# Financial Heterogeneity and Monetary Union

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**Abstract**

We analyze the economic consequences of forming a monetary union among countries with varying degrees of financial distortions, which interact with the firms' pricing decisions because of customer-market considerations. In response to a financial shock, firms in financially weak countries (the periphery) maintain cashflows by raising prices—in both domestic and export markets—while firms in financially strong countries (the core) lower prices, undercutting their financially constrained competitors to gain market share. When the two regions are experiencing different shocks, common monetary policy results in a substantially higher macroeconomic volatility in the periphery, compared with a floating exchange rate regime; this translates into a welfare loss for the union as a whole, with the loss borne entirely by the periphery. While complete risk sharing can improve the periphery's welfare, such an arrangement involves large state-contingent transfers of wealth. By helping firms from the core internalize the pecuniary externality engendered by the interaction of financial frictions and customer markets, a unilateral fiscal devaluation by the periphery can improve the union's overall welfare.

JEL Classification: E31, E32, F44, F45
Keywords: eurozone, financial crisis, monetary union, inflation dynamics, markups, fiscal devaluation, fiscal union

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# 1 Introduction

The consensus in both academic and policy circles is that the eurozone's recent economic woes stem from a classic balance-of-payment crisis, which can be traced to the toxic mix of excessive credit growth and loss of competitiveness in the euro area "periphery." Following the introduction of the euro in early 1999, periphery countries such as Greece, Ireland, Italy, Spain, and Portugal went on a borrowing spree, the proceeds of which were used largely to finance domestic consumption and housing investment. Foreign investors' widespread reassessment of risks during the 2008–2009 global financial crisis, along with a growing recognition of an unsustainable fiscal situation in Greece, precipitated a sharp pullback in private capital from the periphery in early 2010. This further tightening of financial conditions exacerbated the already painful process of deleveraging, through which the periphery economies were attempting to bring domestic spending—both government and private—back into line with domestic incomes.[1]

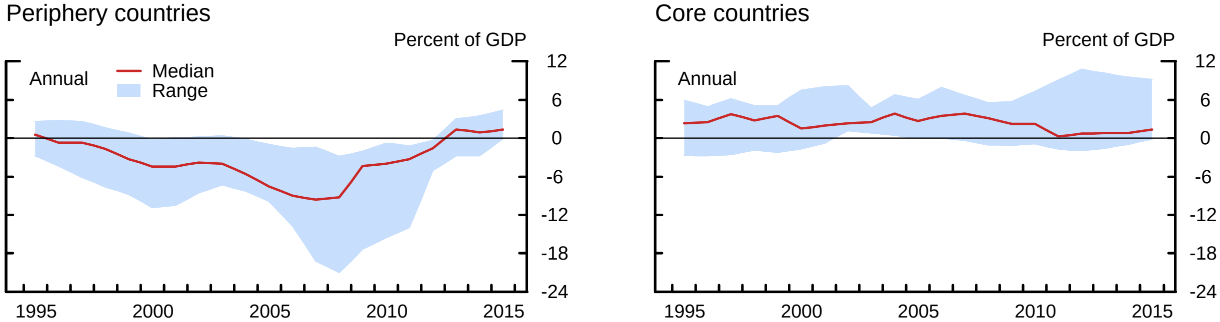
As shown in Figure 1, this narrative accords well with empirical evidence. The median current account deficit in the euro area periphery reached almost 10 percent of GDP on the eve of the global financial crisis, with some countries running current account deficits as high as 15 percent (panel (a)).[2] The next two panels provide evidence of overheating that led to the crisis: Between 1999 and 2007, periphery economies saw their real GDP growing persistently above potential, whereas their counterparts in the core registered a much more balanced pattern of economic growth (panel (b)). As a result, prices in the periphery increased at a much faster pace during this period compared with those in the core countries (panel (c)). Given these developments, real exchange rates in the periphery appreciated substantially (panel (d)), eroding these countries' competitiveness and producing large trade deficits, which were easily financed by foreign capital inflows against the backdrop of the convergence in domestic interest rates across the euro area.

In a monetary union comprised of countries experiencing dramatically different economic conditions—with limited labor mobility and no common fiscal policy—the financial crisis would have to be resolved largely through a downward adjustment of the overvalued real exchange rates in the periphery. In the euro area, however, this adjustment has occurred very slowly. Although the periphery has endured significant disinflation since 2010, a noticeable gap remains, on balance, between the general level of prices in the core and periphery. As a result, real effective exchange rates in the periphery have tended to remain above those of the core euro area countries.

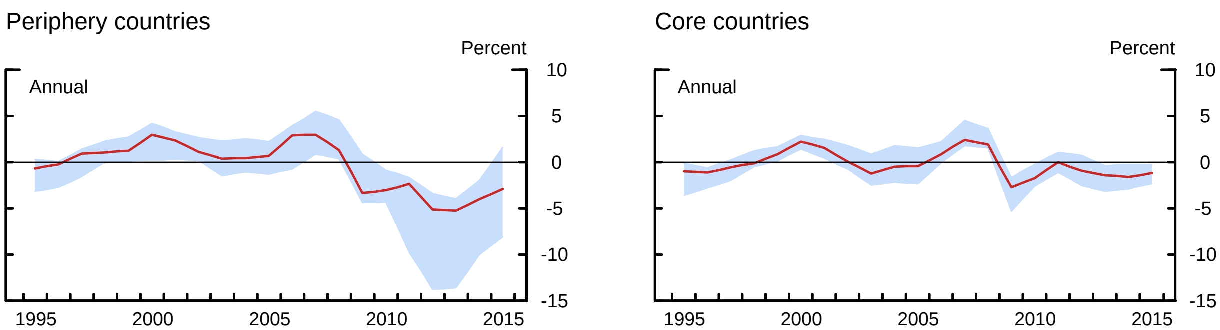
[1] As emphasized by Auer (2014) and Higgins and Klitgaard (2014), the tightening of financial conditions was not as severe as might have been expected given the scale of capital flight from the periphery. The withdrawal of capital was tempered importantly by cross-border credits to central banks in deficit countries, extended by other euro area central banks through the so-called TARGET2 system, a mechanism for managing payment imbalances among eurozone member countries. In combination with policies to supply liquidity to banks in the periphery, this balance of payments financing helped offset the drain of funds abroad.

[2] Throughout the paper, we use the following definition of the euro area core and periphery. Core countries: Austria, Belgium, Finland, France, Germany, and Netherlands. Periphery countries: Greece, Ireland, Italy, Portugal, and Spain. We omit the Eastern European countries (Estonia, Latvia, Lithuania, Slovakia, and Slovenia) from the periphery because they adopted the euro relatively recently. Our analysis also excludes Cyprus, Luxembourg, and Malta because of limited data in some instances and because of their very specialized economies. All told, our sample of countries accounts for about 95 percent of the eurozone's total economic output.

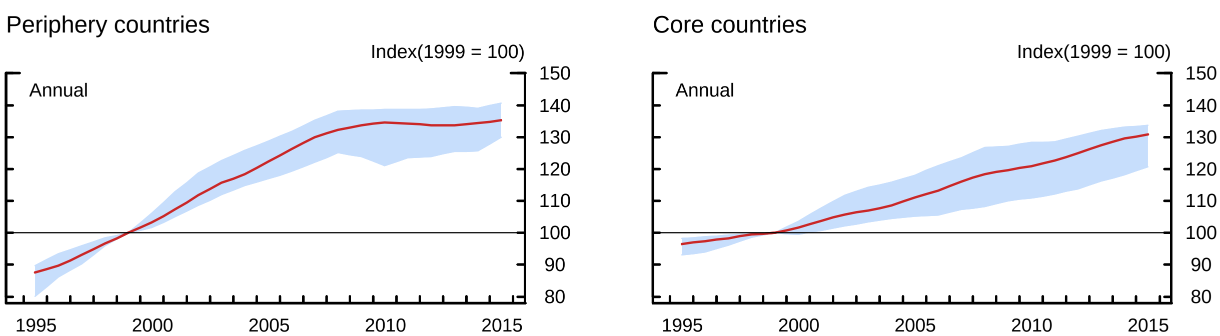
Figure 1 – Selected Macroeconomic Indicators for the Euro Area (1995–2015)



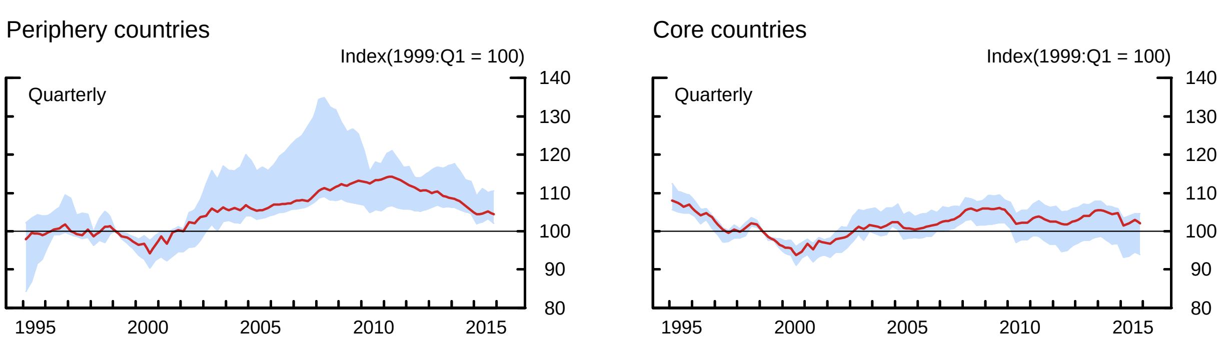

(a) Current accounts




(b) Output gaps




(c) Prices (GDP price deflator)




(d) Real effective exchange rates

Note: The solid lines depict the evolution of the cross-sectional median of the specified macroeconomic series, while the shaded bands denote the corresponding cross-sectional range. Periphery countries: Greece, Ireland, Italy, Portugal, and Spain. Core countries: Austria, Belgium, Finland, France, Germany, and Netherlands.
Source: AMECO database (European Commission); and Bank for International Settlements.

What economic forces are responsible for such a slow adjustment in the price levels between the core and periphery countries? Why have firms in the periphery—given the degree of resource underutilization in these economies—been so slow to cut prices? By the same token, why have firms in the core been reluctant to increase prices, despite an improvement in the economic outlook and highly stimulative monetary policy? In fact, some prominent commentators have argued that it is the core countries that are exporting deflationary pressures into the periphery, a dynamic contrary to that needed to reverse the real exchange rate appreciation that has eroded the periphery's competitiveness (see Krugman, 2014).

To help answer these questions, we build on Gilchrist et al. (2017), GSSZ hereafter, and introduce the interaction of customer markets and financial frictions into an otherwise standard international macroeconomic framework. Specifically, we augment the conventional multi-country model featuring home bias, firms that price to market, and a low Armington elasticity of substitution between foreign and domestic goods with two new assumptions: First, we assume that firms operate in customer markets—both domestically and abroad.[3] And second, we assume that foreign and domestic firms are subject to differing degrees of financial market frictions in the form of costly external (equity) finance.

We show that in such an environment firms from the "core"—that is, firms with a relatively unimpeded access to external finance—have a strong incentive to expand their market share at home and abroad by undercutting prices charged by their "periphery" competitors, especially when the latter are experiencing financial distress. By contrast, firms from the periphery—that is, financially constrained firms—have an incentive to increase markups in order to preserve internal liquidity, even though doing so means forfeiting some of their market share in the near term.

The idea that firms set prices to actively manage current versus expected future demand in an environment of imperfect capital markets is not new to macroeconomics (see Gottfries, 1991; Chevalier and Scharfstein, 1996). Our contribution lies in bringing the interplay of customer markets and financial frictions into the international context and studying the implications of this interaction within a dynamic, two-country stochastic general equilibrium model. As we show below, this pricing mechanism generates time-varying markups and import price dynamics that differ significantly from those in the standard literature (see Dornbusch, 1987; Kimball, 1995; Yang, 1997; Bergin and Feenstra, 2001; Atkeson and Burstein, 2008; Gopinath and Itskhoki, 2010a,b; Burstein and Gopinath, 2014; Auer and Schoenle, 2016).[4] Specifically, this literature

[3] By customer markets, we mean markets in which a customer base is "sticky" and thus an important determinant of firm's assets and its ability to generate profits. Various microeconomic mechanisms that can lead to a sticky customer base include costly switching (Klemperer, 1987), costly search (Hall, 2008), or idiosyncratic preferences (Bronnenberg et al., 2012). As emphasized by Bils (1989), pricing decisions in such an environment are a form of investment that builds the future customer base. Recent work by Foster et al. (2016) and Hottman et al. (2014) shows that customer markets feature prominently in the U.S. manufacturing and retail sectors. Moreover, Roberts et al. (2012), Eaton et al. (2015), and Fitzgerald et al. (2016) document that customer-market considerations importantly shape the pricing decision of exporting firms in both advanced and emerging market economies. The available evidence thus indicates that customer markets are a pervasive feature of the economic landscape.

[4] By exploiting the open economy setting, this literature tries to explain the firms' pricing behavior by analyzing the responsiveness of international prices to fluctuations in exchange rates.

shows that following an adverse exchange rate shock, firms do not fully pass the resulting cost increase into import prices, but instead absorb some of this cost shock in their profits by *lowering* markups. In our model, by contrast, financially constrained firms, when hit by adverse shocks, try to maintain their cashflows by *increasing* markups in both the domestic and export markets, in effect trading off future market shares for current profits.

The interaction of customer markets and financial frictions helps explain several aspects of the eurozone financial crisis that are difficult to reconcile using conventional open-economy macro models. Most importantly, the pricing mechanism implied by this interaction is consistent with our empirical evidence, which shows that the acute tightening of financial conditions in the euro area periphery between 2009 and 2013 significantly attenuated the downward pressure on prices arising from the emergence of substantial and long-lasting economic slack. The tightening of financial conditions during this period is also strongly associated with a notable increase in markups in the periphery, which is exactly the pattern predicted by our model. Thus our framework can explain why the periphery countries have managed to avoid a potentially devastating Fisherian debt-deflation spiral in the face of massive and persistent economic slack. It also helps us to understand the chronic stagnation in the euro area periphery and how the "price war" between the core and periphery has impeded the adjustment process through which the latter economies have been trying to regain external competitiveness.

The general equilibrium nature of our analysis also allows us to compare the economic implications of different currency regimes. As it is well known, with floating exchange rates, monetary authorities in the periphery should be able to largely offset the real economic effects of an asymmetric financial shock by aggressively cutting policy rates, inducing a depreciation of nominal exchange rates in the periphery. And although the price levels between the core and periphery move in opposite directions because of customer-market considerations in our model, the policy-induced currency devaluation can be sufficiently large to cause the real exchange rate to depreciate, thereby boosting exports of firms in the periphery and helping to stabilize the contraction in output.

In a monetary union, this policy option is, of course, not available. The pricing behavior of firms in the core in response to an asymmetric financial shock in the periphery implies a real exchange rate depreciation vis-à-vis the periphery, which causes an export-driven boom in the core countries and a deepening of the recession in the periphery. The divergent economic trajectories between the core and periphery present a dilemma for the union's central bank because monetary policy cannot be targeted to just one region. According to our simulations, common monetary policy in a situation where members of the union are at different phases of the business cycle increases the volatility of consumption and hours worked in the periphery significantly above the levels registered under a floating exchange rate regime. This translates into a welfare loss for the union as a whole, with the loss borne entirely by the periphery.

Given the union's problem with a "one-size-fits-all" monetary policy, we consider two fiscal policy alternatives that have received considerable attention from policymakers: a fiscal union and a unilateral fiscal devaluation by the periphery. First, we show how a complete risk-sharing arrangement can improve the union's overall welfare. In principle, such a cross-country risk-sharing arrangement can be achieved by forming a fiscal union, a point emphasized by Farhi and Werning (2014). However, our simulations indicate that such a union involves large state-contingent transfers of wealth from the core to the periphery, casting doubt on its political feasibility.

As an alternative, we consider the macroeconomic implications of a fiscal devaluation. Recent work by Adao et al. (2009) and Farhi et al. (2014) explores the stabilization properties of certain fiscal policy mixes, intended to replicate the effects of a nominal devaluation in a fixed exchange rate regime. What makes such policies desirable, according to the theory, is the fact that they can be implemented unilaterally by the periphery countries encountering economic weakness. However, it is not clear why the core countries should welcome such unilateral policy interventions—in many instances, core countries have joined the monetary union precisely to avoid the manipulation of nominal exchange rates by the monetary authorities in the periphery.

Thus, a natural question that emerges is whether the periphery can carry out a unilateral fiscal devaluation without worrying about a retaliatory reaction from the core. We show that a fiscal devaluation by the periphery can be beneficial even to the core, provided that the aggregate demand externality generated by the international price war is not remedied by the union's policymakers. When firms in the core countries lower markups to expand market shares, they do not internalize the pecuniary externality, whereby driving out their foreign competitors by undercutting prices to an excessive degree increases macroeconomic volatility in their own country. As shown by Farhi and Werning (2016), a distortionary taxation in such situations can help firms from the core internalize this externality, and fiscal devaluations provide an effective means of achieving this goal.

The remainder of the paper is organized as follows. In Section 2, we provide new empirical evidence, which shows that the dynamics of prices, wages, and markups in the eurozone periphery during the recent financial crisis were influenced importantly by the severe tightening of financial conditions, in a way that is consistent with our model. Section 3 presents the formal model and Section 4 discusses its calibration. Section 5 contains our baseline simulation results, while Section 6 explores the welfare implications of a fiscal union and fiscal devaluations. Section 7 concludes.

## 2 Financial Conditions, Prices, Wages, and Markups

We begin our empirical analysis by examining the extent to which price and wage inflation forecast errors implied by the canonical Phillips curve relationships during the eurozone's financial crisis are systematically related to differences in the tightness of financial conditions across countries. We do so in two steps. First, we use our panel of 11 euro area countries to estimate the following two Phillips curve specifications:

$$\pi_{it} = \alpha_i + \rho\pi_{i,t-1} + \lambda(u_{it} - \bar{u}_{it}) + \phi\Delta\text{VAT}_{it} + \psi\mathbf{1}[i \in \text{€}] + \epsilon_{it}; \quad (1)$$

$$\Delta w_{it} = \alpha_i + \rho\pi_{i,t-1} + \lambda(u_{it} - \bar{u}_{it}) + \phi\Delta\tilde{z}_{it} + \psi\mathbf{1}[i \in \text{€}] + \epsilon_{it}, \quad (2)$$

where $i$ indexes countries and $t$ represents time (in years). In terms of notation, $\pi_{it}$ denotes price inflation measured by the log-difference of the GDP price deflator, while $\Delta w_{it}$ denotes wage inflation measured by the log-difference of nominal compensation per employee. These two specifications are the textbook price and wage Phillips curves, which assume that inflation expectations are proportional to past inflation and where labor market tightness—measured by the difference of the unemployment rate $u_{it}$ from its corresponding natural rate $\bar{u}_{it}$—is a fundamental determinant of price and wage dynamics. The wage Phillips curve (2) also includes the growth rate of trend labor productivity—denoted by $\Delta\tilde{z}_{it}$—thereby allowing for a link between real wage bargaining and labor productivity (see Blanchard and Katz, 1999).

Although "accelerationist" Phillips curves (1) and (2) tend to fit the data quite well, their major theoretical shortcoming involves the assumptions of backward-looking inflation expectations. Accordingly, we also consider a New Keynesian variant of the Phillips curve (NKPC), which incorporates into the process of price inflation determination both rational expectations as well as more explicit microfoundations (see Galí and Gertler, 2000; Galí et al., 2001). In that case, we estimate,

$$\pi_{it} = \alpha_i + \beta_f E_t\pi_{i,t+1} + \beta_b\pi_{i,t-1} + \lambda\widehat{mc}_{it} + \phi\Delta\text{VAT}_{it} + \psi\mathbf{1}[i \in \text{€}] + \epsilon_{it}, \tag{3}$$

where $\widehat{mc}_{it}$ denotes a proxy for marginal cost. Note that in addition to country fixed effects, all three specifications also include $\mathbf{1}[i \in \text{€}]$, an indicator variable that equals one when country $i$ adopts the euro and thereafter; specifications (1) and (3) also control for the pass-through of changes in the effective value-added tax (VAT) rate to aggregate price inflation.

To ensure that our estimates of the Phillips curves are not unduly influenced by the extraordinary events surrounding the eurozone sovereign debt crisis, we estimate all three specifications using data through 2007—that is, our sample ends well before before the onset of the crisis in the euro area.[5] In columns (1) and (4) of Table 1, we report estimates of the coefficients of the standard price and wage Phillips curves, respectively; in columns (2) and (5), we repeat the same exercise, except that we allow the coefficients on the unemployment gap $(u_{it} - \bar{u}_{it})$ to differ across countries. And lastly, column (3) reports coefficient estimates of the NKPC with common coefficients, using the output gap $(y_{it} - \bar{y}_{it})$ as a proxy for marginal cost.[6]

As shown in columns (1), (2), (4), and (5), the degree of labor market slack is an economically and statistically important determinant of price and wage inflation dynamics in all four standard Phillips curve specifications. The estimated sensitivity of both price and wage inflation to tightness of labor market conditions is, on average, somewhat higher in specifications (2) and (5), which allow for a greater degree of heterogeneity in the price and wage inflation processes across countries. All

[5] All data, including the estimates of the natural rate of unemployment and potential GDP, come from the AMECO databases maintained by the European Commission. We estimate trend labor productivity $(\tilde{z}_{it})$ by regressing the log of labor productivity on a constant and a third-order polynomial in time.

[6] Specifications (1), (2), (4), and (5) are estimated by OLS; in the case of specifications (2) and (5), we report the average of the coefficient on the unemployment gap across the 11 countries in our panel. The NKPC is estimated by GMM, treating $(y_{it} - \bar{y}_{it})$ and $E_t\pi_{i,t+1}$ as endogenous and instrumented with lags 1 to 3 of $(y_{it} - \bar{y}_{it})$ and $\pi_{it}$, and lags 0 to 2 of the log-difference of commodity prices.

Table 1 – Price and Wage Phillips Curves in the Euro Area

| | Prices[a] | | | Wages[b] | |
|---|---|---|---|---|---|
| Explanatory Variables | (1) | (2) | (3) | (4) | (5) |
| $(u_{it} - \bar{u}_{it})$ | −0.273 | −0.529 | . | −0.559 | −0.659 |
| | (0.117) | (0.127) | | (0.096) | (0.118) |
| $(y_{it} - \bar{y}_{it})$ | . | . | 0.134 | . | . |
| | | | (0.084) | . | . |
| $\pi_{i,t-1}$ | 0.845 | 0.813 | 0.561 | 0.763 | 0.745 |
| | (0.046) | (0.046) | (0.078) | (0.057) | (0.050) |
| $E_t\pi_{i,t+1}$ | . | . | 0.407 | . | . |
| | | | (0.085) | | |
| $\Delta\tilde{z}_{it}$ | . | . | . | 0.689 | 0.668 |
| | | | | (0.127) | (0.104) |
| $\Delta\text{VAT}_{it}$ | 0.091 | 0.072 | 0.035 | . | . |
| | (0.040) | (0.039) | (0.057) | | |
| $\mathbf{1}[i \in €]$ | −0.631 | −0.657 | −0.315 | −1.529 | −1.230 |
| | (0.300) | (0.298) | (0.202) | (0.358) | (0.286) |
| Adj. $R^2$ | 0.839 | 0.845 | . | 0.858 | 0.872 |
| Pr > $J$[c] | . | . | 0.109 | . | . |
| Equal coeff. on $(u_{it} - \bar{u}_{it})$[d] | . | <.001 | . | . | <.001 |

Note: In columns (1), (2), and (3), the dependent variable is $\pi_{it}$, the log-difference of the GDP price deflator of country $i$ from year $t-1$ to year $t$; in columns (4) and (5), the dependent variable is $\Delta w_{it}$, the log-difference of (nominal) compensation per employee of country $i$ from year $t-1$ to year $t$. Explanatory variables: $(u_{it} - \bar{u}_{it})$ = unemployment gap; $(y_{it} - \bar{y}_{it})$ = output gap; $\Delta\tilde{z}_{it}$ = growth rate of trend labor productivity; $\text{VAT}_{it}$ = effective VAT rate; and $\mathbf{1}[i \in €]$ = indicator variable that equals 1 once country $i$ joined the eurozone. All specifications include country fixed effects; those in columns (1), (2), (4), and (5) are estimated by OLS, while the specification in column (3) is estimated by GMM. In columns (2) and (5), the coefficients on the unemployment gap are allowed to differ across countries, and the entries correspond to the average of the estimated OLS coefficients across the 11 countries. Asymptotic standard errors reported in parentheses are clustered in the time ($t$) dimension.

[a] Sample period: annual data: from 1970 to 2007 ($\bar{T} = 29.7$); No. of countries = 11; Obs. = 327.

[a] Sample period: annual data: 1971 to 2007 ($\bar{T} = 26.1$); No. of countries = 11; Obs. = 287.

[c] $p$-value for the Hansen (1982) $J$-test of the over-identifying restrictions.

[d] $p$-value for the test of equality of country-specific coefficients on $(u_{it} - \bar{u}_{it})$.

four specifications, however, explain about the same proportion of the variability in annual price and wage inflation rates across our sample of 11 euro area countries.

The estimates of the NKPC in column (3) also indicate an economically significant effect of the output gap—our proxy for marginal cost—on inflation outcomes. This effect, however, is estimated with considerably less precision, compared with the estimated sensitivity of inflation to labor market slack implied by the standard Phillips curve specifications. The estimated NKPC assigns a significant role to the forward-looking component of the euro area inflation, though the inflation processes are also characterized by substantial inertial behavior, a result consistent with that of Benigno and López-Salido (2006).

As noted above, our interest is not in these estimates per se. Rather, we are interested in whether deviations of actual price and wage inflation from the trajectories implied by these Phillips curves during the crisis are systematically related to differences in the tightness of financial conditions across countries. To test this hypothesis, we use spreads on *sovereign* credit default swap (CDS) contracts to measure the degree of financial strains in each country and, by extension, the severity of credit constraints faced by companies in the euro area.[7] As emphasized by Lane (2012), the European sovereign debt crisis originated over concerns related to the solvency of national banking systems in the periphery. Accordingly, the level of sovereign CDS spreads likely provides a timely and accurate gauge of pressures faced by the national banking systems in the eurozone during the crisis. Given the bank-centric nature of the euro area financial system, variation in the sovereign CDS spreads across countries should also accurately reflect differences in the tightness of the credit conditions faced by businesses and households.

Panel (a) of Figure 2 shows the evolution of sovereign CDS spreads in the euro area from 2008 to 2015, while panel (b) shows the corresponding country-specific indicators of the change in business credit conditions estimated using the Survey on the Access to Finance of Enterprises.[8] Clearly evident is the severe tightening of business credit conditions in the eurozone periphery in 2011 and early 2012, which coincided with a surge in sovereign CDS spreads in those countries. To stabilize the situation that was beginning to spiral out of control, EU leaders and the ECB responded with a number of aggressive policy measures, and by the end of 2012, the risk of financial contagion that investors thought would have likely led to a break-up of the eurozone receded notably. With CDS spreads continuing to narrow in 2013, credit conditions finally started to ease in 2014.

To gauge the effects of these financial strains on price and wage dynamics during the crisis, we first use the estimates in Table 1 to generate price and wage inflation prediction errors from 2008 to 2013. In the second step, we estimate the following regression:

$$\hat{\epsilon}_{it} = \alpha + \theta_1 \ln \mathrm{CDS}_{i,t+1} + \theta_2 \ln \mathrm{CDS}_{i,t-1} \times \mathbf{1}[i \in \mathrm{P}] + \chi \mathbf{1}[i \in \mathrm{P}] + u_{it}, \tag{4}$$

where $\hat{\epsilon}_{it}$ denotes a residual from one of the estimated Phillips curves in Table 1 and $\mathbf{1}[i \in P]$ is an indicator variable that equals one if country $i$ is in the periphery and zero otherwise.[9] The parameters $\theta_1$ and $\theta_2$ thus measure the extent to which differences in the evolution of financial conditions between the core and periphery countries during the crisis can explain deviations of price and wage inflation trajectories from those implied by the various Phillips curve specifications.

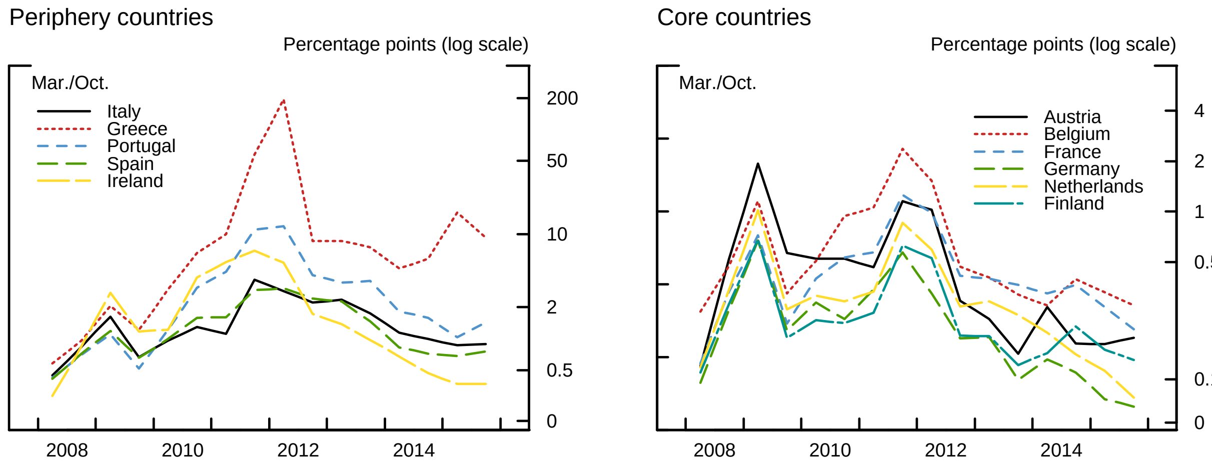
As shown in panel (a) of Table 2, differences in financial conditions across the euro area during the crisis are systematically related to the deviations of price and wage inflation from the dynamics implied by canonical Phillips curve-type relationships. Turning first to prices (rows 1, 2, and 3),

[7] We uses premiums implied by the 5-year, euro-denominated contracts because they are the most liquid segment of the credit derivatives market.

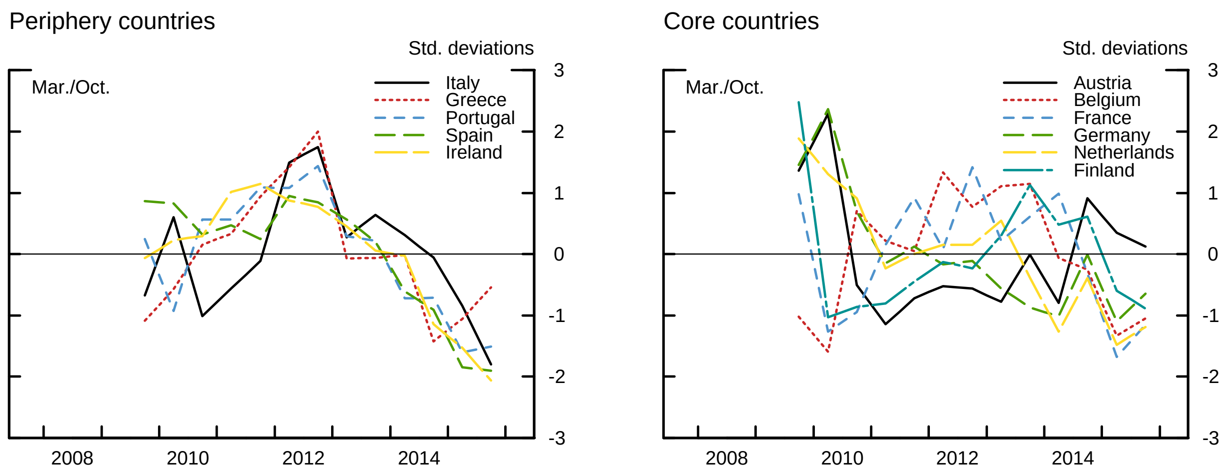
[8] Appendix A describes how we constructed these country-specific indexes; it also documents formally that an increase in sovereign CDS spreads is strongly associated with a subsequent tightening of business credit conditions.

[9] We estimate equation (4) by OLS. However, the associated statistical inference that relies on the usual asymptotic arguments is likely to be unreliable, given a relatively small number of observations, especially in the time-series dimension. Accordingly, we report the 95-percent confidence intervals for coefficients $\theta_1$ and $\theta_2$, based on the time-clustered wild bootstrap procedure of Cameron et al. (2008), which is designed for situations in which the number of clusters or the number of observations within each cluster is relatively small.

Figure 2 – Financial Conditions in the Euro Area (2008–2015)




(a) Sovereign CDS spreads




(b) Changes in business credit conditions

Note: Panel (a) depicts sovereign (5-year) CDS spreads on euro-denominated contracts, while panel (b) depicts the corresponding country-specific indicator of the change in business credit conditions over the 6-month survey period; positive values of the index indicate a tightening of business credit conditions, while negative values indicate an easing of credit conditions (see Appendix A for details).
Source: Markit and Survey on the Access to Finance of Enterprises.

the positive estimates of $\theta_2$, the coefficient on the interaction term $\ln \text{CDS}_{i,t-1} \times \mathbf{1}[i \in \text{P}]$, imply that a widening of sovereign CDS spreads in the eurozone periphery is associated with subsequent inflation rates that exceed those predicted by our various estimated Phillips curves. With regards to wages (rows 4 and 5), on the other hand, negative estimates of $\theta_2$ imply that increased sovereign risk in the periphery leads to subsequent wage growth that is below that predicted by the estimated Phillips curves. The 95-percent confidence intervals bracketing the point estimates of $\theta_2$ exclude zero, an indication that these relationships are statistically significant at conventional levels. For the core euro area countries, by contrast, there appears to be no systematic relationship between

Table 2 – Financial Conditions and Phillips Curve Prediction Errors

| PC Specification | Explanatory Variable: $\ln \text{CDS}_{i,t-1}$ | Explanatory Variable: $\ln \text{CDS}_{i,t-1} \times \mathbf{1}[i \in \text{P}]$ | $R^2$ |
|---|---|---|---|
| (a) *Without time fixed effects* | | | |
| 1. Prices (homogeneous) | 0.043 | 0.601 | 0.198 |
| | [−0.139, 0.227] | [0.218, 0.985] | |
| 2. Prices (heterogeneous) | 0.204 | 0.593 | 0.258 |
| | [0.028, 0.372] | [0.156, 1.030] | |
| 3. Hybrid NK | 0.028 | 0.299 | 0.110 |
| | [−0.100, 0.156] | [0.022, 0.577] | |
| 4. Wages (homogeneous) | −0.008 | −0.776 | 0.254 |
| | [−0.266, 0.251] | [−1.425, 0.100] | |
| 5. Wages (heterogeneous) | 0.085 | −2.075 | 0.425 |
| | [−0.190, 0.360] | [−3.082, −1.069] | |
| (b) *With time fixed effects* | | | |
| 1. Prices (homogeneous) | 0.044 | 0.453 | 0.329 |
| | [−0.239, 0.327] | [0.092, 0.814] | |
| 2. Prices (heterogeneous) | 0.684 | 0.275 | 0.419 |
| | [0.369, 0.999] | [0.031, 0.519] | |
| 3. Hybrid NK | 0.125 | 0.200 | 0.205 |
| | [−0.051, 0.301] | [−0.031, 0.410] | |
| 4. Wages (homogeneous) | −1.364 | −0.495 | 0.352 |
| | [−2.221, −0.506] | [−1.359, 0.369] | |
| 5. Wages (heterogeneous) | −2.196 | −1.469 | 0.542 |
| | [−2.731, −1.661] | [−2.550, −0.389] | |

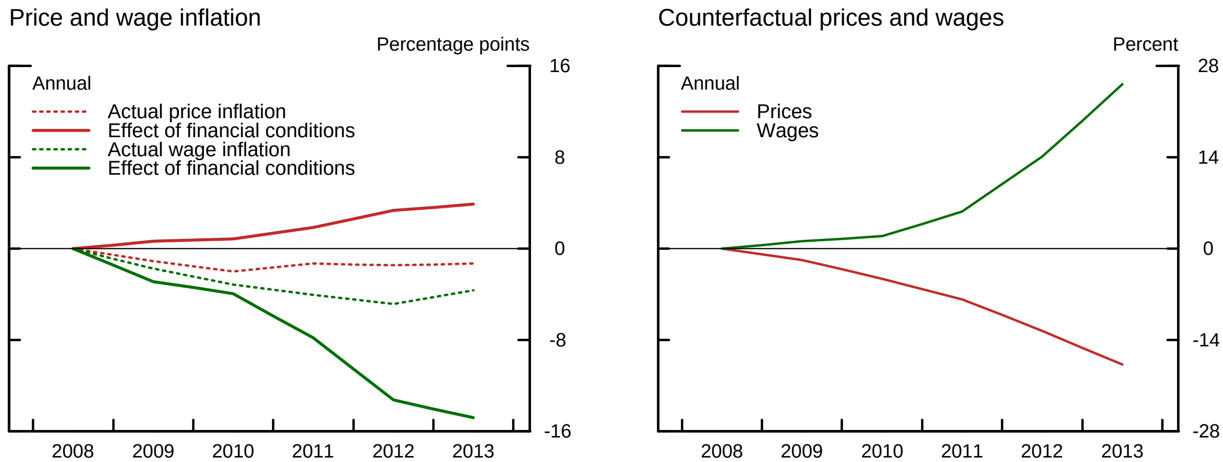
Note: Sample period: annual data from 2008 to 2013; No. of countries = 11; Obs. = 66. The dependent variable is $\hat{\epsilon}_{it}$, a price or wage inflation prediction error of country $i$ in year $t$ implied by the specified Phillips curve (see the text and notes to Table 1 for details). The entries denote the OLS estimates of the coefficients associated with the log-level of sovereign (5-year) CDS spreads at the end of year $t-1$. All specifications include a constant and $\mathbf{1}[i \in \text{P}]$, an indicator for whether country $i$ is in the euro area periphery (not reported). The 95-percent confidence intervals reported in brackets are based on the empirical distribution of coefficients across 5,000 replications, using the wild bootstrap clustered in the time ($t$) dimension (see Cameron et al., 2008).

sovereign credit risk and Phillips curve prediction errors.

In panel (b), we repeat the same exercise, except we add time fixed effects to specification (4)—hence, the parameters $\theta_1$ and $\theta_2$ are identified using only variation between countries. As before, the results indicate that an increase in sovereign CDS spreads in the eurozone periphery—a strong predictor of a subsequent tightening in business credit conditions—is associated with rates of price inflation that lie systematically above those predicted by the estimated Phillips curves, whereas such tightening of credit conditions leads to rates of wage inflation that run systematically below those implied by the corresponding estimated wage Phillips curve.

The economic significance of these results for the euro area periphery is summarized in Figure 3. The dotted red line in the left panel shows the output-weighted average price inflation in the eurozone periphery during the crisis, while the dotted green line shows the corresponding trajectory

Figure 3 – Prices, Wages, and Financial Factors in the Euro Area Periphery (2008–2013)



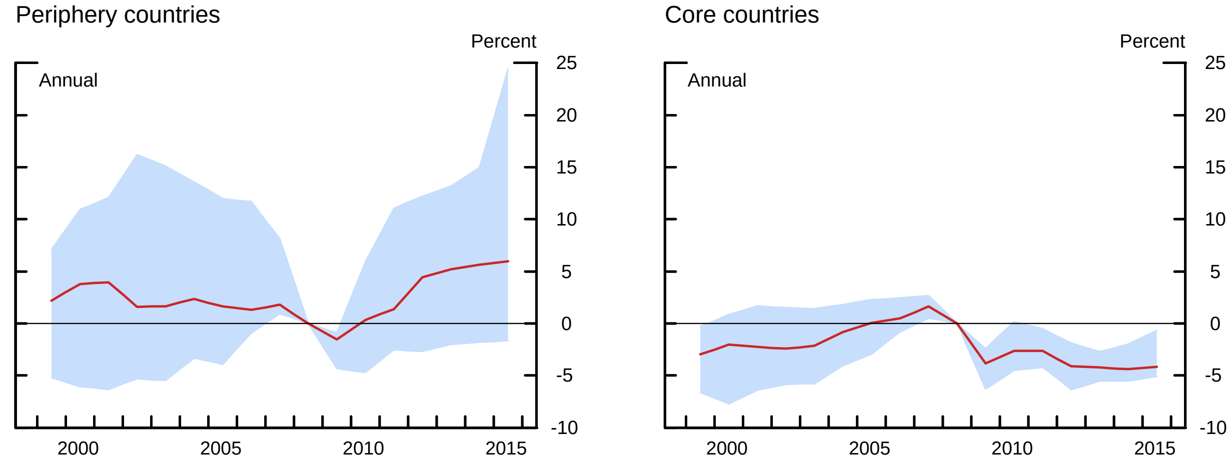

Note: The dotted red (green) line in the left panel depicts the output-weighted average price (wage) inflation across five euro area periphery countries (Greece, Ireland, Italy, Portugal, and Spain), relative to their respective 2008 values. The corresponding red and green solid lines in the same panel show the estimated effects of financial factors on the output-weighted average price and wage inflation, respectively, based on the estimates reported in rows 1 and 5 of panel (b) in Table 2. The lines in the right panel depict the corresponding counterfactual trajectories for the GDP price deflator (Prices) and compensation per employee (Wages).
Source: Authors' calculations using data from Markit and the AMECO database.

for wage inflation. The estimated effects of a tightening in financial conditions—based on the estimates reported in rows 2 and 5 of panel (b) in Table 2—are shown by the corresponding solid lines. According to our calculations, the increase in sovereign CDS spreads at the nadir of the crisis in 2011 and 2012 is estimated to have boosted annual price inflation in the euro area periphery about 4 percentage points, while depressing nominal wage growth nearly 15 percentage points.

In the right panel, we translate these financial effects on the average *level* of prices and wages in the periphery. Our estimates imply that in the absence of this acute financial distress, the average price level in the eurozone periphery would have fallen—relative to its 2008 level—about 15 percent by the end of 2013, a counterfactual outcome entailing a rate of deflation with potentially devastating economic effects given the high levels of private and public debt in the periphery countries. And while the severe tightening of financial conditions may have helped stave off a Fisherian deflationary spiral, the same forces appeared to have had a catastrophic effect on nominal wages: Relative to its 2008 level, compensation per employee would have been about 25 percent higher by the end of the crisis, according to this counterfactual.

We now turn directly to the behavior of markups. As shown by Galí et al. (2007), the price markup can, under reasonable assumptions, be measured (up to an additive constant) as minus the log of real unit labor costs. Figure 4 shows the evolution of price markups in the eurozone periphery (left panel) and in the core countries (right panel) since the introduction of the euro in 1999. The divergence in markups between the core and periphery during the crisis is striking: The median markup in the periphery increased about 5 percentage point between 2009 and 2013, while

Figure 4 – Price Markups in the Euro Area (1999–2015)




Note: The solid lines depict the cross-sectional median of price markups, while the shaded bands denote the corresponding cross-sectional range. The price markup is defined as minus (100 times) the log of real unit labor costs (2008 = 1). Periphery countries: Greece, Ireland, Italy, Portugal, and Spain. Core countries: Austria, Belgium, Finland, France, Germany, and Netherlands.
Source: AMECO database.

in the core, the median markup fell about the same amount during this period.

To examine how differences in financial strains across countries affected the behavior of markups in the euro area during the crisis, we re-estimate regression (4) using the difference in the price markup as the dependent variable. As indicated in panel (a) of Table 3, a widening of sovereign CDS spreads in the euro area periphery is associated with a statistically significant subsequent increase in markups, whereas in the euro area core, such a tightening of financial conditions has no effect on price markups. It is worth noting that this effect is robust to the inclusion of time fixed effects. In panel (b), we improve on the power of this simple test by considering changes in markups at the sectoral level.[10] Adding this dimension to our data further strengthens the relationship between financial conditions and subsequent changes in price markups. Using the "between" estimates in row 2 as a benchmark, a periphery country with CDS spreads at the 90th percentile of the distribution would see its markups increase more than 5.5 percentage points, compared with a country whose CDS spreads are at the 10th percentile of the distribution.

In sum, the results presented in this section add to the growing empirical evidence, which strongly supports the notion that financial conditions of firms in the euro area affected their pricing decisions during the global financial crisis and its aftermath (see Montero and Urtasun, 2014; Antoun de Almedia, 2015; Montero, 2017; Duca et al., 2017).[11] As we show below, combining the theory of customer markets with financial market frictions provides a natural way to understand

[10]For each country in our sample, the AMECO data base contains real unit labor costs for the following five sectors: (1) Agriculture, Forestry & Fishing; (2) Building & Construction; (3) Industry (excl. Building & Construction); (4) Manufacturing; and (5) Services.

[11]Similar evidence for the United States during the Great Recession is provided by GSSZ and Gilchrist and Zakrajšek (2016) and for the 1990s Japan by Kimura (2013).

Table 3 – Financial Conditions and Price Markups

| Specification | Explanatory Variable: $\ln \text{CDS}_{i,t-1}$ | Explanatory Variable: $\ln \text{CDS}_{i,t-1} \times \mathbf{1}[i \in \text{P}]$ | $R^2$ |
|---|---|---|---|
| (a) *Aggregate markups*[a] | | | |
| 1. Without time fixed effects | −0.205 | 1.378 | 0.256 |
| | [−0.944, 0.534] | [0.557, 2.220] | |
| 2. With time fixed effects | −0.312 | 1.148 | 0.681 |
| | [−0.528, −0.095] | [0.926, 1.372] | |
| (b) *Sectoral markups*[b] | | | |
| 1. Without time fixed effects | −0.442 | 2.556 | 0.057 |
| | [−2.135, 1.252] | [0.913, 4.198] | |
| 2. With time fixed effects | −0.331 | 1.974 | 0.152 |
| | [−1.915, 1.254] | [1.244, 2.704] | |

Note: In panel (a), the dependent variable is the change in the aggregate price markup in country $i$ from year $t-1$ to year $t$, while in panel (b) the dependent variable is the change in the country-specific sectoral price markup over the same period; the markup is defined as minus the log of real unit labor costs. The entries denote the OLS estimates of the coefficients associated with the log-level of sovereign (5-year) CDS spreads at the end of year $t-1$. All specifications include a constant and $\mathbf{1}[i \in \text{P}]$, an indicator for whether country $i$ is in the euro area periphery (not reported); specifications in panel (b) also include sector fixed effects. The 95-percent confidence intervals reported in brackets are based on the empirical distribution of coefficients across 5,000 replications, using the wild bootstrap clustered in the time ($t$) dimension (see Cameron et al., 2008).

[a] Sample period: annual data from 2008 to 2013; No. of countries = 11; Obs. = 66.

[b] Sample period: annual data from 2008 to 2013; No. of countries = 11; No. of sectors = 5; Obs. = 328. Sectors: (1) Agriculture, Forestry & Fishing; (2) Building & Construction; (3) Industry; (4) Manufacturing; and (5) Services.

these new findings. The pricing mechanism implied by this interaction predicts exactly the differences in the behavior of prices and markups between the eurozone core and periphery documented above: In response to an adverse financial shock in the periphery, the tightening of credit conditions causes firms—in an effort to preserve internal liquidity—to boost prices by raising markups, thereby losing market share to their financially healthy competitors from the core.

Our last piece of empirical evidence does not involve any econometrics. The following quote from Sergio Marchionne, the CEO of Fiat Chrysler, in mid-2012 paints a visceral picture of the price dynamics implied by our theory:

> Mr. Marchionne and other auto executives accuse Volkswagen of exploiting the crisis to gain market share by offering aggressive discounts. “It’s a bloodbath of pricing and it’s a bloodbath on margins,” he said.
>
> *The New York Times*, July 25, 2012

# 3 Model

## 3.1 Environment

The model consists of two countries—referred to as home ($h$) and foreign ($f$)—and where foreign country variables carry a superscript "*." We think of home and foreign countries as representing the periphery and core countries of the euro area, respectively.

### 3.1.1 Preferences

In each country, there exists a continuum of households indexed by $j \in N_c = [0, 1]$, $c = h, f$. Each household consumes two types, $h$ and $f$, of differentiated varieties of consumption goods, indexed by $i \in N_h = [0, 1]$ in the home country and by $i \in N_f = [1, 2]$ in the foreign country. Consistent with the standard assumption used in international macroeconomics, the home country only produces the $h$-type goods, while the foreign country only produces the $f$-type goods. In this two-country setting, $c^j_{i,f,t}$ denotes the consumption of product $i$ of type $f$ by a home country household $j$, while $c^{j*}_{i,f,t}$ denotes its foreign counterpart—that is, the consumption of product $i$ of type $f$ by a foreign country household $j$.[12]

The preferences of household $j$ in the home country are given by

$$\mathbb{E}_t \sum_{s=0}^{\infty} \delta^s U(x^j_{t+s} - \omega_{t+s}, h^j_{t+s}); \quad (0 < \delta < 1). \tag{5}$$

The household's per-period utility function $U(\cdot, \cdot)$ is strictly increasing and concave in the consumption bundle $x^j_t$ and strictly decreasing and concave in hours worked $h^j_t$. The preference shock $\omega_t$ affects the marginal utility of consuming the bundle $x^j_t$ today and is used to explore the implications of an aggregate demand shock in our framework. We assume that labor is perfectly immobile.

Standard open economy models allow for home-bias in consumption by combining Dixit-Stiglitz preferences with an Armington aggregator of home and foreign goods. We introduce into this framework a sticky customer base via the "deep habits" preference structure of Ravn et al. (2006). This yields the consumption/habit aggregator

$$x^j_t \equiv \left[ \sum_{k=h,f} \Xi_k \left[ \int_{N_k} \left(c^j_{i,k,t}/s^{\theta}_{i,k,t-1}\right)^{1-1/\eta} di \right]^{\frac{1-1/\varepsilon}{1-1/\eta}} \right]^{\frac{1}{1-1/\varepsilon}},$$

where $\eta > 0$ and $\varepsilon > 0$ are the elasticities of substitution *within* a type of goods produced in a given country and *between* the two types of goods, respectively. The parameter $\Xi_k > 0$ governs the degree of home bias in the household's consumption basket in the steady state, with $\sum_{k=h,f} \Xi^{\varepsilon}_k = 1$.

Let $c_{i,k,t} = \int_0^1 c^j_{i,k,t} dj$ denote the average level of consumption of good $i$ in country $k$. As in

[12] In our notation, $c^j_{i,f,t}$ denotes consumption of an imported good by a home country household $j$, while $c^{*j}_{i,f,t}$ denotes consumption of a domestically produced good by a foreign household $j$.

Ravn et al. (2006), let $s_{i,k,t}$ denote the *good-specific* habit, which evolves according to

$$s_{i,k,t} = \rho s_{i,k,t-1} + (1-\rho)c_{i,k,t}; \quad k = h, f \quad (0 < \rho < 1). \tag{6}$$

In this formulation, habits are external to the household and country specific. When $\theta < 0$, the stock of habit formed by past consumption of the average household has a positive effect on the utility derived from today's consumption, making the household desire more of the same good. This creates an incentive for firms to lower prices in order to build customer base.[13]

In equilibrium, all households within a given country choose the same consumption basket. Going forward, we thus omit the household index $j$. The cost minimization associated with equation (5) implies the following demand function for good $i$ (of type $h$ or $f$) in the home country:

$$c_{i,k,t} = \left(\frac{P_{i,k,t}}{\tilde{P}_{k,t}}\right)^{-\eta} s_{i,k,t-1}^{\theta(1-\eta)} x_{k,t}; \quad k = h, f, \tag{7}$$

where the habit-adjusted price index $\tilde{P}_{k,t}$ and the habit-adjusted consumption bundle $x_{k,t}$ are given by

$$\tilde{P}_{k,t} = \left[\int_{N_k} (P_{i,k,t} s_{i,k,t-1}^{\theta})^{1-\eta} di\right]^{\frac{1}{1-\eta}} \text{ and } x_{k,t} = \left[\int_{N_k} (c_{i,k,t}/s_{i,k,t-1}^{\theta})^{1-1/\eta} di\right]^{\frac{1}{1-1/\eta}}; \quad k = h, f.$$

In equilibrium, the consumption/habit basket $x_{k,t}$ is equal to

$$x_{k,t} = \Xi_k^{\varepsilon} \left(\frac{\tilde{P}_{k,t}}{\tilde{P}_t}\right)^{-\varepsilon} x_t; \quad k = h, f, \text{ with } \tilde{P}_t = \left[\sum_{k=h,f} \Xi_k \tilde{P}_{k,t}^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}}, \tag{8}$$

where $\tilde{P}_t$ denotes the welfare-based aggregate price index of the home country. Due to the symmetric structure of the two countries, the foreign country analogues of $c_{i,k,t}$, $x_{k,t}$, and $\tilde{P}_t$ can be expressed simply by adding a superscript "*" to each variable. For later use, we also define the consumer price index (CPI) as

$$P_t = \left[\sum_{k=h,f} \Xi_k P_{k,t}^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}}, \text{ where } P_{k,t} = \left[\int_{N_k} {P_{i,k,t}}^{1-\eta} di\right]^{\frac{1}{1-\eta}}; \quad k = h, f, \tag{9}$$

is the CPI corresponding to a $k$-type category of goods.[14]

[13]Because of external habits, households take the habit stock as given and do not internalize the effect of their own consumption on future demand; see Nakamura and Steinsson (2011) for the analysis of firms' pricing-setting behavior implied by good-specific internal habits.

[14]See Appendix B for the derivations of equations (7)–(9).

### 3.1.2 Technology

The production technologies in the home and foreign countries are given by

$$y_{i,t} = \left(\frac{A_t}{a_{i,t}} h_{i,t}\right)^{\alpha} - \phi \quad \text{and} \quad y_{i,t}^{*} = \left(\frac{A_t^{*}}{a_{i,t}^{*}} h_{i,t}^{*}\right)^{\alpha} - \phi^{*}; \quad (0 < \alpha \leq 1),$$

where $\phi, \phi^{*} > 0$ denote fixed operating costs, which in principle can differ between the two countries; $A_t$ and $A_t^{*}$ are the country-specific aggregate technology shocks, and $a_{i,t}$ and $a_{i,t}^{*}$ are the idiosyncratic "cost" shocks affecting home and foreign firms, respectively. We assume that the idiosyncratic cost shocks are distributed according to identical log-normal distributions: $\ln a_{i,t}, \ln a_{i,t}^{*} \overset{iid}{\sim} N(-0.5\sigma^2, \sigma^2)$. We denote the CDF of the idiosyncratic shocks by $F(a)$. Note that the presence of fixed costs makes it possible for firms to incur operating losses and hence find themselves in a liquidity squeeze if external financing is costly or, as during the height of the eurozone sovereign debt crisis, virtually unavailable.

### 3.1.3 Frictions

For fixed costs to play a role in creating liquidity risk, we introduce several frictions to the firm's flow-of-funds constraint. First, we adopt a timing convention, whereby in the first half of period $t$ firms collect information about the aggregate state of the economy. Based on this aggregate information, firms post prices, take orders from customers, and plan production based on *expected* marginal cost. In the second half of the period, idiosyncratic cost uncertainty is resolved, and firms realize their *actual* marginal cost. They then hire labor to fulfill the agreed-upon orders and produce period-$t$ output.

We also assume that firms pay out all operating profits as dividends within a given period—that is, we rule out corporate savings.[15] Because of fixed costs, the firm's operating profits may, ex post, be too low to cover the total cost of production. In that case, the firm must issue new shares within that period. Because of agency problems in capital markets, such equity financing involves a constant dilution cost per share issued, denoted by $0 < \varphi < 1$ and $0 < \varphi^{*} < 1$. Consistent with the fact that core euro area countries have deeper and more developed capital markets than the eurozone periphery, we assume that the dilution costs in the home country strictly exceed those in foreign country—that is, $0 < \varphi^{*} < \varphi$. This implies that firms in the home country are more exposed to liquidity risk than their foreign counterparts.[16] The dilution cost associated with the newly issued equity implies that when a home country firm issues a notional amount of equity $d_{i,t} < 0$, the actual amount of funds raised is given by $-(1 - \varphi)d_{i,t}$.

[15] Ruling out precautionary savings limits the dimension of the state space. However, this assumption does not mean that firms do not engage in any form of risk management. Rather, as shown below, the firms' liquidity risk management involves the accumulation and decumulation of market shares.

[16] An implicit assumption of our setup is that the equity markets of the two countries are fully segmented—only domestic (foreign) households invest in the shares of domestic (foreign) firms. Empirical evidence of significant home bias in equity holdings is provided by French and Poterba (1991), Tesar and Werner (1995), and Obstfeld and Rogoff (2000).

In addition to financial frictions, we also allow for nominal rigidities by assuming that firms incur costs when adjusting prices. Following Rotemberg (1982), these costs are given by

$$\frac{\gamma_p}{2}\left(\frac{P_{i,h,t}}{P_{i,h,t-1}}-1\right)^2 c_t+\frac{\gamma_p}{2}\frac{Q_t P_t^*}{P_t}\left(\frac{P_{i,h,t}^*}{P_{i,h,t-1}^*}-1\right)^2 c_t^*;\quad (\gamma_p,\gamma_p>0),$$

where $Q_t$ denotes the nominal exchange rate. We assume the same degree of price stickiness ($\gamma_p$) in both countries and let the price adjustment costs be proportional to local consumption—that is, $c_t$ and $c_t^*$.[17] Note also that we assume local currency pricing rather than producer currency pricing.

### 3.2 The Firm's Problem

The firm's objective is to maximize the present value of its dividend flow, $\mathbb{E}_t\left[\sum_{s=0}^{\infty} m_{t,t+s} d_{i,t+s}\right]$, where $d_{i,t}=D_{i,t}/P_t$ is the real dividend payout when positive and real equity issuance when negative. We assume that the firms are owned by households, and that they discount future cashflows using the stochastic discounting factor of the representative household, denoted by $m_{t,t+s}$, in their respective country.

Before formally stating the firm's optimization problem, we define relative prices. The real product prices relative to the CPIs in home and foreign countries can be written as

$$\frac{P_{i,h,t}}{P_t}=\frac{P_{i,h,t}}{P_{h,t}}\frac{P_{h,t}}{P_t}\equiv p_{i,h,t}p_{h,t}\quad\text{and}\quad\frac{P_{i,h,t}^*}{P_t^*}=\frac{P_{i,h,t}^*}{P_{h,t}^*}\frac{P_{h,t}^*}{P_t^*}\equiv p_{i,h,t}^*p_{h,t}^*.$$

Note that $p_{i,h,t}$ and $p_{i,h,t}^*$ are prices charged by home country firm $i$ relative to the average price level chosen by the home country firms in the home and foreign markets, respectively; $p_{h,t}$ and $p_{h,t}^*$, on the other hand, are the average price levels relative to the CPI in the home and foreign markets, respectively and as such are taken as given by individual firms. From the perspective of firms in the foreign country, the relative prices $p_{i,f,t}$, $p_{i,f,t}^*$, $p_{f,t}$, and $p_{f,t}^*$ are interpreted in the same way.

We now turn to the problem of the firm, which to conserve space, we describe from the vantage point of the home country. A home country firm maximizes the present value of real dividends, subject to a flow-of-funds constraint:

$$\begin{aligned}d_{i,t}&=p_{i,h,t}p_{h,t}c_{i,h,t}+q_t p_{i,h,t}^*p_{h,t}^*c_{i,h,t}^*-w_t h_{i,t}+\varphi\min\{0,d_{i,t}\}\\&\quad-\frac{\gamma_p}{2}\left(\frac{p_{i,h,t}}{p_{i,h,t-1}}\pi_{h,t}-1\right)^2 c_t-\frac{\gamma_p}{2}q_t\left(\frac{p_{i,h,t}^*}{p_{i,h,t-1}^*}\pi_{h,t}^*-1\right)^2 c_t^*,\end{aligned}\tag{10}$$

where $w_t=W_t/P_t$ is the real wage, $q_t=Q_tP_t^*/P_t$ is the real exchange rate, and $\pi_{h,t}=P_{h,t}/P_{h,t-1}$ and $\pi_{h,t}^*=P_{h,t}^*/P_{h,t-1}^*$ are the market-specific (gross) inflation rates faced by firms in the home country. The firm's problem is also subject to the law of motion for the habit stock (6), the demand

[17] This latter assumption is made solely to preserve the homogeneity of the firm's problem and has no first-order consequences for dynamics of the model.

constraint (7), and a production constraint:

$$\left(\frac{A_t}{a_{i,t}}h_{i,t}\right)^{\alpha} - \phi \geq c_{i,h,t} + c^*_{i,h,t}. \tag{11}$$

Formally, the firm is choosing the sequence $\{d_{i,t}, h_{i,t}, c_{i,h,t}, c^*_{i,h,t}, s_{i,h,t}, s^*_{i,h,t}, p_{i,h,t}, p^*_{i,h,t}\}_{t=0}^{\infty}$ to maximize the following Lagrangian:

$$\begin{aligned}
\mathcal{L} = \mathbb{E}_0 \sum_{t=0}^{\infty} m_{0,t} \Bigg\{ & d_{i,t} + \kappa_{i,t}\left[\left(\frac{A_t}{a_{i,t}}h_{i,t}\right)^{\alpha} - \phi - (c_{i,h,t} + c^*_{i,h,t})\right] \\
& + \xi_{i,t}\Bigg[p_{i,h,t}p_{h,t}c_{i,h,t} + q_t p^*_{i,h,t}p^*_{h,t}c^*_{i,h,t} - w_t h_{i,t} - d_{i,t} + \varphi \min\{0, d_{i,t}\} \\
& - \frac{\gamma_p}{2}\left(\frac{p_{i,h,t}}{p_{i,h,t-1}}\pi_{h,t} - 1\right)^2 c_t - \frac{\gamma_p}{2}q_t\left(\frac{p^*_{i,h,t}}{p^*_{i,h,t-1}}\pi^*_{h,t} - 1\right)^2 c^*_t\Bigg] \\
& + \nu_{i,h,t}\left[(p_{i,h,t})^{-\eta}\tilde{p}^{\eta}_{h,t}s^{\theta(1-\eta)}_{i,h,t-1}x_{h,t} - c_{i,h,t}\right] + \nu^*_{i,h,t}\left[(p^*_{i,h,t})^{-\eta}\tilde{p}^{*\eta}_{h,t}s^{*\theta(1-\eta)}_{i,h,t-1}x^*_{h,t} - c^*_{i,h,t}\right] \\
& + \lambda_{i,h,t}\left[\rho s_{i,h,t-1} + (1-\rho)c_{i,h,t} - s_{i,h,t}\right] + \lambda^*_{i,h,t}\left[\rho s^*_{i,h,t-1} + (1-\rho)c^*_{i,h,t} - s^*_{i,h,t}\right]\Bigg\},
\end{aligned} \tag{12}$$

where $\tilde{p}_{h,t} = \tilde{P}_{h,t}/P_{h,t}$ and $\tilde{p}^*_{h,t} = \tilde{P}^*_{h,t}/P^*_{h,t}$; $\kappa_{i,t}$ and $\xi_{i,t}$ are the Lagrange multipliers associated with the production constraint (11) and the flow-of-funds constraint (10), respectively; $\nu_{i,h,t}$ and $\nu^*_{i,h,t}$ are the Lagrange multipliers associated with the domestic and foreign demand constraints (equation 7 and its foreign counterpart); and $\lambda_{i,h,t}$ and $\lambda^*_{i,h,t}$ are the multipliers associated with the domestic and foreign habit accumulation processes (equation 6 and its foreign counterpart).

We begin by describing the firms' optimal choice of labor hours and equity issuance, two decisions that are made after the realization of the idiosyncratic cost shock $a_{it}$. We then discuss decisions made prior to the realization of the idiosyncratic cost shock. These include the firm's pricing policies in the domestic and foreign markets, which determine the amount of output sold at home and abroad, as well as the overall level of production. For maximum intuition, we focus on the case without sticky prices. We then discuss the implications of our model for inflation and the Phillips curve in an environment where firms face quadratic costs of changing prices.

The efficiency condition for labor hours in problem (12) is given by

$$a_{i,t}\xi_{i,t}w_t = \kappa_{i,t}\alpha A_t\left(\frac{A_t}{a_{i,t}}h_{i,t}\right)^{\alpha-1}, \tag{13}$$

where given the production function, labor hours satisfy the conditional labor demand:

$$h_{i,t} = \frac{a_{i,t}}{A_t}(\phi + c_{i,h,t} + c^*_{i,h,t})^{\frac{1}{\alpha}}. \tag{14}$$

Our timing assumptions imply that $c_{i,h,t}$ and $c^*_{i,h,t}$ are determined prior to the realization of the idiosyncratic cost shock $a_{i,t}$. Combining equations (13) and (14), applying the expectation operator $\mathbb{E}^a_t[x] \equiv \int x dF(a)$ to both sides of the resulting expression, and dividing through by $\mathbb{E}^a_t[\xi_{i,t}]$ yields the following expression for the expected real marginal cost normalized by the expected shadow value of internal funds:

$$\frac{\mathbb{E}^a_t[\kappa_{i,t}]}{\mathbb{E}^a_t[\xi_{i,t}]} = \frac{\mathbb{E}^a_t[a_{i,t}\xi_{i,t}]}{\mathbb{E}^a_t[\xi_{i,t}]} \frac{w_t}{\alpha A_t}(\phi + c_{i,h,t} + c^*_{i,h,t})^{\frac{1-\alpha}{\alpha}}. \tag{15}$$

To understand the economic content behind the above expression, consider first the case with no financial frictions. Hence, the shadow value of internal funds $\xi_{i,t} = 1$, for all $i$ and $t$, implying that $\mathbb{E}^a_t[\xi_{i,t}] = 1$ and $\mathbb{E}^a_t[a_{i,t}\xi_{i,t}] = \mathbb{E}^a_t[a_{i,t}]\mathbb{E}^a_t[\xi_{i,t}] = 1$. With constant returns-to-scale (i.e., $\alpha = 1$), the expected real marginal cost $\mathbb{E}^a_t[\kappa_{i,t}] = w_t/A_t$, that is, unit labor costs. With the decreasing returns-to-scale (i.e., $\alpha < 1$), the expected real marginal cost is also a function of the firm's output: $\mathbb{E}^a_t[\kappa_{i,t}] = (w_t/\alpha A_t)(\phi + c_{i,h,t} + c^*_{i,h,t})^{\frac{1-\alpha}{\alpha}} = (w_t/\alpha A_t)y_{i,t}^{\frac{1-\alpha}{\alpha}}$.

In the presence of financial frictions, however, the shadow value of internal funds is not always equal to 1 and becomes stochastic, according to the realization of the idiosyncratic cost shock $a_{i,t}$, which influences the liquidity position of the firm. The first-order condition for dividend payouts (or equity issuance) implies that

$$\xi_{i,t} = \begin{cases} 1 & \text{if } d_{i,t} \geq 0; \\ 1/(1-\varphi) & \text{if } d_{i,t} < 0. \end{cases} \tag{16}$$

In other words, the shadow value of internal funds is equal to 1 when the firm's profits are sufficiently high to cover labor and fixed costs and thus pay dividends. If, however, the firm incurs an operating loss, it must issues new equity to cover its costs, and the shadow value of internal funds jumps to $1/(1-\varphi)$. Intuitively, given the equity dilution costs, a firm must issue $1/(1-\varphi)$ units of equity to obtain one unit of cashflow. These conditions imply that $\mathbb{E}^a_t[\xi_{i,t}] > 1$. It is also the case that the realized shadow value of internal funds covaries positively with the idiosyncratic cost shock $a_{i,t}$, as profits and hence dividends are negative when costs are high. As we show below, this implies

$$\frac{\mathbb{E}^a_t[a_{i,t}\xi_{i,t}]}{\mathbb{E}^a_t[\xi_{i,t}]} > 1.$$

Financial frictions, therefore, raise the normalized real marginal cost given by equation (15).

In contrast to the optimal choice of a labor input and dividends (or equity issuance), the optimality conditions for prices, $(p_{i,h,t}, p^*_{i,h,t})$, output, $(c_{i,h,t}, c^*_{i,h,t})$, and habit stocks $(s_{i,h,t}, s^*_{i,h,t})$ in the domestic and foreign markets are determined prior to the realization of the idiosyncratic cost shock. The optimality conditions for prices require firms to set the real relative price in each market equal to a constant multiple of the shadow value of a marginal sale in each market, normalized by the shadow value of internal funds:

$$p_{i,h,t}p_{h,t} = \eta\frac{\mathbb{E}_t^a[\nu_{i,h,t}]}{\mathbb{E}_t^a[\xi_{i,t}]}; \tag{17}$$

$$p_{i,h,t}^* q_t p_{h,t}^* = \eta\frac{\mathbb{E}_t^a[\nu_{i,h,t}^*]}{\mathbb{E}_t^a[\xi_{i,t}]}. \tag{18}$$

The normalized value of marginal sales is given by the first-order condition for sales in each market ($c_{i,h,t}$ and $c_{i,h,t}^*$):

$$\frac{\mathbb{E}_t^a[\nu_{i,h,t}]}{\mathbb{E}_t^a[\xi_{i,t}]} = p_{i,h,t}p_{h,t} - \frac{\mathbb{E}_t^a[\kappa_{i,t}]}{\mathbb{E}_t^a[\xi_{i,t}]} + (1-\rho)\frac{\mathbb{E}_t^a[\lambda_{i,h,t}]}{\mathbb{E}_t^a[\xi_{i,t}]}; \tag{19}$$

$$\frac{\mathbb{E}_t^a[\nu_{i,h,t}^*]}{\mathbb{E}_t^a[\xi_{i,t}]} = p_{i,h,t}^* q_t p_{h,t}^* - \frac{\mathbb{E}_t^a[\kappa_{i,t}]}{\mathbb{E}_t^a[\xi_{i,t}]} + (1-\rho)\frac{\mathbb{E}_t^a[\lambda_{i,h,t}^*]}{\mathbb{E}_t^a[\xi_{i,t}]}. \tag{20}$$

To the firm, the value of a marginal sale is equal to the current marginal profit—price minus normalized marginal cost—and the additional value that the firm receives from increasing its customer base by expanding the habit stock.

Optimality conditions for habit stocks imply that the marginal value of the habit stock in each market satisfies the recursion:

$$\frac{\mathbb{E}_t^a[\lambda_{i,h,t}]}{\mathbb{E}_t^a[\xi_{i,t}]} = \theta(1-\eta)\mathbb{E}_t\left[\tilde{m}_{t,t+1}\frac{\mathbb{E}_{t+1}^a[\nu_{i,h,t+1}]}{\mathbb{E}_{t+1}^a[\xi_{i,t+1}]}\frac{c_{h,t+1}}{s_{h,t}}\right] + \rho\mathbb{E}_t\left[\tilde{m}_{t,t+1}\frac{\mathbb{E}_{t+1}^a[\lambda_{i,h,t+1}]}{\mathbb{E}_{t+1}^a[\xi_{i,t+1}]}\right]; \tag{21}$$

$$\frac{\mathbb{E}_t^a[\lambda_{i,h,t}^*]}{\mathbb{E}_t^a[\xi_{i,t}]} = \theta(1-\eta)\mathbb{E}_t\left[\tilde{m}_{t,t+1}\frac{\mathbb{E}_{t+1}^a[\nu_{i,h,t+1}^*]}{\mathbb{E}_{t+1}^a[\xi_{i,t+1}]}\frac{c_{i,h,t+1}^*}{s_{h,t}^*}\right] + \rho\mathbb{E}_t\left[\tilde{m}_{t,t+1}\frac{\mathbb{E}_{t+1}^a[\lambda_{i,h,t+1}^*]}{\mathbb{E}_{t+1}^a[\xi_{i,t+1}]}\right], \tag{22}$$

where

$$\tilde{m}_{t,t+1} \equiv m_{t,t+1}\frac{\mathbb{E}_{t+1}^a[\xi_{i,t+1}]}{\mathbb{E}_t^a[\xi_{i,t}]}$$

reflects the effect of financial frictions on the firm's internal valuation of a unit of cashflow tomorrow relative to today.

Equations (21) and (22) highlight the forward-looking nature of the firms' price-setting decisions. The marginal value of having a bigger customer base is equal to the present-discounted value of future marginal sales. How fast this present value decays depends on three things: the survival rate of the habit $\rho$; the representative household's discount factor $m_{t,t+1}$; and the firm's liquidity position, as measured by the shadow value of internal funds today relative to tomorrow—that is, $\mathbb{E}_{t+1}^a[\xi_{i,t+1}]/\mathbb{E}_t^a[\xi_{i,t}]$.

## 3.3 Optimal Pricing in a Symmetric Equilibrium

With risk-neutral firms and i.i.d. idiosyncratic costs shocks, our timing assumptions imply that all firms in a given country are identical ex ante. As a result, we focus on an equilibrium that has a number of symmetric features. Specifically, all home country firms choose identical relative prices

($p_{i,h,t} = 1$ and $p^*_{i,h,t} = 1$), scales of production ($c_{i,h,t} = c_{h,t}$ and $c^*_{i,h,t} = c^*_{h,t}$), and habit stocks ($s_{i,h,t} = s_{h,t}$ and $s^*_{i,h,t} = s^*_{h,t}$).

The symmetric equilibrium condition $p_{i,h,t} = p^*_{i,h,t} = 1$ implies that firms in the home country set the same relative prices in domestic and foreign markets vis-à-vis other competitors from the same origin.[18] Similarly, foreign firms make pricing decisions among themselves, both in the domestic and foreign markets, such that $p_{i,f,t} = p^*_{i,f,t} = 1$. The asymmetric nature of financial conditions induces differences in the firms' internal liquidity positions and causes home and foreign firms to adopt different pricing policies. As a result, $p_{h,t} = P_{h,t}/P_t \neq 1$, $p^*_{h,t} = P^*_{h,t}/P^*_t \neq 1$, $p_{f,t} = P_{f,t}/P_t \neq 1$, and $p^*_{f,t} = P^*_{f,t}/P^*_t \neq 1$, implying that $p_{h,t} \neq p_{f,t}$ and $p^*_{h,t} \neq p^*_{f,t}$, in general. As we show below, the relatively weaker financial positions of home firms forces them to maintain higher prices and markups in the neighborhood of the nonstochastic steady state, such that $p_h > p_f$ and $p^*_h > p^*_f$.

Imposing the relevant symmetric equilibrium conditions, the firm's internal funds are given by revenues less production costs:

$$p_{h,t}c_{h,t} + q_t p^*_{h,t}c^*_{h,t} - w_t \frac{a_{i,t}}{A_t}\left(\phi + c_{h,t} + c^*_{h,t}\right)^{\frac{1}{\alpha}},$$

where we substituted the conditional labor demand (14) for $h_t$. The firm resorts to costly external finance—that is, issues new shares—if and only if

$$a_{i,t} > a^E_t \equiv \frac{A_t}{w_t}\left[\frac{p_{h,t}c_{h,t} + q_t p^*_{h,t}c^*_{h,t}}{\left(\phi + c_{h,t} + c^*_{h,t}\right)^{\frac{1}{\alpha}}}\right]. \tag{23}$$

Using the above definition of the equity issuance trigger $a^E_t$, we can rewrite the first-order conditions for dividends (16) as

$$\xi_{i,t} = \begin{cases} 1 & \text{if } a_{i,t} \leq a^E_t; \\ 1/(1-\varphi) & \text{if } a_{i,t} > a^E_t, \end{cases}$$

which states that because of costly external financing, the shadow value of internal funds jumps from 1 to $1/(1-\varphi) > 1$ when the realization of the idiosyncratic cost shock $a_{i,t}$ exceed the threshold value $a^E_t$. Let $z^E_t$ denote the standardized value of $a^E_t$ (i.e, $z^E_t = (\ln a^E_t + 0.5\sigma^2)/\sigma$). Taking expectations, the expected shadow value of internal funds is given by

$$\mathbb{E}^a_t[\xi_{i,t}] = \int_0^{a^E_t} dF(a) + \int_{a^E_t}^{\infty} \frac{1}{1-\varphi} dF(a) = 1 + \frac{\varphi}{1-\varphi}\left[1 - \Phi(z^E_t)\right] \geq 1, \tag{24}$$

where $\Phi(\cdot)$ denotes the standard normal CDF. Thus, the expected shadow value of internal funds is strictly greater than one as long as equity issuance is costly ($\varphi > 0$) and future costs are uncertain ($\sigma > 0$). As emphasized by GSSZ, this makes firms de facto risk averse when making their pricing

[18] Recall that $p_{i,h,t}$ and $p^*_{i,h,t}$ are relative prices measured against average prices charged by firms in the home country. These are different from the relative prices against local and foreign CPIs, which are averages of prices of both domestic and imported goods (see equation 9).

decisions: A policy of setting a low markup and committing to fulfilling the resulting large number of orders exposes the firm to operating losses, which must be covered by issuing costly new equity. A defensive pricing strategy is to choose a higher markup, which would not be optimal in an environment with frictionless financial markets.

In our context, $\mathbb{E}_t^a[\xi_{i,t}]$ directly captures the firm's ex ante valuation of an additional unit of cashflow obtained from increasing marginal revenue. As discussed above, the firm's ex ante internal valuation of marginal cost depends on $\mathbb{E}_t^a[\xi_{i,t}a_{i,t}]$. From the assumption that $\mathbb{E}_t^a[a_{i,t}] = 1$ and properties of the log-normal distribution (see Kotz et al., 2000), it follows that

$$\mathbb{E}_t^a[\xi_{i,t}a_{i,t}] - \mathbb{E}_t^a[\xi_{i,t}] = \text{Cov}[\xi_{i,t}, a_{i,t}] = \frac{\varphi}{1-\varphi}\left[\Phi(z_t^E) - \Phi(z_t^E - \sigma)\right] > 0.$$

Because the realized shadow value of internal funds covaries positively with the cost shock, the ex ante internal valuation of marginal cost exceeds the ex-ante valuation of marginal revenue, so that

$$\frac{\mathbb{E}_t^a[\xi_{it}a_{it}]}{\mathbb{E}_t^a[\xi_{it}]} = \frac{1-\varphi\Phi(z_t^E - \sigma)}{1-\varphi\Phi(z_t^E)} > 1. \tag{25}$$

### 3.3.1 Optimal Pricing

To streamline the notation, we define the markup $\tilde{\mu}_t$ as the inverse of real marginal cost inclusive of financing costs:

$$\tilde{\mu}_t = \left[\frac{\mathbb{E}_t^a[a_{i,t}\xi_{i,t}]}{\mathbb{E}_t^a[\xi_{i,t}]}\frac{w_t}{\alpha A_t}\left(\phi + c_{h,t} + c_{h,t}^*\right)^{\frac{1-\alpha}{\alpha}}\right]^{-1}. \tag{26}$$

Substituting $\mathbb{E}_t^a[\kappa_{i,t}]/\mathbb{E}_t^a[\xi_{i,t}] = \tilde{\mu}_t^{-1}$ and the rational expectation solutions of equations (21) and (22) into equations (19) and (20), we obtain the closed-form solutions for the normalized shadow values of marginal sales in the two markets:

$$\frac{\mathbb{E}_t^a[\nu_{i,h,t}]}{\mathbb{E}_t^a[\xi_{i,t}]} = p_{h,t} - \frac{1}{\tilde{\mu}_t} + \chi\mathbb{E}_t\left[\sum_{s=t+1}^{\infty}\beta_{h,t,s}\frac{\mathbb{E}_s^a[\xi_{i,s}]}{\mathbb{E}_t^a[\xi_{i,t}]}\left(p_{h,s} - \frac{1}{\tilde{\mu}_s}\right)\right]; \tag{27}$$

$$\frac{\mathbb{E}_t^a[\nu_{i,h,t}^*]}{\mathbb{E}_t^a[\xi_{i,t}]} = q_t p_{h,t}^* - \frac{1}{\tilde{\mu}_t} + \chi\mathbb{E}_t\left[\sum_{s=t+1}^{\infty}\beta_{h,t,s}^*\frac{\mathbb{E}_s^a[\xi_{i,s}]}{\mathbb{E}_t^a[\xi_{i,t}]}\left(q_s p_{h,s}^* - \frac{1}{\tilde{\mu}_s}\right)\right], \tag{28}$$

where $\chi = (1-\rho)\theta(1-\eta) > 0$, and the growth-adjusted, compounded discount factors $\beta_{t,s}$ and $\beta_{t,s}^*$ are given by:

$$\beta_{h,t,s} = m_{s,s+1}g_{h,s+1} \times \prod_{j=1}^{s-t}(\rho + \chi g_{h,t+j})m_{t+j-1,t+j}, \quad \text{with } g_{h,t} = \frac{s_{h,t}/s_{h,t-1} - \rho}{1-\rho};$$

$$\beta_{h,t,s}^* = m_{s,s+1}g_{h,s+1}^* \times \prod_{j=1}^{s-t}(\rho + \chi g_{h,t+j}^*)m_{t+j-1,t+j}, \quad \text{with } g_{h,t}^* = \frac{s_{h,t}^*/s_{h,t-1}^* - \rho}{1-\rho}.$$

By substituting equations (27) and (28) into equations (17) and (18) and imposing the symmetric equilibrium conditions, we can express the firm's optimal pricing strategies in the domestic and foreign markets as

$$p_{h,t} = \frac{\eta}{\eta-1}\frac{1}{\tilde{\mu}_t} + (1-\rho)\theta\eta\mathbb{E}_t\left[\sum_{s=t+1}^{\infty}\beta_{h,t,s}\frac{\mathbb{E}_s^a[\xi_{i,s}]}{\mathbb{E}_t^a[\xi_{i,t}]}\left(p_{h,s}-\frac{1}{\tilde{\mu}_s}\right)\right]; \tag{29}$$

$$q_t p_{h,t}^* = \frac{\eta}{\eta-1}\frac{1}{\tilde{\mu}_t} + (1-\rho)\theta\eta\mathbb{E}_t\left[\sum_{s=t+1}^{\infty}\beta_{h,t,s}^*\frac{\mathbb{E}_s^a[\xi_{i,s}]}{\mathbb{E}_t^a[\xi_{i,t}]}\left(q_s p_{h,s}^*-\frac{1}{\tilde{\mu}_s}\right)\right]. \tag{30}$$

In the absence of customer-market relationships (i.e., $\theta = 0$), the second term on the right-hand sides of the above equations disappears, and we obtain the standard pricing equation for a static monopolist facing isoelatic demand: The price is equal to a constant markup $\frac{\eta}{1-\eta}$ over current marginal cost, inclusive of financing costs. Note that with customer markets (i.e., $\theta < 0$), prices are, on average, strictly lower than those that would have been set by the static monopolist because the firms have an incentive to lower prices in order to expand their market shares.

Financial frictions create a tension between the firm's desire to expand its market share and its desire to maintain adequate internal liquidity. Note that the terms inside of the square brackets represent the present values of future profits. When expanding market shares becomes more important, which happens through the increase in the growth-adjusted, compound discount factors $\beta_{t,s}$ and $\beta_{t,s}^*$, the firm has a greater incentive to reduce prices. However, when the firm faces a liquidity problem in the sense that the shadow value of internal funds today is strictly greater than its future values—that is, $\mathbb{E}_t^a[\xi_{i,t}] > \mathbb{E}_t^a[\xi_{i,s}]$, for $s > t$—the firm discounts future profits more heavily. Again, the fact that $(1-\rho)\theta\eta < 0$ implies that the firm is more likely to raise prices in order to increase current cashflows, even though doing so cannibalizes its future market share.

It is worth noting that the terms in the square brackets represent a new price-adjustment channel in the international macroeconomics literature. This literature has emphasized the role of variable markups micro-founded via oligopolistic competition (see Dornbusch, 1987; Yang, 1997; Atkeson and Burstein, 2008; Gopinath and Itskhoki, 2010a; Auer and Schoenle, 2016). In these models, firms absorb adverse shocks by lowering markups, a pricing behavior associated with the preservation of current market shares. As a result, the pass-through of shocks is incomplete. In our model, by contrast, the tension between concerns about future market shares due to customer-market considerations and financial frictions opens up a distinct, though complementary, pass-through channel, one that weakens the degree of pass-through for firms with liquid balance sheets but strengthens the degree of pass-through for their cash-strapped counterparts.

Given the optimal dividend policy specified by equation (23), equations (24), (25), and (26) together determine the ex ante shadow value of internal funds $\mathbb{E}_t^a[\xi_{i,t}]$, along with the financially adjusted markup $\tilde{\mu}_t$, as static functions of aggregate variables $w_t/A_t$, $c_{h,t}$, $c_{h,t}^*$, $p_{h,t}$, $p_{h,t}^*$, and $q_t$. This implies that equations (29) and (30) completely summarize the firm's optimal pricing policies.

In equilibrium, the firm's production and demand constraints imply that

$$\begin{aligned} A_t h_t^\alpha &= c_{h,t} + c_{h,t}^* + \phi; \\ c_{h,t} &= s_{h,t-1}^\theta x_{h,t}, \quad \text{with } s_{h,t} = \rho s_{h,t-1} + (1-\rho)c_t; \\ c_{h,t}^* &= s_{h,t-1}^{*\theta} x_{h,t}^*, \quad \text{with } s_{h,t}^* = \rho s_{h,t-1}^* + (1-\rho)c_t^*, \end{aligned}$$

while the domestic and foreign demands $x_{h,t}$ and $x_{h,t}^*$, respectively, are determined as functions of the aggregate domestic and foreign consumption baskets $x_t$ and $x_t^*$ and their respective relative prices:

$$\begin{aligned} x_{h,t} &= \Xi_h^\varepsilon \left(\frac{\tilde{p}_{h,t}}{\tilde{p}_t}\right)^{-\varepsilon} x_t; \quad \tilde{p}_{k,t} = p_{k,t} s_{k,t-1}^\theta; \quad \text{and } \tilde{p}_t = \left[\sum_{k=h,f} \Xi_k \tilde{p}_{k,t}^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}}; \\ x_{h,t}^* &= \Xi_h^{*\varepsilon} \left(\frac{\tilde{p}_{h,t}^*}{\tilde{p}_t^*}\right)^{-\varepsilon} x_t; \quad \tilde{p}_{k,t}^* = p_{k,t}^* s_{k,t-1}^{*\theta}; \quad \text{and } \tilde{p}_t^* = \left[\sum_{k=h,f} \Xi_k^* \tilde{p}_{k,t}^{*(1-\varepsilon)}\right]^{\frac{1}{1-\varepsilon}}. \end{aligned}$$

In the absence of nominal rigidities, the household equilibrium conditions then allow us to determine aggregate consumption $x_t$, the household's discount factor $m_{t,t+1}$, and the real exchange rate $q_t$.

#### 3.3.2 Inflation Dynamics

Adding nominal rigidities to the model does not alter the nature of the optimal pricing problem in any fundamental way. The inherent tension between the maximization of market shares and the maximization of current profits that arises from the interaction of financial frictions and customer markets is also present in a version of the model with sticky prices. Therefore, instead of repeating the analysis, we simply close this section by showing how the well-known, log-linearized Phillips curve is modified owing to financial frictions and customer-market relationships.

Using equation (9), we can express the log-linearized dynamics of national CPIs as

$$\hat{\pi}_t = \Xi_h p_h(\hat{p}_{h,t-1} + \hat{\pi}_{h,t}) + \Xi_f p_f(\hat{p}_{f,t-1} + \hat{\pi}_{f,t}); \tag{31}$$

$$\hat{\pi}_t^* = \Xi_h^* p_h^*(\hat{p}_{h,t-1}^* + \hat{\pi}_{h,t}^*) + \Xi_f^* p_f^*(\hat{p}_{f,t-1}^* + \hat{\pi}_{f,t}^*), \tag{32}$$

where the variables with the "hat" denote log-linearized deviations from their respective steady-state values, which correspond to variables without the time subscript. Equations (31) and (32) illustrate how import prices affect the inflation dynamics of national CPIs. A full characterization of these dynamics requires a construction of Phillips curves for $\hat{\pi}_{h,t}$, $\hat{\pi}_{f,t}$, $\hat{\pi}_{h,t}^*$, and $\hat{\pi}_{f,t}^*$. For the sake of space, we focus on the first and the third.

The log-linearization of the first-order conditions for $p_{i,h,t}$ and $p^*_{i,h,t}$ implies:

$$\hat{\pi}_{h,t} = \frac{1}{\gamma_p}\frac{p_h c_h}{c}\left[\hat{p}_{h,t} - (\hat{\nu}_{h,t} - \hat{\xi}_t)\right] + \delta\mathbb{E}_t[\hat{\pi}_{h,t+1}]; \tag{33}$$

$$\hat{\pi}^*_{h,t} = \frac{1}{\gamma_p}qp^*_h\frac{c^*_h}{c^*}\left[\hat{q}_t + \hat{p}^*_{h,t} - (\hat{\nu}^*_{h,t} - \hat{\xi}_t)\right] + \delta\mathbb{E}_t[\hat{\pi}^*_{h,t+1}], \tag{34}$$

where $\hat{\nu}_{h,t}$, $\hat{\nu}^*_{h,t}$, and $\hat{\xi}_t$ are the log-deviations of $\mathbb{E}^a_t[\nu_{i,h,t}]$, $\mathbb{E}^a_t[\nu^*_{i,h,t}]$, and $\mathbb{E}^a_t[\xi_{i,t}]$ from their respective steady-state values. In the absence of customer markets, the terms in brackets are exactly equal to the log-deviation of the financially adjusted real marginal cost $\tilde{\mu}_t^{-1}$, and we recover the standard forward-looking Phillips curve for each market.

With customer markets, however, we obtain a considerably richer set of inflation dynamics. Log-linearizing equations (27) and (28) and substituting the resulting expressions into equations (33) and (34) yields the following Phillips curve for the domestic market:

$$\begin{aligned}\hat{\pi}_{h,t} = \frac{1}{\gamma_p}\frac{p_h c_h}{c}&\left[\hat{p}_{h,t} - \eta\left(\hat{p}_{h,t} + \frac{\hat{\mu}_t}{p_h\tilde{\mu}}\right) - \eta\chi\mathbb{E}_t\sum_{s=t+1}^{\infty}\tilde{\delta}^{s-t}\left(\hat{p}_{h,s} + \frac{\hat{\mu}_s}{p_h\tilde{\mu}}\right)\right] \\ &+ \frac{\eta\chi}{\gamma_p}\frac{p_h c_h}{c}\left(1 - \frac{1}{p_h\tilde{\mu}}\right)\mathbb{E}_t\sum_{s=t+1}^{\infty}\tilde{\delta}^{s-t}\left[(\hat{\xi}_t - \hat{\xi}_s) - \hat{\beta}_{h,t,s}\right] + \delta\mathbb{E}_t[\hat{\pi}_{h,t+1}];\end{aligned} \tag{35}$$

and for the foreign market:

$$\begin{aligned}\hat{\pi}^*_{h,t} = \frac{1}{\gamma_p}qp^*_h\frac{c^*_h}{c^*}&\left[\hat{q}_t + \hat{p}^*_{h,t} - \eta\left((\hat{q}_t + \hat{p}_{h,t}) + \frac{\hat{\mu}_t}{qp^*_h\tilde{\mu}}\right) + \chi\mathbb{E}_t\sum_{s=t+1}^{\infty}\tilde{\delta}^{s-t}\left((\hat{q}_s + \hat{p}^*_{h,s}) + \frac{\hat{\mu}_s}{qp^*_h\tilde{\mu}}\right)\right] \\ &+ \frac{\eta\chi}{\gamma_p}qp^*_h\frac{c^*_h}{c^*}\left(1 - \frac{1}{qp^*_h\tilde{\mu}}\right)\mathbb{E}_t\sum_{s=t+1}^{\infty}\tilde{\delta}^{s-t}\left[(\hat{\xi}_t - \hat{\xi}_s) - \hat{\beta}^*_{h,t,s}\right] + \delta\mathbb{E}_t[\hat{\pi}^*_{h,t+1}],\end{aligned} \tag{36}$$

where $\tilde{\delta} = \delta(\rho + \chi)$. Because $\chi > 0$, the firm's heightened concern about its current liquidity position, as manifested by the fact that $\hat{\xi}_t - \hat{\xi}_s > 0$, will result in higher inflation in both markets. In contrast, the increased importance of future market shares at home and abroad, as captured by $\hat{\beta}_{h,t,s} > 0$ and $\hat{\beta}^*_{h,t,s} > 0$, leads to lower inflation in both markets. The terms $(\hat{\xi}_t - \hat{\xi}_s) - \hat{\beta}_{h,t,s}$ and $(\hat{\xi}_t - \hat{\xi}_s) - \hat{\beta}^*_{h,t,s}$, therefore, capture the fundamental tension between the maximization of current profits and the maximization of long-run market shares, a tension that importantly shapes inflation dynamics in periods of financial turmoil.

### 3.4 The Household's Problem

We now turn to the optimization problem of the representative household in the home country. First, we formulate this problem in an environment with incomplete risk sharing and floating exchange rates. We then impose restrictions that deliver the baseline model of a monetary union. We also develop an alternative version of the model that allows for complete risk sharing.

### 3.4.1 Floating Exchange Rates With Incomplete Risk Sharing

The representative household in the home country works $h_t$ hours. It allocates its savings between shares of the home country firms and international bonds that are not state contingent. We denote the home country's holdings of international bonds issued in home and foreign currency units by $B_{h,t+1}$ and $B_{f,t+1}$, respectively, while $B^*_{h,t+1}$ and $B^*_{f,t+1}$ denote their foreign counterparts.[19] The respective (gross) nominal interest rates on these securities are denoted by $R_t$ and $R^*_t$.

We assume that investors in both countries face identical portfolio rebalancing costs, denoted by $\tau$. Focusing on the home country, these costs are given by

$$\frac{\tau}{2}P_t\left[\left(\frac{B_{h,t+1}}{P_t}\right)^2 + q_t\left(\frac{B_{f,t+1}}{P^*_t}\right)^2\right]; \quad (\tau > 0).$$

Under these assumptions, the marginal cost of borrowing in home currency is given by $R_t/(1+\tau B_{h,t+1}/P_t)$, which is strictly greater than $R_t$ if $B_{h,t+1} < 0$. The marginal return on foreign lending in home currency is given by $R_t(Q_t/Q_{t+1})/(1+\tau B^*_{h,t+1}/P^*_t)$, which is strictly less than $R_t(Q_t/Q_{t+1})$ if $B^*_{h,t+1} > 0$. Thus, $(1+\tau B_{h,t+1}/P_t)^{-1}$ represents a welfare loss, not only to the borrowers, but also to the lenders. As pointed out by Ghironi and Melitz (2005), the role of such portfolio rebalancing costs is to pin down the steady-state levels of international bond holdings, as varying $\tau$ does not modify the model dynamics in any significant way.

The number of outstanding shares of home country firm $i$ is denoted by $S_{i,t}$, while $P^S_{i,t-1,t}$ is the period-$t$ per-share value of the shares outstanding as of period $t-1$ and $P^S_{i,t}$ is the (ex-dividend) per-share value of shares in period $t$. Using the fact that $\int_{N_k} P_{i,k,t}c_{i,k,t}di = \tilde{P}_{k,t}x_{k,t}$, for $k = h, f$ (see Appendix B), we can express the household's budget constraint as

$$\begin{aligned} 0 = {} & W_t h_t + R_{t-1}B_{h,t} + Q_t R^*_{t-1}B_{f,t} + \int_{N_h}\left[\max\{D_{i,t}, 0\} + P^S_{i,t-1}\right]S^S_{i,t}di \\ & - \tilde{P}_t x_t - B_{h,t+1} - Q_t B_{f,t+1} - \frac{\tau}{2}P_t\left[\left(\frac{B_{h,t+1}}{P_t}\right)^2 + q_t\left(\frac{B_{f,t+1}}{P^*_t}\right)^2\right] - \int_{N_h} P^S_{i,t}S^S_{i,t+1}di \end{aligned} \tag{37}$$

Note that we have expressed the consumption expenditure problem as purchasing the habit-adjusted consumption bundle $x_t$ using the price index $\tilde{P}_t$, which is possible because $\tilde{P}_t$ is a welfare-based price index.

The representative household maximizes the life-time utility given by equation (5) subject to the budget constraint (37). Letting $\Lambda_t$ denote the Lagrange multiplier associated with the budget constraint, the first-order condition for $x_t$ is then given by $\Lambda_t = U_{x,t}/\tilde{P}_t = U_{x,t}/(\tilde{P}_t/P_t)P_t = (U_{x,t}/\tilde{p}_t)/P_t$. We can then express the first-order condition for hours worked as $U_{x,t}w_t/\tilde{p}_t = -U_{h,t}$.

Note that the two equity valuation terms that appear in the budget constraint are related to each

[19] Note that our notation implies that $B_{h,t+1} + B^*_{h,t+1} = 0$, where $B_{h,t+1}$ and $B^*_{h,t+1}$ are denominated in home currency—as denoted by the subscript $h$—and are held by the home and foreign country residents, respectively. If $B_{h,t+1} < 0$ ($B_{f,t+1} < 0$), the home country borrows money in home currency units (in foreign currency units) from the foreign country, whose claim is $B^*_{h,t+1} > 0$ ($B^*_{f,t+1} > 0$).

other through an accounting identity $P^s_{i,t} = P^s_{i,t-1,t} + E^s_{i,t}$, where $E^s_{i,t}$ is the per-share value of new equity issued by a firm $i$ in period $t$. Because of equity dilution costs, $E^s_{i,t} = -(1-\varphi)\min\{D_{i,t}, 0\}$. Substituting $P^s_{i,t-1,t} = P^s_{i,t} - E^s_{i,t} = P^s_{i,t} + (1-\varphi)\min\{D_{i,t}, 0\}$ into the budget constraint (37), we obtain the optimality conditions governing the household's holdings of international bonds and shares of the firms:

$$1 = \delta \mathbb{E}_t \left[ \frac{U_{x,t+1}/\tilde{p}_{t+1}}{U_{x,t}/\tilde{p}_t} \left( \frac{R_t}{\pi_{t+1}} \frac{1}{1+\tau b_{h,t+1}} \right) \right]; \tag{38}$$

$$1 = \delta \mathbb{E}_t \left[ \frac{U_{x,t+1}/\tilde{p}_{t+1}}{U_{x,t}/\tilde{p}_t} \left( \frac{q_{t+1}}{q_t} \frac{R^*_t}{\pi^*_{t+1}} \frac{1}{1+\tau b_{f,t+1}} \right) \right]; \tag{39}$$

$$1 = \delta \mathbb{E}_t \left[ \frac{U_{x,t+1}/\tilde{p}_{t+1}}{U_{x,t}/\tilde{p}_{t+1}} \frac{1}{\pi_{t+1}} \left( \frac{\mathbb{E}^a_{t+1}[\tilde{D}_{i,t+1}] + P^s_{t+1}}{P^s_t} \right) \right], \tag{40}$$

where $\tilde{D}_{i,t} = \max\{D_{i,t}, 0\} + (1-\varphi)\min\{D_{i,t}, 0\}$, $b_{h,t+1} = B_{h,t+1}/P_t$, and $b_{f,t+1} = B_{f,t+1}/P^*_t$. In deriving the first-order condition (40), we exploited the fact that the ex-ante value of the firm—the value prior to the realization of the idiosyncratic cost shock—is the same for all firms; that is, $\mathbb{E}^a_{t+1}[P^s_{i,t+1}] = P^s_{t+1}$ in the symmetric equilibrium.

The bond market clearing conditions are given by

$$0 = b_{h,t+1} + b^*_{h,t+1} \quad \text{and} \quad 0 = b_{f,t+1} + b^*_{f,t+1}, \tag{41}$$

where foreign holdings of international bonds denominated in home and foreign currencies—$b^*_{h,t+1}$ and $b^*_{f,t+1}$, respectively—satisfy the foreign counterparts of equations (38) and (39):

$$1 = \delta \mathbb{E}_t \left[ \frac{U^*_{x,t+1}/\tilde{p}^*_{t+1}}{U^*_{x,t+1}/\tilde{p}^*_{t+1}} \frac{q_t}{q_{t+1}} \frac{R_t}{\pi_{t+1}} \frac{1}{1+\tau b^*_{h,t+1}} \right]; \tag{42}$$

$$1 = \delta \mathbb{E}_t \left[ \frac{U^*_{x,t+1}/\tilde{p}^*_{t+1}}{U^*_{x,t+1}/\tilde{p}^*_{t+1}} \frac{R^*_t}{\pi^*_{t+1}} \frac{1}{1+\tau b^*_{f,t+1}} \right]. \tag{43}$$

Assuming that the portfolio rebalancing costs are transferred back to the household in a lump-sum fashion, imposing the stock market equilibrium condition $S_{i,t} = S_{i,t+1} = 1$, $i \in N_h$, and dividing the budget constraint through by $P_t$, equation (37) then implies the following law of motion for the bond holdings in the home country:

$$b_{h,t+1} + q_t b_{f,t+1} = \frac{R_{t-1}}{\pi_t} b_{h,t} + \frac{R^*_{t-1}}{\pi^*_t} q_t b_{f,t} + w_t h_t + \tilde{d}_t - \tilde{p}_t x_t, \tag{44}$$

where $\tilde{d}_t = \tilde{D}_t/P_t$; the corresponding law of motion for the bond holdings in the foreign country is given by

$$\frac{1}{q_t} b^*_{h,t+1} + b^*_{f,t+1} = \frac{R_{t-1}}{q_t \pi_t} b^*_{h,t} + \frac{R^*_{t-1}}{\pi^*_t} b^*_{f,t} + w^*_t h^*_t + \tilde{d}^*_t - \tilde{p}^*_t x^*_t, \tag{45}$$

where $\tilde{d}_t^* = \tilde{D}_t^*/P_t^*$. Multiplying equation (45) by $q_t$, subtracting the resulting expression from equation (44), and imposing the bond market clearing conditions given in equation (41) yields

$$b_{h,t+1} + q_t b_{f,t+1} = \frac{R_{t-1}}{\pi_t} b_{h,t} + \frac{R^*_{t-1}}{\pi^*_t} q_t b_{f,t} + \frac{1}{2}(w_t h_t - q_t w_t^* h_t^*) + \frac{1}{2}(\tilde{d}_t - q_t \tilde{d}_t^*) - \frac{1}{2}(\tilde{p}_t x_t - q_t \tilde{p}_t^* x_t^*). \quad (46)$$

This condition, together with bond market clearing conditions (41), should hold for the balance-of-payments between the two countries.

Closing the model requires us to specify a monetary policy rule. In the case of floating exchange rates, we assume that monetary authorities in the home and foreign countries set prices of government bonds in their respective countries using interest-rate rules of the form:

$$R_t = R\left(\frac{y_t}{y}\right)^{\psi_y}\left(\frac{\pi_t}{\pi}\right)^{\psi_\pi} \quad \text{and} \quad R_t^* = R^*\left(\frac{y_t^*}{y^*}\right)^{\psi_y}\left(\frac{\pi_t^*}{\pi^*}\right)^{\psi_\pi}, \quad (47)$$

where the reaction coefficients $\psi_y$ and $\psi_\pi$ are assumed to be the same across the two countries. We do not assume any policy inertia because such an interest-rate smoothing motive is frequently a source of inefficiency in the conduct of monetary policy.

### 3.4.2 Monetary Union With Incomplete Risk Sharing

In a monetary union, all products and financial assets are denominated in units of common currency. As a result, the nominal exchange rate $Q_t$ is not defined. In addition, a single monetary authority sets the interest rate, denoted by $R_t^U$, and all investors, regardless of their country of origin and current location, earn the same nominal return on their bond holdings.[20] We assume that monetary policy in the union is conducted in a manner that reflects the economic fundamentals of both countries:

$$R_t^U = R^U\left(\frac{y_t^U}{y^U}\right)^{\psi_y}\left(\frac{\pi_t^U}{\pi^U}\right)^{\psi_\pi},$$

where the union-wide variables are constructed as weighted averages of country-specific aggregates, with the weights given by the steady-state share of output:[21]

$$y_t^U = y_t\left(\frac{y}{y+qy^*}\right) + q_t y_t^*\left(\frac{qy^*}{y+qy^*}\right) \quad \text{and} \quad \pi_t^U = \pi_t\left(\frac{y}{y+qy^*}\right) + \pi_t^*\left(\frac{qy^*}{y+qy^*}\right).$$

Because there is no longer any distinction between bonds issued in home or foreign currency,

[20] However, the *real* returns on international bond holdings will differ in equilibrium, depending on the reference location of investors. This divergence in real returns reflects two factors. First, the two countries have different consumption baskets in the long run, owing to the presence of home bias in consumption. Second, at any point in time, the law of one price does not hold in the monetary union because two consumers residing in different countries have accumulated different stocks of habit for an identical product. Because firms price their products to markets—the so-called pricing to habits Ravn et al. (2007)—inflation rates are not equalized across countries, despite the adoption of a single currency and common monetary policy.

[21] The model dynamics are robust to alternative weighting schemes (real time or lagged weights, and so on).

we replace the bond market clearing conditions (see equation 41) by

$$b_{t+1} + b_{t+1}^* = 0, \tag{48}$$

where $b_{t+1}$ and $b_{t+1}^*$ denote holdings of international bonds in the single currency units by home and foreign countries, respectively. Now there are two, instead of four, Euler equations characterizing the equilibrium in the international bond market:

$$1 = \delta \mathbb{E}_t \left[ \frac{U_{x,t+1}/\tilde{p}_{t+1}}{U_{x,t+1}/\tilde{p}_{t+1}} \frac{R_t^U}{\pi_{t+1}} \frac{1}{1+\tau b_{t+1}} \right]; \tag{49}$$

$$1 = \delta \mathbb{E}_t \left[ \frac{U_{x,t+1}^*/\tilde{p}_{t+1}^*}{U_{x,t+1}^*/\tilde{p}_{t+1}^*} \frac{q_t}{q_{t+1}} \frac{R_t^U}{\pi_{t+1}^*} \frac{1}{1+\tau b_{t+1}^*} \right]. \tag{50}$$

Note that $q_t/q_{t+1} = (Q_t/Q_{t+1})(\pi_{t+1}/\pi_{t+1}^*) = \pi_{t+1}/\pi_{t+1}^*$ in a monetary union. Finally, a monetary union implies that the combined law of motion for the international bond holdings given in equation (46) can be expressed as

$$b_{t+1} = \frac{R_{t-1}^U}{\pi_t} b_t + \frac{1}{2}(w_t h_t - q_t w_t^* h_t^*) + \frac{1}{2}(\tilde{d}_t - q_t \tilde{d}_t^*) - \frac{1}{2}(\tilde{p}_t x_t - q_t \tilde{p}_t^* x_t^*). \tag{51}$$

### 3.4.3 Monetary Union With Complete Risk Sharing

As a topical policy exercise, we also analyze the welfare gains implied by a complete risk-sharing arrangement in a monetary union. We model such risk sharing by allowing for state-contingent bonds that are traded internationally, along with government bonds that are in zero net supply. With complete risk sharing, we no longer need to rely on imperfections in the government bond market to induce a long-run stationary equilibrium—accordingly, we set $\tau = 0$. The presence of a complete set of state-contingent bonds implies the following risk-sharing condition (see Appendix B):

$$q_t = \varrho_0 \frac{U_{x,t}^*/\tilde{p}_t^*}{U_{x,t}/\tilde{p}_t}, \quad \text{where } \varrho_0 = q_0 \frac{U_{x,0}/\tilde{p}_0}{\tilde{U}_{x,0}^*/\tilde{p}_0^*}. \tag{52}$$

Equation (52) replaces the bond-holding condition (46), which was derived under incomplete markets.

The above risk-sharing condition should hold regardless of the exchange rate arrangement between the two countries (i.e., floating exchange rate regime or common currency). However, in contrast to the case of floating exchange rates, only one of the two consumption Euler equations can be included in the system of equations that characterize the equilibrium in a monetary union. This is because the combination of common monetary policy and the assumption of complete risk sharing introduces linear dependence into the two Euler equations. Hence, only the following efficiency condition enters the equilibrium system of equations in the monetary union with complete risk sharing:

$$1 = \delta \mathbb{E}_t \left[ \frac{U_{x,t+1}/\tilde{p}_{t+1}}{U_{x,t}/\tilde{p}_t} \frac{R_t^U}{\pi_{t+1}} \right].$$

## 4 Calibration

There are three sets of parameters in the model: (1) parameters related to preferences and technology; (2) parameters governing the strength of nominal rigidities and the conduct of monetary policy; and (3) parameters determining the degree of financial market distortions, including portfolio rebalancing costs. In setting their values, our calibration strategy closely follows GSSZ, while expanding the set of parameters to the international environment.

We set the deep habit parameter $\theta$ to $-0.86$, a value similar to that used by Ravn et al. (2006). The key tension between the maximization of a long-run market share and the maximization of current profits does not exist when $\theta = 0$. In such an environment, the financial shock we consider has considerably smaller effect on economic outcomes. It is in this sense that our model owes a lot to customer-market considerations as captured by deep habits. Consequently, we follow Ravn et al. (2006) and choose a fairly persistent habit-formation process, so that only 15 percent of the habit stock depreciates in a given quarter ($\rho = 0.85$), a choice that highlights the firms' incentives to compete for market share. Later, we consider a range of values for parameters $\theta$ and $\rho$ to analyze the sensitivity of our results.

We set the time discount factor equal to 0.996. The CRRA parameter in the household's utility function is set equal to 2, and we set the elasticity of labor supply equal to 5. As we explain below, we specify the same degree of persistence (0.90) for all exogenous shock processes (i.e., aggregate demand shocks, aggregate technology shocks, and financial shocks). We then adjust the volatilities of the shocks to match the shares of output implied by the model's variance decomposition.

The elasticity of substitution $\eta$ is a key parameter in the customer-markets model because the greater the firm's market power, the greater the incentive to invest in customer base. We set $\eta$ equal to 2, a value consistent with Broda and Weinstein (2006), who provide a set of point estimates for the elasticity of substitution for the U.S. economy. Their estimates lie in the range between 2.1 and 4.8, depending on the characteristics of products (commodities vs. differentiated goods) and sub-samples (before 1990 vs. after 1990). Our choice of $\eta = 2$ corresponds closely to the median value of the estimated elasticities for differentiated goods for the post-1990 period, a class of products that is most relevant for the deep habits framework; this choice is also broadly consistent with Ravn et al. (2010), who estimate $\eta$ equal to 2.48 within a context of the deep habits model.

Regarding $\Xi_h$ and $\Xi_f$ (and $\Xi_f^*$ and $\Xi_h^*$), the weights of home and foreign goods in the household's utility function, we choose their values so that the share of imported goods in the steady-state consumption basket is equal to 0.4 in both countries, a value in the middle of the range of the import-to-GDP ratios for the euro area countries since 2000.[22] As for the Armington elasticity,

[22] Note that $\Xi_f$ itself is not equal to the share of imported goods in the GDP of the home country; rather $\Xi_f$ is

we set $\varepsilon$ equal to 1.5, in order to stay close to the near-unit elasticity estimated by Feenstra et al. (2014). As long $\varepsilon > 1$, a value lower than 1.5 does not affect our main results. For example, setting $\varepsilon$ close to 1 reduces the impact of a financial shock on aggregate output in a monetary union to two-thirds of that implied by our baseline calibration. This is because the lower elasticity of cross-border substitution implies a less intense price war between firms of the two countries. However, even in this extreme case, the qualitative features of the equilibrium remain the same.

The fixed operating costs $\phi$ and $\phi^*$ are another two key parameters in our model. In our baseline calibration, we assume $\phi = \phi^*$, which implies that differences in the degree of financial distortions are the sole source of heterogeneity between the two countries. We calibrate $\phi$ in conjunction with the returns-to-scale parameter $\alpha$. Specifically, we set $\alpha$ first and then choose $\phi$ so that the dividend-payout ratio (relative to operating income) hits 2.5 percent, the mean of this ratio in the U.S. since 1945. As noted by GSSZ, decreasing returns-to-scale enhance the link between financial distortions and the firms' pricing decisions; for this reason, GSSZ set $\alpha = 0.8$, a value consistent with the structural estimates based on firm-level data. In this paper, however, we set $\alpha = 1$, a standard value in the international macroeconomics literature. This leads us to set $\phi = 0.1$. With $\alpha = 1$, $\phi = 0.1$, and $\eta = 2$, the average (gross) markup in our model comes out at 1.19.

In calibrating the degree of financial distortions faced by domestic firms, we set the equity dilution cost parameter $\varphi$ equal to 0.2, a value that is in the middle of the range typically used in the corporate finance literature. The degree of financial frictions faced by foreign firms $\varphi^*$ is then calibrated to be one-tenth of $\varphi$ (i.e., $\varphi^* = 0.1\varphi$), implying a considerably more accommodative financial conditions for foreign country firms in the steady state. The volatility of the idiosyncratic cost shock $\sigma$ is set to 0.2 at a quarterly frequency. With $\varphi = 0.2$ and $\phi = 0.1$, this level of idiosyncratic volatility implies that the expected shadow value of internal funds equals 1.16 for home country firms in the steady state. The portfolio rebalancing costs $\tau$ are set equal to 0.15.

For the parameters related to nominal rigidities, we set $\gamma_p$, the adjustment costs of nominal prices in both countries, equal to 10. In presenting the model, we treated wages as completely flexible. In the actual simulations, however, we introduce nominal wage rigidities along the lines of Bordo et al. (2000) and Erceg et al. (2000). In symmetry with our assumptions regarding nominal price rigidities, we assume market power for households that supply labor to production firms and a quadratic cost of adjusting nominal wages. In this case, assuming a separable, constant elasticity of labor supply $U_{h,t} = -h_t^{1/\zeta}$, the efficiency condition for labor hours becomes

$$\begin{aligned} \eta_w \frac{h_t^{1/\zeta}/U_{x,t}}{w_t/\tilde{p}_t} &= \eta_w - 1 + \gamma_w(\pi_{w,t} - \pi_w)\pi_{w,t} \\ &\quad - \delta \mathbb{E}_t \left[ \frac{U_{x,t+1}/\tilde{p}_{t+1}}{U_{x,t}/\tilde{p}_t} \gamma_w(\pi_{w,t+1} - \pi_w)\pi_{w,t+1} \frac{\pi_{w,t+1}}{\pi_{t+1}} \frac{h_{t+1}}{h_t} \right] \end{aligned} \tag{53}$$

where $\pi_{w,t} = W_{t+1}/W_t$, $\gamma_w$ is the coefficient of nominal wage adjustment costs, and $\eta_w$ is the elasticity of substitution of labor.

chosen such that $\Xi_f^{\varepsilon} = p_f c_f / \sum_{k=h,f} p_k c_k = 0.4$.

We set $\zeta = 5$ and $\gamma_w = 30$ in both countries. Our choice for the degrees of price and wage stickiness are comparable to the point estimates of $\gamma_p = 14.5$ and $\gamma_w = 41$ obtained by Ravn et al. (2010), who show that deep habits substantially enhance the persistence of inflation without the need to impose an implausibly large degree nominal price stickiness. It is worth noting that the addition of nominal wage rigidities does not materially modify any of our main results. It does, however, lead to a notably greater volatility of the real exchange rate because the countercyclical markups in the country where firms face acute financial distress are driven more by an increase in product prices as opposed to an immediate decline in nominal wages, which would have occurred in an environment with flexible wages. In the latter case, the more stable final product prices result in a less volatile real exchange rate, which runs counter to intuition, in addition to being at odds with the data.

Finally, we assume that monetary policy is conducted using an interest-rate rule proposed by Taylor (1999). Thus, we are implicitly assuming that the monetary authorities in the two countries operate under a dual mandate of price stability and maximum employment. As shown by GSSZ, the combination of customer markets and financial frictions can break the "divine coincidence," in the sense that an adverse demand shock during the financial crisis can lead to higher inflation, as firms in financial distress find it optimal to raise prices—and sacrifice their market share—in order to preserve internal liquidity. For this reason, we view the interest-rate rule proposed by Taylor (1999) as appropriate and set the inflation reaction coefficient $\psi_\pi$ equal to 1.5 and the coefficient on the output gap $\psi_y$ equal to 1. (Table C-1 in Appendix C conveniently summarizes our baseline calibration of the model.)

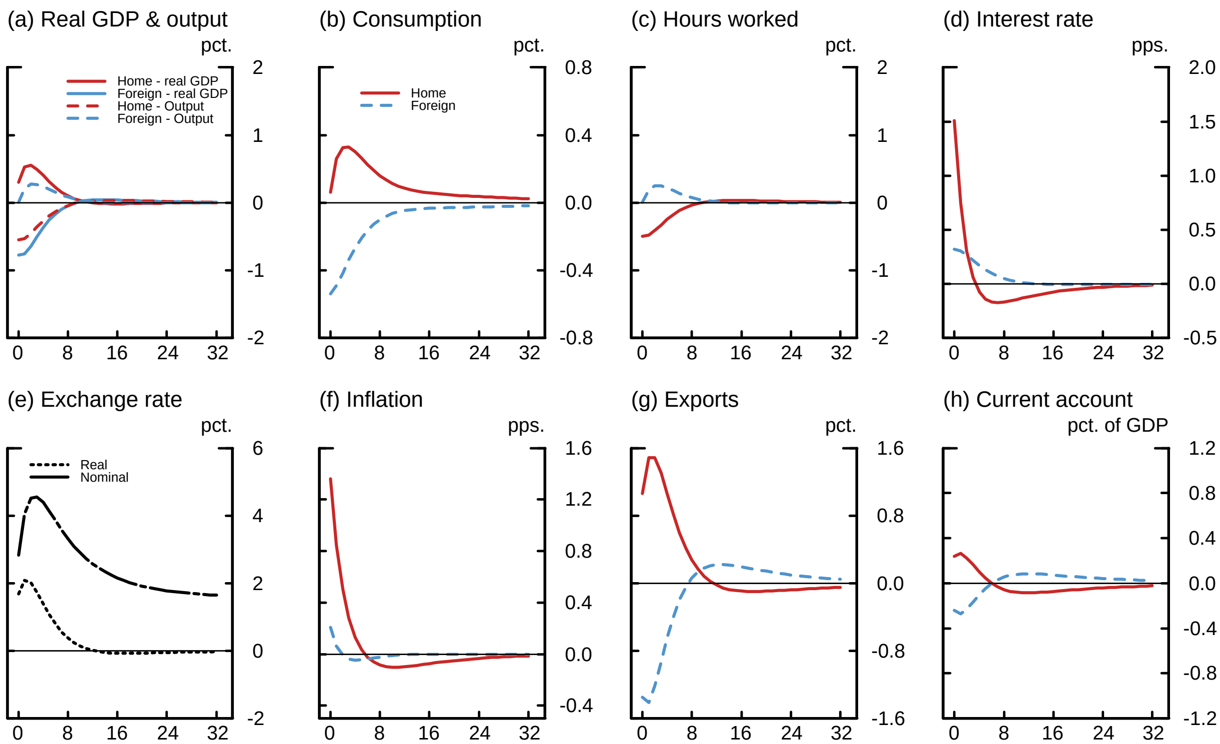
# 5 Model Simulations

## 5.1 Currency Regimes and the Impact of Financial Shocks

In this section, we use our model to analyze quantitatively the macroeconomic implications of home and foreign countries forming a monetary union—that is, adopting a common currency and hence common monetary policy. Motivated by the empirical results reported above, we focus primarily on a setup in which firms in the two countries face differing degrees of financial market frictions and the home country is hit by an adverse financial shock. In conducting these simulations, we rule out any form of a risk-sharing arrangement through cross-border transfers of funds, as well as cross-border labor mobility. Under these assumptions, we compare the international macroeconomic dynamics under a floating exchange rate regime and when the two countries share a common currency.

To analyze the effects of financial instability under various currency regimes, we posit a financial shock, which temporarily raises the cost of outside equity capital for firms in the two countries. Specifically, we assume that the cost of issuing new shares is subject to a random "cost-of-capital"

Figure 5 – An Asymmetric Financial Shock: Floating Exchange Rates




Note: The panels of the figure depict the model-implied responses of selected variables to an adverse financial shock in the home country in period 0 (see the text for details). Unless noted otherwise, the solid lines show responses of variables in the home country, while the dashed lines show those of the foreign country. Exchange rates (panel (e)) are expressed as home currency relative to foreign currency.

shock:

$$\begin{aligned} \varphi_t &= \varphi f_t; \quad \text{where } \ln f_t = \rho_f \ln f_{t-1} + \epsilon_{f,t}, \text{ with } \epsilon_{f,t} \sim N(-0.5\sigma_f^2, \sigma_f^2); \\ \varphi_t^* &= \varphi^* f_t^*; \quad \text{where } \ln f_t^* = \rho_f \ln f_{t-1}^* + \epsilon_{f,t}^*, \text{with } \epsilon_{f,t}^* \sim N(-0.5\sigma_f^2, \sigma_f^2). \end{aligned}$$

We calibrate the size of the shock $\epsilon_{f,t}$ such that $\varphi_t$ jumps to $1.5\varphi$ upon impact and then returns to its normal level of $\varphi = 0.2$, according to the autoregressive dynamics specified above.[23] Because our baseline calibration assumes that $\varphi^* = 0.1\varphi$, the above specification results in asymmetric financial conditions between the two countries, with home country firms facing a significantly higher cost of external finance. To further underscore the effects of differences in financial conditions faced by domestic and foreign firms, we keep the cost of external equity capital in the foreign country at $\varphi_t^* = 0.1\varphi$, for all $t$.

In this experiment, the financial shock increases the expected shadow value of internal funds for firms in the home country from 1.16 to 1.32 upon impact. Figure 5 displays the macroeconomic effects of such an asymmetric financial shock under a floating exchange rate regime and when the

[23] As noted in Section 4, the persistence of all exogenous shock processes is set to 0.9; thus, we set $\rho_f = 0.9$.

monetary authorities in the two countries act independently. As shown in panel (f), home country firms indeed raise prices in response to an adverse financial shock, a result consistent with that reported by GSSZ in a closed-economy setting. Foreign inflation also increases slightly, though not because of a price hike by foreign firms, but rather because of an increase in import prices.

If the nominal exchange rate is unable to respond to the shock, the differential behavior of inflation in the two countries would imply a significant *appreciation* of the real exchange rate. With flexible exchange rates, however, the nominal exchange rate (panel (e)), strongly *depreciates*. In fact, the depreciation is so large that the real exchange rate also depreciates, despite the inflation differential moving in the "wrong" direction. As in the data, therefore, the short-run dynamics of the real exchange rate are dominated by fluctuations in the nominal exchange rate, rather than by changes in the relative price levels.[24]

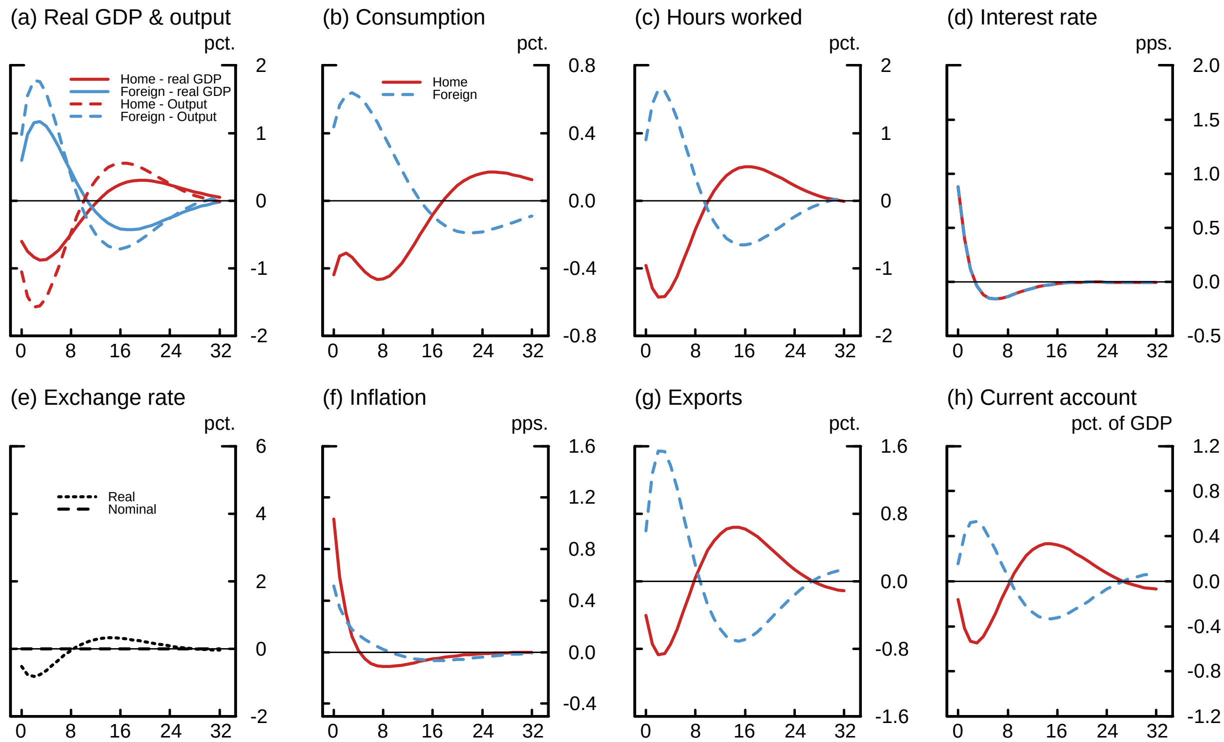
As shown in panels (a) and (g), the near-term depreciation of the real exchange rate leads to an export-driven boom—in terms of real GDP—in the home country. The foreign country, by contrast, experiences a mild recession, according to this metric. However, looking at the responses of real GDP is somewhat misleading. Because firms in the home country respond to the shock by raising prices and are facing downward-sloping demand curves, real output (i.e., production) must fall, which is indeed the case, according to panel (a). Consequently, hours worked in the home country decline (panel (c)), while exactly the opposite happens in the foreign country. In the home country, therefore, the financial shock has real consequences in terms of the foregone output and employment, though the effects are relatively small, given the assumed severity and persistence of financial distress.

As shown by Figure 6, the pattern of international macroeconomic adjustment in response to such a shock looks dramatically different when the two countries are in a monetary union. According to panel (a), real GDP and real output in the home country both decline notably in the immediate aftermath of the shock. The fall in output when the two countries share a common currency and monetary policy is about three times as large as that induced by the same shock in the case of floating exchange rates. Panels (g) and (h) show that exports from the home country drop and the current account deficit worsens in the near term, dynamics that stand in stark contrast to those shown in the corresponding panels of Figure 5. Even more strikingly, the economic downturn in the home country is accompanied by a robust boom in the foreign country: real GDP, output, consumption, hours worked, and exports all increase significantly, and the foreign country registers a sizable current account surplus.

Panels (d)–(f) of Figure 6 illustrate the economic forces responsible for this stark difference in the international macroeconomic dynamics across the two currency regimes. First, note that the behavior of inflation in the two countries when they are in a monetary union (panel (f) of Figure 6)

[24] Note that the nominal exchange rate shown in panel (e) appears to return to its steady-state value in the long run. However, this is simply a coincidence because our New Keynesian framework does not have a prediction for the level of nominal exchange rate, just as it does not have one for the price level. In all simulations, we assume that the initial value of the nominal exchange rate is equal to one, an arbitrary but innocuous assumption, as only changes in the nominal exchange rate are a well-defined concept in our model.

FIGURE 6 – An Asymmetric Financial Shock: Monetary Union




NOTE: The panels of the figure depict the model-implied responses of selected variables to an adverse financial shock in the home country in period 0 (see the text for details). Unless noted otherwise, the solid lines show responses of variables in the home country, while the dashed lines show those of the foreign country. Exchange rates (panel (e)) are expressed as home currency relative to foreign currency.

is quite similar to that under floating exchange rates (panel (f) of Figure 5). This result reflects the fact that regardless of the currency regime, firms in the home country, when confronted with a tightening of financial conditions, have a strong incentive to raise prices compared with their foreign counterparts.

What differs between the two institutional arrangements is the behavior of the real exchange rate. To understand this difference, note that with floating exchange rates, the international bond-holding conditions (38) and (39) imply the following no-arbitrage condition:

$$\tau(b_{h,t+1} - b_{f,t+1}) = \mathbb{E}_t \left[ m_{t,t+1} \left( \frac{R_t}{\pi_{t+1}} - \frac{q_{t+1}}{q_t} \frac{R_t^*}{\pi_{t+1}^*} \right) \right]. \tag{54}$$

In equilibrium with a relatively small portfolio rebalancing costs, the left side of equation (54) is close to zero up to a first-order approximation, which means that $R_t/\pi_{t+1} - (q_{t+1}/q_t)(R_t^*/\pi_{t+1}^*) \approx 0$ in expectation. Given the difference in the behavior of inflation and common monetary policy (see panels (d) and (f) of Figure 6), it is clear that the real interest rate in the home country is lower than in the foreign country in a monetary union. In the absence of capital controls, the real exchange rate should therefore *appreciate* over time (i.e., $q_{t+1}/q_t < 1$), so as to prevent the flight of capital from the home country. This, however, requires the nominal exchange rate to *depreciate* immediately.

With the two countries sharing a common currency, such an adjustment is, of course, not possible. In a monetary union, the bond market efficiency conditions (49) and (50) impose no restrictions on dynamics of the real exchange rate. The real interest rate differential engendered by the difference in the behavior of inflation between the two countries does not have to be compensated by the expected changes in the nominal exchange rate. Adding the efficiency conditions (49) and (50) and imposing the bond market clearing condition yields

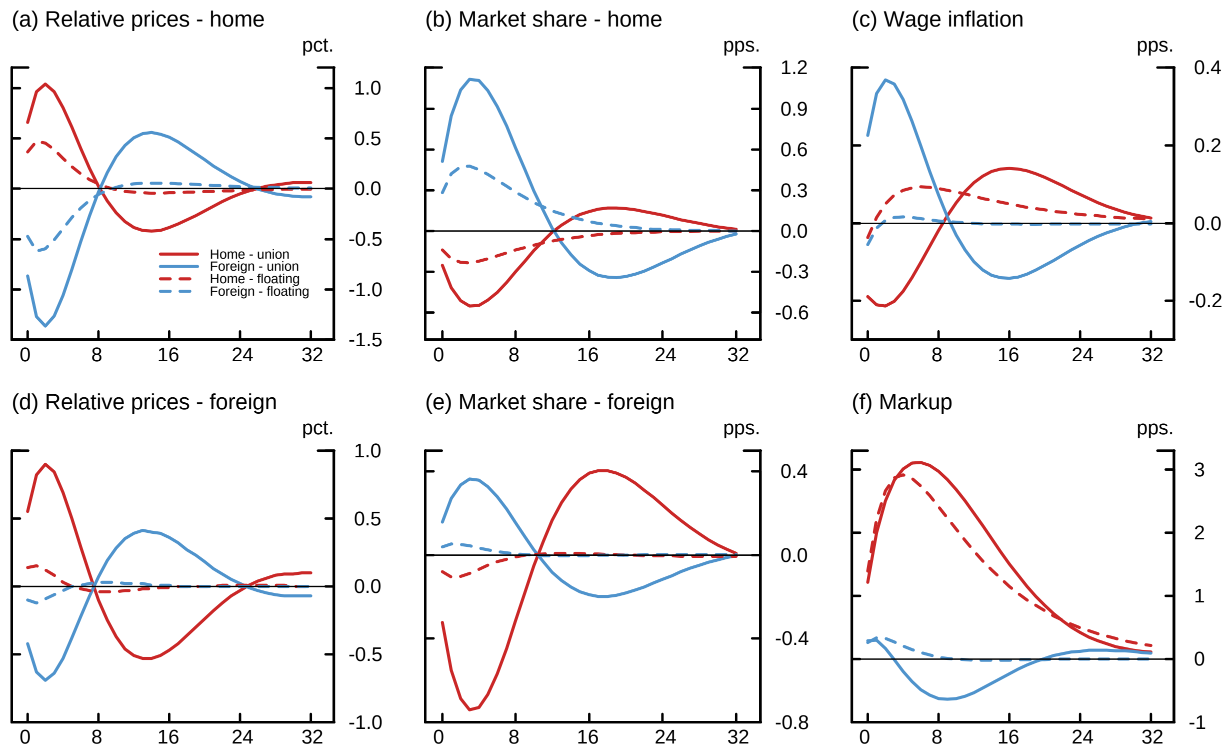
$$1 = \mathbb{E}_t\left[\frac{1}{2}\left(\frac{m_{t,t+1}}{\pi_{t+1}} + \frac{m^*_{t,t+1}}{\pi^*_{t+1}}\right)R^U_t\right], \tag{55}$$

which states the union-wide policy rate $R^U_t$ should be set according to the *average* of the fundamentals of the two economies, regardless of the coefficients of the monetary policy reaction function. As a result, differences in inflation rates translate directly into movements in the real exchange rate. Because firms in the home country optimally choose higher relative prices in response to the tightening of financial conditions, the real exchange rate appreciates substantially and exports of the home country firms drop sharply, as does real GDP and production. In comparison, the decline in consumption is noticeably less severe because international borrowing—while subject to costly portfolio rebalancing—allows consumers in the home country to smooth the effects of the financial shock to a certain extent. The foreign economic boom is simply a mirror image of the home country's economic plight and is reminiscent of the dichotomy in economic outcomes between the eurozone core and periphery during the recent financial crisis.

As noted above, despite the significantly worse economic performance of the home country in a monetary union, inflation dynamics in both countries in those circumstances are very similar to those under floating exchange rates. The similar behavior of home and foreign inflation in response to an asymmetric financial shock across these two very different monetary frameworks reflects the offsetting effects of the international price war, sparked by the interaction of customer markets and financial frictions. As shown in panels (a) and (d) of Figure 7, the financial shock in the home country induces a significant dispersion in relative prices in both countries, regardless of the currency regime. The increase in the cost of external finance causes home country firms to raise relative prices in both their domestic and export markets. Foreign country firms, in contrast, optimally follow the opposite strategy and lower relative prices in both markets in order to steal market share from their financially constrained home country counterparts (panels (b) and (e)). Gauging by the degree of endogenous dispersion in relative prices, this "predatory" price war is noticeably more intense when the two countries share a common currency, as home country firms are unable to rely on the depreciation of their currency to improve their internal liquidity positions.

In a monetary union and with floating exchange rates, foreign firms cut export prices—that is, prices they charge in the home country—notably more than their domestic prices. This pricing

Figure 7 – An Asymmetric Financial Shock and International Price War




Note: The panels of the figure depict the model-implied responses of selected variables to an adverse financial shock in the home country in period 0 (see the text for details). The solid lines show responses when the two countries are in a monetary union, while the dashed lines show responses under floating exchange rates.

behavior lies at the heart of the interaction between customer markets and financial frictions, which provides an especially strong incentive to steal market share from competitors in financial distress. As a result, the large increase in relative prices by firms in the home country (domestic prices in the home country) is partially offset by an aggressive price discount offered by foreign firms (import prices in the home country), which attenuates the upward pressure on the overall inflation in the home country arising from the financial "cost-push" shock. These opposing forces also result in aggregate inflation dynamics that in both countries differ very little between the two institutional frameworks. And lastly, financial distress leads to a strongly countercyclical markup in the home country, irrespective of the currency regime (panel (f)). The model-implied dynamics of markups in the home country in response to a financial shock are thus consistent with the behavior of the price markups in the eurozone periphery during the recent financial crisis and its aftermath shown in Figure 4.[25]

Another important prediction of our model concerns the relative behavior of market shares (panels (b) and (e) in Figure 7). According to these simulations, foreign firms, by undercutting

[25]It is also worth noting that the deleterious effects of a monetary union on the volatilities of macroeconomic variables are not confined to asymmetric financial shocks. As shown in Appendix D, the same conclusion emerges in the case when a home country is hit by an adverse technology shock.

prices charged by their home country counterparts, significantly expand market shares in both the domestic and export markets. To examine whether such patterns are consistent with the data, we use the Eurostat trade data to construct relative import shares by major product groups defined on the basis of Broad Economic Categories (BECs)—our proxy for market shares in different industries—for the eurozone core and periphery.[26]

Specifically, for each eurozone region—that is, core (C) and periphery (P)—and BEC (indexed by $k$), we calculate an import share as

$$\text{ImpShr}^k_{t,P\to C} = \frac{\text{Imp}^k_{t,P\to C}}{\text{Imp}^k_{t,C}} \quad \text{and} \quad \text{ImpShr}^k_{t,C\to P} = \frac{\text{Imp}^k_{t,C\to P}}{\text{Imp}^k_{t,P}},$$

where $\text{Imp}^k_{t,P\to C}$ is the value of imports (in BEC $k$) by the core countries from the periphery ($P \to C$) in year $t$, $\text{Imp}^k_{t,C\to P}$ is the value of imports (in BEC $k$) by the periphery countries from the core ($C \to P$) over the same period, and $\text{Imp}^k_{t,C}$ and $\text{Imp}^k_{t,P}$ denote total imports (in BEC $k$) by the core and periphery countries, respectively. We use the relative growth in import shares between the periphery and core, defined as

$$\Delta \ln \text{RelImpShr}^k_t = \Delta \ln \text{ImpShr}^k_{t,P\to C} - \Delta \ln \text{ImpShr}^k_{t,C\to P},$$

as a proxy for changes in relative market shares between the two regions.

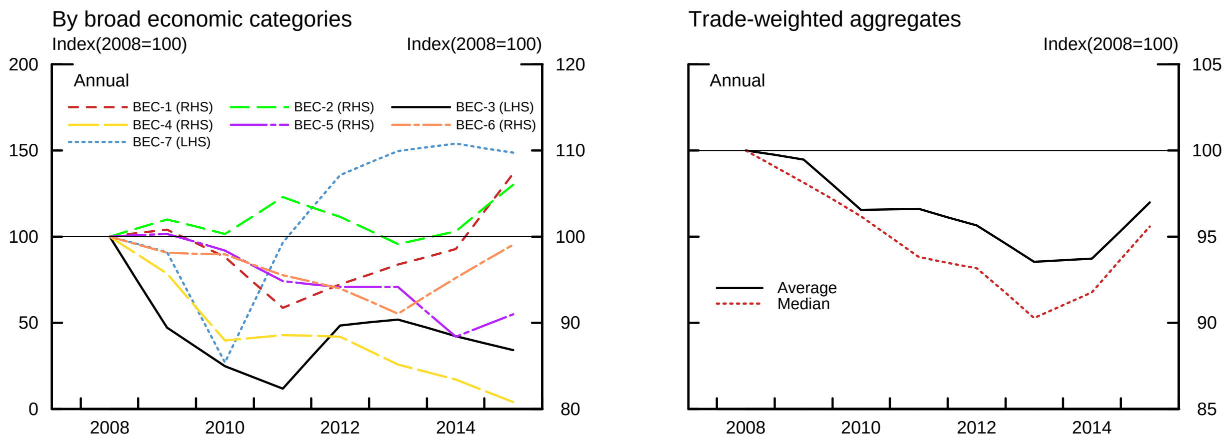
The left panel of Figure 8 shows the cumulative relative growth in import shares between the periphery and core for the seven BECs. With the exception of BEC-2 (Industrial Supplies)—a category of goods for which the relative import share between the eurozone periphery and core was about unchanged—the relative import shares for all other categories declined markedly during the crisis. Although in BEC-7 (Goods, not elsewhere classified), the sharp drop in the relative import share was fairly transient, the relative import shares in the remaining categories registered appreciably more persistent declines.

To gauge the aggregate implications of these trade patterns, the right panel shows the cumulative trade-weighted average and the trade-weighted median of the relative growth in import shares across the seven BECs, using total trade flows between the two regions as weights. Both measures paint the same picture: As the crisis in the euro area unfolded, imports by the periphery countries from the core countries—normalized by the periphery's total imports—declined by considerably more than the imports by the core countries from the periphery, normalized by the total imports of the core countries.[27] Such dynamics in relative import shares are consistent with our model, which predicts that in periods of financial distress, firms in the home country will lose their market share—both at home and abroad—to their financially stronger foreign counterparts.

[26]The seven categories are BEC-1: Food & Beverages; BEC-2: Industrial Supplies; BEC-3: Fuels & Lubricants; BEC-4: Capital Goods (excluding transport equipment); BEC-5: Transport Equipment; BEC-6: Consumer Goods; and BEC-7: Goods, not elsewhere specified.

[27]The aggregate patterns are qualitatively the same if instead of total imports by each region, imports from the periphery and core and vice versa are normalized by the relevant region's nominal GDP.

FIGURE 8 – Relative Import Shares: Periphery and Core Countries (2008–2015)




NOTE: The left panel depicts the behavior of relative import shares between the eurozone periphery in seven broad economic categories (BECs): BEC-1 = Food & Beverages; BEC-2 = Industrial Supplies; BEC-3 = Fuels & Lubricants; BEC-4 = Capital Goods (excluding transport equipment); BEC-5 = Transport Equipment; BEC-6 = Consumer Goods; and BEC-7 = Goods, not elsewhere specified. The right panel depicts the cumulative trade-weighted average and the trade-weighted median of the relative growth in import shares across the seven BECs, using total trade flows between the two regions as weights (see the text for details).
SOURCE: Eurostat.

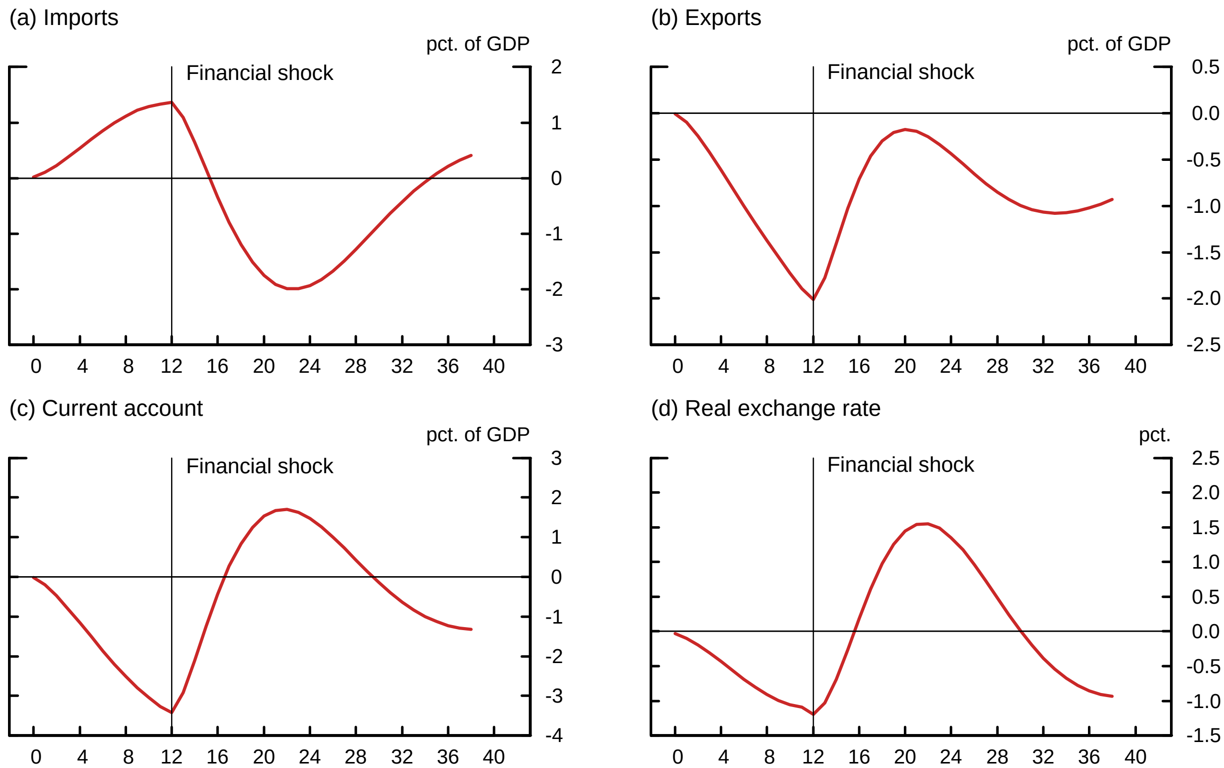
## 5.2 The Boom-Bust Cycle

An aspect of the macroeconomic dynamics shown in Figure 6 that appears at odds with the crisis in the euro area periphery is the fact that following the financial shock, imports to the home country (that is, exports from the foreign country) increase notably (panel (g)), causing a deterioration in the current account deficit of the home country (panel (h)). After about eight quarters, this pattern is reversed, and the home country begins to register an improvement in its external position. The current account deficits in the periphery countries, however, started to improve immediately with the onset of the crisis in 2009 (panel (a) of Figure 1), owing primarily to a sharp decline in imports.

This discrepancy in the timing of external adjustment patterns should not be taken as evidence that the model-implied crisis dynamics are inconsistent with the data. The impulse responses are expressed as deviations from the steady state—that is, our simulations assume that the two economies are at their respective steady states prior to the home country being hit by a shock, a situation that is unlikely to have characterized the euro area on the eve of the crisis. In fact, with our model, it is straightforward to generate external adjustment patterns in the home country that closely resemble those experienced by the eurozone periphery in the period surrounding the sovereign debt crisis.

As noted at the outset, periphery countries borrowed heavily in the years preceding the crisis, primarily to finance domestic consumption and housing investment. As a result, real exchange rates in the eurozone periphery appreciated significantly, eroding these countries' competitiveness. These developments also produced large trade deficits among periphery countries, which in the years

Figure 9 – The Boom-Bust Cycle in the Home Country



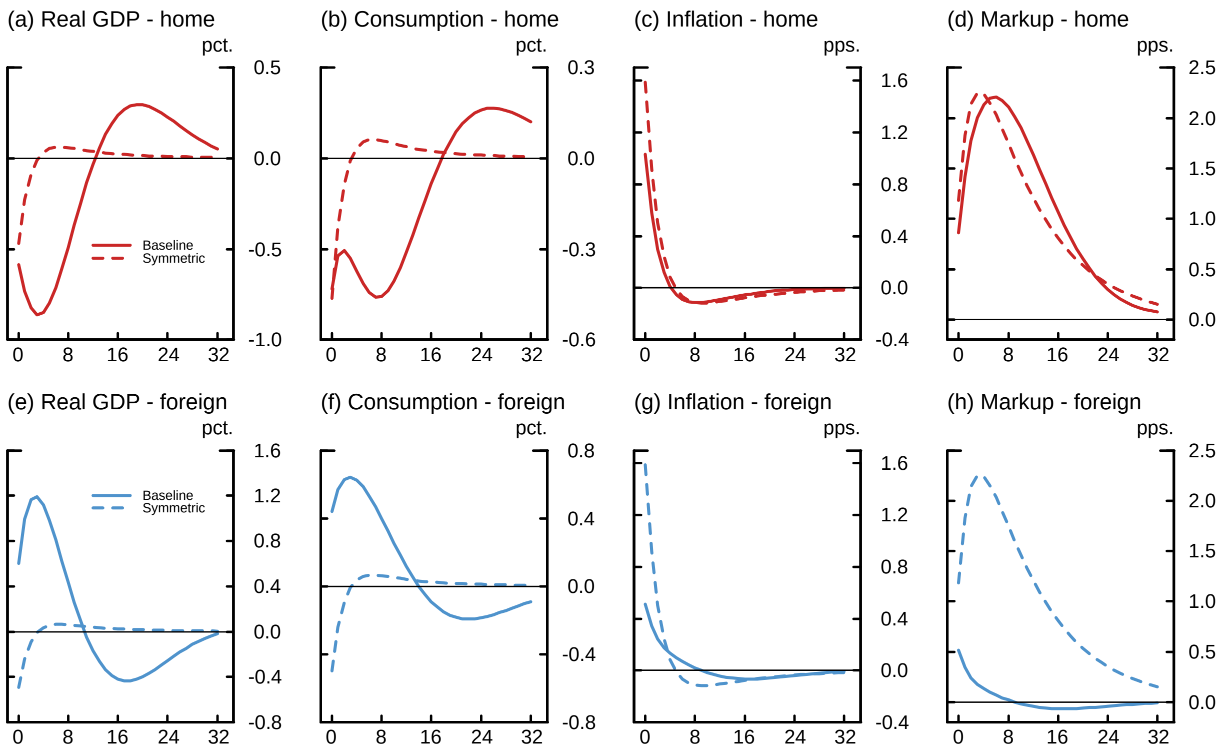

Note: The solid lines depict the model-implied responses of selected variables in the home country, when the country experiences a sequence of positive demand shocks in periods $0, \ldots, 11$ and in period 12 is hit by a financial shock (see the text for details). The real exchange rate is expressed as home currency relative to foreign currency.

leading to the crisis were easily financed by foreign capital inflows, facilitated by the convergence in domestic interest rates across the euro area.

To capture the buoyant economic sentiment that prevailed in the eurozone periphery prior to the crisis, we consider a simulation, whereby the home country first experiences a sequence of gradually increasing positive demand shocks $\omega_t$—the pre-crisis economic boom—which is then followed by an asymmetric financial shock. In implementing this scenario, we assume a sequence of demand shocks in periods $0, 1, \ldots, 11$, such that $\omega_t$ gradually increases to 5 percent of its steady-state value; in period 12, we hit the home country with a large and persistent financial shock, which increases the equity dilution costs $\varphi_t$ from 0.2 to 0.5 upon impact.

As shown in Figure 9, this sequence of events generates external adjustment patterns in the home country that correspond closely to those experienced in the eurozone periphery in the period surrounding the crisis. In the years immediately preceding the financial shock, imports-to-GDP increase notably (panel (a)), while exports-to-GDP fall (panel (b)), trade dynamics that are consistent with the erosion in the home country's competitiveness as evidenced by the appreciation of the real exchange rate during this period (panel (d)). When the home country is hit by the financial shock, these patterns are abruptly reversed: With imports falling and exports rising, the

Figure 10 – Financial Heterogeneity and Monetary Union




Note: The solid lines depict the model-implied responses of selected variables to an adverse financial shock in the home country in period 0 under the baseline calibration of the model. The dashed lines depict the corresponding responses under the alternative calibration of homogeneous financial capacity and when both countries are hit by an adverse financial shock in period 0. Panels (a)–(d) depict responses of selected variables of the home country, while panels (e)–(h) depict the corresponding variables of the foreign country; see the text for details.

current account deficit—which reached about 3 percent of GDP at the eve of the crisis—begins to shrink immediately (panel (c)). Thus with an economically plausible sequence of shocks, the model is able replicate the kind of current account reversal dynamics experienced by the eurozone periphery during the crisis.

## 5.3 Financial Heterogeneity as a Propagation Mechanism

In a monetary union, differences in the degree of financial distortions between the home and foreign countries play a critical role in propagating the effects of a financial shock in the home country. When firms in the home country experience a tightening of financial conditions, the relative financial strength of foreign firms allows them to lower markups in an effort to drive out their home country competitors from both the domestic and foreign markets. A question that then emerges is whether the model can generate the same degree of endogenous propagation if the two countries were identical in terms of financial capacity and subjected to the same degree of financial distress.

To answer this question, we consider an alternative calibration of the model, in which firms in both countries face the same degree of financial frictions in the steady state ($\varphi = \varphi^* = 0.2$) and both economies are perturbed by a financial shock of the type described above ($\epsilon_t = \epsilon_t^* > 0$). The dashed lines in Figure 10 show the impulse responses of selected variables under this alternative calibration, while the solid lines show the corresponding responses under our baseline calibration (see Figure 6).

These simulation clearly indicate that the home country would prefer the alternative economic environment, in which firms in the foreign country also have limited financial capacity and are subject to the same tightening of financial conditions. This result reflects the fact that under the alternative calibration, firms in the foreign country are also experiencing a liquidity squeeze and are thus unable to maintain, in relative terms, lower markups, which exacerbates the downturn in the home country in the baseline case. In response to the deterioration in their own financial conditions, foreign firms raise markups significantly to maintain current cashflows (panels (d) and (h)), and foreign inflation dynamics mirror those in the home country (panels (c) and (g)). Consequently, there is no movement in the real exchange rate, and the foreign country undergoes the same contraction in economic activity as the home country (panels (a) and (e)). This result stands in stark contrast to the baseline case in which the foreign country experiences an export-driven boom, while the home country falls into a recession.

# 6 Welfare Analysis and Policy Implications

## 6.1 Welfare Consequences of a Monetary Union

Simulations in Section 5 show that when financial markets of countries in a monetary union are subject to a differing degree of distortions, the financially weaker members of the union undergo a much more severe recession when hit by an exogenous shock, compared with a floating exchange rate regime. In this section, we examine formally the welfare implications of forming a monetary union among countries with different financial capacities.

To highlight the welfare effects of such a political choice, we adopt a calibration strategy, in which we assume that the home and foreign countries are subject to only two types of aggregate shocks: technology shocks ($\epsilon_{A,t}$ and $\epsilon^*_{A,t}$) and financial shocks ($\epsilon_{f,t}$ and $\epsilon^*_{f,t}$). We set the standard deviation of aggregate technology shocks to 1 percent and then calibrate the standard deviation of financial shocks so that they account for 10 times as much of the variance of real GDP of the home country as technology shocks (see Jermann and Quadrini, 2012).[28] To compare welfare across the different currency regimes, we approximate the value functions of the representative households in the two countries up to a second order and report their analytic first moments in Table 4. In addition to reporting the households' welfare, we also calculate the certainty-equivalent changes in consumption (CE), which are required to make the welfare levels of the households in two countries

[28]Recall that our calibration strategy sets the persistence of all exogenous shock processes to 0.9; thus, we set $\rho_A = \rho_f = 0.9$. With financial shocks playing such an outsized role in economic fluctuations, this calibration clearly does not provide the most realistic representation of the two economies. However, our main conclusions are qualitatively the same under alternative calibrations, whereby the business cycles are driven primarily by aggregate technology shocks.

Table 4 – Welfare Consequences of a Monetary Union

| | Welfare | | |
|---|---|---|---|
| | Monetary Union | Floating FX | CE (pct.) |
| Home country | −259.23 | −254.16 | 2.53 |
| Foreign country | −254.05 | −254.26 | −0.11 |
| *Memo*: Joint welfare | −513.28 | −508.42 | . |

Note: CE denotes the certainty-equivalent change in the average consumption per period (holding hours worked constant) that is required to make the representative household in the specified country no worse off when the two countries choose to abandon floating exchange rates and independent monetary policies and form a monetary union.

under the monetary union equal to those under the floating exchange rate regime.

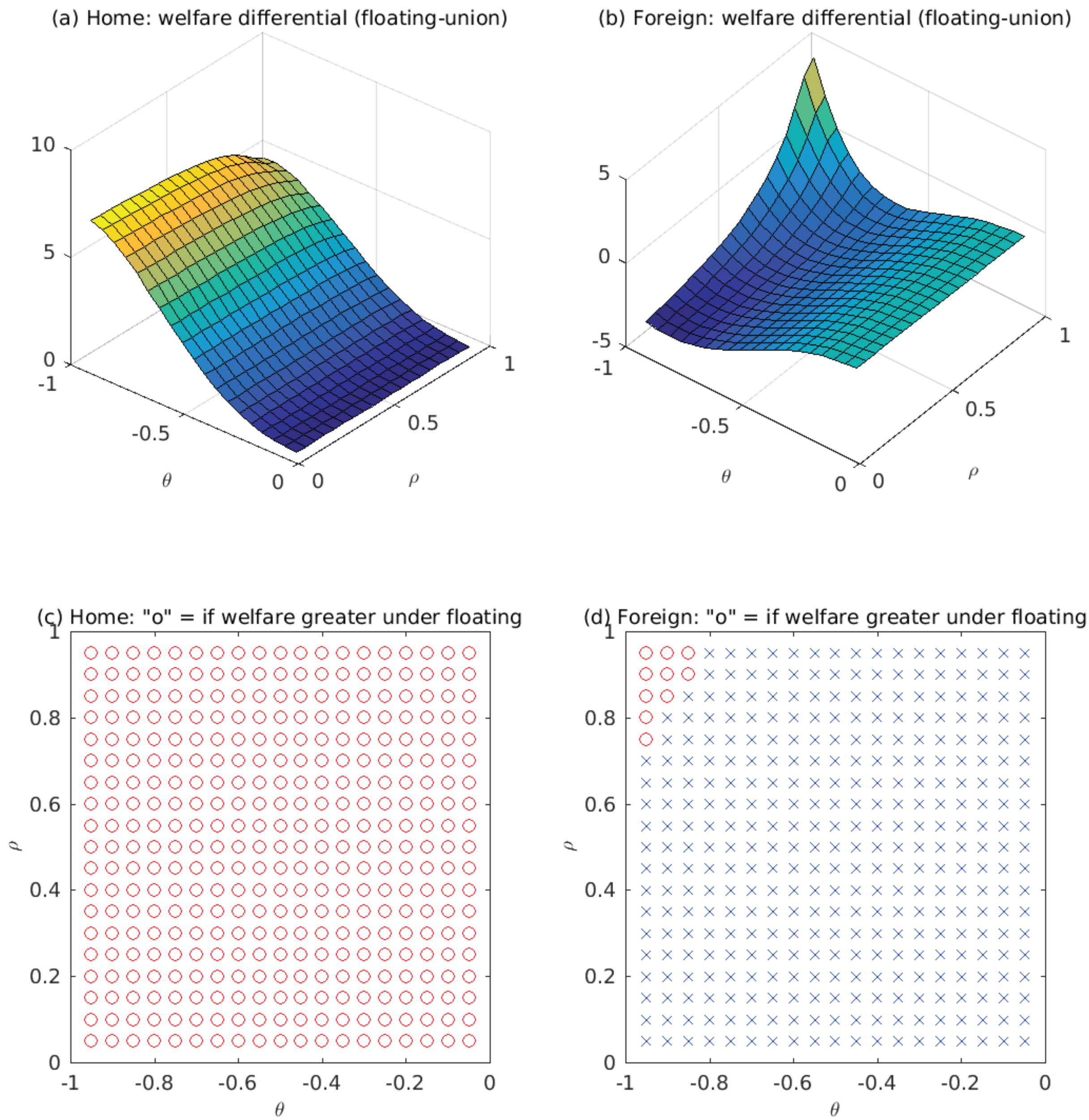
As evidenced by the entries in the table, joining the monetary union results in a significant welfare loss for the home country: The representative household in the home country should be given an increase of 2.5 percent of their steady-state consumption level per quarter in order to be as well off in the monetary union as they were in the floating exchange rate regime. In contrast, the representative household in the foreign country is notably better off in the monetary union, given that typical estimates of the welfare cost of business cycles are on the order of 0.2 percent, according to this metric.

The result that the welfare of the foreign country is higher in the monetary union than with floating exchange rates runs counter to the conventional view in the international finance and macroeconomics literature. This view, however, does not take into account the role that heterogeneity in financial capacities of countries joining the monetary union plays in incentivizing firms to compete for market share by engaging in a "predatory pricing" behavior. When a country with ample financial capacity forms a monetary union with a country with heavily distorted financial markets, the latter is highly vulnerable to the beggar-thy-neighbor pricing policies of firms in the former country, especially in periods of financial distress. This is the main reason why in our model independent monetary policy is such a valuable macroeconomic stabilization tool for the financially weak country and why the welfare of the home country is lower in the monetary union than with floating exchange rates.

Figure 11 explores the robustness of this result across the different combinations of parameters $\theta$ and $\rho$, which govern the strength and persistence of the customer-market relationships. Panels (a) and (b) show the change in welfare for the home and foreign countries, respectively, if the two countries dissolve the union and adopt their own currencies and independent monetary policies. Panels (c) and (d) map these changes in welfare into a binary variable, where the symbol "o" indicates that the welfare of the representative household is greater under a floating exchange rate regime, while the symbol "x" indicates higher welfare when the two countries are in a monetary union.

According to panels (a) and (c), the welfare of the home country improves with the exit from the monetary union for all configurations of the parameters $\theta$ and $\rho$. As shown in panels (b)

Figure 11 – Welfare Gains and Losses from Dissolving the Monetary Union




Note: Panels (a) and (b) show the change in welfare of the representative household—over the relevant $(\theta, \rho)$-space—in the home and foreign countries, respectively, in the case that the two countries dissolve the monetary union and return to a floating exchange rate regime. Panels (c) and (d) map the welfare differential from panels (a) and (b) into a binary variable, with the symbol "o" indicating that the welfare of the representative household is greater under a floating exchange rate regime and with the symbol "x" indicating higher welfare when the two countries are in a monetary union.

and (d), by contrast, the welfare of the foreign country is greater in the monetary union across most of the $(\theta, \rho)$-space, the exception being a small region of the parameter space characterized by a very strong and persistent deep-habit mechanism (roughly $\theta < -0.9$ and $\rho > 0.9$). Thus, the welfare implications of a monetary union under our baseline calibration shown in Table 4 are not a knife-edged result, as they are robust for most of the combinations of the parameters $\theta$ and $\rho$. In general, the cost of adopting a common currency and relinquishing independent monetary policy is for the foreign country more than compensated by the loss of independent monetary policy in the home country, whereas the exactly opposite is true for the home country.

The finding that both countries are better off by dissolving the union in an environment where customer-market considerations (in terms of both $\theta$ and $\rho$) play a very powerful role in the firms' pricing decisions may appear counterintuitive at first glance—after all, it is precisely in this region of the parameter space that foreign firms should benefit the most from undercutting their financially weaker home country competitors and stealing their market share. Figure 12 sheds light on this puzzling result. Panels (a) and (b) show the volatility surface of consumption in the foreign country under the two currency regimes over the relevant $(\theta, \rho)$-space, while panels (c) and (d) show the corresponding volatility surfaces of the hours worked. With very strong and persistent deep habits, the volatility of both consumption and hours worked in the foreign country increases dramatically under both currency regimes. This endogenous increase in macroeconomic volatility, however, is much more pronounced when the two countries share a common currency (panels (a) and (c)). The excessive volatility of foreign aggregate demand in an environment where very strong customer-market considerations interact with financial frictions leads to lower welfare because common, union-wide monetary policy is relatively ineffective in stabilizing the spillover effects of financial shocks originating in the home country.

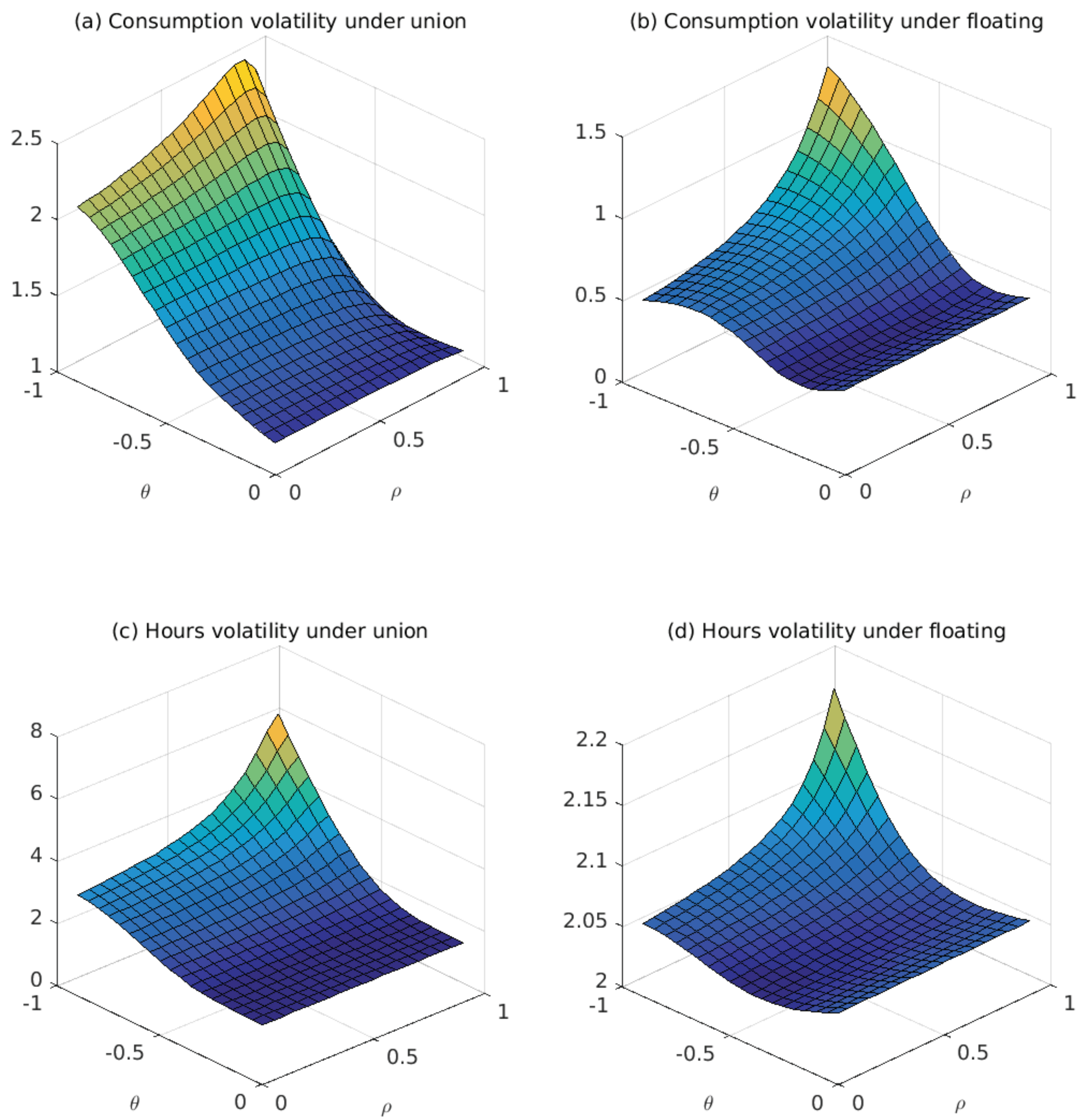
## 6.2 Policy Implications

### 6.2.1 Complete Risk Sharing: A Fiscal Union

The above analysis shows that when countries with heterogeneous financial capacities—and whose firms operate in customer markets—form a monetary union, a country with the most severely distorted financial markets may experience a significant loss of welfare. Absent political considerations that lie beyond the scope of our model, it is difficult to understand why the European periphery—a group of countries with underdeveloped and inefficient financial markets—would relinquish their independent monetary policy and join a monetary union with countries sharing a notably deeper and more sophisticated financial system, especially a union with rather limited labor mobility. The fact, however, is that the Economic and Monetary Union, whose aim is a full integration of the European economies, continues to exist and exiting the monetary union is proving to be a formidable political and economic challenge.

Taking the existence of the eurozone as de facto given, we now examine whether there exist policies that can remedy the problems brought about the one-size-fits-all monetary policy, while preserving the common currency. A standard approach to analyze this question, which also provides a benchmark against which to judge the efficacy of other policy proposals, is by assuming that the households of the two countries in the union can trade a full set of state-contingent bonds—that

FIGURE 12 – Macroeconomic Volatility in the Foreign Country

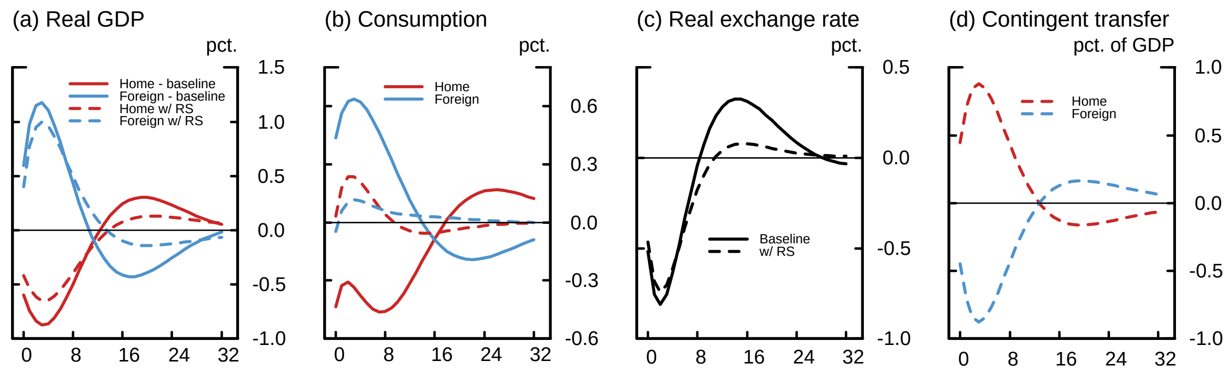



NOTE: Panels (a) and (c) show the volatility of consumption in the foreign country—over the relevant $(\theta, \rho)$-space—in the monetary union and under flexible exchange rates, respectively, while panels (b) and (d) show the corresponding volatilities of hours worked. Volatility is measured by the coefficient of variation and expressed in percent.

is, a complete risk-sharing arrangement or, equivalently, a fiscal union.

The dashed lines in Figure 13 trace out dynamics of the key macroeconomic aggregates in response to an asymmetric financial shock in the home country under such a risk-sharing arrangement. To conserve space, we only show responses of real GDP and consumption (panels (a)

Figure 13 – An Asymmetric Financial Shock: Monetary Union with Risk Sharing




Note: In Panels (a) and (b), the solid lines depict the model-implied responses of selected variables to an adverse financial shock in the home country in period 0 under the baseline calibration of the model (see Figure 6); the dashed lines depict the corresponding responses under the assumption of complete risk sharing between the two countries. Panel (c) depicts the model-implied responses of the real exchange rate (expressed as home currency relative to foreign currency) under the two institutional frameworks, while panel (d) depicts the contingent transfers between the two countries under the risk-sharing arrangement.

and (b)), the real exchange rate (panel (c)), and the state-contingent transfers between the two countries (panel (d)); for comparison purposes, the solid lines in panels (a)–(c) replicate responses from Figure 6, which considers the same experiment, but without complete risk sharing among the union members.

It is well-known that in standard open-economy macro models, the assumption of complete risk sharing has little effect on the behavior of endogenous quantities, including the real exchange rate (see Steinsson, 2008). In our model, by contrast, this is not generally the case. As shown by the consumption trajectories in panel (b), allowing households to trade a full set of state-contingent bonds effectively spreads the deleterious effects of the asymmetric financial shock across the two countries. This is achieved primarily through cross-border wealth transfers, rather than through the changes in production shares, as the dynamics of real GDP are very similar across the two allocations (panel (d)).

In a monetary union with complete risk sharing, production is organized so as to equalize the marginal costs across the two countries. To achieve risk sharing, the proceeds of production are then redistributed between the households of the two countries through the trading of state-contingent bonds. This results in a substantial amount of wealth transfer between the two countries—at the peak of the financial crisis in the home country, the foreign country is transferring almost 1 percent of its real economic output to the home country (panel (d)). Finally, note that the fiscal union does not fully eliminate fluctuations in the real exchange rate, although its volatility is diminished somewhat in those circumstances.

Table 5 compares the representative households' welfare—with and without risk sharing—when the two countries share a common currency. Under our baseline calibration shown in panel (a), both countries can potentially reap large welfare gains by forming a fiscal union, according to

Table 5 – Welfare Consequences of a Fiscal Union

| | Welfare in the Monetary Union | | |
|---|---|---|---|
| | w/o Risk Sharing | w/ Risk Sharing | CE (pct.) |
| (a) $\theta = -0.86$, $\rho = 0.85$, $\phi^* = \phi$ | | | |
| Home country | −259.23 | −257.61 | 0.79 |
| Foreign country | −254.05 | −253.15 | 0.45 |
| *Memo*: Joint welfare | −513.28 | −510.76 | . |
| (b) $\theta = -0.95$, $\rho = 0.95$, $\phi^* = \phi$ | | | |
| Home country | −283.64 | −279.86 | 1.71 |
| Foreign country | −278.47 | −274.94 | 1.66 |
| *Memo*: Joint welfare | −562.11 | −554.80 | . |
| (c) $\theta = -0.86$, $\rho = 0.85$, $\phi^* = 0.9\phi$ | | | |
| Home country | −261.00 | −254.69 | 3.13 |
| Foreign country | −248.73 | −249.81 | −0.56 |
| *Memo*: Joint welfare | −509.73 | −504.50 | . |

Note: CE denotes the certainty-equivalent change in the average consumption per period (holding hours worked constant) that is required to make the representative household in the specified country no worse off when the two countries in the monetary union abandon a complete risk-sharing arrangement. In panels (a) and (b), $\phi^* = \phi = 0.1$, as in our baseline calibration.

the certainty-equivalent changes in consumption, which are required to make the welfare levels of the households in the monetary union with risk sharing equal to those in the union without risk sharing. As shown in panel (b), the potential welfare gains from forming a fiscal union are even larger with very strong and persistent deep habits, an environment where the interaction between customer markets and financial distortions leads to an especially powerful propagation of financial shocks when the two countries share a common currency. Recall that this configuration of $\theta$ and $\rho$ lies in the region of parameter space that is associated with an especially elevated macroeconomic volatility in the foreign country (see Figure 12). Thus in these more extreme circumstances, the macroeconomic stabilization properties of a fiscal union may also confer significant benefits on the financially strong members of the union.

The calibration in panel (c), by contrast, indicates that the formation of a fiscal union when the two countries already share a common currency—a progression envisioned by the European political establishment—is not necessarily Pareto improving. This calibration differs from our baseline in only one dimension: We assume that foreign firms are slightly more efficient—in terms of fixed operating costs—than their domestic counterparts; that is, $\phi^* = 0.9\phi$, where $\phi = 0.1$, our baseline value. In this case, the welfare of the foreign country is significantly lower with complete risk sharing, according to the certainty-equivalent consumption metric.

A useful way to think about this result is to interpret the fixed operating costs as capturing the quality of the firms' balance sheets. That is, these costs can include long-term debt payments, a coupon payment to perpetual bond holders and can thus capture the possibility of a debt overhang.

Under this interpretation, the country with high fixed operating costs can be viewed as highly indebted, as is the eurozone periphery; for instance, the debt-to-GDP ratio averaged 130 percent in the eurozone periphery in 2013, about 55 percentage points higher than the corresponding average for the core. In our model, this differential translates into $\phi^* = 0.6\phi$, and our welfare calculations imply that the representative foreign household would see its steady-state consumption level decline 7 percent per quarter in the fiscal union, compared with a situation in which the two countries only share a common currency. By the same token, the representative home country household would see an increase of 9 percent in certainty-equivalent consumption were the two countries form a fiscal union. While admittedly crude, these welfare calculations underscore the political difficulties of forming a fiscal union, as residents of the foreign country are unlikely to agree with the size of such wealth transfers.

### 6.3 Fiscal Devaluations

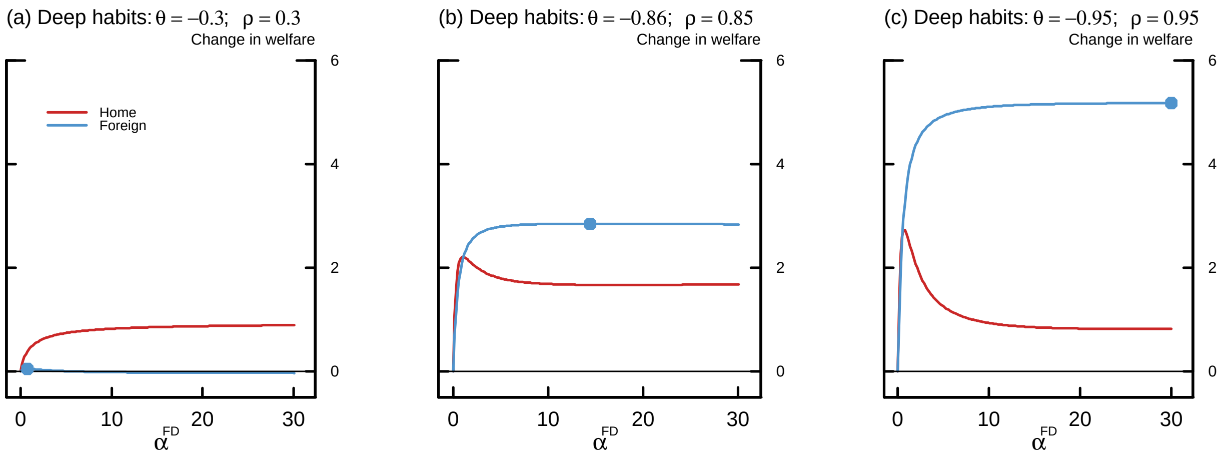
Given the likely political obstacles and the lengthy process of forming a fiscal union, we now turn to a potentially more feasible and frequently advocated policy option in the context of the European sovereign debt crisis: a budget-neutral fiscal devaluation by the periphery countries. As emphasized by Adao et al. (2009) and Farhi et al. (2014), the aim of such policies is to use a mix of fiscal measures to replicate the effects of a nominal exchange rate depreciation in a fixed exchange rate system, in order to improve competitiveness and support the rebalancing of external accounts. Fiscal measures can, for example, include a combination of import tariffs and export subsidies or a shift from labor to consumption taxation.[29]

A particular form of fiscal devaluation that received a lot of attention in policy circles during the crisis involved the following (budget neutral) combination of fiscal measures in the periphery: a reduction in employers' social security contributions, coupled with an increase in the VAT rate (see Puglisi, 2014).[30] Within the eurozone, however, the non-cooperative nature of a unilateral fiscal devaluation by the periphery could potentially have negative spillover effects on the core countries, especially if the latter viewed the adoption of the euro as a way to avoid the manipulation of nominal exchanges rates by the monetary authorities in the periphery. In addition, any short-term improvements in external competitiveness resulting from a fiscal devaluation will be diminished as more periphery countries engage in this policy simultaneously. Hence a natural question to

[29] Farhi et al. (2014) provide an in-depth analysis of various policy mixes that can under various asset market conditions replicate the effects of a given size of nominal exchange rate depreciation.

[30] A reduction in employers' social security contributions would directly lower labor costs of firms in the periphery countries. If lower labor costs were to be passed through to producer prices—and if wages were not to fall—domestically produced goods would become less expensive, which would reduce relative export prices and induce a depreciation of the real effective exchange rate vis-à-vis the core. At the same time, the increase in the VAT rate would not be fully offset by the reduction in labor costs because only final consumption would be taxed at a higher rate. With a decline in relative export prices and an increase in relative import prices, the domestic demand for imports would fall. Because consumer prices for domestically produced goods would have remained essentially unchanged—as the VAT hike and a cut in employers' social security contributions more or less offset each other—the increase in the VAT rate would have fallen primarily on imports, causing a shift towards domestic production. In effect, such fiscal devaluation would stimulate exports and lower domestic import demand, factors that would in the short run improve external competitiveness of the periphery countries and lead to an improvement in the trade balance.

FIGURE 14 – Fiscal Devaluation in a Monetary Union




NOTE: The lines depict changes in welfare for the home and foreign countries as a function of $\alpha^{FD}$, the parameter governing the size of a unilateral fiscal devaluation by the home country (see the text for details). The "•" symbol marks the value of $\alpha^{FD}$ that maximizes the welfare of the foreign country. Welfare differentials are measured relative to a baseline of no fiscal devaluation—that is, $\alpha^{FD} = 0$.

ask is whether the eurozone periphery can carry out such a fiscal devaluation without the fear of retaliation from the core?

To provide a qualitative insight into this question, we consider a situation, whereby the home country introduces a payroll subsidy ($\varsigma_t^P$) that is financed by a VAT ($\tau_t^V$).[31] With these policies, the marginal revenue of a home country firm selling its product in the domestic market becomes $(1-\tau_t^V)p_{h,t}$, while its marginal labor cost is equal to $(1-\varsigma_t^P)w_t$. The marginal revenue of a foreign firm selling its product in the home country is given by $(1-\tau_t^V)p_{f,t}/q_t$. We assume that the home country firms are not subject to the same VAT in the foreign country and that the foreign country does not retaliate in response to the unilateral adoption of these fiscal measures by the home country. In addition, we assume that the government of the home country uses these fiscal policies to stabilize the economy using the following Taylor-type fiscal rule:

$$\tau_t^V = \frac{\Delta_t}{1+\Delta_t}, \quad \text{where } \Delta_t = -\alpha^{FD} \ln\left(\frac{y_t}{y}\right); \quad (\alpha^{FD} > 0).$$

To pin down the level of the payroll subsidy $\varsigma_t^P$, we impose the following revenue-neutrality constraint:

$$\varsigma_t^P w_t h_t = \tau_t^V (p_{h,t} c_{h,t} + p_{f,t} c_{f,t}),$$

where the left side represents fiscal expenditures due to the payroll subsidy and the right side is the fiscal revenue generated by the VAT. Note that when the home country slips into a recession,

[31] We stress the qualitative nature of this exercise because the effectiveness of a fiscal devaluation depends importantly on a variety of country-specific factors: the degree of price and wage rigidities, the degree of price pass-through, the elasticity of labor supply, the size of the economy, its trade openness, and the share of labor as variable production input.

$\Delta_t > 0$, which makes the export sales of foreign country firms and the domestic sales of the home country firms subject to a VAT rate of $\tau_t^V > 0$. At the same time, the revenue-neutrality constraint implies a payroll subsidy $\varsigma_t^P > 0$, which lowers the marginal labor cost for home country firms to a fraction $1 - \varsigma_t^P$ of the level that prevailed before the implementation of these fiscal measures. We then perform an extensive grid search to find the value of $\alpha^{FD}$ that maximizes the second-order approximation of the value function of the representative household in the foreign country and analyze the effect of such a policy on the welfare of the home country.

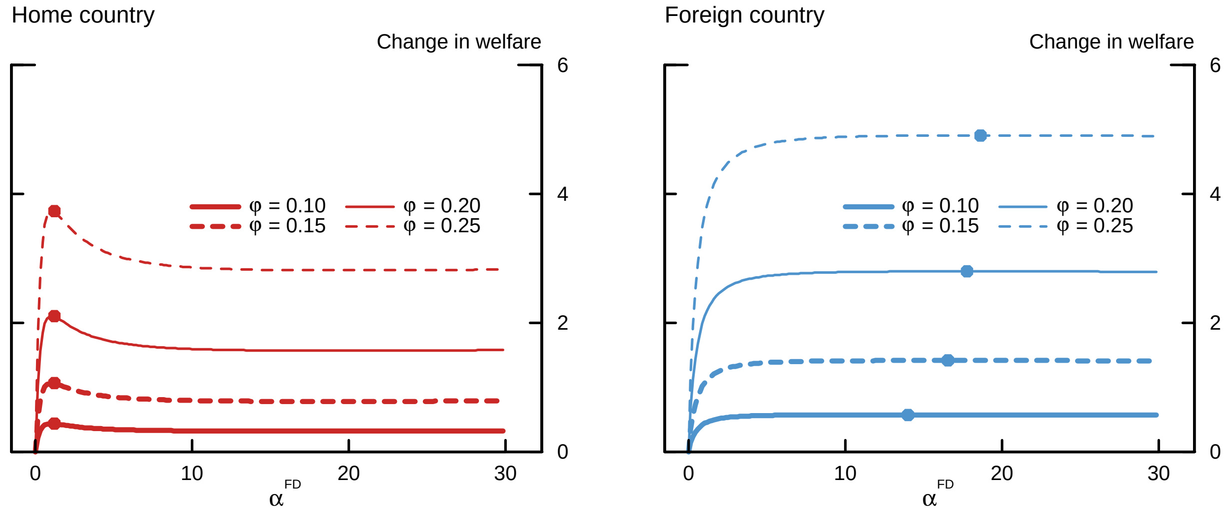
Figure 14 traces out the implications of this exercise on the welfare of the two countries under three different calibrations of the deep-habit mechanism: "weak" deep habits (panel (a)); baseline deep habits (panel (b)); and "strong" deep habits (panel (c)). In general, this analysis indicates that the macroeconomic stabilization benefits brought about a unilateral fiscal devaluation by the home country may be shared by both countries of the union. However, the magnitude of potential welfare gains depends critically, especially for the foreign country, on the strength of customer-market relationships in the two economies.

As shown in panel (a), when the strength and persistence of deep habits are fairly weak, the foreign country realizes only a minuscule welfare gain from such a unilateral fiscal devaluation—in fact, too much fiscal activism may result in a small welfare loss for the foreign country. As indicated by the symbol "•," the foreign welfare reaches the maximum when the home country sets $\alpha^{FD} \approx 1$, and even in this case, the maximal foreign welfare is essentially indistinguishable from the baseline. This result suggests that when foreign firms have relatively little incentive to engage in predatory pricing to capture market share from their home country competitors, it may be difficult for the home country to make a compelling argument for a unilateral fiscal devaluation within the monetary union, even though such a policy is clearly beneficial domestically.

Under our baseline calibration shown in panel (b), by contrast, we reach a very different conclusion. In this case, the fiscal devaluation that maximizes the foreign welfare calls for an aggressive policy setting of $\alpha^{FD} \approx 15$. Even more interestingly, the maximal foreign welfare is attained at the value of $\alpha^{FD}$ that is substantially greater than that preferred by the home country—the latter's welfare is maximized at $\alpha^{FD} \approx 1$. Thus, with a more powerful deep-habit mechanism, the foreign country has a strong incentive to support an aggressive unilateral fiscal devaluation by the home country. The results in panel (c) confirm this conclusion, even for the case when the foreign country is worse off by joining the monetary union (see panels (b) and (d) of Figure 11).

This last result may seem counterintuitive at first glance. Why would the representative foreign household benefit from a unilateral fiscal devaluation by the home country, when such a policy weakens the competitiveness of foreign firms, especially as these firms are aggressively pursuing profit-maximizing pricing strategies that build their customer base? The answer lies in the fact that the interaction of customer markets and financial frictions creates an important pecuniary externality in our model: When foreign firms cut prices at the time when home country firms are experiencing financial distress, they treat aggregate prices and quantities as given—that is, they do not internalize the effects of their pricing behavior on aggregate demand in their own country.

FIGURE 15 – Fiscal Devaluation in a Monetary Union: The Role of Financial Frictions



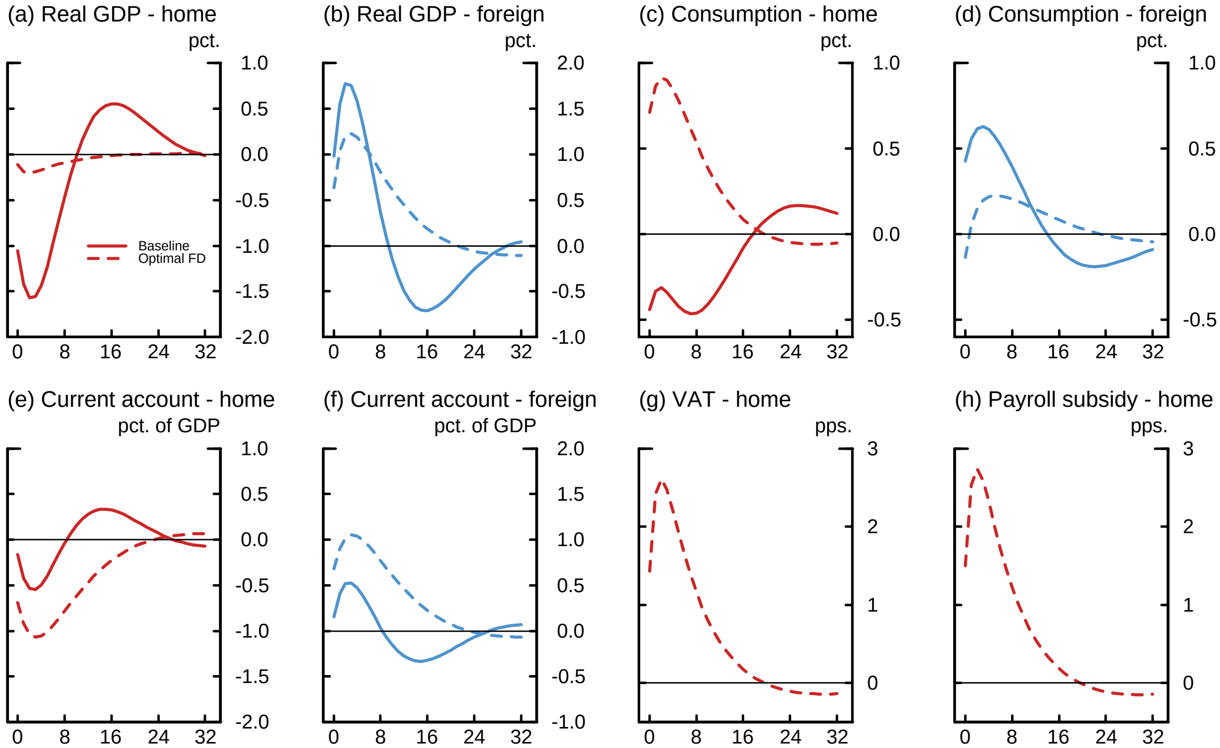

NOTE: The left (right) panel depicts changes in welfare for the home (foreign) country as a function of $\alpha^{FD}$, the parameter governing the size of a unilateral fiscal devaluation by the home country, for different values of the steady-state equity dilution costs $\varphi$ (see the text for details). The "•" symbol in the left panel marks the value of $\alpha^{FD}$ that maximizes the welfare of the home country, while in the right panel, the "•" symbol marks the value of $\alpha^{FD}$ that maximizes the welfare of the foreign country. Welfare differentials are measured relative to a baseline of no fiscal devaluation—that is, $\alpha^{FD} = 0$.

Consequently, foreign firms reduce markups to excessively low levels, behavior that is, of course, individually rational, but one that does not take into account the fact that driving out their home country competitors will significantly boost the volatility of consumption and hours worked in their own country.

In both countries, the magnitude of any potential welfare gains (or losses) arising from a unilateral fiscal devaluation by the home country will also depend on the degree of financial market frictions. Because it is the relatively limited capacity of the financial system in the home country that makes the predatory pricing strategies of foreign firms so profitable, we may expect that the greater the degree of financial market imperfections in the home country, the greater are the potential benefits from pursuing a unilateral fiscal devaluation. The left panel of Figure 15 shows the welfare gains from such a fiscal policy for the home country, as we vary the steady-state value of equity dilution costs $\varphi$, while the right panel displays the same information for the foreign country. As expected, increasing the severity of financial frictions monotonically increases the welfare brought about a unilateral fiscal devaluation for both countries. Moreover, the optimal degree of fiscal activism by the home country—as measured by the coefficient $\alpha^{FD}$ in the fiscal Taylor rule—also increases, as financial distortions become more severe.

It is important to note that the large fiscal policy reaction coefficient $\alpha^{FD}$, which is preferred from the perspective of the foreign country under our baseline calibration of the deep-habit mechanism (see panel (b) of Figure 14), does not necessarily imply large changes in the VAT or payroll subsidy rates. Our posited fiscal policy rule—just like the interest-rate rule governing the conduct of

FIGURE 16 – Macroeconomic Implications of a Fiscal Devaluation in a Monetary Union




NOTE: The solid lines depict the model-implied responses of selected variables to an adverse financial shock in the home country in period 0, when the two countries are in a monetary union (see Figure 6). The dashed lines show the corresponding responses when the home country pursues a unilateral fiscal devaluation, with the fiscal reaction coefficient $\alpha^{FD}$ that maximizes the welfare of the foreign country; see the text for details.

monetary policy—responds to an endogenous variable and to the extent that such fiscal measures are effective in stabilizing the output gap, the effective VAT and payroll subsidy rates will not fluctuate very much in response to changes in the degree of economic slack. A fiscal rule that responds aggressively to the output gap sends a signal to the agents that deviations of real GDP from its potential will be countered by large changes in the tax and subsidy rates. Because such a policy is credible, effective rates do not need to change much in equilibrium and do not result in overly protectionist trade policy.

Figure 16 illustrates this point in the context of our standard asymmetric financial shock in the home country. The solid lines show the impulse responses of selected variables from the simulation reported in Figure 6, a situation where the countries share a common currency and monetary policy. The dotted lines show the corresponding responses in the case when the home country responds to the shock by engaging in a unilateral fiscal devaluation that maximizes the foreign country's welfare. Despite the large reaction coefficient $\alpha^{FD}$ implied by such policy, the home country needs to increase the VAT and payroll subsidy rates only about 2.5 percentage points in response to the financial disturbance (panels (g) and (h)).[32]

Panels (e) and (f) show that this policy actually boosts trade between the two countries, despite the loss of competitiveness by foreign firms engendered by the increase in the domestic VAT rate and the introduction of the payroll subsidy for home country firms. In effect, this relatively modest unilateral fiscal devaluation by the home country is very effective in stabilizing domestic real GDP (panel (a)) and provides significant stimulus for consumption (panel (c)). As a result, the home country's current account deficit actually increases relative to the baseline case. It is worth noting that from the vantage point of the home country, this additional volatility of consumption and current account is likely to be suboptimal, as the fiscal policy rule seeks to optimize the welfare of the foreign country in this simulation.[33]

# 7 Conclusion

In this paper, we present a dynamic, two-country general equilibrium model and use it to analyze the business cycle and welfare consequences of forming a monetary union among countries, whose financial markets are subject to varying degrees of distortions. Because of customer-market considerations, financial shocks significantly affect the firms' pricing decisions, thereby influencing the dynamics of inflation, markups, and market shares—and therefore patterns of external adjustment—across countries. Most importantly, firms from the country with relatively frictionless financial markets have an incentive to expand their market shares at home and abroad by undercutting prices charged by their financially constrained competitors, especially when the latter are experiencing financial distress. Firms located in the country with distorted financial markets, in contrast, increase markups during the crisis in an effort to maintain cashflows, even though doing so means forfeiting some of their market share in the near term.

When applied to the 2009–2013 eurozone crisis, the interaction of customer markets and financial frictions helps explain several phenomena that are difficult to reconcile using conventional open-economy macro models. First, the pricing mechanism implied by this interaction is consistent with our empirical evidence, which shows that the acute tightening of financial conditions in the euro area periphery during this period significantly attenuated the downward pressure on prices arising from the emergence of substantial and long-lasting economic slack. And second, this tightening of financial conditions is strongly associated with a significant increase in price markups in the periphery. Hence our framework can explain why the periphery countries have managed to avoid a debt-deflation spiral in the face of massive and persistent economic slack and how the price war between the core and periphery has impeded the adjustment process through which the latter economies have been trying to regain their external competitiveness.

[32]We assume that both rates are equal to zero in the steady state.

[33]To conserve space, we do not show the results for the case when the fiscal policy rule seeks to maximize the welfare of the home country. These results, which are available upon request, indeed indicate that the fiscal devaluation reported in Figure 16 generates excessive volatility of domestic consumption and current account, compared with the policy that is optimal from the perspective of the home country.

In our model, the pricing behavior of firms in the core in response to a financial shock in the periphery implies a real exchange rate depreciation vis-à-vis the periphery, which causes an export-driven boom in the core and a deepening of the recession in the periphery. The one-size-fits-all aspect of monetary policy—an inherent feature of a monetary union—is especially ill-suited to address such divergent economic outcomes. According to our simulations, when union members are experiencing different economic conditions, common monetary policy aimed at stabilizing inflation and output fluctuations results in a substantially higher macroeconomic volatility compared with a floating exchange rate regime. This translates into a welfare loss for the union as a whole, with the loss borne entirely by the periphery.

To overcome limitations of common monetary policy, we consider two fiscal policy alternatives: a fiscal union and a unilateral fiscal devaluation by the periphery. We show that a complete risk-sharing arrangement among union members—or equivalently, a fiscal union—can significantly improve welfare in both the core and periphery. A fiscal union, however, requires large state-contingent transfers of wealth between the two regions. Given the likely lack of political appetite for such transfers, we then consider the macroeconomic effects of a unilateral fiscal devaluation by the periphery. Our results indicate that such a policy action offers an effective macroeconomic stabilization tool that, in general, is beneficial even to the core. This finding reflects the fact that when firms in the core cut prices to expand their market shares, they do not internalize the pecuniary externality, whereby driving out their foreign competitors by reducing markups to an excessive degree leads to excessive volatility of aggregate demand in their own country. A distortionary taxation in the form a unilateral fiscal devaluation by the periphery helps firms from the core internalize this externality, leading to an improvement in the union's overall welfare.

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# Appendices – For Online Publication

## A Index of Changes in Business Credit Conditions

This appendix describes the construction of the country-specific indexes of changes in business credit conditions shown in panel (b) of Figure 2 in the main text. The indexes are estimated using the Survey on the Access to Finance of Enterprises (SAFE), a comprehensive semi-annual survey of small and medium-sized enterprises (SMEs) in the European Union. Given the SMEs economic relevance in the European Union, the European Central Bank and the European Commission decided in 2008 to collaborate on a survey that is used to assess developments in financing conditions and credit availability for firms in the union countries. Twice a year (in March and October), thousands of businesses in Europe are contacted and asked about their ease of access to various forms of external finance and about financing conditions.[34]

To construct an indicator of the change in business credit conditions, we focus on the following questions regarding how the availability of external financing in each euro area country has changed over the six-month survey period:

(1) What is currently the most pressing problem your firm is facing?
*Answer:* The fraction of respondents that answered "access to finance."

(2) If you applied and tried to negotiate a *bank loan* (new or renewal, but excluding overdraft and credit lines) over the past six months, did you receive all the financing you requested; receive only part of the financing you requested; refuse to proceed because of unacceptable costs or terms and conditions; or have you not received anything at all?
*Answer:* The fraction of respondents that answered "applied but refused because cost too high" and "applied but was rejected."

(3) If you applied and tried to negotiate *trade credit* over the past six months, did you receive all the financing you requested; receive only part of the financing you requested; refuse to proceed because of unacceptable costs or terms and conditions; or have you not received anything at all?
*Answer:* The fraction of respondents that answered "applied but refused because cost too high" and "applied but was rejected."

(4) Would you say that the availability of *bank loans* (excluding overdrafts and credit lines) has improved, remained unchanged, or deteriorated for your enterprise over the past six months?
*Answer:* The net fraction of respondents that indicated that the availability of bank loans improved.[35]

(5) Would you say that the availability of *trade credit* has improved, remained unchanged, or deteriorated for your enterprise over the past six months?
*Answer:* The net fraction of respondents that indicated that the availability of trade credit improved.

(6) Please indicate whether the following terms and conditions of *bank financing* increased, remained unchanged, or decreased in the past six months?

[34] See http://www.ecb.europa.eu/stats/money/surveys/sme/html/index.en.htmlfordetails.

[35] The net fraction is defined as a proportion of respondents that indicated that the availability of financing has improved less the proportion that indicated that the availability of financing deteriorated over the survey period.

(a) Level of interest rates

(b) Level of the cost of financing other than interest rates, such as charges, fees, commissions

(c) Available size of loan or credit line

(d) Available maturity of the loan

(e) Collateral requirements

*Answer:* The net fractions of respondents that indicated that the above loan terms tightened over the survey period.

It is clear that the above survey questions aim to elicit how credit-supply conditions in each EU country have changed over the survey period. To create a country-specific index of changes in business credit conditions—a common component driving changes in these credit-supply indicators—we extract the first principal component from the time series of the 10 answers in each country of our panel. On average, the first principal component explains more than 45 percent of the correlation in these series, and the estimated factor loadings have their expected signs.

Table A-1 – Sovereign Distress and Euro Area Business Credit Conditions

| Explanatory Variables | (1) | (2) |
|---|---|---|
| $\ln \text{CDS}_{i,t-1}$ | 0.734 | 0.686 |
| | [0.546, 0.917] | [0.464, 0.912] |
| Adj. $R^2$ | 0.270 | 0.370 |
| Country fixed effects | Yes | Yes |
| Time fixed effects | No | Yes |
| *Memo*: Euro area sovereign CDS spreads (pps.) | | |
| Core countries: P50 = 0.29, IQR = 0.29 | | |
| Periphery countries: P50 = 1.76, IQR = 3.40 | | |

Note: Sample period: semi-annual (March/October) data from 2009:M3 to 2015:M10; No. of countries = 11; and Obs. = 143. The dependent variable is a SAFE-based indicator of the change in business credit conditions in country $i$ from survey period $t-1$ to survey period $t$ (see the text and Figure 2 for details). The entries denote the OLS estimates of the coefficients associated with the log of sovereign CDS spreads at the end of the survey period $t-1$. The 95-percent confidence intervals reported in brackets are based on the empirical distribution of coefficients across 5,000 replications, using the wild bootstrap clustered in the time ($t$) dimension (see Cameron et al., 2008).

To document the link between sovereign distress and future changes in business credit conditions throughout the euro area, Table A-1 reports the results from regressions, in which the country-specific indicator of the change in business credit conditions over the survey period is regressed on the corresponding sovereign (log) CDS spread at the end of the previous survey period. According to column (1), increases in sovereign CDS spreads are strongly associated with a tightening of business credit conditions over the subsequent six months; moreover, as shown in column (2), this finding is robust to the inclusion of time fixed effects, which control for common shocks in the euro area during this period.

In addition to being statistically significant, the estimated coefficients are quantitatively important. For example, using the estimate from column (2), a widening of sovereign CDS spreads from the 25th percentile to 75th percentile of the distribution in the euro area core is associated with a further tightening of credit conditions of about 0.7 standard deviations over the subsequent six months; in the periphery, by contrast, the same-sized move in CDS spreads is estimated to have tightened business credit conditions by an additional 1.3 standard deviations during the 2009–2015 period.[36]

# B Model Appendix

## B.1 Product Demands and Accounting Identities

### B.1.1 Product Demands

In the symmetric equilibrium, all households choose the same level of consumption. Hence, we omit the household superscript $j$ going forward. The Lagrangian associated with the cost minimization problem faced by the household in the home country is given by

$$\mathcal{L}_c = \sum_{k=h,f} \int_{N_k} P_{i,k,t} c_{i,k,t} di - \lambda_{c,t} \left\{ \left[ \sum_{k=h,f} \Xi_k \left( \int_{N_k} \left( c_{i,k,t} / s^{\theta}_{i,k,t-1} \right)^{1-1/\eta} di \right)^{\frac{1-1/\varepsilon}{1-1/\eta}} \right]^{\frac{1}{1-1/\varepsilon}} - x_t \right\}.$$

The first-order condition for $c_{i,h,t}$ is given by

$$P_{i,h,t} = \Xi_h \lambda_{c,t} \frac{\left( c_{i,h,t} / s^{\theta}_{i,h,t-1} \right)^{1-1/\eta}}{c_{i,h,t}} \left[ \int_{N_h} \left( c_{i,h,t} / s^{\theta}_{i,h,t-1} \right)^{1-1/\eta} di \right]^{\frac{1-1/\varepsilon}{1-1/\eta} - 1} x_t^{1/\varepsilon}, \tag{B.1-1}$$

while the first-order condition for $c_{j,h,t}$ is given by

$$P_{j,h,t} = \Xi_h \lambda_{c,t} \frac{\left( c_{j,h,t} / s^{\theta}_{j,h,t-1} \right)^{1-1/\eta}}{c_{j,h,t}} \left[ \int_{N_h} \left( c_{j,h,t} / s^{\theta}_{j,h,t-1} \right)^{1-1/\eta} dj \right]^{\frac{1-1/\varepsilon}{1-1/\eta} - 1} x_t^{1/\varepsilon}. \tag{B.1-2}$$

Taking the ratio of equations (B.1-1) and (B.1-2) yields

$$\frac{P_{i,h,t}}{P_{j,h,t}} = \frac{c_{j,h,t}}{c_{i,h,t}} \frac{\left( c_{i,h,t} / s^{\theta}_{i,h,t-1} \right)^{1-1/\eta}}{\left( c_{j,h,t} / s^{\theta}_{j,h,t-1} \right)^{1-1/\eta}},$$

or equivalently,

$$\left( c_{j,h,t} / s^{\theta}_{j,h,t-1} \right)^{-1/\eta} = \frac{P_{j,h,t} s^{\theta}_{j,h,t-1}}{P_{i,h,t}} \frac{\left( c_{i,h,t} / s^{\theta}_{i,h,t-1} \right)^{1-1/\eta}}{c_{i,h,t}}.$$

Raising the above expression to the power of $1 - 1/\eta$, integrating the resulting expression with respect to $j$, and finally raising the resulting expression to the power of $1/(1 - 1/\eta)$ yields

$$\begin{aligned} \left[ \int_{N_h} \left( c_{j,h,t} / s^{\theta}_{j,h,t-1} \right)^{1-1/\eta} dj \right]^{1/(1-1/\eta)} &= \left[ \int_{N_h} \left( P_{j,h,t} s^{\theta}_{j,h,t-1} \right)^{1-\eta} dj \right]^{1/(1-1/\eta)} \\ &\quad \times c_{i,h,t} \left( s^{\theta}_{i,h,t-1} \right)^{\eta-1} P^{\eta}_{i,h,t}. \end{aligned} \tag{B.1-3}$$

[36] These effects are evaluated at the median CDS spreads for core and periphery countries, respectively.

Define the following aggregates:

$$x_{h,t} = \left[\int_{N_h} \left(c_{j,h,t}/s^{\theta}_{j,h,t-1}\right)^{1-1/\eta} dj\right]^{1/(1-1/\eta)}; \tag{B.1-4}$$

$$\tilde{P}_{h,t} = \left[\int_{N_h} \left(P_{j,h,t}s^{\theta}_{j,h,t-1}\right)^{1-\eta} dj\right]^{1/(1-\eta)}. \tag{B.1-5}$$

We can then rewrite equation (B.1-3) in terms of the aggregates (B.1-4) and (B.1-5) as

$$c_{i,h,t} = \left(\frac{P_{i,h,t}}{\tilde{P}_{h,t}}\right)^{-\eta} s^{\theta(1-\eta)}_{i,h,t-1} x_{h,t} = \left(\frac{P_{i,h,t}}{P_{h,t}}\right)^{-\eta} \left(\frac{\tilde{P}_{h,t}}{P_{h,t}}\right)^{\eta} s^{\theta(1-\eta)}_{i,h,t-1} x_{h,t}. \tag{B.1-6}$$

Following the same steps, we can derive demand for the foreign product by the home country's household:

$$c_{i,f,t} = \left(\frac{P_{i,f,t}}{\tilde{P}_{f,t}}\right)^{-\eta} s^{\theta(1-\eta)}_{i,f,t-1} x_{f,t} = \left(\frac{P_{i,f,t}}{P_{f,t}}\right)^{-\eta} \left(\frac{\tilde{P}_{f,t}}{P_{f,t}}\right)^{\eta} s^{\theta(1-\eta)}_{i,f,t-1} x_{f,t}, \tag{B.1-7}$$

where

$$x_{f,t} = \left[\int_{N_f} \left(c_{j,f,t}/s^{\theta}_{j,f,t-1}\right)^{1-1/\eta} dj\right]^{1/(1-1/\eta)}; \tag{B.1-8}$$

$$\tilde{P}_{f,t} = \left[\int_{N_f} \left(P_{j,f,t}s^{\theta}_{j,f,t-1}\right)^{1-\eta} dj\right]^{1/(1-\eta)}. \tag{B.1-9}$$

Using equations (B.1-4) and (B.1-8), the consumption/habit aggregator $x_t$ can then be written as

$$x_t = \left[\sum_{k=h,f} \Xi_k x^{1-1/\varepsilon}_{k,t}\right]^{1/(1-1/\varepsilon)}. \tag{B.1-10}$$

We can then think of another cost minimization problem: minimizing the cost of obtaining $x_t$ by choosing $x_{k,t}$, when the unit price of $x_{k,t}$ is given by $\tilde{P}_{k,t}$. The Lagrangian associated with this problem is given by

$$\mathcal{L}_x = \sum_{k=h,f} \tilde{P}_{k,t} x_{k,t} - \tilde{P}_t \left[\left(\sum_{k=h,f} \Xi_k x^{1-1/\varepsilon}_{k,t}\right)^{1/(1-1/\varepsilon)} - x_t\right],$$

where $\tilde{P}_t$ is the Lagrangian multiplier. The first-order conditions for this program are given by

$$x_{h,t} = \Xi^{\varepsilon}_h \left(\frac{\tilde{P}_{h,t}}{\tilde{P}_t}\right)^{-\varepsilon} x_t; \tag{B.1-11}$$

$$x_{f,t} = \Xi^{\varepsilon}_f \left(\frac{\tilde{P}_{f,t}}{\tilde{P}_t}\right)^{-\varepsilon} x_t. \tag{B.1-12}$$

Substituting these conditions into equation (B.1-10) yields:

$$1 = \left[\sum_{k=h,f} \Xi_k \left[\Xi^{\varepsilon}_k \left(\frac{\tilde{P}_{k,t}}{\tilde{P}_t}\right)^{-\varepsilon}\right]^{1-1/\varepsilon}\right]^{1/(1-1/\varepsilon)}.$$

Solving the above expression for $\tilde{P}_t$ gives the expression for the welfare-based aggregate price index:

$$\tilde{P}_t = \left[ \sum_{k=h,f} \Xi_k \tilde{P}_{k,t}^{1-\varepsilon} \right]^{1/(1-\varepsilon)}. \tag{B.1-13}$$

### B.1.2 Accounting Identities

The following accounting identities are used in the main text:

$$\begin{aligned} \int_{N_k} P_{i,k,t} c_{i,k,t} di &= \int_{N_k} P_{i,k,t} \left( \frac{P_{i,k,t}}{\tilde{P}_{k,t}} \right)^{-\eta} s_{i,k,t-1}^{\theta(1-\eta)} x_{k,t} di \\ &= \tilde{P}_{k,t}^{\eta} x_{k,t} \int_{N_k} \left( P_{i,k,t} s_{i,k,t-1}^{\theta} \right)^{1-\eta} di \\ &= \tilde{P}_{k,t} x_{k,t}, \quad k = h, f; \end{aligned} \tag{B.1-14}$$

and

$$\sum_{k=h,f} \tilde{P}_{k,t} x_{k,t} = \sum_{k=h,f} \tilde{P}_{k,t} \Xi_k \left( \frac{\tilde{P}_{k,t}}{\tilde{P}_t} \right)^{-\varepsilon} x_t = \tilde{P}_t^{\varepsilon} x_t \sum_{k=h,f} \Xi_k \tilde{P}_{k,t}^{1-\varepsilon} = \tilde{P}_t x_t. \tag{B.1-15}$$

For $k = h, f$, we define the following relative prices in the home country: $p_{i,k,t} = P_{i,k,t}/P_{k,t}$, $\tilde{p}_{k,t} = \tilde{P}_{k,t}/P_{k,t}$, and $p_{k,t} = P_{k,t}/P_t$; similarly, for the foreign country we define $p^*_{i,k,t} = P^*_{i,k,t}/P^*_{k,t}$, $\tilde{p}^*_{k,t} = \tilde{P}^*_{k,t}/P^*_{k,t}$, and $p^*_{k,t} = P^*_{k,t}/P^*_t$, for $k = h, f$. In a symmetric equilibrium, these definitions then imply

$$\begin{aligned} \tilde{p}_t = \frac{\tilde{P}_t}{P_t} &= \left[ \sum_{k=h,f} \Xi_k \tilde{p}_{k,t}^{1-\varepsilon} p_{k,t}^{1-\varepsilon} \right]^{1/(1-\varepsilon)} \\ &= \left[ \sum_{k=h,f} \Xi_k p_{k,t}^{1-\varepsilon} s_{k,t-1}^{\theta(1-\varepsilon)} \right]^{1/(1-\varepsilon)}. \end{aligned} \tag{B.1-16}$$

## B.2 The Phillips Curve

Using the symmetric equilibrium conditions and dividing the first-order condition for $c_{i,h,t}$ implied by the firm's problem by $\mathbb{E}_t^a[\xi_{i,t}]$, we can express the ratio of the marginal value of an additional sale to the expected marginal value of internal funds as

$$\begin{aligned} \frac{\nu_{h,t}}{\mathbb{E}_t^a[\xi_{i,t}]} &= p_{h,t} - \frac{\mathbb{E}_t^a[\kappa_{i,t}]}{\mathbb{E}_t^a[\xi_{i,t}]} + (1-\rho) \frac{\lambda_{h,t}}{\mathbb{E}_t^a[\xi_{i,t}]} \\ &= p_{h,t} - \frac{\mathbb{E}_t^a[\xi_{i,t} a_{i,t}]}{\mathbb{E}_t^a[\xi_{i,t}]} \frac{w_t}{\alpha A_t} \left( \phi + c_{h,t} + c^*_{h,t} \right)^{\frac{1-\alpha}{\alpha}} + (1-\rho) \frac{\lambda_{h,t}}{\mathbb{E}_t^a[\xi_{i,t}]}. \end{aligned} \tag{B.2-1}$$

Define the aggregate marginal (gross) markup—denoted by $\mu_t$—as

$$\mu_t = \frac{\alpha A_t}{w_t} \left( \phi + c_{h,t} + c^*_{h,t} \right)^{\frac{\alpha-1}{\alpha}};$$

and then let the financially adjusted markup—denoted by $\tilde{\mu}_t$—be given by

$$\tilde{\mu}_t = \frac{\mathbb{E}_t^a[\xi_{i,t}]}{\mathbb{E}_t^a[\xi_{i,t} a_{i,t}]} \mu_t = \frac{\mathbb{E}_t^a[\xi_{i,t}]}{\mathbb{E}_t^a[\xi_{i,t} a_{i,t}]} \frac{\alpha A_t}{w_t} \left( \phi + c_{h,t} + c^*_{h,t} \right)^{\frac{\alpha-1}{\alpha}}.$$

We can then express equation (B.2-1) as

$$\frac{\nu_{h,t}}{\mathbb{E}_t^a[\xi_{i,t}]} = p_{h,t} - \frac{1}{\tilde{\mu}_t} + (1-\rho)\frac{\lambda_{h,t}}{\mathbb{E}_t^a[\xi_{i,t}]}. \tag{B.2-2}$$

Dividing the first-order condition for $s_{i,h,t}$ implied by the firm's problem through by $\mathbb{E}_t^a[\xi_{i,t}]$ and rearranging terms yields

$$\begin{aligned}\frac{\lambda_{h,t}}{\mathbb{E}_t^a[\xi_{i,t}]} &= \rho\mathbb{E}_t\left[m_{t,t+1}\frac{\mathbb{E}_{t+1}^a[\xi_{i,t+1}]}{\mathbb{E}_t^a[\xi_{i,t}]}\frac{\lambda_{h,t+1}}{\mathbb{E}_{t+1}^a[\xi_{i,t+1}]}\right] \\ &\quad + \theta(1-\eta)\mathbb{E}_t\left[m_{t,t+1}\frac{\mathbb{E}_{t+1}^a[\xi_{i,t+1}]}{\mathbb{E}_t^a[\xi_{i,t}]}\frac{\nu_{h,t+1}}{\mathbb{E}_{t+1}^a[\xi_{i,t+1}]}\frac{c_{h,t+1}}{s_{h,t}}\right].\end{aligned} \tag{B.2-3}$$

By substituting equation (B.2-2) into equation (B.2-3) and solving the resulting expression forward, one can verify that

$$\frac{\lambda_{h,t}}{\mathbb{E}_t^a[\xi_{i,t}]} = \theta(1-\eta)\mathbb{E}_t\left[\sum_{s=t+1}^{\infty}\beta_{t,s}\frac{\mathbb{E}_s^a[\xi_{i,s}]}{\mathbb{E}_t^a[\xi_{i,t}]}\left(p_{h,s} - \frac{1}{\tilde{\mu}_s}\right)\right], \tag{B.2-4}$$

where the growth-adjusted, compounded discount factor $\beta_{t,s}$ is given by

$$\beta_{t,s} = m_{s,s+1}g_{h,s+1} \times \prod_{j=1}^{s-t}[\rho + \theta(1-\eta)(1-\rho)g_{h,t+j}]m_{t+j-1,t+j},$$

with $g_{h,t} = c_{h,t}/s_{h,t-1} = (s_{h,t}/s_{h,t-1} - \rho)/(1-\rho)$. Hence,

$$\frac{\nu_{h,t}}{\mathbb{E}_t^a[\xi_{i,t}]} = p_{h,t} - \frac{1}{\tilde{\mu}_t} + (1-\rho)\theta(1-\eta)\mathbb{E}_t\left[\sum_{s=t+1}^{\infty}\beta_{t,s}\frac{\mathbb{E}_s^a[\xi_{i,s}]}{\mathbb{E}_t^a[\xi_{i,t}]}\left(p_{h,s} - \frac{1}{\tilde{\mu}_s}\right)\right]. \tag{B.2-5}$$

## B.3 Nonstochastic Steady State

In the main text, we presented the profit-maximization problem from the vantage point of the firm in the home country. For the sake of completeness, we now state the corresponding problem faced by foreign firms and present the associated first-order conditions—writing down the full set of these conditions helps to understand the derivation of the steady-state relationships of the model. Like a home country firm, a foreign firm chooses the sequence $\{d_{i,t}^*, h_{i,t}^*, c_{i,f,t}^*, c_{i,f,t}, s_{i,f,t}^*, s_{i,f,t}, p_{i,f,t}^*, p_{i,f,t}\}_{t=0}^{\infty}$ to maximize the following Lagrangian:

$$\begin{aligned}\mathcal{L}^* = \mathbb{E}_0\sum_{t=0}^{\infty}m_{0,t}^*\Bigg\{& d_{i,t}^* + \kappa_{i,t}^*\left[\left(\frac{A_t^*}{a_{i,t}^*}h_{i,t}^*\right)^{\alpha} - \phi^* - (c_{i,f,t}^* + c_{i,f,t})\right] \\ & + \xi_{i,t}^*\Bigg[p_{i,f,t}^*p_{f,t}^*c_{i,f,t}^* + q_t^{-1}p_{i,f,t}p_{f,t}c_{i,f,t} - w_t^*h_{i,t}^* - d_{i,t}^* + \varphi^*\min\{0, d_{i,t}^*\} \\ & - \frac{\gamma_p}{2}\left(\frac{p_{i,f,t}^*}{p_{i,f,t-1}^*}\pi_{f,t}^* - 1\right)^2 c_t^* - \frac{\gamma_p}{2}q_t^{-1}\left(\frac{p_{i,f,t}}{p_{i,f,t-1}}\pi_{f,t} - 1\right)^2 c_t\Bigg] \\ & + \nu_{i,f,t}^*\left[(p_{i,f,t}^*)^{-\eta}(\tilde{p}_{f,t}^*)^{\eta}s_{i,f,t-1}^{*\theta(1-\eta)}x_{f,t}^* - c_{i,f,t}^*\right] + \nu_{i,f,t}\left[p_{i,f,t}^{-\eta}\tilde{p}_{f,t}^{\eta}s_{i,f,t-1}^{\theta(1-\eta)}x_{f,t} - c_{i,f,t}\right] \\ & + \lambda_{i,f,t}^*\left[\rho s_{i,f,t-1}^* + (1-\rho)c_{i,f,t}^* - s_{i,f,t}^*\right] + \lambda_{i,f,t}\left[\rho s_{i,f,t-1} + (1-\rho)c_{i,f,t} - s_{i,f,t}\right]\Bigg\},\end{aligned} \tag{B.3-1}$$

where $\tilde{p}^*_{f,t} = \tilde{P}^*_{f,t}/P^*_{f,t}$ and $\tilde{p}_{f,t} = \tilde{P}_{f,t}/P_{f,t}$.

The first-order conditions implied by this program are given by

$$d^*_{i,t}\colon \quad \xi^*_{i,t} = \begin{cases} 1 & \text{if } d^*_{i,t} \geq 0; \\ 1/(1-\varphi^*) & \text{if } d^*_{i,t} < 0. \end{cases} \tag{B.3-2}$$

$$h^*_{i,t}\colon \quad \xi^*_{i,t} w^*_t = \alpha \kappa^*_{i,t} \left( \frac{A^*_t}{a^*_{i,t}} h^*_{i,t} \right)^{\alpha-1}, \quad \text{where } h^*_{i,t} = \frac{a^*_{i,t}}{A^*_t} \left( \phi^* + c^*_{i,f,t} + c_{i,f,t} \right)^{\frac{1}{\alpha}}. \tag{B.3-3}$$

$$c^*_{i,f,t}\colon \quad \nu^*_{i,f,t} = \mathbb{E}^a_t[\xi^*_{i,t}] p^*_{i,f,t} p^*_{f,t} - \mathbb{E}^a_t[\kappa^*_{i,t}] + (1-\rho)\lambda^*_{i,f,t}. \tag{B.3-4}$$

$$c_{i,f,t}\colon \quad \nu_{i,f,t} = \mathbb{E}^a_t[\xi^*_{i,t}] q^{-1}_t p_{i,f,t} p_{f,t} - \mathbb{E}^a_t[\kappa^*_{i,t}] + (1-\rho)\lambda_{i,f,t}. \tag{B.3-5}$$

$$s^*_{i,f,t}\colon \quad \lambda^*_{i,f,t} = \rho \mathbb{E}_t[m^*_{t,t+1} \lambda^*_{i,f,t+1}] + \theta(1-\eta)\mathbb{E}_t \left[ m^*_{t,t+1} \mathbb{E}^a_{t+1} \left[ \nu^*_{i,f,t+1} \frac{c^*_{i,f,t+1}}{s^*_{i,f,t}} \right] \right]. \tag{B.3-6}$$

$$s_{i,f,t}\colon \quad \lambda_{i,f,t} = \rho \mathbb{E}_t[m^*_{t,t+1} \lambda_{i,f,t+1}] + \theta(1-\eta)\mathbb{E}_t \left[ m^*_{t,t+1} \mathbb{E}^a_{t+1} \left[ \nu_{i,f,t+1} \frac{c_{i,f,t+1}}{s_{i,f,t}} \right] \right]. \tag{B.3-7}$$

$$\begin{aligned} p^*_{i,f,t}\colon \quad 0 = {} & \mathbb{E}^a_t[\xi^*_{i,t}] \left[ p^*_{f,t} c^*_{i,f,t} - \gamma_p \frac{\pi^*_{f,t}}{p^*_{i,f,t-1}} \left( \pi^*_{f,t} \frac{p^*_{i,f,t}}{p^*_{i,f,t-1}} - 1 \right) c^*_t \right] - \eta \frac{\nu^*_{i,f,t}}{p^*_{i,f,t}} c^*_{i,f,t} \\ & + \gamma_p \mathbb{E}_t \left[ m^*_{t,t+1} \mathbb{E}^a_{t+1}[\xi^*_{i,t+1}] \pi^*_{f,t+1} \frac{p^*_{i,f,t+1}}{(p^*_{i,f,t})^2} \left( \pi^*_{f,t+1} \frac{p^*_{i,f,t+1}}{p^*_{i,f,t}} - 1 \right) c^*_{t+1} \right]. \end{aligned} \tag{B.3-8}$$

$$\begin{aligned} p_{i,f,t}\colon \quad 0 = {} & \mathbb{E}^a_t[\xi^*_{i,t}] \left[ q^{-1}_t p_{f,t} c_{i,f,t} - \gamma_p \frac{q^{-1}_t \pi_{f,t}}{p_{i,f,t-1}} \left( \pi_{f,t} \frac{p_{i,f,t}}{p_{i,f,t-1}} - 1 \right) c_t \right] - \eta \frac{\nu_{i,f,t}}{p_{i,f,t}} c_{i,f,t} \\ & + \gamma_p \mathbb{E}_t \left[ m^*_{t,t+1} \mathbb{E}^a_{t+1}[\xi^*_{i,t+1}] q^{-1}_{t+1} \pi_{f,t+1} \frac{p_{i,f,t+1}}{p^2_{i,f,t}} \left( \pi_{f,t+1} \frac{p_{i,f,t+1}}{p_{i,f,t}} - 1 \right) c_{t+1} \right]. \end{aligned} \tag{B.3-9}$$

It is also helpful to spell out in more detail how relative prices are determined in various markets and how they are related to each other in the symmetric equilibrium. Recall that the assumptions of risk neutrality, i.i.d. idiosyncratic cost shocks, and our within-period sequence of decisions together imply that all home country firms set identical prices in the domestic and foreign markets: $P_{i,h,t} = P_{h,t}$ and $P^*_{i,h,t} = P^*_{h,t}$, for all $i$. By the symmetry of our setup, $P_{i,f,t} = P_{f,t}$ and $P^*_{i,f,t} = P^*_{f,t}$, for all $i$ as well. However, because both sets of firms "price to market," $P_{i,h,t} \neq Q_t P^*_{i,h,t}$ and $P_{i,f,t} \neq Q^{-1}_t P^*_{i,f,t}$, in general. The symmetric equilibrium implies that $p_{i,h,t} = p^*_{i,h,t} = p_{i,f,t} = p^*_{i,f,t} = 1$ or equivalently, $(P_{i,h,t}/P_{h,t}) = (P^*_{i,h,t}/P^*_{h,t}) = (P_{i,f,t}/P_{f,t}) = (P^*_{i,f,t}/P^*_{f,t}) = 1$.

In any symmetric equilibrium, the type-specific ratios of the habit-adjusted price index ($\tilde{P}_{k,t}$) to the corresponding CPI ($P_{k,t}$) must satisfy the following conditions:

$$\begin{aligned} \tilde{p}_{h,t} &= \tilde{P}_{h,t}/P_{h,t} = s^{\theta}_{h,t-1}; \\ \tilde{p}^*_{h,t} &= \tilde{P}^*_{h,t}/P^*_{h,t} = s^{*\theta}_{h,t-1}; \\ \tilde{p}_{f,t} &= \tilde{P}_{f,t}/P_{f,t} = s^{\theta}_{f,t-1}; \\ \tilde{p}^*_{f,t} &= \tilde{P}^*_{f,t}/P^*_{f,t} = s^{*\theta}_{f,t-1}. \end{aligned}$$

These relative prices can then be used to derive the equilibrium demands for the habit-adjusted consumption baskets. For example, in the home country, the symmetric equilibrium condition $x^j_{h,t} = x_{h,t}$, for all $j$, implies

$$\begin{aligned} x_{h,t} &= \Xi_h^\varepsilon \left(\frac{\tilde{P}_{h,t}}{\tilde{P}_t}\right)^{-\varepsilon} x_t \\ &= \Xi_h^\varepsilon \left(\frac{\tilde{P}_{h,t}}{P_{h,t}} \frac{P_{h,t}}{P_t} \frac{P_t}{\tilde{P}_t}\right)^{-\varepsilon} x_t \\ &= \Xi_h^\varepsilon p_{h,t}^{-\varepsilon} \left(\frac{\tilde{p}_{h,t}}{\tilde{p}_t}\right)^{-\varepsilon} x_t, \end{aligned}$$

where

$$\begin{aligned} \tilde{p}_t &= \left[\sum_{k=h,f} \Xi_k \left(\frac{\tilde{P}_{k,t}}{P_t}\right)^{1-\varepsilon}\right]^{1/(1-\varepsilon)} \\ &= \left[\sum_{k=h,f} \Xi_k \left(\frac{\tilde{P}_{k,t}}{P_{k,t}} \frac{P_{k,t}}{P_t}\right)^{1-\varepsilon}\right]^{1/(1-\varepsilon)} \\ &= \left[\sum_{k=h,f} \Xi_k s_{k,t-1}^{\theta(1-\varepsilon)} p_{k,t}^{1-\varepsilon}\right]^{1/(1-\varepsilon)}. \end{aligned}$$

Similarly, it can be shown that

$$\begin{aligned} x_{f,t} &= \Xi_f^\varepsilon p_{f,t}^{-\varepsilon} \left(\frac{\tilde{p}_{f,t}}{\tilde{p}_t}\right)^{-\varepsilon} x_t; \\ x^*_{h,t} &= \Xi_h^{*\varepsilon} (p^*_{h,t})^{-\varepsilon} \left(\frac{\tilde{p}^*_{h,t}}{\tilde{p}^*_t}\right)^{-\varepsilon} x^*_t; \\ x^*_{f,t} &= \Xi_f^{*\varepsilon} (p^*_{f,t})^{-\varepsilon} \left(\frac{\tilde{p}^*_{f,t}}{\tilde{p}^*_t}\right)^{-\varepsilon} x^*_t; \end{aligned}$$

and

$$\tilde{p}^*_t = \left[\sum_{k=h,f} \Xi^*_k s_{k,t-1}^{*\theta(1-\varepsilon)} p_{k,t}^{*(1-\varepsilon)}\right]^{1/(1-\varepsilon)}.$$

As is typical in such models, only relative prices $p_{h,t}$, $p^*_{h,t}$, $p_{f,t}$, $p^*_{f,t}$, $\tilde{p}_{h,t}$, $\tilde{p}^*_{h,t}$, $\tilde{p}_{f,t}$, $\tilde{p}^*_{f,t}$, $\tilde{p}_t$, and $\tilde{p}^*_t$—along with the real exchange rate $q_t$—are determined in equilibrium.

### B.3.1 Relative Prices and Quantities in the Symmetric Equilibrium

In the steady state, the four Phillips curves implied by our model are given by

$$p_h = \eta \frac{\nu_h}{\mathbb{E}^a_t[\xi_i]}; \quad \text{(B.3-10)}$$

$$q p^*_h = \eta \frac{\nu^*_h}{\mathbb{E}^a_t[\xi_i]}; \quad \text{(B.3-11)}$$

$$p^*_f = \eta \frac{\nu^*_f}{\mathbb{E}^a_t[\xi^*_i]}; \quad \text{(B.3-12)}$$

$$p_f q^{-1} = \eta \frac{\nu_f}{\mathbb{E}^a_t[\xi^*_i]}. \quad \text{(B.3-13)}$$

Equations (B.3-10)–(B.3-13) correspond to the steady-state Phillips curves for the price of home-country goods in the home country, the price of home-country goods in the foreign country, the price of foreign-country goods in the home country, and the price of foreign-country goods in the foreign country, respectively.[37]

The symmetric equilibrium conditions and the law of motion for habit stocks imply that $c_h = s_h$, $c_h^* = s_h^*$, $c_f = s_f$, and $c_f^* = s_f^*$. Using these conditions, together with the first-order conditions for habit stocks, we obtain

$$\frac{\lambda_h}{\mathbb{E}^a[\xi_i]} = \frac{\theta(1-\eta)\delta}{1-\rho\delta}\frac{\nu_h}{\mathbb{E}^a[\xi_i]}; \tag{B.3-14}$$

$$\frac{\lambda_h^*}{\mathbb{E}^a[\xi_i]} = \frac{\theta(1-\eta)\delta}{1-\rho\delta}\frac{\nu_h^*}{\mathbb{E}^a[\xi_i]}; \tag{B.3-15}$$

$$\frac{\lambda_f^*}{\mathbb{E}^a[\xi_i^*]} = \frac{\theta(1-\eta)\delta}{1-\rho\delta}\frac{\nu_f^*}{\mathbb{E}^a[\xi_i^*]}; \tag{B.3-16}$$

$$\frac{\lambda_f}{\mathbb{E}^a[\xi_i^*]} = \frac{\theta(1-\eta)\delta}{1-\rho\delta}\frac{\nu_f}{\mathbb{E}^a[\xi_i^*]}. \tag{B.3-17}$$

Combining equations (B.3-10)–(B.3-13) and equations (B.3-14)–(B.3-17) yields

$$\frac{\lambda_h}{\mathbb{E}^a[\xi_i]} = p_h\frac{\theta(1-\eta)\delta}{\eta(1-\rho\delta)}; \tag{B.3-18}$$

$$\frac{\lambda_h^*}{\mathbb{E}^a[\xi_i]} = qp_h^*\frac{\theta(1-\eta)\delta}{\eta(1-\rho\delta)}; \tag{B.3-19}$$

$$\frac{\lambda_f^*}{\mathbb{E}^a[\xi_i^*]} = p_f^*\frac{\theta(1-\eta)\delta}{\eta(1-\rho\delta)}; \tag{B.3-20}$$

$$\frac{\lambda_f}{\mathbb{E}^a[\xi_i^*]} = \frac{p_f}{q}\frac{\theta(1-\eta)\delta}{\eta(1-\rho\delta)}, \tag{B.3-21}$$

which imply that $qp_h^*/p_h = \lambda_h^*/\lambda_h$ and $qp_f^*/p_f = \lambda_f^*/\lambda_f$. By combining the first-order conditions (B.3-3), (B.3-4), and (B.3-5) with the corresponding first-order conditions for the home country firm, we obtain

$$\frac{\nu_h}{\mathbb{E}^a[\xi_i]} = p_h - \frac{\mathbb{E}^a[\xi_i a_i]}{\mathbb{E}^a[\xi_i]}\frac{w}{\alpha A}\left(\phi + c_h + c_h^*\right)^{\frac{1-\alpha}{\alpha}} + (1-\rho)\frac{\lambda_h}{\mathbb{E}^a[\xi_i]}; \tag{B.3-22}$$

$$\frac{\nu_h^*}{\mathbb{E}^a[\xi_i]} = qp_h^* - \frac{\mathbb{E}^a[\xi_i a_i]}{\mathbb{E}^a[\xi_i]}\frac{w}{\alpha A}\left(\phi + c_h + c_h^*\right)^{\frac{1-\alpha}{\alpha}} + (1-\rho)\frac{\lambda_h^*}{\mathbb{E}^a[\xi_i]}; \tag{B.3-23}$$

$$\frac{\nu_f^*}{\mathbb{E}^a[\xi_i^*]} = p_f^* - \frac{\mathbb{E}^a[\xi_i^* a_i^*]}{\mathbb{E}^a[\xi_i^*]}\frac{w^*}{\alpha A^*}\left(\phi^* + c_f^* + c_f\right)^{\frac{1-\alpha}{\alpha}} + (1-\rho)\frac{\lambda_f^*}{\mathbb{E}^a[\xi_i^*]}; \tag{B.3-24}$$

$$\frac{\nu_f}{\mathbb{E}^a[\xi_i^*]} = \frac{p_f}{q} - \frac{\mathbb{E}^a[\xi_i^* a_i^*]}{\mathbb{E}^a[\xi_i^*]}\frac{w^*}{\alpha A^*}\left(\phi^* + c_f^* + c_f\right)^{\frac{1-\alpha}{\alpha}} + (1-\rho)\frac{\lambda_f}{\mathbb{E}^a[\xi_i^*]}. \tag{B.3-25}$$

In the last step, we substitute equations (B.3-10)–(B.3-13) and equations (B.3-18)–(B.3-21) into

[37]Recall that in our notation, subscripts $h$ and $f$ indicate the origin of the good, while the superscript "*," or its absence, indicate the destination of the good, with "*" indicating the foreign country. For example, $p_f^*$ is the (relative) price of goods produced by foreign firms and sold in the foreign country, whereas $p_f$ is the price of goods produced by foreign firms, but sold in the home country in the local currency—hence the multiplication by $1/q$, which converts it to the currency of the foreign country.

equations (B.3-22)–(B.3-25) and solve for $p_k$ and $p_k^*$, $k = h, f$, which yields:

$$p_h = \frac{\eta(1-\rho\delta)}{(\eta-1)\big[(1-\rho\delta)-\theta\delta(1-\rho)\big]}\frac{\mathbb{E}^a[\xi_i a_i]}{\mathbb{E}^a[\xi_i]}\frac{w}{\alpha A}\big(\phi + c_h + c_h^*\big)^{\frac{1-\alpha}{\alpha}}; \tag{B.3-26}$$

$$p_h^* = \frac{\eta(1-\rho\delta)}{(\eta-1)\big[(1-\rho\delta)-\theta\delta(1-\rho)\big]}q^{-1}\frac{\mathbb{E}^a[\xi_i a_i]}{\mathbb{E}^a[\xi_i]}\frac{w}{\alpha A}\big(\phi + c_h + c_h^*\big)^{\frac{1-\alpha}{\alpha}}; \tag{B.3-27}$$

$$p_f^* = \frac{\eta(1-\rho\delta)}{(\eta-1)\big[(1-\rho\delta)-\theta\delta(1-\rho)\big]}\frac{\mathbb{E}^a[\xi_i^* a_i^*]}{\mathbb{E}^a[\xi_i^*]}\frac{w^*}{\alpha A^*}\big(\phi^* + c_f^* + c_f\big)^{\frac{1-\alpha}{\alpha}}; \tag{B.3-28}$$

$$p_f = \frac{\eta(1-\rho\delta)}{(\eta-1)\big[(1-\rho\delta)-\theta\delta(1-\rho)\big]}q\frac{\mathbb{E}^a[\xi_i^* a_i^*]}{\mathbb{E}^a[\xi_i^*]}\frac{w^*}{\alpha A^*}\big(\phi^* + c_f^* + c_f\big)^{\frac{1-\alpha}{\alpha}}. \tag{B.3-29}$$

Note that the law of one price holds in the nonstochastic steady state: $p_h = qp_h^*$ and $p_f^* = p_f/q$, which imply that $\lambda_h^*/\lambda_h = 1$ and $\lambda_f^*/\lambda_f = 1$. This follows from the assumed symmetry of the two markets, in terms of the elasticity of substitution, the strength of the deep-habit mechanism, and so on. However, the law of one price is generally violated in stochastic simulation because in those circumstances, the two countries experience different sequences of asymmetric shocks, which affect the intensity of customer-market relationships, leading to different long-run demand elasticities in the two countries, as well as to different internal liquidity positions. In general, firms will optimally exploit any differences in customer-market relationships and financial positions between the two countries by price discriminating across the border.

In the steady state, the external financing triggers are given by

$$a^E = \frac{A}{w\big(\phi + c_h + c_h^*\big)^{\frac{1}{\alpha}}}\big(p_h c_h + qp_h^* c_h^*\big); \tag{B.3-30}$$

$$a^{*E} = \frac{A^*}{w^*\big(\phi^* + c_f^* + c_f\big)^{\frac{1}{\alpha}}}\big(p_f^* c_f^* + q^{-1}p_f c_f\big), \tag{B.3-31}$$

which can be used to compute $\mathbb{E}^a[\xi_i]$, $\mathbb{E}^a[\xi_i a_i]$, $\mathbb{E}^a[\xi_i^*]$, and $\mathbb{E}^a[\xi_i^* a_i^*]$:

$$\mathbb{E}^a[\xi_i] = 1 + \frac{\varphi}{1-\varphi}\big[1 - \Phi(z^E)\big]; \tag{B.3-32}$$

$$\mathbb{E}^a[\xi_i a_i] = 1 + \frac{\varphi}{1-\varphi}\big[1 - \Phi(z^E - \sigma)\big]; \tag{B.3-33}$$

$$\mathbb{E}^a[\xi_i^*] = 1 + \frac{\varphi^*}{1-\varphi^*}\big[1 - \Phi(z^{*E})\big]; \tag{B.3-34}$$

$$\mathbb{E}^a[\xi_i^* a_i^*] = 1 + \frac{\varphi^*}{1-\varphi^*}\big[1 - \Phi(z^{*E} - \sigma)\big], \tag{B.3-35}$$

where

$$z^E = \sigma^{-1}\big(\ln a^E + 0.5\sigma^2\big); \tag{B.3-36}$$

$$z^{*E} = \sigma^{-1}\big(\ln a^{*E} + 0.5\sigma^2\big). \tag{B.3-37}$$

Equations (7) and (8) from the main text and their foreign counterparts imply that the following ratios should be satisfied in the steady state:

$$\frac{c_{i,h}}{c_{i,f}} = \frac{p_{i,h}^{-\eta}\tilde{p}_h^{\eta}s_{i,h}^{\theta(1-\eta)}x_h}{p_{i,f}^{-\eta}\tilde{p}_f^{\eta}s_{i,f}^{\theta(1-\eta)}x_f} = \frac{p_{i,h}^{-\eta}\tilde{p}_h^{\eta}s_{i,h}^{\theta(1-\eta)}\Xi_h^{\varepsilon}\tilde{p}_h^{-\varepsilon}p_h^{-\varepsilon}\tilde{p}^{\varepsilon}x}{p_{i,f}^{-\eta}\tilde{p}_f^{\eta}s_{i,f}^{\theta(1-\eta)}\Xi_f^{\varepsilon}\tilde{p}_f^{-\varepsilon}p_f^{-\varepsilon}\tilde{p}^{\varepsilon}x};$$

$$\frac{c_{i,h}^*}{c_{i,f}^*} = \frac{p_{i,h}^{-\eta}\tilde{p}_h^{*\eta}s_{i,h}^{*\theta(1-\eta)}x_h^*}{p_{i,f}^{*-\eta}\tilde{p}_f^{*\eta}s_{i,f}^{*\theta(1-\eta)}x_f^*} = \frac{p_{i,h}^{*-\eta}\tilde{p}_h^{*\eta}s_{i,h}^{*\theta(1-\eta)}\Xi_h^{*\varepsilon}\tilde{p}_h^{*-\varepsilon}p_h^{*-\varepsilon}\tilde{p}^{*\varepsilon}x^*}{p_{i,f}^{*-\eta}\tilde{p}_f^{*\eta}s_{i,f}^{*\theta(1-\eta)}\Xi_f^{*\varepsilon}\tilde{p}_f^{*-\varepsilon}p_f^{*-\varepsilon}\tilde{p}^{*\varepsilon}x^*}.$$

Imposing the symmetric equilibrium conditions and using the fact that $\tilde{p}_k = s_k^{\theta}$, for $k = h, f$, then yields

$$\frac{c_h}{c_f} = \left(\frac{\Xi_h}{\Xi_f}\right)^{\varepsilon}\left(\frac{p_h}{p_f}\right)^{-\varepsilon}\left(\frac{s_h^{\theta}}{s_f^{\theta}}\right)^{1-\varepsilon}; \quad \text{(B.3-38)}$$

$$\frac{c_h^*}{c_f^*} = \left(\frac{\Xi_h^*}{\Xi_f^*}\right)^{\varepsilon}\left(\frac{p_h^*}{p_f^*}\right)^{-\varepsilon}\left(\frac{s_h^{*\theta}}{s_f^{*\theta}}\right)^{1-\varepsilon}. \quad \text{(B.3-39)}$$

Because $c_{i,k} = c_k = s_k = s_{i,k}$ and $c_{i,k}^* = c_k^* = s_k^* = s_{i,k}^*$, equation (8) from the main text and its foreign counterpart imply

$$x = \left[\sum_{k=h,f} \Xi_k (c_k^{1-\theta})^{1-1/\varepsilon}\right]^{1/(1-1/\varepsilon)}; \quad \text{(B.3-40)}$$

$$x^* = \left[\sum_{k=h,f} \Xi_k^* (c_k^{*1-\theta})^{1-1/\varepsilon}\right]^{1/(1-1/\varepsilon)}. \quad \text{(B.3-41)}$$

Aggregate (conditional) labor demands in the home and foreign countries are given by

$$h = \left[\frac{\phi + c_h + c_h^*}{A^{\alpha}\exp\left[0.5\alpha(1+\alpha)\sigma^2\right]}\right]^{\frac{1}{\alpha}}; \quad \text{(B.3-42)}$$

$$h^* = \left[\frac{\phi^* + c_f + c_f^*}{A^{*\alpha}\exp\left[0.5\alpha(1+\alpha)\sigma^2\right]}\right]^{\frac{1}{\alpha}}, \quad \text{(B.3-43)}$$

while the first-order conditions for hours worked from the households' problems can be expressed as

$$h = U_h^{-1}\left[-\frac{w}{\tilde{p}}\frac{\eta_w - 1}{\eta_w}U_x\right], \quad \text{(B.3-44)}$$

$$h^* = U_h^{*-1}\left[-\frac{w^*}{\tilde{p}^*}\frac{\eta_w - 1}{\eta_w}U_x^*\right]. \quad \text{(B.3-45)}$$

Combining these expressions yields the conditions that clear the domestic and foreign labor markets and determine equilibrium wages:

$$U_h^{-1}\left[-\frac{w}{\tilde{p}}\frac{\eta_w - 1}{\eta_w}U_x\right] = \left[\frac{\phi + c_h + c_h^*}{A^{\alpha}\exp\left[0.5\alpha(1+\alpha)\sigma^2\right]}\right]^{\frac{1}{\alpha}}; \quad \text{(B.3-46)}$$

$$U_h^{*-1}\left[-\frac{w^*}{\tilde{p}^*}\frac{\eta_w - 1}{\eta_w}U_x^*\right] = \left[\frac{\phi^* + c_f + c_f^*}{A^{*\alpha}\exp\left[0.5\alpha(1+\alpha)\sigma^2\right]}\right]^{\frac{1}{\alpha}}. \quad \text{(B.3-47)}$$

Finally, equilibrium consistency requires that

$$1 = \left[\sum_{k=h,f} \Xi_k p_k^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}}; \quad \text{(B.3-48)}$$

$$1 = \left[\sum_{k=h,f} \Xi_k^* p_k^{*1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}}. \quad \text{(B.3-49)}$$

### B.3.2 The Real Exchange Rate

In the case of complete risk sharing between the two countries, the real exchange rate—defined by equation (52) in the main text—should at any point in time satisfy

$$q = \varrho_0 \frac{U_x^*}{U_x} \left[ \frac{\sum_{k=h,f} \Xi_k^* (p_k^* s_k^{*\theta})^{(1-\varepsilon)}}{\sum_{k=h,f} \Xi_k (p_k s_k^{\theta})^{(1-\varepsilon)}} \right]^{-1/(1-\varepsilon)}. \tag{B.3-50}$$

We assume that the equilibrium interest rates are determined by the households' rate of time preferences: $r = r^* = \delta^{-1} - 1$. This condition, in the case of incomplete risk sharing, determines the equilibrium holdings of international bonds $B_h = B_f = 0$. Via the bond market clearing conditions $B_h + B_h^* = 0$ and $B_f + B_f^* = 0$, this implies that $B_h^* = B_f^* = 0$.

In the case of incomplete risk sharing, the real exchange rate is determined such that $b_h = b_f = 0$, which, together with equation (46) in the main text, implies

$$0 = wh - qw^*h^* + \tilde{d} - q\tilde{d}^* - (\tilde{p}x - q\tilde{p}^*x^*),$$

or equivalently,

$$q = \frac{wh + \tilde{d} - \tilde{p}x}{w^*h^* + \tilde{d}^* - \tilde{p}^*x^*}. \tag{B.3-51}$$

## B.4 The Log-Linear Phillips Curves

As shown by equations (31) and (32) in the main text, the log-linearized CPI inflation dynamics in the home and foreign country are given by

$$\begin{aligned} \hat{\pi}_t &= \Xi_h p_h \left(\hat{p}_{h,t-1} + \hat{\pi}_{h,t}\right) + \Xi_f p_f \left(\hat{p}_{f,t-1} + \hat{\pi}_{f,t}\right); \\ \hat{\pi}_t^* &= \Xi_h^* p_h^* \left(\hat{p}_{h,t-1}^* + \hat{\pi}_{h,t}^*\right) + \Xi_f^* p_f^* \left(\hat{p}_{f,t-1}^* + \hat{\pi}_{f,t}^*\right), \end{aligned}$$

where the dynamics of $\hat{\pi}_{h,t}$, $\hat{\pi}_{h,t}^*$, $\hat{\pi}_{f,t}^*$, and $\hat{\pi}_{f,t}$ are determined by the four log-linearized Phillips curves.

**Home country:** The Phillips curve governing the behavior of $\hat{\pi}_{h,t}$, the domestic inflation in the home country, is given by

$$\begin{aligned} \gamma_p \pi_{h,t}(\pi_{h,t} - \pi) = {} & p_{h,t} \frac{c_{h,t}}{c_t} - \eta \frac{\mathbb{E}_t^a[\nu_{i,h,t}]}{\mathbb{E}_t^a[\xi_{i,t}]} \frac{c_{h,t}}{c_t} \\ & + \gamma_p \mathbb{E}_t \left[ m_{t,t+1} \frac{\mathbb{E}_{t+1}^a[\xi_{i,t+1}]}{\mathbb{E}_t^a[\xi_{i,t}]} \pi_{h,t+1}(\pi_{h,t+1} - \pi) \frac{c_{t+1}}{c_t} \right]; \end{aligned} \tag{B.4-1}$$

while the Phillips curve governing the behavior of domestic producers' inflation in the foreign country is given by

$$\begin{aligned} \gamma_p q_t \pi_{h,t}^*(\pi_{h,t}^* - \pi^*) = {} & q_t p_{h,t}^* \frac{c_{h,t}^*}{c_t^*} - \eta \frac{\mathbb{E}_t^a[\nu_{i,h,t}^*]}{\mathbb{E}_t^a[\xi_{i,t}]} \frac{c_{h,t}^*}{c_t^*} \\ & + \gamma_p \mathbb{E}_t \left[ m_{t,t+1} \frac{\mathbb{E}_{t+1}^a[\xi_{i,t+1}]}{\mathbb{E}_t^a[\xi_{i,t}]} q_{t+1} \pi_{h,t+1}^*(\pi_{h,t+1}^* - \pi^*) \frac{c_{t+1}^*}{c_t^*} \right]. \end{aligned} \tag{B.4-2}$$

In the steady state, equations (B.4-1) and (B.4-2) reduce to

$$0 = \frac{p_h c_h}{c} - \eta \frac{\mathbb{E}^a[\nu_h]}{\mathbb{E}^a[\xi]} \frac{c_h}{c}; \tag{B.4-3}$$

and

$$0 = \frac{q p_h^* c_h^*}{c^*} - \eta \frac{\mathbb{E}^a[\nu_h^*]}{\mathbb{E}^a[\xi]} \frac{c_h^*}{c^*}, \tag{B.4-4}$$

respectively. Note that the ratios $p_h c_h / c$ and $q p_h^* c_h^* / c^*$ correspond to the market shares of home country firms in the domestic and foreign markets, respectively. Imposing these conditions, one can derive the log-linearized Phillips curves as

$$\hat{\pi}_{h,t} = \frac{1}{\gamma_p} \frac{p_h c_h}{c} \left[\hat{p}_{h,t} - (\hat{\nu}_{h,t} - \hat{\xi}_t)\right] + \delta \mathbb{E}_t[\hat{\pi}_{h,t+1}]; \tag{B.4-5}$$

and

$$\hat{\pi}_{h,t}^* = \frac{1}{\gamma_p} q p_h^* \frac{c_h^*}{c^*} \left[\hat{q}_t + \hat{p}_{h,t}^* - (\hat{\nu}_{h,t}^* - \hat{\xi}_t)\right] + \delta \mathbb{E}_t[\hat{\pi}_{h,t+1}^*]. \tag{B.4-6}$$

As shown in the main text (see equations 27 and 28), the dynamics of the terms $\hat{\nu}_{h,t} - \hat{\xi}_t$ and $\hat{\nu}_{h,t}^* - \hat{\xi}_t$ are governed by

$$\frac{\mathbb{E}_t^a[\nu_{i,h,t}]}{\mathbb{E}_t^a[\xi_{i,t}]} = p_{h,t} - \frac{1}{\tilde{\mu}_t} + \chi \mathbb{E}_t \left[ \sum_{s=t+1}^{\infty} \beta_{h,t,s} \frac{\mathbb{E}_s^a[\xi_{i,s}]}{\mathbb{E}_t^a[\xi_{i,t}]} \left( p_{h,s} - \frac{1}{\tilde{\mu}_s} \right) \right]; \tag{B.4-7}$$

$$\frac{\mathbb{E}_t^a[\nu_{i,h,t}^*]}{\mathbb{E}_t^a[\xi_{i,t}]} = q_t p_{h,t}^* - \frac{1}{\tilde{\mu}_t} + \chi \mathbb{E}_t \left[ \sum_{s=t+1}^{\infty} \beta_{h,t,s}^* \frac{\mathbb{E}_s^a[\xi_{i,s}]}{\mathbb{E}_t^a[\xi_{i,t}]} \left( q_s p_{h,s}^* - \frac{1}{\tilde{\mu}_s} \right) \right], \tag{B.4-8}$$

where

$$\begin{aligned}
\chi &= (1-\rho)\theta(1-\eta); \\
\tilde{\mu}_t &= \frac{\mathbb{E}_t^a[\xi_{i,t}]}{\mathbb{E}_t^a[\xi_{i,t} a_{i,t}]} \frac{\alpha A_t}{w_t} \left(\phi + c_{h,t} + c_{h,t}^*\right)^{\frac{\alpha-1}{\alpha}}; \\
\beta_{h,t,s} &= m_{s,s+1} g_{h,s+1} \times \prod_{j=1}^{s-t} (\rho + \chi g_{h,t+j}) m_{t+j-1,t+j}, \quad \text{with } g_{h,t} = \frac{s_{h,t}/s_{h,t-1} - \rho}{1-\rho}; \\
\beta_{h,t,s}^* &= m_{s,s+1} g_{h,s+1}^* \times \prod_{j=1}^{s-t} (\rho + \chi g_{h,t+j}^*) m_{t+j-1,t+j}, \quad \text{with } g_{h,t}^* = \frac{s_{h,t}^*/s_{h,t-1}^* - \rho}{1-\rho}.
\end{aligned}$$

In the steady state, $g_{h,s+1} = g_{h,s+1}^* = 1$, which implies that $\beta_{h,t,s} = \beta_{h,t,s}^* = [\delta(\rho+\chi)]^{s-t}$. Hence, given that $\mathbb{E}^a[\nu_h]/\mathbb{E}^a[\xi] = p_h/\eta$ and $\mathbb{E}^a[\nu_h^*]/\mathbb{E}^a[\xi] = q p_h^*/\eta$, the log-linear dynamics of $\hat{\nu}_{h,t} - \hat{\xi}_t$ and $\hat{\nu}_{h,t}^* - \hat{\xi}_t$ are given by

$$\begin{aligned}
\hat{\nu}_{h,t} - \hat{\xi}_t = \eta \left( \hat{p}_{h,t} - \frac{\hat{\mu}_t}{p_h \tilde{\mu}} \right) + \eta \chi \mathbb{E}_t \left[ \sum_{s=t+1}^{\infty} \left[\delta(\rho+\chi)\right]^{s-t} \left( \hat{p}_{h,s} - \frac{\hat{\mu}_s}{p_h \tilde{\mu}} \right) \right] \\
+ \eta \chi \left( 1 - \frac{1}{p_h \tilde{\mu}} \right) \mathbb{E}_t \left[ \sum_{s=t+1}^{\infty} \left[\delta(\rho+\chi)\right]^{s-t} \left[\hat{\beta}_{h,t,s} - (\hat{\xi}_t - \hat{\xi}_s)\right] \right]
\end{aligned} \tag{B.4-9}$$

and

$$
\begin{aligned}
\hat{\nu}^*_{h,t} - \hat{\xi}_t = \eta &\left( (\hat{q}_t + \hat{p}_{h,t}) - \frac{\hat{\mu}_t}{q p^*_h \tilde{\mu}} \right) \\
&+ \chi \mathbb{E}_t \left[ \sum_{s=t+1}^{\infty} \big[\delta(\rho+\chi)\big]^{s-t} \left( (\hat{q}_s + \hat{p}^*_{h,s}) - \frac{\hat{\mu}_s}{q p^*_h \tilde{\mu}} \right) \right] \\
&+ \eta\chi \left(1 - \frac{1}{q p^*_h \tilde{\mu}}\right) \mathbb{E}_t \left[ \sum_{s=t+1}^{\infty} \big[\delta(\rho+\chi)\big]^{s-t} \big[\hat{\beta}^*_{h,t,s} - (\hat{\xi}_t - \hat{\xi}_s)\big] \right],
\end{aligned} \tag{B.4-10}
$$

where the steady-state values of $p_h$, $p^*_h$, and $\tilde{\mu}$ satisfy

$$
p_h = \frac{\eta(1-\rho\delta)}{(\eta-1)\big[(1-\rho\delta) - \theta\delta(1-\rho)\big]} \frac{1}{\tilde{\mu}};
$$

and

$$
p^*_h = \frac{\eta(1-\rho\delta)}{(\eta-1)\big[(1-\rho\delta) - \theta\delta(1-\rho)\big]} \frac{1}{q\tilde{\mu}}.
$$

**Foreign country:** The two Phillips curves pertaining to foreign firms are given by

$$
\begin{aligned}
0 = \frac{p^*_{f,t} c^*_{f,t}}{c^*_t} &- \gamma_p \pi^*_{f,t}(\pi^*_{f,t} - \pi^*) - \eta \frac{\mathbb{E}^a_t[\nu^*_{i,f,t}]}{\mathbb{E}^a_t[\xi^*_{i,t}]} \frac{c^*_{f,t}}{c^*_t} \\
&+ \gamma_p \mathbb{E}_t \left[ m^*_{t,t+1} \frac{\mathbb{E}^a_{t+1}[\xi^*_{i,t+1}]}{\mathbb{E}^a_t[\xi^*_{i,t}]} \pi^*_{f,t+1}(\pi^*_{f,t+1} - \pi^*) \frac{c^*_{t+1}}{c^*_t} \right];
\end{aligned} \tag{B.4-11}
$$

and

$$
\begin{aligned}
0 = \frac{q_t^{-1} p_{f,t} c_{i,f,t}}{c_t} &- \gamma_p q_t^{-1} \pi_{f,t}(\pi_{f,t} - \pi) - \eta \frac{\mathbb{E}^a_t[\nu_{i,f,t}]}{\mathbb{E}^a_t[\xi^*_{i,t}]} \frac{c_{f,t}}{c_t} \\
&+ \gamma_p \mathbb{E}_t \left[ m^*_{t,t+1} \frac{\mathbb{E}^a_{t+1}[\xi^*_{i,t+1}]}{\mathbb{E}^a_t[\xi^*_{i,t}]} q_{t+1}^{-1} \pi_{f,t+1}(\pi_{f,t+1} - \pi) \frac{c_{t+1}}{c_t} \right].
\end{aligned} \tag{B.4-12}
$$

In the steady state, these become

$$
0 = \frac{p^*_f c^*_f}{c^*} - \eta \frac{\nu^*_f}{\xi^*} \frac{c^*_f}{c^*}; \tag{B.4-13}
$$

and

$$
0 = \frac{q^{-1} p_f c_f}{c} - \eta \frac{\nu^*_f}{\xi^*} \frac{c_f}{c}. \tag{B.4-14}
$$

Imposing these conditions, one can derive the log-linear versions of the foreign Phillips curves as

$$
\hat{\pi}^*_{f,t} = \frac{1}{\gamma_p} \frac{p^*_f c^*_f}{c^*} \big[\hat{p}^*_{f,t} - (\hat{\nu}^*_{f,t} - \hat{\xi}^*_t)\big] + \delta \mathbb{E}_t[\hat{\pi}^*_{f,t+1}]; \tag{B.4-15}
$$

and

$$
\hat{\pi}_{f,t} = \frac{1}{\gamma_p} \frac{q^{-1} p_f c_f}{c} \big[\hat{p}_{f,t} - \hat{q}_t - (\hat{\nu}_{f,t} - \hat{\xi}^*_t)\big] + \delta \mathbb{E}_t[\hat{\pi}_{f,t+1}]. \tag{B.4-16}
$$

Given our symmetric setup, the log-linear dynamics of the terms $\hat{\nu}^*_{f,t} - \hat{\xi}^*_t$ and $\hat{\nu}^*_{f,t} - \hat{\xi}^*_t$ are governed by

$$\hat{\nu}^*_{f,t} - \hat{\xi}^*_t = \eta \left( \hat{p}^*_{f,t} - \frac{\hat{\mu}^*_t}{p^*_f \tilde{\mu}^*} \right) + \eta\chi \mathbb{E}_t \left[ \sum_{s=t+1}^{\infty} \left[\delta(\rho+\chi)\right]^{s-t} \left( \hat{p}^*_{f,s} - \frac{\hat{\mu}_s}{p^*_f \tilde{\mu}^*} \right) \right] + \eta\chi \left( 1 - \frac{1}{p^*_f \tilde{\mu}^*} \right) \mathbb{E}_t \left[ \sum_{s=t+1}^{\infty} \left[\delta(\rho+\chi)\right]^{s-t} \left[\hat{\beta}^*_{f,t,s} - (\hat{\xi}^*_t - \hat{\xi}^*_s)\right] \right] \tag{B.4-17}$$

and

$$\hat{\nu}_{f,t} - \hat{\xi}^*_t = \eta \left( (\hat{p}_{f,t} - \hat{q}_t) - \frac{\hat{\mu}^*_t}{q^{-1} p_f \tilde{\mu}^*} \right) + \chi \mathbb{E}_t \left[ \sum_{s=t+1}^{\infty} \left[\delta(\rho+\chi)\right]^{s-t} \left( (\hat{p}_{f,s} - \hat{q}_s) - \frac{\hat{\mu}^*_s}{q^{-1} p_f \tilde{\mu}^*} \right) \right] + \eta\chi \left( 1 - \frac{1}{q^{-1} p_f \tilde{\mu}^*} \right) \mathbb{E}_t \left[ \sum_{s=t+1}^{\infty} \left[\delta(\rho+\chi)\right]^{s-t} \left[\hat{\beta}_{f,t,s} - (\hat{\xi}^*_t - \hat{\xi}^*_s)\right] \right], \tag{B.4-18}$$

where $p^*_f$, $p_f$, and $\tilde{\mu}^*$ satisfy the following steady-state relationships:

$$p^*_f = \frac{\eta(1-\rho\delta)}{(\eta-1)\left[(1-\rho\delta) - \theta\delta(1-\rho)\right]} \frac{1}{\tilde{\mu}^*}$$

$$p_f = \frac{\eta(1-\rho\delta)}{(\eta-1)\left[(1-\rho\delta) - \theta\delta(1-\rho)\right]} \frac{q}{\tilde{\mu}^*}.$$

## B.5 Equilibrium System of Equations

There are a total of 71 equations for 71 endogenous variables in the system characterizing the symmetric equilibrium in the case of floating exchange rates and when the two countries have a complete risk-sharing arrangement. We provide these equations in their symmetric equilibrium forms:

$$0 = -\frac{h_t^{1/\zeta}/U_{x,t}}{w_t/\tilde{p}_t} + \frac{\eta_w - 1}{\eta_w} + \frac{\gamma_w}{\eta_w}(\pi_{w,t} - \pi_w)\pi_{w,t} - \delta \frac{\gamma_w}{\eta_w} \mathbb{E}_t \left[ \frac{U_{x,t+1}/\tilde{p}_{t+1}}{U_{x,t}/\tilde{p}_t} (\pi_{w,t+1} - \pi_w)\pi_{w,t+1} \frac{\pi_{w,t+1}}{\pi_{t+1}} \frac{h_{t+1}}{h_t} \right]; \tag{B.5-1}$$

$$0 = -\frac{h_t^{*1/\zeta}/U^*_{x,t}}{w^*_t/\tilde{p}^*_t} + \frac{\eta_w - 1}{\eta_w} + \frac{\gamma_w}{\eta_w}(\pi^*_{w,t} - \pi^*_w)\pi^*_{w,t} - \delta \frac{\gamma_w}{\eta_w} \mathbb{E}_t \left[ \frac{U^*_{x,t+1}/\tilde{p}^*_{t+1}}{U^*_{x,t}/\tilde{p}^*_{w,t+1}} (\pi^*_{w,t+1} - \pi^*_w)\pi^*_{w,t+1} \frac{\pi^*_{w,t+1}}{\pi^*_{t+1}} \frac{h^*_{t+1}}{h^*_t} \right]. \tag{B.5-2}$$

$$0 = -\frac{c_{h,t}}{c_{f,t}} + \left(\frac{\Xi_h}{\Xi_f}\right)^{\varepsilon} \left(\frac{p_{h,t}}{p_{f,t}}\right)^{-\varepsilon} \left(\frac{s^{\theta}_{h,t-1}}{s^{\theta}_{f,t-1}}\right)^{1-\varepsilon}; \tag{B.5-3}$$

$$0 = -\frac{c^*_{h,t}}{c^*_{f,t}} + \left(\frac{\Xi^*_h}{\Xi^*_f}\right)^{\varepsilon} \left(\frac{p^*_{h,t}}{p^*_{f,t}}\right)^{-\varepsilon} \left(\frac{s^{*\theta}_{h,t-1}}{s^{*\theta}_{f,t-1}}\right)^{1-\varepsilon}. \tag{B.5-4}$$

$$0 = -\tilde{p}_{h,t} + s^{\theta}_{h,t-1}; \tag{B.5-5}$$
$$0 = -\tilde{p}_{f,t} + s^{\theta}_{f,t-1}; \tag{B.5-6}$$
$$0 = -\tilde{p}^*_{h,t} + s^{*\theta}_{h,t-1} \tag{B.5-7}$$
$$0 = -\tilde{p}^*_{f,t} + s^{*\theta}_{f,t-1}. \tag{B.5-8}$$

$$0 = -x_{h,t} + \Xi^{\varepsilon}_{h} p^{-\varepsilon}_{h,t} \left( \frac{\tilde{p}_{h,t}}{\tilde{p}_t} \right)^{-\varepsilon} x_t; \tag{B.5-9}$$
$$0 = -x_{f,t} + \Xi^{\varepsilon}_{f} p^{-\varepsilon}_{f,t} \left( \frac{\tilde{p}_{f,t}}{\tilde{p}_t} \right)^{-\varepsilon} x_t; \tag{B.5-10}$$
$$0 = -x^*_{h,t} + \Xi^{*\varepsilon}_{h} (p^*_{h,t})^{-\varepsilon} \left( \frac{\tilde{p}^*_{h,t}}{\tilde{p}^*_t} \right)^{-\varepsilon} x^*_t; \tag{B.5-11}$$
$$0 = -x^*_{f,t} + \Xi^{*\varepsilon}_{f} (p^*_{f,t})^{-\varepsilon} \left( \frac{\tilde{p}^*_{f,t}}{\tilde{p}^*_t} \right)^{-\varepsilon} x^*_t. \tag{B.5-12}$$

$$0 = -\tilde{p}_t + \left[ \sum_{k=h,f} \Xi_k s^{\theta(1-\varepsilon)}_{k,t-1} p^{1-\varepsilon}_{k,t} \right]^{\frac{1}{1-\varepsilon}}; \tag{B.5-13}$$
$$0 = -\tilde{p}^*_t + \left[ \sum_{k=h,f} \Xi^*_k s^{*\theta(1-\varepsilon)}_{k,t-1} p^{*(1-\varepsilon)}_{k,t} \right]^{\frac{1}{1-\varepsilon}}. \tag{B.5-14}$$

$$0 = -\pi_{h,t} + \frac{p_{h,t}}{p_{h,t-1}} \pi_t; \tag{B.5-15}$$
$$0 = -\pi^*_{h,t} + \frac{p^*_{h,t}}{p^*_{h,t-1}} \pi^*_t; \tag{B.5-16}$$
$$0 = -\pi_{f,t} + \frac{p_{f,t}}{p_{f,t-1}} \pi_t; \tag{B.5-17}$$
$$0 = -\pi^*_{f,t} + \frac{p^*_{f,t}}{p^*_{f,t-1}} \pi^*_t. \tag{B.5-18}$$

$$0 = -h^S_t + h^D_t; \tag{B.5-19}$$
$$0 = -h^{*S}_t + h^{*D}_t. \tag{B.5-20}$$

$$0 = -\mathbb{E}^a_t[\kappa_{i,t}] + \mathbb{E}^a_t[\xi_{i,t} a_{i,t}] \frac{w_t}{\alpha A_t} \left( \phi + c_{h,t} + c^*_{h,t} \right)^{\frac{1-\alpha}{\alpha}}; \tag{B.5-21}$$
$$0 = -\mathbb{E}^a_t[\kappa^*_{i,t}] + \mathbb{E}^a_t[\xi^*_{i,t} a^*_{i,t}] \frac{w^*_t}{\alpha A^*_t} \left( \phi^* + c_{f,t} + c^*_{f,t} \right)^{\frac{1-\alpha}{\alpha}}. \tag{B.5-22}$$

$$0 = -\nu_{h,t} + \mathbb{E}_t^a[\xi_{i,t}]p_{h,t} - \mathbb{E}_t^a[\kappa_{i,t}] + (1-\rho)\lambda_{h,t}; \tag{B.5-23}$$

$$0 = -\nu_{h,t} + \mathbb{E}_t^a[\xi_{i,t}]q_t p^*_{h,t} - \mathbb{E}_t^a[\kappa_{i,t}] + (1-\rho)\lambda^*_{h,t}; \tag{B.5-24}$$

$$0 = -\nu^*_{f,t} + \mathbb{E}_t^a[\xi^*_{i,t}]p^*_{f,t} - \mathbb{E}_t^a[\kappa^*_{i,t}] + (1-\rho)\lambda^*_{f,t}; \tag{B.5-25}$$

$$0 = -\nu_{f,t} + \mathbb{E}_t^a[\xi^*_{i,t}]q_t^{-1} p_{f,t} - \mathbb{E}_t^a[\kappa^*_{i,t}] + (1-\rho)\lambda_{f,t}. \tag{B.5-26}$$

$$0 = -\lambda_{h,t} + \rho\mathbb{E}_t[m_{t,t+1}\lambda_{h,t+1}] + \theta(1-\eta)\mathbb{E}_t\left[m_{t,t+1}\mathbb{E}_{t+1}^a\left[\nu_{h,t+1}\frac{c_{h,t+1}}{s_{h,t}}\right]\right]; \tag{B.5-27}$$

$$0 = -\lambda^*_{h,t} + \rho\mathbb{E}_t[m_{t,t+1}\lambda^*_{h,t+1}] + \theta(1-\eta)\mathbb{E}_t\left[m_{t,t+1}\mathbb{E}_{t+1}^a\left[\nu^*_{h,t+1}\frac{c^*_{h,t+1}}{s^*_{h,t}}\right]\right]; \tag{B.5-28}$$

$$0 = -\lambda^*_{f,t} + \rho\mathbb{E}_t[m^*_{t,t+1}\lambda^*_{f,t+1}] + \theta(1-\eta)\mathbb{E}_t\left[m^*_{t,t+1}\mathbb{E}_{t+1}^a\left[\nu^*_{f,t+1}\frac{c^*_{f,t+1}}{s^*_{f,t}}\right]\right]; \tag{B.5-29}$$

$$0 = -\lambda_{f,t} + \rho\mathbb{E}_t[m^*_{t,t+1}\lambda_{f,t+1}] + \theta(1-\eta)\mathbb{E}_t\left[m^*_{t,t+1}\mathbb{E}_{t+1}^a\left[\nu_{f,t+1}\frac{c_{f,t+1}}{s_{f,t}}\right]\right]. \tag{B.5-30}$$

$$\begin{aligned} 0 = &-p_{h,t}\frac{c_{h,t}}{c_t} + \gamma_p\pi_{h,t}(\pi_{h,t}-\pi) + \eta\frac{\nu_{h,t}}{\mathbb{E}_t^a[\xi_{i,t}]}\frac{c_{h,t}}{c_t} \\ &- \gamma_p\mathbb{E}_t\left[m_{t,t+1}\frac{\mathbb{E}_{t+1}^a[\xi_{i,t+1}]}{\mathbb{E}_t^a[\xi_{i,t}]}\pi_{h,t+1}(\pi_{h,t+1}-\pi)\frac{c_{t+1}}{c_t}\right]; \end{aligned} \tag{B.5-31}$$

$$\begin{aligned} 0 = &-q_t p^*_{h,t}\frac{c^*_{h,t}}{c^*_t} + \gamma_p q_t\pi^*_{h,t}(\pi^*_{h,t}-\pi^*) + \eta\frac{\nu^*_{h,t}}{\mathbb{E}_t^a[\xi_{i,t}]}\frac{c^*_{h,t}}{c^*_t} \\ &- \gamma_p\mathbb{E}_t\left[m_{t,t+1}\frac{\mathbb{E}_{t+1}^a[\xi_{i,t+1}]}{\mathbb{E}_t^a[\xi_{i,t}]}q_{t+1}\pi^*_{h,t+1}(\pi^*_{h,t+1}-\pi)\frac{c^*_{t+1}}{c^*_t}\right]; \end{aligned} \tag{B.5-32}$$

$$\begin{aligned} 0 = &-p^*_{f,t}\frac{c^*_{f,t}}{c^*_t} + \gamma_p\pi^*_{f,t}(\pi^*_{f,t}-\pi^*) + \eta\frac{\nu^*_{f,t}}{\mathbb{E}_t^a[\xi^*_{i,t}]}\frac{c^*_{f,t}}{c^*_t} \\ &- \gamma_p\mathbb{E}_t\left[m^*_{t,t+1}\frac{\mathbb{E}_{t+1}^a[\xi^*_{i,t+1}]}{\mathbb{E}_t^a[\xi^*_{i,t}]}\pi^*_{f,t+1}(\pi^*_{f,t+1}-\pi^*)\frac{c^*_{t+1}}{c^*_t}\right]; \end{aligned} \tag{B.5-33}$$

$$\begin{aligned} 0 = &-q_t^{-1}p_{f,t}\frac{c_{f,t}}{c_t} + \gamma_p q_t^{-1}\pi_{f,t}(\pi_{f,t}-\pi) + \eta\frac{\nu_{f,t}}{\mathbb{E}_t^a[\xi^*_{i,t}]}\frac{c_{f,t}}{c_t} \\ &- \gamma_p\mathbb{E}_t\left[m^*_{t,t+1}\frac{\mathbb{E}_{t+1}^a[\xi^*_{i,t+1}]}{\mathbb{E}_t^a[\xi^*_{i,t}]}q_{t+1}^{-1}\pi_{f,t+1}(\pi_{f,t+1}-\pi)\frac{c_{t+1}}{c_t}\right]. \end{aligned} \tag{B.5-34}$$

$$0 = -\mu_t + \frac{\alpha A_t}{w_t}\left(\phi + c_{h,t} + c^*_{h,t}\right)^{\frac{\alpha-1}{\alpha}}; \tag{B.5-35}$$

$$0 = -\mu^*_t + \frac{\alpha A^*_t}{w^*_t}\left(\phi^* + c_{f,t} + c^*_{f,t}\right)^{\frac{\alpha-1}{\alpha}}. \tag{B.5-36}$$

$$0 = -\tilde{\mu}_t + \frac{\mathbb{E}_t^a[\xi_{i,t}]}{\mathbb{E}_t^a[\xi_{i,t}a_{i,t}]}\mu_t; \quad \text{(B.5-37)}$$

$$0 = -\tilde{\mu}_t^* + \frac{\mathbb{E}_t^a[\xi_{i,t}^*]}{\mathbb{E}_t^a[\xi_{i,t}^* a_{i,t}^*]}\mu_t^*. \quad \text{(B.5-38)}$$

$$\begin{aligned} 0 = -a_t^E + &\frac{A_t}{w_t\big(\phi + c_{h,t} + c_{h,t}^*\big)^{\frac{1}{\alpha}}} \\ &\times \left[c_t\left[\frac{p_{h,t}c_{h,t}}{c_t} - \frac{\gamma_p}{2}(\pi_{h,t}-\pi)^2\right] + q_t c_t^*\left[\frac{p_{h,t}^* c_{h,t}^*}{c_t^*} - \frac{\gamma_p}{2}(\pi_{h,t}^*-\pi^*)^2\right]\right]; \end{aligned} \quad \text{(B.5-39)}$$

$$\begin{aligned} 0 = -a_t^{*E} + &\frac{A_t^*}{w_t^*\big(\phi^* + c_{f,t} + c_{f,t}^*\big)^{\frac{1}{\alpha}}} \\ &\times \left[c_t^*\left[\frac{p_{f,t}^* c_{f,t}^*}{c_t^*} - \frac{\gamma_p}{2}(\pi_{f,t}^*-\pi)^2\right] + q_t^{-1} c_t\left[\frac{p_{f,t}c_{f,t}}{c_t} - \frac{\gamma_p}{2}(\pi_{f,t}-\pi)^2\right]\right]. \end{aligned} \quad \text{(B.5-40)}$$

$$0 = -z_t^E + \sigma^{-1}\big(\ln a_t^E + 0.5\sigma^2\big); \quad \text{(B.5-41)}$$

$$0 = -z_t^{*E} + \sigma^{-1}\big(\ln a_t^{*E} + 0.5\sigma^2\big). \quad \text{(B.5-42)}$$

$$0 = -\mathbb{E}_t^a[\xi_{i,t}] + 1 + \frac{\varphi_t}{1-\varphi_t}\big[1 - \Phi(z_t^E)\big]; \quad \text{(B.5-43)}$$

$$0 = -\mathbb{E}_t^a[\xi_{i,t}^*] + 1 + \frac{\varphi_t^*}{1-\varphi_t^*}\big[1 - \Phi(z_t^{*E})\big]. \quad \text{(B.5-44)}$$

$$0 = -\mathbb{E}_t^a[\xi_{i,t}a_{i,t}] + 1 + \frac{\varphi_t}{1-\varphi_t}\big[1 - \Phi(z_t^E - \sigma)\big]; \quad \text{(B.5-45)}$$

$$0 = -\mathbb{E}_t^a[\xi_{i,t}^* a_{i,t}^*] + 1 + \frac{\varphi_t^*}{1-\varphi_t^*}\big[1 - \Phi(z_t^{*E} - \sigma)\big]. \quad \text{(B.5-46)}$$

$$0 = -h_t^D + \left[\frac{\phi + c_{h,t} + c_{h,t}^*}{A_t^\alpha \exp\big[0.5\alpha(1+\alpha)\sigma^2\big]}\right]^{\frac{1}{\alpha}}; \quad \text{(B.5-47)}$$

$$0 = -h_t^{*S} + \left[\frac{\phi^* + c_{f,t} + c_{f,t}^*}{A_t^{*\alpha} \exp\big[0.5\alpha(1+\alpha)\sigma^2\big]}\right]^{\frac{1}{\alpha}}. \quad \text{(B.5-48)}$$

$$0 = -U_{x,t} + (x_t - \omega_t)^{-\gamma_x}; \quad \text{(B.5-49)}$$

$$0 = -U_{x,t}^* + (x_t^* - \omega_t^*)^{-\gamma_x}. \quad \text{(B.5-50)}$$

$$0 = -y_t + \exp\big[0.5\alpha(1+\alpha)\sigma^2\big](A_t h_t)^\alpha - \phi; \quad \text{(B.5-51)}$$
$$0 = -y_t^* + \exp\big[0.5\alpha(1+\alpha)\sigma^2\big](A_t^* h_t^*)^\alpha - \phi^*. \quad \text{(B.5-52)}$$

$$0 = -1 + \mathbb{E}_t\left[\delta \frac{U_{x,t+1}/\tilde{p}_{t+1}}{U_{x,t}/\tilde{p}_t} \frac{R_t}{\pi_{t+1}}\right]; \quad \text{(B.5-53)}$$
$$0 = -1 + \mathbb{E}_t\left[\delta \frac{U^*_{x,t+1}/\tilde{p}^*_{t+1}}{U^*_{x,t}/\tilde{p}^*_t} \frac{R^*_t}{\pi^*_{t+1}}\right]. \quad \text{(B.5-54)}$$

$$0 = -R_t + R\left(\frac{y_t}{y}\right)^{\psi_y}\left(\frac{\pi_t}{\pi}\right)^{\psi_\pi}; \quad \text{(B.5-55)}$$
$$0 = -R^*_t + R^*\left(\frac{y^*_t}{y^*}\right)^{\psi_y}\left(\frac{\pi^*_t}{\pi^*}\right)^{\psi_\pi}. \quad \text{(B.5-56)}$$

$$0 = -c_t + p_{h,t}c_{h,t} + p_{f,t}c_{f,t}; \quad \text{(B.5-57)}$$
$$0 = -c^*_t + p^*_{h,t}c^*_{h,t} + p^*_{f,t}c^*_{f,t}. \quad \text{(B.5-58)}$$

$$0 = -\pi_t + \left[\sum_{k=h,f} \Xi_k (p_{k,t-1}\pi_{k,t})^{1-\varepsilon}\right]^{1/(1-\varepsilon)}; \quad \text{(B.5-59)}$$
$$0 = -\pi^*_t + \left[\sum_{k=h,f} \Xi^*_k (p^*_{k,t-1}\pi^*_{k,t})^{1-\varepsilon}\right]^{1/(1-\varepsilon)}; \quad \text{(B.5-60)}$$
$$0 = -x_t + \left[\sum_{k=h,f} \Xi_k (c_{k,t}^{1-\theta})^{1-1/\varepsilon}\right]^{1/(1-1/\varepsilon)}; \quad \text{(B.5-61)}$$
$$0 = -x^* + \left[\sum_{k=h,f} \Xi^*_k (c_{k,t}^{*1-\theta})^{1-1/\varepsilon}\right]^{1/(1-1/\varepsilon)}. \quad \text{(B.5-62)}$$

$$0 = -s_{h,t} + \rho s_{h,t-1} + (1-\rho)c_{h,t}; \quad \text{(B.5-63)}$$
$$0 = -s_{f,t} + \rho s_{f,t-1} + (1-\rho)c_{f,t}; \quad \text{(B.5-64)}$$
$$0 = -s^*_{h,t} + \rho s^*_{h,t-1} + (1-\rho)c^*_{h,t}; \quad \text{(B.5-65)}$$
$$0 = -s^*_{f,t} + \rho s^*_{f,t-1} + (1-\rho)c^*_{f,t}. \quad \text{(B.5-66)}$$

**Complete risk sharing with floating exchange rates:** With a complete risk-sharing arrangement, the real exchange rate is determined by the risk-sharing condition:

$$0 = -q_t + \varrho_0 \frac{U^*_{x,t}/\tilde{p}^*_t}{U_{x,t}/\tilde{p}_t}, \quad \text{where } \varrho_0 = q_0 \frac{U_{x,0}/\tilde{p}_0}{\tilde{U}^*_{x,0}/\tilde{p}^*_0}. \quad \text{(B.5-67)}$$

**Incomplete risk sharing with floating exchange rates:** With incomplete risk sharing, the following conditions govern the equilibrium in the international capital markets:

$$0 = -(1+\tau b_{h,t+1}) + \delta \mathbb{E}_t\left[\frac{U_{x,t+1}/\tilde{p}_{t+1}}{U_{x,t}/\tilde{p}_t}\frac{R_t}{\pi_{t+1}}\right]; \quad \text{(B.5-68)}$$

$$0 = -(1+\tau b_{f,t+1}) + \delta \mathbb{E}_t\left[\frac{U_{x,t+1}/\tilde{p}_{t+1}}{U_{x,t}/\tilde{p}_t}\frac{R^*_t}{\pi^*_{t+1}}\frac{q_{t+1}}{q_t}\right]; \quad \text{(B.5-69)}$$

$$0 = -(1+\tau b^*_{h,t+1}) + \delta \mathbb{E}_t\left[\frac{U^*_{x,t+1}/\tilde{p}^*_{t+1}}{U^*_{x,t}/\tilde{p}^*_t}\frac{R_t}{\pi_{t+1}}\frac{q_t}{q_{t+1}}\right]; \quad \text{(B.5-70)}$$

$$0 = -(1+\tau b^*_{f,t+1}) + \delta \mathbb{E}_t\left[\frac{U^*_{x,t+1}/\tilde{p}^*_{t+1}}{U^*_{x,t}/\tilde{p}^*_t}\frac{R^*_t}{\pi^*_{t+1}}\right]. \quad \text{(B.5-71)}$$

$$0 = b_{h,t+1} + b^*_{h,t+1}; \quad \text{(B.5-72)}$$

$$0 = b_{f,t+1} + b^*_{f,t+1}. \quad \text{(B.5-73)}$$

$$\begin{aligned}0 = &-(b_{h,t+1} + q_t b_{f,t+1}) + \frac{R_{t-1}}{\pi_t}b_{h,t} + \frac{R^*_{t-1}}{\pi^*_t}q_t b_{f,t}\\ &+\frac{1}{2}(w_t h_t - q_t w^*_t h^*_t) + \frac{1}{2}(\tilde{d}_t - q_t\tilde{d}^*_t) - \frac{1}{2}(\tilde{p}_t x_t - q_t \tilde{p}^*_t x^*_t);\end{aligned} \quad \text{(B.5-74)}$$

where

$$0 = -\tilde{d}_t + \tilde{d}^+_t + (1-\varphi_t)\tilde{d}^-_t; \quad \text{(B.5-75)}$$

$$0 = -\tilde{d}^*_t + \tilde{d}^{*+}_t + (1-\varphi^*_t)\tilde{d}^{*-}_t; \quad \text{(B.5-76)}$$

and

$$\begin{aligned}0 = -\tilde{d}^+_t + \Phi(z^E_t)\Bigg[&p_{h,t}c_{h,t} + q_t p^*_{h,t}c^*_{h,t} - \frac{w_t}{A_t}\frac{\Phi(z^E_t-\sigma)}{\Phi(z^E_t)}\left(\phi + c_{h,t} + c^*_{h,t}\right)^{\frac{1}{\alpha}}\\ &-\frac{\gamma_p}{2}(\pi_{h,t}-\pi)^2 c_t - \frac{\gamma_p}{2}q_t(\pi^*_{h,t}-\pi^*)^2 c^*_t\Bigg];\end{aligned} \quad \text{(B.5-77)}$$

$$\begin{aligned}0 = -\tilde{d}^-_t + \frac{1-\Phi(z^E_t)}{1-\varphi_t}\Bigg[&p_{h,t}c_{h,t} + q_t p^*_{h,t}c^*_{h,t} - \frac{w_t}{A_t}\frac{1-\Phi(z^E_t-\sigma)}{1-\Phi(z^E_t)}\left(\phi + c_{h,t} + c^*_{h,t}\right)^{\frac{1}{\alpha}}\\ &-\frac{\gamma_p}{2}(\pi_{h,t}-\pi)^2 c_t - \frac{\gamma_p}{2}q_t(\pi^*_{h,t}-\pi^*)^2 c^*_t\Bigg];\end{aligned} \quad \text{(B.5-78)}$$

$$\begin{aligned}0 = -\tilde{d}^{*+}_t + \Phi(z^{*E}_t)\Bigg[&q_t^{-1}p_{f,t}c_{f,t} + p^*_{f,t}c^*_{f,t} - \frac{w^*_t}{A^*_t}\frac{\Phi(z^{*E}_t-\sigma)}{\Phi(z^{*E}_t)}\left(\phi^* + c_{f,t} + c^*_{f,t}\right)^{\frac{1}{\alpha}}\\ &-\frac{\gamma_p}{2}q_t^{-1}(\pi_{f,t}-\pi)^2 c_t - \frac{\gamma_p}{2}(\pi^*_{f,t}-\pi^*)^2 c^*_t\Bigg];\end{aligned} \quad \text{(B.5-79)}$$

$$\begin{aligned}0 = -\tilde{d}^{*-}_t + \frac{1-\Phi(z^{*E}_t)}{1-\varphi^*_t}\Bigg[&q_t^{-1}p_{f,t}c_{f,t} + p^*_{f,t}c^*_{f,t} - \frac{w^*_t}{A^*_t}\frac{1-\Phi(z^{*E}_t-\sigma)}{1-\Phi(z^{*E}_t)}\left(\phi^* + c_{f,t} + c^*_{f,t}\right)^{\frac{1}{\alpha}}\\ &-\frac{\gamma_p}{2}q_t^{-1}(\pi_{f,t}-\pi)^2 c_t - \frac{\gamma_p}{2}(\pi^*_{f,t}-\pi^*)^2 c^*_t\Bigg].\end{aligned} \quad \text{(B.5-80)}$$

**Incomplete risk sharing in a monetary union:** In a monetary union with incomplete risk sharing, equations (B.5-69)–(B.5-71) are replaced with

$$
\begin{aligned}
0 &= -(1+\tau b_{h,t+1}) + \delta \mathbb{E}_t \left[ \frac{U_{x,t+1}/\tilde{p}_{t+1}}{U_{x,t+1}/\tilde{p}_{t+1}} \frac{R_t^U}{\pi_{t+1}} \right]; & \text{(B.5-81)} \\
0 &= -(1+\tau b^*_{h,t+1}) + \delta \mathbb{E}_t \left[ \frac{U^*_{x,t+1}/\tilde{p}^*_{t+1}}{U^*_{x,t+1}/\tilde{p}^*_{t+1}} \frac{R_t^U}{\pi^*_{t+1}} \right]. & \text{(B.5-82)}
\end{aligned}
$$

In addition, the bond market clearing condition $0 = b_{f,t+1} + b^*_{f,t+1}$ is deleted, and the following identity is added to the system:

$$
\frac{\mathbb{E}_t[q_{t+1}]}{q_t} = \frac{\mathbb{E}_t[S_{t+1}]}{S_t} \frac{\mathbb{E}_t[\pi^*_{t+1}]}{\mathbb{E}_t[\pi_{t+1}]}. \qquad \text{(B.5-83)}
$$

Note that $S_t$ is not a model variable, as the level of nominal exchange rate cannot be determined in the steady state. However, $\pi^S_{t+1} \equiv S_{t+1}/S_t$ is a well-defined model variable.

**Exogenous variables:** There are six exogenous aggregate variables in the model, all of which are assumed to follow AR(1) processes.

Technology shocks:

$$
\begin{aligned}
0 &= -\ln A_t + \rho_A \ln A_{t-1} + \epsilon_{A,t}; & \text{(B.5-84)} \\
0 &= -\ln A^*_t + \rho_A \ln A^*_{t-1} + \epsilon^*_{A,t}. & \text{(B.5-85)}
\end{aligned}
$$

Demand shocks:

$$
\begin{aligned}
0 &= -\omega_t + \rho_\omega \omega_{t-1} + \epsilon_{\omega,t}; & \text{(B.5-86)} \\
0 &= -\omega^*_t + \rho_\omega \omega^*_{t-1} + \epsilon^*_{\omega,t}. & \text{(B.5-87)}
\end{aligned}
$$

Financial shocks:

$$
\begin{aligned}
0 &= -\ln \varphi_t + (1-\rho_f) \ln \varphi + \rho_f \ln \varphi_{t-1} + \epsilon_{f,t}; & \text{(B.5-88)} \\
0 &= -\ln \varphi^*_t + (1-\rho_f) \ln \varphi^* + \rho_f \ln \varphi^*_{t-1} + \epsilon^*_{f,t}. & \text{(B.5-89)}
\end{aligned}
$$

## C Calibration Summary

The entries in the table denote the values of the model parameters used in the baseline calibration of the model.

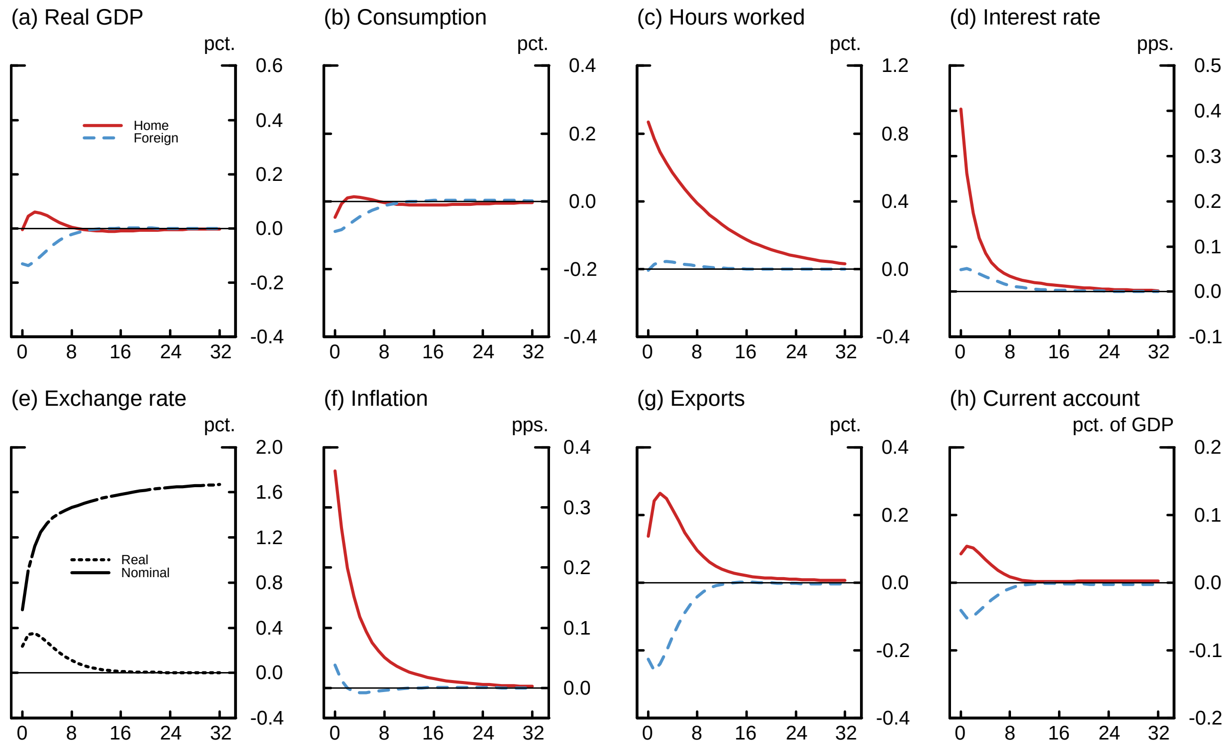
TABLE C-1 – Baseline Calibration

| Model Parameters | Value |
|---|---|
| *Preferences & technology* | |
| time discount factor ($\delta$) | 0.996 |
| constant relative risk aversion ($\gamma_x$) | 2.000 |
| elasticity of labor supply ($\zeta$) | 5.000 |
| strength of deep habits ($\theta$) | $-0.860$ |
| persistence of deep habits ($\rho$) | 0.850 |
| elasticity of substitution ($\eta$) | 2.000 |
| Armington elasticity ($\varepsilon$) | 1.500 |
| home bias ($\Xi_h^{\varepsilon}, \Xi_f^{*\varepsilon}$) | $(0.600, 0.600)$ |
| returns-to-scale ($\alpha$) | 1.000 |
| fixed operating costs ($\phi, \phi^*$) | $(0.10, 0.10)$ |
| *Nominal rigidities & monetary policy* | |
| price adjustment costs ($\gamma_p$) | 10.00 |
| wage adjustment costs ($\gamma_w$) | 30.00 |
| Taylor rule inflation gap coefficient ($\psi_\pi$) | 1.500 |
| Taylor rule output gap coefficient ($\psi_y$) | 1.000 |
| *Financial frictions & shocks* | |
| equity dilution costs ($\varphi, \varphi^*$) | $(0.20, 0.02)$ |
| std. deviation of idiosyncratic cost shock ($\sigma$) | 0.200 |
| portfolio rebalancing costs ($\tau$) | 0.150 |
| persistence of aggregate financial shocks ($\rho_f$) | 0.900 |
| persistence of aggregate technology shocks ($\rho_A$) | 0.900 |
| persistence of aggregate demand shocks ($\rho_\omega$) | 0.900 |

# D The Impact of Technology Shocks

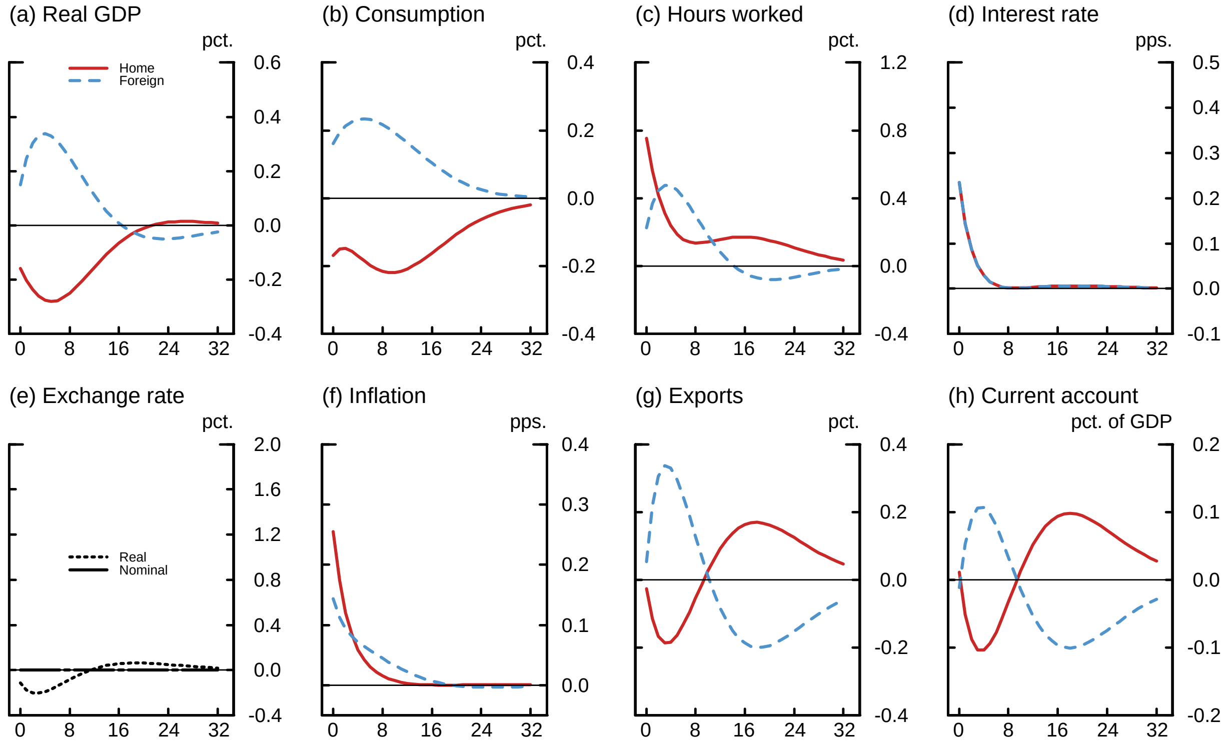
In this section, we examine the model dynamics in response to aggregate technology shocks under different currency regimes. Figure D-1 depicts the dynamics of selected macroeconomic variables in response to an adverse technology shock in the home country under floating exchange rates. Figure D-2, by contrast, shows the dynamics of the same variables in the case when the two countries are in a monetary union.

Figure D-1 – An Asymmetric Technology Shock: Floating Exchange Rates




Note: The panels of the figure depict the model-implied responses of selected variables to an adverse technology shock in the home country in period 0 (see the main text for details). Unless noted otherwise, the solid lines show responses of variables in the home country, while the dashed lines show those of the foreign country. Exchange rates (panel (e)) are expressed as home currency relative to foreign currency.

Figure D-2 – An Asymmetric Technology Shock: Monetary Union




Note: The panels of the figure depict the model-implied responses of selected variables to an adverse technology shock in the home country in period 0 (see the main text for details). Unless noted otherwise, the solid lines show responses of variables in the home country, while the dashed lines show those of the foreign country. Exchange rates (panel (e)) are expressed as home currency relative to foreign currency.