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Jan Berndsen

De Nederlandsche Bank

**Does duration extension enhance
excess returns?**

April 2003

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Please direct any enquiries to
Danmarks Nationalbank, Information Desk, Havnegade 5, DK-1093 Copenhagen K
Denmark
Tel: +45 33 63 70 00 (direct) or +45 33 63 63 63
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Resumé

I dette papir undersøges det historiske afkast ved at øge renterisikoen på en portefølje. Set over historiske perioder er der et højere afkast forbundet med øget renterisiko.

Der er dog ikke en lineær sammenhæng mellem risiko og afkast. Generelt er der et højere afkast forbundet med at renterisiko i korte løbetider, hvor rentestrukturen typisk er stejl. En obligationsinvestor, der ønsker at opnå et højere ekstraafkast i estimationsperioden, vil derfor få det bedste forhold mellem afkast og risiko ved at investere i obligationer med en løbetid på op mod 2 år.

Resultaterne i papiret er stærkt afhængige af den valgte estimationsperiode, og det er naturligvis ingenlunde sikkert, at de historiske resultater gentager sig.

Abstract

Many financial institutions voluntarily undertake additional interest rate exposure, due to their short-term funding and the placements of their assets in longer term bonds. Based on realised total bond returns of the major bond markets this paper assesses whether a fixed-income investor is actually rewarded by taking this additional interest rate risk. Unfortunately, the question raised in the title of this article can not clearly be answered. The outcome of the empirical analysis has shown that returns, return volatilities and their correlations are time-varying. However, some investment policy implications and conclusions can be stated, but caution is warranted when interpreting the empirical findings.

When various bullet-portfolio strategies are observed a concave (excess) risk/return trade-off pattern can be found. This implies that the efficiency of duration extension is limited. The major government bond and US agency markets do reward interest rate risk, but (excess) returns do not increase linearly with return volatility. Generally, the reward for taking additional interest rate risk is the highest at the front end of the curve, and diminishes as the slope of the excess returns curve flattens from the one-to-three-year part of the curve. In other words, if a fixed-income investor wanted to obtain higher excess returns by taking more interest rate risk during this long sample period, he should have invested (more) in bonds up to a maturity of approximately two years.

Interestingly, the observed excess risk/return trade-off is unstable over time. Different risk/reward patterns that hold across periods are found, and seem to be dependent on macro-economic business cycles. In a bearish interest rate climate, taking additional interest rate risk is not rewarded at all. In these circumstances, a fixed-income investor has a strong incentive to lower the long-run benchmark

duration. In addition, it should be noticed that long-duration bonds may be good investments for investors with long-duration liabilities.

Finally, based on historical interdependencies a fixed-income investor can improve the risk/return characteristics of a bullet treasury bond portfolio with a given interest rate exposure, by allocating the interest rate exposure across maturities and/or international government bond markets. The inclusion of US agency bonds in a US treasury portfolio also improved the risk/return characteristics of the overall portfolio, but to a much lesser extent.

Does duration extension enhance excess returns?

Jan Berndsen, De Nederlandsche Bank¹

Abstract

Many financial institutions voluntarily undertake additional interest rate exposure, due to their short-term funding and the placements of their assets in longer term bonds. Based on realised total bond returns of the major bond markets this paper assesses whether a fixed-income investor is actually rewarded by taking this additional interest rate risk. Unfortunately, the question raised in the title of this article can not clearly be answered. The outcome of the empirical analysis has shown that returns, return volatilities and their correlations are time-varying. However, some investment policy implications and conclusions can be stated, but caution is warranted when interpreting the empirical findings.

When various bullet-portfolio strategies are observed a concave (excess) risk/return trade-off pattern can be found. This implies that the efficiency of duration extension is limited. The major government bond and US agency markets do reward interest rate risk, but (excess) returns do not increase linearly with return volatility. Generally, the reward for taking additional interest rate risk is the highest at the front end of the curve, and diminishes as the slope of the excess returns curve flattens from the one-to-three-year part of the curve. In other words, if a fixed-income investor wanted to obtain higher excess returns by taking more interest rate risk during this long sample period, he should have invested (more) in bonds up to a maturity of approximately two years.

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¹ This paper was prepared while Jan Berndsen was on secondment in Danmarks Nationalbank.

Introduction

Just as many other financial institutions, central banks voluntarily undertake additional interest rate exposure, due to their short-term funding and the placements of their assets in longer term bonds. This entails a risk of capital losses if interest rates increase. However, it is generally assumed that such an exposure will normally lead to higher earnings since long-term interest rates, under normal conditions, are higher than short-term interest rates.

The goal of this paper is to assess whether a fixed-income investor is actually rewarded by taking additional interest rate risk. Future risk/return patterns are hard to predict, but history can illustrate whether this strategy was successful in the past or not. This article will examine if excess returns can be expected by taking interest rate and follows some of the concepts of Ilmanen's (1996)² analysis.

The empirical analysis focuses mainly on the US treasury market, based on realised total bond returns over the past 24 years. In addition, government bonds from other, major markets and US agency bonds are examined, as these assets might be relevant investment opportunities for a central bank FX-portfolio. All findings about historical returns depend on the interest rate trend in the sample period. This paper utilizes data from 1978 to 2001. The sample period is considered long enough and without too strong trends to alleviate concerns about sample-specific results of the US treasury market. Furthermore, the historical returns are examined over several subperiods, across markets in order to give as conclusive answers about long-run expected returns as possible.

The report consists of three parts. The first section carries out an empirical analysis about the risk/reward trade-off in the US treasury market. The second part will analyse this pattern in the other bond markets. Finally, the third section investigates the potential benefits of portfolio diversification (thus resulting in a better risk/return trade off at a portfolio level) by allocating across maturities, international bond markets or by adding high-grade spread instruments to a US treasury portfolio.

Part A: Empirical analysis about the risk/reward trade-off in the US treasury market

A.1. Indications from treasury yield curve shapes

Figure 1 displays the path of the short-term and long-term interest rate during the sample period. Up to the beginning of the 1980s both interest rates increased dramatically and the curve was frequently inverted. After 1984 both rates declined equally dramatically, and the yield curve was most of the time steeply upward sloping.

² See A. Ilmanen (1996), *Does duration extension enhance long-term expected returns?* in Journal of Fixed Income.

Figure 1 Yield levels 3-month bill and 10-year note, US treasuries (1978 – September 2001)

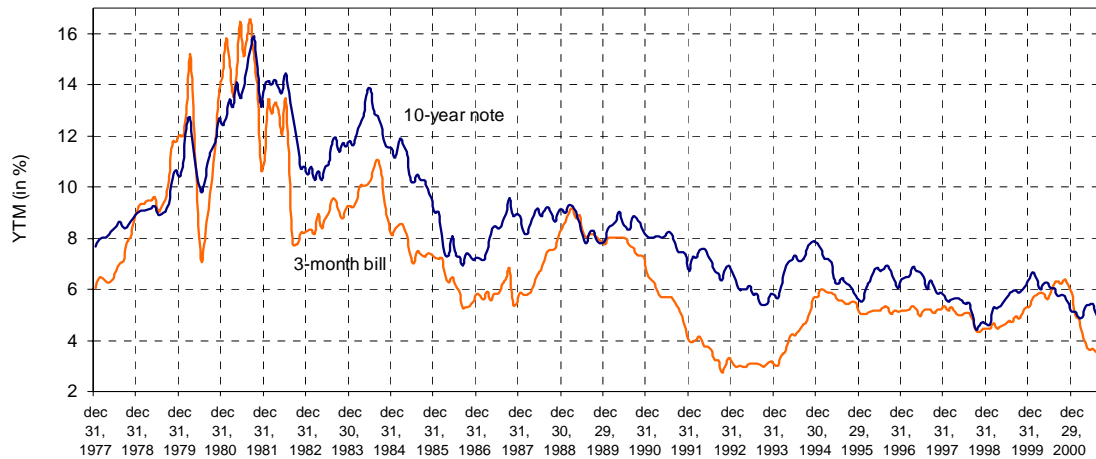


Table 1 presents average yields of generic treasury instruments (extracted from Bloomberg) and yield spreads over the three-month bill rate, as well as the annualised standard deviations of monthly yield changes. The following observations can be noticed:

- Average yields are increasing across the curve. An upward-sloping curve shape probably reflects a positive bond risk premium, but also rising rate expectations.
- The curve is concave in maturity: yields increase at a decreasing rate as a function of maturity;
- The term structure of yield volatilities is inverted.

Table 1 US treasury yields (1978 – September 2001)

	3m bill	6m bill	12m bill	2year note	5year note	10year note	30year note
average yield	7,0%	7,2%	7,5%	7,8%	8,2%	8,4%	8,6%
term spread over 3-month rate	-	0,2%	0,5%	0,8%	1,2%	1,4%	1,6%
standard deviation of yield changes	2,3%	2,2%	2,0%	1,9%	1,6%	1,4%	1,2%

A.2 Indications from bond returns

A.2.1 Data description

Historical average returns can vary dramatically across samples, even when long sample periods are used. Period specificity is a problem that sophisticated econometric techniques cannot overcome. This paper focuses on the longest sample period provided by the data-vendor. We do realise that it is not a complete 'neutral' sample period, and that the beginning and ending yield levels differ slightly.

In this empirical study annual average returns of strategies are analysed, which concentrate portfolio holdings in a certain maturity sector of the US treasury market. As such, these portfolio strategies can be regarded as various bullet-portfolios. The analysis covers the past 24 years, based on the Merrill Lynch's bond index family. The US treasury return series include two treasury bill portfolios (three-month, six-month) and six maturity-subsector treasury bond portfolios (one-to-three years, three- to-five years, five-to-seven years, seven-to-ten years, ten-to-fifteen years, and fifteen years and more).

A.2.2 Main findings

Table 2 shows the annual arithmetic and geometric means (averages) and other statistics for the eight return series described above. The geometric mean reflects the multiperiod compound return that the various strategies would have accumulated over the sample. The arithmetic mean exaggerates the historical performance, but may be a better measure of expected return for central banks as it reflects the return on a yearly rebalanced investment/portfolio (see also box 1).

Table 2 US treasury maturity-subsector annual returns and other statistics (1978 – September 2001)

	3m bill	6m bill	1-3 years	3-5 years	5-7 years	7-10 years	10-15 years	15+ years
average return (arithmetic)	7,6%	7,8%	8,7%	9,4%	9,9%	10,0%	10,3%	10,6%
arithmetic premium		0,2%	1,2%	1,8%	2,3%	2,4%	2,7%	3,0%
average return (geometric)	7,5%	7,8%	8,6%	9,2%	9,6%	9,6%	9,7%	9,9%
geometric premium		0,2%	1,1%	1,7%	2,1%	2,1%	2,2%	2,3%
standard deviation	3,2%	3,3%	4,4%	6,5%	8,0%	9,8%	11,3%	13,3%
average modified duration	0,23	0,46	1,60	3,19	4,43	5,79	7,04	9,77
Sharpe-ratio		0,07	0,25	0,25	0,25	0,20	0,19	0,17
Information ratio (arithmetic)		0,57	0,36	0,29	0,29	0,25	0,24	0,23
Information ratio (geometric)		0,56	0,34	0,26	0,26	0,20	0,19	0,16

BOX 1 Arithmetic versus geometric mean

The arithmetic mean (AM) and geometric mean (GM) are computed using the following equations:

$$AM = (h_1 + \dots + h_N)/N$$

$$GM = [(1 + h_1) * (1 + h_2) * \dots * (1 + h_N)]^{1/N} - 1$$

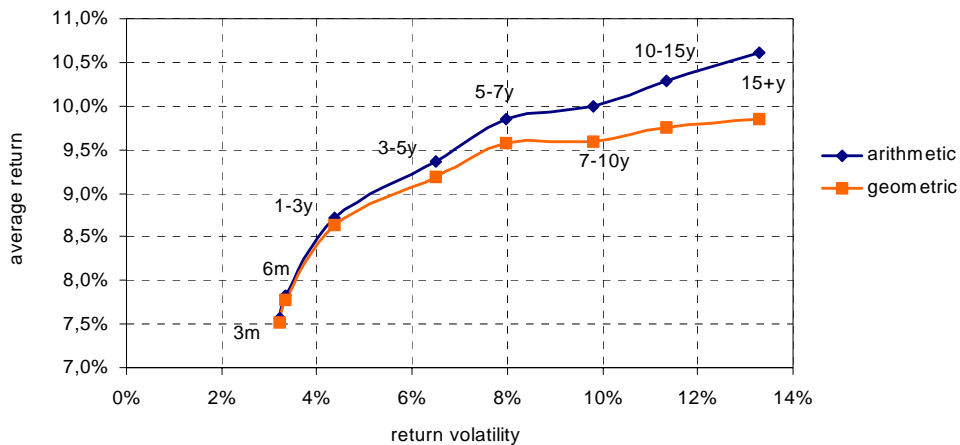
where h_i are one-period holding-period returns and N is the sample size. As Pythagoras already proved, the geometric mean is always less than or equal to the arithmetic mean. The difference between the two means increases with the (return) volatility. To illustrate, assume an investment of \$100, which an investor holds during two one-periods. The return in period 1 is 10% and -10% in the second period. The arithmetic mean calculus suggests that the investor on average would make no profit or loss. However, in period 1 the initial investment increases to \$110, which in period 2 is subject to a 10% decrease resulting in an end value of \$99. Actually, this investor made a loss of \$1, which is reflected by the two-period geometric mean of -0,5%.

Some practitioners argue that the geometric mean is the correct number to use in historical analysis. We believe that the choice between the two means depends on other considerations. If we assume that at the beginning of each one-period holding-period the portfolio has the size of its initial investment, because after each period the holding-period return will be rebalanced (gains are sold, losses replenished), the arithmetic return calculation would be appropriate. However, if we suppose beforehand that all coupon payments and capital gains and losses remain intact up to the ending of the sample period, we should use the geometric mean.

It is harder to say which number is relevant when describing future prospects of a given strategy. It is believed that the arithmetic mean is the mathematically correct measure of expected returns, while the geometric mean better represents a typical outcome (median).

Figure 2 shows the ex-post risk/reward trade-off in the US treasury market by plotting both the annual arithmetic and geometric means for the eight return series on their return volatilities. The arithmetic mean return increases almost monotonically from the one-to-three year sector, suggesting a linear relationship between additional risk and excess return. On the other hand, the geometric mean return curve is quite flat after the five-seven years maturity point.

Figure 2 Risk/return trade-off US treasuries (1978 – September 2001)

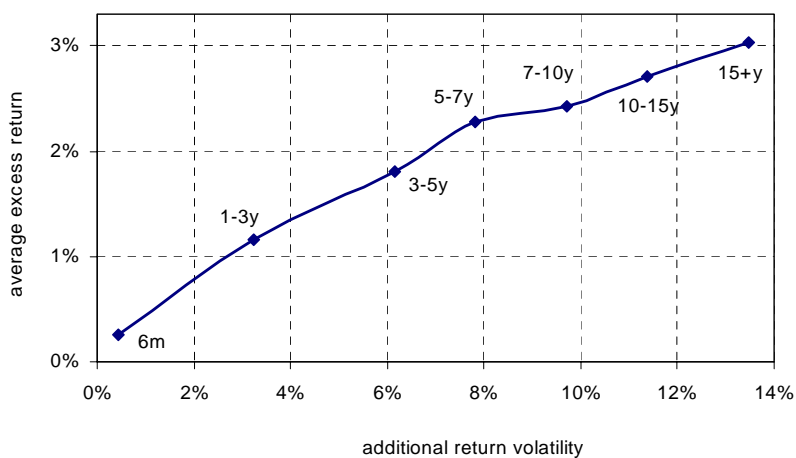


In both cases, there appears to be a bond risk premium during the sample period, but mainly at the front end of the curve. An investor could have earned an additional percentage-point by holding a portfolio with a two-year duration (1-3y) instead of a portfolio with a six-month duration, while an additional 60 basis points premium is found between portfolios with a two- and four-year (3-5y) duration.

From now on, this paper focuses on the arithmetic return averages when historical returns are analysed. The rationale is that in most financial organisations profit allocation and determination usually takes place on an annual basis. In the case of central banks, the vast majority of the profit is yearly transferred to the central government. Investment portfolios also appear to be rebalanced frequently in order to keep the size constant. One can argue that the average long-term return on an FX-portfolio is not a multiple-period compound return and therefore the arithmetic mean gives a better reflection of reality.

Furthermore, it is probably more interesting to analyse the slope of the excess return as a function of additional interest rate risk as many financial institutions fund themselves at short-term rates and invest their assets in bonds. Figure 3 shows the excess risk/return trade-off of the above-described portfolio strategies in comparison to the riskless rate, which is represented by the three-month bill portfolio.

Figure 3 Excess risk/return trade-off US treasuries (1978 – September 2001)



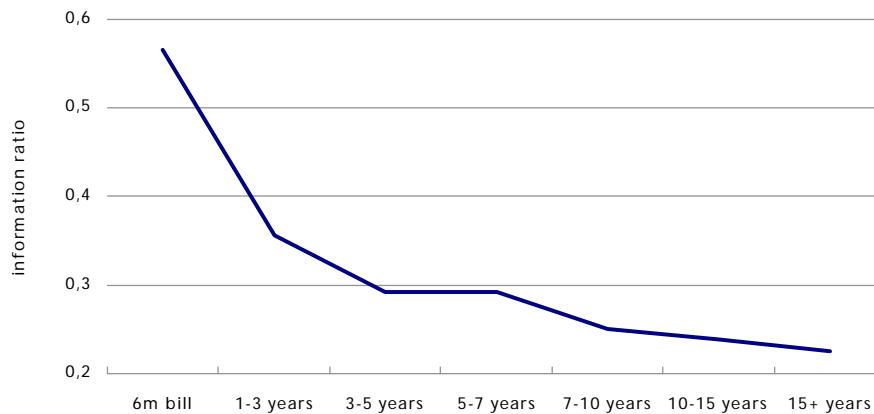
Instead of a Libor-rate, the three-month T-bill rate is used as a reference rate for short-term borrowing. In the first place, we want to ensure that bonds of the same issuer are compared. In this case the use of the Libor-rate would imply the introduction of (some degree of) credit risk at the short end of the curve.

Secondly, it is assumed that the borrowing level of a central bank from a high-grade country is closer to US Treasury than interbank rates.

Figure 3 shows that the slope at the front end of the return curve is slightly steeper than the rest of the curve. This may suggest that the highest excess return per unit of extra risk can be found when the portfolio risk is increased from an average six-month duration to an average two-year duration.

Potential explanations for the slightly concave shape of figure 3 include the convexity advantage of long-term bonds (see paragraph 2.4) and the demand for long-term bonds from the long-horizon investors. The latter explanation is related to the preferred habitat hypothesis. The expected returns of the long-term bonds are 'pulled-down' by the demand of the long-horizon investors, such as pension funds, which perceive the long-term bond as the least risky asset because it best matches the average duration of their liabilities. However, these long-horizon investors can be regarded as a minority in the US treasury market. One could argue whether they really pull down the expected return of the long-term bonds quite as low as that of bonds with a shorter maturity.

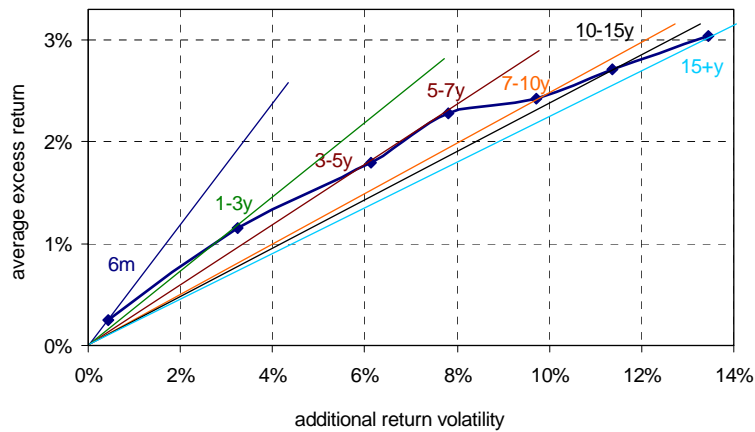
Figure 4 Information ratio's per maturity-subsector (1978 – September 2001)



However, more detailed observations from risk/return ratio's, such as the Sharpe- and information ratio³ presented in table 2, indicate that the excess return does not increase linearly with return volatility or duration. Figure 4 shows that the excess return per unit of risk decreases with maturity. Actually, the information ratio of a particular bond portfolio can be interpreted as the slope of a risk/reward line drawn from the origin that intersects its excess risk/reward value. As figure 5 illustrates, the slopes of these lines slightly flatten with maturity indicating that duration extension does not enhance excess returns.

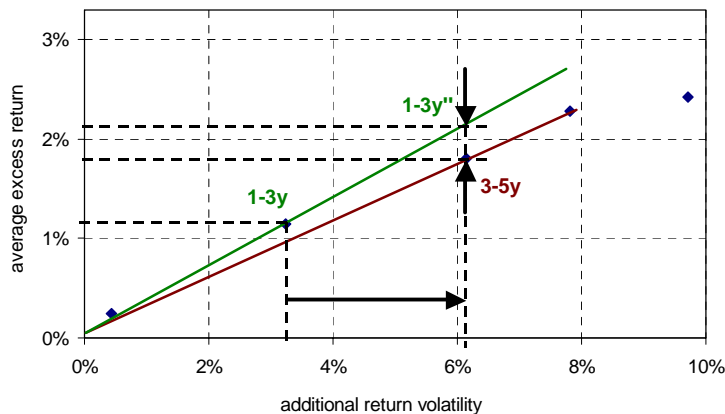
³ The Sharpe and information ratios are mean to volatility ratios of a bond portfolio's excess return. The Sharpe ratio is the bond portfolio's excess return in comparison to the return on the riskless rate (here a three-month bill portfolio), divided by the standard deviation of the individual bond portfolio. This ratio relates thus the additional return to the volatility of the total investment. On the other hand, the information ratio, which is defined by the excess return divided by the standard deviation of the excess return, measures the extra return per unit of additional risk.

Figure 5 Excess risk/return trade-off US treasuries including excess risk/reward lines (1978 – Sep. 2001)



The above-mentioned pattern can also be interpreted in the following way. Assume an investor, who funds himself at a three-month T-bill rate and invests his assets in a bullet-portfolio with an average duration of approximately two (the 1-3y portfolio). Suppose that the investor is willing to double his (additional) portfolio risk. One option is to reshuffle the portfolio by investing in a bullet-portfolio with an average duration of 4 (the 3-5y portfolio) instead of the portfolio with the duration of 2. The other alternative is to increase the leverage and double the risk exposure of the original portfolio (figure 6). In this case the excess risk/return characteristics of the new portfolio would move upwards along the green line to point 1-3y'', resulting in a higher excess return than would be obtained by investing in the bullet-portfolio with an average duration of 4. This observation suggests (again) that the highest excess return per unit of extra risk can be found at the front end of the treasury curve and that this trade-off decreases with maturity.

Figure 6 Excess risk/return trade-offs of two bullet portfolio with the same risk profile (1978 – Sep. 2001)

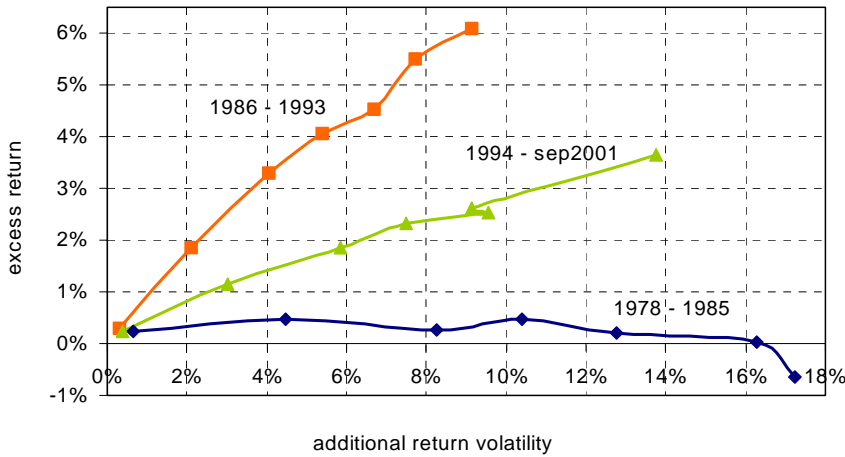


A.2.3 A further analysis of the sample period

To alleviate the problem of period specificity, subperiods of the sample period are analysed. Figure 7 shows separate risk/reward curves for three eight-year subperiods. In different time periodes, different patterns are found. The bond markets were bearish or neutral in the first subperiod and bullish in the last two subperiods.

The chart indicates that the above-described, linear upward sloping relationship between excess returns and additional interest rate risk is unstable over time, because the slopes of the three curves differ significantly. In a bearish interest rate climate, this pattern has a flat, slightly negative slope. This implies that during the 1978 – 1985 period an investor would not be rewarded by extending his interest rate exposure, and in the long-end of the curve even would have lost money.

Figure 7 Excess risk/return trade-off US treasuries in three subperiods (1978 – September 2001)

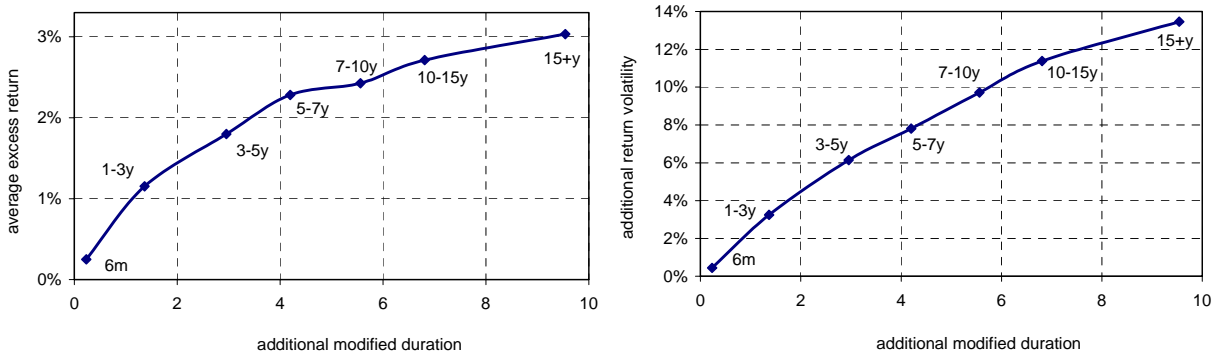


A.2.4 Is duration a good proxy for interest rate risk?

Instead of the return volatility, most practitioners use the duration measure to express the interest rate exposure of a bond (portfolio). The modified duration can be defined as the price change factor of a bond (portfolio) given a one percent-point parallel change of the yield curve. In many ways duration is a more manageable measure than volatility. Is a simple measure to assess the yield sensitivity of most fixed income securities, and to aggregate the yield sensitivity of different bonds into one key number.

In figure 8 the excess return is plotted on the additional modified duration of each maturity-subsector portfolio in comparison to a three-month T-bill portfolio. The shape of the chart illustrates that duration extension enhances returns, but this relationship is visually even more concave shaped than for return volatility (figure 3).

Figures 8 & 9 Excess risk/return trade-off and return volatility/duration linearity, US treasuries ('78 – Sep '01)



The duration measure is also defined as the first-order approximation of the price/yield function of a bond (portfolio). Duration ignores other components of return (volatility), such as convexity. This sec-

ond-order approximation of the price/yield function estimates the degree of non-linearity of the price/yield relationship. Generally all non-callable bonds exhibit positive convexity, which means that their prices rise more for a given yield decline than they fall for a similar yield increase. All else being equal, positive convexity is thus a desirable characteristic because it increases as bond's return⁴. For bonds without embedded options, such as government bonds, convexity increases roughly as a square of duration. Long-term bonds exhibit higher convexity values than bonds with a shorter maturity. Although duration seems to be a good proxy for interest rate risk for short-term and medium-term bond, it is a moderate proxy for return volatility for long-term bonds. This might explain the concave slope of the non-linear relationship of (additional) volatility with duration (see figure 9).

Part B: Empirical analysis about the risk/reward trade-off in other bond markets

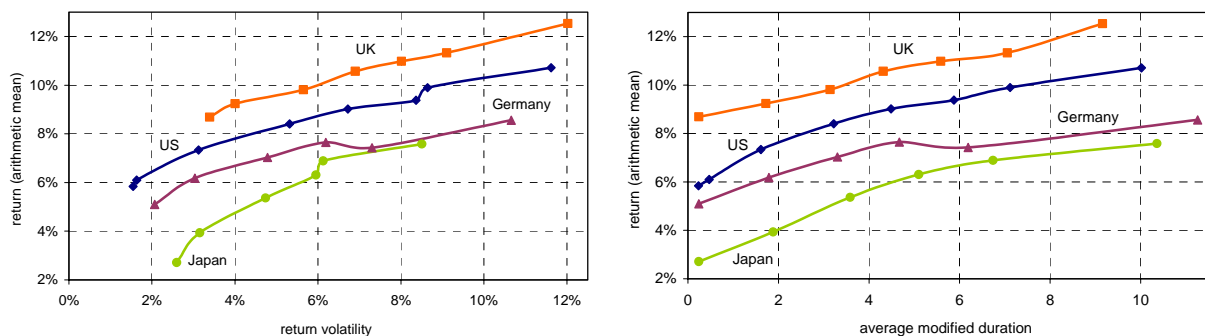
In this section, we examine whether the same excess return patterns documented above can be found in other government bond markets, and in the US agency bond market. The analysis of the German, UK, Japanese and US government bond markets covers the past 16 years, which is the longest available sample period, while the sample period of the US agencies is equal to the period used in section A (the past 24 years).

The two analyses are based on total return indices of the Merrill Lynch's bond index family described above. Exceptions are the UK, German and Japanese three-month T-bill returns, which are estimated by using interest rates extracted from the ECOWIN, a market data system used by Danmarks Nationalbank.

B.1 Findings from other government bond markets

Figures 10 and 11 show the ex-post risk/reward trade-off in the four government bond markets by plotting the average annual means for the various return series on respectively their return volatilities, and average modified durations. The relationship between the excess return and additional interest rate risk is illustrated in figure 12 and 13. The tables with the statistics can be found in annex 1.

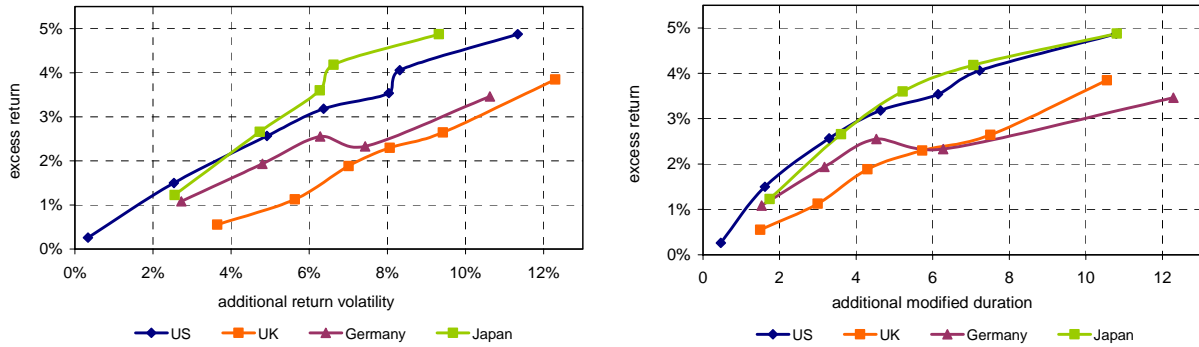
Figures 10 & 11 Risk/return trade-off US, UK, German and Japanese bond markets (1986 – September 2001)



⁴ On the other hand callable bonds, such as most mortgage bonds and asset-backed securities, are characterised by negative convexity. This means that a given yield decline increases the bond price more than a yield increase of equal magnitude reduces it. In a climate of declining interest rates, negative convexity is an undesirable feature for a fixed-income investor.

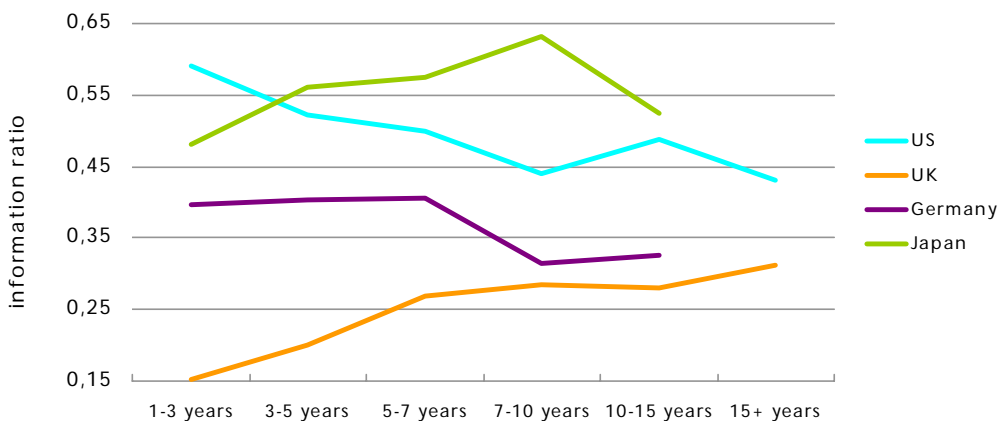
Although the composition of the national return series is not fully identical⁵, the risk/return patterns appear to be quite similar. The mean returns increase almost monotonically with maturity, and in most cases the slope of the return curve flattens from the intermediate part. An exception is the British return curve, which appears to have a linear relationship with return volatility in this sample period.

Figures 12 & 13 Excess risk/return trade-off US, UK, German and Japanese bond markets (1986 – Sep 2001)



At first sight, the reward of taking additional interest rate risk appears to be quite the same for the four treasury markets, as the slopes of the excess return versus additional return volatility curves seem to be identical. However, a better picture of the excess return/trade-off relationship is given by figure 14. This chart plots the information ratios of the various strategies in the four government bond markets. If the excess return increased linearly by taking additional interest rate risk, suggested by the linear looking slopes of figure 11, the information ratio curves should be flat. This is not the case. The interest rate curves of the US and German government bond markets are slightly downward sloping, suggesting that the excess reward to additional return volatility decreases with maturity. An observation identical as found in section A. In contrast, the UK and Japanese return curves seem to reward additional interest rate risk exposure as their information ratios increase slightly with maturity.

Figure 14 Information ratio's per maturity-subsector (1986 – September 2001)



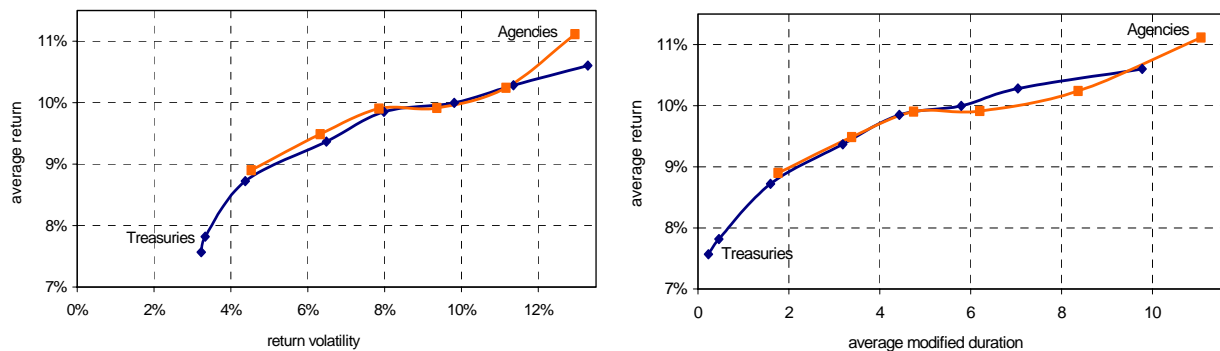
⁵ The US treasury return series include two Treasury bill portfolios (3m, 6m) while the UK, German and Japanese money markets are each represented by a three-month Treasury bill portfolio. The US and UK yield curves include six maturity-subsector Treasury bond portfolios (1-3y 3-5y, 5-7y, 7-10y, 10-15y, and 15+ y). The German and Japanese curves have five maturity-subsector portfolios (1-3y 3-5y, 5-7y, 7-10y, and 10+ y). The indices can be found on Bloomberg (ticker "IND").

Some caution is warranted when interpreting these observations. Although the slope of the British return curve is positive, the information ratios are rather low. This is a result of the relatively high average return and high return volatility of the UK three-month T-bill portfolio, which is used as a proxy for the riskless rate. On the other hand, the high Japanese information ratios are caused by relatively high excess returns due to the huge decrease of Japanese interest rates during the sample period, whereby the reference rate dropped to almost nil percent.

B.2 Findings from US agency bonds

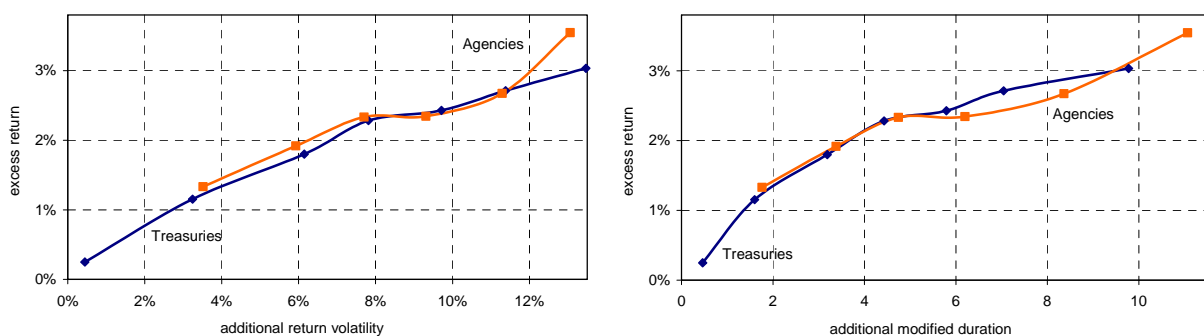
A similar trade-off pattern between mean returns and interest rate risk can be found for non-callable US agency bonds. Figures 15 and 16 show the return curves for agency bond portfolios against treasuries, while figures 17 and 18 illustrate the excess risk/return trade-offs. The statistics can be found in annex 2. It should be noted that the plotted treasury curves are the same as earlier shown in paragraph 2.2.

Figures 15 & 16 Risk/return trade-off US treasuries & agencies (1978 – September 2001)



The returns of various agency bond portfolio strategies seem to be slightly higher or equal to those on treasury portfolios with a comparable maturity. In most cases, the return volatility of the agency bond portfolios is slightly lower than on the treasury bond portfolios.

Figures 17 & 18 Excess risk/return trade-off US treasuries & agencies (1978 – September 2001)



The agency bond portfolios with a short-term and an ultra-long-term maturity provide better excess risk/return-characteristics than similar treasury portfolios. Potential explanations might be that the spread of agencies over treasuries: (1) mainly covers the liquidity premium of treasuries; and (2) is recently more often priced over the relatively stable swap-curve instead of over the more volatile treasury yield curve. On the other hand, the relative underperformance of agencies in the long-end of the curve (7-10y and 10-15y) seems at first sight hard to explain.

Part C: Impact of diversification

Up to now, the analysis focused on portfolio strategies with concentrate holdings in a certain maturity sector of the yield curve. However, a combination of the various bond portfolios might have better (excess) risk/return characteristics if the different risk factors do not change simultaneously and/or with the same magnitude over time⁶.

This third section analyses the impact of portfolio diversification (a) across maturities, (b) across international bond markets and (c) by adding spread instruments (agencies) to a US treasury portfolio. The reference portfolio is fully allocated in the one-to-three year part of the treasury curve. All portfolios have a size of ten billion dollars and a modified duration of 1,6. These virtual characteristics might be considered representative of a central bank FX-portfolio. It is also assumed that the portfolio returns are not compounded over the long-term because the portfolios are 'rebalanced' at the end of each calendar year. Therefore, average arithmetic means have been used in the analysis.

C.1 Investing across maturities

Correlations between the annual returns of various maturity-subsector strategies on the US treasury market are presented in table 3. The returns of these portfolios indicate two properties during this sample period (1978 – September 2001). In the first place, all intermediate- and long-term maturity-subsector returns seem to be strongly correlated. Secondly, the returns of the short-term maturity-subsectors are much less or almost not correlated with the rest of the curve. This latter property indicates that adding T-bills to a US treasury note portfolio might benefit from diversification effects in comparison to a portfolio, which only is allocated to intermediate- and long-term notes.

Table 3 Observed correlations US treasury bonds, based on maturity-subsector annual returns (1978 – September 2001)

	3m bill	6m bill	1-3 years	3-5 years	5-7 years	7-10 years	10-15 year	15+ years
3m bill	1,0							
6m bill	1,0	1,0						
1-3 years	0,7	0,7	1,0					
3-5 years	0,4	0,4	0,9	1,0				
5-7 years	0,3	0,3	0,9	1,0	1,0			
7-10 years	0,2	0,3	0,8	1,0	1,0	1,0		
10-15 years	0,1	0,2	0,8	1,0	1,0	1,0	1,0	
15+ years	0,1	0,1	0,7	0,9	1,0	1,0	1,0	1,0

Note: bold figures are correlations lower than 0,9

To illustrate the impact of diversification, several virtual portfolios with different compositions have been set up (table 4). The 'super bullet' portfolio, which is fully invested in the one-to-three-year maturity-subsector, is the point of reference. Obviously, this portfolio does not benefit from diversification effects with other maturity-subsectors. In order to illustrate these potential benefits, the following tables show two types of volatility results: the portfolio risk including the diversification effects (named 'diversified') and the portfolio risk if the different risk factors would be fully positively correlated ('not-diversified'). In addition, the row 'diversification' presents the diversification benefit of each portfolio strategy (in USD billions) in comparison to the reference portfolio.

⁶ We are aware that correlations can also be unstable over time, but this subject is out of the scope of this article.

Table 4 Risk/return characteristics of various portfolio strategies, US treasury bonds (1978 – September 2001)

Starting points: portfolio size = USD 10 billion; portfolio duration = 1,6						
	Composition	Portfolio strategy				
		Super bullet	Bullet	Eq. Weighted	Barbell	Optimised
	3m bill	-	-	13,0%	42,5%	41,5%
	6m bill	-	35,0%	50,4%	41,5%	42,7%
	1-3 years	100,0%	35,0%	14,5%	-	-
	3-5 years	-	30,0%	7,3%	-	-
	5-7 years	-	-	5,2%	-	-
	7-10 years	-	-	4,0%	-	-
	10-15 years	-	-	3,3%	8,0%	8,7%
	15+ years	-	-	2,4%	8,0%	7,1%
Average return (in USD billion)		0,87	0,84	0,84	0,82	0,81
Volatility of annual returns (in USD billion)	diversified	0,44	0,38	0,38	0,36	0,36
	not-diversified	0,44	0,47	0,47	0,48	0,47
	<i>diversification</i>	-	0,05	0,05	0,07	0,08
Return/volatility-ratio	diversified	2,0	2,18	2,18	2,24	2,27
	not-diversified	2,0	1,78	1,78	1,70	1,73

The reference portfolio (so-called 'super-bullet') has the highest average return and is the most risky strategy, when portfolio risk is measured by the historical return volatility. The low return/volatility ratio confirms that this portfolio strategy is not optimal in comparison to the other portfolio compositions. The bullet, the equally duration-weighted⁷ and the barbell portfolio show quite similar risk/return trade-offs, while their compositions differ substantially!

Of the examined portfolios, the 'optimised' portfolio (last column) has the highest return/volatility ratio, implicating the highest return per unit of risk measured by the return volatility. This 'optimised' portfolio is the outcome of a linear programming problem, which allocates the portfolio over the several maturity-subsectors, given the size and the interest rate exposure of the portfolio. Short-positions are not allowed, but the optimisation is not limited by other constraints. The optimisation was constructed to examine if a portfolio composition could be found that showed better risk/return characteristics than the rather 'traditional' compositions such as the bullet, barbell or equally duration-weighted portfolios. Actually, in this case the optimisation resulted in a barbell type portfolio composition.

C.2 Adding other government bonds

Besides allocating interest rate risk across the curve, the total interest rate risk can also be distributed across several financial markets. In this paragraph some diversification opportunities on the US, UK, German and Japanese yield curves are examined for the sample period 1986 up to present. In order to avoid the interference of exchange-rate risk into this discussion, it is assumed that the foreign-exchange exposure is continuously sold forward and that the hedging costs over this relative large sample period are close to nil.

⁷ Into each maturity-subsector of the 'equally weighted duration' portfolio an equal proportion of the total interest rate exposure is allocated (from 6-month bill up to 15+-years). The remainder cash is allocated at a short-term money market rate, represented by the three-month T-bill.

From table 5 some preliminary conclusions can be drawn. Firstly, in each national treasury market the same aspects can be found as for the US treasury market (table 3). Intermediate- and long-term maturity-subsector returns appear to be correlated, although not so strong as for US treasuries. Also the returns of the money market returns, represented by the three-month T-bill portfolio, are less or not correlated with other maturity-subsectors. This aspect is probably stronger for the UK, German and Japanese markets than for the American market.

Secondly, the data show that the correlation between (the annual returns of) the various financial markets is less than perfect. Thus, diversifying the interest rate risk and placements on several markets, can reduce the risk of capital losses.

Table 5 Observed correlations US, UK, German and Japanese government bonds markets, based on maturity-subsector annual returns (1986 – September 2001)

	TREASURIES					GILTS					BUNDS					JGBs						
	3m	1-3y	3-5y	7-10y	15+y	3m	1-3y	3-5y	7-10y	15+y	3m	1-3y	3-5y	7-10y	10+y	3m	1-3y	3-5y	7-10y	10+y		
TREASURIES	3m	1,00																				
	1-3y	0,59	1,00																			
	3-5y	0,39	0,95	1,00																		
	7-10y	0,30	0,87	0,97	1,00																	
	15+y	0,25	0,76	0,90	0,98	1,00																
GILTS	3m	0,83	0,52	0,35	0,27	0,21	1,00															
	1-3y	0,29	0,60	0,58	0,49	0,43	0,52	1,00														
	3-5y	0,08	0,55	0,61	0,58	0,55	0,31	0,94	1,00													
	7-10y	-0,01	0,49	0,60	0,62	0,61	0,20	0,81	0,95	1,00												
	15+y	-0,16	0,27	0,42	0,48	0,50	0,06	0,53	0,72	0,89	1,00											
BUNDS	3m	0,16	0,35	0,33	0,27	0,19	0,56	0,56	0,43	0,31	0,18	1,00										
	1-3y	-0,26	0,39	0,51	0,50	0,46	0,01	0,75	0,83	0,75	0,54	0,48	1,00									
	3-5y	-0,29	0,41	0,56	0,58	0,57	-0,13	0,68	0,84	0,84	0,67	0,21	0,93	1,00								
	7-10y	-0,30	0,40	0,60	0,68	0,70	-0,20	0,49	0,73	0,84	0,79	0,08	0,75	0,92	1,00							
	10+y	-0,12	0,46	0,63	0,72	0,74	-0,06	0,44	0,67	0,81	0,80	0,11	0,58	0,76	0,93	1,00						
JGBs	3m	0,55	0,40	0,28	0,19	0,11	0,88	0,58	0,36	0,23	0,14	0,76	0,23	0,01	-0,13	-0,06	1,00					
	1-3y	0,01	0,41	0,44	0,40	0,37	0,35	0,69	0,65	0,58	0,49	0,51	0,72	0,62	0,46	0,36	0,62	1,00				
	3-5y	-0,22	0,35	0,45	0,43	0,42	0,02	0,59	0,64	0,63	0,56	0,28	0,78	0,77	0,65	0,50	0,28	0,91	1,00			
	7-10y	-0,30	0,29	0,42	0,43	0,46	-0,16	0,40	0,54	0,61	0,59	0,04	0,65	0,73	0,71	0,57	0,01	0,74	0,93	1,00		
	10+y	-0,39	-0,01	0,12	0,15	0,21	-0,32	0,17	0,35	0,48	0,56	-0,10	0,45	0,55	0,58	0,46	-0,18	0,43	0,67	0,83	1,00	

Note: bold figures are correlations lower than 0,50.

In order to demonstrate these potential diversification effects, table 6 compares several mixed portfolios with a reference portfolio that is 100% invested in one-to-three year US treasury notes. Although the ex-post return of the reference portfolio was higher than of the other portfolios, its portfolio risk (measured by the return volatility) was significantly higher. This explains the low return/volatility value of the reference portfolio.

At first sight, allocating the interest rate risk of a one-to-three year bullet portfolio over several other government bond markets, lowers the expected ex-post portfolio return. However, risk/return characteristics improve significantly due to the lower overall portfolio risk obtained by the less than perfect correlation between the various risk factors. Thus, these findings confirm that any portfolio strategy, which invests in two or more financial markets, benefits significantly of diversification effects given the historical interdependencies.

Table 6 Risk/return characteristics of various portfolio strategies, US, UK, German & Japanese government bonds (1978 – September 2001)

Starting points: portfolio size = USD 10 billion; portfolio duration = 1,6									
	Composition	Portfolio strategy						Eq. Weighted	Optimised
		Reference	1	2	3	4			
	Treasuries	100%	25%	24%	50%	50%	25%	33%	
	Gilts	-	25%	13%	-	50%	25%	15%	
	Bunds	-	25%	50%	50%	-	25%	29%	
	JGBs	-	25%	13%	-	-	25%	23%	
	3m bill	-	12%	6%	6%	6%	50%	10%	
	6m bill	-	-	-	-	-	13%	-	
	1-3 years	100%	88%	94%	94%	94%	15%	90%	
	3-5 years	-	-	-	-	-	8%	-	
	5-7 years	-	-	-	-	-	6%	-	
	7-10 years	-	-	-	-	-	4%	-	
	10-15 years	-	-	-	-	-	3%	-	
	15+ years	-	-	-	-	-	1%	-	
Average return (in USD billion)		0,87	0,69	0,68	0,74	0,89	0,67	0,68	
Volatility of annual returns (in USD billion)	diversified	0,44	0,26	0,26	0,25	0,31	0,24	0,25	
	not-diversified	0,44	0,35	0,34	0,37	0,41	0,39	0,35	
	<i>diversification</i>		<i>0,18</i>	<i>0,18</i>	<i>0,19</i>	<i>0,13</i>	<i>0,20</i>	<i>0,19</i>	
Return/volatility-ratio	diversified	2,0	2,65	2,62	3,02	2,87	2,87	2,76	
	not-diversified	2,0	1,97	2,00	2,03	2,17	1,72	1,92	

C.3 Adding high-grade spread instruments

This last paragraph examines the potential diversification effects by adding non-callable US agency bonds to a US treasury bond portfolio. The benefits will probably be limited as the annual returns of intermediate- and long-term US treasury and agency bonds are highly correlated, as agency bonds are often priced against T-notes. Only US T-bill returns appear to have low interdependencies with the rest of the treasury and agency return curves.

Table 7 Observed correlations US treasury and agency bonds, based on maturity-subsector annual returns (1978 – September 2001)

	US Treasuries					US Agencies			
	<i>3m bill</i>	<i>1-3 years</i>	<i>3-5 years</i>	<i>7-10 years</i>	<i>15+ years</i>	<i>1-3 years</i>	<i>3-5 years</i>	<i>7-10 years</i>	<i>15+ years</i>
US Treasuries <i>3m bill</i>	1,00								
<i>1-3 years</i>	0,67	1,00							
<i>3-5 years</i>	0,35	0,92	1,00						
<i>7-10 years</i>	0,20	0,81	0,97	1,00					
<i>15+ years</i>	0,07	0,70	0,91	0,98	1,00				
US Agencies <i>1-3 years</i>	0,64	1,00	0,93	0,83	0,73	1,00			
<i>3-5 years</i>	0,37	0,93	0,99	0,95	0,89	0,95	1,00		
<i>7-10 years</i>	0,19	0,82	0,97	0,99	0,96	0,85	0,97	1,00	
<i>15+ years</i>	0,09	0,72	0,91	0,97	0,99	0,75	0,90	0,97	1,00

Note: bold figures are correlations lower than 0,90.

The various portfolio strategies indicate that risk/return characteristics of the overall portfolio slightly improve by increasing the share of agency bonds in a treasury portfolio. However, it is difficult to assess whether this relative small risk/return enhancement (1) results from the generally higher information ratio of agencies; or (2) might be the result of diversification benefits between treasury and agency bonds. In most circumstances, probably both aspects might explain this observation.

Table 8 Risk/return characteristics of various portfolio strategies, US treasury and agency bonds (1978 – September 2001)

Starting points: portfolio size = USD 10 billion; portfolio duration = 1,6								
	Composition	Portfolio strategy						
		Super bullet			Eq. Weighted		Barbell	
	3m bill	-	5%	8%	13%	58%	43%	43%
	6m bill	-	-	-	50%	0%	42%	42%
	1-3 years	100%	95%	92%	15%	21%	-	-
	3-5 years	-	-	-	7%	7%	-	-
	5-7 years	-	-	-	5%	5%	-	-
	7-10 years	-	-	-	4%	4%	-	-
	10-15 years	-	-	-	3%	3%	8%	8%
	15+ years	-	-	-	2%	2%	8%	8%
Share of agencies		0%	50%	appr 92%	0%	appr 42%	0%	16%
Average return (in USD billion)		0,87	0,88	0,88	0,84	0,84	0,82	0,82
Volatility of annual returns (in USD billion)	diversified	0,44	0,43	0,43	0,38	0,38	0,36	0,35
	not-diversified	0,44	0,44	0,44	0,47	0,47	0,48	0,47
	<i>diversification</i>	-	0,01	0,01	-	0,00	-	0,01
Return/volatility-ratio	diversified	2,0	2,02	2,03	2,18	2,21	2,24	2,31
	not-diversified	2,0	1,99	1,99	1,78	1,80	1,70	1,75

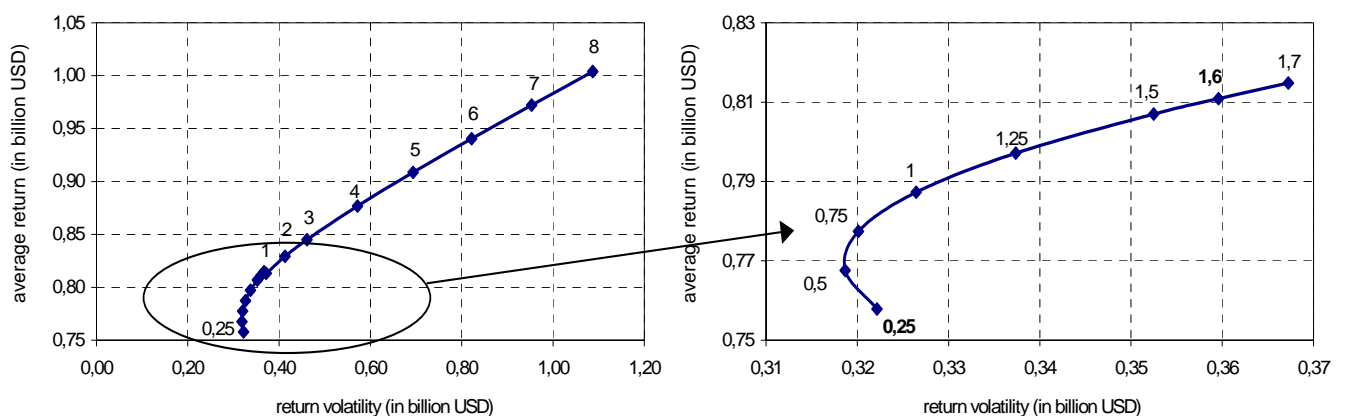
Based on the (diversified) return/volatility measure, the barbell portfolio strategy seems to have the most optimal risk/return ratio. This observation is probably the result of the low interdependencies between the returns of US T-bills and long-term agency bonds.

C.4 Some optimisations as a function of portfolio duration

Although we are aware that many potential caveats, such as time-varying returns, return volatilities and their correlations, prevent an 'optimal' outcome of a mean-variance exercise, the application of this method can be helpful to better understand the problem of portfolio allocation. In the preceding tables results of an 'optimised' portfolio were shown, which was the outcome of a linear programming problem that allocates the portfolio over the several maturity-subsectors, given the size and the interest rate exposure of the portfolio. Except from preventing short-positions, this rather simple optimisation was not limited by other constraints.

In this last paragraph this 'optimiser' is used to analyse the risk/return trade-off of US treasury portfolio with a fixed size of USD 10 billion, but with different portfolio durations (see figures 19 and 20). The charts indicate visually, that (1) the portfolio return increases almost linearly with portfolio risk for

Figures 19 & 20 Risk/return trade-off of various US treasury portfolio strategies (1978 – September 2001)



portfolios with a modified duration of 2 or more; and (2) the minimum risk point of an optimised portfolio is not the portfolio with the lowest portfolio duration, but corresponds to a portfolio duration of approximately 0,6 or 0,7 (see figure 20).

As no limits on maturity-subsectors are imposed, the optimiser generally allocates the bulk of the portfolio into less return-sensitive, short-term instruments (three- and six-month T-bill) and to a small extent into high yielding, but high return risk-sensitive long-term instruments (10+ year bonds) in order to obtain the highest portfolio return and the highest diversification benefits. In all generated scenarios, this barbell allocation prevailed other types of portfolio compositions, because it was more successful during the whole sample period.

Part D: Investment policy implications & conclusions

Sadly, the question raised in the title of this article can not conclusively be answered. The outcome of this data survey shows that returns, return volatilities and their correlations are time-varying. Although caution is warranted when interpreting the empirical findings, some investment policy implications and conclusions can be stated.

The concave (excess) risk/return trade-off pattern of the various bullet-portfolio strategies implies that the efficiency of duration extension is limited. The major government bond and US agency markets do reward interest rate risk, but (excess) returns do not increase linearly with return volatility. Generally, the reward for taking additional interest rate risk is the highest at the front end of the curve, and diminishes as the slope of the excess returns curve flattens from the one-to-three-year part of the curve. This implies that the optimal allocation of a bullet-structured bond portfolio will be at the front end of the curve. If a fixed-income investor wanted to obtain higher excess returns by taking more interest rate risk during this sample period, he should have invested (more) in bonds up to a maturity of approximately two years.

Interestingly, the observed excess risk/return trade-off is unstable over time. Different risk/reward patterns that hold across periods are found, and seem to be dependent on macro-economic business cycles. In a bearish interest rate climate, taking additional interest rate risk is not rewarded at all. In these circumstances, a fixed-income investor has a strong incentive to lower the long-run benchmark duration. In addition, it should be noticed that long-duration bonds may be good investments for investors with long-duration liabilities.

Finally, based on historical interdependencies a fixed-income investor can improve the risk/return characteristics of a bullet treasury bond portfolio with a given interest rate exposure, by allocating the interest rate exposure across maturities and/or international government bond markets. The inclusion of US agency bonds in a US treasury portfolio also improved the risk/return characteristics of the overall portfolio, but to a much lesser extent. Although absolute returns of a diversified portfolio were in the examined sample periods lower than the return of the reference portfolio (bullet-structured and fully invested in 1-3y US treasury bonds), the reduction of the portfolio risk due to diversification benefits was significantly higher. Interestingly, the type of portfolio composition was not decisive during this long sample period, as the risk/return trade-offs of portfolio strategies with different portfolio compositions, such as the bullet, equally duration-weighted and barbell portfolio investing in US treasuries, appeared to be

quite similar. However, if the interest rate exposure is an additional decision parameter, the barbell composition proved to be more successful.

Annex 1 US, UK, German and Japanese maturity-subsector annual returns and other statistics
(1986 – September 2001)

US TREASURIES

	3m bill	6m bill	1-3 years	3-5 years	5-7 years	7-10 years	10-15 years	15+ years
average return (arithmetic)	5,8%	6,1%	7,3%	8,4%	9,0%	9,4%	9,9%	10,7%
arithmetic premium		0,3%	1,5%	2,6%	3,2%	3,5%	4,1%	4,9%
average return (geometric)	5,8%	6,1%	7,3%	8,3%	8,8%	9,1%	9,6%	10,1%
geometric premium		0,3%	1,5%	2,5%	3,0%	3,2%	3,7%	4,3%
standard deviation	1,5%	1,6%	3,1%	5,3%	6,7%	8,4%	8,6%	11,6%
average modified duration	0,23	0,47	1,61	3,22	4,49	5,88	7,12	10,02
Sharpe-ratio		0,16	0,47	0,46	0,45	0,39	0,43	0,37
Information ratio (arithmetic)		0,78	0,59	0,52	0,50	0,44	0,49	0,43
Information ratio (geometric)		0,78	0,58	0,50	0,47	0,40	0,45	0,38

UK GILTS

	3m bill	1-3 years	3-5 years	5-7 years	7-10 years	10-15 years	15+ years
average return (arithmetic)	8,7%	9,2%	9,8%	10,6%	11,0%	11,3%	12,5%
arithmetic premium		0,6%	1,1%	1,9%	2,3%	2,6%	3,8%
average return (geometric)	8,6%	9,2%	9,7%	10,4%	10,7%	11,0%	11,9%
geometric premium		0,5%	1,0%	1,7%	2,1%	2,3%	3,3%
standard deviation	3,4%	4,0%	5,7%	6,9%	8,0%	9,1%	12,0%
average modified duration	0,24	1,72	3,14	4,32	5,59	7,06	9,16
Sharpe-ratio		0,12	0,17	0,24	0,25	0,25	0,26
Information ratio (arithmetic)		0,15	0,20	0,27	0,29	0,28	0,31
Information ratio (geometric)		0,13	0,17	0,24	0,25	0,24	0,26

GERMAN BUNDS

	3m bill	1-3 years	3-5 years	5-7 years	7-10 years	10+ years
average return (arithmetic)	5,1%	6,2%	7,0%	7,7%	7,4%	8,6%
arithmetic premium		1,1%	1,9%	2,6%	2,3%	3,5%
average return (geometric)	5,1%	6,1%	6,9%	7,5%	7,2%	8,1%
geometric premium		1,1%	1,9%	2,4%	2,1%	3,0%
standard deviation	2,1%	3,0%	4,8%	6,2%	7,3%	10,7%
average modified duration	0,24	1,79	3,30	4,67	6,19	11,26
Sharpe-ratio		0,35	0,38	0,38	0,28	0,28
Information ratio (arithmetic)		0,40	0,40	0,41	0,31	0,33
Information ratio (geometric)		0,38	0,38	0,38	0,28	0,28

JAPANESE YEN

	3m bill	1-3 years	3-5 years	5-7 years	7-10 years	10+ years
average return (arithmetic)	2,7%	3,9%	5,4%	6,3%	6,9%	7,6%
arithmetic premium		1,2%	2,7%	3,6%	4,2%	4,9%
average return (geometric)	2,7%	3,9%	5,3%	6,2%	6,7%	7,3%
geometric premium		1,2%	2,6%	3,5%	4,0%	4,6%
standard deviation	2,6%	3,2%	4,7%	5,9%	6,1%	8,5%
average modified duration	0,24	1,88	3,58	5,10	6,74	10,36
Sharpe-ratio		0,38	0,54	0,57	0,65	0,53
Information ratio (arithmetic)		0,48	0,56	0,57	0,63	0,52
Information ratio (geometric)		0,47	0,54	0,55	0,60	0,48

Annex 2 US treasuries, agencies and corporates maturity-subsector annual returns and other statistics
(1978 – September 2001)

US TREASURIES

	3m bill	6m bill	1-3 years	3-5 years	5-7 years	7-10 years	10-15 years	15+ years
average return (arithmetic)	7,6%	7,8%	8,7%	9,4%	9,9%	10,0%	10,3%	10,6%
arithmetic premium		0,2%	1,2%	1,8%	2,3%	2,4%	2,7%	3,0%
average return (geometric)	7,5%	7,8%	8,6%	9,2%	9,6%	9,6%	9,7%	9,9%
geometric premium		0,2%	1,1%	1,7%	2,1%	2,1%	2,2%	2,3%
standard deviation	3,2%	3,3%	4,4%	6,5%	8,0%	9,8%	11,3%	13,3%
average modified duration	0,23	0,46	1,60	3,19	4,43	5,79	7,04	9,77
Sharpe-ratio		0,07	0,25	0,25	0,25	0,20	0,19	0,17
Information ratio (arithmetic)		0,57	0,36	0,29	0,29	0,25	0,24	0,23
Information ratio (geometric)		0,56	0,34	0,26	0,26	0,20	0,19	0,16

US AGENCIES

	1-3 years	3-5 years	5-7 years	7-10 years	10-15 years	15+ years
average return (arithmetic)	8,9%	9,5%	9,9%	9,9%	10,2%	11,1%
arithmetic premium		0,6%	1,0%	1,0%	1,3%	2,2%
average return (geometric)	8,8%	9,3%	9,6%	9,5%	9,7%	10,4%
geometric premium		0,5%	0,8%	0,7%	0,9%	1,6%
standard deviation	4,5%	6,3%	7,9%	9,4%	11,2%	13,0%
average modified duration	1,76	3,38	4,74	6,20	8,36	11,07
Sharpe-ratio	0,28	0,28	0,26	0,21	0,19	0,21
Information ratio (arithmetic)	0,38	0,32	0,30	0,25	0,24	0,27
Information ratio (geometric)	0,36	0,30	0,27	0,21	0,18	0,21