

When we proceed to solve the model we use that the exogenous variable is lognormally distributed. This, in turn, will imply that all endogenous variables are lognormally distributed as well. Exploiting the tractability of

[7]By assuming a sufficiently large number of unions, it is possible to maintain the property that they have market power in the labor market without introducing the possibility that they perceive that they can affect aggregate variables (see Hart, 1982).

[8]We assume that workers are willing to participate in the sense that for any labor demand, the marginal consumption value of the real wage is larger than the marginal disutility of effort (Corsetti and Pesenti, 2001). For small shocks the assumption of demand-determined labor is reasonable as the real consumption value of the wage exceeds the marginal disutility of effort. The process (15) does not rule out large shocks and as such the participation constraint might be violated (see also Obstfeld and Rogoff, 2000). The assumption can be made arbitrarily precise by assuming an ever smaller variance σ_u^2.

the lognormal distribution, we can write (17) as[9]

$$W_t = \frac{\kappa\gamma^{-1}\left[\left(E_{t-1}N_t\right)^{1+\mu} + \delta\left(E_{t-1}N_{t+1}\right)^{1+\mu}\right]\exp\left(\psi\right)}{\left(E_{t-1}C_t\right)^{-\frac{1}{\sigma}} E_{t-1}\left(N_t\right) E_{t-1}\left(\frac{1}{P_t}\right) + \delta\left[\left(E_{t-1}C_{t+1}\right)^{-\frac{1}{\sigma}} E_{t-1}\left(N_{t+1}\right) E_{t-1}\left(\frac{1}{P_{t+1}}\right)\right]}, \quad (18)$$

where

$$\psi = \frac{1+\mu}{2}\mu\sigma_n^2 - \frac{1}{2\sigma}\left(\frac{1}{\sigma}+1\right)\sigma_c^2 + \sigma_{np} + \frac{1}{\sigma}\sigma_{nc} - \frac{1}{\sigma}\sigma_{pc}. \quad (19)$$

The risk term ψ consists of variances and covariances of the endogenous variables.[10]

Two factors emerge in wage determination under imperfect competition and uncertainty; first, there is market power represented by γ^{-1}, and secondly, risk represented by ψ. The striking feature, though, is that these effects *do not affect the dynamic adjustment to shocks.* They constitute a level effect only. Taking logs of (18) and utilizing the properties of the lognormal distribution we find that the log wage set at time $t-1$ can be written

$$w_t = \Xi + \frac{1}{1+\delta}E_{t-1}w_t' + \frac{\delta}{1+\delta}E_{t-1}w_{t+1}', \quad (20)$$

where

$$w_t' = \eta_{wp}p_t^h + \left(1-\eta_{wp}\right)\left(s_t + p_t^{*f}\right) + \eta_{wc}c_t, \quad (21)$$

is the wage which would have prevailed (up to constants of no importance for dynamics) in the case of a competitive labor market. This is useful since the parameters are easier to interpret in terms of labor demand and supply elasticities. We thus have

$$\eta_{wp} = \frac{0.5 - \frac{\eta_{nw}^d}{\eta_{nw}^s}}{1-\frac{\eta_{nw}^d}{\eta_{nw}^s}} \in (0.5, 1), \quad \eta_{wc} = \frac{1}{\sigma\left(1-\frac{\eta_{nw}^d}{\eta_{nw}^s}\right)} > 0, \quad (22)$$

where η_{nw}^s is the "labor supply elasticity". The parameter μ determines the elasticity of individual labor supply with respect to the real wage, and it is given as $\eta_{nw}^s = \mu^{-1}$. The higher μ, the less elastic is labor supply. The income (consumption) elasticity is determined by $\sigma\mu$ as $\eta_{nc}^s = -\left(\sigma\mu\right)^{-1}$. The higher $\sigma\mu$, the less the income elasticity.

[9] Consult Aitchison and Brown (1957) for an introduction to the lognormal distribution.

[10] The conditional variance as of time $t-1$ with respect to the log of variable X_t, $Var_{t-1}(x_t)$, is denoted σ_x^2. Similarly $Cov_{t-1}(x_t z_t)$ is denoted σ_{xz}. The omission of time subscripts indicate that these variances and covariances are constant over time.

The nominal wage expression shows four things. First, level effects from risk and imperfect competition are subsumed in Ξ and of no importance for wage adjustment, which is solely determined by w_t' (cf. below). Thus, we can neglect the constant. Consult Obstfeld and Rogoff (2000) for an excellent introduction to the effects of risk on wage setting and monetary policy in a similar stochastic framework. Notice that the higher the discount factor (δ), the larger the weight to second period consequences in the wage formula

Secondly, wages depend on consumption. This reflects the income/wealth effect; an increase in wealth makes households enjoy more consumption and leisure. The latter leads to a wage increase. Potentially, the assumption of incomplete capital markets can have large effects on wage setting since it allows for wealth and thus consumption changes across countries.

Thirdly, the wage equation implies that nominal wages and thus prices depend on expected exchange rates. This captures a channel through which exchange rates affect the real side of the economy. It follows immediately that price increases on both the Home and the Foreign goods lead to higher wages. Moreover, if the nominal exchange rate changes (endogenized below), it has a direct impact effect on wage demands through the Home-currency price of the Foreign good. Subsequent indirect effects work through the open-economy wage-price spiral.[11] Below we will formalize the sources of exchange-rate changes (nominal shocks) and examine how the open-economy wage-price spiral affects persistence in the adjustment process in the presence of nominal rigidities and asynchronized decision making.

Lastly, the nominal wage is affected by a weighted average of the prices of the Home and Foreign goods, and as is apparent from (22) these weights depend solely on the relative magnitude of the labor-supply and the labor-demand elasticities.[12] The weight on consumption is determined similarly but depends on consumers' willingness to substitute over time (σ). To see how wage responses depend on the underlying labor-market elasticities we

[11] Note that the exchange-rate change triggers a switch in demand. The warranted price adjustment is then determined by the magnitude of this demand switch along with the direct change in the wage. This price change then feeds back into the wage through (21) and the wage-price spiral is up and running.

[12] Basically, this is just a question of the relative magnitudes of the slopes of the labor-demand and labor-supply curves. Remember that the elasticities depend on the structural parameters γ and μ.

note that[13]

$$\frac{\partial \eta_{wp}}{\partial \left(\frac{\eta^d_{nw}}{\eta^s_{nw}}\right)} < 0, \quad \frac{\partial \eta_{wc}}{\partial \left(\frac{\eta^d_{nw}}{\eta^s_{nw}}\right)} > 0. \tag{23}$$

When labor demand becomes more sensitive relative to labor supply, the weight on the Home-good price (η_{wp}) increases, and the one to the Foreign-good price ($1 - \eta_{wp}$) decreases. This just reflects that firms consider the product real wage ($w_t - p^h_t$), whereas consumers consider the consumption real wage

$$w_t - p_t = w_t - \frac{1}{2}\left(p^h_t + p^{*f}_t + s_t\right), \tag{24}$$

which also explains why the lower bound for η_{wp} is one-half. The low weight on wealth (consumption), when labor demand is highly sensitive, reflects the small relative importance of labor supply, and wealth effects enter into wage setting exactly via labor supply. In deriving (24) we have used the log-linearized version of the price index.

The aggregate wage faced by firms in period t, $\overline{w}_t$, can be written (suppressing constants)

$$\overline{w}_t = \frac{1}{2(1+\delta)}\left(E_{t-2}w'_{t-1} + \delta E_{t-2}w'_t + E_{t-1}w'_t + \delta E_{t-1}w'_{t+1}\right). \tag{25}$$

We end the description of the model by noting that Foreign is completely symmetric and that under flexible wages and prices there exists a competitive equilibrium to the model in which money is neutral (see also Obstfeld and Rogoff, 1995).

3 Adjustment to nominal shocks

In this section we offer a characterization of the dynamic adjustment process in the presence of staggered nominal wage contracts. The focus is on the medium-run dynamics, and the conditions for persistent effects on international relative prices.

In Obstfeld and Rogoffs' (1995) seminal work, they present a complete analytical characterization of the open-economy dynamics of relative consumption, the nominal exchange rate, and the terms of trade to monetary

[13] In terms of the structural parameters we have $\frac{\partial \eta_{wp}}{\partial \gamma} > 0$, $\frac{\partial \eta_{wp}}{\partial \mu} > 0$, $\frac{\partial \eta_{wc}}{\partial \gamma} < 0$ and $\frac{\partial \eta_{wc}}{\partial \mu} < 0$. The larger γ, the more sensitive labor demand, and the larger μ, the less sensitive is labor supply.

shocks in the presence of one-period contracts. Their assumption on contract structure allows them to consider only impact and long-run effects. The basic working of the Obstfeld-Rogoff model is well-known. Following a one-time unexpected positive monetary shock the nominal exchange rate depreciates, and the terms of trade deteriorates. The depreciation induces an expenditure switch towards Home goods, and a concomitant wealth increase for Home consumers. This wealth effect leads to an increase in relative Home consumption. The terms-of-trade effect reverses already after one period. The reason for this reversal is that the initial expansionary effect on income induces an increase in consumption which is smoothed over time. Due to the income effect on labor supply, it follows that higher consumption leads to a permanent reduction in labor supply. This induces an increase in the terms of trade in all subsequent periods. The adjustment process is therefore ended after one period (= the length of the contract). Thus, the implied pattern for the terms of trade is not matching the type of dynamics observed in the data with half-lives of the impact effect of up to 3 to 4 years. We now consider how this is affected by introducing the problem of uncoordinated decision-making in a decentralized market economy.

Despite the complications added by staggering and wealth effects, we are able to solve this stochastic model analytically. As a consequence we can not only find closed-form solutions, but pinpoint the exact determinants of persistence in our model. In the appendix we show that there exists an equilibrium in which the nominal exchange rate, the terms of trade, and relative consumption are determined as

$$s_t = \pi_{sc}\left(c_t - c_t^*\right) + \pi_{sm}\left(m_t - m_t^*\right), \tag{26}$$

$$q_t = \pi_{qc}\left(c_{t-1} - c_{t-1}^*\right) + \pi_{qq}q_{t-1} + \pi_{qu}u_t + \pi_{qu}^1 u_{t-1}, \tag{27}$$

$$c_t - c_t^* = c_{t-1} - c_{t-1}^* + \pi_{cu}u_t. \tag{28}$$

Equation (27) reveals the basic difficulty in finding an analytical solution to the model, namely, that the terms of trade depend on relative consumption which in turn depends on wealth and thus the terms of trade. Relative consumption is also crucial to the development of nominal exchange rates.

3.1 The nominal exchange rate

The nominal exchange rate can straightforwardly be found from the money market equilibrium conditions in the two countries. The dynamic equation can easily be solved by the method of undetermined coefficients, where π_{sc}

and π_{sm} are the coefficients to be determined in (26). Taking expectations and inserting we find the following restrictions

$$\pi_{sc} = \eta^1_{mc} - \eta_{mc} = -(\sigma\varepsilon)^{-1} < 0, \quad \pi_{sm} = 1. \tag{29}$$

The determination of the nominal exchange rate is thus equivalent to the monetary approach except that the relevant activity variable is relative consumption rather than output. The elasticity of the exchange rate with respect to relative consumption is determined by money demand's consumption elasticity ($[\sigma\varepsilon]^{-1}$). Using the process for the money stock the nominal exchange-rate equation can be written as

$$s_t = s_{t-1} + (\pi_{sm} + \pi_{sc}\pi_{cu})\, u_t. \tag{30}$$

The nominal exchange rate is seen to follow a random walk. The effect of a monetary expansion on the nominal exchange rate is a depreciation, i.e.

$$\frac{\partial s_t}{\partial u_t} = \pi_{sm} + \pi_{sc}\pi_{cu} > 0. \tag{31}$$

The higher the elasticity of consumption with respect to the money shock the less the variability of the nominal exchange rate relative to the variability of the money shock.[14] The higher the consumption elasticity of money demand ($\pi_{sc} = -[\sigma\varepsilon]^{-1}$), the lower the effect on the nominal exchange rate.

3.2 Relative consumption

The random-walk conjecture for relative consumption (28) follows from the Euler equations implying that relative consumption changes can only be driven by unanticipated shocks,

$$E_t\left(c_{t+1} - c^*_{t+1}\right) = c_t - c^*_t, \tag{32}$$

and that monetary shocks (u) are the only shocks in the model. There exists an equilibrium satisfying (28), where[15]

$$\pi_{cu} \gtreqless 0 \text{ for } \rho \gtreqless 1. \tag{33}$$

[14] Betts and Devereux (2000) show that pricing-to-market lowers the expenditure switching effects of an exchange rate depreciation, but this in turn magnifies the exchange rate responsiveness to a monetary shock.

[15] In the appendix it is shown that $\pi_{cu} < 0$ holds for $\rho \in [\underline{\rho}, 1)$. The reason why $\pi_{cu} < 0$ does not hold generally for $\rho < 1$ is the following. With an inelastic demand a fall in production leads to an increase in income and vice versa. For a low value of ρ the following scenario is possible. A monetary expansion induces an appreciation of the nominal exchange rate because the induced fall in production leads to a large increase in income and thus consumption. The latter is so large that the increase in money demand dominates the increase in money supply and as a consequence the nominal exchange rate appreciates. We consider this case to be extremely implausible and hence the text only discusses the case where $\rho \in [\underline{\rho}, 1)$ for $\rho < 1$.

Betts and Devereux (1999) and Chari, Kehoe, and McGrattan (1998) dismiss incomplete capital markets as being important for the transmission of nominal shocks in a setting with pricing-to-market strategies. In particular, it is argued that the effects of wealth reallocations are mitigated by changes in the terms of trade and domestic consumption so as to leave Foreign consumption unaffected. This depends critically on the assumed pricing behavior: prices are rigid in local currency and following a nominal depreciation the increased export earnings and increased expenditure on Foreign goods net out. In our framework with traditional demand switching effects we find that wealth reallocations arise under nominal wage contracts except in the special case where $\rho = 1$.[16] Below we shall explore further the importance of wealth reallocations for the adjustment process.

3.3 The terms of trade

Going through the tedious procedure of solving (26)-(28) we end up with (27) where

$$\pi_{qc} > 0,\ \pi_{qq} \in (0,1)\,,\ \pi_{qu} < 0 \text{ and } \pi^1_{qu} \lesseqgtr 0. \tag{34}$$

The terms of trade (27) can be rewritten by the use of the expression for relative consumption (28) as

$$\begin{aligned} q_t \;=\; & (1+\pi_{qq})\, q_{t-1} - \pi_{qq} q_{t-2} \\ & + \pi_{qu} u_t + \left(\pi^1_{qu} - \pi_{qu} + \pi_{qc}\pi_{cu}\right) u_{t-1} - \pi^1_{qu} u_{t-2}, \end{aligned} \tag{35}$$

showing that the terms of trade follow an ARIMA(1,1,2) process when the two propagation mechanisms - consumption smoothing and staggering - are merged. Note that consumption smoothing alone generates an ARIMA(0,1,1) (Obstfeld and Rogoff, 1995). This indicates that a more plausible dynamic adjustment path arises when merging the strong permanent effect generated by consumption smoothing with the sluggish adjustment path arising under staggering.

For the terms of trade there is an initial decrease, which is then gradually worked out of the system.[17] The long-run effect remains a decrease if $\rho < 1$

[16] It is easily shown that wealth reallocations are increasing in ρ ($\rho > 1$). Other structural parameters affect wealth as well, but for $\rho = 1$ variations in these are irrelevant; the trade balance is unchanged.

[17] This is always the case when $\rho \geq 1$. If $\rho < 1$ the dynamic adjustment is gradual at least from the period after the shock. More specifically, we cannot rule out that the terms of trade, in the period after the initial fall, moves in the opposite direction of the long-run level and then from there adjusts gradually towards the long-run level. Numerical exercises indicated that this perverse adjustment is highly unlikely.

and vice versa for $\rho > 1$. The model is thus capable of generating a plausible path for the impulse response for the terms of trade qualitatively similar to that observed in the data; whether it generates persistence of sufficient quantitative importance is another question (see below).

It is noted that the solutions found above for the nominal exchange rate, relative consumption, and the terms of trade confirm the earlier made conjecture that all endogenous variables are lognormally distributed.

3.3.1 Persistence

For a closer look at the mechanisms determining the dynamics of the terms of trade it is useful to write the terms-of-trade equation in the following stochastic trend representation

$$q_t = \frac{1}{1-\pi_{qq}L}\left(\pi_{qu}u_t + \pi_{qu}^1 u_{t-1} + \pi_{qc}\pi_{cu}\sum_{j=0}^{\infty} u_{t-1-j}\right), \tag{36}$$

where L is the lag operator. The process has an autoregressive part depending on the parameter π_{qq}, and the larger π_{qq}, the more persistence. Considering the moving-average part we have that the first two terms capture the dynamics induced by staggered two-period contracts while the last term captures the stochastic trend driven by consumption smoothing. The obvious implication being that not only does the dynamics run forever due to staggered contracts, but the effects running via consumption smoothing have both a direct effect (the $\pi_{qc}\pi_{cu}$-terms) as well as an indirect arising from the interaction with the dynamics introduced by staggering (π_{qu} depends on π_{cu}).

3.3.2 The open-economy wage-price spiral

Generating a real impact effect of a nominal shock requires a nominal rigidity. Once the impact effect is generated the issue of persistence becomes a question of how fast nominal variables adjust to the nominal shock so as to reach the long-run equilibrium. In the present model the nominal rigidity arises due to nominal wage contracts, and the key nominal variables are prices and wages. The issue of persistence therefore depends on how fast nominal wages and prices catch up with the nominal change. The speed at which wages and prices adjust depends on the interaction between the two, and to see this, consider the following sequence. When the nominal shock is realized the nominal wage is rigid, but nominal prices can adjust. In the subsequent periods wages adjust (under the staggered contract structure) to take account of changes in prices and other variables, and prices adjust to take account of changes in wages and other variables. This process runs forward until nominal wages and prices eventually are fully adjusted to the nominal shock, and the long run equilibrium has been reached. Suppose the wage-price interdependencies are strong, that is, wages are very sensitive to prices and vice versa. It follows that the adjustment process must be slow. In the first period prices do not adjust by much because wages are rigid (and prices are very sensitive to wages). In the subsequent period wages do not adjust by much because prices did not change much and so on. In this case the adjustment process to the nominal shock displays persistence. The adjustment displays no strong persistence in the opposite case where wage-price interdependencies are weak, since the prices will adjust by much on impact, and similarly wages will change much when contracts are renewed and so forth. This brings out that the wage-price interdependencies or the wage-price spiral must be critical to the persistence in the adjustment process.[18]

This intuition drives the persistence in the present intertemporal open-economy model, although it is embedded in the complicated contract structure and the other intertemporal mechanisms in the model. To see this consider the variable on which we have focussed, the relative price (in common currency) of Foreign and Home commodities, i.e. the terms of trade (27) which can be written as a function of the relative wage measured in common currency

$$q_t = \frac{-\eta_{yw}}{\rho - \eta_{yw}} \left(\overline{w}_t - \overline{w}_t^* - s_t \right). \tag{37}$$

Relative prices depend on the relative wages and the coefficient $\frac{-\eta_{yw}}{\rho-\eta_{yw}}$ determines by how much wage changes feed into price changes. The path from prices to wages is slightly more complicated due to the staggered contracts. The relative wage can by use of (21) and (25) be written

$$\overline{w}_t - \overline{w}_t^* - s_t = \frac{1}{2\left(1+\delta\right)} \left[\sum_{j=1}^{2} \sum_{i=1}^{2} \delta^{i-1} E_{t-j} \left(w'_{t+i-j} - w^{*\prime}_{t+i-j} \right) \right] - s_t, \tag{38}$$

where

$$w'_t - w^{*\prime}_t = (2\eta_{wp} - 1) q_t + \eta_{wc}(c_t - c_t^*) + s_t. \tag{39}$$

[18]It is well-known from the closed-economy literature that a stronger wage-price link tends to reinforce inertia (see, e.g., Ball and Romer, 1990; and Andersen, 1994). The basic intuition is simply that it strengthens strategic complementarity (in the sense of Cooper and John, 1988) between prices and wages. That is, with a strong wage-price link (strong strategic complementarity) inertia in wages tends to imply inertia in prices which in turn reinforces inertia in wages, and so on.

Relative wages are thus driven by relative prices (terms of trade), unexpected exchange-rate changes, and (relative) consumption. A relative price change affects relative wages by $\frac{2\eta_{wp}-1}{2(1+\delta)}$.

Thus, we have a sensitivity of (relative) prices to (relative) wages given by a wage-to-price link

$$\frac{-\eta_{yw}}{\rho-\eta_{yw}}=\frac{1}{1-\frac{\rho}{\eta_{yw}}}. \tag{40}$$

This way of writing the link stresses that sensitivity of prices to wages depends on the relative slopes of the demand (ρ) and supply curves (η_{yw}) in the product market. In the special case of constant returns to scale ($\gamma \rightarrow 1$) we have that the wage-to-price link is unity, and prices respond proportionally to wage changes.

The sensitivity of (relative) wages to (relative) prices is determined by a price-to-wage link

$$\frac{2\eta_{wp}-1}{2\left(1+\delta\right)}=\frac{1}{2\left(1+\delta\right)}\frac{\frac{\eta_{nw}^{d}}{\eta_{nw}^{s}}}{1-\frac{\eta_{nw}^{d}}{\eta_{nw}^{s}}}, \tag{41}$$

where the right-hand side expression again emphasizes, that the link is determined by the relative slopes of the demand and supply curves (in the labor market) and the discounting.[19]

Thus, there is clearly an international relative wage - international relative price spiral in the model, since relative wages affect the terms of trade, which in turn influence relative wages although the mechanism is complicated by the staggered contract structure. We label this link: the open-economy wage-price spiral. The final step is to unravel where this wage-price spiral arises in this intertemporal model. This is straightforward since the autoregressive parameter π_{qq} is determined as

$$\pi_{qq}=\eta_{qq}\left[1+\left(\delta+1\right)\pi_{qq}+\delta\pi_{qq}^{2}\right]. \tag{42}$$

The non-linear relation is due to the staggered contracts and the presence of

[19] The linking of relative labor costs and the terms of trade at the business cycle frequency is different to the Balassa-Samuelson effect, which is a connection between international relative prices and labor costs at a longer horizon. In this case the change in labor costs is driven by technological changes in one sector of the economy, and there is no feed-back from international relative price changes to relative wage changes.

δ reflects the extent to which contracts are forward looking. Using that[20]

$$\eta_{qq} = \frac{-\eta_{yw}}{\rho - \eta_{yw}} \frac{(2\eta_{wp} - 1)}{2(1+\delta)} \in \left(0, \frac{1}{2}\right), \tag{43}$$

and observing that π_{qq} is increasing in η_{qq}, it follows that the stronger the wage-price spiral (measured by the product of the wage-to-price link, and the price-to-wage link) the more persistent is the adjustment process, and vice versa.

The explicit analytical approach taken here yields two major insights. First, despite the complication of having a stochastic intertemporal model we are able to identify the wage-price spiral as the key mechanism generating persistence. Second, the general equilibrium set-up shows that parameters related both to preferences and technology are determining the actual strength of the wage-price spiral. In the particular set-up here, there are four structural parameters of importance for persistence, namely, ρ, γ, μ and δ. It can be shown that $\pi_{qq} \in (0,1)$ and

$$\frac{\partial \pi_{qq}}{\partial \rho} < 0, \ \frac{\partial \pi_{qq}}{\partial \gamma} > 0, \ \frac{\partial \pi_{qq}}{\partial \mu} > 0, \ \frac{\partial \pi_{qq}}{\partial \delta} < 0. \tag{44}$$

It follows that persistence is strengthened the smaller the demand elasticity (ρ) and the smaller the discount factor (δ). Less sensitivity of labor supply (a high μ) tends to strengthen persistence as does more sensitivity of output to labor inputs (a high γ). The intuition is as follows: the lower the demand elasticity (ρ) between Home and Foreign goods, the more sensitive are relative prices to relative wages. The higher γ the more elastic is output supply and labor demand, and the more prices tend to respond to wage changes; the higher μ the less elastic the labor supply and hence the higher the wage response to a price increase; the lower δ, the larger the weight attached to current prices in wage setting, see (25). Whether persistence is quantitatively strong therefore depends on the values of these four structural parameters.

In recent years the persistence issue has been widely discussed in both open as well as closed economies; see Lane, 2001, for an overview and references. Our analysis brings forward two main insights in relation to this literature.

First, it is sometimes stated that one of the conditions for persistence in staggering models is a small response of prices to changes in wages. This is cited as the chief result of Chari, Kehoe, and McGrattan (2000) in their

[20] When we solve for the terms of trade we solve a stochastic difference equation, where the endogenous expectational term is given by $\eta_{qq}(E_{t-2}q_{t-1} + \delta E_{t-2}q_t + E_{t-1}q_t + \delta E_{t-1}q_{t+1})$. Thus the constant η_{qq} determines the degree of backward looking, or inertia.

closed-economy model with staggered price contracts; and it generalizes to open economies (e.g., Chari, Kehoe, and McGrattan, 1998). What we show here is that the condition going from wages to prices is only half the story. The reasoning goes the other way around as well; an appropriate response of wages to prices is also needed in subsequent periods. The adjustment is determined by the workings of the wage-price spiral over time and it depends on both the wage-to-price link and the price-to-wage link. This distinction is not trivial, since different parameters and elasticities determine the two links. Furthermore, the condition is referred to as prices being too sensitive to wages. This is not the case as the problem is that when the shock hits the economy, wages in their model are too insensitive to prices. They change a lot despite prices being locked.

Secondly, it follows that, although open and closed economies share some features, the open-economy adjustment to nominal shocks is different. The difference can be seen in two ways. The specific open-economy feature of expenditure switching plays a potentially important role (captured by ρ), since it is a determinant of how much the Home-good price can change relative to the Foreign-good price in the wake of relative wage changes. Furthermore, the finding that less wage sensitivity strengthens persistence (a high μ, which corresponds to a low labor-supply elasticity in a competitive labor market) brings out a difference in the transmission mechanism between open and closed economies. In closed economies a more elastic labor supply strengthens persistence, see for instance, Ascari (2000) and Chari, Kehoe, and McGrattan (2000), and therefore it is a problem that quantitative important persistence can only be generated if labor-supply elasticities are implausibly large. The difference arises here due to the important distinction between the consumer and producer real wages in open economies; workers consider the former and firms the latter.[21] The upshot is, that a calibration exercise, which delivers endogenous persistence in a closed-economy setting, might not generate persistence in an open economy. Some parameter choices can obstruct endogenous persistence in an open economy, the labor-supply elasticity, and some parameters specific to the open economy needs to be carefully chosen as well, demand elasticities between Foreign and Home goods. Thus, going forward in the analysis of persistence in international relative prices with more complicated market and pricing assumptions, a thorough understanding of

[21] It should be stressed that this is a general equilibrium model, and implications of parameter changes are difficult to explain fully, since they are blurred by the general-equilibrium effects accompanying a given change. For example, a given change in γ alters not only goods market conditions but labor market conditions as well via changes in η_{wp} and η_{wc}. Moreover, changes in γ have also implications for wealth redistributions. See below.

how these assumptions change the open-economy wage-price spiral - if any - is imperative.

3.3.3 Wealth effects

Persistence in this open-economy model is not only dependent on the autoregressive part. If we are after relative-price half-lives of up to 3 to 4 years, then the moving-average part is crucial as well. The moving-average terms represent a specific open-economy feature, namely wealth effects which transmit into relative consumption differences.

We start by noting that changes in relative consumption are only possible because capital markets are incomplete.[22] Moreover, relative consumption follows a random walk, and the size of the wedge between Home and Foreign consumption levels depends on the wealth reallocation, i.e., the fact that capital markets are incomplete. If a relative Home monetary expansion is accompanied by a large wealth reallocation in favor of Home, then this will transmit into a large relative consumption increase and a subsequent large adjustment in relative wages. The effect will be a large jump in the terms of trade which possibly reduces the half-life considerably. Small wage effects of wealth changes can both be obtained by small wealth effects per se and with a small weight attached to consumption in the wage equation (η_{wc}).

Interestingly, the wealth effect on relative consumption is a one-time effect and, hence, is captured in the moving-average terms. With two-period staggering we have second order lags of the moving-average part. Similarly, the nominal exchange rate enters the wage equation, and due to its random-walk nature it affects the wage-setting in the same manner as relative consumption. This suggests that mechanisms inducing richer dynamics for the nominal exchange rate and relative consumption (nontradable goods or habit formation in consumption) might transmit into more complicated dynamics for the terms of trade. In terms of the wage-price spiral, changes in the exchange rate or relative consumption indirectly weakens the link from prices to wages, and thus weakens the spiral, since the *relative* weight on the terms of trade in (39) is lowered. Roughly, this is what happens in Chari, Kehoe, and McGrattans' (2000) sticky-price closed-economy model: wages change a lot due to the change in output. The link from prices to wages is too weak.

[22] With complete capital markets, it follows under purchasing power parity and separable preferences, that consumption evolves symmetrically between the two countries, see, e.g., Chari, Kehoe, and McGrattan (1998).

4 Numerical illustrations

For illustrative purposes we report some numerical examples. Since the model is highly abstract focussing only on a few mechanisms and relying on particular functional forms, we stress that we do not find that such numerical illustrations can be taken as an empirical test of the model, but it can be suggestive on the quantitative importance of the different channels.

Figure 1 shows the impulse-response functions for the terms of trade following a 1 percent increase in the (relative) domestic money supply in period 1. The figure also shows how the impulse-response functions change to variations in: (i) the demand elasticity, ρ; (ii) the technology parameter γ, which governs the labor-demand elasticity and output-supply elasticity; (iii) the parameter μ (the labor-supply elasticity); and the discount factor δ.[23]

Figure 1 about here

The figures build on a baseline case where the parameter values are given in table 1.

Table 1: Baseline parameter values

ρ	γ	μ	σ	ε	δ
2	2/3	10	0.75	9	1/1.05

This implies that we have the following baseline elasticities:

Table 2: Baseline elasticities

η_{yw}	η^d_{nw}	η^s_{nw}
2	3	0.1

The analytical results have already indicated that the parameter γ is critical for the results. One approach is to interpret this as the wage share as done in table 1. However, since there is no real capital in the model, it may be more appropriate to let the parameter equal one to preserve constant returns to scale at the aggregate level. We stick to the first interpretation for the baseline case but this should be thought of as a lower bound, and any $\gamma \in \left(\frac{2}{3}, 1\right)$ should be considered as a reasonable parameter value. The elasticity parameter ρ is chosen to ensure that the Marshall-Lerner condition is fulfilled. In the literature ρ is often between 1 and 2 (Chari, Kehoe, and

[23] For brevity sensitivity analyses for σ and ε are left out as they do not affect persistence via the autoregressive part (π_{qq}).

McGrattan, 1998, choose $\rho = 1.5$). The labor-supply parameter μ is chosen so as to imply an elasticity of 0.1. The three last coefficients correspond to those adopted in Hairault and Portier (1993), and Sutherland (1996).

To assess how persistence depends on the open-economy wage-price link considered above, we report in figure 2 how the autoregressive parameter π_{qq} varies with these key structural parameters. We find that both the labor-supply elasticity (μ) and the discount factor (δ) are quantitatively unimportant for the persistence properties captured by π_{qq}; variations in these parameters have only trivial effects on persistence. Changes in the demand elasticity ρ have some effects, while the persistence properties are very sensitive to variations in γ (η^d_{nw}, η_{yw}). In sum: the largest effects on persistence stem from the goods-market parameters ρ and γ.

Figure 2 about here

The numerical illustrations provided here indicate that the long-run effects are quantitatively small, that is, the long-run non-neutrality of nominal shocks in open economies does not seem to have a quantitative important effect on the terms of trade.[24] However, one cannot infer from this, that wealth reallocation implied by incomplete capital markets does not play any role.[25] It generates a unit-root in the process for the terms of trade. Moreover, it should be noted that the model-specification in a number of respects induces a downward bias in the wealth effect, since we deviate only marginally from a setting with complete capital markets by assuming perfect capital mobility in a real bond, and since infinite horizon for agents implies that wealth changes are distributed over an infinite horizon with consequent small effects within a given period. Home bias is another dormant channel. If there were Home bias in consumption, wealth changes would generate larger effects on the relative price between the Home and the Foreign goods. Lastly, empirical evidence showing low cross-country correlations of consumption indicates that the issue warrants further research in, e.g., overlapping generations models (Ghironi, 2000), where wealth effects would play a larger role. This could potentially increase the importance of μ as well. Our analytical approach - represented by (27) - emphasizes that wealth effects are likely to be important given the empirical evidence, since the terms of trade is a function of relative consumption.

[24] This also means that the potential problem underlying the solution method of a shock-dependent long-run equilibrium ($\rho \neq 1$) is not a major problem.

[25] Betts and Devereux (1999) use that argument, that since relative consumption changes are small under incomplete capital markets, the capital market structure is not important for monetary shocks.

Does the model generate effects of nominal shocks beyond the time period of the exogenously imposed contract length of two periods? It does if the demand elasticity is low, the sensitivity of output with respect to profitability is high, or if labor supply is inelastic. Since all three properties may seem likely to hold (in the short run) this yields support to the view that staggering not only has an important qualitative role in producing a plausible path for the terms of trade (contrary to the one-period contracts) but also that it has quantitative importance.[26] It is particularly interesting that the closer we are to constant returns to scale (for $\gamma \rightarrow 1$, $\eta_{yw} \rightarrow \infty$), the stronger the persistence. Since this case implies that nominal prices are also sticky, it suggests that the combination of nominal wage and price stickiness may generate very strong persistence. However, except in the limiting case where the sensitivity of output to profitability is large, the persistence generated is not as strong as observed in the data. This indicates that staggered two-period nominal wage contracts cannot fully solve the persistence puzzle.

4.1 Three-period contracts

It is a natural next step to analyze how the dynamic properties change when the number of overlapping contracts is extended. Longer duration of staggered contracts has two effects. First, longer nominal contracts prolong the impact real effects of nominal shocks. Secondly, the dynamic adjustment process changes due to the interaction between an increasing number of contracts set at different points in time. On the other hand, by interpreting the period length under three-period contracts as 2/3 of the period length under two-period contracts it is possible to analyze how less synchronization of wage formation affects the dynamic adjustment path for given contract lengths. It can be shown that the terms of trade under this contract structure evolve according to an ARIMA(2,1,3) process, which suggests a qualitative difference compared to two-period staggering. Quantitatively (not reported), though, it seems that the persistence properties are only affected very moderately.

Summarizing our findings we find that the introduction of staggering (one-period contracts versus two-period staggering) has strong qualitative implications, while a strengthening of asynchronization (two-period staggering versus three-period staggering) only has a moderate effect. We interpret this as indicating that the introduction of backward and forward looking elements via staggering is the important mechanism while further asynchro-

[26] In the baseline case 10% of the impact effect remains after all contracts have been renewed once. For the parameter choice $\gamma = 0.9$ and $\rho = 1.5$ the percentage is 32%. Hence, the endogenous persistence found is no less than that reported by Bergin and Feenstra (2001) and stronger than in Chari, Kehoe, and McGrattan (1998).

nization has little effects. This may be explained by the fact that we in the present set-up introduce staggering in a way precluding strategic considerations in wage adjustments.[27]

5 Concluding remarks

The explicit analytical approach taken to the analysis of persistence of nominal shocks in this paper, has shown that the wage-price spiral is the crucial mechanism determining the sluggishness of the adjustment process. The intertemporal general equilibrium approach allowed us to identify the structural parameters determining the wage-price spiral and thus the persistence in the adjustment process to nominal shocks. Numerical illustrations indicated the sensitivity to variations in the various parameters, and in particular the technology parameter (the degree of returns to scale) turned out to be critical. Assessing the quantitative strength of the propagation mechanism in general, we find that staggered nominal wage contracts have an important effect when compared to the effects arising in the standard case of one-period contracts. Though, it is also clear that staggered contracts cannot match a half-life of the effects of shocks at the level of 3-4 years, unless the open economy wage-price spiral is fairly strong (in the present model this is the case if we are close to constant returns to scale in production). This may suggest that adding labor hoarding or a putty-clay model for real capital may strengthen propagation substantially.

The results of the paper suggest two implications concerning various assumptions on wage and price setting. First, whether nominal wages are staggered and nominal prices flexible, or oppositely nominal prices are staggered and nominal wages flexible should not have any implications for the persistence issue, but only for the impact effects. The impact effects obviously differs depending on whether nominal wage or prices are rigid in the short-run. However, the wage-price spiral is independent of this. This conjecture is proved correct by Hansen and Nielsen (2000), who show that the present model with staggered nominal prices instead of wages yields the exact same persistence results. Thus, one should think of conditions for persistence in both closed and open economies in terms of conditions for a strong wage-price spiral.

[27] In an earlier version of their model, Bergin and Feenstra (2001) find that increasing the contract length and number of contracting groups, increase endogenous persistence. The intuition for this is simply that their model contains a strategic element in the sense that the expenditure share for each good is inversely related to its relative price. This makes firms reluctant to alter their price following shocks.

Second, the issue of producer currency pricing and local currency pricing - and, hence, the expenditure switching effect - may in the same vein be of crucial importance for the impact effects of nominal shocks (as shown by Betts and Devereux, 2000), but of no consequences for the persistence issue since the fundamental price-wage interdependencies are the same irrespective of which pricing assumption is made. An interesting issue for further research is to check whether the latter conjecture is correct.

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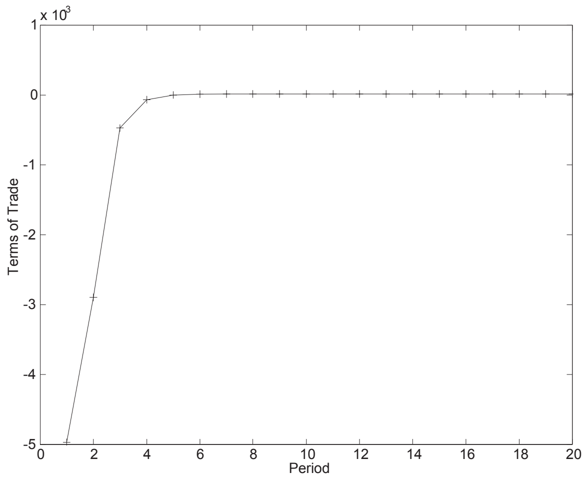
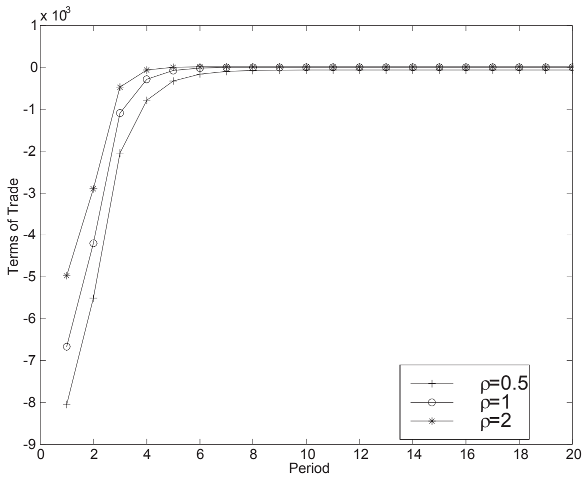
Figure 1

Terms-of-trade impulse-response functions to an expansion in relative Home money supply (1 percent).[28]



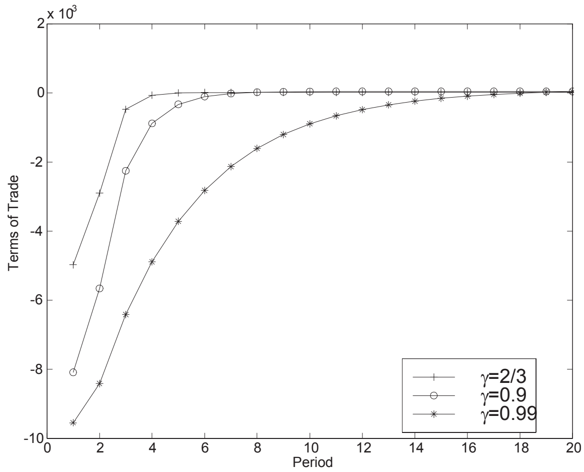
i) Baseline case



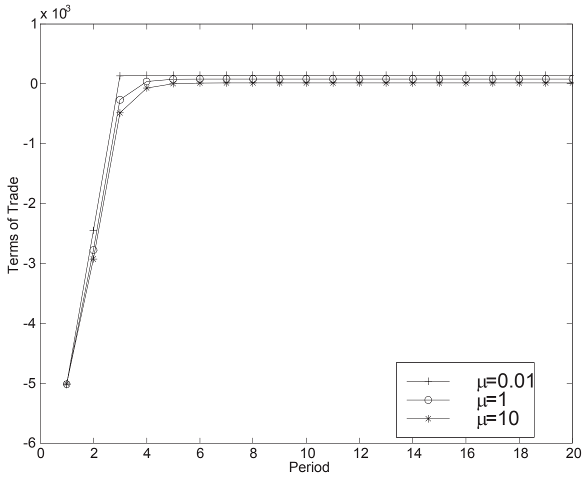
ii) Variations in ρ

[28]Only one parameter varies. The rest are kept on their baseline values. Remember that $\eta_{nw}^{d} = \frac{1}{\gamma-1}$, $\eta_{yw} = \frac{\gamma}{\gamma-1}$ and $\eta_{nw}^{s} = \mu^{-1}$.

Figure 1
Terms-of-trade impulse-response functions to an expansion in relative Home money supply (1 percent).

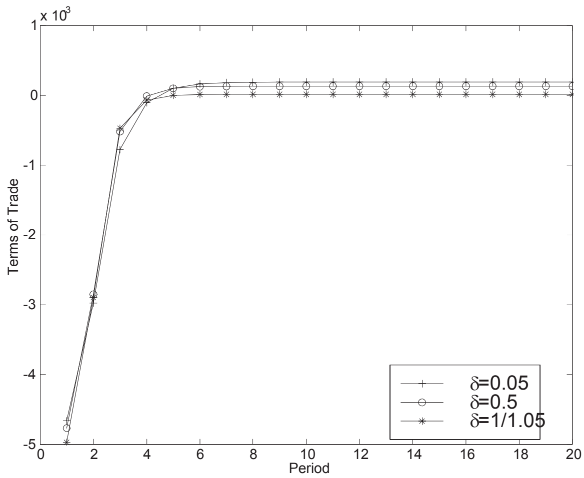


iii) Variations in γ



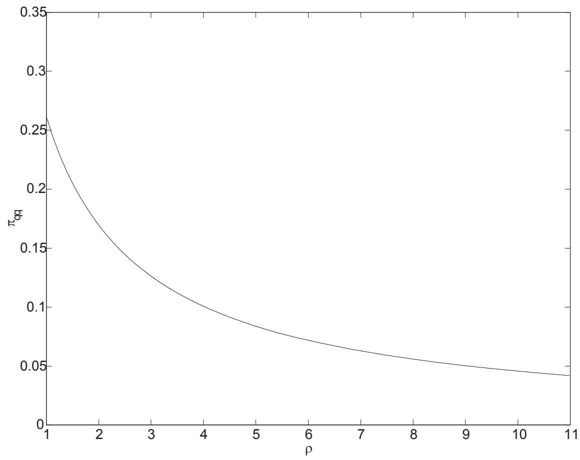
iv) Variations in μ

Figure 1
Terms-of-trade impulse-response functions to an expansion in relative Home money supply (1 percent).

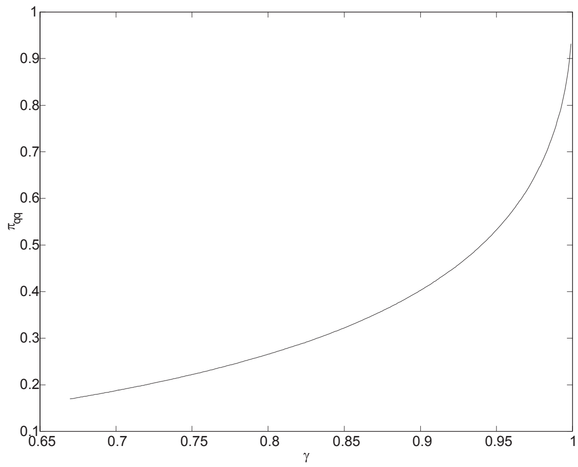


v) Variations in δ

Figure 2
Sensitivity of persistence parameter π_{qq}.[29]



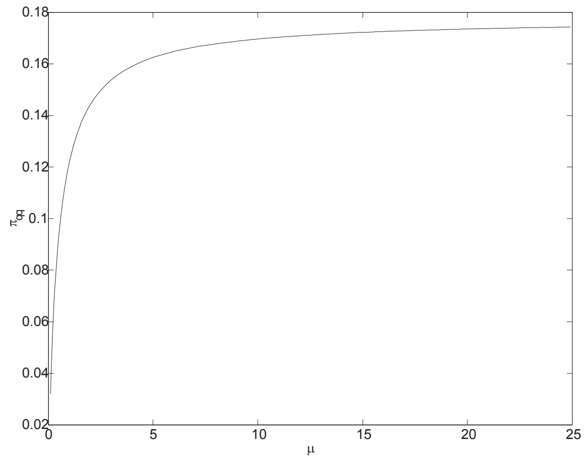
i) Dependence on ρ



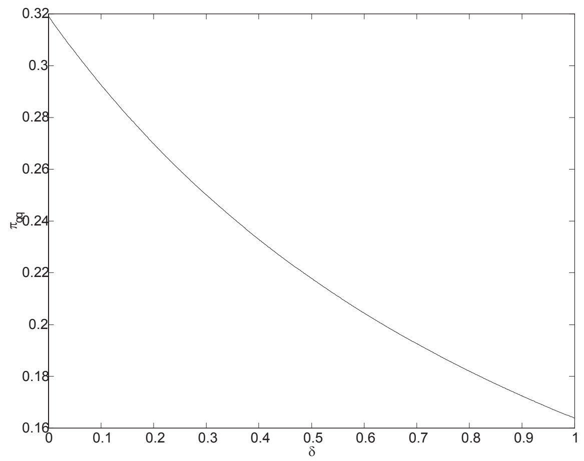
ii) Dependence on γ

[29] Only one parameter varies. The rest are kept on their baseline values.

Figure 2
Sensitivity of persistence parameter π_{qq}.



iii) Dependence on μ



iv) Dependence on δ

APPENDIX

6 Steady state and log-linearization

Our analysis builds on a version of the model set up in section 2 in logarithms. The first-order conditions are

$$C_t^{-\frac{1}{\sigma}} = \delta\left(1+r_t\right)E_t\left(C_{t+1}^{-\frac{1}{\sigma}}\right), \tag{45}$$

$$C_t^{-\frac{1}{\sigma}} = \lambda\left(\frac{M_t}{P_t}\right)^{-\varepsilon} + E_t\left(\delta C_{t+1}^{-\frac{1}{\sigma}}\frac{P_t}{P_{t+1}}\right), \tag{46}$$

As is apparent the money first-order condition is not linear in logs and subsequently we have to approximate. The steady-state version of the model is similar to that analyzed in, for example, Obstfeld and Rogoff (1995). We focus on a symmetric non-stochastic steady state where $B = B^* = 0$, $C = C^* = Y^h = Y^{*f} = Y = Y^*$, $r = \delta^{-1} - 1$, $\frac{P^h}{P} = \frac{P^f}{P} = \frac{P^{*h}}{P^*} = \frac{P^{*f}}{P^*} = 1$, $\frac{W}{P} = \frac{W^*}{P^*}$. Real incomes are $Y = \frac{P^hY^h}{P}$ and $Y^* = \frac{P^{*f}Y^{*f}}{P^*}$. Steady-state values are indicated by omission of time subscripts.

Next step is to log-linearize the first-order conditions arising from consumer optimization (45)-(46). The log-linearized Euler equation (9) is obtained by using the convenient formula for lognormally distributed variables

$$\log E\left(X^f\right) = fE\left[\log\left(X\right)\right] + \frac{f^2}{2}Var\left[\log\left(X\right)\right], \tag{47}$$

where f is a scalar and X is lognormally distributed. The money demand warrants a comment. Taking logs on both sides of (46) yields the log of a sum and it is easy to show that around a steady state (disregarding constants)

$$\log(X_t + Z_t) = \frac{X}{X+Z}\log(X_t) + \frac{Z}{X+Z}\log(Z_t). \tag{48}$$

Using this we get that

$$\begin{aligned}\log\left[\lambda\left(\frac{M_t}{P_t}\right)^{-\varepsilon} + E_t\left(\delta C_{t+1}^{-\frac{1}{\sigma}}\frac{P_t}{P_{t+1}}\right)\right] &= (1-\delta)\log\left[\lambda\left(\frac{M_t}{P_t}\right)^{-\varepsilon}\right] \\ &\quad +\delta\log\left[E_t\left(\delta C_{t+1}^{-\frac{1}{\sigma}}\frac{P_t}{P_{t+1}}\right)\right].\end{aligned}$$

Eq. (10) follows immediately with

$$\eta_{mc} = \frac{1}{\sigma\left(1-\delta\right)\varepsilon}, \qquad \eta_{mc}^1 = \frac{\delta}{\sigma\left(1-\delta\right)\varepsilon}, \qquad \eta_{mp} = \frac{\delta}{\left(1-\delta\right)\varepsilon}. \tag{49}$$

Note, that following a shock the economy moves away from the initial steady state and does not return for $\rho \neq 1$. The log-linearized first-order condition for money demand (10) still holds, though. Log-linearizing (46) around any steady state with no inflation will yield (10) (disregarding constants). The numerical illustrations showed that the long-run real effects are very small. Thus, any error due to linearization is small as well. For $\rho = 1$ the economy returns to its initial steady state.

While the model is specified so as to yield a log-linear structure, we have that the budget constraint is linear in levels, i.e.

$$B_t = (1 + r_{t-1}) B_{t-1} + Y_t - C_t. \tag{50}$$

Subtracting the steady-state version of the budget constraint from (50) and dividing by Y $(= C)$ we get

$$\begin{aligned} \frac{B_t - B}{Y} &= (1+r) \frac{B_{t-1} - B}{Y} + \frac{Y_t - Y}{Y} - \frac{C_t - C}{C} \\ &\quad + [(1 + r_{t-1}) - (1+r)] \frac{B_{t-1} - B}{Y}. \end{aligned}$$

The last term on the right-hand side is negligible as we look at small deviations around steady state. We end up with

$$b_t = \delta^{-1} b_{t-1} + y_t - c_t, \tag{51}$$

as $1 + r = \delta^{-1}$, $\log\left(\frac{Y_t}{Y}\right) \approx \frac{Y_t - Y}{Y}$, $b_t = \frac{B_t}{Y}$ and $\log\left(\frac{C_t}{C}\right) \approx \frac{C_t - C}{C}$.

7 Equilibrium with two-period nominal wage staggering

We conjecture a solution for the exchange rate, the terms of trade, and relative consumption as

$$s_t = \pi_{sc} (c_t - c_t^*) + \pi_{sm} (m_t - m_t^*), \tag{52}$$

$$q_t = \pi_{qc} \left(c_{t-1} - c_{t-1}^*\right) + \pi_{qq} q_{t-1} + \pi_{qu} u_t + \pi_{qu}^1 u_{t-1}, \tag{53}$$

$$\begin{aligned} c_t - c_t^* &= \pi_{cb} \left(b_{t-1} - b_{t-1}^*\right) + \pi_{cc} \left(c_{t-1} - c_{t-1}^*\right) \\ &\quad + \pi_{cq} q_{t-1} + \pi_{cu} u_t + \pi_{cu}^1 u_{t-1}. \end{aligned}$$

Note that the Euler equation implies

$$E_t \left(c_{t+1} - c_{t+1}^*\right) = c_t - c_t^*, \tag{54}$$

which by use of the guess implies that

$$c_{t+1} - c_{t+1}^* = c_t - c_t^* + \pi_{cu} u_{t+1}. \tag{55}$$

7.1 Nominal exchange rate

Equalizing money demands and supplies yields

$$s_t = \eta_{sc}\left(c_t - c_t^*\right) + \eta_{ss} E_t s_{t+1} + \eta_{sm}\left(m_t - m_t^*\right), \tag{56}$$

where,

$$\eta_{sc} = \frac{\eta_{mc}^1 - \eta_{mc}}{1+\eta_{mp}}, \quad \eta_{ss} = \frac{\eta_{mp}}{1+\eta_{mp}}, \quad \eta_{sm} = \frac{1}{1+\eta_{mp}}. \tag{57}$$

Taking expectations and inserting yields

$$\pi_{sc} = \eta_{mc}^1 - \eta_{mc} = -\left(\sigma\varepsilon\right)^{-1}, \quad \pi_{sm} = 1. \tag{58}$$

7.2 Terms of trade

Using the expressions for aggregate wages, product market equilibrium determines the terms of trade as

$$\begin{aligned} q_t \;=\; & \eta_{qc}\left(c_{t-1} - c_{t-1}^*\right) + \eta_{qq}\left(E_{t-2}q_{t-1} + \delta E_{t-2}q_t + E_{t-1}q_t + \delta E_{t-1}q_{t+1}\right) \\ & + \eta_{qu}u_t + \eta_{qu}^1 u_{t-1}, \end{aligned}$$

where

$$\eta_{qq} = \frac{4\eta_{yw}}{4\eta_{yw} - 4\rho}\frac{2\eta_{wp} - 1}{2\left(1+\delta\right)}, \quad \eta_{qu} = -\frac{4\eta_{yw}}{4\eta_{yw} - 4\rho}\left(\pi_{sm} + \pi_{sc}\pi_{cu}\right), \tag{59}$$

$$\eta_{qc} = \frac{4\eta_{yw}\eta_{wc}}{4\eta_{yw} - 4\rho}, \quad \eta_{qu}^1 = -\frac{2\eta_{yw}}{4\eta_{yw} - 4\rho}\left(\pi_{sm} + \pi_{sc}\pi_{cu} + \eta_{wc}\pi_{cu}\right). \tag{60}$$

Using our guess to find $E_{t-2}q_{t-1}$, $E_{t-2}q_t$, $E_{t-1}q_t$, and $E_{t-1}q_{t+1}$ we end up with

$$\begin{aligned} q_t \;=\; & \left[\eta_{qc} + \left(2\delta + 1\right)\eta_{qq}\pi_{qc} + \delta\eta_{qq}\pi_{qq}\pi_{qc}\right]\left(c_{t-1} - c_{t-1}^*\right) \\ & + \eta_{qq}\left[1 + \left(\delta + 1\right)\pi_{qq} + \delta\pi_{qq}^2\right] q_{t-1} + \eta_{qu}u_t \\ & + \left[\eta_{qu}^1 - \eta_{qq}\left(\pi_{qu} + \delta\pi_{qc}\pi_{cu} + \delta\pi_{qq}\pi_{qu} - \pi_{qu}^1 - \delta\pi_{qq}\pi_{qu}^1\right)\right] u_{t-1}. \end{aligned}$$

Hence,

$$\pi_{qc} = \eta_{qc} + \left(2\delta + 1\right)\eta_{qq}\pi_{qc} + \delta\eta_{qq}\pi_{qq}\pi_{qc}, \;\; \pi_{qq} = \eta_{qq}\left[1 + \left(\delta + 1\right)\pi_{qq} + \delta\pi_{qq}^2\right], \tag{61}$$

$$\pi_{qu} = \eta_{qu}, \;\; \pi_{qu}^1 = \eta_{qu}^1 - \eta_{qq}\left(\pi_{qu} + \delta\pi_{qc}\pi_{cu} + \delta\pi_{qq}\pi_{qu} - \pi_{qu}^1 - \delta\pi_{qq}\pi_{qu}^1\right). \tag{62}$$

7.3 Relative consumption

Given that

$$y_t - y_t^* = (1-\rho)\,q_t = (1-\rho)\left[\pi_{qc}\left(c_{t-1}-c_{t-1}^*\right)+\pi_{qq}q_{t-1}+\pi_{qu}u_t+\pi_{qu}^1 u_{t-1}\right], \quad (63)$$

we have (se eq. [51])

$$\begin{aligned} b_t - b_t^* &= \delta^{-1}\left(b_{t-1}-b_{t-1}^*\right)-\left(c_t-c_t^*\right) \\ &\quad +(1-\rho)\left[\pi_{qc}\left(c_{t-1}-c_{t-1}^*\right)+\pi_{qq}q_{t-1}+\pi_{qu}u_t+\pi_{qu}^1 u_{t-1}\right]. \end{aligned}$$

It follows that

$$\begin{aligned} E_t\left(c_{t+1}-c_{t+1}^*\right) &= \pi_{cb}\left[\delta^{-1}\left(b_{t-1}-b_{t-1}^*\right)-\left(c_t-c_t^*\right)\right] \\ &\quad +\pi_{cb}(1-\rho)\left[\pi_{qc}\left(c_{t-1}-c_{t-1}^*\right)\right. \\ &\quad \left.+\pi_{qq}q_{t-1}+\pi_{qu}u_t+\pi_{qu}^1 u_{t-1}\right] \\ &\quad +\pi_{cc}\left(c_t-c_t^*\right) \\ &\quad +\pi_{cq}\left[\pi_{qc}\left(c_{t-1}-c_{t-1}^*\right)+\pi_{qq}q_{t-1}+\pi_{qu}u_t+\pi_{qu}^1 u_{t-1}\right] \\ &\quad +\pi_{cu}^1 u_t. \end{aligned}$$

Using the Euler equation and rearranging yields the following restrictions

$$\pi_{cb} = \frac{\delta^{-1}\pi_{cb}}{1+\pi_{cb}-\pi_{cc}}, \quad \pi_{cu} = \frac{\pi_{cb}(1-\rho)\pi_{qu}+\pi_{cq}\pi_{qu}+\pi_{cu}^1}{1+\pi_{cb}-\pi_{cc}}, \quad (64)$$

$$\pi_{cc} = \frac{\pi_{cb}(1-\rho)\pi_{qc}+\pi_{cq}\pi_{qc}}{1+\pi_{cb}-\pi_{cc}}, \quad \pi_{cq} = \frac{\pi_{cb}(1-\rho)\pi_{qq}+\pi_{cq}\pi_{qq}}{1+\pi_{cb}-\pi_{cc}}, \quad (65)$$

$$\pi_{cu}^1 = \frac{\pi_{cb}(1-\rho)\pi_{qu}^1+\pi_{cq}\pi_{qu}^1}{1+\pi_{cb}-\pi_{cc}}. \quad (66)$$

7.4 Analytical characterization of the solution

Proposition 1 *$\pi_{qq} \in (0,1)$.*

Proof. π_{qq} is determined by: $\eta_{qq}\delta\pi_{qq}^2+\left[(\delta+1)\eta_{qq}-1\right]\pi_{qq}+\eta_{qq}=0$. It is seen that $\eta_{qq}=\frac{4\eta_{yw}}{4\eta_{yw}-4\rho}\frac{2\eta_{wp}-1}{2(1+\delta)}\in(0,\frac{1}{2})\Rightarrow\pi_{qq}\in(0,1)$. ■

Proposition 2 *Persistence as measured by π_{qq} is: (i) decreasing in ρ, and increasing in γ and μ; (ii) decreasing in δ.*

Proof. Writing out the expression for η_{qq} we get $\eta_{qq} = \frac{4\eta_{yw}\left(2\eta_{wp}-1\right)}{4\eta_{yw}-4\rho}\frac{1}{2(1+\delta)} = \frac{\gamma\mu}{(1-\gamma+\mu)[\gamma+\rho(1-\gamma)]}\frac{1}{2(1+\delta)} \Rightarrow \frac{\partial\eta_{qq}}{\partial\rho} < 0$, $\frac{\partial\eta_{qq}}{\partial\gamma} > 0$, $\frac{\partial\eta_{qq}}{\partial\mu} > 0$, $\frac{\partial\eta_{qq}}{\partial\delta} < 0$ and (i) follows straightforwardly as $\frac{d\pi_{qq}}{d\eta_{qq}} > 0$.

Note that $\eta_{qq} = \frac{k}{1+\delta}$, where $k = \frac{4\eta_{yw}}{4\eta_{yw}-4\rho}\frac{2\eta_{wp}-1}{2} \in (0,1)$. The quadratic can then be written $\frac{1}{1+\delta}k\left(\delta\pi_{qq}^2+1\right) = (1-k)\,\pi_{qq}$. Now assume that $\delta' < \delta''$ and $\frac{1}{1+\delta''}k\left[\delta''\left(\pi''_{qq}\right)^2+1\right] = (1-k)\,\pi''_{qq}$. Then it has to be the case that $\frac{1}{1+\delta'}k\left[\delta'\left(\pi''_{qq}\right)^2+1\right] < (1-k)\,\pi''_{qq}$ since both terms on the left-hand side decrease and the right-hand side is unchanged. Increasing π_{qq} slightly increases the right-hand side more than the left-hand side ($2\frac{\delta}{1+\delta}k\pi_{qq} < 1-k\ \forall\delta, k, \pi_{qq}$). In sum; $\pi'_{qq} < \pi''_{qq}$ has to hold and since δ' and δ'' were arbitrary (ii) follows. ■

Proposition 3 $\pi_{qc} > 0$.

Proof. From the restriction governing π_{qc} we find $\pi_{qc} = \left[1-(2\delta+1)\,\eta_{qq} - \delta\eta_{qq}\pi_{qq}\right]^{-1}\eta_{qc}$. The result follows as $\pi_{qq} \in (0,1)$, $\eta_{qq} \in (0,\frac{1}{2})$ and $\eta_{qc} = \frac{\eta_{yw}\eta_{wc}}{\eta_{yw}-\rho} = \frac{\eta_{yw}}{\eta_{yw}-\rho}\frac{1}{\left(1-\mu\eta_{nw}^{d}\right)\sigma} > 0$. ■

Proposition 4 $\pi_{cc} \lesseqqgtr 0$ *if* $\rho \gtreqqless 1$.

Proof. By substitution we find $\pi_{cc} = \frac{(1-\delta)(1-\rho)\frac{1}{1-\delta\pi_{qq}}\pi_{qc}}{1-\delta(1-\rho)\frac{1}{1-\delta\pi_{qq}}\pi_{qc}}$ and it follows directly that $\rho > 1 \Rightarrow \pi_{cc} < 0$ and $\rho = 1 \Rightarrow \pi_{cc} = 0$.We can find $\frac{\partial\pi_{cc}}{\partial\rho}$ to be $\left[(1-\rho)\frac{\partial\pi_{qc}}{\partial\rho} - \pi_{qc}\right](1-\delta)\left(1-\delta\pi_{qq}\right) + \delta\,(1-\delta)\,(1-\rho)\,\pi_{qc}\frac{\partial\pi_{qq}}{\partial\rho} \leq 0$ for $\rho \leq 1$. Since $\pi_{cc} = 0$ if $\rho = 1$ it follows that $\pi_{cc} > 0$ if $\rho \in (0,1)$. ■

For later reference it will be useful to define the following constants

$$K_1 = \frac{\pi_{cb}\,(1-\rho) + \pi_{cq}}{1+\pi_{cb}-\pi_{cc}} = \delta\left[\pi_{cb}\,(1-\rho) + \pi_{cq}\right] = \frac{\pi_{cc}}{\pi_{qc}}, \tag{67}$$

$$K_2 = \frac{\frac{1}{2} - \eta_{qq}\left(1+\delta\pi_{qq}\right)}{1-\eta_{qq}\left(1+\delta\pi_{qq}\right)} > 0, \quad K_3 = \frac{\frac{2\eta_{yw}}{4\eta_{yw}-4\rho}\eta_{wc} + \delta\eta_{qq}\pi_{qc}}{1-\eta_{qq}\left(1+\delta\pi_{qq}\right)} > 0. \tag{68}$$

Lemma 5 $\frac{K_3}{\pi_{qc}} = \frac{1}{2}$.

Proof. $\frac{K_3}{\pi_{qc}} = \frac{1}{\pi_{qc}}\frac{\frac{2\eta_{yw}}{4\eta_{yw}-4\rho}\eta_{wc}+\delta\eta_{qq}\pi_{qc}}{1-\eta_{qq}(1+\delta\pi_{qq})} = \left[1-\eta_{qq}\left(1+\pi_{qq}\right)\right]^{-1}\left[\frac{2\eta_{yw}}{4\eta_{yw}-4\rho}\frac{\eta_{wc}}{\pi_{qc}} + \delta\eta_{qq}\right]$

$= \left[1-\eta_{qq}\left(1+\delta\pi_{qq}\right)\right]^{-1}\left[\frac{1}{2}\frac{4\eta_{yw}}{4\eta_{yw}-4\rho}\frac{\eta_{wc}\left[1-(2\delta+1)\eta_{qq}-\delta\eta_{qq}\pi_{qq}\right]}{\eta_{wc}\frac{4\eta_{yw}}{4\eta_{yw}-4\rho}} + \delta\eta_{qq}\right]$

$= \left[1-\eta_{qq}\left(1+\delta\pi_{qq}\right)\right]^{-1}\left\{\frac{1}{2}\left[1-\eta_{qq}\left(1+\delta\pi_{qq}\right)\right]\right\} = \frac{1}{2}$ ■

Lemma 6 $1 + \delta K_1 K_3 > 0$.

Proof. From $\pi_{cc} = K_1 \pi_{qc}$ it is seen that $\rho < 1 \Rightarrow K_1 > 0 \Rightarrow 1 + \delta K_1 K_3 > 0$ since $\pi_{cc} > 0$ and $\pi_{qc} > 0$. For $\rho = 1$ we have that $K_1 = \frac{\pi_{cc}}{\pi_{qc}} = 0 \Rightarrow 1 + \delta K_1 K_3 > 0$. For $\rho > 1$ ($\Rightarrow K_1 < 0$) we simply insert the expressions for K_1 and K_3: $1 + \delta K_1 K_3 > 0 \Rightarrow \delta \frac{\pi_{cc}}{\pi_{qc}} K_3 > -1 \Rightarrow \delta \pi_{cc} > -2 \Rightarrow -\frac{(1-\delta)(1-\rho)\frac{\pi_{qc}}{1-\delta\pi_{qq}}}{1-\delta(1-\rho)\frac{\pi_{qc}}{1-\delta\pi_{qq}}} < \frac{2}{\delta} \Rightarrow (1+\delta)(1-\rho)\frac{\pi_{qc}}{1-\delta\pi_{qq}} < \frac{2}{\delta}$ and this is always fulfilled when $\rho > 1$. ■

Proposition 7 $\begin{cases} \pi_{cu} > 0 \ \textit{if} \ \rho > 1 \\ \pi_{cu} = 0 \ \textit{if} \ \rho = 1 \\ \pi_{cu} < 0 \ \textit{if} \ \underline{\rho} \leq \rho < 1, \quad \underline{\rho} \in (0,1). \end{cases}$

Proof. Substituting the restriction governing π^1_{cu} into π_{cu} we obtain

$$\pi_{cu} = K_1 \left(\pi_{qu} + \delta \pi^1_{qu} \right). \tag{69}$$

Using that $\eta^1_{qu} = \frac{1}{2}\pi_{qu} - \frac{2\eta_{yw}}{4\eta_{yw}-4\rho}\eta_{wc}\pi_{cu}$ we find

$$\pi^1_{qu} = K_2 \pi_{qu} - K_3 \pi_{cu} \ \Rightarrow \ (1 + \delta K_1 K_3)\pi_{cu} = K_1 (1 + \delta K_2) \pi_{qu}, \tag{70}$$

or substituting in for π_{qu}

$$K_4 \pi_{cu} = K_5, \tag{71}$$

$$K_4 = 1 + \delta K_1 K_3 - K_1 (1 + \delta K_2) \frac{\eta_{yw}}{\eta_{yw} - \rho} \frac{1}{\sigma\varepsilon}, \ K_5 = -\frac{\eta_{yw} K_1 (1 + \delta K_2)}{\eta_{yw} - \rho}. \tag{72}$$

Again we have three cases
$\rho > 1 \Rightarrow K_1 < 0, 1 + \delta K_1 K_3 > 0 \Rightarrow K_4 > 0, K_5 > 0 \Rightarrow \pi_{cu} > 0$,
$\rho = 1 \Rightarrow K_1 = 0 \Rightarrow K_4 > 0, K_5 = 0 \Rightarrow \pi_{cu} = 0$,
$\underline{\rho} \leq \rho < 1 \Rightarrow K_1 > 0 \Rightarrow K_4 > 0, K_5 < 0 \Rightarrow \pi_{cu} < 0$,
where $\underline{\rho}$ is defined such that $K_4 > 0$ if $\rho \geq \underline{\rho}$. Note that $K_4 \to 1$ as $\rho \to 1$. ■

Proposition 8 *The terms of trade fall on impact* ($\pi_{qu} < 0$).

Proof. π_{qu} is given as $-\frac{\eta_{yw}}{\rho+\eta_{yw}}\left[1 - (\sigma\varepsilon)^{-1}\pi_{cu}\right]$ and the result follows trivially for $\rho \leq 1$. For $\rho > 1$ note that $\pi_{cc} < 0 \Rightarrow \pi_{qu} = \frac{\pi_{qc}\pi_{cu}}{\pi_{cc}} < 0$ as $\pi_{cu} > 0$ and $\pi_{qc} = \left(1 - \eta_{qq}\right)^{-1}\eta_{qc} > 0$. ■

Proposition 9 *The nominal exchange rate depreciates on impact* $\left(\frac{\partial s_t}{\partial u_t} > 0\right)$.

Proof. $\pi_{qu} < 0 \Leftrightarrow 0 < 1 - (\sigma\varepsilon)^{-1}\pi_{cu} = \frac{\partial s_t}{\partial u_t}$. ■

Lemma 10 *The terms of trade can be written as* $q_t = \sum_{j=0}^{\infty} \pi^j u_{t-j}$, *where,* $\pi^0 = \pi_{qu}$, $\pi^1 = \pi_{qu}^1 + \pi_{qq}\pi_{qu} + \pi_{cu}\pi_{qc}$ *and* $\pi^{j+1} = \pi_{qq}\pi^j + \pi_{qc}\pi_{cu}$ *for* $j = 1, 2, ...$

Proof. From $q_t = (1 + \pi_{qq})\, q_{t-1} - \pi_{qq} q_{t-2} + \pi_{qu} u_t + \left(\pi_{qu}^1 - \pi_{qu} + \pi_{qc}\pi_{cu}\right) u_{t-1} - \pi_{qu}^1 u_{t-2}$ the result follows straightforwardly by recursive substitution. ■

Proposition 11 *The terms of trade adjust gradually to its long-run value if* $\rho \geq 1$.

Proof. $\rho > 1$: We know that $\pi_{cu} > 0$, $\pi_{qu} < 0$ and $\pi_{qu}^1 = K_2\pi_{qu} - K_3\pi_{cu} < 0$. Furthermore, the long-run value $\frac{\pi_{qc}\pi_{cu}}{1-\pi_{qq}}$ is strictly positive. The basic strategy of the proof is to show $\pi^j < \pi^{j+1}$, $j = 0, 1, 2, ...$First, let us show that $\pi^0 < \pi^1$. The expression for π^1 is $\pi_{qu}^1 + \pi_{qq}\pi_{qu} + \pi_{cu}\pi_{qc}$ which can be written as $(K_2 + \pi_{qq})\, \pi_{qu} + (\pi_{qc} - K_3)\, \pi_{cu}$. Since $(K_2 + \pi_{qq}) \in \left(\frac{1}{2}, 1\right)$ and $\pi_{qc} - K_3 > 0$ (by Lemma 9) π^1 has to be strictly greater that π^0. Next, we will show that $\pi^j < \pi^{j+1}$ for $j = 1, 2, ...$If $\pi^j < 0$, then π^{j+1} has to be larger than π^j as π^{j+1} is some fraction of π^j $[\pi_{qq} \in (0, 1)]$ plus $\pi_{qc}\pi_{cu} > 0$. If $\pi^j > 0$ the result follows from observing that if $\pi^{j+1} < \pi^j$ then q_t would converge to zero as $\pi^{j+1} < \pi^j \Rightarrow \pi^{j+2} < \pi^{j+1} \Rightarrow \pi^{j+3} < \pi^{j+2}$.....and we know q_t converges to $\frac{\pi_{qc}\pi_{cu}}{1-\pi_{qq}} > 0$. Note that π^j would never exceed the long-run value as this would imply π^{j+1} would be greater than the long-run value as well.

$\rho = 1$: When the demand elasticity is one we have $\pi_{cu} = 0$, $\pi_{qu} < 0$ and $\pi_{qu}^1 = K_2\pi_{qu} \in \left(\frac{1}{2}\pi_{qu}, 0\right)$. It is easily seen that $\pi^1 = (K_2 + \pi_{qq})\, \pi_{qu} > \pi_{qu} = \pi^0$. Furthermore, $\pi^{j+1} > \pi^j$ for all $j = 1, 2, ...$ as π^{j+1} is some fraction $\pi_{qq} \in (0, 1)$ of π^j. ■

Proposition 12 *If* $\underline{\rho} \leq \rho < 1$ *the terms of trade adjust gradually to its long-run value at least from the period after the shock.*

Proof. With $\rho \in [\underline{\rho}, 1)$ we have that $\pi_{cu} < 0$, $\pi_{qu} < 0$ and $\frac{\pi_{qc}\pi_{cu}}{1-\pi_{qq}} < 0$. Lets split this case into two subcases: one (plausible), where the long-run effect is larger ($\pi_{qu} < \frac{\pi_{qc}\pi_{cu}}{1-\pi_{qq}}$), and one, where the long-run effect is smaller than the impact effect.

In the first subcase $\pi_{qu} < \frac{\pi_{qc}\pi_{cu}}{1-\pi_{qq}} < 0$. First notice that π^j, $j = 2, 3, ...$ would never exceed the long-run value ($\frac{\pi_{qc}\pi_{cu}}{1-\pi_{qq}}$) if $\pi^1 < \frac{\pi_{qc}\pi_{cu}}{1-\pi_{qq}}$as this would imply π^{j+1} also would exceed the long-run value contradicting that q converges to $\frac{\pi_{qc}\pi_{cu}}{1-\pi_{qq}}$. Lastly, notice that if $\pi^1 < \frac{\pi_{qc}\pi_{cu}}{1-\pi_{qq}}$ then $\pi^j < \pi^{j+1}$ for $j = 1, 2, ...$ has to be the case as the q would otherwise diverge from the long-run value. Similar arguments apply if $\pi^1 > \frac{\pi_{qc}\pi_{cu}}{1-\pi_{qq}}$.

In the second subcase $0 > \pi_{qu} > \frac{\pi_{qc}\pi_{cu}}{1-\pi_{qq}}$. First notice that π^j, $j = 2, 3, ...$would never dip below the long-run value $(\frac{\pi_{qc}\pi_{cu}}{1-\pi_{qq}})$ if $\pi^1 > \frac{\pi_{qc}\pi_{cu}}{1-\pi_{qq}}$ as this would imply π^{j+1} would be further below the long-run value contradicting that q converges to it. Lastly, notice that if $\pi^1 > \frac{\pi_{qc}\pi_{cu}}{1-\pi_{qq}}$ then $\pi^j > \pi^{j+1}$ for $j = 1, 2, ...$ has to be the case as q would otherwise diverge from the long-run value. Similar arguments apply if $\pi^1 < \frac{\pi_{qc}\pi_{cu}}{1-\pi_{qq}}$. ■

One implication of this proposition is that we cannot rule out, in the subcase of $\rho < 1$ and a long-run effect of the terms of trade numerically larger than the impact effect, that after the initial deterioration in the period of the shock the terms of trade deteriorate further in the period after and from there gradually adjusts to its long-run level. Similarly, we cannot rule out, in the other subcase of $\rho < 1$ and a long-run effect of the terms of trade smaller (numerically larger) than the impact effect, that after the initial deterioration of the terms of trade, the terms of trade actually improve in the period after (relative to the impact level) and from then on gradually deteriorates to its long-run level which is below the impact level. That said, our numerical exercises showed that only for extreme and implausible parameter values would these cases arise, as well as the long-run effect of the terms of trade being smaller than the impact effect (numerically larger) seemed to be highly unlikely.