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PRECAUTIONARY BORROWING AND THE CREDIT CARD DEBT PUZZLE

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PRECAUTIONARY BORROWING AND THE CREDIT CARD DEBT PUZZLE

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ABSTRACT

This paper explores a rational economic explanation for the much discussed credit card debt puzzle. We set-up and simulate a generalization of the buffer-stock consumption model with long-term revolving debt contracts. In line with US credit card law, lenders can always deny households access to new debt, but they cannot demand immediate repayment of the outstanding balance. Under this assumption it is indeed optimal for households to simultaneously hold positive gross debt and positive gross assets even when the interest rate on the debt is much higher than the return rate on the assets. When the risk of being excluded from new borrowing is positively correlated with unemployment, we are able to explain a substantial share of the observed borrower-saver group and match a high level of liquid net worth.

Precautionary Borrowing and the Credit Card Debt Puzzle*

Jeppe Druedahl† and Casper Nordal Jørgensen‡

August 30, 2015

Abstract

This paper addresses the credit card debt puzzle using a generalization of the buffer-stock consumption model with long-term revolving debt contracts. Closely resembling actual US credit card law, we assume that card issuers can always deny their cardholders access to new debt, but that they cannot demand immediate repayment of the outstanding balance. Hereby, current debt can potentially soften a household's borrowing constraint in future periods and thus provides extra liquidity. We show that for some intermediate values of financial net worth it is indeed optimal for households to simultaneously hold positive gross debt and positive gross assets even though the interest rate on the debt is much higher than the return rate on the assets. Including a risk of being excluded from new borrowing which is positively correlated with unemployment, we are able to simultaneously explain a substantial share of the observed borrower-saver group and match a high level of liquid net worth.

Keywords: Credit Card Debt Puzzle, Precautionary Saving, Consumption.

JEL-Codes: E21, D14, D91.

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1 Introduction

Beginning with Gross and Souleles (2002) it has been repeatedly shown that many households persistently have both expensive credit card debt and hold low return liquid assets. This apparent violation of the no-arbitrage condition has been termed the "credit card debt puzzle", and no resolution has yet been generally accepted (see e.g. the surveys by Tufano (2009) and Guiso and Sodini (2013)).

This paper suggests a new explanation of the puzzle based on *precautionary borrowing*. We begin from the observation that credit card debt is actually a long-term revolving debt contract. Specifically under current US law the card issuer can cancel a credit card at any time, and thus instantly stop the card holder from accumulating additional debt. Contrarily the card issuer *cannot* force the card holder to immediately pay back the remaining balance. Depending on the specific credit card agreement the issuer might be able to increase the minimum payment somewhat, but basically the credit card debt is transformed into an installment loan.[1]

We add such long-term revolving debt contracts, which are partially irrevocable from the lender side, to an otherwise standard buffer-stock consumption model a la Carroll (1992, 1997, 2012). Hereby households gain a motive for *precautionary borrowing* because current debt can potentially relax the borrowing constraint in future periods. For equal (and risk-less) interest rates on debt and assets, the households will therefore always accumulate as much debt as possible maximizing the option value of having a large gross debt. In the more plausible case of a higher interest rate on debt than on assets, there is a trade-off between the benefit of the extra liquidity provided by the debt, and the net cost of the balance sheet expansion.

We further amplify the motive for precautionary borrowing by including credit risk in the model. Specifically we assume that households in any given period might be excluded from new borrowing, and that the risk of this increases under unemployment. The US *Consumer Financial Protection Bureau (CFBP)* shows in its "CARD Act Report" that "over 275 million accounts were closed from July 2008 to December 2012, driving a $1.7 trillion reduction in total [credit] line" (p. 56, October 2013). It is not clear to which extend this was a demand or supply effect, but anecdotal evidence suggests that the credit card companies unilaterally

[1] We thank the *National Consumer Law Center* and the *Consumer Financial Protection Bureau* for help in clarifying the rules for us.

changed their lending during the Great Recession, and that the supply effect thus dominated. Consequently, getting a credit card closed seems to be something a rational household should fear. Naturally, households might have an outside option of getting a new credit card at another issuer, but if a household is simultaneously hit by unemployment this might prove impossible.

Based on a careful calibration, we show numerically that there exists a range of intermediate values of net worth for which it is indeed optimal for the households to simultaneously hold positive gross debt and positive gross assets, even though the interest rate on the debt is much higher than the return rate on the assets. This is especially true when we assume that bad income shocks are positively correlated with a high risk of a fall in the availability of new credit. Beyond this, the parametric robustness of our results are rather strong, and we can explain a large part of the observed puzzle group of borrower-savers while matching central moments from the U.S. *Survey of Consumer Finance* (SCF) including a high level of liquid net worth. This indicates that precautionary borrowing is central in understanding the credit card debt puzzle.

We are somewhat cautious in precisely quantifying the importance of precautionary borrowing, because our model for computational reasons does not include illiquid assets (e.g. houses). It is thus not able to match the empirical facts on *total* net worth without muting the precautionary motive completely. Note, however, that Kaplan and Violante (2014) have recently shown that a buffer-stock model with an illiquid asset, subject to transaction costs, can generate a significant share of *wealthy hand-to-mouth* households while still matching total net worth moments. We hypothesize that both poor and wealthy hands-to-mouth households would also rely on precautionary borrowing, and that our results are thus at least qualitatively robust to extending our model in this direction.

The importance of going beyond one-period debt contracts has naturally been noted before. Closest to our paper are Attanasio, Leicester and Wakefield (2011), Attanasio, Bottazzi, Low, Nesheim and Wakefield (2012), Chen, Michaux and Roussanov (2013) and Halket and Vasudev (2014) who all introduce long-term mortgage contracts, and Alan, Crossley and Low (2012) who model the "credit crunch" of 2008 in terms of a drying up of new borrowing (a flow constraint) instead of a recall of existing loans (the typical change in the stock constraint).[2]

[2] Note that Alan, Crossley and Low (2012) use the term "precautionary borrowing" (borrowing for a rainy day) in a somewhat different fashion than we do because the second asset in their model is a high return risky asset. This e.g. implies that wealthy households also blow up

To the best of our knowledge, Fulford (2015) is the only other paper investigating the importance of multi-period debt contracts for the credit card debt puzzle.[3] Our approach differs from his in a number of important ways. Firstly his model does not include any forced repayment schedule and households are thus (unrealistically) allowed to hold on to once accumulated debt forever. Secondly our formulation of the income process better mimics reality by taking into account *permanent* shocks and *non-zero* income growth which both usually are important in models with a precautionary motive. Consequently our model nests the standard buffer-stock model as a limiting case, while his does not. Thirdly we allow the risk of losing access to the credit market to be positively correlated with unemployment. We show that this is empirically relevant and quantitatively important for explaining a size-able puzzle group. This is especially relevant because introducing permanent shocks strengthens the general precautionary saving motive making the households accumulate a precautionary fund, which diminishes the need for precautionary borrowing and reduces the size of the puzzle group. Fourthly, we are able to explain a large part of the observed puzzle group and simultaneously match a high level of mean liquid net worth, and do so with a more plausible discount factor of 0.90 while Fulford (2015) use a very low discount factor of 0.794.

The paper is structured as follows. Section 2 discusses the related literature. Section 3 presents the model and describes the the solution algorithm briefly. Some stylized facts are presented in section 4 to which the model is calibrated in section 5. Section 6 presents the central results. The welfare gain of the potential for precautionary borrowing is quantified in section 7 and various robustness checks are performed in section 8. Section 9 concludes. Some details are relegated to the appendices A and B.

2 Related Literature

2.1 Empirical Evidence

Gross and Souleles (2002) showed that in the 1995 *Survey of Consumer Finance* (SCF), and in a monthly sample of credit card holders from 1995-98, almost all

their balance sheet by taking loans to invest in the risky asset.

[3] We were only made aware of the working paper version of his paper after writing the first draft of the present paper.

households with credit card debt held low return liquid assets (e.g. they had funds in checking or saving accounts). In itself this might not be an arbitrage violation, but could be a pure timing issue if the interview took place just after pay day and just before the credit card bill was due. However, a third of their sample held liquid assets larger than one month's income; without any further explanation this certainly seems to be an arbitrage violation.

Their result has been found to be robust to alternative definitions of the puzzle group[4] and stable across time periods (see Telyukova and Wright (2008), Telyukova (2013), Bertaut, Haliassos and Reiter (2009), Kaplan, Violante and Weidner (2014) and Fulford (2015)). Telyukova (2013) e.g. utilizes certain questions in the SCF to ensure that the households in the puzzle group had credit card debt left over after the last statement was paid, and that they either only occasionally or never repay their balance in full. Recently Gathergood and Weber (2014) has shown that the puzzle is also present in UK data, and that the puzzle group also has many and large expensive installment loans (e.g. car loans).[5]

Across samples and time periods the interest rate differential between the credit card debt and the liquid assets considered has typically been around 8-12 percentage points, and thus economically very significant. Depending on the correction for timing this implies that the net cost of the expanded balance sheets of the puzzle group has been calculated to be in the range of 0.5-1.5 percent of household income.

2.2 Other Theoretical Explanations

A number of different rational and behavioral explanations of the credit card debt puzzle has been suggested in the literature. First, Gross and Souleles (2002) informally suggested that a behavioral model of either self/spouse-control or mental accounting might be necessary to explain the puzzle.[6] Bertaut and Haliassos (2002),

[4] We denote the group of households simultaneously holding both liquid assets and credit card debt as *the puzzle group*.

[5] Looking over the life-cycle the puzzle group is smallest among the young (below 30) and old (above 60). Puzzle households are typically found to be in the middle of the income distribution and have at least average education and financial literacy. Many have sizeable illiquid wealth (e.g. housing and retirement accounts). There is also some evidence of persistence in puzzle status, and in total it thus seems hard to explain the puzzle as a result of simple mistakes or financial illiteracy.

[6] Note that behavioral models with hyperbolic discounting and a present bias such as Laibson, Repetto and Tobacman (2003) can explain that households with credit card debt has illiquid

Haliassos and Reiter (2007) and Bertaut, Haliassos and Reiter (2009) formalized this insight into an accountant-shopper model where a fully rational accountant tries to control an impulsive (i.e. more impatient) fully rational shopper (a different self or a spouse). The shopper can only purchase goods with the credit card which has an upper credit limit, and the accountant thus has a motive to not use all liquid assets to pay off the card balance in order to limit the consumption possibilities of the shopper. Gathergood and Weber (2014) provides some empirical evidence that a large proportion of households in the puzzle group appears to be impulsive spenders and heavy discounter of the future. A fundamental problem with this solution of the puzzle, however, is that it is not clear why the accountant cannot utilize cheaper control mechanisms such as adjusting the credit limit or limiting the shopper's access to credit cards. Furthermore many households with credit cards also have debit cards, which imply that the shopper in practice has direct access to at least some of the household's liquid assets.

Second, beginning with Lehnert and Maki (2007), and continuing with Lopes (2008) and latest Mankart (2014), it has suggested that US bankruptcy laws might make it optimal for households to strategically accumulate credit card debt in order to purchase exemptible assets in the run up to a bankruptcy filing. Even though state level variation in the size of the puzzle group and exemption levels seems to support this explanation, the empirical power seems limited because it is only relevant relatively shortly before a filling.[7] Moreover many households in the puzzle group have both significant financial assets (e.g. bonds and stocks) and non-financial assets (e.g. cars and houses), and generally few households ever file for bankruptcy. Finally it is far from obvious that such a motive for strategic accumulation of exemptible assets can explain the evidence from the UK (see Gathergood and Weber (2014)) which generally has more creditor friendly bankruptcy laws.

A third resolution of the puzzle has been presented by Telyukova (2013) (see also Telyukova and Wright (2008) and Zinman (2007)). She argues that many expenditures (e.g. rents and mortgage payments) can only be paid for by using cash, and that households thus have a classical Hicksian motive for holding liquid assets despite having expensive credit card debt. The strength of this demand for

assets, but not that they hold fully liquid assets.

[7] Mankart (2014) notes that debt and cash-advances made shortly before the bankruptcy filling (60 or 90 days depending on the time period) are not dischargeable above a rather low threshold.

liquidity is amplified in her model by rather volatile taste shocks for goods that can only be paid for with cash (e.g. many home and auto repairs). It is naturally hard to identify these fundamentally unobserved shocks and their size in the data. A more serious empirical problem is that the use of credit cards has become much more widespread in the last 20 years; in the model this should imply a fall in the size of the puzzle group not seen in the data. Adding a (costly) cash-out option on the credit card to the model, as is now common, could also further reduce the implied size of the puzzle group. In total, this demand for cash might certainly be a contributing factor, but it seems unlikely that it is the central explanation of the credit card debt puzzle. Finally, note that in a model with both a Hicksian motive for holding liquid assets and a precautionary borrowing motive, the two would reinforce each other.

3 Model

3.1 Bellman Equation

We consider potentially infinitely lived households characterized by a vector, $\mathbf{S}_t$, of the following state variables: end-of-period gross debt (D_{t-1}), end-of-period gross assets (A_{t-1}), market income (Y_t), permanent income (P_t), an unemployment indicator, $u_t \in \{0, 1\}$, and an indicator for whether the household is currently excluded from new borrowing, $x_t \in \{0, 1\}$. In each period the households choose *consumption*, C_t, and *debt*, D_t, to maximize expected discounted utility.

Postponing the specification of the exogenous and stochastic income process to section 3.3, the household optimization problem is given in recursive form by

$$
\begin{aligned}
V(\mathbf{S}_t) &= \max_{D_t, C_t} \frac{C_t^{1-\rho}}{1-\rho} + \beta \cdot \mathbb{E}_t \left[V(\mathbf{S}_{t+1}) \right] && (3.1) \\
& \text{s.t.} \\
A_t &= (1 + r_a) \cdot A_{t-1} + Y_t - C_t && (3.2) \\
& \quad - \underbrace{r_d \cdot D_{t-1}}_{\text{interest}} - \underbrace{\lambda \cdot D_{t-1}}_{\text{installment}} + \underbrace{(D_t - (1-\lambda) \cdot D_{t-1})}_{\text{new debt}} \\
N_t &= A_t - D_t && (3.3) \\
D_t &\leq \max \left\{ \underbrace{(1-\lambda) \cdot D_{t-1}}_{\text{old contract}}, \mathbf{1}_{x_t = 0} \cdot \underbrace{(\eta \cdot N_t + \varphi \cdot P_t)}_{\text{new contract}} \right\} && (3.4) \\
A_t, D_t, C_t &\geq 0 && (3.5)
\end{aligned}
$$

where ρ is the risk aversion coefficient, β is the discount factor, r_a is the (real) interest rate on *assets*, r_d is the (real) interest rate on *debt* and $\lambda \in [0,1]$ is the *minimum payment due rate*. Equation (3.2) is the budget constraint, (3.3) defines end-of-period (financial) net worth, and (3.4) is the borrowing constraint. The model is closed by assuming that the households are required to "die without debt" (i.e $N_T \geq 0$ in some infinitely distant terminal period $T \to \infty$). We only cover the case $r_d > r_a$. We denote the optimal debt and consumption functions by $D^\star(\mathbf{S}_t)$ and $C^\star(\mathbf{S}_t)$.

We assume that x_t transitions according to a first order Markov process. The (unconditional) risk of losing access to the credit market is given by $\pi^{lose}_{x,*}$, and the chance of re-gaining access is given by $\pi^{gain}_{x,*}$. Conditional on unemployment we assume that the risk of losing access to the credit market is given by $\pi^{lose}_{x,u} = \chi_{lose} \cdot \pi^{lose}_{x,w}$, where $\pi^{lose}_{x,w}$ is the risk of losing access conditional on employment (in our calibration we choose χ_{lose} and let $\pi^{lose}_{x,w}$ adjust to match the chosen unconditional transition probabilities).

3.2 The Borrowing Constraint

Our specification of the debt contract is obviously simplistic, but it serves our purpose, and only add one extra state variable to the standard model. If $\eta > 0$ asset-rich households are allowed to take on more debt even though there is no formal collaterization. We allow *gearing* in this way to be as general as possible, and we use end-of-period timing and update the effect of income on the borrowing constraint period-by-period following the standard approach in buffer-stock models.[8]

The crucial departure from the canonical buffer-stock model is that we assume that the debt contract is *partially irrevocable from the lender side.* This provides the first term ("old contract") in the maximum operator in borrowing constraint (3.4), implying that the households can always continue to borrow up to the *remaining principal* of their current debt contract (i.e. $(1-\lambda) \cdot D_{t-1}$). The second term ("new contract") is a more standard borrowing constraint and only needs to be satisfied if the households want to take on *new debt* ($D_t > (1-\lambda) \cdot D_{t-1}$). Hereby current debt can potentially relax the households borrowing constraint in future

[8] Note than a borrowing constraint such as $D_t \leq A_t + \alpha \cdot P_t$ would be problematic because it would allow the households to take on infinitely much debt for a given level of consumption. A similar problem would also arise with $D_t \leq A_{t-1} + \alpha \cdot P_t$ in the time limit if $r_d = r_a$.

periods and it thus provides extra liquidity. This implies that it might be optimal for the households to make choices such that both $D_t > 0$ and $A_t > 0$; i.e to simultaneously be a borrower and a saver.

If there was only one-period debt (i.e. $\lambda = 1$) it would never be optimal for the households to simultaneously have both positive assets and positive debt because the option value of borrowing today would disappear. Consequently it would not be necessary to keep track of assets and debts separately and the model could be written purely in terms of net worth.[9] This would also imply that (3.4) could be rewritten as

$$N_t \geq -\frac{\mathbf{1}_{x_t=0} \cdot \varphi}{1+\eta} \cdot P_t \tag{3.6}$$

showing that our model nests the canonical buffer-stock consumption model a la Carroll (1992, 1997, 2012) as a limiting case for $\lambda \to 1$.

3.3 Income

The income process is given by

$$\begin{aligned}
Y_{t+1} &= \tilde{\xi}\left(u_{t+1}, \xi_{t+1}\right) \cdot P_{t+1} \\
P_{t+1} &= \Gamma \cdot \psi_{t+1} \cdot P_t \\
\tilde{\xi}\left(u_{t+1}\right) &\equiv \begin{cases} \mu & \text{if}\, u_{t+1} = 1 \\ \frac{\xi_{t+1} - u_* \cdot \mu}{1-u_*} & \text{if}\, u_{t+1} = 0 \end{cases} \\
u_{t+1} &= \begin{cases} 1 & \text{with probability } u_* \\ 0 & \text{else} \end{cases}
\end{aligned}$$

where ξ_t and ψ_t are respectively *transitory* and *permanent* mean-one log-normal income shocks[10] (with finite lower and upper supports), and u_* is the unemployment rate.[11] Because we have fully permanent shocks, we introduce a small constant mortality rate in the simulation exercise to keep the distribution of income finite.

[9] If $N_t \geq 0$ then $A_t = N_t$ and $D_t = 0$, and if $N_t < 0$ then $D_t = -N_t$ and $A_t = 0$.

[10] Note that the *unconditional* expectation of Y_{t+1} thus is $\Gamma \cdot P_t$.

[11] Throughout the paper we will continue to interpret u_t as unemployment, but it could also proxy for a range of other large shocks to both income and consumption. This would relax the model's tight link between unemployment and a higher risk of a negative shock to the availability of new borrowing.

3.4 Solution Algorithm

As the model has four continuous states, two discrete states and two continuous choices it is not easy to solve, even numerically. We use a novel trick by defining the following helping variables,

$$M_t \equiv (1+r_a)\cdot A_{t-1} - (r_d+\lambda)\cdot D_{t-1} + Y_t \tag{3.7}$$

$$\overline{D}_t \equiv (1-\lambda)\cdot D_{t-1} \tag{3.8}$$

$$\overline{N}_t \equiv N_{t\,|\,C_t=0} = M_t - \overline{D}_t \tag{3.9}$$

where M_t is *market resources*, $\overline{D}_t$ is the *beginning-of-period debt principal,* and $\overline{N}_t$ is *beginning-of-period net worth.* Also using the standard trick of normalizing the model by permanent income[12] denoting normalized variables with lower cases, we make $\overline{n}_t$ a *state variable* instead of m_t (the standard choice). This speeds up the solution algorithm substantially because a change in $\overline{d}_t$ then only affects the set of feasible debt choices; we hereby get that if the optimal debt choice is smaller than the current debt principal, then all households with smaller debt principals will make the same choice if it is still feasible, i.e.

$$k<1: \;\; d^{\star}\left(\overline{d}_t,\overline{n}_t\right) = d \le k\cdot\overline{d}_t \;\;\Rightarrow\;\; \forall \overline{d} \in \left[k\cdot\overline{d}_t,\overline{d}_t\right] : \; d^{\star}\left(\overline{d},\overline{n}_t\right) = d$$

One further complicating issue in solving the model, is that if $\eta \neq 0$ then the choice set might be non-convex as illustrated in figure 3.1 using the following characterization of the choice set

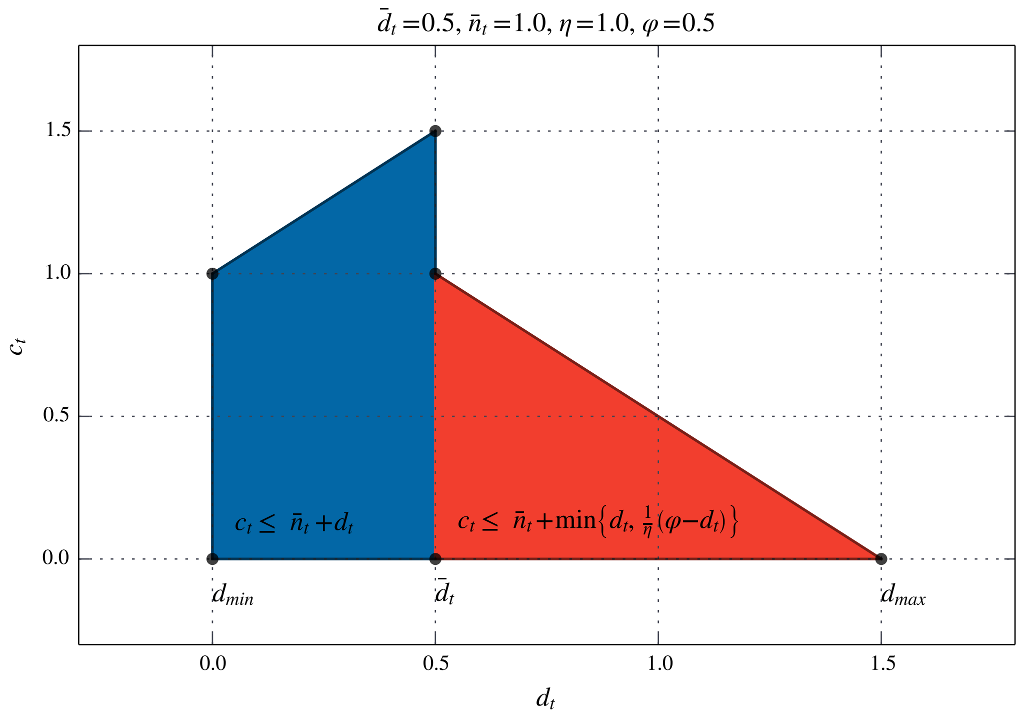
$$d_t \in \left[\max\left\{-\overline{n}_t, 0\right\}, \max\left\{\overline{d}_t, \eta\cdot\overline{n}_t + \mathbf{1}_{x_t=0}\cdot\varphi\right\}\right] \tag{3.10}$$

$$c_t \in \left[0\,,\, \overline{c}\left(x_t,\overline{d}_t,\overline{n}_t,d_t\right)\right] \tag{3.11}$$

$$\overline{c}\left(x_t,\overline{d}_t,\overline{n}_t,d_t\right) \equiv \begin{cases} \overline{n}_t + d_t & \text{if } d_t \le \overline{d}_t \\ \overline{n}_t + \min\left\{d_t, \frac{1}{\eta}\left(\mathbf{1}_{x_t=0}\cdot\varphi - d_t\right)\right\} & \text{if } d_t > \overline{d}_t \end{cases}$$

[12] See appendix B for the normalized model equations and details on the solutions algorithm for the discretized model.

Figure 3.1: Choice Set (*example of non-convexity*)



This possible *non-convexity* of the choice set and the general *non-concavity* of the value function due to the maximum operator in the borrowing constraint (3.4), imply that many of the standard results do not apply directly. Using a recent result from Clausen and Strub (2013) it can, however, be proven[13] that the optimal consumption choice, $c_t^\star\left(u_t, x_t, \bar{d}_t, \overline{n}_t\right)$, *conditional on the debt choice*, still needs to satisfy the standard Euler-equality, i.e.

$$\left(c_t^\star(\bullet)\right)^{-\rho} = \beta \cdot (1 + r_a) \cdot \mathbb{E}_t\left[\left(\Gamma \cdot \psi_{t+1} \cdot c_{t+1}^\star(\bullet)\right)^{-\rho} \mid d_t = d\right]$$

This makes the Euler-equation a *necessary* condition for an interior solution. *Sufficiency* can then be ensured by numerically checking that the Euler-equation does not have multiple solutions.

Similar to Barillas and Fernández-Villaverde (2007), Hintermaier and Koeniger (2010), Kaplan and Violante (2014), Iskhakov, Jørgensen, Rust and Schjerning (2015) and especially Fella (2014), the endogenous grid points method originally developed by Carroll (2006) can thus be *nested* inside a *value function iteration* algorithm with a grid search for the optimal debt choice further speeding up the

[13] See appendix A.

solution algorithm.[14] The full solution algorithm is presented in appendix B.

4 Stylized Facts

Table 4.1: Stylized Facts

	Puzzle	Borrower	Saver	All
Share	27 %	5 %	68 %	100 %
U.S. Dollars 2001		*mean / median*		
Credit Card Debt	5,766	5,172	317	2,050
	3,800	3,340	0	0
Liquid Assets	7,237	227	17,386	13,734
	3,000	200	3,200	2,800
Liquid Net Worth[1]	1,471	-4,945	17,069	11,684
	-270	-3,200	3,000	1,700
Total After-Tax Income (annual)	52,114	28,032	64,331	59,116
	43,600	25,350	39,950	39,950
Installment Loans[2]	10,957	8,216	5,889	7,386
	6,100	3,600	0	600
Total Net Worth	187,912	36,231	466,463	368,367
	84,650	9,450	104,830	86,480
Relative to quarterly income		*mean / median*		
Credit Card Debt	0.44	0.74	0.02	0.14
	0.35	0.53	0.00	0.00
Liquid Assets	0.56	0.03	1.08	0.93
	0.28	0.03	0.32	0.28
Liquid Net Worth[1]	0.11	-0.71	1.06	0.79
	-0.02	-0.50	0.30	0.17

Source: 2001 SCF, all households with heads of age 25-64. Weighted averages within subgroups.
[1] Defined as liquid assets − credit card debt.
[2] Mortgages are not included.

For comparison between our model and the data, table 4.1 presents the central stylized facts on the credit card debt puzzle using the exact same methodology and 2001 *Survey of Consumer Finance* (SCF) data as Telyukova (2013). The facts are very similar to what other papers has found. *Credit card debt* is measured as

[14]On the precision and speed-up benefits of using EGM see Jørgensen (2013).

the balance due on the credit card left over *after* the last statement was paid, and *liquid assets* includes checking and savings accounts plus idle money in brokerage accounts, but not cash.[15]

All working age households are divided into *three* subgroups. Households are included in the *"puzzle"* group (or interchangeably the *"borrower-saver"* group) if they have more than $500 in both credit card debt and liquid assets, and report repaying their balance off in full *only sometimes or never*. On the contrary households are denoted as pure *"borrowers"* if they have more than $500 in credit card debt, but less than $500 in liquid assets. Finally households with less than $500 in credit card debt are denoted pure *"savers"*.

Approximately one in four households are measured to be in the puzzle group. For the median puzzle household both gross debt and gross assets equals about one month's of after-tax income implying zero *liquid net worth* (liquid assets minus credit card debt). The distribution of liquid net worth is, however, somewhat right skewed in the sense that the mean household in the puzzle group has significantly larger gross assets than gross debts. Income wise, the average puzzle household has *less* income than the average income of the total population, but the median puzzle household has *more* income than the median income of the total population.

The borrower group has mean credit card debt equal to about two month's income, and an income level significantly below the average for both the mean and median household. Finally the distribution of gross assets in the saver group is highly right skewed with the mean household holding liquid assets worth more than one quarter's income, but the median holding less than than one month's. Including money market funds, and directly held mutual funds, stocks, bonds and T-bills in the measure of liquid assets would amplify this unbalancedness even further.

A novel fact presented in table 4.1 is that the puzzle households also hold many *installment loans,* most of which are *car loans*. The interest rates on such loans are typically significantly lower than on credit cards, and there can be some contractual terms that disincentivize premature repayment. Nonetheless it is an indication that the puzzle households are also using other precautionary borrowing channels than credit cards.

Finally, as also noted by Telyukova (2013), the puzzle households are often rather wealthy measured in *total net worth* (thus also including illiquid assets). This

[15] See Telyukova (2013) for more details on the data, and a discussion of alternative procedures to quantify the credit card debt puzzle.

is to a large degree explained by *housing equity*. For computational reasons our model does not include an illiquid asset, but as shown in Kaplan and Violante (2014), a buffer stock model with an illiquid asset, and a transaction cost for tapping into this wealth, can imply that households between adjustments act as hand-to-mouth households. In a similar way *hyperbolic discounting* such as in Laibson, Repetto and Tobacman (2003) might further imply that households *"over"*-accumulate illiquid assets in order to strengthen their self-control abilities and better counteract the present bias of their future selves.

5 Calibration

Table 5.1: Calibration

Parameter	Value	Note / Source
income		
Γ (annual)	1.02	Avg. US GDP per capita growth rate 1947-2014.
u_*	0.07	Carroll, Slacalek and Tokuoka (2015).
σ_ψ^2	$0.01 \cdot \frac{4}{11}$	Carroll, Slacalek and Tokuoka (2015).
σ_ξ^2	$0.01 \cdot 4$	Carroll, Slacalek and Tokuoka (2015).
μ	0.30	Martin (1996).
borrowing and saving		
r_a (annual)	-1.48 %	Kaplan and Violante (2014).
$r_d - r_a$ (annual)	12.36 %	Telyukova (2013) and Edelberg (2006).
φ	0.74	Kaplan and Violante (2014).
η	0.00	Standard buffer-stock model.
λ	0.03	Standard credit card contract.
credit risk		
$\pi_{x,*}^{lose}$	2.63 %	Fulford (2015).
$\pi_{x,*}^{gain}$	6.07 %	Fulford (2015).
χ_{lose}	4	See text.
preferences		
β (annual)	0.90	Matched to empirical moments. See text.
ρ	3.00	Matched to empirical moments. See text.

The calibrated parameters are presented in table 5.1. The model is simulated at a *quarterly* frequency, but we discuss discount and interest rates in annualized terms. In section 8 we present a detailed discussion of how robust the results are to changing each single parameter.

The gross income growth factor $\Gamma = 1.02$ is chosen to match U.S. trend growth in GDP per capita. The variances of the income shocks and the unemployment rate are all taken from Carroll, Slacalek and Tokuoka (2015) who show that they parsimoniously match central empirical facts from the literature on estimating uncertain income processes. In annual terms the variance of both the permanent and the transitory shock are 0.01.[16] The unemployment replacement rate μ is set to 0.30 as documented in Martin (1996); we find the choice of $\mu = 0.15$ in Carroll, Slacalek and Tokuoka (2015) to be too extreme.

Regarding *borrowing and saving* we first follow Kaplan and Violante (2014), who based on SCF data, set the real interest rate on liquid wealth to -1.48 percent (annually) and find that the borrowing constraint binds at 74 percent of quarterly income. We thus set $\varphi = 0.74$, and choose $\eta = 0$ to stay as close as possible to these results (and the standard parametrization of the buffer-stock model). The interest rate on credit card debt is taken from Telyukova (2013); she finds that the mean nominal interest rate in the borrower-saver group is 14 percent which we then adjust for 2.5 percentage points of inflation and a 0.62 percentage points default risk (see Edelberg (2006)). In total this implies an interest rate spread of 12.4 percent, which is a bit lower than the 13.2 percent spread in Fulford (2015), but larger than the 10.0 percent spread in Telyukova (2013). We set $\lambda = 0.03$ because many credit card companies use a minimum payment rate of 1 percent on a monthly basis.

For the *credit risk* we set the *un*conditional probabilities equal to the empirical results in Fulford (2015) who utilize a proprietary data set containing a representative sample of 0.1 percent of all individuals with a credit report at the credit-reporting agency *Equifax* from 1999 to 2013. Each quarter the risk of losing access to credit is thus 2.63 percent, while the chance of regaining access is 6.07 percent. Unfortunately Fulford is only able to condition on general covariates such as age, year, credit risk, geographical location and reported number of cards; specifically he is not able to say anything on the relationship between credit risk and income risk or unemployment. To calibrate χ_{lose} we therefore instead turn to the *Survey of Consumer Finance* (SCF) 2007-2009 panel where households were asked whether

[16]For the transitory shock the variance at a quarterly frequency is simply $4 \times$ annual transitory variance, while Carroll, Slacalek and Tokuoka (2015) show that for the permanent shock the conversion factor should be $\frac{4}{11}$.

or not they have a credit card both in 2007 and then again in 2009. This measure of credit card access is inferior to Fulford's, but we believe that the two measures are rather closely related. We restrict attention to all stable couples between age 25 and 59, with positive income, and who in 2007 had and used a credit card. Table 5.2 shows that 7.7 percent of these household when re-interviewed in 2009 reported *not* having a credit card anymore; we denote this as having "lost access". Conditional on experiencing any weeks of unemployment the fraction of those who have lost access increases to 15.2 percent. Like Fulford we have no way of determining whether this indicates voluntary choices made by the households, but table 5.3 reports the odds-ratios from logit estimations controlling for both various *background* variables (age, age squared, minority, household size) and *economic* variables (homeownership, log (normal) income, liquid assets, self-employment, education). The effect from unemployment remains significant even when all controls are used though the odds-ratios falls a bit. This is also in line with Crossley and Low (2013) who showed using the 1995 *Canadian Out of Employment Panel* that current unemployment was important for explaining the share of households answering "no" to the question "[i]f you needed it, COULD you borrow money from a friend, family, or a financial institution in order to increase your household expenditures".

To choose an extract number for χ_{lose} we use that the *theoretical* odds-ratio of losing access to new borrowing if treated with some unemployment in the last year (four quarters), and conditional on having had access two years (eight quarters) ago, is given by

$$\text{Odds-Ratio} = \frac{f(1,1)/f(0,1)}{f(1,0)/f(0,0)} \tag{5.1}$$

where we have defined

$$f(\hat{x},\hat{u}) \equiv \mathbb{E}\left(x_8 = \hat{x} \,|\, x_1 = 0,\, \exists k \in \{5,6,7,8\} : u_k = \hat{u}\right) \tag{5.2}$$

which can easily be calculated for $\hat{x} \in \{0,1\}$ and $\hat{u} \in \{0,1\}$ given the Markov processes of x_t and u_t.

Setting $\chi_{lose} = 4$ we hereby get an odds-ratio of 1.8. This is somewhat below the odds-ratios we find in the data, but due to Fulford's very low estimate of gaining access, we still have that the risk of losing access conditional on being affected by unemployment is 19.9 percent; if not affected by unemployment the probability is 12.4 percent and in percentage points the increase is thus 7.5 similar to what we see in table 5.2. We thus stick with the choice of $\chi_{lose} = 4$ and perform an extensive robustness analysis of these calibrations in section 8.

Finally we calibrate the discount factor β and relative risk aversion ρ to match central moments from table 4.1. Our first target is that the mean level of liquid net worth across all households should be equal to about *two month's of income*. The exact data counterpart is a bit higher at 79 percent of quarterly income; we choose the lower target because the *median* level is always only a bit below the *mean* level in the model while the median level in the data is only 17 percent of quarterly income and thus substantially below the mean level. Our second target is that the median puzzle household should have approximately zero liquid net worth as we see in the data. Increasing either β or ρ both increases the liquid net worth of the full population, but β only marginally affects the net worth of the puzzle group whereby we use the second moment to identify ρ. We obtain $\beta = 0.90$ and $\rho = 3$. The discount factor is including an exogenous quarterly death probability of 1 percent; having mortality is technically necessary to ensure that the cross-sectional distribution of income is finite.[17]

Table 5.2: Lost Access and Unemployment - Raw

	Lost Access[1]	Share of Sample
	percent	
All	7.7	-
No Unemployment over last year[2]	5.6	78.9
Any Unemployment	15.2	21.1
Some Unemployment ($\geq$ 1 month)	15.2	19.3
Deep Unemployment ($\geq$ 3 months)	15.9	14.0

Source: SCF panel 2007-2009; Households between age 25 and 59, positive income, had and used a credit card in 2007 (X410=1 and X09205>0). Adjusted for survey weights and multiple imputations.
[1] *Lost Access*: Report not having a credit card in 2009 (P410 = 5).
[2] *Unemployment*: Sum of head and spouse over the last 12 months (P6781 and P6785).

[17] When a household dies it is replaced with a new household without any debt and assets equal to one week's permanent income, and with the same lagged permanent income as the *mean* of the current population. See e.g. McKay (2015) for a similar approach. The assets of the household is taxed away.

Table 5.3: Lost Access and Unemployment - Logit

	Any Unemployment			Deep Unemployment		
	(1)	(2)	(3)	(1)	(2)	(3)
	odds-ratio (s.e.)					
Unemployment	3.00*** (0.93)	2.98*** (0.94)	2.34** (0.83)	2.82*** (0.95)	2.75*** (0.98)	2.27** (0.89)
Background Controls[1]		✓	✓		✓	✓
Economic Controls[2]			✓			✓
Observations	1,079	1,079	1,079	1,079	1,079	1,079

Source: See table 5.2. *: p<0.10, **: p<0.05, ***: p<0.01.
[1] *Background controls*: Age, age squared, minority, household size.
[2] *Economic controls*: Homeownership, log (normal) income, liquid assets, self-employment, education (none, high school, college).

6 Results

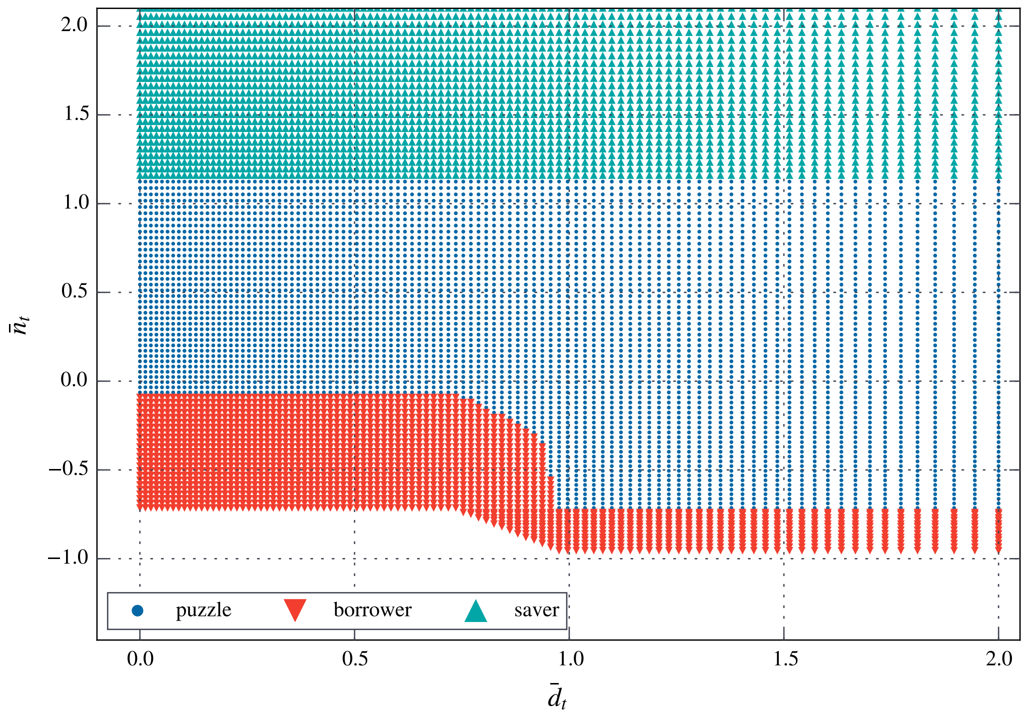
6.1 Policy Functions

Based on the converged policy functions, figure 6.1 shows in which disjoint sets of states the households choose to respectively be a borrower, a saver and a borrower-saver.

The general conclusion is that households always choose to be *savers* if their beginning-of-period net worth ($\overline{n}_t$) is high enough, and *borrowers* if it is low enough. For the wealthy households the option value of holding debt is zero because they have no liquidity problems. In contrast, poor households are already borrowing so much that they either cannot borrow any more, or the option value of more debt is not large enough to cover the net cost of expanding the balance sheet.

The households choose to be in the puzzle group if their beginning-of-period net worth is in between the two extremes mentioned above. If the beginning-of-period debt principal ($\overline{d}_t$) is high, a household can easily accumulate more debt in excess of what it needs to accumulate for consumption purposes. Hence, for a given (low) beginning-of-period net worth it might therefore be optimal for households with a *high* debt principal to be a borrower-saver, while it is optimal to be a borrower for households with a *low* debt principal.

Figure 6.1: **Implied Group Choice** ($u_t = 0$, $x_t = 0$)



6.2 Simulation

Given the converged policy functions it is straightforward to simulate the model. Table 6.1 presents the cross-sectional results from a simulation with 100.000 households (after an initial burn-in period).

We see that the model under the chosen parametrization can explain that 16.6 percent of households choose to be borrower-savers. This is a bit below the empirical estimate of 27 percent (see table 4.1) but still a large share. This shows that precautionary borrowing is at least one of the central explanations of the credit card debt puzzle. It is especially important that such a large proportion of the puzzle group can be explained even when the model also implies that the level of mean net worth is above two months income. Furthermore the implied size of the balance sheets of the puzzle households are also rather large; the median puzzle household e.g. has an asset to income ratio of 0.24, while the data counterpart is 0.28.

On the other hand the size of the borrower group is much too small in the simulation, and the model generally has a hard time explaining why some households decide to go so deeply into debt. Consequently it also overshoot the median net

Table 6.1: Results

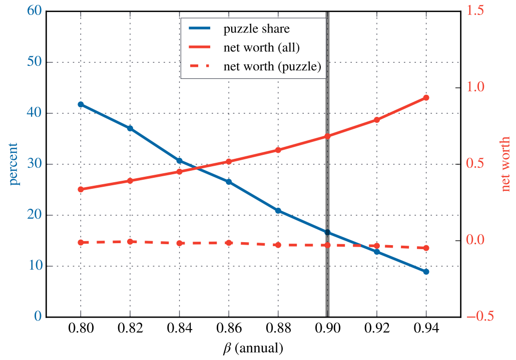
	Puzzle	Borrower	Saver	All
		percent		
Share	16.6	0.5	82.8	100.0
u_t (4 qrt.)	16.6	43.7	4.8	7.0
x_t	13.9	79.4	33.2	30.2
Relative to income		*mean / median*		
d_t	0.30	0.21	0.00	0.05
	0.24	0.01	0.00	0.00
a_t	0.27	0.00	0.84	0.74
	0.24	0.00	0.73	0.61
n_t	-0.03	-0.21	0.84	0.68
	-0.00	-0.01	0.73	0.59
Y_t (4 qrt.)	0.88	0.66	1.03	1.00
	0.80	0.60	0.92	0.89
P_t (4 qrt.)	1.03	1.01	1.00	1.00
	0.94	0.92	0.90	0.91

"4. qrt.": Average of the last four quarters.
Puzzle group definition: $d_t, a_t > 0.04$.

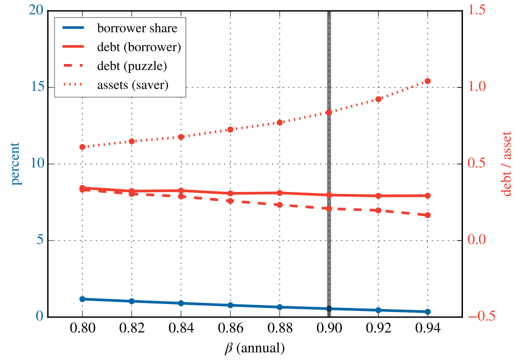
worth of the full population substantially. We do not worry too much about these two shortcomings of the model because introducing heterogeneous impatience and risk aversion would probably be a simple cure. Figure 6.2 thus shows that a large borrower share can be explained if some of the households have a relative risk aversion coefficient below 2. Our calibration implies a somewhat higher degree of risk aversion in order to match the observed net worth of the puzzle households (which are *increasing* in ρ, while the share of borrowers are *decreasing*). Such heterogeneous risk preferences or heterogeneous impatience would also improve the model's ability to match a lower median net worth among savers, which it currently overshoots. In general, both lower ρ and β increases the puzzle group, but make it harder for the model to match the observed level of net worth.

Figure 6.2: Alternative Preferences

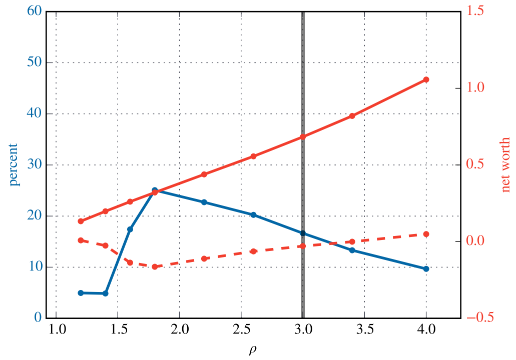
(a) β - puzzle share and net worth



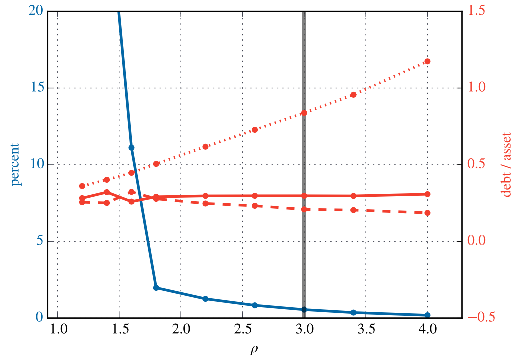
(b) β - borrower share and gross stocks



(c) ρ - puzzle share and net worth



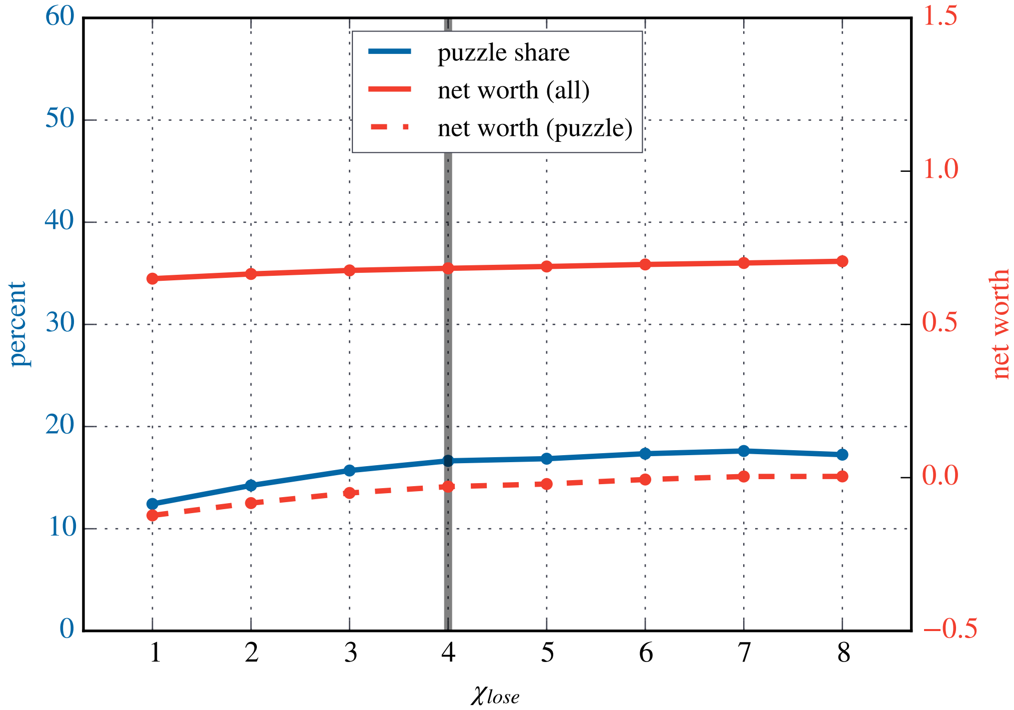
(d) ρ - borrower share and gross stocks



Note: Vertical gray line represents baseline parameter value.

Figure 6.3 shows that the assumed positive correlation between unemployment and losing access to the credit markets is rather important for the quantitative results. Removing the extra risk of losing access when unemployed (setting $\chi_{lose} = 1$ for unchanged $\pi_{x,*}^{lose}$) reduces the puzzle group by a fourth. Increasing χ_{lose} above the calibration value of 4 also increases the size of the puzzle group somewhat further.

Figure 6.3: The importance of $\chi_{lose} > 1$



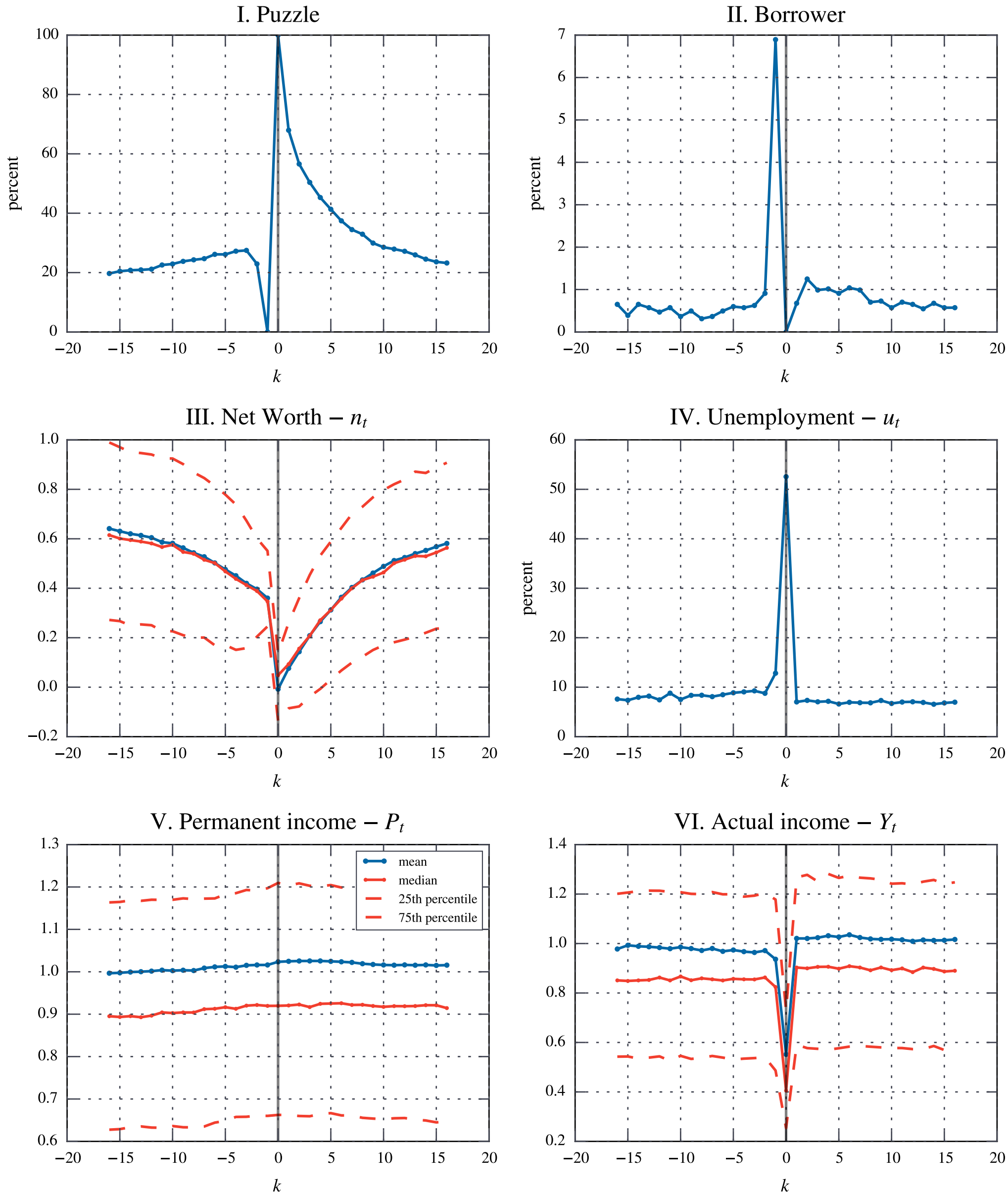
Note: Vertical gray line represents baseline parameter value.

6.3 Before/After

Figure 6.4 provides further details on what happens before and after a household transitions into the puzzle group (i.e. the household is in the puzzle group at time $k = 0$ but not at $k = -1$). We see that persistence is limited as just above 40 percent of the households are still in the puzzle group one year on (graph I), and that most of the puzzle households (over 90 percent) were savers in the period before their transition (graph II). In more general terms, the third graph shows that the households are deaccumulating net worth at an accelerating speed in the quarters before joining the puzzle group.

Looking at the income dimension of the simulation, we see that the yearly income of the puzzle households is a bit below the total mean and median; in the data this was only true for the mean. Note however, that the permanent income of the puzzle households is actually slightly *above* both the total mean and median. The average unemployment rate of the puzzle households over the last four quarters is 17 percent, but looking at figure 6.4 (graph IV) we see that about 50 percent of the puzzle households are unemployed at transition. This shows that continuing large falls in transitory income is necessary to make households choose to be borrower-savers (see also graph VI). On the other hand graph V in figure 6.4 shows that falls in permanent income are *not* necessary; the reason is that such shocks also lowers the optimal consumption level of the household and thus does not induce precautionary borrowing.

Figure 6.4: Before and After Transition to Puzzle Group



Sample: Households who are in the puzzle group at $k = 0$, but were not so at $k = -1$.

7 The Welfare Gain of Precautionary Borrowing

The welfare of the households can be measured as the ex ante discounted expected utility seen from an initial period. The simulation analog of this measure can be calculated taking the average over a sample of households experiencing different draws of shocks,

$$\begin{aligned} \mathcal{U}_0\left(P_0\right) &= P_0^{1-\rho} \cdot \frac{1}{N} \cdot \sum_{i=0}^{N} \sum_{t=0}^{T} \cdot \beta^t \cdot \frac{\left(c^\star\left(s_{it}\right) \cdot \Gamma^t \cdot \Pi_{j=1}^{t} \psi_{ij}\right)^{1-\rho}}{1-\rho} \\ & \mathcal{T}\left(s_{it}, d^\star\left(s_{it}\right), c^\star\left(s_{it}\right)\right) \Rightarrow s_{i,t+1} \end{aligned} \tag{7.1}$$

where $s_{i,t}$ is the vector of normalized state variables of household i and $\mathcal{T}(\bullet)$ is the stochastic transition function.[18]

We are now interested in the level of welfare across different values of λ, remembering that as $\lambda \rightarrow 1$ we return to the canonical buffer-stock model which does not allow precautionary borrowing. Facilitating these comparisons, we can analytically derive the compensation in terms of a percentage increase (τ) in initial permanent income, and thus the average future path of permanent income, a household needs to receive in order to be indifferent to a change in λ relative to the baseline:

$$\mathcal{U}_0\left(P_0, \lambda_0\right) = \mathcal{U}_0\left(P_0 \cdot \left(1 + \frac{\tau_j}{100}\right), \lambda_j\right) \Leftrightarrow \frac{\tau_j}{100} = \left(\frac{\mathcal{U}_0\left(P_0, \lambda_0\right)}{\mathcal{U}_0\left(P_0, \lambda_j\right)}\right)^{\frac{1}{1-\rho}} - 1 \tag{7.2}$$

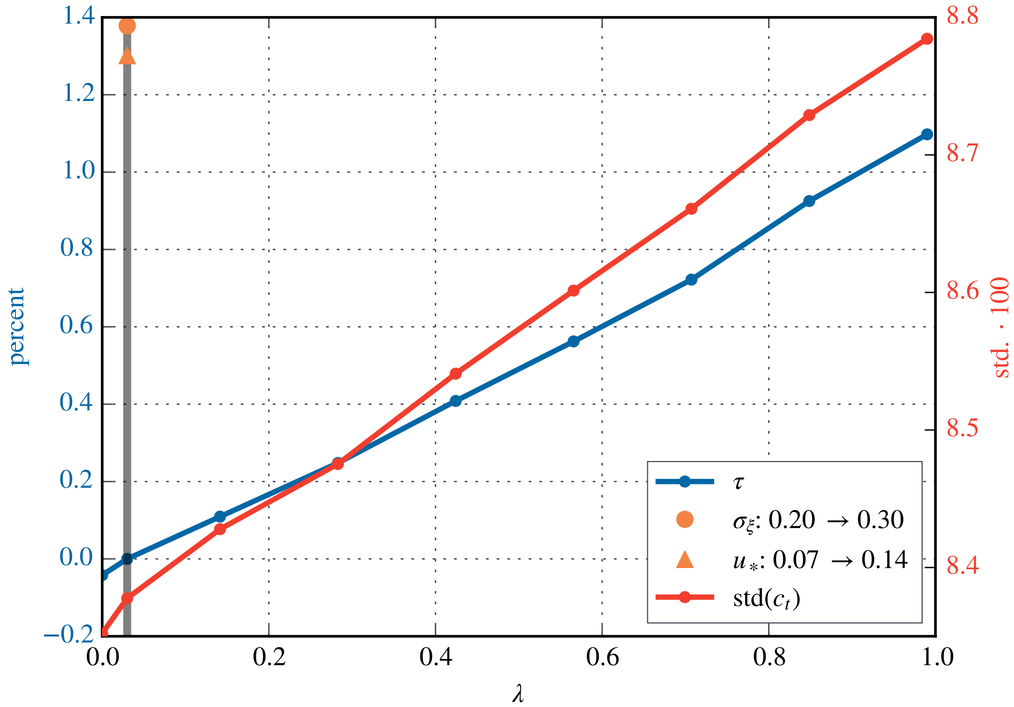
The results are plotted in figure 7.1; as λ increases the required compensation (the blue line) naturally increases as the choice set of the households only shrinks and the scope for precautionary borrowing becomes more limited. In total, the households needs a compensating increase in the path of permanent income of 1.10 percent to be indifferent between $\lambda = 0.03$ (the baseline) and $\lambda = 0.99$.

To ease comparison, figure 7.1 also depicts the compensating equivalents for changes in respectively the variance of the transitory income shock and steady state unemployment: Increasing σ_ξ from 0.20 to 0.30 implies $\tau = 1.37$, while increasing u_* from 7 to 14 percent implies $\tau = 1.30$. The households welfare loss of losing access to precautionary borrowing is thus only a bit smaller than a doubling of the unemployment rate.

[18] The average is calculated conditional on P_0, but not on the other initial states

The red line in figure 7.1 shows that a central underlying reason for the loss of welfare when the household's access to precautionary borrowing is limited is an *increase* in the standard deviation of normalized consumption, which the households dislike because of the concavity of the utility function.

Figure 7.1: Welfare



Note: Vertical gray line represents baseline λ value.

8 Robustness

8.1 Growth Impatience

Figure 8.1 shows how the size of the puzzle group (*blue line*) and the average net worth of both all households (*full red line*) and the puzzle group (*dashed red line*) are affected by changes in r_a and Γ.

In understanding the figure it is useful to consider *the growth impatience factor* as defined in Carroll (2012)

$$\overline{\beta} \equiv (\beta \cdot (1 + r_a))^{\frac{1}{\rho}} \cdot \Gamma^{-1} \tag{8.1}$$

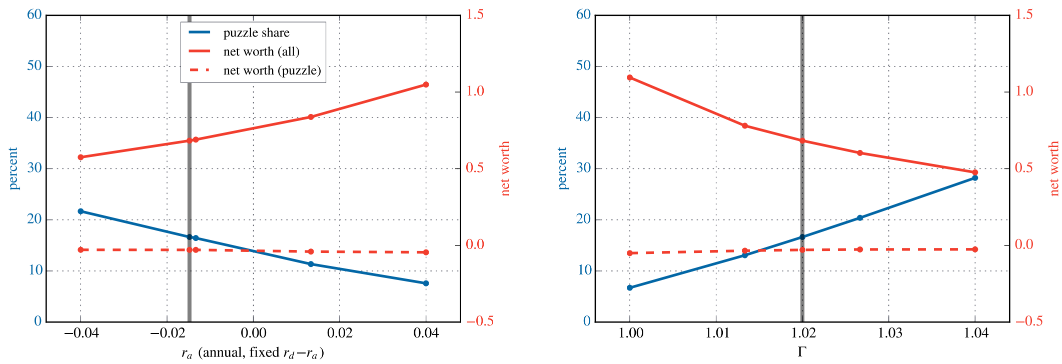
In the perfect foresight case a growth impatience factor *less than one* implies that for an unconstrained consumer the ratio of consumption to permanent income will *fall* over time. In general, a larger growth impatience factor induces saving; these savings also satisfy the household's precautionary motive making costly *precautionary borrowing* less needed. Consequently, the puzzle group is *increasing* in Γ, and *decreasing* in r_a.

In figure 6.2 we likewise saw that the puzzle group was *decreasing* in patience β and eventually in risk aversion ρ (we always have $\beta \cdot (1 + r_a) < 1$). Initially, however, an increase in the curvature of the utility function (ρ) expands the puzzle group because it implies a stronger incentive to smooth consumption, which makes it relatively more worthwhile for the households to pay the costs of precautionary borrowing.

Summing up, the model can explain a large puzzle group if households are impatient enough, in a growth corrected sense, and are neither too risk neutral nor too risk averse.

Figure 8.1: Growth Impatience

(a) r_a (annual, fixed $r_d - r_a$) (b) Γ



Note: Vertical gray line represents baseline parameter value.

8.2 Income Uncertainty

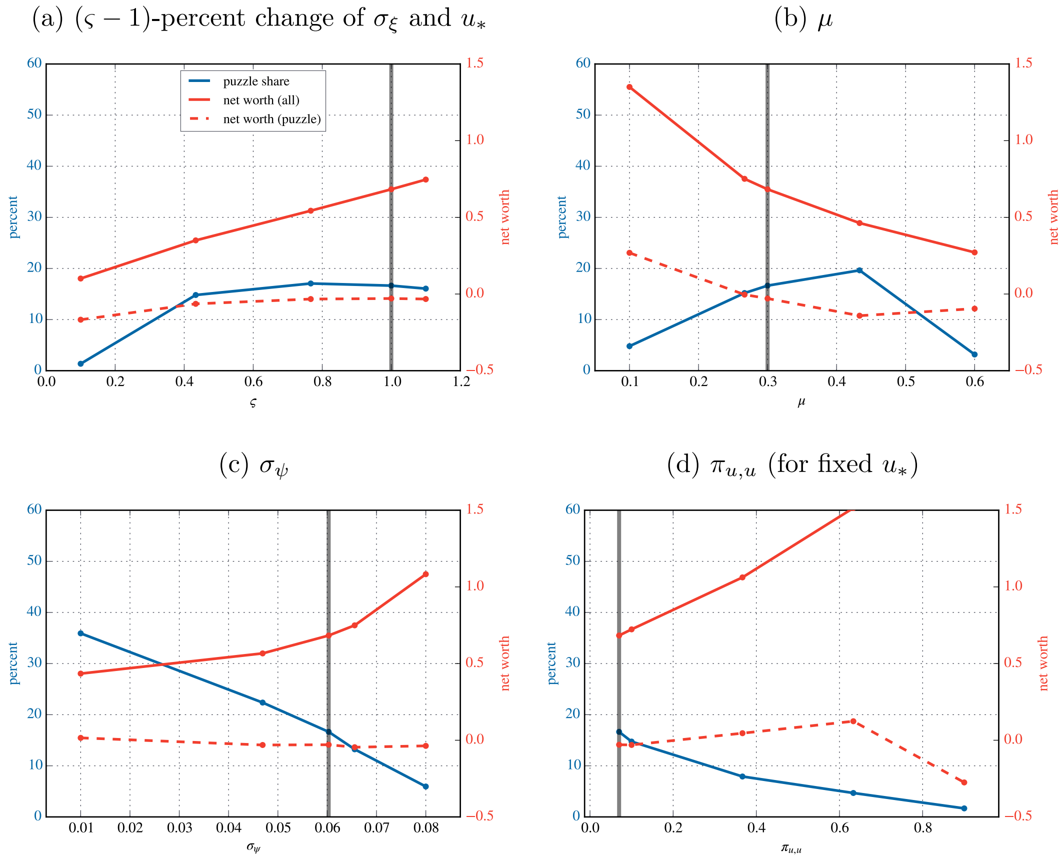
The underlying motive for precautionary borrowing is insurance against transitory income losses. We therefore see in figure 8.2 that the size of the puzzle group is first increasing in the variance of the *transitory* income shock and risk of unemployment (higher σ_ξ and u_*). At some point, however, larger transitory shocks does not increase the puzzle group because they induce too much precautionary saving. Lowering the unemployment benefits (lower μ) thus also only increase the puzzle group if the initial level is rather high.

A larger variance of the *permanent* shock (higher σ_ψ) on the contrary shrinks the puzzle group because the incentive to accumulate precautionary funds imply that the average net worth increases so much that the households do not need to rely on precautionary borrowing. This can also be understood as the consequence of an increase in the *uncertainty adjusted* growth impatience factor,

$$\tilde{\beta} \equiv (\beta \cdot (1 + r_a))^{\frac{1}{\rho}} \cdot \Gamma^{-1} \cdot \mathbb{E}\left[\psi_{t+1}^{-1}\right] = \overline{\beta} \cdot \mathbb{E}\left[\psi_{t+1}^{-1}\right] \tag{8.2}$$

where the last term is increasing in the variance of the permanent shock due to Jensen's Inequality. The same mechanism moreover also implies that the puzzle group is *decreasing* in adding unemployment persistence, where $\pi_{u,u}$ is the unemployment risk for the unemployed.

Figure 8.2: Income Uncertainty



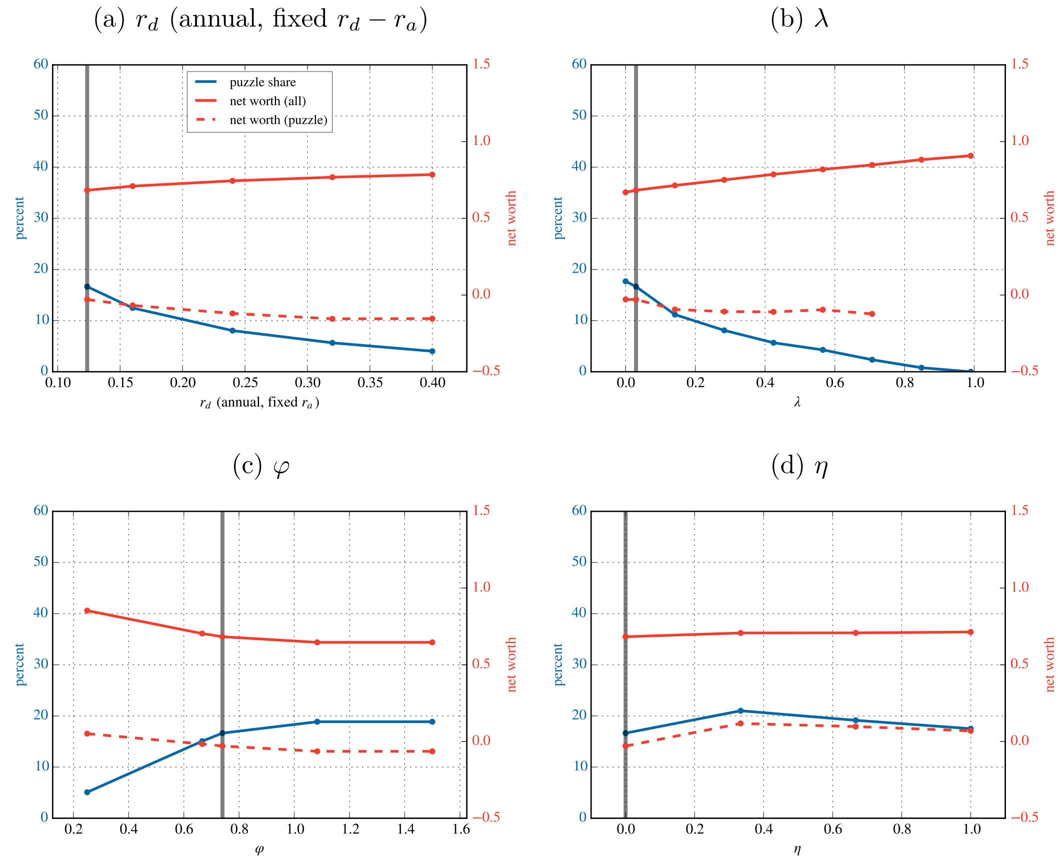
Note: Vertical gray line represents baseline parameter value.

8.3 Terms of Borrowing

Naturally the size of the puzzle group is decreasing if either the cost of borrowing increases (higher $r_d - r_a$, fixed r_a) or the repayment rate increases (higher λ).

This is shown in the two first graphs in figure 8.3. Furthermore the puzzle group is relatively small if φ is too small, as the extensive potential for precautionary borrowing is then limited. When φ reaches one this positive effect on the size of the puzzle group more or less disappears. Allowing for gearing in the form of a $\eta > 0$ does almost not affect the results, and our formulation is thus robustness in this regard.

Figure 8.3: Terms of Borrowing



(a) r_d (annual, fixed $r_d - r_a$)

(b) λ

(c) φ

(d) η

Note: Vertical gray line represents baseline parameter value. The net worth of the puzzle group is not shown if the puzzle group is too small.

8.4 Credit Risk

Figure 8.4 presents the effects of changing the *unconditional* probabilities for losing $(\pi_{x,*}^{lose})$ and gaining $\left(\pi_{x,*}^{gain}\right)$ access to new debt. In the first graph we see that the puzzle group is naturally increasing in the risk of losing access, but that the effect is highly non-linear as a higher risk also induces more saving. Specifically we see that size of the puzzle group stops increasing at $\pi_{x,*}^{lose} = 0.01$ indicating that our results quantitatively are very robust to a lower estimate of $\pi_{x,*}^{lose}$.

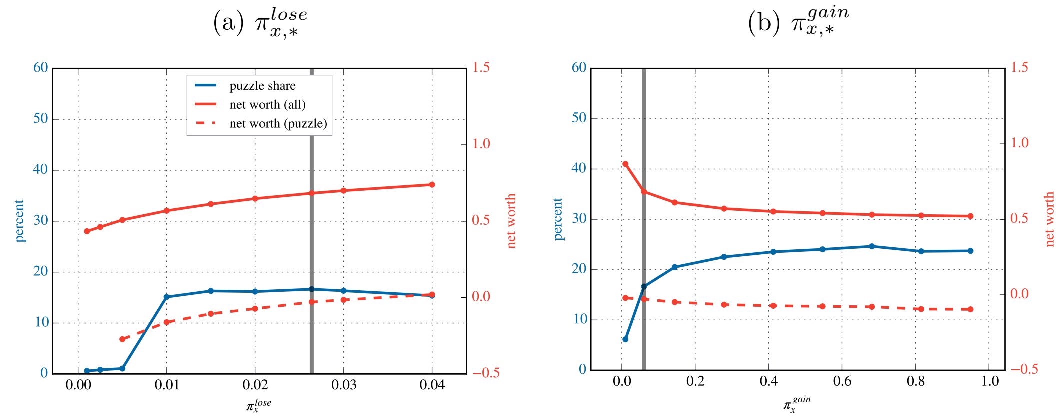
Table 8.1: Results ($\pi_{x,*}^{gain} = 0.5$, $\beta = 0.905$, $\rho = 3.3$)

	Puzzle	Borrower	Saver	All
		percent		
Share	19.2	0.3	80.3	100.0
u_t (4 qrt.)	18.0	42.9	4.1	7.0
x_t	3.5	68.7	5.0	5.0
Relative to income		*mean / median*		
d_t	0.30	0.33	0.00	0.06
	0.24	0.23	0.00	0.00
a_t	0.24	0.00	0.86	0.73
	0.23	0.00	0.74	0.58
n_t	-0.06	-0.32	0.85	0.67
	-0.01	-0.23	0.74	0.57
Y_t (4 qrt.)	0.86	0.69	1.03	1.00
	0.79	0.62	0.92	0.89
P_t (4 qrt.)	1.03	1.03	0.99	1.00
	0.94	0.93	0.90	0.91

"4. qrt.": Average of the last four quarters.
Puzzle group definition: $d_t, a_t > 0.04$.

The second graphs shows that the puzzle group is (perhaps surprisingly) also increasing in the probability of re-gaining access to credit when it is lost; the intuition is that long expected exclusion spells induce more prior saving diminishing the need for precautionary borrowing. Table 8.1 show that we reach the same conclusion when we choose an expected duration of the exclusion spell of about two quarters (setting $\pi_{x,*} = 0.5$) and re-calibrate β and ρ to match the same targets as in the baseline parametrization. It is thus clear that our results does not hinge on the assumption of a very low probability of re-gaining access.

Figure 8.4: Credit Risk



Note: Vertical gray line represents baseline parameter value. The net worth of the puzzle group is not shown if the puzzle group is too small.

9 Conclusion

We have shown that precautionary borrowing can explain a large part of the puzzle group of households who simultaneously has expensive credit card debt and hold low-return liquid assets. We have moreover shown that no knife-edge assumptions on preferences or income uncertainty are needed for this result. However, the power of the precautionary borrowing channel is strongest if households are relatively impatient in a growth and uncertainty adjusted sense, are neither too risk neutral nor too risk averse, and are subject to sizable transitory income shocks.

The strongest assumption we need in order to amplify our results, is that bad income shocks are perceived to be positively correlated with a higher risk of a fall in the availability of credit. This is not an implausible assumption, and we provide some indicative empirical evidence adding to that in Fulford (2015). More work on disentangling demand and supply effects in these estimates are, however, needed. Nevertheless, we show that only a very small risk of losing access to new borrowing is needed for our results to be quantitatively robust, and that the results are actually stronger if the chance of re-gaining access once lost is larger than the current estimate.

A natural extension of our model would be to include an illiquid asset subject to transaction costs as in Kaplan and Violante (2014). We conjecture that in such a model precautionary borrowing will still be an important tool for both poor and wealthy hand-to-mouth households. Together with a detailed life-cycle setup such an extension is probably necessary to empirically estimate the importance of precautionary borrowing with precision. This we leave for future work. Extending the model in this direction would also make it possible to study the implications of precautionary borrowing for the average marginal propensity to consume out of both income and credit shocks. Finally, the concept of precautionary borrowing is also relevant for understanding households utilization of other forms of consumer loans, including car loans and mortgages.[19]

[19] See e.g. Druedahl (2015).

A The Euler-Equation

The purpose of the present appendix is to show that *conditional on the debt choice* the standard *Euler-equation* is *necessary* at all *interior optimal consumption choices.* This is shown for a slightly simplified version of the model from the main text using lemmas from Clausen and Strub (2013); the results can easily be extended to the full model. Using a method along the lines of Fella (2014) (building on Edlin and Shannon (1998)), we furthermore show that the *debt-contingent savings correspondence* is *monotonically increasing* in a specific sense, which is a necessary condition for the *endogenous grid point method* (EGM), developed in Carroll (2006), to work.

A.1 Lemmas from Clausen and Strub (2013)

Using

Definition A.1. $F : X \to \mathbb{R}$ is *differentiable sandwiched* between the lower and upper support functions $L, U : X \to \mathbb{R}$ at $\widehat{x} \in \text{int}\,(X)$ if

$$\forall x \in X: \quad L \text{ and } U \text{ are differentiable} \tag{A.1}$$

$$L\,(x) \le F\,(x) \tag{A.2}$$

$$U\,(x) \ge F\,(x) \tag{A.3}$$

$$x = \widehat{x}: \quad L\,(x) = F\,(x) = U\,(x) \tag{A.4}$$

Clausen and Strub (2013) prove that

Lemma A.1. *(Differentiable Sandwich Lemma). If F is differentiable sandwiched between L and U at $\widehat{x}$ for an $\mathcal{X} \subseteq X$ with $\widehat{x} \in \text{int}(\mathcal{X})$ then F is differentiable at $\widehat{x}$ with $F'\,(\widehat{x}) = L'\,(\widehat{x}) = U'\,(\widehat{x})$.*

and

Lemma A.2. *(Reverse Calculus). Suppose $F : X \to \mathbb{R}$ and $G : X \to \mathbb{R}$ have differentiable lower support functions at $\widehat{x}$ then*

1. *If $H\,(x) = F\,(x) + G\,(x)$ is differentiable at $\widehat{x}$, then F is differentiable at $\widehat{x}$.*
2. *If $H\,(x) = F\,(x)\,G\,(x)$ is differentiable at $\widehat{x}$ and $F\,(\widehat{x}) > 0$ and $G\,(\widehat{x}) > 0$, then F is differentiable at $\widehat{x}$.*
3. *If $H\,(x) = \max\{F\,(x)\,, G\,(x)\}$ is differentiable at $\widehat{x}$ and $F\,(\widehat{x}) = H\,(\widehat{x})$ then F is differentiable at $\widehat{x}$.*

A.2 Simplified Model

The simplified model is written in recursive form as

$$\begin{aligned}
v\left(\overline{d}_t, \overline{n}_t\right) &= \max_{d_t, n_t} u\left(c_t\right) + \beta \cdot \sum_{\Xi \times \Psi} \cdot \psi^{1-\rho} \cdot v\left(\overline{d}_+\left(\bullet\right), \overline{n}_+\left(\bullet\right)\right) \qquad \text{(A.5)} \\
&\text{s.t.} \\
u\left(c_t\right) &= \frac{c_t^{1-\rho}}{1-\rho} \Rightarrow u_c'\left(c_t\right) = c_t^{-\rho} \\
c_t &= \overline{n}_t - n_t \\
a_t &= \overline{n}_t + d_t - c_t = n_t + d_t \\
d_t &\leq \max\left\{\overline{d}_t, \eta \cdot n_t + \varphi\right\} \\
\overline{d}_+\left(d_t; \psi\right) &= \psi^{-1} \cdot \left(1-\lambda\right) \cdot d_t \\
\overline{n}_+\left(d_t, n_t; \psi, \xi\right) &= \psi^{-1} \cdot \left[\left(1+r_a\right) \cdot n_t - \left(r_d - r_a\right) \cdot d_t\right] + \xi \\
d_t, c_t, a_t &\geq 0 \\
\Psi \times \Xi &\equiv \left\{\psi_b, \psi_g\right\} \times \left\{\xi_b, \xi_g\right\} \\
\sum_{\Psi \times \Xi} &\equiv \sum_{(\psi,\xi) \in \Psi \times \Xi} p\left(\psi, \xi\right) = 1
\end{aligned}$$

We denote the *optimal choice functions* by $d^\star\left(\overline{d}_t, \overline{n}_t\right)$ and $n^\star\left(\overline{d}_t, \overline{n}_t\right)$. Furthermore we can define the *consumption function*

$$c^\star\left(\overline{d}_t, \overline{n}_t\right) \equiv \overline{n}_t - n^\star\left(\overline{d}_t, \overline{n}_t\right) \qquad \text{(A.6)}$$

Conditional on d_t, we have that the choice of n_t is constrained by

$$\begin{aligned}
n_t &\in \left[\underline{n}\left(\overline{d}_t, d_t\right), \overline{n}_t\right] \qquad \text{(A.7)} \\
\underline{n}\left(\overline{d}_t, d_t\right) &\equiv \begin{cases} -d_t & \text{if } d_t \leq \overline{d}_t \\ -\min\left\{d_t, \frac{1}{\eta}\left(\varphi - d_t\right)\right\} & \text{if } d_t > \overline{d}_t \end{cases}
\end{aligned}$$

Noting that $n_t = \overline{n}_t$ implies $c_t = 0$, we can conclude that $n^\star\left(\overline{d}_t, \overline{n}_t\right) < \overline{n}_t$.

A.3 "Lazy" Household

Consider a "lazy" household who only "knows" the optimal choice functions $d^\star\left(\overline{d}_t, \overline{n}_t\right)$ and $n^\star\left(\overline{d}_t, \overline{n}_t\right)$ in the particular point $\left(\hat{\overline{d}}, \hat{\overline{n}}\right)$. Due to its laziness it also chooses $d_t = d^\star\left(\hat{\overline{d}}, \hat{\overline{n}}\right)$ and $n_t = n^\star\left(\hat{\overline{d}}, \hat{\overline{n}}\right)$ for all $\left(\overline{d}_t, \overline{n}_t\right) \neq \left(\hat{\overline{d}}, \hat{\overline{n}}\right)$ whenever that it feasible.

If $n^{\star}\left(\widehat{\overline{d}},\widehat{\overline{n}}\right)>\underline{n}\left(\widehat{d},d^{\star}\left(\widehat{\overline{d}},\widehat{\overline{n}}\right)\right)$ then because $n^{\star}\left(\widehat{\overline{d}},\widehat{\overline{n}}\right)<\widehat{\overline{n}}$ this lazy behavior is at least feasible in a *small open interval around* $\widehat{\overline{n}}$, $\mathcal{O}\left(\widehat{\overline{n}}\right)$. Hereby we can define the *"lazy" household value function*

$$
\begin{aligned}
\forall\overline{n}_t\in\mathcal{O}\left(\widehat{\overline{n}}\right):\,L\left(\overline{n}_t;\widehat{\overline{d}},\widehat{\overline{n}}\right) &= u\left(\overline{n}_t-n_L^{\star}\left(\widehat{\overline{d}},\widehat{\overline{n}}\right)\right) \\
&\quad+\beta\cdot\sum_{\Psi\times\Xi}\psi^{1-\rho}\cdot v\left(\overline{d}_{+}\left(d_L^{\star};\psi\right),\overline{n}_{+}\left(d_L^{\star},n_L^{\star};\psi,\xi\right)\right) \\
&\quad\text{where} \\
&\quad d_L^{\star}\equiv d^{\star}\left(\widehat{\overline{d}},\widehat{\overline{n}}\right) \\
&\quad n_L^{\star}\equiv n^{\star}\left(\widehat{\overline{d}},\widehat{\overline{n}}\right)
\end{aligned}
\tag{A.8}
$$

where the *continuation value* $v\left(\bullet\right)$ is a *constant* depending on $\left(\widehat{\overline{d}},\widehat{\overline{n}}\right)$.

Note that $L\left(\overline{n}_t;\widehat{\overline{d}},\widehat{\overline{n}}\right)$ is a *differentiable lower support function* for $v\left(\widehat{\overline{d}},\overline{n}_t\right)$ at $\widehat{\overline{n}}$ as

$$\forall\overline{n}_t\in\mathcal{O}\left(\widehat{\overline{n}}\right):\quad L\ \text{is differentiable.}\tag{A.9}$$

$$\forall\overline{n}_t\in\mathcal{O}\left(\widehat{\overline{n}}\right):\quad L\left(\overline{n}_t;\widehat{\overline{d}},\widehat{\overline{n}}\right)\leq v\left(\widehat{\overline{d}},\overline{n}_t\right)\tag{A.10}$$

$$\overline{n}_t=\widehat{\overline{n}}:\quad L\left(\overline{n}_t;\widehat{\overline{d}},\widehat{\overline{n}}\right)=v\left(\widehat{\overline{d}},\overline{n}_t\right)\tag{A.11}$$

For later use we note

$$
\begin{aligned}
L'\left(\overline{n}_t;\widehat{\overline{d}},\widehat{\overline{n}}\right) &= u_c'\left(\overline{n}_t-n^{\star}\left(\widehat{\overline{d}},\widehat{\overline{n}}\right)\right) \\
&= \left(\overline{n}_t-n^{\star}\left(\widehat{\overline{d}},\widehat{\overline{n}}\right)\right)^{-\rho}
\end{aligned}
\tag{A.12}
$$

A.4 Euler-Equation

Proposition A.1. *Conditional on $d_t=\widehat{d}$ an interior optimal consumption choice $c_t^{\star,\widehat{d}}\equiv c^{\star}\left(\overline{d}_t,\overline{n}_t;\widehat{d}\right)$ must satisfy the Euler-equation*

$$
\begin{aligned}
u_c'\left(c_t^{\star,\widehat{d}}\right) &= (1+r_a)\cdot\beta\cdot\sum_{\Psi\times\Xi}\cdot\psi^{-\rho}\cdot u_c'\left(c_{t+1}^{\star}\right)\Leftrightarrow \\
c_t^{\star,\widehat{d}} &= \left[(1+r_a)\cdot\beta\cdot\sum_{\Psi\times\Xi}\cdot\left(\psi\cdot c_{t+1}^{\star}\right)^{-\rho}\right]^{-\frac{1}{\rho}}
\end{aligned}
\tag{A.13}
$$

where $c^{\star}_{t+1} \equiv c^{\star}\left(\overline{d}_{+}\left(\widehat{d};\psi\right),\overline{n}_{+}\left(\widehat{d},n_t^{\star,\widehat{d}};\psi,\xi\right)\right)$ *with* $n_t^{\star,\widehat{d}} \equiv n^{\star,\widehat{d}}\left(\overline{d}_t,\overline{n}_t;\widehat{d}\right)$ *as the corresponding optimal (net) savings choice.*

Proof. Define the *value-of-choice function* conditional on the debt choice as

$$\begin{aligned}\phi\left(n_t;\overline{d}_t,\overline{n}_t,\widehat{d}\right) &\equiv u\left(\overline{n}_t-n_t\right) \\ &\quad+\beta\cdot\sum_{\Psi\times\Xi}\cdot v\left(\overline{d}_{+}\left(\widehat{d};\psi\right),\overline{n}_{+}\left(\widehat{d},n_t;\psi,\xi\right)\right)\end{aligned}\tag{A.14}$$

Then consider the two functions:

$$\begin{aligned}\overline{\phi}\left(n_t;\overline{d}_t,\overline{n}_t,\widehat{d}\right) &\equiv u\left(\overline{n}_t-n_t^{\star,\widehat{d}}\right) \\ &\quad+\beta\cdot\sum_{\Psi\times\Xi}\cdot\psi^{1-\rho}\cdot v\left(\overline{d}_{+}\left(\widehat{d};\psi\right),\overline{n}_{+}\left(\widehat{d},n_t^{\star,\widehat{d}};\psi,\xi\right)\right) \\ &\quad\text{such that} \\ &\quad\overline{\phi}\left(n_t;\overline{d}_t,\overline{n}_t,\widehat{d}\right)=\phi\left(n_t^{\star,\widehat{d}};\overline{d}_t,\overline{n}_t,\widehat{d}\right) \\ &\quad\overline{\phi}'_n\left(n_t;\overline{d}_t,\overline{n}_t,\widehat{d}\right)=0\end{aligned}\tag{A.15, A.16}$$

$$\begin{aligned}\underline{\phi}\left(n_t;\overline{d}_t,\overline{n}_t,\widehat{d}\right) &= u\left(\overline{n}_t-n_t\right) \\ &\quad+\beta\cdot\sum_{\Psi\times\Xi}\cdot\psi^{1-\rho}\cdot L\left(\overline{n}_{+}\left(\widehat{d},n_t;\psi,\xi\right)\,;\,\widehat{\overline{d}},\widehat{\overline{n}}\right) \\ &\quad\text{where} \\ &\quad\widehat{\overline{d}}\equiv d_{+}\left(\widehat{d};\psi\right) \\ &\quad\widehat{\overline{n}}\equiv\overline{n}_{+}\left(\widehat{d},n_t^{\star,\widehat{d}};\psi,\xi\right)\end{aligned}\tag{A.17}$$

where (A.15) is clearly a differentiable *upper* support function for (A.14) at $n_t = n_t^{\star,\widehat{d}}$, and (A.17) is a differentiable *lower* support function for (A.14) at $n_t = n_t^{\star,\widehat{d}}$ because the first terms are the same in both equations, and because we showed in section A.3 that

$$L\left(\overline{n}_{+}\left(\widehat{d},n_t;\psi,\xi\right);\overline{d}_{+}\left(\widehat{d};\psi\right),\overline{n}_{+}\left(\widehat{d},n_t^{\star,\widehat{d}};\psi,\xi\right)\right)$$

is a differentiable *lower* support function for

$$v\left(\overline{d}_{+}\left(\widehat{d};\psi\right),\overline{n}_{+}\left(\widehat{d},n_t;\psi,\xi\right)\right)\ \text{at}\ n_t=n_t^{\star,\widehat{d}}$$

Using the *differentiable sandwich lemma* A.1 we can now conclude that $\phi\left(n_t;\overline{d}_t,\overline{n}_t,\widehat{d}\right)$ is differentiable at $n_t = n_t^{\star,\widehat{d}}$, and by using the *reverse calculus lemma* A.2 repeatedly we can then conclude that $v\left(\overline{d}_{t+1}, \overline{n}_{t+1}\right)$ is differentiable in $\overline{n}_{t+1}$ at $n_t = n_t^{\star,\widehat{d}}$. Finally the *differentiable sandwich lemma* A.1 also implies that the derivatives of the endogenous functions at $n_t = n_t^{\star,\widehat{d}}$ is equal to the derivatives of both their upper and lower support functions. This implies

$$
\begin{aligned}
\phi_n'\left(n_t^{\star,\widehat{d}}; \overline{d}_t, \overline{n}_t, \widehat{d}\right) &= 0 \Leftrightarrow \\
u_c'\left(\overline{n}_t - n_t^{\star,\widehat{d}}\right) &= \beta \cdot \sum_{\Xi\times\Psi} \psi^{1-\rho} \cdot v_{\overline{n}}'\left(\overline{d}_+\left(\bullet\right), \overline{n}_+\left(\bullet\right)\right) \cdot \frac{\partial \overline{n}_+\left(\widehat{d}, n_t; \psi, \xi\right)}{\partial n_t} \Leftrightarrow \\
u_c'\left(c_t^{\star,\widehat{d}}\right) &= \beta \cdot \sum_{\Xi\times\Psi} \psi^{1-\rho} \cdot L'\left(\overline{n}_+\left(\bullet\right); \overline{d}_+\left(\bullet\right), \overline{n}_+\left(\bullet\right)\right) \cdot \frac{1+r_a}{\psi} \\
&= (1+r_a) \cdot \beta \cdot \sum_{\Xi\times\Psi} \psi^{-\rho} \cdot u_c'\left(c^\star\left(\overline{d}_+\left(\widehat{d};\psi\right), \overline{n}_+\left(\widehat{d}, n_t^{\star,\widehat{d}}; \psi, \xi\right)\right)\right)
\end{aligned}
$$

where first (A.16) and secondly (A.12) were used. Simple insertions now imply equation (A.13). □

A.5 Monotonicity of the Savings Correspondence

Fella (2014) presents the following lemma from Edlin and Shannon (1998):

Lemma A.3. *If $g(x,z)$ is a function where $\frac{\partial g}{\partial x}$ is strictly increasing in z at $x^\star(z) \in$ $\arg\max_x g(x,z)$, then $x^\star(z)$ is strictly increasing in z.*

To use this result we first define an *inner value function* conditional on the d_t-choice:

$$
\begin{aligned}
w\left(\overline{d}_t, \overline{n}_t, d_t\right) \equiv\ & \max_{n_t} u\left(\overline{n}_t - n_t\right) \\
& + \beta \cdot \sum_{\Xi\times\Psi} \cdot \psi^{1-\rho} \cdot v\left(\overline{d}_+\left(d_t;\psi\right), \overline{n}_+\left(d_t, n_t; \psi, \xi\right)\right)
\end{aligned} \tag{A.18}
$$

Hereby we have

$$
n_t^{\star,d_t}\left(\overline{d}_t, \overline{n}_t; d_t\right) = \arg\max_{n_t} w\left(\overline{d}_t, \overline{n}_t, d_t\right) \tag{A.19}
$$

and using proposition A.1 we get

$$
\begin{aligned}
\frac{\partial w}{\partial n}_{|n_t = n_t^{\star,d_t}} =\ & -u_c'\left(\overline{n}_t - n_t^{\star,d_t}\right) \\
& + (1+r_a) \cdot \beta \cdot \sum_{\Xi\times\Psi} \psi^{-\rho} \cdot v_{\overline{n}}'\left(\overline{d}_+\left(d_t;\psi\right), \overline{n}_+\left(d_t, n_t^{\star,d_t}; \psi, \xi\right)\right)
\end{aligned} \tag{A.20}
$$

which is clearly *increasing* in $\overline{n}_t$ due to the *concavity of the utility function.* Consequently lemma A.3 applies, and we get the following proposition

Proposition A.2. *If $\overline{n}_H > \overline{n}_L$ then for any $n_L \in n^{\star,\widehat{d}}\left(\overline{d}_t, \overline{n}_L; d_t\right)$ and any $n_H \in n^{\star,\widehat{d}}\left(\overline{d}_t, \overline{n}_H; \widehat{d}\right)$ we have $n_H \geq n_L$.*

This further implies that the inverse of $n^{\star,d_t}\left(\overline{d}_t, \overline{n}_t; d_t\right)$ with respect to $\overline{n}_t$ is a function, which is a necessity for the EGM-algorithm to work as explained in more detail by Fella (2014). Fundamentally we now know that as $\overline{n}_t$ increases, there *cannot* be any *upward* jumps in $c_t^{\star,d_t}\left(\overline{d}_t, \overline{n}_t; d_t\right)$. As we discuss in more detail in appendix B, we can therefore establish a *numerical criterion* for *"practical sufficiency"* of the Euler-equation, which we for "high enough" degrees of uncertainty always find to be satisfied.

B Solution Algorithm

The purpose of the present appendix is to describe the *solution algorithm* in detail.

B.1 Discretization

To facilitate solving the model, we consider a *discretized* version with *finite-horizon*:

$$
\begin{aligned}
v_t\left(u_t, x_t, \overline{d}_t, \overline{n}_t\right) &= \max_{d_t, c_t} u(c_t) + \beta \cdot \sum \Omega_{t+1}(\bullet) \\
&\text{s.t.} \\
n_t &= \overline{n}_t - c_t \\
\Omega_{t+1}(d_t, n_t; u_+, x_+, \psi, \xi) &= (\Gamma\psi)^{1-\rho} \cdot v_{t+1}\left(u_+, x_+, \overline{d}_+(\bullet), \overline{n}_+(\bullet)\right) \\
\overline{d}_+(d_t; \psi) &= \arg\min_{z \in \overline{\mathcal{D}}} \left| z - \frac{1}{\Gamma\psi} \cdot (1-\lambda) \cdot d_t \right| \\
&\quad \overline{\mathcal{D}} = \{0, \dots, \Upsilon\}, \ \left|\overline{\mathcal{D}}\right| = N_{\overline{d}} \in \mathbb{N}, \ \Upsilon > 0 \\
\overline{n}_+(d_t, n_t; u_+, \psi, \xi) &= \frac{1}{\Gamma\psi} \cdot [(1+r_a) \cdot n_t - (r_d - r_a) \cdot d_t] + \tilde{\xi}(u_+, \xi) \\
d_t &\in \mathcal{D}\left(u_t, x_t, \overline{d}_t, \overline{n}_t\right) \\
c_t &\in \mathcal{C}\left(u_t, x_t, \overline{d}_t, \overline{n}_t, d_t\right) \\
v_T(\overline{n}_t) &= u(\max\{\overline{n}_t, 0\}) \\
\sum &\equiv \sum_{U \times X \times \Psi \times \Xi} p(u_+, x_+, \psi, \xi \,|\, u_t, x_t) = 1
\end{aligned}
$$

where $\mathcal{D}\left(u_t, x_t, \overline{d}_t, \overline{n}_t\right)$ is the choice set for d_t and $\mathcal{C}\left(u_t, x_t, \overline{d}_t, \overline{n}_t, d_t\right)$ is the choice set for c_t:

$$
d_t \in \left[\max\{-\overline{n}_t, 0\}\ ,\ \max\left\{\overline{d}_t, \eta \cdot \overline{n}_t + \mathbf{1}_{x_t=0} \cdot \varphi\right\}\right] \tag{B.1}
$$

$$
c_t \in \left[0\ ,\ \overline{c}\left(\overline{d}_t, \overline{n}_t, d_t\right)\right] \tag{B.2}
$$

$$
\overline{c}\left(\overline{d}_t, \overline{n}_t, d_t\right) \equiv \begin{cases} \overline{n}_t + d_t & \text{if } d_t \leq \overline{d}_t \\ \overline{n}_t + \min\left\{d_t, \frac{1}{\eta}\left(\mathbf{1}_{x_t=0} \cdot \varphi - d_t\right)\right\} & \text{if } d_t > \overline{d}_t \end{cases}
$$

The critical step is discretizing the $\overline{d}_+(\bullet)$-function, but we can easily verify that both a higher Υ and/or a higher $N_{\overline{d}}$ do *not* change the *optimal choice functions* $d_t^\star\left(u_t, x_t, \overline{d}_t, \overline{n}_t\right)$ and $c_t^\star\left(u_t, x_t, \overline{d}_t, \overline{n}_t\right)$.

The shocks are discretized using *Gauss-Hermite quadrature* with node sets $\Psi =$

$\Psi(N_\psi)$ and $\xi = \xi(N_\xi)$, where N_ψ and N_ξ are the number of nodes for each shock. The lower and upper supports are $\underline{\psi} \equiv \min(\Psi)$, $\overline{\psi} \equiv \max(\Psi)$, $\overline{\xi} \equiv \max(\Xi)$, and $\underline{\xi} \equiv \min(\Xi)$. The shock probabilities naturally sum to one, and are conditional on the u_t and x_t states.

B.2 State Space

The discretization allows us to construct the *state space* starting from the the *terminal period*

$$\begin{aligned} \mathcal{S}_T(u_T, x_T) &= \left\{\left(\overline{d}_T, \overline{n}_T\right) : \overline{d}_T \in \overline{\mathcal{D}}, \, \overline{n}_T \geq \kappa_T\left(u_T, x_T, \overline{d}_T\right)\right\} \\ & \kappa_T\left(u_T, x_T, \overline{d}_T\right) = 0 \end{aligned} \tag{B.3}$$

and using the *recursion*

$$\begin{aligned} \mathcal{S}_t(u_t, x_t) &= \left\{\left(\overline{d}_t, \overline{n}_t\right) : \overline{d}_t \in \overline{\mathcal{D}}, \, \overline{n}_t \geq \kappa_t\left(u_t, x_t, \overline{d}_t\right)\right\} \\ & \kappa_t\left(u_t, x_t, \overline{d}_t\right) = \min(\mathcal{Z}) \\ & \mathcal{Z} = \left\{ z : \exists d_t \begin{array}{c} d_t \in \mathcal{D}\left(u_t, x_t, \overline{d}_t, z\right) \text{ and} \\ \forall(\psi, \xi, u_+, x_+) : \\ \overline{n}_+(d_t, z; u_+, \psi, \xi) \geq \kappa_{t+1}\left(u_+, x_+, \overline{d}_+(d_t, \psi)\right) \end{array} \right\} \end{aligned} \tag{B.4}$$

This procedure ensures that there for all *interior* points in the state space *exists* a set of choices such that the *value function is finite*. On the contrary such a set of choices does not exist on the *border* of the state space, and the value function therefore approaches $-\infty$ as $\overline{n}_t \to \kappa_t\left(u_t, x_t, \overline{d}_t\right) \geq -\max\left\{\overline{d}_t, \frac{\mathbf{1}_{x_t=0}\cdot\varphi}{1+\eta}\right\}$.
A corollary is that the households will always choose d_t and c_t such that

$$n_t > \underline{n}_t(d_t) = \max_{x_+ u_+, \psi, \xi} \frac{\Gamma\psi \cdot \left[\kappa_{t+1}\left(x_+, u_+, \overline{d}_+(d_t, \psi)\right) - \tilde{\xi}(u_+, \xi)\right] + (r_d - r_a)\cdot d_t}{1 + r_a} \tag{B.5}$$

Figure B.1: State Space Border, $\kappa_t\left(0, 0, \bar{d}_t\right)$

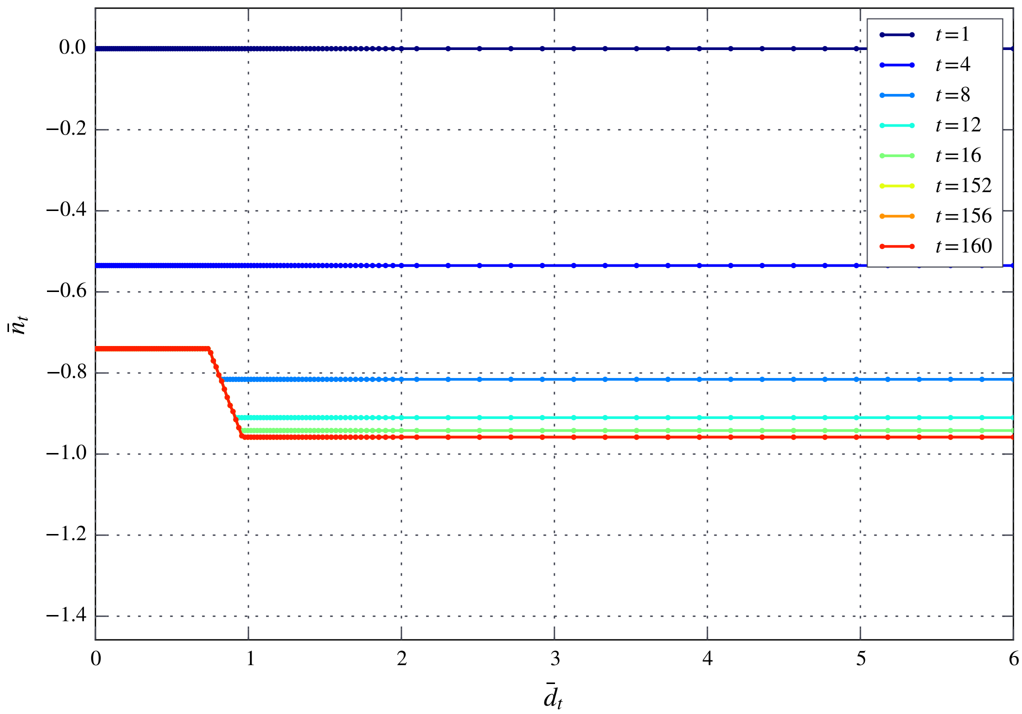
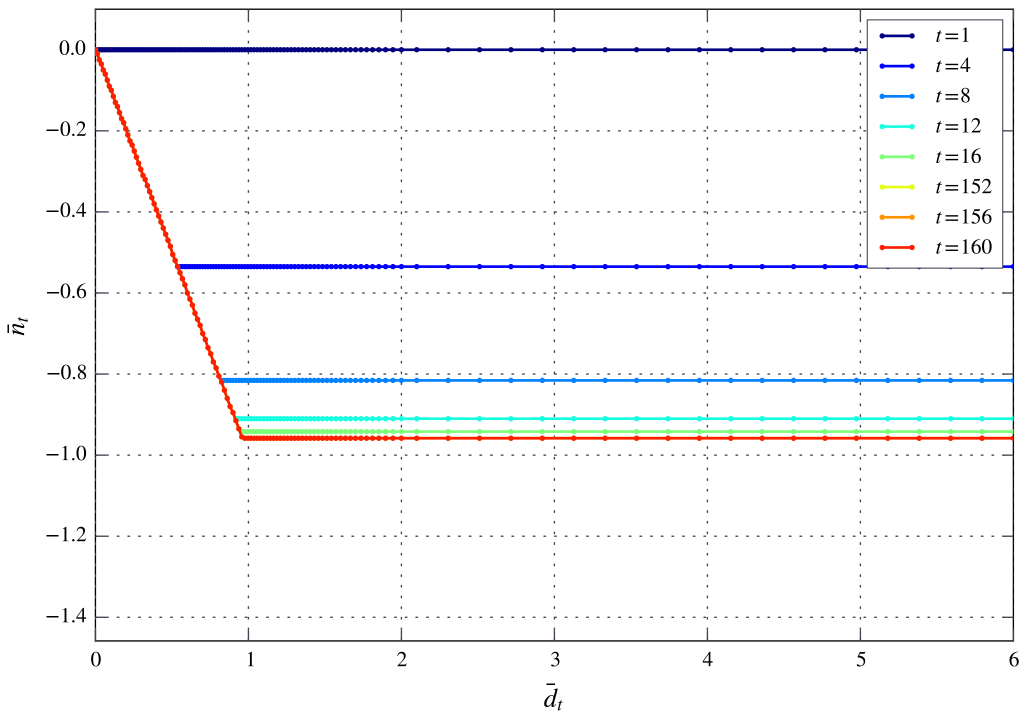


Figure B.2: State Space Border, $\kappa_t\left(1, 1, \bar{d}_t\right)$



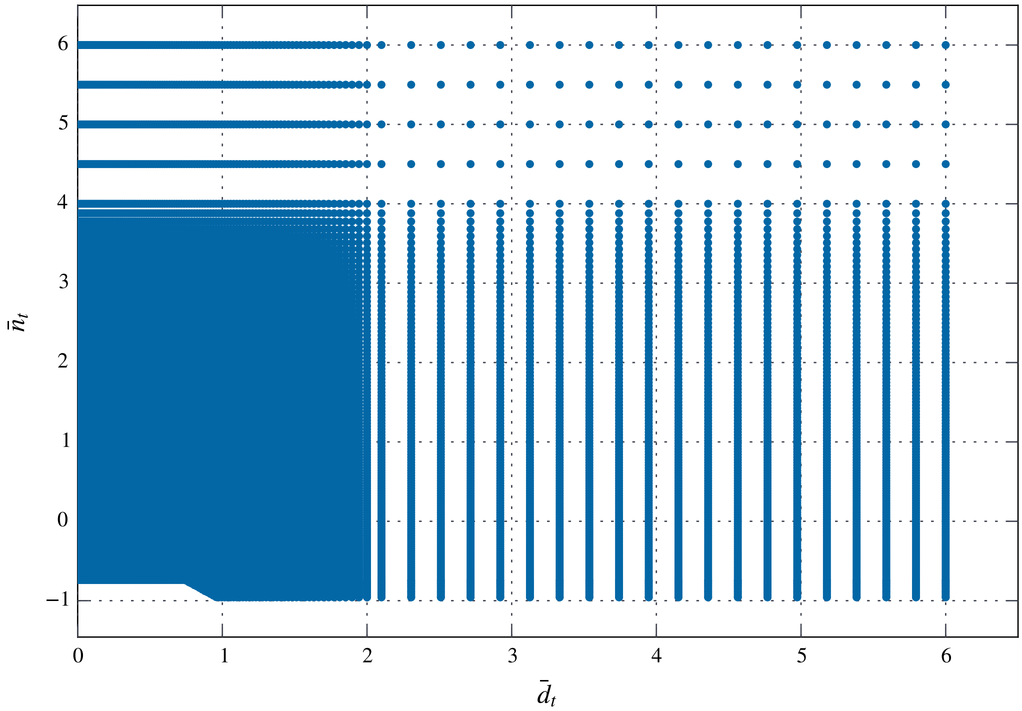
Note that the state space does not seem to have an *analytical* form, but in the limit must satisfy

$$\begin{aligned} \mathcal{S}_{-\infty}\left(u_t, x_t\right) \;&\subseteq\; \mathcal{S}_L \bigcap \mathcal{S}_S \\ \mathcal{S}_L &= \left\{\left(\overline{d}, \overline{n}\right) \,:\, \overline{n} > -\max\left\{\overline{d}, \frac{\mathbf{1}_{x_t=0}\cdot\varphi}{1+\eta}\right\}\right\} \\ \mathcal{S}_S &= \left\{\left(\overline{d}, \overline{n}\right) \,:\, \overline{n} > -\left(\phi + \phi^2 \ldots\right)\min\left\{\mu, \underline{\xi}\right\}\right\} \\ \phi &\equiv \frac{\Gamma\underline{\psi}}{1+r_d} < 1 \end{aligned} \tag{B.6}$$

Outside $\mathcal{S}_L$ the household lacks *liquidity* in the current period, and outside $\mathcal{S}_S$ it is *insolvent* under *worst case expectations*. This is also clear from figure B.1 and B.2.

The state space grid is constructed beginning with an universal $\overline{d}_t$-vector with $N_{\overline{d}}$ nodes chosen such that there are relative more nodes closer to zero. For each combination of u_t and x_t, we hereafter construct a t-specific $\overline{n}_t$-vector as the union of *a)* all unique $\kappa_t\left(u_t, x_t, \overline{d}_t\right)$-values, and *b)* a $\overline{n}_t$-vector with $N_{\overline{n}}$ nodes beginning in the largest $\kappa_t\left(u_t, x_t, \overline{d}_t\right)$-value and chosen such that there are relative more nodes closer to this minimum. The grid values of $\overline{n}_t$ conditional on $\overline{d}_t$ is then the t-specific $\overline{n}_t$-vector excluding all $\overline{n}_t < \kappa_t\left(u_t, x_t, \overline{d}_t\right)$, implying a total maximum of $N_{\overline{d}} + N_{\overline{n}}$ nodes in the $\overline{n}_t$-dimension. The grid is illustrated in figure B.3.

Figure B.3: State Space Grid ($u_t = 0$, $x_t = 0$, $t = 0$)



B.3 Value Function Iteration

The *value function iteration* is now given by $\forall \left(u_t, x_t\right), \forall \left(\overline{d}_t, \overline{n}_t\right) \in \mathcal{S}_t\left(u_t, x_t\right)$

$$v_t\left(u_t, x_t, \overline{d}_t, \overline{n}_t\right) = \max_{d_t, c_t} \frac{c_t^{1-\rho}}{1-\rho} + \beta \cdot \sum \Omega_{t+1}\left(d_t, n_t; u_+, x_+, \psi, \xi\right) \quad \text{(B.7)}$$

where when t is so low that $\mathcal{S}_t \approx \mathcal{S}_{-\infty}$, we could implement the following *stopping criterion*

$$\nexists \left(u, x, \overline{d}, \overline{n}\right) \in \mathcal{S}_{-\infty} : \left|v_t\left(u, x, \overline{d}, \overline{n}\right) - v_{t+1}\left(u, x, \overline{d}, \overline{n}\right)\right| \geq \zeta \quad \text{(B.8)}$$

where ζ is a *tolerance parameter*. To simplify matters we instead always iterate T-periods and check that our results are unchanged when increasing T.

B.4 Unconstrained Consumption Function

Assuming that the debt choice, $d_t = d$, the employment status, $u_t = u$, and the credit market access status, $x_t = x$, are given, the *Euler-equation* (see appendix

A) for the consumption choice, c_t, is

$$c_t = \left[(1+r_a)\cdot\beta\cdot\sum\left(\Gamma\psi\cdot c^{\star}_{t+1}\right)^{-\rho}\right]^{-\frac{1}{\rho}} \tag{B.9}$$

where $c^{\star}_{t+1} = c^{\star}_{t+1}\left(u_+, x_+, \overline{d}_+(d,\psi), \overline{n}_+(d, n_t, u_+, \psi, \xi)\right)$.

Assuming that the $c^{\star}_{t+1}$-function is known from earlier iterations, the *endogenous grid point method* can now be used to construct an *unconstrained consumption function*. The steps are:

1. Construct a **grid vector** of n_t-values denoted $\overrightarrow{n}$ with the minimum value $\underline{n}_t(d)+\epsilon$ (see equation (B.5)) where ϵ is a small number (e.g. 10^{-8}) and of length N_n with more values closer to the minimum.

2. Construct an ***associated*** **consumption vector**

$$\overrightarrow{c} = \left((1+r_a)\cdot\beta\cdot\sum\left(\Gamma\psi\cdot c^{\star}_{t+1}\left(u_+, x_+, \overline{d}_+(d,\psi), \overline{n}_+(d, \overrightarrow{n}, \psi, \xi)\right)\right)^{-\rho}\right)^{-\frac{1}{\rho}}$$

3. Construct an ***endogenous*** **grid vector** of $\overline{n}_t$-values by

$$\overrightarrow{\overline{n}} = \overrightarrow{n} + \overrightarrow{c}$$

4. The **unconstrained consumption function**, $c^{\circ}_{u,x,d}(\overline{n}_t)$ can now be constructed from the association between $\left\{\underline{n}_t, \overrightarrow{\overline{n}}\right\}$ and $\{0, \overrightarrow{c}\}$ together with *linear interpolation*.

Note that this can be done independently across d_t's and does *not* depend on the states, except for u_t and x_t which affects the expectations. This step speeds up the algorithm tremendously because it avoids root finding completely.

Note that because we lack a proof of sufficiency of the Euler-equation, we cannot be certain that $\overrightarrow{\overline{n}}$ will be *increasing* and thus only have *unique values*. If the same value is repeated multiple times in $\overrightarrow{\overline{n}}$ the EGM-algorithm breaks down, but in practice we find that this is never the case as long as the degree of uncertainty is "large enough".

B.5 Choice Functions

The consumption choice can now be integrated out, and the household problem written purely in terms of the debt choice, i.e.

$$
\begin{aligned}
v\left(u_t, x_t, \overline{d}_t, \overline{n}_t\right) &= \max_{d_t \in \mathcal{D}\left(u_t, x_t, \overline{d}_t, \overline{n}_t\right)} \frac{\left(c^{\bullet}\left(\bullet\right)\right)^{1-\rho}}{1-\rho} \\
&\quad +\beta \cdot \sum \Omega_{t+1}\left(d_t, n_t; u_+, x_+, \psi, \xi\right) \\
&\text{s.t.} \\
n_t &= \overline{n}_t - c^{\bullet}\left(\bullet\right) \\
c^{\bullet}\left(u_t, x_t, \overline{d}_t, \overline{n}_t, d_t\right) &= \min\left\{c^{\circ}_{u_t, x_t, d_t}\left(\overline{n}_t\right), \overline{c}\left(u_t, x_t, \overline{d}_t, \overline{n}_t, d_t\right)\right\} \\
\overline{c}\left(u_t, x_t, \overline{d}_t, \overline{n}_t, d_t\right) &= \begin{cases} \overline{n}_t + d_t & \text{if } d_t \leq \overline{d}_t \\ \overline{n}_t + \min\left\{d_t, \frac{1}{\eta} \cdot \left(\mathbf{1}_{x_t=0} \cdot \varphi - d_t\right)\right\} & \text{if } d_t > \overline{d}_t \end{cases}
\end{aligned}
\tag{B.10}
$$

This problem can be solved using a *grid search algorithm* over a *fixed* d_t-grid with step-size d_{step}, such that $c^{\circ}_{u_t, x_t, d_t}\left(\overline{n}_t\right)$ is a simple look-up table. This has to be done for all possible states, but it is possible to speed this up by utilizing some bounds on the optimal debt choice function. Specifically we use that given

$$d^{\star}\left(u_t, x_t, \Upsilon, \overline{n}_t\right) = d_{\Upsilon} \tag{B.11}$$

$$d^{\star}\left(u_t, x_t, 0, \overline{n}_t\right) = d_0 \tag{B.12}$$

$$d^{\star}\left(u_t, x_t, \overline{d}_{d=d_0}, \overline{n}_t\right) = d_0 \tag{B.13}$$

we must have

$$\forall \overline{d}_t \in \left[d_{\Upsilon} : \Upsilon\right] : \ d^{\star}\left(u_t, x_t, \overline{d}_t, \overline{n}_t\right) = d_{\Upsilon} \tag{B.14}$$

$$\forall \overline{d}_t \in \left[d_0 : d_{\Upsilon}\right), \epsilon \geq 0 : \ d^{\star}\left(u_t, x_t, \overline{d}_t + \epsilon, \overline{n}_t\right) \geq d^{\star}\left(u_t, x_t, \overline{d}_t, \overline{n}_t\right) \tag{B.15}$$

$$\forall \overline{d}_t \in \left(0, d_0\right) : \ d^{\star}\left(u_t, x_t, \overline{d}_t, \overline{n}_t\right) \leq d_0 \tag{B.16}$$

$$\forall \overline{d}_t \in \left[0 : \overline{d}_{d=d_0}\right] : \ d^{\star}\left(u_t, x_t, \overline{d}_t, \overline{n}_t\right) = d_0 \tag{B.17}$$

Over u_t, x_t and $\overline{n}_t$ the problem is jointly parallelizable. The value function is evaluated in the $\overline{n}_{t+1}$-dimension[20] by "negative inverse negative inverse" linear interpolation, where the negative inverse value function is interpolated linearly and the negative inverse of the result is then used; this is beneficial because the

[20]The other dimensions are fully discretized.

value function is then equal to zero on the border of the state space.

Note that the grid search needs to be *global* because we otherwise might find multiple *local* extrema and because there might be *discontinues* due to the non-convex choice set. This directly give us $d^\star\left(u_t, x_t, \overline{d}_t, \overline{n}_t\right)$ and therefore also

$$c^\star\left(u_t, x_t, \overline{d}_t, \overline{n}_t\right) = c^\bullet\left(u_t, x_t, \overline{d}_t, \overline{n}_t, d^\star\left(u_t, x_t, \overline{d}_t, \overline{n}_t\right)\right) \tag{B.18}$$

B.6 Implementation

The algorithm is implemented in *Python 2.7,* but the core part is written in *C* parallelized using *OpenMP* and called from Python using *CFFI*. Only free open source languages and programs are needed to run the code. The code-files are available from the authors upon request.

Table B.1 shows the parametric settings we use. Our results are robust to using even finer grids.

Table B.1: Algorithm Settings

Parameter	Value
Nodes for transitory income shock, N_ξ	8
Nodes for permanent income shock, N_ψ	8
Nodes for beginning-of-period debt, $N_{\overline{d}}$	80
Nodes for beginning-of-period net wealth, $N_{\overline{n}}$	100
Nodes for net wealth grid vector ($\overrightarrow{n}$), N_n	40
Value used to calculate mininum of net wealth grid vector, ϵ	10^{-8}
Step-size of fixed debt grid, d_{step}	$5 \cdot 10^{-3}$
Number of iterations, T	160

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