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Identifying monetary policy in a small open economy under fixed exchange rates∗

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Abstract
We demonstrate how to identify monetary policy under fixed exchange rates in a structural vector autoregression (SVAR) using Denmark as a case study. The identifying restrictions are compared to SVARs for flexible exchange-rate regimes. Our basic model generates a plausible central-bank reaction function, and the responses to monetary shocks are in accordance with theory. We extend the basic model and econometric approach to incorporate the central bank’s interventions on the foreign-exchange market.

Keywords: Foreign exchange intervention; monetary policy; structural VAR; exchange rate; reaction function; fixed exchange rate

JEL classification: F31; E52; C32

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1 Introduction

Considerable progress has been made in analyzing monetary policy empirically with the introduction of structural vector autoregressive (SVAR) modelling. This research strand uses "theoretical or institutional knowledge" to place plausible identifying restrictions on the contemporaneous interactions between variables in a VAR (Stock and Watson, 2001). Not least has there been a wave of research for open economies where these methods have solved puzzles present in the early open-economy VAR-literature (e.g. Cushman and Zha, 1997; Smets, 1997; and Kim and Roubini, 2000). Typically, though, these open-economy studies have been conducted for countries with flexible exchange rates, often the non-US G7 countries. The present paper discusses how to identify monetary policy in a small open (industrialized) country under fixed exchange rates using Denmark as a case study. It highlights the differences and similarities in the "theoretical and institutional knowledge" used to identify exogenous monetary-policy shocks and the central bank's reaction function compared to the existing studies for flexible exchange-rate regimes.

Monetary policy has by some observers been labeled a "dark art" (e.g. Blinder, 1997), and although transparency has increased significantly in recent years, monetary policy still has opaque elements in many countries. In particular, the relationship between the central bank's policy instrument and the variables which the central bank consider when setting the policy instrument is not precisely known outside the central bank. The uncertainty relating to this central bank "reaction function" pertains both to included variables and the relative weights of variables, which might even change over time. Relevant variables like the output gap are often subject to substantial revisions and measurement problems, which poses significant challenges to later empirical studies (Orphanides, 2001). These "dark art" elements of monetary policy imply that empirical analysis of monetary policy is no simple task.

Characterizing monetary policy under pure exchange-rate targeting in a small open economy is different from studying monetary policy under flexible exchange rates in a number of ways and is - at least in some respects - simpler. The single monetary-policy objective is to stabilize the exchange rate around the chosen target. Monetary-policy authorities change the main policy instrument, usually a short-term interest rate, only if there is exchange-rate pressure or a change of the anchor country's monetary-policy stance. Deviations from the announced policy is easily detected by the private sector as the market exchange rate is continuously observable. A pure exchange-rate targeting regime like this is very transparent and a candidate central bank (interest-rate) reaction function can quite straightforwardly be pinned down as depending on two financial variables (the targeted exchange rate and the
anchor country’s interest rate), which are observable in real time and free of future revisions. There is no monetary-policy reaction to the real economy or commodity prices. In VARs for countries with flexible exchange rates the latter variable has played a central role, since leaving it out of the reaction function has produced a price puzzle, where the price level increases following a monetary tightening. Presumably the central banks in these countries react endogenously to commodity-price increases, since they signal future inflation. We give a detailed account of the differences in identification of monetary policy across regimes below.

The overall stability and credibility of the Danish fixed exchange-rate regime over a relatively long period makes it a prima facie case for empirical analysis of monetary policy under an exchange-rate peg. In autumn 1982 the incoming government announced that it would cease to use changes of the central parity of the Danish krone in the ERM\(^1\) to serve economic-policy purposes as had been the case at several occasions in the 1970s. Between 1983 and 1987 there were three minor central-parity adjustments (where the D-mark was revalued against a number of currencies, including the Danish krone) and the regime gained considerable credibility over time, cf. figures 1 and 2. The interest-rate spread for 10-year government bonds, which in 1982 was around 10 per cent, has gradually disappeared, and at the time of writing the spread between the monetary-policy rates in Denmark and the euro area - is a mere 25 basis points.\(^2\) The period after the turbulence of the 1992-93 ERM system-wide crisis has in particular seen a stable monetary-policy regime in Denmark.\(^3\)

The literature includes a number of papers that have used SVAR analysis to shed light on the monetary transmission mechanism using information and restrictions on the short-run interaction of the variables. Examples include Cushman and Zha (1997), Smets (1997) and Kim and Roubini (2000), which followed up on early open-economy papers by e.g. Eichenbaum and Evans (1995) and Sims (1992). All abandon the often used recursive ordering, which for a range of countries with varying degrees of exchange-rate flexibility has turned out to produce the so-called exchange-rate puzzle, where the currency depreciates following a monetary-policy contraction (e.g., Grilli and Roubini, 1996; Sims, 1992). The exchange-rate puzzle may result from confusing a monetary-policy shock with the endogenous interest-rate response to an exchange-rate depreciation, which is not captured by the recursive ordering as the exchange rate typically is ordered after the interest rate. By allowing for simultaneity

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\(^1\)The exchange-rate mechanism (ERM) was the framework for fixed exchange rates within the European Monetary System.

\(^2\)When the euro was introduced in 1999 it replaced the D-mark as the anchor for the Danish fixed exchange-rate policy.

\(^3\)Since 1993 there has been three major episodes, where Danmarks Nationalbank unilaterally has raised interest rates markedly: in connection with unrest on global financial markets in 1995 and 1998, and in 2000 around the referendum on joining the euro, which resulted in a no-vote majority. For a non-technical description of Danish monetary policy, see Danmarks Nationalbank (2003).
Figure 1: The Danish krone vis-à-vis the D-mark/euro, 1982-2005

Figure 2: 10-year government bond yields in Denmark and Germany, 1982-2005
between the interest rate and the exchange rate all three analyses retrieve plausible reactions to monetary policy shocks. In a fixed exchange-rate setting, where the exchange rate is the primary variable in the interest-rate reaction function, it is evident that a proper analysis must allow the interest rate to react immediately to changes in the exchange rate.

In recent years several open-economy SVAR papers have followed. A short, non-exhaustive list includes Bjørnland (2005), Faust and Rogers (2003), Kim (2002, 2003, 2005), Mojon (1999), Mojon and Peersman (2003), Scholl and Uhlig (2005), Smets and Wouters (1999) and Zettelmeyer (2004). For our purposes the interesting point is that most papers have focused on flexible exchange-rate regimes or countries which have had varying degrees of exchange-rate flexibility. A few papers, though, have focussed on fixed exchange-rate regimes, notably Kim (2002), Mojon (1999), Mojon and Peersman (2003) and Smets (1997). Kim (2002) and Smets (1997) estimated SVARs for Denmark, France and Germany, and Italy, France and Germany, respectively. Although both allow for simultaneity between the interest rate and the exchange rate, the pegging countries are assumed not to react contemporaneously to the anchor country’s (Germany) interest rate. This is not a credible identifying assumption as it is strongly at odds with the "institutional and theoretical knowledge" of fixed exchange-rate regimes. Furthermore, the estimation samples includes a period where central-parity changes served as an active policy tool. Mojon (1999) analyzes French monetary policy from 1987 to 1993 but his model does not de facto allow for simultaneity between the interest rate and the exchange rate. Mojon and Peersman (2003) explicitly take monetary regimes into account when comparing monetary transmission mechanisms across the euro area countries (i.e. not Denmark) by estimating country SVARs for the period 1980-1998. For the fixed exchange-rate countries, though, the analysis boils down to a description of the endogenous effects of German monetary-policy shocks, as no pure domestic shocks are identified. Although monetary autonomy is limited under fixed exchange rates, it may still be possible to identify monetary-policy shocks. First, there is some limited room for policy if the exchange rate is allowed to fluctuate within a band around the target. Second, when stabilizing the exchange rate the central bank might occasionally and inadvertently deviate from its average response within the band. Another side-effect of the approach of Mojon and Peersman (2003) is that no explicit monetary-policy reaction function is estimated.

In our basic model we identify monetary shocks in a SVAR with production, prices, the exchange rate and the monetary-policy interest rate. The important identifying restrictions relate to the interest rate and exchange rate equations. The monetary-policy interest rate is assumed to react only to the exchange rate and the (exogenous) anchor country interest rate, since the central bank does not react to the real economy. In addition to reacting to the interest rates, the exchange rate is also allowed to react contemporaneously to production and
prices. This difference should allow us to separate the two equations. It turns out, however, that there is not sufficient short-term variation in the exchange rate to news from the real economy to separate the two types of financial shocks. This can be interpreted as a reflection of the credibility of the regime. Instead, we apply the GMM method of Smets (1997), using information from outside the model (innovations in the US-dollar/D-mark exchange rate and the US short-term interest rate) to extract information about exchange-rate shocks. The impulse-response functions we obtain for the structural shocks are well-behaved and in accordance with standard theory.

Although a short-term interest rate is generally the most important policy instrument also for exchange-rate targeting central banks, usually foreign-exchange market intervention is also used to stabilize the exchange rate in the short run. Generally, interventions are used both in isolation and in combination with interest-rate changes. To shed light on the role of this policy instrument we extend the basic model. Not only does this allow us to estimate the central bank’s reaction function for interventions, but its inclusion may well be necessary to pin down the true magnitude of the effects of interest-rate shocks on the exchange rate in so far as the two policy instruments interact non-trivially.

To estimate our SVAR with interventions we derive an extended GMM-method, which again use information from the US-dollar/D-mark exchange rate and the US short-term interest rate to extract intervention, interest rate and exchange-rate shocks. Our model complements the work of Kim (2003, 2005), who set up SVARs with interventions for the flexible exchange-rate countries USA and Canada. The impulse response of an intervention shock, where the central bank buys Danish kroner and sells foreign currency, is as expected a strengthening of the currency. Following an interest-rate shock the impulse responses indicate a leaning-against-the-wind intervention strategy, and following an exchange-rate shock interventions act to stabilize the exchange rate.

Overall, the contribution of the paper is twofold. First of all, we demonstrate how to identify monetary-policy shocks under fixed exchange rates in a SVAR taking the "institutional and theoretical knowledge" given by the regime seriously. Secondly, we extend the framework and the GMM approach to include interventions.

The structure of the paper is the following. In section 2 the basic framework is set up and identifying restrictions are presented and discussed in relation to restrictions typically used for flexible exchange-rate analyses. Section 3 contains the results for the basic model. In section 4 we extend the econometric approach to incorporate interventions, and present the results of a model including these. Section 5 concludes.
2 Identifying monetary policy under fixed exchange rates

2.1 General framework

This section provides a very brief introduction to the SVAR approach before we discuss the particular identifying restrictions relevant for fixed exchange-rate countries. Consider a \((n \times 1)\) vector of endogenous variables \(y\) that follows a \(p\)th-order Gaussian vector autoregression

\[
y_t = c + \Phi_1 y_{t-1} + \Phi_2 y_{t-2} + \cdots + \Phi_p y_{t-p} + \varepsilon_t, \quad (1)
\]

with \(\varepsilon_t \sim \text{i.i.d. } N(0, \Omega)\). This VAR may be estimated using maximum likelihood to obtain estimates of the coefficients \(c, \Phi_1, \Phi_2, \ldots, \Phi_p\) and of the variance-covariance matrix \(\Omega\). But we are interested in estimating a structural model of the following general form

\[
B_0 y_t = k + B_1 y_{t-1} + B_2 y_{t-2} + \cdots + B_p y_{t-p} + u_t, \quad (2)
\]

which allows for contemporaneous interaction between the endogenous variables, and where the equations may be interpreted as parts of a structural economic model. The structural shocks are related to the reduced-form errors by

\[
\varepsilon_t = B_0^{-1} u_t, \quad (3)
\]

and are usually assumed to be mutually uncorrelated in which case the variance-covariance matrix \(D\) of \(u\) will be diagonal.

One common way of obtaining identification is to assume that there is a recursive structure in the relationship between the endogenous variables. If there exists an ordering of the endogenous variables such that the first variable is not affected contemporaneously by the other variables, while the second variable is only affected contemporaneously by the first variable and the third variable is only affected contemporaneously by the first and the second variables, and so forth, then \(B_0\) will be lower triangular. The structural model can then easily be retrieved from the reduced form.

Often theoretical models or institutional knowledge do not allow a recursive structure. In this case it may still be possible to identify the model if sufficient restrictions are placed on \(B_0\) (see e.g. Kim and Roubini, 2000 for details). Alternatively, if the identifying restrictions on \(B_0\) given by theory are not enough - that is, the model is unidentified - they can be combined with information outside the model to pin down the the structural shocks and parameters cf. Smets (1997).\footnote{It is also possible to use assumptions about long-run multipliers to achieve identification, but in this paper...}
2.2 Identifying monetary shocks in a fixed exchange-rate regime

Our basic model contains four endogenous variables, industrial production \((IP)\), consumer prices \((CPI)\), the monetary-policy interest rate \((R)\) and the targeted exchange rate \((E)\), and one exogenous variable, the anchor country monetary-policy interest rate \((R^*)\). In structural terms the model is given by

\[
\begin{pmatrix}
1 & 0 & 0 & 0 \\
b_{21} & 1 & 0 & 0 \\
b_{31} & b_{32} & 1 & b_{34} \\
0 & 0 & b_{43} & 1 \\
\end{pmatrix}
\begin{pmatrix}
IP_t \\
CPI_t \\
E_t \\
R_t \\
\end{pmatrix}
= \begin{pmatrix}
a_1 \\
a_2 \\
a_3 \\
a_4 \\
\end{pmatrix}
R^*_t + \text{lags} + \begin{pmatrix}
u_{IP,t} \\
u_{CPI,t} \\
u_{E,t} \\
u_{R,t} \\
\end{pmatrix}
\tag{4}
\]

where "lags" refers to terms involving lags of the endogenous and exogenous variables and "\(c\)" is a vector of constants. There are four structural shocks: IP shocks \((u_{IP})\), CPI shocks \((u_{CPI})\), monetary-policy shocks \((u_{R})\) and exchange-rate shocks \((u_{E})\).

Our baseline model consists of two main blocks: a real-economic part comprising the equations for output and prices, and a financial part containing the monetary-policy interest rate and the exchange-rate equations. The real-economic part is assumed not to react contemporaneously - i.e. within the month - to the endogenous financial variables. The argument for this common assumption is that it takes time for changes in financial markets to transmit into changes in the real economy. Within the real-economic block there is a recursive structure. Although this feature is shared by most studies, there is no consensus in the literature as to whether output or prices should be ordered first, but we follow, for example, Eichenbaum and Evans (1995) and Smets and Wouters (1999) in placing production first.

It is in the specification of the financial block that a fixed exchange-rate SVAR is fundamentally different from a flexible exchange-rate SVAR.

A common objective in VAR studies of monetary policy is to separate exogenous monetary-policy shocks from changes in monetary policy that correspond to the central bank’s endogenous response to shocks originating elsewhere in the economy. To facilitate this separation a well-specified reaction function for the central bank should be included in the model. In some flexible exchange-rate countries, it might be sufficient to let the central bank react to output, prices and the exchange rate in setting the monetary-policy stance. For others it may be necessary to include other variables, e.g. a money-demand type equation to ensure that interest-rate changes that are due to shifts in money demand are not inadvertently interpreted as monetary-policy shocks. Likewise a lot of models include a price of raw materials - e.g. the oil price - in the reaction function as well. Central banks are forward looking and the we will only be using restrictions on the matrix of structural coefficients \(B_0\).
inclusion of the oil price has lead to the solution of the so-called price puzzle, where contractionary monetary-policy shocks lead to an increase in prices. The explanation for this result is that the identified shocks in models without the oil price are in fact endogenous responses to future inflationary pressure. Typically, the inclusion of extra variables is initiated if the impulse-responses to identified shocks are not in accordance with theory. The identifying restrictions in the equation for the (monetary-policy) interest rate for flexible exchange-rate countries primarily rely on timing assumptions. Financial market variables are known continuously (exchange rate and oil price) whereas real-economic variables are only known with a lag.

A fixed exchange-rate regime works differently. The central bank’s reaction function includes the targeted exchange rate and the anchor country interest rate contemporaneously. (In Denmark’s case this is the exchange rate vis-à-vis the D-mark (euro) and the German (euro area) interest rate.) Output and prices are not - and should not be - included contemporaneously, but this is due to the fact that the central bank does not react to these real-economic variables at all, not due to delays in information dissemination. Similarly, commodity prices are not incorporated into the model since they do not play any role for the central bank’s reaction. Rising oil prices may signal future inflation, but a fixed exchange-rate central bank does not react to this unless the anchor country’s monetary authorities do.

As is clear for anyone following the actual conduct of monetary policy in a fixed exchange-rate regime, movements in the anchor country’s monetary policy is mirrored immediately (in the case of Denmark, usually within the day). This is also evident in figure 3. The obvious implication is that including the anchor country’s exchange rate in the reaction function is indispensable. Failing to do so is likely to produce erroneous magnitudes of the impulse-response for the exchange rate. An equally sized monetary-policy change in the two countries should in most cases leave the exchange rate unaffected, since the spread is preserved. In a VAR without reaction to the anchor country’s interest rate within the month, the domestic interest-rate change would be identified as a shock rather than an endogenous reaction, and the corresponding response of the exchange rate would be nil. The average impulse response of the exchange rate would then be muted as it would be an average of the response to proper shocks and the endogenous reaction to changes in the anchor country’s interest rate.7

5 See also Giordiano (2004) for an alternative explanation, and Hanson (2004) for a different view on the price puzzle for USA.
6 Even if not imposed a priori the coefficients on IP and CPI should be zero in sufficient large samples. But the basic idea of structural VAR modelling is exactly to impose the restrictions given by theory or institutional knowledge.
7 This is indeed supported by our data for the Danish case.
The exchange rates in Kim and Roubini’s (2000) SVAR for the non-US G7 countries are bilateral US-dollar exchange rates, and therefore they include the federal funds rate in their model. Importantly, the non-US central banks are assumed not to react to the federal funds rate within the month. The argument goes that the exchange rate contains all the relevant information, i.e. the non-US central banks are not interested in the federal funds rate per se, but only on its effect on their currencies. Although this identifying restriction is debatable in a flexible exchange-rate setting (Faust and Rogers, 2003), it is clear that for a fixed exchange-rate SVAR leaving out the contemporaneous anchor country’s interest rate out of the interest-rate equation is not reasonable, cf. figure 3.\footnote{When Kim and Roubini (2000) include the Fed’s fund rate contemporaneously, they do not get sensible results. Likewise Kim (2002) does not get sensible results when letting Danmarks Nationalbank react to the German interest rate within the month, and thus he leaves it out. We encounter problems as well (non-convergence) but rather than imposing a dubious identifying restriction, we look for information outside the model, see below. In Cushman and Zha’s (1997) block recursive model Bank of Canada is allowed to react within the month to the Fed’s fund rate.}

For most fixed exchange-rate regimes, it is reasonable to assume that the (usually large) anchor country’s economy is not affected by the small open economy that pegs its currency to it. In our model, we include the anchor country interest rate as an exogenous variable. The advantage of this approach is that there are less parameters to be estimated and more degrees of freedom. Alternatively, one could set up a block recursive model like Cushman and Zha (1997), who have a Canadian block and a US block (see also Mojon and Peersman, 2003). In their model US variables affect Canadian variables, but not vice versa. Within the US block equations for production, prices and so forth are estimated, and thus, US shocks are identified as well. In our approach we do not identify anchor-country monetary shocks, but for a small open economy with a fixed exchange rate, one may argue that any movement in the anchor country interest rate is relevant, irrespective of whether it is a monetary-policy shock or not. In addition, information on the cyclical stance of the foreign economy is captured by the anchor country’s monetary-policy stance.

The other key variable in the interest-rate equation is the exchange rate. The exchange rate should be included since that is what a fixed exchange-rate policy is all about: reacting to and stabilizing a given exchange rate. In flexible exchange-rate regimes failing to let monetary policy react contemporaneously to exchange-rate changes has produced the so-called exchange-rate puzzle, where positive interest-rate shocks are associated with a weakening of the exchange rate, contrary to theory. The explanation is probably that central banks react to the exchange rate and that the identified shocks are instead endogenous reactions to developments in the currency. In fixed exchange-rate regimes it is even more likely that an exchange-rate puzzle would be present if simultaneity between the interest rate and the
exchange rate were not allowed for, since the exchange rate is basically the *only* thing the central bank is interested in.9

In sum, a central bank with an exchange-rate target reacts to the exchange rate and the anchor country’s monetary-policy interest rate when setting its interest rate, and monetary-policy shocks may be retrieved as deviations from this reaction function.

The last equation in our model is the exchange-rate equation. We allow all other variables to influence it contemporaneously since it is a financial-market variable that incorporates all available information. The fact that output and prices are excluded from the interest-rate equation, but included in the exchange-rate equation ensures that the model is formally identified and should allow us to separate the interest-rate and the exchange-rate equations (see below though). Although the exchange rate is determined by the interest-rate spread, we choose to include the two interest rates separately, as it allows us to estimate an interest-rate reaction function in levels.10

All in all, our SVAR with four endogenous variables and one exogenous variable is fairly small compared to recent models, which often include 7-9 endogenous variables. This greatly

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9 When estimating VARs for Denmark without simultaneity, we have indeed found an exchange-rate "puzzle" to be present.

10 In any case, the impulse responses were the same in a model including the spread instead of the two interest rates separately.
reduces the number of coefficients to be estimated, and increases the degrees of freedom. One general feature for fixed exchange-rate regime SVARs seems to be that the set of relevant variables is simply smaller, and perhaps more clearly defined than in the case of flexible exchange-rate regimes (see below on interventions, though).11

2.3 Data and some other modelling choices

Any VAR analysis needs to consider the estimation period carefully. There is a trade-off between degrees of freedom and stability of the parameters of the model. Changes in the policy regime are a particular concern because they often lead to structural breaks. Classic examples are the US monetary-policy change with increased focus on monetary aggregates introduced by Paul Volcker in the beginning of the 1980s and the ERM crisis in 1993 where some European countries abandoned their fixed exchange-rate regimes. But breaks can also be less visible as when relative weights shift in the policy reaction function and stable estimation periods are difficult to pin down.

In a fixed exchange-rate regime transparent candidates for a stable estimation period can be obtained by looking at the central parity, the exchange rate and the interest-rate differential vis-à-vis the anchor country. In an initial phase of the regime it might take time to gain credibility (learning) and there may even be small adjustments in the central parity. As credibility is gained risk premiums are gradually reduced. Criteria for a stable estimation period are: no parity adjustment and a (small and) relatively stable risk premium as indicated by the interest-rate differential.

In choosing the estimation period for our particular case study, Denmark, we need to elaborate a bit on the short discussion of Danish monetary policy in the introduction. It is clear that the period before 1982 should be excluded, since the announcement of the fixed exchange-rate policy was a clear break in monetary policy. But it is also clear that it took a while for the private sector to put faith in the regime, and spreads went only slowly in.12 Furthermore, there were three minor parity adjustments in the years after the announcement of the fixed exchange-rate policy. In August 1993 - following a turbulent year - the fluctuation bands were widened to ± 15 per cent. The krone depreciated considerably on impact, but then strengthened to within the narrow bands (± 2.25 per cent) at the turn of the year. In

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11 A comment on the omission of some measure of the money stock in the VAR is warranted. Often this variable is included in order to separate monetary-policy shocks and money demand shocks. We leave it out of our model since the money stock has not played any role in Danish monetary policy in the period under review.

12 Incidentally, learning also took place in the 1970s where devaluations were increasingly used as an active policy tool. The first devaluations did not lead to any reaction on interest-rate spreads, but when the private sector eventually learned that more were to come, risk premiums increased violently.
the beginning of 1997 the krone reached the central parity again and it stayed close to that until the ERM was replaced by ERM II following the introduction of the euro. Since 1999 Denmark has participated in ERM II at a central parity that is a conversion of the central parity of the krone vis-à-vis the D-mark in the previous ERM and with narrow bands (± 2.25 per cent).

In our view there are then three candidates for the estimation period, the full period from 1982-83, a sub-period starting in 1987 at the time of the last parity change, and after 1993 where the ERM was suspended. We focus on the shortest and most stable period 1994-2005 because we are particularly interested in investigating the interplay between the variables in the financial block, and a necessary condition for doing so is a stable central bank reaction function.\footnote{Note also that our relatively small model allows us to choose a short estimation period.} Results for the full period 1983-2005 are not clear-cut. This is not surprising, since identified monetary-policy shocks (deviations from the reaction function over the full period) to a certain extent reflect the substantial credibility-induced fall in the interest rate (spreads/risk premium) without exchange-rate effects and thus do not reflect true shocks.\footnote{It turns out it is also important to exclude the months where turbulence where at its maximum in 1993 and fluctuation bands were widened. If included, results are distorted. The intuition is clear: during these months exchange rates were allowed to weaken substantially without any reaction from monetary authorities. Hence, these months were extreme and not representative for the "normal" reaction of the central bank. Due to the sizeable movement in the exchange rate, the observations for these months are clearly outliers and they are influential enough to affect e.g. impulse responses. This demonstrates the need for a careful assessment of the estimation period combined with the "institutional and theoretical" knowledge of the monetary-policy regime. As a footnote to this footnote, this lesson also applies to the period before 1982. During the late 1970s there were sizeable movements in the exchange rate due to devaluations. This led to expectations of further devaluations, which increased risk premiums and thus monetary-policy rates significantly. Hence, this period were characterized by a (sizeable) weakening of the krone and an increasing interest rate (spread), i.e. a completely different monetary-policy environment than after 1982. Estimating a reaction function for periods spanning both before and after 1982 makes no sense.}

In order to obtain as much information as possible about the link between the financial variables in the model, we prefer to use high frequency data. However we are limited to the monthly frequency since output and price measures are not available more often.

Turning next to our specific choice of data, we use (seasonally adjusted) log industrial production (denoted $IP$) as our measure of output since broader measures like GDP are only available at lower frequency and we measure prices as year-on-year percentage changes in the consumer price index ($CPI$). The monetary-policy interest rate used is the Nationalbank’s lending rate ($R$), and the Bundesbank/ECB’s lending rate ($R^*$). Finally, we use the (log) D-mark/krone exchange rate ($E$), since the Danish monetary policy during the estimation period has been directed towards keeping the krone stable vis-à-vis the core currencies in the ERM, of which the D-mark is, of course, the prime representative.\footnote{Note that the exchange rate included is the bilateral nominal exchange rate vis-à-vis the D-mark, not an} On 1 January 1999 the
D-mark was replaced by the euro, so we have constructed an artificial D-mark rate for the period since then using the euro/krone exchange rate and the official euro/D-mark conversion rate. A constant is included in the model.\textsuperscript{16} We follow the literature and include the variables in levels except for prices.\textsuperscript{17} The lag length is determined by Akaike’s information criterion (AIC), but if autocorrelation is present the lag length is increased.

3 Results

We first estimate the reduced-form model by maximum likelihood on data from 1994m1 to 2005m11. AIC indicates that two lags should be included in the model, but based on autocorrelation tests, we choose a specification with four lags on both endogenous and exogenous variables.

Next we turn to the structural model (4). Given the restrictions we have put on \( B_0 \) the model is identified and we should be able to recover the remaining parameters by maximizing the associated log-likelihood function (see Hamilton 1994 for details). However, it turns out that numerical convergence cannot be obtained and a maximum of the log-likelihood function can therefore not be found. To gain intuition for this result, it is useful to take a closer look at the restrictions in (4). When leaving \( b_{34} \) and \( b_{43} \) unrestricted the distinction between the monetary-policy shock (\( u_R \)) and the foreign-exchange market shock (\( u_E \)) must be based on information across the two blocks of the model. This information is contained in the covariances between on the one hand the innovations to the real economic variables and on the other hand the innovations to the financial variables. In a credible fixed-exchange-rate regime, however, the exchange rate depends only very weakly on current \( IP \) and \( CPI \). Presumably the restrictions that the interest rate does not depend on current \( IP \) and \( CPI \) (\( b_{41} = b_{42} = 0 \)) are then too weak to separate the two types of financial shocks.

Since the information within the model is not sufficient to identify the structural shocks, we follow Smets (1997) in including additional information. The starting point is the relationship between the structural shocks and the residuals from the reduced form (3), which

\textsuperscript{16} Data are available on request.
\textsuperscript{17} In any case if the variables are non-stationary and co-integrated the estimation would still be efficient (see Hamilton, 1994; and Sims et al, 1990).
when adapted to our model (cf. equation 4) reads

$$
\begin{pmatrix}
\varepsilon_{IP} \\
\varepsilon_{CPI} \\
\varepsilon_E \\
\varepsilon_R
\end{pmatrix}
= \begin{pmatrix}
1 & 0 & 0 & 0 \\
-b_{21} & 1 & 0 & 0 \\
\frac{b_{34}}{b_{32}} & \frac{b_{32}}{b_{31}} & -\frac{1}{\beta} & \frac{b_{34}}{b_{32}} \\
-(\frac{b_{31}}{\beta} - b_{32})b_{43} & \frac{b_{32}}{b_{31}}b_{43} & \frac{b_{43}}{b_{32}} & -\frac{1}{\beta}
\end{pmatrix}
\begin{pmatrix}
u_{IP} \\
u_{CPI} \\
u_E \\
u_R
\end{pmatrix},
$$

(5)

where we have defined $\beta = b_{34}b_{43} - 1$.

The structural $IP$ shock is easily found as the residual in the first equation of the reduced-form VAR

$$u_{IP} = \varepsilon_{IP}. \quad (6)$$

The structural shock to $CPI$ can be retrieved as the error term in the following regression

$$\varepsilon_{CPI} = \beta u_{IP} + u_{CPI}. \quad (7)$$

The error terms of the last two VAR-equations are related to the structural shocks as follows

$$\varepsilon_E = \phi_1 u_{IP} + \phi_2 u_{CPI} + \phi_3 u_E + \phi_4 u_R, \quad (8)$$

$$\varepsilon_R = \delta \phi_1 u_{IP} + \delta \phi_2 u_{CPI} + \delta \phi_3 u_E + \phi_3 u_R, \quad (9)$$

where $\delta = -b_{43}$. Noting that

$$\phi_3 = \delta \phi_4 + 1, \quad (10)$$

we get

$$\varepsilon_R = \delta \varepsilon_E + u_R. \quad (11)$$

An estimate of the structural monetary-policy shock can be obtained by estimating this equation. Note, however, that the residual in the exchange-rate equation of the reduced-form VAR ($\varepsilon_E$) is likely to be correlated with the monetary-policy shock ($u_R$). Equation (11) will, therefore, be estimated using GMM. Finally, having identified $u_{IP}$, $u_{CPI}$ and $u_R$, the structural exchange-rate shock can be found from the residual of the regression (8) $\phi_3 u_E$, using the relationship (10) to obtain an estimate of $\phi_3$.

To estimate equation (11) we need to find instruments that are correlated with the innovation in the krone/D-mark exchange rate, but not with the Danish structural monetary-policy shock. Inspired by Smets (1997) we use shocks to the US-dollar/D-mark exchange rate and the US short-term interest rate (Fed funds target rate).18

18 The shocks are obtained as the residuals in regressions of the instruments on their own lags, lags of the
The impulse responses to the identified shocks are presented in figure 4. A positive IP-shock has no significant effect on the rate of inflation, but it keeps up industrial production for a number of months. While insignificant (at the chosen 10 per cent level), the point estimates of the financial variable responses to a IP-shock point to a slight appreciation of the exchange rate on impact that triggers a short-lived decline in the interest rate. A CPI-shock keeps inflation up for about six months, but it does not have any significant effects on the other variables. In general the two real-economy shocks have only marginal effects on the financial variables.

A negative exchange rate shock (corresponding to a depreciation of the krone vis-à-vis the D-mark/euro) leads to an immediate increase in the interest rate. This is consistent with a central bank reaction function in which the interest rate is raised immediately to counter endogenous variables and the exogenous variable in (2) and the estimated IP and CPI shocks.
negative shocks to the exchange rate. The exchange rate is back to its pre-shock level as early as the second period after the shock, while the interest rate stays above its pre-shock level for several months. The swift return of the exchange rate shows that the fixed exchange-rate policy has been able to quickly eliminate the exchange-rate effect of exchange-rate shocks. Furthermore, the dynamics of the interest-rate response - an increase followed only by a gradual return - is remarkably consistent with the Nationalbank’s description of the fixed exchange-rate policy (Danmarks Nationalbank, 2003, p. 24 ff). The point estimates of the impulse response functions indicate hump-shaped declines in production and inflation, but only the peak effect on production is significant.

Turning to the monetary-policy shock, an increase in the interest rate has no significant effect on industrial production to begin with. Inflation falls in the months following the shock, but the effect is not significant at the 10 per cent level. When considering the limited responses of the real-economic variables, it should be borne in mind that the interest-rate effect of the monetary-policy shock is very short-lived. The exchange rate appreciates significantly and immediately in response to the interest rate increase. In contrast to other studies, we do not find evidence of "delayed overshooting" (see e.g. Eichenbaum and Evans, 1995 and Kim and Roubini, 2000). The exchange rate response is substantial in the month of the shock and the response increases only slightly in the following month before it gradually falls back.

Following a monetary-policy shock the interest rate declines rapidly and after six months it has fallen significantly below its pre-shock level. The exchange-rate response is quite persistent - the appreciation remains significant for more than a year - despite the eventual interest-rate decline. Also, ten months after the shock and following the decline in the interest rate, industrial production increases above its pre-shock level.

More generally, the estimated impulse responses shed light on the validity of our identification scheme. The exchange rate response to a monetary-policy shock indicates that we have indeed been able to separate a true shock from the endogenous monetary-policy response to e.g. an increase in the anchor-country interest rate. Also the fact that the exchange rate appreciates following an interest-rate increase suggests that the monetary-policy shock has not been confused with the reaction to a shock originating in the foreign-exchange market.

A different perspective on the estimated model is given by the forecast error variance decomposition in table 1. Shocks to the exchange rate and monetary policy explain very little of the forecast error variance of the real-economic variables. This indicates that the Danish real economy has been more or less insulated from financial market shocks in the sample period. Exchange-rate shocks and monetary-policy shocks explain the bulk of the forecast error variance of the exchange rate. In the very short run, the exchange-rate shock explains
Forecast error at horizon is due to variance of IP shock CPI shock E shock R shock

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Table 1: Forecast error variance decomposition

slightly more than fifty per cent of the forecast error variance of the exchange rate, while the monetary-policy shock explains slightly less than that. However, the relative contribution of the two shocks shifts very quickly, so that the exchange rate shock only explains about one quarter of the three-months ahead forecast error variance of the exchange rate, while the monetary-policy shock explains close to three-quarters. In the longer run the contribution of the monetary-policy shock falls slightly, while the CPI-shock eventually explains close to one-sixth of the forecast error variance of the exchange rate. The IP-shock does not contribute to explaining the forecast error variance of the exchange rate at any horizon.

The forecast-error variance decomposition of the monetary-policy interest rate is particularly simple. The exchange-rate shock explains more than eighty per cent of the forecast error variance at all horizons, while the contribution of the monetary-policy shock is less than twenty per cent. Again, this points to the importance of shocks to the exchange rate in explaining the unexpected movements of the monetary-policy interest rate. The real-economic shocks do not contribute to explaining the forecast-error variance of the interest rate.

The impulse response functions and the forecast-error variance decompositions summarize the dynamic relations between the four endogenous variables in the VAR. But further insight on the endogenous monetary-policy response can be gained by looking at the estimated structural monetary-policy reaction function. Focusing on the contemporaneous part of the
reaction function, we have

\[ R_t = -0.71 E_t + 0.77 R^*_t + \cdots , \]

where current industrial production and inflation are absent as a result of the identifying assumption that the central bank does not react to the real economy variables when setting the monetary-policy interest rate. A one per cent depreciation of the krone is seen to lead to an average interest rate increase of around 0.7 per cent.\textsuperscript{19} The second important argument in the reaction function is the anchor country monetary-policy interest rate \( R^*_t \). We find that an increase in the German (euro area) interest rate of e.g. 25 basis points leads to an increase of the Danish monetary-policy interest rate of close to 20 basis points on average.\textsuperscript{20} This could reflect that even though anchor country interest-rate changes are normally followed exactly by the Danish central bank, there has been instances in which a change in the interest rate in the anchor country has been used by Danmarks Nationalbank as an opportunity to adjust the interest-rate spread.

## 4 An extended model: Interventions

Although monetary policy under fixed exchange rates can be captured nicely in a 4-variable model, one obvious extension to do is the inclusion of the central bank’s interventions in the foreign-exchange market. Whereas interventions in flexible exchange-rate regimes are used infrequently, they are an integrated part of the ’nuts and bults’ of (short-run) monetary policy with fixed exchange rates, cf. figure 5.\textsuperscript{21} Typically, there is a chronological ordering of the two instruments (Danmarks Nationalbank, 2003). Initially pressure on the currency is counteracted by the first instrument: interventions. If this is not sufficient, the second instrument is called upon: an interest-rate hike. But the Danish experience shows that even in periods of calm currency markets, intervention amounts can be significant and have in recent years reached levels previously only seen in connection with actual currency crisis. This reflects the free movement of capital and the increase in the international diversification of investors’ portfolios.

Precursors to our work is Kim (2003, 2005) who estimates SVARs for USA and Canada, respectively. In addition to being flexible exchange-rate regime models, the model for USA

\textsuperscript{19}This parameter is retrieved from the GMM-estimation of equation (11). Its standard error is 0.34.

\textsuperscript{20}The bootstrapped 90 per cent confidence interval for this parameter is (0.58; 0.96).

\textsuperscript{21}The figure shows the Nationalbank’s net purchases of foreign exchange which includes intervention in the foreign exchange market but also purchases of foreign exchange related to the central government’s foreign-currency income and payments. When estimating the model below, we use actual intervention data.
suffers from non-identification. This point has been stressed by Neely (2005) who demonstrates that the model does not fulfill the rank condition for identification despite satisfying the order condition. The intuition behind Neely’s result is that simultaneity between the variables in the financial block de facto reduces the number of independent conditions from which the parameters of interest have to be found, since the parameters from the simultaneously interacting equations must be pinned down by the same information - covariances (see Neely, 2005, for details). In spite of the obvious problems with interpreting a model which is not identified, Kim’s idea of analyzing both monetary-policy instruments jointly in a unified framework is, of course, worthwhile pursuing.

4.1 Identification, estimation, and other modelling choices

Our extended 5-variable structural model is given by

\[
\begin{pmatrix}
1 & 0 & 0 & 0 & 0 \\
b_{21} & 1 & 0 & 0 & 0 \\
0 & 0 & 1 & b_{34} & 0 \\
b_{41} & b_{42} & b_{43} & 1 & b_{45} \\
0 & 0 & 0 & b_{54} & 1 \\
\end{pmatrix}
\begin{pmatrix}
IP_t \\
CPI_t \\
INT_t \\
E_t \\
R_t \\
\end{pmatrix}
= c +
\begin{pmatrix}
a_1 \\
a_2 \\
a_3 \\
a_4 \\
a_5 \\
\end{pmatrix}
R^* + lags +
\begin{pmatrix}
u_{IP,t} \\
u_{CPI,t} \\
u_{INT,t} \\
u_{E,t} \\
u_{R,t} \\
\end{pmatrix},
\]

(12)
where we have included an equation for interventions (INT) that for now is to be interpreted as an "intervention reaction function". Central-bank interventions do not react to the real-economic block. Within the financial block interventions react contemporaneously to the exchange rate (and vice versa), but not directly to interest-rate changes (and vice versa). The reason being that interventions react only to interest-rate changes in so far as there is an effect on the exchange rate. Likewise during periods with pressure on the exchange rate the interest rate only reacts in so far as interventions are not successful in stabilizing the exchange rate. Thus, there is within-the-month interaction of the monetary-policy interest rate and foreign-exchange market interventions but it runs indirectly via the exchange rate.

Adding an equation to the financial block without contemporaneous interaction with the real-economic block but with contemporaneous interaction within the block does not alleviate our information problem, on the contrary. Thus, to credibly estimate our model we have to extend the GMM method used above in the 4-variable case.\(^{22}\)

The relationships between the structural shocks and the reduced-form residuals are given by

\[
\begin{pmatrix}
\varepsilon_{IP,t} \\
\varepsilon_{CPI,t} \\
\varepsilon_{INT,t} \\
\varepsilon_{E,t} \\
\varepsilon_{R,t}
\end{pmatrix} =
\begin{pmatrix}
1 & 0 & 0 & 0 & 0 \\
-b_{21} & 1 & 0 & 0 & 0 \\
b_{41}b_{34}-b_{31}b_{41}b_{42} & b_{42}b_{34} & -b_{45}b_{54}+1 & 0 & 0 \\
b_{42}b_{34} & -b_{43} & 0 & 0 & 0 \\
b_{43}b_{54} & b_{45}b_{54} & b_{42}b_{34} & -b_{44} & 0 \\
b_{45}b_{54} & b_{43}b_{54} & b_{42}b_{34} & b_{44} & 0 \\
\end{pmatrix}
\begin{pmatrix}
u_{IP,t} \\
u_{CPI,t} \\
u_{INT,t} \\
u_{E,t} \\
u_{R,t}
\end{pmatrix},
\] (13)

where \(\beta = 1 - b_{34}b_{43} - b_{45}b_{54}\). The IP and CPI shocks are found as above, and going through a few steps in the same vein as above (see appendix) we can retrieve the structural shocks from the financial block by running the following regressions

\[
\varepsilon_{INT,t} = \delta_1 \varepsilon_{E,t} + u_{INT,t}, \quad \delta_1 = -b_{34},
\] (14)

\[
\varepsilon_{R,t} = \delta_2 \varepsilon_{E,t} + u_{R,t}, \quad \delta_2 = -b_{54},
\] (15)

\[
\varepsilon_{E,t} = \phi_1 u_{IP,t} + \phi_2 u_{CPI,t} + \phi_3 u_{INT,t} + \phi_4 u_{E,t} + \phi_5 u_{R,t},
\] (16)

where

\[
\phi_1 = (b_{21}b_{42} - b_{41})\beta^{-1}, \quad \phi_2 = -b_{42}\beta^{-1}, \quad \phi_3 = -b_{43}\beta^{-1}, \quad \phi_4 = \beta^{-1}, \quad \phi_5 = -b_{45}\beta^{-1}.
\] (17)

\(^{22}\)The model (12) is formally identified, but it is still the case that the contemporaneous covariance between the exchange rate and the real-economic block is too weak, presumably due to the credibility of the regime.
The relations in (17) can be used to extract the structural coefficients from the exchange-rate equation. The first two regressions are estimated by GMM using innovations in the US-dollar/D-mark exchange rate and the US Fed funds target rate as instruments.

Estimating a stable foreign exchange-market intervention reaction function requires a stable estimation period. Over time the intervention strategy of Danmarks Nationalbank has changed for various reasons. Liberalized capital markets have increased the need for interventions. The credibility of the Danish regime has increased and arguably affected the impact of interventions. In addition, the central bank even stopped intervening for a period after the 1993-crisis and the subsequent central-bank activity in the foreign-exchange market in 1995 were primarily a period of buying back some of the foreign currency which was sold during the 1993-crisis (Danmarks Nationalbank, 1996). Thus, we estimate the 5-variable model on an even briefer period than our previous model namely 1996m1-2005m11. Our model is estimated using 1 lag only, indicating a stable and well specified model.

4.2 Results
The impulse-response functions following structural shocks in the financial block are given in figure 6.

Following a contractionary interest-rate shock the exchange-rate appreciates on impact, and as the shock dies out the exchange rate returns to the pre-shock level. After approximately 6 months the effects become insignificant. Interestingly, the interventions follow a leaning-against-the-wind strategy. Particularly on impact, the strengthening of the currency is counteracted by net-sales of Danish kroner. It is also worth noting that the exchange rate does not follow a delayed-overshooting pattern. This is contrary to results for Canada, where Kim (2005) stresses intervention as an explanation of the delayed-overshooting puzzle. In his model the Canadian exchange rate reaches its peak after almost two years following a positive interest-rate shock, and presumably two years' sale of Canadian dollars is instigated to mitigate the effects of the initial appreciation, although at a declining pace, which then leads to the hump-shaped response of the Canadian dollar. We feel much more at ease with the relative short-lived nature of interventions in our model. As practitioners it is difficult to gain intuition for prolonged one-sided reactions of foreign exchange market intervention. Of course, Kim (2005) estimates a model for the flexible exchange-rate regime Canada, but perhaps more importantly, his model spans a very long period with structural changes. At

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23 Unsurprisingly in a model spanning 1994-1995 interventions' reaction to the exchange rate ($b_{34}$) is much lower than in the chosen model from 1996 and onwards.

24 Although the delayed overshooting response is not a robust phenomenon (Faust and Rogers, 2001), it can be generated in theoretical models with learning (see e.g. Andersen and Beier, 2005, and Gourinchas and
least for our case study Denmark it would be hazardous to estimate a stable intervention reaction function spanning both the 1970s, the 1980s and the 1990s and this "institutional" knowledge is at least as important as the identifying restrictions on $B_0$. In the interest-rate reaction function the coefficient on the exogenous anchor country rate is 0.89. In reaction to an exchange-rate shock, an exogenous depreciation of the krone, the central bank reacts with an increase in the monetary-policy rate and net buying of kroner in the foreign-exchange market. Both monetary-policy instruments act to support the currency - a 0.05 per cent depreciation has on average given rise to net buying of approximately 5 billion kroner and a hike of 3 basis points. The reaction of interventions though, dies out very fast, whereas the interest-rate response is persistent. Actually the interest rate increases further in the period after the shock, indicating that the initial combined response of an increased interest rate and intervention is replaced by an interest-rate-only response in the following periods. This is probably due to the fact the interest-rate response is an average of the responses to small and large exchange-rate shocks. In case of a small shock intervention is perhaps enough to stabilize the exchange-rate - i.e. an interest-rate response of nil - whereas an interest-rate response is needed in case of a larger shock. After 6 months the exchange-rate response becomes insignificant, whereas the interest rate only returns after almost one year. Thus, the response of the exchange rate is slightly more persistent than in our 4-variable model, but it is still the case that the interest rate is the most persistent. The response of inflation is not significant, whereas industrial production falls with a lag with a peak effect of about 6 months (not shown).

An intervention shock (net sale of kroner) leads to a weakening of the currency, which in turn induces an interest-rate tightening. Both interventions and the interest-rate responses are short-lived, whereas it takes a while for the exchange rate to return to its pre-shock level.

At this point it might be useful to digress on the interpretation of an intervention shock or more generally, how interventions work in a fixed exchange-rate regime. Interventions are used for short-run management of the exchange rate. If there is a slight pressure on the exchange rate it is not necessarily stabilized fully in the sense that it is pushed back to its pre-shock value or the central parity. Mostly, the purpose is only to stop or dampen a given movement to stabilize the exchange rate within the fluctuation bands. Thus, a small exchange-rate shock (which does not call upon the interest rate to be used) will result in a long-lived change in the exchange rate and a short-run change in intervention. This behavior

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25 Incidentally, when estimating the 5-variable model for Denmark for 1983-2005, the responses become much more persistent.  
26 The bootstrapped 90 per cent confidence bounds are (0.76:1.00).
is most probably underlying the persistent response of the exchange rate and is also reflected in the forecast error variance decomposition in table 2.

Intervention and exchange-rate shocks explain by far the largest share of the forecast error variance of the exchange rate also at long horizons. This just reflects that most of the action takes place within the fluctuation bands and that intervention policy for small shocks within the band is simply to stop or dampen a given exchange-rate movement, not to reverse it. Interest-rate shocks only account for a small share of the variance at most horizons as would be expected.

Controlling for interventions in a fixed exchange-rate SVAR is important since it is an integral part of the conduct of actual day-to-day monetary policy. But the intervention series behaves differently and more erratically than other macro time series. The amount of variation explained in the reduced-form VAR equation for interventions is merely 12 per cent,
Forecast error variance at horizon is due to variance of IP shock, CPI shock, INT shock, E shock, R shock.

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Table 2: Forecast error variance decomposition financial block - 5-variable model

much lower than for the other endogenous variables. With this caveat in mind it is striking how the impulse-response functions match a priori expectations and how, in addition, the exchange rate enters the intervention equation highly significantly and vice versa.

5 Conclusion

Monetary policy in a fixed exchange-rate regime is distinctly different from that under a floating exchange rate. For a fixed exchange rate to be credible, the central bank must give overriding importance to stabilizing the exchange rate around the chosen target due to the forward-looking nature of the exchange rate. Interest-rate setting in a fixed exchange-rate regime should therefore focus on exchange-rate movements and the monetary-policy stance of the anchor country. Stabilization of the real economy cannot be an objective in interest-rate setting. In contrast, reduction of fluctuations in inflation and output belong to the core monetary-policy objectives under floating exchange rates and interest-rate setting by the central bank depends on a much broader analysis of the medium-term outlook. These differences between regimes have important implications for monetary-policy analysis.

In this paper we have presented a SVAR for a small open industrialized economy under fixed exchange rates. We have given a detailed account of the "institutional and theoretical knowledge" needed to correctly identify monetary shocks and shown the significant differences to SVARs for flexible exchange-rate regimes. One the one hand setting up a SVAR is simpler since the rules of engagement are very clear and transparent, but on the other hand it makes estimation harder as the simultaneity between financial variables complicates identification.
To overcome the estimation challenge we used and extended the GMM-method of Smets (1997) to include both interventions, the two interest rates and the exchange rate, and the impulse response functions were well-behaved and in accordance with theory.
6 Literature


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7 Appendix

7.1 Data

- \( IP \): Log industrial production, seasonally adjusted. Source: Statistics Denmark.

- \( CPI \): Year-on-year percentage changes in the CPI. Source: Statistics Denmark.

- \( E \): Log D-mark/krone exchange rate, monthly average. Synthetic rate after 1999 as explained in the main text. Source: Danmarks Nationalbank.

- \( R \): Danmarks Nationalbank’s lending rate (before June 1999 known as the rate of interest for repurchase agreements). Source: Danmarks Nationalbank.

- \( R^* \): Bundesbank’s (ECB’s) lending rate (Bundesbank repurchase rate until end of 1998, from 1999 to June 2000 the fixed rate in the ECB’s main refinancing operations, since June 2000 the minimum bid rate). Source: Danmarks Nationalbank.

- \( INT \): Log net intervention sale of kroner, in billions. Source: Danmarks Nationalbank.

7.2 Retrieving shocks for the model including interventions

The reduced-form residuals and the structural shocks are linked in the following way

\[ u_t = B_0 \varepsilon_t, \]

thus

\[
\begin{pmatrix}
  u_{IP} \\
  u_{CPI} \\
  u_{INT} \\
  u_E \\
  u_R
\end{pmatrix} =
\begin{bmatrix}
  1 & 0 & 0 & 0 & 0 \\
  b_{21} & 1 & 0 & 0 & 0 \\
  0 & 0 & 1 & b_{34} & 0 \\
  b_{41} & b_{42} & b_{43} & 1 & b_{45} \\
  0 & 0 & 0 & b_{54} & 1
\end{bmatrix}
\begin{pmatrix}
  \varepsilon_{IP} \\
  \varepsilon_{CPI} \\
  \varepsilon_{INT} \\
  \varepsilon_E \\
  \varepsilon_R
\end{pmatrix}
\]

\[
\begin{pmatrix}
  \varepsilon_{IP} \\
  \varepsilon_{CPI} \\
  \varepsilon_{INT} \\
  \varepsilon_E \\
  \varepsilon_R
\end{pmatrix} =
\begin{bmatrix}
  1 & 0 & 0 & 0 & 0 \\
  -b_{21} & 1 & 0 & 0 & 0 \\
  b_{34}b_{43}-b_{34}b_{44} & b_{43}b_{44} & 1-b_{43}b_{54} & -b_{34} & b_{34}b_{45} \\
  b_{43}b_{54}-b_{43}b_{44} & b_{54} & b_{43}b_{54} & -b_{54} & -b_{45} \\
  b_{43}b_{54}-b_{43}b_{44} & b_{44} & b_{43}b_{54} & b_{54} & 1-b_{44}b_{54}
\end{bmatrix}
\begin{pmatrix}
  u_{IP} \\
  u_{CPI} \\
  u_{INT} \\
  u_E \\
  u_R
\end{pmatrix},
\]

29
where
\[ \beta = 1 - b_{34}b_{43} - b_{45}b_{54}. \]

First observe that
\[ \epsilon_E = \phi_1 u_{IP} + \phi_2 u_{CPI} + \phi_3 u_{INT} + \phi_4 u_{E} + \phi_5 u_{R} \]
\[ \epsilon_{INT} = \delta_1 \phi_1 u_{IP} + \delta_1 \phi_2 u_{CPI} + \gamma_3 u_{INT} + \delta_1 \phi_4 u_{E} + \delta_1 \phi_5 u_{R}, \]
where \( \delta_1 = -b_{34} \),

implying
\[
\begin{align*}
\epsilon_{INT} - \delta_1 \epsilon_E &= (\gamma_3 - \delta_1 \phi_3) u_{INT} \\
&= \left( \frac{-b_{45}b_{54} + 1}{\beta} + b_{34} \frac{-b_{43}}{\beta} \right) u_{INT} \\
&= -b_{45}b_{54} + 1 - b_{43}b_{34} u_{INT} \\
&= \frac{1}{1 - b_{34}b_{43} - b_{45}b_{54}} u_{INT} = u_{INT}.
\end{align*}
\]

Thus - if we can find reasonable instruments - we can run the following regression
\[ \epsilon_{INT} = \delta_1 \epsilon_E + u_{INT}. \]

Now observe that
\[ \epsilon_E = \phi_1 u_{IP} + \phi_2 u_{CPI} + \phi_3 u_{INT} + \phi_4 u_{E} + \phi_5 u_{R} \]
\[ \epsilon_{R} = \eta_1 u_{IP} + \delta_2 \phi_2 u_{CPI} + \eta_3 u_{INT} + \delta_2 \phi_4 u_{E} + \eta_5 u_{R}, \]
where \( \delta_2 = -b_{54} \).

Thus
\[
\begin{align*}
\epsilon_{R} - \delta_2 \epsilon_E &= (\eta_1 - \delta_2 \phi_1) u_{IP} + (\eta_3 - \delta_2 \phi_3) u_{INT} + (\eta_5 - \delta_2 \phi_5) u_{R} \\
&= u_{R},
\end{align*}
\]
since
\[
\begin{align*}
\eta_1 - \delta_2 \phi_1 &= \frac{b_{41} b_{54} - b_{21} b_{42} b_{54}}{\beta} + b_{54} \frac{b_{21} b_{42} - b_{41}}{\beta} \\
&= 0, \\
\eta_3 - \delta_2 \phi_3 &= \frac{b_{43} b_{54}}{\beta} + b_{54} \frac{b_{43}}{\beta} \\
&= 0, \\
\eta_5 - \delta_2 \phi_5 &= \frac{-b_{34} b_{43} + 1}{\beta} + b_{54} \frac{-b_{45}}{\beta} \\
&= \frac{1 - b_{34} b_{43} - b_{45} b_{54}}{1 - b_{34} b_{43} - b_{45} b_{54}} \\
&= 1.
\end{align*}
\]
and
\[
\begin{align*}
\eta_3 - \delta_2 \phi_3 &= \frac{b_{43} b_{54}}{\beta} + b_{54} \frac{b_{43}}{\beta} \\
\end{align*}
\]
All in all we have
\[
\begin{align*}
\varepsilon_{INT} &= \delta_1 \varepsilon_E + u_{INT}, \quad \delta_1 = -b_{34} \\
\varepsilon_R &= \delta_2 \varepsilon_E + u_R, \quad \delta_2 = -b_{54}.
\end{align*}
\]
Finally, from the third equation we have
\[
\varepsilon_E = \phi_1 u_{IP} + \phi_2 u_{CPI} + \phi_3 u_{INT} + \phi_4 u_E + \phi_5 u_R,
\]
where
\[
\phi_3 = \frac{-b_{43}}{1 - b_{34} b_{43} - b_{45} b_{54}}, \quad \phi_5 = \frac{-b_{45}}{1 - b_{34} b_{43} - b_{45} b_{54}}
\]
or
\[
\begin{align*}
b_{45} &= \frac{\phi_5}{b_{34} \phi_3 + b_{54} \phi_5 - 1}, \quad b_{43} = \frac{\phi_3 + b_{53} \phi_5}{b_{34} \phi_3 + b_{54} \phi_5 - 1}.
\end{align*}
\]