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# 2004 • 18


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# Money Demand in Denmark 1980-2002


Juni 2004

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ISSN (trykt/print) 1602-1185

ISSN (online) 1602-1193

# Money Demand in Denmark 1980 – 2002

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21 June 2004

[1]The author wish to thank collegues at Danmarks Nationalbank for helpful discussions. All remaining errors are mine.

**Abstract**

This paper investigates the money demand in Denmark in the period 1980 – 2002 using quarterly data. Within the framework of a cointegrated vector autoregression model an empirical long-run money demand relation is identified and analysed. Nominal money demand is shown to be a function of domestic demand and the difference between money's own rate of return and a long-term government bond yield. Contrary to earlier findings the return on short-term foreign placements is no longer significant in the long-run relation. This result can be interpreted as being a consequence of increasing credibility of the fixed exchange rate system pursued by the Danish monetary authorities during the period under consideration.

**Resumé**

Papiret anayserer den danske pengeefterspørgsel i perioden 1980 – 2002. Den langsigtede pengeefterspørgsel identificeres og analyseres inden for rammerne af en cointegreret vektor autoregressiv model. Den nominelle pengeefterspørgsel afhænger af den indenlandske efterspørgsel og forskellen på pengemængdens afkast og den lange obligationsrente. I modsætning til tidligere studier findes, at afkastet på udenlandske placeringer ikke længere er signifikant i den langsigtede pengeefterspørgsel. Dette fortolkes i sammenhæng med den stigende troværdighed af Danmarks fastkurspolitik i perioden.

*Key words:* Money demand, co-integration

# 1 Introduction

This paper investigates the empirical money demand for Denmark 1980 – 2002 within the framework of a cointegrated vector autoregression model (cointegrated VAR).

Estimation of money demand functions in an European context have lately enjoyed increasing attention, not least because the European Central Bank puts special emphasis on M3 growth as an indicator of long-run inflation, arguing that inflation is ultimately a nominal phenomenon. Indeed, in a classic monetary approach with exogenous supply of money coupled with purchasing power parity and uncovered interest parity, the demand for money is the sole determinant of exchange rates, the interest rate, and inflation.

Denmark is a small open economy with fixed exchange rate against the euro. With free and liberalized cross-border capital movements, the main monetary policy instrument – the short term interest rate – is devoted to exchange rate considerations. Thus, the Danish short term interest rate is never used to pursue other goals, such as inflation or output. In the same vein, the stock of money is not under control by the monetary authorities, but responds endogenously to any amount of money the public wishes to hold.

Although the stock of money does not play any active role in the Danish monetary policy it is nevertheless of interest to investigate whether the demand for money develops according to commonly accepted hypotheses. In addition, persistent deviations from (a measure of) a steady state equilibrium development may be a sign of an altered trend driving the money demand, due to factors which are of interest to policy makers. One example of this is the recent year's deregulation of the Danish mortgage institution's credit giving to households, which has made it possible for households obtaining loans from mortgage institutions for other purposes than the financing of real estate. It is likely that this has increased the money demand, as mortgage loans' larger transaction costs compared to traditional bank loans give an incentive to "over-finance" the immediate need for liquidity, thus increasing the households' bank deposits. Such a change will show up as a large and prolonged deviation from the steady state path of aggregate money demand, determined by fundamental factors such as the total amount of transactions in the economy and the relative return on money, or as a change to the steady state equilibrium itself, i.e. a structural break.

The analysis presented in this paper builds on especially two previous studies of the Danish money demand. To my knowledge, Christensen & Jensen (1987) were the first to apply an error correction model to the Danish money demand.[1] They obtained a specification in which money demand depended on a scale-measure of the volume of transactions (final domestic demand with slight alterations) and the relative return on money measured as the difference between the deposit rate and a long term interest rate on government bonds. A special feature of money demand in small open economies is the need to consider the returns on alternative foreign assets. A portfolio balance argument suggests that foreign returns are likely to affect the desired holding of domestic money. Thus, Christensen & Jensen (1987) included a German money market interest rate and the D-mark return.

Hansen (1996) found that the specification in Christensen and Jensen was still a reasonable description of the Danish money demand. Contrary to Christensen & Jensen (1987), Hansen's (1996) analysis was performed in an explicit cointegration framework. This paper can be seen as a follow-up study of Hansen (1996), and finds that the Danish money demand today can be described slightly simpler than the two earlier studies suggest, in that the foreign return variables are no longer significant. This result should be interpreted against the background of increasing credibility of the Danish fixed exchange rate system over the period considered.

The paper is organized as follows. Section 2 lays out the basic money demand model and motivates the selected econometric framework and spec-

[1] The error correction model has become a very common tool for the empirical modeling of money demand. Studies of money demand using this approach are surveyed by Sriram (2001).

ification. Section 3 describes the data, and results are presented in section 4. Section 5 contains the conclusions.

# 2 Money demand and the econometric specification

## 2.1 Money demand

The basic relationship on a quarterly basis between money, transaction volume, and interest rates is illustrated in Figure 1. The cash-ratio $M/Y$ is depicted together with the interest rate differential $r^{dif} = r^{dep} - r^{gb}$, where $M$ denotes the aggregate M2-measure of the nominal money stock, $Y$ is the nominal domestic demand according to national accounts, $r^{dep}$ is the banks' average deposit rate, and $r^{gb}$ is the average interest rate on a 10-year government bond. The increasing tendency in the cash ratio from 1983 to 1994 was associated with a corresponding narrowing in of the interest rate differential. The dive in the interest rate differential 1994 – 1998 was associated with a more muted dip in the cash ratio. One explanation is the deregulation of the Danish mortgage institution's credit giving to households, cf. above, which might have dampened the reaction of the money demand to the widening interest differential. In the last part of the sample the interest rate differential is affected by increasing short interest rates in connection with the Russian default incident in 1998 and the Danish referendum on euro-adoption in 2000, while the cash ratio in that period followed a more dampened and almost constant course. The more formal analysis below aims at exploring the relationship between these variables more rigorously.

A simple and common way of expressing the money demand function in a small open economy is

$$\frac{M}{P} = M\left(\frac{Y}{P}, r^{dep}, r^{gb}, r^{*}, \Delta s, \Delta p\right),$$

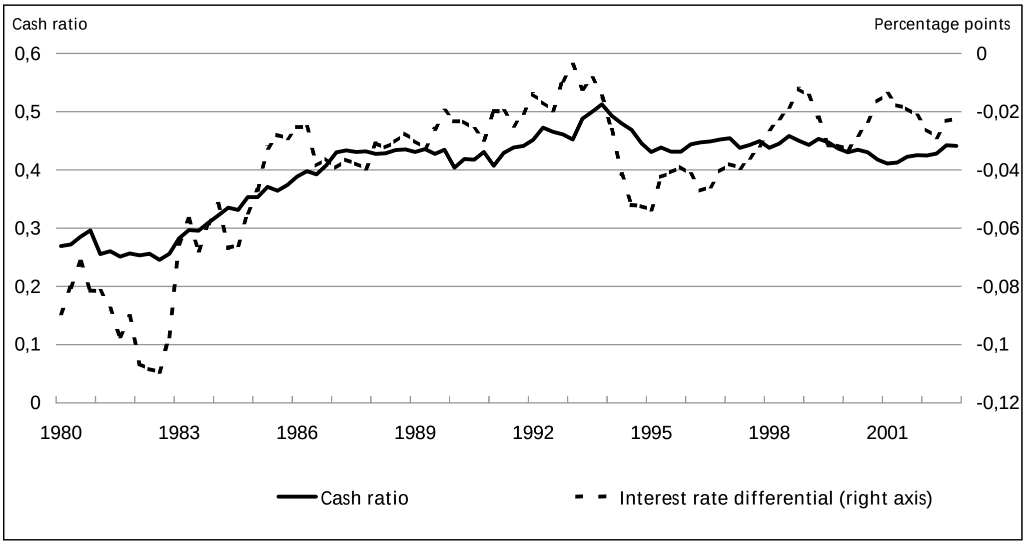


Figure 1: Cash ratio and interest rate differential

where $\Delta$ denotes the first difference operator, $P$ is the price level, $r^*$ is the foreign (German) interest rate, $\Delta s$ is the (log-) change in the DKK/EUR exchange rate, and are included in order to capture the effects of possible alternative foreign investment opportunities relative to domestic government bonds, $\Delta p$ is the (log-) change in the price level, i.e. the rate of inflation. The demand for real balances is expected to depend positively on real domestic demand (as a proxy for real transaction volume), positively on the deposit rate (proxy for money's own rate of return), negatively on the measures for alternative domestic and foreign returns $r^{gb}$ and $r^*$, negatively on the first (log-) difference of the exchange rate (when the exchange rate is expressed as the amount of Danish kroner needed to buy 1 euro), and negatively in the rate of inflation (as a proxy for the return on real assets). A proxy for the total domestic currency return on a foreign investment would be $(1 + r^*)\frac{S}{S_{-1}}$. Thus, a starting point for the estimation of the long run real money demand could be

$$m - p = \beta_0 + \beta_1(y - p) + \beta_2 r^{dep} + \beta_3 r^{gp} + \beta_4 r^* + \beta_5 \Delta s + \beta_6 \Delta p, \qquad (1)$$

where $m = \ln M$, $y = \ln Y$.

The specification in (1) forms the basis for an enormous literature on money demand estimation, cf. e.g. Goldfeld & Sichel (1990) and Sriram (2001).

## 2.2 Econometric specification

An important issue relates to whether the money demand should be formulated in real (as in equation (1)) or in nominal terms. Note that if $\beta_1 = 1$ and $\beta_6 = 0$ then a nominal demand function would be appropriate.

There are several reasons for concluding that a nominal specification is appropriate in the Danish case. First, due to almost two decades of very stable price development in Denmark, there is reason to doubt that the price level contain one or more unit roots. Indeed, simple Dickey-Fuller tests reject the hypothesis of a unit root against a trend stationary alternative. Second, the same type of tests suggest that the variables $y, m, y-p$,and $m-p$ all can be described as $I(1)$ processes. This can be so only if 1) $y, m, p$ are all $I(1)$, but prices and the nominal income and the monetary aggregate variables do not cointegrate without inclusion of further $I(1)$-variables, or 2) $y, m$ are $I(1)$, but $p$ is (trend) stationary.[2] Given the results from the Dickey-Fuller tests and the sensible economic interpretation one can make of it, the latter option seems the more plausible.[3]

If prices are trend stationary, neither the price level nor the rate of inflation can be a part of long run equilibrium in the typical Engle-Granger sense. So even without formal testing on a relationship like (1), we may choose to drop prices altogether. However, in an early stage of the analysis

[2] The measure of prices is the deflator for domestic demand.

[3] This is of course a controversial statement, as many authors often assume that price levels contain even more than one unit root. However, the evidence of trend stationary prices may be more in line with economic intuition, as trend stationary prices is consistent with the goal pursued by Denmark's nominal anchor, i.e. the ECB's definition of price stability.

the hypothesis was tested formally, and indeed the joint restriction $\beta_1 = 1$ and $\beta_6 = 0$ could not be rejected, supporting the initial suspicion. Hansen (1996) came to a similar conclusion.

On top of trend stationary prices, also the exchange rate return $\Delta s$ is possible stationary according to the Dickey-Fuller tests. This is not surprising given the success of the Danish fixed exchange rate regime, which has manifested itself and resulted in a very stable exchange rate in the last 3-4 years of the sample. Economically, it does not make much sense to model the Danish exchange rate vis-a-vis the euro as a variable containing a stochastic trend. Thus, as with prices we might suspect that the exchange rate return will be insignificant in the long run relationship. However, $\Delta s$ is included in the analysis due to the theoretical role it plays in measuring the return on foreign alternatives to Danish securities.

When an $I(0)$ variables is included in a cointegrated VAR (which apart from the $I(0)$ variable consists only of $I(1)$ variables), the dimension of the cointegrating space is theoretically increased by one, because a "cointegrating" vector with weights zero with respect to the $I(1)$ variables and non-zero weight with respect to the stationary variable will be stationary. Hence, the tests for the number of cointegration vectors should be interpreted with due care, and one possible approach (which will be followed below) is simply to tentatively reduce the number of cointegrating relations by one, if one suspects that the system may contain a stationary variable. Although the stationary variable is irrelevant for the long run properties of the model, it may well play a role in the short run dynamics and contribute to the congruence of the system.

Attempting to estimate the vector autoregression consisting of $m$, $y$, $r^{dep}$,$r^{gb}$,$r^{*}$, and $\Delta s$ reveals that a relatively large number of lags are needed in order to leave the estimated residuals without detectable signs of autocorrelation. Independent residuals are a prerequisite for the consistency of the Johansen reduced rank test for cointegration. Therefore the following restrictions are imposed ex ante: $\beta_3 = -\beta_2$, i.e. the interest differential $r^{dif}$

is included instead of $r^{dep}$ and $r^{gb}$ individually.[4] Furthermore, the German interest rate is included as an exogenous variable. Obviously, it is very unlikely that any of the Danish variables under consideration should be able to explain the German interest rate, due to the different size of the economies. Therefore, in a VAR model, the only variable that can be expected to have explanatory power with respect to the German interest rate would be lagged values of the German interest rate itself, accounting for why a large number of lags are needed to reach serially uncorrelated residuals when $r^*$ is included as an endogenous variable. Finally, $\Delta s$ is included as an exogenous variable as well. Indeed, if the exchange rate return is close to $I(0)$, then any $I(1)$ variables should have no explanatory power whatsoever with respect to $\Delta s$.[5] A discussion of the relevance of imposing ex ante exogeneity in cointegrated VAR systems is provided in Pesaran & Smith (1998) and Pesaran, Shin & Smith (2000).

# 3 Data

The sample is 1980q1 – 2002q4. The samples of the two earlier studies started in 1975q1. However, the latest revisions to consistent data for the monetary aggregates are only available from 1980q1 onwards.

In this study, the focus is on the monetary aggregate M2, whereas M3 seems to be various authors' preferred choice, cf. Sriram (2001). However, in the Danish case M3 is fluctuating in recent years due to technical factors, which are unrelated to the underlying determinants of the money demand. Specifically, when the short term adjustable-rate mortgage loans are rolled over each year, new bonds are issued to replace the bonds that matures. Whether these maturing and/or newly issued bonds are a part of M3 depends on the bond series' original maturity. Hence, if mortgage institutions are

[4] Hansen (1996) could not reject this restriction.

[5] This goes the other way around too, of course. However, the inclusion of $I(0)$ variables in a cointegrated VAR system does not hamper correct inference with respect to the cointegrating vectors.

issuing bonds in a bond series with, say, a long original maturity (outside M3), and these are replacing bonds with a short original maturity (inside M3), then the M3 measure of the money supply will show a large drop, although the structural money demand may well have been completely unchanged. [6]

As the scale variable $y$, the seasonal corrected domestic demand in nominal terms according to the national accounts statistics is used, and accordingly the price variable used to determine the relevant specification of the system is the deflator for domestic demand. The time series for the interest rates and exchange rate data are calculated as quarterly averages. The exchange rate return is the annualized quarterly log-change of the Danish exchange rate vis-a-vis the euro. This prevents overlapping observations although at the cost of a more volatile time series.

# 4 Results

As discussed above, the money demand function is formulated in nominal terms, and the cointegrated VAR model is formulated in three endogenous variables: $m$, $y$, and $r^{dif}$, and two exogenous variables $r^*$ and $\Delta s$, both restricted to the cointegrating space. This allows for a comparison with the earlier study of the Danish money demand.

Deterministic components play an important role in cointegrated VAR models. In order to allow for deterministic trends in the data, the VAR was initially formulated with an unrestricted constant and a deterministic trend restricted to the cointegrating space, which implies a symmetric assumption about the possible trends in the data inside and outside the cointegrating space. However, the trend in the cointegrating space turned out to be insignificant across various specifications, although the test for whether the unrestricted constant could be restricted to cointegrating space was rejected. This could be the result of more than one deterministic trend component

[6] Euro deposits in Denmark make up less than 5% of of M2, and is assumed not to dominate the development in M2.

cancelling out in the long run relation. Thus, we focus here on a model with no deterministic trends, but with unrestricted constant terms included.

The consistency of the Johansen test for the number of cointegrating relations is particularly sensible to serial correlation in the error terms, see e.g. Johansen (1995). Hence, the lag-length of the VAR model is chosen such as to remove evidence of remaining autocorrelation in the estimated error terms, while at the same time keeping an eye on the information criteria. An overall assessment lead to a VAR model with one lag, although there are weak signs of autocorrelation, which mainly can be attributed to a few outliers. It requires a relatively large number of lags to significantly improve on the diagnostic tests. At the same time, the associated consumption of degrees of freedom left for the variance in the system when estimating high order VAR models will reduce the power of the misspecification tests. Thus, a first order VAR is considered appropriate.

When the model contains exogenous $I(1)$ variables the relevant asymptotic distribution under the null hypothesis for the $\lambda-$ max and the Trace statistic is altered. The critical values for the standard Johansen test can therefore not be used. The relevant asymptotic distribution is instead taken from Pesaran et al. (2000), Table T.3, which contain the relevant critical values for tests at 5 pct. and 10 pct. critical values. The results of the Johansen test is presented in Table 1.

Johansen tests — Table 1

| Rank | $\lambda$-max | 5 pct. crit.val. | 10 pct. crit.val. | Trace | 5 pct. crit.val. | 10 pct. crit.val. |
|---|---|---|---|---|---|---|
| 0 | 24.31 | 24.59 | 22.15 | 44.50 | 38.93 | 35.88 |
| 1 | 17.03 | 18.06 | 15.98 | 20.19 | 23.32 | 20.75 |
| 2 | 3.06 | 11.47 | 9.53 | 3.16 | 11.47 | 9.53 |

Note: Critical values taken from Pesaran et al. (2000), Table T.3 with $k = 1$ and $n = 3$.

The $\lambda$-max tests point at two cointegrating relations at 10 pct. significance level, while at the 5 pct. level the hypotheses are borderline accepted (i.e. no cointegration). The Trace tests point at one (at 5 pct. significance level) or two (at 10 pct. significance level) cointegration relations. As discussed above, an explanation for the second potential cointegrating relationship is probably due to the stationarity of the exchange rate return. Moreover, estimating the number of cointegrating relationships too high (i.e. including a relationship which is perceived stationary, but is not) is damaging the consistence of the statistical inference based on the system, while estimating the number of cointegration relationships too low (i.e. excluding a true cointegrating relation) does not. In the former case, a non-stationary component will be present in the stationary ECM representation of the system, while in the latter case empirically relevant information is forgone, but the assumptions about the statistical properties of the system remain correct. The cointegrating rank is therefore fixed at one.

With this restriction the unrestricted estimates of the long run coefficients become

$$m_t = \beta_0 + \underset{(0.15)}{0.84} y_t + \underset{(1.6)}{4.7} r_t^{dif} - \underset{(1.4)}{3.4} r_t^* - \underset{(0.65)}{0.51} \Delta s_t,$$

with the asymptotic standard errors in parentheses. All variables enters the relationship with the correct signs. For example, an increase in the foreign interest rate (the opportunity cost of holding money) by one percentage point corresponds to a decrease in the nominal money demand of roughly $3\frac{1}{2}$ pct. The exchange rate return is the only insignificant variable according to the asymptotic standard errors, which is in accordance with the suspicion that $\Delta s$ is a stationary variable.

Estimates of the factor loadings — Table 2

| Equation: | $m$ | $y$ | $r^{dif}$ |
|---|---|---|---|
| | $-0.13$ (0.025) | $-0.024$ (0.013) | $-0.0076$ (0.0060) |

Note: Asymptotic standard errors in parentheses.

The free estimate of the vector of factor loadings is given in Table 2. The point estimates show that the model does not "error correct" in the equations for $y$ and $r^{dif}$. For instance, if $y$ is greater than the level suggested by the long run equilibrium relation, the residual from that long run level will be negative, ceteris paribus. With a negative estimate of the factor loading in the equation for $y$, this implies a positive contribution to the change in $y$ in the next quarter, and therefore does not help in correcting for the "error". The same is the case in the equation for $r^{dif}$, but not $m$.

However, the asymptotic standard deviations of the estimates of the factor loadings suggest that the factor loadings for $y$ and $r^{dif}$ may not be significant. If this is the case, then $y$ and $r^{dif}$ are weakly exogenous with respect to the long run parameters in the equation for $m$. This would strengthen the case for interpreting the cointegration relationship as a money demand function. A likelihood ratio test for the weak exogeneity of $y$ and $r^{dif}$ is given in Table 3, which also reports the results from some further hypotheses testing on the system.

| Results from hypothesis tests | | | Table 3 |
|---|---|---|---|
| Hypothesis | Test statistic | Distrib. $\sim H_0$ | $p$-value |
| 1: $y$ and $r^{dif}$ weakly exogenous | 2.19 | $\chi^2(2)$ | 0.33 |
| 2: 1 and $\beta_3 = \beta_4 = 0$ | 9.04 | $\chi^2(4)$ | 0.06 |
| 3: 2 and $\beta_1 = 1$ | 9.05 | $\chi^2(5)$ | 0.11 |
| 4: 1 and $\beta_1 = \beta_2 = 0$ | 15.63 | $\chi^2(4)$ | 0.004 |
| 5: $m$ weakly exogenous | 7.00 | $\chi^2(1)$ | 0.008 |

Note: Long run relation: $m_t = \beta_0 + \beta_1 y_t + \beta_2 r_t^{dif} + \beta_3 r_t^* + \beta_4 \Delta s_t$.

The hypotheses 1, 2, and 3 aim at reaching a parsimonious description of the long run money demand relation, while the hypotheses 4 and 5 are testing the significance of the core parameters and the validity of the system as a representation of a money demand function. Hypothesis 4 ($y$ and $r^{dif}$ are insignificant) and hypothesis 5 ($m$ exogenous) are both rejected at conventional significance levels, but not by a large margin. Hypothesis 3 represents the most parsimonious model that cannot be rejected, and implies that in the long run the exchange rate return and the German interest rate have no explanatory power with respect to the monetary aggregate. Moreover, the stochastic trends in $y$ and $r^{dif}$ are driving the system, supporting that we are in fact estimating a money demand relation. Hence, the long run money demand can be described in a very simple form,

$$m_t = \beta_0 + y_t + \underset{(0.92)}{6.5}\, r_t^{dif}. \tag{2}$$

The unit long run elasticity of the domestic demand with respect to the stock of money resemble the results of the earlier studies. The novel feature of this long run relation compared to the previous studies of the Danish money demand is that the terms representing the return on an alternative foreign investment are insignificant. From an econometric point of view, it is not surprising that the exchange rate return is insignificant, as it is best described as a stationary variable. Economically, this stationarity arises from the continued success of the Danish fixed exchange rate system, which in recent years has virtually eliminated volatility in the exchange rate compared to previous levels. This is the single most important difference between the present data set and the ones previously used: In large parts of the sample the fixed exchange rate system has enjoyed a large and increasing degree of credibility. Hence, the by now almost non-existing exchange rate risk implies that with the present data set, the Danish euro-exchange rate plays no significant role in the portfolio decisions envisaged in the present context.

At a first thought it might be more surprising that the German interest rate is also insignificant. However, as exchange rate risk diminished Danish and German interest rates converged, and ended up moving in tandem. Such development follows from simple international interest parity conditions, and it is exemplified in Figure 2, which compares the Danish and German 3-months' money market interest rates.

The interest rate convergence in the period after 1994 is immediately clear. Thus, in the sample the dynamics of the German interest rate is to an increasing degree captured by the Danish interest rates, such that

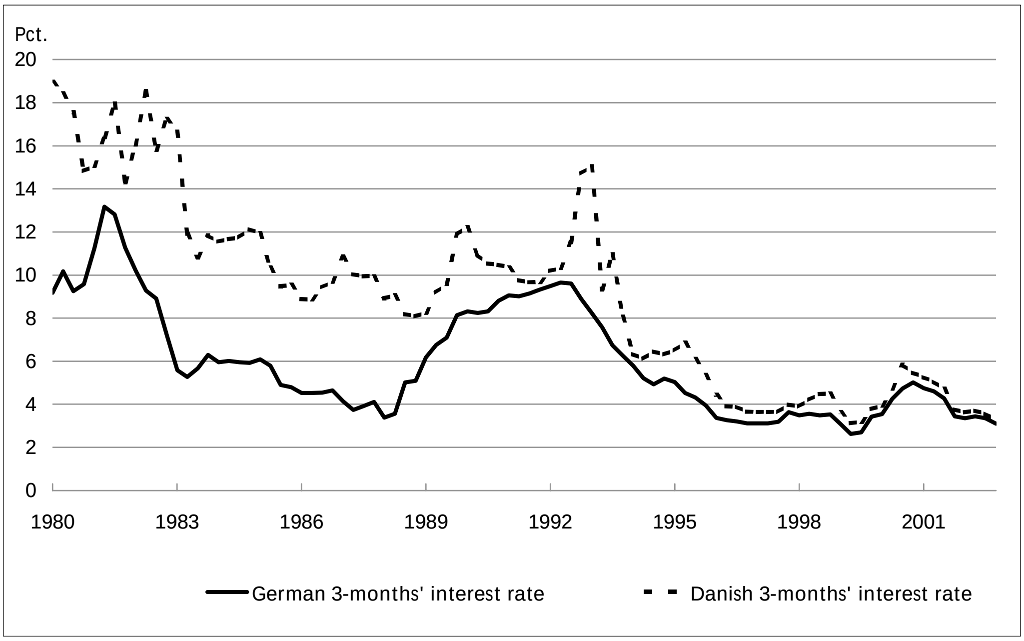


Figure 2: Danish and German money market interest rates

the possibility of large differences in the expected returns on investments in Danish and German debt securities have vanished.

The credibility of the Danish fixed exchange rate system has thus had a significant impact on the structural money demand in Denmark, and is effectively responsible for the very simple specification in (2).

An impression of how well the estimated long run money demand fits the data is given in figure 3. The adjustment of the money stock towards its model-implied level is very sluggish in some periods. In particular, the actual money stock was greater than the level suggested by the its determinants from 1993 to 1998. This is the period where the gaining credibility of the fixed exchange rate system kicked in, and caused the short term interest rate to drop sharply, especially in 1993 – 1995. In 1995 – 1998 the long term interest rate more gradually dropped too, such that the spread between the short and long term interest rate returned to its pre–1990 level, cf. figure 1. Hence, investors chose to hold money although it was relatively expensive in

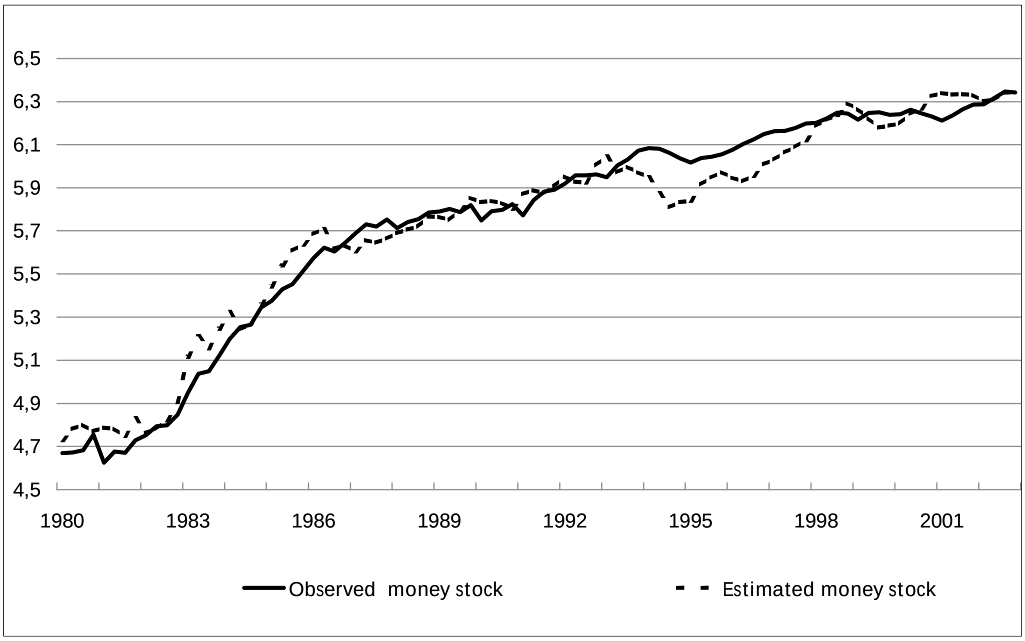


Figure 3: Observed and estimated money stock

terms of opportunity costs. One explanation might be that since such sharp falls in interest rates are rare, agents might have believed that the drop would be temporary, and hence they did not swiftly adjust their portfolios. In other words one interpretation of the large deviation between actual and fitted money supply in the mid-90's is that of a learning process on behalf of investors.

Figure 4 also compares the actual and estimated money stock, but allows the short run dynamics to work on top of the long run relation. The impression is that of a very tight in-sample fit and is a common feature of this type analysis where a rather rich lag structure is allowed for.

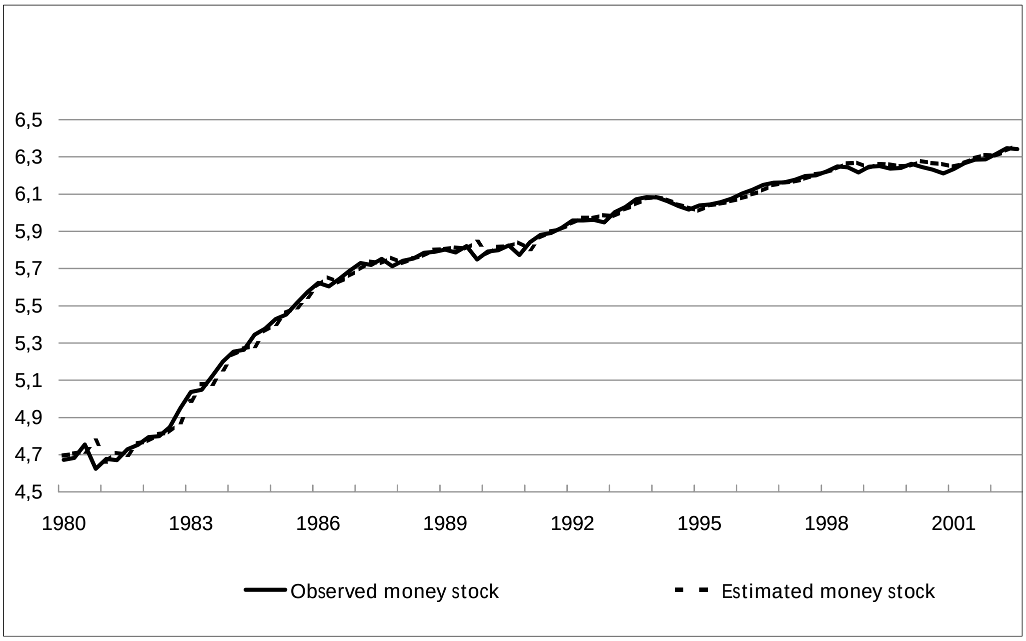


Figure 4: Observed and actual money stock including short run dynamics

## 4.1 Statistical properties of the model

### 4.1.1 Misspecification tests

Results from the usual misspecification tests are presented in appendix A. The tests show that there are some problems with deviation from normality in the residuals. This can be attributed to a few outliers, namely the first quarter of 1981 and the first quarter in 1983. Taking these observation out by including dummy variables enables the model to pass the tests at a 1 percent significance level, and most of them at a 5 percent significance level. However, since the dummies do not alter nor the parameter estimates, neither the conclusions from the various hypothesis tests in any significant way the preferred model is without the dummy variables. A special problem concerning evidence of some remaining autocorrelaton in the residuals is discussed in the appendix.

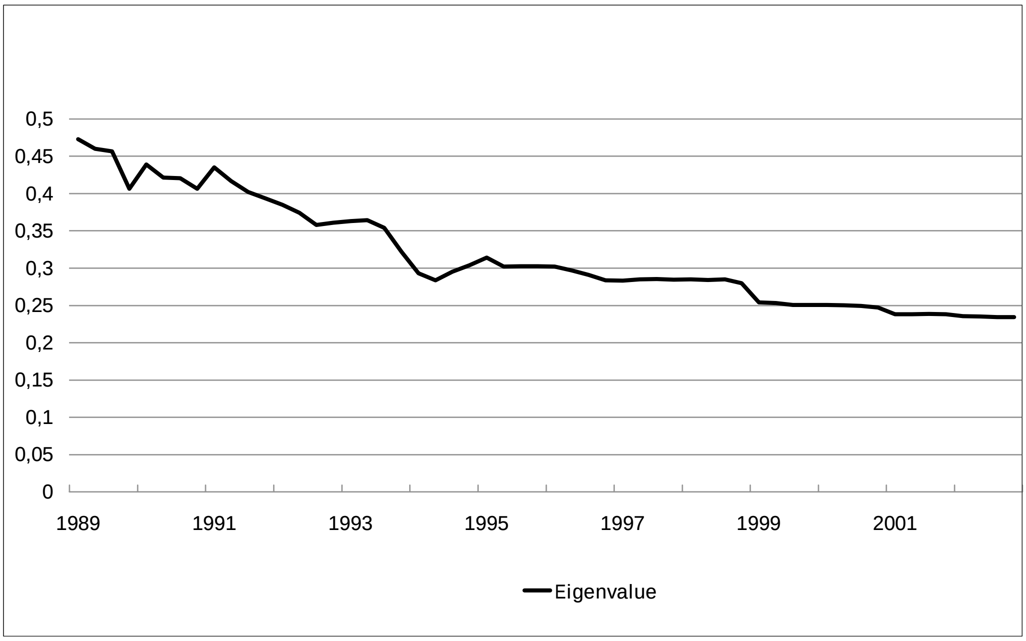


Figure 5: Recursive estimates of the eigenvalue

### 4.1.2 Stability properties

In this section results from recursive estimation are presented.

First, the stability of the eigenvalue corresponding to the cointegrating relation is described. The square root of the eigenvalue has the interpretation of a canonical correlation, and is a measure of how strong the cointegrating relationship is: The higher the eigenvalue the stronger the cointegrating relationship. Recursive estimates of the eigenvalue, i.e. the eigenvalue recalculated successively with a gradually increasing sample, is presented in Figure 5.

It appears that the relation becomes gradually weaker, although the estimates of the eigenvalue level off towards the end of the sample. It is helpful to interpret this result together with figure 6, which plots the recursive test statistic from hypothesis number 2 in Table 3, i.e. the test for the significance of the exchange rate return and the German interest rate.

Figure 6 confirms that the ability of the exchange rate return and the

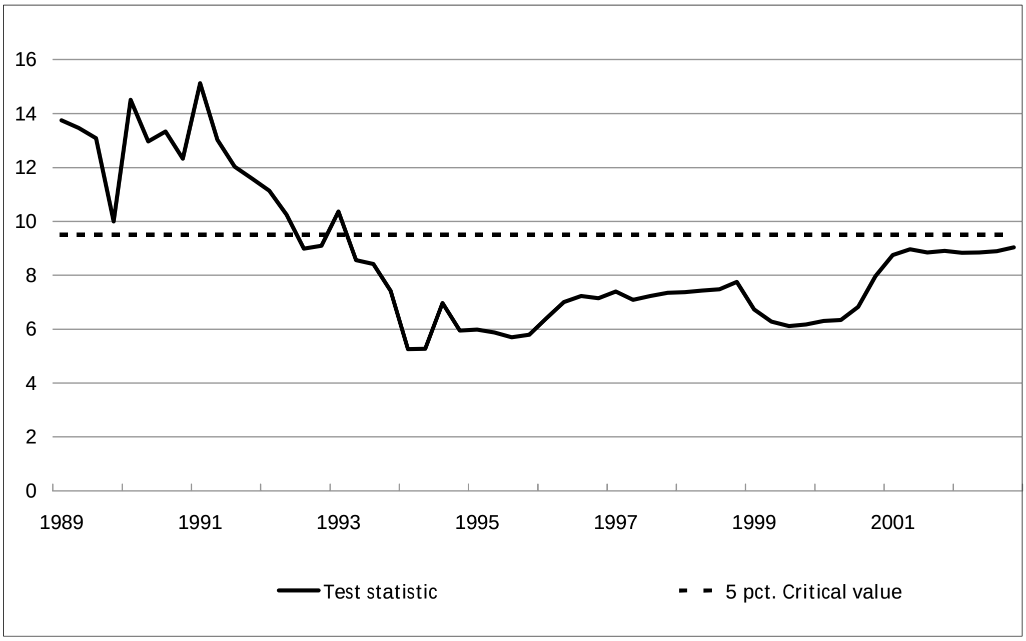


Figure 6: Recursive estimates of test statistic for hypothesis 2

German interest rate to explain the money stock disappears with the fall in the interest rates after 1993 following the stable development in the exchange rate. This can help explain why the cointegrating relationship weakens in the same period, as the number of significant variables in the relation drops from four to two.

A suspicion that naturally arises from these figures is that the model is infected with a structural break around 1993. Interestingly, figures 7 and 8, which show the recursive estimates of the parameters $\beta_1$ and $\beta_2$,[7] do show signs of large problems in this respect. The recursive estimates are shown with $\pm$ two times their asymptotic standard errors, and it seems that when parameter uncertainty is taken into account, there are no major changes in the parameter estimates as the sample increases. The income elasticity, though, drops from about 1.2 to 1 during the last couple of years in the sample and just then reach the lower border of the error-band in some earlier periods.

[7] The recursive estimates are based on a specification where $\beta_1$ is allowed to vary freely.

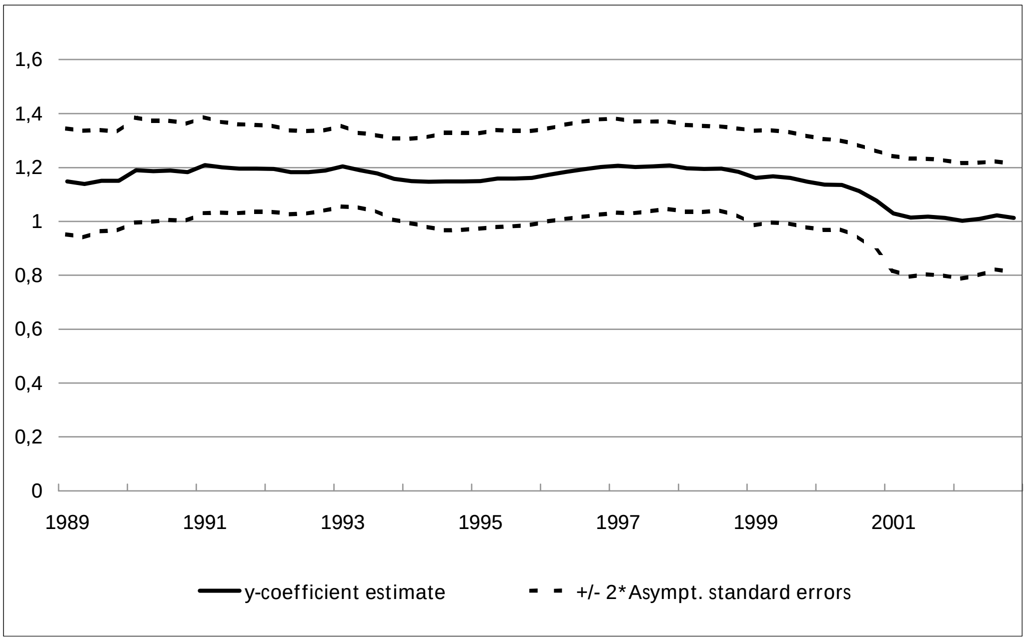


Figure 7: Recursive estimates of $\beta_1$

One can also note a slight widening of the interest rate semi-elasticity towards the end of the sample, but the interest rate differential remains strongly significant. The widening of the error band can probably be attributed to increasing short term interest rates around some exogenous events like the Russian default in 1998 and the euro referendum in 2000, which lead to narrowing of the interest rate differential. In so far that these incidents were (rightly) perceived as temporary phenomena, reflecting periods with e.g. temporarily higher risk premia, investors might have abstained from reshuffling their money demand although the opportunity costs for holding money were relatively low. This can also be seen from Figure 3: The model-implied money demand is slightly greater that the observed money demand in 1998 and even more so in 2000 – 2001.

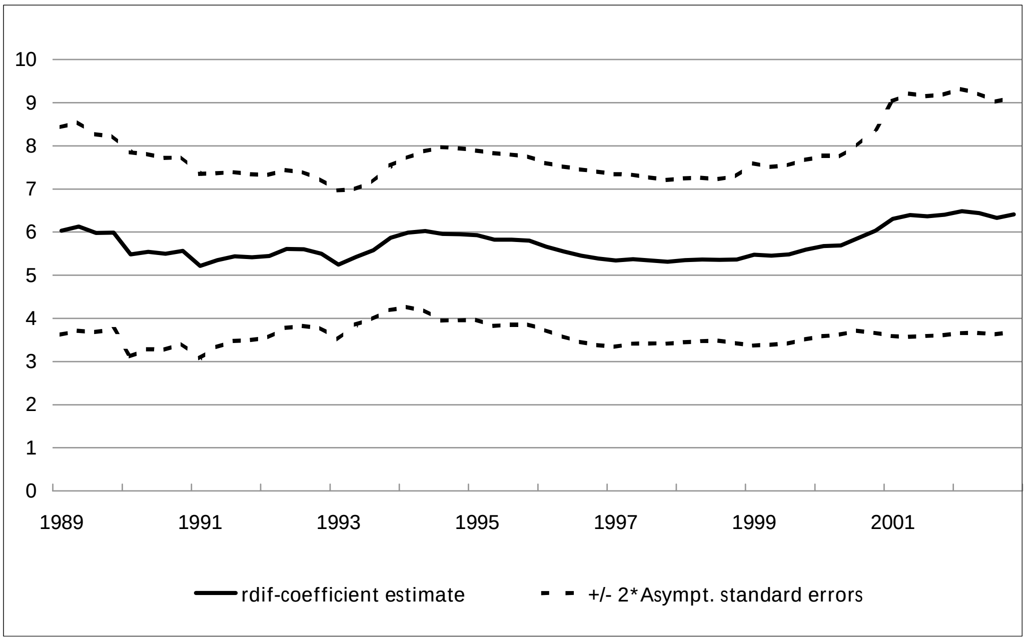


Figure 8: Recursive estimate of $\beta_2$

#### 4.1.3 Forecast properties

The main focus of this study has not been on out-of-sample performance but rather on the properties of the structural money demand. However, sometimes an analysis of the forecast performance of an empirical model can detect whether a model has been "over-fitted", i.e. whether any tight in-sample fit has been reached by using an uncomfortable large number of parameters, leaving very few degrees of freedom left for the variance. If the estimation has consumed too many degrees of freedom, then this will show up as very poor post-sample performance.

It is not very likely that the model presented above will suffer from over-fitting as the initial cointegrated VAR model relatively small to begin with: The appropriate number lags was one, and we started out with a conditional model. The analysis enabled a reduction of the model, by conditioning further down to a single equation, and leaving out the insignificant variables. In fact the final model has only four parameters.

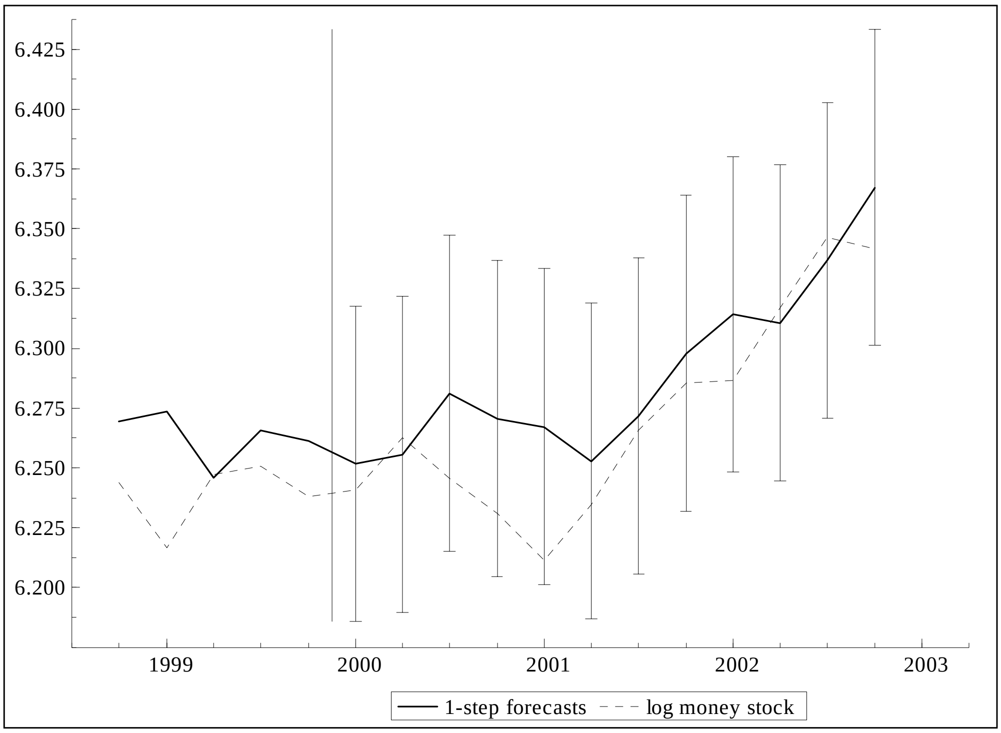


Figure 9: 1-step forecasts and log money stock

A an analysis of the robustness of the model is performed in the following manner. The model was reestimated saving the last three years (12 quarters) of observations for the forecast analysis. Then the model predicts one quarter ahead using the actual values of the explanatory variables, and the predicted values are compared with the actual values. No reestimation takes place between the forecasts. The results is shown in Figure 9.

All the forecasts lie within the estimated error-bars, which are calculated taking into account both the residual variance and the parameter uncertainty. The model does not capture the development in the second half of 2000, but tracks the development fairly precisely in the rest of the forecasting period. In the second half of 2001 the development in the short term interest rates were affected by the Danish euro referendum, cf. above. Thus, the conclusion is that there are no apparent signs of over-fitting.

## 5 Conclusion

The estimated money demand function for Denmark shows that the Danish money demand still develops according to commonly accepted theories, i.e. the money demand is proportional to the a scale measure of transactions, here the domestic demand, and positively related to the difference between the money stock's own rate of return and the interest rate on a long term government bond.

Given the very parsimonious specification the model gives a reasonable statistical description of the data, shows fairly stable parameter estimates, and has an acceptable forecast performance.

The biggest change from earlier studies is that the measures of the returns from investments in Germany (German interest rate and exchange rate return) are no longer significant. The interpretation is that the gradually increasing credibility of the Danish fixed exchange rate system through the sample has eliminated signs of any stochastic trends in the exchange rate return, and caused the German and Danish interest rate to converge such that the explanatory power of the German interest rate documented in earlier studies no longer is present. In effect, the credible fixed exchange rate has caused the Danish and German interest rates to converge, why the potential gain in terms of risk diversification by investing in both Danish and German debt instruments has disappeared. Such diversification gains may have been relevant considerations in the 70's and 80', but much less so in present days where German and Danish debt securities have become close substitutes. This can explain why the money demand is today can be described by domestic interest rates alone.

One interesting hypothesis to investigate would be whether other foreign return variables, e.g. the US interest rate, would challenge the present specification. This is left for further research, however.

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# Appendix A: Misspecification tests

Misspecification tests are shown for two models: The model presented in the main text and an identical model, except that two dummies are included (unrestricted), namely for the first quarters of 1981 and 1983.

The results for the model discussed in the main text is presented in Table A1. The model fails all the misspecification tests except the test for ARCH effects in the estimated residuals. However, the results in Table A2 with the two dummies included reveals that the failing of the tests can be almost entirely attributed to the two outliers in the beginning of the 80's.

Misspecification tests for model without dummy variables — Table A1

| | AR 1-4 | Normality | ARCH 1-4 | Hetero | Hetero-X |
|---|---|---|---|---|---|
| Test statistic | 5.15 | 22.45 | 0.084 | 2.91 | 3.97 |
| Distr. $\sim H_0$ | $F(4,82)$ | $\chi^2(2)$ | $F(4,82)$ | $F(10,73)$ | $F(20,63)$ |
| $p$-value | 0.00 | 0.00 | 0.99 | 0.00 | 0.00 |

Misspecification tests for model with dummy variables — Table A2

| | AR 1-4 | Normality | ARCH 1-4 | Hetero | Hetero-X |
|---|---|---|---|---|---|
| Test statistic | 5.70 | 8.39 | 1.03 | 0.68 | 0.42 |
| Distr. $\sim H_0$ | $F(4,80)$ | $\chi^2(2)$ | $F(4,80)$ | $F(10,69)$ | $F(20,59)$ |
| $p$-value | 0.00 | 0.02 | 0.40 | 0.73 | 0.98 |

Note: Dummy variables are for 1981q1 and 1983q1.

Even in the model with the dummy variables remaining autocorrelation in the estimated residuals are present. A more detailed analysis revealed that the sign of autocorrelation is due to a slightly significant fourth order autocorrelation, which suggest that the are some remaining seasonal effects at play. Indeed, including seasonal dummies in the model to begin with removes all signs of autocorrelation.

In order to investigate the robustness of the model to these results, the entire analysis was repeated with a model including the seasonal and regular dummies. All results carry through virtually unchanged. The most significant change is the $p$-values for tests in Table 3 which are reduced slightly, e.g. for hypothesis 2 the $p$-value is reduced from 0.06 to 0.03. The point estimates of the parameters are almost identical, and the conclusions as regards the stability properties and forecast performance are unchanged.

The conclusion is that the remaining autocorrelation in the preferred model has no economic significance. Furthermore, there is some evidence that the detected seasonal pattern is most pronounced in the beginning of the sample. Reestimating the model starting the sample in 1987 leaves no detectable systematic pattern in the residuals. Hence, since one of the purposes of the model is to follow the structural part of the Danish money demand more closely in the future, it seems safer to base such forward-looking analyses without dummies in order to account for an ancient and possible spurious seasonal pattern.