Estimating Macroeconomic Models of Financial Crises: An Endogenous Regime Switching Approach*

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February 28, 2018

Abstract

We develop a novel approach to specifying, solving and estimating Dynamic Structural General Equilibrium (DSGE) models of financial crises. We first propose a new specification of the standard Kiyotaki-Moore type collateral constraint where the movement from the unconstrained state of the world to constrained state is a stochastic function of the endogenous leverage ratio in the model. This specification results in an endogenous regime switching model. Next, we develop perturbation methods to solve this model. Using the second order solution of the model, we then design an algorithm to estimate the parameters of the model with full-information Bayesian methods. Applying the framework to quarterly Mexican data since 1981, we find that the model's estimated crisis regime probabilities correspond closely with narrative dates for Sudden Stops in Mexico. Our results also shows that fluctuations in the non-crisis regime of the model are driven primarily by real shocks, while leverage shocks are the prime driver of the crisis regime. The paper provides the first set of structural estimates of financial shocks stressed in the normative literature and consistent with available reduced form evidence finding that financial/credit shocks only matter in crisis periods.

Keywords: Financial Crises, Regime Switching, Bayesian Estimation, Leverage Shocks. **JEL Codes**: G01, E3, F41, C11.

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Motivation

- Global financial crisis proved very costly to resolve
- Long history of painful financial crises in emerging markets
- A large theoretical literature has emerged in response
 - ▶ Models of collateral constraints for amplification of shocks
 - Normative analyses of inefficiencies associated with collateral constraints
 - ▶ Debate over *ex-ante* versus *ex-post* policies
 - Debate over which instruments are most effective

Missing piece in financial crisis literature

occasionally binding constraints

Quantitative analysis of financial crises in estimated models with

- ▶ Which shocks drive crises? Are they the same that drive normal cycles?
- Is there time variation in the importance of those shocks?
- How do the dynamic responses to shocks change when collateral constraints bind?
- One can then return to the theoretical questions of when should policy makers intervene and with which instruments?
 - Does it matter what shocks drive crises?
 - Which instruments best address which shocks?

Pre-Crisis and Post-Crisis Consensus on Methodology

- Pre-crisis: Medium scale estimated linear DSGE models
 - Estimate importance of shocks and frictions
 - ► Analyze policy questions in this fully specified empirical framework
 - ▶ Non-linearity restricted to 2nd order solution of model
- Post-Crisis: Events studies with calibrated models featuring non-linear dynamics
 - Non-linearity often in the form of occasionally binding borrowing constraints
- This paper bridges the two approaches by providing an empirical framework that allows for estimation of shocks and frictions while at the same time incorporating the nonlinearities associated with financial crises

Overview

- New approach to specifying, solving and estimating models of financial crises
 - lacktriangle Financial crises are rare but large events ightarrow model must be non-linear
 - Non-linearity poses computational problems
 - We provide a tractable formulation of collateral constraint and then develop methods to solve and estimate a model with such a constraint
- We set up a model with a Kiyotaki-Moore type of collateral constraint
 - ► The constraint limits total debt to a fraction of the market value of physical capital (it is a limit on leverage)
 - Constraint imposed on the agents as in Kiyotaki and Moore (1997),
 Aiyagari and Gertler (1999), Kocherlakota (2000) and Mendoza (2010)
 - ► Constraint is not derived from an optimal contract, but is motivated by the optimal contracting literature

Overview (Cont.)

- We propose a new specification of such a collateral constraint
 - We model the movement from unconstrained state of the world to constrained state as a stochastic function of the LTV ratio (or leverage ratio)
 - ★ We can then write constraint as a regime switching process
 - ★ One regime in which the constraint binds (a crisis regime)
 - ★ One regime in which it does not bind (normal regime)
 - ▶ Probability of the collateral constraint binding rises with leverage
 - ★ This captures the fact that the likelihood of a crisis raises with leverage, without requiring a crisis to occur
 - Agents in the model know that higher leverage levels (and lower collateral values) increase the probability of a financial crisis
- Our constraint specification is an endogenous regime switching model

Overview-cont.

- We develop a solution method for endogenous regime switching models
 - ► The solution is an approximation around a steady state which is the average of the deterministic steady state in the two regimes, weighting with ergodic probabilities
- Some features of our solution method
 - We solve with perturbation methods: we can handle multiple state variables and many shocks
 - ► Second order approximation: Capture impact of risk on decision rules
 - ightharpoonup Fast solution method ightarrow non-linear filters can be used to calculate the likelihood function
 - Structural model would allow us to perform policy counterfactuals (future work)

Outline of Talk

- Model
- Collateral Constraint Formulation
- Solution
- Properties of Model Solution
- Estimation Procedure
- Empirical Results

Structure and Utility

- Small open economy
 - ▶ Bianchi and Mendoza (2015) and Huo and Rios-Rull (2016) apply this setup even to the United States
- Model very similar to Mendoza (2010), but different specification of the borrowing constraint and set of shocks considered
- Consumer Utility with GHH preferences

$$U \equiv \mathbb{E}_0 \sum_{t=0}^{\infty} \left\{ \beta^t \frac{1}{1-\rho} \left(C_t - \frac{H_t^{\omega}}{\omega} \right)^{1-\rho} \right\}$$

Production and Constraints

Production uses capital, labor and imported intermediate goods

$$Y_t = A_t K_{t-1}^{\eta} H_t^{\alpha} V_t^{1-\alpha-\eta}$$

There is a working capital requirement

$$\phi r_t \left(W_t H_t + P_t V_t \right)$$

Investment subject to adjustment costs

$$I_{t} = \delta K_{t-1} + \left(K_{t} - K_{t-1}\right) \left(1 + \frac{\iota}{2} \left(\frac{K_{t} - K_{t-1}}{K_{t-1}}\right)\right)$$

Budget constraint

$$C_t + I_t = Y_t - P_t V_t - \phi r_t (W_t H_t + P_t V_t) - \frac{1}{(1 + r_t)} B_t + B_{t-1}$$

• $B_t < 0$ denotes the debt position at the end of period t

- The agent faces a regime specific collateral constraint
- In regime 1 (the crisis regime) the constraint binds, and total borrowing is equal to a fraction of the value of collateral:

$$\frac{1}{\left(1+r_{t}\right)}B_{t}-\phi\left(1+r_{t}\right)\left(W_{t}H_{t}+P_{t}V_{t}\right)=-\kappa_{t}q_{t}K_{t}$$

- ▶ Both debt and and working capital are restricted
- ► Collateral in the model is defined over the value of capital
- ► The price and quantity of collateral are endogenous (since this relative price is in the constraint, there is the pecuniary externality emphasized in the normative literature)

- In regime 0 (the normal regime) the constraint is slack, and collateral value is sufficient for international lenders to finance all desired borrowing
 - ► There is no explicit constraint on borrowing in this regime
 - A small debt elastic interest rate premium prevents infinite debt
- Define a new variable (the "borrowing cushion") which measures the distance between debt and the value of collateral that constraints borrowing

$$B_t^* = rac{1}{\left(1+r_t
ight)}B_t - \phi\left(1+r_t
ight)\left(W_tH_t + P_tV_t
ight) + \kappa_t q_t K_t$$

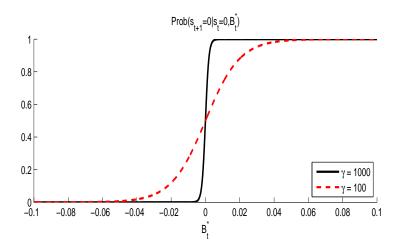
• When the borrowing cushion is small the leverage ratio is high

• In regime 0 (non-binding) the probability that constraint binds the next period depends the borrowing cushion

$$\Pr\left(s_{t+1} = 1 | s_t = 0\right) = \frac{\exp\left(\gamma_{0,0} - \gamma_{0,1} B_t^*\right)}{1 + \exp\left(\gamma_{0,0} - \gamma_{0,1} B_t^*\right)}$$

- The logistic function reformulates the Kiyotaki-Moore idea that increased leverage leads to binding collateral constraints as a probabilistic statement
- ullet The transition probability from regime 0 to regime 1 is a function of all endogenous variables in B_t^*

As parameter γ_0 gets large we recover the deterministic step function of the literature as a special case



Empirical motivation for our formulation

- Borrowing constraints don't bind at any particular LTV ratio in the real world, they are are stochastic functions of LTV ratios
- When a borrower hits the LTV limit, expenditure is adjusted gradually because other source of financing such as cash, precautionary credit lines, asset sales, etc. can be tapped into
 - Capello, Graham, and Harvey (JFE, 2010) survey information on behavior of financially constrained firms
 - Ivashina and Scharfstein (JFE, 2010) loan level data show credit origination dropped during the crisis because firms drew down from pre-existing credit lines in order to satisfy their liquidity

- Our model has the usual slackness condition $B_t^* \lambda_t = 0$
- In the binding Regime 1 the Lagrange multiplier λ_t , associated with the constraint is strictly positive. The transition probability to go back to regime 0 is given by

$$\Pr\left(s_{t+1} = 0 | s_t = 1\right) = \frac{\exp\left(\gamma_{1,0} - \gamma_{1,1}\lambda_t\right)}{1 + \exp\left(\gamma_{1,0} - \gamma_{1,1}\lambda_t\right)}$$

- As the multiplier approaches 0, the probability of transitioning back to the non-binding state rises
- \bullet Prior on $\gamma_{1,0}$ can impose that probability of negative multiplier is very small

Shocks

- There are 5 shocks (2 real, 3 financial): productivity, terms of trade, and leverage, risk premium, world interest rate,
- Interest rate process has endogenous and exogenous components

$$r_t = \left(\psi_r + \sigma_r arepsilon_{r,t}
ight) \left(e^{\overline{B}-B_t} - 1
ight) + \left(r^* + \sigma_w arepsilon_{w,t}
ight)$$

• The TFP and TOT processes:

$$\log A_{t} = (1 - \rho_{A(s_{t})})a(s_{t}) + \rho_{A}(s_{t})\log A_{t-1} + \sigma_{A}(s_{t})\varepsilon_{A,t}$$
$$\log P_{t} = (1 - \rho_{P(s_{t})})p(s_{t}) + \rho_{P}(s_{t})\log P_{t-1} + \sigma_{P}(s_{t})\varepsilon_{P,t}$$

▶ The means and persistence of these shocks are regime dependent

Leverage Shocks

- Restrictions on leverage are stochastic, and may depend on regime
- Binding regime

$$\frac{1}{\left(1+r_{t}\right)}B_{t}-\phi\left(1+r_{t}\right)\left(W_{t}H_{t}+P_{t}V_{t}\right)=-\kappa_{t}q_{t}K_{t}$$

Nonbinding regime

$$B_t^* = \frac{1}{\left(1 + r_t\right)} B_t - \phi \left(1 + r_t\right) \left(W_t H_t + P_t V_t\right) + \kappa_t q_t K_t$$

Where

$$\kappa_{t} = (1 - \rho_{\kappa(s_{t})})\kappa(s_{t}) + \rho_{\kappa}(s_{t})\kappa_{t-1} + \sigma_{\kappa}(s_{t})\varepsilon_{\kappa,t}$$

Solution

- The FOCs, constraints and shocks yield 16 equilibrium conditions
 - ▶ This is the full set of structural equations of the model
- The model as written is a nonlinear model similar to the literature and can in principle be solved with global solution methods
- We compute an approximate solution by solving the model around a steady state
 - ► This steady state is an average of the steady states associated with the two regimes weighting with ergodic probabilities
 - ► The perturbation solution includes a term that corrects for the fact that the switching model is either above or below this approximation point

Markov Switching DSGE Literature

- Large literature on Markov-switching linear rational expectations models (MSLREs) with exogenous switching
 - ► Leeper and Zha (2003), Davig and Leeper (2007), Farmer, Waggoner, and Zha (2009)
- Foerster et al (2016) developed a perturbation method for constructing first and second order solution of exogenous Markov-switching DSGE models (MSDSGE)
 - ► A key innovation in their paper is to work with the original MSDSGE model directly
- Small literature on endogenous switching DSGE models
 - ▶ Davig and Leeper (2006), Lind (2014) Barthelemy and Marx (2016)

Regime Switching: Approximation

- We introduce two indicator variables that allow the regime switching to affect both the slope and intercept of the decision rules:
 - ▶ The variables $\varphi(s_t) = \gamma(s_t) = s_t$ turn "on" and "off" the collateral constraint, depending on the regime
- The borrowing constraint for the two regimes can then be written:

$$\varphi\left(s_{t}\right)B_{ss}^{*}+\gamma\left(s_{t}\right)\left(B_{t}^{*}-B_{ss}^{*}\right)=\left(1-\varphi\left(s_{t}\right)\right)\lambda_{ss}+\left(1-\gamma\left(s_{t}\right)\right)\left(\lambda_{t}-\lambda_{ss}\right)$$

- Where SS denotes a steady state
- With this function:
 - ▶ When $s_t = 0$, then $\varphi(0) = \gamma(0) = 0$ and the equation simplifies to

$$\lambda_t = 0$$

• When $s_t = 1$, then $\varphi(1) = \gamma(1) = 1$ and the equation simplifies to

$$B_{t}^{*}=0$$

Regime Switching: Approximation

Approximate constraint again:

$$\varphi\left(s_{t}\right)B_{ss}^{*}+\gamma\left(s_{t}\right)\left(B_{t}^{*}-B_{ss}^{*}\right)=\left(1-\varphi\left(s_{t}\right)\right)\lambda_{ss}+\left(1-\gamma\left(s_{t}\right)\right)\left(\lambda_{t}-\lambda_{ss}\right)$$

- This equation also pins down the correct steady state values
 - Following FRWZ (2014) only the switching variable $\varphi\left(s_{t}\right)$ is perturbed
 - The steady state slackness condition then satisfies

$$\bar{\varphi}B_{ss}^* = (1 - \bar{\varphi})\,\lambda_{ss}$$

- ightharpoons $\bar{\varphi}$ is the ergodic mean of $\varphi(s_t)$
- If only the non-binding regime occurs, then $\bar{\varphi}=0$ and

$$\lambda_{ss} = 0$$

lacksquare If only the binding regime occurs then $ar{arphi}=1$ and

$$B_{ss}^{*} = 0$$

Regime Switching: Equilibrium

Recall that the model has 16 equilibrium conditions, in vector form:

$$\mathbb{E}_{t} f\left(\mathbf{y}_{t+1}, \mathbf{y}_{t}, \mathbf{x}_{t}, \mathbf{x}_{t-1}, \chi \varepsilon_{t+1}, \varepsilon_{t}, \theta_{t+1}, \theta_{t}\right) = 0$$

where θ_t is the vector of model parameters and the subset for the stochastic processes is regime dependent $(\theta_{S,t})$

Predetermined variables:

$$\mathbf{x}_{t-1} = [K_{t-1}, B_{t-1}, \kappa_{t-1}, A_{t-1}, P_{t-1}]$$

Non-predetermined variables:

$$\mathbf{y}_{t} = [C_{t}, H_{t}, V_{t}, I_{t}, k_{t}, r_{t}, q_{t}, W_{t}, \mu_{t}, \lambda_{t}, B_{t}^{*}]$$

• 5 shocks:

$$\varepsilon_t = [\varepsilon_{A,t}, \varepsilon_{w,t}, \varepsilon_{P,t}, \varepsilon_{\kappa,t}, \varepsilon_{r,t}]$$

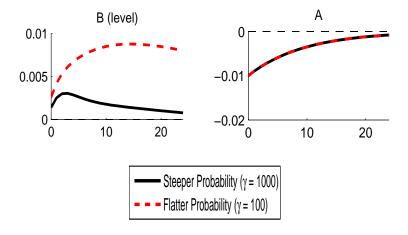
Regime Switching: Perturbation Solution

- The perturbation solution takes the stacked equilibrium conditions and differentiates with respect to $(\mathbf{x}_{t-1}, \varepsilon_t, \chi)$, producing a complicated polynominal system
- We solve this polynomial system by finding a fixed point of a sequence of eigenvalue problems
- This procedure finds a single solution, but does not guarantee uniqueness
- If desired second order system can also be solved
- Second order solution is critical for endogenous switching model:
 - ▶ We show that the first order solution of the endogenous switching is identical to the first order solution of an exogenous switching model, but the second order solution differs
 - ► Interpretation: precautionary behavior in the second order solution is critical for endogenous switching to matter

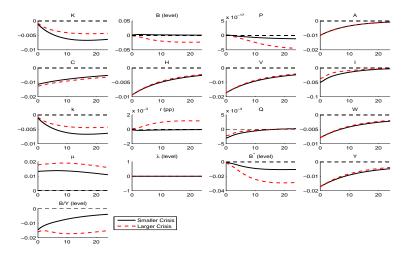
Solution Results

- IRF to shocks for different parameterizations of the model
- We compute solution for endogenous switching, exogenous switching and no switching
 - Does endogenous switching matter?
- IRF is computed assuming we stay in the state (binding or not)

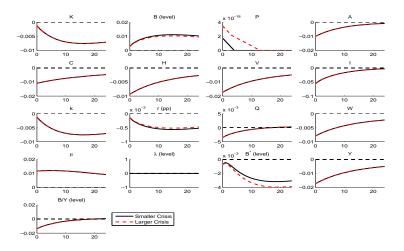
Probability Functions and IRF to TFP shock



IRF in Non-binding state: large versus small crisis (TFP Shock)



IRF in Non-binding state: large versus small crisis (1st order solution, TFP Shock)



Bayesian Full Information Likelihood Methods for Nonlinear Models

- We cannot assume that the parameters in one regime are independent of parameters of the other regime → two step procedures are inappropriate in our case (e.g. Aruoba, Cuba-Borda, Schorfheide (2014), Bocola (2016))
 - Agents in the model fully understand that a crisis may occur, and adjust their behavior accordingly
 - Our estimated model is useful for normative analysis precisely because of this feature of the model solution/estimation
- We need a procedure for simultaneous estimation of regime switch and parameters in each regime
- Second order solution needed
 - ▶ Bianchi (2014) estimates MSLRE with first order solution

Estimation: Bayesian Full Information Likelihood Methods

- We use a Metropolis-in-Gibbs Sampling procedure
 - Conditional on regimes, draw parameters using the standard MH algorithm
 - Given parameters, regimes, data, the value of the likelihood function is computed with a Sigma Point Filter
 - The Sigma Point Filter is used in conjunction with the second order solution of the model
 - ★ The value of the posterior is then computed after evaluating priors
 - Conditional on parameters, data, draw regimes

Estimation from 1981.Q1 to 2016.Q1

- Real GDP Growth, Investment growth, Consumption Growth, Import Price Growth
- Interest rate: (EMBI Global + world interest rate (3month T-Bill rate
 US expected inflation))

Basic Calibration

Table: Basic Calibration

Parameter		Calibrated Value
Discount Factor	β	0.97959
Risk Aversion	ρ	2
Labor Share	α	0.592
Capital Share	η	0.306
Wage Elasticity of Labor Supply	ω	1.846
Capital Depreciation	δ	0.022766
Debt to Output Ratio	$\frac{B}{Y}$	-0.86
Interest Rate Elasticity	$\dot{\psi}_{r}$	0.001
Mean of TFP Process, Normal Regime	a(0)	0
Mean of Import Price Process, Normal Regime	p(0)	0
Mean of Leverage Process, Normal Regime	$\kappa(0)$	0.15

Prior and Posterior: Preliminary Estimation Results

Table: Some key structural parameters

Parameter	Prior	Posterior mean	q5	q95
γ_0	Uniform(0,1000)	163.8018	162.0918	166.0587
γ_1	Uniform(0,1000)	111.8985	107.9163	114.5517
ι	Uniform(0,100)	2.6520	2.6490	2.6557
ϕ	Uniform(0,100)	0.2588	0.2572	0.2608

Posterior of Logistic Function

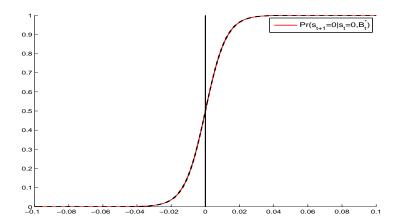


Figure: Transition prob. of nonbinding conditional on nonbinding

Prior and Posterior: Preliminary Estimation Results

Table: Shock Standard Deviations

Parameter	Prior	Posterior mean	q5	q95
$\sigma_r(0)$	Uniform(0,1)	0.0023	0.0014	0.0031
$\sigma_r(1)$	Uniform(0,1)	0.0104	0.0098	0.0109
$\sigma_w(0)$	Uniform(0,1)	0.0017	0.0014	0.0019
$\sigma_w(1)$	Uniform(0,1)	0.0181	0.0175	0.0189
$\sigma_a(0)$	Uniform(0,1)	0.0065	0.0056	0.0075
$\sigma_{\sf a}(1)$	Uniform(0,1)	0.0077	0.0069	0.0084
$\sigma_p(0)$	Uniform $(0,1)$	0.0207	0.0201	0.0214
$\sigma_p(1)$	Uniform(0,1)	0.0005	0.0001	0.0009
$\sigma_{\kappa}(0)$	Uniform(0,1)	0.0132	0.0114	0.0145
$\sigma_{\kappa}(1)$	Uniform(0,1)	0.0070	0.0066	0.0073

Prior and Posterior: Preliminary Estimation Results

Table: Shock Persistence and Means

Parameter	Prior	Posterior mean	q5	q95
$\rho_a(0)$	Uniform(0,1)	0.8330	0.7719	0.8665
$\rho_p(0)$	Uniform $(0,1)$	0.6764	0.6028	0.7701
$ ho_{\kappa}(0)$	Uniform $(0,1)$	0.9826	0.9733	0.9885
$ ho_{\sf a}(1)$	Uniform $(0,1)$	0.8930	0.8665	0.9342
$\rho_{p}(1)$	Uniform $(0,1)$	0.6196	0.5806	0.6504
$ ho_{\kappa}(1)$	Uniform $(0,1)$	0.6990	0.6649	0.7350
a(1)	Uniform(-10,0)	-0.0004	-0.0004	-0.0003
p(1)	Uniform(0,10)	0.0001	0.0000	0.0002
$\kappa(1)$	Uniform(0,1)	0.2078	0.2046	0.2107

Probability of a Binding Regime: Reinhart-Rogoff Currency Crisis in Gray

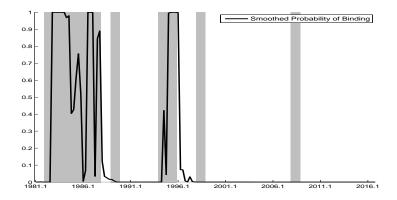


Figure: Smoothed Probability of Binding

Probability of a Binding Regime: OECD Recession Indicator for Mexico in Gray

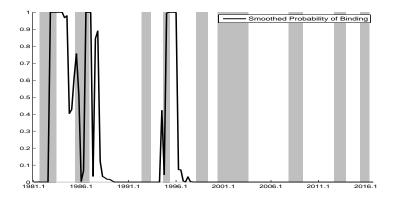


Figure: Smoothed Probability of Binding

Binding Regime Estimates

- Mexico had financial crises in 1982, 1994, both of which show up as binding regimes
- The results show that collateral constraints in the 1980s binded outside the crisis
- We find no crisis for Mexico in 2007
- Recession does not mean binding collateral constraint

Importance of Shocks

Table: Variance Decomposition

			С	ı	r
Risk Premium Shock	$\varepsilon_{r,t}$	Non-Binding	0.0000	0.0000	0.0000
World Interst Rate Shock	$\varepsilon_{w,t}$	Non-Binding	0.0091	0.2430	0.9977
Technology Shock	$\varepsilon_{a,t}$	Non-Binding	0.9068	0.6943	0.0001
Import Price Shock	$\varepsilon_{p,t}$	Non-Binding	0.0840	0.0575	8000.0
Leverage Shock	$\varepsilon_{\kappa,t}$	Non-Binding	0.0001	0.0051	0.0015
Risk Premium Shock	$\varepsilon_{r,t}$	Binding	0.0000	0.0000	0.0003
World Interst Rate Shock	$\varepsilon_{w,t}$	Binding	0.0115	0.0369	0.7192
Technology Shock	$\varepsilon_{a,t}$	Binding	0.2582	0.0136	0.0001
Import Price Shock	$\varepsilon_{m{p},m{t}}$	Binding	0.0000	0.0000	0.0000
Leverage Shock	$\varepsilon_{\kappa,t}$	Binding	0.7303	0.9496	0.2804

Conclusion

- A new approach to specifying, solving, and estimating models of financial crises nested in regular business cycles
- Probability of a change in regime depends on the state of the economy
- For the occasionally binding constraint model we find:
 - ► The endogenous nature of the regime switch impacts in a qualitative and quantitative manner the decisions of agents in the economy
 - A second order solution is needed for endogenous switching to matter economically
 - ► Leverage shocks drive fluctuations during financial crises
 - Real shocks that have beens studied for decades still matter outside of crisis!
- Conditional policy counterfactuals are future work

What is the key difference with respect to the literature?

• The typical specification of the constraint in this class of models is:

$$\frac{1}{\left(1+r_{t}\right)}B_{t}-\phi\left(1+r_{t}\right)\left(W_{t}H_{t}+P_{t}V_{t}\right)\geq\kappa_{t}q_{t}K_{t}$$

- When the left hand side is greater than the right $(B_t^* > 0$ in our notation) the constraint is slack
- When the left hand side is exactly equal to the right $(B_t^* = 0)$ in our notation) the constraint binds
- Our assumptions turn the deterministic relationship between borrowing and collateral into a stochastic one
- High leverage leads to a crisis, but with some uncertainty rather than in a deterministic manner at a given and fixed LTV ratio

A Theoretical motivation for our formulation

• Take the standard borrowing constraint in the literature and add a stochastic monitoring (or enforcement) shock ϵ_t^M .

$$\frac{1}{(1+r_t)}B_t - \phi(1+r_t)(W_tH_t + P_tV_t) \ge -\kappa_t q_t K_t + \epsilon_t^M$$

- Shock has two interpretations, based on the sign of the shock
 - Negative shock: The LHS is then greater than the value of collateral but the lender monitors and decides to impose a borrowing constraint
 - ▶ Positive shock: The LHS is then less than the value of collateral but the constraint does not bind because the lender does not audit
- Distribution of ϵ_t^M is such that when borrowing is much less than the value of collateral the probability of drawing a monitoring shock that leads to a binding constraint is 0. When borrowing exceeds the value of collateral by a large amount the probability of drawing a monitoring shock is such that the probability the lender audits goes to 1.

Endogenous Regime Switching vs. OccBin

- OccBin (Guerrieri and Iacoviello 2015) is an alternative solution method for occasionally binding constraint models
- Their solution is a certainty equivalent method which requires agents to know precisely how long regime will last if there are no shocks
- This is functionally quite similar to the perfect foresight methods used in the ZLB literature
- Their method rules out precautionary effects, which drive the economic behavior in this class of models
- It is not clear how to extend OccBin to quadratic approximations, which seem important for this type of model