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WORKING PAPERS
2006 • 40

Allan Bødskov Andersen

and

Jakob Lage Hansen

Financial Markets
Danmarks Nationalbank

Risk and return in the bond markets – past developments and future prospects

June 2006

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Please direct any enquiries to
Danmarks Nationalbank, Information Desk, Havnegade 5, DK-1093 Copenhagen K
Denmark
Tel.: +45 33 63 70 00 (direct) or +45 33 63 63 63
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ISSN (trykt/print) 1602-1185

ISSN (online) 1602-1193

Working Papers 40 – 2006

Risk and return in the bond markets – past developments and future prospects[1]

Allan Bødskov Andersen

and

Jakob Lage Hansen

Financial Markets

Danmarks Nationalbank

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[1] Comments from colleagues at Danmarks Nationalbank are gratefully acknowledged. All remaining errors are ours. Contact information: Allan Bødskov Andersen: call@dadlnet.dk ; Jakob Lage Hansen: jakoblage@hotmail.com

Abstract

The paper examines the past risk and return trade-off on the US bond market, and uses this as a basis for developing a flexible tool based on simulation of principal components to evaluate future prospects for risk and returns for investors. The principal components of the yield curve are related to a few main macro-economic drivers (inflation and real GDP-gap). This allows simulation of yields curves based on expectations about the future behaviour of the macro-economic drivers, rather than relying solely on parameters estimated from historic data. The overall conclusion is that since 1960 the US-bond market has rewarded risk in the sense that more volatile bond returns has been associated with higher average realized returns. However, this covers very large variation over sub-periods, and in periods with increasing yield trend extra risk has not always been rewarded. In the simulations of the future risk-return trade-off, there are lower excess returns from enhancing the duration exposure in the future relative to the near past. However, the risk associated with duration enhancing is also lower.

Resumé

I papiret undersøges det historiske trade-off mellem risiko og afkast på det amerikanske obligationsmarked, og et værktøj til simulering af fremtidige scenarier for risiko og afkast baseret på simulering af principale komponenter udvikles. Rentekurvens principale komponenter relateres til to fundamentale makroøkonomiske variable, inflation og realt BNP-gab. Dette tillader simulering af rentekurver baseret på forventet udvikling i de fundamentale makrovariable i stedet for udelukkende at relatere sig til parametre estimeret på historiske data. Konklusionen er, at siden 1960 er risiko blevet belønnet i den forstand, at mere volatile obligationsafkast har været ledsaget af højere gennemsnitlige realiserede afkast. Men dette dækker over store variationer i delperioder, og i perioder med stigende trend i renterne er ekstra risiko ikke altid blevet belønnet med højere afkast. I simuleringerne af det fremtidige risiko-afkast forhold er der mindre merafkast ved øget varighedseksponering, men risikoen ved den øgede eksponering er også mindre.

1. Introduction

The paper examines the past risk and return trade-off on the US bond market, and uses this as a basis for developing a flexible tool based on simulation of principal components to evaluate future prospects for risk and returns for investors. The principal components of the yield curve are related to a few main macro-economic drivers (inflation and real GDP-gap). This allows simulation of yields curves based on expectations about the future behaviour of the macro-economic drivers, rather than relying solely on parameters estimated from historic data. We illustrate the approach by a simulation study that builds on a scenario for the future outcome of the macro-drivers that is not directly reflected in the historical data. Such an approach especially useful if the economy is on the brink of a new regime, and for general scenario analysis. Indeed, one can argue that the future yield curves in the US might deviate from the pattern seen in past 20-30 years in several important ways, because inflation fighting over the past decades has gained priority in the economic policies, resulting in strongly anchored inflation expectations and subdued actual inflation rates.

The overall conclusion is that since 1960, the US-bond market has rewarded risk in the sense that more volatile bond returns have been associated with higher average realized returns. However, this covers very large variation over sub-periods, and in periods with increasing yield trend extra risk has not always been rewarded. The variations in risk and return can be explained by variations in inflation and the GDP-gap. In the simulations of the future risk-return trade-off, there are lower excess returns from enhancing the duration exposure in the future relative to the near past. However, the risk associated with duration enhancing is also lower. Hence, the simulations suggest that a US-bond simply is a less risky asset in a scenario with inflation under control and well anchored long term inflation expectations. Furthermore, smaller correlation between yields going forward also reduces the risk of a portfolio of bonds.

The trade-off between risk and return has been subject for intense attention from both academia and practitioners, since it lies at the core of portfolio management, and is a central decision for many businesses and institutions, including Danmarks Nationalbank. From preferences concerning risk-tolerance and investment horizon coupled with information and analysis of how and when different asset classes reward their inherent risks ultimately leads to the actual strategic and tactical asset allocation and the final composition of a portfolio. Few papers (Ilmanen (1995) and Bieri and Chincarini (2005) being examples) have, however, focused on the practical implications for the investment decision, which is the focus of this paper.

The focus here is on the US-bond market. Not only is the US bond market the largest in the world, the US yield curve is also the centre point for global bond prices including those of the euro-area. The determinants of the yield curve have been studied in detail in both the finance and macroeconomic literature. However, only recently has the insights from the two disciplines been combined (see for instance Evans and Marshall (2002), Ang and Piazessi (2003) and Rudebusch and Wu (2004)). Our understanding of the yield curve is very much along the lines of this literature, as we relate the shape of the yield curve to developments in macroeconomic conditions and monetary policy. However, our study differs by also considering the yield curve beyond 5 years.

The trade-off between risk and return in bond markets has recently been examined in Berndsen (2003). The present analysis focuses more narrowly on the risk-return trade-off in the US bond market, which allows for an analysis over a longer period. This facilitates an illustration of risk and return in regimes with both upward and downward trending yields. Another consequence is that estimates of average returns are unbiased, as yields by the end of the period are at similar levels as in the beginning of the period. The study also differs from Berndsen (2003) (and most others) by tying past developments in risk and return to macroeconomic developments, and by basing future scenarios for risk and return on explicit assumptions about macroeconomic conditions.

The paper is organized as follows. In section 2 we give an interpretation of the overall development of yields in the US over the past 45 years, which serves as a basis for the forward-looking analysis. Section 3 describes the risks associated with bond investments and explores the past development in risk and returns across the yield curve. Section 4 lays out the principal component analysis of the yield curve and the principal components are related to simple underlying measures of the state of the economy: Inflation and the real GDP-gap. The principal components are then used to simulate future paths of yields under plausible assumptions about the future macroeconomic developments. This allows us to lay out a scenario for the future risk-return trade-off in the bond markets, as well as the uncertainty surrounding this scenario. Section 5 concludes.

2. Bond yields

2.1. Determinants of bond yields

There are two dominant approaches to analyzing development in bond yield: The Fisher theory and the expectation hypothesis.

The *Fisher theory* decomposes the nominal yield (ex ante) into a real interest rate, expected inflation until the time of maturity, and a risk premium: $i = r + \pi^e + rp$. Often real interest rates are assumed to be (almost) constant. As bonds give a nominal return, actual inflation erodes the real ex post return from holding a given nominal bond. Investors therefore seek a compensation for expected inflation. Lastly, as actual future inflation is uncertain investors generally demand a risk premium as a compensation for the uncertainty regarding the real ex post return over the investment horizon.

Based on no-arbitrage arguments, the *expectation hypothesis* holds that the return over a given horizon from rolling a short term bond is expected to be the same as holding a long term bond. Hence, in its purest form, yields on longer maturities reflect expectations of short term yields. E.g. an upward sloping yield curve reflects an expectation of higher yields at shorter maturities in the future. The theory is often amended with risk premia (hence relaxing the strict no-arbitrage condition), because rolling a short bond is generally viewed as less risky than holding a long bond.

As the aim of this analysis is to reach applicable results, not to test theories, the approach will be pragmatic. As the two theories are by no means incompatible, and as empirical work shows that real interest rates do vary and that risk premiums exist and are time varying (see e.g. Best, Byrne and Ilmanen, 1998), the analysis will draw on insights from both approaches.

Yields are affected by many factors. Shorter maturities are dominated by monetary policy as the Fed sets short term nominal yields. Typically a GDP-growth below potential growth is accompanied by decreasing real short interest rates to stimulate demand, while GDP-growth above potential results in increasing real short interest rates to prevent inflationary pressure, cf. discussion below. Inflation uncertainty is lower and returns less volatile at shorter maturities, hence inflation risk premiums tend to be lowest in the short end.

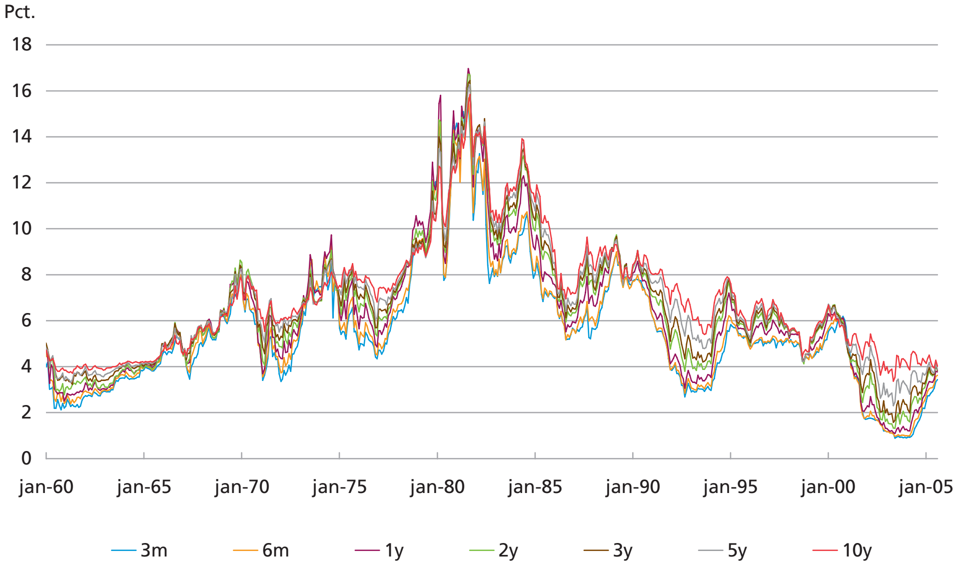
The real rate of return of long maturities is less affected by actual monetary policy. Growth theory determines the real interest by more static fundamental factors, such as productivity and demographics. However, real rates can be affected by changes in supply and demand. Changes in supply can arise due to developments in the Federal budget, while changes in demand can inter alia be due to regulation of pension funds or exchange rate regimes of Asian countries, which alters the real rate they demand for holding longer maturities. Long term inflation expectations are affected by other factors than actual inflation and can be persistent. Lastly, the risk premiums of long maturities are generally positive (and non-constant) due to uncertainty regarding the inflation outlook and the greater volatility of returns at longer maturities.

Maturities at the middle of the yield curve will be determined by some combination of the mentioned factors. The following analysis will build and expand on these considerations.

2.2. The developments in yields

There has been a dramatic development in interest rates since 1960, cf. chart 1. In the beginning of the period yields hovered around 4 pct. before rising sharply in the 1970s. There was a peak in the early 1980s, after which yields have declined to levels seen at the beginning of the period.

US YIELDS Chart 1



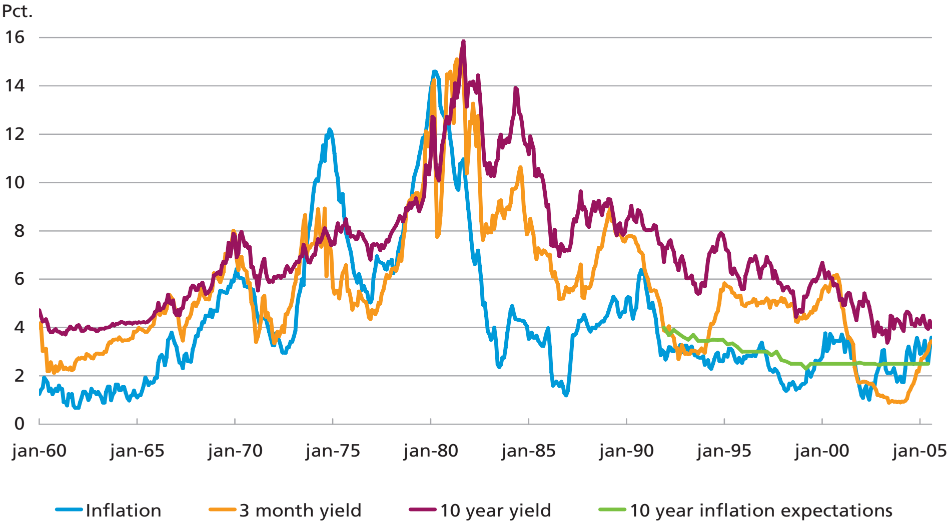
Source: Global Financial Data

2.2.1. The level of yields and inflation

Chart 1 illustrates the high correlation between yields of different maturities. There is a close relationship between the level of the interest rates and inflation in the period, cf. chart 2. In the first half of the 1960s inflation was moderate. At that time inflation had only been experienced in times of war, cf. chart 3. This was in large part due to fixed exchange rates serving as a nominal anchor for prices. It is therefore likely that inflation expectations were subdued, which explains the low level of interest rates (Survey of Professional Forecasters (SPF) and Consensus data on long term inflation expectations first became available in 1992).

As the Bretton Woods system broke down and the first oil crisis evolved, inflation increased sharply in the mid-1970s. However, the large increase in inflation was not matched by an equivalent rise in interest rates, which suggests that the inflation was unexpected. This probably also reflects the historic lesson that such spikes had tended to be short-lived, and that market participants' inflation expectations accordingly remained low despite high actual inflation. However, as the market experienced another sharp rise in inflation later in the 1970s, interest rates rose to new highs – presumably due to upward revisions in inflation expectations. As exchange rates were now floating, they no longer served as a nominal anchor.

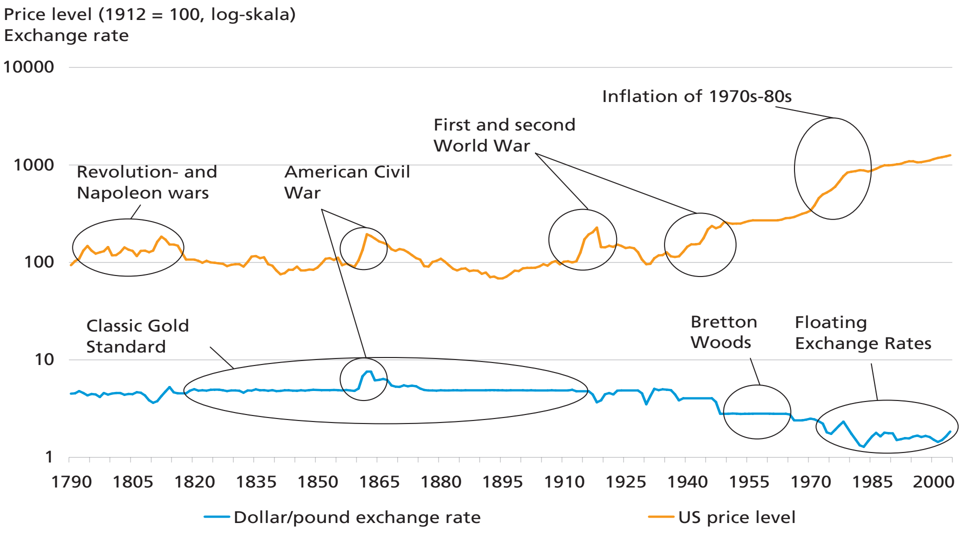
INFLATION AND US YIELDS Chart 2



Note: Inflation is year-to-year changes in the consumer price index.
Source: Global Financial Data, Ecowin and own calculations.

As inflation came back down in the 1980s, interest rates remained at levels well above actual inflation for the rest of the decade. The likely explanation is that market participants, in light of recent developments, were not convinced that inflation would remain at the lower levels.

HISTORIC PRICE-LEVEL IN THE US Chart 3



Source: Lothian and Taylor (1996), EcoWin and own calculations.

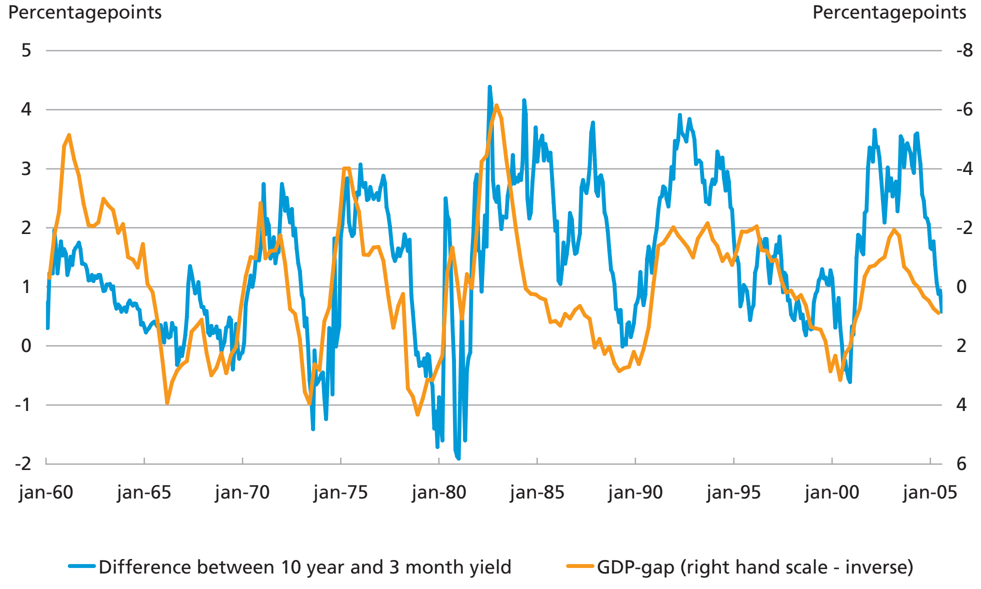
In the 1990s and onward inflation has remained relatively low, eventually convincing investors that inflation probably will remain subdued. Consequently, risk premiums and yields have been pushed back to levels not seen since the beginning of the 1960s. It seems probable that the Fed's focus on price stability has taken over the exchange rates role as a nominal anchor, and that the development in the 1970s and 1980s in retrospect can be seen as the exception rather than the norm. However, there has been a change from a stable price level to stable inflation.

2.2.2. The slope of the yield curve, real rates and risk premiums

Despite the close correlation in yields, there is divergence between the level of short and long yields, cf. chart 4. The Fed typically reacts to low growth by cutting nominal rates, thereby reducing real interest rates, to stimulate demand, while raising nominal rates (and thereby the real interest rate) to curb inflationary pressures when growth is high. This should have a stronger influence on shorter maturities than longer maturities as discussed above. As documented in Christensen (2002), it indeed seems that the spread between yields at long and short maturities co-moves rather closely with changes in the GDP-gap.

GDP-GAP AND US YIELDS

Chart 4



Note: The GDP-gap is measured as the percentage deviation of actual real GDP from its HP-filter trend.
Source: Global Financial Data, Ecowin and own calculations

There is a deviation in the pattern in the 1980s, where the increase in the GDP-gap was not associated with a flattening yield curve. The likely explanation is that market participants, in the light of the negative real returns experienced in the 70's, were concerned about whether the fall in inflation was temporary or permanent. This, coupled with the high volatility of inflation, probably raised uncertainty about future inflation rates, resulted in inflation risk premium at longer maturities remaining at high levels.[1]

Summing up, developments in inflation seem to explain the changes in the overall level of yields, while developments in the GDP-gap and the inflation risk premium

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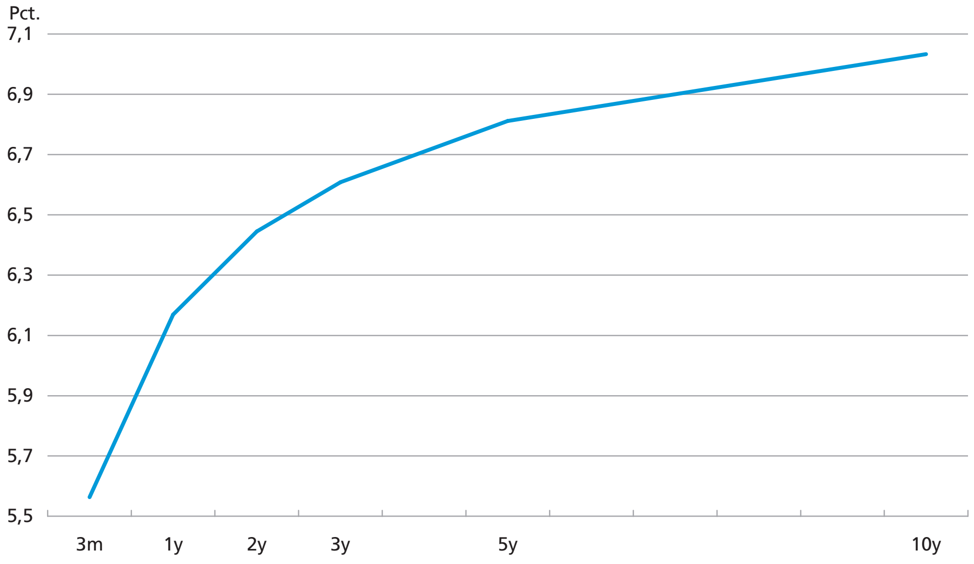
[1] See Buraschi and Jiltsov (2005) for a detailed analysis of the inflation risk premia in the US bond market.

coincides with changes in the steepness of the yield curve. Rudebusch and Wu (2004) find similar results in the period 1988-2000 by modelling the yield curve by the standard finance model and relating this explicitly to monetary policy, output and inflation. The established relationships between the shape of the yield curve and macroeconomic factors will be used as a basis for implementing the simulation study later.

2.2.3. The yield curve

Yields of different maturities are often illustrated by a yield curve. On average, yields at longer maturities have been higher than at shorter maturities, resulting in an upward sloping yield curve, cf. chart 5. The chart also illustrates, that the curve on average has been concave. The shape of the yield curve will be related to the risk at different maturities in section 3.

THE AVERAGE YIELD CURVE Chart 5



Note: The yield at each maturity is the average monthly yield at annual rates in the period 1960-2005.
Source: Global Financial Database and own calculations.

3. Risk and returns

3.1. Risk along the yield curve

The nominal return of a (zero-coupon) bond is certain if the bond is held until maturity (ignoring credit risk, which is irrelevant for US treasuries). However, if the investment horizon is shorter or longer, the return is uncertain, and depends on the developments in yields. Additionally, the real return of a bond investment is generally uncertain. Due to these factors realised (ex post) return will more often than not differ from the expected (ex ante) return, which constitutes risk. Typically longer maturities are viewed as more risky than shorter maturities, which has generally been reflected in an upward sloping yield curve, cf. chart 5. The following elaborates more on the risk at different maturities and relates it to the shape of the yield curve.

3.1.1. Riskless investments and investments horizons

The risk associated with an investment in a bond of a particular duration also depends crucially on the duration of the investors liabilities (debt etc.), that is, how far into the future the liabilities are due on average. The asset an investor considers risk-less should match the duration of the liabilities. Some investors are funded with short maturity debt, which makes the horizon of their risk-less horizon short, while others, such as pension funds, have liabilities far into the future, and hence have a longer riskless horizon. If the assets of an investor match his liabilities, changes in market conditions that affect the value of assets (for instance, a rise in the yield level that depresses the immediate returns on bonds) will be matched by an equivalent change in the value of liabilities. On the other hand, if the assets of the investor have a different duration than the liabilities, unanticipated development in yields will affect assets and liabilities differently and hence affect the investor's equity.

Investors' investment horizons differ and depend on many factors. This study examines monthly returns and therefore in principle a monthly investment horizon for investors marking-to-market. The monthly frequency allows for a study of the risk-return trade-off in sub-periods, as well as more detailed forward analysis. The monthly results are compared to annual returns in the next section. This reveals that the investment horizon does not affect expected returns, but does affect the level of risk. Qualitatively the results based on monthly and annual returns are similar.

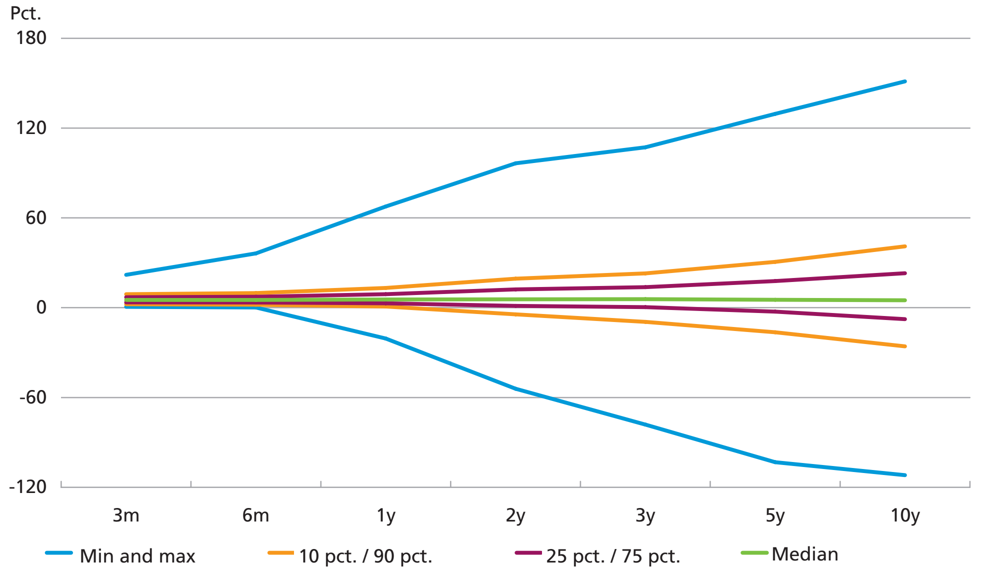
3.1.2. Real returns

It has been laid out above how inflation has varied over time. Since a bond gives a nominal return, an unexpected increase in inflation will erode the real return of the bond. Longer maturity bonds are exposed to changes in inflation over longer horizons and are therefore more risky. Shorter maturities, on the other hand, are not as risky as they can be reinvested at the level of yields that reflect updated information about actual and expected inflation. Real returns are, for instance, relevant for people saving for retirement, as higher real returns makes higher consumption possible.

3.1.3. Volatility of returns

The change in bond price when the yield changes by one percentage point, is measured by a bond's duration. Longer maturities have larger durations than shorter maturities, which makes their price more exposed to a given change in the level of yields. Chart 6 illustrates the greater volatility of monthly returns[2] at longer maturities. This makes longer maturities riskier than shorter maturities if one has a short investment horizon (and marks-to-market).

DISTRIBUTION OF MONTLY RETURNS (ANNUALIZED) 1960 – 2005.8 Chart 6



Note: Monthly returns are computed at annual rates. x pct. denotes x pct. fractile. Min is smallest return and max is largest return.
Source: Global Financial Data and own calculations.

It is worth noting that the greater volatility of returns at longer maturities is due to greater duration and not larger volatility of yields at longer maturities, cf. chart 7. In fact, yields at longer maturities have generally been more stable than at shorter maturities. This may reflect the greater exposure of shorter maturities to activist monetary policy, while longer maturities reflect more anchored long term expectations.

If the investment horizon is longer, say 3 years (meaning that investors are indifferent to volatility of returns during the 3 years), then investing in a 3-year bond is (in nominal terms) riskless, as the nominal return is known (assuming it is a zero-coupon loan). The return of longer maturities is still prone to changes in the interest rate level, and is therefore risky. But also the return of shorter maturities, say a 1-year bond, entails some risk, as the investment will have to be reinvested after 1

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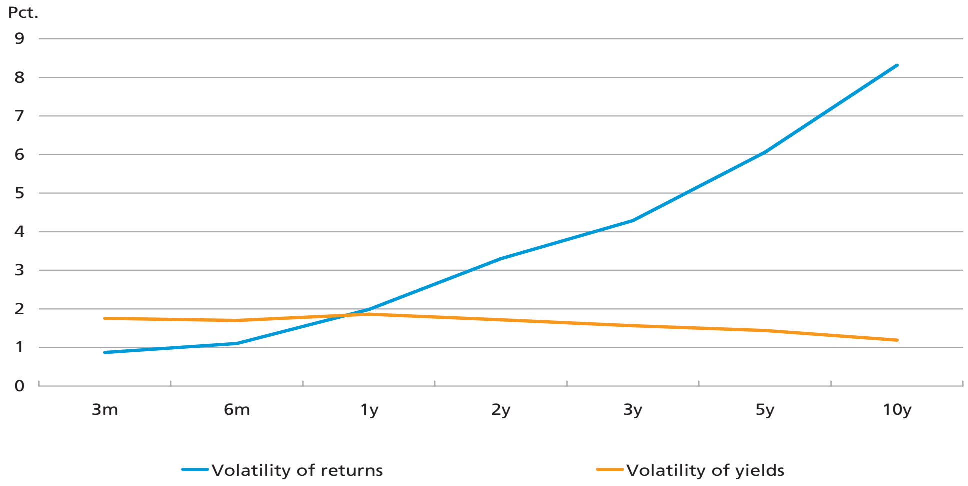
[2] Monthly returns are calculated using Babcock's (1984) formula, whereby the return r is $r = i + (1-(D/h))di$, where D is duration, h is the investment horizon (here 1/12 years) and i is the yield. It is asumed that di is small. In the calculation of D it is assumed that there is one coupon per year, and that the coupon equals the yield. When computing di, roll-down is taken into account such that the monthly change in the e.g. 5-year yield is measured as a synthetic 4-year-and-11-month's yield today minus the 5-year yield one month earlier. The former is computed by linearly interpolation the yield curve. The value of convexity is taken into account by adding to the return the term $½*C*di^2$, where C is convexity. C is approximated by D^2. Together, these simplifying assumptions allow making the return calculation without considering the payment schemes of the bonds in detail, hence speeding the calculations.

year at unknown market conditions. However, this risk is relatively small compared to longer maturities, as the investor is guaranteed that the investment does not fall below par – i.e. that there is a positive return on the investment. Hence, although the maturity that an investor considers risk-free might not be the shortest maturity, the general picture of longer maturities being more risky than shorter maturities is confirmed. However, if a large part of the investor community has long investment horizons, this can have an effect on the yield curve by depressing yields at longer maturities. The so-called preferred habitat theory emphasizes this point.

VOLATILITY OF YIELDS AND RETURNS Chart 7



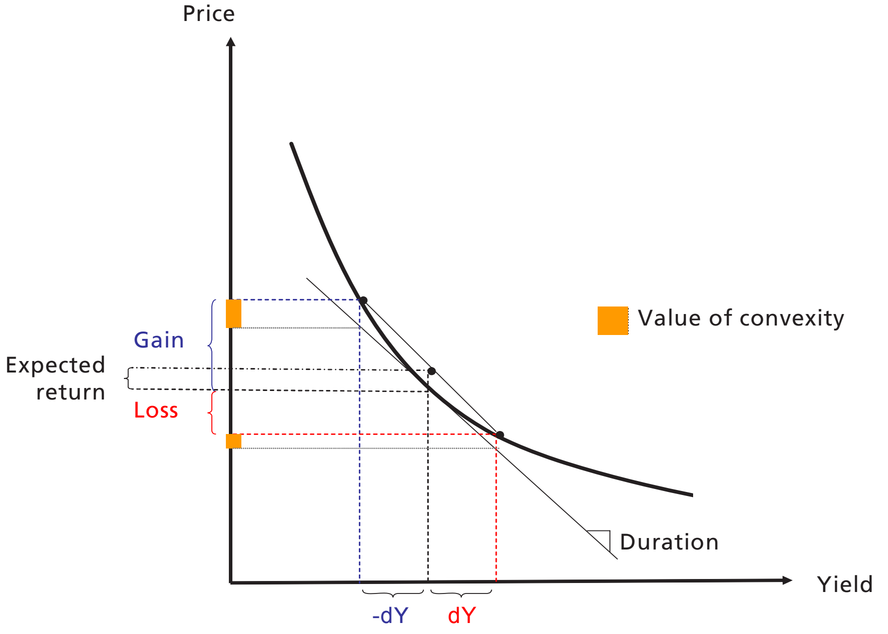
Note: Calculations are made on monthly observations. Annualized.
Source: Global Financial Data and own calculations.

3.1.4. Convexity

The relationship between price and yield on a bond is convex, cf. chart 8. This entails that the loss from an increase in yield (dY) is smaller than the gain from a similar decrease in yield (-dY). This is not captured by duration, which is the slope of the straight line in the chart. If yields are static, convexity has no value. On the other hand, changes in the level of yield (in either direction) give value to convexity. For instance, if investors assign equal probability to dY and –dY, the expected return will be positive, as the average of the two changes in price is positive, even though the yield on average will be unchanged (Jensen's inequality). A more convex bond will increase the expected return (the gain from –dY will be larger and the loss from dY will be smaller). Investors therefore (should) assign value to convexity if the future development in yields is uncertain. The value of convexity is thus a product of the amount of convexity and the expected volatility of yields.

As chart 7 showed, the volatility of yields at longer maturities have been smaller than at shorter maturities. As a rule of thumb, convexity of zero-coupon bonds increases with duration squared (Ilmanen 1996). Therefore longer maturities have much larger convexity than shorter maturities, which dominates in the calculation of the average value of convexity. In isolation this (should) reduce the risk premium demanded for longer maturities. Therefore convexity impacts yields at longer maturities more than yields at shorter maturities, which helps explain that the yield curve is generally concave. However, even though convexity augments the expected return at longer maturities, ceteris paribus, the greater volatility of return still makes longer maturities more risky than shorter maturities.

CONVEXITY OF A BOND Chart 8



3.2. The risk – return trade-off

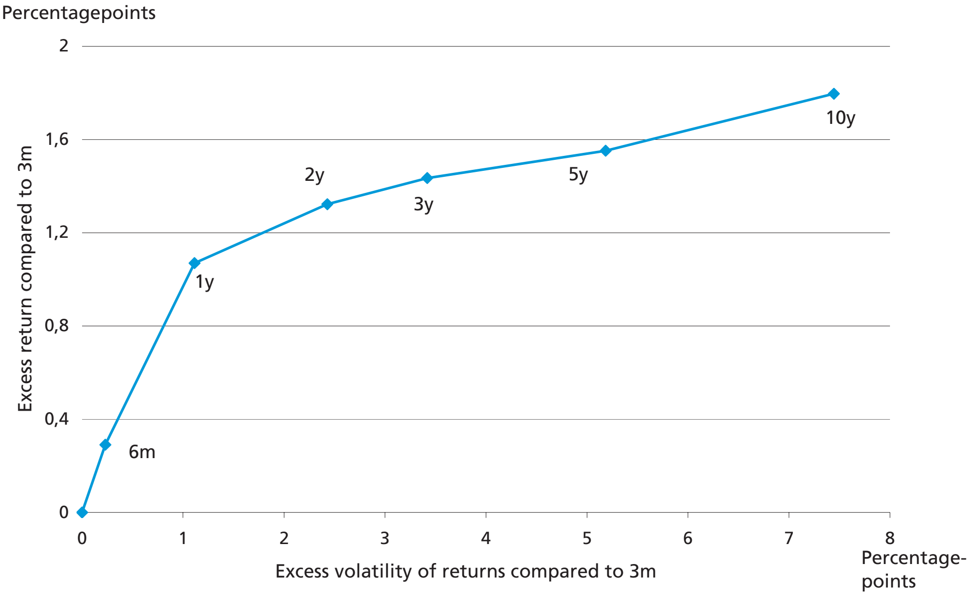
It has been discussed how longer maturities are more risky than shorter maturities due to both uncertainty about realized real returns over longer horizons, as well as due to the higher vulnerability of returns to changes in the level of yields in the short run. This section analyses whether investors have been compensated for the higher risk of longer maturities in the past.

Past risk is measured by the volatility of realised (ex post) returns. We will take the decision to invest in the US bond market for given and focus on relevant trade-offs between US bonds at different maturities. These trade-offs are central for investors like Danmarks Nationalbank. Hence, the volatility of longer maturities will be compared to the volatility of a 3 month bill, and the difference will be denoted excess volatility. Along the same lines, we will compare the return of longer maturities to the return of a 3 month bill, and denote the difference excess return. This focuses the discussion on relative risk and relative return which are the relevant criteria for choosing between maturities.

3.2.1. The whole period

Seen over the whole period, greater risk has on average been rewarded with higher return, cf. chart 9. In other words, risk premiums at longer maturities appear on average to have been higher. For instance, a 3 year bond has on average yielded 1.4 percentage points more than a 3 month bill while having a volatility of return that is 3.4 percentage points higher.

RISK AND RETURN (MONTHLY RETURNS) Chart 9



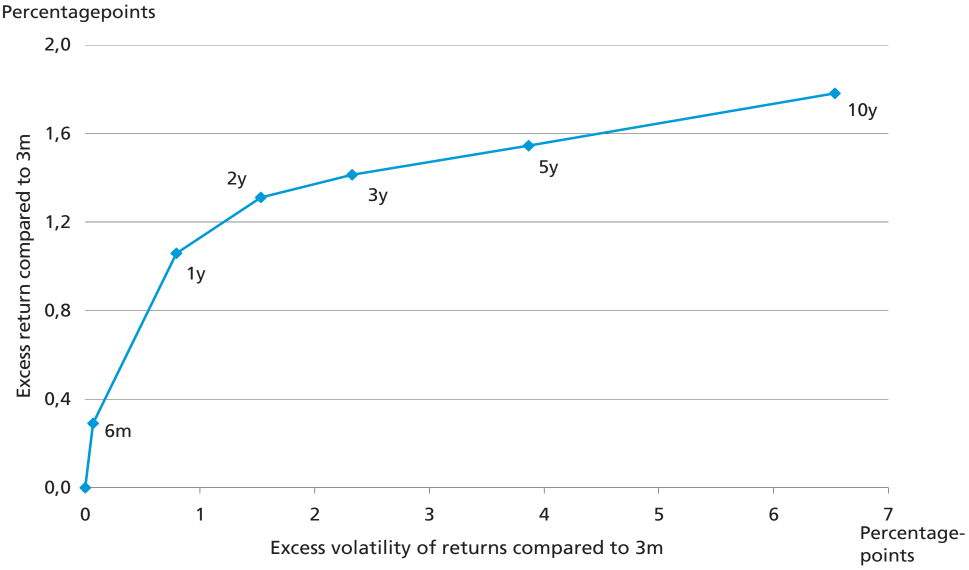
Note: Returns and volatilities are calculated at annual rates. Annual volatilities are found by multiplying the monthly volatility by the square root of 12.
Source: Global Financial Data and own calculations.

It can be seen that the risk-return figure is concave. As Berndsen (2003) also noted, the excess return relative to excess risk seems to be a better trade-off at the shorter end of the curve. For instance, investing in a 10-year bond rather than a 3-year bond increases the risk by 4.0 percentage points while only increasing the excess return by 0.4 percentage points – that is, the return pr. risk unit extra is a lot lower than when increasing the risk by investing in a, say, 3-year bond rather than a 3-month bond.

The concave shape of the risk-return trade-off is related to the concave shape of the yield curve, cf. chart 5. In the long run returns are determined largely by the average yield, if there is no trend in yields. In the observed period the yield curve has (almost) returned to its starting point, resulting in (almost) no bias in the calculations.

The fundamental risk-return trade-off is not altered by an analysis of yearly returns and yearly volatilities, cf. chart 10. Higher risk is rewarded with higher returns, and the risk-return trade-off is concave. However, volatilities are lower reflecting the fact that the chance of extreme returns decreases when the investment horizon is extended. On the other hand, the average returns are the same as at the monthly horizon. In other words, lengthening the investment horizon reduces the volatility of returns, while keeping expected returns constant.

RISK AND RETURN (YEARLY RETURNS) Chart 10

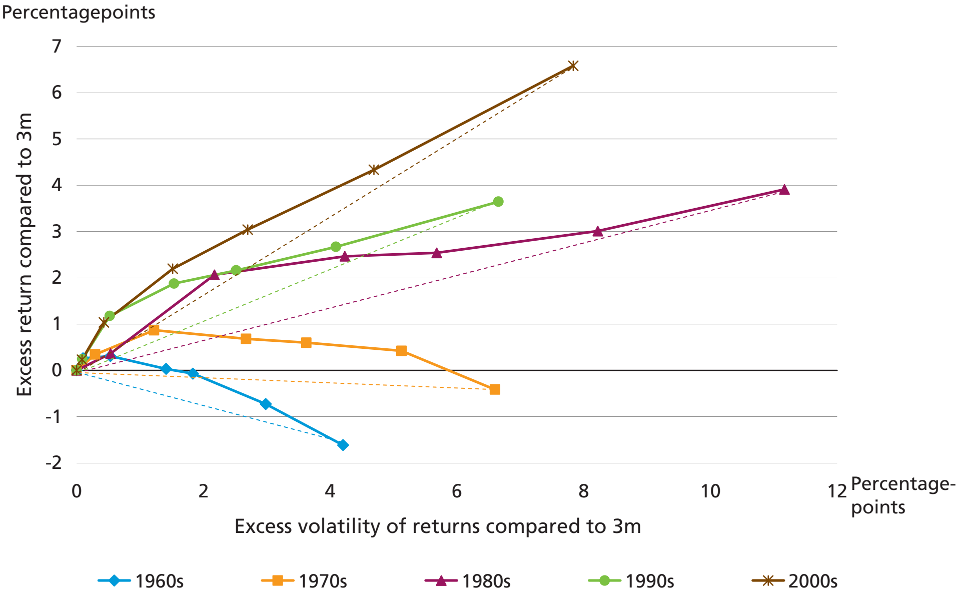


Note: The return for 2005 is based on observations until august.
Source: Global Financial Data and own calculations.

3.2.2. Sub-periods

The risk-return trade-off has changed over the period. In the beginning of the period, higher risk was not matched by higher returns, cf. chart 11.

RISK AND RETURN IN SUBPERIODS Chart 11



Note: Volatility is calculated as the standard deviation of monthly returns. Returns and volatility are calculated at annual rates. The maturities denoted by the dots are at 3m (at origo), 6m, 1y, 2y, 3y, 5y and 10y. The dotted lines are drawn from origo to the coordinate of the 10 year bonds, and the slope of these lines express the information ratio – the excess return pr. excess unit of risk.
Source: Global Financial Data and own calculations.

As the general level of interest rates rose in the 60s and 70s, bonds with longer maturities experienced negative excess returns on average while having excess volatility of returns at the same time. In other words, at longer maturities capital losses due to rising yields dominated the generally higher yields due to the positive slope of the yield curve. As explained earlier, the development was in large part due to *unexpected* inflation. Hence, during this period investors are bound to have expected higher excess returns than were realised.

This changed in the 80s where volatility increased, but at the same time falling interest rates and the positive sloping yield curve compensated the holders of longer maturity bonds. The excess volatility decreased in the 90s, but excess return remained elevated. Since 2000 excess volatility has increased along with the excess returns. The increase in excess returns was both due to falling yields and a relative steep yield curve. In this period, investors have probably been rewarded with higher excess returns than expected, as inflation expectations repeatedly turned out to be too high.

The slope of a line from origo to the coordinate of a 10-year bond indicates the extra return from taken one unit of extra risk in the 10-year segment (the so-called information ratio). This number has increased every sub-period, from a negative value in the 1960s (-1,6/4,2 = -0,4) to a very positive value since 2000 (7,8/6,6 = 1,2). That is, investors have increasingly been rewarded more for taken on a unit of risk in the 10-year segment. A similar picture is seen at other maturities. The outlook for the information ratio is laid out below.

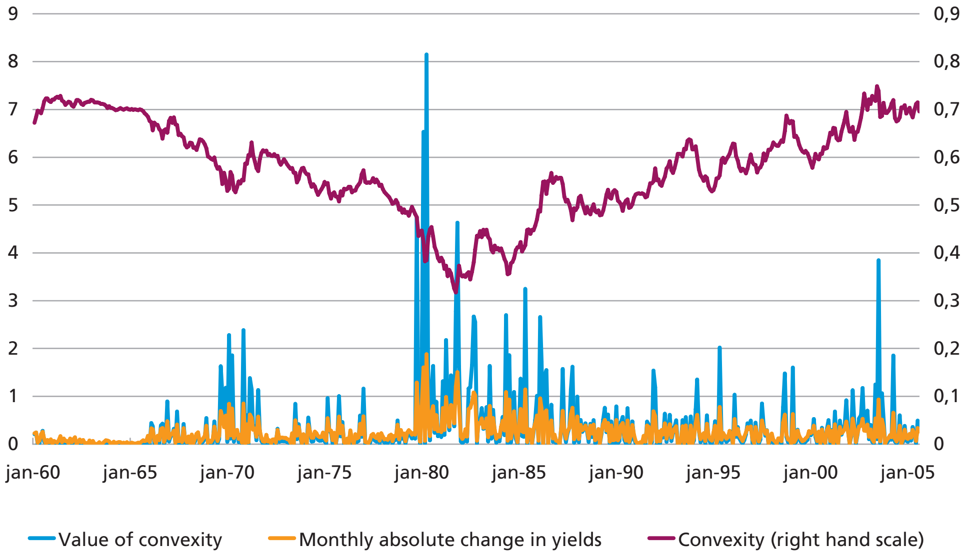
The chart illustrates, that longer maturities are not only more risky on a month to month basis, but that a rising yield environment can depress the excess returns for prolonged periods of time. It points to the danger of relying on past returns for calculating the future risk-return trade-off and to the risks associated with regime-shifts. This adds another '"layer" to the risk, namely the uncertainty regarding the distribution of returns in the future. In the 1960s and 1970s one can say there was a change to a regime associated with higher volatility and small returns, which has since shifted back. These considerations are relevant for the simulations below.

The greater volatility of returns in the 1980s, visible in chart 11, was in large part due to volatile interest rates during that period, cf. chart 12. This volatility has since fallen to a lower level, although a number of shocks have created some spikes since 2000, pushing the volatility of returns above the levels seen in the 1990s. The shocks to the economy experienced since 2000 are different from the shocks in the 1970s and 80s, however, as they have been due to more isolated, short-term events, and they have not affected long-term inflation expectations, cf. chart 2.

Convexity increases in maturity and the value of convexity depends on the level of convexity and the volatility of interest rates, as explained earlier. Typically higher yields reduce the convexity – in chart 8 it can be seen how the convexity gets smaller moving to the right. Therefore convexity fell from the beginning of the period until the beginning of the 1980s, and has since increased again. However, the value of convexity was still higher in the middle of the period due to the volatility of yields. The value of convexity is generally high when volatility, and hence risk, is high – that is, it moderates risk at longer maturities, but longer maturities are still riskier relative to shorter maturities due to their larger duration.

CONVEXITY AND VOLATILITY OF 10-YEAR BONDS

Chart 12



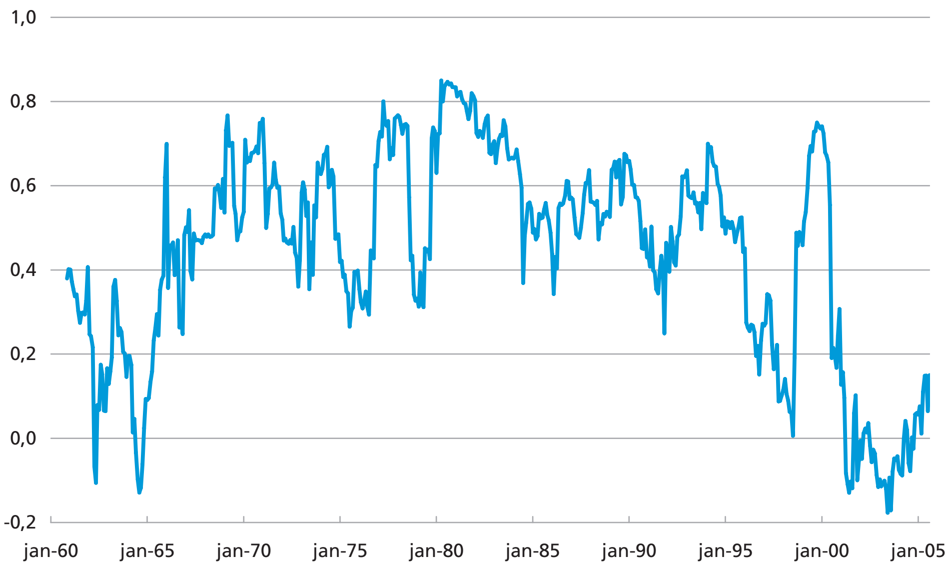
Note: The calculation of value of convexity is described in footnote 3, page 12.
Source: Global Financial Data and own calculations.

3.2.3. Diversification

Up to this point investments at different maturities have been considered individually. However, an important aspect of risk is diversification – the smaller the correlation between the return of two assets, the larger is the diversification from holding both assets since the volatility of aggregate returns will be smaller. Although the returns of different maturities are generally highly correlated, the extent of the correlation has changed during the period, exemplified by the correlation between 3-month and 10-year yields in chart 13.

The correlations increased in the beginning of the period, stayed at a high level in the middle of the period, before declining again. This means that the benefit from diversification from spreading ones position along the yield curve has been higher since 2000 than in the 1980s, for instance. One possible explanation for the development is the evolution of inflation. In the middle of the period inflation was the prime driver of interest rates causing a close correlation between different maturities. Lately, inflation expectations have become more anchored, which limits the impact of inflation on longer maturities. These might be increasingly dominated by supply (government deficits) and demand (pension funds) effects. On the other hand, the short end of the curve is driven to a large extent by expectations of monetary policy, thus making it vulnerable to changes in short term inflation expectations given the Fed's focus on price stability.

CORRELATION BETWEEN 3-MONTH AND 10-YEAR YIELD CHANGES Chart 13



Note: Correlations are calculated over 12 month's monthly changes in yields at a rolling window.
Source: Global FInancial Data and own calculations.

Generally correlations between different maturities have been substantially less than one, and spreading a position along the yield curve thus reduces risk by increasing diversification. However, the magnitudes of the correlations have changed and the expected correlations are important when evaluating the expected risk of a position.

4. Principal components analysis and simulations

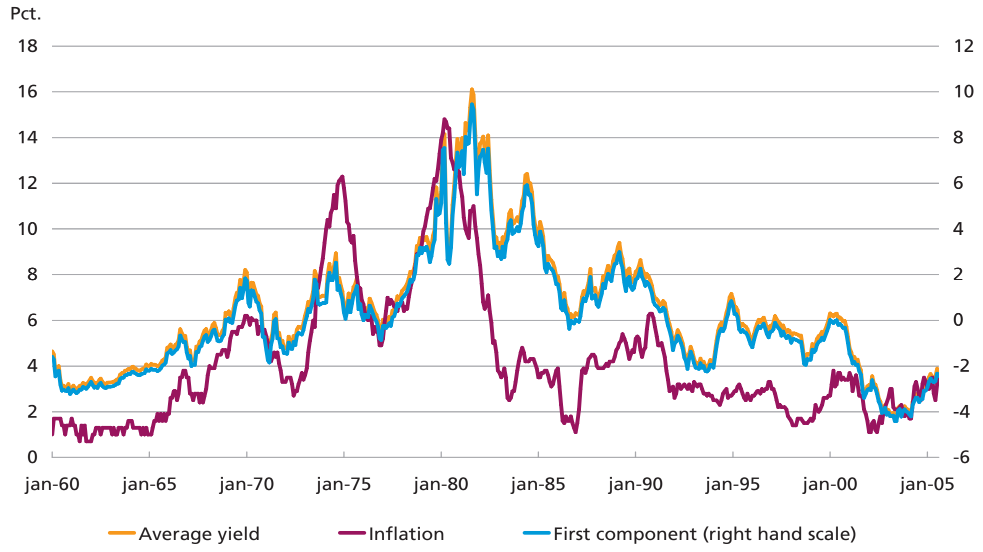
4.1. Introduction

The main purpose of this section is to lay out a framework for simulation of the future risk-return trade-off. The basic idea is to summarize the historic variation in the yield curve by a few (two) principal components, which by construction are orthogonal (that is, uncorrelated). Hence, the system is reduced from seven time series of yield observations with different time to maturity, to a system of two constructed principal components, and a corresponding transition matrix that allows a reconstruction of the original data (except for the variation not captured by the two principal components). Two components explain more than 99 pct. of the variation in all the yields. In other words, only limited information is lost by focusing on the first two principal components. The details of the analysis are given in Appendix A.

The principal components analysis forms a simple and flexible framework for simulating yields. First simple autoregressive time series models are used to describe the dynamics of the principal components. Then with these models, a large number of trajectories for the two components are simulated, and used (via the transition matrix) to construct simulated realizations of the yield curve.

Before computing the principal components, the individual time series of yields are standardized, i.e. the yields are demeaned and divided by their empirical standard deviations. Hence, after simulation and transformation, the simulated yields are "destandardized" by multiplying with their standard deviation and adding their means. This provides flexibility, as the mean and volatility of the simulated yields are under direct control. For instance, if one has certain beliefs about how future interest rates might deviate from the historical pattern, such scenarios can be implemented by adjusting either the estimated parameters in the processes for the principal components (thereby affecting yields at all maturities), or by adjusting the means and volatilities of the individual yields.

AVERAGE YIELDS AND FIRST COMPONENT Chart 14

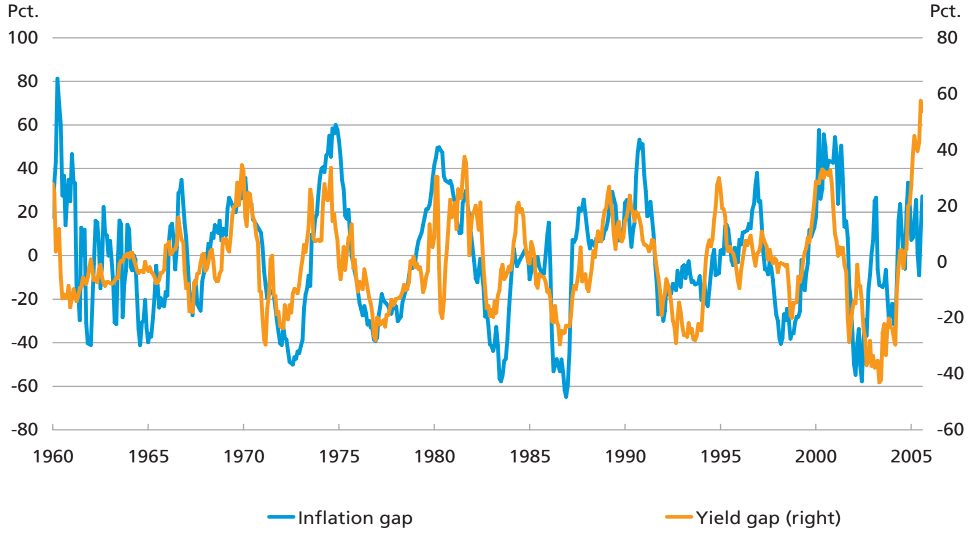


Source: Global Financial Data and own calculations.

The novel feature of the implementation is that the principal components are related to simple macroeconomic measures of the state of the economy: Inflation and the real GDP-gap. It is a robust result that the first principal component captures information about the level of interest rates, which is illustrated in chart 14. The chart leads to the same conclusion as in chart 2: Apart from the periods with unexpected high or unexpected low inflation, c.f. section 2, the first principal component is reasonably well described by the level of inflation. If one (tentatively) corrects for these periods with unexpected high or unexpected low inflation the fit is even tighter. E.g. the interest rate less its HP-filtered trend (the interest rate gap) follows the corresponding inflation gap rather closely, cf. chart 15.

INFLATION GAP AND YIELD GAP

Chart 15

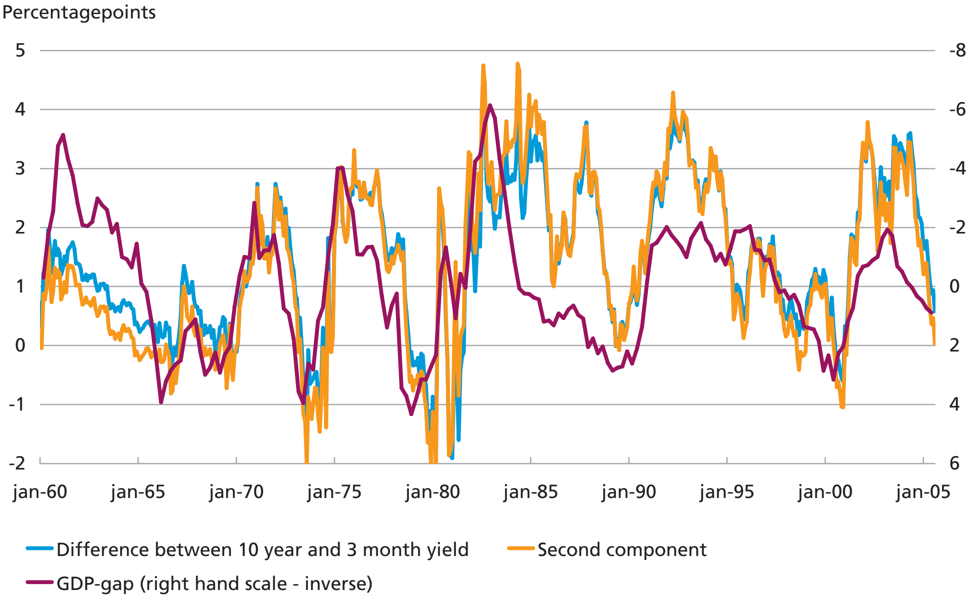


Note: Inflation-gap and yield-gap is measured as the percentage deviation of actual real GDP from its HP-filter trend.
Source: Global Financial Data and own calculations.

It is also a robust result that the second component captures the slope of the yield curve. The present analysis overwhelmingly confirms that finding, cf. chart 16.

SLOPE OF YIELD CURVE , SECOND COMPONENT AND GDP-GAP

Chart 16



Note: GDP-gap is measured as the percentage deviation of actual real GDP from its HP-filter trend.
Source: Global Financial Data and own calculations.

Thus, the second component captures important information about the US business cycle, as the slope of the yield curve is intimately connected to the US real GDP-gap. Simple regression confirms statistical significant relationships between the principal components and the macro variables.

4.2. Construction of scenarios to be simulated

The construction of scenarios for the future can be based on beliefs about the future overall development in inflation and the GDP-gap. As an illustration of this approach, one might argue that the present low interest rates in the US could be a permanent phenomenon. Such arguments are presented in e.g. Andersen, Hydeskov and Sand (2005). It can also be argued, that the slope of the yield curve will be flatter, as long term inflation expectations have only recently become anchored, cf. the discussion above. At the same time inflation volatility has decreased considerably since the times of high inflation in the 70's and 80's, and has since become more stable. If these findings are results of structural changes, such as higher emphasis on inflation-fighting in the economic policies, then one would expect reasonably low and stable levels of interest rates in the future, particularly in the long end of the yield curve. This suggests that the volatility of the simulated first component might be a little lower in the future compared to the (at least the far) past. In addition, one might adjust downward the level and volatility of the long term interest rates (5- and 10 year) in the simulations, to reflect the new nominal anchor and well anchored long term inflation expectations.

On the other hand, the yields in short end will probably be at least as volatile (but have a lower level) as in the past, since the monetary authorities use the short interest rate to keep inflation expectations anchored, and smooth the business cycle. As the slope of the yield curve is well described by the business cycle, then unless one has strong views about lower business cycle volatility in the future, there are no immediate reasons for altering the volatility of the second component or level and volatility of the short-end yields directly. Hence we leave the process for the second component unchanged with the parameters estimated from data.

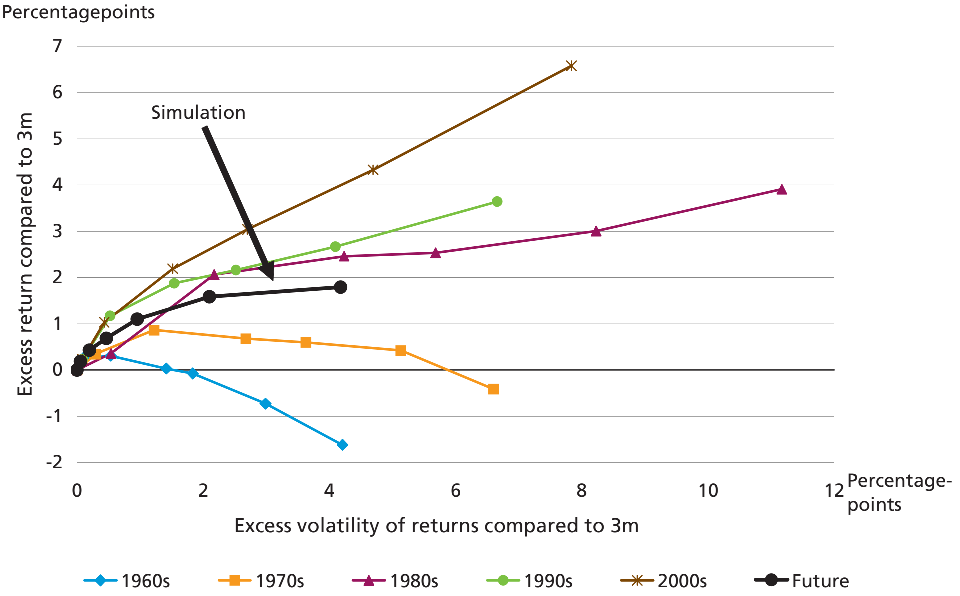
4.3. Results

The simulations are based on a principal components analysis of the period 1993.1 – 2005.8. This choice reflects the need for sufficient data to obtain reliable estimates, and the need for focusing on a period where the macro-economic environment in terms of inflation and business fluctuations arguable is closer to a plausible pattern for the future than a period including the 1970's and 1980's. With this picture of the future, 10,000 yield curves are simulated, and the mean monthly excess return and excess volatility are calculated. The result is presented in chart 17.

The smaller volatility and the more anchored long term interest rates relative to the past do not qualitatively alter the trade-off between return and risk: Longer duration enhances expected returns. But both the reward from longer duration and the risk of longer duration have decreased. So this scenario reflects that US government bonds simply is a less risky asset when inflation is under control. Having said this, the reward pr. unit risk is lower in the 10-year segment than in the 1990s and

2000s – the information ratio is 0.4, compared to 0.6 and 1.2 in the earlier periods. Excess return of longer maturities has declined due to the downward adjustment of the yield curve at the long end. Also the lower volatility of yields, particularly at the long end, has reduced the value of convexity.

SIMULATED RISK-REWARD TRADE-OFF Chart 17



Source: Global Financial Data and own calculations.

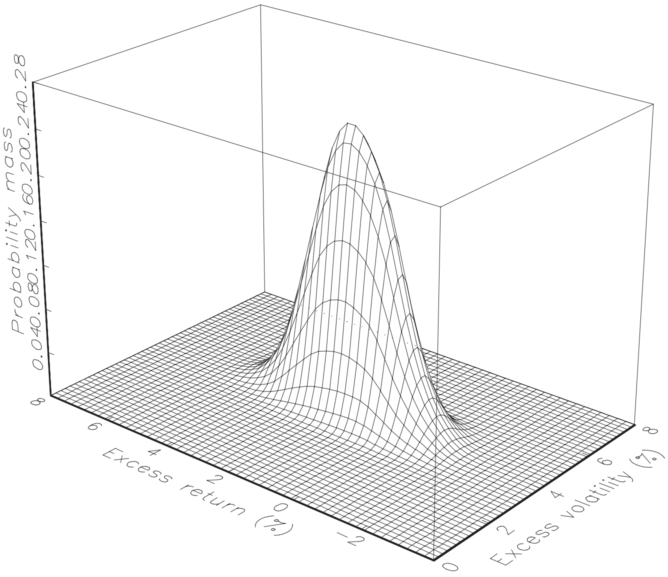
This illustrates how arguments about the fundamental macro-economy can be used to construct scenarios for the future yield curve that are consistent with the historical relation to the macro-economy, and ones own beliefs about future inflation and business cycle development. It is arguably easier to formulate views about the range of plausible future values for a few core macro-variables, than developing strong views about the speed of mean-reversion in the conditional volatility of an N-factor term structure model (we made this up, but you get the picture). Pure time series term structure models can be interesting for a number of reasons, but often their parameters are difficult to interpret in a pragmatic context, such as the present one.

Of course, it is possible to think of less optimistic scenarios for the future. If central banks loose credibility and inflation expectations and/or inflation uncertainty increase dramatically, yields will increase accordingly. In such a scenario the risk-reward trade-off might duplicate that of the 1960's, where a prolonged period with an increasing trend in yields gave rise to negative excess returns. On the other hand, the regime from the 1980s and onwards cannot be repeated, as the yields are already at low levels. Hence, the "regime risk" is to the downside (with respect to returns).

4.4. Uncertainty about the risk-reward trade-off

The framework also allows an assessment of how uncertain the risk-reward trade-off itself is under the assumed regime. This is illustrated here by evaluating the uncertainty about the excess volatility and excess return in the 10-year segment at a 10 year horizon. Simulation of 5000 trajectories over a 10-year period (120 observations in each trajectory) gives 5000 simulated observations of excess return and excess volatility. A joint distribution of these points is then estimated by non-parametric kernel-techniques (see Appendix B for details). The result is given in chart 18.

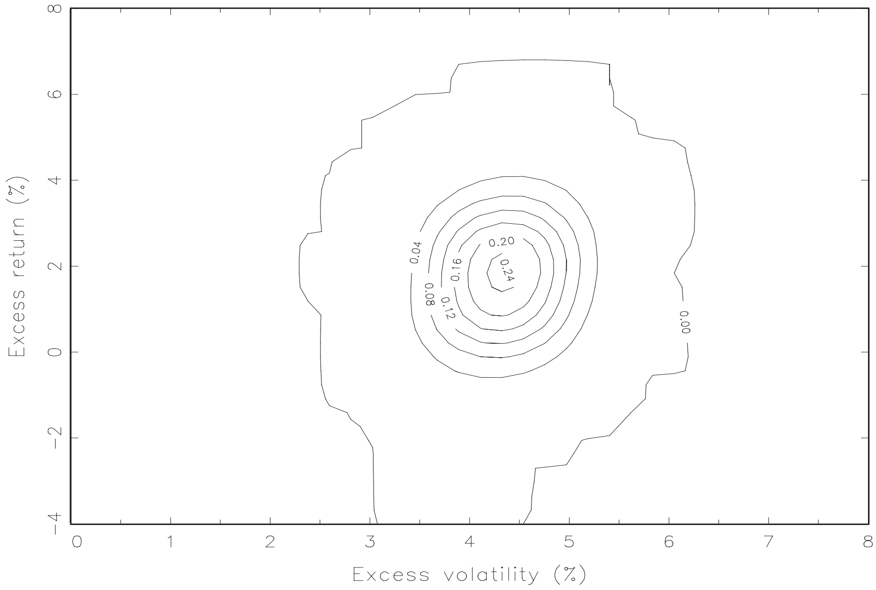
JOINT DISTRIBUTION OF EXCESS RETURN AND EXCESS VOLATILITY IN 10-YEAR SEGMENT Chart 18



Source: Global Financial Data and own calculations.

The 10-year point in chart 17 corresponds approximately to the top of the distribution, which reflects the most likely outcome given the model. Moreover, most of the probability mass lies in the range of 3.1 and 5.2 percent for excess volatility, and 0 and 4 percent for the excess return. Hence, the probability of negative excess returns is low – but not zero - under the assumptions of the model. Furthermore, the probability of extreme values of excess return and excess volatility happening at the same time is very low (horizontal slices of the joint distribution are circular). This is more visible in the contour diagram, Chart 19.

CONTOUR DIAGRAM OF JOINT PROBABILITY FUNCTION Chart 19

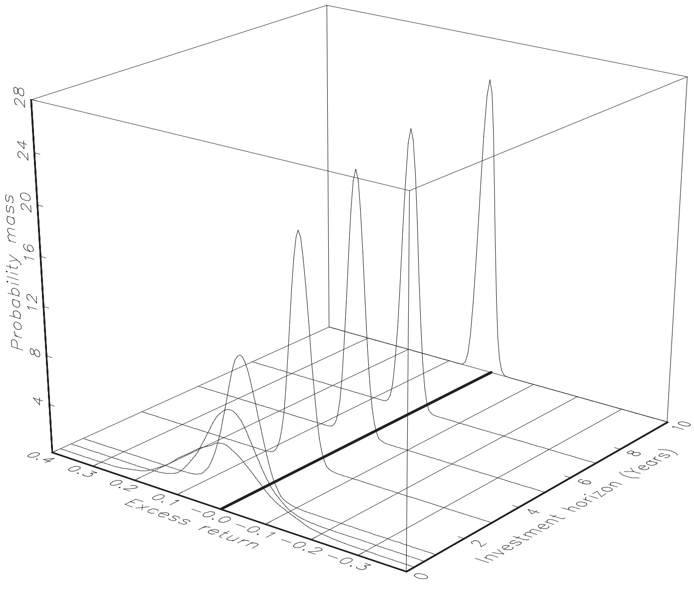


Source: Global Financial Data and own calculations.

4.5. Other investment horizons

The 10-year horizon for the simulations was chosen to allow comparison with earlier 10-year periods, cf. chart 17, and illustrates the uncertainty about the risk-reward trade-off at long horizons.

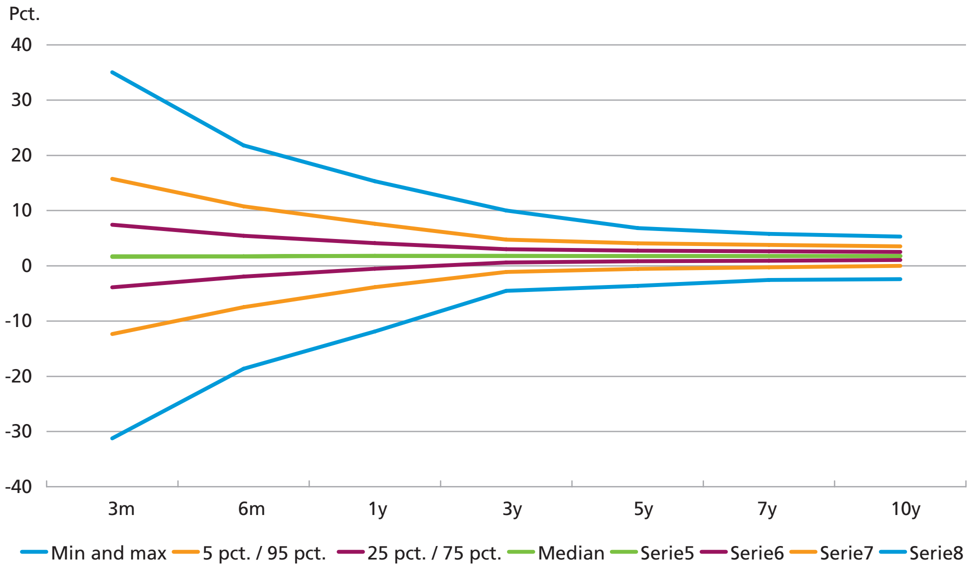
DISTRIBUTION OF RETURNS AT DIFFERENT HORIZONS Chart 20



Source: Global Financial Data and own calculations.

However, the volatility of the return of a 10-year bond is much more volatile over shorter horizons. Chart 20 and 21 illustrates the distribution at horizons from 3 months to 10 years of (excess) returns from buying a 10-year bond, holding it for one month and sell it, and buying a new 10-year bond to hold for one month etc. The 10-year point thus corresponds to the analysis in chart 18 and 19.

DISTRIBUTION OF RETURNS AT DIFFERENT HORIZONS Chart 21



Source: Global Financial Data and own calculations.

The return at short horizons is dominated by the capital loss/gain following the immediate yield changes. As the predictable component in yield changes is small, there is a considerable risk of negative returns at the shorter horizons. However, as the horizon increases the distributions narrow in, since positive and negative returns cancel out and eventually the level of yields dominate the return.

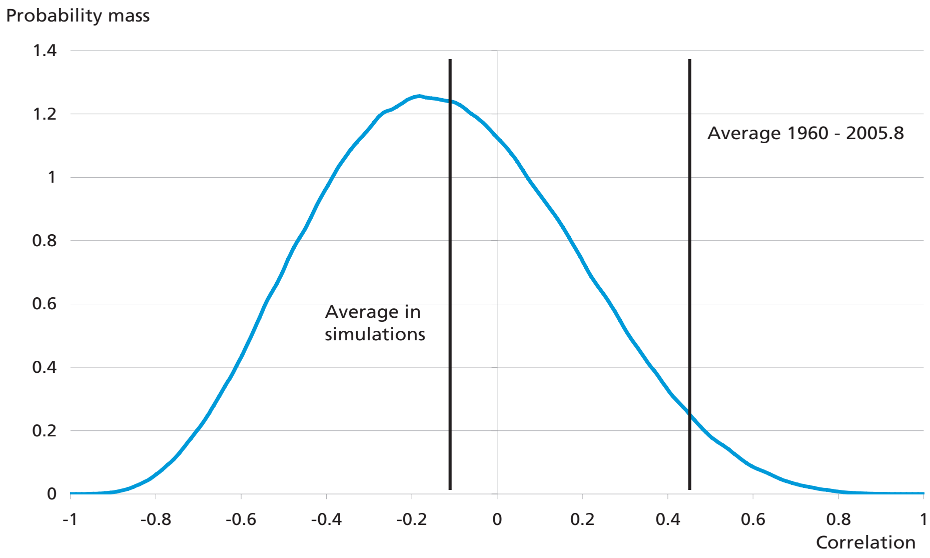
4.6. Correlations between short and long term yields

For each trajectory of the simulated yield curve the correlation between the 3-month yield and the 10-year yield is calculated. The correlations are computed in the same way as the correlations in the historical period (chart 13). The estimated distribution of the correlations is shown in chart 22.

Three features stand out from the distribution of simulated correlations: They are very volatile with the majority of the probability mass in the range from -0.7 to 0.5, they are skewed to the right, and the mean simulated correlation is negative and lower than the historical average of correlations, cf. also chart 13. It is worth noting that the historical correlations also show a large degree of volatility and decreased towards the end of the period.

The simulations are performed based on a principal components analysis of the period since 1993 (although with some adjusted parameters) – a period where the correlations are lower than for the whole period. The high probability of negative correlations in the simulations arises because the second component (i.e. the slope) has been given more weight in the simulations relative to the first component (the level). Positive shocks to the first component lift both short term and long term yields. However, a positive shock to the second component will lower the short yield relative to long – enough to more often cause negative correlations than positive.

DISTRIBTION OF CORRELATIONS BETWEEN 3M AND 10Y IN THE SIMULATIONS Chart 22



Source: Global Financial Data and own calculations.

The volatile correlations mean that conclusions about future diversification gains must be drawn with care. There is some probability of similar correlations as seen historically. However, the likelihood of lower correlations in the future than in the past is high, hence, gains from diversification are likely greater in the future than the pure historical experience suggests.

5. Conclusion

Simulations are always based on assumptions, either implicitly or explicitly. This paper has tried to highlight the advantages of relying on explicit assumptions when forecasting future risk and return rather than uncritically using past data from an arbitrary period.

The risk-reward trade-off on the US bond market has shown considerable variation over past 45 years, from two decades with negative excess returns to the more recent past with healthy excess returns from enhancing duration. Simulations of yield curves based on a plausible scenario for the future inflation and GDP-gap behaviour implies that the bond market will reward extra (duration) risk. However, the volatility of returns over the next decade or so is also likely to be lower than seen in the past. This reflects that nominal bonds are less risky assets when inflation is under control.

6. Literature

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7. Appendiks A: The principal components analysis

Principal components (PC) analysis amounts to an eigenvalue analysis of the variance-covariance matrix of the time series under consideration. The time series analysed in this paper is monthly observations of 3-months, 6-months, 1-year, 2-year, 3-year, 5-year and 10-year annualized yields taken from the Global Financial Database, e.g. 7 time series.

The time series is standardized before the PC analysis, e.g. for each time series the empirical mean is subtracted and the result is divided by the empirical standard deviation. Hence, if $x_{3m,t}, x_{6m,t}, \ldots, x_{10y,t}$, t = 1,2,…,T are the original time series of yields then

$$y_{it} = \frac{x_{it} - \mu_i}{\sigma_i}, \quad i = 3m, 6m, \ldots, 10y, \quad t = 1, 2, \ldots, T$$

are the standardized yields with obvious notation. Y denotes the T×7 matrix with the standardised interest rates as columns.

Ω denotes the 7×7 variance-covariance matrix of Y and λ is a 7×2 matrix where the first column is the eigenvector associated with the largest eigenvalue of Ω, and the second column is the eigenvector associated with the second-largest eigenvalue of Ω (we focus on the two largest eigenvalues only, as the two associated principal components accounts for more than 99 pct. of the variation in the y's). The T×2 matrix PCA with columns containing the two principal components are then given by

$$PCA' = \lambda' Y'.$$

The original data can be computed (except for the information lost by considering two principal components instead of the full dataset) as

$$Y' = (\lambda')^{-1} PCA'. \tag{1}$$

The principle in the simulations is to fit the PCA's with a simple first-order autoregressive model. Using those models the PCA's are simulated, and the simulated yields are deducted from destandardising the Y's computed from (1).

7.1. Values of parameters used in simulations

The μ's and σ's for the period 1993.1 – 2005.8 are given in Table A1, and the adjusted values used in the simulation are given in Table A2.

Means and standard deviations 1993.1 – 2005.8					Table A1	
	x_{3m}	x_{6m}	x_{1y}	x_{2y}	x_{3y}	x_{5y}	x_{10y}
μ	0.0374	0.0385	0.0418	0.0454	0.0476	0.0511	0.0551
σ	0.0164	0.0165	0.0172	0.0165	0.0153	0.0131	0.0106

Adjusted values used in simulation					Table A2	
	x_{3m}	x_{6m}	x_{1y}	x_{2y}	x_{3y}	x_{5y}	x_{10y}
μ	0.0374	0.0385	0.0418	0.0454	0.0476	0.0501	0.0521
σ	0.0164	0.0165	0.0172	0.0165	0.0153	0.0111	0.0096

The results from the PC analysis on Y from 1993.1 – 2005.8 gives the following transition matrix:

$$\lambda = \begin{bmatrix} -0.372 & -0.463 \\ -0.378 & -0.382 \\ -0.385 & -0.227 \\ -0.388 & -0.029 \\ -0.388 & 0.112 \\ -0.380 & 0.358 \\ -0.353 & 0.668 \end{bmatrix}.$$

The autoregressive parameter is 0.99 and standard deviation of the residuals 0.42 for the first principal component. The corresponding values for the second principal component are 0.92 and 0.23. These simple first order autoregressive models pass all standard misspecification tests (for the period 1993 onwards). The standard deviation of the residuals from the modelling of the first principal component is set to 0.32 in the simulations, to reflect more stable inflation rates in the considered scenario, cf. the main text.

8. Appendiks B: Non-parametric kernel-estimation

The literature on kernel density estimation and nonparametric regression is very large. Useful references include Wand and Jones (1995) and Silverman (1986).

The nonparametric kernel estimate of the density p(z) of returns at any point z is given by

$$\hat{p}(z) = \frac{1}{Th}\sum_{i=1}^{T} K\left(\frac{z-r_i}{h}\right)$$

where K is the kernel function, T is the sample size (here the number of monte carlo trajectories), r_i is i'th return and h is the bandwidth. The kernel density estimate can be viewed as a smooth histogram of the bond returns. Kernel estimates are generally consistent, but biased, and the choice of the bandwidth value h represents a 'classic' trade-off between the biasedness of the estimator on the one hand, and the variance of the estimator on the other. Fortunately, the choice of kernel function is less important in that respect.

For the estimations in this paper, the bandwidths were chosen according to the 'over-smoothed' bandwidth selection procedure described in Wand and Jones (1995). This amounts to calculating an upper bound for an asymptotically mean integrated standard error optimal bandwidth choice and then successively lowering the bandwidth value until a reasonable estimate is reached based on visual inspections of plots of the estimated density functions. This procedure was chosen for its simplicity, but more advanced and computationally intensive bandwidth selection routines are available.

For the nonparametric estimate of the joint density of r_i and σ_i at any point (z, v) the kernel estimator

$$\hat{p}(z,v) = \frac{1}{Th^2}\sum_{i=1}^{T} K\left(\frac{z-r_i}{h}\right)K\left(\frac{v-\sigma_i}{h}\right)$$

was used.

The Epanechnikov kernel function

$$K(x) = ¾(1-x^2) \text{ for } |x| \le 1, 0 \text{ elsewhere},$$

was used in all estimations.