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Systemic Risk in the Danish Interbank Netting System*

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November 7, 2002

Abstract

Central banks have become increasingly worried about systemic risks to the financial market and infrastructure stemming from payment systems. Failure to settle by a participant in a netting system can potentially jeopardize the settlement of other participants. The fear of a systemic crisis has been one of the primary motivations for the introduction of Real Time Gross Settlement systems around the world. This paper provides an assessment of the systemic risk inherent in the Danish interbank netting system. In accordance with similar empirical studies conducted in other European countries, the risk is found to be low.

Keywords: Payment system, systemic risk, netting.

Resume

Den systemiske risiko i det danske detailbetalingssystem, Sumclearingen undersøges. På basis af daglige data for deltagernes faktiske positioner i Sumclearingen simuleres effekten af, at deltageren med den største multilaterale nettoposition henlægges, dvs. ikke kan gennemføre afviklingen. Systemiske effekter opstår i det omfang, en deltagers henlæggelse betyder, at andre deltagere også henlægges. Analysen viser i lighed med tilsvarende studier i andre europæiske lande, at den systemiske risiko er meget lav. Der reserveres væsentligt mere likviditet til afviklingen i Sumclearingen end nødvendigt. På de fleste dage er mindre end en femtedel af den reserverede likviditet tilstrækkelig til at undgå systemiske effekter af, at den største enkelt deltager ikke kan gennemføre afviklingen.

*We thank Bent Overgaard, PBS, for providing data from the Danish interbank netting system. The views expressed in this paper do not necessarily reflects those of Danmarks Nationalbank, the Danish Bankers Association, Federal Reserve Bank of New York or the Federal Reserve System.

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1 Introduction

The role of the central bank can be described as the trilogy of monetary policy, financial stability[1] and payment systems. Monetary policy is the paramount objective, but in recent years the emphasis on financial stability and payment systems has increased remarkably. In particular, central banks have put additional emphasis on the interrelation between financial stability and financial market infrastructure. Risks stemming from the payment system in its capacity as the backbone of the financial market have thus become a concern for central banks. Of principal importance are risks that lend to failure of the financial system.

Systemic risk considerations have been one of the primary motivations for the introduction of real time gross settlement (RTGS) systems around the world (see e.g. Bank for International Settlements (BIS) [8], p. 11). Moreover, the Committee on Payment and Settlement Systems under the auspices of BIS has recently published the report: "Core Principles for Systemically Important Payment Systems" (Core Principles), which lists a set of standards to be met by systemically important payment systems. The purpose of the standards is to achieve a dual objective of safety and efficiency. The report lays out the responsibilities of the central bank in that regard, and many central banks around the world are in the process of evaluating payment systems within their sphere of influence according to these standards.

An extensive literature exists on the issue of (systemic) risk in payment systems. Berger et al [13] provides a graphical framework for analyzing risk, cost and efficiency in payment systems. Rochet and Tirole [23] discusses how to control risk by combining the respective advantages of net and gross settlement. Chakravorti [14] presents a theoretical model of the systemic risk in multilateral net settlement systems and analyzes the prudent response of the central bank in the event of a systemic crisis, in light of the fact that central bank

[1] Several central banks, including Danmarks Nationalbank, now publish periodical assessments of the stability of the financial sector.

involvement is likely to create moral hazard problems.

Humphrey [18] appears to be the first attempt to quantify systemic risk in a payment system. The paper analyzes the U.S. banking sector and uses data from Clearing House Interbank Payment System (CHIPS) on two randomly selected business days in January 1983. CHIPS is a wholesale payment netting system and has historically been used to settle large value dollar payments e.g. the dollar leg of FX trades. The study finds that the failure of the bank with the largest settlement obligation would have had a major disruptive system-wide effect in the U.S.

In contrast to this study, Angelini et al [1], [2] reach an almost opposite conclusion using Italian data. They simulate the default of any bank and not only the bank with the largest settlement obligation. The authors attribute the difference to the fact that the total amount of transactions in the Italian clearing system was relatively small compared to the U.S. at the time. These empirical studies have since been supplemented by analysis conducted by the Bank of Finland (see Kuussaari [20]) and the results are comparable to the findings of the Italian study.

McAndrews and Wasilyew [21] undertake a similar study but utilize synthetic data in order to quantify the underlying drivers of the size of the systemic risk. They find that systemic risk is proportional to the size of the underlying payments. Furthermore, they find that greater likelihood of interaction between two banks results in greater losses to the system. A highly concentrated banking system is thus likely to have a higher degree of systemic risk. Finally, they argue that ceteris paribus the systemic risk is increasing with the number of banks in the system.

A related line of research is found in Sheldon and Maurer [24] and Furfine [16]. They both analyze the interbank exposure from over-night interbank loans. These loans are often arranged through the RTGS system and are given on an unsecured basis. As such, the source of the interbank exposure is different but the methodology used to quantify the inherent systemic risk is similar to the studies discussed above.

The focus of this paper is to assess systemic risks to the financial sector stemming from the payment system and, in particular, how to quantify these risks. Section 2 introduce a general framework for analyzing systemic risk and section 3 presents the methodology employed in the paper. An overview of the Danish payment system is provided in section 4 and the systemic risk inherent in the Danish interbank netting system is quantified by way of simulation in the following section. Section 6 concludes.

2 Systemic Risk

There is no general and universal definition of systemic risk in the economic or financial market literature.[2] Here, we follow the framework of De Bandt and Hartmann [4] who define a systemic event as an event where a shock to either a set of financial institutions or markets (e.g. their failure or crash) lead to considerable adverse effects on other financial institutions or markets. Based on this definition of a systemic event, systemic risk can be defined as the risk of such an event. A systemic event consists of two parts: the shock and the propagation mechanism. Systemic risk thus has two dimensions: the severity of the systemic event and the likelihood of it occurring. Within this framework, a systemic crisis can be defined as a state of the world where the systemic event has severely impaired the functioning of the financial market or an important part here of, i.e., the payment system.

We take the set of financial institutions to be the participants of a payments system and

[2] Mishkin [22] suggests that: "Systemic risk is the likelihood of a sudden, usually unexpected event that disrupts information in financial markets, making them unable to effectively channel funds to those parties with the most productive investment opportunities". Kaufmann [19] writes: "To me systemic or contagion risk is the probability that cumulative losses will occur from an event that sets in motion a series of successive losses along a chain of institutions of markets comprising a system". In reference to a payment system the Angell Report (BIS [5]) defines "[System risk is] the risk that the inability of one institution within a payments system, as in the financial market generally, to meet its obligations when due will cause other participants or financial firms to be unable to meet their obligations when due".

define a systemic event as the event where the settlement failure of one institution leads to the settlement failure of at least one other institution. The trigger of the systemic event is not explicitly modeled in what follows below but rather taken to be exogenous. In general, the trigger or the initial shock can either be specific to an institution, a set of institutions or of a system-wide nature. Institution specific or idiosyncratic shocks include poor management, computer problems and bankruptcy. System-wide or systematic shocks include various types of macroeconomic shocks such as a stock market crash, currency devaluation or sovereign debt moratorium. Other types of shocks, such as terrorist attacks, can be either specific to institutions or system-wide.

3 Methodology[3]

Netting systems operate either on a bilateral or multilateral basis. Here, we are interested in multilateral netting, but the bilateral net positions emerge as a by-product from the calculations of the multilateral net positions. A multilateral netting system requires an operator to calculate the net positions and an agent to facilitate settlement.[4] The central bank is often the settlement agent but not, in general, the operator of the system.[5]

Let n denote the number of banks participating in the netting system. Let z_{ij} denote the

[3] A numerical example of the calculations presented in section 3 can be found in the appendix.

[4] The process of transmitting and reconciling obligations and the calculation of the positions to be settled is usually referred to as "clearing".

[5] Core principle VI states: "Assets used for settlement should preferably be a claim on the central bank; where other assets are used, they should carry little or no credit risk and little and or no liquidity risk" (BIS [9]).

gross amount to be paid from bank i to bank j as reported by bank i.[6] The matrix

$$\underset{n\times n}{\mathbf{Z}} = \begin{bmatrix} 0 & z_{12} & \cdots \\ z_{21} & 0 & \\ \vdots & & \ddots \end{bmatrix} \tag{1}$$

is a convenient way of representing all the gross settlement obligations. We define the aggregate gross settlement obligation (GSO) as

$$\text{GSO} \equiv \sum_i \sum_j |z_{ij}| \,. \tag{2}$$

Let $\mathbf{B}$ denote the matrix of bilateral net positions between bank i and bank j.

$$\underset{n\times n}{\mathbf{B}} = \mathbf{Z} - \mathbf{Z}' = \begin{bmatrix} 0 & z_{12} - z_{21} & \cdots \\ z_{21} - z_{12} & 0 & \\ \vdots & & \ddots \end{bmatrix} = \begin{bmatrix} 0 & b_{12} & \cdots \\ b_{21} & 0 & \\ \vdots & & \ddots \end{bmatrix} \tag{3}$$

and define the aggregate bilateral net position (BNP) as

$$\text{BNP} \equiv \sum_i \sum_{j\geq i} |b_{ij}| \tag{4}$$

since the lower triangular of $\mathbf{B}$ is equal to the negative of the upper triangular. The multilateral net positions are given by

$$\underset{n\times 1}{\mathbf{d}} = \mathbf{B} \cdot \mathbf{1} \tag{5}$$

where $\mathbf{1}$ is an unit vector. If $d_i > 0$ then bank i owes funds to the settlement agent whereas bank i will receive funds from the settlement agent if $d_i < 0$. The position of the settlement agent is zero, i.e., $\mathbf{d}'\mathbf{1} = 0$. We define the aggregate multilateral net position (MNP) as

$$\text{MNP} \equiv \sum_i d_i I(d_i > 0) = \frac{1}{2} \sum_i |d_i| \tag{6}$$

where $I()$ is the indicator function.

[6] z_{ij} reflects the net amount of credit and debit requests vis-à-vis bank j that bank i has received. z_{ij} can be negative if, for example, the amount of checks drawn on bank j and deposited at bank i is sufficiently large.

3.1 Netting Effect

By reducing the number and overall value of payments between financial institutions, netting can enhance the efficiency of payment systems.[7] Gross settlement requires as many as $n(n-1)$ transactions, bilateral net settlement requires up to $n(n-1)/2$ and multilateral net settlement requires no more than n transactions. We measure the increase in efficiency by comparing the reduction in the overall value of payments relative to the gross settlement of obligations.[8] We define the bilateral netting effect (BNE) as

$$\text{BNE} \equiv \frac{\text{GSO} - \text{BNP}}{\text{GSO}} \tag{7}$$

and the multilateral netting effect (MNE) as

$$\text{MNE} \equiv \frac{\text{GSO} - \text{MNP}}{\text{GSO}}. \tag{8}$$

3.2 Quantifying systemic risk

We focus narrowly on the severity of the systemic event given a shock and do not attempt to explain nor estimate the likelihood of an event. Furthermore, we focus on institution specific shocks. We envision a financial failure scenario where a participant enters into bankruptcy. Specifically, we assume that bank k (e.g. the bank with the largest multilateral net debit position) defaults on its settlement obligations, $z_{kj} = 0 \ \forall \ j$, and that all other participants can renege on their obligations towards this bank, $z_{ik} = 0 \ \forall \ i$. In other words, the gross payment matrix is perturbed by deleting all gross payments to and from bank k. The

[7] BIS [5] lists the following purposes for banks to enter netting arrangements: reduction: 1) in the number of payment messages, 2) in credit and liquidity risk due to a legal “right to set off” claims, 3) in the need for intraday liquidity to bridge timing gaps and 4) in capital requirements.

[8] “Although the effects depend upon settlement and trading patterns, private payment netting systems can evidently reduce the value of settlement payments and the number of payments by 80% or more from what would be needed for gross settlements in certain cases.” (BIS [6], p. 9).

perturbed gross obligations matrix is given by

$$\underset{n\times n}{\mathbf{Z}^{k,1}} = \begin{bmatrix} 0 & z_{12} & \cdots & z_{1k-1} & 0 & z_{1k+1} & \cdots \\ z_{21} & 0 & & & 0 & & \\ \vdots & & \ddots & & \vdots & & \\ z_{k-11} & & & \ddots & \vdots & & \\ 0 & 0 & \cdots & \cdots & 0 & \cdots & 0 \\ z_{k+11} & & & & \vdots & \ddots & \\ \vdots & & & & 0 & & 0 \end{bmatrix} \tag{9}$$

where the first part of the superscript denotes the index of the epicentral bank and the latter is an iteration index to be explained below. The resulting bilateral and multilateral net positions ($\mathbf{B}^{k,1}$ and $\mathbf{d}^{k,1}$) can be calculated by applying (3) and (5).

3.3 Contagion and domino effect

The failure of a participant will, in general, change the multilateral net positions and each participant will face a different demand for liquidity in order to settle its obligations. Let

$$\underset{n\times 1}{\mathbf{t}} = [t_1 \quad t_2 \quad ... \quad t_n]' \tag{10}$$

denote the levels of liquidity readily available or reserved in advance by the participants for the netting process. The aggregate amount of liquidity (LIQ) available for the netting process is given by

$$\text{LIQ} \equiv \sum_i t_i \tag{11}$$

If the new multilateral net position exceeds this threshold (i.e., $d_i^{k,1} > t_i$), a participant is assumed to default on its settlement obligations.[9] The multilateral net positions are recal-

[9] In the literature cited above, the threshold has typically been a fraction of the capital of the participating banks. The size of the fraction has depended on whether the particular focus of the analysis was either credit or liquidity risk. Credit risk is the risk that a counterpart will not settle an obligation for full value, either when due or at any time thereafter. Liquidity risk is the risk that a counterpart will not settle an obligation for full value when due (see BIS [10]).

culated excluding defaulting banks and the iterations continue until either all net positions are below the thresholds or all banks have been excluded from the settlement process.[10] We denote the final iteration by $\hat{\mathbf{Z}}^k$, $\hat{\mathbf{B}}^k$ and $\hat{\mathbf{d}}^k$.

We refer to the number of iterations needed until a stable set of banks is reached, as the duration of the systemic event. We measure the severity of the systemic event by the number of failing banks and the value of payments not settled due to the unwinding. We split the severity into two parts: the initial failure and subsequent failures. We refer to the former as the *initial* effect, the latter as the *domino* effect and the combined effect as the *total* effect.

In the case of failing banks, the initial effect is one bank by definition and the domino effect is thus equal to the total number of failing banks minus one. In the case of payments not settled, we measure all effects relative to GSO. Let

$$\widehat{\mathrm{GSO}} \equiv \sum_i \sum_j |\hat{z}_{ij}| \,. \tag{12}$$

The total effect ($\widehat{\mathrm{TE}}$) in terms of payments not settled is given by

$$\widehat{\mathrm{TE}} \equiv \frac{\mathrm{GSO} - \widehat{\mathrm{GSO}}}{\mathrm{GSO}}. \tag{13}$$

The initial effect (IE) is

$$\mathrm{IE} \equiv \frac{\sum_j |z_{kj}| + \sum_i |z_{ik}|}{\mathrm{GSO}} \tag{14}$$

and the domino effect ($\widehat{\mathrm{DE}}$) is given by the difference between the total and initial effect

$$\widehat{\mathrm{DE}} \equiv \widehat{\mathrm{TE}} - \mathrm{IE}. \tag{15}$$

[10] In each iteration, all banks failing on their settlement obligations are excluded simultaneously. No attempt is made to exclude banks one by one, even though this in some cases would reduce the systemic spill-overs. The primary motivation is the difficulty in designing a fair rule by which to exclude banks.

3.4 Trade-off between risk and threshold

The severity of the systemic event depends on the amount of liquidity reserved. We propose to look at a range of thresholds to gauge the sensitivity of our measure of systemic risk to changes in the liquidity available. We define a lower bound for the thresholds as equal to the minimal amount of liquidity that allow all obligations to be settled under normal circumstances

$$t_i^{LB} = \max(0, d_i) \quad \forall\ i \tag{16}$$

Using the multilateral net positions as thresholds yields a measure of the intrinsic risk arising from the gross settlement positions and as such a worst case estimate of the systemic risk inherent in the payment system. We use a measure of the liquidity available to each bank, t_i^{UB}, as an upper bound and interpolate different threshold levels using

$$t_i(\alpha) = t_i^{LB} + \alpha(t_i^{UB} - t_i^{LB}) \quad \forall\ i \tag{17}$$

where $\alpha \in [0, 1]$. The iterative procedure outlined above is repeated for each threshold level in order to trace out the trade-off between liquidity and severity of systemic events.

4 The Payment System in Denmark

The payment system in Denmark is described in detail in European Central Bank [17]. Denmark has an RTGS system and one interbank netting system (Sumclearing). The RTGS system is operated by the central bank[11] and it handles time critical payments and typically all interbank payments in excess of 100 million DKK. In 2001, the turnover was approximately three times the gross domestic product (GDP) in Sumclearing and 30 times GDP in the RTGS system.

[11] See Angelius and Henneberg [3] for a description of the Danish RTGS system (Kronos).

The netting system handles the clearing of all retail payments such as direct debit, payment cards, checks and credit transfers. Sumclearing is owned by the Danish Bankers Association and is operated by a company (PBS) jointly owned by members of the association. Sumclearing has 67 participants, including the central bank, and operates on a multilateral basis.[12] The system is described in detail in Danish Bankers Association [15].

The netting process begins with banks reserving a monetary amount for Sumclearing by transferring funds to a designated settlement account at the central bank without knowing their exact net positions.[13] The central bank reports these liquidity lines to PBS.[14] Banks transmit their gross settlement obligations vis-à-vis the other participants to PBS. PBS then calculates the multilateral net positions and compares them to the liquidity lines received from the central bank. Banks are excluded if their net position exceeds the liquidity line and the multilateral net positions are recalculated until all remaining banks are within their line. The final multilateral net positions are reported to the central bank and settled across the settlement accounts at the central bank.

In order to settle any remaining obligations the whole process is repeated, starting with banks reserving liquidity, until every obligation is settled or ultimately annulled. However, in succeeding iterations banks are provided with information on their multilateral net positions based on all outstanding obligations prior to reserving funds.

The Danish Bankers Association levies fines on participants that are not able to cover their net position. Banks thus have an incentive to include a buffer amount when reserving funds for Sumclearing.

[12] The Danish central bank serves as a bank for the government. It is thus at the same time a participant in and the clearing bank for Sumclearing.

[13] The uncertainty is primarily due to the fact that settlement happens over night since the Danish payment infrastructure normally operates on a (t+1) standard.

[14] The central bank reports an unlimited line on itself and was thus excluded from the calculation of LIQ in (11).

5 Simulations

The objective of the simulation exercise was to assess the severity of systemic events in the Danish interbank netting system following the financial failure of a participant.[15]

In all scenarios, the participant with the largest multilateral net debit position was assumed to fail.[16] However, an exception was made if the central bank (as a participant) had the largest net debit position since a default is deemed highly unlikely.[17] In that case, the private bank with the largest position was chosen instead.

The actual amount of liquidity reserved by the banks were used as upper bounds for the threshold levels. The choice of upper bounds does not reflect that banks presumably would be able to raise additional liquidity if needed. This implies that the focus of the analysis is (systemic) liquidity risk (see footnote 10).

5.1 Data

The data utilized was provided by PBS for the period ranging from December 21st, 2001 to January 25th, 2002. Figure 2 shows the daily aggregate gross settlement position (GSO), bilateral net position (BNP) and multilateral net position along with the bilateral and multilateral netting effects (BNE, MNE) as defined in (7) and (8). Summary statistics are shown in Table 1.

On average, obligations worth 12.3 billion DKK were reported to PBS with a minimum of 4.9 billion DKK and a maximum of 49.5 billion DKK. The average aggregate bilateral and multilateral net positions were 6.7 and 4.7 billion DKK, respectively. The multilateral

[15] Standard V of Core Principles explicitly deals with the risk inherent in netting systems if one or more participants are unable to meet their settlement requirement. The standard requires that: "A system in which multilateral netting takes place should at a minimum be capable of ensuring the timely completion of daily settlements in the event of an inability to settle by the participant with the largest settlement obligation". The standard is a minimum one that the systems in question should seek to exceed.

[16] Simulations (not reported) where the epicentral bank was the bank with the largest bilateral net position yielded qualitatively similar but quantitatively smaller estimates in terms of systemic risk.

[17] On 17 of the 22 days in the sample period the central bank had the largest multilateral net debit position.

netting effect ranges from 44% to 85% with an average of 69% while the mean bilateral netting effect is 16 percentage points lower.

However, the reported averages are influenced by three large observations around New Year (see Figure 1), due to seasonality in the underlying payment flow. The equivalent median numbers are lower for the turnover measure and netting positions, but higher for the netting effects (see Table 1).

The total value of the liquidity reserved (LIQ) for Sumclearing over the period ranges between 17.9 and 46.8 billion DKK with an average of 22.8 billion DKK per day. A combination of uncertainty with respect to the multilateral net positions, the fines levied by the Danish Bankers Association, reputational risk and a low cost of mobilizing liquidity induce banks to reserve funds for Sumclearing in excess of the actual requirement as shown in Figure 2. The figure depicts the aggregate liquidity line of the participants along with the aggregate gross settlement position and the actual amount of liquidity required in order to facilitate settlement of all outstanding interbank obligations, i.e., MNP. The amount of liquidity reserved exceeds actual liquidity requirement by a factor of 20 on an average day and on all days, except one, the total amount of liquidity reserved is actually larger than the aggregate gross settlement obligation.

	GSO	LIQ	BNP	MNP	BNE	MNE	IE
	Billion DKK				% of GSO		
Mean	12.3	22.8	6.7	4.7	52.9	68.9	25.9
Median	7.2	19.6	3.2	2.0	55.6	73.5	17.2
Min.	4.9	17.9	1.8	1.0	21.7	43.9	2.3
Max.	49.5	46.8	39.3	27.8	70.4	84.7	72.1
25th percentile	5.2	18.3	16.4	1.2	32.9	45.3	6.6
75th percentile	34.6	38.7	38.2	15.4	66.2	81.5	35.9

Table 1: Summary statistics

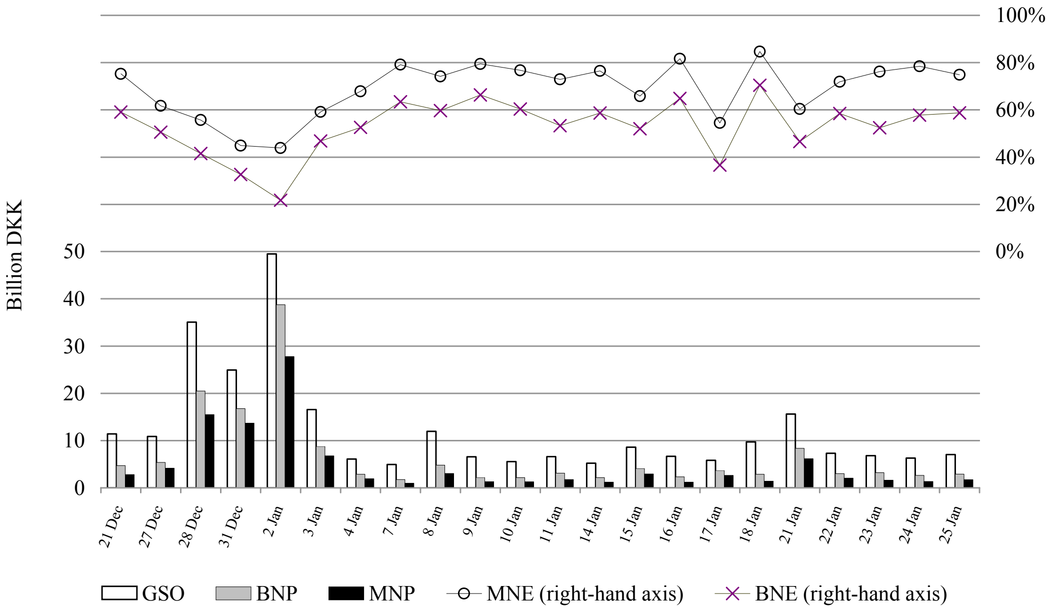

Figure 1: Settlement obligations and nettings effects

5.2 Financial failure

Eight different banks had the largest net debit positions over the period analyzed. The settlement obligations of the epicentral bank accounted for 26% of GSO on average. On one particular day the initial effect was as a high as 72%. However, only on one of the 22 days covered in the analysis did the financial failure of a participant result in a systemic event when the actual liquidity lines were used as thresholds. Moreover, the contagion was limited to one other participant on this particular day. The domino effect in terms of obligations not settled was 7% of GSO, whereas the total effect amounted to 13% of GSO (see Table 2).

With the thresholds reduced to the level of the multilateral net positions ($t = t^{LB}$) the failure of the bank with the largest net debit position resulted in a systemic events on every day in the data set. As illustrated in Figure 3, the initial financial failure led to an average of 25 subsequent settlement failures with a minimum of three and a maximum of 48 banks.

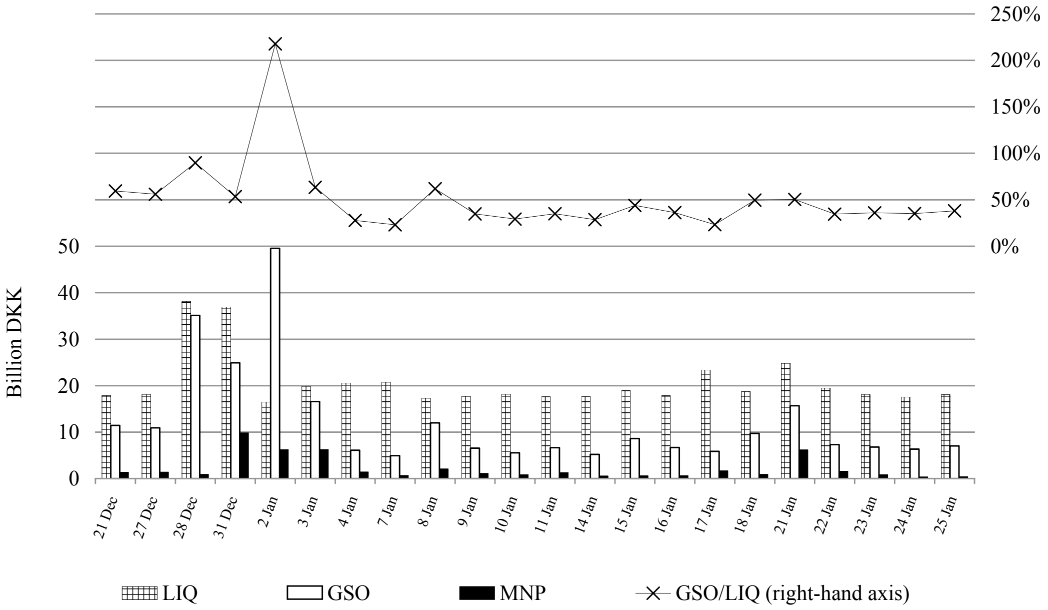

Figure 2: Liquidity reserved, aggregate gross settlement position and multilateral net position

On average, the systemic event resulted in obligations worth 6.1 billion DKK (52% of GSO) not being settled where of the domino effect accounted for about half. The worst systemic event with the thresholds at the lower bounds resulted in 93% of GSO not being settled. The duration of the systemic events ranged from 1 to 4 iterations, with a mean of 2.5 iterations.

In order to gauge the sensitivity of the results to the threshold level, the simulation exercise was repeated for $\alpha \in \{.25, .5, .75\}$ in (17). The results are presented in Table 2. At $\alpha = .25$ systemic events occur on three days in the sample period. The duration of the systemic event was 1 iteration on all days and the domino effect in terms of failing banks was 24, 27 and 31, respectively. However, the domino effect in terms of unsettled payments was relatively small on all days ranging from 0.1% to 7.4% of GSO. At the higher threshold levels, systemic events occur only on one day in the sample and the results are thus the same as for $t = t^{UB}$.

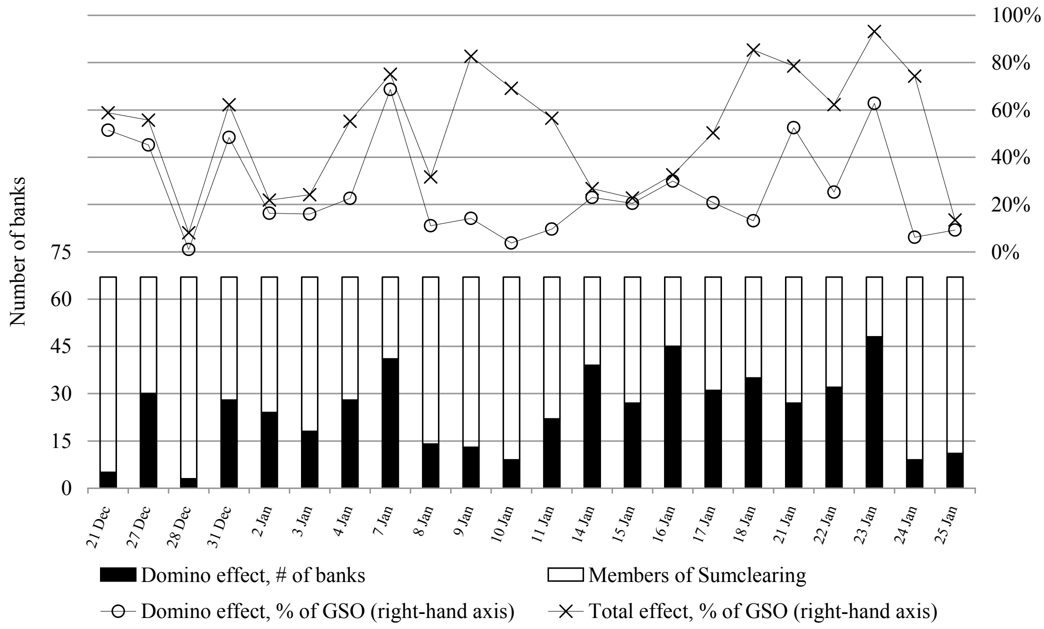

Figure 3: Severity of systemic event (thresholds at the lower bound)

An iterative search was conducted for each day in order to determine the minimum α (α^*) for which systemic spill overs no longer occur. The average α^* was found to be .14 and the median .08. Hence, on most days, less than one fifth of the liquidity reserved for Sumclearing is sufficient to in order to avoid systemic events.

6 Conclusion

Systemic risk in the Danish interbank netting system is low. A financial failure of a participant is unlikely to affect the ability to settle of other participants. In retrospect, the result is perhaps not surprising given that the system primarily settles small value payments. Moreover, the finding is comparable to related studies done in other European countries.

Banks reserve liquidity in excess of the actual need. This behavior is due to the low cost of mobilizing intraday liquidity, reputational risk and to the fines imposed by the Danish

Threshold	t^{LB}	$t(.25)$	$t(.5)$	$t(.75)$	t^{UB}
Days with systemic event*	22	3	1	1	1
Duration**	no. of iterations				
- average	2.5	1	1	1	1
- minimum	1	1	-	-	-
- maximum	4	1	-	-	-
Domino effect (DE)**	no. of banks				
- average	25	27	1	1	1
- minimum	3	24	-	-	-
- maximum	48	31	-	-	-
Domino effect (DE)**	% of GSO				
- average	26.0	4.4	7.4	7.4	7.4
- minimum	1.0	0.1	-	-	-
- maximum	68.7	7.4	-	-	-
Total effect (TE)**	% of GSO				
- average	51.8	24.7	12.9	12.9	12.9
- minimum	8.0	12.9	-	-	-
- maximum	93.1	35.1	-	-	-

Notes: *The data set consists of 22 days.
**Calculated over days with systemic events.

Table 2: Severity of systemic events

Bankers Association in the case of lack of liquidity. It is striking that at the aggregate level banks have sufficient liquidity on all but one day to settle their gross obligations in Sumclearing. Sensitivity analysis showed that the result of low systemic risk is robust to large changes in the liquidity available to the participants.

The paper only provides a partial picture of systemic risk in the Danish financial system and future research should take into consideration potential spill-overs vis-à-vis other payment and security settlement systems and different interbank exposures in order to provide a more complete picture.

A Appendix

A.1 Numerical example

In this appendix, we provide a simple numerical example of the calculations outlined in section 3. Let there be 4 banks participating in the netting scheme and assume that the gross settlement obligations are given by

$$\mathbf{Z} = \begin{bmatrix} 0 & z_{12} & z_{13} & z_{14} \\ z_{21} & 0 & z_{23} & z_{24} \\ z_{31} & z_{32} & 0 & z_{34} \\ z_{41} & z_{42} & z_{43} & 0 \end{bmatrix} = \begin{bmatrix} 0 & -5 & 5 & 8 \\ 10 & 0 & 2 & -3 \\ 8 & -4 & 0 & 5 \\ 10 & 5 & 3 & 0 \end{bmatrix} \tag{18}$$

Bank 1 is reporting that the total amount of debit requests, e.g. checks, it has received from customers of bank 2 exceeds the amount of credit transfers, it has with the beneficiary being a customer of bank 2, by 5 currency units. In the case of bank 3 and 4, bank 1 is reporting that the credits exceeds the debits by 5 and 8, respectively. The matrix of bilateral net positions between the banks is

$$\mathbf{B} = \mathbf{Z} - \mathbf{Z}' = \begin{bmatrix} 0 & b_{12} & b_{13} & b_{14} \\ b_{21} & 0 & b_{23} & b_{24} \\ b_{31} & b_{32} & 0 & b_{34} \\ b_{41} & b_{42} & b_{43} & 0 \end{bmatrix} = \begin{bmatrix} 0 & -15 & -3 & -2 \\ 15 & 0 & 6 & -8 \\ 3 & -6 & 0 & 2 \\ 2 & 8 & -2 & 0 \end{bmatrix} \tag{19}$$

Since bank 2 reports a net credit obligation worth 10 towards bank 1, the bilateral positions between the two banks is 15 in favor of bank 1. The multilateral net positions are

$$\mathbf{d} = \mathbf{B} \cdot \mathbf{1} = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \\ d_4 \end{bmatrix} = \begin{bmatrix} -20 \\ 13 \\ -1 \\ 8 \end{bmatrix} \tag{20}$$

In other words, bank 1 will receive 20 from the central counterpart, bank 2 has to pay 13 to the central counterpart, bank 3 will receive 1 and bank 4 has to pay 8. The aggregate gross settlement obligation (GSO) is 68 and the aggregate bilateral (BNP) and multilateral net position (MNP) is 36 and 21, respectively. The bilateral netting effect (BNE) is thus $\frac{68-36}{68} = 47\%$ and the multilateral netting effect (MNE) is $\frac{68-21}{68} = 69\%$.

If we assume that the bank with the largest multilateral net debit positions (bank 2), defaults on its settlement obligations due to a financial failure (see section 3.2), the gross obligations matrix is pertubed to

$$\mathbf{Z}^{2,1} = \begin{bmatrix} 0 & 0 & 5 & 8 \\ 0 & 0 & 0 & 0 \\ 8 & 0 & 0 & 5 \\ 10 & 0 & 3 & 0 \end{bmatrix} \tag{21}$$

The default of bank 2 leads to a reduction of 29 in the amount of obligations being settled. The new bilateral position matrix is

$$\mathbf{B}^{2,1} = \begin{bmatrix} 0 & 0 & -3 & -2 \\ 0 & 0 & 0 & 0 \\ 3 & 0 & 0 & 2 \\ 2 & 0 & -2 & 0 \end{bmatrix} \tag{22}$$

and the new multilateral net positions are

$$\mathbf{d}^{2,1} = \begin{bmatrix} -5 \\ 0 \\ 5 \\ 0 \end{bmatrix} \tag{23}$$

Assuming that the banks are able to perfectly forecast the liquidity requirement from the netting process and that they only allocate the liquidity required under normal circumstances the thresholds for liquidity available on short notice are given by

$$\mathbf{t} = \begin{bmatrix} t_1 \\ t_2 \\ t_3 \\ t_4 \end{bmatrix} = \max(\mathbf{0}, \mathbf{d}) = \begin{bmatrix} 0 \\ 13 \\ 0 \\ 8 \end{bmatrix} \tag{24}$$

This implies that bank 3 is forced to default on its settlement obligation due to the failure of bank 2, since it under the revised schedule has to pay 5 instead of receiving 1. The revised multilateral net positions are

$$\mathbf{d}^{2,2} = \begin{bmatrix} -2 \\ 0 \\ 0 \\ 2 \end{bmatrix} \tag{25}$$

and it is thus possible for bank 1 and 4 to settle their obligations given the liquidity available. Taking into account the default of bank 3, the aggregate gross settlement obligation falls to 18. The total effect from the default of bank 2 is thus a $\frac{68-18}{68} = 74\%$ reduction in terms the gross settlement obligations being settled. The initial failure of bank 2, the initial effect, accounts for 43 percentages points of this reduction and the subsequent failure of bank 3, i.e., the domino effect, accounts for the remaining 31 percentage points.

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