DANMARKS NATIONALBANK

DANMARKS NATIONALBANK WORKING PAPERS

2004 • 21

Dan Knudsen

Danmarks Nationalbank, Copenhagen

and

Faisel Sethi

Danmarks Nationalbank, Copenhagen

Chain indexing in a macro model – Aggregation and irreversibility

December 2004

The Working Papers of Danmarks Nationalbank describe research and development, often still ongoing, as a contribution to the professional debate.

The viewpoints and conclusions stated are the responsibility of the individual contributors, and do not necessarily reflect the views of Danmarks Nationalbank.

As a general rule, Working Papers are not translated, but are available in the original language used by the contributor.

Danmarks Nationalbank's Working Papers are published in PDF format at www.nationalbanken.dk. A free electronic subscription is also available at this Web site.

The subscriber receives an e-mail notification whenever a new Working Paper is published.

Please direct any enquiries to
Danmarks Nationalbank, Information Desk, Havnegade 5, DK-1093 Copenhagen K Denmark
Tel.: +45 33 63 70 00 (direct) or +45 33 63 63 63
Fax : +45 33 63 71 03
E-mail:info@nationalbanken.dk

Nationalbankens Working Papers beskriver forsknings- og udviklingsarbejde, ofte af foreløbig karakter, med henblik på at bidrage til en faglig debat.

Synspunkter og konklusioner står for forfatternes regning og er derfor ikke nødvendigvis udtryk for Nationalbankens holdninger.

Working Papers vil som regel ikke blive oversat, men vil kun foreligge på det sprog, forfatterne har brugt.

Danmarks Nationalbanks Working Papers er tilgængelige på Internettet www.nationalbanken.dk i pdf-format. På webstedet er det muligt at oprette et gratis elektronisk abonnement, der leverer en e-mail notifikation ved enhver udgivelse af et Working Paper.

Henvendelser kan rettes til :
Danmarks Nationalbank, Informationssektionen, Havnegade 5, 1093 København K.
Telefon: 33 63 70 00 (direkte) eller 33 63 63 63
E-mail: info@nationalbanken.dk

ISSN (trykt/print) 1602-1185

ISSN (online) 1602-1193

Chain indexing in a macro model – Aggregation and irreversibility

Dan Knudsen and Faisel Sethi
Danmarks Nationalbank
Havnegade 5
DK-1093 Copenhagen K
Denmark
Phone:+45 33 63 63 63
E-mail: Dkn@nationalbanken.dk or Fas@nationalbanken.dk

December 2004

Chain indexing in a macro model – Aggregation and irreversibility

Abstract

We investigate a switch from fixed-weighted national accounts data to chain-type data in the quarterly macroeconometric model Mona. As in other countries the idea behind this change in the national accounts data has been to reduce the substitution bias, which has grown with the amount of high-technology goods with a weak price development deviating from the average.

In line with Lasky (1998) we demonstrate that the use of the chain index can also reduce some of the typical aggregation problems in macro econometric models based on traditional fixed-weight relations. Chain-type national accounts data are not perfect as e.g. their lack of additivity makes them more complex to work with. We demonstrate that also the lack of reversibility in the chain index may complicate some model results by leading to permanent effects from temporary chocks. It is not clear a priori if a switch to chain weights in the definitional relations on balance improve the model.

Key words: Chain index; National Accounts; Macro model Mona; Composition effects; Aggregation problem; Lack of reversibility.

Resumé

Vi belyser, hvilken betydning en overgang fra det traditionelle fastvægtsindeks til kædeindeks i nationalregnskabet har for den kvartalsvise makroøkonometriske model, Mona. Som i andre lande er ideen med at anvende kædeindeks i nationalregnskabet, at man vil reducere fastvægtsindeksets substitutionsbias, som er vokset i takt med et stigende antal højteknologiske produkter med særlig svag prisudvikling. Den internationale nationalregnskabsmanual anbefaler også kædeindeks.

På linje med Lasky (1998) vises, at anvendelsen af kædeindeks også kan reducere de sammensætningseffekter, som kan genere i simple aggregerede makromodeller, der er bygget op om det traditionelle fastvægts-nationalregnskab. Anvendelsen af kædeindeks er dog ikke uden problemer. En konsekvens er, at modellens simple nationalregnskabsidentiteter ikke længere holder i faste priser. Desuden introduceres kædeindeksets irreversibilitet, som kan komplicere modeleksperimenter, fx ved at midlertidige stød giver permanente effekter. Sammenfattende er det ikke på forhånd klart, at anvendelse af kædeindeks i modellens fastprisidentiteter vil forbedre modellen. Man skal formentlig gøre sig nogle erfaringer, og afprøve kædeindekset, før man beslutter sig.

Det meste af indholdet i dette papir findes i en artikel i Nationaløkonomisk Tidsskrift 2004, nr. 2.

Table of contents

1. Introduction

Starting in the nineties there has been a renewed interest in measurement problems concerning the price/quantity split of economic variables like private consumption, investment and GDP. One of the most cited contributions has been the so-called Boskin-report, Boskin et al. (1996), which argued that the CPI exaggerated the increase of American consumer prices.

Any problem with the measurement of prices implies a problem with the measurement of volumes, and also the compilation of fixed prices within the national account system has been discussed. Among the arguments in Boskin et al. was that technical progress was not taken sufficiently into account when assessing price developments. Computers and similar high tech products were not given the significant price fall that these products should be given considering the significant rise in quality.

Introducing such a price fall gives rise to a significant trend in relative prices between traditional goods on the one hand side and high tech goods on the other, and this trend in relative prices has been an important argument for introducing a current updating of the weights used in fixed price calculations, cf. e.g. Landefeld and Parker (1995) on the new American national accounts based on chain indexing. The explicit recommendation of chain indexing in national accounts is also incorporated in the SNA93 manual (1993). Inspired by the SNA93 manual and by the significant technological impact on relative prices more and more countries have started to use the chain index formula.

Adopting the chain index formula means that macro data are aggregated in a new way and this change has implications for macro models like the quarterly Mona model of the Danish Central bank. Firstly, a revision of all fixed price series means that all behavioural equations must be re-estimated and consequently their coefficients may change. Secondly, a number of definitional equations concerning fixed price entities have to be re-formulated if they are to reflect the chain index formula and that re-formulation would affect the model properties.

This paper focuses on the second point i.e. on the consequences of re-formulating definitional relations. Thus, we keep the behavioural equations of the model as they are but we introduce new definitional equations, which base the model's GDP and non-farm GVA on chain-index formulas, and we do a couple of model experiments to illustrate the consequences of applying the chain index formula. Let us mention three consequences of chain indexing in the model.

The first consequence to mention is that with the chain index the standard national account identities of the model no longer hold in fixed prices, cf. Vavares, Prakken og Guirl (1998). Simple additivity breaks down and the new definitional relations in your model are more complicated and less user friendly.

The second consequence of chain indexing in the model is more favourable. The chain index formula reduces the composition effects, which constitute a nuisance in a macro model built around the traditional fixed weight index, cf. Lasky (1998). The occurrence of composition effects reflects the potential price/cost inconsistency of having several demand components but only one output-producing sector in a model. Substituting, for instance, a component with a low deflator for a component with a high deflator can reduce the value added per unit produced without changing the model's cost per unit. One way to address this aggregation problem, without disaggregating the producing sector in the model, is to introduce the chain formula where the weights for the demand components are updated from year to year.

The third consequence is potentially bad. The current updating of weights, which reduces the compositional problems, also makes the chain index irreversible and path dependent. This lack of reversibility may keep the endogenous variables from returning to the base line after a temporary shock.

We can safely say that it is non-trivial to establish a best practice for handling the price/volume split in applied macro analysis, and the problem is not new. Keynes (1936) suggested measuring in wage units to facilitate analysis of aggregate demand and aggregate supply.

In the following section 2 we illustrate fixed weight indices versus chain indices by national account series from Statistics Denmark. Readers interested in the model-related illustrations may proceed directly to section 3 where we introduce the necessary definitional equations to calculate GDP and non-farm GVA as chain indices on the basis of Mona variables. We explain the potential aggregation problem in simple macro models in section 4, and in section 5 we illustrate the beneficial impact of chain indexing on the aggregation problem. In section 6 we introduce the irreversibility problems of chain indices, and in section 7 we illustrate the possible consequences of chain index irreversibility for the model. Section 8 concludes.

2. Chain indexing and Danish national accounts

The official Danish national account series in fixed prices are in 2004 still based on the Laspeyres fixed weight index with 1995 as price base. However, alongside the official national account series Statistics Denmark already calculates and publishes annual series in fixed prices based on the Laspeyres chain index, and the plan is to make these chain-index based series the official national account series in 2005.

The traditional Laspeyres fixed weight volume index aggregates volumes by weighting them with the prices of a certain price base period. The Laspeyres fixed weight index Q^{LA} for period t over the base period 0 is given by:

$$Q_{t:0}^{LA} = \frac{\sum_i p_0^i q_t^i}{\sum_i p_0^i q_0^i} \qquad (1)$$

The denominator of the index does not depend on the current time period *t*, and it is standard to let the nominator of the index indicate national account components in fixed prices.

With a chain index the price base changes over time, and the Laspeyres chain index, Q^{CLA}, can be written as a multiplicative chain where every link is a Laspeyres fixed weight index, Q^{LA}, with price base in the previous period:

$$Q_{t:0}^{CLA} = Q_{t:t-1}^{LA} \times Q_{t-1:t-2}^{LA} \times \ldots\ldots \times Q_{1:0}^{LA} = Q_{t:t-1}^{LA} \times Q_{t-1:0}^{CLA} \qquad (2)$$

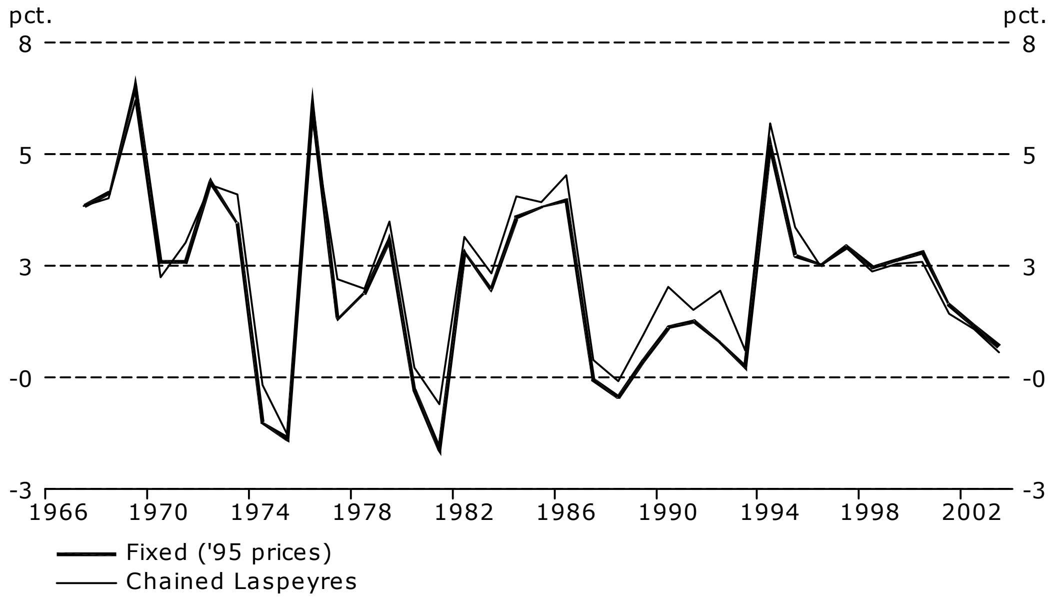
As indicated in (2), the chain-index formula can be expressed cumulatively using the chain index of the previous period and a one-period Laspeyres index. While the index in (1) is 1 in period 0, the level of the chain index in (2) remains to be chosen and there is no self-evident way of scaling national account components in chain index based fixed prices. One standard is to set the components to their current value in a certain year.

It is old news that the Laspeyres fixed weight index in (1) suffers from substitution bias when relative prices change, cf. Diewert (2004). The interest in chain indices like (2) has increased in recent years because the substitution bias has been highlighted by the growing amount of high tech products with significantly falling prices and significantly increasing volumes. Measuring at the prices of a fixed year implies that eventually the high tech products will dominate completely in your fixed price component although the same products may only account for a moderate share of the component in current prices.

In this connection it is important that the technology-induced shift in relative prices constitutes a monotonic development in one direction. Because just as the fixed-weight index suffers from substitution bias the chain index suffers from irreversibility. If prices and volumes for instance tend to fluctuate with negative correlation it will give the Laspeyres chain index for volumes a systematic drift upwards implying that real growth is overrated, cf. Diewert.

As expected the Danish GDP growth differs when measured with a chain index instead of a fixed weight index, but the correlation is significant, cf. chart 1.

Chart 1: Danish annual GDP Growth rate, 1966-2003

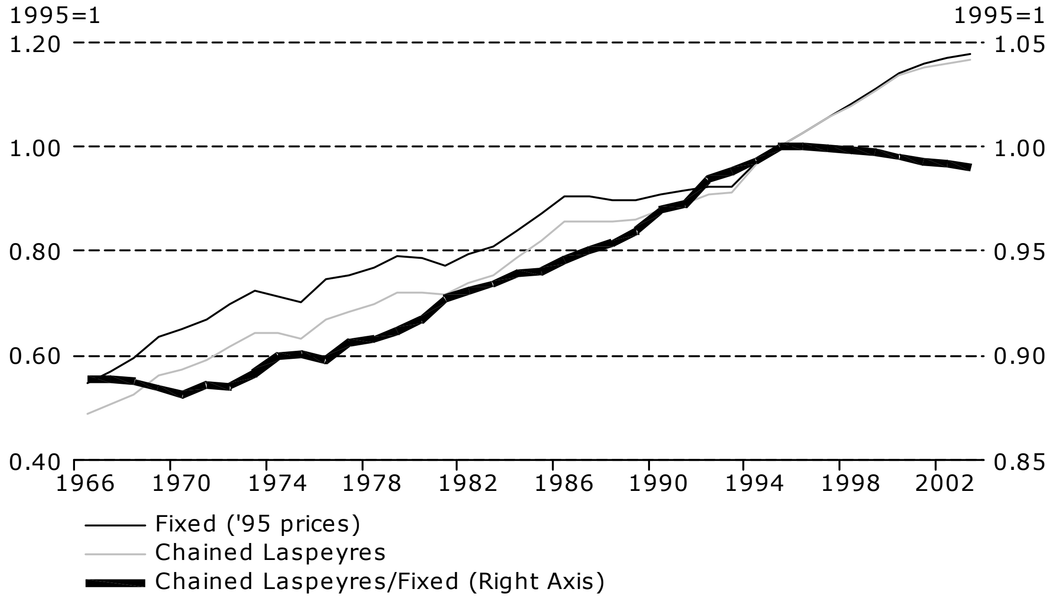


Source: Statistics Denmark

Scaling the GDP Laspeyres chain index to coincide with GDP in current prices in 1995 implies that this chain index will coincide with the traditional GDP in fixed 1995 prices in 1995 and also in 1996, cf. chart 2. This coincidence in 1996 reflects that having the previous year as price base corresponds to having 1995 as price base. The development in the ratio between the two fixed price GDP measures demonstrates that GDP as a chain index tends to grow stronger before the base year 1995 of the fixed weight index and, marginally, weaker after 1995. It is normal to see this relation, which reflects a negative correlation between volume and price fluctuations in the data.

The relation between chain and fixed weight GDP is 0,89 in 1966. Emphasising the relative virtues of the chain index we could argue that the official fixed weight based GDP series underestimates growth by 11 per cent over the 29-year period before 1995 and overestimates it marginally after 1995.

Chart 2. GDP Chain index and the traditional fixed-weight index



Source: Statistics Denmark

The growth difference between the two GDP measures is only marginal after 1995. In general terms this may reflect that the Danish production of high tech products is relatively low and the strong volume growth of those products affects imports more than GDP.

Moreover, it can be argued that negatively correlated fluctuations in prices and volumes creates a positive bias in the growth of the Laspeyres chain index, cf. our earlier remarks on the irreversibility of chain indices and underlying positive drift in the chain index. Such a drift would precisely strengthen the tendency of faster growth in the chain index before 1995 and weaken the tendency of slower growth in the chain index after 1995. This potential drift problem is one possible argument in favour of the slightly more complicated Fisher chain index preferred by the American statistical bureau. However, Dalgaard (1994) finds for Danish national account figures no significant difference between the Laspeyres and the Fisher chain index.

One advantage of the simple fixed weight Laspeyres index is its additivity; identities in current prices hold also in fixed prices. The same identities do not hold in fixed prices when the fixed price items are based on the chain index. This is illustrated in table 1, which gives the main fixed price items based on the Laspeyres chain index. Statistics Denmark has applied current 1995-values as base values, so the GDP-identity holds in 1995 and 1996 but not for other years, where the (modest) discrepancy has both positive and negative sign.

Table 1: National account figures fixed prices Mio. DKK chain indices with base value in 1995								
	Consump-tion	Fixed invest ments	Stock change	Export	Import	1+2+3 +4–5	GDP	Discre pancy 6 – 7
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
1994	754,062	167,452	1,558	345,674	292,194	976,552	976,056	496
1995	769,850	189,298	9,298	357,454	316,144	1,009,756	1,009,756	0
1996	791,245	196,799	1,602	372,900	327,357	1,035,188	1,035,188	0
1997	808,462	217,337	7,109	388,835	359,322	1,062,421	1,065,526	-3,105
1998	828,116	237,925	6,449	404,924	389,677	1,087,737	1,090,508	-2,771
1999	836,182	236,449	-1,437	452,301	403,637	1,119,858	1,118,048	1,810
Source: Statistics Denmark								

While additivity fails for the levels in table 1, there is full additivity in all years for the contributions to GDP growth, cf. table 2.

Table 2: GDP-contribution in per cent, chain indices								
	Consump-tion	Fixed invest ments	Stock change	Export	Import	1+2+3 +4–5	GDP	Discre pancy 6 – 7
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
1994	4.43	1.34	1.01	2.88	3.65	6.01	6.01	0
1995	1.61	2.27	0.83	1.21	2.47	3.45	3.45	0
1996	2.12	0.74	-0.76	1.53	1.11	2.52	2.52	0
1997	1.66	1.95	0.8	1.53	3.01	2.93	2.93	0
1998	1.84	1.87	-0.09	1.51	2.78	2.34	2.34	0
1999	0.74	-0.13	-1.07	4.19	1.21	2.53	2.53	0
Source: Statistics Denmark and own calculations.								

In principle, we only get the growth rates from a chain index based calculation. With chain index based national accounts we must think less in terms of levels and more in terms of growth contributions.

3. Introducing chain indices in Mona

The quarterly model Mona, Danmarks Nationalbank (2003), has been built around the official national accounts based on the Laspeyres fixed weight index. When the Danish statistical bureau switches to chain indices in their national accounts it is to improve the measure of economic growth etc. This kind of data revision will enter a macro econometric model like Mona via revised estimation results. One example is that you would expect to estimate another time trend in the production function.

However, we have not yet re-estimated anything in the quarterly Mona model as Statistics Denmark has so far only published annual national account chain indices, not quarterly. It would of course be interesting to see the result of a re-estimation. However, it is also interesting to see if another set up for the definitional equations in the model changes the marginal properties of the model. Thus, we take the behavioural relations as they are including the pre-set coefficients for import contents, for indirect tax contents etc. but we shall change the definition of output in the model and thus close the model differently.

To do this we re-formulate the definitional equations for fixed price GDP and on top of that for fixed price GVA for the non-farm business sector in the model. This business sector, which excludes farming, oil extraction and housing, is the big endogenous sector in the model and its production function translates output to demand for labour and capital inputs. When we re-formulate the business sector output, i.e. its GVA in fixed prices, as a function of the Mona variables we get another output response and hence another response in the demand for inputs.

We introduce both Laspeyres and Paasche chain indices to calculate the Fisher chain index, which may be more robust to reversibility problems. Thus, we are looking at principles. We are not trying to mimick the precise quarterly chain index that Statistics Denmark is expected to calculate.

To express GDP in fixed prices as a Laspeyres chain index, fY^{CLA}, we have the end-uses and import categories in Mona. That makes 39 components of which several are, however, rather small. Each of the 39 components has a price and a volume and these 78 variables make up the basic information for calculating GDP in the framework of Mona. The principle is the cumulative formulation from (2):

$$fY_t^{CLA} = fY_{t-1}^{CLA}\left(\frac{fC_t^{Car}}{fC_{t-1}^{Car}} \times \frac{C_{t-1}^{Car}}{Y_{t-1}} + \frac{fC_t^{Other}}{fC_{t-1}^{Other}} \times \frac{C_{t-1}^{Other}}{Y_{t-1}} + ...\right) \quad (3)$$

We have only written explicitly the contributions from the purchase of cars, C^{Car}, and from other consumption, C^{Other}. The factors representing volume growth from t-1 to t are weighted by GDP-share in current prices. The applied volume variables with prefix *f* are the 39 relevant fixed price variables of Mona. If these fixed price variables were on an official chain index basis the stipulated equation (3) would in principle give the official chain index based GDP. Now, (3) can only act as an approximation, but the equation introduces the fundamental effects of basing the model on the chain index formula and that is what we are after.

There is, however, one caveat with (3). We cannot use the corresponding expression for the GDP contribution of stock changes, *L*,

$$\frac{fL_t}{fL_{t-1}} \times \frac{L_{t-1}}{Y_{t-1}} \quad (4)$$

The problem with (4) is that stock changes in fixed 1995 prices, which is what we have, may be very close to zero without stock changes in current prices being close to zero, and in that case the contribution in (4) will be enormous. Instead we calculate the GDP contribution from stock changes in current prices

$$\frac{L_t}{L_{t-1}} \times \frac{L_{t-1}}{Y_{t-1}} = \frac{L_t}{Y_{t-1}} \quad (5)$$

Whereby a numerically small denominator will cancel an identical nominator.

Alongside (3), which expresses GDP as a Laspeyres chain index, there is a similar equation for GDP as a Paasche chain index, fY^{CPA}

$$fY_t^{CPA} = fY_{t-1}^{CPA} \times \frac{1}{\frac{fC_{t-1}^{Car}}{fC_t^{Car}} \times \frac{C_t^{Car}}{Y_t} + \frac{fC_{t-1}^{Other}}{fC_t^{Other}} \times \frac{C_t^{Other}}{Y_t} + \ldots\ldots} \quad (6)$$

GDP as a Fisher chain index, fY^{CFI}, is the geometrical average

$$fY_t^{CFI} = (fY_t^{CLA})^{½}(fY_t^{CPA})^{½} \quad (7)$$

To calculate the business sector GVA as a chain index we need to bridge from the GDP by excluding from the GDP: Net indirect taxes on products plus the GVA in the public sector, in agriculture, in energy extraction and in housing. As a Laspeyres chain index the business sector GVA is given by

$$fGVA_t^{bu,CLA} = fGVA_{t-1}^{bu,CLA} \left(\frac{fY_t^{CLA}}{fY_{t-1}^{CLA}} \times \frac{Y_{t-1}}{GVA_{t-1}^{bu}} - \frac{fGVA_t^{pu}}{fGVA_{t-1}^{pu}} \times \frac{GVA_{t-1}^{pu}}{GVA_{t-1}^{bu}} - \right.$$
$$\left. \frac{fIND_t}{fIND_{t-1}} \times \frac{IND_{t-1}}{GVA_{t-1}^{bu}} - \ldots\right) \quad (8)$$

We have in (8) written explicitly the contributions from GDP, from public GVA, $fGVA^{pu}$, and from net indirect product taxes, $fIND$. GVA for other sectors excluded enter like public GVA. With a similar equation for business sector GVA expressed as a Paasche chain index we can determine the business sector GVA as a Fisher index.

Inserting the chain index equations discussed constitutes the simple adaptation of Mona to chain indexing. In the next section we use a multiplier experiment to illustrate the impact of chain indexing on the consistency between costs and revenues.

4. The aggregation problem in Mona

The aggregation problem we are addressing here concerns the consistency between revenues and costs. It stems from Mona having basically only one endogenous production sector and several demand components with different deflators. With a fixed weight Laspeyres representation of volume, the price index is implicitly a Paasche index, which can move with the composition of demand without reflecting the cost side.

It may be helpful to give a simple numerical example of this composition effect. We have two goods, A and B, in two years, 0 and 1, and the problem is how to represent aggregate volume and price change. The example is shown in table 3.

Table 3: Aggregate development in volume and price			
	Value	Volume	Price
	EUR	1995-prices	1995=1
Year 0			
Good A	100	100	1
Good B	150	100	1,5
Year 1			
Good A	125	125	1
Good B	112.5	75	1,5
Aggregated development year 0 to year 1			
1995-prices	-5%	0%	-5%
0-prices	-5%	-5%	0%

In the example total value drops 5 per cent, from 250 to 237.5 EUR, between the two years. Measured in 1995 prices there is no volume change, only price change, and the price change is a pure composition effect as the price of both goods remain unchanged. Measured in year 0 prices, as in a chain index, there is only volume change and no price change.

The latter result may be preferable. Good b with decreasing volume represents the highest value, and may quite likely also represent the highest costs and the highest use of inputs. Thus, a fall in volume for output may work well in an aggregate production function.

It is beyond the scope of this paper to offer a thorough presentation of aggregation theory. Suffice it to say that a basic idea in most aggregation is that your weights should reflect a relevant marginal rate of substitution between the components to be aggregated. Moreover these marginal rates of substitution should preferably be constant or invariant to other factors to facilitate aggregation, cf. Fisher (1987). In relation to the aggregated production and cost side of our model this means that we should refer to the marginal costs, the MCs, cf. Lasky (1998). For instance, the total quantity of three products measured in "product 1 units" should be given by

$$x_1 + x_2 \frac{MC_2}{MC_1} + x_3 \frac{MC_3}{MC_1} \tag{9}$$

In a traditional fixed price calculation we measure the total quantity by using the prices of a certain year like 1995. This corresponds to using fixed weights for measuring in "product 1 units", and implicitly we have thereby assumed that the relative marginal costs in (9) remain constant in all years.

Taken in isolation this might be an excellent simplifying assumption. However, the problem here is that the marginal revenue, MR, when selling a demand component in Mona is close to the deflator of the component[1] and some deflators of the demand components do move significantly relative to each other over time. Consequently, the MRs will also move significantly relative to each other, and if we keep assuming that the relative MCs remain unchanged it becomes difficult to establish an economically sound relation between MR and MC of a given product.

In practical model applications the lack of economic relation between MR and MC could imply that the wage share of the aggregate production sector reacts because demand components have different deflators, just as in the numerical example above. Such composition effects constitute an old problem for models built around the traditional national account system

In some cases we might want to justify the composition effect by referring to different degrees of competition for different goods. However, there is no corresponding mechanism in Mona, and, more importantly, it seems artificial to argue that product prices in general drift apart while product costs move in parallel. In reality a different development of prices will to a large extent reflect a different development of costs.

Taking this point seriously may simply lead us to expand the number of production sectors in the model. With, say five demand components produced by one sector, we need to know the magnitudes of all five components to say what and how much to produce in the sector. With five sectors we know a priori what to produce in each. Thus, if we have one sector per demand component we have the necessary degrees of freedom to explicitly model all MR/MC relations in the model.

If, however, we are not interested in switching to larger and more disaggregated models it is a possibility to apply chain indexing to reduce the composition effects. With the chain index formula the price base is constantly updated. This means that when measuring in "product 1 units" the weights move over time with the relative prices so we are not longer implicitly assuming that the relative MCs are constant. Instead we are assuming that relative MCs move with relative prices, and if the prices are close to the MRs we are effectively assuming that relative MCs move with relative MRs, component by component. This makes it easier to maintain an economic relation between MC and MR regardless of changes in the composition of the demand components in the model. We shall now illustrate this point by model experiment.

---

[1] To be more precise, we are focusing on Danish value added and exclude the content of imports and indirect taxes from both costs and revenues.

5. Illustrating chain indexing and the aggregation problem in Mona

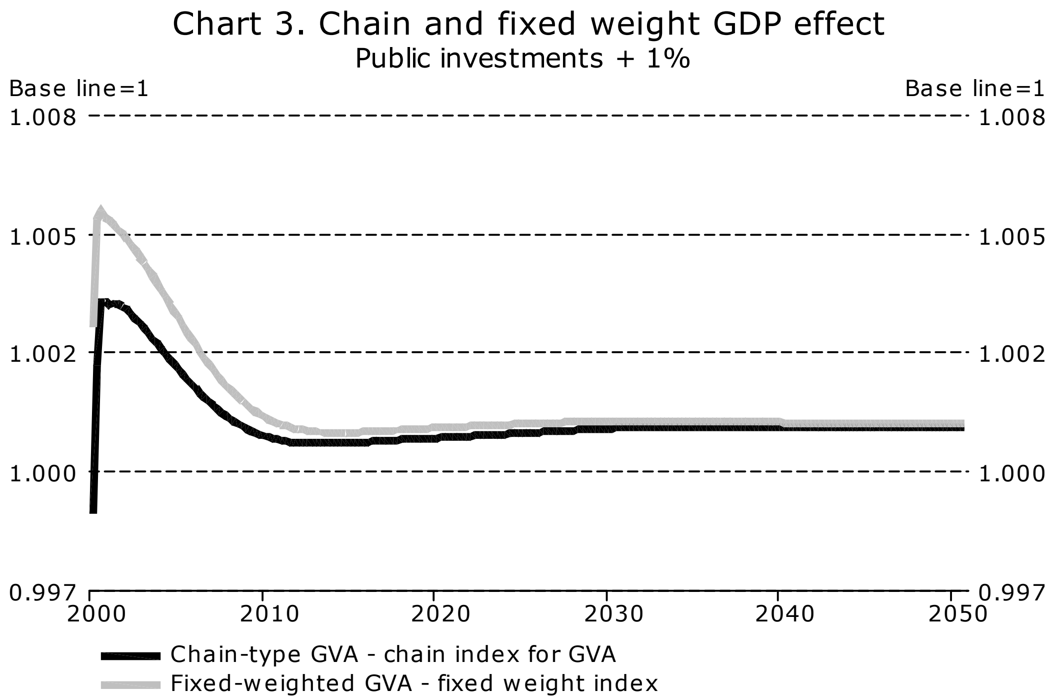
To illustrate the composition effect and its interaction with the choice of index formula we make a simple multiplier experiment with a permanent increase in public investments in machinery. We have chosen public machinery investments, which by the way include computers, because that demand component now has one of the smallest deflators in the model with the official price base year 1995. Thus, we should expect to see a clear composition effect.

In our model set-up the chosen public investment component constitutes one good with one price just as the 38 other components do. The 39 components constitute the detailed level in the model and the increase in the chosen component is the same irrespective of the choice of index. The shock equals 1 per cent of GDP in 1995-prices.

First, we simulate the model with the traditional fixed weight equations defining the model's business sector GVA. Second, we simulate a chain index version of Mona with business sector GVA being based on the chain index equations presented in section 3.

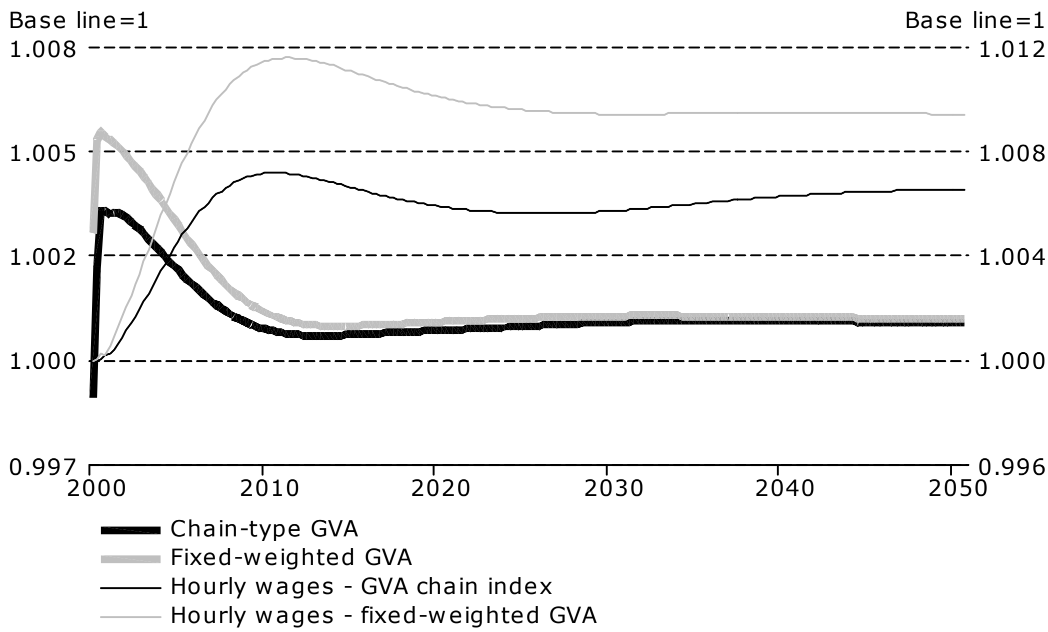
The immediate GVA response in per cent is highest with the traditional model version. That is no surprise. If the 1995-based deflator of the shock is small the shock is larger relative to GVA when measured in 1995-prices than when measured in current prices.

Moreover, we note that this difference in GVA impact mainly concerns the first years of the experiment. Once the GVA effect starts coming down due to the crowding out response in the model the difference between the two model versions narrows, and we end up with an almost identical impact on GVA, cf. chart 3. This reflects that the available labour force is the same whether we calculate output in the one way or the other. Either way we have to return to the same steady state unemployment where our wage rate grows in parallel to our currency anchor.



When the immediate response of output and hence also of employment is larger when output is measured by a fixed weight index we also need a stronger crowding out reaction in that case. The crowding out mechanism works via the labour market. Increasing the wage position relative to our competitors reduces exports, which dampens activity and employment. As expected the wage reaction in the model is larger when measuring GVA by a fixed weight index. The wage reaction in the model overshoots a bit but it should be evident that there is also a permanent effect on relative wages as the shock is assumed to be permanent, cf. chart 4. The higher price of labour gives rise to a higher labour productivity, which in turn explains that the output effect remains slightly higher with the fixed weight index.

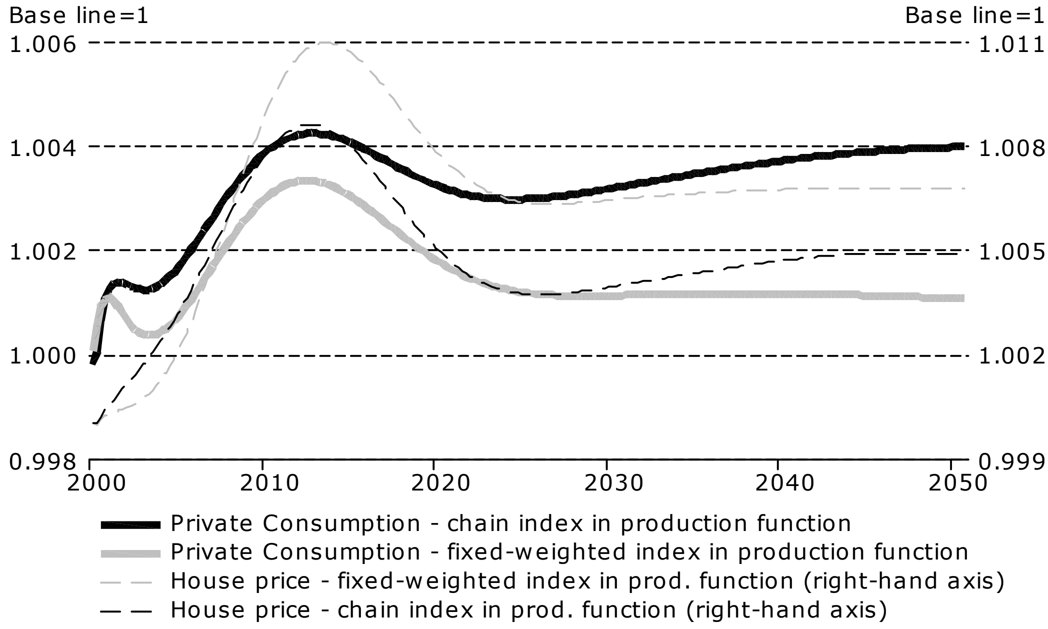
Chart 4. GDP and wage effect



As the wage effect is stronger in the fixed weight simulation, terms of trade will also increase more in that simulation and we might have expected the highest increase in real income as well. However, with fixed weights we also get a negative composition effect on profit income, which dampens private purchasing power, and it turns out that the effect on consumption is lower in the fixed weight simulation, cf. chart 5.

The general observation is that the fixed weight simulation entails a sharper reaction in activity and activity driven variables. The need to accelerate and thereafter to decelerate is larger in that experiment.

Chart 5. Effect on Private Consumption in fixed prices



Last but not least we shall see the composition effects on the business sector GVA deflator and on the wage share. In Mona marginal wage costs are supposed to drive the price component in GVA. Specifically, the price relations of the model are set up to explain the GVA price component by marginal wage costs with long-term elasticity of 1.

Nevertheless, with the fixed weight model version the GVA deflator increases less than marginal wage costs, and also the wage share in the business sector grows significantly. This happens because public investment in machinery has a low deflator.

With the chain index based model the relation between GVA deflator and marginal wage costs is more stable, and also the wage share grows considerably less. Actually, the moderate wage share impact (less than 0.1 per cent) is relatively close to a theoretically consistent response of one minus the elasticity of substitution (1-0.63) multiplied by the average capital share (0.33) and multiplied by the increase (0.25 per cent) in the wage over user cost ratio. This means that with the chain index we have come closer to getting the result of the theoretical framework in the model rather than the result of composition problems.

Chart 6a. Wage share in non-farm sector

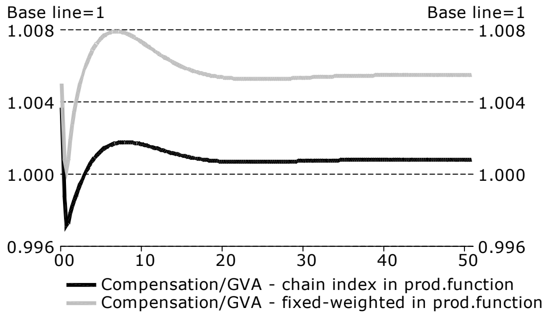
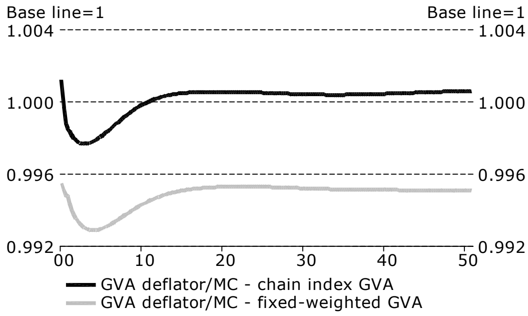


Chart 6b. Effect on deflator/MC ratio



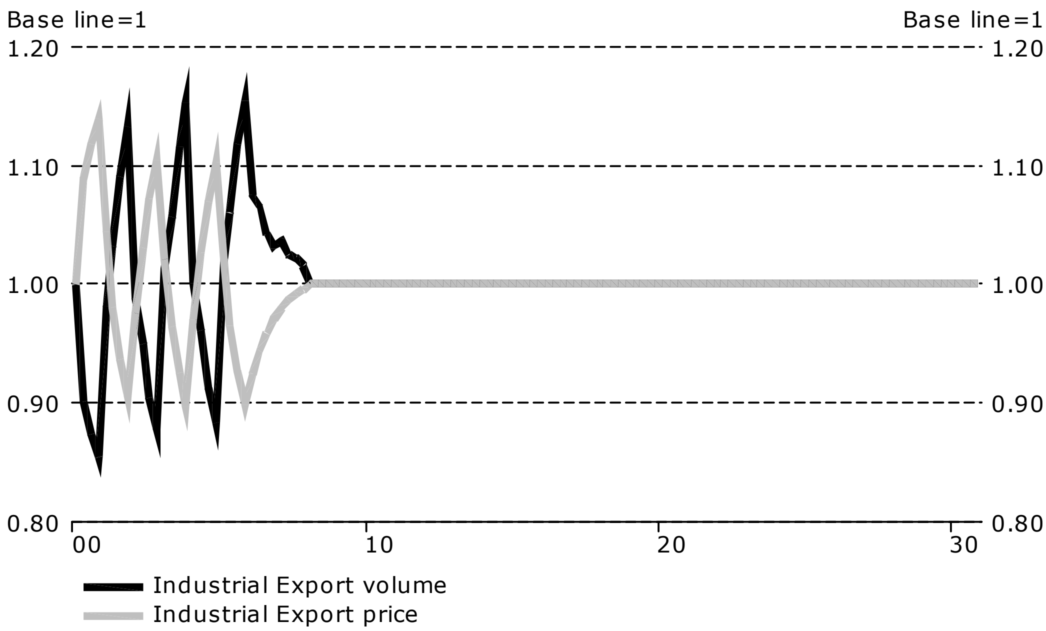
6. The irreversibility of chain indices

It is well established that a chain index, volume as well as price index, does not necessarily assume the same value for the same set of quantities and prices for the individual goods. The chain index formula is irreversible or path dependent.

In relation to Mona the individual goods of the business GVA index are represented by the 39 demand components in the model plus the indirect tax and non-business GVA components. To focus on this business GVA index we exogenise prices and quantities of all components by closing down the behavioural relations of the model.

A simple illustration of the irreversibility of the chain-index formula is now to move the price and quantity of one component, here industrial exports, up and down for some periods and return to the base line values, cf. chart 7.

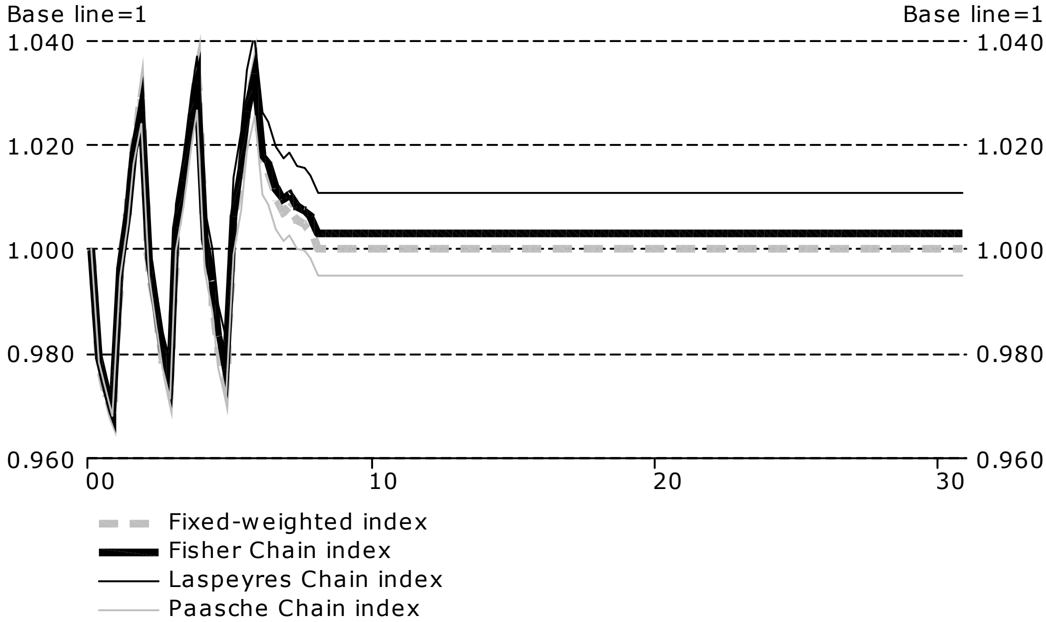
Chart 7. Input of fluctuating volume and price



On the basis of this input the GVA fixed weight index will reflect the movements of the export volume and return to the base line when exports stops moving. The assumed price movement is of course irrelevant to the fixed weight index. The stylised fluctuations of the indices are shown in chart 8.

Chart 8. Effect on GDP, no model

Experiment: Temporarily shocking export quantities and prices



On the other hand, the GVA Laspeyres chain index drifts upward over the fluctuations and does not return to base. This result reflects the imposed negative correlation between quantity and price changes. Movements from quantity peaks are weighted by prices at their trough and therefore dominated by movements from quantity troughs weighted by peak prices. By a similar argument the Paasche chain index drifts downward, and the Fisher chain index is somewhere in the middle but does not quite return to base line. Another example with fully symmetrical price- and volume changes in a component could make the Fisher chain index return to the base line. However, the basic point is that the Fisher chain index will not automatically return to base line.

We shall now illustrate the impact of this irreversibility in business GVA within the whole model.

7. Illustrating chain indexing and irreversibility in Mona

We now open up the behavioural equations and do the same kind of experiment on the full model. The now endogenous export quantities and prices are shocked by letting their add factors change between positive and negative values.

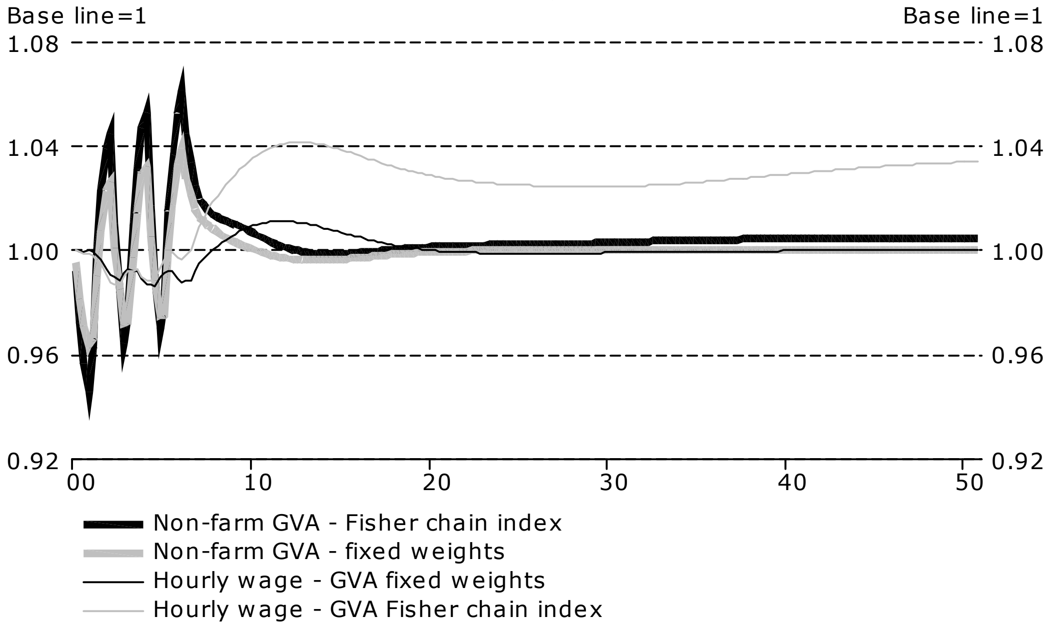
The persistence in the behavioural equations of the model implies that business GVA does not return to the base line as soon as we stop shocking industrial exports. However, with a fixed weight index to determine GVA all variables, including business GVA, do return to base line after some years. That is also what we would expect in theory in case of temporary shocks, as there is no hysteresis built into the behavioural equations of the model.

With a chain index formula to determine output, however, the outcome of this experiment becomes more complicated, because output and all other variables cannot return to the base line simultaneously. We have seen that without the behavioural equations it is only GVA (and GDP), which deviate permanently from the base line. Within the full model several variables give in and deviate permanently from base line. The wage rate will end up being above the baseline permanently and exports will be below.

This happens because the crowding out mechanism tends to control the output activity in the model by controlling employment. The modest permanent effect on GVA reflects that the permanent wage effect triggers a substitution effect on labour productivity.

Chart 9. Model-based GVA and wage effect

Experiment: Temporarily shocking export quantities and prices



The model experiment shows how the chain index can introduce a hysteresis effect in our model, and the magnitude of this effect depends on the nature of the model experiment. This general conclusion is easy to understand if we consider the mathematics of a constantly re-basing chain index.

However, the irreversibility is here of a technical nature and not based on economic considerations as we find them in formal presentations of hysteresis or path dependency. It would hardly make the specific chain index based result more comprehensible to give a full account of all endogenous variables forced away from the base line. If the irreversibility of the chain index is significantly felt in a model calculation it simply constitutes a back draw to using the chain index in our macro model.

8. Conclusions

The introduction of chain indexing in national accounts is intended to improve the data quality, and ideally we should be able to estimate a better quarterly model. We have not yet looked into this re-estimation, as the chain-index based quarterly national accounts are not yet available. There are, however, other model-related issues, which can be addressed before the new data are available.

Specifically, a number of definitional equations in the model will have to be formulated differently if the output aggregation in the model is to be based on the chain index formula. This definitional re-formulation in the model raises a couple of issues. We have illustrated both advantages and disadvantages by tentatively introducing such new definitional relations in Mona.

We have so far always had the official fixed price aggregation in the model. If it turns out that there are difficulties in implementing the new chain indexing in the model we may consider an alternative. An easy alternative, already applied in some models in countries with chain indexing in national accounts, is simply to stick to the additive fixed-weight formula and introduce a discrepancy term. Another alternative to full chain indexing is to introduce the chain index formula but to stop updating the prices of the formula in the forecast period. That would effectively give us the chain index result for the first year of the forecast reducing the composition problems, as all prices are 1 to start with, and at the same time avoid the irreversibility problem in experiments over the forecast period.

We are not yet in a position to assess the practical relevance of potential advantages and disadvantages of having the chain index formula in the model. It may be too difficult to be worthwhile or it may be a good supplement. In any case the introduction of the chain index in the official national accounts can be taken as an opportunity to consider the problem of fixed price calculation in your model.

Literature

Boskin, M.J. *et al.* (1996), Toward a More Accurate Measure of The Cost of Living, Final Report to the Senate Finance Commitee.

Commission of the European Communities, IMF, OECD, UN, and World Bank (1993), System of National Accounts 1993.

Dalgaard, E. (1994), Implementing the revised SNA – recommendations on price and volume measures, Statistics Denmark.

Danmark Nationalbank (2003), MONA – a quarterly model of the Danish economy.

Diewert, Erwin W. (2004), Basic index number theory, Chapter 15 in Consumer price index manual: Theory and practice, ILO.

Fisher, F.M. (1987), Aggregation problem, in The New Palgrave (ed. Eatwell, Milgate and Newman), Macmillan Press.

Keynes, J.M (1936), The General Theory of Employment, Interest and Money, London: Macmillan.

Landefeld, J.S. og R.P. Parker (1995), Preview of the comprehensive revision of the national income and product accounts; BEA's new featured measures pf output and prices. Survey of current business, Bureau of economic analysis.

Lasky, M.J. (1998), Chain-Type Data and Macro Model Properties: The DRI/McGraw-Hill Experience, Journal of Economic and Social Measurement, 24(2).

Vavares, C., J. Prakken og L. Guirl (1998), Macro Modelling with chain-type GDP, Journal of Economic and Social Measurement, 24(2).