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2012 • 77

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Danmarks Nationalbank

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Januar 2012

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ISSN (trykt/print) 1602-1185

ISSN (online) 1602-1193

Resumé

I kølvandet på den finansielle krise er der opstået fornyet fokus på potentielle asymmetrier i forholdet mellem aktiemarkedet og pengepolitikken, og på de mulige problemer disse kan medføre. Nyere empiriske studier tyder på, at den amerikanske centralbank netop førte en asymmetrisk rentepolitik overfor aktiemarkedet i perioden op til krisen. I denne artikel undersøges effekterne af en sådan asymmetrisk politik i en dynamisk, stokastisk, generel ligevægtsmodel (DSGE-model). Den asymmetriske politik medfører en vigtig ikke-linearitet i modellen: Højkonjunkturer vil blive forstærket, mens recessioner vil blive dæmpet som følge af politikken. Det undersøges endvidere, i hvilket omfang en asymmetrisk pengepolitik kan ses som en centralbanks forsøg på at udligne eller korrigere for allerede eksisterende asymmetrier i den måde, hvorpå aktiepriser påvirker makroøkonomien.

Asymmetric Monetary Policy Towards the Stock Market: A DSGE Approach*

Søren Hove Ravn†

January 2012

Abstract

In the aftermath of the financial crisis, it has been argued that a guideline for future policy should be to take the 'a' out of 'asymmetry' in the way monetary policy deals with asset price movements. Recent empirical evidence has suggested that the Federal Reserve may have followed an asymmetric policy towards the stock market in the pre-crisis period. The present paper studies the effects of such a policy in a DSGE model. The asymmetric policy rule introduces an important non-linearity into the model: Booms in output and inflation will tend to be amplified, while recessions will be dampened. I further investigate to what extent an asymmetric stock price reaction could be motivated by the desire of policymakers to correct for inherent asymmetries in the way stock price movements affect the macroeconomy.

Keywords: Asymmetries, Monetary Policy, Asset Prices, DSGE Modelling.

JEL classification: E13, E32, E44, E52, E58.

*The author wishes to thank Henrik Jensen for competent and inspiring guidance, and Niels Blomquist, Martin Ellison, Adalheidur Gudlaugsdottir, Antonio Mele, Tommaso Monacelli, Gert Peersman, Jesper Rangvid, Emiliano Santoro, and Morten Spange for useful comments and discussions. Also thanks to seminar participants at the University of Oxford, Danmarks Nationalbank. All remaining errors are my own. The views expressed are those of the author, and do not necessarily correspond to those of Danmarks Nationalbank.

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1 Introduction

While the recent financial and economic crisis does not invalidate everything we have learned about macroeconomics since 1936, as Barro (2009) eloquently puts it, it has led economists to reconsider some ideas that once were common sense. As one example, the crisis has led to a revival of the debate about the role of asset prices in monetary policy; see Kuttner (2011) for an overview.[1] Despite some enduring disagreement, a certain degree of consensus had been reached before the crisis, according to which central banks should not lean against asset price movements; if not for other reasons then because of the practical problems in doing so. Instead, they should stand ready to cut the interest rate in response to drops in asset prices.[2] The aftermath of the crisis has witnessed an emerging appreciation and critique of an inherent asymmetry in this approach to monetary policy (see White (2009), Mishkin (2010), and Issing (2011), among others). In the words of Stark (2011), the consensus implied that '*monetary policy should react to asset price busts; not to asset price booms*'. Issing (2011) points to the risk that such a policy might lead to moral hazard problems by covering part of the downside risk faced by investors in the stock market. Two recent studies seem to lend empirical support to the existence of an asymmetric monetary policy towards the stock market in the US before the crisis. Ravn (2011) finds that during the period 1998-2008, a 5 % drop in the S&P 500 index increased the probability of a subsequent 25 basis point interest rate cut by 33 %. On the other hand, he finds no significant policy reaction to stock price increases. Hall (2011) studies a Taylor rule augmented with (lagged) stock price deflation. For the period 1987-2008, she finds that stock price deflation leads to a highly significant cut in the interest rate, and that the inclusion of stock price deflation improves the fit of the Taylor rule.

I contribute to this recent debate by examining the effects of an asymmetric monetary policy in general equilibrium. I build a Dynamic Stochastic General Equilibrium (DSGE) model with an explicit role for asset prices through the financial accelerator of Bernanke *et al.* (1999). I then allow the central bank to follow a monetary policy rule with an asymmetric reaction to stock prices. This introduces an important discontinuity into the model that cannot be 'log-linearized away'. As a result, it is not possible to solve the model using standard techniques. Instead, I apply a numerical solution method which exploits the piecewise linearity of the model. Essentially, the model consists of two linear systems; one when stock prices are increasing (or constant), and another when they are decreasing. I construct a shooting algorithm to detect the switching points between these systems in

[1] This debate goes back at least to Bernanke and Gertler (1999, 2001), who argue that monetary policy should not react to asset prices *per se*. This has been supported by, among others, Gilchrist and Leahy (2002), Tetlow (2005), and Faia and Monacelli (2007), as well as in speeches by leading Federal Reserve officials (Kohn, 2006; Mishkin, 2008). In contrast, Cecchetti *et al.* (2000) find that the optimal monetary policy rule does include a reaction to the stock market. This position has received support from Bordo and Jeanne (2002), Borio and White (2003), and recently Pavasuthipaisit (2010) and Leduc and Natal (2011).

[2] This view has been coined the 'pre-crisis consensus' by Bini Smaghi (2009), and the 'Jackson Hole consensus' by Issing (2009).

order to solve the model. In this sense, I make a methodological contribution to the sparse literature on endogenous regime switching in monetary policy initiated by Davig and Leeper (2006). The solution method is similar to the one used by Bodenstein *et al.* (2009) to deal with the zero lower bound on interest rates, which in turn builds on work by Eggertson and Woodford (2003) and Christiano (2004).

The analysis uncovers some interesting implications of the asymmetric policy. By reacting only to stock price drops, the central bank induces an outcome where booms in output and inflation are amplified, while recessions are dampened. In other words, the asymmetric policy translates into an asymmetric business cycle. I briefly relate this finding to the existing literature on asymmetric business cycles. In addition, the asymmetric policy gives rise to what I call an *anticipation boom* in asset prices. In the wake of an expansionary shock, the asset price jumps up. It turns out that this jump is larger than in a model with no reaction to stock price changes, despite the fact that in both cases, the actual policy reaction to stock prices is zero during the asset price boom. The anticipation boom, which measures the *additional* rise in asset prices when the asymmetric policy is introduced, can be attributed to forward-looking agents anticipating that whenever stock prices start falling, the central bank will cut the interest rate. This implicit, partial insurance against asset price drops amplifies the rise in asset prices immediately after the shock.

If the asymmetric policy reaction to stock prices is of the magnitude found in the recent empirical studies, these effects are quantitatively quite small. In the literature, a remarkable divergence exists between the magnitude of the reaction to asset prices found in empirical studies, which is often quite small, and the values used in theoretical investigations, which are usually a lot larger. To bridge this gap, I therefore also employ a value of the reaction parameter which is more in accordance with the values in other theoretical contributions. When this is done, the above effects are sizeable.

I further examine how an asymmetric monetary policy interacts with other potential asymmetries in the economy, in particular in the way stock prices influence the macroeconomy. Ravn (2011) suggests that such a policy could be an attempt by the central bank to 'correct for' other asymmetries, and points to the financial accelerator of Bernanke *et al.* (1999) and to the stock wealth effect on consumption as potential sources of such an asymmetry. I demonstrate that if the financial accelerator is assumed to be stronger when net worth of firms is low, as has been suggested in the literature, the asymmetric policy is able to 'cancel out' this asymmetry in the case of supply shocks, but not after demand shocks. A similar conclusion is reached under the assumption of asymmetric wealth effects.

The remainder of the paper is structured as follows. Section 2 describes the DSGE workhorse model. Section 3 illustrates the dynamics of the model and the implications of introducing an asymmetric reaction to stock prices. In section 4, I discuss possible explanations for the asymmetric policy within the model framework. Section 5 concludes. The appendix contains details about the model and the solution method.

2 The Model

The general equilibrium model is a version of the standard New-Keynesian sticky-price model with capital; augmented with the financial accelerator of Bernanke *et al.* (1999) in order to introduce a role for asset prices. An additional feature is that contracts are written in terms of the nominal interest rate as in Christensen & Dib (2008), introducing the debt-deflation channel of Fisher (1933). Christiano *et al.* (2010) find that this channel is empirically relevant. The model is in large part similar to that of Christensen and Dib (2008) or Gilchrist and Saito (2008). This has the advantage that the dynamics of this class of models is well described in the literature, allowing me to isolate the effects of the asymmetric monetary policy rule. Moreover, this allows me to calibrate the model using the parameter values estimated by Christensen and Dib for the US economy for most of the parameters. Finally, this class of models is typically used in the literature on the role of asset prices in monetary policy cited above. The stochastic part of the model is quite parsimonious, as only two shocks are included: a technology shock and a monetary policy shock. These two shocks, which can loosely be interpreted as a supply and a demand shock, are sufficient to highlight the effects of the asymmetric policy.

2.1 Entrepreneurs

Entrepreneurs produce the intermediate goods that the final goods producers take as input. Each entrepreneur employs labor H_t and capital K_t, and produces output Y_t according to the following production technology:

$$Y_t \leq (A_t H_t)^{1-\alpha} K_t{}^{\alpha}. \tag{1}$$

The technology level A_t evolves according to

$$\ln(A_t) \equiv (1-\rho_a) A + \rho_a \ln(A_{t-1}) + \varepsilon_t^a, \tag{2}$$

where ε_t^a is a normally distributed shock to technology with mean zero. In each period, entrepreneurs face a constant probability $(1-\nu)$ of leaving the economy. As described by Bernanke *et al.* (1999), this assumption is made in order to ensure that entrepreneurs do not eventually accumulate enough capital to be able to finance their own activities entirely. I follow Christensen and Dib (2008) in allowing newly entering firms to inherit a portion of the net worth of those firms who exit the economy. This assumption is made in order to ensure that new entrepreneurs start out with non-zero net worth. In contrast, Bernanke *et al.* (1999) ensure this by assuming that entrepreneurs also work. This difference is of little importance for the results.

Entrepreneurs choose the inputs of capital and labor to maximize their profits, subject to the production technology. As there is perfect competition in the entrepreneurial sector, the price which they receive for their products will be equal to the marginal cost of producing the intermediate good. This gives rise to the following first-order conditions:

$$mp_t = \alpha \frac{Y_t}{K_t} mc_t, \tag{3}$$

$$w_t = (1 - \alpha) \frac{Y_t}{H_t} mc_t, \tag{4}$$

where mp_t denotes the real marginal productivity of capital, and mc_t is the real marginal production cost of entrepreneurs.

Each entrepreneur can obtain the capital needed for production in two ways: He can issue equity shares (internal financing), or he can borrow the money from a financial intermediary (external financing). Because internal financing is cheaper, as discussed below, entrepreneurs use all of their net worth, and borrow the remainder of their funding needs from the financial intermediary. The total funding needed by an entrepreneur is $q_t K_{t+1}$, where q_t is the real price of capital as measured in units of consumption. In order to ensure that any financial constraint faced by the entrepreneur applies to the capital stock as such, and not just to the investment in any given period, I assume that the entrepreneur must refinance his entire capital stock each period. If n_t denotes the net worth of the entrepreneur, the amount he needs to borrow is then $q_t K_{t+1} - n_{t+1}$. Letting f_t denote the external financing cost of one extra unit of capital, the demand for external finance must satisfy the following condition in optimum:

$$E_t [f_{t+1}] = E_t \left[\frac{mp_{t+1} + (1 - \delta) q_{t+1}}{q_t} \right]. \tag{5}$$

The numerator on the right-hand side is the marginal productivity of a unit of capital plus the value of this unit of capital (net of depreciation) in the next period. If this condition was not satisfied, the capital demand of entrepreneurs would be either zero or infinite. Note that I interpret the price of capital q_t as the stock price in the model economy.[3] Equity shares are ultimately claims to the assets of firms, which in this model amounts to their capital stock. Therefore, in a model of this type, q_t is the relevant variable to enter the central bank's reaction function in order to model a reaction to stock prices.[4]

As in Bernanke *et al.* (1999), the existence of an agency problem between borrower and lender renders external finance more costly than internal finance. While entrepreneurs observe the outcome of their investments costlessly, the financial intermediary must pay an auditing cost to observe this outcome. Entrepreneurs must decide - after observing the outcome - whether to report a success or a failure of the project, i.e. whether to repay or default on the loan. If they default, the financial intermediary pays the auditing cost, and

[3] In the rest of the paper, I will use the terms *price of capital*, *asset price* and *stock price* interchangeably.

[4] This is standard in the literature; see for instance Tetlow (2005) or Gilchrist and Saito (2008). In Bernanke and Gertler (1999, 2001), the central bank reacts to the fundamental price of capital plus a bubble term.

then claims the returns to the investment. Bernanke *et al.* (1999) demonstrate that the optimal financial contract involves an external finance premium (the difference between the cost of external and internal finance) which depends on the entrepreneur's net worth, and show that the marginal external financing cost is equal to the external finance premium times the opportunity cost of the investment; given by the risk-free real interest rate (the reader is referred to Bernanke *et al.* (1999) for details):

$$E_t\left[f_{t+1}\right] = E_t\left[\Psi\left(\frac{n_{t+1}}{q_t K_{t+1}}\right)\frac{R_t}{\pi_{t+1}}\right], \tag{6}$$

where the function $\Psi(\cdot)$ describes how the external finance premium depends on the financial position of the firm. $\frac{n_{t+1}}{q_t K_{t+1}}$ denotes the ratio of the firm's internal financing to its total financing, and is thus a measure of the leverage ratio. Equation (6) is the key to the financial accelerator mechanism. Bernanke *et al.* (1999) demonstrate that $\Psi'(\cdot) < 0$, implying that if firms' net worth goes up (or, equivalently, their leverage ratio goes down), the external finance premium falls, and firms get cheaper access to credit. The reason is that as the entrepreneur puts more of his own money behind the project, thus lowering the leverage ratio, the agency problem between borrower and lender is alleviated. The entrepreneur's incentive to undertake projects with a high probability of success increases, and as a result, the lender demands a lower return on the loans he makes. The drop in the external finance premium leads to an increase in the firm's demand for external finance, which in turn causes an increase in the firm's stock of capital in the next period, and thus its production level. In this way, to the extent that movements in net worth are procyclical, the financial accelerator works to amplify business cycle movements.

The net worth of entrepreneurs consists of the financial wealth they have accumulated (i.e., profits earned in previous periods) plus the bequest Υ_t they receive from entrepreneurs leaving the economy:

$$n_{t+1} = \nu\left[f_t q_{t-1} K_t - E_{t-1} f_t\left(q_{t-1} K_t - n_t\right)\right] + (1-\nu)\,\Upsilon_t. \tag{7}$$

2.2 Households

A continuum (of unit length) of households derive utility from an index of the final consumption goods produced by the retailers (C_t) and leisure ($1 - H_t$), and decide how much labor to supply to entrepreneurs producing intermediate goods. As all households are identical, they each solve the following utility maximization problem:

$$\max_{C_t, H_t, D_t} U = E_0 \sum_{t=0}^{\infty} \beta^t u\left(C_t, H_t\right), \tag{8}$$

with instantaneous utility function:

$$u\left(C_t, H_t\right) = \frac{\gamma}{\gamma - 1} \ln\left(C_t^{\frac{\gamma-1}{\gamma}}\right) + \eta \ln\left(1 - H_t\right), \tag{9}$$

subject to the relevant budget constraint:

$$C_t + \frac{D_t - R_{t-1}D_{t-1}}{P_t} \leq \frac{W_t}{P_t} H_t + \Omega_t. \tag{10}$$

D_t are deposits which are stored at a financial intermediary at the risk-free rate of interest R_t. Ω_t denotes dividend payments deriving from households' ownership of retail firms. The first-order conditions of the household are presented in the appendix.

2.3 Capital Producers

The role of capital producers is to construct new capital K_{t+1} from invested final goods I_t and existing capital. As in Bernanke *et al.* (1999), it is implicitly assumed that capital producers rent existing capital from entrepreneurs within each period at a rental rate of zero. They face capital adjustment costs, implying a non-constant price of capital q_t. I use the same quadratic functional form for the capital adjustment costs as Christensen and Dib (2008): $\frac{\chi}{2}\left(\frac{I_t}{K_t} - \delta\right)^2 K_t$. Profits of capital producers are then:

$$\Pi_t^c = q_t I_t - I_t - \frac{\chi}{2}\left(\frac{I_t}{K_t} - \delta\right)^2 K_t. \tag{11}$$

Choosing the level of investment that maximizes this expression results in the following equilibrium condition:

$$q_t - \chi\left(\frac{I_t}{K_t} - \delta\right) = 1. \tag{12}$$

Note that in the absence of adjustment costs, the parameter χ equals zero, so the optimality condition collapses to $q_t = 1$.[5] This illustrates that capital adjustment costs are necessary to create a time-varying price of capital. Moreover, the condition is essentially a Tobin's q-relation, ensuring that the investment level is chosen so that the 'effective' price of capital (i.e., net of capital adjustment costs) is equal to 1.

2.4 Retailers

Firms in the retail sector take intermediate goods as inputs, repackage these costlessly, and sell them. The retail sector is included in the model with the single purpose of creating price stickiness. Following Calvo (1983), price rigidity is introduced by assuming that in

[5] Recall that q_t is a real price measured in units of consumption. Hence, $q_t = 1$ will hold in the absence of adjustment costs, irrespective of the fact that the price level on consumption goods fluctuates due to the price stickiness faced by retailers.

each period, only a fraction $(1-\xi)$ of firms in the retail sector are allowed to change their price. The price of firms who are not allowed to change their price is indexed with the steady state inflation rate π. This problem gives rise to the following first-order condition for the optimal price $P_t^n(i)$ set by firm i:

$$P_t^n(i) = \frac{\epsilon^p}{\epsilon^p - 1} \frac{E_t \left\{ \sum_{s=0}^{\infty} (\beta\xi)^s \lambda_{t+s} Y_{t+s}(i) P_{t+s} mc_{t+s} \right\}}{E_t \left\{ \sum_{s=0}^{\infty} (\beta\xi)^s \lambda_{t+s} Y_{t+s}(i) \pi^s \right\}}. \tag{13}$$

Here, λ_t is the Lagrange multiplier associated with the budget constraint in households' optimization, and the parameter ϵ^p measures the elasticity of substitution between different intermediate goods. The evolution of the aggregate price level is a weighted average of the price of those firms who are allowed to change their price in a given period, and all set the same new price, and of those who are not; whose prices are therefore indexed:

$$P_t = \left[(1-\xi) (P_t^n)^{1-\epsilon^p} + \xi (P_{t-1}\pi)^{1-\epsilon^p} \right]^{1/(1-\epsilon^p)}. \tag{14}$$

In the appendix, I demonstrate how the log-linearized versions of (13) and (14) can be combined to yield a standard version of the New-Keynesian Phillips Curve.

2.5 Monetary Policy

To introduce an asymmetric policy reaction to stock prices, I assume that the central bank follows a Taylor rule augmented with a term that captures a reaction to stock price drops. This is in line with the specification in Hall (2011). Moreover, Ravn (2011) attempts to control for the movements in the interest rate that are driven by macroeconomic variables such as output and inflation. Therefore, also his result is interpretable as a reaction to stock prices *on top of* the reaction to those variables, in line with the implicit assumption behind an augmented Taylor rule. I further add interest rate smoothing, as this tends to improve the empirical performance of Taylor rules (Clarida *et al.*, 1999; Christiano *et al.*, 2010). This gives rise to the following monetary policy rule:

$$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R}\right)^{\rho_r} \left[\left(\frac{\pi_t}{\pi}\right)^{\phi_\pi} \left(\frac{Y_t}{Y}\right)^{\phi_y} \left\{ \left(\frac{\Delta q_t}{q}\right)^{\phi_q} \right\}^{1[\Delta q_t < 0]} \right]^{(1-\rho_r)} e^{\varepsilon_t^r}, \tag{15}$$

where $1[X]$ is the indicator function; equal to 1 if X is true and zero otherwise. This captures that the central bank is reacting to the change in stock prices only when this change is negative. ε_t^r is a normally distributed monetary policy shock with mean zero. The stated monetary policy rule allows for interest rate smoothing, as measured by the parameter ρ_r. The parameters ϕ_π and ϕ_y measure the monetary policy reaction to deviations of inflation from its target level, and of output from its steady state level, respectively. Note that the steady state or natural level of output (Y) is below the efficient level of output (Y^*) due to the presence of monopolistic competition.

While this paper is the first to consider a Taylor rule with a reaction to stock price drops, Taylor rules augmented with a symmetric reaction to stock price changes have been studied by Tetlow (2005) and Gilchrist and Saito (2008) in models largely similar to the one outlined above. A similar type of 'speed limit'-rule is also studied by Leduc and Natal (2011). The rule above is essentially a speed-limit rule with no upper speed limit.

2.6 Equilibrium and Model Solution

The model consists of 15 equilibrium conditions in 15 variables, as described in the appendix. The equilibrium of the model consists of a vector of allocations $(C_t, H_t, Y_t, K_t, n_t, I_t)$ and prices $\left(\pi_t, R_t, w_t, mc_t, mp_t, q_t, f_t, \lambda_t, \left(\frac{P_t^n}{P_t}\right)\right)$ such that those 15 equations are satisfied. In the appendix, I further present the steady state, around which the model is log-linearized. However, the non-linear monetary policy rule implies that even after log-linearization, an important non-linearity remains in the model. As a result, the model cannot be solved with standard techniques.

Instead, I solve the model using a numerical solution method which exploits the piecewise linearity of the model. This method follows the approach taken by Bodenstein *et al.* (2009) in order to deal with problems where the zero lower bound on interest rates is binding in a number of periods. While Bodenstein *et al.* study a one-off switch, I generalize the solution method to handle ongoing switches between policy regimes. As the only non-linearity in the present model is the monetary policy reaction to asset prices, the model in effect consists of two linear systems; one for when asset prices are decreasing, and one for when they are non-decreasing. Following Bodenstein *et al.* (2009), I first build a shooting algorithm in order to identify the 'turning points' in the evolution of the asset price following a shock; i.e. when the sign of Δq_t, and thus the monetary policy regime, shifts. For any initial guess of the turning points, the model is then solved using backward induction. If the initial guess turns out not to be consistent with the sign of Δq_t shifting at that time, the guess is adjusted accordingly, and the process is repeated until the shifting criteria are satisfied. Details of the solution method are outlined in the appendix.

It should be noted that this approach to endogenous regime switching is somewhat different from that of Davig and Leeper (2006). They solve their model, in which the monetary policy reaction to inflation depends on the lagged level of inflation, numerically over a discrete partition of the state space. However, applying this method to the model of the present paper, which is considerably larger than that of Davig and Leeper, involves substantial computational problems, as their approach suffers heavily from the curse of dimensionality. The ability to handle endogenous switching even in a medium-scale DSGE model is thus an advantage of the shooting method employed in the present paper.[6] On

[6] The shooting method is in fact more similar in spirit to the 'guess-and-verify' method used by Máckowiak (2007) in order to study the outbreak of currency crises. However, common to all these studies is the use of numerical methods. The task of developing analytical tools to deal with endogenous regime switching is an

the other hand, the shooting method in effect combines two approximations, as the model is linearized under each regime. This is a potential drawback, albeit a small one, as the two systems are almost identical. A more substantial disadvantage of using a numerical, non-linear solution method is that it renders welfare calculations unfeasible, and thus prevents an investigation of whether the asymmetric policy is optimal in terms of welfare.

2.7 Calibration

As already mentioned, I obtain most of the parameter values from Christensen and Dib (2008), who estimate a model largely similar to the one outlined above using US data for the sample period 1979-2004. The parameters that were not estimated by Christensen and Dib are instead calibrated. With a few minor exceptions described in the appendix, I follow the calibration in Christensen and Dib (2008). The reader is therefore referred to Christensen and Dib and to the appendix for a more detailed discussion of the parameter values. All parameter values are presented in Table A1.

The parameter measuring the elasticity of the external finance premium with respect to changes in firms' leverage position deserves special mention. I use the value $\psi = 0.042$ as estimated by Christensen and Dib. This value is somewhat smaller than the calibrated value used by Bernanke *et al.* (1999) and Gilchrist and Saito (2008) of $\psi = 0.05$. This implies that the financial accelerator mechanism is less strong in the present paper.

As for monetary policy, the policy rule in my model differs substantially from that of Christensen and Dib (2008). Therefore, I do not use their parameter estimates. Instead, I set $\phi_\pi = 1.5$ as suggested by Taylor (1993). Furthermore, I set $\phi_y = 0.2$, as recent empirical studies with US data seem to suggest that Taylor's suggested value of 0.5 is perhaps too high (see e.g. Christiano *et al.* 2010). The interest rate smoothing parameter is set at 0.67, indicating a degree of interest rate smoothing around 2/3 as suggested by, among others, Clarida *et al.* (1999). Finally, a value must be assigned to the parameter ϕ_q, the reaction to stock price drops. Hall (2011) directly estimates this parameter in a comparable Taylor rule with interest rate smoothing.[7] In her baseline specification, she finds an estimate for the period 1987-2008 of 0.139 when $\phi_q < 0$. To cast the result of Ravn (2011) in terms of the Taylor rule, the point estimate of that study needs to be transformed. The interpretation offered by Ravn relies on the fact that the Federal Open Market Committee meets once every six weeks. The model of the present paper is formulated (and calibrated) in quarterly terms. This involves an implicit assumption that monetary policy can only be changed every 12 weeks; once per quarter. Thus, following the same line of argument as Ravn (2011), his estimated result implies a value of $\phi_q = 0.0246$ whenever $\Delta\widehat{q}_t < 0$.

obvious next step, but beyond the scope of this study.

[7] To be exact, Hall finds a significant reaction to the *lagged* stock price change relative to the change in their fundamental value, as measured by dividend yields.

These numbers are quite low. As the studies by Hall (2011) and Ravn (2011) are the first to identify a specific reaction to stock price drops, the literature in general offers little guidance on the magnitude of this parameter. However, some information can be obtained from the contributions of Tetlow (2005) and Gilchrist and Saito (2008), who augment the Taylor rule with a symmetric reaction to the change in stock prices. Tetlow evaluates a rule with a stock price reaction that is quite large; always bigger than 1. Gilchrist and Saito allow the parameter to take on values between 0.1 and 2.0. In other words, there seems to be a severe divergence between estimated and calibrated values of this parameter.[8] To bridge this gap, I therefore perform most of the simulations below for three different values of ϕ_q; the estimates obtained from Hall (2011) and Ravn (2011), and a calibrated value of 0.5 which is more in line with the values used in the theoretical literature.

3 Dynamics of the Model

In this section, I investigate the dynamics of the model when the asymmetric monetary policy rule is in place. In linear models, the impulse response to a positive shock is by construction the mirror image of the response to a negative shock of the same type and size. In this model, instead, positive and negative shocks have different dynamic effects. As the central bank reacts only to falling asset prices, a shock that drives asset prices down will induce a stronger monetary policy reaction than a shock which leads to higher asset prices. Further, the adjustment back to the steady state will also differ, depending on whether asset prices are approaching their steady state value from above or below.

Before looking into the effects of the asymmetric policy, it is useful to study the effects of each shock in the model without an asymmetric policy. Figure 1 and 2 display the impulse responses of some key endogenous variables to an orthogonalized unit shock to technology and monetary policy when the policy reaction to stock prices is always zero; $\phi_q = 0$. Following a positive technology shock, Figure 1 illustrates that output rises, as does consumption and investment (not shown). The hump-shaped pattern of output is generated by the real and nominal rigidities in the model. Inflation and the nominal interest rate both fall in response to this positive supply shock. The drop in the inflation rate is the source of the drop in net worth. Lower inflation implies a higher real cost of repaying outstanding debt, depressing the net worth of firms. This is the debt-deflation channel. The consumer price index is the relevant price index for 'deflating' net worth, since firms are eventually owned by households. As net worth goes down, the external finance premium increases due to more severe agency problems between borrower and lender, as described

[8] Indeed, if one were to use the result of Rigobon and Sack (2003) in the present setting, this would imply a (symmetric) value of $\phi_q = 0.0428$. One potential explanation of this divergence is that in theoretical investigations, the researcher is often interested only in policy reactions to stock price changes larger than some threshold value, while the empirical contributions considered here measure the (presumably smaller) reaction to stock price changes of any size.

above. In turn, this dampens economic activity. Thus, the term financial accelerator is in fact misleading in the case of shocks originating from the supply side when the debt-deflation channel is included, as in this case the fluctuations in output are actually attenuated. This was already noted by Iacoviello (2005). The presence of the debt-deflation channel is crucial for this result, as also demonstrated by Christiano *et al.* (2010). In a similar model, they find that the debt-deflation channel and the financial accelerator mechanism reinforce each other in the wake of shocks that drive output and inflation in the same direction, whereas they counteract each other after shocks that, like the technology shock, drive output and inflation in different directions. A related point is made by Liu *et al.* (2009), who argue that credit constraints in general do not amplify technology shocks.

The technology shock leads to a boom-bust cycle in the asset price. The initial rise and fall in the price of capital is due to the investment boom following the technology shock. However, the price of capital 'undershoots' its steady state level for a number of periods. This undershooting is again due to the debt-deflation channel, as the persistent drop in net worth leads to a persistent rise in the price of external funding, lowering the demand for capital (and thus, the asset price) even many periods after the shock. It may seem counterintuitive that net worth and the price of capital move in different directions. The explanation is that the initial (and numerically quite small) increase in the price of capital is the result of two opposing effects: While the positive technology shock increases investment and the price of capital; the resulting rise in the external finance premium has the exact opposite effect.

Figure 2 illustrates the dynamics after a one-time positive innovation to monetary policy. As expected, the nominal interest rate jumps up, and then falls back gradually due to interest rate smoothing. In this case, the financial accelerator does work to amplify business cycle fluctuations. As output and inflation move in the same direction, this is in line with the predictions of Christiano *et al.* (2010). The higher interest rate depresses economic activity and in particular investment, reducing the price of capital. This leads to a drop in the net worth of firms, which is further enhanced by the drop in inflation through the debt-deflation channel. Lower net worth increases the external finance premium, which further depresses investment and output. These dynamics explain why this mechanism is referred to as the financial accelerator.

In order to isolate the effects of an asymmetric policy, a useful next step is to outline the implications of reacting symmetrically to stock price changes. Figures 3 and 4 illustrate the effects of a moderate but symmetric monetary policy reaction to asset price changes. Here, the central bank reacts to both positive and negative stock price changes with a reaction parameter of 0.5. It is clear that a symmetric reaction of this size does not lead to major changes in the dynamics of the model. The effects of technology shocks are qualitatively similar to those presented in figure 1 when no stock price reaction was present. The drop in the interest rate is somewhat smaller, as the increase in the asset price now calls for a tighter monetary policy. However, the quantitative difference is small. For monetary policy shocks, the reaction to stock prices has a larger impact. The drop in the asset price following a monetary contraction implies that the increase in the interest rate is almost cut in half relative to the scenario with no stock price reaction. This in turn dampens the drop in output.

As an alternative to stock price *changes*, monetary policymakers could instead be interested in stock price *deviations* from their steady state level, as is the case for output and inflation. Such a policy rule has been studied by Faia and Monacelli (2007), Gilchrist and Saito (2008), and Pavasuthipaisit (2010), among others. As in Faia and Monacelli (2007), the fact that the efficient level of the asset price is always 1 implies that in the present setup, a reaction to the asset price *gap* is equivalent to a reaction to the asset price *level*. In figure 5 and 6, I illustrate the effects of having the central bank react to stock price deviations with an unchanged reaction parameter of 0.5. In the wake of a technology shock, the interest rate and inflation return to their steady state values much faster than when policymakers are following a 'speed limit' rule (figure 3). This is not surprising, given the more sluggish nature of policy rules with a reaction to lagged values. For monetary policy shocks, the same pattern is observed for inflation. Moreover, the effect of the shock is smaller, as agents anticipate that the low stock price will continue to call for a more lenient monetary policy many periods ahead, as opposed to the scenario with a reaction to stock price changes (figure 4). Overall, however, this modification of the nature of the policy rule does not significantly alter the transmission of shocks within the model. In the following, I therefore stick to the original assumption of a reaction to stock price *changes*; partly because this seems to better capture the recent theoretical discussion as well as the empirical findings of Hall (2011) and Ravn (2011), and partly because the quantitative difference is modest. A final advantage of reacting to changes in the stock price rather than its deviations from a steady state level is that the latter can be quite difficult to determine in practice.

3.1 Dynamics under Asymmetric Policy

Having discussed the effects of each shock in the absence of asymmetric policy, I now turn to the study of how these effects are altered when an asymmetric monetary policy rule is introduced. When computing impulse responses, I use the calibrated value of $\phi_q = 0.5$ in order to clearly illustrate the effects of the asymmetric policy. For each shock, I compare the effects of positive and negative shocks on the dynamics of key endogenous variables. Consider first the effects of a technology shock. Figure 7 illustrates what happens after positive and negative technology shocks. The 'mirror image' of a negative shock is just the impulse responses of the negative shock multiplied by -1; facilitating comparison. As illustrated, the asymmetric policy has a dampening effect on contractions in output relative to expansions. A positive technology shock causes output to increase by more than it decreases following a similar-sized negative shock. The explanation is that in the wake of a negative technology shock, the asset price is pushed down for a number of periods (except for the effect on impact, when the asset price actually rises). Under the asymmetric policy, this drop in asset prices is met with an interest rate cut (although this cut is dominated by the increase in the interest rate as a reaction to the jump in inflation), spurring economic activity and thus dampening the initial economic slowdown. On the other hand, as asset prices rise following a positive technology shock, this induces no increase in the interest rate *per se*. In other words, output contractions following technology shocks are mitigated by an interest rate reaction to asset prices, while output expansions are not. Also for inflation, increases will be larger than drops, as the interest rate reaction to asset prices exerts an upward pressure on inflation following a negative shock, but no corresponding downward pressure after a positive shock. While the asset price still displays a boom-bust cycle, the asymmetric policy implies that the decline following a negative shock is less severe than the boom following a positive shock. It thus seems that the policy reaction to asset price drops succeeds in mitigating these drops. The quantitative importance of the asymmetric policy is limited, however, as indicated by the small absolute distance between the impulse responses for the positive and (mirrored) negative shocks.

It is interesting to compare the effects on the asset price to the effects of a similar-sized shock with no stock price reaction (Figure 1). As the negative shock induces a monetary policy reaction to the drop in stock prices, it is not surprising that the effects of a negative shock (Figure 7) are numerically smaller than the effects of a positive shock under no stock price reaction at all. However, we also observe that the increase in the asset price following a positive shock is larger under the asymmetric policy than in the absence of an asset price reaction. As the asset price increases immediately after a positive technology shock, both models imply no reaction of monetary policy to this increase. Under the asymmetric policy, however, agents realize that whenever asset prices start to fall, this drop will be alleviated by a monetary policy reaction. This expectation drives up the asset price more than in the case where the reaction to asset prices is always zero, giving rise to an 'anticipation boom'. This anticipation boom measures the additional increase of the asset price under asymmetric policy, relative to its increase in the case of no stock price reaction following a positive shock. Quantitatively, the anticipation boom is quite substantial under the calibration with $\phi_q = 0.5$; amounting to 23.9 % when evaluated two periods after the shock; the last period before the asset price starts to fall and monetary policy actually starts reacting to asset price changes. On the other hand, with the the estimated values of $\phi_q = 0.139$ (Hall, 2011) or $\phi_q = 0.0246$ (Ravn, 2011), the number is reduced to only 6.5% or 1.1 %, respectively.

Consider finally the asymmetric effects on the two financial variables, net worth and the external finance premium. Recall that because of the debt-deflation channel, net worth is depressed after a positive technology shock, as the drop in inflation increases the real burden of firms' debt repayments. However, it is apparent that the effect on net worth is much larger following a negative shock. After a positive shock, the drop in net worth is counteracted by the rise in the asset price. In the case of a negative shock, this effect is much weaker, as the drop in asset prices is much smaller. Indeed, after a negative shock, the asset price rises in the first period, which is exactly where most of the difference arises in the effects on net worth. As net worth is highly persistent, so is this difference. In turn, also the external finance premium is affected more by a negative shock, which is unsurprising given the movements in net worth.

Figure 8 illustrates the asymmetric effects of contractionary and expansionary monetary policy shocks. Once again, output and inflation both drop following a contractionary monetary policy shock. An expansionary shock, however, induces an even larger increase in output and inflation. As was the case for technology shocks, then, the asymmetric policy implies that when the economy is hit by monetary policy shocks, booms become larger than recessions, once again creating an asymmetric business cycle. The explanation is again linked to the movements in the asset price. Following a contractionary shock, the asset price goes down, inducing the central bank to cut the interest rate. This mitigates the initial economic downturn caused by the shock, and also pushes inflation up. On the other hand, the rise in asset prices following an expansionary shock is not met with any monetary policy reaction, so the counteracting effect is not present in that case. Furthermore, adding to the asymmetric effects on output and inflation stemming from the monetary policy reaction to asset prices, the increase in the external finance premium during expansions is much larger than the drop during contractions. In turn, this implies cheaper access to credit for firms, increasing the demand for capital, the investment level, and eventually output. Note that while the nominal interest rate does not display a large, numerical difference, the real interest rate, which matters for consumption and investment decisions, is affected differently during expansionary and contractionary phases, as implied by the impulse responses for inflation.

As in the case of technology shocks, an expansionary shock to monetary policy leads to an anticipation boom in asset prices. This is evident when comparing the effects of an expansionary shock under asymmetric policy (Figure 8) to the effects in the case of no stock price reaction (Figure 2). In the case of monetary policy shocks, the anticipation boom is evaluated one period after the shock; the last period before the asset price starts declining. The extra rise in asset prices is substantial, 28.1 %, when ϕ_q is set to 0.5. Using instead the estimated values from Hall (2011) and Ravn (2011), the number drops to 6.7 % and 1.1 %, respectively.

The emergence of the anticipation boom can be related to what Davig and Leeper (2006) call the *preemption dividend*. In their model, the central bank is assumed to react stronger to inflation if the lagged inflation level is above a certain threshold (the inflation target). Rational agents will embed this non-linearity in their inflation expectations. As a consequence, monetary policy will be more effective in bringing down inflation in the wake of an inflationary shock, compared to a situation with a linear reaction to inflation. As the central bank is able to successfully manage expectations, the actual increase in the interest rate does not have to be very large. In my setup, agents embed the monetary policy reaction to stock price drops in their expectations, leading to a larger increase in asset prices immediately after a positive shock. This happens despite the fact that when asset prices are increasing, as in the first period(s) after the shock, the actual monetary policy reaction to asset prices is zero under the asymmetric policy as well as with no reaction to asset prices at all. As the preemptive dividend of Davig and Leeper (2006), the anticipation boom arises solely due to the central bank's ability to manage the expectations of private agents. In this way, the asymmetric monetary policy amplifies the boom-bust cycle in asset prices following a shock to the economy, thereby creating additional volatility in asset prices. Finally, it is worth mentioning that similarly to Davig and Leeper (2006), I find substantial differences between the impulse responses shown above, which take into account that agents anticipate the possibility of future regime switches, and the impulse responses (not shown, but available upon request) obtained when agents naively expect the present regime to be in place forever.

The results above can be related to some of the results from the empirical literature on asymmetric business cycles. The finding that the asymmetric policy amplifies booms relative to recessions seems to contradict a number of empirical studies which tend to find that recessions are bigger than booms (Neftci, 1984; Acemoglu and Scott, 1997). This suggests that an asymmetric policy of the type investigated above has not historically been driving the business cycle. For several reasons, this is not particularly surprising. First, the findings of Hall (2011) and Ravn (2011) are obtained only for relatively short samples. Second, these results are of too little quantitative importance to be a dominant driver of the business cycle. On the other hand, Beaudry and Koop (1993) find that negative shocks to the economy are much less persistent than positive ones, implying that recessions should be shorter than booms. This is more in line with the effects of an asymmetric policy shown above, even if the quantitative differences between booms and recessions are too small to match the findings of Beaudry and Koop. Finally, the implications of an asymmetric policy are also consistent with the results of Cukierman and Muscatelli (2008) and Wolters (2011), who find that the Federal Reserve has displayed a recession avoidance preference in the recent past.[9] According to these studies, estimated reaction functions of the Federal Reserve indicate that US monetary policymakers tend to react more strongly to the output gap during recessions than during expansions. This creates outcomes that are in line with the impulse responses displayed above, suggesting that an asymmetric reaction to stock prices can be rationalized by recession avoidance preferences. This is further discussed in the next section.

4 Potential Motivations for an Asymmetric Policy

As demonstrated by the impulse responses in the previous section, reacting asymmetrically to asset prices can lead to a situation in which recessions are attenuated relative to expan-

[9] On the contrary, Surico (2007) finds no evidence of a recession avoidance preference in the US.

sions. This raises the question of whether one could think of the central bank as aiming to obtain exactly such an asymmetric outcome. This would then have to show up in the central bank's underlying loss function. Usually, it is assumed that the central bank (implicitly or explicitly) minimizes a loss function where deviations of output and inflation from their target values are punished in a fully symmetric way (see, e.g., Woodford, 2003). Given a mapping from the parameter governing the central bank's preference for output stability relative to inflation stability to the parameters of the Taylor rule, one can think of the Taylor rule as a tool used by the central bank to minimize a loss function of this type. It is, however, not given that the objective of the central bank should be perfectly symmetric, as also observed above. Among others, Blinder (1997), Ruge-Murcia (2004), and Surico (2003, 2007) suggest that the central bank could be seeking to minimize an asymmetric loss function. For example, Ruge-Murcia assumes that the loss arising from inflation fluctuations is symmetric, but that social loss is higher when unemployment (which he allows to enter the loss function in lieu of output) is above its natural level, compared to when it is below.

If the loss function of the central bank is of such an asymmetric type, this could serve as the motivation for an asymmetric stock price reaction. Indeed, the central bank could adjust the parameters in its asymmetric Taylor rule (15) to obtain the outcome that minimizes the asymmetric loss function. In section 3, we saw how the asymmetric policy implied that booms not only in output, but also in inflation, tended to be stronger and longer than recessions. This would be consistent with a central bank that has a preference for booms and high inflation over recessions and low inflation.

Woodford (2003) shows that a symmetric loss function approximates the negative of the utility of the representative household in the basic New-Keynesian model, so that minimizing such a loss function is equivalent to maximizing the utility of the representative household. Accordingly, if the central bank minimizes an asymmetric loss function, this would also require a micro-foundation in order to be optimal.Ruge-Murcia (2004) suggests that the motivation for the asymmetric loss function could be concerns about the costs of high unemployment. Another way to micro-found an asymmetric loss function is to assume that agents are loss averse with respect to changes in financial wealth. This possibility is discussed at the end of subsection 4.2. Surico (2007) discusses other possible sources of asymmetric welfare losses. The model outlined above, however, does not include any features that could serve as a welfare-based motivation for an asymmetric loss function, and therefore is unable to explain why the central bank would adopt such a loss function.

If one is not willing to accept the notion of an asymmetric loss function, it is still possible to think of potential motivations for an asymmetric policy. One potential motivation for the central bank to obtain outcomes such as the ones illustrated in section 3 could be the fact that natural or steady state output is lower than the efficient level of output. This gives the central bank an incentive to try to push output above its natural level, as in the well-known model of Barro and Gordon (1983). Other rationalizations derive from the fact that specifying a loss function of the central bank of the usual, symmetric form not necessarily implies that the tools of the central bank should also be symmetric. Indeed, if the central bank believes that certain asymmetries exist in the economy, for example that stock price drops and increases have asymmetric macroeconomic effects, an asymmetric policy might be seen as an attempt to correct for this inherent asymmetry, and in turn obtain a symmetric outcome. Ravn (2011) acknowledges this possibility, and points out two potential sources of asymmetric effects of stock price. In the following, I study each of them in more detail.

4.1 Asymmetric Financial Accelerator

One channel which may give rise to asymmetric effects of stock price movements is the financial accelerator included in the model above. The possibility of non-linear balance sheet effects has received some attention in the literature, and was discussed by, among others, Bernanke and Gertler (1989), Gertler and Gilchrist (1994) and Bernanke *et al.* (1996). During a recession, when asset prices tend to be falling, more firms are likely to be liquidity constrained and in need of external financing. Moreover, small changes in the net worth of firms are likely to be more costly when the collateral value of firms is already low, and the agency costs of borrowing are already large. A final reason why the financial accelerator might be stronger when net worth is low is that ultimately, as firms' net worth becomes 'low enough', a credit crunch might result. Peersman and Smets (2005) assess the empirical transmission effects of monetary policy in the euro area, and find that the financial accelerator effect does indeed seem to be stronger in recessions. Gertler and Gilchrist (1994) provide empirical evidence that the performance of small firms are more sensitive to interest rate changes during economic downturns than in booms, suggesting that financial factors are more important in bad times. As discussed by Peersman and Smets (2005), such an asymmetry could potentially explain why monetary policy exerts a stronger effect on output during recessions than in booms.

In the model of this paper, the strength of the financial accelerator is measured by the parameter ψ in equation (51) in the appendix, which is in turn the log-linearized version of equation (6) above. ψ measures the elasticity of the external finance premium with respect to the net worth of firms, so the larger is ψ, the stronger is the effect on the business cycle of a given change in net worth. In other words, an asymmetric financial accelerator can be modelled by assuming different values of ψ. In particular, in light of the above discussion, I allow ψ to take on one value (ψ_L) for the case when net worth is above its steady state value, i.e., $\widehat{n}_t > 0$, and a higher value (ψ_H) when $\widehat{n}_t < 0$. This reflects that when net worth of firms is already low, the external finance premium is more sensitive to small changes in net worth. In this way, the financial accelerator becomes a source of asymmetric business cycle fluctuations by amplifying bad economic shocks more than good ones. As described in the previous section, the asymmetric policy reaction to stock prices had the exact opposite effects, suggesting that these two asymmetries might 'cancel each other out'.

To investigate this possibility in detail, consider first the effects of technology shocks. After a positive shock, net worth drops below its steady state value, implying that the elasticity of the external finance premium becomes high. This exerts a downward pressure on output through the accelerator effect, dampening the initial boom, while the drop in inflation is amplified. In the case of a negative technology shock, net worth instead rises, so the dampening of the initial downturn in output is small. Hence, the drop in output is large, which in turn mitigates the increase in inflation. In consequence, the effects of positive and negative shocks are asymmetric.[10] By following an asymmetric policy and cutting the interest rate in response to the drop in stock prices after a negative shock, the central bank can drive up output and inflation, and thereby mimic (the mirror image of) a positive shock. In fact, for a given 'degree' of asymmetry of the financial accelerator, there exists a magnitude of the asymmetric policy reaction (ϕ_q) that exactly eliminates the initial asymmetry after supply-side shocks.

It turns out that the same is not true after shocks originating from the demand side. An expansionary shock to monetary policy pushes up net worth, so that the balance-sheet effect is relatively weak. The resulting amplification of the initial boom in output is limited, while the increase in inflation is relatively large. On the other hand, the financial accelerator is much stronger following a contractionary monetary policy shock due to the drop in net worth, resulting in a large drop in output and a strong dampening of the initial drop in inflation. An asymmetric policy induces an interest rate cut in response to the stock price drop after the contractionary shock. While this dampens the drop in output, again mimicking the mirror image of a positive shock, it also mitigates further the drop in inflation, which was already 'too small' compared to the relatively large increase in inflation after a positive shock. In other words, a trade-off arises between bringing output or inflation to their 'symmetric' values. While the asymmetric policy might alleviate the effects of a non-linear financial accelerator, it never obtains the fully symmetric outcome.[11]

To shed light on the empirical relevance of these issues, it seems natural to ask: How severe should the asymmetry of the financial accelerator be in order to 'rationalize' the recent results of Hall (2011) or Ravn (2011) as the policy that 'cancels out' the asymmetric financial accelerator under supply shocks, or obtains the most favorable trade-off under demand shocks? In order to quantify the necessary degree of asymmetry, I fix the elasticity of the external finance premium at the baseline value of $\psi_L = 0.042$ when net worth is above its steady state value. I then use impulse response matching of output and inflation responses

[10] Note that the debt-deflation channel is not critical for this conclusion. Without the debt-deflation channel, net worth would be procyclical after technology shocks (Gilchrist and Saito; 2008). An asymmetric financial accelerator would then amplify recessions more than booms.

[11] To visualize these scenarios, observe that an asymmetric financial accelerator induces a 'kink' in both the aggregate demand (AD) and aggregate supply (AS) curves, as firms are on the demand side of the market for financing, but on the supply side of the goods market. Through an asymmetric policy reaction of the 'right' size, the central bank can eliminate the kink in the AD curve, but not in the AS curve. As a consequence, shocks to aggregate supply shifts the AS curve along the resulting, potentially linear AD curve, giving rise to symmetric outcomes of positive and negative shocks. On the other hand, the AD curve will intersect steep or flat areas of the non-linear AS curve in response to positive or negative shocks, respectively.

for positive and negative shocks to calibrate the 'optimal' value of ψ_H.[12] This value can then be compared to ψ_L. Table 1 shows the degree of asymmetry needed to optimally match the impulse responses of output and inflation to technology shocks for different values of ϕ_q, the reaction coefficient of monetary policy to stock price changes. As the table illustrates, the degree of asymmetry in the financial accelerator needed to match impulse responses is quite sensitive to the choice of ϕ_q. For the values found by Ravn (2011) or Hall (2011), the balance-sheet channel needs to be only slightly asymmetric (2 or 13% stronger when net worth is low, compared to when it is high) in order for the two asymmetries to 'cancel each other out' under supply shocks. If instead ϕ_q is set at 0.50, this number rises to 40 %.

For the same grid of values of ϕ_q, table 2 shows the degree of asymmetry of the financial accelerator needed to obtain the most favorable trade-off between symmetry in output and in inflation after monetary policy shocks. If the policy reaction to stock price changes is set at $\phi_q = 0.50$, the balance-sheet effect has to be much stronger (77%) during periods of low net worth in order to minimize the distance to the symmetric outcome. For a policy reaction of the size estimated by Hall (2011) and Ravn (2011), the numbers are 44% and 8%, respectively.

To put these numbers in perspective, I look to the empirical study of asymmetric balance-sheet effects by Peersman and Smets (2005). They show that a positive innovation of 1 %-point to the interest rate causes a drop in the growth rate of output of 0.22 %-points during a boom, but a much larger drop of 0.66 %-points during a recession. They then estimate how various measures of firms' financial position contribute in explaining this asymmetry. They find that if firms' leverage ratio increases by 5 % of its average value, the *difference* between the effect on output growth of a monetary policy shock in booms and in recessions increases by 0.14 %-points; i.e. from the original 0.44 %-points to 0.58 %-points. In other words, the financial position of firms is able to account for substantial asymmetries over the business cycle, indicating that the financial accelerator effect is considerably stronger in recessions than in booms. In this light, the degrees of asymmetry computed above to 'rationalize' the results of Hall (2011) and Ravn (2011) do not seem unrealistic.

4.2 Asymmetric Wealth Effects and Loss Aversion

Another possible source of asymmetric macroeconomic effects of stock price movements is the wealth effect on consumption. Shirvani and Wilbratte (2000) and Apergis and Miller (2006) provide empirical evidence that the wealth effect of stock prices is stronger when stock prices are declining than when they are increasing. One possible, theoretical explanation for this finding is provided by prospect theory (Kahneman and Tversky, 1979). Prospect

[12] More specifically; for each of the two types of shocks, I focus on the impulse responses of output and inflation. I then compute the sum of squared errors (SSE) between the impulse response to a positive shock and the mirror image of the impulse response to a negative shock. For this, I use the values in the first 16 periods after the shock. Finally, I solve for the value of ψ_H that minimizes the sum of the SSE's.

theory introduces an inherent asymmetry in agents' preferences, as the utility loss from bad outcomes is assumed to be larger than the utility gain from good outcomes. If agents display such loss aversion in consumption, as suggested by, among others, Koszegi and Rabin (2009), this might give rise to non-linear effects on consumption from asset price movements. If asset prices decline, so does financial wealth and permanent income, and agents will have to cut their consumption level, painful as it is. On the other hand, following a rise in asset prices, loss averse agents are likely not to increase their consumption level by as much, but instead engage in precautionary savings to cushion themselves against the risk of a future drop in asset prices. As a result, increases in asset prices have smaller effects on consumption, and hence on the macroeconomy, than asset price declines.

Gaffeo *et al.* (2010) show how loss aversion in consumption can be introduced into a Markov-switching DSGE model with state-dependent preferences. However, augmenting the non-linear model of the present paper to include state-dependent preferences and regime-switching in consumption is not tractable. Instead of a formal analysis, I therefore resort to an intuitive discussion of how such asymmetric wealth effects would affect the dynamics of the present model, and whether these effects might serve as a motivation for the asymmetric policy reaction. Indeed, Gaffeo *et al.* (2010) demonstrate that in their setup, the optimal monetary policy is asymmetric in order to make up for the asymmetry introduced by loss aversion in consumption. To see how asymmetric wealth effects alter the dynamics of the present model, first observe that loss aversion in consumption implies a state-dependent rate of intratemporal substitution between consumption and leisure (Gaffeo *et al.*, 2010). In particular, after a drop in asset prices and consumption, households increase their labor supply so as to compensate for this loss, giving rise to a more favorable trade-off between output and inflation.

Consider first what happens under demand shocks. A positive innovation to monetary policy drives up inflation, output and the asset price. However, the weak wealth effect attenuates the boom in output. At the same time, more labor is needed to satisfy the extra demand. However, because consumption is rising, the labor supply of households is relatively low, resulting in a large increase in inflation. Instead, after a monetary contraction the wealth effect is strong, so output falls by a lot. Due to the more favorable trade-off between output and inflation, however, the drop in the latter is small. A reaction to the drop in stock prices is able to offset the direct wealth effect, but not the effect on the labor-leisure decision.

As for supply shocks, the initial rise in asset prices following a positive innovation to technology leads to only a small wealth effect, moderating the boom in output and amplifying the drop in inflation. A negative technology shock instead causes a large drop in output through a strong, negative wealth effect. At the same time, the spike in inflation is modest. An asymmetric reaction to the stock price drop will tend to push up inflation and output, bringing both variables closer to the mirror image of a positive shock. In sum, if the stock wealth effect is assumed to be asymmetric over the business cycle, an asymmetric policy is able to 'correct for' this asymmetry and obtain symmetric outcomes only in the case of shocks to the supply side, while a trade-off arises after demand shocks. This is similar to what was found in the previous subsection when the financial accelerator was the source of the underlying asymmetry.[13]

According to the above explanation, asymmetric wealth effects arise through the effect of stock wealth on consumption. A related line of argument, also deriving from prospect theory, is that gains and losses in financial wealth might have direct, asymmetric effects on utility. Barberis *et al.* (2001) assume that agents display loss aversion with respect to fluctuations in their financial wealth. Thus, the loss in utility following from a drop in asset prices and financial wealth is larger than the utility gain from a similar-sized increase. As illustrated in section 3, the introduction of an asymmetric policy rule implies a dampening of the drops in asset prices and an amplification of the increases. If the central bank believes that agents have preferences of the type suggested by Barberis *et al.* (2001), the asymmetric policy could therefore be an attempt to cushion agents from the utility losses when asset prices decline. As agents are assumed to derive utility from *changes* in asset prices (as opposed to the level), this story would be consistent with the result that the central bank is reacting to *changes* in stock prices. Note the distinction that in this case, changes in asset prices would be entering the reaction function of the central bank not because of their effects on other variables of interest, such as output and inflation, but as a separate target variable entering the underlying loss function of the central bank. Loss aversion with respect to changes in financial wealth could therefore serve as a potential welfare-based motivation for an asymmetric loss function.

5 Concluding Remarks

The present paper provides some theoretical inputs to the recent debate concerning a potentially asymmetric reaction of monetary policy to stock prices. I demonstrate that an asymmetric policy towards the stock market will translate into an asymmetric business cycle. Booms in output following expansionary shocks will tend to be amplified, while recessions will be dampened. A similar pattern emerges for inflation. This could be motivated by assuming that the desire of the policymaker is to minimize an asymmetric loss function, or by the existence of other asymmetries in the economy. I show that if the financial accelerator or the stock wealth effect is assumed to be non-linear over the business cycle, an asymmetric monetary policy can obtain symmetric outcomes in response to supply shocks, but only partly alleviate such asymmetries after demand shocks.

Although an asymmetric policy reaction to stock prices might be useful in order to eliminate or mitigate other asymmetries, it also implies a risk of creating moral hazard

[13] Similar to the explanation in footnote 18 for the financial accelerator, asymmetric wealth effects induce a kink in both the AD curve (through the direct wealth effect) and the AS curve (through the intratemporal labor-leisure choice). An asymmetric policy can eliminate the kink in the AD curve but not in the AS curve.

problems by effectively insulating stock market investors from part of their downside risk. As a matter of fact, this has been at the heart of the critique of the pre-crisis consensus and the plea to take the 'a' out of 'asymmetry' in the policy approach to asset prices (Mishkin, 2010; Issing, 2011). The potential moral hazard problems of an asymmetric monetary policy towards the stock market was already analyzed by Miller *et al.* (2001). In the present paper, this issue is linked to the anticipation boom in asset prices that arises following expansionary shocks as a result of the asymmetric policy. However, I have not attempted to analyze the potential moral hazard problems in detail. A comprehensive study of how an asymmetric monetary policy can cause moral hazard problems by distorting the incentives of the individual investor would require an even richer microfoundation than that of the present paper, explicitly modelling the investment decision. While this is surely an interesting idea for future research, it is beyond the scope of this paper.

The present paper follows most of the modern macroeconomic literature by log-linearizing the equilibrium conditions around a steady state. Thus, by construction, the economy eventually returns to the same steady state following a shock. This inherent limitation implies that it is not possible to study whether the asymmetric policy might push the economy to a new steady state. For example, if economic booms are consistently stronger and longer than recessions, as suggested by the impulse responses, one would eventually expect a 'level' effect on output. Another question is whether the asymmetric policy will sooner or later drive the interest rate to its zero lower bound. Due to the limitations of the log-linear approach, the model above does not have much to say about such issues.

Another question left for future research is how this type of monetary policy alters the conditions for equilibrium determinacy. The numerical solution method does not allow an immediate answer to this question, unlike in standard models. Moreover, as discussed by Davig and Leeper (2007), the introduction of regime-switching has implications for equilibrium determinacy through its effects on expectations formation. It would be interesting to conduct a formal analysis of this question when the monetary policy rule contains an asymmetric stock price reaction.

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Impulse Responses

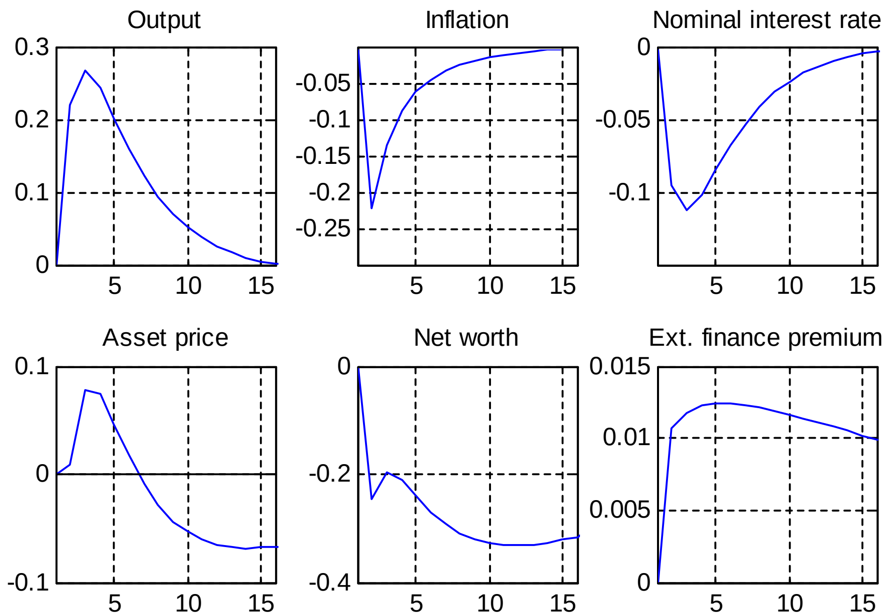

Figure 1: Effects of a positive technology shock, no policy reaction to asset prices

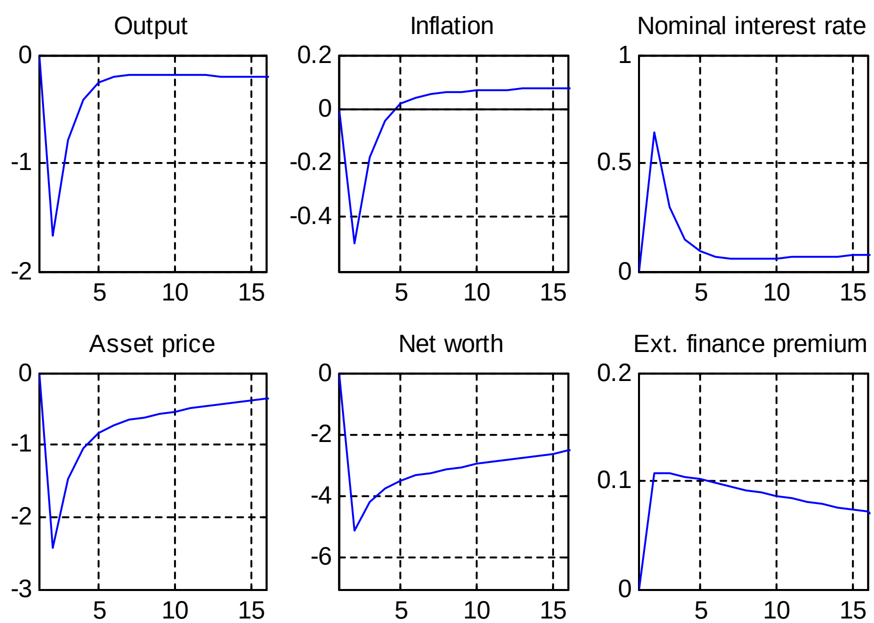

Figure 2: Effects of a contractionary monetary policy shock, no reaction to asset prices

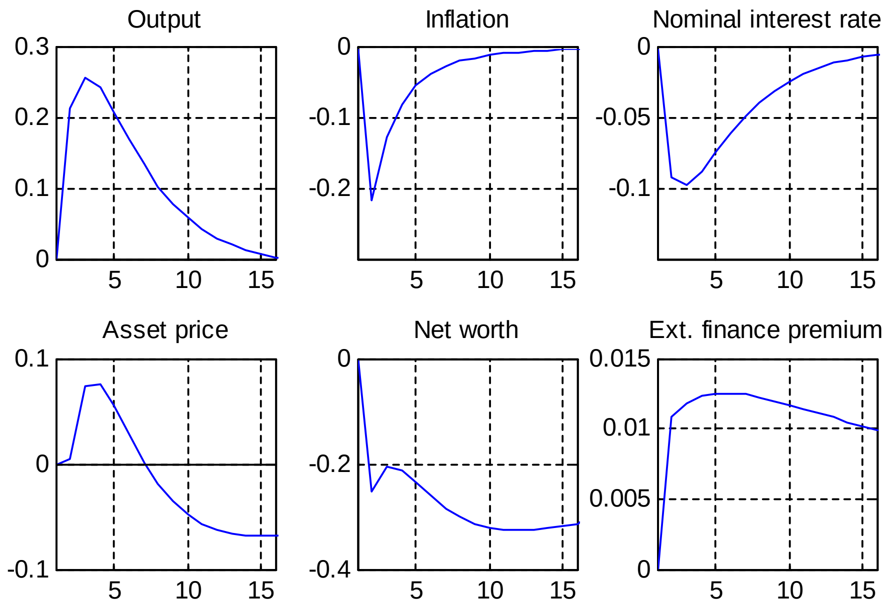

Figure 3: Positive technology shock, symmetric policy reaction to asset prices $(\phi_q = 0.5)$

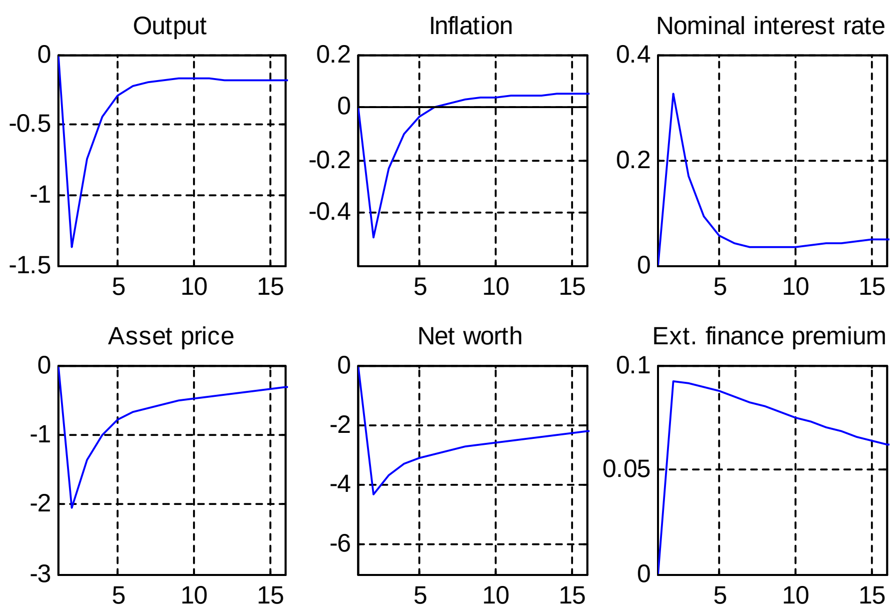

Figure 4: Contractionary monetary policy shock, symmetric reaction to asset prices $(\phi_q = 0.5)$

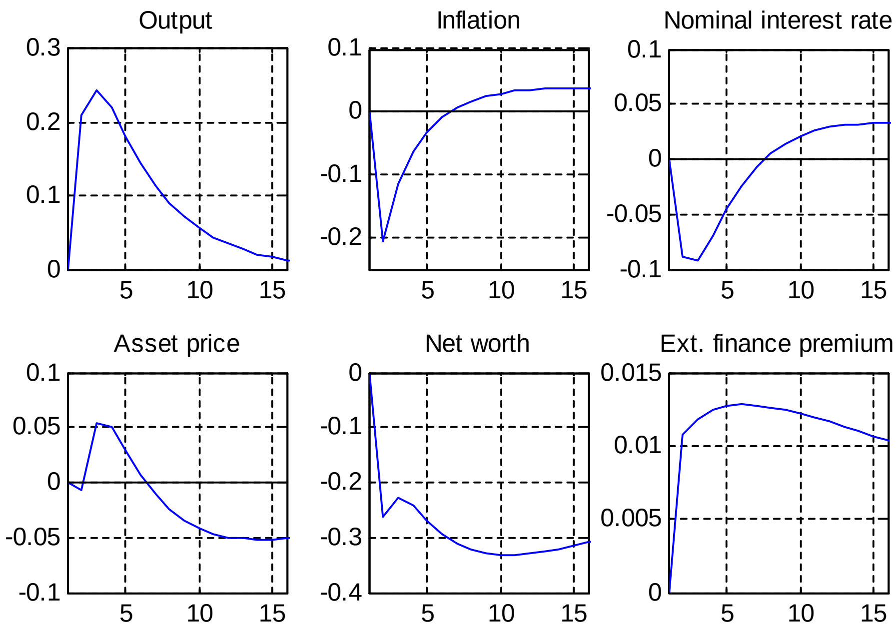

Figure 5: Positive technology shock, symmetric reaction to asset price deviation $(\phi_q = 0.5)$

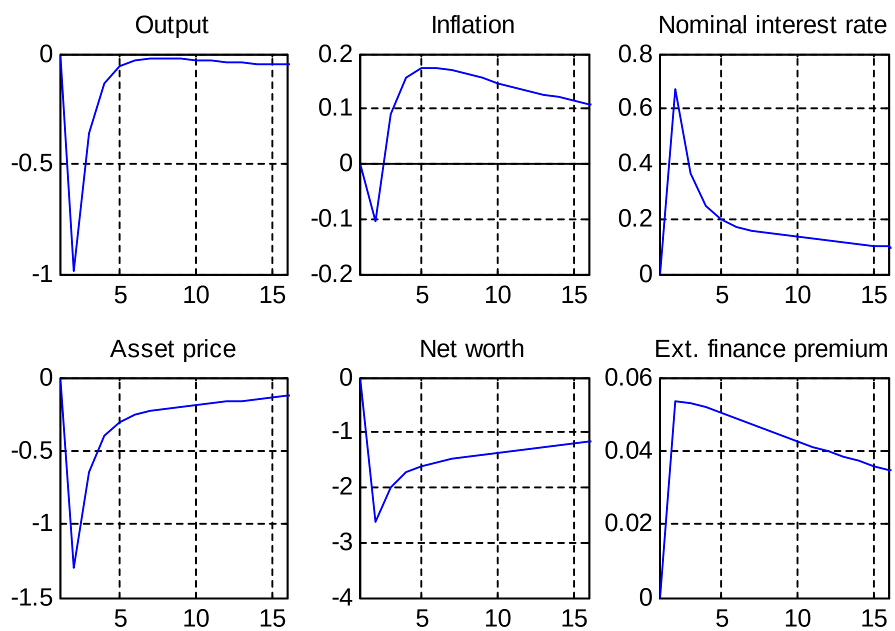

Figure 6: Contractionary monetary policy shock, symmetric reaction to asset price deviation $(\phi_q = 0.5)$

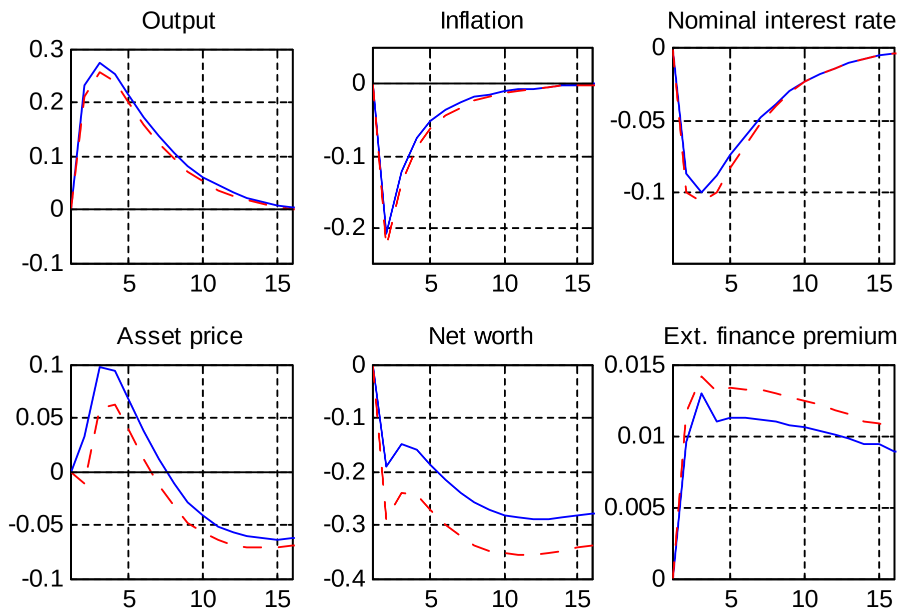

Figure 7: Effects of a technology shock, asymmetric model. *Solid blue line:* Positive shock. *Dashed red line:* Mirror image of negative shock.

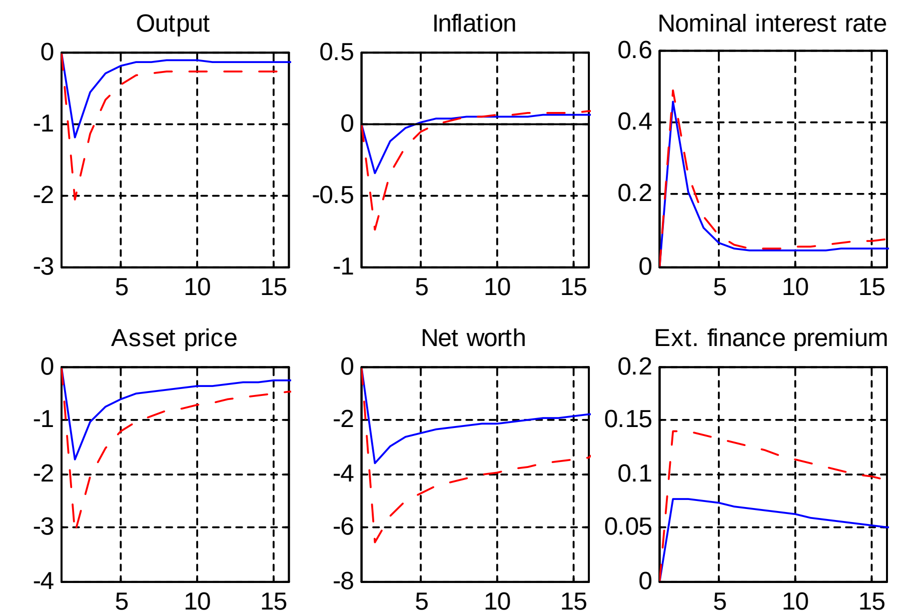

Figure 8: Effects of a monetary policy shock, asymmetric model. *Solid blue line:* Contractionary shock. *Dashed red line:* Mirror image of expansionary shock.

Table 1: Asymmetric financial accelerator, technology shocks

Value of ϕ_q	**Value of** ψ_L	**Calibrated value of** ψ_H	**Ratio** $\frac{\psi_H}{\psi_L}$
0.0246	0.042	0.043	1.02
0.0139	0.042	0.0475	1.13
0.50	0.042	0.059	1.40

Table 2: Asymmetric financial accelerator, monetary policy shocks

Value of ϕ_q	**Value of** ψ_L	**Calibrated value of** ψ_H	**Ratio** $\frac{\psi_H}{\psi_L}$
0.0246	0.042	0.0455	1.08
0.139	0.042	0.0605	1.44
0.50	0.042	0.0745	1.77

6 Appendix: Model Details

This appendix describes the calibration, the equilibrium conditions, and the solution method.

6.1 Equilibrium Conditions

The first step is to present the conditions which must hold in equilibrium, and the details underlying a few of them.

6.1.1 Household First-order Conditions

As described in the main paper, the problem of the representative household is the following:

$$\max_{C_t,H_t,D_t} U = E_0 \sum_{t=0}^{\infty} \beta^t u\left(C_t, H_t\right),$$

where the instantaneous utility function is given by

$$u\left(C_t, H_t\right) = \frac{\gamma}{\gamma-1} \ln\left(C_t^{\frac{\gamma-1}{\gamma}}\right) + \eta \ln\left(1-H_t\right),$$

and subject to the following budget constraint:

$$C_t + \frac{D_t - R_{t-1}D_{t-1}}{P_t} \leq \frac{W_t}{P_t} H_t + \Omega_t.$$

This problem gives rise to the following first-order conditions:

$$\lambda_t = C_t^{-1}, \tag{I}$$

$$\frac{\eta}{1-H_t} = \lambda_t w_t, \tag{II}$$

$$\frac{\lambda_t}{R_t} = \beta E_t \frac{\lambda_{t+1}}{\pi_{t+1}}, \tag{III}$$

Conditions (I), (II), and (III) are the first-order equations describing optimal behaviour by the representative household.

6.1.2 Optimal Pricing Behaviour of Retail Firms

The problem of retail firm i is to set the optimal price $P_t^n\left(i\right)$ so as to maximize its profits:

$$\max_{P_t^n(i)} \; E_0 \left\{ \sum_{s=0}^{\infty} \left(\beta\xi\right)^s \frac{\lambda_{t+s}}{\lambda_t} Y_{t+s}\left(i\right)\left[P_t^n\left(i\right)\pi^s - P_{t+s} mc_{t+s}\right] \right\},$$

subject to the demand function

$$Y_{t+s}(i) = \left[\frac{P_{t+s}^n(i)}{P_{t+s}}\right]^{-\epsilon^p} Y_{t+s},$$

with aggregate demand for the final good given by:

$$Y_t = \left[\int_0^1 Y_t(i)^{(\epsilon^p-1)/\epsilon^p} di\right]^{\epsilon^p/(\epsilon^p-1)}.$$

Note that as in the main text, mc_t denotes the real marginal cost of the entrepreneurs. Since these operate under perfect competition, they set their output price equal to their marginal cost, which therefore becomes the input price faced by retailers. As all entrepreneurs are identical, their marginal cost is the same. The first-order condition with respect to the choice of $P_t^n(i)$ becomes:

$$E_t \sum_{s=0}^{\infty} (\beta\xi)^s \frac{\lambda_{t+s}}{\lambda_t} \Theta_{t+s}^p = 0,$$

where $\Theta_{t+s}^p =$

$$(-\epsilon^p) Y_{t+s} \left[\frac{P_{t+s}^n(i)}{P_{t+s}}\right]^{-\epsilon^p-1} \frac{1}{P_{t+s}} \left[P_t^n(i)\pi^s - P_{t+s}mc_{t+s}\right] + Y_{t+s}\pi^s \left[\frac{P_{t+s}^n(i)}{P_{t+s}}\right]^{-\epsilon^p}.$$

Using the definition of $Y_{t+s}(i)$ from the demand function above, this expression can be rewritten as:

$$0 = E_t \sum_{s=0}^{\infty} (\beta\xi)^s \frac{\lambda_{t+s}}{\lambda_t} \left[(-\epsilon^p) Y_{t+s}(i) \frac{1}{P_t^n(i)} \left[P_t^n(i)\pi^s - P_{t+s}mc_{t+s}\right] + Y_{t+s}(i)\pi^s\right] \quad (\Leftrightarrow)$$

$$0 = E_t \sum_{s=0}^{\infty} (\beta\xi)^s \frac{\lambda_{t+s}}{\lambda_t} \left[(1-\epsilon^p) Y_{t+s}(i)\pi^s + \epsilon^p Y_{t+s}(i) \frac{P_{t+s}mc_{t+s}}{P_t^n(i)}\right] \quad (\Leftrightarrow)$$

$$\frac{1}{\lambda_t} E_t \sum_{s=0}^{\infty} (\beta\xi)^s \lambda_{t+s} (\epsilon^p - 1) Y_{t+s}(i)\pi^s = \frac{1}{\lambda_t} E_t \sum_{s=0}^{\infty} (\beta\xi)^s \lambda_{t+s}\epsilon^p Y_{t+s}(i) \frac{P_{t+s}mc_{t+s}}{P_t^n(i)} \quad (\Leftrightarrow)$$

$$P_t^n(i) = \frac{\epsilon^p}{\epsilon^p - 1} \frac{E_t\left\{\sum_{s=0}^{\infty} (\beta\xi)^s \lambda_{t+s} Y_{t+s}(i) P_{t+s}mc_{t+s}\right\}}{E_t\left\{\sum_{s=0}^{\infty} (\beta\xi)^s \lambda_{t+s} Y_{t+s}(i)\pi^s\right\}},$$

which is the expression for the first-order condition presented in the main paper. Finally, since each firm that is allowed to change its price in a given period will set the same price, I can drop the index i to obtain an expression for the new price being set in any period:

$$P_t^n = \frac{\epsilon^p}{\epsilon^p - 1} \frac{E_t \left\{ \sum_{s=0}^{\infty} (\beta\xi)^s \lambda_{t+s} Y_{t+s} P_{t+s} mc_{t+s} \right\}}{E_t \left\{ \sum_{s=0}^{\infty} (\beta\xi)^s \lambda_{t+s} Y_{t+s} \pi^s \right\}},$$

which is the expression used to derive the equilibrium of the model.

6.1.3 Equilibrium Conditions

The model consists of 15 equations in 15 variables.The equilibrium conditions are summarized below, and the variables are: $\left(C_t, H_t, Y_t, K_t, n_t, I_t, \pi_t, R_t, w_t, mc_t, mp_t, q_t, f_t, \lambda_t, \left(\frac{P_t^n}{P_t} \right) \right)$. In equilibrium, the production technology constraint (eq. (19) below) will hold with equality. Moreover, with respect to the main paper, the law of motion for capital (eq. (25) below) and the aggregate resource constraint (30) are needed to fully describe the equilibrium. The remaining conditions have all been described in the main paper or above.

$$\lambda_t = C_t^{-1} \tag{16}$$

$$\frac{\eta}{1 - H_t} = \lambda_t w_t \tag{17}$$

$$\frac{\lambda_t}{R_t} = \beta E_t \frac{\lambda_{t+1}}{\pi_{t+1}} \tag{18}$$

$$Y_t = (A_t H_t)^{1-\alpha} K_t{}^{\alpha} \tag{19}$$

$$mp_t = \alpha \frac{Y_t}{K_t} mc_t \tag{20}$$

$$w_t = (1 - \alpha) \frac{Y_t}{H_t} mc_t \tag{21}$$

$$E_t [f_{t+1}] = E_t \left[\frac{mp_{t+1} + (1 - \delta) q_{t+1}}{q_t} \right] \tag{22}$$

$$E_t [f_{t+1}] = E_t \left[\Psi \left(\frac{n_{t+1}}{q_t K_{t+1}} \right) \frac{R_t}{\pi_{t+1}} \right] \tag{23}$$

$$n_{t+1} = \nu \left[f_t q_{t-1} K_t - E_{t-1} f_t (q_{t-1} K_t - n_t) \right] + (1 - \nu) \Upsilon_t \tag{24}$$

$$K_{t+1} = I_t + (1 - \delta) K_t \tag{25}$$

$$q_t - \chi \left(\frac{I_t}{K_t} - \delta \right) = 1 \tag{26}$$

$$P_t^n = \frac{\epsilon^p}{\epsilon^p - 1} \frac{E_t \left\{ \sum_{s=0}^{\infty} (\beta\xi)^s \lambda_{t+s} Y_{t+s} mc_{t+s} P_{t+s} \right\}}{E_t \left\{ \sum_{s=0}^{\infty} (\beta\xi)^s \lambda_{t+s} Y_{t+s} \pi^s \right\}} \tag{27}$$

$$P_t = \left[(1-\xi) \left(P_t^n\right)^{1-\epsilon^p} + \xi \left(P_{t-1}\pi\right)^{1-\epsilon^p} \right]^{1/(1-\epsilon^p)} \tag{28}$$

$$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R}\right)^{\rho_r} \left[\left(\frac{\pi_t}{\pi}\right)^{\phi_\pi} \left(\frac{Y_t}{Y}\right)^{\phi_y} \left\{ \left(\frac{\Delta q_t}{q}\right)^{\phi_q} \right\}^{1[\Delta q_t < 0]} \right]^{(1-\rho_r)} e^{\varepsilon_t^r} \tag{29}$$

$$Y_t = C_t + I_t \tag{30}$$

I further need to assume a functional form for how the external finance premium depends on firms' net worth, i.e. the function $\Psi(\cdot)$. I specify the following functional form:

$$\Psi\left(\frac{n_{t+1}}{q_t K_{t+1}}\right) = \left(\frac{n_{t+1}}{q_t K_{t+1}}\right)^{-\psi}, \tag{$\lozenge$}$$

where $\psi > 0$ measures the elasticity of the external finance premium with respect to the capital position of the firms. This specification satisfies $\Psi'(\cdot) < 0$ and follows Christensen and Dib (2008) and Gilchrist and Saito (2008).

6.2 The Steady State

The steady state of the model requires that all the endogenous variables are constant, giving rise to the following conditions:

$$\lambda = C^{-1} \tag{31}$$

$$\frac{\eta}{1-H} = \lambda w \tag{32}$$

$$R = \frac{\pi}{\beta} \tag{33}$$

$$Y = (AH)^{1-\alpha} K^\alpha \tag{34}$$

$$\frac{mp}{mc} = \alpha \frac{Y}{K} \tag{35}$$

$$\frac{w}{mc} = (1-\alpha)\frac{Y}{H} \tag{36}$$

$$f = mp + 1 - \delta \tag{37}$$

$$f = \left(\frac{n}{qK}\right)^{-\psi}\frac{R}{\pi} \tag{38}$$

$$1 = \nu f \tag{39}$$

$$I = \delta K \tag{40}$$

$$q = 1 \tag{41}$$

$$mc = \frac{\epsilon^p - 1}{\epsilon^p} \tag{42}$$

$$Y = C + I \tag{43}$$

In addition, recall that I calibrated the steady state values of the variable π and the ratio $\frac{K}{n}$. Equation (38) imposes the functional form for $\Psi(\cdot)$ specified in equation ($\Diamond$) above. In the steady state version of the law of motion of net worth (39), I have assumed that bequests from entrepreneurs leaving the economy (Υ) are small and drop out of the model. This follows the related literature, see Christensen and Dib (2008) or Gilchrist and Saito (2008).

6.3 Calibration

The model is calibrated using the estimated values from Christensen and Dib (2008). For those parameters that were not estimated in that study, I use the calibrated values from that study to the extent possible. As described in the main text, exceptions include the parameters of the monetary policy rule, as the rule estimated by Christensen and Dib differs substantially from that of the present model.

Moreover, Christensen and Dib (2008) do not impose steady state conditions (38) and (39) presented above. In fact, with their choice of the relevant parameters (which are calibrated, not estimated, in their study), these conditions do not hold. On the contrary, Gilchrist and Saito (2008) do impose these conditions. I therefore follow Gilchrist and Saito and set $\frac{K}{n} = 1.8$ and $\beta = 0.984$. Keeping the estimated value of the key parameter

$\psi = 0.042$ found by Christensen and Dib (2008), this yields a steady state value of the external finance premium of $\Psi\left(\frac{n}{qK}\right) = \left(\frac{n}{qK}\right)^{-\psi} = 1.0250$. Equation (38) then implies a steady state external financing *cost* of $f = 1.0417$, and (39) then in turn implies that the survival rate of entrepreneurs must be set to $\nu = 0.960$, i.e. slightly lower than the value of 0.9728 chosen by Christensen and Dib (2008).

Parameter	**Interpretation**	**Value**
α	Capital share in production	0.3384
β	Discount factor	0.984
γ	Preference for consumption	0.0598
δ	Depreciation rate	0.025
ϵ^p	Elasticity of substitution between final goods	6
η	Preference for leisure	1.315
ν	Entrepreneurs' survival rate	0.9600
ξ	Probability of not adjusting price	0.7418
ρ_a	Persistence in technology process	0.7625
χ	Importance of capital adjustment cost	0.5882
ψ	Elasticity of ext. fin. premium wrt. leverage	0.042
$\overline{\Psi}$	Steady state external finance premium	1.0250
π	Steady state inflation rate	1
$\frac{K}{n}$	Rate of capital to net worth in steady state	1.8
ρ_r	Degree of interest rate smoothing	0.67
ϕ_π	Monetary policy reaction to inflation	1.5
ϕ_y	Monetary policy reaction to output	0.2
ϕ_q	Monetary policy reaction to stock price drops (estimated)	0.0246
ϕ_q	Monetary policy reaction to stock price drops (calibrated)	0.5

6.4 Log-linearizing the Equilibrium Conditions

The next step is to log-linearize the conditions describing the equilibrium; (16)-(30), around the steady state described above. For details about log-linearization, see for example Uhlig (1999) or Woodford (2003). In the following, $\widehat{x}_t$ will denote the log-deviation of variable x_t from its value in the nonstochastic steady state; denoted x.

Below, I derive the log-linearized equations, presenting the calculations as I find necessary. First, log-linearize (16):

$$\lambda\left(1+\widehat{\lambda}_t\right) = C\left(1-\widehat{C}_t\right) \Leftrightarrow$$

$$\widehat{\lambda}_t = -\widehat{C}_t \tag{44}$$

To log-linearize (17), first rewrite it as $\eta = \lambda_t w_t - \lambda_t w_t H_t$. Then log-linearize to get:

$$\eta = \lambda w \left(1 + \widehat{\lambda}_t + \widehat{w}_t\right) - \lambda w H \left(1 + \widehat{\lambda}_t + \widehat{w}_t + \widehat{H}_t\right).$$

Now use (32) to substitute in for η, cancel out terms, and rearrange to get:

$$H\widehat{H}_t = (1 - H)\left(\widehat{\lambda}_t + \widehat{w}_t\right). \tag{45}$$

From (18), I get:

$$\frac{\lambda}{R}\left(1 + \widehat{\lambda}_t - \widehat{R}_t\right) = \beta\frac{\lambda}{\pi}E_t\left(1 + \widehat{\lambda}_{t+1} - \widehat{\pi}_{t+1}\right).$$

Using the steady state condition that $\pi = \beta R$, I get:

$$\widehat{\lambda}_t = E_t\widehat{\lambda}_{t+1} - E_t\widehat{\pi}_{t+1} + \widehat{R}_t. \tag{46}$$

The log-linearization of (19) results in:

$$Y\left(1 + \widehat{Y}_t\right) = (AH)^{1-\alpha}K^{\alpha}\left(1 + (1-\alpha)\widehat{A}_t + (1-\alpha)\widehat{H}_t + \alpha\widehat{K}_t\right).$$

Recall from (34) that in steady state, we have: $Y = (AH)^{1-\alpha}K^{\alpha}$. Use this to get:

$$\widehat{Y}_t = (1-\alpha)\widehat{A}_t + (1-\alpha)\widehat{H}_t + \alpha\widehat{K}_t. \tag{47}$$

From (20), and using (35), it is straightforward to get:

$$\widehat{mp}_t = \widehat{Y}_t + \widehat{mc}_t - \widehat{K}_t. \tag{48}$$

Similarly, log-linearize (21) and use (36) to get:

$$\widehat{w} = \widehat{Y}_t + \widehat{mc}_t - \widehat{H}_t. \tag{49}$$

From (22), I get (lagging the equation by one period for convenience; recognizing that it must hold in every period):

$$fq\left(1 + \widehat{f}_t + \widehat{q}_{t-1}\right) = mp\left(1 + \widehat{mp}_t\right) + (1-\delta)\,q\left(1 + \widehat{q}_t\right).$$

Using (37) and the fact that in steady state; $q = 1$:

$$\widehat{f}_t = \frac{mp}{f}\widehat{mp}_t + \frac{1-\delta}{f}\widehat{q}_t - \widehat{q}_{t-1}. \tag{50}$$

Equation (23) log-linearized becomes:

$$f\left(1+E_t\widehat{f}_{t+1}\right)=\left(\frac{n}{qK}\right)^{-\psi}\frac{R}{\pi}\left[1+\widehat{R}_t-E_t\widehat{\pi}_{t+1}-\psi\left(\widehat{n}_{t+1}-\widehat{q}_t-\widehat{K}_{t+1}\right)\right],$$

which can be rewritten as:

$$E_t\widehat{f}_{t+1}-\left(\widehat{R}_t-E_t\widehat{\pi}_{t+1}\right)=-\psi\left(\widehat{n}_{t+1}-\widehat{q}_t-\widehat{K}_{t+1}\right). \tag{51}$$

Note that in deriving (51), I use the functional form for $\Psi(\cdot)$ imposed in ($\Diamond$). Recall that the parameter ψ (which measures the elasticity of that function) is larger than zero.

The log-linearization of (24) is not entirely straightforward and deserves some attention. First, substitute in for $E_{t-1}f_t$ by lagging (23) one period:

$$n_{t+1}=\nu\left[f_tq_{t-1}K_t-E_{t-1}\left[\Psi\left(\frac{n_t}{q_{t-1}K_t}\right)\frac{R_{t-1}}{\pi_t}\right](q_{t-1}K_t-n_t)\right]+(1-\nu)\Upsilon_t.$$

As mentioned, I follow the literature and assume that bequests (Υ_t) are small and drop out of the model. Log-linearizing then yields:

$$\begin{aligned}
n\left(1+\widehat{n}_{t+1}\right) &= \nu fqK\left[1+\widehat{f}_t+\widehat{q}_{t-1}+\widehat{K}_t\right]\\
&\quad-\nu\left(\frac{n}{qK}\right)^{-\psi}\frac{R}{\pi}qK\left[1+\widehat{R}_{t-1}-\widehat{\pi}_t-\psi\left(\widehat{n}_t-\widehat{q}_{t-1}-\widehat{K}_t\right)+\widehat{q}_{t-1}+\widehat{K}_t\right]\\
&\quad+\nu\left(\frac{n}{qK}\right)^{-\psi}\frac{R}{\pi}n\left[1+\widehat{R}_{t-1}-\widehat{\pi}_t-\psi\left(\widehat{n}_t-\widehat{q}_{t-1}-\widehat{K}_t\right)+\widehat{n}_t\right].
\end{aligned}$$

Now use the steady state conditions (38) and (41) to get:

$$\begin{aligned}
\frac{n}{\nu}\left(1+\widehat{n}_{t+1}\right) &= fK\left[1+\widehat{f}_t+\widehat{q}_{t-1}+\widehat{K}_t\right]\\
&\quad-fK\left[1+\widehat{R}_{t-1}-\widehat{\pi}_t-\psi\left(\widehat{n}_t-\widehat{q}_{t-1}-\widehat{K}_t\right)+\widehat{q}_{t-1}+\widehat{K}_t\right]\\
&\quad+fn\left[1+\widehat{R}_{t-1}-\widehat{\pi}_t-\psi\left(\widehat{n}_t-\widehat{q}_{t-1}-\widehat{K}_t\right)+\widehat{n}_t\right].
\end{aligned}$$

Next, cancel out terms and simplify, and obtain:

$$\begin{aligned}
\frac{1}{\nu f}\left(1+\widehat{n}_{t+1}\right) &= \frac{K}{n}\widehat{f}_t+\left(1-\frac{K}{n}\right)\left(\widehat{R}_{t-1}-\widehat{\pi}_t\right)+\left(1-\frac{K}{n}\right)\left(\widehat{q}_{t-1}+\widehat{K}_t\right)\\
&\quad+\left[1+\psi\left(\frac{K}{n}-1\right)\right]\widehat{n}_t+1.
\end{aligned}$$

Finally, from (39) we have that in steady state, $\nu f = 1$ must hold. Imposing this condition yields the log-linearized equation:

$$\widehat{n}_{t+1} = \frac{K}{n}\widehat{f}_t + \left(1 - \frac{K}{n}\right)\left(\widehat{R}_{t-1} - \widehat{\pi}_t\right) + \left(1 - \frac{K}{n}\right)\left(\widehat{q}_{t-1} + \widehat{K}_t\right) + \left[1 + \psi\left(\frac{K}{n} - 1\right)\right]\widehat{n}_t. \tag{52}$$

From (25), and using steady state relation (40), I get:

$$\widehat{K}_{t+1} = \frac{I}{K}\widehat{I}_t + (1-\delta)\widehat{K}_t. \tag{53}$$

The log-linear version of (26) is:

$$\widehat{q}_t = \chi\left(\widehat{I}_t - \widehat{K}_t\right). \tag{54}$$

The log-linearized version of the monetary policy rule (29) is:

$$\widehat{R}_t = \rho_r \widehat{R}_{t-1} + (1-\rho_r)\left[\phi_\pi \widehat{\pi}_t + \phi_y \widehat{Y}_t + \phi_q \left[\Delta\widehat{q}_t < 0\right]\Delta\widehat{q}_t\right] + \varepsilon_t^r. \tag{55}$$

- where $\Delta\widehat{q}_t = \frac{\Delta q_t}{q}$, and where $[\Delta\widehat{q}_t < 0]$ is the indicator function; equal to 1 if the change in stock prices is negative, and zero otherwise.

In log-linear terms, (30) becomes:

$$Y\widehat{Y}_t = C\widehat{C}_t + I\widehat{I}_t. \tag{56}$$

Finally, I show below how (27) and (28) can be combined to yield a log-linear version of the so-called New-Keynesian Phillips Curve (Woodford, 2003). Start by log-linearizing (28). Recall that I calibrated the steady state value of the gross inflation rate to $\pi = 1$, which I impose in the following calculations.

$$P^{1-\epsilon^p}\left(1 + (1-\epsilon^p)\widehat{P}_t\right) = (1-\xi)(P^n)^{1-\epsilon^p}\left(1 + (1-\epsilon^p)\widehat{P_t^n}\right) + \xi P^{1-\epsilon^p}\left(1 + (1-\epsilon^p)\widehat{P}_{t-1}\right).$$

Recognizing that in steady state, it must hold that $P^n = P$, I can cancel out terms and rewrite as:

$$(1-\epsilon^p)\widehat{P}_t = (1-\xi)(1-\epsilon^p)\widehat{P_t^n} + \xi(1-\epsilon^p)\widehat{P}_{t-1},$$

which then further collapses to:

$$\widehat{P}_t = (1-\xi)\widehat{P_t^n} + \xi\widehat{P}_{t-1}. \tag{\#}$$

Next, define $\theta \equiv \frac{\epsilon^p}{\epsilon^p - 1}$, and rewrite (27) as:

$$P_t^n E_t \left\{ \sum_{s=0}^{\infty} (\beta\xi)^s \lambda_{t+s} Y_{t+s} \pi^s \right\} = \theta E_t \left\{ \sum_{s=0}^{\infty} (\beta\xi)^s \lambda_{t+s} Y_{t+s} mc_{t+s} P_{t+s} \right\}. \qquad (\#\#)$$

For the sake of tractability, let us first consider only the left hand side of (##). Writing out the sum, I get:

$$LHS = P_t^n E_t \left[\lambda_t Y_t + \beta\xi\lambda_{t+1} Y_{t+1} \pi + ... \right].$$

Log-linearize this expression to get:

$$LHS = P^n \lambda Y \left(1 + \widehat{P_t^n} + \widehat{\lambda}_t + \widehat{Y}_t \right) + \beta\xi P^n \lambda Y \pi \left(1 + \widehat{P_t^n} + \widehat{\lambda}_{t+1} + \widehat{Y}_{t+1} \right) + ...$$

Recollect the sums:

$$LHS = P^n \lambda Y \sum_{s=0}^{\infty} (\beta\xi)^s \widehat{P_t^n} + P^n \lambda Y E_t \sum_{s=0}^{\infty} (\beta\pi\xi)^s \widehat{\lambda}_{t+s} + \widehat{Y}_{t+s}.$$

Using the formula for an infinite sum, and the condition $\pi = 1$, this gives:

$$LHS = P^n \lambda Y \frac{1}{1 - \beta\xi} \widehat{P_t^n} + P^n \lambda Y E_t \sum_{s=0}^{\infty} (\beta\xi)^s \widehat{\lambda}_{t+s} + \widehat{Y}_{t+s}. \qquad (\Delta)$$

Now, consider the right hand side of (##). Importantly, this features the *real* marginal cost. Proceeding as above, I can write this as:

$$RHS = \theta E_t \left[\lambda_t Y_t mc_t P_t + \beta\xi\lambda_{t+1} Y_{t+1} mc_{t+1} P_{t+1} + ... \right].$$

In log-linear terms, this becomes (imposing $\pi = 1$):

$$RHS = \theta\lambda Y mc P \left(1 + \widehat{\lambda}_t + \widehat{Y}_t + \widehat{mc}_t + \widehat{P}_t \right) + \theta\beta\xi\lambda Y mc P \left(1 + \widehat{\lambda}_{t+1} + \widehat{Y}_{t+1} + \widehat{mc}_{t+1} + \widehat{P}_{t+1} \right) + ...$$

Now use the steady state condition that $mc = \frac{\epsilon^p - 1}{\epsilon^p} = \frac{1}{\theta}$, and recollect the sum:

$$RHS = \lambda Y P E_t \left[\sum_{s=0}^{\infty} (\beta\xi)^s \left(\widehat{\lambda}_{t+s} + \widehat{Y}_{t+s} + \widehat{mc}_{t+s} + \widehat{P}_{t+s} \right) \right]. \qquad (\Delta\Delta)$$

Now I am ready to combine the LHS and the RHS of the original equation (##). First, use that $P = P^n$ to cancel out terms:

$$\frac{1}{1-\beta\xi}\widehat{P_t^n} + E_t\sum_{s=0}^{\infty}(\beta\xi)^s\,\widehat{\lambda}_{t+s} + \widehat{Y}_{t+s} = E_t\left[\sum_{s=0}^{\infty}(\beta\xi)^s\left(\widehat{\lambda}_{t+s} + \widehat{Y}_{t+s} + \widehat{mc}_{t+s} + \widehat{P}_{t+s}\right)\right].$$

This immediately collapses to:

$$\widehat{P_t^n} = (1-\beta\xi)\,E_t\left[\sum_{s=0}^{\infty}(\beta\xi)^s\left(\widehat{mc}_{t+s} + \widehat{P}_{t+s}\right)\right].$$

The next step is to rewrite this condition as a first-order difference equation. This gives:

$$\begin{aligned}\widehat{P_t^n} &= (1-\beta\xi)\left(\widehat{mc}_t + \widehat{P}_t\right) + (1-\beta\xi)\,\beta\xi E_t\left(\widehat{mc}_{t+1} + \widehat{P}_{t+1}\right) \\ &= (1-\beta\xi)\left(\widehat{mc}_t + \widehat{P}_t\right) + \beta\xi E_t\widehat{P_{t+1}^n}.\end{aligned}$$

Leading eq. (#) by one period, and isolating for $\widehat{P_{t+1}^n}$, I can substitute in the resulting expression:

$$\widehat{P_t^n} = (1-\beta\xi)\left(\widehat{mc}_t + \widehat{P}_t\right) + \beta\xi E_t\left(\frac{\widehat{P}_{t+1} - \xi\widehat{P}_t}{(1-\xi)}\right).$$

The final step is then to insert this expression for $\widehat{P_t^n}$ into the log-linearized price level equation (#), which yields:

$$\widehat{P}_t = (1-\xi)(1-\beta\xi)\left(\widehat{mc}_t + \widehat{P}_t\right) + (1-\xi)\,\beta\xi E_t\left(\frac{\widehat{P}_{t+1} - \xi\widehat{P}_t}{(1-\xi)}\right) + \xi\widehat{P}_{t-1}.$$

Rewrite this:

$$\widehat{P}_t - \xi\widehat{P}_{t-1} = (1-\xi)(1-\beta\xi)\left(\widehat{mc}_t + \widehat{P}_t\right) + \beta\xi E_t\widehat{P}_{t+1} - \beta\xi^2\widehat{P}_t \qquad (\Leftrightarrow)$$

$$\xi\left(\widehat{P}_t - \widehat{P}_{t-1}\right) = -(1-\xi)\,\widehat{P}_t + (1-\xi)(1-\beta\xi)\,\widehat{mc}_t + \beta\xi E_t\widehat{P}_{t+1} + (1-\beta\xi-\xi)\,\widehat{P}_t \qquad (\Leftrightarrow)$$

$$\xi\left(\widehat{P}_t - \widehat{P}_{t-1}\right) = (1-\xi)(1-\beta\xi)\,\widehat{mc}_t + \beta\xi E_t\left(\widehat{P}_{t+1} - \widehat{P}_t\right).$$

Using the fact that $\left(\widehat{P}_t - \widehat{P}_{t-1}\right) = \widehat{\pi}_t$, this can then be rewritten:

$$\widehat{\pi}_t = \beta E_t\widehat{\pi}_{t+1} + \frac{(1-\xi)(1-\beta\xi)}{\xi}\widehat{mc}_t. \qquad (57)$$

This is the log-linearized New-Keynesian Phillips Curve that enters the set of log-linearized equations used to solve the model.

6.5 Summarizing the Linearized Equilibrium Conditions

The log-linearized version of the model consists of equations (44) – (57).[14] Note that the monetary policy condition (55) is not linear, as the value of the parameter ϕ_q depends on the sign of $\Delta\widehat{q}_t$. However, the model is piecewise linear, in the sense that given one of the two possible values of ϕ_q, all equations are linear. This is the key insight underlying the solution method. I can represent each of the two linear systems in the following way, stacking the 14 equations and 14 variables:

$$0 = AE_t s_{t+1} + Bs_t + Cs_{t-1} + D\varepsilon_t \tag{58}$$

Here, the vector s contains all the relevant variables, as measured in log-deviations from their steady state values: $s_t = \left[\widehat{K}_t, \widehat{n}_t, \widehat{q}_t, \widehat{R}_t, \widehat{C}_t, \widehat{H}_t, \widehat{\lambda}_t, \widehat{f}_t, \widehat{Y}_t, \widehat{I}_t, \widehat{w}_t, \widehat{\pi}_t, \widehat{mp}_t, \widehat{mc}_t\right]'$. The matrices A, B, and C are $N \times N$ coefficient matrices, where $N = 14$ is the number of variables. Finally, $\varepsilon_t = [\varepsilon_t^a, \varepsilon_t^r]'$ is the vector of shocks, and D is a $N \times M$ coefficient matrix, with $M = 2$ representing the number of shocks. The elements of the coefficient matrices derive from the log-linear system of equations derived above.

Each of the two systems summarized as in (58) can then be solved using standard methods for solving linear rational expectations models. These methods include the Toolkit of Uhlig (1999) and the Gensys method of Sims (2002), but many other methods exist. I use Uhlig's method to solve each of the systems. This gives me a solution that can be written on the form:

$$s_t = Ps_{t-1} + Q\varepsilon_t \tag{59}$$

This illustrates that at any point in time, the set of values of the endogenous variables is fully described by the set of lagged values (in particular, the lagged values of the state variables) and the realization of the shocks in that period. This explains the appeal of the state space representation. See the subsection about the solution method for details about how to solve the overall model, given the solution to each of the two linear systems that it consists of.

Finally, following Uhlig (1999), I can establish a link between the exposition of each of the two linear systems above and that in Blanchard and Kahn (1980). To do this, I need to reformulate the second-order difference equation (58) as a first-order difference equation. The method employed by Blanchard and Kahn implicitly constructs the stacked vector $x_t' = \left[s_t', s_{t-1}'\right]$, and then proceeds by analyzing the first-order difference equation:

[14] While the model originally consisted of 15 equations in 15 variables, the log-linearized model has only 14 equations in 14 variables, as equations (27) and (28) were combined to yield one log-linearized equation; (57), making the variable $\left(\frac{P_t^n}{P_t}\right)$ redundant.

$$E_t x_{t+1} = \Omega x_t + \Xi \varepsilon_t \tag{60}$$

- where the matrices Ω and Ξ are mappings of the matrices A, B, C, and D:

$$\underset{2N\times 2N}{\Omega} = \begin{bmatrix} \underset{N\times N}{-A^{-1}B} & \underset{N\times N}{-A^{-1}C} \\ \underset{N\times N}{0} & \underset{N\times N}{0} \end{bmatrix}, \quad \underset{2N\times N}{\Xi} = \begin{bmatrix} \underset{N\times N}{-A^{-1}D} \\ \underset{N\times N}{0} \end{bmatrix}$$

Uhlig (1999) demonstrates the equivalence between solving model (58) and (60), and further discusses pro's and con's of each of the two formulations.

According to Proposition 1 in Blanchard and Kahn (1980), there exists a unique solution to the problem, and hence a determinate equilibrium, if and only if the number of non-predetermined variables in the model exactly corresponds to the number of eigenvalues of the matrix Ω that lie outside the unit circle. Hence, examining the determinacy properties of each of the two linear systems that my model comprises would be straightforward. However, as the model itself is non-linear, it cannot be represented on the form (60). As a consequence, as also pointed out in the main text, it is not possible to proceed with a traditional, formal analysis of the determinacy properties of the equilibrium of the model.

6.6 The Solution Method

Below, I present the details of the solution method used to solve the model outlined above. The method exploits the fact that while the model is not linear, it is piecewise linear; consisting of two linear systems. A number of authors have used solution methods that rely on piecewise linearity, see for instance Eggertson and Woodford (2003) or Christiano (2004). As the solution method I use follows the work of Bodenstein *et al.* (2009), this section builds on their Appendix A.

Assuming that the model starts out in steady state, the initial regime for monetary policy involves a zero reaction to stock price changes. As discussed above, the log-linearized conditions describing the equilibrium can be written on matrix form.

$$0 = \overline{A} E_t s_{t+1} + \overline{B} s_t + \overline{C} s_{t-1} + \overline{D} \varepsilon_t. \tag{61}$$

In this system, s is the vector containing all the endogenous variables as described above, $\overline{A}$, $\overline{B}$, $\overline{C}$ and $\overline{D}$ are coefficient matrices describing the dynamics of the system, and ε is the vector of shocks.

Similarly, whenever the asset price is decreasing, the dynamics of the system is described by the following set of equations, including a non-zero reaction to asset price changes:

$$0 = A^* E_t s_{t+1} + B^* s_t + C^* s_{t-1} + D^* \varepsilon_t. \tag{62}$$

Note, however, that the only difference between the two systems is the reaction of monetary policy to asset price changes; i.e., whether $\phi_q = 0$ in equation (55) or not. This affects only the matrices multiplying s_t and s_{t-1}. In other words, $\overline{A} = A^*$, and $\overline{D} = D^*$. Further, the matrices $\overline{B}$ and B^* differ in only one entry, and the same is true for $\overline{C}$ and C^*: If the monetary policy reaction function is listed as the n'th equation in the system, and the price of capital appears as the m'th variable in the vector s, then these matrices differ only in the (n,m)'th entry.

As each of these two systems are linear, they can be solved separately using well-known methods such as the Toolkit method of Uhlig (1999) or the Gensys method of Sims (2002). The solutions can then also be written on matrix form, as the evolution of the endogenous variables are fully described by the lagged values of the state variables and the realizations of the shocks. Hence, the solutions to the above systems are, respectively:

$$s_t = \overline{P} s_{t-1} + \overline{Q} \varepsilon_t, \tag{63}$$

$$s_t = P^* s_{t-1} + Q^* \varepsilon_t. \tag{64}$$

Assume that a shock hits the economy in period 0. As the economy starts out in the regime with no reaction to stock price changes, the first regime change will occur the first time the change in the asset price ($\Delta q_t = q_t - q_{t-1}$) becomes negative. Depending on the shock, this may happen on impact or after a number of periods.[15] Once the regime has shifted, it may shift back, or it may remain in the new regime.[16] In principle, an arbitrary number of regime shifts might take place, depending on the evolution of the asset price.

In order to illustrate the idea behind the solution method, consider the evolution of the asset price following a positive technology shock. This impulse response is repeated here for convenience:[17]

[15] Unless the asset price remains forever constant, however, it will happen sooner or later, as the asset price must return to its initial value.

[16] Of course, the economy will eventually return to its steady state, where the regime is always that of a zero reaction to stock price changes.

[17] The figure shows the impulse response of the asset price in the model without asymmetric policy. I first assume that the turning points under this policy are unchanged when the asymmetric policy is introduced. I then later verify that this is in fact the case.

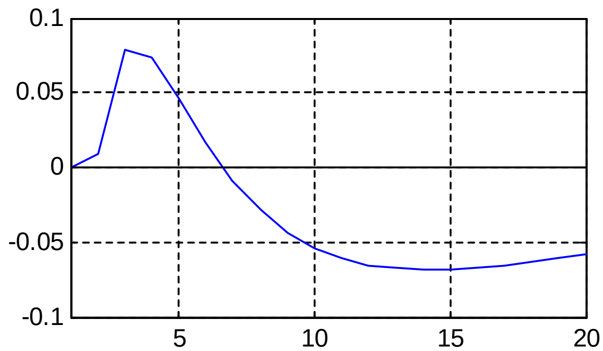

Figure B1: Impulse response of asset price to positive technology shock

Evidently, this impulse response involves two turning points; called T_1 and T_2, i.e. points where the sign of the change in the asset price switches. After the second turning point, the stock price is increasing, so the dynamics of the economy are described by the solution to the model with no reaction to asset prices (and no further shocks):

$$s_t = \overline{P} s_{t-1}, \; t > T_2. \tag{65}$$

Consider now the dynamics for $T_1 < t \leq T_2$, for which the monetary policy reaction to asset prices is non-zero. I use backward induction to trace out the evolution of the endogenous variables in these periods. As no shocks are assumed to hit the economy outside period 0, it follows from (65) that $s_{T_2+1} = \overline{P} s_{T_2}$. This is useful in the last period before the shift ($t = T_2$), where the following is true:

$$0 = \overline{A} E_t s_{T_2+1} + B^* s_{T_2} + C^* s_{T_2-1} \qquad \Leftrightarrow$$

$$0 = \left(\overline{AP} + B^*\right) s_{T_2} + C^* s_{T_2-1} \qquad \Leftrightarrow$$

$$s_{T_2} = -\left(\overline{AP} + B^*\right)^{-1} C^* s_{T_2-1} \qquad \Leftrightarrow$$

$$s_{T_2} = \Gamma_1 s_{T_2-1} \;, \; \Gamma_1 \equiv -\left(\overline{AP} + B^*\right)^{-1} C^*. \tag{66}$$

In similar fashion, I can derive an expression for the second-last period before the shift ($t = T_2 - 1$). Let $A = -\left(B^*\right)^{-1} \overline{A}$, and $C = -\left(B^*\right)^{-1} C^*$. Then;

$$0 = \overline{A} E_t s_{T_2} + B^* s_{T_2-1} + C^* s_{T_2-2} \qquad \Leftrightarrow$$

$$s_{T_2-1} = A \Gamma_1 s_{T_2-1} + C s_{T_2-2} \qquad \Leftrightarrow$$

$$s_{T_2-1} = (I - A\Gamma_1)^{-1} C s_{T_2-2}. \tag{67}$$

Thus, by recursive substitutions, I can express the endogenous variables at any point in this interval as a function of their 1-period lagged values. In the general case, I get:

$$s_t = (I - A\Gamma_{T_2-t})^{-1} C s_{t-1} \qquad \Leftrightarrow$$

$$s_t = \Gamma_{T_2-t+1} s_{t-1} \ , \ T_1 < t \leq T_2, \tag{68}$$

where, for each t;

$$\Gamma_{T_2-t+1} = (I - A\Gamma_{T_2-t})^{-1} C,$$

recalling the definition of $\Gamma_1 \equiv \left(\overline{AP} + B^*\right)^{-1} C^*$. In fact, the recursivity of the problem allows me to write s_t for each period in this interval as a function of s_{T_1+1}; the first period in this interval:

$$s_t = \left(\prod_{i=1}^{t-1} \Gamma_{T_2-i}\right) s_{T_1+1}. \tag{69}$$

In period T_1+1, the values of the endogenous variables are 'inherited' from the dynamics in the previous interval. For $t \leq T_1$, when the policy reaction to asset prices is again zero, I can similarly compute the value of s_t in each period recursively as a function of s_1. From (68), I get the following expression, which is needed to describe the last period before this first shift:

$$s_{T_1+1} = \Gamma_{T_2-T_1} s_{T_1}. \tag{70}$$

Performing recursive operations in a similar fashion to above provides me with the following expression for s_t:

$$s_t = \left(I - \widehat{A}\Theta_{T_1-t}\right)^{-1} \widehat{C} s_{t-1} \qquad \Leftrightarrow$$

$$s_t = \Theta_{T_1-t+1} s_{t-1} \ , \ 2 \leq t \leq T_1, \tag{71}$$

where, for each t;

$$\Theta_{T_1-t+1} = \left(I - \widehat{A}\Theta_{T_1-t}\right)^{-1} \widehat{C},$$

and where $\widehat{A} = -\left(\overline{B}\right)^{-1} \overline{A}$; $\widehat{C} = -\left(\overline{B}\right)^{-1} \overline{C}$; and:

$$\Theta_1 \equiv -\left(\overline{A}\Gamma_{T_2-T_1} + \overline{B}\right)^{-1}\overline{C}.$$

Finally, the special case where $t = 1$ is the only time at which the shocks take on non-zero values. I use (61) and (71) as well as the assumption that the economy starts out in steady state in period 0, implying that $s_0 = 0$. I then obtain an expression for s_1 as a function of the time 1-innovations:

$$0 = \overline{A}s_2 + \overline{B}s_1 + \overline{C}s_0 + \overline{D}\varepsilon_1 \qquad \Leftrightarrow$$

$$s_1 = \left(I - \widehat{A}\Theta_{T_1-1}\right)^{-1}\widehat{D}\varepsilon_1, \tag{72}$$

where $\widehat{D} = -\left(\overline{B}\right)^{-1}\overline{D}$. Finally, I then obtain:

$$s_t = \left(\prod_{i=1}^{t-1}\Theta_{T_1-i}\right)s_1 \qquad \Leftrightarrow$$

$$s_t = \left(\prod_{i=1}^{t-1}\Theta_{T_1-i}\right)\left(I - \widehat{A}\Theta_{T_1-1}\right)^{-1}\widehat{D}\varepsilon_1 \ , \ 2 \leq t \leq T_1. \tag{73}$$

As mentioned in the main text, the model is solved in practice by making use of a shooting algorithm to find the turning points. An initial guess for each of the turning points is needed. Given the initial guess, I then solve for s_t, $\forall t$. It is then easy to verify whether this initial guess was correct or not by simply checking whether the sign of Δq_t actually does shift for $t = T_{initial\ guess}$. If this is the case, I keep the solution. If not, I adjust my initial guess, and I 'shoot' again, until the condition is satisfied.