PhD Dissertation

Essays on Monetary and Fiscal Policy Interactions

Author:

Gitte Yding Salmansen

Supervisor:

Professor Henrik Jensen

Submitted to the Department of Economics,
University of Copenhagen
in partial fullfillment of the requirements for the degree of
PhD in Economics

August 29, 2014

Contents

Acknowledgements

I began my PhD studies at the University of Copenhagen in September 2010. These studies have been financed by Danmarks Nationalbank, for which I am most obliged. I am grateful to a number of people for supporting, guiding and supervising me through these studies. Without their inputs and helpful comments this dissertation would not have been the same. First, I would like to thank my supervisor Professor Henrik Jensen for his initial encouragement to apply for the PhD and later for his helpful comments and insights, especially during the last critical phase of my thesis writing. A meeting with Henrik always involved an extra dimension of fun non-academic conversations about music and life in general. I have appreciated these talks very much.

My journey into the field of monetary macroeconomics started when I followed Henrik's course in Macroeconomic theory, and as I spent a year of my Master visiting New York University following their first year PhD courses in Macro. This gave me the desire to pursue this PhD. During my PhD studies I had the privilege of spending a semester at two other world class institutions. I am grateful to Tomasso Monacelli from Universitá Bocconi and Stephanie Schmitt-Grohe from Columbia University in the City of New York for welcoming me at their institutions.

I have had the pleasure of having an office at Danmarks Nationalbank, and thereby access to some of the best macro- and monetary economists in the country. A warm thanks to my colleagues at the Research Division Asger Lau Andersen, Jesper Pedersen, Kim Abildgren, Mark Strøm Kristoffersen, and Søren Hove Ravn for always being very open for discussing my work and offering advice both regarding my academic work and the PhD process.

My friends and family deserve a warm thank you for supporting and encouraging me, even when my frustration with a technical problem momentarily overshadowed my general excitement about the research. Particularly, I owe a warm thanks to my parents Connie and Erik, and my sisters Anne and Mette for always reaching out, even when I buried myself in the dissertation work for extended periods. Last, but certainly not least I am deeply grateful to my husband Henrik for always listening patiently and somehow always managing to offer a positive outlook and tons of loving support.

Gitte Yding Salmansen
Copenhagen, August 2014

Summary

This dissertation consists of three self-contained chapters on monetary policy and fiscal policy. Each paper can be read separately, which can lead to some overlap and possible repetition of arguments, just as the notation changes slightly from the first paper to the two last.

All three papers concern the effects of monetary and fiscal policy in the presence of financial frictions. In the first paper this friction is a zero lower bound on the nominal interest rate, which can leave monetary policymaker unable to stimulate the economy through the traditional interest rate channel. In the two last papers, the friction is limited asset market participation of households, in the sense that a fraction of households have no access to asset markets and thus consume their disposable income every period.

The topics of my research bears witness of the turbulent times in which I started my PhD. The recent financial crisis has proved the zero lower bound (or some slightly negative lower bound) to be more than just a theoretical phenomenon. Instead, this technical limitation suddenly constrained policymakers from stimulating the economy through the usual interest rate channel. Further, the financial crisis made it painfully clear, that households' access to financial markets could be an important driver of the aggregate outcome of the economy and the economic effects of fiscal stimulus.

The first paper 'Fiscal Multipliers in a Liquidity Trap: The Effects of Endogenous Persistence' analyzes an economy, where the nominal interest rate is constrained by the zero lower bound, a situation also referred to as a liquidity trap. This means that fiscal policy can take the stage and stimulate the economy without generating a response of the nominal interest rate. I introduce a persistence channel, in which the expected duration of the liquidity trap depends negatively on output. This persistence channel makes fiscal stimulus much more potent, as any stimulus of output will reduce expected duration, which in turn makes output rise, etc.

I calculate the effect of the American Recovery and Reinvestment Act and show that introducing the persistence channel makes the output effect of this stimulus package increase from 2.31 percentage points of natural output to 2.62. This change in the output-response is substantial, given that the ARRA reduces expected duration by less than a month, indicating that our calibration of the duration channel is very conservative. The duration effect could very well be considerably stronger, highlighting the importance of the persistence channel, when considering the output effects of fiscal stimulus in a liquidity trap.

In the second paper 'Monetary Policy in a Small Open Economy with Liquidity Constrained Households' I introduce limited asset market participation (LAMP) in a small open economy. Having a fraction of households that cannot borrow nor save was shown by Galí et al. [2007] to improve the fit on their model on US consumption data.

I consider a positive technology shock and find the introduction of LAMP implies the negative response of hours is halved under a domestic PPI inflation-based Taylor rule (DITR) and hardly affected under a CPI inflation-based Taylor rule (CITR) or a fixed exchange rate (PEG). This stands in contrast to much more persistent decline in hours under LAMP, which Furlanetto and Seneca [2012] find in a closed economy.

I derive the Ramsey policy. That is, the optimal monetary policy given the agents' optimizing behavior. I then proceed to a welfare comparison which shows that the welfare effects of DITR are very similar to those of the Ramsey policy. Welfare is lowest under a fixed exchange rate. However, this should not be taken as a definitive argument against a fixed exchange rate regime, as there are many other advantages from such a regime that are not captured by our model - increased trade, a lower interest rate spread to mention a few.

In the final paper 'Government Spending in a Small Open Economy with Liquidity Constrained Households' I analyze how the introduction of LAMP affects the propagation of a positive government spending shock. Galí et al. [2007] find that LAMP makes the effects of fiscal stimulus much larger. However, this result does not carry over to our small open economies, where the introduction of LAMP only has minor effects on the output response to a government spending shock.

The mechanism driving this result is that liquidity constrained households, who increase consumption immediately due to higher disposable income, will increase domestic demand. This causes domestic prices to inflate, thereby worsening terms of trade. As a result net exports will drop, and the total effect on output is limited. We find that under a fixed nominal exchange rate the introduction of LAMP will have a small positive effect on the response of output to a government spending shock. For a floating exchange rate, however, LAMP will have a negative effect on the output response.

When the exchange rate is fixed, it cannot respond to the rise in domestic demand caused by an introduction of LAMP. This implies that net exports will drop less than consumption increases will dominate, so that the output response is increased by the introduction of LAMP. Under a floating exchange rate the introduction of LAMP implies a stronger initial nominal appreciation of the domestic currency. This appreciation generates a larger drop in net exports, which dominates the higher consumption response to the chock. As a result response of output to become lower when we introduce LAMP in a flexible exchange rate regime. To the best of my knowledge, the fact that exchange rate movements can cause a negative effect of LAMP on the output response is a new result in the literature.

Resumé

Denne afhandling består af tre selvstændige artikler om penge- og finanspolitik. Hver artikel kan læses særskilt, hvilket kan medføre en vis grad af overlap og gentagelse af argumenter, ligesom notationen skifter lidt fra den første artikel til de to sidste.

Alle tre artikler vedrører virkningerne af penge- og finanspolitik under finansielle friktioner. Den første artikel undersøger en friktion i form af rentens nedre nulgrænse, som kan efterlade pengepolitikken ude af stand til at stimulere økonomien gennem den traditionelle rentekanal. I de sidste to artikler er friktionen begrænset deltagelse i de finansielle markeder i den forstand, at en andel af husstandene ikke har adgang til handel med værdipapirer og dermed forbruger hele deres disponible indkomst hver periode.

Emnerne i min forskning vidner om de turbulente tider, hvori jeg startede min ph.d. De seneste års finansielle krise har vist, at rentens nedre nulgrænse er mere end blot et teoretisk fænomen. I stedet har denne tekniske begrænsning forhindret centralbankerne i at stimulere økonomien via den sædvanlige rentekanal. Desuden har finanskrisen gjort det tydeligt, at husholdningernes adgang til finansielle markeder kan være en vigtig drivkraft for økonomiens tilstand og for de økonomiske effekter af finanspolitik.

Den første artikel 'Fiscal Multipliers in a Liquidity Trap: The Effects of Endogenous Persistence' analyserer en økonomi, hvor den nominelle rente er begrænset af rentens nedre nulgrænse, en situation som traditionelt også kaldes en likviditetsfælde. Det betyder, at finanspolitikken kan indtage scenen og stimulere økonomien uden at generere den sædvanlige reaktion fra den nominelle rente. Jeg introducerer en varighedskanal, hvor den forventede varighed af likviditetsfælden afhænger negativt af produktionsniveauet i økonomien. Det vil sige, at jo værre en krise er, jo længere forventes den at vare. Denne varighedskanal gør finanspolitisk stimulans langt mere potent, eftersom et øget output vil reducere den forventede varighed, hvilket igen får produktionen til at stige osv.

Jeg beregner effekten af the American Recovery and Reinvestment Act og viser, at indførelsen af varighedskanalen øger output-effekten af hjælpepakken fra 2,31 procentpoint af naturligt output til 2,62 procentpoint. Denne ændring er væsentlig, eftersom hjælpepakken kun reducerer den forventede varighed med mindre end én måned, hvilket indikerer, at kalibreringen af varighedskanalen er meget konservativ. Varighedseffekten kan meget vel være betydeligt stærkere, hvilket understreger betydningen af varighedskanalen, når en finanspolitisk hjælpepakke overvejes i en likviditetsfælde.

I den anden artikel 'Monetary Policy in a Small Open Economy with Liquidity Constrained Households' introducerer jeg begrænset adgang til de finansielle markeder (LAMP) i en lille åben økonomi. Derved vil en brøkdel af alle husstande i økonomien hverken foretage indlån eller udlån. Gali et al. (2007) har vist, at denne antagelse forbedrer modellens forklaringsgrad for amerikanske forbrugsdata.

Jeg betragter et positivt teknologichok og finder, at indførelsen af LAMP indebærer,

at faldet i arbejdstimer halveres under en Taylor-regel baseret på indenlandske priser (DITR), mens LAMP har forsvindende effekt under en Taylor-regel baseret på forbrugerpriser (CITR) eller fast valutakurs (PEG). Dette står i kontrast til et langt mere vedvarende fald i arbejdstimer ved indførelsen af LAMP i en lukket økonomi, jf. Furlanetto og Seneca (2012).

Jeg udleder efterfølgende Ramsey-politikken for den lille åbne økonomi. Det vil sige, den optimale pengepolitik givet agenternes optimerende adfærd. Jeg fortager derefter en velfærdssammenligning, som viser, at de velfærdsmæssige virkninger af DITR er meget lig dem vi ser under Ramsey-politikken. Velfærden er lavest under i det faste valutakursregime. Dette bør dog ikke tages som et endeligt argument imod et fastkurs-regime, da der er mange andre fordele ved denne pengepolitik, som ikke opfanges af min model - herunder øget handel og et lavere rentespænd for at nævne et par stykker.

I den sidste artikel 'Government Spending in a Small Open Economy with Liquidity Constrained Households' analyserer jeg, hvordan indførelsen af LAMP påvirker effekterne af en stigning i det offentlige forbrug. Galí et al. (2007) finder, at LAMP øger effekten af finanspolitiske vækstpakker. Dette resultat holder dog ikke i vores lille åbne økonomi, hvori indførelsen af LAMP kun har en mindre indvirkning på den offentlige forbrugsmultiplikator.

Mekanismen bag dette resultat er, at likviditetsbegrænsede husstande, der øjeblikkeligt øger deres forbrug når de får en højere disponibel indkomst, vil øge den indenlandske efterspørgsel. Dette forårsager indenlandsk inflation, og dermed forværrede bytteforhold i udenrigshandlen. Dette får nettoeksporten til at falde, og den samlede effekt på produktionen forbliver derfor begrænset. Jeg finder således, at indførelsen af LAMP under en fast valutakurs vil have en lille positiv effekt på reaktionen af output ved et stød til det offentlige forbrug. For en flydende valutakurs vil LAMP dog have en negativ virkning på output-effekten af forbrugsstødet.

Under en fast valutakurs, kan kursen ikke reagere på stigningen i den indenlandske efterspørgsel som følge af en indførelse af LAMP. Dette indebærer, at nettoeksporten vil falde mindre end hvad forbruget stiger, således at outputstigningen øges ved indførelsen af LAMP. Under en flydende valutakurs vil indførelsen af LAMP indebære en nominel appreciering af den indenlandske valuta. Denne appreciering medfører et større fald i nettoeksporten, som vil dominere forbrugsstigningen. Samlet set vil outputstigningen derfor være lavere, når vi indfører LAMP under en flydende valutakurs. Efter min bedste overbevisning er afhandlingens sidste artikel det første papir som påpeger, at valutakursbevægelser kan forårsage en negativ output-effekt af LAMP.

References

Francesco Furlanetto and Martin Seneca. Rule-of-thumb consumers, productivity, and hours. *The Scandinavian Journal of Economics*, 114(2):658–679, 2012.

J. Galí, J.D. López-Salido, and J. Vallés. Understanding the effects of government spending on consumption. *Journal of the European Economic Association*, 5(1):227–270, 2007.

Fiscal Multipliers in a Liquidity Trap: The Effects of Endogenous Persistence*

Gitte Yding Salmansen

August 29, 2014

Abstract

We introduce a new channel through which fiscal policy can stimulate the economy in a liquidity trap. Data suggests that the level of output in a liquidity trap is correlated with the expected duration of the trap, but existing literature assumes a fixed expected duration. We instead assume that expected duration depends on output. This generates a persistence augmented multiplier which can be considerably larger than the multipliers under fixed expected duration.

We apply our model to the American Recovery and Reinvestment Act and show that for this fiscal stimulus the persistence effect increases the output effect from 2.32 percent of GDP to 2.61 percent.

*The author wishes to thank Henrik Jensen, Mark Strøm Kristoffersen, and Søren Hove Ravn for useful comments and discussions. Also thanks to seminar participants at University of Copenhagen, Danmarks Nationalbank, and 2011 Jamboree at Università Bocconi. All remaining errors are my own. The views expressed are those of the author, and do not necessarily correspond to those of Danmarks Nationalbank.

1 Introduction

In the wake of the recent financial crisis a heated debate has erupted over the efficacy of fiscal policy as a means to stimulate an economy that is in a liquidity trap.[1] As many of the large economies found their monetary policy rates close to or at the zero lower bound, and thus incapable of stimulating the economy through the traditional interest rate channel, an alternative is to turn to fiscal stimulus.

Being in the liquidity trap implies that the interaction between fiscal policy and monetary policy is muted, so there are potentially very different mechanisms at play. Under an unconstrained monetary policy following an active Taylor rule, the real and nominal interest rate will be increased following a fiscal stimulus that puts an upward pressure on prices, causing households to reduce their consumption and save more. If the economy is brought to the zero lower bound by a fundamental shock, nominal interest rate cannot respond, and the fiscal stimulus results in an erosion of the real rate, so that households increase consumption.[2] This difference between crowding out and in of households' consumption means that fiscal policy can potentially be very powerful in the liquidity trap.[3]

This new situation has caused a large debate amongst economists on the size of fiscal multipliers in the current crisis.[4] Fiscal output multipliers quantifying the general equilibrium effect of a given fiscal policy tool on the level of output in the economy. These are often used to debate the efficacy of fiscal policy tools.

As there does not exist a large amount of data that has the feature of a zero interest rate, most empirical estimates of fiscal multipliers might not apply to the current crisis, given that there are potentially different effects in the liquidity trap. One branch of the literature has tried to approximate the multipliers in the liquidity trap by estimating fiscal multipliers that depend on the level of the economic activity, see Auerbach and Gorodnichenko (2012) and Bachmann and Sims (2012). These papers find that the

[1] In New Keynesian economics the term liquidity trap refers to a situation where monetary policy cannot be used to stimulate the economy. In this paper we will use the term liquidity trap and nominal interest rate being at the zero lower bound interchangeably.

[2] This argument assumes that the economy is in the trap not at the threshold, as the latter point has an asymmetry - interest rates can rise but not contract.

[3] Mertens and Ravn (2014) show that if the liquidity trap is instead caused by a self-fulfilling state of low confidence, a rise in government spending will cause deflationary effects, that cause output response to be smaller in the liquidity trap than outside the trap.

[4] For an overview of empirical literature on fiscal stimulus see Coenen et al. (2012) and Hebous (2011).

government spending multipliers are significantly higher in an economic contraction than in an expansion.

Another branch of the literature has incorporated the zero bound in microfounded models with optimizing agents, see Woodford (2011), Eggertsson (2011), Christiano et al. (2011), and Cogan et al. (2010). The first three papers investigate a temporary stimulus that lasts while the economy is in the liquidity trap and find that the government expenditure multiplier is considerably larger in a liquidity trap than outside the trap. Cogan et al. (2010) consider a permanent stimulus and find that the multiplier is smaller in a liquidity trap. This highlights that the timing of stimulus is crucial for the size of the government spending multiplier.

One common feature of these models is that they either have no uncertainty or assume that the shock which drives the economy into the liquidity trap will be completely exogenous, i.e. unaffected by the state of the economy. Both the size and the persistence of the shock that causes the liquidity trap will affect the contraction of output and prices, but there is no feedback. It is however not a well established fact that the causality should only go in this direction, see the discussion in Section 2.

This paper contributes to the current literature by introducing an effect from the level of output to the expected duration of the crisis in a model similar to Woodford (2011) and Eggertsson (2011). We allow the persistence of the shock (and thus the probability of remaining in the liquidity trap) to depend on the state of the economy, more specifically on the level of output. This new channel will be referred to as the persistence channel.

We analyze the case of a persistence channel, where a higher level of output causes the expected duration of the liquidity trap to drop, as this direction is supported by data. This creates a new persistence augmented multiplier in the economy: A higher output will imply a lower persistence and expected duration, which implies an even higher output etc. The effect of this new channel is that there is no longer a single value for the fiscal multiplier in the liquidity trap, as this will depend on the level of output. We show that the persistence channel can make fiscal policy more potent, especially government spending and a sales tax cut.

The rest of the paper is structured as follows. We begin by reviewing the evidence for an endogenous persistence of the liquidity trap in Section 2. In Section 3 we set up the model except for the shock process, which is specified separately in Section 4. We derive analytical multipliers in Section 5, but as these include the output level, which cannot be solved analytically, we proceed with a graphical analysis of fiscal stimulus. Section 5.2 discusses robustness of the results, and in Section 6 the effect of the American Recovery and Reinvestment Act is evaluated in our endogenous persistence model. Section 7 concludes.

2 Evidence of Endogenous Persistence

Baldacci et al. (2010) investigate the effect of fiscal stimulus during a financial crisis. They use the crisis resolution database of Laeven and Valencia (2008), and have a sample consisting of 118 episodes of financial crisis covering 99 countries in the period 1980-2008.[5] They find a significant duration effect from fiscal stimulus.[6] The authors conclude that an '*increase of 1 percent of GDP in the fiscal deficit reduced the duration of the crisis by almost two months. This suggests that fiscal expansion of the size similar to the one adopted on average by G-20 countries during the current global financial crisis may cut the length of the recession by almost one year, compared to a baseline situation in which the budget deficits remained the same as in the pre-crisis period*' (Baldacci et al., 2010, p. 5).

Cecchetti et al. (2009) consider 40 systemic banking crises from the Laeven and Valencia (2008) database and find that the correlation between duration and depth of the crisis (peak to trough percentage decline in GDP) have a correlation of 0.7.[7]

Craigwell et al. (2013) use a sample of 79 financial crises from the Demirgüç-Kunt and Detragiache (2005) and Laeven and Valencia (2008) databases. Regressing recovery probability on macroeconomic variables, they find that consumption and GDP per capita growth were negatively correlated with the duration of crises. GDP per capita growth has the largest absolute influence: A one percentage point increase in the growth doubles the probability of having exited the crisis in the subsequent period. Adding government expenditures and fiscal shocks (the residual from regressing real government spending on

[5] The Laeven and Valencia (2008) data set has 124 crisis episodes. Baldacci et al. (2010) drop ten of these due to insufficient data on fiscal policy and reclassify 4 episodes from currency to financial crises.

[6] In a standard OLS (Ordered Logit) model, the expected duration of a financial crisis is increased by 0.072 (0.122) years when the fiscal deficit in percent of GDP is increased by one percent. All these numbers are significant at the 1 percent level.

[7] Cecchetti et al. (2009) exclude 84 crises that occur in Africa and other emerging countries, and the crises in the U.K and U.S. in 2007.

GDP), only the latter has a statistically significant (and positive) effect on crisis duration.

Thus all three papers indicate that there is a link between the output contraction and the duration of financial crises. The data only covers periods with positive nominal interest rates, so they are only an indication of the correlation in the liquidity trap.

Bachmann and Sims (2012) estimate the relationship between government spending, output, and confidence (using the Michigan Consumer Survey). They use a structural VAR model and split the effect of government spending on output into a direct and indirect effect. The former is the output effect ceteris paribus, and in the latter government spending affects consumer confidence and thereby output. They estimate a nonlinear vector autoregressive model, where the effects of government spending depend on the state of the economy as in Auerbach and Gorodnichenko (2012). Bachmann and Sims (2012) find that spending multipliers are significantly larger during periods of economic distress (just above 2 in contrast to around one in expansions). The direct effects are very similar in the two states, and they show that it is the indirect effect that drives the high total multiplier in a downturn. The authors investigate whether this effect is driven by a sentiments shock or a fundamental shock, and find that fundamental shocks are the primary driver of the large output effects of government spending in a downturn.

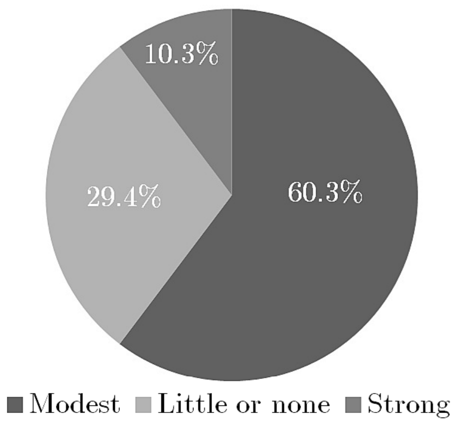

Figure 1: **Impact of ARRA:** Projected impact of the American Recovery and Reinvestment Act on shortening U.S. recession, NABE Economic Survey, March 2009.

Surveys conducted around the passing of the American Recovery and Reinvestment Act (ARRA, signed into law on February 17 2009) gives an indication of an endogenous expected duration at a time, where the monetary policy was in a liquidity trap. A National Association for Business Economics survey in February 2009 showed that 70 percent of the respondents projected the impact of the recently passed ARRA on shorten-

Expected time to recovery		
	Dec 12-14, 2008	Feb 20-22, 2009
Less than one year	6	8
One year	10	16
Two years	27	30
Three years	15	13
Four years	11	9
Five years or more	28	19

Table 1: **Expected Time to Recovery:** Result of Gallup Survey before and after the ARRA was passed. Each survey consisted of at least 1000 persons selected randomly.

ing the recession to be 'modest' or 'strong', see Figure 1.[8] An advantage of this survey is the stated causal structure from the fiscal stimulus to expected duration. Furthermore, a Gallup survey on expected time to recovery also shows that there was a drop in expected time to recovery around the time ARRA was passed, see Table 1. This change could reflect other things than ARRA, but considering the NABE survey, it seems likely that at least some of the drop in expected duration was due to ARRA.

To our knowledge Erceg and Lindé (2014) is the only existing theoretical paper that endogenizes the duration channel. An autoregressive process of order one (AR(1)) demand shock brings the economy into the liquidity trap, and thus at some point following the initial shock, demand shock will be small enough to imply a positive nominal interest rate. Government spending can stimulate output so that the economy has a positive interest rate sooner, thus reducing the duration of the trap.

3 Model

We extend Eggertsson (2011) by letting the shock that triggered the liquidity trap have a state dependent persistence. The liquidity trap is driven by a fundamental shock. The economy consists of households who consume and supply labor, and intermediate goods firms who use this labor as input in production. The intermediate firms operate under monopolistic competition, and final goods producers operating under perfect competition aggregate the intermediate goods into a consumption good, which is either consumed by households or the government. Monetary policy follows an active Taylor rule, and is constrained by a zero lower bound on the nominal interest rate.

[8] The NABE Economic Policy Survey presents the consensus of a panel of 252 of its members. The March 2009 survey was taken Feb. 3-17, 2009.

3.1 Households

We consider an economy made up of a large number of identical, infinitely lived households each of which seeks to maximize

$$U = E_o \left[\sum_{t=0}^{\infty} \beta^t \xi_t \left[u(C_t) - v\left(\int_0^1 l_t(j)\, dj \right) \right] \right]$$

where C_t is consumption in period t of the economy's final good, $l_t(j)$ is hours of labor supplied to industry j by the household in period t. The instantaneous utility functions satisfy $u' > 0, u'' < 0, v' > 0, v'' > 0$ and the discount factor β satisfies $0 < \beta < 1$. Consumption and labor are additively separable in the utility function.[9] As in Eggertsson (2011) the source of uncertainty in our economy stems from a discounting shock ξ_t. The households will take the stochastic properties of this shock as given.

The budget constraint of the household is

$$\begin{aligned}&(1+\tau_t^s) P_t C_t + B_t = \\ &\quad \left(1-\tau_{t-1}^A\right)(1+i_{t-1}) B_{t-1} + \left(1-\tau_t^P\right) \int_0^1 Z_t(i) di + (1-\tau_t^w) \int_0^1 W_t(j) l_t(j)\, dj - T_t\end{aligned}$$

where P_t is the price of the final good, B_t is a one period risk free bond which has unity price at time t and pays $1 + i_t$ at time $t + 1$, and which we assume is the unique asset in the economy. $Z_t(i)$ is the profit of intermediate firm i and $W_t(j)$ is the wage paid in industry j, thus all profits are paid to the households. Finally, taxes consist of a sales tax on the consumption good τ_t^s, a tax on financial assets τ_t^A, a tax on intermediate firm profits τ_t^P, a labor tax τ_t^w and a lump sum tax T_t.[10]

The household maximizes its expected utility subject to the budget constraint and a standard no-Ponzi game condition by choosing optimal levels of C_t, B_t and $l_t(j)$ for all i, j and all t.

[9] Christiano et al. (2011) estimate their model with and without additively seperable utility and find that "across a wide set of parameter values, dY/dG is always less than one with this preference specification", whereas they get a multiplier larger than one when they assume complementarity of consumption and leisure in preferences. Thus, the choice of additive seperability represents a conservative choice.

[10] The different tax rates for return on risk-free bond holdings and profits are allowed in order to have a clear interpretation of the taxes when we study these in Section 4.

The optimal consumption path satisfies the Euler equation

$$u'(C_t) = \left(1-\tau_t^A\right)(1+i_t)\,\beta E_t\left[u'(C_{t+1})\frac{\xi_{t+1}}{\xi_t}\frac{P_t}{P_{t+1}}\frac{1+\tau_t^s}{1+\tau_{t+1}^s}\right], \tag{1}$$

and the labor supply satisfies

$$\frac{v'(l_t(j))}{u'(C_t)} = \frac{1-\tau_t^w}{1+\tau_t^s}\frac{W_t(j)}{P_t} \tag{2}$$

for all sectors j. Finally, in optimum household consumption and debt must satisfy the transversality condition

$$\lim_{T\to\infty} E_0 \frac{B_T}{P_T\left(1+\tau_T^s\right)} u'(C_T) = 0.$$

The Euler equation implies that in a zero inflation steady state $\left(1-\tau^A\right)(1+i)\,\beta = 1$.

3.2 Firms

3.2.1 Final Goods Producers

The final good is produced from a continuum of measure one of differentiated intermediate goods through a constant elasticity of substitution (CES) technology

$$Y_t = \left(\int_0^1 Y_t(i)^{\frac{\theta-1}{\theta}}\,di\right)^{\frac{\theta}{\theta-1}}, \tag{3}$$

where $Y_t(i)$ is the quantity used of intermediate good i, and $\theta > 1$ is the elasticity of substitution between the intermediate goods. The price of the final good is fully flexible, and since there are no adjustment costs, these firms maximization problem reduces to maximizing profits in each period. This yields

$$Y_t(i) = Y_t\left(\frac{p_t(i)}{P_t}\right)^{-\theta}, \tag{4}$$

where $p_t(i)$ is the price of the intermediate good i.

The final good goes to households' consumption or government spending, G_t, so our goods market equilibrium is

$$Y_t = C_t + G_t. \tag{5}$$

3.2.2 Intermediate Firms

Intermediate firms have a linear production function, where one unit of labor produces one unit of output, and we assume that firm i sets an optimal price and then hires the amount of labor required to meet the output demand at that price and takes the industry wage $W(j)$ as given. By inserting the demand function from (4) we then have that pre-tax profit of firm i operating in industry j can be written as

$$Z_t(i) = p_t(i) Y_t \left(\frac{p_t(i)}{P_t}\right)^{-\theta} - W_t(j) Y_t \left(\frac{p_t(i)}{P_t}\right)^{-\theta}.$$

Prices are subject to a friction as in Calvo (1983), where each intermediate firm in any period with a probability $1-\alpha$ can freely reset its price but with the probability α has to maintain the price at what it was in the previous period. The probabilities are exogenous.[11] Since price resetting firms in period t face the same demand function and technology, they all choose the same price p_t^*. This price is exclusive of the sales tax.[12] A firm that resets its price will maximize the expected present value of future post-tax profits in periods, where the reset price is still in effect, i.e. choose p_t^* to maximize

$$E_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \lambda_T \left(1-\tau_T^P\right) \left(p_t^* Y_T \left(\frac{p_t^*}{P_T}\right)^{-\theta} - W_T(j) Y_T \left(\frac{p_t^*}{P_T}\right)^{-\theta}\right). \tag{6}$$

Since the intermediate firms are owned by the households, profits are weighted by the marginal utility of nominal income, λ_T, and discounted by β. The optimality condition for the reset price is derived in Appendix C and is

$$E_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \lambda_T \left(1-\tau_T^P\right) Y_T \left(\frac{p_t^*}{P_T}\right)^{-\theta-1} \left(\frac{p_t^*}{P_T} - \frac{\theta}{\theta-1} W_T(j) p_t^{*-1} \left(\frac{p_t^*}{P_T}\right)\right) = 0.$$

This is the classic result that firms set p_t^* as a markup over probability-weighted expected future marginal costs when facing a Calvo-friction.[13] If prices were fully flexible, each

[11] The Calvo-exogeneity greatly reduces the state space of the model, but disregards 'selection effects', i.e. that firms changing their prices will be those with prices furthest away from equilibrium. However Midrigan (2011) and show that microfounded model can have predictions close to those of a Calvo-model.

[12] Assuming prices are exclusive of sales tax implies that a change in the sales tax will have a one-to-one effect on the after-tax price of the goods, whose prices remained fixed. A sales tax thus has a much larger effect than under the assumption that prices are quoted including the tax.

[13] Other papers such as Woodford (2011) and Christiano et al. (2011) assume a subsidy to offset the inefficiency, thus making the steady state efficient. We will follow Eggertsson (2011) and allow for the inefficiency.

firm will in every period set its price as a constant markup, $\frac{\theta}{\theta-1}$, over real marginal cost. With price rigidities, however, there will be a variation in the price markup, and thus a role for demand (current and expected future levels) in determining output. This means that demand side effects will matter for the size of the fiscal multipliers.

Using the intertemporal optimality conditions of the household in equation (1)-(2) and the fact that the Lagrange multiplier from the household optimization problem is $\lambda_T = \frac{u'(C_T)\xi_T}{\left(1+\tau_T^s\right)P_T}$, the firm's optimality condition becomes

$$E_t \sum_{T=t}^{\infty} \left[\begin{array}{c} (\alpha\beta)^{T-t} \frac{u'(C_T)\xi_T}{P_T} \frac{\left(1-\tau_T^P\right)}{\left(1+\tau_T^s\right)} Y_T \left(\frac{p_t^*}{P_T}\right)^{-\theta-1} \\ \cdot \left(\frac{p_t^*}{P_T} - \frac{\theta}{\theta-1} \frac{1+\tau_t^s}{1-\tau_t^w} \frac{v'\left(Y_T\left(\frac{p_t^*}{P_T}\right)^{-\theta}\right)}{u'(C_t)} \right) \end{array} \right] = 0. \tag{7}$$

From the technology defined in (3) combined with the demand function (4) we get the aggregate price level

$$P_t = \left(\int_0^1 p_t\left(i\right)^{1-\theta} di\right)^{\frac{1}{1-\theta}}, \tag{8}$$

which is the standard CES index for the aggregate price level. As each intermediate price is Bernoulli distributed with outcome $p_{t-1}(i)$ and p_t^* with probability α and $1-\alpha$ respectively, we can use equation (8) to write the following law of motion for the aggregate price level

$$P_t = \left[(1-\alpha)\left(p_t^*\right)^{1-\theta} + \alpha P_{t-1}^{1-\theta}\right]^{\frac{1}{1-\theta}}. \tag{9}$$

3.3 Monetary and Fiscal Policy

Monetary policy consists of a rule for the nominal interest rate, which is subject to a zero lower bound (ZLB)

$$i_t = \max\left\{0, \mathcal{M}\left(\frac{P_t}{P_{t-1}}, Y_t, \xi_t, \psi_t\right)\right\} \tag{10}$$

The monetary policy rule $\mathcal{M}$ will be specified after our log-linearization in Section 3.4.1.

The fiscal policy in our model consists of a policy rule for each of the fiscal policy instruments $\left\{\tau_t^s, \tau_{t-1}^A, \tau_t^P, \tau_t^w, G_t, T_t\right\}$ to be specified in Section 4. Throughout we assume that changes in fiscal policy are financed by lump sum taxes, such that the government satisfies a transversality condition. We disregard the timing of these lump sum taxes given that Ricardian equivalence holds in the model.[14]

3.4 Solving the Model

We proceed by defining an equilibrium for the model and showing the steady state.

Definition 1 *A rational expectations equilibrium in the model consists of stochastic processes for the endogenous aggregate variables in our model, $\{p_t^*, P_t, Y_t, C_t, i_t\}$ for $t \geq t_0$ that satisfy the optimality conditions (1) and (7), the aggregate constraints (5) and (8), and the monetary policy rule in (10) given the initial price index P_{t_0-1}, an exogenous sequence of the discounting shock $\{\xi_t\}$, and the sequence of fiscal policy variables $\left\{\tau_t^s, \tau_{t-1}^A, \tau_t^P, \tau_t^w, T_t, G_t\right\}$.*

There exists a unique zero inflation deterministic steady state with steady state output $\overline{Y}$ determined by

$$\frac{\theta}{\theta-1}\frac{1+\overline{\tau}^s}{1-\overline{\tau}^w}\frac{v'\left(\overline{Y}\right)}{u'\left(\overline{Y}-\overline{G}\right)}=1,$$

see Appendix D. We see that the distortionary sales and labor taxes and the price markup drive a wedge between the marginal rate of substitution and the marginal rate of transformation, causing the steady state to be inefficient.

3.4.1 Log-linearization

In order to get approximations of the variables in a neighborhood of the steady state, we log-linearize the model, except the zero lower bound constraint in (10). This follows the tradition of Eggertsson and Woodford (2003), Christiano et al. (2011), and Bilbiie et al. (2012). Recent literature has documented some challenges arising with this approach at the ZLB, see Braun et al. (2012). However, recent papers by Braun and Waki (2010) and Fernández-Villaverde et al. (2012) suggest, that the qualitative findings are robust to applying other solution methods. In Section 5.2 we will briefly discuss how the last two papers relate to our analysis.

We start by defining $\widehat{Y}_t \equiv \ln \frac{Y_t}{\overline{Y}}$ and $\widehat{G}_t \equiv \ln \frac{G_t}{\overline{Y}}$. This ensures comparability in units between the two relative deviations and furthermore that $\widehat{G}_t$ is defined even when the steady state value of government spending is zero. Unless otherwise stated, log-deviations from steady state are defined as $\widehat{x}_t \equiv \ln\left(\frac{x_t}{\overline{x}}\right)$ for the remaining variables.

[14] Ricardian equivalence holds because all households have access to asset markets, and thus the timing of taxes will not affect their consumption choice.

In the log-linear version of the model we will consider the goods market equilibrium and the aggregate supply equations, these are derived in Appendix E. The goods market equilibrium describes the states of the economy that satisfy the households' Euler equation (1) and the aggregate resource constraint (5)

$$\widehat{Y}_t - \widehat{G}_t = E_t\left(\widehat{Y}_{t+1} - \widehat{G}_{t+1}\right) - \sigma\left(i_t - E_t\pi_{t+1} - r_t^e\right) + \sigma\chi^s E_t\left(\widehat{\tau}_{t+1}^s - \widehat{\tau}_t^s\right) + \sigma\chi^A\widehat{\tau}_{t.}^A \quad (11)$$

where $\pi_t \equiv \ln\left(\frac{P_{t-1}}{P_t}\right)$ is inflation, and $\sigma \equiv -\frac{u'\left(\overline{Y}-\overline{G}\right)}{u''\left(\overline{Y}-\overline{G}\right)\overline{Y}} > 0$ reflects the curvature of the utility function. The two auxiliary tax parameters are defined as $\chi^A \equiv \frac{1-\beta}{1-\overline{\tau}^A} > 0$, and $\chi^s \equiv \frac{1}{1+\overline{\tau}^s} > 0$. r_t^e is the value of the real rate that is consistent with an expected constant consumption if fiscal policy instruments are at their steady state levels. It is defined as $r_t^e \equiv \overline{r} + E_t\left(\widehat{\xi}_t - \widehat{\xi}_{t+1}\right)$, where $\overline{r} \equiv \ln\beta^{-1} + \ln\left(1-\overline{\tau}^A\right)^{-1}$.[15] The shock-adjusted rate r_t^e thus only depends on the steady state value $\overline{r}$ and the expected process for the discounting shock $\widehat{\xi}_t$.

The monetary policy function (10) has the log-linear form

$$i_t = \max\left\{0, r_t^e + \phi_\pi\pi_t + \phi_y\widehat{Y}_t\right\} \quad (12)$$

where $\phi_\pi > 1, \phi_y > 0$ so that the Taylor principle holds.[16] When discussing aggregate demand we mean the goods market equilibrium in (11) combined with the interest rate rule in (12).

The aggregate supply curve represents the outcomes consistent with the optimal decisions of the intermediate firm (7) and the resulting price index (9). Aggregate supply is

$$\pi_t = \kappa\widehat{Y}_t + \kappa\psi\left(\chi^s\widehat{\tau}_t^s + \chi^w\widehat{\tau}_t^w - \sigma^{-1}\widehat{G}_t\right) + \beta E_t\pi_{t+1} \quad (13)$$

where $\kappa \equiv \frac{(1-\alpha)(1-\alpha\beta)}{\alpha(1+\omega\theta)}\left(\omega+\sigma^{-1}\right)$, $\omega \equiv \frac{v''\left(\overline{Y}\right)\overline{Y}}{v'\left(\overline{Y}\right)}$, $\chi^w \equiv \frac{1}{1-\overline{\tau}^w}$, and $\psi \equiv \frac{1}{(\omega+\sigma^{-1})}$ are all positive constants.

[15] Our definition is different from Eggertsson (2011), where $\overline{r} = \ln\beta^{-1}$. The former is consistent with steady state for any $\overline{\tau}^A$, while the latter can only be a steady state solution if $\overline{\tau}^A = 0$.

[16] We use the definition of the Taylor principle in Woodford (2001) that $\phi_\pi + \frac{1-\beta}{\kappa}\phi_y > 1$. This ensures determinacy in our forward looking AD-AS model if there is no zero lower interest bound.

This log-linear approximation leads to a new equilibrium definition:

Definition 2 *The steady state of the model is a set of constants* $\left\{\overline{p^*}, \overline{P}, \overline{Y}, \overline{C}\right\}$ *shown in Appendix D. An approximate equilibrium, which is accurate up to a first order, is a collection of stochastic processes* $\left\{\widehat{\pi}_t, , \widehat{Y}_t, i_t\right\}$ *for* $t \geq t_0$ *that satisfy equation (11)-(13) given a path for the fiscal policy variables* $\left\{\widehat{\tau}_t^s, \widehat{\tau}_t^A, \widehat{\tau}_t^P, \widehat{\tau}_t^w, \widehat{T}_t, \widehat{G}_t\right\}$.

The deterministic steady state described in the previous section is the equilibrium where $\pi_t = \widehat{Y}_t = 0$ and $i_t = \overline{r}$ for all t.

4 Endogenous Persistence

In this model the driving shock will be a change in the households' discount factor, causing the efficient real rate r_t^e, the real interest rate consistent with a constant level of consumption in equation (11), to deviate from its steady state value $\overline{r}$. If the shock is positive or only has a very small negative value, monetary policy can offset this shock by adjusting the interest rate and bring output and inflation back to their steady state values. In the case of a negative shock, however, the zero lower bound can be reached without stabilization being complete, and the lower bound on the nominal interest rate causes the economy to experience a negative output gap and deflation.

This shock is the driving factor behind the occurrence of the liquidity trap and will be key in our endogenous persistence extension of the model. It is therefore worth considering where this shock comes from and what could cause it to return to its steady state value faster. Looking at the goods market equilibrium in (11), a drop in the discount rate of the households and consequently of the rate r_t^e bears a large resemblance to the classical Keynesian negative demand shock. Thus, this shock can be used to capture a number of exogenous reasons for a drop in demand.

It is possible, however, to have a deeper microfounded interpretation of the shock. Cúrdia and Woodford (2009) show that a DSGE model with financial frictions in the form of non-perfect adjustment of insurance payments to heterogenous agents can be reduced to the form of the AS and AD curves presented in equation (11) and (13). In that case the negative interest rate shock reflects an exogenous increase in the probability of a default event for each borrower.

The interpretation of the shock in Cúrdia and Woodford (2009) is both plausible given the events of the recent financial crisis and it allows us to obtain data for the shock, by making r_t^e equal to the wedge between the risk free nominal rate and an interest rate paid on risky loans. For these reasons Eggertsson (2011) proceeds with an analysis of an economy in the liquidity trap with the interpretation that it is brought on by an increased default probability by borrowers. What could cause this financial friction to revert back to its long run value? It is very likely that a higher level of output in the economy and thus higher income of households will improve the outlook for future defaults of borrowers. This would mean an impact from higher output to a lower persistence of the shock.

We will refer to r_t^e as the fundamental shock or demand shock rather than the financial friction, as Cúrdia and Woodford (2009) coin the shock.

4.1 Introducing Endogenous Persistence in the Model

We now introduce a process for the demand shock, where the duration of the shock depends on the level of output. The shock is an absorbing Markov process. With probability $\mu \in (0,1)$ the shock continues to have the same level in the following period and with probability $1-\mu$ the shock returns to its steady state level (so that $r_t^e = \overline{r}$) and remains at this level in all subsequent periods. The expected duration of a shock is $1/(1-\mu)$, which is strictly increasing in the continuation probability μ. If the probability is fixed, this model is in line with Eggertsson (2011) and Christiano et al. (2011), however we allow the probability to depend on the state of the economy, a feature which we will refer to as *endogenous persistence*. Specifically, we assume that the Markov probability μ depends on the output gap $\widehat{Y}_t$. Given the infinite number of households and intermediate firms, all forward-looking agents will take the probability as given but realize how this depends on the output gap. We assume the linear form

$$\mu_t = \widetilde{\mu} - \varphi \widehat{Y}_t \quad , \quad \varphi \geq 0 \tag{14}$$

where $\widetilde{\mu} \in (0,1)$ and the bounds of the persistence sensitivity φ will be specified in (C3). The optimization and log-linearization were done without specifying the formation of expectations. Since the agents take the distribution of the fundamental shock to be independent of their own decisions, the equations (11)-(13) still characterize the model, but they should be augmented by equation (14).

We will refer to the special case $\varphi = 0$ as the simple Markov (SM) model and $\varphi > 0$ as the endogenous Markov (EM) model. When $\varphi > 0$, an increase in the output (or equivalently a smaller contraction) will reduce the expected duration of the shock. This is the sign consistent with the the duration effect found in Erceg and Lindé (2014) and with the NABE and Gallup Surveys presented in Section 2.

The fact that μ is a probability, limits the parameter φ. Assume that the support of $\widehat{Y}_t$, $\widehat{Y}_t \in \left[\widehat{Y}^{\min}, \widehat{Y}^{\max}\right]$, is a bounded interval. Combining these two constraints, we have that the new endogeneity parameter φ must satisfy the following condition

Condition 3

$$\varphi \widehat{Y}^{\max} < \widetilde{\mu} < 1 + \varphi \widehat{Y}^{\min} \tag{C3}$$

See derivation in Appendix G.

Given our log-linearization around $\widehat{Y}_t$, our model is only a good description in a neighborhood of the steady state. This means that condition (C3) can have a looser version which only requires that for the parameter φ, $0 < \widetilde{\mu} - \varphi \widehat{Y}_t < 1$ for reasonable values of $\widehat{Y}_t$. We check the value of μ for all our numerical simulations. However, condition (C3) is the more stringent approach.

We will initially consider fiscal stimulus that is immediately implemented and lasts only while the shock occurs. Given the absorbing nature of the deterministic state, we can define T^e as the time, where the shock disappears. Thus from the time T^e and forward there is no remaining uncertainty, so the economy will return to the deterministic steady state as soon as the shock disappears in period T^e, and remains in this state.[17]

Proposition 4 *Once the demand shock disappears, $t \geq T^e$, there is a unique bounded solution, in which $\pi_t = \widehat{Y}_t = 0$ and $i_t = \overline{r}$.*

Proof. *See Appendix G.* ■

The constant policy path $\left\{\widehat{\tau}^s_t, \widehat{\tau}^A_t, \widehat{\tau}^w_t, \widehat{G}_t\right\}$ for $t < T^e$ implies that, the agents will face the same distribution of future economic outcomes while in the liquidity trap. Thus for a

[17] This is due to our assumption of no capital and due to the log-linearization. In the non-linearized model, price dispersion causes the convergence to steady state to depend on the outcome in the short run.

given size of the shock, the outcome will be identical for all $t < T^e$.[18] This implies that as we can represent this infinite horizon model by a two-period model, where we will denote the periods the long run ($t \geq T^e$) and the short run ($t < T^e$). Using the subscript S to denote the short run, we thus have that for any variable $\widehat{x}_S$, $E_t \widehat{x}_{t+1} = \mu \widehat{x}_S + (1-\mu) \cdot 0 = \mu \widehat{x}_S$, where μ is given by equation (14). Inserting the expectations in equations (11)-(13) we have the following three-equation model

$$
\begin{aligned}
\widehat{Y}_S &= \left(\widehat{G}_S - \sigma \chi^s \widehat{\tau}_S^s\right) \\
&\quad - \frac{\sigma}{\left(1 - \left(\widetilde{\mu} - \varphi \widehat{Y}_S\right)\right)} \left(\widehat{i}_S - \left(\widetilde{\mu} - \varphi \widehat{Y}_S\right) \pi_S - r_S^e - \chi^A \widehat{\tau}_{S.}^A\right) && (15) \\
\pi_S &= \frac{\kappa}{\left(1 - \beta \left(\widetilde{\mu} - \varphi \widehat{Y}_S\right)\right)} \widehat{Y}_S + \frac{\kappa \psi}{\left(1 - \beta \left(\widetilde{\mu} - \varphi \widehat{Y}_S\right)\right)} \chi^w \widehat{\tau}_S^w && (16) \\
&\quad - \frac{\kappa \psi \sigma^{-1}}{\left(1 - \beta \left(\widetilde{\mu} - \varphi \widehat{Y}_S\right)\right)} \left(\widehat{G}_S - \sigma \chi^s \widehat{\tau}_S^s\right) \\
i_S &= \max \left\{0, r_S^e + \phi_\pi \pi_S + \phi_y \widehat{Y}_S\right\}. && (17)
\end{aligned}
$$

The slope of the AS curve is $\frac{d\widehat{Y}_S}{d\pi_S} = \frac{\kappa - \beta \varphi \widehat{\pi}_S}{(1-\beta\mu)}$ while the slope of the AD curve is

$$
\begin{aligned}
i &> 0 : \frac{d\widehat{Y}_S}{d\pi_S} = -\frac{(1-\mu) + \varphi \widehat{Y}_S - \varphi \left(\widehat{G}_S - \sigma \chi^s \widehat{\tau}_S^s\right) + \sigma \left(\varphi \pi_S + \phi_y\right)}{\sigma \left(\phi_\pi - \mu\right)} \\
i &= 0 : \frac{d\widehat{Y}_S}{d\pi_S} = \frac{(1-\mu) + \varphi \widehat{Y}_S - \varphi \left(\widehat{G}_S - \sigma \chi^s \widehat{\tau}_S^s\right) + \sigma \varphi \pi_S}{\sigma \mu}
\end{aligned}
$$

In the SM model ($\varphi = 0$) the AD and AS are linear. The AS curve has a positive slope, while the slope of the AD curve is negative for $i > 0$ and positive when $i = 0$. This is because the Taylor principle holds when the ZLB is not binding, so that a rise in inflation will cause the nominal interest rate to increase by even more, causing the real interest rate to increase, and all else equal the optimizing household will have an incentive to save more and consume less, and aggregate demand falls. If the interest rate is at the ZLB, a rise in inflation erodes the real interest rate, causing households to consume more and aggregate demand to increase.

In our EM model the two curves are non-linear. When φ is positive, a higher output

[18] This is a result of the forward-looking nature of the system of equations in (11)-(13), the absorbing Markov process and a constant fiscal policy path.

makes the persistence μ lower, which causes the AS curve to flatten and the AD curve to be steeper (locally): A drop in μ means that the effect of the inflation level on expected inflation, $\mu\pi_S$, is lower, and an optimizing firm will not adjust as much for future inflation when setting the prices at the given output, so the AS curve is flatter in $(\widehat{Y}_S,\pi_S)$-space. For aggregate demand the lower sensitivity of expected inflation means that the real rate is also less sensitive, and thus demand will respond less to a change in inflation. Further, expected consumption is lower, dampening demand even more. Both channels cause the AD curve to be steeper in $(\widehat{Y}_S,\pi_S)$-space.[19] Given the functional form in equation (14), these slope effects become more pronounced as output increases, and the AD curve is convex and the AS curve is concave in an endogenous Markov (EM) model with $\varphi > 0$.

These effects can be seen in Figure 2, which contains AD-AS diagrams for an economy that is in the liquidity trap. The curvature is much more pronounced for the AD curve than the AS curve, reflecting that the former is affected by the persistence effect both through expected inflation and expected consumption.

As Figure 2 shows, uniqueness and existence is not guaranteed in our model. Panel (a) shows how a very large drop in aggregate demand can imply that equilibrium does not exist, if the duration effect is too strong. In panel (b) the drop in demand is smaller and the knife edge case, where the two curves share a tangent. If the negative shock is more moderate and the duration effect is strong, there can be two equilibria, as shown in panel (c). In Panel (d) there are two equilibria, but the second one is outside the support of $\widehat{Y}_S$.

The existence of multiple rational expectations equilibria in our model introduces an additional source of fluctuations, so that the outcome of an economy is not uniquely determined by fundamentals. This presents a complication for policy analysis as a policy change can have many different effects depending on the initial and following equilibrium. As the focus of this paper is on the effect under a real demand shock, we will restrict our attention to the case where a unique equilibrium exists. This is ensured under the

[19] The expected inflation effect is seen in the term $\left(\widetilde{\mu} - \varphi\widehat{Y}_S\right)\pi_S$ in eq. (15) and (16). For the AD curve there is another effect through expected demand, $\mu\widehat{Y}_S$, which responds less to an increase in $\widehat{Y}_S$ if μ is lower. So a given change in the real rate will not cause as large an effect on $\widehat{Y}_S$, since a change in current consumption $\widehat{Y}_S$ will have a lower effect on expected future consumption, thus the relative expected marginal utilities are affected faster. This effect is seen in the fraction $\left(1 - \left(\widetilde{\mu} - \varphi\widehat{Y}_S\right)\right)^{-1}$ on the right side of equation (15), and it makes the total effect of inflation π_S on output $\widehat{Y}_S$ even smaller, so the drop in μ makes the change of slope (and thus the convexity) of the AD curve more pronounced.

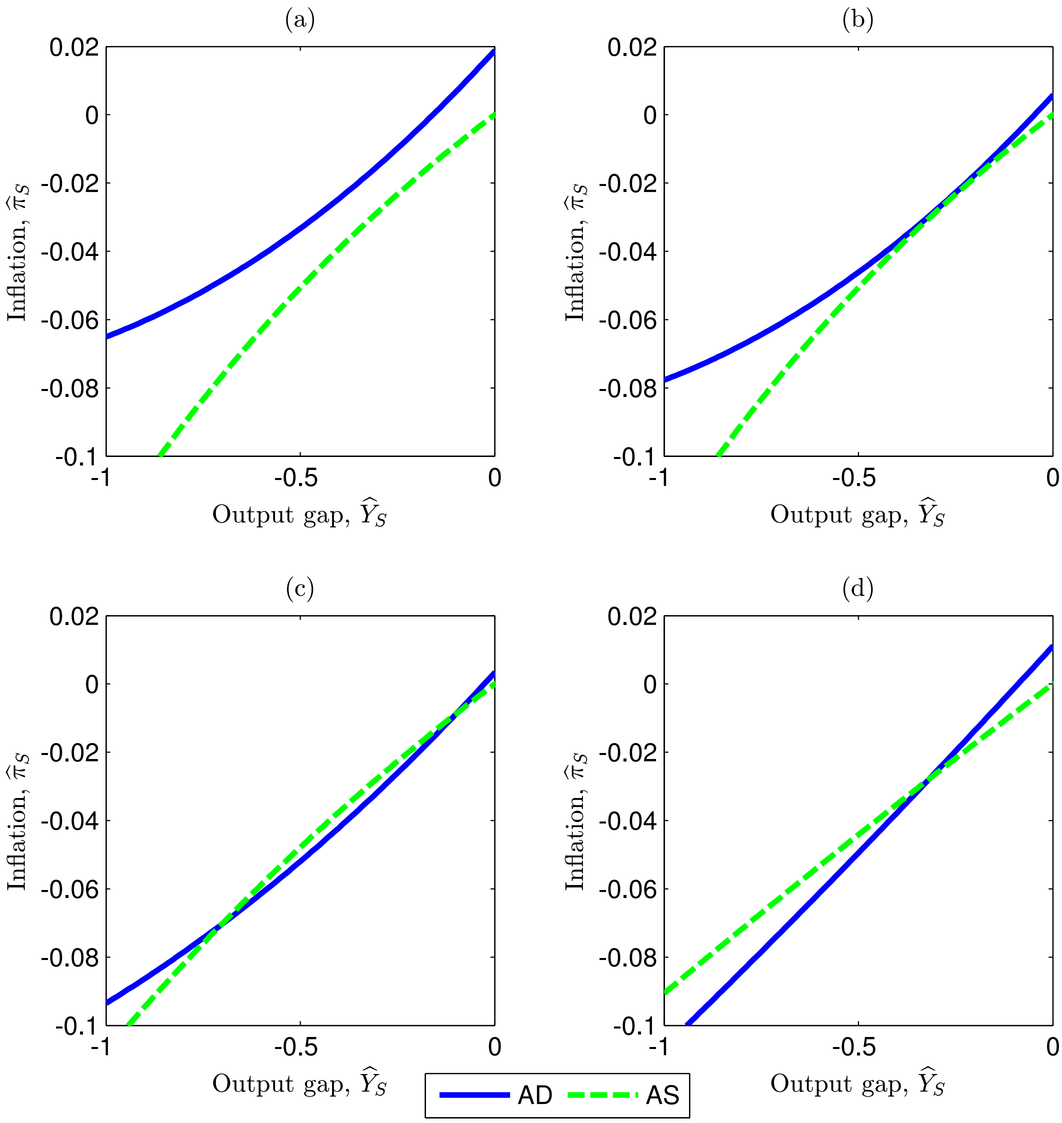

Figure 2: **AD-AS curves:** Examples of AD-AS curves at zero interest rate for different sign of φ in the endogenous Markov model.

following conditions:

Condition 5

$$\widetilde{r}^{exist} < r^e_S < \widetilde{r}^{ZLB} \tag{C5.1}$$

$$0 < \varphi < \widetilde{\varphi} \tag{C5.2}$$

The upper bound in equation (C5.1) ensures that the shock pushes the nominal interest to the ZLB, while the lower bound on r^e_S ensures the AD does not drop so much that there is not a bounded equilibrium, cf. the discussion of Figure 2 panel (a). Condition (C5.2) ensures uniqueness of our equilibrium by limiting the strength of the duration effect. Due to the endogenous persistence, all these bounds will depend on the parameters of the model and equilibrium value of $\widehat{Y}_S$, the latter implying a dependence on the fiscal policy stance. As there is no closed form solution for $\widehat{Y}_S$, we cannot state closed form solutions for $\widetilde{r}^{exist}$, $\widetilde{r}^{ZLB}$ or $\widetilde{\varphi}$ either.

4.2 Calibration of the Model

In our EM model, output is the solution to a third order polynomial that cannot be solved analytically, see the equations for $\widehat{Y}_S$ in Appendix H.1. In order to solve the model numerically, we will now proceed with calibrating our model.

Our model nests the SM models in Eggertsson (2011) and Denes et al. (2013). The former calibrate the model to match a Great Depression (GD) scenario while the latter calibrate the model to a Great Recession (GR) scenario, by using a constructed data point, which in the GD scenario is $(\widehat{Y}_{GD}; \pi_{GD}; i_{GD}) = (-0.30; -0.10; 0)$ and in the GR scenario is $(\widehat{Y}_{GR}; \pi_{GR}; i_{GR}) = (-0.10; -0.02; 0)$.[20]

The GD scenario can be seen as a worst case scenario, however, choosing such a big demand shock will imply that the estimates of the multipliers are increased, as we will see. Thus, we use the GR scenario as our baseline calibration and use GD calibration for robustness.

The parameters found for the two scenarios are shown in Table 2. In the GR case, the real interest rate is minus 5 percent p.a. and expected duration is 7 quarters. In order to account for the large contraction in the GD scenario, these numbers are minus 4 percent p.a. and ten quarters. Even though the shock generating the GD scenario is smaller (in absolute terms), the higher persistence in the GD calibration will drive the larger output contraction in the GD calibration.

As the SM model was used to calibrate these parameters, we must take a stand on how to calibrate the Markov probability in the EM model. Particularly, the two scenarios are not equilibria in our model, if we keep all parameters and introduce a $\varphi > 0$. We therefore proceed with a calibration, where $\mu_{GR} = \widetilde{\mu}_{GR} - \varphi_{GR}\widehat{Y}_{GR}$ and $\mu_{GD} = \widetilde{\mu}_{GD} - \varphi_{GD}\widehat{Y}_{GD}$, as this enables us to replicate the two scenarios. We set $\varphi_{GR} = 0.1$ and $\varphi_{GD} = 0.005$, as this is close to but not at the limits $\widetilde{\varphi}$ from condition (C5.2). This allows us to capture a considerable duration effect but at the same time it ensures that the economy is still

[20] Denes and Eggertsson (2009) is a technical paper describing the Bayesian estimation of the parameters used in Eggertsson (2011) and Denes et al. (2013). As only Eggertsson (2011) states confidence bands, we will use estimates from this paper for the GD scenario.

	σ^{-1}	β	ω	α	θ
Parameters GD (mode)	1.1599	0.9970	1.5692	0.7747	12.7721
Parameters GR (mode)	1.22	0.997	1.69	0.784	13.22
	ϕ_π	ϕ_y	$\overline{\tau}^s$	$\overline{\tau}^w$	$\overline{\tau}^A$
Parameters GD (calibrated)	1.5	0.125	0.05	0.2	0
Parameters GR (calibrated)	1.5*	0.125*	0.1	0.3	0**
	r_S^e	μ		$\widetilde{\mu}$***	φ***
Shocks GD (mode)	-0.0104	0.9030		0.9015	0.005
Shocks GR (mode)	-0.0128	0.8570		0.8470	0.1

Table 2: Calibration. * Parameters not stated in paper, assumed same value as in GD calibration. ** Tax on financial assets is not included in Denes et al. (2013), but we include it for comparison between the two scenarios. ***Own Calibration

well-behaved for any shock in the neighborhood of r_S^e.

5 Effects of Fiscal Stimulus

We will now analyze the effects of fiscal policies in the case where a unique equilibrium exists. The government spending multiplier is the amount of dollars output will increase given a one dollar increase in government spending, and for the tax multipliers the interpretation is how many percent output will rise given a one percentage point reduction of the given tax rate. An advantage of the "perfect timing" of fiscal policy is that the stimulus and output effect will be constant values while the shock is present, thus the multipliers found using short run levels are the same as the net present value multipliers suggested by Uhlig (2010).

We derive the partial fiscal multipliers by taking total derivatives to our third order polynomials in $\widehat{Y}_S$.

Proposition 5 *In the short run, $t < T^e$, there are two cases for the fiscal multipliers in the economy. These are calculated under the assumption that only one fiscal stimulus tool is used at a time, all others are at their steady state values .*

Case 1. The economy has a positive nominal interest rate

$$\frac{d\widehat{Y}_S}{d\widehat{G}_S} = \frac{\left(1-\left(\widetilde{\mu}-\varphi\widehat{Y}_S\right)\right)\left(1-\beta\left(\widetilde{\mu}-\varphi\widehat{Y}_S\right)\right)+\kappa\psi\left(\phi_\pi-\left(\widetilde{\mu}-\varphi\widehat{Y}_S\right)\right)}{H_1\left(\widehat{Y}_S\right)-\varphi\left[\left(1+\beta-2\beta\left(\widetilde{\mu}-\varphi\widehat{Y}_S\right)\right)+\kappa\psi\right]\widehat{G}_S} \tag{18}$$

$$\frac{d\widehat{Y}_S}{d\widehat{\tau}^s_S} = -\frac{\left[\left(1-\left(\widetilde{\mu}-\varphi\widehat{Y}_S\right)\right)\left(1-\beta\left(\widetilde{\mu}-\varphi\widehat{Y}_S\right)\right)+\kappa\psi\left(\phi_\pi-\left(\widetilde{\mu}-\varphi\widehat{Y}_S\right)\right)\right]\sigma\chi^s}{H_1\left(\widehat{Y}_S\right)-\varphi\left[\left(1+\beta-2\beta\left(\widetilde{\mu}-\varphi\widehat{Y}_S\right)\right)+\kappa\psi\right]\sigma\chi^s\widehat{\tau}^s_S} \tag{19}$$

$$\frac{d\widehat{Y}_S}{d\widehat{\tau}^w_S} = -\frac{\kappa\sigma\psi\left(\phi_\pi-\left(\widetilde{\mu}-\varphi\widehat{Y}_S\right)\right)\chi^w}{H_1\left(\widehat{Y}_S\right)+\kappa\sigma\psi\varphi\chi^w\widehat{\tau}^w_S} \tag{20}$$

$$\frac{d\widehat{Y}_S}{d\widehat{\tau}^A_S} = \frac{\sigma\left(1-\beta\left(\widetilde{\mu}-\varphi\widehat{Y}_S\right)\right)\chi^A}{H_1\left(\widehat{Y}_S\right)-\sigma\beta\varphi\chi^A\widehat{\tau}^A_S} \tag{21}$$

where the second order polynomial $H_1\left(\widehat{Y}_S\right)$ *is defined as*

$$H_1\left(\widehat{Y}_S\right)=\varphi\left(1-\beta\left(\widetilde{\mu}-\varphi\widehat{Y}_S\right)\right)\widehat{Y}_S$$
$$+\left(1-\left(\widetilde{\mu}-\varphi\widehat{Y}_S\right)+\sigma\phi_y\right)\left(1-\beta\left(\widetilde{\mu}-2\varphi\widehat{Y}_S\right)\right)+\kappa\sigma\left(\phi_\pi-\left(\widetilde{\mu}-2\varphi\widehat{Y}_S\right)\right)$$

Case 2. The nominal interest rate is at the zero lower bound

$$\frac{d\widehat{Y}_S}{d\widehat{G}_S} = \frac{\left(1-\beta\left(\widetilde{\mu}-\varphi\widehat{Y}_S\right)\right)\left(1-\left(\widetilde{\mu}-\varphi\widehat{Y}_S\right)\right)-\kappa\psi\left(\widetilde{\mu}-\varphi\widehat{Y}_S\right)}{H_2\left(\widehat{Y}_S\right)-\beta\varphi\sigma r^e_S-\varphi\left[\kappa\psi+\left(1+\beta-2\beta\left(\widetilde{\mu}-\varphi\widehat{Y}_S\right)\right)\right]\widehat{G}_S} \tag{22}$$

$$\frac{d\widehat{Y}_S}{d\widehat{\tau}^s_S} = \frac{\left[\kappa\psi\left(\widetilde{\mu}-\varphi\widehat{Y}_S\right)-\left(1-\beta\left(\widetilde{\mu}-\varphi\widehat{Y}_S\right)\right)\left(1-\left(\widetilde{\mu}-\varphi\widehat{Y}_S\right)\right)-\right]\sigma\chi^s}{H_2\left(\widehat{Y}_S\right)-\beta\varphi\sigma r^e_S+\varphi\left[\kappa\psi+\left(1+\beta-2\beta\left(\widetilde{\mu}-\varphi\widehat{Y}_S\right)\right)\right]\sigma\chi^s\widehat{\tau}^s_S} \tag{23}$$

$$\frac{d\widehat{Y}_S}{d\widehat{\tau}^w_S} = \frac{\kappa\psi\sigma\left(\widetilde{\mu}-\varphi\widehat{Y}_S\right)\chi^w}{H_2\left(\widehat{Y}_S\right)-\beta\varphi\sigma r^e_S+\kappa\psi\sigma\varphi\chi^w\widehat{\tau}^w_S} \tag{24}$$

$$\frac{d\widehat{Y}_S}{d\widehat{\tau}^A_S} = \frac{\left(1-\beta\left(\widetilde{\mu}-\varphi\widehat{Y}_S\right)\right)\sigma\chi^A}{H_2\left(\widehat{Y}_S\right)-\beta\varphi\sigma r^e_S-\beta\varphi\sigma\chi^A\widehat{\tau}^A_S} \tag{25}$$

where the second order polynomial $H_2\left(\widehat{Y}_S\right)$ *is defined as*

$$H_2\left(\widehat{Y}_S\right)=\varphi\left(1-\beta\left(\widetilde{\mu}-\varphi\widehat{Y}_S\right)\right)\widehat{Y}_t$$
$$+\left(1-\left(\widetilde{\mu}-\varphi\widehat{Y}_S\right)\right)\left(1-\beta\left(\widetilde{\mu}-2\varphi\widehat{Y}_S\right)\right)-\kappa\sigma\left(\widetilde{\mu}-2\varphi\widehat{Y}_S\right).$$

Proof. See Appendix H.4 and H.5. ■

Note that for $\varphi = 0$ we have $H_1\left(\widehat{Y}_S\right) = \left(1-\widetilde{\mu}+\sigma\phi_y\right)\left(1-\beta\widetilde{\mu}\right)+\kappa\sigma\left(\phi_\pi-\widetilde{\mu}\right)$ and $H_2\left(\widehat{Y}_S\right)-\beta\varphi\sigma r^e_S = \left(1-\widetilde{\mu}\right)\left(1-\beta\widetilde{\mu}\right)-\kappa\sigma\widetilde{\mu}$, so the multipliers are constant and identical to those in Eggertsson (2011).

The multipliers in the more general case, where we do not look at one fiscal policy at a time, can be found in Appendix H.4 and H.5.

As the multipliers still contain $\widehat{Y}_S$, which we do not have an analytical solution for, we proceed with a numerical analysis of the multipliers. The solution for $\widehat{Y}_S$ used in Figures 3-6 has been solved numerically via the *fsolve* function in Matlab.

Figure 3 panel (a) shows the equilibrium output for the simple and endogenous Markov model as a function of the demand shock r_S^e, when there is no fiscal stimulus. There is no difference in the outcome of the two models as long as $r_S^e \geq 0$. At these values the nominal interest rate is positive, and the output gap is stabilized at zero, thus muting the endogeneity channel in the EM model.[21] When there is a sufficiently negative shock, so the nominal interest rate is constrained by the ZLB, the two models yield different outcomes, and the difference in slope is increasing in the size of negative output gap. This is because a larger output drop, and the higher persistence μ causes expected deflation and expected negative output gap to increase. These sinister expectations cause aggregate supply and demand to drop, generating a more severe output contraction in the EM model.

The mechanism just described reveals a new multiplier effect in the liquidity trap, as more output generates lower persistence of the shock, leading to higher output and even lower persistence etc. We will call this multiplier the *persistence augmented multiplier*, and as argued the absolute value of this multiplier effect is increasing in the absolute size of the output gap.

Figure 3 panel (b) shows the output effect of increasing the level of government spending from $\widehat{G}_S = 0$ to $\widehat{G}_S = 0.1$.[22] This has an unambiguously positive effect on output. At positive interest rates the stimulated output makes the expected duration of the stimulus lower, which partially dampens the initial stimulus of $\widehat{Y}_S$. The figure also shows that the higher output level due to the stimulus means that it takes a larger negative demand shock for the ZLB to become binding. This is because the stimulated output implies a higher nominal interest rate due to the Taylor rule. This effect is stronger, the more monetary policy reacts to output, so a higher ϕ_π would move the kink further left.

[21] The fact that the graphs in Figure 3 panel (a) are identical for the flat part of the curve and share a kink, is due to setting $\widetilde{\mu} = \mu_{SM}$.

[22] Increasing government spending by ten percent is perhaps not a realistic scenario, but it has been chosen to make the effect more visible in the figure.

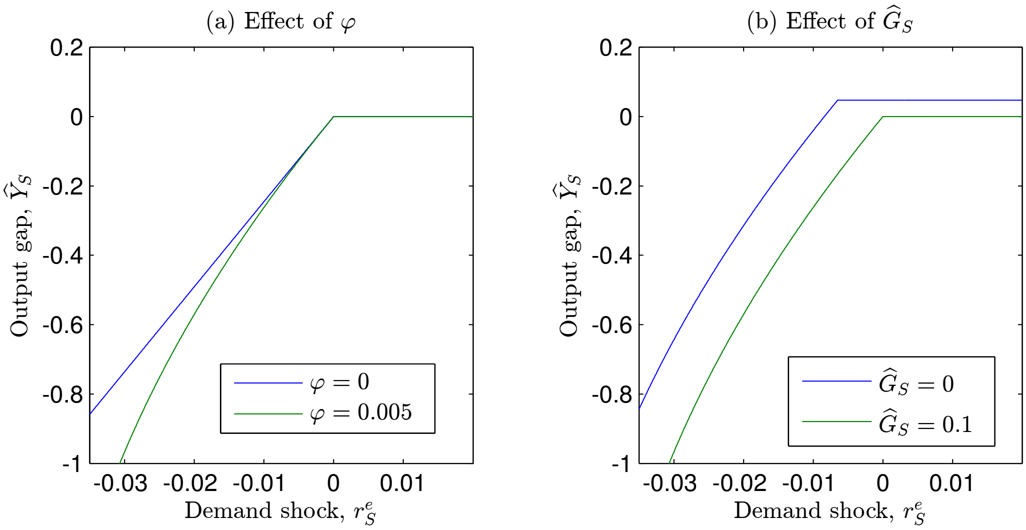

Figure 3: **Equilibrium in the EM model:** Panel (a) shows the equilibrium for different values sizes of the demand shock in the SM and EM model. Panel (b) shows the effect of a rise in government spending in the EM model.

The vertical distance between the curves is larger than to the left of the kink, indicating that government spending has a larger output effect in the ZLB in the EM model. Figure 3 only shows the aggregate effect of the stimulus. This is only an indication of the marginal effects, and we therefore proceed to analyze the aggregate and marginal effects of fiscal stimulus in the EM model.

5.1 Government Spending

In Figure 4 we show the aggregate and marginal effects of fiscal stimulus using the Great Recession (GR) scenario. The GR-value of the shock is $r_S^e = -0.0128$, so the the starting point of the fiscal stimulus is an economy experiencing a severe output contraction. A similar figure for the Great Depression (GD) scenario is in Appendix B.

The relationship between $\widehat{Y}_S$ and $\widehat{G}_S$ is illustrated in panel (a) and (b). In panel (a), we vary $\widehat{G}_S$ and solve for $\widehat{Y}_S$ numerically. For the SM model there is a piecewise linear relationship, and for both the SM and EM model the graph has a kink exactly where the zero lower bound on the interest rate becomes binding. For the endogenous model we show the outcome from setting $\widetilde{\mu} = \mu_{SM}$ (the green line, from now on referred to as calibration A) and from setting $\widetilde{\mu} = \mu_{SM} - \varphi\,(-0.1)$ (the red line, will be referred to as calibration B). The EM curves cross the SM curve where the conditional duration of the respective curves are equal. For calibration A this happens at $\widehat{Y}_S = 0$, for calibration B

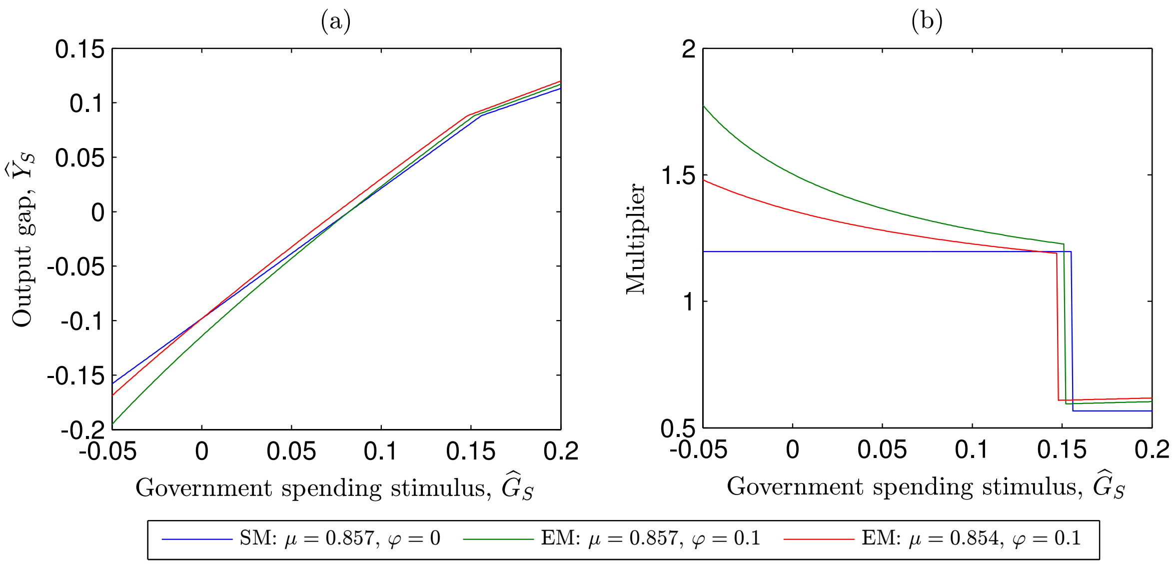

Figure 4: **Effect of government spending:** The output effect and marginal multipliers for different levels of government spending in the Great Recession Scenario.

this is when $\widehat{G}_S = 0$. Both EM curves in panel (a) are concave to the left of the kink, for the same reasons described in relation to Figure 3.

The decreasing EM multipliers in panel (b) capture the essence of the persistence augmented multiplier. The EM multipliers are higher because government spending will decrease the persistence through a higher output. This increases expected inflation and output ceteris paribus, as these expectations are $\mu\pi_S$ and $\mu\widehat{Y}_S$ respectively, and as a result equilibrium output $\widehat{Y}_S$ rises. The reason the EM multipliers are declining is the following: When a higher level of government spending has stimulated inflation and output more, that is π_S and $\widehat{Y}_S$ are closer to zero, a change in the persistence will not change expected values as much. As a result aggregate demand and aggregate supply change less, and the persistence channel is weaker when government spending is higher (and output is contracted less). In that sense, the success of government spending in stimulating output, means that a the duration-channel is weakened, leaving fiscal policy less potent on the margin.

In panel (b), all multipliers drop when the economy reaches a positive interest rate. These points coincide with the kinks in panel (a): Once the interest rates are not constrained by the ZLB, the nominal interest rate will increase when government spending increases output and inflation, thus dampening the initial stimulus. We see for the EM models, the drop occurs at slightly lower levels of stimulus, again reflecting that the duration channel stimulates output further, thus yielding a higher nominal interest rate under the Taylor rule.[23] The curvature of the multipliers is hardly visible when the interest rate is positive. This is because the level of the multiplier is much smaller, thus rendering the duration channel very small, and because small output gap leaves the effect of a change in persistence on aggregate demand and supply very small.

Overall we see that the ability of government spending to affect output *and* the expected duration causes the fiscal multipliers to increase considerably. Considering the GR calibration in Figure 4 (b), the multiplier rises from 1.20 in the SM model to 1.36 at zero stimulus in the endogenous Markov model under calibration B (and 1.50 with calibration A). In the GD scenario the government spending multiplier changes from a SM value of 2.29 to 2.50 in the EM model under calibration B, see Figure 4 in Appendix B.

Comparing the multipliers in the two scenarios, we see that the qualitative effects are the same in the GR and GD scenarios, but that the multipliers are numerically larger in the GD scenario despite the much smaller size of φ ($\varphi_{GR} = 0.1$ while $\varphi_{GD} = 0.05$). This is partly due to the higher value of the multipliers in the SM model calibrated to the GD scenario, but it is also due to the severe output contraction in the GD scenario. As the persistence augmented multiplier is non-linear in output, the severe GD contraction will cause the duration effect to gain momentum, leaving fiscal policy more potent across the board.

5.2 Robustness

A word on the log-linear approach we use. Braun and Waki (2010) use non-linear methods (a variant of extended shooting) and find that log-linearizing around the zero-inflation steady state can exaggerate the size of the multiplier under realistic parameter values. However, they also find that the conclusion that the government spending is comfortably above one in the liquidity trap is robust. Fernández-Villaverde et al. (2012) also apply nonlinear-methods (using Chebyshev polynomials for projections) and find that when the economy is hit by a discount factor shock that sends the interest rate to the ZLB for an average of four quarters, the impact multiplier of government expenditure is

[23] The monetary policy is out of the liquidity trap at a lower amount of stimulus for the B calibration than the A calibration of the EM model. This is because the expected duration is always lower in the A calibration for a given level of stimulus.

approximately three times larger than when the economy is outside the trap. Thus we conclude, that the results that governments spending has a larger impact in the liquidity trap is robust to other solution methods than the log-linear approach we have chosen.

The assumption that government spending is additively separable from consumption is an important driver of our results. In the extreme polar case of where government spending is a perfect substitute for private consumption, the multiplier will be zero regardless of the interest rate, as this type of government spending will crowd out consumption one to one, cf. Eggertsson (2011). However, in the intermediate case of non-perfect substitutability, the result that the multiplier is larger in the liquidity trap will still hold, although the difference will be dampened, cf. Christiano et al. (2011). In this case our persistence augmented multiplier will still be larger than the simple multiplier.

5.2.1 The Timing of Stimulus

We have shown that government spending can potentially be very effective at stimulating an economy which is in a liquidity trap, given that the spending is implemented immediately and only lasts as long as the zero lower bound is binding. We now discuss the effects of relaxing these assumptions.

Christiano et al. (2011) introduce a time lag from the moment the economy enters the liquidity trap, t_0, and the time the stimulus is implemented, t_1. The level of stimulus is known the minute the shock occurs, so there is no recognition or decision lag, only an implementation lag. We will look at government consumption, $\widehat{G}_S$, and an implementation lag of one quarter. From the time the policy is implemented, the economy will be back in the short run solution, so government consumption will stimulate output and reduce the deflationary pressure from t_1 and onwards. This reduction of expected deflation from t_1 and onwards, causes aggregate supply and demand to increase already at time t_0. Christiano et al. (2011) find that the t_0 effect of one percent rise in $\widehat{G}_S$ at t_1 drops from 3.7 (no lag) to 2.4 for a one period lag.[24] For stimulus via tax instruments their result are qualitatively the same.

What happens if fiscal stimulus remains at the same level permanently? In the long (and short) run the permanent stimulus imply that government spending (as well as the sales tax) cancels out in the Euler equation, and there is no direct AD effect in the

[24] Assuming a two period lag this t_0 effect drops to 2.38, so the multiplier is not very sensitive to the length of the lag after period one.

long run. The increased government spending increases long run labor supply, which is the driving force behind the long run output boost and deflation arising. For a deeper analysis, see Eggertsson (2011).

In the short run aggregate supply is stimulated by the government spending and the lower expected future inflation. Higher expected future output stimulates short run demand, while lower expected inflation contracts this demand. The total effect is ambiguous, but Eggertsson (2011) states that with the GD parameters, the demand increases by 0.030. The short run equilibrium effects depend on the slope of the AD curve. If the interest rate is positive in the short run, output will increase. However, in the liquidity trap, there are opposing effects from supply and demand. If the feedback from future conditions into aggregate demand dominates, then output is stimulated, but if the aggregate supply effects dominate, the increased government spending has a contractionary short run effect. Using the GD parameters, Eggertsson (2011) finds that for positive nominal interest rate the mode of the short run multiplier is 0.02, which is considerably lower than 0.37 in the SM model using the GD calibration. For the liquidity trap, the mode of the short run multiplier is -2.41, which is quite different from the SM multiplier of 2.29 in the case for a temporary stimulus.

While our initial analysis indicated that government spending can potentially be very potent in fighting a recession, this result relies heavily on the fact that the stimulus is not permanent, since a permanent stimulus can be less effective and even potentially worsen the contraction. These results explain why Cogan et al. (2010) find a much smaller multiplier in a model similar to Eggertsson (2011), as they assume permanent fiscal stimulus.

5.2.2 Comparing the Multipliers with Erceg and Lindé (2014)

We compare our results with the endogenous duration model of Erceg and Lindé (2014). The model is similar to the SM model, but the household demand shock and government spending shocks are AR(1) processes, so at some point following the initial shock, the size of the aggregate demand shock r_t^e will be small enough to imply a positive nominal interest rate. A government spending shock then stimulates output and causes the interest rate to turn positive sooner. This gives a negative effect of government spending on duration (through output): A 5 percent government consumption rise reduces the duration from

8 to 5 quarters. This large duration effect implies that their (instantaneous) government spending multiplier is 2.1, which is a bit larger than our multiplier of 1.36 in the GR scenario.

The equilibrium determined duration of the liquidity trap is an attractive feature, but the AR(1) process raises some other issues. Erceg and Lindé (2014) find that the multiplier is lower in a more shallow recession, so their fiscal stimulus is less effective over time as the shock diminishes. Furthermore, the fiscal stimulus continues after the nominal interest rate becomes positive, i.e. at a time when fiscal policy is less effective than in the liquidity trap. The instantaneous representation thus yields a higher number than is to expected of the net present value multipliers. Their interpretation of fiscal stimulus in the liquidity trap is quite different from ours, as their stimulus remains active when the economy is no longer in the trap.

Government spending does not affect aggregate supply in their model, leading to a larger multiplier given the positive slope of the AD curve. Combined with their large duration effect and choice of the instantaneous multiplier we would expect that their model yields a larger multiplier. Adjusting for these differences in the models could very likely yield more similar multipliers despite the different approaches to modeling the duration channel.

A final note on the comparison: In our model the large (expected) duration effect would be generated with a value of $\varphi = 0.37$. This would by coincidence imply a multiplier of 2.1 in our model, however such a strong duration effect will imply problems with multiple equilibria in our policy experiments, which is why we do not use this value for the graphs in Figures 4 and 6.

5.3 Tax Rates

Panel (a) and (b) in Figure 5 shows the expansionary effects of changing the sales tax. Given that the tax and government spending enter the AD and AS curves in a parallel manner, it is not surprising that a sales tax reduction shares the qualitative features of the government spending stimulus. As the tax cut stimulates output, it will gradually reduce expected duration and thus dampen its own stimulus potential, which is why the multiplier in panel (b) is declining. The graph is capped at a 5 percentage point tax cut, as this is the size of the tax in steady state. Consequently this stimulus tool will be fully

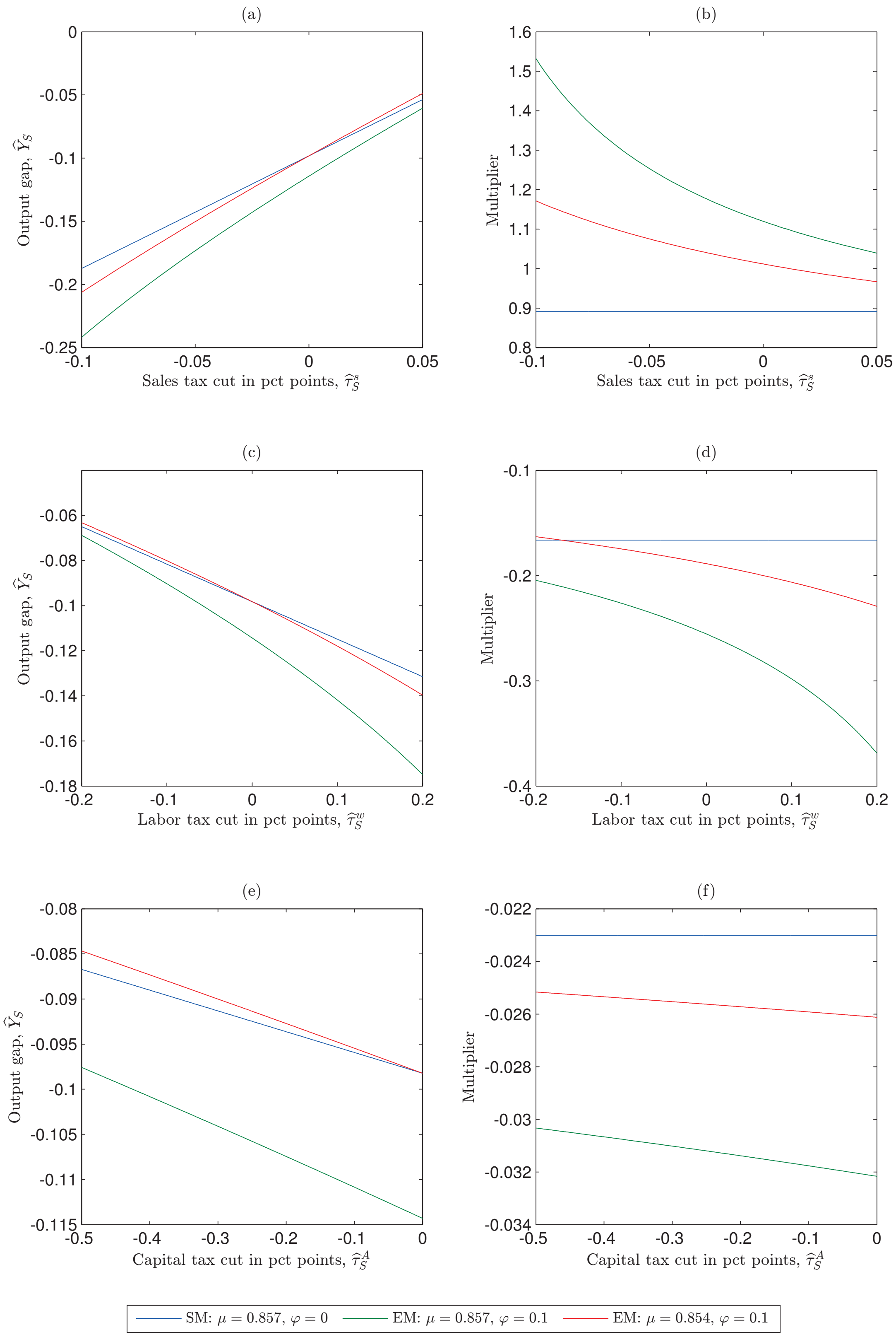

Figure 5: **Effect of tax cuts:** The output effect and marginal multipliers for different levels of tax cuts in the Great Recession Scenario.

exhausted before it can bring the economy out of the liquidity trap in any of our models, which is seen by the absence of a kink and drop in the output curves in panel (a) and (b). Introducing endogenous persistence increases the initial multiplier (at zero stimulus) from 0.89 to 0.97 (1.04 in case B), so again fiscal stimulus becomes more potent.

Panel (c) and (d) in Figure 5 shows the contractionary effects of a labor tax cut. A labor tax cut is expansionary at a positive interest rate but contractionary when the interest rate at the ZLB. The tax cut increases the after tax wage and hence labor supply will increase, causing the equilibrium real wage to drop. Firms thus supply a larger amount at a given price, and deflationary pressure arises in the economy. An active monetary policy will respond by reducing the nominal (and real) interest rate, thus causing the households to save less and consume more. The outcome is an increase in aggregate demand. In a liquidity trap the monetary policy does not respond to the deflationary pressure, so the real interest rate rises, leading to a fall in demand.[25]

The effect of an output contraction and thus a lower duration causes the persistence augmented multiplier to be greater as the tax cut increases. The output curves for the EM model are again concave, but at negative slopes as opposed to the expansionary sales tax cut. The duration channel will cause the multipliers to be numerically larger than in the simple model, the multiplier drops from -0.167 to -0.173 (-0.227 in calibration A). This example highlights a feature of the liquidity trap: The demand side is the main determinant of output, so that the supply side mainly has an effect on expected inflation. This implies that stimulating the supply will only increase the deflationary pressure in the liquidity trap, and hence cause a drop in output.

Panel (e) shows the effects of the tax on financial wealth. Reducing the tax will increase the after tax real interest, thus reducing current demand and causing a larger deflationary pressure. With an unconstrained nominal interest rate the monetary policy response is to decrease the nominal and real interest rate, which dampens the contraction of demand and inflation. At a zero interest rate the initial drop in demand and inflationary pressure is not dampened, thus causing a stronger contraction. Thus cutting taxes on financial wealth will increase the incentive to save, which further depresses the inadequate demand. We therefore only show a tax increase in panel (e) and (f). Since the multiplier

[25] The fact that if everybody wants to work more, there will be less labour used in the aggregate when the economy in at the zero lower bound, is by Eggertsson (2011) coined the paradox of toil. This refers to the Keynesian paradox of thrift; where everybody trying to save more leads to less savings in the aggregate (Keynes, 1936).

of this tax is so little, the duration effect is extremely small, and the curvature is hardly visible in the output curves in panel (e). The duration channel still makes the absolute size of the multipliers increasing in the tax cut, but the effects are numerically small and do not change the conclusion that an asset tax is not a very relevant stimulus tool in our model without capital accumulation.

We have seen that the EM model shares the conclusion of the simple Markov model that the most potent stimulus tools are still the government spending and sales tax, as these are the only ones that are successful in stimulating the depressed demand, which is the main problem in the liquidity trap. Further, as these tools no longer just affects output but also expected duration of the shock, they can potentially be even more efficient stimulus tools when the economy is in the liquidity trap.

6 The American Recovery and Reinvestment Act

In January 2009 the American Recovery and Reinvestment Act (ARRA) was passed with a fiscal stimulus of $787 bn to be spent in 2008 and 2009. Romer and Bernstein (2009) estimate that a stimulus of $775 bn would cause an output boost of 3.7 percent of GDP. This number has been heavily debated, and we will now investigate the effect of ARRA in our model. These calculations are not meant as an exact measure of the economic effect of ARRA, but rather as a way to get an impression of the importance of the duration effect when applied to actual policies.

In line with a similar case study in Denes and Eggertsson (2009), we assume that 2/3 of ARRA consisted of government spending and the remaining 1/3 was a decrease labor tax revenue. Denes and Eggertsson (2009) find that ARRA has an output effect of 3.3 percent. This is the same order magnitude as Romer and Bernstein (2009), but whereas they find that both government spending and the labor tax cut contribute positively to this number, Denes and Eggertsson (2009) find that this number would actually have been higher without the tax cut. This is due to the contractionary effect of a labor tax in the liquidity trap in their SM model.

We assume the stimulus is extended until the crisis is over.[26] Based on potential output estimates from the Congressional Budget Office, we find that the government

[26] The ARRA covers 8 quarters, which is between the expected duration in the GR scenario (7 quarters) and the GD scenario (10 quarters).

	$\widehat{G}_S$	$\widehat{\tau}_S$	$\widehat{Y}_S^{\text{no stim}}$	$\widetilde{\mu}$	φ	Expected duration		ARRA's effect on output	
	Pct.	Pct. pts.	Pct.			No stim.	ARRA	Full package	Without tax cut
Romer and Bernstein (2009)	-	-	-		-	-		3.7%	
Great Depression scenario									
Denes and Eggertsson (2009)	-	-	-	0.9030	-	10.31	10.31	3.3%	3.9%
Simple Markov model	2.09	-1.129	-29.92	0.9030	0	10.31	10.31	3.63%	4.79%
Endogenous Markov model									
- matching constant duration *	2.09	-1.116	-33.15	0.9030	0.005	10.48	10.47	4.27%	5.67%
- matching duration in GD scenario **	2.09	-1.113	-29.92	0.9015	0.005	10.31	10.28	3.97%	5.18%
Great Recession scenario									
Simple Markov model	2.09	-1.075	-9.82	0.8570	0	6.993	6.993	2.32%	2.51%
Endogenous Markov model									
- matching constant duration *	2.09	-1.075	-11.43	0.8570	0.1	7.601	7.441	2.83%	3.08%
- matching duration in GR scenario **	2.09	-1.068	-9.80	0.8470	0.1	6.983	6.858	2.61%	2.80%

*Deep parameters kept at values from Table 2 (Calibration A).

**Deep parameters chosen to match output drop and expected duration before stimulus (Calibration B).

Table 3: Calculations of the output effect of ARRA

spending is 2.09 percent of potential output. For the labor tax the revenue depends on general equilibrium effects, we calibrate the tax rate for each scenario, see Appendix H.6.

Table 3 shows our calculations on the effects of ARRA. In the GR scenario the SM model yields an output effect of 2.32 percent (2.51 without the tax cut), which only two thirds of the estimated effect in the GD scenario (3.63 percent and 4.79 percent). This clearly shows that the choice of scenario has large policy implications. In our EM model with calibration B, ARRA has a total effect of 2.61 percent of GDP (2.80 with only the spending leg).[27] Introducing endogenous persistence increases the output effect by 0.31 percentage points.

The expected duration changes due to the ARRA are less than a month, which tells us that the chosen value of φ is not extreme in the context of a great financial crisis, where the Gallup survey asked about quarters and years to expected recovery. Our model yields an output effect of 2.61 percent of steady state GDP, which is not too far from the 3.3 percent found in Denes and Eggertsson (2009). This is not due to an extremely severe depressionary starting point as in their paper, rather we assume only a severe recession but then get a further stimulus effect via the slightly shorter duration caused by the ARRA.

7 Concluding Remarks

The present paper considers the effect of fiscal stimulus in a New Keynesian DSGE model with an explicit role of the zero lower bound for the nominal interest rate. We introduce a state-dependent Markov probability of remaining in the liquidity trap, which depends negatively in the output gap in the economy, a feature which is supported by survey data.

Introducing endogenous persistence of the shock causes all fiscal policy multipliers to be scaled up, as the effects in the simple Markov model are strengthened via the persistence channel, both in and outside the liquidity trap. As the multiplier responds to the Markov probability in a non-linear way, the multipliers in the endogenous Markov model are greater the larger the output contraction, while they are almost constant when the nominal interest rate is positive. Evaluating the multipliers in the point of

[27] To check for robustness, we have adjusted the value of φ and recalculated our GR results under calibration B. For $\varphi = 0.08$ we have that the ARRA boosts output 2.56 percent (2.74 without the tax cut). For $\varphi = 0.12$ we find that this effect is 2.68 percent (2.87 without the tax cut). Thus the output results in our preferred calibration seem fairly robust.

zero stimulus we have that the government spending multiplier at the zero lower bound is increased from 1.19 in the simple Markov model to 1.36 in our endogenous Markov model, and that the spending tax multiplier equivalently increased from 0.89 to 0.96. A smaller multiplier in the simple version of the model will cause a smaller change in the persistence and thus a weaker persistence channel. This is why the labor tax multiplier only changes from -0.166 to -0.174 and the capital tax multiplier changes from -0.023 to -0.026.

We use the model to evaluate the expected output of the American Recovery and Reinvestment Act from February 2009 and find that under our preferred conservative calibration in the endogenous Markov model the stimulus package increases output by 2.61 percent, compared to 2.32 percent in a simple Markov model. This calibration implies that ARRA decreased expected duration by less than a month, thus we have that introducing even a small duration effect can affect the size of the multipliers considerably.

Our analysis of the endogenous Markov model and the case study of ARRA show that endogenizing the persistence of the demand shock can have considerable effects on the efficacy of fiscal policy in a liquidity trap. Hence, this mechanism should not be disregarded when considering the effect of fiscal policies when the nominal interest rate is at the zero lower bound.

Introducing capital accumulation would be interesting. Eggertsson (2011) and Christiano et al. (2011) make robustness checks and find that capital accumulation does not change their conclusion that the output effect of government spending is large in a liquidity trap. Endogenous persistence could, however, interact with capital accumulation, as the investment decision depends on future expected payoff dependent on expected duration of the downturn.

An important expansion of the DSGE literature has been the introduction of non-Ricardian households. López-Salido and Rabanal (2006), Galí et al. (2007) and Coenen and Straub (2005) incorporate these in different DSGE models that do not model the zero lower bound. Liquidity constrained households do not respond to the real interest rate, which is at the heart of the different dynamics in and outside the zero lower bound. Thus, the introduction of these could have interesting implications for the effect of the zero lower bound on fiscal multipliers.

References

A. Auerbach and Y. Gorodnichenko. Fiscal multipliers in recession and expansion. 2012.

Rüdiger Bachmann and Eric R Sims. Confidence and the transmission of government spending shocks. *Journal of Monetary Economics*, 59(3):235–249, 2012.

Emanuele Baldacci, Sanjeev Gupta, and Carlos Mulas-Granados. Getting it right: How fiscal response can shorten crisis length and raise growth. *Fiscal Policy: Lessons from the Crisis*, page 365, 2010.

Florin Bilbiie, Tommaso Monacelli, and Roberto Perotti. Tax cuts vs. government spending: Welfare and the zero lower bound. 2012.

O.J. Blanchard and C.M. Kahn. The solution of linear difference models under rational expectations. *Econometrica: Journal of the Econometric Society*, pages 1305–1311, 1980.

R Anton Braun, Lena Mareen Körber, and Yuichiro Waki. Some unpleasant properties of log-linearized solutions when the nominal rate is zero. 2012.

R.A. Braun and Y. Waki. On the fiscal multiplier when the zero bound on nominal interest rates is binding. Technical report, 2010.

G.A. Calvo. Staggered prices in a utility-maximizing framework. *Journal of monetary Economics*, 12(3):383–398, 1983.

CBO. Historical and projected estimates of potential gdp and the nairu - january 2009. Technical report, Congressional Budget Office, January 2009. URL `http://cbo.gov/publication/42344`.

Stephen G. Cecchetti, Marion Kohler, and Christian Upper. Financial crises and economic activity. Technical report, National Bureau of Economic Research, 2009.

Lawrence Christiano, Martin Eichenbaum, and Sergio Rebelo. When is the government spending multiplier large? *Journal of Political Economy*, 119(1):pp. 78–121, 2011.

G. Coenen and R. Straub. Does government spending crowd in private consumption? theory and empirical evidence for the euro area. *International Finance*, 8(3):435–470, 2005.

G. Coenen, C.J. Erceg, C. Freedman, D. Furceri, M. Kumhof, R. Lalonde, D. Laxton, J. Lindé, A. Mourougane, D. Muir, et al. Effects of fiscal stimulus in structural. *American Economic Journal: Macroeconomics*, 4(1):22–68, 2012.

J.F. Cogan, T. Cwik, J.B. Taylor, and V. Wieland. New keynesian versus old keynesian government spending multipliers. *Journal of Economic Dynamics and Control*, 34(3): 281–295, 2010.

Roland Craigwell, Troy Lorde, and Winston Moore. Fiscal policy and the duration of financial crises. *Applied Economics*, 45(6):793–801, 2013.

Vasco Cúrdia and Michael Woodford. Credit frictions and optimal monetary policy. 2009.

Asli Demirgüç-Kunt and Enrica Detragiache. Cross-country empirical studies of systemic bank distress: a survey. *National Institute Economic Review*, 192(1):68–83, 2005.

M. Denes and G.B. Eggertsson. A bayesian approach to estimating tax and spending multipliers. *Federal Reserve Bank of New York Staff Report*, 403, 2009.

Matthew Denes, Gauti B Eggertsson, and Sophia Gilbukh. Deficits, public debt dynamics and tax and spending multipliers. *The Economic Journal*, 123(566):F133–F163, 2013.

Gauti B Eggertsson. What fiscal policy is effective at zero interest rates? In *NBER Macroeconomics Annual 2010, Volume 25*, pages 59–112. University of Chicago Press, 2011.

G.B. Eggertsson and M. Woodford. The zero bound on interest rates and optimal monetary policy. *Brookings Papers on Economic Activity*, 2003(1):139–211, 2003.

Christopher Erceg and Jesper Lindé. *Is there a Fiscal Free Lunch in a Liquidity Trap?*, volume 12. 2014.

Jesús Fernández-Villaverde, Grey Gordon, Pablo A Guerrón-Quintana, and Juan Rubio-Ramírez. Nonlinear adventures at the zero lower bound. 2012.

J. Galí, J.D. López-Salido, and J. Vallés. Understanding the effects of government spending on consumption. *Journal of the European Economic Association*, 5(1):227–270, 2007.

M. Gertler and John Leahy. A phillips curve with an ss foundation. *Journal of Political Economy*, 116:533–572, 2008.

S. Hebous. The effects of discretionary fiscal policy on macroeconomic aggregates: a reappraisal. *Journal of Economic Surveys*, 25(4):674–707, 2011.

J.M. Keynes. *The general theory of interest, employment and money.* London: Macmillan, 1936.

L. Laeven and F. Valencia. *Systemic banking crises: a new database.* Number 2008-2224. International Monetary Fund, 2008.

J.D. López-Salido and P. Rabanal. Government spending and consumption-hours preferences. *Mimeograph, Federal Reserve Bank*, 2006.

Karel R. S. M. Mertens and Morten O. Ravn. Fiscal policy in an expectations-driven liquidity trap. *The Review of Economic Studies*, 2014.

Virgiliu Midrigan. Menu costs, multiproduct firms, and aggregate fluctuations. *Econometrica*, 79(4):pp. 1139–1180, 2011.

Christina Romer and Jared Bernstein. The job impact of the american recovery and reinvestment plan. *Washington, DC*, 2009.

H. Uhlig. Some fiscal calculus. *American Economic Review*, 100(2):30–34, 2010.

Michael Woodford. The taylor rule and optimal monetary policy. *American Economic Review*, pages 232–237, 2001.

Michael Woodford. *Interest and prices: Foundations of a theory of monetary policy.* Princeton University Press, 2003.

Michael Woodford. Simple analytics of the government expenditure multiplier. *American Economic Journal: Macroeconomics*, 3(1):1–35, 2011.

A Parameters in the Model

For the convenience of the reader, the parameters of the model are listed in the table below

α	Calvo parameter; probability of having your price fixed in a given period
β	Discounting rate of the household
ξ	Shock to discounting rate of the household
θ	The elasticity of substitution between the intermediate goods
ω	Curvature of the labor disutility function
σ	Curvature of the utility function for consumption
κ	Steepness of the Phillips curve, depends on other deep parameters
ϕ_π	Monetary policy response to inflation
ϕ_y	Monetary policy response to output gap
$\overline{\tau}^s, \overline{\tau}^w, \overline{\tau}^A$	Steady state rates for taxes on sales, wage income and assets.
μ	Probability of remaining in shock-state
$\widetilde{\mu}$	Constant part of Markov probability μ
φ	Duration parameter
r_S^e	The size of the real interest rate consistent with constant consumption

B Fiscal Multipliers in the Great Depression Scenario

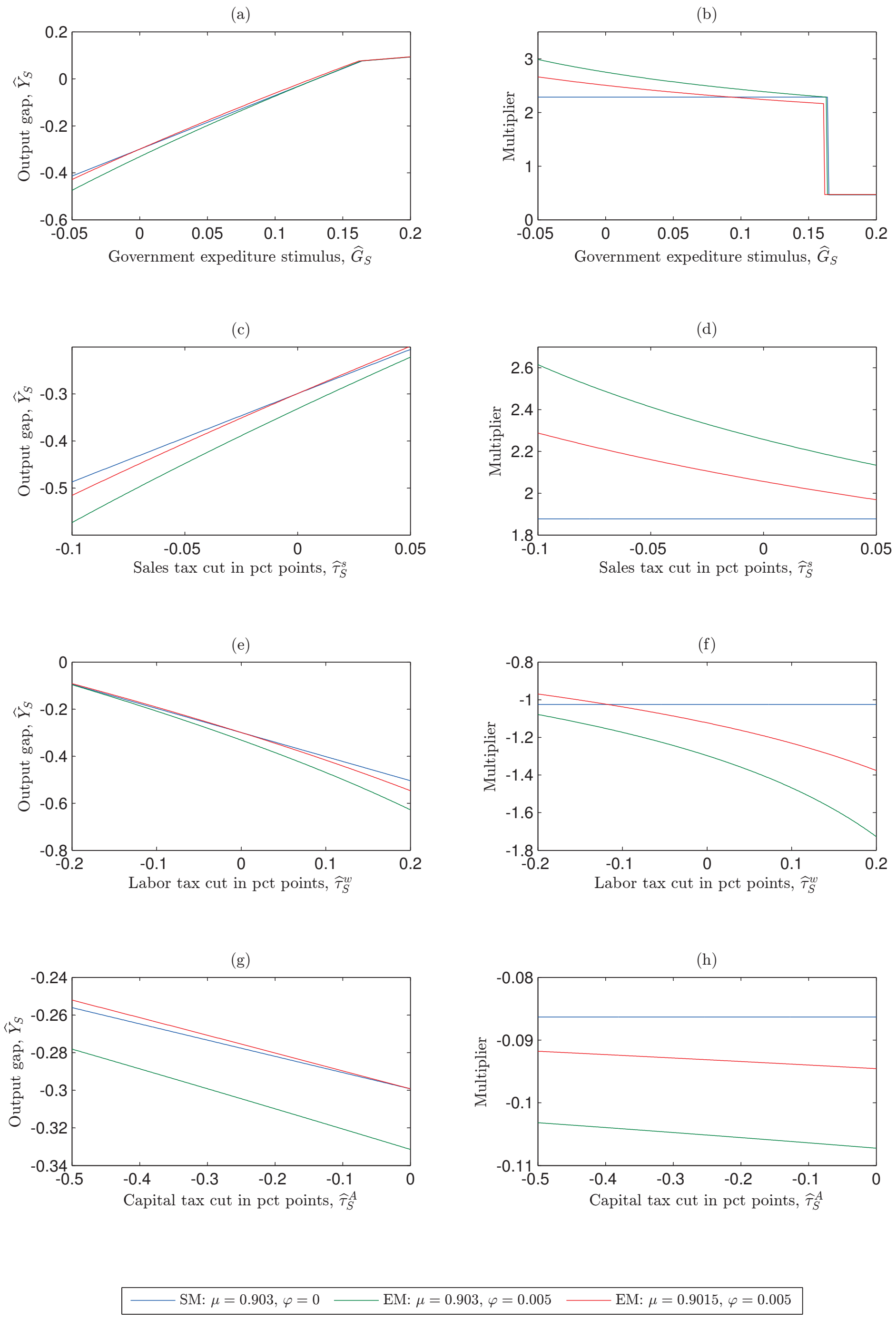

Figure 6: **Fiscal stimulus in the Great Depression Scenario.**

C Optimal Reset Price

Given the maximization problem in (6), the first order condition of the intermediate firm is

$$E_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \lambda_T \left(1-\tau_T^P\right) Y_T \left(\frac{1}{P_T}\right)^{-\theta} \left((1-\theta)\, p_t^{*-\theta} - (-\theta)\, W_T(j)\, p_t^{*-\theta-1}\right) = 0 \Leftrightarrow$$

$$E_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \lambda_T \left(1-\tau_T^P\right) Y_T \left(\frac{p_t^*}{P_T}\right)^{-\theta-1} \left(\frac{p_t^*}{P_T} - \frac{\theta}{\theta-1} W_T(j)\, p_t^{*-1} \left(\frac{p_t^*}{P_T}\right)\right) = 0$$

If we insert the intratemporal and intertemporal optimality conditions of the household (1)-(2), the fact that the Lagrange multiplier from the household optimization problem is $\lambda_T = \frac{u'(C_T)\xi_T}{\left(1+\tau_T^s\right)P_T}$, and $l_T(i) = Y_T \left(\frac{p_t^*}{P_T}\right)^{-\theta}$, then the firms optimality condition becomes

$$E_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \frac{u'(C_T)\,\xi_T}{\left(1+\tau_T^s\right) P_T} \left(1-\tau_T^P\right) Y_T \left(\frac{p_t^*}{P_T}\right)^{-\theta-1}$$
$$\left(\frac{p_t^*}{P_T} - \frac{\theta}{\theta-1} \frac{v'\left(Y_T \left(\frac{p_t^*}{P_T}\right)^{-\theta}\right)}{u'(C_t)} \frac{1+\tau_t^s}{1-\tau_t^w} P_t p_t^{*-1} \left(\frac{p_t^*}{P_T}\right)\right) = 0 \Leftrightarrow$$

$$E_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \frac{u'(C_T)\,\xi_T}{P_T} \frac{\left(1-\tau_T^P\right)}{\left(1+\tau_T^s\right)} Y_T \left(\frac{p_t^*}{P_T}\right)^{-\theta-1}$$
$$\left(\frac{p_t^*}{P_T} - \frac{\theta}{\theta-1} \frac{1+\tau_t^s}{1-\tau_t^w} \frac{v'\left(Y_T \left(\frac{p_t^*}{P_T}\right)^{-\theta}\right)}{u'(C_t)}\right) = 0.$$

D Existence of Deterministic Equilibrium

In this appendix we prove the existence of the deterministic steady state. We specifically consider the case of constant fiscal policy variables, so that $\tau_t^s = \overline{\tau}^s, \tau_t^A = \overline{\tau}^A, \tau_t^P = \overline{\tau}^P, \tau_t^w = \overline{\tau}^w, T_t = \overline{T}$, and $G_t = \overline{G}$ for $t \geq t_0$,where the budget constraint for the government holds for these values in steady state. [28]

The real economic variables are constant, so $Y_t = \overline{Y}$, $C_t = \overline{C} = \overline{Y} - \overline{G}$, and $l_t(i) = y_t(i) = \overline{y}(i)$. It then follows that the optimal reset price in equation (7) is a constant, and hence so is the aggregate price level. We thus have $\Pi_t = \frac{P_t}{P_{t-1}} = \frac{p_t^*}{P_t} = 1$, i.e. we are in a zero inflation steady state. This finally implies that $l_t(i) = y_t(i) = \overline{y}(i) = \overline{Y}$ and from the households Euler equation (1) it follows that the post- tax real interest rate must equal the inverse of the after tax stochastic discount factor, $i_t = \bar{i} = \beta^{-1}\left(1 - \overline{\tau}^A\right)^{-1} - 1 > 0$.

Finally, $\overline{Y}$ is given as the solution to the steady state version of the intermediate firm's optimality condition

$$\sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \frac{u'\left(\overline{Y} - \overline{G}\right)}{\overline{P}} \frac{\left(1 - \overline{\tau}^P\right)}{\left(1 + \overline{\tau}^s\right)} \overline{Y} \left(1 - \frac{\theta}{\theta - 1} \frac{1 + \overline{\tau}^s}{1 - \overline{\tau}^w} \frac{v'\left(\overline{Y}\right)}{u'\left(\overline{Y} - \overline{G}\right)}\right) = 0 \Leftrightarrow$$

$$\frac{\theta}{\theta - 1} \frac{1 + \overline{\tau}^s}{1 - \overline{\tau}^w} \frac{v'\left(\overline{Y}\right)}{u'\left(\overline{Y} - \overline{G}\right)} = 1$$

Note that the output level is independent of the tax on financial assets $\overline{\tau}^A$ and further does not have to be the efficient level of output. Woodford (2003) (in Appendix A) uses the inverse function theorem to show existence of a locally unique deterministic steady state in this nonlinear model, and Eggertsson and Woodford (2003) show that this also holds in the zero lower bound case of the model.

[28] Another way of ensuring this is to determine $\overline{T}$ residually, once the steady state is solved. The resulting steady states are identical for the two approaches.

E Linearizing the model

E.1 Aggregate Demand

We first find the aggregate demand by inserting the aggregate resource constraint (5) in the household's Euler equation (1)

$$u'\left(Y_t - G_t\right) = \left(1-\tau_t^A\right)\left(1+i_t\right)\beta E_t u'\left(Y_{t+1} - G_{t+1}\right)\frac{\xi_{t+1}}{\xi_t}\frac{1}{1+\pi_{t+1}}\frac{1+\tau_t^s}{1+\tau_{t+1}^s},$$

We then log-linearize around the zero inflation deterministic steady state. First we take natural logarithms on both sides

$$\begin{aligned}\ln u'\left(Y_t - G_t\right) = {} & \ln\left(1-\tau_t^A\right) + \ln\left(1+i_t\right) + \ln\beta \\ & + \ln E_t u'\left(Y_{t+1} - G_{t+1}\right) - \ln E_t\frac{\xi_t}{\xi_{t+1}} + \ln E_t\frac{1}{1+\pi_{t+1}} + \ln E_t\frac{1+\tau_t^s}{1+\tau_{t+1}^s},\end{aligned}$$

Next we make a first order Taylor approximation around the deterministic steady state, here shown after cancelling the logarithmic steady state terms

$$\begin{aligned}\frac{u''\left(\overline{Y}-\overline{G}\right)}{u'\left(\overline{Y}-\overline{G}\right)}\left(Y_t-\overline{Y}\right) - \frac{u''\left(\overline{Y}-\overline{G}\right)}{u'\left(\overline{Y}-\overline{G}\right)}\left(G_t-\overline{G}\right) = {} & -\frac{1}{1-\overline{\tau}^A}\left(\tau_t^A-\overline{\tau}^A\right) + \frac{\left(i_t-\overline{i}\right)}{\left(1+\overline{i}\right)} \\ & + \frac{u''\left(\overline{Y}-\overline{G}\right)}{u'\left(\overline{Y}-\overline{G}\right)}E_t\left(Y_{t+1}-\overline{Y}\right) - \frac{u''\left(\overline{Y}-\overline{G}\right)}{u'\left(\overline{Y}-\overline{G}\right)}E_t\left(G_{t+1}-\overline{G}\right) + \frac{1}{\overline{\xi}}\left(\xi_{t+1}-\overline{\xi}\right) \\ & - \frac{1}{\overline{\xi}}E_t\left(\xi_t-\overline{\xi}\right) - E_t\left(\pi_{t+1}-\overline{\pi}\right) + \frac{1}{1+\overline{\tau}^s}(\tau_t^s-\overline{\tau}^s) + \frac{1}{1+\overline{\tau}^s}E_t(\tau_{t+1}^s-\overline{\tau}^s)\end{aligned}$$

By defining the deviations from the deterministic steady state measured relative to output, $\widehat{Y}_t \equiv \ln\frac{Y_t}{\overline{Y}}$ and $\widehat{G}_t \equiv \ln\frac{G_t}{\overline{Y}}$, and $\widehat{\tau}_t^s \equiv \tau_t^s - \overline{\tau}^s$ and using the approximation that $\widehat{x}_t \equiv \ln\left(\frac{x_t}{x}\right) = \ln\left(1+\frac{x_t-x}{x}\right) \approx \frac{x_t-x}{x}$ for $\frac{x_t-x}{x}$ close to zero, we get the following expression

$$\begin{aligned}\frac{u''\left(\overline{Y}-\overline{G}\right)\overline{Y}}{u'\left(\overline{Y}-\overline{G}\right)}\left(\widehat{Y}_t-\widehat{G}_t\right) = {} & \frac{\left(i_t-\overline{i}\right)}{\left(1+\overline{i}\right)} - \frac{\left(\tau_t^A-\overline{\tau}^A\right)}{1-\overline{\tau}^A} \\ + \frac{u''\left(\overline{Y}-\overline{G}\right)\overline{Y}}{u'\left(\overline{Y}-\overline{G}\right)}E_t\left(\widehat{Y}_{t+1}-\widehat{G}_{t+1}\right) & - \frac{\xi_t-\overline{\xi}}{\overline{\xi}} + E_t\frac{\xi_{t+1}-\overline{\xi}}{\overline{\xi}} - E_t\pi_{t+1} + \frac{1}{1+\overline{\tau}^s}E_t\left(\widehat{\tau}_t^s-\widehat{\tau}_{t+1}^s\right)\end{aligned}$$

Using the parameter definitions $\sigma \equiv -\frac{\overline{u}_c}{\overline{u}_{cc}\overline{Y}}$, $\chi^A \equiv \frac{1-\beta}{1-\overline{\tau}^A}$, $\chi^w \equiv \frac{1}{1-\overline{\tau}^w}$, $\chi^s \equiv \frac{1}{1+\overline{\tau}^s}$, and $\widehat{\xi}_t \equiv \ln(\xi_t/\overline{\xi})$ we have

$$-\sigma^{-1}\left(\widehat{Y}_t - \widehat{G}_t\right) = -\sigma^{-1}E_t\left(\widehat{Y}_{t+1} - \widehat{G}_{t+1}\right) + \frac{\left(i_t - \overline{i}\right)}{\left(1+\overline{i}\right)} - \frac{\left(\tau_t^A - \overline{\tau}^A\right)}{1-\overline{\tau}^A} - E_t\pi_{t+1}$$
$$-\widehat{\xi}_t + E_t\widehat{\xi}_{t+1} + \chi^s E_t\left(\widehat{\tau}_t^s - \widehat{\tau}_{t+1}^s\right)$$

In steady state $\left(1+\overline{i}\right)\left(1-\overline{\tau}^A\right) = \beta^{-1}$. Using this and defining $\widehat{\tau}_t^A \equiv (1-\beta)^{-1}\left(\tau_t^A - \overline{\tau}^A\right)$ gives $\frac{\left(\tau_t^A - \overline{\tau}^A\right)}{1-\overline{\tau}^A} = \frac{1-\beta}{1-\overline{\tau}^A}(1-\beta)^{-1}\left(\tau_t^A - \overline{\tau}^A\right) = \chi^A\widehat{\tau}_t^A$.

Using the ln-approximation rule we have $\frac{\left(i_t - \overline{i}\right)}{\left(1+\overline{i}\right)} = \frac{\left(1+i_t-\left(1+\overline{i}\right)\right)}{\left(1+\overline{i}\right)} \approx \ln\left(\frac{1+i_t}{1+\overline{i}}\right) = \ln\left(1+i_t\right) - \ln\left(1+\overline{i}\right) = \ln\left(1+i_t\right) - \ln\beta^{-1} - \ln\left(1-\overline{\tau}^A\right)^{-1} \equiv \ln\left(1+i_t\right) - \overline{r}$.

Further, recalling that $i \approx \ln\left(1+i_t\right)$ for i_t close to zero, and defining $r_t^e \equiv \overline{r} + E_t\left(\widehat{\xi}_t - \widehat{\xi}_{t+1}\right)$ we have that

$\frac{\left(i_t - \overline{i}\right)}{\left(1+\overline{i}\right)} + E_t\left(\widehat{\xi}_{t+1} - \widehat{\xi}_t\right) \approx \ln\left(1+i_t\right) - \overline{r} - E_t\left(\widehat{\xi}_t - \widehat{\xi}_{t+1}\right) \approx i_t - \overline{r} - E_t\left(\widehat{\xi}_t - \widehat{\xi}_{t+1}\right) = i_t - r_t^e$

so that the log–linearized goods market equilibrium becomes

$$\widehat{Y}_t - \widehat{G}_t = E_t\left(\widehat{Y}_{t+1} - \widehat{G}_{t+1}\right) - \sigma\left(i_t - E_t\pi_{t+1} - r_t^e\right) + \sigma\chi^s E_t\left(\widehat{\tau}_{t+1}^s - \widehat{\tau}_t^s\right) + \sigma\chi^A\widehat{\tau}_{t-1}^A.$$

E.2 Aggregate Supply

The aggregate supply is determined by the optimal price equation (7) and the aggregate price equation. We first restate the aggregate price equation

$$P_t^{1-\theta} = (1-\alpha)\left(p_t^*\right)^{1-\theta} + \alpha P_{t-1}^{1-\theta}$$

Taking logs gives

$$(1-\theta)\ln P_t = \ln\left[(1-\alpha)\left(p_t^*\right)^{1-\theta} + \alpha P_{t-1}^{1-\theta}\right]$$

and the first order Taylor expansion becomes

$$\ln \overline{P}^{(1-\theta)} + \frac{(1-\theta)\,\overline{P}^{(-\theta)}}{\overline{P}^{(1-\theta)}}\left(P_t - \overline{P}\right) = \ln\left[(1-\alpha)\,\overline{P}^{1-\theta} + \alpha\overline{P}^{1-\theta}\right]$$
$$+ \frac{(1-\alpha)(1-\theta)\,\overline{P}^{-\theta}}{(1-\alpha)\,\overline{P}^{1-\theta} + \alpha\overline{P}^{1-\theta}}\left(p_t^* - \overline{P}\right) + \frac{\alpha(1-\theta)\,\overline{P}^{-\theta}}{(1-\alpha)\,\overline{P}^{1-\theta} + \alpha\overline{P}^{1-\theta}}\left(P_{t-1} - \overline{P}\right)$$

$$\frac{P_t - \overline{P}}{\overline{P}} = (1-\alpha)\,\frac{p_t^* - \overline{P}}{\overline{P}} + \alpha\frac{P_{t-1} - \overline{P}}{\overline{P}}$$

This finally gives us

$$\widehat{P}_t = (1-\alpha)\,\widehat{p}_t^* + \alpha\widehat{P}_{t-1}.$$

We now turn to the optimal reset pice of the intermediate firm, and state this with the aggregate resource constraint and demand function substituted in

$$E_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t}\, \frac{u'(Y_T - G_T)\,\xi_T}{P_T}\frac{\left(1-\tau_T^P\right)}{(1+\tau_T^s)} Y_T \left(\frac{p_t^*}{P_T}\right)^{-\theta-1}\left(\frac{p_t^*}{P_T} - \frac{\theta}{\theta-1}\frac{1+\tau_t^s}{1-\tau_t^w}\frac{v'\left(Y_T\frac{p_t^*}{P_T}\right)}{u'(Y_T - G_T)}\right) = 0$$

$$E_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t}\, \frac{u'(Y_T - G_T)\,\xi_T}{P_T}\frac{\left(1-\tau_T^P\right)}{(1+\tau_T^s)} Y_T \left(\frac{p_t^*}{P_T}\right)^{-\theta}$$
$$= \frac{\theta}{\theta-1} E_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t}\, \frac{\xi_T}{P_T}\frac{\left(1-\tau_T^P\right)}{(1-\tau_T^w)} Y_T \left(\frac{p_t^*}{P_T}\right)^{-\theta-1} v'\left(Y_T\left(\frac{p_t^*}{P_T}\right)^{-\theta}\right)$$

So the log-linear approximation to the LHS is (excl. the logarithmic terms that will cancel out with the logarithmic terms on the RHS)

$$E_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t}(1-\alpha\beta)\left[\sigma^{-1}+1\right]\frac{(Y_T - \overline{Y})}{\overline{Y}}$$
$$- E_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t}(1-\alpha\beta)\,\sigma^{-1}\frac{(G_T - \overline{G})}{\overline{Y}} + E_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t}(1-\alpha\beta)\,\frac{1}{\overline{\xi}}(\xi_T - \overline{\xi})$$
$$- E_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t}(1-\alpha\beta)\,\frac{\widehat{\tau}_T^s}{(1+\tau_T^s)} - E_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t}(1-\alpha\beta)\,\frac{\left(\tau_T^P - \tau_T^P\right)}{\left(1-\overline{\tau}^P\right)}$$
$$- E_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t}(1-\alpha\beta)\,\theta\frac{p_t^* - \overline{P}}{\overline{P}} + E_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t}(1-\alpha\beta)(\theta-1)\,\frac{P_t - \overline{P}}{\overline{P}}$$

Finally, using he definition of the hat-variables, the log-linear approximation to LHS is

$$E_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} (1-\alpha\beta) [\sigma^{-1}+1]\widehat{Y}_T - E_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} (1-\alpha\beta) \sigma^{-1}\widehat{G}_T$$
$$+ E_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} (1-\alpha\beta) \frac{\xi_T - \overline{\xi}}{\overline{\xi}} - E_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} (1-\alpha\beta) \left(\chi^s \widehat{\tau}_T^s - \chi^P \widehat{\tau}_T^P\right)$$
$$+ E_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} (1-\alpha\beta) \left[-\theta \widehat{p}_t^* + (\theta - 1) \widehat{P}_T\right]$$

Now turn to RHS, where the log-linear approximation (excl. logarithmic terms) is

$$E_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} (1-\alpha\beta) \left[1 + \frac{\overline{Y} v''\left(\overline{Y}\right)}{v'\left(\overline{Y}\right)}\right] \frac{\left(Y_T - \overline{Y}\right)}{\overline{Y}}$$
$$+ E_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} (1-\alpha\beta) \left[\frac{(\xi_T - \overline{\xi})}{\overline{\xi}} + \frac{(\tau_T^w - \tau_T^w)}{(1-\overline{\tau}^w)} - \frac{\left(\tau_T^P - \overline{\tau}^P\right)}{\left(1-\overline{\tau}^P\right)}\right]$$
$$+ E_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} (1-\alpha\beta) \frac{1}{\overline{P}} \left((-\theta - 1) - \frac{v''\left(\overline{Y}\right)\overline{Y}\theta}{v'\left(\overline{Y}\right)}\right) \left(p_t^* - \overline{P}\right)$$
$$+ E_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} (1-\alpha\beta) \frac{1}{\overline{P}} \left(1 - \frac{v''\left(\overline{Y}\right)\overline{Y}\theta}{v'\left(\overline{Y}\right)}\right) \left(\widehat{P}_T - \overline{P}\right)$$

So define $\omega \equiv \frac{\overline{v}_l}{\overline{v}_{ll}\overline{Y}}$, the RHS linearization is

$$+E_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} (1-\alpha\beta) \left[1+\omega^{-1}\right] \widehat{Y}_T + E_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} (1-\alpha\beta) \left(\frac{\xi_T - \overline{\xi}}{\overline{\xi}} + \chi^w \widehat{\tau}_T^w - \chi^P \widehat{\tau}_T^P\right)$$
$$+ E_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} (1-\alpha\beta) \left[(-\theta - 1 - \omega\theta) \frac{p_t^* - \overline{P}}{\overline{P}} + (1-\omega\theta) \frac{\widehat{P}_T - \overline{P}}{\overline{P}}\right]$$

Setting the RHS equal to the LHS and cancelling terms gives us

$$E_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} [(1+\omega\theta) \left(\widehat{p}_t^* - \widehat{P}_T\right) - \chi^s \widehat{\tau}_T^s - \chi^w \widehat{\tau}_T^w - \left(\omega + \sigma^{-1}\right) \widehat{Y}_T + \sigma^{-1}\widehat{G}_T] = 0 \Leftrightarrow$$

$$\widehat{p}_t^* = E_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} [(1-\alpha\beta)\widehat{P}_T + \frac{(1-\alpha\beta)}{(1+\omega\theta)}\chi^s\widehat{\tau}_T^s + \frac{(1-\alpha\beta)}{(1+\omega\theta)}\chi^w\widehat{\tau}_T^w + \frac{(\omega+\sigma^{-1})(1-\alpha\beta)}{(1+\omega\theta)}\widehat{Y}_T - \frac{(1-\alpha\beta)\sigma^{-1}}{(1-\omega\theta)}\widehat{G}_T]$$

Writing this as a first order difference equation we have

$$\widehat{p}_t^* = (1-\alpha\beta)\widehat{P}_t + \frac{(1-\alpha\beta)}{(1+\omega\theta)}\left(\chi^s\widehat{\tau}_t^s + \chi^w\widehat{\tau}_t^w + \left(\omega+\sigma^{-1}\right)\widehat{Y}_t - \sigma^{-1}\widehat{G}_t\right) + \alpha\beta E_t\widehat{p}_{t+1}^*$$

Now using that $\widehat{p}_{t+1}^* = \frac{\widehat{P}_{t+1}-\alpha\widehat{P}_t}{1-\alpha}$ and inserting this, we get

$$\widehat{p}_t^* = (1-\alpha\beta)\widehat{P}_t + \alpha\beta E_t\left(\frac{\widehat{P}_{t+1}-\alpha\widehat{P}_t}{1-\alpha}\right) + \frac{(1-\alpha\beta)}{(1+\omega\theta)}\left(\chi^s\widehat{\tau}_t^s + \chi^w\widehat{\tau}_t^w + \left(\omega+\sigma^{-1}\right)\widehat{Y}_t - \sigma^{-1}\widehat{G}_t\right)$$

Inserting this into $\widehat{P}_t = (1-\alpha)\widehat{p}_t^* + \alpha\widehat{P}_{t-1}$ then gives

$$\widehat{P}_t = (1-\alpha)(1-\alpha\beta)\widehat{P}_t + \alpha\beta E_t\left(\widehat{P}_{t+1} + \alpha\widehat{P}_t\right) - \alpha\widehat{P}_{t-1} + \frac{(1-\alpha)(1-\alpha\beta)}{(1+\omega\theta)}\left(\chi^s\widehat{\tau}_t^s + \chi^w\widehat{\tau}_t^w + \left(\omega+\sigma^{-1}\right)\widehat{Y}_t - \sigma^{-1}\widehat{G}_t\right) \Leftrightarrow$$

$$\widehat{P}_t - \alpha\widehat{P}_{t-1} = (1-\alpha)(1-\alpha\beta)\widehat{P}_t + \alpha\beta E_t\widehat{P}_{t+1} - \alpha^2\beta E_t\widehat{P}_t + \frac{(1-\alpha)(1-\alpha\beta)}{(1+\omega\theta)}\left(\chi^s\widehat{\tau}_t^s + \chi^w\widehat{\tau}_t^w + \left(\omega+\sigma^{-1}\right)\widehat{Y}_t - \sigma^{-1}\widehat{G}_t\right) \Leftrightarrow$$

$$\left(\widehat{P}_t - \widehat{P}_{t-1}\right) = \frac{(1-\alpha)(1-\alpha\beta)}{\alpha(1+\omega\theta)}\left(\chi^s\widehat{\tau}_t^s + \chi^w\widehat{\tau}_t^w + \left(\omega+\sigma^{-1}\right)\widehat{Y}_t - \sigma^{-1}\widehat{G}_t\right) + \beta\left(E_t\widehat{P}_{t+1} - \widehat{P}_t\right)$$

Finally, using the definition that $\left(\widehat{P}_t - \widehat{P}_{t-1}\right) = \pi_t$ and defining $\kappa = \frac{(1-\alpha)(1-\alpha\beta)}{\alpha(1+\omega\theta)}\left(\omega+\sigma^{-1}\right)$ and $\psi = \frac{1}{(\omega+\sigma^{-1})}$ we have our aggregate supply, which is also the standard New Keynesian

Phillips curve

$$\widehat{\pi}_t = \kappa \widehat{Y}_t + \kappa \psi \left(\chi^s \widehat{\tau}_t^s + \chi^w \widehat{\tau}_t^w - \sigma^{-1} \widehat{G}_t \right) + \beta E_t \pi_{t+1}.$$

F Deriving Condition C3

We assume that the support of $\widehat{Y}_t$ is a bounded interval $\left[\widehat{Y}^{\min}, \widehat{Y}^{\max}\right]$. If we want to keep the probability interpretation of μ, we must have

$$
\begin{aligned}
\widetilde{\mu} - \varphi \widehat{Y}^{\min} &< 1 \\
0 &< \widetilde{\mu} - \varphi \widehat{Y}^{\max}
\end{aligned}
$$

Combining these two constraints gives

$$\varphi \widehat{Y}^{\max} < \widetilde{\mu} < 1 + \varphi \widehat{Y}^{\min}.$$

G Proof of Proposition 2

This proof is based on Blanchard and Kahn (1980) Proposition 1 and follows the approach of (Woodford, 2003, Appendix A) and Eggertsson and Woodford (2003).[29] Consider the linear system of equations

$$E_t z_{t+1} = \mathbf{A} z_t + \mathbf{a} e_{t+1} \tag{26}$$

where z_t for our purpose is a two-dimensional vector of forward looking (non-predetermined) variables, e_{t+1} is a vector of n shocks, the coefficient matrices $\mathbf{A}$ and $\mathbf{a}$ are 2x2 and 2xn respectively. Applying the Blanchard-Kahn conditions, there exists a unique stationary solution if and only if both eigenvalues for $\mathbf{A}$ are outside the unit circle (since the BK-condition states that the number of eigenvalues with modulus larger than one must exactly equal the number of forward-looking variables in z_t). Using the fact that $\mathbf{A}$ is a 2x2 matrix, and letting $tr(\mathbf{A})$ denote the trace of $\mathbf{A}$ and $det(\mathbf{A})$ denote the denominator of $\mathbf{A}$, this requirement boils down to the following proposition.

Proposition 6 *There exists a unique solution to the system in equation (26) if and only if one of the two cases below holds.*

Case 1: *(a)* $det(\mathbf{A}) > 1$ *(b)* $det(\mathbf{A})$ - $tr(\mathbf{A}) > -1$ *(c)* $det(\mathbf{A})$ + $tr(\mathbf{A}) > -1$

Case 2: *(d)* $det(\mathbf{A})$ - $tr(\mathbf{A}) < -1$ *(e)* $det(\mathbf{A})$ + $tr(\mathbf{A}) < -1$

Proof. *See Blanchard and Kahn (1980) and (Woodford, 2003, Appendix A).* ■

We will now consider the Blanchard-Kahn conditions for existence of a unique deterministic solution of our model. Given the absence of shocks for all $t > T^e$ we have that $\tau_t^s, \tau_{t-1}^A, \tau_t^P, \tau_t^w, G_t$ and T_t equal their steady state values. As noted in the model section, in a steady state, it must hold that $\widehat{i}_t = \overline{r} > 0$.This implies that for all $t > T^e$, the model in equations (11), (13) and (12) may be written on the form

$$E_t \begin{bmatrix} \widehat{Y}_{t+1} \\ \pi_{t+1_t} \end{bmatrix} = \begin{bmatrix} 1 + \sigma\left(\phi_y + \frac{\kappa}{\beta}\right) & \sigma\left(\phi_\pi - \frac{1}{\beta}\right) \\ -\frac{\kappa}{\beta} & \frac{1}{\beta} \end{bmatrix} \begin{bmatrix} \widehat{Y}_t \\ \pi_t \end{bmatrix} = A \begin{bmatrix} \widehat{Y}_t \\ \widehat{\pi}_t \end{bmatrix}, \tag{27}$$

where $\det(A) = \frac{1}{\beta}\left(1 + \sigma\left(\phi_y + \kappa\phi_\pi\right)\right) > 0$ and $tr(A) = 1 + \sigma\left(\phi_y + \frac{\kappa}{\beta}\right) + \frac{1}{\beta} > 0$. This immediately implies that *(e)* is violated, so we can rule out case 2. Let us turn to case 1.

[29] Woodford (2003) shows the posposition for a similar nodel without a lower bound for the interest rate. Eggertsson and Woodford (2003) expand this to the case of a lower bound on the interest rate.

(a) $\det(A) > 1 \Leftrightarrow 1+\sigma\left(\phi_y + \kappa\phi_\pi\right) > \beta$, which holds as $\sigma, \phi_y, \kappa, \phi_\pi > 0$ and $0 < \beta < 1$.

(b) $\det(A) - tr(A) > -1 \Leftrightarrow \frac{(1-\beta)}{\kappa}\phi_y + \phi_\pi > 1$, which holds as $\phi_\pi > 1$, $0 < \beta < 1$, and $\phi_y, \kappa > 0$.

(c) $\det(A) + tr(A) > -1$ holds as both $\det(A)$ and $tr(A)$ are positive.

Thus there exists a unique bounded solution to the model for all $t > T^e$. It can easily be seen form equation (27) that $\begin{bmatrix}\widehat{Y}_t\\ \pi_t\end{bmatrix} = \begin{bmatrix}0\\0\end{bmatrix}$ is a solution to the model.

H The Endogenous Markov Model

H.1 The Model

Restating the equilibrium conditions

$$
\begin{aligned}
(1-\mu)\widehat{Y}_S &= (1-\mu)\left(\widehat{G}_S - \sigma\chi^s\widehat{\tau}_S^s\right) - \sigma\left(\widehat{i}_t - \mu\pi_S - r_S^e\right) + \sigma\chi^A\widehat{\tau}_S^A \\
\widehat{\pi}_S &= \kappa\widehat{Y}_t + \kappa\psi\left(\chi^s\widehat{\tau}_S^s + \chi^w\widehat{\tau}_S^w - \sigma^{-1}\widehat{G}_S\right) + \beta\mu\pi_S \\
i_S &= \max\left\{0, r_S^e + \phi_\pi\pi_S + \phi_y\widehat{Y}_S\right\} \\
\mu &= \widetilde{\mu} - \varphi\widehat{Y}_S
\end{aligned}
$$

which by substitution of the function for μ yields

$$
\pi_S = \frac{\kappa}{1-\beta\left(\widetilde{\mu}-\varphi\widehat{Y}_S\right)}\widehat{Y}_S + \frac{\kappa\psi}{1-\beta\left(\widetilde{\mu}-\varphi\widehat{Y}_S\right)}\left(\chi^s\widehat{\tau}_S^s + \chi^w\widehat{\tau}_S^w - \sigma^{-1}\widehat{G}_S\right) \tag{28}
$$

$$
\begin{aligned}
\left(1-\left(\widetilde{\mu}-\varphi\widehat{Y}_S\right)\right)\widehat{Y}_S = \left(1-\left(\widetilde{\mu}-\varphi\widehat{Y}_S\right)\right)&\left(\widehat{G}_S - \sigma\chi^s\widehat{\tau}_S^s\right) \\
&- \sigma\left(\widehat{i}_t - \left(\widetilde{\mu}-\varphi\widehat{Y}_S\right)\pi_S - r_S^e\right) + \sigma\chi^A\widehat{\tau}_{S.}^A
\end{aligned} \tag{29}
$$

H.2 Slope of the AD and AS curves

When $i > 0$ the AD curve is

$$
(1-\mu)\widehat{Y}_S = (1-\mu)\left(\widehat{G}_S - \sigma\chi^s\widehat{\tau}_S^s\right) + \sigma\left(\mu\pi_S + r_S^e\right) + \sigma\chi^A\widehat{\tau}_S^A
$$

$$
f\left(\right) = (1-\mu)\widehat{Y}_S - (1-\mu)\left(\widehat{G}_S - \sigma\chi^s\widehat{\tau}_S^s\right) - \sigma\mu\pi_S - \sigma r_S^e - \sigma\chi^A\widehat{\tau}_S^A = 0
$$

$$
\begin{aligned}
\frac{df\left(\right)}{d\widehat{Y}_S} &= (1-\mu) + \varphi\widehat{Y}_S - \varphi\left(\widehat{G}_S - \sigma\chi^s\widehat{\tau}_S^s\right) + \sigma\varphi\pi_S \quad \wedge \quad \frac{df\left(\right)}{d\pi_S} = -\sigma\mu \\
\frac{d\widehat{Y}_S}{d\pi_S} &= \frac{(1-\mu) + \varphi\widehat{Y}_S - \varphi\left(\widehat{G}_S - \sigma\chi^s\widehat{\tau}_S^s\right) + \sigma\varphi\pi_S}{\sigma\mu}
\end{aligned}
$$

When $i = 0$ the AD curve is

$$(1-\mu)\widehat{Y}_S = (1-\mu)\left(\widehat{G}_S - \sigma\chi^s\widehat{\tau}_S^s\right) - \sigma\left((\phi_\pi - \mu)\pi_S + \phi_y\widehat{Y}_S\right) + \sigma\chi^A\widehat{\tau}_S^A$$

$$f^p() = (1-\mu)\widehat{Y}_S - (1-\mu)\left(\widehat{G}_S - \sigma\chi^s\widehat{\tau}_S^s\right) + \sigma\left((\phi_\pi - \mu)\pi_S + \phi_y\widehat{Y}_S\right) - \sigma\chi^A\widehat{\tau}_S^A = 0$$

$$\begin{aligned}\frac{df()}{d\widehat{Y}_S} &= (1-\mu) + \varphi\widehat{Y}_S - \varphi\left(\widehat{G}_S - \sigma\chi^s\widehat{\tau}_S^s\right) + \sigma\left(\varphi\pi_S + \phi_y\right) \quad \wedge \quad \frac{df()}{d\pi_S} = \sigma(\phi_\pi - \mu) \\ \frac{d\widehat{Y}_S}{d\pi_S} &= -\frac{(1-\mu) + \varphi\widehat{Y}_S - \varphi\left(\widehat{G}_S - \sigma\chi^s\widehat{\tau}_S^s\right) + \sigma\left(\varphi\pi_S + \phi_y\right)}{\sigma(\phi_\pi - \mu)}.\end{aligned}$$

The AS curve is $\widehat{\pi}_S = \kappa\widehat{Y}_t + \kappa\psi\left(\chi^s\widehat{\tau}_S^s + \chi^w\widehat{\tau}_S^w - \sigma^{-1}\widehat{G}_S\right) + \beta\mu\pi_S$ which implies the slope

$$(1-\beta\mu)\widehat{\pi}_S = \kappa\widehat{Y}_t + \kappa\psi\left(\chi^s\widehat{\tau}_S^s + \chi^w\widehat{\tau}_S^w - \sigma^{-1}\widehat{G}_S\right)$$

$$f^{as}() = \kappa\widehat{Y}_t + \kappa\psi\left(\chi^s\widehat{\tau}_S^s + \chi^w\widehat{\tau}_S^w - \sigma^{-1}\widehat{G}_S\right) - (1-\beta\mu)\widehat{\pi}_S$$

$$\begin{aligned}\frac{df()}{d\widehat{Y}_S} &= \kappa - \beta\varphi\widehat{\pi}_S \quad \wedge \quad \frac{df()}{d\pi_S} = -(1-\beta\mu) \\ \frac{d\widehat{Y}_S}{d\pi_S} &= \frac{\kappa - \beta\varphi\widehat{\pi}_S}{(1-\beta\mu)}\end{aligned}$$

H.3 Equilibrium for φ=0

First we introduce the following two conditions

$$
\begin{aligned}
r_L^e \quad < \quad & -\frac{\kappa\left(1-\mu\right)\left(1-\psi\sigma^{-1}\right)\phi_\pi+\phi_y\left(1-\mu\right)\left(1-\beta\mu\right)-\phi_y\kappa\psi}{\left(1-\mu+\sigma\phi_y\right)\left(1-\beta\mu\right)+\kappa\sigma\left(\phi_\pi-\mu\right)}\left(\widehat{G}_L-\sigma\chi^s\widehat{\tau}_L^s\right) \\
& -\frac{\kappa\sigma\phi_\pi+\sigma\left(1-\beta\mu\right)\phi_y}{\left(1-\mu+\sigma\phi_y\right)\left(1-\beta\mu\right)+\kappa\sigma\left(\phi_\pi-\mu\right)}\chi^A\widehat{\tau}_L^A \\
& -\frac{\left(1-\mu\right)\kappa\psi\phi_\pi+\sigma\kappa\psi\phi_y}{\left(1-\mu+\sigma\phi_y\right)\left(1-\beta\mu\right)+\kappa\sigma\left(\phi_\pi-\mu\right)}\chi^w\widehat{\tau}_L^w
\end{aligned}
\tag{C1_SM}
$$

$$
\sigma\mu\kappa<\left(1-\mu\right)\left(1-\beta\mu\right) \tag{C2_SM}
$$

When $\varphi = 0$ we have closed form solutions for $\widehat{Y}_S$ and π_S:

Proposition 7 *In the short run, $t < T^e$, we consider two cases for the equilibrium outcome of the economy.*

Proposition 8 Case 1. The economy has a positive nominal interest rate

If (C1_SM) then a unique bounded solution exists outside the zero lower bound, where

$$
\begin{aligned}
\widehat{Y}_S = & \frac{\left(1-\mu\right)\left(1-\beta\mu\right)+\kappa\psi\left(\phi_\pi-\mu\right)}{\left(1-\mu+\sigma\phi_y\right)\left(1-\beta\mu\right)+\kappa\sigma\left(\phi_\pi-\mu\right)}\left(\widehat{G}_S-\sigma\chi^s\widehat{\tau}_S^s\right) \\
& -\frac{\kappa\sigma\psi\left(\phi_\pi-\mu\right)}{\left(1-\mu+\sigma\phi_y\right)\left(1-\beta\mu\right)+\kappa\sigma\left(\phi_\pi-\mu\right)}\chi^w\widehat{\tau}_S^w \\
& +\frac{\sigma\left(1-\beta\mu\right)}{\left(1-\mu+\sigma\phi_y\right)\left(1-\beta\mu\right)+\kappa\sigma\left(\phi_\pi-\mu\right)}\chi^A\widehat{\tau}_S^A
\end{aligned}
\tag{30}
$$

$$
\begin{aligned}
\pi_S = & \frac{\kappa\left[\left(1-\mu\right)-\psi\sigma^{-1}\left(1-\mu+\sigma\phi_y\right)\right]}{\left(1-\mu+\sigma\phi_y\right)\left(1-\beta\mu\right)+\kappa\sigma\left(\phi_\pi-\mu\right)}\left(\widehat{G}_S-\sigma\chi^s\widehat{\tau}_S^s\right) \\
& +\frac{\left(1-\mu+\sigma\phi_y\right)\kappa\psi}{\left(1-\mu+\sigma\phi_y\right)\left(1-\beta\mu\right)+\kappa\sigma\left(\phi_\pi-\mu\right)}\chi^w\widehat{\tau}_S^w \\
& +\frac{\kappa\sigma}{\left(1-\mu+\sigma\phi_y\right)\left(1-\beta\mu\right)+\kappa\sigma\left(\phi_\pi-\mu\right)}\chi^A\widehat{\tau}_{S.}^A
\end{aligned}
\tag{31}
$$

Case 2. The economy is at the zero lower bound

If the conditions (C1_SM) and (C2_SM) are satisfied, there exists a unique bounded *solution, where the zero lower bound is active and where*

$$\widehat{Y}_t = \frac{(1-\beta\mu)\,\sigma}{(1-\mu)(1-\beta\mu)-\mu\kappa\sigma} r_S^e + \frac{(1-\mu)(1-\beta\mu)-\kappa\mu\psi}{(1-\mu)(1-\beta\mu)-\mu\kappa\sigma}\left(\widehat{G}_S - \sigma\chi^s\widehat{\tau}_S^s\right) + \frac{\sigma\mu\kappa\psi}{(1-\mu)(1-\beta\mu)-\mu\kappa\sigma}\chi^w\widehat{\tau}_S^w + \frac{\sigma(1-\beta\mu)}{(1-\mu)(1-\beta\mu)-\mu\kappa\sigma}\chi^A\widehat{\tau}_{S.}^A \tag{32}$$

$$\pi_S = \frac{\kappa\sigma}{(1-\mu)(1-\beta\mu)-\kappa\mu\sigma} r_S^e + \frac{\kappa(1-\mu)(1-\psi\sigma^{-1})}{(1-\mu)(1-\beta\mu)-\kappa\mu\sigma}\left(\widehat{G}_S - \sigma\chi^s\widehat{\tau}_S^s\right) + \frac{\kappa\sigma}{(1-\mu)(1-\beta\mu)-\kappa\mu\sigma}\chi^A\widehat{\tau}_{S.}^A + \frac{(1-\mu)\kappa\psi}{(1-\mu)(1-\beta\mu)-\kappa\mu\sigma}\chi^w\widehat{\tau}_S^w. \tag{33}$$

This allows us to solve for the threshold re

$$r_S^e + \phi_\pi\pi_S + \phi_y\widehat{Y}_S < 0 \Longleftrightarrow r_S^e < -\phi_\pi\pi_S - \phi_y\widehat{Y}_S$$

inserting the solutions for $\pi_S + \widehat{Y}_S$ for our model

$$r_S^e < -\frac{\phi_y(1-\mu)(1-\beta\mu)+\kappa\psi(\phi_\pi-\mu)+\phi_\pi\kappa\left[(1-\mu)-\psi\sigma^{-1}\left(1-\mu+\sigma\phi_y\right)\right]}{\left(1-\mu+\sigma\phi_y\right)(1-\beta\mu)+\kappa\sigma(\phi_\pi-\mu)}\left(\widehat{G}_S - \sigma\chi^s\widehat{\tau}_S^s\right) + \frac{\phi_y\kappa\sigma\psi(\phi_\pi-\mu)-\phi_\pi\left(1-\mu+\sigma\phi_y\right)\kappa\psi}{\left(1-\mu+\sigma\phi_y\right)(1-\beta\mu)+\kappa\sigma(\phi_\pi-\mu)}\chi^w\widehat{\tau}_S^w - \frac{\phi_y\sigma(1-\beta\mu)+\phi_\pi\kappa\sigma}{\left(1-\mu+\sigma\phi_y\right)(1-\beta\mu)+\kappa\sigma(\phi_\pi-\mu)}\chi^A\widehat{\tau}_S^A$$

H.4 Equilibrium at a Positive Interest Rate

We use the solution from the SM model to write our equilibrium output in the EM model: In equation (30) we multiply by $\left(1-\mu+\sigma\phi_y\right)(1-\beta\mu)+\kappa\sigma\left(\phi_\pi-\mu\right)$ and insert $\mu=\widetilde{\mu}-\varphi\widehat{Y}_S$ to get

$$\begin{aligned}
&\left[\left(1-\left(\widetilde{\mu}-\varphi\widehat{Y}_S\right)+\sigma\phi_y\right)\left(1-\beta\left(\widetilde{\mu}-\varphi\widehat{Y}_S\right)\right)+\kappa\sigma\left(\phi_\pi-\left(\widetilde{\mu}-\varphi\widehat{Y}_S\right)\right)\right]\widehat{Y}_S\\
&=\left[\left(1-\left(\widetilde{\mu}-\varphi\widehat{Y}_S\right)\right)\left(1-\beta\left(\widetilde{\mu}-\varphi\widehat{Y}_S\right)\right)+\kappa\psi\left(\phi_\pi-\left(\widetilde{\mu}-\varphi\widehat{Y}_S\right)\right)\right]\left(\widehat{G}_S-\sigma\chi^s\widehat{\tau}_S^s\right)\\
&\qquad-\kappa\sigma\psi\left(\phi_\pi-\left(\widetilde{\mu}-\varphi\widehat{Y}_S\right)\right)\chi^w\widehat{\tau}_S^w+\sigma\left(1-\beta\left(\widetilde{\mu}-\varphi\widehat{Y}_S\right)\right)\chi^A\widehat{\tau}_S^A.
\end{aligned}$$

Now define the auxiliary function h

$$\begin{aligned}
&h\left(\widehat{Y}_S,\widehat{G}_S,\widehat{\tau}_S^s,\widehat{\tau}_S^A,\widehat{\tau}_S^w\right)=\left(1-\left(\widetilde{\mu}-\varphi\widehat{Y}_S\right)+\sigma\phi_y\right)\left(1-\beta\left(\widetilde{\mu}-\varphi\widehat{Y}_S\right)\right)\widehat{Y}_S\\
&+\kappa\sigma\left(\phi_\pi-\left(\widetilde{\mu}-\varphi\widehat{Y}_S\right)\right)\widehat{Y}_S+\kappa\sigma\psi\left(\phi_\pi-\left(\widetilde{\mu}-\varphi\widehat{Y}_S\right)\right)\chi^w\widehat{\tau}_S^w-\sigma\left(1-\beta\left(\widetilde{\mu}-\varphi\widehat{Y}_S\right)\right)\chi^A\widehat{\tau}_S^A\\
&-\left[\left(1-\left(\widetilde{\mu}-\varphi\widehat{Y}_S\right)\right)\left(1-\beta\left(\widetilde{\mu}-\varphi\widehat{Y}_S\right)\right)+\kappa\psi\left(\phi_\pi-\left(\widetilde{\mu}-\varphi\widehat{Y}_S\right)\right)\right]\left(\widehat{G}_S-\sigma\chi^s\widehat{\tau}_S^s\right)=0
\end{aligned}$$

We will find the partial derivatives to h, as these will be used in our total derivatives

$$\begin{aligned}
\frac{\partial h}{\partial\widehat{Y}_S}&=\varphi\left(1-\beta\left(\widetilde{\mu}-\varphi\widehat{Y}_S\right)\right)\widehat{Y}_S+\left(1-\left(\widetilde{\mu}-\varphi\widehat{Y}_S\right)+\sigma\phi_y\right)\left(1-\beta\left(\widetilde{\mu}-2\varphi\widehat{Y}_S\right)\right)\\
&\qquad+\kappa\sigma\left(\phi_\pi-\left(\widetilde{\mu}-2\varphi\widehat{Y}_S\right)\right)+\kappa\sigma\psi\varphi\chi^w\widehat{\tau}_S^w-\sigma\beta\varphi\chi^A\widehat{\tau}_S^A\\
&\qquad-\left[\varphi\left(1-\beta\left(\widetilde{\mu}-\varphi\widehat{Y}_S\right)\right)+\left(1-\left(\widetilde{\mu}-\varphi\widehat{Y}_S\right)\right)\beta\varphi+\kappa\psi\varphi\right]\left(\widehat{G}_S-\sigma\chi^s\widehat{\tau}_S^s\right)=0
\end{aligned}$$

Setting $H_1\left(\widehat{Y}_S\right)=\varphi\left(1-\beta\left(\widetilde{\mu}-\varphi\widehat{Y}_S\right)\right)\widehat{Y}_S+\left(1-\left(\widetilde{\mu}-\varphi\widehat{Y}_S\right)+\sigma\phi_y\right)\left(1-\beta\left(\widetilde{\mu}-2\varphi\widehat{Y}_S\right)\right)+\kappa\sigma\left(\phi_\pi-\left(\widetilde{\mu}-2\varphi\widehat{Y}_S\right)\right)$ the partial derivative with respect to $\widehat{Y}_S$ is

$$\begin{aligned}
\frac{\partial h}{\partial\widehat{Y}_S}&=H_1\left(\widehat{Y}_S\right)+\kappa\sigma\psi\varphi\chi^w\widehat{\tau}_S^w-\sigma\beta\varphi\chi^A\widehat{\tau}_S^A\\
&\qquad-\varphi\left[\left(1+\beta-2\beta\left(\widetilde{\mu}-\varphi\widehat{Y}_S\right)\right)+\kappa\psi\right]\left(\widehat{G}_S-\sigma\chi^s\widehat{\tau}_S^s\right).
\end{aligned}$$

We also have the partial derivatives for the fiscal policy instruments

$$\frac{\partial h}{\partial \widehat{G}_S} = -\left[\left(1-\left(\widetilde{\mu}-\varphi\widehat{Y}_S\right)\right)\left(1-\beta\left(\widetilde{\mu}-\varphi\widehat{Y}_S\right)\right)+\kappa\psi\left(\phi_\pi-\left(\widetilde{\mu}-\varphi\widehat{Y}_S\right)\right)\right]$$
$$\frac{\partial h}{\partial \widehat{\tau}_S^s} = \left[\left(1-\left(\widetilde{\mu}-\varphi\widehat{Y}_S\right)\right)\left(1-\beta\left(\widetilde{\mu}-\varphi\widehat{Y}_S\right)\right)+\kappa\psi\left(\phi_\pi-\left(\widetilde{\mu}-\varphi\widehat{Y}_S\right)\right)\right]\sigma\chi^s$$
$$\frac{\partial h}{\partial \widehat{\tau}_S^w} = \kappa\sigma\psi\left(\phi_\pi-\left(\widetilde{\mu}-\varphi\widehat{Y}_S\right)\right)\chi^w$$
$$\frac{\partial h}{\partial \widehat{\tau}_S^A} = -\sigma\left(1-\beta\left(\widetilde{\mu}-\varphi\widehat{Y}_S\right)\right)\chi^A$$

As we do not have a closed form solution for $\widehat{Y}_S$, we will find the multiplier by using total differentiation. Our definition $h\left(\widehat{Y}_S,\widehat{G}_S,\tau_S^s,\tau_S^A,\tau_S^w\right)=0$ implies that

$$dh(\cdot)=\frac{\partial h(\cdot)}{\partial\widehat{Y}_S}d\widehat{Y}_S+\frac{\partial h(\cdot)}{\partial\widehat{G}_S}d\widehat{G}_S+\frac{\partial h(\cdot)}{\partial\widehat{\tau}_S^s}d\widehat{\tau}_S^s+\frac{\partial h(\cdot)}{\partial\widehat{\tau}_S^A}d\widehat{\tau}_S^A+\frac{\partial h(\cdot)}{\partial\widehat{\tau}_S^w}d\widehat{\tau}_S^w=0 \tag{34}$$

Looking at only one active fiscal policy, here the government spending, total differentiation implies the following formula for the multiplier

$$dh\left(\widehat{Y}_S,\widehat{G}_S\right)=\frac{\partial h(\cdot)}{\partial\widehat{Y}_S}d\widehat{Y}_S+\frac{\partial h(\cdot)}{\partial\widehat{G}_S}d\widehat{G}_S=0\Rightarrow\frac{d\widehat{Y}_S}{d\widehat{G}_S}=-\frac{\partial h(\cdot)}{\partial\widehat{G}_S}/\frac{\partial h(\cdot)}{\partial\widehat{Y}_S} \tag{35}$$

and the same formula applies for the three remaining multipliers. Therefore the multipliers in the endogenous Markov model are

$$\frac{\partial\widehat{Y}_S}{\partial\widehat{G}_S}=\frac{-\left(1-\left(\widetilde{\mu}-\varphi\widehat{Y}_S\right)\right)\left(1-\beta\left(\widetilde{\mu}-\varphi\widehat{Y}_S\right)\right)-\kappa\psi\left(\phi_\pi-\left(\widetilde{\mu}-\varphi\widehat{Y}_S\right)\right)}{H_1\left(\widehat{Y}_S\right)-\varphi\left[\left(1+\beta-2\beta\left(\widetilde{\mu}-\varphi\widehat{Y}_S\right)\right)+\kappa\psi\right]\widehat{G}_S}$$
$$\frac{\partial\widehat{Y}_S}{\partial\widehat{\tau}_S^s}=\frac{\left[\left(1-\left(\widetilde{\mu}-\varphi\widehat{Y}_S\right)\right)\left(1-\beta\left(\widetilde{\mu}-\varphi\widehat{Y}_S\right)\right)+\kappa\psi\left(\phi_\pi-\left(\widetilde{\mu}-\varphi\widehat{Y}_S\right)\right)\right]\sigma\chi^s}{H_1\left(\widehat{Y}_S\right)-\varphi\left[\left(1+\beta-2\beta\left(\widetilde{\mu}-\varphi\widehat{Y}_S\right)\right)+\kappa\psi\right]\sigma\chi^s\widehat{\tau}_S^s}$$
$$\frac{\partial\widehat{Y}_S}{\partial\widehat{\tau}_S^w}=\frac{\kappa\sigma\psi\left(\phi_\pi-\left(\widetilde{\mu}-\varphi\widehat{Y}_S\right)\right)\chi^w}{H_1\left(\widehat{Y}_S\right)+\kappa\sigma\psi\varphi\chi^w\widehat{\tau}_S^w}$$
$$\frac{\partial\widehat{Y}_S}{\partial\widehat{\tau}_S^A}=\frac{-\sigma\left(1-\beta\left(\widetilde{\mu}-\varphi\widehat{Y}_S\right)\right)\chi^A}{H_1\left(\widehat{Y}_S\right)-\sigma\beta\varphi\chi^A\widehat{\tau}_S^A}$$

when we restrict ourselves to looking at one fiscal stimulus tool at a time.

H.5 Equilibrium at the Zero Lower Bound

The solution $\widehat{Y}_t$ from the simple Markov Model is

$$\widehat{Y}_t = \frac{(1-\beta\mu)\,\sigma}{(1-\mu)(1-\beta\mu)-\mu\kappa\sigma} r_S^e + \frac{(1-\mu)(1-\beta\mu)-\kappa\mu\psi}{(1-\mu)(1-\beta\mu)-\mu\kappa\sigma}\left(\widehat{G}_S - \sigma\chi^s\widehat{\tau}_S^s\right) \\ + \frac{\sigma\kappa\mu\psi}{(1-\mu)(1-\beta\mu)-\mu\kappa\sigma}\chi^w\widehat{\tau}_S^w + \frac{\sigma(1-\beta\mu)}{(1-\mu)(1-\beta\mu)-\mu\kappa\sigma}\chi^A\widehat{\tau}_{S.}^A$$

We multiply by $(1-\mu)(1-\beta\mu)-\mu\kappa\sigma$ and insert $\mu = \widetilde{\mu} - \varphi\widehat{Y}_S$

$$\left(\left(1-\left(\widetilde{\mu}-\varphi\widehat{Y}_S\right)\right)\left(1-\beta\left(\widetilde{\mu}-\varphi\widehat{Y}_S\right)\right) - \kappa\sigma\left(\widetilde{\mu}-\varphi\widehat{Y}_S\right)\right)\widehat{Y}_t = \left(1-\beta\left(\widetilde{\mu}-\varphi\widehat{Y}_S\right)\right)\sigma r_S^e \\ + \left[\left(1-\left(\widetilde{\mu}-\varphi\widehat{Y}_S\right)\right)\left(1-\beta\left(\widetilde{\mu}-\varphi\widehat{Y}_S\right)\right) - \kappa\psi\left(\widetilde{\mu}-\varphi\widehat{Y}_S\right)\right]\left(\widehat{G}_S - \sigma\chi^s\widehat{\tau}_S^s\right) \\ + \sigma\kappa\psi\left(\widetilde{\mu}-\varphi\widehat{Y}_S\right)\chi^w\widehat{\tau}_S^w + \sigma\left(1-\beta\left(\widetilde{\mu}-\varphi\widehat{Y}_S\right)\right)\chi^A\widehat{\tau}_{S.}^A$$

we then set this equation equal to zero and define $f(\cdot)$ as

$$f\left(\widehat{Y}_S, \widehat{G}_S, \widehat{\tau}_S^s, \widehat{\tau}_S^A, \widehat{\tau}_S^w\right) = \left(\left(1-\left(\widetilde{\mu}-\varphi\widehat{Y}_S\right)\right)\left(1-\beta\left(\widetilde{\mu}-\varphi\widehat{Y}_S\right)\right) - \kappa\sigma\left(\widetilde{\mu}-\varphi\widehat{Y}_S\right)\right)\widehat{Y}_t \\ - \left[\left(1-\left(\widetilde{\mu}-\varphi\widehat{Y}_S\right)\right)\left(1-\beta\left(\widetilde{\mu}-\varphi\widehat{Y}_S\right)\right) - \kappa\psi\left(\widetilde{\mu}-\varphi\widehat{Y}_S\right)\right]\left(\widehat{G}_S - \sigma\chi^s\widehat{\tau}_S^s\right) \\ - \left(1-\beta\left(\widetilde{\mu}-\varphi\widehat{Y}_S\right)\right)\sigma r_S^e - \sigma\kappa\psi\left(\widetilde{\mu}-\varphi\widehat{Y}_S\right)\chi^w\widehat{\tau}_S^w \\ - \sigma\left(1-\beta\left(\widetilde{\mu}-\varphi\widehat{Y}_S\right)\right)\chi^A\widehat{\tau}_S^A = 0$$

We first solve for the partial derivatives of f

$$\frac{\partial f}{\partial \widehat{Y}_S} = \varphi\left(1-\beta\left(\widetilde{\mu}-\varphi\widehat{Y}_S\right)\right)\widehat{Y}_t + \left(1-\left(\widetilde{\mu}-\varphi\widehat{Y}_S\right)\right)\left(1-\beta\left(\widetilde{\mu}-2\varphi\widehat{Y}_S\right)\right) - \kappa\sigma\left(\widetilde{\mu}-2\varphi\widehat{Y}_S\right) \\ - \left[\varphi\left(1-\beta\left(\widetilde{\mu}-\varphi\widehat{Y}_S\right)\right) + \left(1-\left(\widetilde{\mu}-\varphi\widehat{Y}_S\right)\right)\beta\varphi + \kappa\psi\varphi\right]\left(\widehat{G}_S - \sigma\chi^s\widehat{\tau}_S^s\right) \\ - \beta\varphi\sigma r_S^e + \sigma\kappa\psi\varphi\chi^w\widehat{\tau}_S^w - \sigma\beta\varphi\chi^A\widehat{\tau}_S^A = 0$$

Let $H_2\left(\widehat{Y}_S\right) = \varphi\left(1-\beta\left(\widetilde{\mu}-\varphi\widehat{Y}_S\right)\right)\widehat{Y}_t + \left(1-\left(\widetilde{\mu}-\varphi\widehat{Y}_S\right)\right)\left(1-\beta\left(\widetilde{\mu}-2\varphi\widehat{Y}_S\right)\right) -$

$\kappa\sigma\left(\widetilde{\mu}-2\varphi\widehat{Y}_S\right)$. Then

$$\begin{aligned}
\frac{\partial f}{\partial \widehat{Y}_S} &= H_2\left(\widehat{Y}_S\right)-\beta\varphi\sigma r_S^e+\sigma\kappa\psi\varphi\chi^w\widehat{\tau}_S^w-\sigma\beta\varphi\chi^A\widehat{\tau}_S^A \\
&\quad -\varphi\left[\left(1+\beta-2\beta\left(\widetilde{\mu}-\varphi\widehat{Y}_S\right)\right)+\kappa\psi\right]\left(\widehat{G}_S-\sigma\chi^s\widehat{\tau}_S^s\right) \\
\frac{\partial f}{\partial \widehat{G}_S} &= \kappa\psi\left(\widetilde{\mu}-\varphi\widehat{Y}_S\right)-\left(1-\beta\left(\widetilde{\mu}-\varphi\widehat{Y}_S\right)\right)\left(1-\left(\widetilde{\mu}-\varphi\widehat{Y}_S\right)\right) \\
\frac{\partial f}{\partial \widehat{\tau}_S^s} &= \left[\left(1-\beta\left(\widetilde{\mu}-\varphi\widehat{Y}_S\right)\right)\left(1-\left(\widetilde{\mu}-\varphi\widehat{Y}_S\right)\right)-\kappa\psi\left(\widetilde{\mu}-\varphi\widehat{Y}_S\right)\right]\sigma\chi^s \\
\frac{\partial f}{\partial \widehat{\tau}_S^w} &= -\kappa\psi\sigma\left(\widetilde{\mu}-\varphi\widehat{Y}_S\right)\chi^w \\
\frac{\partial f}{\partial \widehat{\tau}_S^A} &= -\left(1-\beta\left(\widetilde{\mu}-\varphi\widehat{Y}_S\right)\right)\sigma\chi^A
\end{aligned}$$

Using the formula (35), we thus have that multipliers in the endogenous Markov model are

$$\begin{aligned}
\frac{d\widehat{Y}_S}{d\widehat{G}_S} &= -\frac{\kappa\psi\left(\widetilde{\mu}-\varphi\widehat{Y}_S\right)-\left(1-\beta\left(\widetilde{\mu}-\varphi\widehat{Y}_S\right)\right)\left(1-\left(\widetilde{\mu}-\varphi\widehat{Y}_S\right)\right)}{H_2\left(\widehat{Y}_S\right)-\beta\varphi\sigma r_S^e-\varphi\left[\kappa\psi+\left(1+\beta-2\beta\left(\widetilde{\mu}-\varphi\widehat{Y}_S\right)\right)\right]\widehat{G}_S} \\
\frac{d\widehat{Y}_S}{d\widehat{\tau}_S^s} &= -\frac{\left[\left(1-\beta\left(\widetilde{\mu}-\varphi\widehat{Y}_S\right)\right)\left(1-\left(\widetilde{\mu}-\varphi\widehat{Y}_S\right)\right)-\kappa\psi\left(\widetilde{\mu}-\varphi\widehat{Y}_S\right)\right]\sigma\chi^s}{H_2\left(\widehat{Y}_S\right)-\beta\varphi\sigma r_S^e+\varphi\left[\kappa\psi+\left(1+\beta-2\beta\left(\widetilde{\mu}-\varphi\widehat{Y}_S\right)\right)\right]\sigma\chi^s\widehat{\tau}_S^s} \\
\frac{\partial\widehat{Y}_S}{\partial\widehat{\tau}_S^w} &= \frac{\kappa\psi\sigma\left(\widetilde{\mu}-\varphi\widehat{Y}_S\right)\chi^w}{H_2\left(\widehat{Y}_S\right)-\beta\varphi\sigma r_S^e+\kappa\psi\sigma\varphi\chi^w\widehat{\tau}_S^w} \\
\frac{\partial\widehat{Y}_S}{\partial\widehat{\tau}_S^A} &= \frac{\left(1-\beta\left(\widetilde{\mu}-\varphi\widehat{Y}_S\right)\right)\sigma\chi^A}{H_2\left(\widehat{Y}_S\right)-\beta\varphi\sigma r_S^e-\beta\varphi\sigma\chi^A\widehat{\tau}_S^A}
\end{aligned}$$

when we restrict ourselves to looking at one fiscal stimulus tool at a time.

H.6 Calculation Concerning the ARRA

We are interested in the effects of a 775 bn stimulus, where 2/3 is channeled through increased government spending and 1/3 as decreased wage tax revenues. As this stretches over two years we have that the increased spending per quarter is 64.5833 bn and the tax revenues decrease 32.2917 bn USD.

According to the CBO's potential output estimated from January 2009 (CBO (2009)), the average potential quarterly output for 2009 and 2010 was 8083,74 bn chained 2000-dollars. This implies that the ARRA spending leg is represented by $\widehat{G}_S$=2.0945 percent.

When doing our graphical analysis we found that $\varphi = 0.1$ was close to the upper bound for a well-behaved model in the case of a GD scenario, while $\varphi = 0.005$ was close to this bound in the GR scenario. We will thus proceed with these values in our case study of the ARRA.

We now calculate the tax rates needed to obtain the intended change in labor tax revenue. As there are dynamic effects from the change in output, we use a shooting mechanism approach (manually) to find the difference in total wage tax revenue before and after the tax cut. This is done under the assumption that the increase in government spending is done before we start considering the change in tax revenue (so $\widehat{G}_S$=2.0945 percent is assumed before and after the tax cut).

The case of the EM model is divided into two approaches: In the first approach we keep $\widetilde{\mu}$ equal to μ_{SM}, so that the two models are the same for $\varphi = 0$. This is referred to as "matching constant duration" and is equivalent of calibration A. In the second approach we match the persistence μ (and hence duration) of the two models in the relevant scenario ($\widehat{Y}_S = -0.3$ for the GD and $\widehat{Y}_S = -0.1$ for the GR scenario), which is referred to as scenario B in Section 5.

Monetary Policy in a Small Open Economy with Liquidity Constrained Households*

Gitte Yding Salmansen

August 29, 2014

Abstract

This paper analyses the effect of liquidity constrained households in a DSGE model for a small open economy. As liquidity constrained households stand out by not responding to the interest rate, we would expect monetary policy to be important for determining the effect of such households.

We find however, that the effects of openness are even stronger, as liquidity constrained households have only mall effects on the propagation of the shock. Further, under a CPI-based Taylor rule, the presence of these households dampens the contraction of output and working hours slightly in response to positive technology shock. This is in contrast to the result in closed economies.

We solve the Ramsey planner's problem and show, that the presence of liquidity constrained households implies a better scope for price stabilization.

*The author wishes to thank Henrik Jensen, Mark Strøm Kristoffersen, Jesper Pedersen, and Søren Hove Ravn for useful comments and discussions. Also thanks to seminar participants at University of Copenhagen and Danmarks Nationalbank. All remaining errors are my own. The views expressed are those of the author, and do not necessarily correspond to those of Danmarks Nationalbank.

1 Introduction

This paper investigates the effects of introducing liquidity constrained households in a DSGE model for a small open economy. A fraction of households behave in the classical manner, i.e. optimizing over an infinite horizon with unrestricted access to international asset markets, while the remaining households do not have access to asset markets, and hence consume their entire disposable income each period. We will refer to the first as Ricardian households and the latter as liquidity constrained households in line with Horvath (2009).[1] The presence of these households is referred to limited asset market participation (LAMP). There can be many reasons for households to consume their entire disposable income each periods, for instance myopia, no access to asset markets, heterogeneous transaction costs, or continuously binding borrowing constraints.[2] This paper does not take a stand on the microfoundations but rather focus on the effect of LAMP in open economies.

This paper supplements the existing literature by introducing liquidity constrained households in a small open economy. This setup allows us to analyze the interplay between LAMP, trade channels, and the monetary policy. Although many papers have introduced LAMP, this is often done in the context of investigating the effects of fiscal stimulus (see e.g. Galí et al. (2007), Natvik (2012) and Corsetti et al. (2012)). Furlanetto and Seneca (2012) stand out by investigating the effects of liquidity constrained households on the propagation of technology shocks in a closed economy. They find that LAMP increases the contraction of hours following a positive technology shock, thus improving the models fit with data.[3]

As liquidity constrained households stand out by not responding to the interest rate, we would expect monetary policy to be important for determining the effect of introducing these households. Indeed, we find that this is the case.

We analyze the propagation of technology shocks under standard monetary policy rules. These policy rules imply very diverse effects on Ricardian and liquidity constrained households. Our inflation-based Taylor rules (using consumer prices or domestic producer

[1] The liquidity constrained households have also been referred to as rule-of-thumbers, hand-to-mouth consumers, and non-asset holders.

[2] In a working paper version of Bilbiie (2008), the author investigates the transaction costs, that would generate liquidity constrained consumption, see Bilbiie (2005).

[3] A negative response of hours following a technology shock is found in Francis and Ramey (2005) and Basu et al. (2006).

prices) imply that the Ricardian households increase consumption following a positive technology shock, as is standard. Liquidity constrained households on the other hand have a hump-shaped response. Their response is initially negative due to the drop in hours, but as the real wage increases over time, their consumption response is positive in the medium run. The drop in total hours worked is muted under LAMP. In a fixed exchange rate regime the forward-looking Ricardian households have a more muted (and hump-shaped) consumption response, while liquidity constrained households experience a bust-boom path of consumption. As a result, hours respond negatively in the short run and marginally positive in the medium run. LAMP has a negligible effect on hours.

We proceed to an analysis of the optimal policy in an open economy with LAMP. Our Ramsey planner has an incentive to minimize the real costs from wage and price inflation, but also stabilize consumption and to stimulate terms of trade.[4] We find that the Ramsey policy responds more aggressively to a technology shock under LAMP. The liquidity constrained households dampen the wage (and price) deflation, leaving the Ramsey planner more wiggle room to stimulate terms of trade and output.

Our welfare comparisons show that the welfare effects of DITR are very similar to those of the Ramsey policy. Welfare is lowest under a fixed exchange rate. However, this should not be taken as a definitive argument against a fixed exchange rate regime, as there are many other advantages from such a regime that are not captured by our model - increased trade, a lower interest rate spread to mention a few.

The rest of the paper is organized as follows. In Section 2 we review the key literature on optimal policy and LAMP, before we set up the model in Section 3. In Section 4 we discuss the method used to solve the model and our benchmark parameterization. Section 5 analyses determinacy in the model. In Section 6 we analyze technology shocks under simple policy rules. In Section 7 we derive optimal monetary policy, and Section 8 analyzes the Ramsey policy via technology shocks. In Section 9 we discuss welfare, and Section 10 concludes.

[4] The last incentive only arises in an open economy with home bias and price rigidity, as these enable the monetary policy to affect terms of trade.

2 Literature on Optimal Policy and LAMP

To the best of our knowledge, we are the first to consider optimal policy in a small open economy with LAMP. This overview will therefore consist of papers that discuss optimal policy either under LAMP or in an open economy.

Auray et al. (2011) find optimal policies in a small open economy with capital, and no LAMP or wage rigidities. Optimal monetary and tax policies under sticky prices replicates the outcome under flexible prices, but at the cost of very volatile tax rates. We do not focus on fiscal policies, hence tax rates are constant in our model.

De Paoli (2009) derives optimal policy in a small open economy and shows that the loss function can be represented by a quadratic expression of domestic inflation, output gap and the real exchange rate. When there is a domestic monopolistic distortion and a terms of trade externality (that is, a benefit from manipulating terms of trade) the optimal policy will move away from domestic price stabilization. Closely related, Faia and Monacelli (2008) derive the optimal policy in a small open economy using the Marcet-Marimon approach, that we also apply. They find that the presence of home bias in preferences changes the Ramsey policy from strict domestic price stabilization to balancing this against exchange rate stabilization.

Galí et al. (2007) acknowledge that there is great scope for investigating the implication of liquidity constrained households for optimal monetary policy. Horvath (2009) answers this call and analyzes welfare in a closed economy with liquidity constrained households. Horvath (2009) derives optimal stabilization policies (both fiscal and monetary), and show that under the optimal policy, expected future inflation is zero. Further, the optimal policy is generally associated with crowding out of consumption, unless households have a high labor elasticity and low risk aversion, and there is a great share of liquidity constrained households. The model has a fixed labor supply of liquidity constrained households, whereas we assume that wage setting unions require that both Ricardian and liquidity constrained households meet their share of firms' labor demand, and hence they have the same labor supply.

Ascari et al. (2013) derive the optimal policy for a closed economy with LAMP, and consider the average welfare under different monetary policies. We consider the individual welfare of both types of households, as this gives us more information on the welfare effects. In the second order approximation to the Ramsey planners objective function, Ascari et al. (2013) find the classical quadratic form with output gap, inflation and a real wage gap. The weight of the latter is increasing in the degree of LAMP, showing that the weighting of the trade-off in Erceg et al. (2000) is shifted by LAMP.

Amato and Laubach (2003) analyze an economy where each household has a fixed probability of having liquidity constrained consumption in the following period.[5] This is different, because the (future) rule-of-thumb part of consumption will thus feed back into the intertemporally optimizing household's first order condition. They find that the policy objective has a LAMP-term that is based on the persistence in output (and hence consumption).

3 Model

The model builds on Galí and Monacelli (2005) and is augmented with a wage friction and liquidity constrained households as in Galí et al. (2007). We will consider a continuum of small open economies, each of measure zero. There will not be a strategic element to monetary policy, as a given economy's policy choices will not have an impact on the rest of the economies.[6] We will focus on one country, the home economy, which is identical to all other economies with regard to preferences, pricing restrictions, market structure as well as the distribution of the individual productivity shocks. Our home economy will stand out in one important way - it is the only economy with liquidity constrained households.[7]

Each economy consists of households, unions, firms, and a government. Households choose optimal consumption subject to their respective budget constraints. Nominal wages are set by monopolistically competitive unions, and households supply the demanded amount of hours at this wage. Nominal wages and prices are sticky due to an adjustment cost on both. Firms set prices under monopolistic competition, and use labor to produce their good. The government runs a passive fiscal policy, where net revenue

[5] There are two main differences between our model and Amato and Laubach (2003). The latter has one representative agent, who occasionally behaves as a constrained household. Further, the constraint on consumption is that it is the same as last period's consumption rather than the current period's disposable income. In that sense, their model has path dependence.

[6] For a model where the economy has a non-zero weight in a monetary union and the subsequent strategic considerations, see for instance Beetsma and Jensen (2005).

[7] Ricardian and liquidity constrained households have the same level of steady state consumption. As our analysis of domestic shocks and policies will not affect the world economy, the inclusion of liquidity constrained foreign households would not change our results.

from subsidies are transferred lump sum to households, and the monetary policy can be either a simple interest rate rule or a Ramsey optimal monetary policy.

3.1 Households

The household sector consists of two types of households. A share $1-\lambda$ of households have perfect access to international asset markets. We refer to these as Ricardian households (R). The remaining share λ have neither wealth nor debt and thus consume their disposable income every period. These are referred to as liquidity constrained households (L).

The following properties hold for both types of households. Let $h \in \{R; L\}$ be the type of a given household. All households can be represented by an optimizing household with an infinite horizon, each maximizing the expected sum of discounted utilities from feasible future paths of consumption and labor supply

$$E_0 \sum_{t=0}^{\infty} \beta^t U\left(C_t^h, N_t^h\right), \tag{1}$$

where N_t^h is the number of hours worked and C_t^h is the composite consumption index

$$C_t^h \equiv \left[(1-\alpha)^{\frac{1}{\eta}} \left(C_{H,t}^h\right)^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} \left(C_{F,t}^h\right)^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}}. \tag{2}$$

The index consists of a bundle of domestically produced goods $C_{H,t}^h$ and a bundle of imported goods $C_{F,t}^h$. The parameter $\eta > 0$ is the substitutability between foreign and domestic goods. The weight of imported goods $\alpha \in [0,1]$ will serve as a measure of openness in the model. Given the infinitesimal weight of the home economy, any $\alpha < 1$ reflects home bias in the households' preferences.[8]

The index of domestic consumption is of the CES form

$$C_{H,t}^h \equiv \left(\int_0^1 C_{H,t}^h(j)^{\frac{\varepsilon-1}{\varepsilon}} dj\right)^{\frac{\varepsilon}{\varepsilon-1}} \tag{3}$$

where $j \in [0,1]$ is the type of good and $\varepsilon > 1$ is the elasticity of substitution between

[8] As we will see under the Ramsey policy, home bias combined with sticky prices means that the Ramsey planner can manipulate the terms of trade.

domestic goods.[9]

Foreign consumption is an aggregate of imports from all foreign countries

$$C_{F,t}^h \equiv \left(\int_0^1 \left(C_{i,t}^h \right)^{\frac{\gamma-1}{\gamma}} di \right)^{\frac{\gamma}{\gamma-1}} \quad ; \quad C_{i,t}^h \equiv \left(\int_0^1 C_{i,t}^h \left(j \right)^{\frac{\varepsilon-1}{\varepsilon}} dj \right)^{\frac{\varepsilon}{\varepsilon-1}} \tag{4}$$

where the parameter γ measures the substitutability between goods from different countries. The imports from a given country i, $C_{i,t}^h$, has the same elasticity of substitution between goods as domestic goods, ε.

Optimal allocation of expenditures within each category yields

$$C_{H,t}^h \left(j \right) = \left(\frac{P_{H,t} \left(j \right)}{P_{H,t}} \right)^{-\varepsilon} C_{H,t}^h \quad ; \quad C_{i,t}^h \left(j \right) = \left(\frac{P_{i,t} \left(j \right)}{P_{i,t}} \right)^{-\varepsilon} C_{i,t}^h \tag{5}$$

for all $i, j \in [0,1]$, where $P_{H,t} = \left(\int_0^1 P_{H,t} \left(j \right)^{1-\varepsilon} dj \right)^{\frac{1}{1-\varepsilon}}$ is the demand-weighted domestic producer price index and $P_{i,t} = \left(\int_0^1 P_{i,t} \left(j \right)^{1-\varepsilon} dj \right)^{\frac{1}{1-\varepsilon}}$ is the price index for the bundle of goods imported from country i, but denominated in the domestic currency, for all $i \in [0,1]$, also consistent with demand weights. Here, $P_{i,t} \left(j \right)$ is the price of good j in country i, denoted in our home country's currency.

The optimal level of imports from country i are

$$C_{i,t}^h = \left(\frac{P_{i,t}}{P_{F,t}} \right)^{-\gamma} C_{F,t}^h \tag{6}$$

for all $i \in [0,1]$, where $P_{F,t} \equiv \left(\int_0^1 P_{i,t}^{1-\gamma} di \right)^{\frac{1}{1-\gamma}}$ is the price index for imported goods, again expressed in domestic currency. This allows us to determine the total expenditures on imported goods as $\int_0^1 P_{i,t} C_{i,t}^h di = P_{F,t} C_{F,t}^h$.

The optimal consumption shares for domestic and foreign goods respectively are

$$C_{H,t}^h = (1-\alpha) \left(\frac{P_{H,t}}{P_t} \right)^{-\eta} C_t^h \quad ; \quad C_{F,t}^h = \alpha \left(\frac{P_{F,t}}{P_t} \right)^{-\eta} C_t^h \tag{7}$$

where $P_t \equiv \left[(1-\alpha) \left(P_{H,t} \right)^{1-\eta} + \alpha \left(P_{F,t} \right)^{1-\eta} \right]^{\frac{1}{1-\eta}}$ is the consumer price index (CPI).[10] Thus

[9] When we specify the production side, each country has a continuum of firms (measure zero) each producing a differentiated good, indexed by j.

[10] Given the identical optimal consumption weights for any given level of expenditures, the CPI will be the same for both types of domestic households.

the total consumption expenditures are $P_{H,t}C_{H,t}^h + P_{F,t}C_{F,t}^h = P_tC_t^h$.[11]

All households have isoelastic preferences that can be represented by the instantaneous utility function

$$U\left(C_t^h, N_t^h\right) = \frac{\left(C_t^h\right)^{1-\sigma}}{1-\sigma} - \frac{\left(N_t^h\right)^{1+\varphi}}{1+\varphi} \tag{8}$$

where $\sigma > 0$ is the coefficient of risk aversion for consumption, while $\varphi > 0$ is the inverse of the Frisch labor supply elasticity.

Finally, aggregate consumption is a weighted average of the two types of households' consumption:

$$C_t = \lambda C_t^L + (1-\lambda) C_t^R. \tag{9}$$

3.1.1 Ricardian Households

Ricardian households have access to a complete set of internationally traded contingency claims, thus they have perfect risk sharing with foreign households.[12] Under the optimal consumption weights in (5) and (6), they face the following series of budget constraints

$$P_tC_t^R + E_t\left[Q_{t,t+1}D_{t+1}\right] \leq D_t + (1+\tau_w) W_tN_t^R - P_tT^R - \int_0^1 F_t(u)\,du \tag{10}$$

where τ_w is a labor subsidy, W_t is the nominal wage, T^R is a lump sum tax on Ricardian households and $\int_0^1 F_t(u)\,du$ is spending on union fees. D_{t+1} is the nominal pay-off in period $t+1$ of the market portfolio (including shares in firms) held at the end of period t. $Q_{t,t+1}$ is the stochastic discount factor for one period ahead nominal payoffs relevant to participants in international asset markets.

The access to asset markets implies the following intertemporal optimality condition for the households

$$\beta\left(\frac{C_{t+1}^R}{C_t^R}\right)^{-\sigma}\left(\frac{P_t}{P_{t+1}}\right) = Q_{t,t+1}. \tag{11}$$

[11] The price indices ensure the aggregate multiplicative quality that $\int_0^1 P_{H,t}(j)\,C_{H,t}^h(j)\,dj + \int_0^1\int_0^1 P_{i,t}(j)\,C_{i,t}^h(j)\,dj\,di = P_tC_t^h$ under optimal consumption.

[12] In a closed economy model, capital serves as the consumption smoothing channel for Ricardian households, see Galí et al. (2007). International asset markets provide this smoothing channel in our model.

Taking conditional expectations we have the stochastic Euler equation

$$\beta R_t E_t\left[\left(\frac{C_{t+1}^R}{C_t^R}\right)^{-\sigma}\left(\frac{P_t}{P_{t+1}}\right)\right]=1$$

where $R_t = E_t\left[Q_{t,t+1}\right]^{-1}$ is the gross return on a risk-free one-period bond, which returns one unit of domestic currency a time $t+1$, and trading at the price $E_t\left[Q_{t,t+1}\right]$.[13]

3.1.2 Liquidity Constrained Households

Liquidity constrained households consume their entire disposable income, which is equal to the real labor income net of (real) taxes

$$P_t C_t^L = (1+\tau_w) W_t N_t^L - T^L - \int_0^1 F_t(u)\, du \tag{12}$$

reflecting that they do not have access to the financial markets. Note that we allow the lump sum tax for the liquidity constrained households, T^L, to differ from that of the Ricardian households.

3.2 Unions

Nominal wages are set by a continuum of unions, indexed by $u \in [0,1]$, all operating under monopolistic competition. We follow Schmitt-Grohé and Uribe (2006) and Colciago (2011) in assuming that each household supplies labor to each union.[14]

For each firm j, all labor types are imperfect substitutes, and effective labor is generated through a CES aggregator. Firms are cost minimizing, implying that any union u faces the demand schedule

$$N_t(u) = \left(\frac{W_t(u)}{W_t}\right)^{-e_w} N_t^d \tag{13}$$

where e_w is the elasticity of substitution between different types of labor and N_t^d is total demand for labor. This combined with the aggregator for household hours $N_t \equiv \int_0^1 N_t^d(u)\, du$ implies that the total amount of hours supplied for any household is $N_t = N_t^d \int_0^1 \left(\frac{W_t(u)}{W_t}\right)^{-e_w} du$.

[13] Due to the assumption of Arrow securities, equation (11) does not merely hold in expectation but holds for any possible state in period $t+1$.

[14] This limits the source of heterogeneity in households to stem only from LAMP.

Each union maximizes the weighted utility of its members, and require them to meet their share of the resulting labor demand.[15] Each union has a share λ of liquidity constrained households among its members, implying that labor is identical across all households

$$N_t = N_t^L = N_t^R. \tag{14}$$

The unions face a convex wage adjustment cost following Rotemberg (1982). This cost is covered by the union fee $F_t(u)$

$$F_t(u) = \frac{\phi_w}{2} \left(\Pi_t^w(u) - 1\right)^2 W_t N_t \quad , \quad \phi_w > 0 \tag{15}$$

where $\Pi_t^w(u) \equiv \frac{W_t(u)}{W_{t-1}(u)}$ is the nominal gross wage inflation in union u. The wage adjustment cost is scaled by the aggregate nominal wage income. The objective function of any union u is

$$\max_{W_t(u)} E_t \sum_{k=0}^{\infty} \beta^k \left[(1-\lambda) U\left(C_{t+k}^R, N_{t+k}\right) + \lambda U\left(C_{t+k}^L, N_{t+k}\right)\right] \tag{16}$$

subject to the two budget constraints (10) and (12), firms' labor demand (13), and the aggregate union fee $F_t \equiv \int_0^1 F_t(u)\, du$, where $F_t(u)$ is given by equation (15). Finding the optimality condition for each firm and realizing they are all identical,[16] we have that the (optimal) nominal wage W_t satisfies the condition

$$\begin{aligned} 0 \;=\;& \left(MRS_t^A\right)^{-1} \frac{W_t}{P_t} \left[(1+\tau_w)(1-e_w) - \phi_w \left(\Pi_t^w - 1\right) \Pi_t^w\right] N_t^{1+\varphi} + e_w N_t^{1+\varphi} \\ & + \beta \left(MRS_{t+1}^A\right)^{-1} \frac{W_{t+1}}{P_{t+1}} \phi_w \left(\Pi_{t+1}^w - 1\right) \Pi_{t+1}^w N_{t+1}^{1+\varphi} \end{aligned} \tag{17}$$

where MRS_t^h is the marginal rate of substitution between labor and consumption of households of type h and MRS_t^A is the weighted average of the marginal rates of substitution,[17] see the derivation of equation (17) in Appendix A. Given that each union represents all households, its must balance setting the real wage equal to (a markup over) the average marginal rates of substitution, while at the same smooth out the path of

[15] In a model with steady state wealth, the unions might set wages below the marginal rate of substitution of Ricardian household. In that case it is Pareto improving to shift labour supply towards liquidity constrained households, see Natvik (2012)

[16] This also implies that $F_t = \frac{\phi_w}{2}\left(\Pi_t^w - 1\right)^2 W_t N_t$.

[17] that is, we define $\left(MRS_t^A\right)^{-1} \equiv (1-\lambda)\left(MRS_t^R\right)^{-1} + \lambda \left(MRS_t^L\right)^{-1}$. If there is no adjustment cost we have $\frac{W_t}{P_t} = \frac{e_w(1+\tau_w)}{e_w - 1} MRS_t^A$ in every period, thus the (potential) wedge stemming from the monopolistic competition does not depend on this adjustment cost.

inflation.

3.3 Prices, Exchange Rates, and International Risk Sharing

The behavior of households and unions depends on domestic wages and consumer prices. Before proceeding to the behavior of the firms, and thus producer prices, we will establish key relations for the different price indices, exchange rates and international risk sharing.

Bilateral terms of trade between the domestic economy and country i is defined as $S_{i,t} \equiv \frac{P_{i,t}}{P_{H,t}}$. The domestic effective terms of trade is defined as

$$S_t \equiv \frac{P_{F,t}}{P_{H,t}} = \left(\int_0^1 S_{i,t}^{1-\gamma} di \right)^{\frac{1}{1-\gamma}}, \tag{18}$$

By defining PPI inflation as $\Pi_{H,t} = \frac{P_{H,t}}{P_{H,t-1}}$ it follows directly from the definition of the price index P_t that domestic PPI inflation is linked to CPI inflation, $\Pi_t = \frac{P_t}{P_{t-1}}$, and wage inflation as follows

$$\Pi_t = \frac{g(S_t)}{g(S_{t-1})} \Pi_{H,t} \quad ; \quad \Pi_t^w - \frac{\frac{W_t}{P_t} g(S_t)}{\frac{W_{t-1}}{P_{t-1}} g(S_{t-1})} \Pi_{H,t} \tag{19}$$

where $g(S_t) \equiv \frac{P_t}{P_{H,t}} = \left[(1-\alpha) + \alpha (S_t)^{1-\eta} \right]^{\frac{1}{1-\eta}}$ is introduced to ease notation. When we have wage and price frictions in our model, law of motion for the nominal wage does not trivially hold in our model. This is because none of the three variables in the equation are free to adjust residually: Nominal wage and price inflation are chosen by optimizing agents in the economy and the terms of trade S_t is determined by the equilibrium. Hence, the wage inflation equation changes from being an identity to a law of motion with last periods real wage and terms of trade being state variables (see Chugh (2006) for a discussion).

Let $\varepsilon_{i,t}$ be the bilateral nominal exchange rate.[18] Given the presence of complete international financial markets, the price of a risk-free bond denominated in foreign currency will be $\varepsilon_t^i (R_t^i)^{-1} = E_t \left[Q_{t,t+1} \varepsilon_{t+1}^i \right]$. Subtracting the nested domestic bond pricing

[18] Here we define the bilateral exchange rate as the price of country i's currency in terms of the domestic currency. An appreciation of the home currency thus implies a fall in $\varepsilon_{i,t}$.

equation $(R_t)^{-1} = E_t[Q_{t,t+1}]$ gives the uncovered interest parity (UIP)

$$E_t\left[Q_{t,t+1}\left(R_t - R_t^i\left(\varepsilon_{t+1}^i/\varepsilon_t^i\right)\right)\right] = 0. \tag{20}$$

Recalling that all other countries only have Ricardian households, the consumption C_t^i in any foreign country i will satisfy an intertemporal optimality condition of the form

$$\beta\left(\frac{C_{t+1}^i}{C_t^i}\right)^{-\sigma}\left(\frac{P_t^i}{P_{t+1}^i}\right)\left(\frac{\varepsilon_t^i}{\varepsilon_{t+1}^i}\right) = Q_{t,t+1}. \tag{21}$$

We define the bilateral real exchange rate, $\mathcal{Q}_{i,t}$, as the ratio of home and country i's CPI denominated in the home currency; $\mathcal{Q}_{i,t} \equiv \frac{\varepsilon_{i,t}P_t^i}{P_t}$. Combining this definition with the domestic Euler equation (11) and equation (21) we have that

$$C_t^R = \vartheta^i C_t^i \mathcal{Q}_{i,t}^{\frac{1}{\sigma}} \tag{22}$$

for all t and all i, and where ϑ^i is a constant that will depend on the countries initial net asset positions. As we are interested in the effect of the domestic liquidity constrained households we will proceed with the assumption that the Ricardian households are initially symmetric across countries, so that $\vartheta^i = \vartheta = 1$ for all $i \in [0,1]$.

3.4 Firms

In the home economy there is a continuum of firms, indexed by $j \in [0,1]$, each producing a differentiated good with the linear technology

$$Y_t(j) = A_t N_t(j) - AC_t(j) \tag{23}$$

in which $a_t = \log A_t$ follows the process $a_t = \rho_a a_{t-1} + \omega_t^a$, where $\rho_a < 1$ and the productivity shock ω_t^a is $N(0, \omega_a^2)$. $AC_t(j)$ is an adjustment cost of changing the price of good j, which is a convex function of the price increase and increasing in the producer price level

$$AC_t(j) = \frac{\phi_p}{2}\left(\frac{P_t(j)}{P_{t-1}(j)} - 1\right)^2 P_{H,t}. \tag{24}$$

Aggregate domestic output is defined by the index $Y_t \equiv \left(\int_0^1 Y_t(j)^{\frac{\varepsilon-1}{\varepsilon}} dj\right)^{\frac{\varepsilon}{\varepsilon-1}}$, i.e. the

the same aggregation weights as in the consumption index. This gives the following demand for good j

$$Y_t(j) = \left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\varepsilon} Y_t \tag{25}$$

which is scaled by the level of output. All firms receive a constant employment subsidy τ_p, making the nominal marginal cost equal to all firms given by $MC_t = (1-\tau_p)\frac{W_t}{A_t}$.

The objective function of firm j is

$$\max_{P_t(z)} E_0 \sum_{k=0}^{\infty} Q_{t,t+k}\left[P_{H,t}(j)\,Y_t(j) - (1-\tau_p)\,W_t N_t^h(j) - \frac{\phi_p}{2}\left(\frac{P_t(z)}{P_{t-1}(z)} - 1\right)^2 P_{H,t}\right]$$

subject to the demand in equation (25). Where the stochastic discount factors satisfy $Q_{t,t+k} = Q_{t,t+k-1}Q_{k-1,k}$ to rule out arbitrage. Assuming that τ_p removes the markup due to monopolistic competition, we show in Appendix B that the optimal price satisfies

$$\begin{aligned}(\Pi_{H,t}-1)\,\Pi_{H,t} &= \beta E_t\left[\frac{\left(C_{t+1}^R\right)^{-\sigma}}{\left(C_t^R\right)^{-\sigma}}\frac{g(S_t)}{g(S_{t+1})}(\Pi_{H,t+1}-1)\,\Pi_{H,t+1}\right] \\ &\quad + \frac{\varepsilon-1}{\phi_p}Y_t\left[\frac{W_t^{\text{real}}}{A_t}g(S_t) - 1\right].\end{aligned} \tag{26}$$

Given that all firms choose the same inflation and hence the same level of labor input $N_t(j) = N$ and output $Y_t(j) = Y_t$, we have from equations (23) and (24) that aggregate output satisfies

$$Y_t = A_t N_t - \frac{\phi_p}{2}(\Pi_{H,t}-1)^2. \tag{27}$$

3.5 Fiscal Policy

The fiscal policy is passive; the government runs a balanced budget and has zero debt. Government spending consists only of expenses to union and firm subsidies and this is financed through the lump sum taxes on households

$$(1-\lambda)\,T_t^R + \lambda T_t^L = \tau_w W_t N_t + \tau_p P_{H,t} Y_t. \tag{28}$$

The households' lump sum taxes are set according to the policy rules

$$T_t^R = \tau_w W_t N_t + \frac{1}{(1-\lambda)} \tau_p P_{H,t} Y_t \tag{29}$$

$$T_t^L = \tau_w W_t N_t. \tag{30}$$

These rates imply that the firm subsidy is financed only by Ricardian households and the wage subsidy is financed proportionally by all households. This ensures that households have the same level of steady state consumption and that the firm subsidy will not be a transfer between households via firm profits. Note that the fiscal policy implies that $P_t C_t^L = W_t N_t - F_t$. We analyze the effects of government spending in Salmansen (2014).

3.6 Domestic Goods Markets

Equilibrium in all domestic goods markets implies

$$\begin{aligned} Y_t(j) &= (1-\lambda) C_{H,t}^R(j) + \lambda C_{H,t}^L(j) + \int_0^1 C_{H,t}^i(j)\, di \\ &= \left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\varepsilon} \left[(1-\alpha)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t\right] \\ &\quad + \alpha \int_0^1 \left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\varepsilon} \left(\frac{P_{H,t}}{\varepsilon_{i,t} P_{F,t}^i}\right)^{-\gamma} \left(\frac{P_{F,t}^i}{P_t}\right)^{-\eta} C_t^i di \end{aligned}$$

for all j and all t, where $C_{H,t}^i(j)$ is country i's demand for good j produced in the home country. See derivation hereof in Appendix C. The openness parameter α plays two roles: More openness increases exports and reduces domestic demand for home goods.

Using the aggregation index for output we have

$$Y_t = g(S_t)^\eta (1-\alpha)\left[(1-\lambda) C_t^R + \lambda C_t^L\right] + \alpha S_t^\gamma C_t^* \tag{31}$$

which uses the fact that our home economy has an infinitesimal weight and that all foreign economies are identical, so that $S_t^i = 1$, $S_{i,t} = S_t$ and $\mathcal{Q}_{i,t}$=$\mathcal{Q}_t$, see Appendix C. C_t^* is the aggregate world consumption level.

3.7 Trade Balance

Let the net exports in terms of domestic output as a fraction of steady state GDP be defined as

$$\begin{aligned} nx_t &\equiv \frac{1}{Y}\left(Y_t - \frac{P_t}{P_{H,t}}C_t\right) \\ &= \frac{1}{Y}\left[\alpha S_t^{\gamma}\mathcal{Q}_t^{-\frac{1}{\sigma}}C_t^R + \left(g\left(S_t\right)^{\eta}\left(1-\alpha\right) - g\left(S_t\right)\right)\left(\left(1-\lambda\right)C_t^R - \lambda C_t^L\right)\right] \end{aligned} \tag{32}$$

where Y is the deterministic steady state level of output. Equation (32) reveals that the net exports vary with the terms of trade, and the sign of the relationship is ambiguous, depending on the parameters γ, η, α and σ. In the purely Ricardian model, where $C_t = C_t^R$, there is balanced trade, if $\gamma = \eta = \sigma = 1$. However, our model with liquidity constrained households does not necessarily experience balanced trade in this case

$$\begin{aligned} nx_t &= \frac{1}{Y}\alpha g\left(S_t\right)\left[C_t^R - C_t\right] \\ &= \frac{1}{Y}\alpha\lambda\frac{P_t}{P_{H,t}}\left[C_t^R - C_t^L\right] \end{aligned}$$

as the risk sharing does not include all households.[19] Thus, even in this knife edge case, the trade balance will depend on the consumption difference between Ricardian and liquidity constrained households. Risk sharing ensures that Ricardian consumption is perfectly aligned with worlds consumption (generally adjusted for the effective real exchange rate $\mathcal{Q}_t$). Thus, if there is a positive (negative) consumption gap between Ricardian and liquidity constrained consumption, this reflects that foreign households' consumption is higher (lower) than the average demand in our home economy. This will result in exports that are higher than imports, i.e. positive net exports.

We still consider the case $\gamma = \eta = \sigma = 1$. The effect of the terms of trade on net exports will also depend on this domestic consumption gap.[20] An increase in terms of trade increases exports, that are equal to $\alpha g\left(S_t\right)C_t^R$. For domestic households, however, a wealth effect arises because increased terms of trade enhances domestic consumption measured in terms of output, thereby boosting demand for imports. While the substi-

[19] We use the fact that $S_t^{-1}\mathcal{Q}_t = g\left(S_t\right)$.

[20] In the limiting case $\eta = 1$ the price index is defined $P_t \equiv P_{H,t}^{1-a}P_{F,t}^{\alpha}$, implying that $\frac{P_t}{P_{H,t}} = S_t^{\alpha}$ and $\frac{\partial NX_t}{\partial S_t} = \frac{1}{Y}\alpha^2\lambda\left[C_t^R - C_t^L\right]S_t^{\alpha-1}$, which has the same sign as $\left[C_t^R - C_t^L\right]$.

tution effect increases imports, the wealth effect dominates and imports, $\alpha g(S_t) C_t$, are increasing in terms of trade. Thus, the rise in exports will dominate when Ricardian consumption is higher than liquidity constrained consumption. A large degree of openness, α, will exacerbate these international demand effects. Likewise, the larger a fraction of liquidity constrained households, the more imports will lag.

The above discusses a knife edge case, and from (32) it follows that in general, the effect of terms of trade on net exports is ambiguous and depends on the parameters γ, η, σ and α, as well as the difference between Ricardian and liquidity constrained consumption.

3.8 Equilibrium

We can now define the rational expectations equilibrium in our model.

Definition 1 *A rational expectations equilibrium consists of a sequence* $\{C_t^R, C_t^L, N_t^R, N_t^L, N_t, Y_t, nx_t, W_t^{\text{real}}, S_t, \Pi_{H,t}, \Pi_t, T_t^R, T_t^L\}$ *satisfying equations (11), (12), (14), (17), (22), (26), (27), (29), (30), (31), (32), and the two price indices in (19), given the policy* $\{R_t\}$ *and the exogenous processes* $\{A_t, C_t^*\}$.

We now consider the perfect-foresight steady state of the model, which exist and is unique as proved in Appendix E.

In the model with only Ricardian households, their only wealth consists of shares in firms that generate profits.[21] These profits together with the wage subsidy are exactly offset by the lump sum tax, $T^R = (\tau_w + \tau_p)$, so we have that $C^R = C^* = N = Y = W^{\text{real}} = 1$.

When we introduce LAMP, the steady state consumption of liquidity constrained and Ricardian households will be equal to the one (Ricardian) consumption in the no-LAMP steady state. Under LAMP the perfect-foresight steady state is given by

$$T^L = \tau_w \tag{33}$$

$$T^R = \tau_w + \frac{1}{(1-\lambda)}\tau_p \tag{34}$$

$$C^R = C^L = N = Y = W^{\text{real}} = 1. \tag{35}$$

The real wage, hours and output are thus unaffected by the introduction of LAMP.

[21] This is due to the symmetry between all countries and the absence of the public debt.

3.9 Simple Monetary Policy Rules

In our analysis we consider four different monetary policies. We will derive the Ramsey monetary policy (RP) given the presence of liquidity constrained households. We will compare this to the classical monetary policy rules in the small open economy literature, namely a domestic inflation-based Taylor rule (DITR, for short), a CPI inflation-based Taylor rule (CITR) and an exchange rate peg (PEG). In our model these policies are defined as follows:

$$\begin{aligned} DITR &: \widehat{r}_t = \varphi_\pi \widehat{\pi}_{H,t} && (36) \\ CITR &: \widehat{r}_t = \varphi_\pi \widehat{\pi}_t && (37) \\ PEG &: \widehat{e}_t = 0 && (38) \end{aligned}$$

In the two Taylor rules, the nominal interest rate is adjusted according to the equilibrium level of inflation. Under the exchange rate peg the nominal interest rate is the same as the world nominal interest rate level due to UIP.

We proceed by solving the model for the three simple rules in equations (36)-(38) in order to analyze the effects of introducing LAMP under these policies, before we turn to our analysis of the Ramsey policy.

4 Solution Method and Parameterization

We solve the model by log-linearizing the equilibrium conditions around the perfect-foresight steady state presented in Section 3.8. The log-linearized version of the equilibrium conditions for the model are shown in Appendix F.

We use DYNARE to solve the log-linearized model and then proceed to discuss determinacy, perform impulse response analysis, and simulate the equilibrium paths.

Although our steady state is unique, this is not ensured for the dynamic equilibrium for all possible parameters of the model. We consider determinacy using the approach of Blanchard and Kahn (1980). We can write our log-linearized model on the form

$$E_t z_{t+1} = \mathbf{A} z_t + \mathbf{a} e_{t+1}$$

where z_t is an 11-dimensional vector including four forward-looking variables, e_{t+1} is a vector of 2 shocks, the coefficient matrices $\mathbf{A}$ and $\mathbf{a}$ are 13×13 and 13×2 respectively. There exists a unique solution if and only if four eigenvalues for $\mathbf{A}$ are outside the unit circle. We will analyze the determinacy features in Section 5 and verify that our calibrations are within the uniqueness region of the parameter space.

Given the presence of future expectations of control variables in the wage and price Phillips curve, the Ramsey planner's optimization problem is not recursive in nature.[22] We deviate from other optimal policy papers using the timeless perspective approach (e.g. Colciago (2011), Albonico and Rossi (2014), and Auray et al. (2011)) by applying the method outlined in Marcet and Marimon (2011) to formulate the problem on recursive form. The difference between these two approaches is the following:[23] The timeless perspective solves the optimality problem for all periods t and then imposes that the initial period is in the distant past in order to remove the initial-period problem that arises in the initial period (see Woodford (2003)). The Marcet-Marimon approach instead imposes the time consistency before solving for the optimal policy.

4.1 Parameterization

Before we proceed, we assign values to the parameters of the model in order to be able to carry out determinacy analysis and numerical simulations.

Degree of LAMP: Empirical estimates of the share of liquidity constrained households, λ, are in the range 0.3-0.5. Using the Galí et al. (2004) parameterization of 0.5 to inform their prior, Forni et al. (2009) use Bayesian methods to estimate λ to be 0.37 with labor unions (0.35 if they assume no unions). Also using Bayesian methods Bartolomeo and Rossi (2007) find that for the G7 countries, the share of liquidity constrained households is 0.26.[24] Using Danish household data Chetty et al. (2014) find that the share of liquidity constrained households is 0.85, considerably higher than the findings for macro data. We use the results based on macro data and set the benchmark share of liquidity constrained households, λ, to be 0.35.[25]

[22] The utilitarian Ramsey planner optimizes welfare taking the technology, resource constraints, and optimizing behavior of the other agents in the economy as given.

[23] For a thorough analysis of the two approaches to optimal policy see Brendon (2009).

[24] This covers a variety of country specific estimates ranging from about 0.09 for Japan and Italy to roughly 0.4 for France and UK.

[25] $\lambda = 0.35$ is partly due to determinacy problems arising at much higher values, cf. graphs in Section 5.

Parameter	Value	Description
λ	0.35	Share of domestic households that are liquidity constrained
α	0.3	Degree of home bias
β	0.99	Discount factor
σ	2	Relative risk aversion
φ	3	Inverse of the Frish elasticity of labor supply
η	1.5	Substitutability between foreign and domestic goods
γ	1	Elasticity of substitution between goods from different countries
ϵ_p	6	Elasticity of substitution between differentiated domestic goods
ϵ_w	21	Elasticity of substitution between labor inputs
ϕ_p	58	Rotemberg price adjustment cost parameter
ϕ_p	19	Rotemberg wage adjustment cost parameter
ρ_a , ρ_{y*}	0.9	Persistence in shocks to technology and foreign output
σ_a , σ_{y*}	0.007	Variance of shocks to technology and foreign output
$\sigma_{a,y*}$	0.3	Correlation of shocks to technology and foreign output
τ_w,τ_p		Pay-roll tax rate and sales tax rate (endogenous)
γ_c	0.8	Consumption share of steady state output
ϕ_π	1.5	Coefficient in inflation based Taylor rules

Table 1: **Benchmark Parametrization.**

Preferences and technology: The substitution of domestic and foreign goods is $\eta = 1.5$, as in Auray et al. (2011). $\gamma = 1$, so goods from different foreign countries are perfect substitutes, as in Galí and Monacelli (2005). Interpreting one period as a quarter and setting $\beta = 0.99$ implies a steady state (risk-free) interest rate of approximately 4 percent p.a.. The degree of openness α is 0.3, as in Auray et al. (2011) and between the values in Galí and Monacelli (2005) and Pappa and Vassilatos (2007).[26] The relative risk aversion parameter is $\sigma = 2$ in line with Corsetti et al. (2008) and $\varphi = 3$, implying a Frisch labor supply elasticity of 1/3.

The substitution between differentiated goods is $\varepsilon_p = 6$ and the price adjustment cost is $\phi_p = 58$, so that we obtain a Phillips curve with the same output-response as a classical Calvo-model (Calvo, 1983) with an average price duration of four quarters (and hence a Calvo parameter of $\theta_p = 0.75$).[27] The labor demand elasticity is $\varepsilon_w = 21$ and the adjustment cost $\phi_w = 19$. This is in line with Schmitt-Grohé and Uribe (2006), who have Calvo version of our union setup with a Calvo parameter of 0.64, following the estimates in Christiano et al. (2005).[28]

[26] Galí and Monacelli (2005) have $\alpha = 0.4$ using Canada as their example of a SOE, while Pappa and Vassilatos (2007) have a related two country model and choose $\alpha = 0.2$ based on France ad Germany as a case of symmetric home bias.

[27] In a Calvo price friction model for our economy, the slope of the log-linearized Phillips curve is $\kappa_p = \frac{(1-\beta\theta_p)(1-\theta_p)}{\theta_p}$, see Galí and Monacelli (2005).

[28] As pointed out by Schmitt-Grohé and Uribe (2006) and elaborated in Colciago (2011), assuming that households are *not* monopolistic supliers of a given labour input means that the slope in a log-linearized Calvo model is given by $\kappa_w = \frac{(1-\beta\theta_w)(1-\theta_w)}{\theta_w}$, where assuming that the each households is a monopolistic supplier implies $\kappa_w = \frac{(1-\beta\theta_w)(1-\theta_w)}{\theta_w(1+\varphi\varepsilon_w)}$ (that is, κ_w is smaller). In order to match data for the wage Philips curve we therefore set ε_w higher than classical calibrations of around 5-6 (see Erceg et al. (2000) and

Shocks: For the domestic technology shock, we have $\rho_a = \rho_{c*} = 0.9$ and $\omega_a = \omega_{c*} = 0.007$. The two shocks have correlation 0.3. These numbers are consistent with Auray et al. (2011) and very close to what Galí and Monacelli (2005) find for Canadian data.

5 Determinacy

The insight that limited participation in asset or money markets can lead to indeterminacy under an otherwise stabilizing Taylor rule, dates back to Christiano and Gust (1999), who consider limited participation.[29] Galí et al. (2004) build on this insight and discuss how LAMP together with price rigidities can imply that the Taylor principle seizes to be a sufficient condition for a unique equilibrium. In our model we define the Taylor principle as the case where $\varphi_\pi > 1$, see Woodford (1999) and Taylor (1999). This principle ensures a unique equilibrium in the model with Ricardian households and price rigidities, however, it seizes to be a sufficient condition, when we have liquidity constrained households in the economy.

Bilbiie (2008) shows that for a sufficiently high degree of LAMP, aggregate demand becomes inverted - demand rises when the real interest rate is increased. If this contraction becomes large, sunspot equilibria are possible. Colciago (2011) shows that the presence of wage rigidity will increase the model's determinacy region.

In line with the closed economy in Galí et al. (2004), we find that LAMP can significantly change the determinacy properties of the small open economy model.

5.1 Monetary Policy Regime

Figure 1 shows the uniqueness and indeterminacy regions in the (θ_p, λ) space, where each line represents the boundary between the two regions.[30] Panel (a) shows that under all three simple monetary policies, a high degree of price stickiness and a large share of liquidity constrained households results in indeterminacy, whereas a sufficiently low stickiness or share of liquidity constrained households ensures the existence of a unique

Furlanetto and Seneca (2012)).

[29] In Christiano and Gust (1999) households must deposit their (monetary) savings with a financial intermediary before a periods monetary supply shock is realized, and hence they cannot adjust immediately.

[30] Translating the Rotemberg price adjustment cost, ϕ_p, into the corresponding Calvo price friction parameter θ_p (as described in the Parametrization section), transforms our parameter space from $\mathbb{R}^+$ to the bounded interval $[0, 1]$, and further allows an easier comparison with the existing literature.

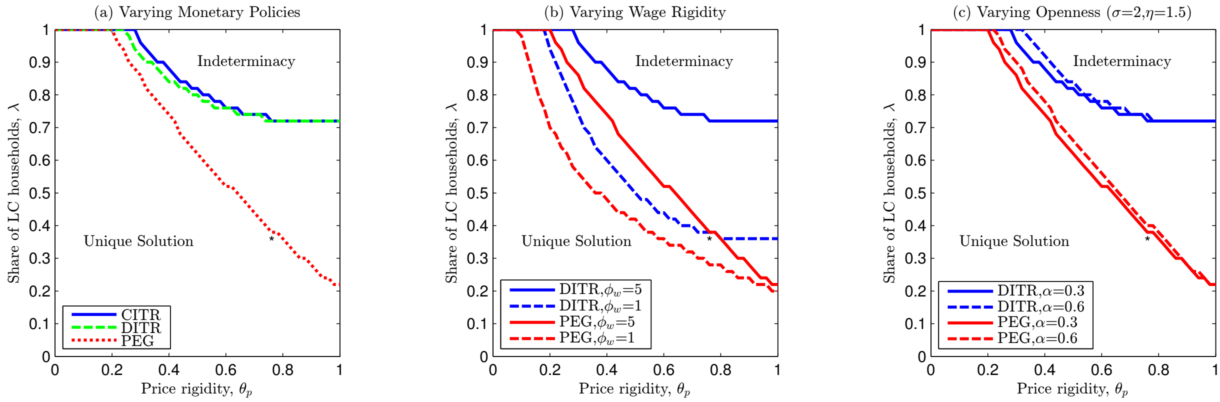

Figure 1: **Determinacy regions:** Each line represents the boundary between the uniqueness region (southwest) and the indeterminacy region (northeast).

equilibrium. Just as for the closed economy, liquidity constrained households cannot single-handedly overturn the result on the sufficiency of the Taylor principle; it is the combination of LAMP and a price rigidity that together drive the potential indeterminacy.

To explain why the indeterminacy occurs, imagine that households expect higher output. This would come about through more hours worked, higher real wages according to the unions' optimal wage setting, and higher inflation in nominal wages and domestic goods prices. Higher domestic inflation in turn worsens terms of trade, causing Ricardian households to reduce consumption, and both domestic and foreign households to substitute away from the domestic economy's goods. As a result, demand for domestic goods is lower and cannot support the expected output increase. Thus, if there are only Ricardian households, the outcome cannot be a rational expectations equilibrium (REE). The liquidity constrained households, on the other hand, merely turn their increased real income into a higher consumption. If the price rigidity (and thus the real wage increase) and the share of liquidity constrained households is sufficiently large, the aggregate consumption increases so much that the outcome is consistent with a REE.[31]

In the mechanisms above, there is no nominal interest rate response, which is consistent with the fixed exchange rate regime. Under the two Taylor rules, the persistent inflationary pressure lead to a higher nominal and real interest rate, causing Ricardian households to substitute savings for consumption. This will coincide with a lower terms

[31] Bilbiie (2008) discusses inverted aggregate demand - the case where a real interest rate rise has a positive demand effect. This can occur, if the demand drop of Ricardian households results in such a large real wage increase, that liquidity constrained demand increases enough to dominate the Ricardian consumption decline.

of trade according to the international risk sharing, and for the same reasons as before this yields an even lower total demand. Thus, in a purely Ricardian economy, the gap between demand and expected output is larger under a Taylor rule than a PEG. It therefore takes a larger share of liquidity constrained households or a higher wage rigidity to increase demand enough to support the outcome as a REE, which is why the indeterminacy region is smaller for the two Taylor rules in Figure 1 panel (a). As domestic inflation only feeds partially into the consumer price index, the interest rate response will be higher under DITR than under CITR given the same policy parameter φ_π. This is why the indeterminacy region under DITR is slightly smaller than this region under CITR.

5.2 Wage Rigidity

Panel (b) in Figure 1 shows that the determinacy problem is shrinking in the degree of wage rigidity, which is in line with Colciago (2011). The wage rigidity decreases the nominal (and real) wage increase expected under the sunspot, and disposable income of the liquidity constrained households increases less.[32] Hence, it takes a larger share of liquidity constrained households to make demand high enough to support the output as a REE. Colciago (2011) finds that, in his model with liquidity constrained households, merely a small degree of wage rigidity will restore the Taylor principle as a sufficient condition for determinacy for reasonable ranges of the model's parameters.

5.3 Openness

Panel (c) in Figure 1 shows that a larger degree of openness, α, implies a smaller indeterminacy region under our benchmark parameterization. The inflationary pressure in the discussed sunspot will deteriorate the terms of trade and reduce net exports and thus total demand. This leads to a larger gap between the output and total demand with merely Ricardian households (due to net exports). This gap is increasing in openness. A higher share of liquidity constrained households or a larger price friction must be present to support the sunspot, and as a result the uniqueness region is larger for a higher degree of openness α (for a given monetary policy regime).

[32] If sufficiently large, the wage rigidity can cause a decline in the real wage, see Galí (2013) for a thorough discussion.

6 Technology Shocks under Simple Rules

We now analyze how LAMP affects the propagation of technology shocks in the economy. We compare our results to Furlanetto and Seneca (2012), who consider a technology shock in a closed economy version of our model.

Figure 2 - 4 show the impulse responses to a technology shock of 1 percent under the respective monetary policy. The solid blue line represents the model with only Ricardian households, while the dashed green line represents our benchmark model with liquidity constrained households. All stated numbers are percentage deviation from steady state.

6.1 Floating Exchange Rate

6.1.1 DITR

Figure 2 shows the propagation of a positive technology shock under the domestic (PPI) inflation-based Taylor rule, DITR. The difference between the two lines indicate that the presence of liquidity constrained households boosts the output slightly in the short run (from a 0.82 increase to 0.88). This reflects larger movements in the components of output, as LAMP decreases aggregate domestic consumption (from 0.46 to 0.22) but reduces the drop in net exports by slightly more (from -0.97 to -0.77). Further, when we introduce liquidity constrained households, their consumption decline (-0.30) is very different from the consumption rise of Ricardian households (0.50).

The mechanisms in a model with no LAMP are the following. First, we consider an economy with only Ricardian households. A positive technology shock lowers the firms marginal cost ceteris paribus. This implies an increased supply and persistent reductions in producer prices (due to the adjustment cost). The deflation implies a reduction of the nominal (and real) interest rate according to the Taylor rule (DITR). The interest rate reduction and the increase in permanent income, caused by the technology shock, both cause Ricardian households to increase consumption on impact. Furthermore, the deflation improves the terms of trade, and the net exports are stimulated considerably. Now we turn to the case with LAMP. The decrease in hours (and ultimately disposable income) makes the liquidity constrained households reduce consumption. This in turn causes unions to set a lower wage, and firms to demand more hours. The total effect is a drop in current income and liquidity constrained consumption. As the PPI contracts

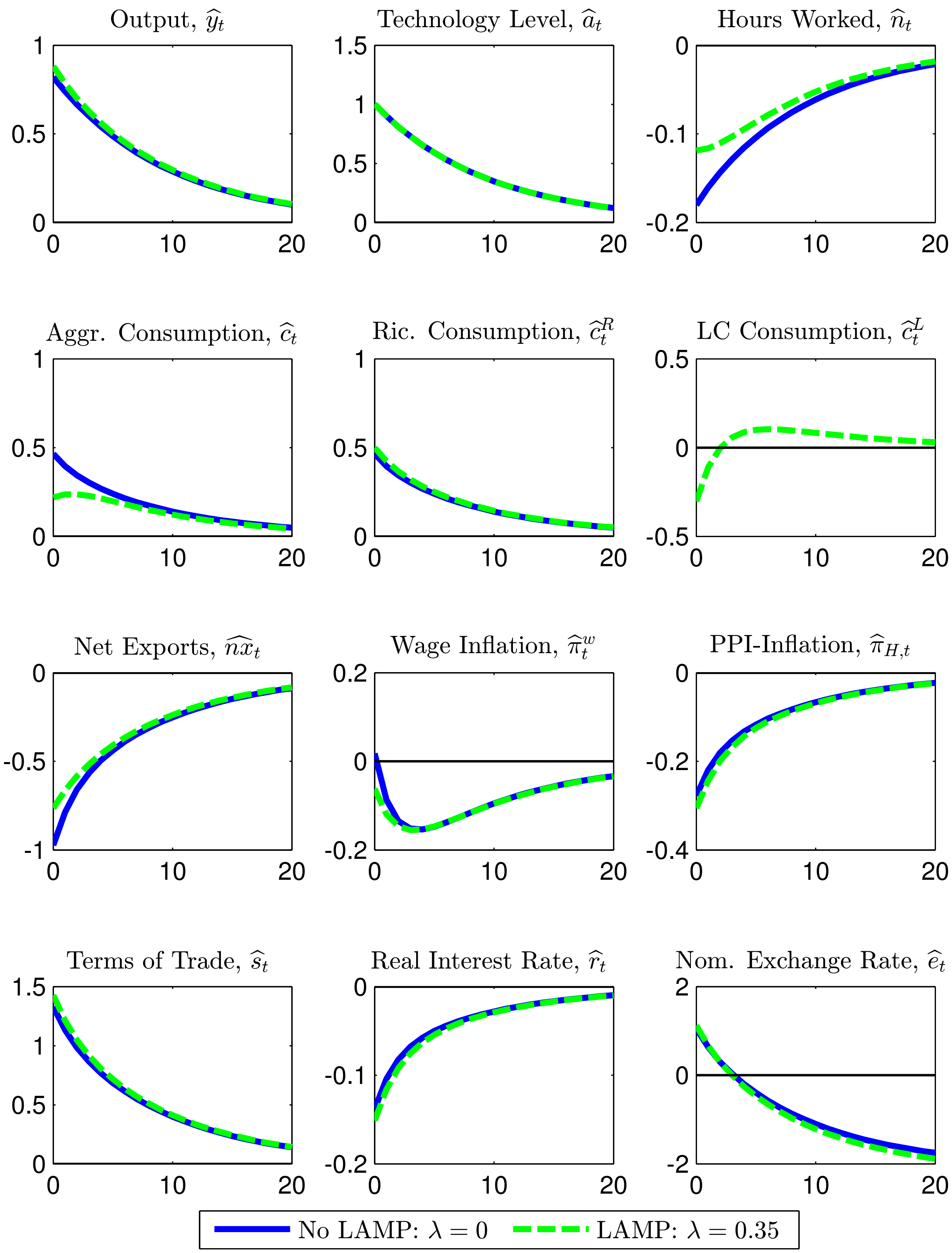

Figure 2: **DITR:** Response to a technology shock under a domestic PPI-inflation based Taylor rule.

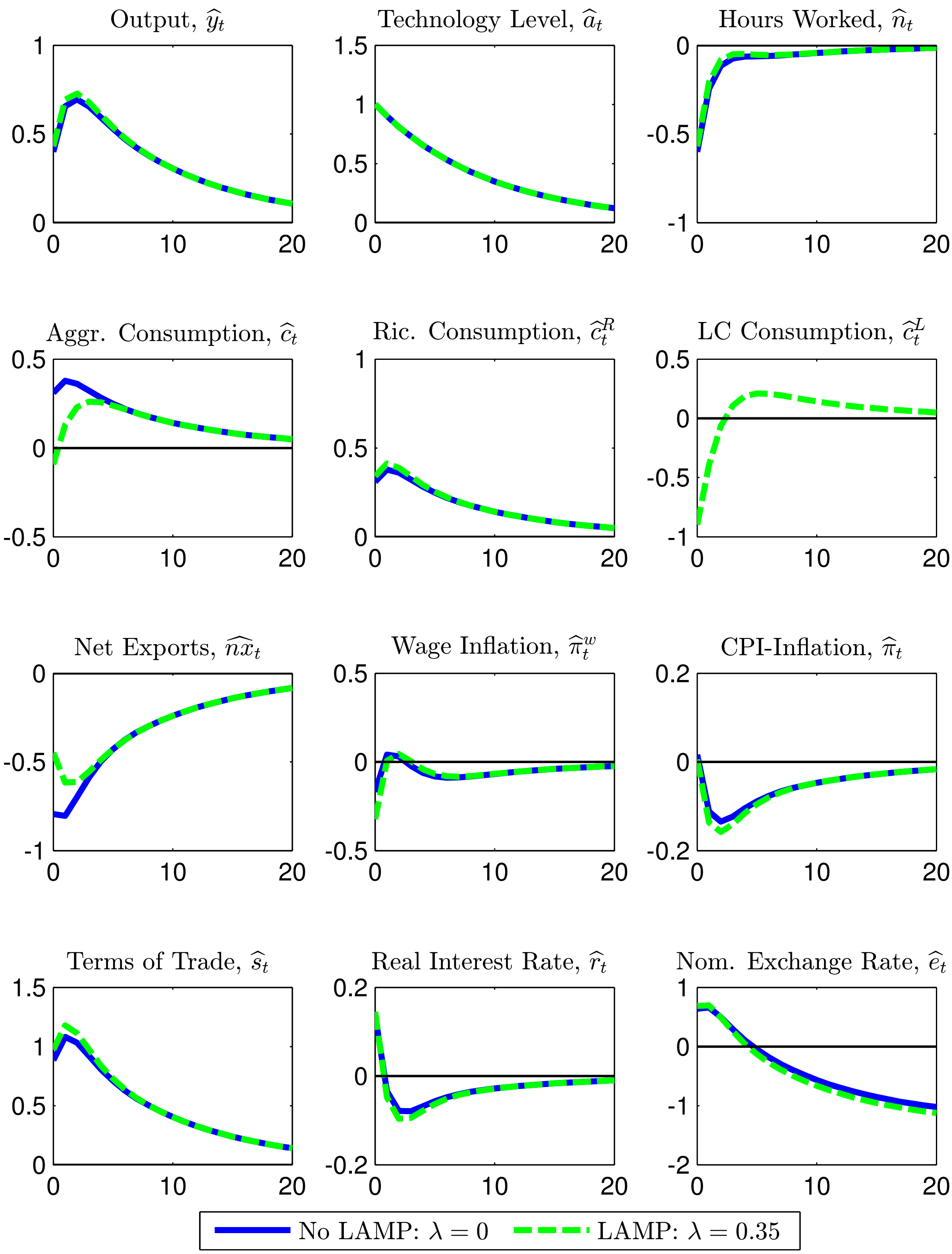

Figure 3: **CITR:** Response to a technology shock under a CPI-inflation based Taylor rule.

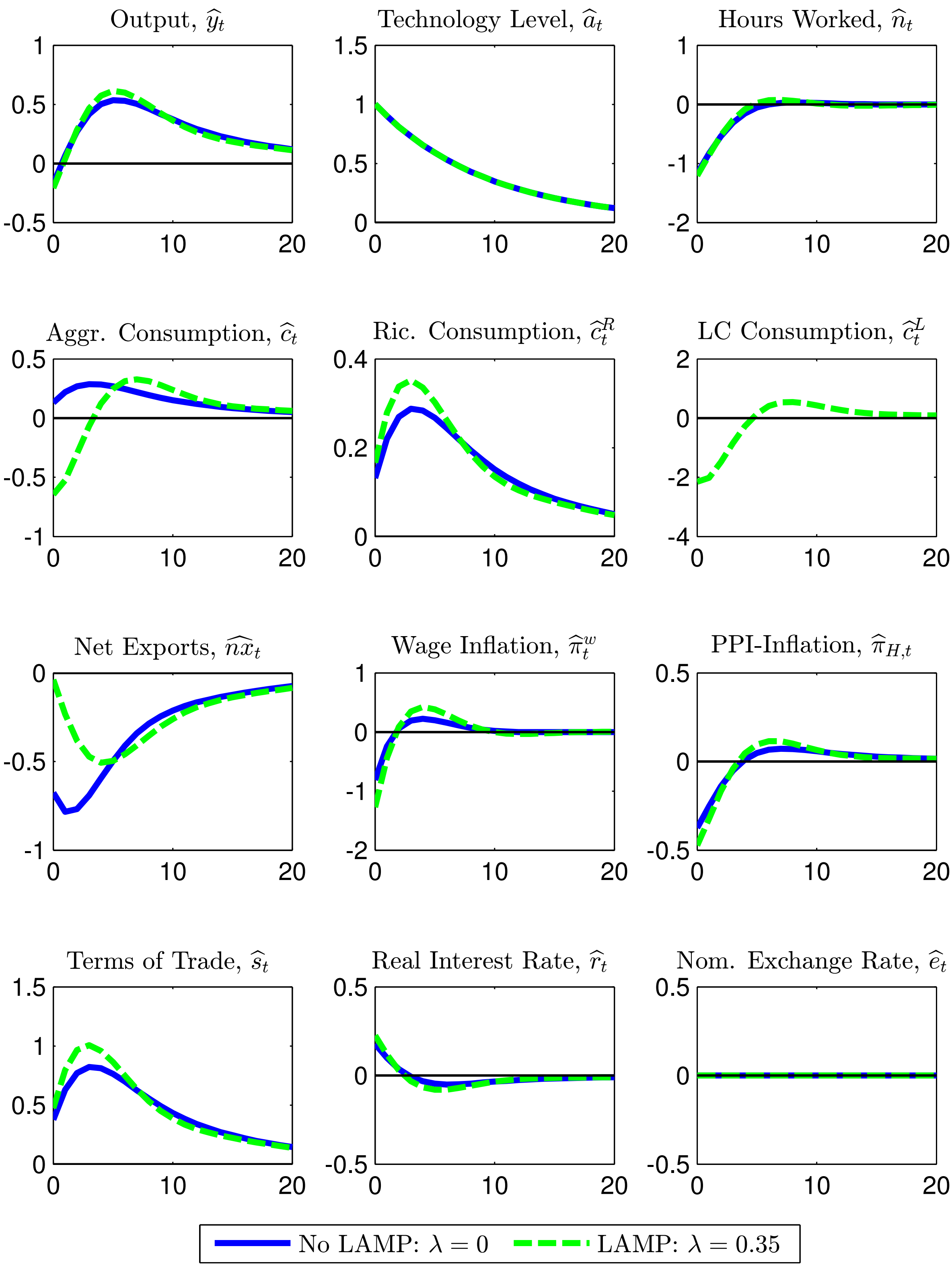

Figure 4: **PEG:** Response to a technology shock under a fixed nominal exchange rate.

more rapidly than the nominal wages, the liquidity constrained households will experience a rise in their real income after a few periods. Finally, we note that the initial boost in domestic demand for foreign goods makes the domestic currency depreciate on impact. As the shock is only transitory, the domestic currency will appreciate in the long run, thus restoring the relative prices of domestic and foreign goods.

6.1.2 CITR

Figure 3 shows the impulse responses under the consumption (CPI-) inflation-based Taylor rule, CITR. The mechanisms are very similar to the ones described under DITR policy. The gradual downward adjustment of producer prices is initially dominated by the instantaneous depreciation of the exchange rate, causing the consumer price index to increase slightly on impact (0.02 percent). Therefore, the real interest rate is increased in the initial period. From the following quarter, the PPI is reduced sufficiently to contract the CPI, and hence the real rate response also turns negative. The real rate path means that the Ricardian households do not increase consumption as much initially, making their consumption path hump-shaped. This implies more wage deflation and a larger drop in labor demand. The nominal exchange rate depreciates less, and net exports drop by less.

Introducing LAMP in this environment, the disposable income of liquidity constrained households has taken a larger hit than under DITR and they contract consumption three times more (-0.89) in this regime. The effect on total consumption is so large that it becomes negative in the first period (-0.09).

6.2 Fixed Exchange Rate

Figure 4 shows the same shock under a fixed exchange rate policy (PEG). Now the nominal interest rate is fixed due to the UIP. Further, the transitory shock does not change the relative prices in the long run, so any short run increase in domestic prices (and wages) must be reversed by subsequent inflation. This causes the real interest rate to increase in the short run and decrease in the medium/long run, and the long run real interest rate drops by exactly as much as the initial unexpected drop in consumer prices (see proof in Corsetti et al. (2011)).[33] The permanent income still rises, so Ricardian consumption is still rising (0.13) but not by as much as under the Taylor rules, and now there is a pronounced hump-shape. As a result, unions drop nominal wages more aggressively, and labor demand drops more under the PEG. Even though demand is considerably lower, deflation is only slightly higher under the peg due to the expectation of inflation and price reversal in the medium run. The real rate mirrors the CPI path, and thus first rises and then contracts. The inability of the exchange rate to adjust means that net exports are smaller than under CITR. Adding this to the smaller consumption rise implies that on impact, the output contracts (0.17), and although it becomes positive, the output is never boosted as much as under the Taylor rules.

Introducing LAMP under the PEG, the larger contraction of hours and the nominal wage, means that the liquidity constrained households contract their consumption much more (2.14). The resulting large drop in aggregate consumption (-0.64) makes terms of trade depreciate more, and net exports rise to almost zero (0.04), so that output is barely affected (from -0.17 to -0.21). As the shock reverts back to its steady state, the real wage increases rapidly and is even boosted in the medium run. This means that Ricardian households substitute towards more consumption in the short run and even have a negative response in the medium run. As a result, hours worked contract slightly more in the short run but is slightly more positive in the medium run. Overall, liquidity constrained households amplify the boom-bust effects of technology shocks on the economy under a PEG.

In a closed economy model where monetary policy follows a Taylor rule, Furlanetto and Seneca (2012) find that the presence of LAMP implies a much more persistent decline in hours. In the open economy this effect is not present. Rather, the decline in hours is halved under DITR and hardly affected under CITR and PEG. This suggests that the effects of LAMP are different in the open economy compared to closed economy, as trade effects will reduce or even offset the lower domestic demand caused by the presence of liquidity constrained households.

[33] The result in Corsetti et al. (2011) carries through to our model, as they only use a log-linear Euler equation, similar to the one Ricardian households satisfy in our model, and the fact that price levels return to their initial values under a peg.

7 Optimal Policy

We now turn to the welfare implications of the presence of LAMP. We initially consider the social planner's problem, i.e. we solve for the optimal allocation by a social planner, who puts an equal weight on every household in the economy and is only subject to technology and resource constraints. Then we turn to the Ramsey planners problem, where we solve for the optimal allocation given the optimizing behavior of all agents in the economy.

7.1 Social Planners Problem

We start by finding the optimal allocation from the view of a utilitarian social planner[34]. We allow the hours worked by Ricardian and liquidity constrained households to differ.

The social planner's problem is

$$\max_{C_t^R, C_t^L, N_t^h, S_t, P_{H,t}} \sum_{t=0}^{\infty} \beta^t \left((1-\lambda) U \left(C_t^R, N_t^R \right) + \lambda U \left(C_t^L, N_t^L \right) \right) \tag{39}$$

subject to the technology constraint (27), the market clearing condition (31) , and the risk sharing condition (22). The full formulation of the problem and the optimality conditions are in Appendix G.

The first two results for the solution are that hours for the two types of households are equal and that there is zero PPI inflation

$$\begin{aligned} N_t^R &= N_t^L = N_t \\ \Pi_{H,t} &= 1. \end{aligned}$$

This is because the households have the same productivity and disutility from labor. Zero inflation removes the resources spent on the adjustment cost.

We now turn to consumption, which differs for the two types of households

$$\left(C_t^L \right)^{-\sigma} = \frac{N_t^{\varphi}}{A_t} g_t^{\eta} (1-\alpha) \quad ; \quad \left(C_t^R \right)^{-\sigma} = \frac{N_t^{\varphi}}{A_t} g_t^{\eta} (1-\alpha) - \frac{f_t^{Risk}}{1-\lambda} \tag{40}$$

[34] The weighting of utility of different households is not without difficulty. However, if we assume that the optimal policy is chosen ex ante, before the type of any households is revealed, then giving them weights according to their share of the population amounts to maximizing any household's expected welfare.

where ${f_t}^{Risk}$ is the Lagrange multiplier associated with the international risk sharing condition.

The reason the consumption levels differ are the following: The marginal utility gain from more consumption is identical, but the marginal cost differs. For the liquidity constrained households this cost is the marginal welfare loss of producing the domestic part, $(1-\alpha)$, of this unit of consumption. For Ricardian consumption the marginal cost has an extra term, reflecting that a rise in consumption implies a coinciding increase of the terms of trade.

We now analyze this wedge between the two levels of consumption. To ease the notation we define $\mathcal{Q}_t = \frac{S_t}{g(S_t)} \equiv q(S_t) \equiv q_t$. From the first order condition for S_t we have that ${f_t}^{Risk} = -\frac{1}{q_{s,t}} \frac{(N_t)^{\varphi}}{A_t} K(\bullet)$, and the function $K(\bullet) > 0$ for $\alpha \in (0,1]$, and $K(\bullet) = 0$ for $\alpha = 0$, see Appendix G. Given that $q_{s,t} = \frac{1-\alpha q_t^{1-\eta}}{g_t}$, it follows that ${f_t}^{Risk} < 0$ (and thus from (40) that $C_t^R < C_t^L$) for

$$1 > \alpha q_t^{1-\eta} \tag{41}$$

This is because a rise in Ricardian consumption can only occur through a rise in the real exchange rate due to the international risk sharing condition. If $q_{s,t} > 0$ this comes about through an increase in the terms of trade, which requires that foreign goods have a very small weight in the CPI (low α) or $\frac{P_t}{P_{H,t}}$ responds a lot to changes in S_t (that is, if $q_t^{1-\eta}$ is low).

If we assume that terms of trade is unity (and hence $q_t = 1$), we have that (41) holds as long as there is some degree of home bias ($\alpha \in (0,1)$), thus in steady state the social planner would want to set $C^R < C^L$. Consequently, in a neighborhood of the steady state, Ricardian consumption is lower than liquidity constrained consumption. In general this is the case if the degree of openness, α, is sufficiently small and/or the real exchange rate has appreciated sufficiently (q_t is low).

7.2 Ramsey Policy

The utilitarian Ramsey planner solves for the optimal monetary policy further subject to the budget constraints and optimizing behavior of the agents in the economy.

The incentives guiding the optimal monetary policy in our model are fourfold: Stabilizing domestic producer prices, manipulating the terms of trade, stabilizing nominal wages, and finally smoothing the consumption of the liquidity constrained households.[35]

Rotemberg and Woodford (1999) show that in a closed New Keynesian economy with sluggish price adjustment and a subsidy τ_p that neutralizes the price markup, it is optimal that the markup does not fluctuate from its efficient level, i.e. to have zero inflation.[36]

In an open economy with home bias and sticky prices there is further an incentive to influence the terms of trade, as monetary policy can affect terms of trade.[37,38] If the tax rate is set only to cancel the price markup, it is no longer optimal to have zero PPI inflation. Zero PPI inflation can however be optimal under a different tax rate: Galí and Monacelli (2005) show that in the knife edge case, where $\eta = \gamma = \sigma = 1$, a fixed price equilibrium is optimal, if the tax satisfies $(1 - \tau_p)(1 - \alpha) = \left(1 - \frac{1}{\varepsilon}\right)$.[39] The incentives to manipulate the terms of trade are increasing in the degree of home bias. By decreasing the subsidy accordingly, the effects of market power offset the terms of trade distortions, so that zero inflation is optimal.[40]

Faia and Monacelli (2008) investigate the effect of home bias on optimal policy in a model similar to ours (although only Ricardian households and with flexible wages).[41] They find that home bias renders a constant markup policy optimal only if the elasticity of substitution and the risk aversion are unity ($\eta = \sigma^{-1} = 1$). We show in Section 8 that this condition is not sufficient for price stabilization in our model.

Finally, Erceg et al. (2000) show that for a closed economy with both price and wage rigidities, the Pareto optimal outcome is not feasible, and the policymaker is left with a trade-off between stabilizing price inflation, wage inflation, and the output gap.[42] This trade-off is also present in our open economy.

[35] The first two mechanisms are pointed out in Galí and Monacelli (2005).

[36] Rotemberg and Woodford (1999) have a Calvo price friction. As their result builds on the log-linearized model, and the two price rigidities yield the same log-linearized Phillips curve, the result is still relevant for our model.

[37] See for instance Corsetti and Pesenti (2001) and Galí and Monacelli (2005).

[38] This non-neutrality of monetary policy sets us apart from Goodfriend and King (2001) and Kristoffersen, who assume that the price of tradeable goods (in domestic currency) is given by the world price(s).

[39] Benigno and Benigno (2003) show a similar result in a two country model.

[40] For the calibration in Galí and Monacelli (2005), the zero-inflation subsidy is indeed a tax.

[41] Beetsma and Giuliodori (2011) investigate the optimal monetary and fiscal policies for a two country model, both under coordination and non-coordination. Their model does not have LAMP and there is no home bias in preferences.

[42] The Pareto optimum involves a zero output gap as well as fully stabilized prices and nominal wages. However this is at odds with the fact that the Pareto-optimal real wages moves with technology shocks.

7.2.1 The Ramsey Problem

We now solve the Ramsey Problem for our model with home bias and liquidity constrained households. The problem of the Ramsey planner is to maximize

$$\max_{\left\{N_t,,C_t^R,C_t^L,S_t,\Pi_{H,t},\Pi_t^w\right\}_{t=0}^{\infty}} E_0\sum_{t=0}^{\infty}\beta^t\left((1-\lambda)U\left(C_t^R,N_t\right)+\lambda U\left(C_t^L,N_t\right)\right)$$

subject to the technology constraint (27), market clearing (31), international risk sharing (22), the wage and Phillips curve in equations (17) and (26), the budget constraint of the liquidity constrained households (12), and finally the law of motion for the real wage in equation (19).

The Marcet-Marimon formulation of the problem and first order conditions guiding optimal policy are shown in Appendix G. Since we derive the Ramsey policy in our levels-model, the adjustment costs of wages and prices will enter the Ramsey planner's first order conditions for producer price inflation and wage inflation:[43]

$$\begin{aligned}
\lambda_t^{GM}\phi_p\left(\Pi_{H,t}-1\right) &= \left(\lambda_t^{PC}-\lambda_{t-1}^{PC}\right)\frac{\left(C_t^R\right)^{-\sigma}}{g\left(S_t\right)}\left(2\Pi_{H,t}-1\right)-\lambda_t^{RW}\frac{\Pi_t^w}{\Pi_{H,t}}\\
\lambda_t^{LAMP}W_t^{\text{real}}N_t\phi_w\left(\Pi_t^w-1\right) &= -\left(\lambda_t^{WC}-\lambda_{t-1}^{WC}\right)\left(MRS_t^A\right)^{-1}W_t^{\text{real}}\left(2\Pi_t^w-1\right)N_t^{1+\varphi}-\lambda_t^{RW}.
\end{aligned}$$

Thus, the Ramsey planner has an incentive to minimize the wedges these inflation rates generate, although these wedges are zero in our log-linear approximation of the other equilibrium conditions.[44] This can possibly create some distortions in our welfare ranking of monetary policies, which we will return to in our welfare analysis in Section 9.

We do not derive a functional form for the policy instrument i_t. Rather, we find the optimality conditions describing the optimal allocations (including inflation rates), and the interest rate is determined in the resulting equilibrium. Rather than stating these optimality conditions here, we will graphically present the Ramsey policy by showing the propagation of a technology shock under this monetary policy in the next section.

[43] The log-linear approximations of the two equations are $\lambda^{GM}\phi_p\widehat{\pi}_{H,t}=C^{-\sigma}\lambda^{PC}\left(\widehat{\lambda_t^{PC}}-\widehat{\lambda_{t-1}^{PC}}\right)-\lambda_t^{RW}$ and $\lambda^{LAMP}N\phi_w\widehat{\pi}_t^w=-\lambda^{WC}\left(\widehat{\lambda_t^{WC}}-\widehat{\lambda_{t-1}^{WC}}\right)N^{1+\varphi}-\lambda_t^{RW}$.

[44] Wage inflation generates a wedge between real income and consumption of liquidity constrained households, and price inflation generates a wedge between direct production output and output available for consumption.

8 Technology Shocks and Ramsey Policy

Figure 5 shows the Ramsey optimal monetary policy for our model with full and limited asset market participation. In the wake of a technology shock, the Ramsey planner increases the (real) interest rate, thereby resembling DITR policy response more than CITR and PEG policies.

8.1 The effect of Liquidity Constrained Households

In the Ricardian economy the Ramsey planner contracts the interest rate slightly less than under DITR (-0.10 compared to -0.14) but the reduction is more persistent. The more muted interest rate contraction stimulates Ricardian consumption less (through a higher optimal consumption growth and a lower permanent income). This feeds into the pricing behavior of firms, who now discount less hard and therefore set a lower inflation (higher deflation). This increases the real wage, and the unions respond by setting a lower nominal wage. Both inflation rates are down -0.5 percent in equilibrium, so the marginal cost and producer prices have adjusted the same (under DITR the real wage drops). This equal change in wages and prices combined with a slightly lower demand (due to higher domestic consumption) and a higher adjustment cost from PPI-inflation makes firms demand less labor, and the drop in hours is doubled compared to under DITR. Combining these factors, means that the technology shock increases output less under the Ramsey policy (0.63 compared to 0.82 under DITR).

When we introduce liquidity constrained households, the difference becomes much more pronounced. The real interest rate responds more to the introduction of LAMP under the Ramsey policy; the real rate is decreased from -0.10 to -0.13. This dampens the wage and PPI inflation through reversing the mechanism just described.

The presence of liquidity constrained households helps the Ramsey planner obtain two of her goals. The liquidity constrained households' immediate drop in consumption, makes the unions cut wages by much less. As a result, the firms are not able to reduce output prices as much, and stabilization of wages and prices is obtained to a much higher extent. The large drop in hours (twice as large a sunder DITR) can be obtained at a very similar liquidity constrained consumption path, due to the stabilized nominal and real wages. This gives the Ramsey planner more wiggle room, and the real interest rate can

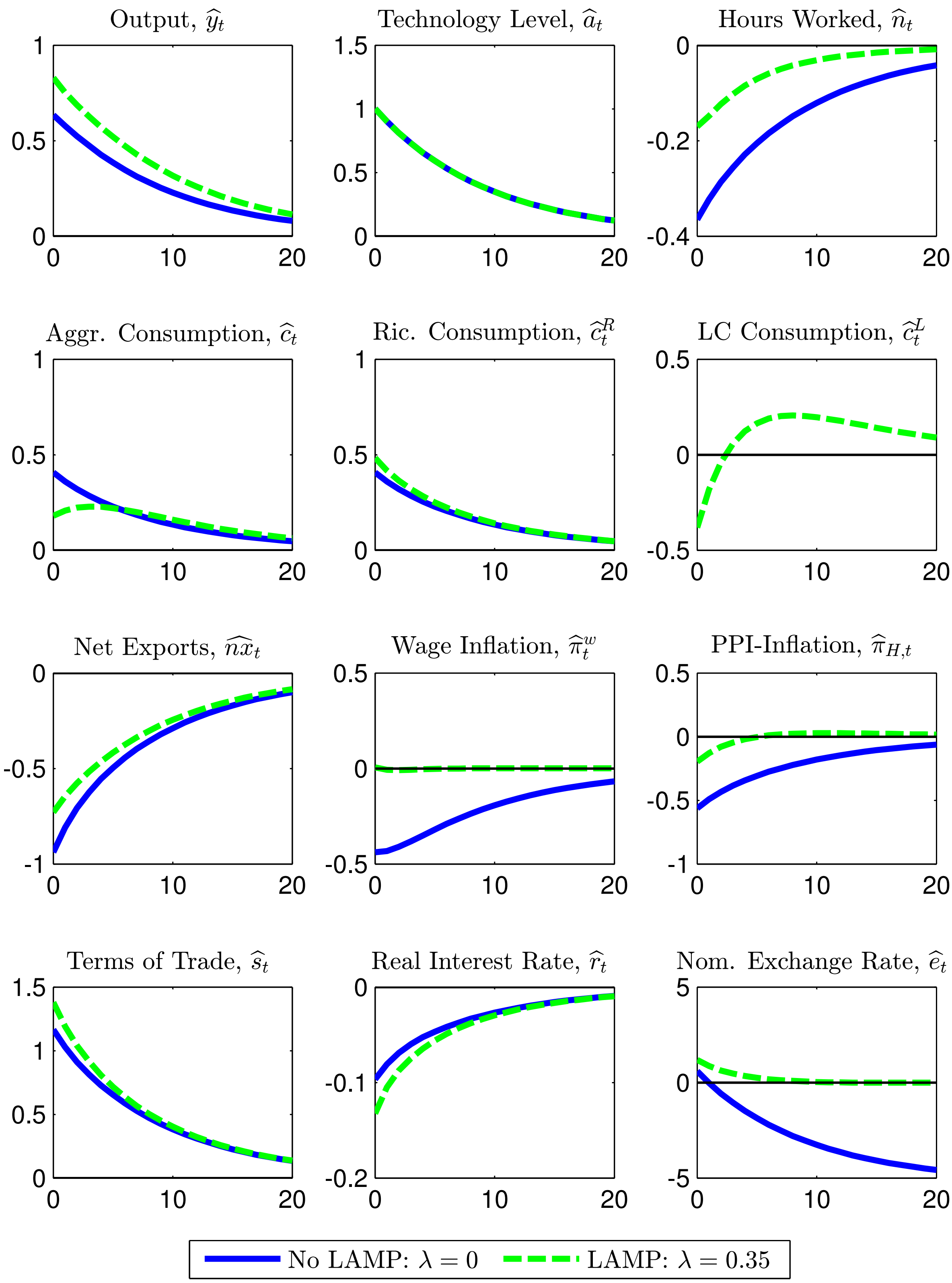

Figure 5: **Ramsey policy:** Response to a technology shock under the Ramsey policy in a model with and without LAMP..

be dropped more to increase Ricardian demand and improve the terms of trade.[45] The result is that the output response is increased from 0.63 to 0.83 by the introduction of liquidity constrained households (the numbers changed from 0.82 to 0.88 under DITR).

In line with Erceg et al. (2000), the Ramsey policy does not imply full price stabilization. For our benchmark parameterization of 35 percent liquidity constrained households, nominal wages and producer price depreciations are dampened, but not fully. The higher price stabilization comes at the cost of more output volatility.

Overall, we see that our small open economy model cannot replicate the finding in Furlanetto and Seneca (2012) that LAMP increase the drop in hours following a technology shock. Rather, under CITR and PEG, there is hardly an effect, and under DITR and Ramsey optimal monetary policy, the decline in hours is dampened by the presence of liquidity constrained households.

8.2 The Effect of Openness

Figure 6 shows how the propagation of a technology shock under the Ramsey policy depends on the degree of openness. Here we see that the incentive to improve terms of trade is larger, when there is more home bias (openness, α, is lower). Hence, in a more open economy (going from the green dashed to red dotted line) sees the Ramsey planner boost terms of trade less. This is done through a lower real interest rate, thereby dampening the rise in Ricardian consumption. The result through the change in domestic demand and the higher share of trade, means that the net exports contracts much more. The total effect is that a technology shock increases output slightly more, when the openness share of trade is increased, in our example doubled.

Figure 6 also includes the closed economy version of our model, and we see that the negative response of hours worked is still present in the closed economy.

8.3 The Effect of Wage and Price Rigidities

Figure 7 shows to which extent the wage and price frictions are driving our results. A striking result is that the real interest rate drop is reduced significantly, when either of

[45] The lower deflation rates also imply that the exchange rate is hardly affected under LAMP in the long run. In contrast, in the Ricardian model the Ramsey policy implies a long run appreciation of the real exchange rate.

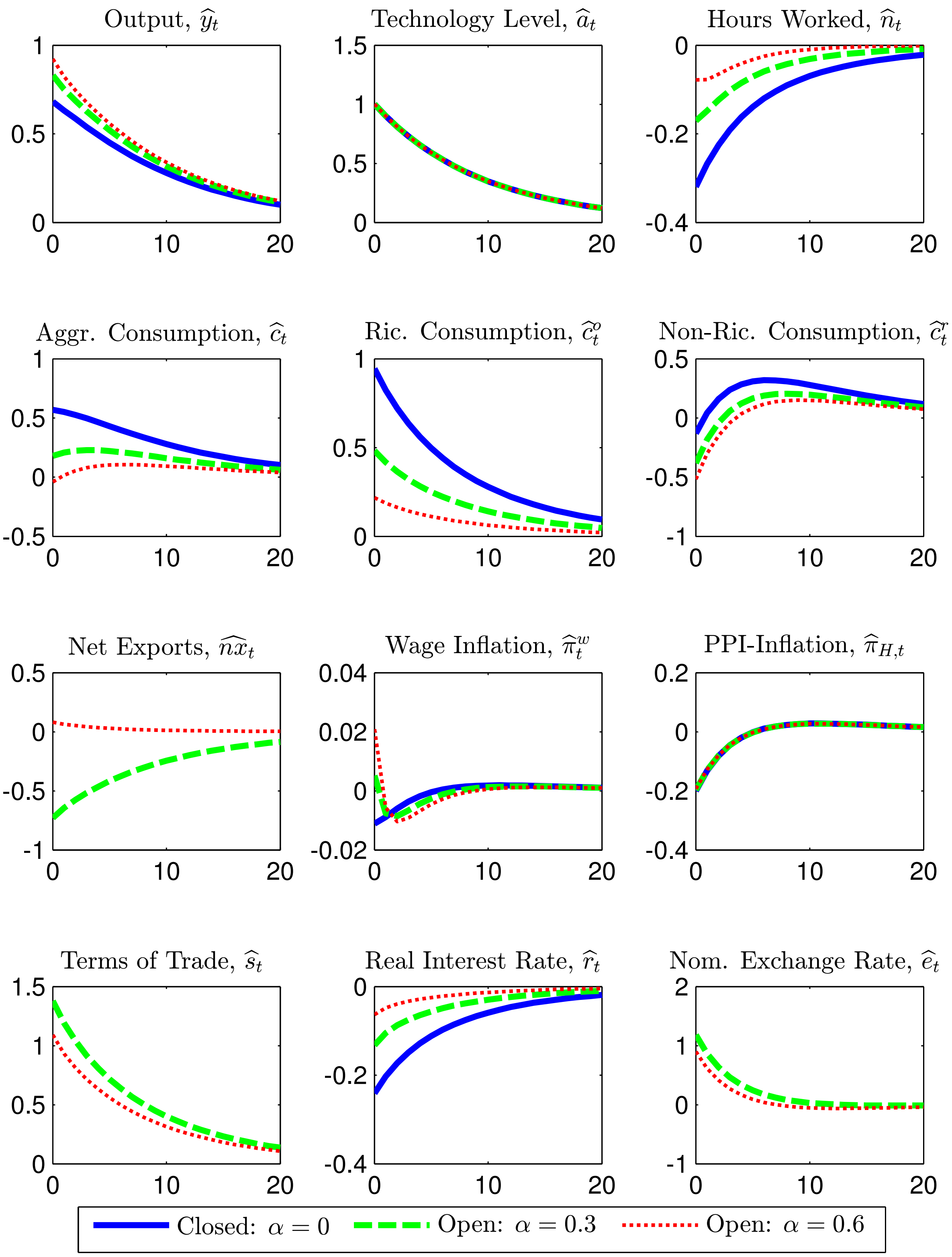

Figure 6: **Openness:** Response to a technology shock for varying degrees of openness under the Ramsey policy in an economy with LAMP.

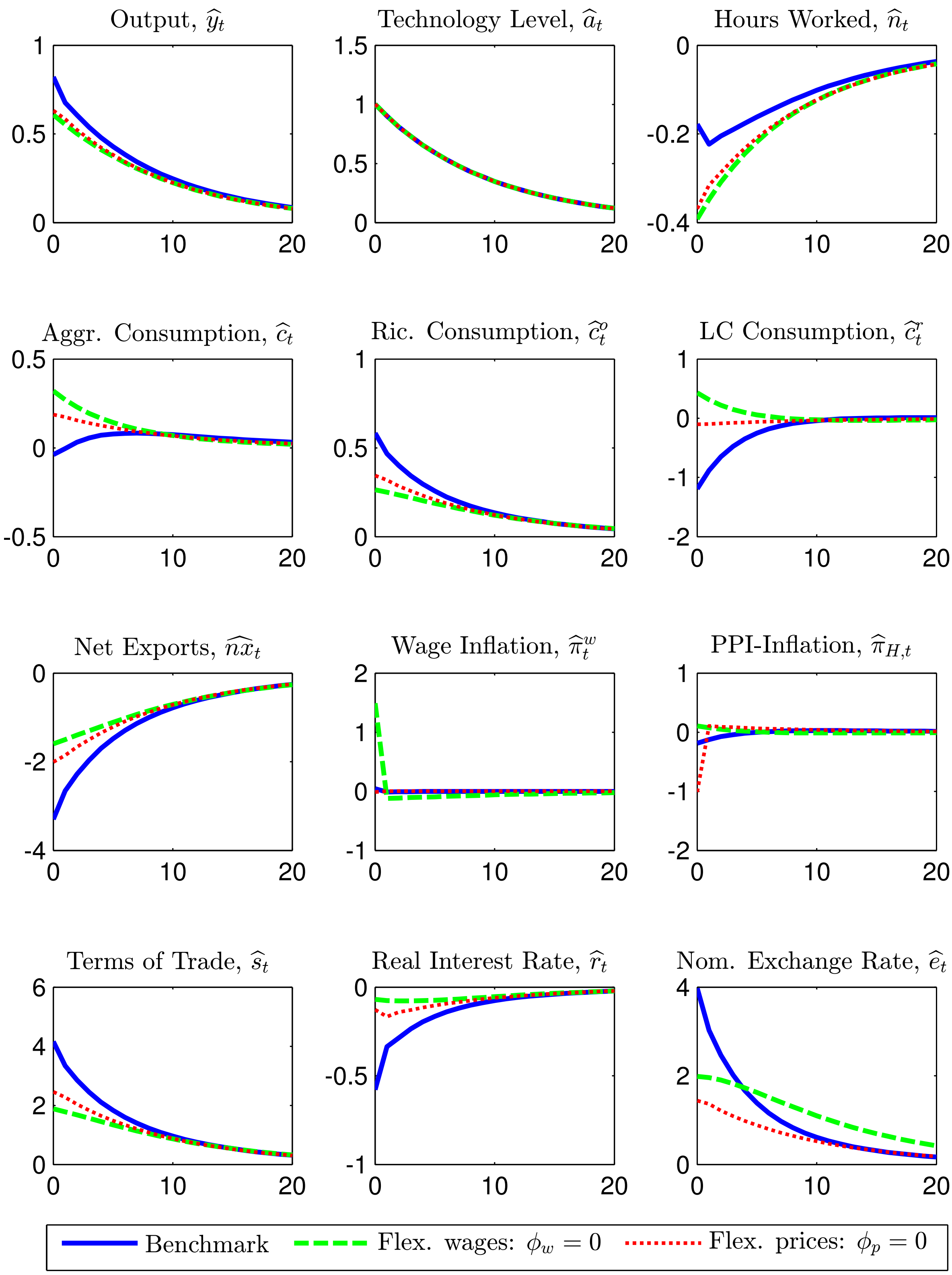

Figure 7: **Removing rigidities:** Response to a technology shock the Ramsey policy for our benchmark and the case with flexible wages and flexible prices respectively.

the frictions is removed. In this case, the Ramsey planner no longer faces the trilemma between price and wage stability and stabilizing output at its natural level. Now the flexible price/wage can take the full adjustment on impact, and as a result we have that the output response is very similar under both the flex-wage and the flex-price economy.

First, we consider the switch to flexible wages (the dashed green line). The higher real rate lowers Ricardian demand (compared to the dual-friction benchmark), and the firms respond by demanding less hours. As a result, wages negotiated by unions increase by 2 percent, dominating the drop in hours so that liquidity constrained households have an increased income and thus their demand rises. The higher real rate path implies that the exchange rate depreciates less, terms of trade are improved less, and net exports drop less under flexible wages. The sum of the demand shifts is that the output is stimulated less by a technology shock, when wages can adjust fully.

When we instead consider flexible prices (the red dotted line), the Ramsey planner still reduces the real rate contraction, but not as much as under the flex wage-economy. This reflects a key difference between the two frictions: When there is a nominal wage friction, the Ramsey planner is constrained by the sticky wages, and the inefficiency arising from the wedge between the marginal rate of transformation and the marginal rate of substitution. With a price friction, the skewed allocations, caused by a wedge between marginal cost and the price, are supplemented by a direct adjustment cost, taking up real resources that could have been consumed by households. This last cost is not present under the wage friction, as the adjustment cost is purely monetary and paid by union members/households. For this reason the Ramsey planner can pay more attention to the terms of trade incentive, allowing Ricardian consumption to increase, when the price rigidity is removed (rather than the wage rigidity). Therefore the real rate is lowered more than in the flex-wage case, improving terms of trade and slightly increasing demand. As a result, the real income of liquidity constrained households is higher - and we see that their demand actually rises following a technology shock.

Overall the change in the impulse responses of real variables is very similar, when we remove one of the frictions, and it is the presence of both frictions, and hence the gradual adjustment of the real wage, that has the largest impact on the economy under the Ramsey optimal policy.

9 Welfare Effects of Monetary Policy

Following our analysis of the Ramsey policy, a natural next step is to consider the welfare-ranking of the three simple policy rules. We define welfare of households type h, W^h as

$$W^h \equiv E_0 \sum_{t=0}^{\infty} \beta^t U\left(C_t^h, N_t^h\right) \tag{42}$$

Figure 8 recaps the utility effects of a technology shock in Figure 2-5. The graphs show that both types of households experience a much more volatile utility path under a PEG, causing us to expect that this policy will yield the lowest welfare for both types. For the Ricardian households the utility under DITR shows the least volatility and is practically coinciding with the Ramsey path. For liquidity constrained households DITR again has the smallest deviations and initially coincides with the Ramsey-path. This causes us to expect that DITR yields highest welfare of the three simple rules for both types of households. On a final note, the utility of the Ricardian agents take a (relatively) large jump initially, where the liquidity constrained households have a muted oscillating response. This can potentially lead to a higher expected welfare of the constrained households.

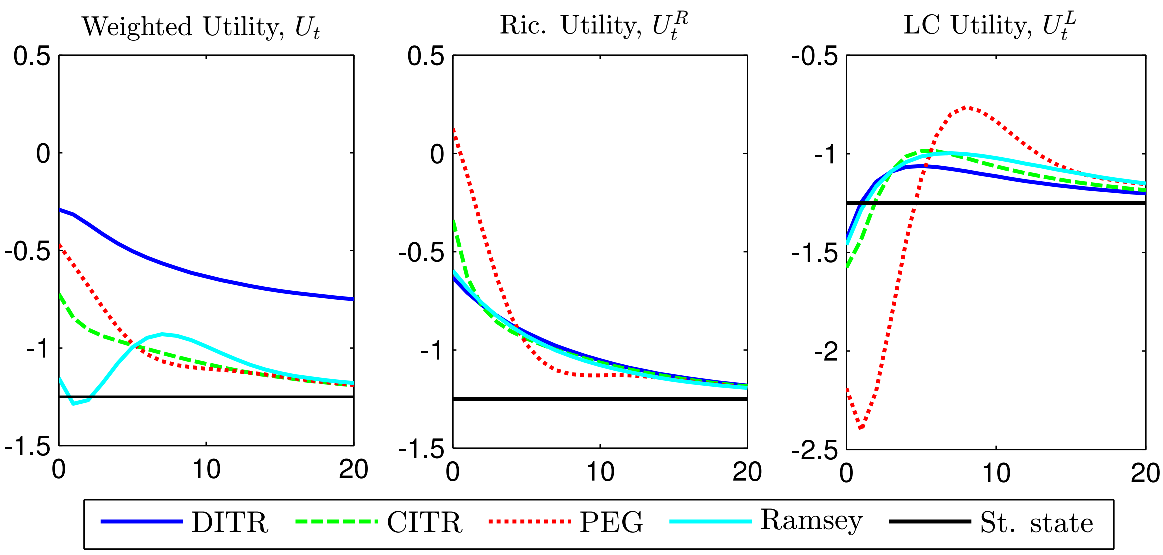

Figure 8: **Utility:** Effect of a technology shock on the path of household's in-period utility for different monetary policies.

The utilities in Figure 8 are only indicative, as the economy experiences foreign demand shocks as well as technology shocks. In order to capture the full dynamics of

	Taylor Rule, DITR		Taylor Rule, CITR		PEG		Ramsey Policy	
	Mean	s.d.	Mean	s.d.	Mean	s.d.	Mean	s.d.
C_t^R	1	0.0122	1	0.0123		0.0126	1	0.0121
C_t^L	1	0.0043	1	0.0097		0.0367	1	0.0075
N_t	1	0.0039	1	0.0054		0.0130	1	0.0037
U_t^R	-1.2502	0.0161	-1.2502	0.0162	-1.2504	0.0171	-1.2502	0.0158
U_t^L	-1.2500	0.0074	-1.2501	0.0091	-1.2516	0.0268	-1.2501	0.0098
$corr(U_t^R, U_t^L)$	0.8699		0.6642		-0.0551		0.8065	
Π_t^w	1	0.0040	1	0.0048	1	0.0048	1	0.0002
$\Pi_{H,t}$	1	0.0046	1	0.0044	1	0.0044	1	0.0019
	W^h	$\mu^h(pct)$	W^h	$\mu^h(pct)$	W^h	$\mu^h(pct)$	W^h	$\mu^h(pct)$
Ricardian	-125.0230	0.0230	-125.0248	0.0248	-125.0408	0.0408	-125.0223	0.0223
Liq.Constrained	-125.0074	0.0074	-125.0159	0.0160	-125.1694	0.1691	-125.0109	0.0109
Weighted	-125.0175	0.0175	-125.0217	0.0217	-125.0858	0.0858	-125.0184	0.0184

Table 2: **Welfare in simulated model:** This table reports the distrubutions for key variables under our benchmark calibration. The model is simulated for 10000 periods (dropping the 100 first periods) 500 times.

the model, we perform a simulation study and compare the welfare of the two types of household and a weighted welfare (consistent with the objective of the Ramsey planner). We follow the tradition of Lucas (1987) and calculate a consumption-equivalent welfare measure that is the share of steady state consumption μ^h that the household would give up (holding the steady state labor supply fixed) and still have the same welfare as in the dynamic economy we are considering.[46] For a given monetary policy regime we define this share for household h, μ^h, by the equation

$$E_0 \sum_{t=0}^{\infty} \beta^t U\left(\left(1-\mu^h\right) C, N\right) = E_0 \sum_{t=0}^{\infty} \beta^t U\left(C_t^h, N_t^h\right).$$

Table 2 shows the distributions of consumption, hours, and wage and PPI inflation under the different regimes for a simulated economy that is hit by technology and foreign demand shocks continuously. The table also shows the welfare function W^h and translates this welfare level into consumption shares μ^h.

In our simulation we solve for the equilibrium in the log-linearized equilibrium, derive consumption and labor in levels and calculate the utility using the function U. Even though there is certainty equivalence in the log-linearized model, volatilities will still matter for the welfare, as U reflects is risk aversion in the underlying preferences.

Table 2 reveals that under our benchmark parameterization the Ricardian households

[46] As the agents have the same steady state consumption, this measure can be used to compare welfare for both types of households.

are better off under the Ramsey policy ($\mu^R = 0.0223$ percent), very closely followed by the DITR policy ($\mu^R = 0.0230$ percent), the CITR policy ($\mu^R = 0.0248$ percent) and finally by a larger margin the PEG yields the largest welfare loss ($\mu^R = 0.0408$ percent). For the constrained households the DITR takes first place ($\mu^R = 0.0074$ percent), followed by the RP ($\mu^R = 0.0109$ percent), CITR ($\mu^R = 0.0160$ percent), and finally the PEG ($\mu^R = 0.1691$ percent).

Between the three simple policy rules, DITR yields the highest weighted welfare amongst the three simple policies. Further, we have the peculiar result that weighted welfare under DITR is higher than under the Ramsey policy. This is a result of the log-linearization: The Ramsey planner takes inflation wedges into consideration, but these are zero to a first order approximation and therefore not present in our log-linearized model. This approximation error gives an upward bias of the welfare calculations, which is increasing in the volatility of wage and domestic inflation. DITR yields more volatile inflation than the Ramsey policy, so the upward bias would overstate the relative welfare under DITR .

Table 2 also shows that for the Taylor rules and the Ramsey policy, liquidity constrained households have a higher welfare than Ricardian households. We conjecture that Liquidity constrained households are more exposed to the upward bias: Both households will be subject to the bias stemming from the price wedge in the labor market. As the cost of the wage inflation feed directly into consumption on liquidity constrained households via the union fee, their consumption will potentially be more volatile than the log-linear model captures. The volatilities are somewhat stable across the three simple regimes. We therefore conclude that there is potentially a change in ranking of the cost of business cycles for each of the two types of households, when we switch from a Taylor rule to a PEG.

Table 3 shows the welfare under the four monetary policy regimes, when each of the two shocks in turn is the only stochastic driver. We see that the rankings from the full dynamics in Table 2 carry through under both shocks.

A quick note on the size of the welfare effects - we have that the consumption equivalent welfare loss for the Ricardian households is $0.02 - 0.04$ percent. This is almost the same order of magnitude that Lucas (1987) found for $\sigma = 1$, but slightly lower than

	Only Technology Shocks				Only Foreign demand shocks			
	DITR	CITR	PEG	RP	DITR	CITR	PEG	RP
C_t^R (s.d.)	0.0075	0.0071	0.0066	0.0073	0.0082	0.0087	0.0097	0.0082
C_t^L (s.d.)	0.0032	0.0080	0.0260	0.0058	0.0030	0.0077	0.0347	0.0045
N_t (s.d.)	0.0023	0.0045	0.0114	0.0023	0.0027	0.0035	0.0104	0.0024
Π_t^w	0.0034	0.0028	0.0109	0.0001	0.0016	0.0040	0.0147	0.0002
$\Pi_{H,t}$	0.0041	0.0030	0.0046	0.0019	0.0015	0.0029	0.0049	0.0001
Cons-equiv. welfare loss (pct)								
Ricardian	0.0107	0.0125	0.0287	0.0103	0.0128	0.0142	0.0276	0.0123
Liq.Constrained	0.0037	0.0115	0.0831	0.0070	0.0047	0.0102	0.1371	0.0063
Weighted	0.0083	0.0122	0.0477	0.0092	0.0100	0.0128	0.0660	0.0102

Table 3: **Welfare statistics:** This table reports the distributions for key variables, when each shock is considered isolated. The model is simmuled for 10000 periods (dropping the 100 first periods) 500 times.

what he found for $\sigma = 5$.[47] For the liquidity constrained households, μ is in the interval $0.007 - 0.169$. The efficient steady state makes the welfare effects in our model small. Had we instead set labor and wage taxes to zero, thereby leaving the steady state output inefficiently low, the business cycle fluctuations would imply larger welfare losses.

9.1 Increasing Asset Market Participation

In Table 4 the share of liquidity constrained households is reduced from 35 to 20 percent. As the weight in the objective of the wage setting unions is reduced accordingly, there will be a higher variance of hours and a lower variance in Ricardian consumption in all regimes but the PEG. The same holds for the variance of the liquidity constrained households' consumption. As the share of liquidity constrained households drops, so does their weight in the objective function of the Ramsey planner, and the stabilization of Ricardian households' utility takes center stage. As the outcome favors the Ricardian households more, we find that under 20 percent LAMP, the liquidity constrained households are better off under DITR than under the Ramsey policy by a larger margin ($\mu^L = 0.0079$ percent compared to 0.0206 under the Ramsey policy) .

Comparing the dynamic outcome under the two degrees of LAMP, we see that the increased degree of asset market participation in Table 4 only affects the Ricardian welfare to a very limited extent. The liquidity constrained households' welfare loss is hardly affected under the Taylor rules DITR and CITR. Under the PEG, they experience a reduc-

[47]For $\sigma = 1$ Lucas (1987) estimates that the business cycle fluctuation implies a welfare loss of $\mu = 0.072$ percent (for $\sigma_a = 0.039$), and for $\sigma = 5$ he finds that these shires is 0.38 percent .

	Taylor Rule, DITR		Taylor Rule, CITR		PEG		Ramsey Policy	
	Mean	s.d.	Mean	s.d.	Mean	s.d.	Mean	s.d.
C_t^R	1	0.0120	1	0.0121	1	0.0127	1	0.0118
C_t^L	1	0.0042	1	0.0093	1	0.0331	1	0.0073
N_t	1	0.0044	1	0.0058	1	0.0128	1	0.0045
U_t^R	-1.2502	0.016	-1.2502	0.0164	-1.2504	0.0168	-1.2502	0.0163
U_t^L	-1.2500	0.0078	-1.2501	0.0091	-1.2513	0.0235	-1.2501	0.0096
$corr(U_t^R, U_t^L)$	0.8946		0.7235		0.0478		0.7987	
Π_t^w	1	0.0039	1	0.0046	1	0.0133	1	0.0005
$\Pi_{H,t}$	1	0.0044	1	0.0043	1	0.0051	1	0.0021
	$E\left[V_0^h\right]$	$\mu^h(pct)$	$E\left[V_0^h\right]$	$\mu^h(pct)$	$E\left[V_0^h\right]$	$\mu^h(pct)$	$E\left[V_0^h\right]$	$\mu^h(pct)$
Ricardian	-125.0231	0.0231	-125.0251	0.0251	-125.0407	0.0407	-125.0228	0.0228
Liq.Constrained	-125.0079	0.0079	-125.0156	0.0156	-125.1431	0.1429	-125.0116	0.0116
Weighted	-125.0201	0.0201	-125.0232	0.0232	-125.0612	0.0612	-125.0206	0.0206

Table 4: **Welfare under 20 pct. LAMP:** This table reports the distributions for key variables when setting the degree of LAMP to 20 pct. The model is simulated for 10000 periods (dropping the 100 first periods) 500 times.

tion in the welfare loss - as the boom-bust fluctuations of the real wage and hence income are reduced, they have a more stable consumption and hence higher welfare. Under the Ramsey policy their reduced share implies a welfare loss for a given liquidity constrained household, which has less weight in both the unions' and the Ramsey planner's objective function, and is thus less favoured by the equilibrium responses to shocks.

Hence, if a policy maker believes that increasing the degree of asset market participation will make the households better off, this is never the case for all households at the same time in our simulated log-linear model. When turning to weighted welfare, increasing asset market participation will reduce weighted welfare under DITR, CITR and Ramsey policy, but increase welfare under a PEG.

10 Conclusion

This paper adds to the growing literature on optimal monetary policy under limited asset market participation (LAMP) by considering a (New Keynesian) small open economy model. Our model nests three widely used models for analyzing monetary policy and the effect of LAMP: The sticky price and wage model of Erceg et al. (2000), the small open economy of Galí and Monacelli (2005) and the closed economy with LAMP and price frictions by Galí et al. (2007).

For each monetary policy we investigate the effect of LAMP on the propagation of technology shocks. we find that the decline in hours in response to a technology shock is halved under DITR and the Ramsey policy, while the response of hours is hardly affected under CITR and PEG. This suggests that the effects of LAMP are different in the open economy compared to closed economy, as trade effects will reduce or even offset the lower domestic demand caused by the presence of liquidity constrained households. Thus evaluating LAMP under PEG or CITR, if policy is in fact optimal, will lead to an underestimation of their impact on the economy .

Indeterminacy of the equilibrium arising under LAMP and price frictions is a well established fact in the literature. We show that for a small open economy the indeterminacy region is largest for a fixed exchange rate policy, followed by a domestic PPI inflation-based Taylor rule and finally a consumer price inflation-based Taylor rule.

Our welfare comparisons show that the welfare effects of DITR are very similar to those of the Ramsey policy. Welfare is lowest under a fixed exchange rate. However, this should not be taken as a definitive argument against a fixed exchange rate regime, as there are many other advantages from such a regime that are not captured by our model - increased trade, a lower interest rate spread to mention a few. Comparing the welfare of the two types of households, there is an indication that their relative welfare is switched if the monetary policy is changed from a Taylor rule to a PEG. Our welfare analysis highlights the limitations of using a log-linear framework and a levels welfare function. Our welfare measure has upward bias, which is increasing in the volatilities of wage inflation and PPI inflation. That being said, we find that welfare is considerably higher under a DITR than under the Taylor rules, suggesting that DITR is preferred if the choice is limited to simple rules.

In future work, it would be interesting to consider a non-constant share of liquidity constrained households - either by introducing an occasionally binding credit constraint or, as a first approximation, by letting the share of liquidity constrained households depend on the business cycle. We have not introduced capital accumulation in our model, but rather used the international risk sharing as a means of consumption smoothing for Ricardian households. Looking at fiscal policy under the different monetary policies in this open economy model is a natural next step and the author is currently working on this, see Salmansen (2014).

References

Albonico, Alice and Rossi, Lorenza. Policy games, distributional conflicts and the optimal inflation. *Macroeconomic Dynamics*, page Forthcoming, 2014.

Amato, Jeffery D and Laubach, Thomas. Rule-of-thumb behaviour and monetary policy. *European Economic Review*, 47(5):791–831, 2003.

Ascari, Guido; Colciago, Andrea, and Rossi, Lorenza. Limited asset market participation and optimal monetary policy. 2013.

Auray, Stéphane; de Blas, Beatriz, and Eyquem, Aurélien. Ramsey policies in a small open economy with sticky prices and capital. *Journal of Economic Dynamics and Control*, 35(9):1531–1546, 2011.

Bartolomeo, Giovanni Di and Rossi, Lorenza. Effectiveness of monetary policy and limited asset market participation: Neoclassical versus keynesian effects. *International Journal of Economic Theory*, 3(3):213–218, 2007.

Basu, Susanto; Fernald, John G., and Kimball, Miles S. Are technology improvements contractionary? *American Economic Review*, 96(5):1418–1448, 2006.

Beetsma, R. and Giuliodori, M. The effects of government purchases shocks: Review and estimates for the eu*. *The Economic Journal*, 121(550):F4–F32, 2011.

Beetsma, Roel MWJ and Jensen, Henrik. Monetary and fiscal policy interactions in a micro-founded model of a monetary union. *Journal of International Economics*, 67(2): 320–352, 2005.

Benigno, Gianluca and Benigno, Pierpaolo. Price stability in open economies. *The Review of Economic Studies*, 70(4):743–764, 2003.

Bilbiie, F. (2005). Limited asset markets participation, monetary policy and (inverted) keynesian logic. *Working Paper, Nuffield College, Oxford.*, 2005.

Bilbiie, Florin O. Limited asset markets participation, monetary policy and (inverted) aggregate demand logic. *Journal of Economic Theory*, 140(1):162–196, 2008.

Blanchard, O.J. and Kahn, C.M. The solution of linear difference models under rational expectations. *Econometrica: Journal of the Econometric Society*, pages 1305–1311, 1980.

Brendon, Charles. *Optimal Time-Invariant Monetary Policy.* Department of Economics, University of Oxford, 2009.

Calvo, G.A. Staggered prices in a utility-maximizing framework. *Journal of monetary Economics*, 12(3):383–398, 1983.

Chetty, Raj; Friedman, John N; Leth-Petersen, Soren; Nielsen, Torben, and Olsen, Tore. Active vs. passive decisions and crowdout in retirement savings accounts: Evidence from denmark. *Quarterly Journal of Economics*, Forthcoming, 2014.

Christiano, Lawrence J and Gust, Christopher J. Taylor rules in a limited participation model. *De Economist*, 147(4):437–460, 1999.

Christiano, Lawrence J; Eichenbaum, Martin, and Evans, Charles L. Nominal rigidities and the dynamic effects of a shock to monetary policy. *Journal of political Economy*, 113(1):1–45, 2005.

Chugh, Sanjay K. Optimal fiscal and monetary policy with sticky wages and sticky prices. *Review of Economic Dynamics*, 9(4):683–714, 2006.

Colciago, Andrea. Rule-of-thumb consumers meet sticky wages. *Journal of money, credit and banking*, 43(2-3):325–353, 2011.

Corsetti, Giancarlo and Pesenti, Paolo. Welfare and macroeconomic interdependence. *The Quarterly Journal of Economics*, 116(2):421–445, 2001.

Corsetti, Giancarlo; Dedola, Luca, and Leduc, Sylvain. High exchange-rate volatility and low pass-through. *Journal of Monetary Economics*, 55(6):1113–1128, 2008.

Corsetti, Giancarlo; Kuester, Keith, and Müller, Gernot J. *Floats, pegs and the transmission of fiscal policy.* Centre for Economic Policy Research, 2011.

Corsetti, Giancarlo; Meier, André, and Müller, Gernot J. What determines government spending multipliers? *Economic Policy*, 27(72):521–565, 2012. ISSN 1468-0327.

De Paoli, Bianca. Monetary policy and welfare in a small open economy. *Journal of International Economics*, 77(1):11–22, 2009.

Erceg, Christopher J; Henderson, Dale W, and Levin, Andrew T. Optimal monetary policy with staggered wage and price contracts. *Journal of monetary Economics*, 46 (2):281–313, 2000.

Faia, Ester and Monacelli, Tommaso. Optimal monetary policy in a small open economy with home bias. *Journal of Money, Credit and Banking*, 40(4):721–750, 2008.

Forni, Lorenzo; Monteforte, Libero, and Sessa, Luca. The general equilibrium effects of fiscal policy: Estimates for the euro area. *Journal of Public Economics*, 93(3):559–585, 2009.

Francis, Neville and Ramey, Valerie A. Is the technology-driven real business cycle hypothesis dead? shocks and aggregate fluctuations revisited. *Journal of Monetary Economics*, 52(8):1379–1399, 2005.

Furlanetto, Francesco and Seneca, Martin. Rule-of-thumb consumers, productivity, and hours. *The Scandinavian Journal of Economics*, 114(2):658–679, 2012.

Galí, J.; López-Salido, J.D., and Vallés, J. Understanding the effects of government spending on consumption. *Journal of the European Economic Association*, 5(1):227–270, 2007.

Galí, Jordi. Notes for a new guide to keynes (i): Wages, aggregate demand, and employment. *Journal of the European Economic Association*, 11(5):973–1003, 2013.

Galí, Jordi and Monacelli, Tommaso. Monetary policy and exchange rate volatility in a small open economy. *The Review of Economic Studies*, 72(3):707–734, 2005.

Galí, Jordi; López-Salido, J David, and Vallés, Javier. Rule-of-thumb consumers and the design of interest rate rules. Technical report, National Bureau of Economic Research, 2004.

Goodfriend, Marvin and King, Robert G. The case for price stability. Technical report, National Bureau of Economic Research, 2001.

Horvath, Michal. The effects of government spending shocks on consumption under optimal stabilization. *European Economic Review*, 53(7):815–829, 2009.

Kristoffersen, Mark Strøm. Liquidity constraints and fiscal stabilization policy.

Lucas, Robert E. *Models of business cycles*, volume 26. Basil Blackwell Oxford, 1987.

Marcet, Albert and Marimon, Ramon. Recursive contracts. 2011.

Natvik, Gisle James. Government spending shocks and rule-of-thumb consumers with steady-state inequality. *The Scandinavian Journal of Economics*, 114(4):1414–1436, 2012.

Pappa, Evi and Vassilatos, Vanghelis. The unbearable tightness of being in a monetary union: Fiscal restrictions and regional stability. *European Economic Review*, 51(6): 1492–1513, 2007.

Rotemberg, Julio J. Monopolistic price adjustment and aggregate output. *The Review of Economic Studies*, 49(4):517–531, 1982.

Rotemberg, Julio J and Woodford, Michael. Interest rate rules in an estimated sticky price model. In *Monetary policy rules*, pages 57–126. University of Chicago Press, 1999.

Salmansen, G.Y. Fiscal multipliers in a small open economy with limited asset market participation. *mimeo.*, 2014.

Schmitt-Grohé, Stephanie and Uribe, Martin. Optimal fiscal and monetary policy in a medium-scale macroeconomic model. *NBER Macroeconomics Annual 2005*, pages 383–462, 2006.

Taylor, John B. A historical analysis of monetary policy rules. In *Monetary policy rules*, pages 319–348. University of Chicago Press, 1999.

Woodford, M. Price-level determination under interest-rate rules. *unpublished, Princeton University*, 1999.

Woodford, Michael. *Interest and prices: Foundations of a theory of monetary policy.* Princeton University Press, 2003.

A Deriving the Wage Phillips Curve

The maximization problem of any union at time t is

$$\max_{W_t(z)} \sum_{k=0}^{\infty} \beta^k \left[(1-\lambda) U\left(C_{t+k}^R, N_{tt+k}^R\right) + \lambda U\left(C_{t+k}^r, N_{t+k}^r\right)\right]$$

$$st: P_{t+k} C_{t+k}^r = (1+\tau_w) N_t^d \int_0^1 W_{t+k}(j)^{1-e_w} W_{t+k}{}^{e_w} dj - P_{t+k} T^r$$

$$-\frac{\phi_w}{2}\left(\frac{W_t(z)}{W_{t-1}(z)} - 1\right)^2 W_t N_t^h v$$

$$P_{t+k} C_{t+k}^R = (1+\tau_w) N_t^d \int_0^1 W_{t+k}(j)^{1-e_w} W_{t+k}{}^{e_w} dj - \frac{\phi_w}{2}\left(\frac{W_t(z)}{W_{t-1}(z)} - 1\right)^2 W_t N_t^h + K$$

$$N_t^h = N_t^d \int_0^1 W_t(j)^{-e_w} W_t^{e_w} dj$$

The first order condition is

$$0 = \sum_{k=0}^{\infty} \beta^k (1-\lambda) \frac{\partial U_{t+k}^o}{\partial C_{t+k}^R} \frac{\partial C_{t+k}^R}{\partial W_t(z)} + \lambda \frac{\partial U_t^r}{\partial C_t^L} \frac{\partial C_t^L}{\partial W_t(z)} + \left[(1-\lambda) \frac{\partial U_t^o}{\partial N_t^h} + \lambda \frac{\partial U_t^r}{\partial N_t^h}\right] \frac{\partial N_t}{\partial W_t(z)}$$

where

$$\begin{aligned}
\frac{\partial C_t^h}{\partial W_t(z)} &= (1+\tau_w) \frac{N_t^d}{P_t} (1-e_w) W_t(z)^{-e_w} W_t^{e_w} - \frac{\phi_w}{P_t}\left(\frac{W_t(z)}{W_{t-1}(z)} - 1\right) \frac{W_t N_t}{W_{t-1}(z)} \\
&= \frac{(1+\tau_w)}{P_t} (1-e_w) N_t(z) - \frac{\phi_w}{P_t}\left(\frac{W_t(z)}{W_{t-1}(z)} - 1\right) \frac{W_t N_t}{W_{t-1}(z)} \\
&= \frac{(1+\tau_w)}{P_t} (1-e_w) N_t - \frac{\phi_w}{P_t}\left(\frac{W_t}{W_{t-1}} - 1\right) \frac{W_t}{W_{t-1}} N_t
\end{aligned}$$

$$\begin{aligned}
\frac{\partial C_{t+1}^h}{\partial W_t(z)} &= -\frac{\phi_w}{P_{t+1}}\left(\frac{W_{t+1}(z)}{W_t(z)} - 1\right)\left(-\frac{W_{t+1}(z)}{W_t(z)^2}\right) W_{t+1} N_{t+1} \\
&= \frac{\phi_w}{P_{t+1}}\left(\frac{W_{t+1}}{W_t} - 1\right) \frac{W_{t+1}^2}{W_t^2} N_{t+1}
\end{aligned}$$

$$\frac{\partial N_t^h(z)}{\partial W_t(z)} = -e_w N_t^d W_t(z)^{-e_w-1} W_t^{e_w} = -e_w N_t^h(z) W_t^{-1}.$$

as the unions are identical so that $W_t(z) = W$ and $N_t^h(z) = N_t^h$, we have the first order condition :

$$\begin{aligned}
0 &= (1-\lambda)\frac{\partial U_t^o}{\partial C_t^R}\frac{\partial C_t^R}{\partial W_t(z)} + \lambda\frac{\partial U_t^r}{\partial C_t^L}\frac{\partial C_t^L}{\partial W_t(z)} + \left[(1-\lambda)\frac{\partial U_t^o}{\partial N_t^h} + \lambda\frac{\partial U_t^r}{\partial N_t^h}\right]\frac{\partial N_t^h}{\partial W_t(z)} \\
&\quad + \beta\left[(1-\lambda)\frac{\partial U_{t+1}^o}{\partial C_{t+1}^R}\frac{\partial C_{t+1}^R}{\partial W_t(z)} + \lambda\frac{\partial U_{t+1}^r}{\partial C_{t+1}^r}\frac{\partial C_{t+1}^r}{\partial W_t(z)}\right] \\
&= \left[(1-\lambda)\left(C_t^R\right)^{-\sigma} + \lambda\left(C_t^L\right)^{-\sigma}\right]\frac{1}{P_t}N_t\left[(1+\tau_w)(1-e_w) - \phi_w(\Pi_t^w - 1)\Pi_t^w\right] + e_w N_t^{\varphi+1}W_t^{-1} \\
&\quad + \beta\left[(1-\lambda)\left(C_{t+1}^R\right)^{-\sigma} + \lambda\left(C_{t+1}^r\right)^{-\sigma}\right]\frac{\phi_w}{P_{t+1}}\left(\Pi_{t+1}^w - 1\right)\left(\Pi_t^w\right)^2 N_{t+1}
\end{aligned}$$

Rearranging we finally have

$$\begin{aligned}
\left(MRS_t^A\right)^{-1} W_t^{\text{real}}\phi_w(\Pi_t^w - 1)\Pi_t^w N_t^{1+\varphi} &= \left[\left(MRS_t^A\right)^{-1} W_t^{\text{real}}(1+\tau_w)(1-e_w) + e_w\right] N_t^{1+\varphi} \quad (43)\\
&\quad + \beta\left(MRS_{t+1}^A\right)^{-1} W_{t+1}^{\text{real}}\phi_w\left(\Pi_{t+1}^w - 1\right)\Pi_{t+1}^w N_{t+1}^{1+\varphi}
\end{aligned}$$

where $\left(MRS_t^A\right)^{-1} \equiv \left[(1-\lambda)\left(MRS_t^R\right)^{-1} + \lambda\left(MRS_t^L\right)^{-1}\right] = \left[(1-\lambda)\left(C_t^R\right)^{-\sigma} + \lambda\left(C_t^L\right)^{-\sigma}\right] N_t^{-\varphi}$.

Assuming that the subsidy is set to offset the markup due to monopolistic competition, that is $(1+\tau_w)(e_w - 1) = e_w$ (see E), we have

$$\begin{aligned}
\left(MRS_t^A\right)^{-1} W_t^{\text{real}}(\Pi_t^w - 1)\Pi_t^w N_t^{1+\varphi} &= \beta\left(MRS_{t+1}^A\right)^{-1} W_{t+1}^{\text{real}}\left(\Pi_{t+1}^w - 1\right)\Pi_{t+1}^w N_{t+1}^{1+\varphi} \quad (44)\\
&\quad - \frac{e_w}{\phi_w}\left[\left(MRS_t^A\right)^{-1} W_t^{\text{real}} - 1\right] N_t^{1+\varphi} \quad (45)
\end{aligned}$$

The log-linear approximation to this is using $(1+\tau_w)(e_w - 1) = e_w$ so that

$$\widehat{\pi}_t^w = \beta E_t \widehat{\pi}_{t+1}^w - \frac{e_w}{\phi_w}\left(\widehat{mrs}_t^A + \widehat{w}_t^{\text{real}}\right)$$

where

$$\widehat{mrs}_t^A = -\sigma\widehat{c}_t - \varphi\widehat{n}_t.$$

B Deriving the Phillips Curve

The optimal price $\overline{P}_{H,t}$ of any resetting firm satisfies the optimality condition

The objective function of union z that can renegotiate wages at time t is

$$\max_{P_t(z)} E_t \sum_{k=0}^{\infty} Q_{t,t+k} \left[P_{H,t+k}(j) Y_{t+k}(j) - (1-\tau_p) W_{t+k} N^h_{t+k}(j) - \frac{\phi_p}{2} \left(\frac{P_{t+k}(z)}{P_{t+k-1}(z)} - 1 \right)^2 P_{H,t+k} \right]$$

subject to the demand $Y_t(j) = \left(\frac{P_{H,t}(j)}{P_{H,t}} \right)^{-e_p} Y_t$. This is equivalent to

$$\max_{P_t(z)} E_t \sum_{k=0}^{\infty} Q_{t,t+k} \left[\begin{array}{c} P_{H,t+k}(j) \left(\frac{P_{H,t+k}(j)}{P_{t+k}} \right)^{-e_w} Y_{t+k} \\ -W_{t+k} \frac{1}{A_{t+k}} \left(\frac{P_{H,t+k}(j)}{P_{t+k}} \right)^{-e_w} Y_{t+k} - \frac{\phi_p}{2} \left(\frac{P_{t+k}(z)}{P_{t+k-1}(z)} - 1 \right)^2 P_{H,t+k} \end{array} \right]$$

FOC

$$\begin{aligned} 0 &= \left[\begin{array}{c} (1-e_w) P_{H,t}(z)^{-e_w} P_t^{e_w} Y_t + e_w (1-\tau_p) \frac{Y_t}{A_t} \left(\frac{P_{H,t}(z)}{P_{H,t}} \right)^{-e_w-1} \frac{W_t}{P_{H,t}} \\ -\phi_p \left(\frac{P_{H,t}(z)}{P_{H,t-1}(z)} - 1 \right) \frac{1}{P_{H,t-1}(z)} P_{H,t} \end{array} \right] \\ &\quad + Q_{t,t+1} \phi_p E_t \left[\left(\frac{P_{H,t+1}(z)}{P_{H,t}(z)} - 1 \right) \frac{P_{H,t+1}(z)}{P_{H,t}(z)^2} P_{H,t+1} \right] \end{aligned}$$

So balances the present value of the sum of changes in revenue, lower wage costs and adjustment costs all in period t and the expected effect on next periods adjustment cost.

Using that all firms set the same price, we have $P_t(z) = P_{H,t}$ and $Y_t(z) = Y_t$ and defining$\Pi_{H,t} = \frac{P_{H,t}}{P_{H,t-1}}$we have

$$\begin{aligned} \phi_p (\Pi_{H,t} - 1) \Pi_{H,t} &= \left[e_w (1-\tau_p) \frac{Y_t}{A_t} W_t^{\text{real}} \frac{P_t}{P_{H,t}} + (1-e_w) Y_t \right] \\ &\quad + Q_{t,t+1} \phi_p E_t \left[(\Pi_{H,t} - 1) \Pi_{H,t} \frac{P_{H,t+1}}{P_{H,t}} \right] \end{aligned}$$

finally , noting that $Q_{t,t+k}=\frac{U^o_{c,t+1}}{U^o_{c,t}}\frac{P_t}{P_{t+1}}$

$$\begin{aligned}\left(\Pi_{H,t}-1\right)\Pi_{H,t} &= \frac{e_w Y_t}{\phi_p}\left[\frac{(1-\tau_p)}{A_t}W_t^{\text{real}}g\left(S_t\right)-\frac{e_w-1}{e_w}\right] \\ &+\beta E_t\left[\frac{U^o_{c,t+1}}{U^o_{c,t}}\frac{P_t}{P_{t+1}}\frac{P_{H,t+1}}{P_{H,t}}\left(\Pi_{H,t+1}-1\right)\Pi_{H,t+1}\right]\end{aligned}$$

$$\begin{aligned}\left(\Pi_{H,t}-1\right)\Pi_{H,t} &= \beta E_t\left[\frac{\left(C^R_{t+1}\right)^{-\sigma}}{\left(C^R_t\right)^{-\sigma}}\frac{g\left(S_t\right)}{g\left(S_{t+1}\right)}\left(\Pi_{H,t+1}-1\right)\Pi_{H,t+1}\right] \\ &+\frac{e}{\phi_p}Y_t\left[\frac{(1-\tau_p)}{A_t}W_t^{\text{real}}g\left(S_t\right)-\frac{e-1}{e}\right].\end{aligned}$$

Assuming $(1-\tau_p)=\frac{e-1}{e}$we have

$$\begin{aligned}\left(\Pi_{H,t}-1\right)\Pi_{H,t} &= \beta E_t\left[\frac{\left(C^R_{t+1}\right)^{-\sigma}}{\left(C^R_t\right)^{-\sigma}}\frac{g\left(S_t\right)}{g\left(S_{t+1}\right)}\left(\Pi_{H,t+1}-1\right)\Pi_{H,t+1}\right] \\ &+\frac{e-1}{\phi_p}Y_t\left[\frac{W_t^{\text{real}}}{A_t}g\left(S_t\right)-1\right].\end{aligned} \tag{46}$$

The log-linear approximation of the Phillips curve in equation (46) is

$$\widehat{\pi}_{H,t}=\beta E_t\widehat{\pi}_{H,t+1}+\frac{e-1}{\phi_p}N\left(\widehat{w}_t^{\text{real}}+g_{s,t}\widehat{s}_t-\widehat{a}_t\right).$$

Note, for the Marcet-Marimon formulation of our Ramsey problem, we note that this can also be written as

$$\begin{aligned}0 &= \frac{\left(C^R_t\right)^{-\sigma}}{g\left(S_t\right)}\left(\Pi_{H,t}-1\right)\Pi_{H,t}-\frac{e}{\phi_p}Y_t\left[\frac{(1-e)}{e}\frac{\left(C^R_t\right)^{-\sigma}}{g\left(S_t\right)}-\frac{(1-\tau_p)}{A_t}W_t^{\text{real}}\left(C^R_t\right)^{-\sigma}\right] \\ &-\beta E_t\left[\frac{\left(C^R_{t+1}\right)^{-\sigma}}{g\left(S_{t+1}\right)}\left(\Pi_{H,t+1}-1\right)\Pi_{H,t+1}\right]\end{aligned}$$

C Goods Market equilibrium

For good j:

$$Y_t(j) = (1-\lambda) C^R_{H,t}(j) + \lambda C^L_{H,t}(j) + \int_0^1 C^i_{H,t}(j)\, di$$

Splitting it up

$$\begin{aligned}
& (1-\lambda) C^o_{H,t}(j) + \lambda C^r_{H,t}(j) && (47)\\
= & \left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\varepsilon} \left((1-\lambda) C^o_{H,t} + \lambda C^r_{H,t}\right) \\
= & \left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\varepsilon} \left[(1-\alpha)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} \left((1-\lambda) C^o_t + \lambda C^r_t\right)\right] \\
= & \left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\varepsilon} \left[(1-\alpha)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t\right] && (48)
\end{aligned}$$

$$\begin{aligned}
\int_0^1 C^i_{H,t}(j)\, di &= \int_0^1 \left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\varepsilon} C^i_{H,t} di && (49)\\
&= \int_0^1 \left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\varepsilon} \left(\frac{P_{H,t}/\varepsilon_{i,t}}{P^i_{F,t}}\right)^{-\gamma} C^i_{F,t} di \\
&= \int_0^1 \left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\varepsilon} \left(\frac{P_{H,t}/\varepsilon_{i,t}}{P^i_{F,t}}\right)^{-\gamma} \alpha \left(\frac{P^i_{F,t}}{P_t}\right)^{-\eta} C^i_t di \\
&= \alpha \int_0^1 \left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\varepsilon} \left(\frac{P_{H,t}}{\varepsilon_{i,t} P^i_{F,t}}\right)^{-\gamma} \left(\frac{P^i_{F,t}}{P_t}\right)^{-\eta} C^i_t di && (50)
\end{aligned}$$

Combining the expressions yields

$$\begin{aligned}
Y_t(j) &= \left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\varepsilon} \left[(1-\alpha)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t\right] && (51)\\
& \quad + \alpha \left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\varepsilon} \int_0^1 \left(\frac{P_{i,t}}{\varepsilon_{i,t} P^i_{F,t}}\right)^{-\gamma} \left(\frac{P^i_{F,t}}{P_t}\right)^{-\eta} C^i_t di \\
&= \left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\varepsilon} K_t && (52)
\end{aligned}$$

where K_t is the same for all j.

Using our aggregate measure of Y_t we have

$$\begin{aligned} Y_t &= \left(\int_0^1 Y_t(j)^{\frac{\varepsilon-1}{\varepsilon}} dj\right)^{\frac{\varepsilon}{\varepsilon-1}} = \left(\int_0^1 \left(\left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\varepsilon} K_t\right)^{\frac{\varepsilon-1}{\varepsilon}} dj\right)^{\frac{\varepsilon}{\varepsilon-1}} \\ &= (P_{H,t})^{\varepsilon} \left(\int_0^1 (P_{H,t}(j))^{1-\varepsilon} dj\right)^{\frac{\varepsilon}{\varepsilon-1}} K_t = (P_{H,t})^{\varepsilon} (P_{H,t})^{-\varepsilon} K_t = K_t \end{aligned} \tag{53}$$

Then

$$\begin{aligned} & \alpha \int_0^1 \left(\frac{P_{H,t}}{\varepsilon_{i,t} P_{F,t}^i}\right)^{-\gamma} \left(\frac{P_{F,t}^i}{P_t^i}\right)^{-\eta} C_t^i di \\ &= \alpha \int_0^1 \left(\frac{\varepsilon_{i,t} P_{F,t}^i}{P_{H,t}}\right)^{\gamma-\eta} \left(\frac{P_{F,t}^i}{P_t^i} \frac{P_{H,t}}{\varepsilon_{i,t} P_{F,t}^i}\right)^{-\eta} C_t^i di \\ &= \alpha \int_0^1 \left(\frac{\varepsilon_{i,t} P_{F,t}^i}{P_{i,t}} \frac{P_{i,t}}{P_{H,t}}\right)^{\gamma-\eta} \left(\frac{P_{H,t}}{\varepsilon_{i,t} P_t^i}\right)^{-\eta} C_t^i di \\ &= \left(\frac{P_{H,t}}{P_t}\right)^{-\eta} \alpha \int_0^1 \left(\frac{\varepsilon_{i,t} P_{F,t}^i}{P_{i,t}} \frac{P_{i,t}}{P_{H,t}}\right)^{\gamma-\eta} \mathcal{Q}_{i,t}^{\eta} C_t^i di \\ &= \left(\frac{P_{H,t}}{P_t}\right)^{-\eta} \alpha \int_0^1 \left(S_t^i S_{i,t}\right)_{i,t}^{\gamma-\eta} \mathcal{Q}_{i,t}^{\eta-\frac{1}{\sigma}} C_t^o di \\ &= \left(\frac{P_{H,t}}{P_t}\right)^{-\eta} \alpha C_t^o \int_0^1 \left(S_t^i S_{i,t}\right)^{\gamma-\eta} \mathcal{Q}_{i,t}^{\eta-\frac{1}{\sigma}} di \end{aligned}$$

where $\mathcal{Q}_{i,t} \equiv \frac{\varepsilon_{i,t} P_t^i}{P_t}$ and $S_t^i \equiv \frac{P_{F,t}^i}{P_{H,t}^i} = \frac{P_{F,t}^i}{P_{i,t}/\varepsilon_{i,t}} = \frac{\varepsilon_{i,t} P_{F,t}^i}{P_{i,t}}$. Note that this holds because the two price indices $P_{H,t}$ and $P_{i,t}$ use the same weights (ε).

Finally we have

$$\begin{aligned} Y_t &= (1-\alpha) \left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t + \left(\frac{P_{H,t}}{P_t}\right)^{-\eta} \alpha C_t^o \int_0^1 \left(S_t^i S_{i,t}\right)^{\gamma-\eta} \mathcal{Q}_{i,t}^{\eta-\frac{1}{\sigma}} di \\ &= \left(\frac{P_{H,t}}{P_t}\right)^{-\eta} \left[(1-\alpha) C_t + \alpha \int_0^1 \left(S_t^i S_{i,t}\right)^{\gamma-\eta} \mathcal{Q}_{i,t}^{\eta-\frac{1}{\sigma}} di C_t^o\right] \\ &= \left(\frac{P_{H,t}}{P_t}\right)^{-\eta} \left[(1-\alpha)(1-\lambda) + \alpha \int_0^1 \left(S_t^i S_{i,t}\right)^{\gamma-\eta} \mathcal{Q}_{i,t}^{\eta-\frac{1}{\sigma}} di\right] C_t^o \\ &\quad + \left(\frac{P_{H,t}}{P_t}\right)^{-\eta} (1-\alpha) \lambda C_t^r \end{aligned}$$

If all foreign countries are symmetrical this becomes

$$Y_t = \left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t + \alpha \left(\frac{P_{H,t}}{P_t}\right)^{-\eta} \int_0^1 \left(S_t^i S_{i,t}\right)^{\gamma-\eta} \mathcal{Q}_{i,t}^{\eta} di C_t^*$$

D Equilibrium conditions

The equilibrium conditions of the model are listed for allowing the reader a greater overview

$$
\begin{aligned}
\text{Euler equation} &: \quad \beta R_t E_t \left[\left(\frac{C_{t+1}^R}{C_t^R} \right)^{-\sigma} \Pi_t^{-1} \right] = 1 \\
\text{Disposable income} &: \quad P_t C_t^L = W_t N_t - F_t \\
\text{Risk sharing} &: \quad \left(C_t^R\right)^{\sigma} = \left(C_t^*\right)^{\sigma} \left[(1-\alpha)(S_t)^{-(1-\eta)} + \alpha \right]^{-\frac{1}{1-\eta}} \\
\text{Inflation rates} &: \quad \Pi_t = \frac{\left[(1-\alpha) + \alpha (S_t)^{1-\eta}\right]^{\frac{1}{1-\eta}}}{\left[(1-\alpha) + \alpha (S_{t-1})^{1-\eta}\right]^{\frac{1}{1-\eta}}} \Pi_{H,t} \\
\text{Union Fee} &: \quad F_t = \frac{\phi_w}{2} \left(\Pi_t^w - 1\right)^2 W_t N_t \\
\text{Wage setting} &: \quad 0 = \left(MRS_t^A\right)^{-1} \frac{W_t}{P_t} \left[(1+\tau_w)(1-e_w) - \phi_w \left(\Pi_t^w - 1\right) \Pi_t^w\right] N_t^{1+\varphi} \\
&\quad\ + e_w N_t^{1+\varphi} + \beta \left(MRS_{t+1}^A\right)^{-1} \frac{W_{t+1}}{P_{t+1}} \phi_w \left(\Pi_{t+1}^w - 1\right) \Pi_{t+1}^w N_{t+1}^{1+\varphi} \\
\text{Price setting} &: \quad \left(\Pi_{H,t} - 1\right) \Pi_{H,t} = \beta E_t \left[\frac{\left(C_{t+1}^R\right)^{-\sigma}}{\left(C_t^R\right)^{-\sigma}} \frac{g(S_t)}{g(S_{t+1})} \left(\Pi_{H,t+1} - 1\right) \Pi_{H,t+1} \right] \\
&\quad\ + \frac{e}{\phi_p} Y_t \left[\frac{(1-\tau_p)}{A_t} W_t^{\text{real}} g(S_t) - \frac{e-1}{e} \right] \\
\text{Fiscal policy} &: \quad T_t^R = \tau_w W_t N_t + \frac{\tau_p}{1-\lambda} P_{H,t} Y_t \\
&: \quad T_t^L = \tau_w W_t N_t \\
\text{Aggregate production} &: \quad Y_t = A_t N_t - \frac{\phi_p}{2} \left(\Pi_{H,t} - 1\right)^2 \\
\text{Good market clearing} &: \quad Y_t = g(S_t)^{\eta} (1-\alpha) \lambda C_t^r + g(S_t)^{\eta} (1-\alpha)(1-\lambda) C_t^R + \alpha S_t^{\gamma} C_t^*
\end{aligned}
$$

$$
\begin{aligned}
NX_t &\equiv \frac{1}{Y} \left(Y_t - \frac{P_t}{P_{H,t}} C_t \right) \\
&= \frac{1}{Y} \left(Y_t - g(S_t) C_t \right) && (54) \\
&= \frac{1}{Y} \left(\alpha S_t^{\gamma} C_t^* + g(S_t)^{\eta} (1-\alpha) \lambda C_t^r + g(S_t)^{\eta} (1-\alpha)(1-\lambda) C_t^R - g(S_t) C_t \right) && (55) \\
&= \frac{1}{Y} \left(\alpha S_t^{\gamma} C_t^* - g(S_t) C_t + g(S_t)^{\eta} (1-\alpha) C_t \right) && (56)
\end{aligned}
$$

$$\begin{aligned}
NX_t &= \frac{1}{Y}\alpha\left(S_t C_t^* - g\left(S_t\right) C_t\right) &(57)\\
&= \frac{1}{Y}\alpha g\left(S_t\right)\left(C_t^R - C_t\right) &(58)\\
&= \frac{1}{Y}\alpha g\left(S_t\right)\lambda\left(C_t^R - C_t\right) &(59)
\end{aligned}$$

E Steady State

E.1 Steady State without LAMP

The model has a unique steady state, if we consider a steady state with zero inflation, domestically as well as abroad. This implies that $F_t = 0$. For a model without liquidity constrained households, symmetry between the domestic and foreign economy implies that $S = 1$. Then our equilibrium conditions just stated in Appendix D (excluding the variables C^L and T^L and the liquidity constrained households budget constraint and tax rule fir T^L) reduces to

$$\begin{aligned}
R &= \beta^{-1}\\
C^R &= C^*\\
0 &= MRS^A W^{\text{real}}\left[\left(1+\tau_w\right)\left(1-e_w\right)\right] + e_w\\
0 &= \frac{e}{\phi_p} Y\left[\left(1-\tau_p\right) W^{\text{real}} - \frac{e-1}{e}\right]\\
T^R &= \tau_w W^{\text{real}} N + \tau_p Y\\
Y &= N\\
Y &= \left(1-\alpha\right) C^R + \alpha C^*
\end{aligned}$$

Inserting the definition $MRS_t^A \equiv (1-\lambda) MRS_t^R + \lambda MRS_t^L$ where $MRS_t^h \equiv \left(C_t^h\right)^{-\sigma} N_t^{-\varphi}$ and rearranging we have

$$
\begin{aligned}
R &= \beta^{-1} \\
Y &= \left[W^{\text{real}} (1+\tau_w) \left(1 - \frac{1}{e_w}\right)\right]^{\frac{1}{\sigma+\varphi}} \\
W^{\text{real}} &= \frac{1-\frac{1}{e}}{(1-\tau_p)} \\
T^R &= \left(\tau_w W^{\text{real}} + \tau_p\right) Y \\
Y &= N = C^R = C^*
\end{aligned}
$$

Setting the subsidies τ_p and τ_w to the value that exactly offsets the distortions from monopolistic competition, that is $\tau_p = \frac{1}{e}$ and $\tau_w = \frac{1}{e_w - 1}$, we have

$$
\begin{aligned}
R &= \beta^{-1} \\
T^R &= (\tau_w + \tau_p) \\
C^R &= C^* = N = Y = W^{\text{real}} = 1
\end{aligned}
$$

E.2 Steady State with LAMP

When the home economy has liquidity constrained households, reintroduce we must adjust the tax rates to account for the fact that there are now fewer Ricardian households who receive the dividends, and that Liquidity constrained households do not receive any dividends. To obtain the same economic outcome with adjusted taxes we must have

$$
\begin{aligned}
C^L &= (1+\tau_w) W^{\text{real}} N - T^L \\
(1-\lambda) T_t^R + \lambda T_t^L &= \tau_w N + \tau_p Y
\end{aligned}
$$

Inserting the steady state values for N, C^R, W^{real}, and Y, we have that

$$
\begin{aligned}
T^L &= \tau_w \\
T^R &= \tau_w + \frac{1}{(1-\lambda)} \tau_p.
\end{aligned}
$$

Thus we have that the following perfect-foresight steady state is consistent with the steady state equations in Appendix D.

$$
\begin{aligned}
R &= \beta^{-1} \\
T^L &= \tau_w \\
T^R &= \tau_w + \frac{1}{(1-\lambda)}\tau_p \\
C^R &= C^* = N = Y = W^{\text{real}} = 1
\end{aligned}
$$

F Log-linear version of the model

The equilibrium conditions of the model are listed for allowing the reader a greater overview. The log linearized model where lower case variables represents the logarithmic deviation of that respectable variable, that is $\widehat{x}_t \equiv \frac{X_t - X}{X}$ and inflation rates are defined as $\pi_t = p_t - p_{t-1}$.

$$
\begin{aligned}
\text{Euler equation} &: \quad \widehat{c}_t^R = E_t\left[\widehat{c}_{t+1}^R\right] - \frac{1}{\sigma}\left(\widehat{i}_t - E_t\left[\widehat{\pi}_{t+1}\right]\right) \\
\text{Risk sharing} &: \quad \widehat{c}_t^R = \widehat{c}_t^* + \frac{1-\alpha}{\sigma}\widehat{s}_t && (60) \\
\text{Disposable income} &: \quad \widehat{c}_t^L = \left(\widehat{w}_t - \widehat{p}_t + \widehat{n}_t\right) \\
\text{Inflation rates} &: \quad \widehat{\pi}_{H,t} = \widehat{\pi}_t - \alpha\Delta\widehat{s}_t \\
\text{Union Fee} &: \quad f_t = 0 && (61) \\
\text{Wage setting} &: \quad \widehat{\pi}_t^w = \beta E_t \widehat{\pi}_{t+1}^w - \frac{e_w - 1}{\phi_w}\left(\widehat{w}_t^{\text{real}} - \widehat{mrs}_t^A\right) && (62) \\
&: \quad \widehat{mrs}_t^A = \sigma\left(\left(1-\lambda\right)\ \widehat{c}_t^o + \lambda\ \widehat{c}_t^r\right) + \varphi\widehat{n}_t && (63) \\
\text{Price setting} &: \quad \widehat{\pi}_{H,t} = \beta E_t \widehat{\pi}_{H,t+1} + \frac{e-1}{\phi_p}\left(\widehat{w}_t^{\text{real}} + \alpha\widehat{s}_t - \widehat{a}_t\right) && (64) \\
\text{Fiscal policy} &: \quad \widehat{t}_t^R = \tau_w\left(\widehat{w}_t + \widehat{n}_t\right) + \frac{\tau_p}{1-\lambda}\left(\widehat{p}_{H,t} + \widehat{y}_t\right) && (65) \\
&: \quad \widehat{t}_t^L = \tau_w\left(\widehat{w}_t + \widehat{n}_t\right) && (66) \\
\text{Aggregate production} &: \quad \widehat{y}_t = \widehat{a}_t + \widehat{n}_t \\
\text{Good market clearing} &: \quad \widehat{y}_t = \gamma_c\left(1-\alpha\right)\left(\left(1-\lambda\right)\ \widehat{c}_t^o + \lambda\ \widehat{c}_t^r\right) + \alpha\gamma_c\widehat{c}_t^* \\
&\quad\; + \gamma_c\left(\eta\alpha\left(1-\alpha\right) + \alpha\gamma\right)\widehat{s}_t && (67)
\end{aligned}
$$

Note that the union membership is zero at a first order approximation, due to the assumption that $\Pi^w = 0$. We thus omit the union membership fee in our DYNARE code for the log-linear model.

Monetary policy rules

$$
\begin{aligned}
\text{DITR} &: \quad \widehat{r}_t = \varphi_\pi \widehat{\pi}_{H,t} && (68) \\
\text{CITR} &: \quad \widehat{r}_t = \varphi_\pi \widehat{\pi}_t && (69) \\
\text{PEG} &: \quad \widehat{e}_t = 0 && (70)
\end{aligned}
$$

The following are not equilibrium conditions but variables we will consider in our discussion

$$\text{Marginal cost} \quad : \quad \widehat{mc}_t = \widehat{w}_t - \widehat{p}_{H,t} - \widehat{a}_t$$

$$\text{Net exports} \quad : \quad \widehat{nx}_t = \widehat{y}_t - \widehat{c}_t - \alpha\widehat{s}_t$$

using that $g_{s,t} = \alpha\left(\frac{g(S_t)}{S_t}\right)^{\eta}$, so that $g_s = \alpha\left(\frac{g(S)}{S}\right)^{\eta} = \alpha$.

$$nx_t = \frac{1}{Y}\left(Y_t - g\left(S_t\right)C_t\right) \tag{71}$$

$$nx_t \approx \frac{1}{Y}\left(Y_t - Y\right) - \frac{1}{Y}g\left(S\right)\left(C_t - C\right) - \frac{1}{Y}g_S C\left(S_t - S\right) \tag{72}$$

$$= \widehat{y}_t - \widehat{c}_t - \alpha\widehat{s}_t \tag{73}$$

$$nx_t = \alpha\frac{1}{Y}\left(S_t^{\gamma}C_t^{*} - g\left(S_t\right)^{\eta}C_t\right) \tag{74}$$

G Optimal Policy

G.1 Social Planners Problem

The Social Planner is not subject to the optimizing behavior of the households. We solve the problem without assuming identical labor supply of the two types of households. If we assume the Social Planner places equal weight on all households, the optimization problem is

$$\max_{C_t^R, C_t^L, N_t^h, S_t, P_{H,t}} \sum_{t=0}^{\infty} \beta^t \left((1-\lambda) U\left(C_t^R, N_t^R\right) + \lambda U\left(C_t^L, N_t^L\right) \right)$$
$$+ \sum_{t=0}^{\infty} \beta^t f_{GM}\left(h^t\right) \left(\begin{array}{c} A_t \left[(1-\lambda) N_t^R + \lambda N_t^r \right] - \frac{\phi_p}{2} \left(\frac{P_{H,t}}{P_{H,t-1}} - 1 \right)^2 \\ -g_t^\eta (1-\alpha) \left[(1-\lambda) C_t^R + \lambda C_t^r \right] - \alpha S_t^\gamma C_t^* \end{array} \right)$$
$$+ \sum_{t=0}^{\infty} \beta^t f_{Fee}\left(h^t\right) \left((1-\lambda) C_t^R + \lambda C_t^r - \frac{\phi_w}{2} \left(\Pi_t^w - 1\right)^2 W_t N_t \right)$$
$$+ \sum_{t=0}^{\infty} \beta^t f_{Risk}\left(h^t\right) \left(C_t^R - C_t^* \mathbb{Q}\left(S_t\right)^{\frac{1}{\sigma}} \right)$$

where $f_{GM}\left(h^t\right)$ and $f_{Risk}\left(h^t\right)$ are the Lagrange multipliers in the state with the history of shocks h^t. We have used the auxiliary function $\mathbb{Q}_t = \frac{S_t}{g(S_t)} \equiv q\left(S_t\right) \equiv q_t$.

Letting $f_{1,t} \equiv f_1\left(h^t\right)$, the optimality conditions are

$$0 = (1-\lambda) \left(C_t^o\right)^\sigma - f_{GM,t} (1-\lambda) g_t^\eta (1-\alpha) - f_{Risk,t} \tag{75}$$
$$0 = \lambda \left(C_t^L\right)^{-\sigma} - f_{GM,t} \lambda g_t^\eta (1-\alpha) \tag{76}$$
$$0 = -(1-\lambda) \left(N_t^R\right)^\varphi + f_{GM,t} (1-\lambda) A_t \tag{77}$$
$$0 = -\lambda \left(N_t^L\right)^\varphi + f_{GM,t} \lambda A_t \tag{78}$$
$$0 = -f_{GM,t} \left[\eta g_t^{\eta-1} g_{s,t} (1-\alpha) C_t + \alpha\gamma S_t^{\gamma-1} C_t^* \right] - f_{Risk,t} \frac{1}{\sigma} q\left(S_t\right)^{\frac{1}{\sigma}} q_{s,t} C_t^* \tag{79}$$
$$0 = -f_{GM,t} \phi_p \left(\Pi_{H,t} - 1\right) \tag{80}$$

We note that $A_t f_{GM,t} = (N_t^o)^{\varphi} = (N_t^r)^{\varphi} = N_t^{\varphi}$ implying that

$$
\begin{aligned}
N_t^R &= N_t^L = N_t = (A_t f_{GM,t})^{\frac{1}{\varphi}} \\
\left(C_t^R\right)^{-\sigma} &= \left(C_t^L\right)^{-\sigma} + \frac{f_{Risk,t}}{1-\lambda} \\
&= f_{GM,t} g_t^{\eta} (1-\alpha) + \frac{f_{Risk,t}}{1-\lambda} \\
\Pi_{H,t} &= 1
\end{aligned}
$$

The shadow price of the risk sharing condition is

$$
\begin{aligned}
f_t^{\,Risk} &= -\frac{1}{q_{s,t}} f_{GM,t} \sigma q \left(S_t\right)^{-\frac{1}{\sigma}} \left[\eta g_t^{\eta-1} g_{s,t} (1-\alpha) \frac{C_t}{C_t^*} + \alpha \gamma S_t^{\gamma-1}\right] \\
&= -\frac{1}{q_{s,t}} \frac{(N_t)^{\varphi}}{A_t} \alpha \sigma q \left(S_t\right)^{-\frac{1}{\sigma}} \left[\eta g_t^{\eta-1} q_t^{-\eta} (1-\alpha) \frac{C_t}{C_t^*} + \gamma S_t^{\gamma-1}\right] \\
&= -\frac{1}{q_{s,t}} K(\bullet)
\end{aligned}
$$

where $K(\bullet) = \frac{(N_t)^{\varphi}}{A_t} \sigma q \left(S_t\right)^{-\frac{1}{\sigma}} \left[\eta g_t^{\eta-1} g_{s,t} (1-\alpha) \frac{C_t}{C_t^*} + \alpha \gamma S_t^{\gamma-1}\right] > 0$ for $\alpha \in (0,1)$, $K(\bullet) = 0$ for $\alpha = 0$, and $K(\bullet) = \frac{1}{A_t} q_t^{1-\frac{1}{\sigma}} \sigma \gamma S_t^{\gamma-1} > 0$ for $\alpha = 1$.

Recalling that $q_{s,t} = \frac{1-\alpha q_t^{1-\eta}}{g_t}$ we have that $f_t^{\,Risk} < 0$ for

$$1 > \alpha q_t^{1-\eta} \tag{81}$$

Thus, even though the labor supply of each type is identical, the two types' consumption can differ. If we assume that terms of trade are unity, we have that (81) holds, as long as there is some degree of home bias ($\alpha \in (0,1)$), thus in steady state $C^R < C^L$. Thus, for small fluctuations in the real exchange rate $C_t^R < C_t^L$, but if the degree of openness (α) is sufficiently large and the real exchange rate is appreciated sufficiently.

We normalize the PPI to one. Linking this to our marginal cost we have

$$f_{GM,t} \left(C_t^L\right)^{\sigma} = \frac{(N_t)^{\varphi} \left(C_t^L\right)^{\sigma}}{A_t} = \frac{W_t}{A_t P_t} = \frac{MC_t^{\text{real}}}{1+\tau} \equiv \frac{\Phi_t^{SP}}{g_t}$$

Thus we can conclude the following or the social planner equilibrium: 1) The two labor supplies are equal in optimum 2) the PPI-inflation is zero 3) due to the risk sharing condition the two households differ in consumption and 4) the markup is not constant.

Further we have that

$$\begin{aligned} \frac{\left(C_t^L\right)^{-\sigma}}{g_t^{\eta}\left(1-\alpha\right)} &= f_{GM,t} = \frac{\left(N_t^L\right)^{\varphi}}{A_t} \Rightarrow \\ \left(C_t^L\right)^{-\sigma}\left(N_t^L\right)^{-\varphi} &= \frac{g_t^{\eta}\left(1-\alpha\right)}{A_t} \end{aligned}$$

G.2 Ramsey Policy

G.2.1 The Ramsey Planners Problem

The Ramsey planner maximizes the following objective function

$$
\begin{aligned}
&\max_{\left\{N_t^h,,C_t^R,C_t^L,S_t,\Pi_{H,t},\Pi_t^w\right\}_{t=0}^{\infty}} \sum_{t=0}^{\infty}\beta^t\left((1-\lambda)\frac{\left(C_t^R\right)^{1-\sigma}}{1-\sigma}+\lambda\frac{\left(C_t^L\right)^{1-\sigma}}{1-\sigma}-\frac{\left(N_t^h\right)^{1+\varphi}}{1+\varphi}\right)\\
&+E_0\sum_{t=0}^{\infty}\beta^t\lambda_t^{GM}\left(\begin{array}{c} A_tN_t^h-g_t^{\eta}(1-\alpha)\left((1-\lambda)C_t^R+\lambda C_t^r\right)-G\\ -\alpha S_t^{\gamma}C_t^*-\frac{\phi_p}{2}\left(\Pi_{H,t}-1\right)^2\end{array}\right)\\
&+E_0\sum_{t=0}^{\infty}\lambda_t^{PC}\left(\begin{array}{c}\left(\Pi_{H,t}-1\right)\Pi_{H,t}-\beta E_t\left[\frac{\left(C_{t+1}^R\right)^{-\sigma}}{\left(C_t^R\right)^{-\sigma}}\frac{g(S_t)}{g(S_{t+1})}\left(\Pi_{H,t+1}-1\right)\Pi_{H,t+1}\right]\\ -\frac{e_w}{\phi_p}Y_t\left[\frac{(1-\tau_p)}{A_t}W_t^{\text{real}}g\left(S_t\right)-\frac{e-1}{e}\right]\end{array}\right)\\
&+E_0\sum_{t=0}^{\infty}\lambda_t^{WC}\left(\begin{array}{c} MRS_t^AW_t^{\text{real}}\left[\left(\Pi_t^w-1\right)\Pi_t^w-\frac{(1+\tau_w)(1-e_w)}{\phi_w}\right]N_t^{1+\varphi}\\ -\beta E_t\left[MRS_{t+1}^A\frac{W_{t+1}}{P_{t+1}}\left(\Pi_{t+1}^w-1\right)\Pi_{t+1}^wN_{t+1}^{1+\varphi}\right]-\frac{e_w}{\phi_w}N_t^{1+\varphi}\end{array}\right)\\
&+E_0\sum_{t=0}^{\infty}\beta^t\lambda_t^R\left(C_t^R-q\left(S_t\right)^{\frac{1}{\sigma}}C_t^*\right)\\
&+E_0\sum_{t=0}^{\infty}\beta^t\lambda_t^{LAMP}\left(C_t^L-W_t^{\text{real}}N_t\left(1-\frac{\phi_w}{2}\left(\Pi_t^w-1\right)^2\right)\right)
\end{aligned}
$$

We derive the Ramsey policy by using the method in Marcet and Marimon (2011) . Our two forward-looking constraints are

$$
h_{1,t}^1+E_th_{o,t+1}^1=h_{1,t}^2+E_th_{o,t+1}^2=0
$$

Where the auxiliary functions for the Phillips curve and the Unions' wage setting curve are:

$$
\begin{aligned}
&h_{1,t}^1\equiv\frac{\left(C_t^R\right)^{-\sigma}}{g(S_t)}\left(\Pi_{H,t}-1\right)\Pi_{H,t}-\frac{e}{\phi_p}Y_t\left[\frac{1}{A_t}W_t^{\text{real}}\left(C_t^R\right)^{-\sigma}-\frac{e-1}{e}\frac{\left(C_t^R\right)^{-\sigma}}{g(S_t)}\right]\\
&h_{o,t+1}^1\equiv-\frac{\left(C_{t+1}^R\right)^{-\sigma}}{g(S_{t+1})}\left(\Pi_{H,t+1}-1\right)\Pi_{H,t+1}\\
&h_{1,t}^2\equiv MRS_t^AW_t^{\text{real}}\left[\left(\Pi_t^w-1\right)\Pi_t^w-\left(1+\tau_w\right)\frac{(1-e_w)}{\phi_w}\right]N_t^{1+\varphi}-\frac{e_w}{\phi_w}N_t^{1+\varphi}\\
&h_{o,t+1}^2\equiv-MRS_{t+1}^A\frac{W_{t+1}}{P_{t+1}}\left(\Pi_{t+1}^w-1\right)\Pi_{t+1}^wN_{t+1}^{1+\varphi}.
\end{aligned}
$$

Let $\mu_{t+1}^1=\lambda_t^{PC}$ and $\mu_{t+1}^2=\lambda_t^{PW}$. Then Marcet and Marimon (2011) prove that our maximization problem is equivalent to the problem

$$
\begin{aligned}
W_t &= \inf_{\left\{\mu_t^1,\mu_t^2\right\}} \sup_{\left\{N_t^h,C_t^R,C_t^L,S_t,W_t^{\text{real}}\Pi_{H,t},\Pi_t^w\right\}_{t=0}^{\infty}} \left\{ \begin{array}{c} (1-\lambda)\frac{\left(C_t^R\right)^{1-\sigma}}{1-\sigma}+\lambda\frac{\left(C_t^L\right)^{1-\sigma}}{1-\sigma}-\frac{N^{1+\varphi}}{1+\varphi} \\ +\mu^1 h_{o,t}^1++\mu^2 h_{o,t}^2+\beta E_t W_{t+1} \end{array} \right\} \\
&+E_0\sum_{t=0}^{\infty}\beta^t\lambda_t^{GM}\left(\begin{array}{c} A_t N_t^h - g_t^{\eta}(1-\alpha)\left[(1-\lambda)C_t^R+\lambda C_t^r\right]-G \\ -\alpha S_t^{\gamma}C^* - \frac{\phi_p}{2}\left(\Pi_{H,t}-1\right)^2 \end{array}\right) \\
&+E_0\sum_{t=0}^{\infty}\lambda_t^{PC}\left(h_{1,t}^1\right)+E_0\sum_{t=0}^{\infty}\lambda_t^{WC}\left(h_{1,t}^2\right) \\
&+E_0\sum_{t=0}^{\infty}\beta^t\lambda_t^{R}\left(C_t^R-q\left(S_t\right)^{\frac{1}{\sigma}}C_t^*\right) \\
&+E_0\sum_{t=0}^{\infty}\beta^t\lambda_t^{LAMP}\left(C_t^L-W_t^{\text{real}}N_t^L+T_t\right) \\
\mu_{t+1}^1 &= \lambda_t^{PC} \quad ; \quad \mu_{t+1}^2=\lambda_t^{WC}
\end{aligned}
$$

Inserting yields

$$
\begin{aligned}
W &= \inf_{\left\{Z_t^1,Z_t^2\right\}} \sup_{\left\{N_t,C_t^R,C_t^L,S_t,W_t^{\text{real}}\Pi_{H,t},\Pi_t^w\right\}_{t=0}^{\infty}} \left\{ \begin{array}{c} (1-\lambda)\frac{\left(C_{t+1}^R\right)^{1-\sigma}}{1-\sigma}+\lambda\frac{\left(C_t^L\right)^{1-\sigma}}{1-\sigma}-\frac{N^{1+\varphi}}{1+\varphi} \\ -\mu^1\frac{\left(C_t^R\right)^{-\sigma}}{g(S_t)}\left(\Pi_{H,t}-1\right)\Pi_{H,t} \\ -\mu^2 MRS_t^A W_t^{\text{real}}\left(\Pi_t^w-1\right)\Pi_t^w N_t^{1+\varphi}+\beta E_t W\left(\right) \end{array} \right\} \\
&E_0\sum_{t=0}^{\infty}\beta^t\lambda_t^{GM}\left(\begin{array}{c} A_t N_t - g_t^{\eta}(1-\alpha)\left[(1-\lambda)C_t^R+\lambda C_t^r\right]-\alpha S_t^{\gamma}C_t^* \\ -\frac{\phi_p}{2}\left(\Pi_{H,t}-1\right)^2 \end{array}\right) \\
&+E_0\sum_{t=0}^{\infty}\lambda_t^{PC}\left(\frac{\left(\Pi_{H,t}-1\right)\Pi_{H,t}}{g\left(S_t\right)}-\frac{e_w}{\phi_p}Y_t\left[\frac{\left(1-\tau_p\right)}{A_t}W_t^{\text{real}}-\frac{e-1}{e}\frac{1}{g\left(S_t\right)}\right]\right)\left(C_t^R\right)^{-\sigma} \\
&+E_0\sum_{t=0}^{\infty}\lambda_t^{WC}\left(MRS_t^A W_t^{\text{real}}\left[\left(\Pi_t^w-1\right)\Pi_t^w+\frac{\left(1+\tau_w\right)\left(e_w-1\right)}{\phi_w}\right]N_t^{1+\varphi}-\frac{e_w}{\phi_w}N_t^{1+\varphi}\right) \\
&+E_0\sum_{t=0}^{\infty}\beta^t\lambda_t^{R}\left(C_t^R-q\left(S_t\right)^{\frac{1}{\sigma}}C_t^*\right) \\
&+E_0\sum_{t=0}^{\infty}\beta^t\lambda_t^{LAMP}\left(C_t^L-W_t^{\text{real}}N_t\left(1-\frac{\phi_w}{2}\left(\Pi_t^w-1\right)^2\right)\right) \\
&+E_0\sum_{t=0}^{\infty}\beta^t\lambda_t^{RW}\left(\Pi_t^w-\frac{W_t^{\text{real}}g\left(S_t\right)}{W_{t-1}^{\text{real}}g\left(S_{t-1}\right)}\Pi_{H,t}\right) \\
\mu_{t+1}^1 &= \lambda_t^{PC} \quad ; \quad \mu_{t+1}^2=\lambda_t^{WC}
\end{aligned}
$$

G.2.2 First order conditions

The optimality condition of the reformulated Ramsey planners problem for $C_t^L, C_t^R, N_t, S_t, W_t^{\text{real}}, \Pi_{H,t}$, and Π_t^w are

$$\begin{aligned}
0 \;=\;& (1-\lambda)\left(C_{t+1}^R\right)^{-\sigma} - \lambda_t^{GM} g_t^{\eta}(1-\alpha)(1-\lambda) \\
& - \left(\lambda_t^{PC} - \lambda_{t-1}^{PC}\right)\sigma \frac{\left(C_t^R\right)^{-\sigma-1}}{g(S_t)}\left(\Pi_{H,t}-1\right)\Pi_{H,t} \\
& + \sigma \lambda_t^{PC}\frac{e-1}{\phi_p}\left(C_t^R\right)^{-\sigma-1} N_t\left[W_t^{\text{real}} - \frac{A_t}{g(S_t)}\right] \\
& - \left(\lambda_t^{WC} - \lambda_{t-1}^{WC}\right)\sigma(1-\lambda)\left(C_t^R\right)^{-\sigma-1} N_t W_t^{\text{real}}\left(\Pi_t^w - 1\right)\Pi_t^w \\
& - \lambda_t^{WC}\frac{e_w}{\phi_w}\sigma(1-\lambda)\left(C_t^R\right)^{-\sigma-1} N_t W_t^{\text{real}} + \lambda_t^R
\end{aligned}$$

$$\begin{aligned}
0 \;=\;& \lambda\left(C_t^L\right)^{-\sigma} - \lambda_t^{GM} g_t^{\eta}(1-\alpha)\lambda \\
& - \left(\lambda_t^{WC} - \lambda_{t-1}^{WC}\right)\sigma\lambda\left(C_t^L\right)^{-\sigma-1} N_t W_t^{\text{real}}\left[\left(\Pi_t^w - 1\right)\Pi_t^w\right] \\
& - \lambda_t^{WC}\sigma\lambda\frac{e_w}{\phi_w}\left(C_t^L\right)^{-\sigma-1} N_t W_t^{\text{real}} + \lambda_t^{LAMP}
\end{aligned}$$

$$\begin{aligned}
0 = N_t^{\varphi} \quad & + \lambda_t^{GM} A_t - \lambda_t^{PC}\frac{e-1}{\phi_p}\left[W_t^{\text{real}} - \frac{A_t}{g(S_t)}\right]\left(C_t^R\right)^{-\sigma} \\
& + \left(\lambda_t^{WC} - \lambda_{t-1}^{WC}\right) MRS_t^A N_t^{\varphi} W_t^{\text{real}}\left(\Pi_t^w - 1\right)\Pi_t^w \\
& + \lambda_t^{WC}\frac{e_w}{\phi_w}\left(MRS_t^A W_t^{\text{real}} - (1+\varphi)\right) N_t^{\varphi} - \lambda_t^{RoT} W_t^{\text{real}}\left(1 + \frac{\phi_w}{2}\left(\Pi_t^w - 1\right)^2\right)
\end{aligned}$$

$$\begin{aligned}
0 \;=\;& -\lambda_t^{GM}\left(\eta g_t^{\eta-1} g_{s,t}(1-\alpha) C_t + \alpha\gamma C_t^* S_t^{\gamma-1}\right) - \lambda_t^{PC}\frac{e_p-1}{\phi_p} Y_t \frac{g_{s,t}}{g(S_t)^2}\left(C_t^R\right)^{-\sigma} \\
& - \left(\lambda_t^{PC} - \lambda_{t-1}^{PC}\right)\frac{\left(C_t^R\right)^{-\sigma}}{g(S_t)^2} g_{s,t}\left(\Pi_{H,t}-1\right)\Pi_{H,t} \\
& - \lambda_t^R \frac{1}{\sigma} q(S_t)^{\frac{1}{\sigma}-1} q_{s,t} C_t^* - \left(\lambda_t^{RW}\Pi_t^w - \beta\lambda_{t+1}^{RW}\Pi_{t+1}^w\right)\frac{g_{s,t}}{g(S_t)}
\end{aligned}$$

$$
\begin{aligned}
0 &= -\lambda_t^{PC}\frac{e-1}{\phi_p}N_t\left(C_t^R\right)^{-\sigma}+\lambda_t^{WC}\frac{e_w}{\phi_w}MRS_t^A N_t^{1+\varphi} \\
&\quad+\left(\lambda_t^{WC}-\lambda_{t-1}^{WC}\right)MRS_t^A\left(\Pi_t^w-1\right)\Pi_t^w N_t^{1+\varphi} \\
&\quad-\lambda_t^{RoT}N_t\left(1+\frac{\phi_w}{2}\left(\Pi_t^w-1\right)^2\right)-\frac{\lambda_t^{RW}\Pi_t^w-\beta\lambda_{t+1}^{RW}\Pi_{t+1}^w}{W_t^{\text{real}}}
\end{aligned}
$$

$$
\lambda_t^{GM}\phi_p\left(\Pi_{H,t}-1\right)=\left(\lambda_t^{PC}-\lambda_{t-1}^{PC}\right)\frac{\left(C_t^R\right)^{-\sigma}}{g\left(S_t\right)}\left(2\Pi_{H,t}-1\right)-\lambda_t^{RW}\frac{\Pi_t^w}{\Pi_{H,t}}
$$

$$
\begin{aligned}
0 &= \left(\lambda_t^{WC}-\lambda_{t-1}^{WC}\right)MRS_t^A W_t^{\text{real}}\left(2\Pi_t^w-1\right)N_t^{1+\varphi} \\
&\quad+\lambda_t^{LAMP}W_t^{\text{real}}N_t\phi_w\left(\Pi_t^w-1\right)+\lambda_t^{RW}
\end{aligned}
$$

G.2.3 Steady State value of Lagrange Multipliers

$$
\begin{aligned}
0 &= C^{-\sigma}-\lambda^{GM}(1-\alpha)-\lambda^{WC}\frac{e_w}{\phi_w}\sigma C^{-\sigma-1}N+\frac{\lambda^R}{(1-\lambda)} \\
0 &= C^{-\sigma}-\lambda^{GM}(1-\alpha)-\lambda^{WC}\sigma\frac{e_w}{\phi_w}C^{-\sigma-1}N+\frac{\lambda^{LAMP}}{\lambda} \\
0 &= N^{\varphi}+\lambda^{GM}-\lambda^{WC}\varphi\frac{e_w}{\phi_w}N_t^{\varphi}-\lambda^{LAMP} \\
0 &= -\lambda_t^{GM}\left(\eta\alpha(1-\alpha)+\alpha\gamma\right)C-\lambda^{PC}\alpha\frac{e-1}{\phi_p}YC^{-\sigma-1} \\
&\quad-\lambda^R\frac{1-\alpha}{\sigma}-(1-\beta)\lambda^{RW}\Pi^w \\
0 &= -\lambda^{PC}\frac{e-1}{\phi_p}+\lambda^{WC}\frac{e_w}{\phi_w}-\lambda^{LAMP}C^{\sigma} \\
0 &= \lambda^{GM}\phi_p\left(\Pi_H-1\right)+\lambda^{RW} \\
0 &= \lambda^{LAMP}W^{\text{real}}N\phi_w\left(\Pi^w-1\right)+\lambda^{RW}
\end{aligned}
$$

We show that the optimal policy implies zero price and wage inflation in the steady state; as the inefficiency from monopolistic competition is offset by the subsidies τ_p and τ_w there is no need for the Ramsey Planner create surprise inflation at zero inflation.

In the zero inflation steady state we have

$$
\begin{aligned}
0 &= C^{-\sigma}-\lambda^{GM}(1-\alpha)-\lambda^{WC}\frac{e_w}{\phi_w}\sigma C^{-\sigma-1}N+\frac{\lambda^{R}}{(1-\lambda)} \\
0 &= C^{-\sigma}-\lambda^{GM}(1-\alpha)-\lambda^{WC}\sigma\frac{e_w}{\phi_w}C^{-\sigma-1}N+\frac{\lambda^{LAMP}}{\lambda} \\
0 &= N^{\varphi}+\lambda^{GM}-\lambda^{WC}\varphi\frac{e_w}{\phi_w}N_t^{\varphi}-\lambda^{LAMP} \\
0 &= -\lambda_t^{GM}(\eta\alpha(1-\alpha)+\alpha\gamma)C-\lambda^{PC}\alpha\frac{e-1}{\phi_p}YC^{-\sigma-1}-\lambda^{R}\frac{1-\alpha}{\sigma} \\
0 &= -\lambda^{PC}\frac{e-1}{\phi_p}+\lambda^{WC}\frac{e_w}{\phi_w}-\lambda^{LAMP}C^{\sigma} \\
0 &= \lambda^{RW}
\end{aligned}
$$

as $\Pi^{w}=\Pi_H$ in steady state

G.2.4 Log-linearizing the FOC's of the Ramsey Planner

The log-linearized first order conditions for $C_t^R, C_t^L, N_t^h, S_t, W_t^{\text{real}}, \Pi_{H,t}$ and Π_t^w are respectively

$$
\begin{aligned}
0 \;=\; & -\sigma(1-\lambda)C^{-\sigma}\left[1-\lambda^{WC}(1+\sigma)\frac{e_w}{\phi_w}C^{-1}N\right]\widehat{c}_t^{R} \\
& +\left[\lambda^{PC}\sigma\frac{e-1}{\phi_p}C^{-\sigma-1}N-\eta\lambda^{GM}(1-\alpha)(1-\lambda)\right]g_s\widehat{s}_t \\
& -\lambda^{WC}\frac{e_w}{\phi_w}\sigma(1-\lambda)C^{-\sigma-1}N\widehat{n}_t \\
& +\sigma\left[\lambda^{PC}\frac{e-1}{\phi_p}-\lambda^{WC}\frac{e_w}{\phi_w}(1-\lambda)\right]C^{-\sigma-1}N\widehat{w}_t^{\text{real}} \\
& -\sigma\lambda_t^{PC}\frac{e-1}{\phi_p}C^{-\sigma-1}N\widehat{a}_t \\
& -(1-\alpha)(1-\lambda)\lambda^{GM}\widehat{\lambda_t^{GM}}-\frac{e_w}{\phi_w}\sigma(1-\lambda)C^{-\sigma-1}N\lambda^{WC}\widehat{\lambda_t^{WC}}+\lambda^{R}\widehat{\lambda^{R}}
\end{aligned}
$$

$$
\begin{aligned}
0 &= -\sigma\lambda C^{-\sigma}\left[1-\lambda_t^{WC}\left(\sigma+1\right)\frac{e_w}{\phi_w}C^{-1}N\right]\widehat{c}_t^{L} \\
&\quad -\eta\lambda^{GM}\left(1-\alpha\right)\lambda g_s\widehat{s}_t \\
&\quad -\lambda^{WC}\sigma\lambda\frac{e_w}{\phi_w}C^{-\sigma-1}N\left(\widehat{n}_t+\widehat{w}_t^{\mathrm{real}}\right) \\
&\quad -\lambda\left(1-\alpha\right)\lambda^{GM}\widehat{\lambda_t^{GM}}-\sigma\lambda\frac{e_w}{\phi_w}C^{-\sigma-1}N\lambda^{WC}\widehat{\lambda_t^{WC}}+\lambda^{LAMP}\widehat{\lambda_t^{LAMP}}
\end{aligned}
$$

$$
\begin{aligned}
0 &= \left[1-\lambda^{WC}\frac{e_w}{\phi_w}\varphi\right]\varphi N^{\varphi}\widehat{n}_t \\
&\quad -\left[\lambda^{PC}\frac{e-1}{\phi_p}C^{-\sigma}-\lambda^{WC}\frac{e_w}{\phi_w}N^{\varphi}+\lambda^{LAMP}\right]\widehat{w}_t^{\mathrm{real}} \\
&\quad +\left[\lambda^{GM}+\lambda^{PC}\frac{e-1}{\phi_p}C^{-\sigma}\right]\widehat{a}_t \\
&\quad -\lambda_t^{PC}\frac{e-1}{\phi_p}C^{-\sigma}g_s\widehat{s}_t+\lambda^{WC}\frac{e_w}{\phi_w}N^{\varphi}\widehat{mrs}_t^{A} \\
&\quad +\lambda^{GM}\widehat{\lambda_t^{GM}}-\frac{e_w}{\phi_w}\varphi N^{\varphi}\lambda^{WC}\widehat{\lambda_t^{WC}}-\lambda^{LAMP}\widehat{\lambda_t^{LAMP}}
\end{aligned}
$$

$$
\begin{aligned}
0 &= -\lambda_t^{GM}\eta g_s\left(1-\alpha\right)\left(1-\lambda\right)C\widehat{c}_t^{R}+\lambda_t^{PC}\sigma\frac{e-1}{\phi_p}g_sNC^{-\sigma}\widehat{c}_t^{R} \\
&\quad -\lambda^{GM}\eta g_s\left(1-\alpha\right)\lambda C\widehat{c}_t^{r} \\
&\quad -\lambda^{GM}\left[\left(1-\alpha\right)\eta\left(\left(\eta-1\right)g_s^2+g_{ss}\right)+\alpha\gamma\left(\gamma-1\right)\right]C\widehat{s}_t \\
&\quad +\lambda^{PC}\frac{e-1}{\phi_p}N\left[2g_s^2-g_{ss}\right]C^{-\sigma}\widehat{s}_t \\
&\quad -\lambda^{R}\frac{1}{\sigma}\left(\left(\frac{1}{\sigma}-1\right)q_s^2+q_{ss}\right)C^{*}\widehat{s}_t \\
&\quad -\lambda^{PC}\frac{e-1}{\phi_p}NC^{-\sigma}\left(\widehat{n}_t+\widehat{a}_t\right) \\
&\quad -\left(\eta g_s\left(1-\alpha\right)+\alpha\gamma\right)C\lambda^{GM}\widehat{\lambda_t^{GM}} \\
&\quad -\frac{e-1}{\phi_p}g_sC^{-\sigma}N\lambda^{PC}\widehat{\lambda_t^{PC}} \\
&\quad -\frac{1}{\sigma}q_sC\lambda^{R}\widehat{\lambda_t^{R}}-g_{s,t}\left(\lambda_t^{RW}+\beta\lambda_{t+1}^{RW}\right)
\end{aligned}
$$

$$
\begin{aligned}
0 \;=\;& \lambda_t^{PC}\frac{e-1}{\phi_p}\sigma N C^{-\sigma}\widehat{c}_t^{R} \\
& +\left[\lambda_t^{WC}\frac{e_w}{\phi_w}\left(1+\varphi\right)N^{1+\varphi}-\lambda_t^{PC}\frac{e-1}{\phi_p}NC^{-\sigma}-\lambda^{LAMP}N\right]\widehat{n}_t \\
& +\lambda_t^{WC}\frac{e_w}{\phi_w}N_t^{1+\varphi}\widehat{mrs}_t^{A} \\
& -\lambda^{PC}\frac{e-1}{\phi_p}NC^{-\sigma}\widehat{\lambda_t^{PC}}+\lambda^{WC}\frac{e_w}{\phi_w}N^{1+\varphi}\widehat{\lambda_t^{WC}} \\
& -\lambda^{LAMP}N\widehat{\lambda_t^{LAMP}}-\lambda_t^{RW}+\beta\lambda_{t+1}^{RW}
\end{aligned}
$$

$$
\lambda^{GM}\phi_p\widehat{\pi}_{H,t}=C^{-\sigma}\lambda^{PC}\left(\widehat{\lambda_t^{PC}}-\widehat{\lambda_{t-1}^{PC}}\right)-\lambda_t^{RW}
$$

$$
0=\lambda^{LAMP}N\phi_w\widehat{\pi}_t^{w}+\lambda^{WC}\left(\widehat{\lambda_t^{WC}}-\widehat{\lambda_{t-1}^{WC}}\right)N^{1+\varphi}+\lambda_t^{RW}
$$

where $g_s=\alpha$, $g_{ss}=-\alpha\eta\left(1-\alpha\right)$, $q_s=1-a$, $q_{ss}=-1-\alpha\left(\alpha\left(2-\eta\right)+\left(1-\eta\right)\right)$.

G.3 Welfare Comparisons

I compute the consumption-equivalent welfare loss of each Policy. This is done for each type of household and for an aggregated household, consistent with an ex ante expectation before the household type is revealed. The consumption-equivalent welfare loss of deviating from the Ramsey policy, μ^h, as satisfying the equation

$$V_o^h = E_0 \sum_{t=0}^{\infty} \beta^t U\left(C_t^h, N_t^h\right) = E_0 \sum_{t=0}^{\infty} \beta^t U\left(\left(1-\mu^h\right) C, N\right)$$

where $h \in \{o; r; b\}$ corresponding to a Ricardian optimizing household, a rule-of-thumb household or a weighted average of both types.

G.3.1 Log-utility

If we assume log-utility, the equation reduces to

$$\begin{aligned} V_o^h &= \frac{1}{1-\beta}\left[\log\left(\left(1-\mu^h\right) C\right) - v\left(N\right)\right] = \frac{1}{1-\beta}\left[\log\left(1-\mu^h\right) + \log\left(C\right) - v\left(N\right)\right] \\ &= \frac{1}{1-\beta}\log\left(1-\mu^h\right) + V^{SS} \end{aligned}$$

Which gives us the closed form solution

$$\mu^h = 1 - \exp\left[-\left(1-\beta\right)\left(V^{SS} - V_o^h\right)\right]$$

G.3.2 CRRA Utility

Assuming that our consumption risk aversion is not unity, we have that the consumption share must satisfy

$$V_o^h = \frac{1}{1-\beta}\left\{\left(1-\mu^h\right)^{1-\sigma} v\left(C\right) - v\left(N\right)\right\}$$

$$\left[\frac{\left(1-\beta\right) V_o^h + v\left(N\right)}{v\left(C\right)}\right]^{\frac{1}{1-\sigma}} = 1-\mu^h$$

Which gives us the closed form solution

$$\mu^h = 1 - \left[\frac{\left(1-\beta\right) V_o^h + v\left(N\right)}{\frac{C^{1-\sigma}}{1-\sigma}}\right]^{\frac{1}{1-\sigma}}$$

Government Spending in a Small Open Economy with Liquidity Constrained Households*

Gitte Yding Salmansen

August 29, 2014

Abstract

In this paper we analyze the effects of introducing liquidity constrained households in a DSGE model for a small open economy under three different monetary policy regimes. We consider a government spending shock and show that the presence of liquidity constrained households only has a very small effect on the output multiplier under all three regimes. Thus their potentially large effect in a closed economy does not carry through to the open economy. In fact, the output multiplier is reduced under consumer and producer price inflation-based Taylor rules, while it is marginally increased under a fixed exchange rate regime.

*The author wishes to thank Henrik Jensen, Mark Strøm Kristoffersen, Jesper Pedersen and Søren Hove Ravn for useful comments and discussions. Also thanks to seminar participants at University of Copenhagen, Danmarks Nationalbank, and the 2013 Jamboree at Cambridge University. All remaining errors are my own. The views expressed are those of the author, and do not necessarily correspond to those of Danmarks Nationalbank.

1 Introduction

The effect of fiscal stimulus is at the core of macroeconomic theory. The massive fiscal stimulus packages following the credit crisis of 2008 has revived a heated debate on the output effects of fiscal stimulus (see Hall (2009) and Cogan et al. (2010) for a survey). One of the key components of output is private consumption, thus understanding households' consumption behavior is critical for analyzing the output effects of fiscal stimulus.

This paper analyzes the effects of government spending. In the literature, the debate on the effect of a government spending shock is still open. On one side, Blanchard and Perotti (2002) use a structural VAR model and find that a government spending shock crowds in private consumption. On the other side, using a narrative approach, Ramey and Shapiro (1998) and Ramey (2011) finds a negative response of consumption to a government spending shock.

Adding to this debate, Galí et al. (2007) performs an SVAR analysis of US data and finds crowding in of government spending. They proceed to introduce households that consume their entire disposable income every period in a New Keynesian model for a closed economy. These households cause a debt-financed government spending shock to generate a rise in aggregate consumption, and generate a nice fit with their data. The finding of Galí et al. (2007) is now celebrated as a well established result: The introduction of consumer that consume their disposable income every period (in a closed economy), will cause the output effects of a debt-financed government spending shock to be higher on impact, as these households will disregard the negative wealth effect of the financing of these expenditures.

Our paper investigate the robustness of these results in an open economy. We assume that a fraction of households do not have access to asset markets, and therefore consume their entire disposable income each period as in Galí et al. (2007). We will refer to these as liquidity constrained households, and the presence of these is referred to as limited asset market participation (LAMP).[1]

We investigate the robustness of the positive effect of LAMP on the government spending multiplier, by introducing LAMP in a DSGE model for a small open economy. More specifically, we augment Galí and Monacelli (2005) with a fiscal sector and liquidity

[1]These households have also been referred to as hand-to-mouth consumers, rule-of-thumb consumers and non-asset holders.

constrained households. We show that the presence of LAMP raises the response of private consumption to a positive government spending shock under both fixed and floating exchange rate policies. However, due to diverse exchange rate and net export effects, the introduction of LAMP has ambiguous effects on output. While LAMP increase the output multiplier under a fixed nominal exchange policy, government spending's effect on output is reduced on impact when LAMP is introduced in a floating exchange rate regime. The fact that LAMP can reduce the output response to a government spending shock is a new insight, to the best of our knowledge.

The intuition for our results are as follows. The introduction of liquidity constrained households, who increase consumption immediately due to higher disposable income, will increase aggregate household consumption. Under a floating exchange rate, these households cause a stronger initial nominal appreciation of the domestic currency. This exchange rate effect of LAMP causes net export to drop more aggressively, and this dominates the consumption rise, causing the impact response of output to be lower under a flexible exchange rate regime. When the exchange rate is fixed net exports will drop less and the consumption increase will dominate, so that the output response on impact is increased by the introduction of LAMP. We show that by calculating net present value (NPV) multipliers as suggested by Uhlig (2010), LAMP will unambiguously increase the NPV output multipliers in all monetary policy regimes considered.

We find that the effects of openness on the size of fiscal multipliers depends on the monetary policy regime. Under a Taylor rule where the interest rate responds to the domestic producer price inflation, and under a fixed nominal exchange rate, more openness will decrease the response of output to a government spending shock. If the monetary policy is a Taylor rule based on consumer price inflation, the openness will instead increase the multiplier. The latter effect occurs because more openness also increases the weights of foreign goods in the consumer price index. Openness thus reduces the interest rate path and the terms of trade contraction, causing a higher consumption and thus a higher output effect of government spending. This highlights that the monetary policy in place is crucial, when discussing the effect of openness on fiscal multipliers.

The paper is organized as follows. In Section 2 we briefly discuss the empirical evidence regarding LAMP and open economy government spending multipliers respectively. In Section 3 we set up the model, and the parametrization is presented in Section 4. In

Section 5 we analyze the effects of LAMP on the propagation of a government spending shock under different monetary policy regimes. In Section 6 we investigate the robustness of our results, and Section 7 concludes.

2 Overview of Empirical Evidence

2.1 Limited Asset Market Participation

The empirical investigation of LAMP goes back to Campbell and Mankiw (1989, 1990). The authors assume that a share λ of income is spent completely on consumption, and the remaining share $(1 - \lambda)$ is consumed in accordance with the permanent income hypothesis. Using various estimation techniques, they conclude that λ is approximately 0.5 in US data. Muscatelli et al. (2004) estimate $\lambda = 0.37$ using general method of moments on US data.[2]

Coenen and Straub (2005) extend the Smets and Wouters (2003) model of the Euro Area with LAMP. They find that the posterior mean of λ is 0.25 under lump sum taxes (0.37 when including disortionary taxes). LAMP incases the level of consumption in response to a government spending shock, but the aggregate consumption response is still not very likely to become positive. Ratto et al. (2009) estimate an open economy model of the Euro Area and find that the posterior mean of λ is 0.35. Forni et al. (2009) find that the posterior mean of λ is 0.34 (0.37 with labor unions) in their Euro Area model. For the G7 countries Bartolomeo and Rossi (2007) estimate $\lambda = 0.26$.[3] Thus Euro Area and G7 data suggest a share of liquidity constrained households in the range 0.25-0.40.

Finally, using Danish household data Chetty et al. (2014) find that the degree of LAMP is 0.85, considerably higher than the findings for macro data.

2.2 Fiscal Multipliers in Small Open Economies

Beetsma and Giuliodori (2011) use SVAR methods for the 14 EU member countries and find that an increase in government spending raises output, consumption and investment,

[2] Muscatelli et al. (2004) and Ratto et al. (2009) have habits in comsumption.

[3] This covers a variety of country specific estimates ranging from about 0.09 for Japan and Italy to roughly 0.4 for France and UK.

but that this stimulating effect is decreasing in the openness of the economy.

Ilzetzki et al. (2013) use a dataset for 44 countries and find that open economies have an impact multiplier on GDP of -0.07 and a long run multiplier of -0.46.[4] For a closed economy these numbers are 0.61 and 1.1. Thus, in their sample openness reduces the output effect of government spending. Further, a country with a fixed exchange rate have impact multipliers of 0.15 and long run multipliers of 1.4, which is much higher than the multipliers under a floating exchange rate: -0.14 and -0.69. All are statistically different from zero.

Corsetti et al. (2012a) use a panel of 17 OECD countries and find that government spending shocks imply that output effects under an exchange rate peg are (significantly) positive, while they are statistically insignificant under a flexible exchange rate regime.[5] In contrast, the impact response of consumption does not change significantly. Unlike Ilzetzki et al. (2013), they find that the monetary policy is less accommodating and that the exchange rate appreciates under an exchange rate peg.

Ravn and Spange (2014) use an SVAR analysis of Danish data and find that a government spending shock has a relatively large effect on GDP on impact (the multiplier is 1.1) but after a year the impulse response becomes insignificant. They also find a borderline significant drop in consumption on impact. Denmark has a fixed exchange rate against the Euro, so the numbers are consistent with the findings in Ilzetzki et al. (2013).

Nakamura and Steinsson (2014) use US regions to estimate the effects of government spending in a monetary region and find an "open economy relative multiplier" of roughly 1.5.[6] They also find that while a closed economy aggregate multiplier is very sensitive to the degree by which tax and monetary policy lean against the wind, the open economy relative multiplier is not sensitive to these, when different regions share the tax burden.

García and Restrepo (2007) is very similar to our model, but the paper focuses on government budget rules, and they have a role for distortionary taxation. Monetary policy is a Taylor rule with consumer price inflation and output. They find that the instantaneous output multiplier of government spending is always positive, except under a balanced government budget rule. Further, they find that the consumption multiplier

[4] Open economies are defined as having the sum of imports and exports exceeding 60 percent of GDP.

[5] They use two-stage estimation. Government spending shocks are defined as the residuals of a estimated country-specific government spending rule.

[6] The authors define "open economy relative multiplier" as the effect that government spending in one region relative to another region has on relative output.

of government spending is increasing in the share of liquidity constrained households.

3 Model

Our model builds on Galí and Monacelli (2005) augmented with a nominal wage friction and liquidity constrained households as in Galí et al. (2007). Further we introduce government spending, which can be (partially) debt-financed.[7] We will consider a continuum of small open economies. As each economy is of measure zero there will not be a strategic element to fiscal and monetary policy, as a given economy's policy-choices will not have an impact on the rest of the economies. We will focus on one country, the home economy, which will be identical to all other countries except that it will be the only economy with LAMP.[8]

3.1 Households

The household sector consists of two types of households. A share $1 - \lambda$ have perfect access to asset markets and behave as Ricardian (R) households performing intertemporal optimization. The remaining share λ can neither save nor lend and thus consume their entire disposable income every period. These will be referred to as liquidity constrained (L) households.

The in-period utility function and thus the optimal weighting of consumption bundles will be identical for the two types of households. We therefore start with some properties that hold for both types of households, before we proceed with the individual characterization of the two types. Let $h \in \{R, L\}$ be the type of a given household.

All households can be represented by an optimizing agent with an infinite horizon, each maximizes its expected sum of discounted utilities of feasible future paths of consumption and labor supply

$$E_0 \sum_{t=0}^{\infty} \beta^t U\left(C_t^h, N_t^h\right), \tag{1}$$

[7] A similar model is used in Salmansen (2014b), but that paper does not have government spending or the possibility of government debt financing.

[8] Due to our tax rules, all households have the same level of steady state consumption so foreign LAMP will not affect the steady state. The home economy does not affect foreign economies, so introducing LAMP in foreign economies will not affect our results.

where N_t^h is the number of hours worked and C_t^h is the composite consumption index

$$C_t^h = \left[(1-\alpha)^{\frac{1}{\eta}} \left(C_{H,t}^h \right)^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} \left(C_{F,t}^h \right)^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}}. \tag{2}$$

The index consists of a bundle of domestically produced goods $C_{H,t}^h$ and a bundle of imported goods $C_{F,t}^h$. The parameter $\eta > 0$ is the degree of substitutability between foreign and domestic goods, given the utility weight of imported goods, $\alpha \in [0,1]$.

The index of domestic consumption is of the CES form

$$C_{H,t}^h \equiv \left(\int_0^1 C_{H,t}^h (j)^{\frac{\varepsilon-1}{\varepsilon}} dj \right)^{\frac{\varepsilon}{\varepsilon-1}}, \tag{3}$$

where $j \in [0,1]$ is the type of good and $\varepsilon > 1$ is the elasticity of substitution between domestic goods.[9]

Foreign consumption is an aggregate of imports from all foreign countries

$$C_{F,t}^h \equiv \left(\int_0^1 \left(C_{i,t}^h \right)^{\frac{\gamma-1}{\gamma}} di \right)^{\frac{\gamma}{\gamma-1}} \quad ; \quad C_{i,t}^h \equiv \left(\int_0^1 C_{i,t}^m (j)^{\frac{\varepsilon-1}{\varepsilon}} dj \right)^{\frac{\varepsilon}{\varepsilon-1}}, \tag{4}$$

where the parameter γ measures the substitutability between goods from different countries. The imports from a given country i, $C_{i,t}^h$, has the same elasticity of substitution between these imported goods as domestic goods, ε.

Optimal allocation of expenditures within each category yields

$$C_{H,t}^h (j) = \left(\frac{P_{H,t}(j)}{P_{H,t}} \right)^{-\varepsilon} C_{H,t}^h \quad ; \quad C_{i,t}^h (j) = \left(\frac{P_{i,t}(j)}{P_{i,t}} \right)^{-\varepsilon} C_{i,t}^h \tag{5}$$

for all $i, j \in [0,1]$, where $P_{H,t} = \left(\int_0^1 P_{H,t}(j)^{1-\varepsilon} dj \right)^{\frac{1}{1-\varepsilon}}$ is the domestic price index and $P_{i,t} = \left(\int_0^1 P_{i,t}(j)^{1-\varepsilon} dj \right)^{\frac{1}{1-\varepsilon}}$ is the price index for the bundle of goods imported from country i, but denominated in the domestic currency. Here, $P_{i,t}(j)$ is the price of good j in country i, denoted in our home country's currency.

The import share of goods from country i is

$$C_{i,t}^h = \left(\frac{P_{i,t}}{P_{F,t}} \right)^{-\gamma} C_{F,t}^h \tag{6}$$

[9] When we specify the production side, each country has a continuum of firms producing a differentiated good, indexed by $j \in [0,1]$.

for all $i, j \in [0,1]$, where $P_{F,t} \equiv \left(\int_0^1 P_{i,t}^{1-\gamma} di \right)^{\frac{1}{1-\gamma}}$ is the price index for imported goods in our domestic currency. As all price indices are consistent with the demand weights, the total expenditures on imported goods satisfies $\int_0^1 P_{i,t} C_{i,t}^m di = P_{F,t} C_{F,t}^m$.

The optimal consumption shares for domestic and foreign goods respectively are

$$C_{H,t}^h = (1-\alpha) \left(\frac{P_{H,t}}{P_t} \right)^{-\eta} C_t^h \quad ; \quad C_{F,t}^h = \alpha \left(\frac{P_{F,t}}{P_t} \right)^{-\eta} C_t^h, \tag{7}$$

where $P_t = \left[(1-\alpha)(P_{H,t})^{1-\eta} + \alpha (P_{F,t})^{1-\eta} \right]^{\frac{1}{1-\eta}}$ is the consumer price index (CPI). Thus the total consumption expenditures are $P_{H,t} C_{H,t}^h + P_{F,t} C_{F,t}^h = P_t C_t^h$.[10] Equation (7) shows that α will serve as a measure of openness in our model: Given the infinitesimal weight of the home economy, any $\alpha \neq 1$ reflects home bias in the households' preferences, and $(1-\alpha) \in [0,1]$ determines the degree of home bias.

Aggregate consumption is given by

$$C_t \equiv (1-\lambda) C_t^R + \lambda C_t^L. \tag{8}$$

All households have isoelastic preferences, and the instantaneous utility is given by

$$U(C,N) = \frac{C^{1-\sigma}}{1-\sigma} - \frac{N^{1+\varphi}}{1+\varphi}, \tag{9}$$

where σ is the coefficient of relative risk aversion for consumption and $\varphi > 0$ is the inverse of the Frisch labor supply elasticity.

3.1.1 Ricardian Households

The Ricardian households are optimizing agents with an infinite horizon and unlimited access to asset markets. Under the optimal consumption weights in (5) and (6), they face the following budget constraint

$$P_t C_t^R + E_t \left[Q_{t,t+1} D_{t+1} \right] \leq D_t + (1+\tau_w) W_t N_t^R - P_{H,t} T_t^R - F_t \tag{10}$$

[10] The price indices ensure the aggregate multiplicative property that $\int_0^1 P_{H,t}(j) C_{H,t}^m(j) dj + \int_0^1 \int_0^1 P_{i,t}(j) C_{i,t}^m(j) dj di = P_t C_t^m$ under optimal consumption.

where D_{t+1} is the nominal pay-off in period $t+1$ of the market portfolio (including shares in intermediate firms) held at the end of period t. $Q_{t,t+1}$ is a stochastic discount factor for one period ahead nominal payoffs relevant to all participants in asset markets. τ_w is a pay-roll subsidy, T_t^R is real (in PPI indexation) lump sum taxes paid by Ricardian households, and F_t is a union fee.

All households delegate the wage negotiation to unions, and are required to meet the labor demand at the given wage. Therefore, households must choose between consumption and savings.

The access to a complete set of internationally traded contingency claims implies the following intertemporal optimality conditions for the Ricardian households

$$\beta \left(\frac{C_{t+1}^R}{C_t^R} \right)^{-\sigma} \left(\frac{P_t}{P_{t+1}} \right) = Q_{t,t+1}. \tag{11}$$

Taking conditional expectations yields the Euler equation.[11]

$$\beta R_t E_t \left[\left(\frac{C_{t+1}^R}{C_t^R} \right)^{-\sigma} \left(\frac{P_t}{P_{t+1}} \right) \right] = 1,$$

where $R_t = E_t \left[Q_{t,t+1} \right]^{-1}$ is the gross nominal return on a riskfree one-period bond paying off one unit of domestic currency a time $t+1$, and trading at the price $E_t \left[Q_{t,t+1} \right]$.

3.1.2 Non-Ricardian Households

The liquidity constrained households consume their entire disposable income according to the budget constraint

$$P_t C_t^L = (1 + \tau_w) W_t N_t^L - T^L - \int_0^1 F_t(u) \, du. \tag{12}$$

Note that we allow the lump sum tax for the liquidity constrained households T_t^L to differ from that of the Ricardian households.

[11] Due to the assumption of Arrow Securities the identity does not merely hold in expectation, but must hold for any possible state in period $t+1$.

3.2 Unions

Nominal wages are set by a continuum of unions, indexed by $u \in [0,1]$, all operating under monopolistic competition. Each household supplies labor to each union as in Schmitt-Grohé and Uribe (2006) and Colciago (2011).

For each firm j, all labor types are imperfect substitutes, and effective labor is generated through a CES aggregator. Firms are cost minimizing, implying that any union u faces the demand schedule

$$N_t(u) = \left(\frac{W_t(u)}{W_t}\right)^{-e_w} N_t^d \tag{13}$$

where e_w is the elasticity of substitution between different types of labor and N_t^d is total demand for labor. This combined with the aggregator for household hours $N_t \equiv \int_0^1 N_t^d(u)\,du$ implies that the total amount of hours supplied for any household is $N_t = N_t^d \int_0^1 \left(\frac{W_t(u)}{W_t}\right)^{-e_w} du$.

Each union maximizes the weighted utility of its members, and require them to meet their share of the resulting labor demand.[12] Each union has a share λ of liquidity constrained households among its members, implying that labor is identical across all households

$$N_t = N_t^L = N_t^R. \tag{14}$$

The unions face a convex wage adjustment cost following Rotemberg (1982). The convex adjustment cost of changing nominal wages, is covered by the union's fee $F_t(u)$, so that

$$F_t(u) = \frac{\phi_w}{2}\left(\Pi_t^w(u) - 1\right)^2 W_t N_t,$$

where $\Pi_t^w(u) \equiv \frac{W_t(u)}{W_{t-1}(u)}$is the nominal wage inflation in union u. The cost is scaled by the aggregate wage income. The objective function of union u is

$$\max_{W_t(u)} E_t \sum_{k=0}^{\infty} \beta^k \left[(1-\lambda)\,U\left(C_{t+k}^R, N_{t+k}\right) + \lambda U\left(C_{t+k}^L, N_{t+k}\right)\right] \tag{15}$$

subject to the two budget constraints (10) and (12) and the firms' labor demand. As every firm faces the same problem, they will all set the same price.[13] This price yields an

[12] In a model with steady state wealth, the unions might set wages below the marginal rate of substitution of Ricardian household. In that case it is Pareto improving to shift labour supply towards liquidity constrained households, see Natvik (2012)

[13] This also implies that $F_t = \frac{\phi_w}{2}\left(\Pi_t^w - 1\right)^2 W_t N_t$.

(optimal) wage inflation, which satisfies

$$\begin{aligned} 0 &= \frac{1}{MRS_t^A}\frac{W_t}{P_t}\left[(1+\tau_w)(1-e_w)-\phi_w(\Pi_t^w-1)\Pi_t^w\right]N_t^{1+\varphi}+e_wN_t^{1+\varphi} \\ &\quad +\beta\frac{1}{MRS_{t+1}^A}\frac{W_{t+1}}{P_{t+1}}\phi_w\left(\Pi_{t+1}^w-1\right)\Pi_{t+1}^wN_{t+1}^{1+\varphi}, \end{aligned} \tag{16}$$

where $MRS_t^h=\left(C_t^L\right)^\sigma N_t^\varphi$ is the marginal rate of substitution between labor and consumption of households of type h and MRS_t^A is the weighted average of these marginal rates of substitution.[14] Thus, because each union represents all households, it will set the real wage equal to (a markup over) the average MRS. Given the adjustment cost, present unions must balance off current inflation and future inflation. For a derivation of equation (16) see technical appendix in Salmansen (2014b).

3.3 Prices and Exchange Rates

Bilateral terms of trade between the domestic economy and country i is defined as $S_{i,t}\equiv\frac{P_{i,t}}{P_{H,t}}$, and the effective terms of trade is defined as

$$S_t\equiv\frac{P_{F,t}}{P_{H,t}}=\left(\int_0^1 S_{i,t}^{1-\gamma}di\right)^{\frac{1}{1-\gamma}}. \tag{17}$$

Domestic producer price (PPI) inflation as $\Pi_{H,t}\equiv\frac{P_{H,t}}{P_{H,t-1}}$, consumer price (CPI) inflation is $\Pi_t\equiv\frac{P_t}{P_{t-1}}$, and nominal wage inflation is given by $\Pi_t^w\equiv\frac{W_t}{W_{t-1}}$. Then the definition of the CPI index means that these three inflation rates satisfy the following identities

$$\Pi_t=\frac{g(S_t)}{g(S_{t-1})}\Pi_{H,t} \quad ; \quad \Pi_t^w=\frac{\frac{W_t}{P_t}g(S_t)}{\frac{W_{t-1}}{P_{t-1}}g(S_{t-1})}\Pi_{H,t}, \tag{18}$$

where $g(S_t)\equiv\frac{P_t}{P_{H,t}}=\left[(1-\alpha)+\alpha(S_t)^{1-\eta}\right]^{\frac{1}{1-\eta}}$ is used to ease notation, following Faia and Monacelli (2008).

We now turn to the link between the terms of trade and the real exchange rate. Let $\varepsilon_{i,t}$ be the bilateral nominal exchange rate and $P_{i,t}^i(j)$ be the price of country i's good j in country i's currency.[15] Inserting this in the definition of $P_{i,t}$ yields $P_{i,t}=\varepsilon_{i,t}P_{i,t}^i$, where

[14] That is, $\left(MRS_t^A\right)^{-1}\equiv(1-\lambda)\left(MRS_t^R\right)^{-1}+\lambda\left(MRS_t^L\right)^{-1}$. If there is no adjustment cost we have $\frac{W_t}{P_t}=\frac{e_w(1-\tau_w)}{e_w-1}MRS_t^A$ in every period, thus the (potential) wedge stemming from the monopolistic competition does not depend on this adjustment cost.

[15] Here we define the bilateral exchange rate as the price of country i's currency in terms of the domestic

$P_{i,t}^{i} = \left(\int_0^1 P_{i,t}^{i}(j)^{1-\varepsilon}\, dj\right)^{\frac{1}{1-\varepsilon}}$. The law of one price does not ensure that the purchase power parity holds, as this also requires no home bias. The measure of openness, α, will determine how strongly the domestic inflation responds to the terms of trade.

We first define the bilateral real exchange rate $\mathcal{Q}_{i,t} \equiv \frac{\varepsilon_{i,t} P_t^i}{P_t}$,which is the ratio of home and country i's CPI denominated in the home currency. Since all other countries have only Ricardian households,[16] the consumption in any foreign country i will satisfy an Euler equation of the form

$$\beta \left(\frac{C_{t+1}^i}{C_t^i}\right)^{-\sigma} \left(\frac{P_t^i}{P_{t+1}^i}\right)\left(\frac{\varepsilon_t^i}{\varepsilon_{t+1}^i}\right) = Q_{t,t+1} \tag{19}$$

where C_t^i is the consumption of the representative consumer in the foreign country i. Combining equation (11) and (19) with our definition of the bilateral real exchange rate, we have that

$$C_t^R = \vartheta^i C_t^i \mathcal{Q}_{i,t}^{\frac{1}{\sigma}} \tag{20}$$

for all t and all i, and where ϑ^i is constant that depends on the country's initial net asset position. As we are interested in the effect of the domestic LAMP we will proceed with the assumption that Ricardian households are initially symmetric across countries, so that $\vartheta^i = 1$ for all $i \in [0,1]$. The risk sharing shows that when converted to the same currency, the marginal utility of consumption of all market participants must be equal, otherwise there would be unexploited gains from trade in the asset market.

Given the presence of complete international financial markets, the price of a riskfree bond denominated in foreign currency will be $\varepsilon_t^i \left(R_t^i\right)^{-1} = E_t\left[Q_{t,t+1}\varepsilon_{t+1}^i\right]$. Subtracting the domestic bond pricing equation from this yields the uncovered interest parity (UIP)

$$E_t\left[Q_{t,t+1}\left(R_t - R_t^i \frac{\varepsilon_{t+1}^i}{\varepsilon_t^i}\right)\right] = 0. \tag{21}$$

currency. An appreciation of the home currency thus implies a fall in $\varepsilon_{i,t}$.

[16] The assumption that all other countries only has Ricardian households does not have an effect, when we consider only domestic government consumptions shocks.

3.4 Firms

In the home economy, there is a continuum of measure one of firms, indexed by $j \in [0,1]$, each producing a differentiated good j with the linear technology

$$Y_t(j) = A_t N_t(j) \tag{22}$$

in which $a_t = \log A_t$ follows the process $a_t = \rho_a a_{t-1} + \omega_t^a$, where $\rho_a < 1$ and ω_t^a is $N(0, \omega_a^2)$. All firms receive a constant employment subsidy τ_p, which ensures that the zero inflation steady state will be efficient, as it offsets the markup of the monopolistically competing firms.[17]

$AC_t(j)$ is a convex adjustment cost of changing the price of good j

$$AC_t(j) = \frac{\phi_p}{2}\left(\frac{P_t(j)}{P_{t-1}(j)} - 1\right)^2.$$

Aggregate domestic output is defined by the index $Y_t \equiv \left(\int_0^1 Y_t(j)^{\frac{\varepsilon-1}{\varepsilon}} dj\right)^{\frac{\varepsilon}{\varepsilon-1}}$, where the weights are the same as in the consumption index. This gives the following demand for good j

$$Y_t(j) = \left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\varepsilon} Y_t. \tag{23}$$

The objective function of firm j is

$$\max_{P_t(z)} E_0 \sum_{k=0}^{\infty} Q_{t,t+k}\left[P_{H,t}(j) Y_t(j) - (1-\tau_p) W_t N_t^h(j) - \frac{\phi_p}{2}\left(\frac{P_t(z)}{P_{t-1}(z)} - 1\right)^2 P_{H,t}\right]$$

subject to equation (23). Arbitrage implies that the stochastic discount factors satisfy $Q_{t,t+k} = Q_{t,t+k-1}Q_{k-1,k}$. Each firm faces the same optimization problem and hence chooses the same price $P_{H,t}$, which implies the following real wage and inflation trade-off

$$\begin{aligned}(\Pi_{H,t} - 1)\Pi_{H,t} &= \beta E_t\left[\frac{\left(C_{t+1}^R\right)^{-\sigma}}{\left(C_t^R\right)^{-\sigma}} \frac{g(S_t)}{g(S_{t+1})}(\Pi_{H,t+1} - 1)\Pi_{H,t+1}\right] \\ &\quad + \frac{\varepsilon - 1}{\phi_p} Y_t\left[\frac{W_t^{\text{real}}}{A_t} g(S_t) - 1\right],\end{aligned} \tag{24}$$

where we have assumed that τ_p removes the markup due to monopolistic competition.

[17] The nominal marginal cost is common to all firms and given by $MC_t = (1-\tau_p)\frac{W_t}{A_t}$.

The firms also have identical levels of labor input $N_t(j) = N_t$ and output $Y_t(j) = Y_t$, so the aggregate output is given by

$$Y_t = A_t N_t - \frac{\phi_p}{2} \left(\Pi_{H,t} - 1\right)^2 . \tag{25}$$

It is the presence of nominal rigidities that enables a demand effect in our model. If prices were fully flexible, there would be a constant markup, μ, over marginal cost, and there would be no price wedge effects on the government spending multiplier. By introducing price rigidities, we get a variation in the price markup. This gives a role for demand in determining output, so that demand side effects will matter for the output effect of fiscal stimulus.[18]

3.5 Fiscal Policy

Aggregate government spending, G_t, is defined as

$$G_t = \left(\int_0^1 G_t(j)^{\frac{\varepsilon-1}{\varepsilon}} dj \right)^{\frac{\varepsilon}{\varepsilon-1}} , \tag{26}$$

in which $g_t = \log G_t$ follows the process $g_t = \rho_g g_{t-1} + \omega_t^g$, where $\rho_g < 1$ and ω_t^g is $N\left(0, \omega_g^2\right)$. Cost minimization will yield the same demand functions as derived for C_t^h. Government demand is solely directed towards domestically produced goods, so there are no leakage effects in the model. This is an extreme assumption, but it allows a comparison with the closed economy model, where liquidity constrained households receive a share λ of the income generated by government spending.

The government finances its spending through one-period risk free bonds B_t and tax revenues. Its revenue is generated by the lump sum taxes on Ricardian and liquidity constrained households, while the expenditures consist of the wage and labor subsidies and government spending on domestic goods. The government budget constraint is therefore

$$P_{H,t} G_t + B_t = P_{H,t} T_t + \tau_w W_t N_t + \tau_p P_{H,t} Y_t + R_t^{-1} B_{t+1} \quad , \tag{27}$$

in which T_t is the weighted sum of household lump sum taxes, $T_t = \lambda T_t^R + (1-\lambda) T_t^L$. The tax $T_t^h = \overline{T}_t^h - \widetilde{T}_t^h$ consists of two parts. A tax $\widetilde{T}_t^h$, which covers the expenses on wage

[18] See Woodford (2011) for more details.

and labor subsidies, and a tax $\overline{T}_t^h$, which covers the tax payments related to spending and debt. The first category is set according to the policy rules

$$P_{H,t}\widetilde{T}_t^R = \tau_w W_t N_t + \frac{1}{(1-\lambda)}\tau_p P_{H,t} Y_t \quad ; \quad P_{H,t}\widetilde{T}_t^L = \tau_w W_t N_t, \tag{28}$$

so that the firm subsidy is financed by Ricardian households only and the wage subsidy is financed proportionally by all households. This implies that the two households have the same level of consumption in steady state and further that the labor subsidy τ_p does not imply a transfer from one household to another via profits. The wage subsidy τ_w will affect the unions and remove the wage markup, but the wage subsidy and the tax $\widetilde{T}_t^L$ will net out in the liquidity constrained households' budget constraint, thus not affecting their consumption level.[19] Likewise, $\widetilde{T}_t^R$ and income from wage the subsidy and the firm subsidy will net out, removing these parts of the fiscal policy from the Ricardian households' budget constraint.

The presence of LAMP implies that the timing of the remaining taxes $\overline{T}_t$, and hence government debt, will matter for the evolution of the domestic economy. In order to pin down the path of government debt and to ensure that government debt does not explode, we assume, in line with Galí et al. (2007) and Coenen and Straub (2005), that the tax $\overline{T}_t$ follows the policy rule

$$\overline{T}_t = B_t^{\phi_b} G_t^{\phi_g} \tag{29}$$

in which ϕ_b, $\phi_g \geq 0$ are the elasticities of the aggregate lump sum taxes to real government debt and real government spending. ϕ_b reflects how much of existing debt will be collected in taxes in a given period, whereas ϕ_g reflects the degree to which government spending is debt financed. $\phi_g = 0$ corresponds to complete debt financing. As shown in Appendix D, $\phi_b > 1 - \beta$ is a necessary and sufficient condition for non-explosive government debt paths.

This tax is levied equally on both types of domestic households, so that $\overline{T}_t^h = \overline{T}_t$, ensuring that government spending does not imply a redistribution between the two types. Coenen and Straub (2005) show that going from this assumption to the extreme assumption that liquidity constrained households are completely exempt from paying taxes, the impact multiplier becomes considerably larger. This is because the negative

[19] The budget constraint reduces to $P_t C_t^L = W_t N_t - \overline{T}_t^L - F_t$.

wealth effect of the Ricardian households is smoothed over a long period, while the income effect for the liquidity constrained households immediately feeds into their demand one-for-one.[20]

The tax rules $\widetilde{T}_t^R$, $\widetilde{T}_t^L$, and $\overline{T}_t$ ensure that the two households have equal steady state consumption, and that changing the degree of LAMP will not change the wealth and income of any of the two representative households.

3.6 Equilibrium in Domestic Goods Markets

The equilibrium condition for goods market j in the home economy is

$$Y_t(j) = (1-\lambda) C_{H,t}^R(j) + \lambda C_{H,t}^L(j) + G_t(j) + \int_0^1 C_{H,t}^i(j)\, di$$

for all j, and where $C_{H,t}^i(j)$ is country i's demand for good j produced in our home economy. Foreign households are all Ricardian, and foreign governments do not demand the home country's goods. Openness, α, determines the amount of exports and the share of domestic consumption which is directed towards the domestic good. A larger degree of openness increases exports, but reduces domestic demand for the domestic good.

Using the aggregation index for output we have

$$\begin{aligned} Y_t &= g(S_t)^\eta \left[(1-\alpha)(1-\lambda) + \alpha S_t^{\gamma-\eta} \mathcal{Q}_t^{\eta-\frac{1}{\sigma}}\right] C_t^R \\ &\quad + g(S_t)^\eta (1-\alpha) \lambda C_t^L + G_t, \end{aligned} \tag{30}$$

which uses the fact that our home economy has infinitesimal weight and that all foreign economies are identical, so that $S_t^i = 1$, $S_{i,t} = S_t$ and $\mathcal{Q}_{i,t} = \mathcal{Q}_t$.

Let the net exports in terms of domestic output as a fraction of steady state GDP be defined as $NX_t \equiv \frac{1}{Y}\left(Y_t - \frac{P_t}{P_{H,t}} C_t - G_t\right)$. Inserting Y_t from (30) yields

$$NX_t = \frac{1}{Y}\left\{\alpha S_t^\gamma C_t^* + g(S_t)^\eta (1-\alpha) C_t - g(S_t) C_t\right\}.$$

The first two terms represent output produced to meet private demand for domestic goods. The last term represents the degree of private demand adjusted for the difference

[20] In our model Ricardian households have no income effect, as income risk is hedged in international asset markets. Therefore,putting the full tax burden on liquidity constrained households would have an even greater effect in our model.

between the CPI and the PPI. Government spending nets out and does not affect net export directly (although it will through households' demand, as we will see in Section 5). An improvement of the terms of trade will cause foreign and domestic households to substitute towards domestic goods. There is also a wealth effect, as the improved terms of trade increase the value of domestic consumption adjusted to output prices, and hence reduces net exports.

3.7 Equilibrium

We now turn to equilibrium in the economy

Definition 1 *A rational expectations equilibrium consists of a sequence* $\left\{C_t^R, C_t^L, N_t^R, N_t^L, N_t, Y_t, W_t^{\text{real}}, S_t, \Pi_{H,t}, \Pi_t, \widetilde{T}_t^R, \widetilde{T}_t^L, \overline{T}_t^R, \overline{T}_t^L\right\}$ *satisfying equations (11), (12), (16), (20), (24), (25), (30), (28) and the two equations in each of (14), (18), and (29), given the policy* $\{R_t\}$ *and the exogenous processes* $\{A_t, G_t, C_t^*\}$*.*

The equilibrium conditions are restated in Appendix B, and the unique perfect foresight steady state with zero inflation is derived in Appendix B.1. The wage and price subsidies are set to offset the inefficiencies arising from monopolistic competition in steady state, that is $\tau_p = \frac{1}{e}$ and $\tau_w = \frac{1}{1-e_w}$. Then steady state satisfies

$$Y = N = (\gamma_c)^{-\frac{\sigma}{\varphi+\sigma}} \quad ; \quad C = (\gamma_c)^{\frac{\varphi}{\varphi+\sigma}} \quad ; \quad W_t^{\text{real}} = 1.$$

where γ_c is the steady state consumption share of output, $\gamma_c = \frac{C}{Y}$. We see that the steady state level of government spending drives a wedge between output and working hours on the one side and consumption on the other side, causing steady state level of hours to be increasing and households' consumption will be decreasing in the share of government spending $(1-\gamma_c)$. This wedge between hours and consumption distorts the marginal rate of substitution of the households, and labor supply will be lower. Combined with the direct effect of government spending on aggregate demand, the equilibrium effects is an increase in hours and a reduction in consumption in the steady state, when government spending is increased.

3.8 Monetary policy

We will analyze the effects of fiscal policy under three standard monetary policy rules in the small open economy literature, namely a domestic inflation-based Taylor rule (DITR, for short), a CPI inflation-based Taylor rule (CITR) and an exchange rate peg (PEG). In our model these policies are defined as follows:

$$
\begin{aligned}
DITR &: \widehat{r}_t = \varphi_\pi \widehat{\pi}_{H,t} \\
CITR &: \widehat{r}_t = \varphi_\pi \widehat{\pi}_t \\
PEG &: \widehat{e}_t = 0
\end{aligned}
$$

where $\widehat{r}_t = \log(R_t) - \ln(\beta^{-1})$, $\widehat{\pi}_t = \log(\Pi_t)$, and $\widehat{e}_t = \frac{\varepsilon_t - \varepsilon}{\varepsilon}$. Under the two Taylor rules, the nominal interest rate responds to equilibrium inflation, while the PEG implies a constant nominal interest rate via the UIP.

4 Parametrization

We solve our model by log-linearizing the equilibrium conditions around the perfect foresight steady state presented in Section 3.7. The log-linearized version of the equilibrium conditions for the model are shown in Appendix C.

We use DYNARE to solve the log-linearized model and to perform impulse response analysis. In order to do this we assign numerical values to the parameters.

Degree of LAMP: The share of liquidity constrained households λ is set to 0.35, which is in the range found for the Euro Area and G7-countries, see Section 2.

Preferences and technology: The substitution of domestic and foreign goods is $\eta = 1.5$, as in Auray et al. (2011), and substitution between supplier countries $\gamma = 1$ as in Galí and Monacelli (2005). Setting $\beta = 0.99$ and interpreting our period as a quarter, the steady state (risk-free) interest rate is approximately 4% p.a. The degree of openness α is 0.3, as in Auray et al. (2011), this is between the values in Galí and Monacelli (2005) and Pappa and Vassilatos (2007).[21] The relative risk aversion parameter is $\sigma = 2$ in line with Corsetti et al. (2008) and $\varphi = 3$ implies a labor supply elasticity of 1/3.

[21] Galí and Monacelli (2005) have $\alpha = 0.4$ using Canada as their example of a SOE, while Pappa and Vassilatos (2007) have a related two country model and choose $\alpha = 0.2$ based on France ad Germany as a case of symmetric home bias.

Parameter	Value	Description
λ	0.35	Share of domestic households, that are liquidity constrained
α	0.3	Degree of home bias
β	0.99	Discount factor
σ	2	Relative risk aversion
φ	3	Inverse of the Frish elasticity of labor supply
η	1.5	Substitutability between foreign and domestic goods
γ	1	Elasticity of substitution between goods from different countries
ϵ_p	6	Elasticity of substitution between differentiated domestic goods
ϵ_w	21	Elasticity of substitution between labor inputs
ϕ_p	58	Rotemberg price adjustment cost parameter
ϕ_p	19	Rotemberg wage adjustment cost parameter
ρ_g	0.9	Persistence in shocks to government consumption
τ_w , τ_p		Pay-roll tax rate and sales tax rate (endogenous)
γ_c	0.8	Consumption share of steady state output
ϕ_b	0.33	Elasticity of the aggregate lump sum taxes to the real government debt
ϕ_g	0.10	Elasticities of the aggregate lump sum taxes to the real government debt
ϕ_π	1.5	Coefficient in inflation based Taylor rules

Table 1: **Benchmark Parametrization.**

The substitution between differentiated goods is $\varepsilon_p = 6$ and the price adjustment cost parameter is $\phi_p = 58$, so that we obtain a Phillips curve with the same output-response as in a Calvo-model (Calvo, 1993) with an average price duration of four quarters (and hence a price continuation probability $\theta_p = 0.75$)[22]. The labor demand elasticity is $\varepsilon_w = 21$ and the wage adjustment cost parameter is $\phi_w = 19$. This is in line with Schmitt-Grohé and Uribe (2006) who have a Calvo version of our union setup with $\theta_w = 0.64$, following the estimates in Christiano et al. (2005).[23]

Policy: The interest rate elasticity φ_π is set to 1.5, as suggested by Taylor (1993). The steady state government spending share is 20 percent, $\gamma_c = 0.8$. Tax elasticities are $\phi_b = 0.33$, $\phi_g = 0.10$ in line with Galí et al. (2007), as is the persistence of the fiscal shock $\rho_g = 0.9$. Thus, their government spending is to a large degree financed by debt.

5 Effects of Government Spending Shocks

We now analyze the effect of a government spending shock. We consider an unexpected rise in government spending of one percent of steady state output and all responses of

[22]In a Calvo price friction model for our economy the slope of the log-linearized Phillips curve is $\kappa_p = \frac{(1-\beta\theta_p)(1-\theta_p)}{\theta_p}$, see Galí and Monacelli (2005).

[23]As pointed out by Schmitt-Grohé and Uribe (2006) and elaborated in Colciago (2011), assuming that households are *not* monopolistic suppliers of a given labour input means that the slope in a log-linearized Calvo model is given by $\kappa_w = \frac{(1-\beta\theta_w)(1-\theta_w)}{\theta_w}$, where assuming that the each households is a monopolistic supplier implies $\kappa_w = \frac{(1-\beta\theta_w)(1-\theta_w)}{\theta_w(1+\varphi\varepsilon_w)}$ (that is, κ_w is smaller). In order to match data for the wage Philips curve we therefore set ε_w higher than classical calibrations of around 5-6 (see Erceg et al. (2000) and Furlanetto and Seneca (2012)).

other economic variables will be in percentage deviation from their steady state values, except taxes and net exports that are in percent of steady state output. Figures 1-3 compare the economy with only Ricardian households ($\lambda = 0$, the solid blue line) and the economy with LAMP ($\lambda = 0.35$, the dashed green line).

The direct consumption responses to the shock are similar under all three monetary policy regimes, and we will briefly review these before turning to each monetary regime.

Consider an economy with Ricardian households only. Their consumption does not change in direct relation to a government spending shock, as their access to state contingent claims implies that they do not experience any income effects. However, as government spending only consists of domestically consumed goods, the shock implies a shift in relative aggregate demand, making domestic goods relatively more expensive and terms of trade worse. The latter implies that Ricardian consumption drops, following the international risk sharing condition.

Now we introduce LAMP. As real disposable income rises under the debt-financed spending shock, liquidity constrained households increase consumption. The introduction of LAMP thus increases the initial response of total household consumption following a government spending increase. As taxes gradually increase due to the growing debt, liquidity constrained households will experience a decline in disposable income, and hence their consumption contracts in the longer run.

Wage and price effects are key drivers of the size of the two opposite consumption responses, and therefore of the response of total demand, that the monetary policy regime will have. This leads us to proceed with the analysis for each monetary policy regime separately.

5.1 Floating Exchange Rate and Taylor rules

5.1.1 Domestic Inflation-Based Taylor Rule

Figure 1 shows the propagation of a government spending shock under a domestic producer inflation-based Taylor rule (DITR). Unlike much of the literature, we see that LAMP reduces the response of output from 0.60 to 0.48, when we introduce LAMP. This result is due the liquidity constrained households' effect on wages.

The increase in output and thus working hours following a rise in government spending

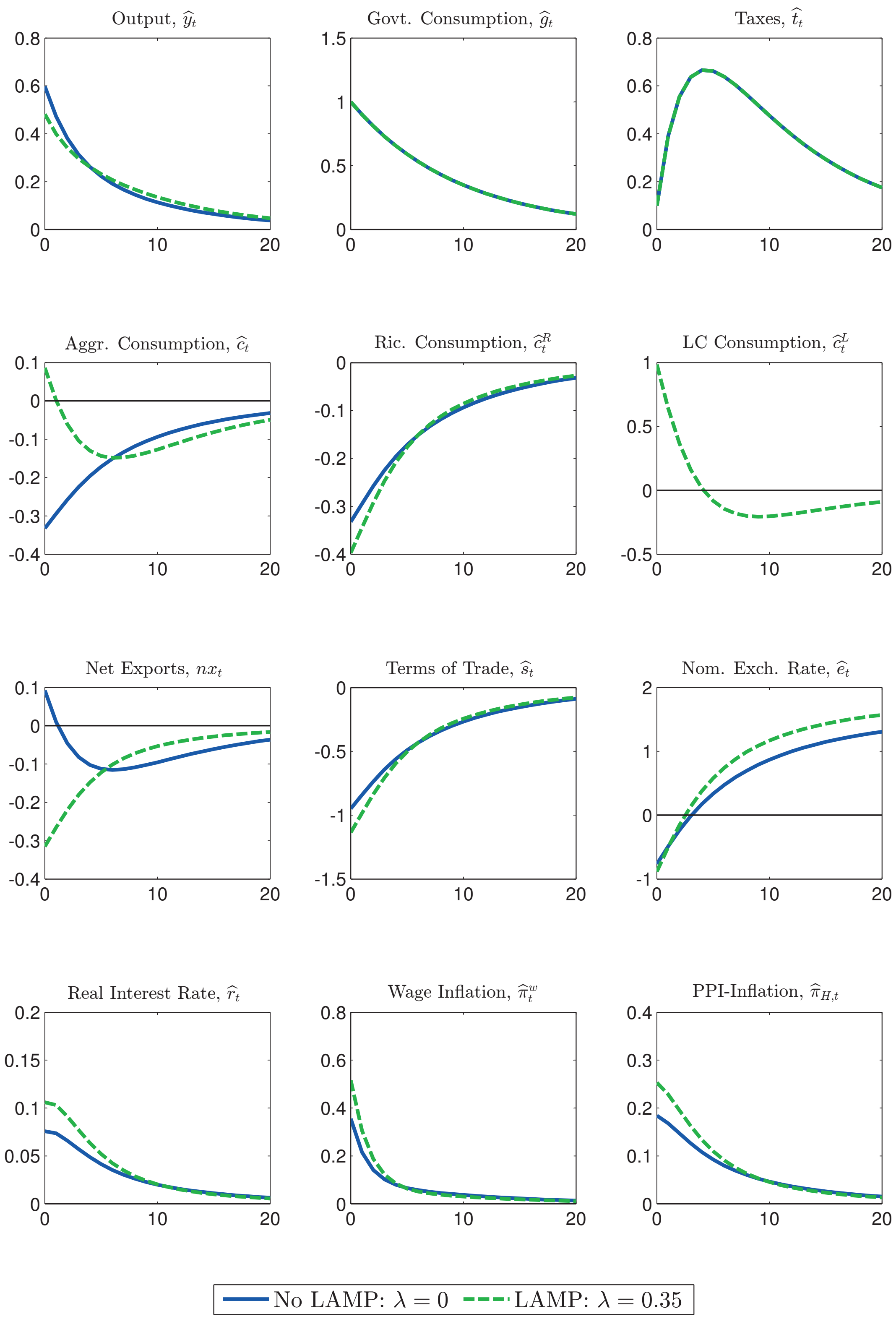

Figure 1: **Effect of LAMP under DITR:** Effect of a government spending shock under a domestic PPI-based Taylor rule in the model with and without LAMP.

will ceteris paribus increase the wage set by unions. In a purely Ricardian economy, the decrease in household consumption puts a downward pressure on wages, but the effect from working hours dominates and real wages increase 0.46. With LAMP, the increase in liquidity constrained consumption will put a further upward pressure on wages, so that the real wage rises 0.60 on impact. The higher wages are transmitted into higher producer prices, and the nominal interest rate is increased according to the Taylor rule.

The higher total domestic demand appreciates the domestic currency initially. The UIP implies that the increase in the nominal interest rate makes the nominal exchange rate gradually depreciate to a new and higher steady state level. As LAMP increases the wage and PPI inflation, monetary policy will respond more in an economy with LAMP, and the new nominal exchange rate steady state level is depreciated more.

The initial PPI inflation outweighs the exchange rate appreciation, and terms of trade worsen, causing Ricardian households to contract consumption by 0.33. The drop in terms of trade is increased by LAMP, causing Ricardian consumption to contract slightly more (0.39) when we introduce LAMP. Liquidity constrained consumption increases by 0.98, and total household consumption goes from a decrease of 0.22 to a small crowding in of 0.08 with LAMP.

The drop in terms of trade further reduces the value consumption measured in output prices. This will reduce imports and thereby increase net exports. However, the drop in terms of trade will also make foreign and domestic households substitute away from domestic goods, reducing net exports. The wealth effect dominates, and net exports rise by 0.09 in the Ricardian economy. When we introduce LAMP, the higher household consumption and even worse terms of trade will reduce net exports that become -0.17.

The introduction of LAMP boosts household consumption, but this is dominated by a drop in net exports, causing the instantaneous output multiplier of government spending to drop from 0.60 to 0.48.

5.1.2 Consumer Price Inflation-Based Taylor Rule

Figure 2 shows the impulse responses for a CPI based Taylor rule (CITR). Changing the base of the Taylor rule from PPI to CPI implies that the output response to the shock is 0.91 without and 0.85 with LAMP. The output-effect of LAMP under the shock is halved from -0.12 under DITR to -0.06 under CITR.

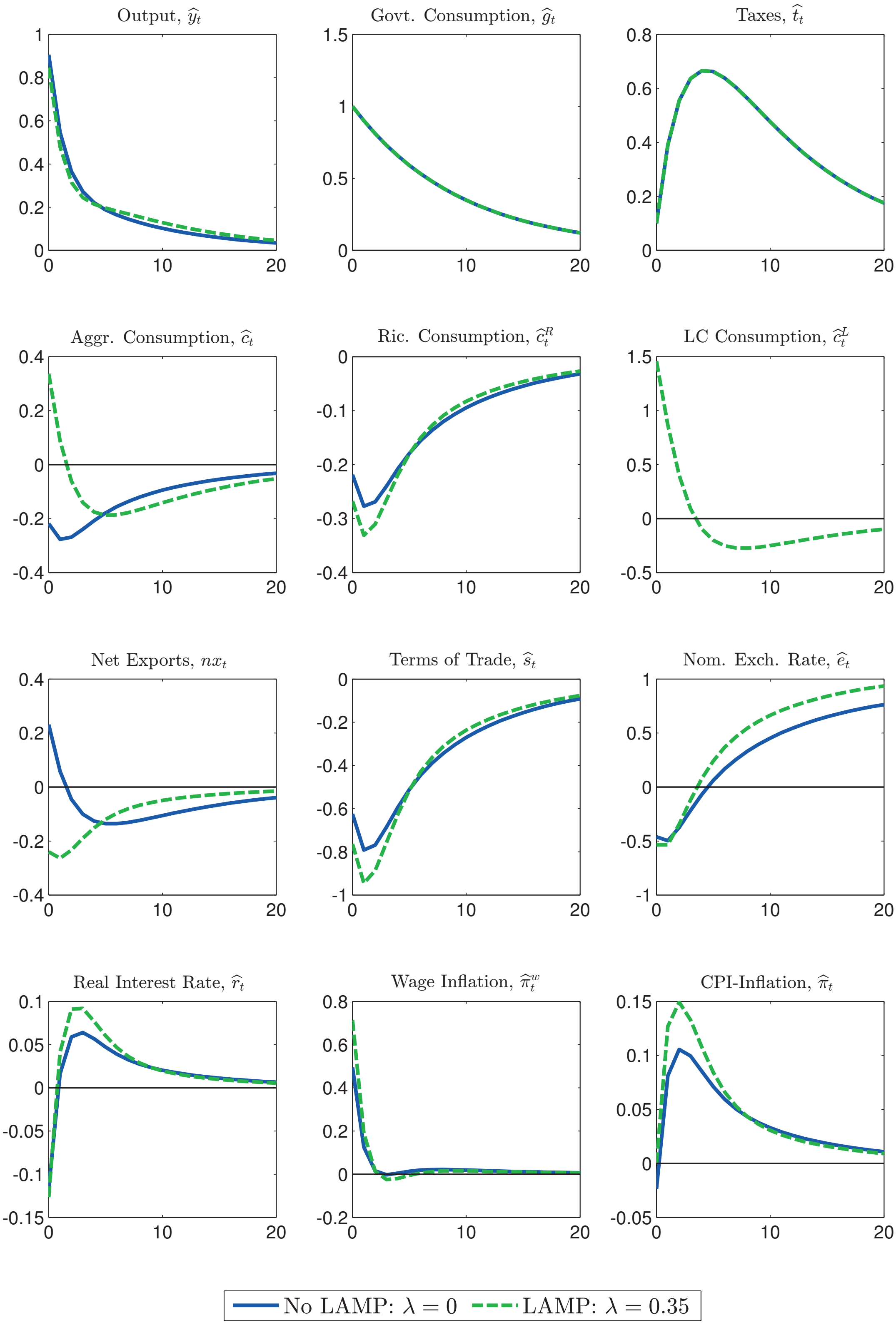

Figure 2: **Effect of LAMP under CITR:** Effect of a government spending Shock under a CPI-based Taylor rule in the model with and without LAMP.

The initial appreciation of the nominal exchange rate, due to higher demand for domestic goods, causes import prices and CPI to drop initially. Under CITR the nominal interest rate is reduced, causing the real interest rate to contract initially, and the nominal exchange rate to appreciate by only half as much as under DITR. As a result, in the Ricardian model terms of trade drop less and Ricardian households' initial consumption response is only 0.22 (this is -0.33 under DITR), causing wages to be increased more than under DITR. Further, the dampened substitution effect makes net exports twice as high as under DITR. The combined effect is crowding out of total private sector demand for domestic goods, but this is smaller than under DITR, which is why the instantaneous output multiplier is higher under CITR than under DITR.

When we introduce LAMP, the disposable income and thus consumption of liquidity constrained households becomes higher (1.46 compared to 0.98 under DITR). Total household consumption goes from contracting by 0.22 to a positive response of 0.34 under LAMP. Therefore, LAMP increases the consumption response more under CITR, as the effect is 0.56 percentage points under CITR compared to 0.30 under DITR.

The higher domestic demand makes terms of trade contract more, and net export to drop to -0.24. The LAMP-effects on consumption are again dominated by net exports, so that the instantaneous output multiplier is 0.85 under LAMP. Therefore, switching to the CPI base in the Taylor rule greatly reduces the effect of LAMP on the output response to a government spending shock in our model.

5.2 Fixed Nominal Exchange Rate

Figure 3 shows the propagation of a government spending shock, when the nominal exchange rate is fixed. Under the peg, output multipliers are much larger than under the floating regimes, and further, LAMP increases the output response slightly from 1.31 to 1.34. The effects of LAMP are still numerically small, but their effect on multipliers change sign compared to the Taylor rules.

Given the peg, the central bank cannot increase the nominal interest rate, as this would cause the domestic currency to appreciate according to the UIP. The drop in terms of trade caused by PPI inflation is therefore not exacerbated by an exchange rate appreciation, and hence the drop in terms of trade is more modest than under the Taylor rules. Thus in the Ricardian model the size of the Ricardian consumption drop is more

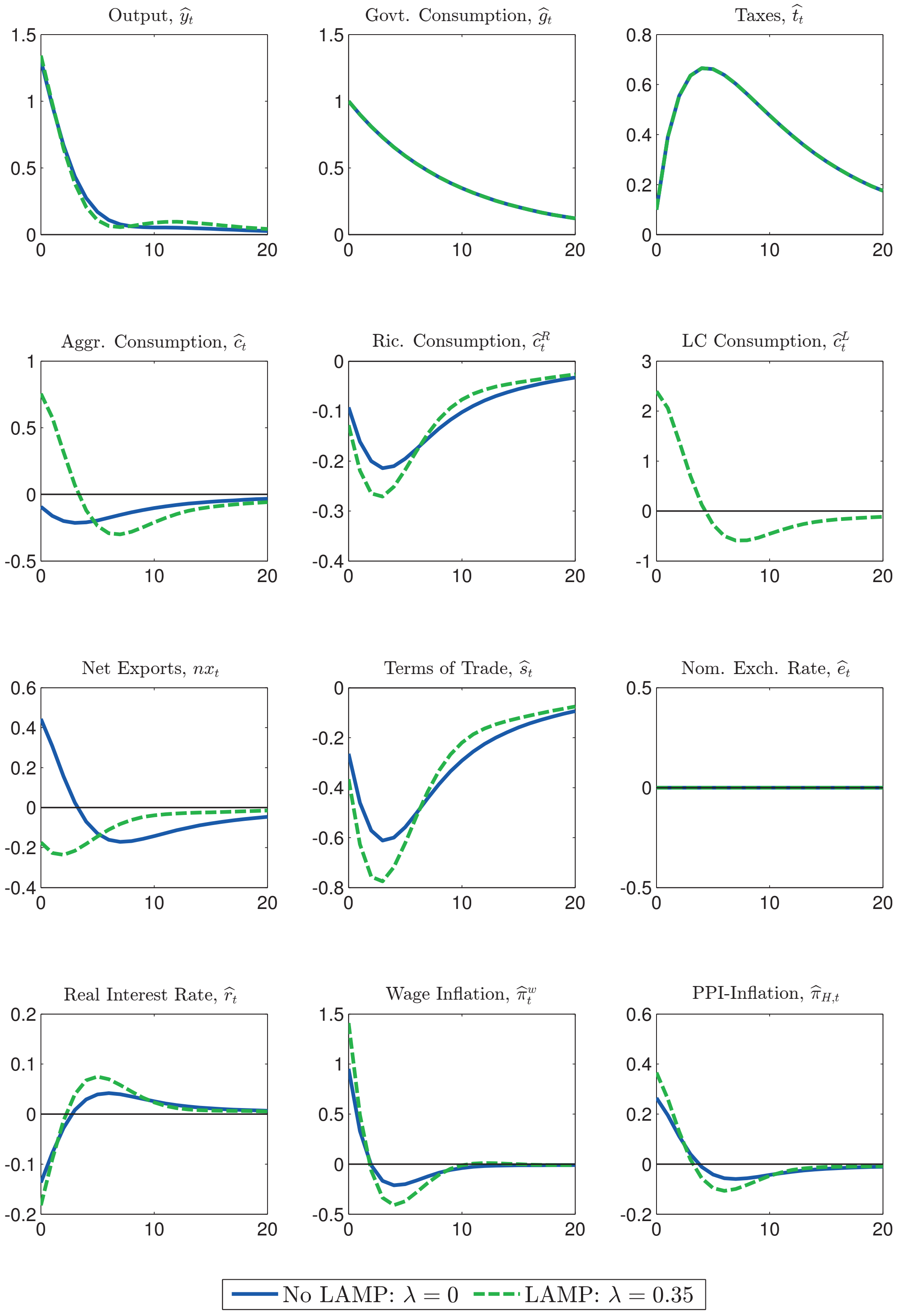

Figure 3: **Effect of LAMP under a PEG:** Effect of a government spending shock under a fixed nominal exchange rate in the model with and without LAMP.

modest, only 0.09. This implies that unions negotiate higher wages initially, but as the shock is transitory it does not change the relative prices in the long run. Under a fixed exchange rate, any short run increase in domestic wages and prices must therefore be reversed by a fall in these prices at a later time.[24] This future readjustment causes the forward-looking wage and price setters to reduce their current inflation rates somewhat, and hence dampens the amplitude of the boom-bust path. As PPI inflation now is the only driver of terms of trade, the latter will experience a bust-boom path, as will Ricardian consumption.

Now we introduce LAMP. The real wage income and thus liquidity constrained consumption will in contrast to Ricardian consumption have a strong boom-bust path. The hump-shaped response of taxes magnify this (as in the other monetary regimes), and liquidity constrained consumption rises 2.40 on impact (this is only 0.98 under DITR). The liquidity constrained response is so strong that aggregate consumption rises by 0.76 initially and then contracts in the medium run. The effect of LAMP is therefore a magnification of the boom-bust effect on nominal wages, prices and demand.

As the nominal exchange rate does not adjust to changes in demand, introducing LAMP will imply a contraction of net exports, from 0.44 to -0.17, which is a larger change than under the Taylor rules. The missing nominal exchange rate response implies that the rise in domestic consumption induced by LAMP is almost offset by the drop in net exports. The output response only increases from 1.31 to 1.34 when we introduce LAMP. This positive, although modest effect, is in contrast to the contractionary output effect of LAMP under the Taylor rules.

The model is not able to generate the negative output response found for floating exchange rate regimes in Ilzetzki et al. (2013), not even when we introduce LAMP and thereby have lower output effects responses to a government spending shock.

Our positive output multiplier under the peg is consistent with the positive multipliers in Ilzetzki et al. (2013) and Corsetti et al. (2012a). However, the output multiplier of 1.3 is considerably higher than the multiplier of 0.15 in Ilzetzki et al. (2013), and LAMP makes this discrepancy slightly larger.

[24] As the nominal interest rate is constant under a fixed exchange rate, it seems natural to equate this with an economy being at the zero lower bound. These two cases differ, as prices need not revert to initial levels when considering the ZLB, and thus an erosion of both the short and long run interest rates is possible (see Salmansen (2014a)). For a comparison of the two cases, see Erceg and Lindé (2012) and Farhi and Werning (2012).

5.3 Comparing Regimes

In order to compare the effect of LAMP on the propagation of a government pending shock across the three monetary regimes, Figure 4 shows the change in consumption and output response induced by a change from $\lambda = 0$ to $\lambda = 0.35$.

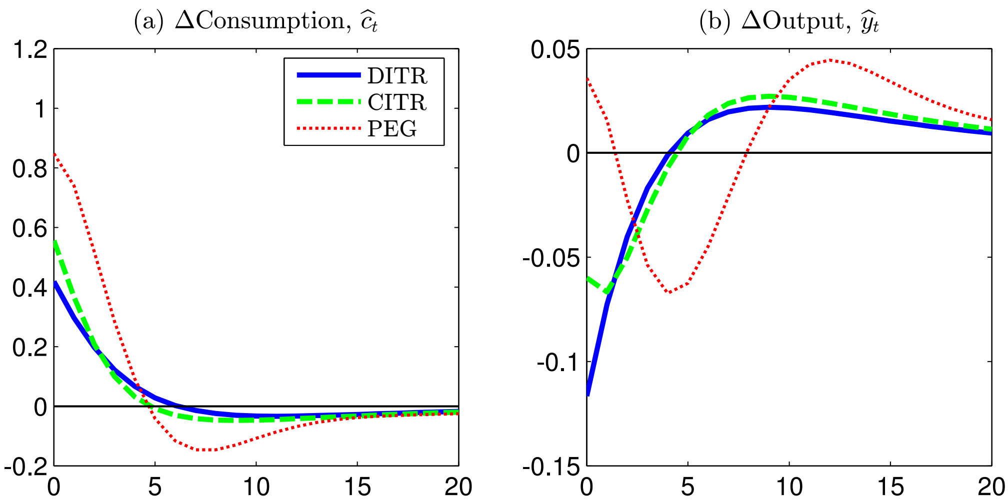

Figure 4: **Effect of LAMP:** Change in consumption and output responses to a government consumption shock, when LAMP is introduced.

Figure 4 panel (a) shows that the instantaneous consumption response is largest under the PEG, and although LAMP makes consumption contract in the medium run for all three regimes, the contractionary effect is more pronounced under the PEG. Panel (b) shows that LAMP under the two Taylor rules has a contractionary effect initially and then an expansionary effect on output. Under the PEG, LAMP implies an initial stimulation of output followed by a large contraction and finally yet a stimulating effect.

Given the very different paths of both the impulse responses in figures 1-3 and the LAMP-effects in Figure 4, it is hard to quantify, under which regime fiscal stimulus has the largest effect, and whether LAMP makes government spending more effective in stimulating consumption and output. We therefore follow the approach suggested in Uhlig (2010) and compare net present value (NPV) fiscal multipliers. The NPV multiplier for consumption is defined as the NPV of consumption responses (discounted by the steady state interest rate) divided by the NPV of the government spending shock. The size of this ratio will depend on the horizon, and the subscript T thus refers to the number of quarters included

$$\Psi_T^C \equiv \frac{\sum_{t=0}^{T} R^{-t}\left(C_t - C\right)}{\sum_{t=0}^{T} R^{-t}\left(G_t - G\right)} = \gamma_c \frac{\sum_{k=0}^{t} \beta^t \widehat{c}_t}{\sum_{k=0}^{t} \beta^t \widehat{g}_t}.$$

This means that Ψ_3^C is the ratio of total consumption effects in the first four quarters over total government spending in the first four quarters in NPV terms. The NPV fiscal multiplier on output is defined correspondingly, so that $\Psi_T^Y = \frac{\sum_{t=0}^{T} \beta^t \widehat{y}_t}{\sum_{t=0}^{T} \beta^t \widehat{g}_t}$. Note that if we set $T = 0$, the NPV fiscal multiplier is the impact multiplier.

Figure 5 shows the NPV fiscal multipliers Ψ_t^C and Ψ_t^Y for all three monetary policy regimes. Each panel shows the multiplier for an economy without LAMP, with LAMP, and the difference between the two cases, which we will refer to as the NPV effect of LAMP.

Figure 5 panel (a) - (d) show that the NPV multipliers are highest under PEG for any horizon both in the model with and without the LAMP. With LAMP the NPV consumption multiplier eventually turns negative for all regimes, which reflects that the tax burden reduces liquidity constrained households' disposable income. Panel (c) shows that the effect of LAMP on NPV consumption multipliers is also highest under PEG. This is because their consumption rise is not offset by a nominal exchange rate movement, and hence terms of trade do not generate as large an output drop as under the Taylor rules.

For the output the picture is more ambiguous. Panel (f) shows that under the two Taylor rules the negative NPV effect of LAMP is dampened as the horizon is expanded, and for CITR it even turns positive after 16 quarters. However for the PEG, the NPV multiplier first contracts and then expands. This is due to the exacerbation of the boom-bust path of the economy that the liquidity constrained households cause. A longer horizon monotonically reduces the NPV output multiplier for both models, however this reduction happens at varying rates, and the difference in these multipliers i.e. the NPV effect of LAMP is oscillating.

Generally the graphs show that across all three monetary policies, using the instantaneous multiplier will greatly overstate the effect of LAMP compared to the five year horizon. This is again due to the debt financing of government spending, implying that liquidity constrained households consume immediately rather than smoothing consumption as Ricardian households.

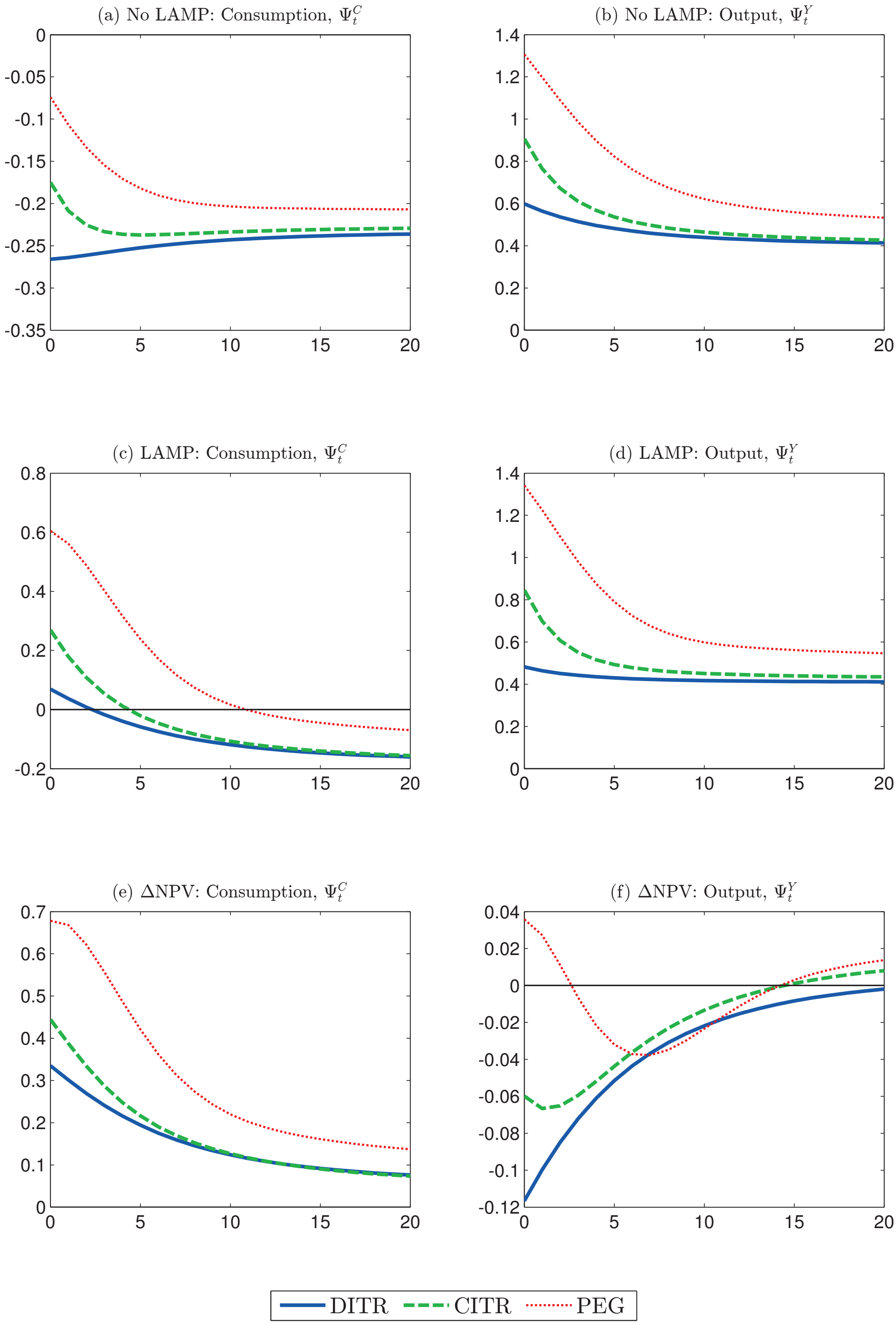

Figure 5: **NPV Fiscal Multipliers:** The Net Present Value multipliers for consumption and output of a government spending shock in our benchmark model for increasing horizon T for all three monetary policy regimes. Top panels show these multipliers for a model without LAMP, middle panels show them for a model with LAMP, and the bottom panels shows the difference between the mulipliers in the two models.

6 Robustness

We now consider how some key features affect the economic response to a government spending shock in our model with LAMP. We start by considering the effect of increasing openness, then proceed to discuss alternative tax rules, and finally consider an alternative formulation of the model, in which labor markets have perfect competition.

6.1 Openness

We start by considering the effects of increasing openness in our model with LAMP. Figure 6-8 in Appendix A shows the effect of increasing the degree of openness in the model (from $\alpha = 0.3$ to $\alpha = 0.6$) for each of the three monetary policy regimes.

In general, increasing the degree of openness, and thus the size of trade flows, will make substitution effects stronger. A drop in the terms of trade generated by the government spending shock, will imply a larger substitution towards foreign goods, and net exports respond to the government shock with a larger contraction in all three regimes.

This implies that a smaller contraction in terms of trade is necessary to obtain equilibrium in the goods markets following the government spending shock, and as a direct result Ricardian consumption contracts less. This implies that aggregate consumption is higher in all three regimes, when there is more openness. Thus the effect of openness on output depends on the relative sizes of the drop in net exports and increase in consumption.

Under DITR and PEG the drop in net exports dominate, and more openness implies a lower output response to the government spending shock.

Under CITR the higher degree of openness will imply a lower CPI inflation path, due to the higher weight of foreign goods, and thus a lower interest rate path. According to the UIP this implies a lower rate of appreciation of the domestic currency, and the drop in terms of trade is dampened more by the increased openness under CITR than under DITR. This both implies that the response of net exports is hardly affected by openness on impact (however, the drop is increased from the second quarter) and that the Ricardian households' consumption contracts much less. The total effect is a larger response of output to the government spending shock, when openness is increased (however, this response is smaller from the third quarter)[25].

Ilzetzki et al. (2013) find that openness decreases the instantaneous output multiplier. Our model has this property for DITR and PEG, but under CITR this multiplier is increasing in openness. Ilzetzki et al. (2013) pools all 44 countries when estimating the effect of openness. Based on our results, we should be careful in such a pooled analysis, as the pool of countries potentially contain heterogeneity in the effects of openness due to monetary policy differences.

6.2 Balanced Government Budget

The response of the liquidity constrained households is extremely sensitive to the chosen tax rule. The benchmark tax rule has $\phi_b = 0.33$ and $\phi_g = 0.10$, so that the instantaneous tax financing is very limited, and the response of the lump sum tax is hump-shaped.

In Figure 9-11 in Appendix A we show the effect of changing to a fully balanced budget, $\phi_g = 1$.[26] Switching to a balanced budget has large effects of the propagation of our government spending shock. Given a balanced budget rule, the increased labor income and the tax burden coincide, thereby removing the rise in disposable income that occurs under a debt-financed government spending shock. Apart from general equilibrium effects on terms of trade, Ricardian consumption is not affected by the timing of lump sum taxes, so the direct effects of government spending on the two households' demand is very similar.

As previously explained, the drop in terms of trade implies a drop in Ricardian demand and net exports in the Ricardian model. When we introduce LAMP, the liquidity constrained households will have a similar drop in consumption due to the crowding out of domestic demand and net exports, so the effects of introducing LAMP are very small. Under DITR the response of output is 0.36, and this is 0.48 and 0.60 under debt financing with and without LAMP.

Under CITR and PEG the introduction of LAMP increases the response of consumption, however, this is offset by a drop in net exports, so that the output effect are the same before and after LAMP. The output response under CITR and PEG are 0.64 and

[25]The increase in output increases disposable income of liquidity constrained households, who have a higher consumption, when there is a larger degree of openness.

[26]The case of pure debt financing, $\phi_g = 0$, does not change the dynamic response much, reflecting that our benchmark is already very close to full debt financing.

0.99, which is below the respective values of 0.9 and 1.3 under the debt-financed shock.

Therefore, moving to a balanced budget means that LAMP no longer affects the output multiplier of government spending, although it does increase the response of aggregate consumption under CITR and PEG. For all three monetary policy regimes the level of the output multiplier drops when we switch to a balanced budget.

6.3 Perfectly Competitive Labor Market

The labor market assumptions are important for the effects of LAMP, as first pointed out in Galí et al. (2007) and elaborated in Colciago (2011) and Furlanetto (2011). We therefore repeat the impulse response analyzes for an economy under the assumption of a perfectly competitive labor market in Figures 12-14 in Appendix A.

We no longer have the adjustment costs of union-wages, and households do not meet their share of the aggregate labor demand, but instead supply labor until their marginal rate of substitution equals the equilibrium determined real wage. For the liquidity constrained households this implies that their labor supply is determined solely by the real wage

$$\left(\frac{W_t}{P_t}\right)^{\frac{1}{\sigma}} \left(N_t^L\right)^{-\frac{\varphi}{\sigma}} = (1+\tau_w)\frac{W_t}{P_t}N_t^L - \frac{P_{H,t}}{P_t}T_t^L - \frac{F_t}{P_t}. \tag{31}$$

Therefore, when a government spending shock hits the economy and the increased demand puts an upward pressure on the real wage, households respond by increasing consumption and reducing their labor supply, as leisure is a normal good. Thus wages must spike and then contract in order to increase Ricardian labor supply enough to reestablish equilibrium in the labor market. This real wage increase further reduces liquidity constrained labor supply,[27] and the equilibrium response is much higher than we saw under our unionized labor market: $\widehat{n}_0^L$ is -0.15 (DITR), -0.86 (CITR) and -2.42 (PEG). Constrained consumption responds even more: $\widehat{c}_0^L$ is 0.71 (DITR), 3.54 (CITR) and 9.81 (PEG). As the wage hike is only initial, forward-looking firms do not adjust prices very much due to the adjustment cost. This implies that terms of trade responds very little, as do net exports.[28] As Ricardian consumption only responds to these terms

[27] This follows from equation (31) and our parametrization ($\sigma > 1$).

[28] Unalmis (2012) analyzes a similar shock in a model with a perfectly competitive labor market and a CITR monetary policy. The author assumes identical home bias in government and private consumption, so the shock is not a shift towards more domestic demand. Consequently, terms of trade improve and net exports drop following the fiscal shock.

of trade, the response is much more subdued: $\widehat{c}_0^R$ is -0.40 (DITR), -0.39 (CITR) and -0.38 (PEG).

The liquidity constrained consumption rise dominates under CITR and PEG, where LAMP increases the response of households consumption and output to our fiscal shock. Upon the introduction of LAMP output changes from 0.45 to 0.40 under DITR, from 0.83 to 1.11 under CITR and from 1.36 to 2.67 under the PEG.

Galí et al. (2007) find that labor market frictions are necessary for obtaining a positive consumption response. As the drop in permanent income following the shock will cause Ricardian households to supply more labor, they will meet much of the extra demand caused by a government spending shock. This means that liquidity constrained households have a lower rise in hours and thus disposable income. This income effect is not present in our model, as the complete asset markets provide Ricardian households an insurance against such income losses. This is why we find that in our specification with perfectly competitive labor markets, consumption rises for both CITR (from -0.28 to 0.59) and PEG (-0.18 to 2.16) in our model with LAMP.

7 Concluding Comments

This paper finds that the introduction of LAMP has very small effects in a small open economy. This is in contrast to the closed economy, where LAMP has a considerable effect on the output multiplier, see Galí et al. (2007).

Although the size of the effects is not very large for any of the monetary policy regimes, the direction of the change in the output multiplier differs for the monetary regimes. Even though introducing LAMP in the model will increase to households' consumption response to a government spending shock, this does not carry through to a higher output response under a floating exchange rate regime, where the introduction of LAMP reduced the output multiplier. The reason is that higher consumption under LAMP leads to more wage and price inflation, causing net exports to rise. Under a floating exchange rate with a domestic inflation-based (DITR) or consumer inflation-based Taylor rule (CITR) the latter effect dominates, and LAMP causes a drop in output response. Under a fixed nominal exchange rate (PEG), LAMP increases output when the shock hits but contracts output in the medium run.

We performed robustness checks that indicated that our finding of a small effect of LAMP on output is even more pronounced, when we assume a balanced budget. We further change the labor market assumption to perfect competition, and find that LAMP increases the response of output under CITR and PEG, but reduces this under DITR. As an extension it could be interesting to consider government spending reversals as in Corsetti et al. (2012b), and investigate whether this would affect the implications of LAMP.

We consider the effects of openness in our model with LAMP, and find that this depends on the monetary policy regime. Under a DITR and a PEG more openness decreases the output multiplier, while this is increased under CITR. These results suggest that going forward, the monetary policy regime should be considered, when analyzing and estimating the effects of openness on fiscal multipliers.

References

Stéphane Auray, Beatriz de Blas, and Aurélien Eyquem. Ramsey policies in a small open economy with sticky prices and capital. *Journal of Economic Dynamics and Control*, 35(9):1531–1546, 2011.

Giovanni Di Bartolomeo and Lorenza Rossi. Effectiveness of monetary policy and limited asset market participation: Neoclassical versus keynesian effects. *International Journal of Economic Theory*, 3(3):213–218, 2007.

R. Beetsma and M. Giuliodori. The effects of government purchases shocks: Review and estimates for the eu*. *The Economic Journal*, 121(550):F4–F32, 2011.

Olivier Blanchard and Roberto Perotti. An empirical characterization of the dynamic effects of changes in government spending and taxes on output. *The Quarterly Journal of Economics*, 117(4):1329–1368, 2002.

G.A. Calvo. Staggered prices in a utility-maximizing framework. *Journal of monetary Economics*, 12(3):383–398, 1983.

John Y Campbell and N Gregory Mankiw. Consumption, income and interest rates: Reinterpreting the time series evidence. In *NBER Macroeconomics Annual 1989, Volume 4*, pages 185–246. MIT Press, 1989.

John Y Campbell and N Gregory Mankiw. Permanent income, current income, and consumption. *Journal of Business & Economic Statistics*, 8(3):265–279, 1990.

Raj Chetty, John N Friedman, Soren Leth-Petersen, Torben Nielsen, and Tore Olsen. Active vs. passive decisions and crowdout in retirement savings accounts: Evidence from denmark. *Quarterly Journal of Economics*, Forthcoming, 2014.

Lawrence J Christiano, Martin Eichenbaum, and Charles L Evans. Nominal rigidities and the dynamic effects of a shock to monetary policy. *Journal of political Economy*, 113(1):1–45, 2005.

G. Coenen and R. Straub. Does government spending crowd in private consumption? theory and empirical evidence for the euro area. *International Finance*, 8(3):435–470, 2005.

J.F. Cogan, T. Cwik, J.B. Taylor, and V. Wieland. New keynesian versus old keynesian government spending multipliers. *Journal of Economic Dynamics and Control*, 34(3): 281–295, 2010.

Andrea Colciago. Rule-of-thumb consumers meet sticky wages. *Journal of money, credit and banking*, 43(2-3):325–353, 2011.

Giancarlo Corsetti, Luca Dedola, and Sylvain Leduc. High exchange-rate volatility and low pass-through. *Journal of Monetary Economics*, 55(6):1113–1128, 2008.

Giancarlo Corsetti, André Meier, and Gernot J. Müller. What determines government spending multipliers? *Economic Policy*, 27(72):521–565, 2012a. ISSN 1468-0327.

Giancarlo Corsetti, André Meier, and Gernot J Müller. Fiscal stimulus with spending reversals. *Review of Economics and Statistics*, 94(4):878–895, 2012b.

Christopher J Erceg and Jesper Lindé. Fiscal consolidation in an open economy. 2012.

Christopher J Erceg, Dale W Henderson, and Andrew T Levin. Optimal monetary policy with staggered wage and price contracts. *Journal of monetary Economics*, 46(2):281–313, 2000.

Ester Faia and Tommaso Monacelli. Optimal monetary policy in a small open economy with home bias. *Journal of Money, Credit and Banking*, 40(4):721–750, 2008.

Emmanuel Farhi and Ivan Werning. Fiscal multipliers: Liquidity traps and currency unions. Working Paper 18381, National Bureau of Economic Research, September 2012.

Lorenzo Forni, Libero Monteforte, and Luca Sessa. The general equilibrium effects of fiscal policy: Estimates for the euro area. *Journal of Public Economics*, 93(3):559–585, 2009.

Francesco Furlanetto. Fiscal stimulus and the role of wage rigidity. *Journal of Economic Dynamics and Control*, 35(4):512–527, 2011.

Francesco Furlanetto and Martin Seneca. Rule-of-thumb consumers, productivity, and hours. *The Scandinavian Journal of Economics*, 114(2):658–679, 2012.

J. Galí, J.D. López-Salido, and J. Vallés. Understanding the effects of government spending on consumption. *Journal of the European Economic Association*, 5(1):227–270, 2007.

Jordi Galí and Tommaso Monacelli. Monetary policy and exchange rate volatility in a small open economy. *The Review of Economic Studies*, 72(3):707–734, 2005.

Carlos J. García and Jorge Restrepo. How effective is government spending in a small open economy with distortionary taxes. Technical report, Ilades-Georgetown University, Universidad Alberto Hurtado/School of Economics and Bussines, 2007.

R.E. Hall. By how much does gdp rise if the government buys more output? *Brookings Papers on Economic Activity*, 2009.

Ethan Ilzetzki, Enrique G. Mendoza, and Carlos A. VÃľgh. How big (small?) are fiscal multipliers? *Journal of Monetary Economics*, 60(2):239 – 254, 2013.

V Anton Muscatelli, Patrizio Tirelli, and Carmine Trecroci. Fiscal and monetary policy interactions: Empirical evidence and optimal policy using a structural new-keynesian model. *Journal of Macroeconomics*, 26(2):257–280, 2004.

Emi Nakamura and Jon Steinsson. Fiscal stimulus in a monetary union: Evidence from us regions. *American Economic Review*, 104(3):753–792, 2014.

Gisle James Natvik. Government spending shocks and rule-of-thumb consumers with steady-state inequality. *The Scandinavian Journal of Economics*, 114(4):1414–1436, 2012.

Evi Pappa and Vanghelis Vassilatos. The unbearable tightness of being in a monetary union: Fiscal restrictions and regional stability. *European Economic Review*, 51(6): 1492–1513, 2007.

Valerie A. Ramey. Identifying government spending shocks: It's all in the timing*. *The Quarterly Journal of Economics*, 126(1):1–50, 2011.

Valerie A Ramey and Matthew D Shapiro. Costly capital reallocation and the effects of government spending. In *Carnegie-Rochester Conference Series on Public Policy*, volume 48, pages 145–194. Elsevier, 1998.

Marco Ratto, Werner Roeger, and Jan in 't Veld. Quest iii: An estimated open-economy dsge model of the euro area with fiscal and monetary policy. *Economic Modelling*, 26 (1):222–233, 2009.

Søren Hove Ravn and Morten Spange. The effects of fiscal policy in a small open economy with a fixed exchange rate: The case of denmark. *Open Economies Review*, 25(3):451–476, 2014.

Julio J Rotemberg. Monopolistic price adjustment and aggregate output. *The Review of Economic Studies*, 49(4):517–531, 1982.

G.Y. Salmansen. Government expenditure multiplier in an endogenous switching model. *mimeo.*, 2014a.

G.Y. Salmansen. Determinacy in a small open economy with non-ricardian agents. *mimeo.*, 2014b.

Stephanie Schmitt-Grohé and Martin Uribe. Optimal fiscal and monetary policy in a medium-scale macroeconomic model. *NBER Macroeconomics Annual 2005*, pages 383–462, 2006.

Frank Smets and Raf Wouters. An estimated dynamic stochastic general equilibrium model of the euro area. *Journal of the European economic association*, 1(5):1123–1175, 2003.

John B. Taylor. Discretion versus policy rules in practice. *Carnegie-Rochester Conference Series on Public Policy*, 39(0):195 – 214, 1993.

H. Uhlig. Some fiscal calculus. *American Economic Review*, 100(2):30–34, 2010.

Ibrahim Unalmis. Composition of the government spending and behaviour of the real exchange rate in a small open economy. *Central Bank Review*, 10(1):1–27, 2012.

Michael Woodford. Simple analytics of the government expenditure multiplier. *American Economic Journal: Macroeconomics*, 3(1):1–35, 2011.

A Graphs

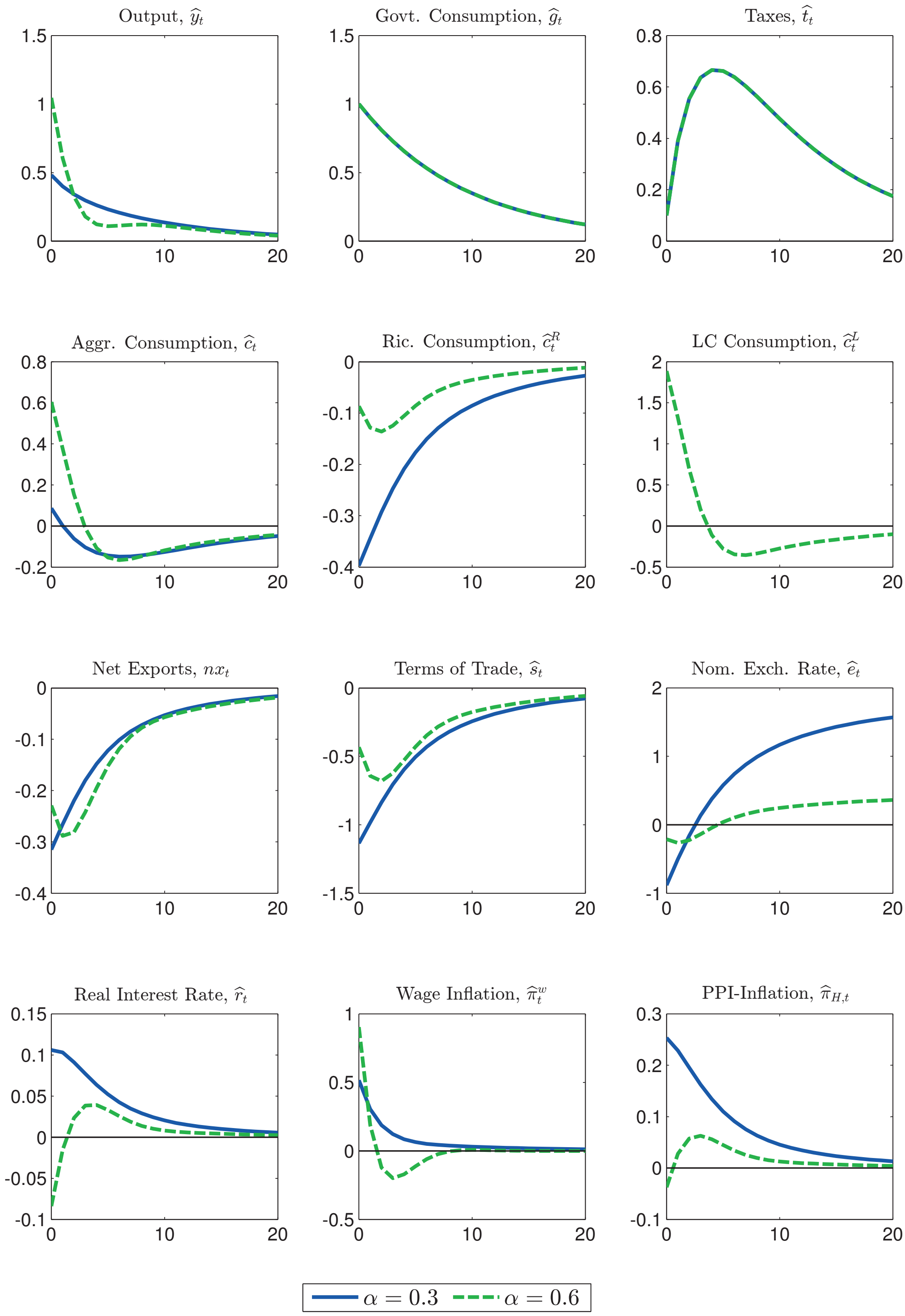

Figure 6: **Effect of openness under DITR:** Effect of a government spending shock under a domestic PPI-based Taylor rule for different degrees of openness in the model with LAMP.

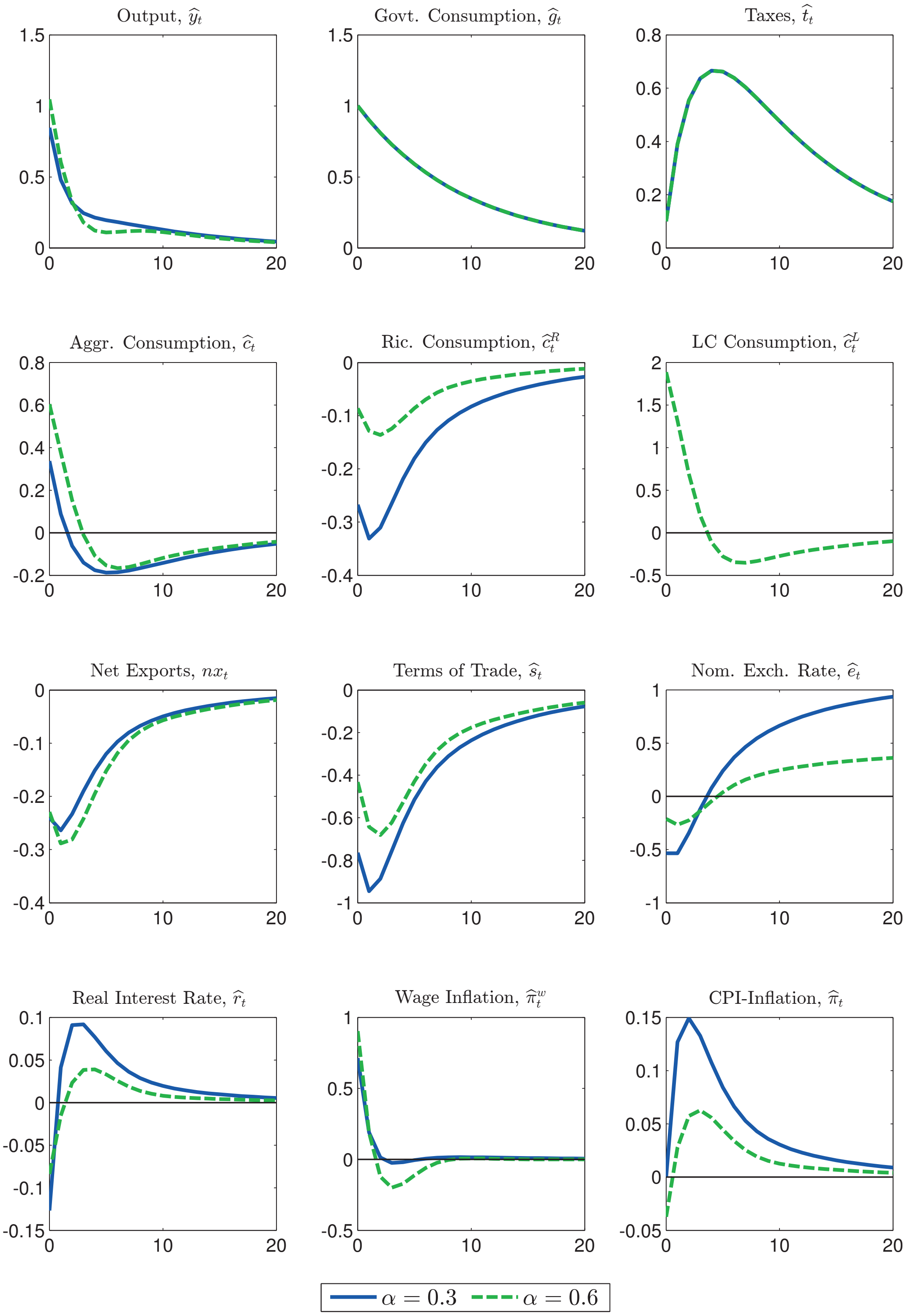

Figure 7: **Effect of openness under CITR:** Effect of a government spending shock under a CPI-based Taylor rule for different degrees of openness in the model with LAMP.

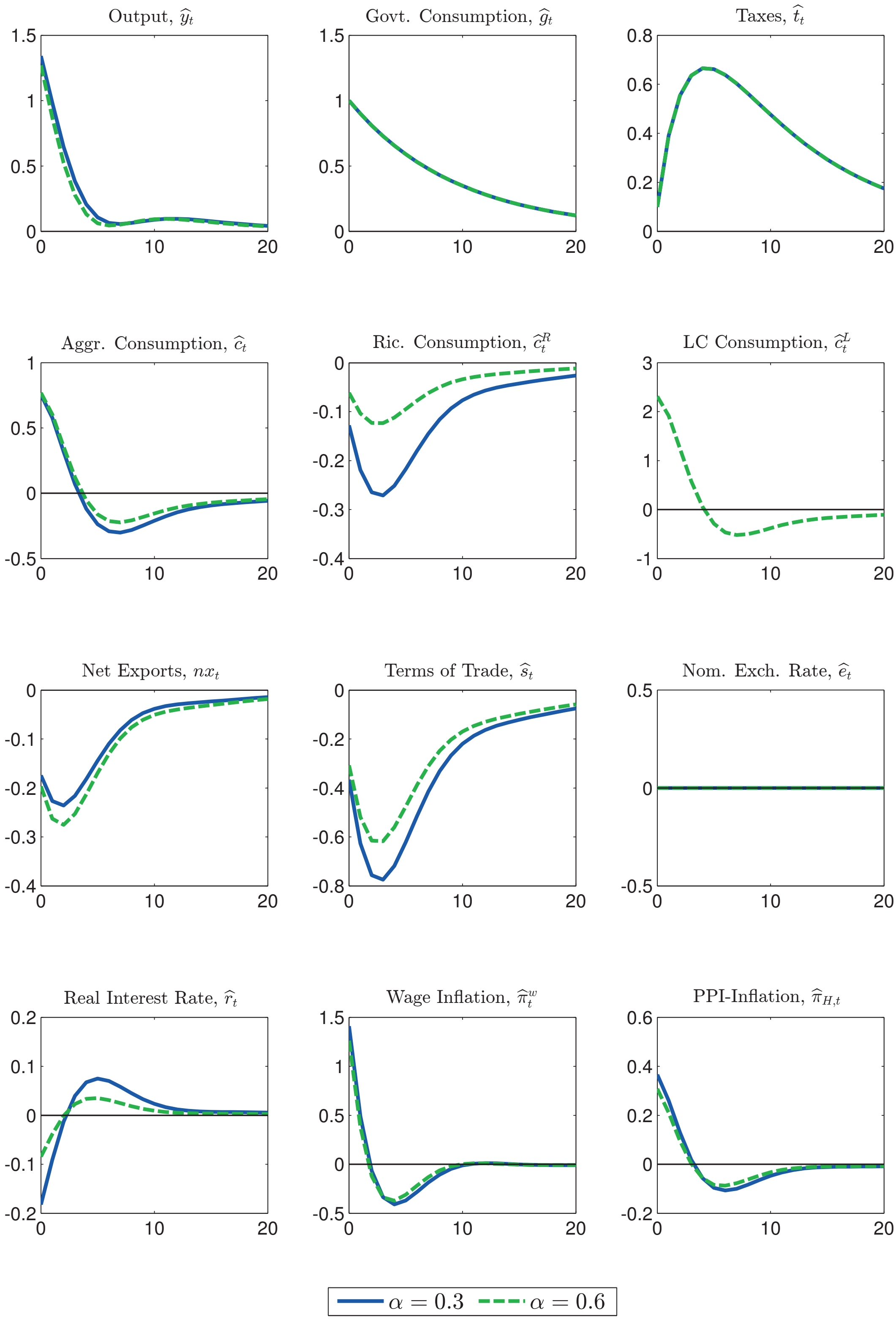

Figure 8: **Effect of openness under a PEG:** Effect of a government ppending shock under a fixed nominal exchange rate for different degrees of trade in the model with LAMP.

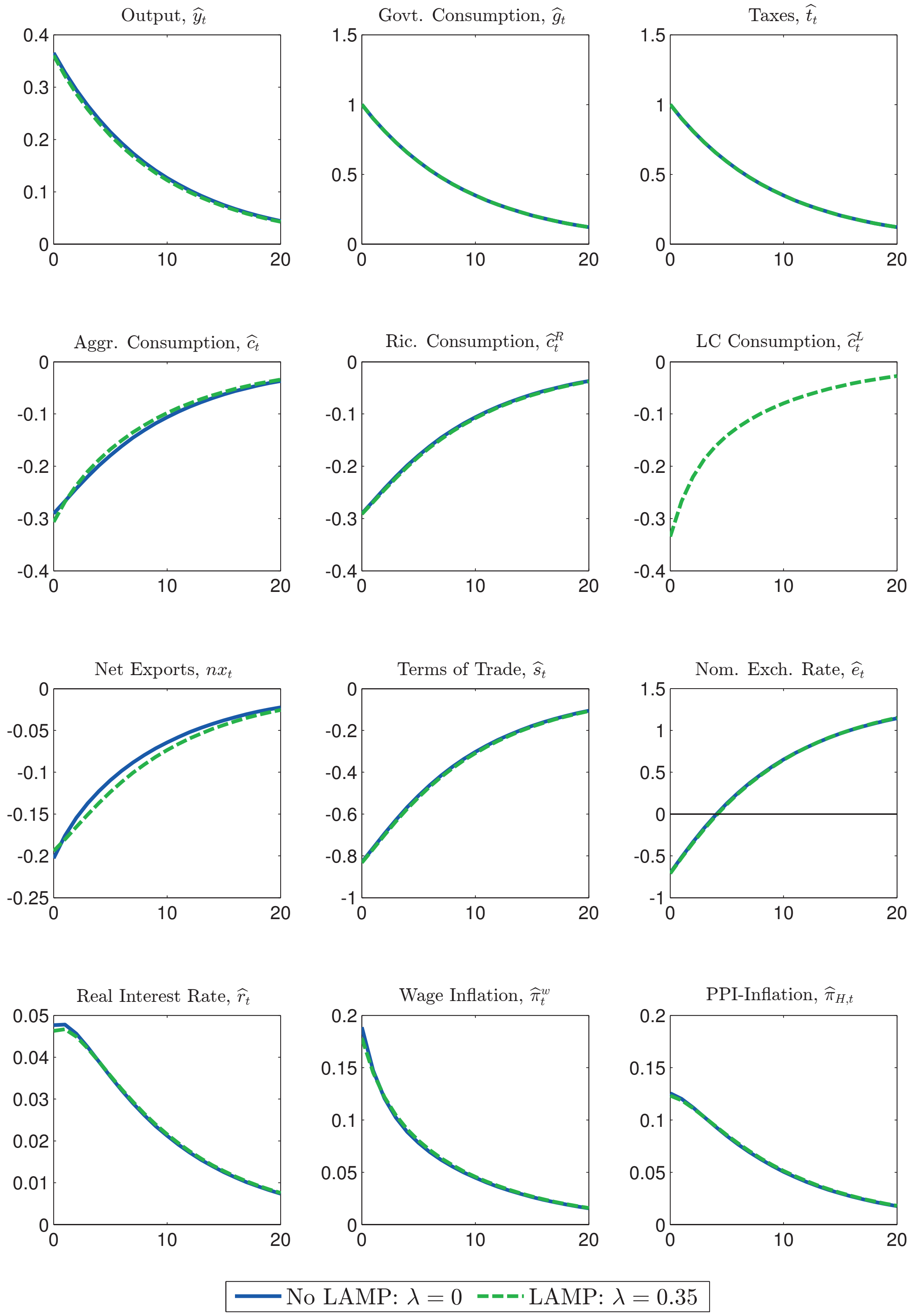

Figure 9: **Effect of LAMP under DITR and a balanced budget:** Effect of a government spending shock under domestic PPI-based inflation in a model with and without LAMP.

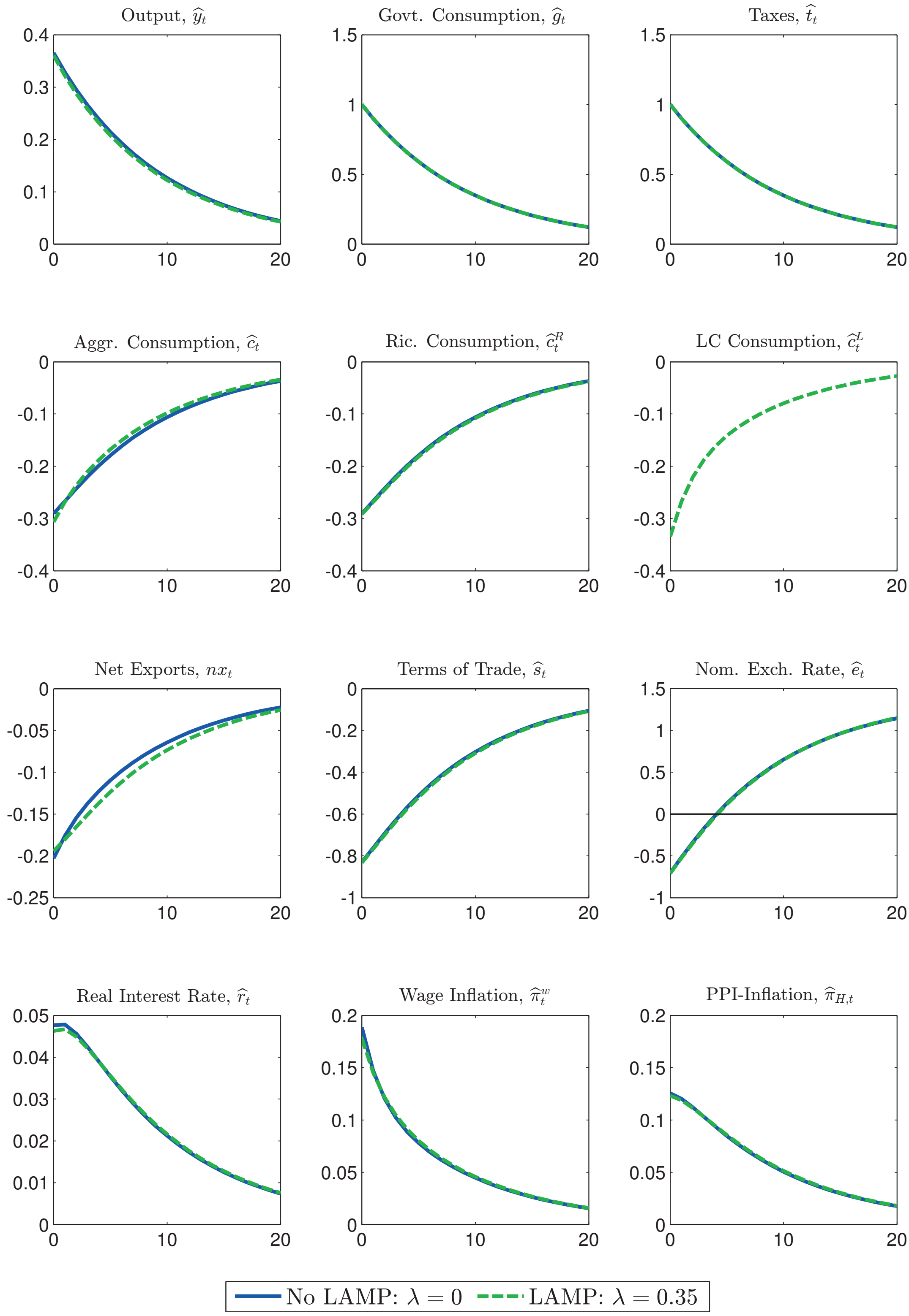

Figure 10: **Effect of LAMP under CITR and a balanced budget:** Effect of a government spending shock under CPI-based inflation in a model with and without LAMP.

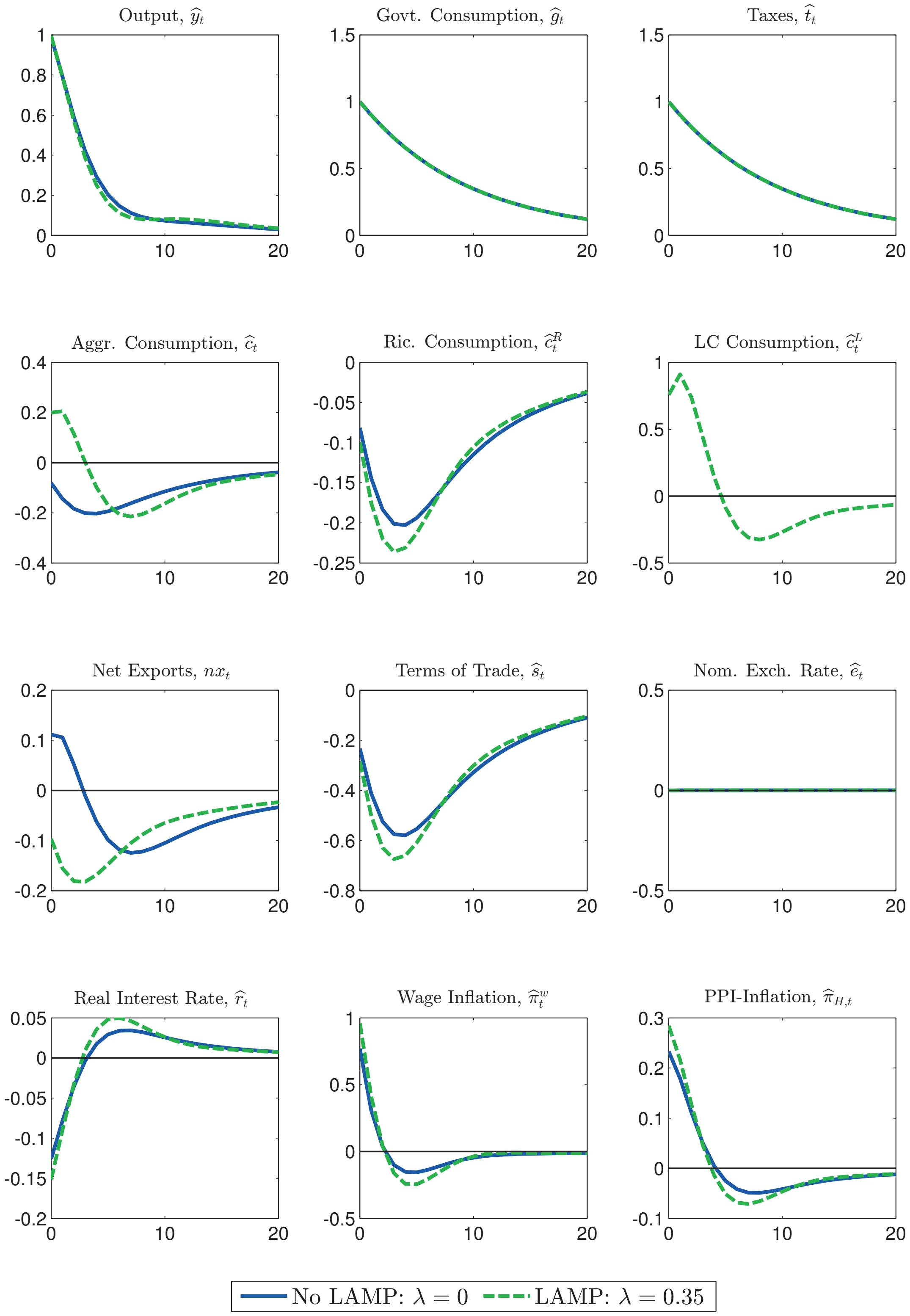

Figure 11: **Effect of LAMP under a PEG and a balanced budget:** Effect of a government spending shock under a fixed nominal exchange rate in a model with and without LAMP.

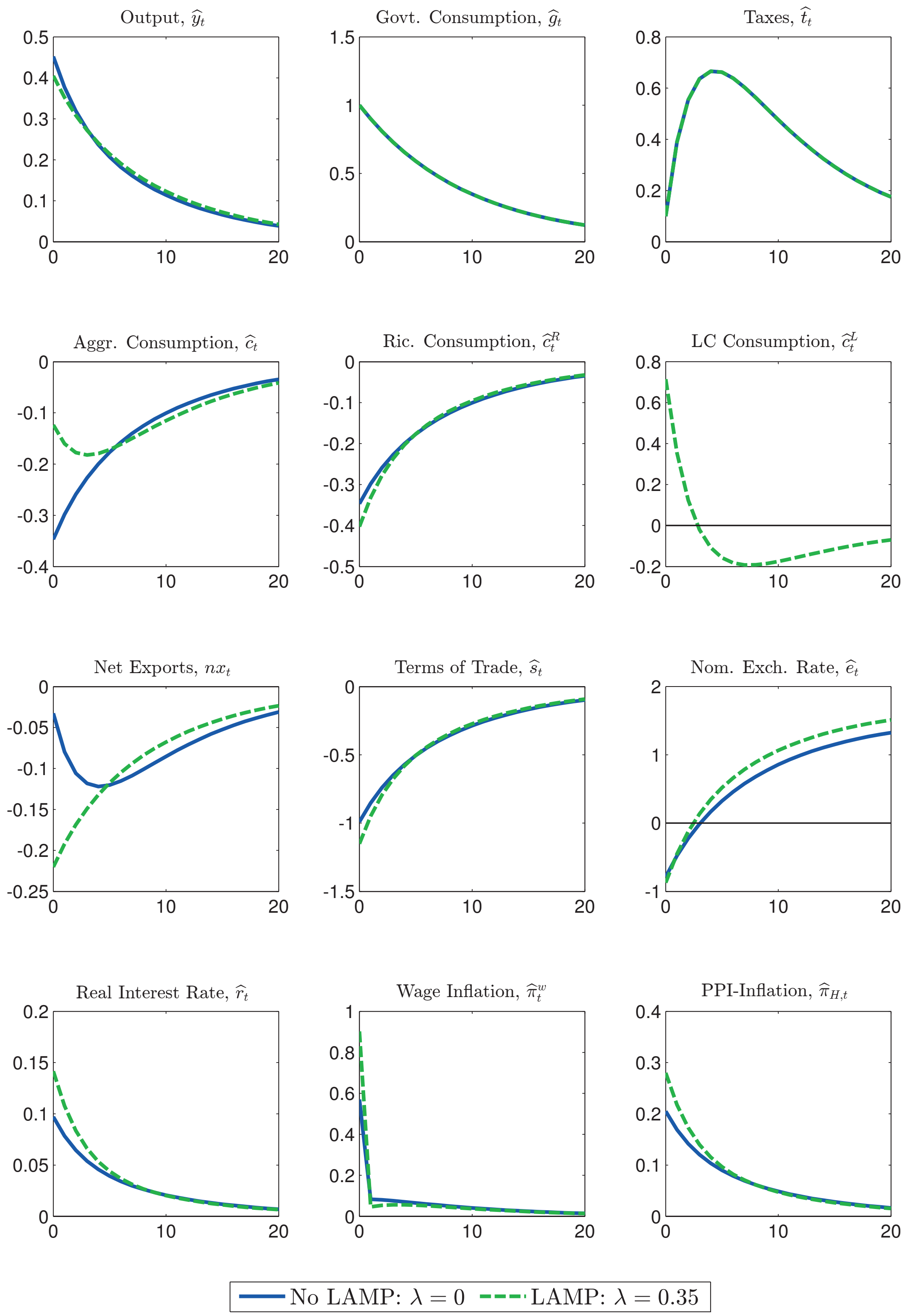

Figure 12: **Effect of LAMP under DITR and perfect competition i the labor market:** Effect of a government spending shock under domestic PPI-based Taylor rule with and without LAMP.

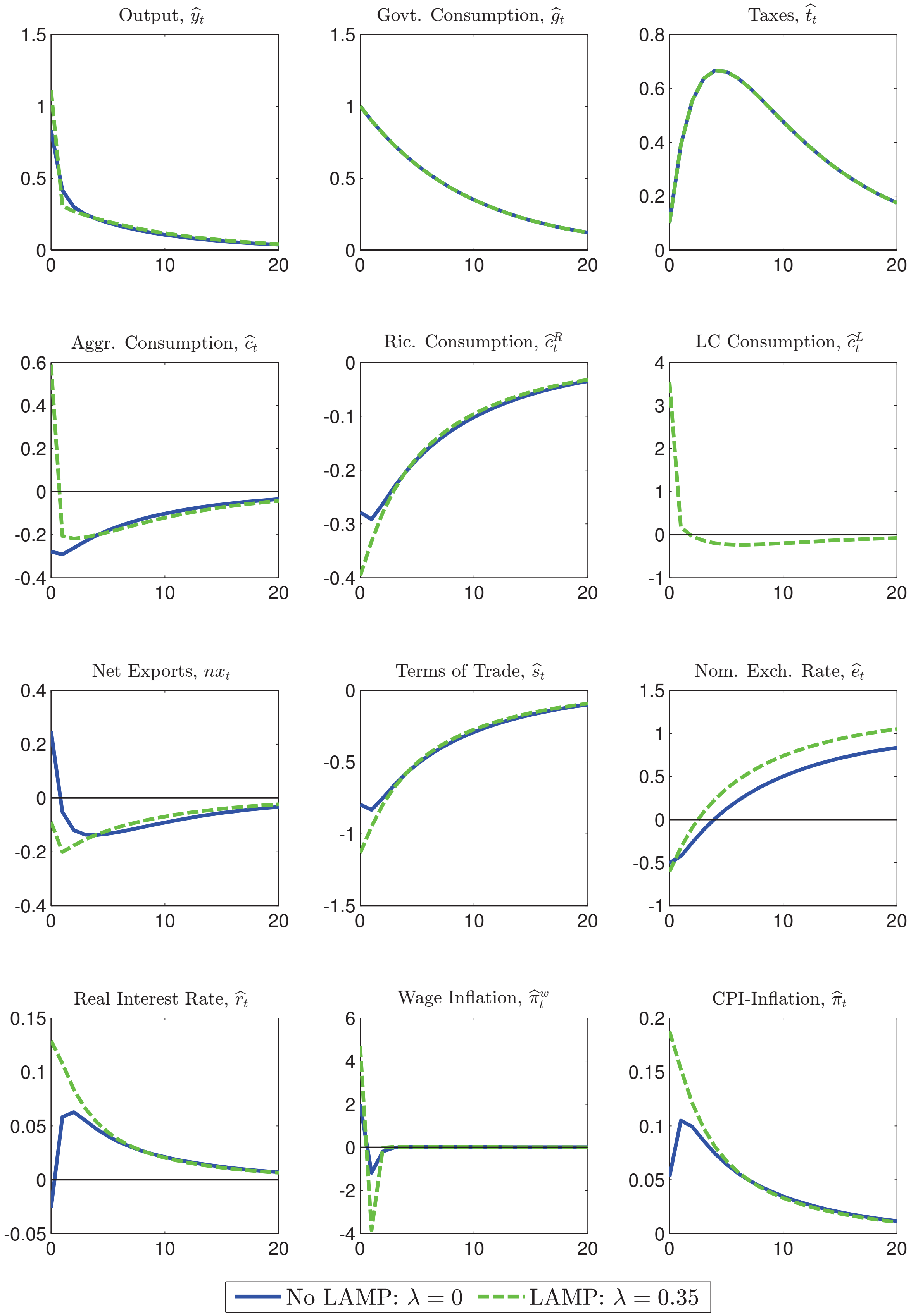

Figure 13: **Effect of LAMP under CITR and perfect competition i the labor market:** Effect of a government spending shock under CPI-based Taylor rule with and without LAMP.

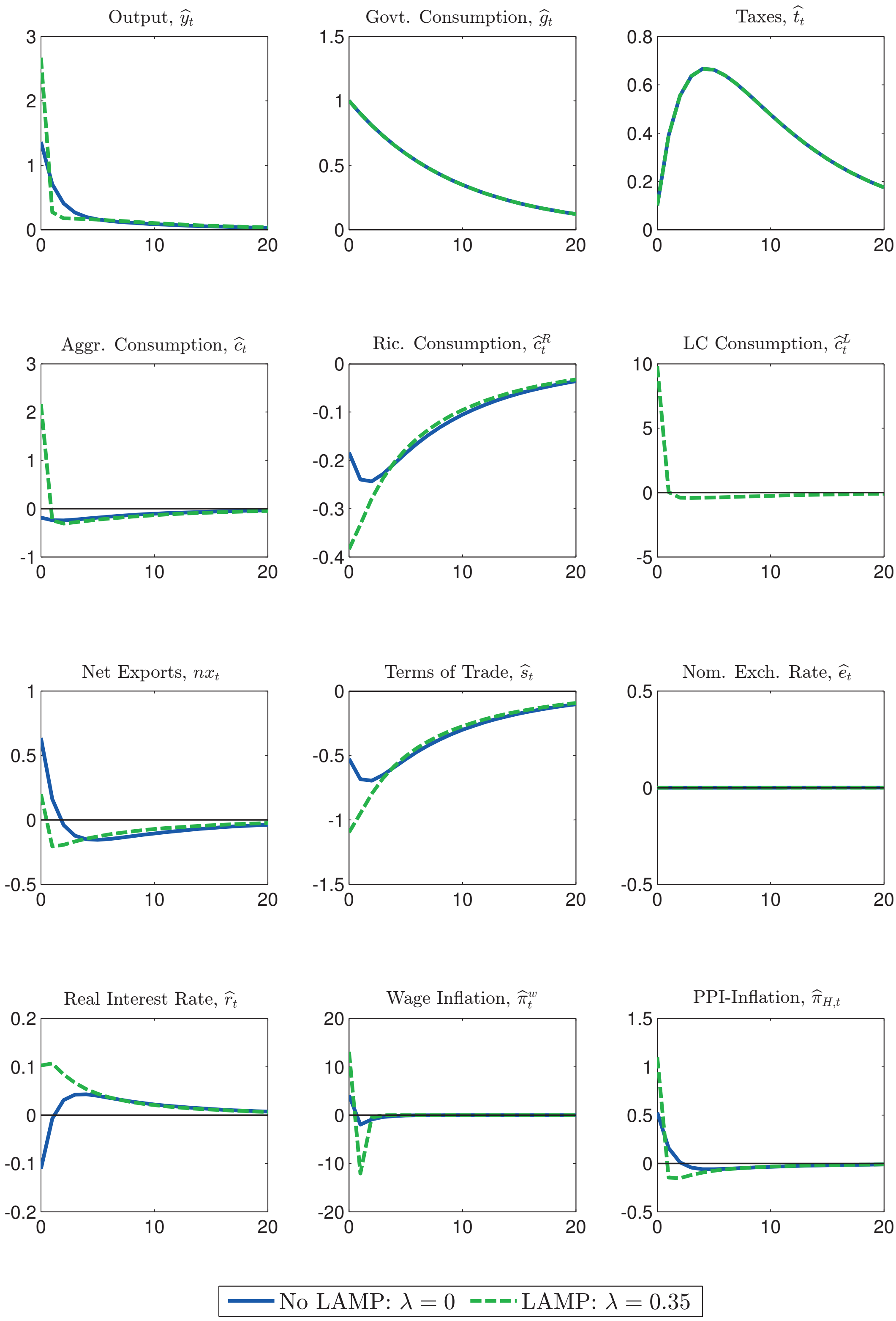

Figure 14: **Effect of LAMP under a PEG and perfect competition i the labor market:** Effect of a government spending shock under a fixed nominal exchange rate with and without LAMP.

B Equilibrium conditions

The equilibrium conditions of the model are listed for allowing the reader a greater overview

$$
\begin{aligned}
\text{Euler equation} &: \quad \beta R_t E_t \left[\left(\frac{C_{t+1}^R}{C_t^R} \right)^{-\sigma} \Pi_t^{-1} \right] = 1 \\
\text{Disposable income} &: \quad P_t C_t^L = W_t N_t + \overline{T}_t - F_t \\
\text{Risk sharing} &: \quad \left(C_t^R\right)^\sigma = \left(C_t^*\right)^\sigma \left[(1-\alpha)(S_t)^{-(1-\eta)} + \alpha \right]^{-\frac{1}{1-\eta}} \\
\text{Inflation rates} &: \quad \Pi_t = \frac{\left[(1-\alpha) + \alpha (S_t)^{1-\eta}\right]^{\frac{1}{1-\eta}}}{\left[(1-\alpha) + \alpha (S_{t-1})^{1-\eta}\right]^{\frac{1}{1-\eta}}} \Pi_{H,t} \\
\text{Union Fee} &: \quad F_t = \frac{\phi_w}{2} \left(\Pi_t^w - 1\right)^2 W_t N_t \\
\text{Wage setting} &: \quad 0 = \left(MRS_t^A\right)^{-1} \frac{W_t}{P_t} \left[(1+\tau_w)(1-e_w) - \phi_w \left(\Pi_t^w - 1\right) \Pi_t^w\right] N_t^{1+\varphi} + e_w N_t^{1+\varphi} \\
& \quad +\beta \left(MRS_{t+1}^A\right)^{-1} \frac{W_{t+1}}{P_{t+1}} \phi_w \left(\Pi_{t+1}^w - 1\right) \Pi_{t+1}^w N_{t+1}^{1+\varphi} \\
\text{Price setting} &: \quad \left(\Pi_{H,t} - 1\right) \Pi_{H,t} = \beta E_t \left[\frac{\left(C_{t+1}^R\right)^{-\sigma}}{\left(C_t^R\right)^{-\sigma}} \frac{g(S_t)}{g(S_{t+1})} \left(\Pi_{H,t+1} - 1\right) \Pi_{H,t+1} \right] \\
& \quad + \frac{e}{\phi_p} Y_t \left[\frac{(1-\tau_p)}{A_t} W_t^{\text{real}} g(S_t) - \frac{e-1}{e} \right] \\
\text{Fiscal policy} &: \quad \overline{T}_t = B_t^{\phi_b} G_t^{\phi_g} \\
&: \quad R_t^{-1} B_{t+1} = B_t + P_{H,t} G_t - P_{H,t} \overline{T}_t \\
\text{Aggregate production} &: \quad Y_t = A_t N_t - \frac{\phi_p}{2} \left(\Pi_{H,t} - 1\right)^2 \\
\text{Good market clearing} &: \quad Y_t = g(S_t)^\eta (1-\alpha) \lambda C_t^r + g(S_t)^\eta (1-\alpha)(1-\lambda) C_t^R + \alpha S_t^\gamma C_t^* + G_t
\end{aligned}
$$

B.1 Steady State

B.1.1 Economy without LAMP

Assuming a zero inflation steady state and recalling that government debt is zero in steady state, the equilibrium conditions become

$$
\begin{aligned}
\text{Euler equation} &: \quad \beta R = 1 \\
\text{Disposable income} &: \quad C^L = W^{\text{real}} N + \overline{T} \\
\text{Risk sharing} &: \quad C^R = C^* \\
\text{Inflation rates} &: \quad \Pi = \Pi_H = 1 \\
\text{Union Fee} &: \quad F = 0 \\
\text{Wage setting} &: \quad W^{\text{real}} \left[(1+\tau_w)(e_w - 1) \right] = e_w MRS^A \\
\text{Price setting} &: \quad (1-\tau_p) W_t^{\text{real}} = \frac{e-1}{e} \\
\text{Fiscal policy} &: \quad \overline{T} = G \\
\text{Aggregate production} &: \quad Y = N \\
\text{Good market clearing} &: \quad Y_t = (1-\alpha) C + \alpha C_t^* + G
\end{aligned}
$$

Further, the budget constraint of the Ricardian households collapses to be identical to the ones for liquidity constrained households, hence $C^R = C^L = C = C^*$.

The markup-removing taxes, $\tau_p = \frac{1}{e}$ and $\tau_w = \frac{1}{e_w - 1}$, imply the following steady state values of C, N, Y and W^{real}.

$$
\begin{aligned}
Y &= N = (\gamma_c)^{-\frac{\sigma}{\varphi+\sigma}} \\
C &= (\gamma_c)^{\frac{\varphi}{\varphi+\sigma}} \\
W_t^{\text{real}} &= 1
\end{aligned}
$$

using that $MRS^A = C^\sigma N^\varphi$.

C Log-linear version of the model

The equilibrium conditions of the model are listed for allowing the reader a greater overview. The log linearized model where lower case variables represents the logarithmic deviation of that respectable variable, that is $\widehat{x}_t \equiv \frac{X_t - X}{X}$ and inflation rates are defined as $\pi_t = p_t - p_{t-1}$. For the fiscal policies, we normalize by steady state output so that $\bar{t}_t \equiv \left(\overline{T}_t - \overline{T}\right)/Y$, $\widehat{g}_t \equiv (G_t - G)/Y$, and $\widehat{b}_t \equiv (B_t/P_t - B/P)/Y$

$$
\begin{aligned}
\text{Euler equation} &: \quad \widehat{c}_t^R = E_t\left[\widehat{c}_{t+1}^R\right] - \frac{1}{\sigma}\left(\widehat{i}_t - E_t\left[\widehat{\pi}_{t+1}\right]\right) \\
\text{Risk sharing} &: \quad \widehat{c}_t^R = \widehat{c}_t^* + \frac{1-\alpha}{\sigma}\widehat{s}_t && (32) \\
\text{Disposable income} &: \quad \gamma_c \widehat{c}_t^L = (\widehat{w}_t - \widehat{p}_t + \widehat{n}_t) - \widehat{t}_t \\
\text{Inflation rates} &: \quad \widehat{\pi}_{H,t} = \widehat{\pi}_t - \alpha \Delta \widehat{s}_t \\
\text{Union Fee} &: \quad f_t = 0 && (33) \\
\text{Wage setting} &: \quad \widehat{\pi}_t^w = \beta E_t \widehat{\pi}_{t+1}^w - \frac{e_w - 1}{\phi_w}\left(\widehat{w}_t^{\text{real}} - \widehat{mrs}_t^A\right) && (34) \\
&: \quad \widehat{mrs}_t^A = \sigma\left((1-\lambda)\ \widehat{c}_t^o + \lambda\ \widehat{c}_t^r\right) + \varphi \widehat{n}_t && (35) \\
\text{Price setting} &: \quad \widehat{\pi}_{H,t} = \beta E_t \widehat{\pi}_{H,t+1} + \frac{e-1}{\phi_p}\left(\widehat{w}_t^{\text{real}} + \alpha \widehat{s}_t - \widehat{a}_t\right) && (36) \\
\text{Fiscal policy} &: \quad \bar{t}_t = \phi_b b_t + \phi_g \widehat{g}_t && (37) \\
&: \quad \widehat{b}_{t+1} = R\left(\widehat{b}_t + \widehat{g}_t - \bar{t}_t\right) && (38) \\
\text{Aggregate production} &: \quad \widehat{y}_t = \widehat{a}_t + \widehat{n}_t \\
\text{Good market clearing} &: \quad \widehat{y}_t = \gamma_c (1-\alpha)\left((1-\lambda)\ \widehat{c}_t^R + \lambda\ \widehat{c}_t^L\right) + \alpha \gamma_c \widehat{c}_t^* \\
&: \quad + \widehat{g}_t + \alpha \gamma_c (\eta(1-\alpha) + \gamma)\,\widehat{s}_t && (39)
\end{aligned}
$$

Note that the union membership is zero at a first order approximation, due to the assumption that $\Pi^w = 0$. We thus omit the union membership fee in our DYNARE code for the log-linear model.

Taking a first order approximation to net export gives

$$
nx_t = \widehat{y}_t - \gamma_c \widehat{c}_t - \widehat{g}_t - \gamma_c \alpha \widehat{s}_t \tag{40}
$$

Inserting the good market equilibrium, we have

$$
\widehat{nx}_t = \gamma_c (\alpha\lambda)\left(\widehat{c}_t^o - \widehat{c}_t^r\right) + \gamma_c \alpha\left(\frac{\omega}{\sigma} - 1\right)\widehat{s}_t. \tag{41}
$$

D Government debt

We now derive conditions for a non-explosive debt, following Galí et al. (2007). Substituting the tax rule into the government budget constraint, we have

$$b_{t+1} = R\left(1-\phi_b\right) b_t + R\left(1-\phi_g\right) \widehat{g}_t \tag{42}$$

Thus, in our log-linear model, a necessary and sufficient condition for a non-explosive debt is $R\left(1-\phi_b\right) < 1$ or, as $R = \beta^{-1}$this condition is equivalent to

$$\phi_b > 1 - \beta$$