

# Leverage and Deepening Business Cycle Skewness\*

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## Abstract

Since the mid 1980s the US economy has displayed a marked reduction in the volatility of business fluctuations, a phenomenon labeled as the Great Moderation. We document that, over the same period, the business cycle has also been characterized by an increasingly negative skewness. This finding can be explained by the concurrent increase in leverage of both households and firms. To demonstrate this point, we devise a DSGE model with collateralized borrowing and occasionally binding credit constraints. Looser credit increases the likelihood that constraints become slack in the face of expansionary shocks, while contractionary shocks are further amplified due to tighter constraints. As a result, booms gradually become smoother than busts. Based on the differential impact that occasionally binding constraints exert on the shape of expansions and contractions, we are able to reconcile a more negatively skewed business cycle with a moderation in its volatility. Finally, our model can account for an intrinsic feature of economic downturns preceded by private credit build-ups: Financially driven expansions lead to deeper contractions, as compared with equally-sized non-financial expansions.

*Keywords:* Credit constraints, business cycles, skewness, deleveraging.

*JEL:* E32, E44.

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# 1 Introduction

Economic fluctuations across the industrialized world are typically characterized by asymmetries in the shape of expansions and contractions in aggregate activity. A prolific literature has extensively studied the statistical properties of this empirical regularity, reporting that, relative to expansions, contractions are periods of larger output fluctuations; see, among others, [Neftci \(1984\)](#), [Hamilton \(1989\)](#), [Sichel \(1993\)](#) and, more recently, [Morley and Piger \(2012\)](#). While these studies have generally indicated that business fluctuations are negatively skewed, the possibility that business cycle asymmetry may have changed over time has been overlooked. Yet, we know that the shape of the business cycle has evolved over the last three decades: For instance, since the mid 1980s the U.S. economy has displayed a marked decline in macroeconomic volatility, a phenomenon dubbed as the Great Moderation.<sup>1</sup> This paper documents that, over the same period, the skewness of the U.S. business cycle has become increasingly negative. Our key contribution is to show that occasionally binding financial constraints, combined with a sustained increase in private debt, allow us to capture several factors underlying the transformation of business cycle asymmetry.

[Figure 1](#) reports the post-WWII rate of growth of real GDP, together with the 68% and 90% confidence intervals from a Gaussian density fitted on pre- and post-1984 data points. Three facts stand out: First, as discussed above, the U.S. business cycle has become less volatile in the second part of the sample, even if we account for the major turmoil induced by the Great Recession. Second, real GDP growth displays large swings in either direction during the first part of the sample, while in the post-1984 period this is the case only during the three recessionary episodes. In fact, if we examine the size of economic contractions in connection with the drop in volatility occurring since the mid 1980s, it appears that recessions have become relatively more ‘violent’, whereas the ensuing recoveries have become smoother, as recently pointed out by [Fatás and Mihov \(2013\)](#). Third, recessionary episodes have become less frequent, thus implying more prolonged expansions.

[Insert [Figure 1](#)]

All in all, these properties translate into the U.S. business cycle becoming more negatively skewed over the last three decades. Explaining this pattern represents a challenge for existing business cycle models. To meet this, a theory is needed that involves both non-linearities and a secular development of the underlying mechanism, so as to shape the evolution in the

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<sup>1</sup>See, among others, [Kim and Nelson \(1999\)](#) and [McConnell and Perez-Quiros \(2000\)](#).

skewness of the business cycle. As for the first prerequisite, the importance of borrowing constraints as a source of business cycle asymmetries has long been recognized in the literature; see, e.g., the survey by [Brunnermeier \*et al.\* \(2013\)](#). In expansions credit constraints tend to relax, so that financially-constrained households and firms may find themselves temporarily unconstrained. By contrast, financial constraints tighten during recessions. The resulting non-linearity inevitably translates into a negatively skewed business cycle. As for the second prerequisite of the mechanism we are in the quest of, the past decades have witnessed a massive deregulation of financial markets, with one result being a substantial increase in the degree of leverage of advanced economies. To see this, [Figure 2](#) reports the credit-to-GDP and the loan-to-asset (LTA) ratios of both households and the corporate sector in the US.<sup>2</sup> This leveraging process is also confirmed by [Jordà \*et al.\* \(2017\)](#), who examine a large cross-section of countries and report a positive correlation between the skewness of real GDP growth and the credit-to-GDP ratio.

[Insert [Figure 2](#)]

To account for these facts we devise a dynamic stochastic general equilibrium (DSGE) model that allows for the collateral constraints faced by the firms and a fraction of the households not to bind at all points in time. We show that an increase in leverage raises the likelihood of financial constraints becoming slack in the face of expansionary shocks, dampening the magnitude of the resulting boom. By contrast, in the face of contractionary shocks borrowers tend to remain financially constrained, making their debt reduction more burdensome. In light of this mechanism, the skewness of the business cycle becomes increasingly negative. As in the data, the model also predicts that the duration of business cycle contractions does not change much as leverage increases, while the duration of expansions almost doubles.

Our findings carry important information about recent changes in the shape of business fluctuations. To elaborate on this, we juxtapose the drop in the skewness of the business cycle with the Great Moderation in macroeconomic volatility. While increasing LTV ratios cannot be pointed to as a main driver of the Great Moderation, our analysis shows that the increase in the asymmetry of the business cycle is compatible with a drop in its volatility. In line with recent empirical evidence reported by [Gadea-Rivas \*et al.\* \(2014, 2015\)](#), neither changes to the depth nor to the frequency of recessionary episodes account for the stabilization

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<sup>2</sup>As we discuss in Appendix A, the aggregate loan-to-asset ratios reported in [Figure 2](#) are likely to understate the actual LTV ratios requirements faced by the marginal borrower. While alternative measures may yield higher LTV ratios, they point to the same behavior of leverage over time (see also [Graham \*et al.\*, 2014](#), and [Jordà \*et al.\*, 2017](#)).

of macroeconomic activity. In fact, the adjustment in macroeconomic volatility mostly rests on the characteristics of the expansions, whose magnitude declines as an effect of collateral constraints becoming increasingly non-binding in the face of higher credit limits.

Recently, increasing attention has been devoted to the connection between the driving factors behind business cycle expansions and the extent of the subsequent contractions. [Jordà \*et al.\* \(2013\)](#) report that more credit-intensive expansions tend to be followed by deeper recessions – irrespective of whether the latter are accompanied by a financial crisis. Our model accounts for this feature along two dimensions. First, we show that contractions become increasingly deeper as the average LTV ratio increases, even though the boom-bust cycle is generated by the same combination of expansionary and contractionary shocks. Second, financially-driven expansions lead to deeper contractions, as compared with similar-sized expansions generated by non-financial shocks. Both exercises emphasize that, following a contractionary shock, the repercussion of constrained agents’ deleveraging increases in the size of their debt. As a result, increasing leverage makes it harder for savers to compensate for the drop in consumption and investment of constrained agents. This narrative of the boom-bust cycle characterized by debt overhang is consistent with the results of [Mian and Sufi \(2010\)](#), who identify a close connection at the county level in the US between pre-crisis household leverage and the severity of the Great Recession.

The observation that occasionally binding credit constraints may give rise to macroeconomic asymmetries is not new. [Mendoza \(2010\)](#) explores this idea in the context of a small open economy facing a constraint on its access to foreign credit. As this constraint becomes binding, the economy enters a ‘sudden stop’ episode characterized by sharp declines in economic activity. Importantly, correspondingly sharp increases in output neither arise in the model nor are observed in the data. In related work, albeit in a closed-economy setup, [Maffezzoli and Monacelli \(2015\)](#) have recently shown that the aggregate implications of financial shocks are state-dependent, with the economy’s response being greatly amplified in situations where agents switch from being financially unconstrained to being constrained. A recent study by [Guerrieri and Iacoviello \(2017\)](#) focus on the boom-bust cycle in the U.S. housing market and its connection with private consumption before and during the Great Recession. They show that non-binding credit constraints were prevalent during the pre-crisis boom in the US, thus lending support to the hypothesis that non-binding credit constraints are likely to have become a more salient feature of the macroeconomy in recent decades; a key implication of our study. While all these studies focus on specific economic disturbances and/or historical

episodes, we consider the impact of a secular process of financial liberalization – as reflected into an increase in credit limits – on the shape of business fluctuations that arise from both financial and non-financial disturbances.

Regarding the connection between financial liberalization and business cycle asymmetry, our paper is related to a recent empirical literature that focuses on the connection between leverage and the shape of the business cycle. [Popov \(2014\)](#) studies business cycle asymmetry in a large panel of developed and developing countries. Two main results are documented. First, the average business cycle skewness across all countries became markedly negative after 1991, consistent with our findings for the US. Second, this pattern is particularly distinct in countries that liberalized their financial markets. Also [Bekaert and Popov \(2015\)](#) examine a large cross-section of countries, reporting that more financially developed economies have more negatively skewed business cycles. By contrast, [Ordoñez \(2013\)](#) documents that countries with more developed financial markets display less asymmetry than others with a more rudimentary financial system. Finally, [Rancièrè \*et al.\* \(2008\)](#) establish a cross-country link between real GDP growth and the skewness of credit growth – a link which is stronger in financially liberalized countries. While we focus on the asymmetry of output, we observe a similar pattern for credit, making our results comparable with their findings. On a more general note, all of these cross-country studies focus on the connection between business cycle skewness and financial factors within samples that include both developing and developed countries, whereas we examine a secular development along various dimensions of business cycle asymmetry in a single advanced economy.

The rest of the paper is organized as follows. In [Section 2](#) we report evidence on the connection between leverage and changes in the shape of the business cycle in the US. [Section 3](#) inspects the key mechanisms at play in our narrative within a simple two-period model. [Section 4](#) presents the DSGE model to be estimated. [Section 5](#) discusses the solution and estimation of the model. [Section 6](#) reports the main results and connects our findings to the Great Moderation in economic volatility. [Section 7](#) shows that the model is capable of producing the type of debt overhang recession emphasized in recent empirical studies. [Section 8](#) concludes. The Appendices contain supplementary material concerning the model solution and various empirical and computational details.

## 2 Empirical evidence

This section presents a set of stylized facts that motivate the analysis. We first examine various aspects of business cycle asymmetry and how they changed over the last three decades. We then take advantage of cross-sectional variation across the U.S. States to document an empirical relationship between household leverage and the deepness of state-level contractions during the Great Recession.

### 2.1 Changing business cycle asymmetry

A number of empirical studies have widely documented a major reduction in the volatility of the U.S. business cycle since the mid 1980s. In this section we document changes in the asymmetry of the cycle that have occurred over the same timespan. [Table 1](#) reports the skewness of the rate of growth of different macroeconomic aggregates in the pre- and post-1984 period.

[Insert [Table 1](#)]

The skewness is typically negative and not too distant from zero in the first part of the sample – in fact, zero is generally comprised within the 68% confidence interval – while decreasing significantly thereafter. The tendency is particularly marked for the annualized growth rates.<sup>3</sup> To corroborate these findings, [Figure 3](#) reports the histogram of GDP growth, as well as the corresponding fitted normal density over the two subsamples. Two things stand out: first, the histogram referring to second subsample is much less dispersed – implying greater concentration of probability mass in the central part of the distribution – as compared with the histogram obtained from the previous sample period; second, as the density gets squeezed around its mean in the second part of the sample, more probability mass accumulates in the left tail, as implied by the decline in the skewness coefficient. To dig deeper into this aspect, we employ the nonparametric test of [Anderson and Darling \(1954\)](#), with the null hypothesis being that real GDP growth data in either of the two periods are drawn from a Normal distribution: This is strongly rejected for the second subsample (p-value=0.0086), whereas it cannot be rejected in the first one (p-value=0.7236).<sup>4,5</sup>

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<sup>3</sup>The drop in skewness of output growth has also been pointed out in recent work by [Garín \*et al.\* \(2017\)](#).

<sup>4</sup>This result is also confirmed by additional normality tests reported in Appendix B.

<sup>5</sup>We also check that the drop in the skewness does not result from a moderate asymmetry in the first part of the sample being magnified by a fall in the volatility, such as that occurred during the Great Moderation. The skewness of a random variable is defined as  $m_3/\sigma^3$ , where  $m_3$  is the third central moment of the distribution and  $\sigma$  denotes its standard deviation: Therefore, an increase in the absolute size of the skewness could merely reflect a fall in  $\sigma$ , with  $m_3$  remaining close to invariant. However, this does not seem to be the case, as  $m_3 = -2.8169$  for the year-on-year growth rate of real GDP in the pre-1984 sample, while it equals  $-6.8755$  afterwards.

[Insert [Figure 3](#)]

Another way to highlight changes in the shape of the business cycle is to compare the upside and the downside semivariances over the two subsamples.<sup>6</sup> The overall volatility of the business cycle during the Great Moderation has dropped by more than 40% compared to the pre-Moderation period (1.74% vs. 3.04% when calculated on year-on-year GDP growth). However, the drop is not symmetric. In fact, whereas the (square root of) the upside and downside semivariance are roughly equal in the pre-Moderation sample, in the post-1984 sample the downside semivariance is roughly 40% larger than its upside counterpart (1% vs 1.41%, when calculated on year-on-year GDP growth). As highlighted in [Figure 1](#), this implies an increase in the smoothness of the expansions, indicating that the unraveling of the Great Moderation mostly rests on the characteristics of the upsides of the cycle (see [Gadea-Rivas et al., 2014, 2015](#)).

All in all, this evidence suggests that the US business cycle has become less volatile and more asymmetric in the last three decades. The next step in the analysis consists of translating changes in the business cycle asymmetry into some explicit measure of the deepness of economic contractions, while accounting for time-variation in the dispersion of the growth rate process. To this end, [Table 2](#) reports the fall of real GDP during a given recession, divided by the duration of the recession itself: A measure that [McKay and Reis \(2008\)](#) label as the violence of a recessionary episode.

[Insert [Table 2](#)]

In line with anecdotal evidence, it is confirmed that the 1991 and 2001 recessions have been rather mild in terms of violence, as compared with both the Great Recession and several pre-1984 recessions. However, to compare the relative magnitude of different recessions over a period that displays major changes in the volatility of the business cycle, it is appropriate to control for the average variability of the cycle around a given recessionary episode. To this end, the last column of [Table 2](#) reports a measure of *standardized violence*, obtained by normalizing *violence* by the variability of year-on-year GDP growth in the 5 years prior to the recession.<sup>7</sup> Under this metric we get a rather different picture. The three recessionary episodes occurred

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<sup>6</sup>The upside (downside) semivariance is obtained as the average of the squared deviation from the mean of observations that are above (below) the mean. Semivariances are reported in Appendix B.

<sup>7</sup>The volatility is calculated as the standard deviation of the year-on-year growth rate of real GDP over a 5-year window. We also exclude the period running up to the recession by calculating the standard deviation up to a year before the recession begins. Weighting violence by various alternative measures of business cycle volatility returns a qualitatively similar picture: Appendix B reports additional robustness evidence on the violence of the recessions in the US.

during the Great Moderation are substantially deeper than the pre-1984 ones: averaging out the first seven recessionary episodes returns a standardized violence of 1.22%, against an average of 2.90% for the post-1984 period (2.70%, if we exclude the Great Recession). It is also important to highlight that the duration of business cycle contractions does not change much between the two samples, while the duration of the expansions doubles. This contributes to picturing the business cycle in the post-1984 sample as consisting of more smoothed and prolonged expansions, interrupted by shorter – yet, more abrupt – contractionary episodes.

## 2.2 Leverage and business cycle asymmetry: cross-state evidence

So far we have established that the post-1984 period is characterized by a smoother path of the expansionary periods and a stronger standardized violence of the recessionary episodes, as compared with the pre-Great Moderation period. In addition, over the same time window the process of financial deregulation has been associated with a sizeable increase in leverage of both households and firms. Relying on county-level US data, [Mian and Sufi \(2010\)](#) have identified a strong causal link between pre-crisis household leverage and the severity of the Great Recession. We now produce related evidence based on state-level data. Specifically, we take data on quarterly real Gross State Product (GSP) from the BEA Regional Economic Accounts and compute both the skewness of GSP growth and the violence of the Great Recession in the U.S. states.<sup>8</sup> [Figure 4](#) correlates the resulting statistics to the average debt-to-income ratio prior to the recession.<sup>9</sup> Notably, states where households were more leveraged not only have witnessed more severe GSP contractions during the last recession, but have also displayed a more negatively skewed GSP growth.

[Insert [Figure 4](#)]

To gain further insights into the cross-sectional connection between the magnitude of the Great Recession and business cycle dynamics, we order the U.S. states according to households' average debt-to-income ratio.<sup>10</sup> We then construct two synthetic series, computed as the growth rates of the median real GSP of the top and the bottom ten states in terms of leverage,

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<sup>8</sup>To account for the possibility that the recession does not begin/end in the same period across the US, we define the start of the recession in a given state as the period with the highest level of real GSP in the window that goes from 5 quarters before the NBER peak date to one quarter after that. Similarly, the end of the recession is calculated as the period with the lowest real GSP in the window from one quarter before to 5 quarters after the NBER trough date.

<sup>9</sup>To construct this variable, we rely on the State Level Household Debt Statistics produced by the New York Fed.

<sup>10</sup>See [Figure B.1](#) in [Appendix B](#).

respectively. According to [Figure 5](#), there are no noticeable differences in the performance of the two groups before and after the Great Recession, while the drop in real activity has been much deeper for relatively more leveraged states. Altogether, this evidence points to a close link between leverage and business cycle asymmetries. The remainder of the analysis consists of showing how looser credit conditions may generate increasingly negative skewness within models with occasionally binding collateral constraints.

[Insert [Figure 5](#)]

### 3 A partial equilibrium model

The basic intuition behind the analysis below can be preliminarily conveyed through a textbook two-period model with debt. To keep things simple, we work under certainty equivalence, assuming that the representative household has quadratic preferences defined over a nondurable consumption good,  $C_t$ . In period 1 households' budget constraint is  $C_1 + B_0 = Y_1 + B_1$ , where  $Y_1$  is a stochastic endowment,  $B_0$  is the initial stock of debt and  $B_1$  is the level of debt at the end of the first period. As for period 2, the constraint reads as  $C_2 = Y_2 - RB_1$ , where  $R$  is a constant gross real rate of interest. Furthermore, we assume that credit in period 1 is a fraction of the expected income in period 2:

$$B_1 \leq sE_1 \{Y_2\}, \quad s \in [0, 1]. \quad (1)$$

where  $E_1$  is a conditional expectation operator and  $s$  denotes the loan-to-income ratio. We assume that income follows a random walk process. Thus, if the constraint (1) binds with equality,  $B_1 = sY_1$  and consumption can be determined from the budget constraint:

$$C_1 = (1 + s)Y_1 - B_0. \quad (2)$$

In alternative, if the credit constraint is slack consumption behaves in accordance with:

$$C_1 = \frac{\beta R (1 + R)}{1 + \beta R^2} Y_1 - \frac{\beta R^2}{1 + \beta R^2} B_0, \quad (3)$$

where  $0 < \beta < 1/R$  denotes households' discount factor. Two implications can be drawn from the analysis so far: first, when the constraint binds with equality – and consumers behave according to a hand-to-mouth protocol – the response of  $C_1$  to marginal variations in  $Y_1$  (i.e., the

marginal propensity to consume), increases in the credit limit; second, the marginal propensity to consume when the financial constraint binds is always greater than that observed when the constraint does not bind and households behave as standard consumption smoothers.

The next step in the analysis consists of showing how the critical value of  $Y_1$  that makes consumption at time 1 the same under a binding and a non-binding constraint is affected by the credit limit. To this end, it is possible to retrieve  $\bar{Y}_1$  by equalizing (2) and (3), so that

$$\bar{Y}_1 = \frac{1}{(1+s)(1+\beta R^2) - \beta R(1+R)} B_0. \quad (4)$$

We denote the probability that the credit constraint binds with  $F(\bar{Y}_1)$ , where  $F(\cdot)$  is the cumulative distribution function of  $Y_1$ . In line with our arguments, lower credit limits increase the probability of the constraint being binding, *ceteris paribus*, as  $F' > 0$ .

The next section introduces a dynamic general equilibrium economy where the mechanisms we have just described are at play and produce negative asymmetry due to the financial constraints faced by the borrowers being occasionally binding. Broadly speaking, in such a model the stochastic process accounting for aggregate dynamics emerges as a mixture of the behavioral rules governing consumption and investment decisions under different regimes. In line with (4), the probability that the constraints become non-binding will be a positive function of the average LTV ratios faced by different types of borrowers. In turn, a higher probability of non-binding financial constraints will be associated with a more marked asymmetry.

## 4 A general equilibrium model

We now devise a structural model that can explain the empirical evidence described in the previous section. We adopt a standard real business cycle model augmented with collateral constraints along the lines of [Kiyotaki and Moore \(1997\)](#), [Iacoviello \(2005\)](#), [Liu et al. \(2013\)](#), and [Justiniano et al. \(2015\)](#); *inter alia*.<sup>11</sup> The economy is populated by three types of agents, each of mass equal to one. These agents differ by their discount factors, with the so-called patient households displaying the highest degree of time preference, while impatient households and entrepreneurs have relatively lower discount factors. As a result, patient households will be acting as lenders. Moreover, patient and impatient households supply labor, consume nondurable goods and land. Entrepreneurs only consume nondurable goods, and accumulate

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<sup>11</sup>[Jensen et al. \(2016\)](#) employ this framework to examine the impact of secular changes in credit limits on business cycle volatility and comovement between private debt and consumption/investment dynamics.

both land and physical capital, which they rent to firms. The latter are of unit mass and operate under perfect competition, taking labor inputs from both types of households, along with capital and land from the entrepreneurs. The resulting gross product may be used for investment and nondurable consumption.

## 4.1 Patient households

The utility function of patient households is given by:

$$\mathbb{E}_0 \left\{ \sum_{t=0}^{\infty} (\beta^P)^t \left[ \log (C_t^P - \theta^P C_{t-1}^P) + \varepsilon_t \log (H_t^P) + \frac{\nu^P}{1 - \varphi^P} (1 - N_t^P)^{1 - \varphi^P} \right] \right\}, \quad (5)$$

$$0 < \beta^P < 1, \varphi^P \geq 0, \varphi^P \neq 1, \nu^P > 0, 0 \leq \theta^P < 1$$

where  $C_t^P$  denotes nondurable consumption,  $H_t^P$  is the stock of land, and  $N_t^P$  denotes the fraction of time devoted to labor. Moreover,  $\beta^P$  is the discount factor,  $\theta^P$  measures the degree of habit formation in nondurable consumption and  $\varphi^P$  is the coefficient of relative risk aversion pertaining to leisure. Finally,  $\varepsilon_t$  is a land-preference shock satisfying

$$\log \varepsilon_t = \log \varepsilon + \rho_\varepsilon (\log \varepsilon_{t-1} - \log \varepsilon) + u_t, \quad 0 < \rho_\varepsilon < 1, \quad (6)$$

where  $\varepsilon > 0$  denotes the steady-state value and where  $u_t \sim \mathcal{N}(0, \sigma_\varepsilon^2)$ . Utility maximization is subject to the following budget constraint

$$C_t^P + Q_t (H_t^P - H_{t-1}^P) + R_{t-1} B_{t-1}^P = B_t^P + W_t^P N_t^P, \quad (7)$$

where  $B_t^P$  denotes the stock of one-period debt held at the end of period  $t$ ,  $R_t$  is the gross real interest rate on debt,  $Q_t$  is the price of land in units of consumption goods, and  $W_t^P$  is the real wage.

## 4.2 Impatient households

The utility of impatient households takes the same form as that of patient households:

$$\mathbb{E}_0 \left\{ \sum_{t=0}^{\infty} (\beta^I)^t \left[ \log (C_t^I - \theta^I C_t^I) + \varepsilon_t \log (H_t^I) + \frac{\nu^I}{1 - \varphi^I} (1 - N_t^I)^{1 - \varphi^I} \right] \right\}, \quad (8)$$

$$0 < \beta^I < \beta^P, \varphi^I > 0, \varphi^I \neq 1, \nu^I > 0, 0 \leq \theta^I < 1$$

where, as for the patient households,  $C_t^I$  denotes nondurable consumption,  $H_t^I$  is the stock of land, and  $N_t^I$  denotes the fraction of time devoted to labor. Households' difference in the degree of time preference is captured by imposing  $\beta^P > \beta^I$ . This ensures that, in the steady state, patient and impatient households act as lenders and borrowers, respectively. Impatient households are also subject to the following budget constraint

$$C_t^I + Q_t (H_t^I - H_{t-1}^I) + R_{t-1} B_{t-1}^I = B_t^I + W_t^I N_t^I. \quad (9)$$

Moreover, impatient households are subject to a collateral constraint, according to which their borrowing  $B_t^I$  is bounded above by a fraction  $s_t$  of the expected present value of durable goods holdings at the beginning of period  $t + 1$ :

$$B_t^I \leq s_t^I \frac{\mathbb{E}_t \{Q_{t+1}\} H_t^I}{R_t}, \quad (10)$$

This constraint can be rationalized in terms of limited enforcement, as in [Kiyotaki and Moore \(1997\)](#) and [Iacoviello \(2005\)](#). The loan-to-value (LTV) ratio (or credit limit),  $s_t^I$ , is stochastic and aims at capturing financial shocks (see, e.g., [Jermann and Quadrini, 2012](#) and [Liu \*et al.\*, 2013](#)):

$$\log s_t^I = \log s^I + \log s_t \quad (11)$$

$$\log s_t = \rho_s \log s_{t-1} + v_t, \quad 0 < \rho_s < 1, \quad (12)$$

where  $v_t \sim \mathcal{N}(0, \sigma_s^2)$  and  $s^I$ , the steady-state LTV ratio, is a proxy for the *average* stance of credit availability to the impatient households.

### 4.3 Entrepreneurs

Entrepreneurs have preferences over nondurables only (cf. [Iacoviello, 2005](#); [Liu \*et al.\*, 2013](#)), and maximize

$$\mathbb{E}_0 \left\{ \sum_{t=0}^{\infty} (\beta^E)^t \log (C_t^E - \theta^E C_{t-1}^E) \right\}, \quad 0 < \beta^I < \beta^E, \quad 0 \leq \theta^E < 1, \quad (13)$$

where  $C_t^E$  denotes entrepreneurial nondurable consumption. Utility maximization is subject to the following budget constraint

$$C_t^E + I_t + Q_t (H_t^E - H_{t-1}^E) + R_{t-1} B_{t-1}^E = B_t^E + r_{t-1}^K K_{t-1} + r_{t-1}^H H_{t-1}^E, \quad (14)$$

where  $I_t$  denotes investment in physical capital,  $K_{t-1}$  is the physical capital stock rented to firms at the end of period  $t - 1$ , and  $H_{t-1}^E$  is the stock of land rented to firms. Finally,  $r_{t-1}^K$  and  $r_{t-1}^H$  are the rental rates on capital and land, respectively. Capital accumulation is given by the law of motion

$$K_t = (1 - \delta) K_{t-1} + \left[ 1 - \frac{\Omega}{2} \left( \frac{I_t}{I_{t-1}} - 1 \right)^2 \right] I_t, \quad 1 > \delta > 0, \quad \Omega > 0, \quad (15)$$

whereby quadratic investment adjustment costs are assumed. Like impatient households, entrepreneurs are credit constrained, but they are able to use both capital and their holdings of land as collateral:<sup>12</sup>

$$B_t^E \leq s_t^E \mathbf{E}_t \left\{ \frac{Q_{t+1}^K K_t + Q_{t+1} H_t^E}{R_t} \right\}, \quad (16)$$

where  $Q_t^K$  denotes the price of installed capital in consumption units and  $s_t^E$  behaves in accordance with

$$\log s_t^E = \log s^E + \log s_t, \quad (17)$$

where  $s^E$  denotes entrepreneurs' steady-state LTV ratio.

## 4.4 Firms

Firms operate under perfect competition, employing a constant-returns-to-scale technology. They rent capital and land from the entrepreneurs and hire labor from both types of households in order to maximize their profits. The production technology for output,  $Y_t$ , is given by:<sup>13</sup>

$$Y_t = A_t \left[ (N_t^P)^\alpha (N_t^I)^{1-\alpha} \right]^\gamma \left[ (H_{t-1}^E)^\phi K_{t-1}^{1-\phi} \right]^{1-\gamma}, \quad 0 < \alpha, \phi, \gamma < 1, \quad (18)$$

<sup>12</sup>The importance of real estate as collateral for business loans has recently been emphasized by [Chaney et al. \(2012\)](#) and [Liu et al. \(2013\)](#).

<sup>13</sup>The assumption of imperfect substitutability between labor types follows [Iacoviello \(2005\)](#) and [Justiniano et al. \(2015\)](#), among others. [Iacoviello and Neri \(2010\)](#) note that perfect substitutability complicates the solution of their model substantially, but yields similar results.

with total factor productivity  $A_t$  evolving according to

$$\log A_t = \log A + \rho_A (\log A_{t-1} - \log A) + z_t, \quad 0 < \rho_A < 1, \quad (19)$$

where  $A > 0$  is the steady-state value of  $A_t$ , and  $z_t \sim \mathcal{N}(0, \sigma_A^2)$ .

## 4.5 Market clearing

Aggregate supply of land is fixed at  $H$ , implying that land-market clearing is given by

$$H = H_t^P + H_t^I + H_t^E. \quad (20)$$

The economy-wide net financial position is zero, such that

$$B_t^P + B_t^I + B_t^E = 0. \quad (21)$$

Finally, the aggregate resource constraint can be written as

$$Y_t = C_t^P + C_t^I + C_t^E + I_t. \quad (22)$$

## 5 Equilibrium, solution and estimation

An equilibrium is defined as a sequence of prices and quantities which, conditional on the sequence of shocks  $\{A_t, \varepsilon_t, s_t\}_{t=0}^{\infty}$  and the initial conditions, satisfy the optimality conditions, the budget and credit constraints, as well as the technological constraints and the market-clearing conditions.<sup>14</sup> Due to the assumptions about the discount factors,  $\beta^P < \beta^I$  and  $\beta^P < \beta^E$ , both collateral constraints are binding in steady state. The steady-state real interest rate is pinned down by patient households' Euler equation, whereas impatient households and entrepreneurs have a higher subjective real rate of interest. However, the optimal level of debt of one or both agents may fall short of the credit limit when the model is not at its steady state, in which case the collateral constraint will be non-binding.

To account for the occasionally binding nature of the collateral constraints, our solution method follows [Laséen and Svensson \(2011\)](#) and [Holden and Paetz \(2012\)](#). The idea is to introduce a set of (anticipated) “shadow value shocks” to ensure that the shadow values associated with each of the two collateral constraints remain non-negative at all times. For first-order

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<sup>14</sup>We provide the optimality conditions in Appendix D.

perturbations, we have verified that our solution method produces similar simulated moments as the method of [Guerrieri and Iacoviello \(2015, 2017\)](#); cf. also [Holden and Paetz \(2012\)](#). We present the technical details of the method in Appendix E.

## 5.1 Calibration and estimation

In the remainder we aim at assessing the extent to which a relaxation of the credit limits faced by the borrowers can account for the evolution of the asymmetry of the business cycle. With this in mind, we assign parameter values that allow us to match a set of characteristics of the U.S. business cycle in the decades preceding the Great Moderation. We do this by calibrating a subset of the parameters, while estimating the remaining ones using the simulated method of moments (SMM). Thus, we simulate the model for progressively higher average LTV ratios faced by households and firms and track the implied changes in the skewness of output and other macroeconomic variables, as well as other business cycle statistics.<sup>15</sup>

### 5.1.1 Calibrated parameters

We choose to calibrate a subset of the model parameters that can be pinned down using a combination of existing studies and first moments of the data. We interpret one period as a quarter. Therefore, we set  $\beta^P = 0.99$ , implying an annualized steady-state rate of interest of about 4%. Moreover, as we assume that both impatient households and entrepreneurs have lower discount factors than patient households, we set  $\beta^I = \beta^E = 0.96$ , in the ballpark of the available estimates; see, e.g. [Iacoviello, 2005](#) and references therein. The Frisch elasticity of labor supply is given by the inverse of  $\varphi^i$ , multiplied by the steady-state ratio of leisure to labor hours. Calibrating the latter to 3 for both types of households implies  $\varphi^i = 9$ ,  $i = \{P, I\}$ . Therefore, we set  $\nu^i = 0.27$  for  $i = \{P, I\}$ , so as to induce households to work around 1/4 of their time in the steady state. Broadly in line with [Iacoviello \(2005\)](#) and [Iacoviello and Neri \(2010\)](#), we set the income share of labor supplied by patient households,  $\alpha$ , to 0.7.

The remaining part of the calibration ensures that the model reproduces a set of ‘big ratios’ of the U.S. economy for the period from World War II until the onset of the Great Moderation. First, we follow [Elsby \*et al.\* \(2013\)](#) and use the official estimate of the Bureau of Labor Statistics to pin down the labor income share. The average value for the years 1948-1983 implies  $\gamma = 0.6355$ . We set the difference between the steady-state LTV ratios of entrepreneurs

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<sup>15</sup>The aim of the exercise is not to account for the process of financial innovation and liberalization lying behind the increase in leverage in the last decades – a task the model is not suitable for. Instead, we take this increase for granted and examine how it affected the shape of the business cycle.

and impatient households,  $s^E - s^I$ , to the sample average of the difference between the two loan-to-asset series shown in the right panel of [Figure 2](#), which amounts to 0.09. We then calibrate the remaining four parameters –  $s^I, \delta, \varepsilon, \phi$  – to jointly match the following four ratios (all at the annual frequency): A steady-state ratio of residential land to output of 1.10, a ratio of commercial land to output of 0.63, an average capital-output ratio of 1.11, and an average ratio of private nonresidential investment to output of 0.23.<sup>16</sup> The calibrated parameters are summarized in Panel A of [Table 3](#). The depreciation rate of capital is 0.0518, somewhat higher than standard values, as it reflects that our measure of capital excludes residential capital and structures, which have lower depreciation rates than, e.g., intellectual properties. The implied value of  $\phi = 0.1340$ , which multiplied by  $(1 - \gamma)$  measures land’s share of inputs, which is somewhat higher than [Liu et al. \(2013\)](#). Finally, our calibration implies average LTV ratios of 0.62 for impatient households, and thus 0.71 for entrepreneurs. These values are lower than those typically employed in models calibrated over the Great Moderation sample,<sup>17</sup> as our calibration covers the period before the concurrent financial liberalization.

### 5.1.2 Estimated parameters

We rely on the SMM to estimate the remaining model parameters. This method is particularly well-suited for DSGE models involving non-binding constraints or other non-linearities, as these preclude the use of the Kalman filter, thus making the application of Bayesian methods particularly cumbersome.<sup>18</sup> [Ruge-Murcia \(2012\)](#) studies the properties of SMM estimation of non-linear DSGE models, and finds that this method is computationally efficient and delivers accurate parameter estimates. Moreover, [Ruge-Murcia \(2007\)](#) performs a comparison of the SMM with other widely used estimation techniques applied to a basic RBC model, showing it fares quite well in terms of accuracy and computing efficiency, along with being less prone to misspecification issues than Maximum Likelihood-based methods.

We estimate the following parameters: The investment adjustment cost parameter ( $\Omega$ ), the

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<sup>16</sup>Our computations of these ratios largely follow those of [Liu et al. \(2013\)](#). For residential land, we use owner-occupied real estate from the Flow of Funds tables. For commercial land, [Liu et al. \(2013\)](#) use Bureau of Labor Statistics data on land inputs in production, which is not available for the sample period we consider. Instead, we compute the sum of the real estate holdings of nonfinancial corporate and nonfinancial noncorporate businesses from the Flow of Funds, and then follow [Liu et al. \(2013\)](#) in multiplying this number by a factor of 0.5 to impute the value of land. For capital, we compute the sum of the annual stocks of equipment and intellectual property products of the private sector and consumer durables. We use the corresponding flow variables to measure investment. Finally, we measure output as the sum of investment (as just defined) and private consumption expenditures on nondurable goods and services.

<sup>17</sup>See, e.g., [Calza et al. \(2013\)](#), [Liu et al. \(2013\)](#) or [Justiniano et al. \(2014\)](#).

<sup>18</sup>See, however, [Fernández-Villaverde et al. \(2016\)](#) for a discussion of the particle filter as a potential replacement for the Kalman filter.

parameters measuring habit formation in consumption ( $\theta^P$ ,  $\theta^I$ , and  $\theta^E$ ), and the parameters governing the persistence and volatility of the shocks ( $\rho_A, \rho_s, \rho_\varepsilon, \sigma_A, \sigma_s, \sigma_\varepsilon$ ).<sup>19,20</sup> In the estimation, we use five macroeconomic time series for the U.S. economy spanning the sample period 1952:I–1984:II: The growth rates of real GDP, real private consumption, real non-residential investment, real house prices, and the average of the deviations from trend of the two LTA series reported in the right panel of [Figure 2](#), where the trend is computed using a multivariate Beveridge-Nelson decomposition.<sup>21</sup> The beginning of the sample is dictated by the availability of quarterly Flow of Funds data.<sup>22</sup> The end of the sample coincides with the onset of the Great Moderation. In the estimation, we match the following empirical moments: The standard deviations and first-order autoregressive parameters of each of the five variables, the correlation of consumption, investment, and house prices with output, and the skewness of output, consumption, and investment. This gives a total of 16 moment conditions to estimate nine parameters. We provide more details about the data and our estimation strategy in [Appendix F](#).

The estimated parameters are reported in [Panel B of Table 3](#). The estimate of  $\Omega$  is in line with existing results from estimated DSGE models; see, e.g., [Justiniano \*et al.\* \(2013\)](#). Likewise, the degree of habit formation of impatient households and entrepreneurs is close to the estimates of [Justiniano \*et al.\* \(2013\)](#) and [Guerrieri and Iacoviello \(2017\)](#), whereas the estimated habit parameter for patient households is virtually zero. The volatility and persistence parameters of the technology shock are in line with those typically found in the real business cycle literature; see, e.g., [Mandelman \*et al.\*, 2011](#). The finding of quite large and persistent land-demand shocks is consistent with the results of [Iacoviello and Neri \(2010\)](#) and [Liu \*et al.\* \(2013\)](#). Finally, while the financial shocks in our model are more volatile than found by [Jermann and Quadrini \(2012\)](#) and [Liu \*et al.\* \(2013\)](#), they are also less persistent, implying an unconditional standard deviation of the process in line with the estimates from these studies. We report the implied business cycle moments along with their empirical counterparts in [Appendix F](#).

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<sup>19</sup>Unlike other estimated parameters,  $\theta^i$ ,  $\{i = P, I, E\}$ , also affects the steady state of the model. To account for this, we rely on the following iterative procedure: We first calibrate the model based on the starting values for  $\theta^i$  used to initiate the estimation (i.e.,  $\theta^i = 0.7$ ,  $\{i = P, I, E\}$ ), cf. [Appendix F](#). Upon estimation, but before simulating the model, we recalibrate it for the estimated values of the habit parameters. This leads only to a small change in the value of  $\varepsilon$ , while the remaining parameters are unaffected.

<sup>20</sup>As in this class of models the consumption of the patient households displays perfect comovement with the price of the durable asset ([Barsky \*et al.\*, 2007](#)), we allow for different habit parameters between constrained and unconstrained agents, while imposing that the degree of habit formation of the entrepreneurs is equal to that of the impatient households.

<sup>21</sup>Specifically, since the LTV ratios for households and firms share a common trend, we follow [Robertson \*et al.\* \(2006\)](#) in computing the multivariate [Beveridge and Nelson \(1981\)](#) trends for the two series.

<sup>22</sup>In fact, house prices are only available starting in 1963:I. We choose not to delay the beginning of other data series to this date.

[Insert Table 3]

## 6 Asymmetric business cycles and collateral constraints

We are now ready to explore the ability of our model to generate stronger business cycle asymmetry as leverage increases. We do so in three steps. First, we inspect a set of impulse responses to build intuition around the non-linear transmission of different shocks. Next, we present various business cycle statistics obtained from simulating the model at different degrees of leverage. Finally, we examine the behavior of business cycle asymmetry in conjunction with the Great Moderation in macroeconomic volatility.

### 6.1 Impulse-response functions

To gain a preliminary insight into the nature of the asymmetry generated by our framework, and how this evolves under different LTV ratios, we study the propagation of different shocks. Figure 6 displays the response of output to a set of positive shocks, as well as the mirror image of the response to equally-sized negative shocks, under different credit limits. Looking at the first row of the figure, technology shocks of either sign produce symmetric responses under the calibrated LTV ratios for impatient households and entrepreneurs. By contrast, at higher credit limits a positive technology shock renders the borrowing constraint of the entrepreneurs slack for three quarters, while impatient households remain constrained throughout.<sup>23</sup> Entrepreneurs optimally choose to borrow less than they are able to: This attenuates the expansionary effect on their demand for land and capital, dampening the boom in aggregate economic activity. On the contrary, following a negative technology shock, the borrowing constraints remain binding throughout. As a result, impatient households and entrepreneurs are forced to cut back on their borrowing in response to the drop in the value of their collateral assets. This produces a stronger output response. In other words, under relatively high LTV ratios a negative technology shock has a larger impact on output than a similar-sized positive shock.

[Insert Figure 6]

As for stochastic shifts in household preferences, the second row of Figure 6 indicates that entrepreneurs' collateral constraint becomes non-binding for two quarters after a positive land demand shock in the scenario with high LTV ratios, while impatient households remain

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<sup>23</sup>In our stochastic simulations, instead, combinations of the estimated shocks will be sufficient to observe occasionally binding constraints for both types of borrowers.

constrained throughout. Therefore, entrepreneurs have no incentive to expand their borrowing capacity by increasing their stock of land. In fact, they lower their land holdings on impact, allowing patient and impatient households to increase their stock of land at the expense of nondurable consumption. By contrast, there is no attenuation of large negative shocks to the economy. In that case, both collateral constraints remain binding, giving rise to a large output drop. Similar observations apply to the transmission of the financial shock, with the main difference being that upward shifts in the credit limits bear a greater potential of rendering the financial constraints non-binding, as they exert a direct impact on the borrowing limit. In fact, under high average LTV ratios the entrepreneurs are unconstrained during the first five periods following a positive shock. For the reasons discussed above, this leads to a smooth response of output, as compared with what happens following a negative shock: In this case entrepreneurs are forced into a sizeable deleveraging, reducing the stock of land available for production. Simultaneously, also impatient households deleverage and bring down their stock of land, which further depresses the land price, and thus the borrowing capacity of both types of constrained agents. The result is a large drop in output.

## 6.2 Asymmetry and credit limits

The impulse responses in the previous subsection offer a clear message: As leverage increases economic expansions tend to become smoother than contractions, paving the way for a negatively skewed business cycle. This is broadly consistent with the observation of lower volatility of the upside of the business cycle, as compared with its downside. Moreover, the three types of shock we consider exert similar effects on business cycle asymmetry, so that their relative contribution is not crucial to our qualitative findings.

To deepen our understanding of the properties of the model in connection with the degree of leverage, we conduct a large set of stochastic simulations and report a number of statistics associated with different values of the average LTV ratios.<sup>24</sup> In line with the partial equilibrium economy of Section 3, Figure 7 shows that the occurrence of episodes of non-binding constraints increases with the degree of leverage.<sup>25</sup> In fact, entrepreneurs go from being uncon-

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<sup>24</sup>Specifically, we retrieve each statistic as the median from 501 simulations each running for 2000 periods. Unless stated otherwise, from now on we report the variable of interest for different average LTV ratios faced by the impatient households. In each simulation the entrepreneurial average LTV ratio is adjusted so as to be 9 basis points greater than any value we consider for impatient households' credit limits, in line with the baseline calibration of the model.

<sup>25</sup>It is important to stress that a process of leveraging in our artificial economy only reflects into an increase in the access to credit at the intensive margin (i.e., how much a given agent can borrow). In practice, financial innovation is likely to have also affected credit access at the extensive margin (i.e., who can borrow). In future work, we plan to investigate both dimensions within an incomplete markets setting.

strained about 20% of the time at the baseline calibration, to as much as 95% of the time at higher values of the average LTV ratios. As for the impatient households, they only start experiencing instances of non-binding financial constraints at a 25% average downpayment, finding themselves financially unconstrained about 22% of the time at the upper end of the interval for the average LTV ratios. Given these figures, in light of the impulse-response analysis of the previous section we should expect the increasing relevance of periods of lax credit constraints to be associated with an increasingly negative asymmetry of the resulting macroeconomic aggregates.

[Insert [Figure 7](#)]

The left panel of [Figure 8](#) displays the skewness of the growth rate of output, aggregate consumption and investment: All statistics start from being negative at our calibrated average LTV ratios and decline as leverage increases.<sup>26</sup> This property has sizeable implications for the size of the recessions in our artificial economy, as indicated by the right panel of [Figure 8](#). At the baseline calibration, the standardized violence of the recessions computed from the simulated time series of gross output is quantitatively in line with its data analogue reported in [Table 2](#) for the pre-1984 sample. As leverage rises, the standardized violence increases, up to the point it doubles at the upper end of the interval of average LTV ratios, being broadly in line with what is observed in the post-1984 sample.

[Insert [Figure 8](#)]

It is also important to highlight that the model is capable of reproducing relative changes in the duration of contractions and expansions that are similar to those reported in [Table 2](#). As leverage increases expansions tend to last longer – as indicated by the left panel of [Figure 9](#) – while the duration of the contractions displays a pattern that is virtually unchanged between the pre- and post-financial liberalization scenario.

[Insert [Figure 9](#)]

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<sup>26</sup>In our dynamic simulations, impatient households and entrepreneurs may sometimes find themselves unconstrained even during economic downturns as a result of, say, a positive credit limit shock and a concurrent negative non-financial shock. In such situations – which are most likely to occur at high LTV ratios – even recessions may be dampened, thereby mitigating business cycle skewness. This explains the small reversal of the skewness of the growth of consumption and investment at relatively high LTV ratios.

### 6.3 Skewness and volatility

Recent statistical evidence has demonstrated that the Great Moderation was never associated with smaller or less frequent downturns, but has been driven exclusively by the characteristics of the expansions, whose magnitude has declined over time (Gadea-Rivas *et al.*, 2014, 2015). We now examine this major statistical development in conjunction with the change in the skewness of the business cycle, which has largely occurred over the same time span.

[Insert Figure 10]

The left panel of Figure 10 reports the standard deviation of output growth as a function of the average LTV ratios. As discussed in Jensen *et al.* (2016), macroeconomic volatility displays a hump-shaped pattern. Starting from low credit limits, higher availability of credit allows financially constrained agents to engage in debt-financed consumption and investment, as dictated by their relative impatience, thus reinforcing the macroeconomic repercussions of shocks that affect their borrowing capacity. This pattern eventually reverses, as higher LTV ratios increase the likelihood that credit constraints become non-binding. In such cases, the consumption and investment decisions of households and entrepreneurs tend to delink from changes in the value of their collateral assets, dampening the volatility of aggregate economic activity. In fact, at the upper end of the range of average LTV ratios we consider, volatility drops below the value we match under the baseline calibration.

A key property of a model with occasionally binding constraints is that the volatility reversal is much stronger for positive than for negative shocks, in the face of which financial constraints tend to remain binding. This inherent property of our framework indicates that the drop in output volatility observed beyond  $s^I \approx 0.75$  is mostly connected with expansionary periods, as in the evidence reported by Gadea-Rivas *et al.* (2014, 2015). The right panel of Figure 10 confirms this view: Here we compare the volatility of expansionary and contractionary episodes, respectively, as a function of the average LTV ratios. The volatility of expansions is always lower than that of contractions, and declines over most of the range of average credit limits. The volatility of contractions, on the other hand, initially increases and then reverts at a relatively high degree of leverage: This drop is due to financial constraints being potentially non-binding even during economic contractions. Such situations may arise if, e.g., a negative technology shock coincides with a positive credit limit shock: By virtue of its direct impact on the borrowing limit, the latter is highly effective at rendering the financial constraints temporarily non-binding.

While our framework points to a hump-shaped relationship between credit limits and macroeconomic volatility, the key driver of business cycle asymmetry – occasionally binding credit constraints – in itself works as an impetus of lower macroeconomic volatility, *ceteris paribus*. Thus, despite our analysis does not warrant the claim that the empirical developments in the volatility and skewness of the business cycle necessarily have the same origin, higher credit limits do eventually lead to a drop in the overall volatility of our model economy by making financial constraints increasingly slack.<sup>27</sup>

Notably, the increasing prevalence of non-binding credit constraints allows the model to account for different correlations between the volatility and the skewness of output growth, conditional on different credit limits. Based on the comparison between [Figure 8](#) and the left panel of [Figure 10](#), this correlation is increasingly negative until  $s^I \approx 0.75$ , thus becoming positive as financial deepening reaches very advanced stages. These results are reminiscent of the evidence reported by [Bekaert and Popov \(2015\)](#), who document a positive long-run correlation between the volatility and skewness of output growth in a large cross-section of countries, but also a negative short-run relationship in financially developed economies.<sup>28</sup>

## 7 Debt overhang and business cycle asymmetries

Several authors have recently pointed to the nature of the boom phase of the business cycle as a key determinant of the subsequent recession. Using data for 14 advanced economies for the period 1870–2008, [Jordà et al. \(2013\)](#) find that more credit-intensive expansions tend to be followed by deeper recessions, whether or not the recession is accompanied by a financial crisis. This evidence is consistent with our cross-state evidence and the results of [Mian and Sufi \(2010\)](#), who identify a close connection at the county level in the US between pre-crisis household leverage and the severity of the Great Recession.

In this section we demonstrate that our model is capable of reproducing these empirical facts. [Figure 11](#) reports the results of the following experiment: Starting in the economy’s steady state, we generate a boom-bust cycle for different steady-state debt levels, as reflected

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<sup>27</sup>A large literature suggests that innovation in the credit market – especially in consumer credit and home mortgages – have played a role in the Great Moderation; see [den Haan and Sterk \(2010\)](#) for a review. A related question is whether our main finding of increasingly negative business cycle skewness would survive in the presence of an exogenous reduction in macroeconomic volatility of the magnitude observed during the Great Moderation. Appendix G documents that this is indeed the case.

<sup>28</sup>Clearly, our model cannot account for the positive link between the skewness and volatility of output growth in economies at early stages of their financial development. As pointed out by [Bekaert and Popov \(2015\)](#), while occasionally hit by crises and sudden stops, these countries experience periods of rapid economic growth that tend to generate high volatility along with positive skewness.

by different average LTV ratios. We first feed the economy with a series of positive shocks of all three types in the first five periods (up to period 0 in the figure). During the boom phase, we calibrate the size of the expansionary shocks hitting the economy so as to make sure that the boom in output is identical across all the experiments. Hereafter, starting in period 1 in the figure, we shock the economy with contractionary shocks of all three types for two periods, after which the negative shocks are ‘phased out’ over the next three periods. Crucially, the contractionary shocks are identical across the calibrations. This ensures that the severity of the recession is determined by the endogenous response of the model at each different LTV ratio.<sup>29</sup> As the figure illustrates, the deepness of the contraction increases with the steady-state LTV ratios. A boom of a given size is followed by a more severe recession when debt is relatively high, as compared with the case of a more scarce credit availability. At higher average LTV ratios, households and entrepreneurs are more leveraged during the boom, and they therefore need to face a more severe process of deleveraging when the recession hits. By contrast, when credit levels are relatively low, financially constrained agents face lower credit availability to shift consumption and investment forward in time during booms, and are therefore less vulnerable to contractionary shocks.

[Insert [Figure 11](#)]

We next focus on the nature of the boom and how this spills over to the ensuing contraction. The left panel of [Figure 12](#) compares the path of output in two different boom-bust cycles, while the right panel shows the corresponding paths for aggregate debt. In each panel, the dashed line represents a non-financial boom generated by a combination of technology and land-demand shocks, while the solid line denotes a financial boom generated by credit limit shocks.<sup>30</sup> We calibrate the size of the expansionary shocks so as to deliver an identical increase in output during each type of boom (which lasts for five periods, up until period 0 in the figure). As in the previous experiment, we then subject the economy to identical sets of contractionary shocks of all three types, so as to isolate the role played by the specific type of boom in shaping the subsequent recession. The contractionary shocks hit in periods 1 and 2 in the figure, and are then ‘phased out’ over the next three periods. While the size of the expansion in output is

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<sup>29</sup>During both the boom and the bust we keep the relative size of the three shocks fixed, in accordance with their standard deviations estimated in Subsection 5.1.2. However, we set their persistence parameters to zero, in order to avoid that the shape of the recession may be determined by lagged values of the shocks during the boom. Finally, we make sure that impatient households and entrepreneurs remain constrained in all periods of each of the cases, so as to enhance comparability.

<sup>30</sup>In the non-financial boom we keep the relative size of the technology and land-demand shocks in line with the values estimated in subsection 5.1.2. As in the previous experiment, we set the persistence parameters of all the shock processes to zero.

identical in each type of boom, the same is not the case for total debt, which increases by more than twice as much during the financial boom. The consequences of this build-up of credit show up during the subsequent contraction, which is much deeper following the financially fueled expansion, in line with the empirical findings of [Jordà \*et al.\* \(2013\)](#). Further, as in [Mian and Sufi \(2010\)](#), this exercise confirms that the macroeconomic repercussions of constrained agents' deleveraging is increasing in the size of their debt.

[Insert [Figure 12](#)]

## 8 Concluding comments

We have documented how different dimensions of business cycle asymmetry in the U.S. have changed over the last few decades, and pointed to the concurrent increase in private debt as a potential driver of these phenomena. We have presented a dynamic general equilibrium model with credit-constrained households and firms, in which increasing leverage translates into a more negatively skewed business cycle. This finding relies on the occasionally binding nature of financial constraints: As their credit limits increase, households and firms are more likely to become temporarily unconstrained during booms, while credit constraints tend to remain binding during downturns.

The Great Moderation is widely regarded as the main development in the statistical properties of the U.S. business cycle since the 1980s. Our contribution points to a simultaneous change in the shape of the business cycle closely connected with financial factors. Enhanced credit access as observed over the last few decades implies both a prolonging and a smoothing of expansionary periods as well as less frequent – yet, more dramatic – economic contractions, exacerbated by deeper deleveraging episodes. As for the first part of this story, several contributions have pointed to the attenuation of the upside of the business cycle as the main statistical driver of the Great Moderation (see, e.g., [Gadea-Rivas \*et al.\*, 2014, 2015](#)). Nevertheless, our second insight implies that contractionary episodes of the proportions of the Great Recession, albeit less frequent, might represent a ‘new normal’.

In light of these considerations, our results are of interest to macroprudential policymakers, mainly for two reasons. First, we add to a recent literature emphasizing that the seeds of the recession are sown during the boom: The nature of the boom phase, as much as its size, is an important determinant of the ensuing downturn, and policymakers should pay close attention to the build-up of credit during expansions in macroeconomic activity. Indeed, [Mian \*et al.\*](#)

(2017) find that IMF and OECD forecasts made after large increases in household debt tend to overestimate subsequent output growth, and that those forecasts could be improved by adjusting them downwards to account for past increases in household as well as firm credit. Second, one focus of macroprudential policies has typically been to reduce LTV ratios in order to curb macroeconomic volatility. According to our results, a blunt reduction of credit limits – while likely to dampen the size of economic expansions – may actually result in deeper contractions. Therefore, macroprudential policies need to be designed with a specific focus on avoiding large output drops. In fact, in related work (see [Jensen \*et al.\*, 2016](#)), we have demonstrated that countercyclical LTV ratios can be particularly effective in this respect.

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# Tables and Figures

	QoQ		YoY	
	1947:I-1984:II	1984:III-2016:II	1947:I-1984:II	1984:III-2016:II
GDP	-0.1178 [0.5708]	-1.2122 [0.6218]	-0.0983 [0.6241]	-1.3037 [0.6998]
Consumption	-0.5064 [0.5694]	-0.4678 [0.6299]	-0.2023 [0.6183]	-1.0005 [0.7372]
Investment	-0.2100 [0.5658]	-0.8271 [0.6161]	-0.0067 [0.6004]	-1.3994 [0.6707]

Notes: In the ‘QoQ’ (‘YoY’) column we report, for different macroeconomic aggregates, the coefficient of skewness computed on the quarter-on-quarter (year-on-year) growth rate of real GDP over the 1947:I-1984:II and 1984:III-2016:II samples. Standard errors are reported in brackets.

	Duration		Violence	Std. Violence
	Contractions	Expansions		
1953:II – 1954:II	4	–	3.4106	0.9488
1957:III – 1958:II	3	13	7.3088	2.5419
1960:II – 1961:I	3	8	1.8009	0.5718
1969:IV – 1970:IV	4	35	0.4710	0.2674
1973:IV – 1975:I	5	12	2.5293	1.2275
1980:I – 1980:III	2	20	4.4006	1.7747
1981:III – 1982:IV	5	4	2.6785	1.1896
1990:III – 1991:I	2	31	2.6511	3.6238
2001:I – 2001:IV	3	40	1.2671	1.7850
2007:IV – 2009:II	6	24	2.8909	3.2966
Average				
Pre-1984	3.7143	15.3333	3.2285	1.2174
Post-1984	3.6667	31.6667	2.2697	2.9018

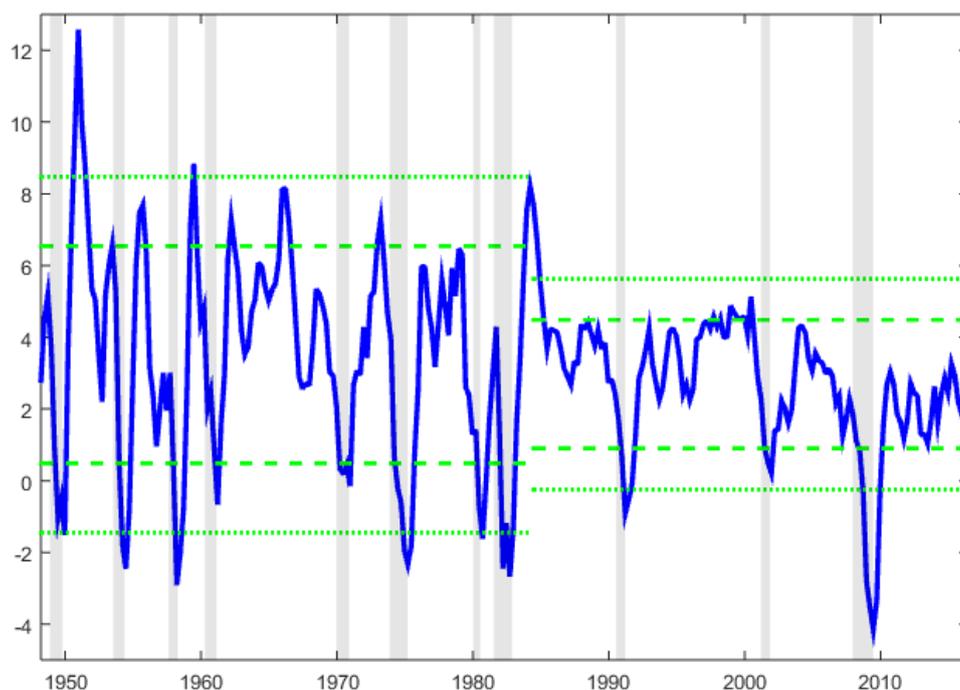
Notes: For every recession we calculate ‘Violence’ as the annualized fall of real GDP from the peak to the trough of the contractionary episode, divided by the length of the recession; ‘Std. Violence’ standardizes the violence of the recession by the average business cycle volatility prior to the recession itself. The latter is calculated as the standard deviation of the year-on-year growth rate of real GDP over a 5-year window. We exclude the period running up to the recession by calculating the standard deviation up to a year before the recession begins.

Table 3: Parameter values

<i>Panel A: Calibrated parameters</i>		
Parameter	Description	Value
$\beta^P$	Discount factor, patient households	0.99
$\beta^i, i = \{I, E\}$	Discount factor, impatient agents	0.96
$\varphi^i, i = \{P, I\}$	Curvature of utility of leisure	9
$\nu^i, i = \{P, I\}$	Weight of labor disutility	0.27
$\varepsilon$	Weight of housing utility	0.0763
$\phi$	Non-labor input share of land	0.1340
$\gamma$	Labor share of production	0.6355
$\delta$	Capital depreciation rate	0.0518
$\alpha$	Income share of patient households	0.7
$s^I$	Initial loan-to-value ratio, impatient households	0.6239
$s^E$	Initial loan-to-value ratio, entrepreneurs	0.7139
<i>Panel B: Estimated parameters</i>		
Parameter	Description	Value
$\Omega$	Investment adjustment cost parameter	2.9827 (0.9585)
$\theta^P$	Habit formation, patient households	0.0031 (0.0526)
$\theta^I$	Habit formation, impatient households + entrepreneurs	0.7723 (0.1380)
$\rho_A$	Persistence of technology shock	0.9894 (0.0326)
$\rho_s$	Persistence of credit-limit shock	0.8873 (0.0658)
$\rho_\varepsilon$	Persistence of land-demand shock	0.9893 (0.0527)
$\sigma_A$	Std. dev. of technology shock	0.0080 (0.0012)
$\sigma_s$	Std. dev. of credit-limit shock	0.0345 (0.0011)
$\sigma_\varepsilon$	Std. dev. of land-demand shock	0.0361 (0.1156)

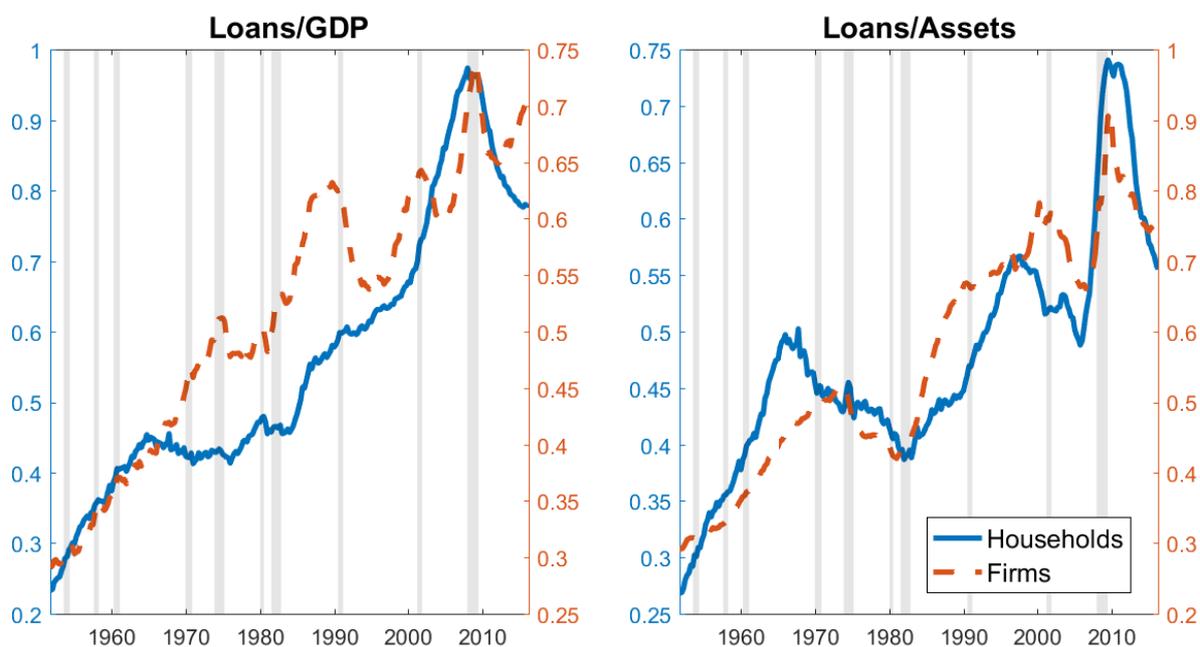
Note: The standard errors of the estimated parameters are reported in brackets.

Figure 1: Growth of real GDP.



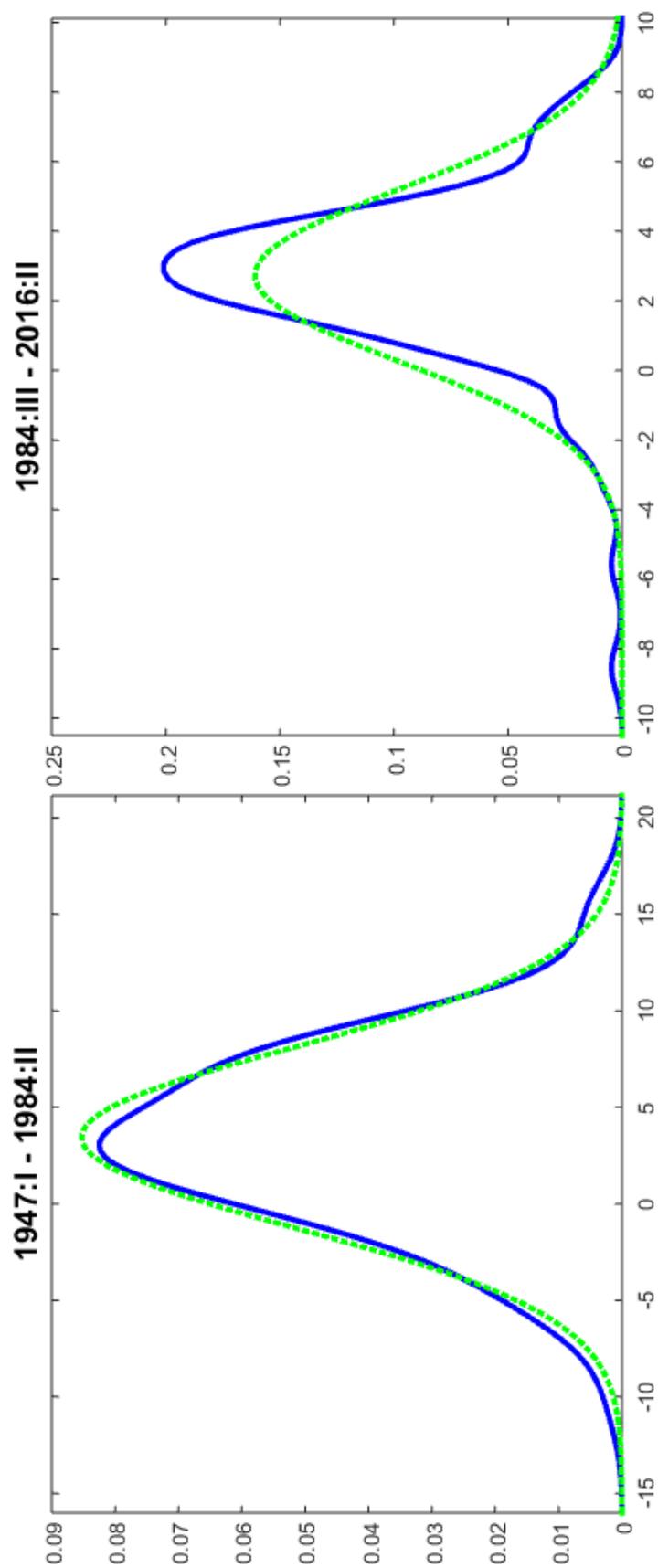
Notes: Figure 1 reports the year-on-year rate of growth of real GDP over the 1947:I-2016:II sample. The bands correspond to the 68% and 90% confidence intervals from a Gaussian density fitted to the 1947:I-1984:II and 1984:III-2016:II samples. The vertical shadowed bands denote the NBER recession episodes.

Figure 2: Household and corporate leverage in the US.



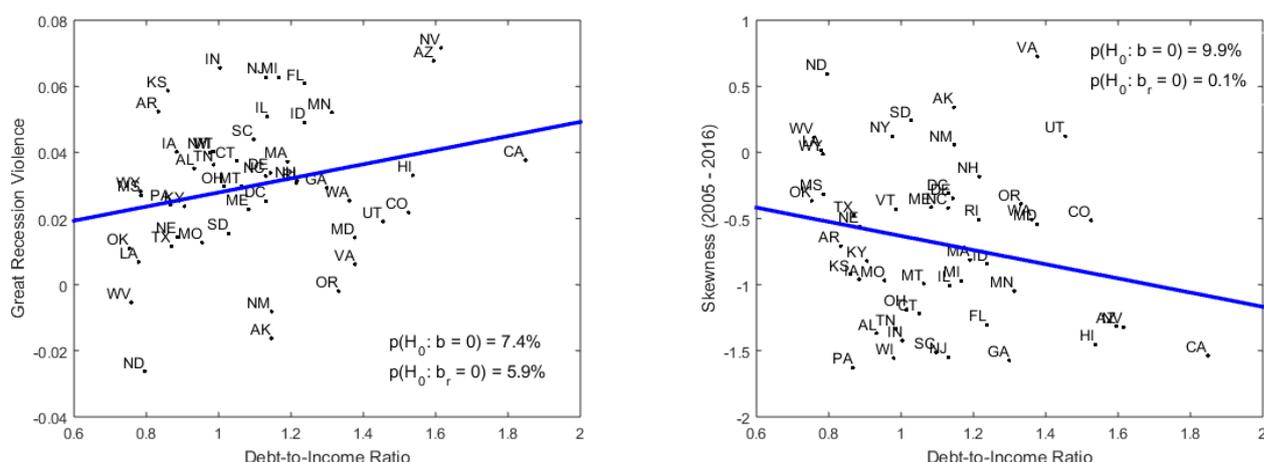
Notes: Left panel: the solid-blue line graphs the ratio between loans to households and GDP, while the dashed-red line reports the same variable at the corporate level. Right panel: the solid-blue line graphs the ratio between households' liabilities and assets, while the dashed-red line reports the same variable at the corporate level. The vertical shadowed bands denote the NBER recession episodes.

Figure 3: GDP growth: 1947:I-1984:II vs. 1984:III-2016:II.



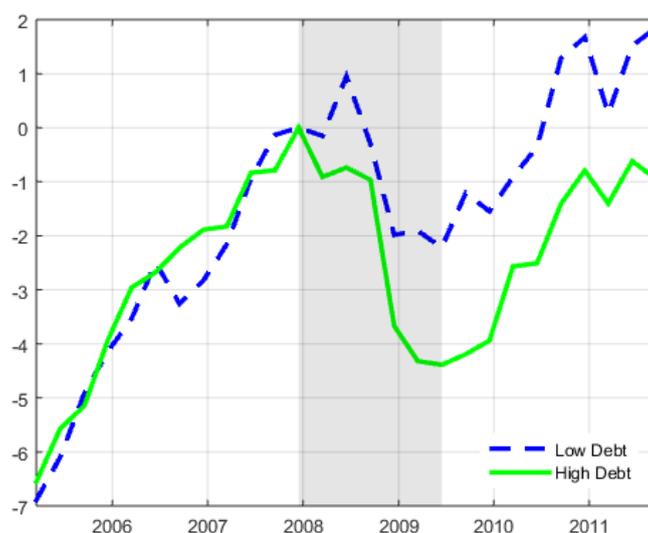
Notes: Figure 3 reports the histogram of the quarter-on-quarter growth rate of real GDP (solid-blue line), as well as the corresponding fitted normal density (dotted-green line) over the 1947:I-1984:II and 1984:III-2016:II samples.

Figure 4: Leverage and asymmetry across U.S. States.



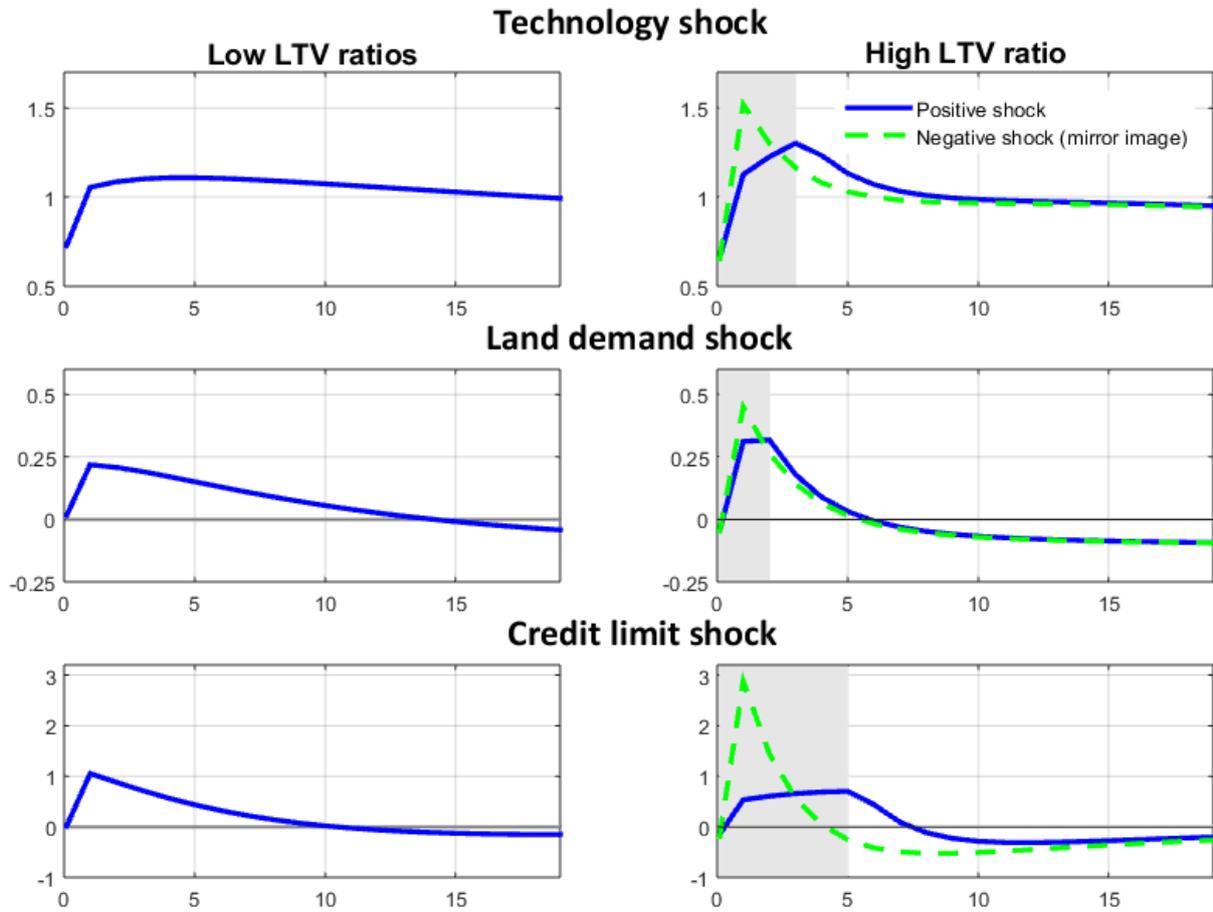
The left panel plots the violence of the Great Recession in each U.S. State against the average debt-to-income ratio at the household-level over the period 2003-07. To allow for the fact that the recession does not begin/end at the same time throughout the US, we calculate the start (end) of the recession in a given state as the period with the highest (lowest) level of real Gross State Product (GSP) in a window that goes from 5 quarters before (after) to one quarter after (before) the NBER dates. The right-hand panel plots the skewness of year-on-year real GSP growth over the 2005-2016 period against the average debt-to-income ratio. In each panel we report the p-values associated with the slope coefficient: the first p-value is calculated on the slope coefficient estimated by OLS, while the second p-value refers to the slope estimated by excluding outliers (i.e., the observations whose standardized residuals from a first stage OLS regression are classified as being out of the 5/95% Gaussian confidence interval). In both cases we compute [White \(1980\)](#) heteroskedasticity-robust standard errors.

Figure 5: GSP dynamics and household leverage.



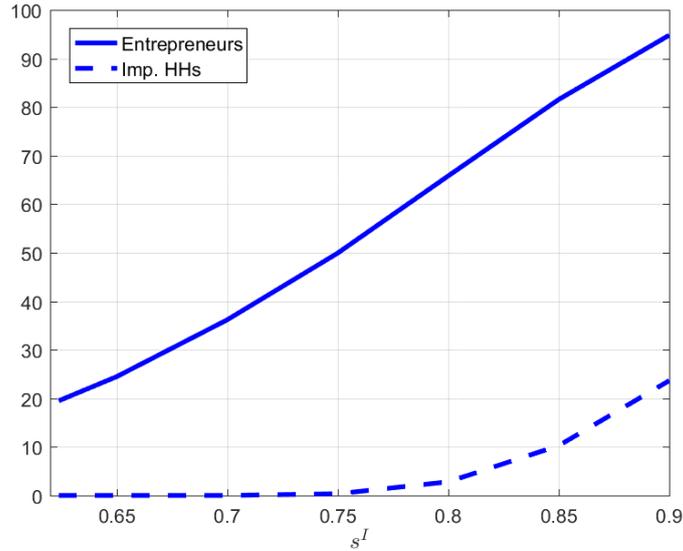
Notes: Figure 5 reports the growth rates of two synthetic GSP series obtained by ranking the U.S. States according to their average debt-to-income ratio in the 5 years before the Great Recession. The dashed-blue line is calculated from the median real GSP of the top 10 states, while the solid-green line is obtained from the median for the bottom 10 states. The resulting statistics have been normalized to zero at the beginning of the Great Recession (i.e., 2007:IV). The vertical shadowed band denotes the 2007:IV-2009:II recession episode.

Figure 6: Impulse responses.



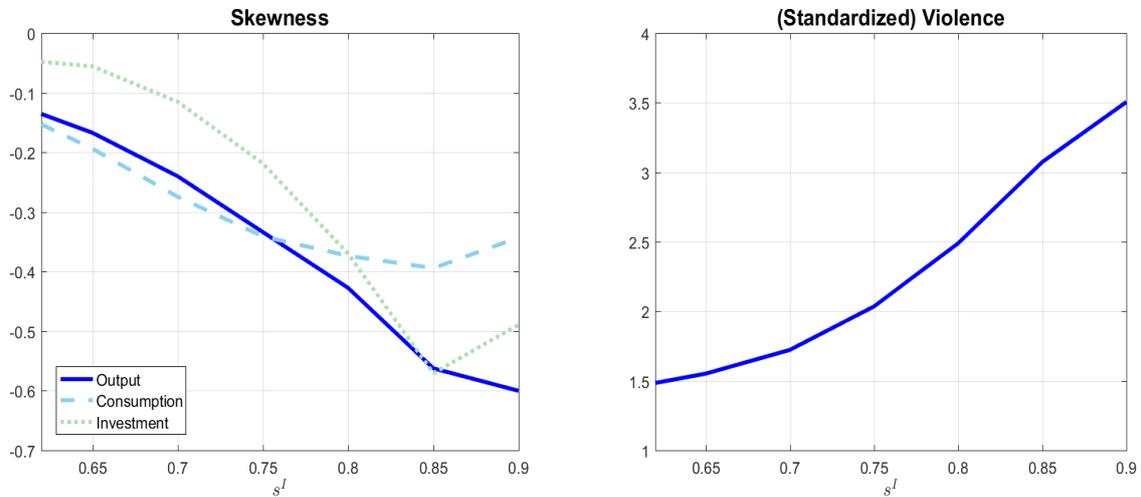
Notes: Figure 6 reports the impulse responses of gross output to a one-standard deviation shock to technology (row 1), land demand (row 2), and credit limits (row 3). Left column:  $s^I = 0.62$ ,  $s^E = 0.71$ ; right column:  $s^I = 0.85$ ,  $s^E = 0.94$ . The shadowed bands indicate the periods in which the entrepreneurs are financially unconstrained.

Figure 7: Leverage and frequency of non-binding collateral constraints.



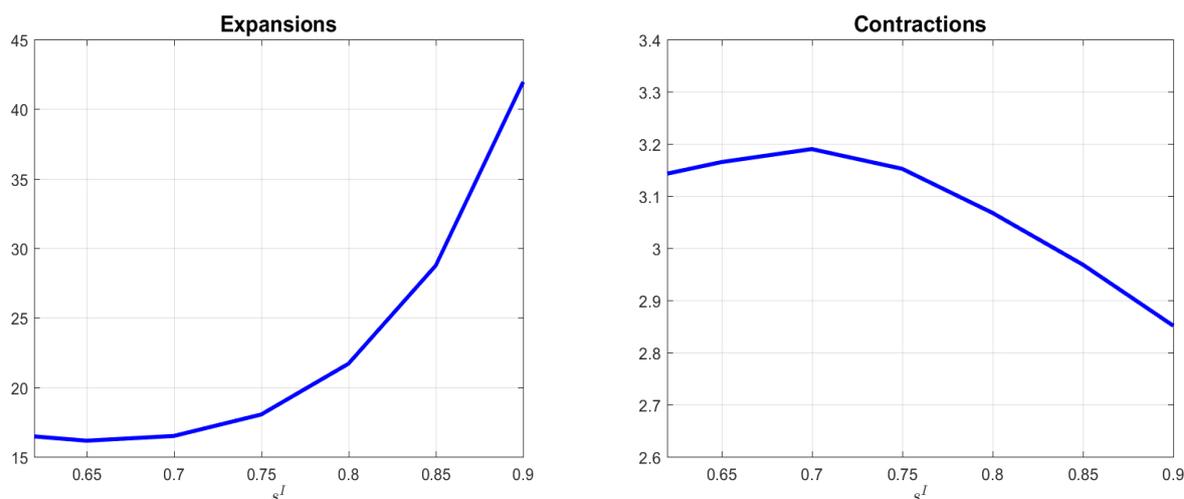
Notes: Frequency of non-binding constraints for impatient households (solid line) and entrepreneurs (dashed line). Both statistics are graphed for different average LTV ratios faced by the impatient household. Across all the simulations the entrepreneurial average LTV ratio is adjusted so as to be 9 basis points greater than any value we consider for impatient households' credit limits, in line with the baseline calibration of the model.

Figure 8: Business cycle asymmetry



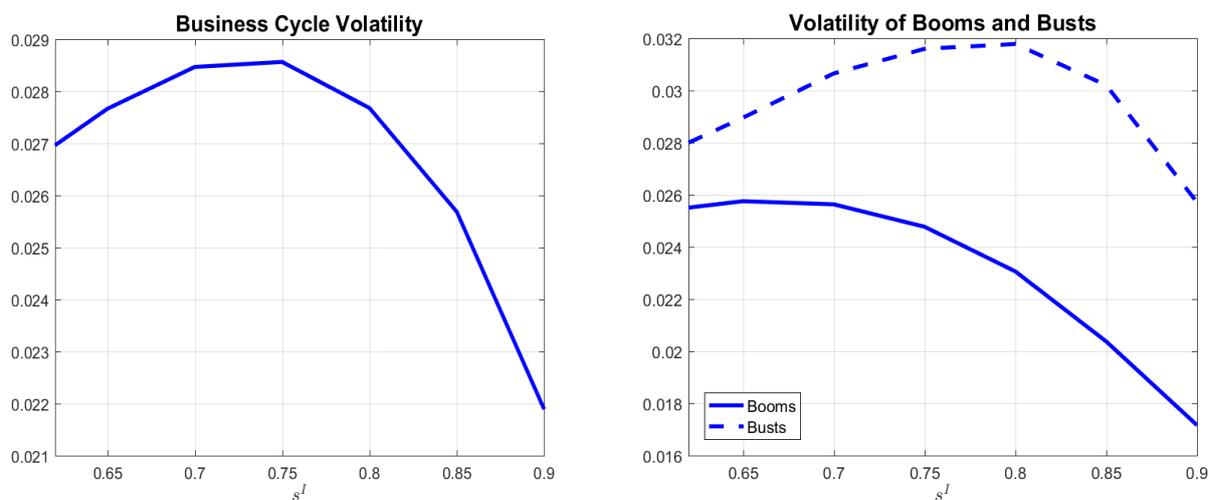
Notes: Figure 8 graphs the skewness of the year-on-year growth rate of output, consumption and investment (left panel), and the standardized violence of the recessions (right panel), for different average LTV ratios faced by the impatient household. To identify the recessionary episodes in our simulated gross output series, we use the [Harding and Pagan \(2002\)](#) algorithm. We then compute violence as the average fall of output over a given recessions, divided by the length of the recession itself. Finally, we standardize violence by means of the volatility of year-on-year output growth over the five years prior to the recession. Across all the simulations the entrepreneurial average LTV ratio is adjusted so as to be 9 basis points greater than any value we consider for impatient households' credit limits, in line with the baseline calibration of the model.

Figure 9: Average duration of expansions and contractions.



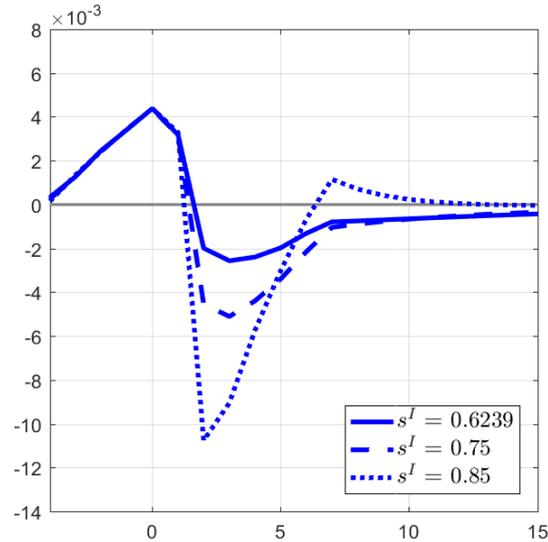
Notes: To identify expansions and contractions in our simulated gross output series, we use the [Harding and Pagan \(2002\)](#) algorithm. The duration of both cyclical phases is graphed for different average LTV ratios faced by the impatient household. Across all the simulations the entrepreneurial average LTV ratio is adjusted so as to be 9 basis points greater than any value we consider for impatient households' credit limits, in line with the baseline calibration of the model.

Figure 10: Leverage and volatility.



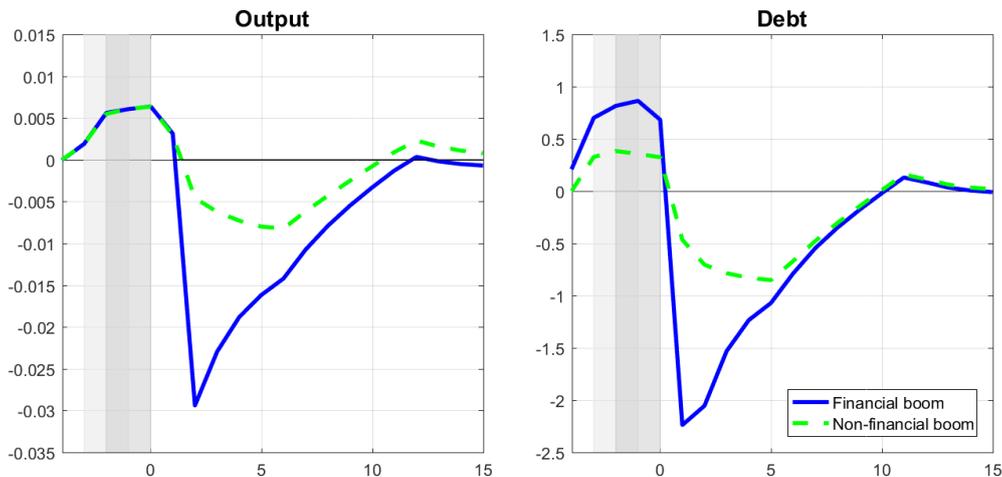
Notes: The left panel reports the standard deviation of output growth, while the right panel reports the standard deviation of booms (solid line) and busts (dashed line) in economic activity. These are determined based on whether output is above or below its steady-state level. Across all the simulations the entrepreneurial average LTV ratio is adjusted so as to be 9 basis points greater than any value we consider for impatient households' credit limits, in line with the baseline calibration of the model.

Figure 11: Boom-bust cycles and leverage.



Notes: Starting in the economy’s steady state, we generate a boom-bust cycle for different steady-state debt levels, as implied by different average LTV ratios. We first feed the economy with a series of positive shocks during the first five periods. The size of the expansionary shocks is set so as to make sure that the boom is identical across all the calibrations. Thus, we shock the economy with identical contractionary shocks for two periods, after which the negative shocks are ‘phased out’ over the next three periods.

Figure 12: Financial vs. non-financial recessions.



Notes: The solid-blue line represents a financial boom, while the dashed-green line represents a non-financial boom. The light-grey area denotes periods in which the entrepreneur becomes financially unconstrained in the financial boom, while the darker grey area denotes periods in which the entrepreneur becomes unconstrained in the non-financial boom. The darkest grey area thus represents periods in which the entrepreneur becomes unconstrained in both types of boom (and the areas overlap). Impatient households remain constrained throughout in both types of booms. We calibrate the size of the expansionary shocks so as to deliver an identical increase in output during each type of boom (which lasts for five periods, up until period 0). We then subject the economy to identical sets of contractionary shocks of all three types. The contractionary shocks hit in periods 1 and 2, and are then ‘phased out’ over the next three periods.

## Appendix A: Assets and liabilities in the US

Figure 2 shows the ratio of liabilities to assets for households and firms in the United States, respectively. All data are taken from FRED (Federal Reserve Economic Data), Federal Reserve Bank of St. Louis. The primary source is Flow of Funds data from the Board of Governors of the Federal Reserve System. For business liabilities we use the sum of debt securities and loans of nonfinancial corporate and noncorporate businesses. As for the assets we follow Liu *et al.* (2013) and use data on both sectors' equipment and software as well as real estate at market value. For households and nonprofit organizations, we again use the sum of debt securities and loans as data for liabilities and use as assets both groups' real estate at market value and equipment and software of nonprofit organizations.<sup>31</sup>

The ratios reported in Figure 2 are aggregate measures, and may therefore not reflect actual loan-to-value (LTV) requirements for the marginal borrower. Nonetheless, we report these figures since the flow of funds data delivers a continuous measure of LTV ratios covering the entire period 1952–2016. For households, the aggregate ratio of credit to assets in the economy is likely to understate the actual down-payment requirements faced by households applying for a mortgage loan, since loans and assets are not uniformly distributed across households. In our model we distinguish between patient and impatient households, and we assume that only the latter group is faced with a collateral constraint. In the data we do not make such a distinction, so that the LTV ratio for households reported in Figure 2 represents an average of the LTV of patient households (savers), who are likely to have many assets and small loans, and that of impatient households (borrowers), who on average have larger loans and fewer assets. Justiniano *et al.* (2014) use the Survey of Consumer Finances and identify borrowers as households with liquid assets of a value less than two months of their income. Based on the surveys from 1992, 1995, and 1998, they arrive at an average LTV ratio for this group of around 0.8, while our measure fluctuates around 0.5 during the 1990s. Following Duca *et al.* (2011), an alternative approach is to focus on first-time home-buyers, who are likely to fully exploit their borrowing capacity. Using data from the American Housing Survey, these authors report LTV ratios approaching 0.9 towards the end of the 1990s; reaching a peak of almost 0.95 before the onset of the recent crisis. While these alternative approaches are likely to result in higher levels of LTV ratios, we are especially interested in the development of these ratios over a rather long time span. While we believe the Flow of Funds data provide the most comprehensive and consistent time series evidence in this respect, substantial increases over time in the LTV ratios faced by households have been extensively documented; see, e.g., Campbell and Hercowitz (2009), Duca *et al.* (2011), Favilukis *et al.* (2017), and Boz and Mendoza (2014). It should be noted that for households, various government-sponsored programs directed at lowering the down-payment requirements faced by low-income or first-time home buyers have been enacted by different administrations (Chambers *et al.*, 2009). These are likely to have contributed to the increase in the ratio of loans to assets illustrated in the left panel of Figure 2

Likewise, the aggregate ratio of business loans to assets in the data may cover for a disparate distribution of credit and assets across firms. In general, the borrowing patterns and conditions of firms are more difficult to characterize than those of households, as their credit demand is more volatile, and their assets are less uniform and often more difficult to assess. Liu *et al.* (2013) also use Flow of Funds data to calibrate the LTV ratio of the entrepreneurs, and arrive at a value of 0.75. This ratio is based on the assumption that commercial real estate enters with a weight of 0.5 in the asset composition of firms. The secular increase in firm leverage over the second half of the 20<sup>th</sup> century has also been documented by Graham *et al.* (2014)

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<sup>31</sup>Until 2015, debt securities and loans were aggregated under the title 'Credit market instruments' for businesses as well as households in the Financial Accounts of the United States.

using data from the Compustat database.<sup>32,33</sup> These authors report loan-to-asset ratios that are broadly in line with those we present. More generally, an enhanced access of firms to credit markets over time has been extensively documented in the literature. This involves, for instance, the emergence of a market for high-risk, high-yield bonds (Gertler and Lown, 1999), increased flexibility in firms’ financing decisions, and the resulting immoderation in financial quantities (Jermann and Quadrini, 2009).

## Appendix B: Additional empirical evidence

### Normality tests

Table B1. Normality tests		
	GDP growth (QoQ)	
	1947:I-1984:II	1984:III-2016:II
AD	0.534	0.000
KS	0.638	0.002
SW	0.507	0.000
JB	$\gg 0.50$	$\ll 0.001$

	GDP growth (YoY)	
	1947:I-1984:II	1984:III-2016:II
AD	0.060	0.000
KS	0.289	0.004
SW	0.091	0.000
JB	$\gg 0.50$	$\ll 0.001$

Notes. Table B1 reports the p-values of a battery of tests assuming the null hypothesis that real GDP growth is normally distributed in a given sample. AD refers to the test of Anderson and Darling (1954), KS refers to Kolmogorov-Smirnov test with estimated parameters (Liliefors, 1967), SW refers to the Shapiro-Wilk test (Shapiro and Wilk, 1965) with p-values calculated as outlined by Royston (1992); JB refers to the Jarque-Bera test for normality (Jarque and Bera, 1987).

### Asymmetries in the variance of the business cycle

Table B2. Semivariances		
	GDP growth (QoQ)	
	1947:I-1984:II	1984:III-2016:II
$\sigma$	4.701	2.358
$\sigma_U/\sigma_D$	0.966	0.775

	GDP growth (YoY)	
	1947:I-1984:II	1984:III-2016:II
$\sigma$	3.070	1.747
$\sigma_U/\sigma_D$	0.942	0.732

<sup>32</sup>It should be mentioned that they also show a Flow of Funds-based measure of debt to total assets at historical cost (or book value) for firms. The increase over time in this measure is smaller. However, we believe that the ratio of debt to *pledgeable* assets at market values (as shown in Figure 2) is the relevant measure for firms’ access to collateralized loans, and hence more appropriate for our purposes.

<sup>33</sup>We emphasize that Figure 2 reports a *gross* measure of firm leverage. Bates *et al.* (2009) report that firm leverage *net of* cash holdings has been declining since 1980, but that this decline is entirely due to a large increase in cash holdings.

Notes. Table B2 reports the volatility of different business cycle indicators and its decomposition into upside and downside volatility. Specifically  $\sigma = \sqrt{\sum_{t=1}^T (x_t - \bar{x})^2 / T} = \sqrt{\sigma_U^2 + \sigma_D^2}$ , with the upside and downside volatility defined as  $\sigma_U = \sqrt{\sum_{t=1}^T (x_t - \bar{x})^2 \mathbf{1}(x_t \geq \bar{x}) / T}$  and  $\sigma_D = \sqrt{\sum_{t=1}^T (x_t - \bar{x})^2 \mathbf{1}(x_t < \bar{x}) / T}$  respectively, where  $\mathbf{1}(z)$  is an indicator function taking value 1 when condition  $z$  is verified and 0 otherwise.

## Additional evidence on the violence of the US business cycle

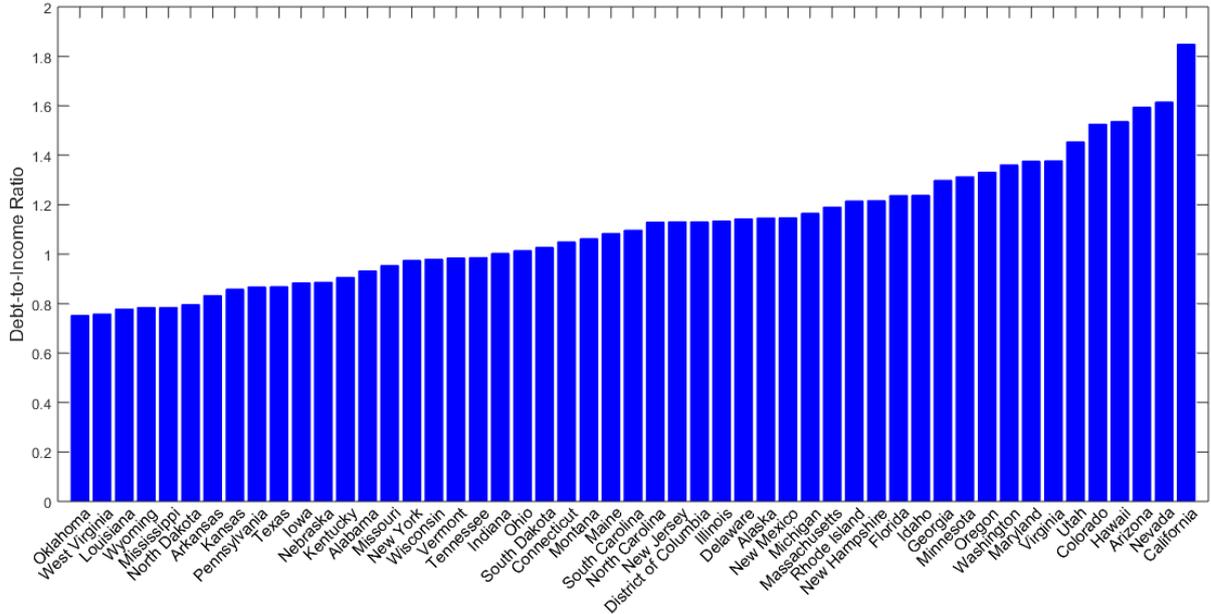
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
1953:II – 1954:II	0.6353	1.1270	0.7337	0.7313	0.5989	0.6644	0.5835
1957:III – 1958:II	1.6292	2.4152	1.5723	1.7671	1.7163	1.6875	1.7846
1960:II – 1961:I	0.3512	0.5951	0.3874	0.4288	0.3074	0.4636	0.3038
1969:IV – 1970:IV	0.1631	0.1556	0.1013	0.2476	0.1548	0.2398	0.1584
1973:IV – 1975:I	0.6618	0.8358	0.5441	0.8468	0.5417	0.8329	0.5086
1980:I – 1980:III	0.9991	1.4542	0.9467	1.2388	0.9719	1.2342	0.8633
1981:III – 1982:IV	0.5977	0.8851	0.5762	0.6452	0.4481	0.6211	0.3995
1990:III – 1991:I	1.9098	1.5271	1.1319	1.3633	1.2550	1.2294	1.3007
2001:I – 2001:IV	0.7295	0.7299	0.5410	0.7551	0.4630	0.7010	0.4191
2007:IV – 2009:II	1.8467	1.6652	1.2343	2.0200	1.6071	1.9154	1.5710
Average							
Pre-84	0.7196	1.0669	0.6945	0.8437	0.6770	0.8205	0.6574
Post-84	1.4953	1.3074	0.9691	1.3795	1.1083	1.2819	1.0969

Notes: Table B3 reports different measures of standardized violence that change depending on the business cycle volatility employed in the denominator. ‘QoQ’ and ‘YoY’ indicate that the volatility has been computed over annualized quarter-on-quarter growth and year-on-year growth, respectively. Column (1) follows the same procedure employed to obtain the standardized violence in Table 2, but calculates the volatility on QoQ data. Columns (2) and (3) calculate the volatility by splitting the data between pre- and post-Great Moderation. In columns (4) and (5) the standardization is operated by considering the following stochastic volatility model for real GDP growth:  $y_t = \rho_0 + \rho_1 y_{t-1} + \rho_2 y_{t-2} + \sigma_t \varepsilon_t$ , where  $\sigma_t^2 = \sigma_{t-1}^2 + \kappa \sigma_t^2 (\varepsilon_t^2 - 1)$  and  $\varepsilon_t \sim N(0, 1)$ . In columns (6) and (7) the standardization is operated by considering a time-varying AR model for real GDP growth with stochastic volatility similar to that of [Stock and Watson \(2007\)](#), where all the time-varying parameters follow random walk laws of motion (as in [Delle Monache and Petrella, 2017](#)).

## Household leverage in the US

[Figure B1](#) displays the ranking of most highly leveraged US states used in subsection 2.2.

Figure B1: U.S. States ordered by households' average debt-to-income ratio.



Notes. The US States are ordered by the average debt-to-income ratio in the household sector, over the period 2003-2007.

## Time-varying volatility and skewness

In the main text we report evidence on the skewness of real GDP growth changing between the pre- and post-Great Moderation sample. The choice of a cut-off date is inspired by a large literature that has documented a drop in the volatility over the two samples. This exercise entails a possible drawback: The estimates of the skewness can be biased by the first and second moment of the business cycle changing over time. In particular: i) There is now ample evidence that the volatility of the business cycle displays a cyclical behavior (see, e.g., [Kim and Nelson, 1999](#); and [McConnell and Perez-Quiros, 2000](#)) and ii) the long-run growth rate of the economy since around 2000 is substantially lower than the average for the entire sample (see, e.g., [Antolin-Diaz et al., 2017](#)). To account for these issues we report a measure of time-varying skewness of real GDP growth for the entire sample, relying on a nonparametric estimator. To this end, take a generic time series,  $y_t$ , so that its variance and skewness can be respectively calculated as

$$\sigma^2 = \text{Var}(y_t) = \frac{1}{T} \sum_{t=1}^T (y_t - \mu)^2,$$

$$\varrho = \text{Skew}(y_t) = \left\{ \frac{1}{T} \sum_{t=1}^T (y_t - \mu)^2 \right\}^{-3/2} \left\{ \frac{1}{T} \sum_{t=1}^T (y_t - \mu)^3 \right\},$$

where  $T$  denotes the number of observations in the sample and  $\mu = E(y_t) = T^{-1} \sum_{t=1}^T y_t$  is the sample average. Define the sample autocovariance and autocorrelation as

$$\begin{aligned}\gamma_\tau &= \frac{1}{T} \sum_{t=1}^{T-|\tau|} (y_{t-|\tau|} - \mu)(y_t - \mu), \\ \rho_\tau &= \frac{\gamma_\tau}{\sigma^2}.\end{aligned}$$

When  $y_t$  is a Gaussian process with absolutely summable autocovariances, it can be shown that the standard errors associated with the two measures are:<sup>34</sup>

$$\begin{aligned}\text{Var}(\sigma^2) &= \frac{2}{T} \left( \sum_{\tau=-\infty}^{\infty} \gamma_\tau \right)^2, \\ \text{Var}(\varrho) &= \frac{6}{T} \sum_{\tau=-\infty}^{\infty} \rho_\tau^3.\end{aligned}$$

In practice the two summations are truncated at some appropriate (finite) lag  $k$ .

The framework we follow in order to account for time-variation in the variance and skewness has a long pedigree in statistics, starting with the work of [Priestley \(1965\)](#), who introduced the concept of slowly varying process. This work suggests that time series may have time-varying spectral densities which change slowly over time, and proposed to describe those changes as the result of a non-parametric process. This work has more recently been followed up by [Dahlhaus \(1996\)](#), as well as [Kapetanios \(2007\)](#) and [Giraitis \*et al.\* \(2014\)](#) in the context of time-varying regression models and economic forecasting, respectively. Specifically, the time-varying variance and skewness are calculated as

$$\begin{aligned}\sigma_t^2 &= \text{Var}_t(y_t) = \sum_{j=1}^t \omega_{j,t} (y_j - \mu_t)^2, \\ \varrho_t &= \text{Skew}_t(y_t) = \left\{ \sum_{j=1}^t \omega_{j,t} (y_j - \mu_t)^2 \right\}^{-3/2} \left\{ \sum_{j=1}^t \omega_{j,t} (y_j - \mu_t)^3 \right\},\end{aligned}$$

where  $\mu_t = \sum_{j=1}^t \omega_{j,t} y_j$ . Thus, the sample moments are discounted by the function  $\omega_{j,t}$ :

$$\omega_{j,t} = cK\left(\frac{t-j}{H}\right),$$

where  $c$  is an integration constant and  $K\left(\frac{T-t}{H}\right)$  is the kernel function determining the weight of each observation  $j$  in the estimation at time  $t$ . This weight depends on the distance to  $t$  normalized by the bandwidth  $H$ . [Giraitis \*et al.\* \(2014\)](#) show that the estimator has desirable frequentist properties. They suggest using Gaussian kernels with the optimal bandwidth value  $H = T^{1/2}$ .

Similarly, we can compute the time-varying standard deviation of variance and skewness

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<sup>34</sup>The first expression computes the variance as the Newey-West variance of the squared residuals, in order to account for the autocorrelation of the errors. The second equality follows from [Gasser \(1975\)](#) and [Psaradakis and Sola \(2003\)](#).

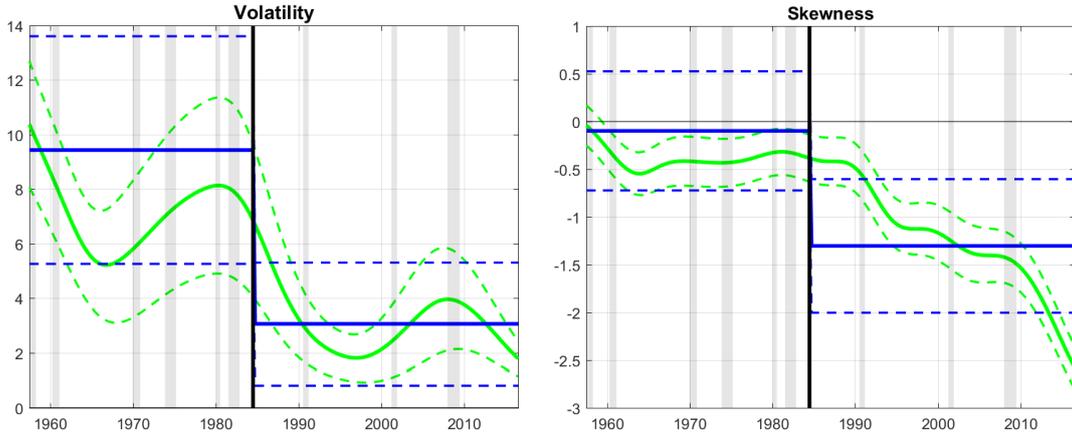
estimates using time-varying estimates of the sample autocovariance and autocorrelations:

$$\gamma_{\tau,t} = \sum_{j=1}^{t-|\tau|} \omega_{j,t} (y_{j-|\tau|} - \mu_t) (y_j - \mu_t),$$

$$\rho_{\tau,t} = \frac{\gamma_{\tau,t}}{\sigma_t^2}.$$

Based on this, [Figure B2](#) reports time-varying measures of volatility and skewness of GDP growth. Clearly, the skewness drops in the second subsample, with a first drop being identified after the 1991 recession and a further one after the Great Recession.

Figure B2: Time-varying volatility and skewness



Notes. Figure B2 reports the time-varying volatility and skewness of year-on-year growth of real GDP – obtained by using a nonparametric estimator in the spirit of [Giraitis et al. \(2014\)](#) – as well as the associated 68% confidence interval. The vertical shadowed bands denote the NBER recession episodes. Sample: 1947:I-2016:II. The first 10 years of data are dropped to initialize the algorithm.

## Appendix C: Details of the partial equilibrium economy

## Appendix D: First-order conditions of the DSGE model

This appendix reports the first-order conditions for each agent in the model.

### Patient households

Patient households' optimal behavior is described by the following first-order conditions:

$$\frac{1}{C_t^P - \theta^P C_{t-1}^P} - \frac{\beta \theta^P}{\mathbf{E}_t \{C_{t+1}^P\} - \theta^P C_t^P} = \lambda_t^P, \quad (23)$$

$$\nu^P (1 - N_t^P)^{-\sigma_N^P} = \lambda_t^P W_t^P, \quad (24)$$

$$\lambda_t^P = \beta^P R_t \mathbf{E}_t \{ \lambda_{t+1}^P \}, \quad (25)$$

$$Q_t = \frac{\varepsilon_t}{\lambda_t^P H_t^P} + \beta^P \mathbf{E}_t \left\{ \frac{\lambda_{t+1}^P}{\lambda_t^P} Q_{t+1} \right\}, \quad (26)$$

where  $\lambda_t^P$  is the multiplier associated with (7) for  $i = P$ .

## Impatient households

The first-order conditions of the impatient households are given by:

$$\frac{1}{C_t^I - \theta^I C_{t-1}^I} - \frac{\beta\theta^I}{\mathbf{E}_t \{C_{t+1}^I\} - \theta^I C_t^I} = \lambda_t^I, \quad (27)$$

$$\nu^I (1 - N_t^I)^{-\sigma^I} = \lambda_t^I W_t^I, \quad (28)$$

$$\lambda_t^I - \mu_t^I = \beta^I R_t \mathbf{E}_t \{ \lambda_{t+1}^I \}, \quad (29)$$

$$Q_t = \frac{\varepsilon_t}{\lambda_t^I H_t^I} + \beta^I \mathbf{E}_t \left\{ \frac{\lambda_{t+1}^I}{\lambda_t^I} Q_{t+1} \right\} + s_t^I \frac{\mu_t^I}{\lambda_t^I} \frac{\mathbf{E}_t \{ Q_{t+1} \}}{R_t}, \quad (30)$$

where  $\lambda_t^I$  is the multiplier associated with (9) for  $i = I$ , and  $\mu_t^I$  is the multiplier associated with (10). Additionally, the complementary slackness condition

$$\mu_t^I \left( B_t^I - s_t^I \frac{\mathbf{E}_t \{ Q_{t+1} \} H_t^I}{R_t} \right) = 0, \quad (31)$$

must hold along with  $\mu_t^I \geq 0$  and (10).

## Entrepreneurs

The optimal behavior of the entrepreneurs is characterized by:

$$\frac{1}{C_t^E - \theta^E C_{t-1}^E} - \frac{\beta\theta^E}{\mathbf{E}_t \{C_{t+1}^E\} - \theta^E C_t^E} = \lambda_t^E, \quad (32)$$

$$\lambda_t^E - \mu_t^E = \beta^E R_t \mathbf{E}_t \{ \lambda_{t+1}^E \}, \quad (33)$$

$$\lambda_t^E = \psi_t^E \left[ 1 - \frac{\Omega}{2} \left( \frac{I_t}{I_{t-1}} - 1 \right)^2 - \Omega \frac{I_t}{I_{t-1}} \left( \frac{I_t}{I_{t-1}} - 1 \right) \right] + \beta^E \Omega \mathbf{E}_t \left\{ \psi_{t+1}^E \left( \frac{I_{t+1}}{I_t} \right)^2 \left( \frac{I_{t+1}}{I_t} - 1 \right) \right\}, \quad (34)$$

$$\psi_t^E = \beta^E r_t^K \mathbf{E}_t \{ \lambda_{t+1}^E \} + \beta^E (1 - \delta) \mathbf{E}_t \{ \psi_{t+1}^E \} + \mu_t^E s_t^E \frac{\mathbf{E}_t \{ Q_{t+1}^K \}}{R_t}, \quad (35)$$

$$Q_t = \beta^E r_t^H \mathbf{E}_t \left\{ \frac{\lambda_{t+1}^E}{\lambda_t^E} \right\} + \beta^E \mathbf{E}_t \left\{ \frac{\lambda_{t+1}^E}{\lambda_t^E} Q_{t+1} \right\} + s_t^E \frac{\mu_t^E}{\lambda_t^E} \frac{\mathbf{E}_t \{ Q_{t+1} \}}{R_t}, \quad (36)$$

where  $\lambda_t^E$ ,  $\mu_t^E$  and  $\psi_t^E$  are the multipliers associated with (14), (15), and (16), respectively. Moreover,

$$\mu_t^E \left( B_t^E - s_t^E \mathbf{E}_t \left\{ \frac{Q_{t+1}^K K_t + Q_{t+1} H_t^E}{R_t} \right\} \right) = 0, \quad (37)$$

holds along with  $\mu_t^E \geq 0$  and (16). Finally, the definition of  $Q_t^K$  implies that

$$Q_t^K = \psi_t^E / \lambda_t^E. \quad (38)$$

## Firms

The first-order conditions for the firms determine the optimal demand for the input factors:

$$\alpha\gamma Y_t/N_t^P = W_t^P, \quad (39)$$

$$(1-\alpha)\gamma Y_t/N_t^I = W_t^I, \quad (40)$$

$$(1-\gamma)(1-\phi)\mathbb{E}_t\{Y_{t+1}\}/K_t = r_t^K, \quad (41)$$

$$(1-\gamma)\phi\mathbb{E}_t\{Y_{t+1}\}/H_t^E = r_t^H. \quad (42)$$

## Appendix E: Solution method

As discussed in the main text, we treat the collateral constraints as inequalities when we solve the model, accounting for two complementary slackness conditions:

$$\begin{aligned} \mu_t^I \left( B_t^I - s_t^I \frac{\mathbb{E}_t\{Q_{t+1}\} H_t^I}{R_t} \right) &= 0, \\ \mu_t^E \left( B_t^E - s_t^E \mathbb{E}_t \left\{ \frac{Q_{t+1}^K K_t + Q_{t+1} H_t^E}{R_t} \right\} \right) &= 0. \end{aligned}$$

We then adopt the solution method of [Holden and Paetz \(2012\)](#), on which this appendix builds. In turn, [Holden and Paetz \(2012\)](#) expand on previous work by [Laséen and Svensson \(2011\)](#). With first-order perturbations, this solution method is equivalent to the piecewise linear approach used in [Guerrieri and Iacoviello \(2017\)](#), as discussed by [Guerrieri and Iacoviello \(2015\)](#) and the references therein. We have verified that the solution method described in [Guerrieri and Iacoviello \(2015\)](#) does indeed produce identical results. Furthermore, [Holden and Paetz \(2012\)](#) and [Guerrieri and Iacoviello \(2015\)](#) evaluate the accuracy of their respective methods against a global solution based on projection methods. This is done for a very simple model with a borrowing constraint, for which a highly accurate global solution can be obtained and used as a benchmark. They find that the non-linear local approximations are very accurate. For the model used in this paper, the large number of state variables (14 endogenous state variables and three shocks) renders the use of global solution methods impractical due to the curse of dimensionality typically associated with such methods.

The collateral constraints put an upper bound on the borrowing of each of the two constrained agents. While the constraints are binding in the steady state, this may not be the case outside the steady state, where the constraints may not bind. Observe that we can reformulate the collateral constraints in terms of restrictions on each agent's shadow value of borrowing;  $\mu_t^j$ ,  $j = \{I, E\}$ : We know that  $\mu_t^j \geq 0$  if and only if the optimal debt level of agent  $j$  is exactly at or above the collateral value. In other words, we need to ensure that  $\mu_t^j \geq 0$ . If this restriction is satisfied with inequality, the constraint is binding, so the slackness condition is satisfied. If it holds with equality, the collateral constraint becomes non-binding, but the slackness condition is still satisfied. If instead  $\mu_t^j < 0$ , agent  $j$ 's optimal level of debt is lower than the credit limit, so that treating his collateral constraint as an equality implies that we are forcing him to borrow 'too much'. In this case, the slackness condition is violated. We then need to add shadow price shocks so as to 'push'  $\mu_t^j$  back up until it exactly equals its lower limit of zero and the slackness condition is satisfied. To ensure compatibility with rational expectations, these shocks are added to the model as "news shocks". The idea of adding such shocks to the model derives from [Laséen and Svensson \(2011\)](#), who use such an approach to deal with pre-announced paths for the interest rate setting of a central bank. The contribution of [Holden and Paetz \(2012\)](#) is to develop a numerical method to compute the size of these shocks that are required to obtain the desired level for a given variable in each period, and to

make this method applicable to a general class of potentially more complicated problems than the relatively simple experiments conducted by [Laséen and Svensson \(2011\)](#).

We first describe how to compute impulse responses to a single generic shock, e.g., a technology shock. The first step is to add independent sets of shadow price shocks to each of the two log-linearized collateral constraints. To this end, we need to determine the number of periods  $T$  in which we conjecture that the collateral constraints may be non-binding. This number may be smaller than or equal to the number of periods for which we compute impulse responses;  $T \leq T^{IRF}$ . For each period  $t \leq T$ , we then add shadow price shocks which hit the economy in period  $t$  but become known at period 0, that is, at the same time the economy is hit by the technology shock. In other words, the log-linearized collateral constraints now become:

$$\frac{nY}{n_I B^I} \widehat{B}_t^I = \widehat{s}_t^I + E_t \left\{ \widehat{Q}_{t+1} \right\} + \widehat{H}_t^I - \beta^P \widehat{R}_t - \sum_{s=0}^{T-1} \varepsilon_{s,t-s}^{SP,I},$$

$$\frac{nY}{n_E B^E} \widehat{B}_t^E = \widehat{s}_t^E - \beta^P \widehat{R}_t + \frac{K}{K + QH^E} \left( E_t \left\{ \widehat{Q}_{t+1}^K \right\} + \widehat{K}_t \right) + \frac{QH^E}{K + QH^E} \left( E_t \left\{ \widehat{Q}_{t+1} \right\} + \widehat{H}_t^E \right) - \sum_{s=0}^{T-1} \varepsilon_{s,t-s}^{SP,E},$$

where  $\varepsilon_{s,t-s}^{SP,j}$  is the shadow price shock that hits agent  $j$  in period  $t = s$ , and is anticipated by all agents in period  $t = t - s = 0$  ensuring consistency with rational expectations. We let all shadow price shocks be of unit magnitude. We then need to compute two sets of weights  $\alpha_{\mu_I}$  and  $\alpha_{\mu_E}$  to control the impact of each shock on  $\mu_t^I$  and  $\mu_t^E$ . The ‘optimal’ sets of weights ensure that  $\mu_t^I$  and  $\mu_t^E$  are bounded below at exactly zero. The weights are computed by solving the following quadratic programming problem:

$$\begin{aligned} \alpha^* &\equiv \left[ \alpha_{\mu_I}^* \quad \alpha_{\mu_E}^* \right]' \\ &= \arg \min \left[ \alpha_{\mu_I}' \quad \alpha_{\mu_E}' \right] \left[ \begin{array}{c} \left[ \mu^I + \widetilde{\mu}^{I,A} \right] \\ \left[ \mu^E + \widetilde{\mu}^{E,A} \right] \end{array} + \left[ \begin{array}{cc} \widetilde{\mu}^{I,\varepsilon^{SP,I}} & \widetilde{\mu}^{I,\varepsilon^{SP,E}} \\ \widetilde{\mu}^{E,\varepsilon^{SP,I}} & \widetilde{\mu}^{E,\varepsilon^{SP,E}} \end{array} \right] \left[ \begin{array}{c} \alpha_{\mu_I} \\ \alpha_{\mu_E} \end{array} \right] \right], \end{aligned}$$

subject to

$$\alpha_{\mu_j}' \geq 0,$$

$$\mu^j + \widetilde{\mu}^{j,A} + \widetilde{\mu}^{j,\varepsilon^{SP,j}} \alpha_{\mu_j} + \widetilde{\mu}^{j,\varepsilon^{SP,k}} \alpha_{\mu_k} \geq 0,$$

$j = \{I, E\}$ . Here,  $\mu^j$  and  $\widetilde{\mu}^{j,A}$  denote, respectively, the steady-state value and the unrestricted relative impulse response of  $\mu^j$  to a technology shock, that is, the impulse-response of  $\mu^j$  when the collateral constraints are assumed to always bind. In this respect, the vector  $\begin{bmatrix} \mu^I + \widetilde{\mu}^{I,A} \\ \mu^E + \widetilde{\mu}^{E,A} \end{bmatrix}$  contains the absolute, unrestricted impulse responses of the two shadow values stacked. Further, each matrix  $\widetilde{\mu}^{j,\varepsilon^{SP,k}}$  contains the relative impulse responses of  $\mu^j$  to shadow price shocks to agent  $k$ 's constraint for  $j, k = \{I, E\}$ , in the sense that column  $s$  in  $\widetilde{\mu}^{j,\varepsilon^{SP,k}}$  represents the response of the shadow value to a shock  $\varepsilon_{s,t-s}^{SP,j}$ , i.e. to a shadow price shock that hits in period  $s$  but is anticipated at time 0, as described above.<sup>35</sup> The off-diagonal elements of the matrix  $\begin{bmatrix} \widetilde{\mu}^{I,\varepsilon^{SP,I}} & \widetilde{\mu}^{I,\varepsilon^{SP,E}} \\ \widetilde{\mu}^{E,\varepsilon^{SP,I}} & \widetilde{\mu}^{E,\varepsilon^{SP,E}} \end{bmatrix}$  take into account that the impatient household may be affected if the collateral constraint of the entrepreneur becomes non-binding, and *vice versa*. Following the discussion in [Holden and Paetz \(2012\)](#), a sufficient condition for the existence of a unique

<sup>35</sup>Each matrix  $\widetilde{\mu}^{j,\varepsilon^{SP,k}}$  needs to be a square matrix, so if the number of periods in which we guess the constraints may be non-binding is smaller than the number of periods for which we compute impulse responses,  $T < T^{IRF}$ , we use only the first  $T$  rows of the matrix, i.e., the upper square matrix.

solution to the optimization problem is that the matrix  $\begin{bmatrix} \tilde{\mu}_{I,\varepsilon}^{SP,I} & \tilde{\mu}_{I,\varepsilon}^{SP,E} \\ \tilde{\mu}_{E,\varepsilon}^{SP,I} & \tilde{\mu}_{E,\varepsilon}^{SP,E} \end{bmatrix} + \begin{bmatrix} \tilde{\mu}_{I,\varepsilon}^{SP,I} & \tilde{\mu}_{I,\varepsilon}^{SP,E} \\ \tilde{\mu}_{E,\varepsilon}^{SP,I} & \tilde{\mu}_{E,\varepsilon}^{SP,E} \end{bmatrix}'$  is positive definite. We have checked and verified that this condition is in fact always satisfied.

We can explain the nature of the optimization problem as follows. First, note that  $\mu^j + \tilde{\mu}^{j,A} + \tilde{\mu}^{j,\varepsilon^{SP,j}} \alpha_{\mu_j} + \tilde{\mu}^{j,\varepsilon^{SP,k}} \alpha_{\mu_k}$  denotes the combined response of  $\mu_t^j$  to a given shock (here, a technology shock) *and* a simultaneous announcement of a set of future shadow price shocks for a given set of weights. Given the constraints of the problem, the objective is to find a set of optimal weights so that the impact of the (non-negative) shadow-price shocks is exactly large enough to make sure that the response of  $\mu_t^j$  is never negative. The minimization ensures that the impact of the shadow price shocks will never be larger than necessary to obtain this. Finally, we only allow for solutions for which the value of the objective function is zero. This ensures that at any given horizon, positive shadow price shocks occur if and only if at least one of the two constrained variables,  $\mu_t^I$  and  $\mu_t^E$ , are at their lower bound of zero in that period. As pointed out by [Holden and Paetz \(2012\)](#), this can be thought of as a complementary slackness condition on the two inequality constraints of the optimization problem. Once we have solved the minimization problem, it is straightforward to compute the bounded impulse responses of all endogenous variables by simply adding the optimally weighted shadow price shocks to the unconstrained impulse responses of the model in each period.

We rely on the same method to compute dynamic simulations. In this case, however, we need to allow for more than one type of shock. For each period  $t$ , we first generate the shocks hitting the economy. We then compute the unrestricted path of the endogenous variables given those shocks and given the simulated values in  $t - 1$ . The unrestricted paths of the bounded variables ( $\mu_t^I$  and  $\mu_t^E$ ) then take the place of the impulse responses in the optimization problem. If the unrestricted paths of  $\mu_t^I$  and  $\mu_t^E$  never hit the bounds in future periods, our simulation for period  $t$  is fine. If the bounds are hit, we follow the method above and add anticipated shadow price shocks for a sufficient number of future periods. We then compute restricted values for all endogenous variables, and use these as our simulation for period  $t$ . Note that, unlike the case for impulse responses, in our dynamic simulations not all anticipated future shadow price shocks will eventually hit the economy, as other shocks may occur before the realization of the expected shadow price shocks and push the restricted variables away from their bounds.

## Appendix F: Data description and estimation strategy

As described in the main text, we use data for the following five macroeconomic variables of the U.S. economy spanning the period 1952:I–1984:II: The growth rates (in log-differences) of real GDP, real private consumption, real non-residential investment, and real house prices, and the average of the deviations from trend of the two LTA series reported in the right panel of [Figure 2](#). All data series are taken from the Federal Reserve’s FRED database, with the exception of the house price, which is provided by the US Census Bureau. The series are the following:

- Growth rate of *Real Gross Domestic Product*, billions of chained 2009 dollars, seasonally adjusted, annual rate (FRED series name: GDPC1).
- Growth rate of *Real Personal Consumption Expenditures*, billions of chained 2009 dollars, seasonally adjusted, annual rate (FRED series name: PCECC96).
- Growth rate of *Real private fixed investment: Nonresidential* (chain-type quantity index), index 2009=100, seasonally adjusted (FRED series name: B008RA3Q086SBEA).
- Growth rate of *Price Index of New Single-Family Houses Sold Including Lot Value*, index 2005=100, not seasonally adjusted. This series is available only from 1963:Q1 onwards.

- To obtain the house price in real terms, this series is deflated using the GDP deflator (*Gross Domestic Product: Implicit Price Deflator*, index 2009=100, seasonally adjusted, FRED series name: GDPDEF).
- LTA data: See Appendix A. We use the average of the cyclical components – obtained through a multivariate Beveridge-Nelson decomposition – of the series in the right panel of [Figure 2](#) for the period up until 1984:II.

## Estimation

We use 16 empirical moments in the SMM estimation: The standard deviations and first-order autoregressive parameters of each of the five variables described above, the correlation of consumption, investment, and house prices with output, and the skewness of output, consumption, and investment. These moments are matched to their simulated counterparts from the theoretical model. Our estimation procedure seeks to minimize the sum of squared deviations between empirical and simulated moments. As some of the moments are measured in different units (e.g., standard deviations vs. correlations), we use the percentage deviation from the empirical moment in each case. In order for the minimization procedure to converge, it is crucial to use the same set of shocks repeatedly, making sure that the only change in the simulated moments from one iteration to the next is that arising from updating the parameter values. In practice, since the list of parameter values to be estimated includes the variance of the shocks in the model, we draw from the standard normal distribution with zero mean and unit variance, and then scale the shocks by the variance of each of the three shock processes, allowing us to estimate the latter. We use a draw of 2000 realizations of each of the three shocks in the model, thus obtaining simulated moments for 2000 periods.<sup>36</sup> To make sure that the draw of shocks used is representative of the underlying distribution, we make 501 draws of potential shock matrices, rank these in terms of the standard deviations of each of the three shocks, and select the shock matrix closest to the median in all cases. This matrix of shocks is then used in the estimation. In the estimation, we impose only very general bounds on parameter values: All parameters are bounded below at zero, and the habit formation parameters along with all AR(1)-coefficients are bounded above at 0.99.

To initiate the estimation procedure a set of initial values for the estimated parameters are needed. These are chosen based on values reported in the existing literature. It is important to state that the estimation results proved robust to changes in the set of initial values, as long as these remain within the range of available estimates. Based on the empirical estimates of [Justiniano \*et al.\* \(2013\)](#), we set the initial values of the investment adjustment cost parameter ( $\Omega$ ) and the parameters governing habit formation in consumption for the three agents to 4 and 0.7, respectively. For the technology shock, we choose values similar to those used in most of the real business cycle literature,  $\rho_A = 0.97$  and  $\sigma_A = 0.005$  (see, e.g., [Mandelman \*et al.\*, 2011](#)). For the credit limit shock, we set the persistence parameter  $\rho_s = 0.98$ , while the standard deviation is set to  $\sigma_s = 0.01$ , consistent with the values estimated by [Jermann and Quadrini \(2012\)](#) and [Liu \*et al.\* \(2013\)](#). Finally, for the land-demand shock, we set  $\rho_\varepsilon = 0.96$  and  $\sigma_\varepsilon = 0.06$ , in line with [Iacoviello and Neri \(2010\)](#) and [Liu \*et al.\* \(2013\)](#).

We abstain from using an optimal weighting matrix in the estimation. This choice is based on the findings of [Altonji and Segal \(1996\)](#), who show that when GMM is used to estimate covariance structures and, potentially, higher-order moments such as variances, as in our case, the use of an optimal weighting matrix causes a severe downward bias in estimated parameter

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<sup>36</sup>Our simulated sample is thus more than 15 times longer than the actual dataset (which spans 130 quarters). [Ruge-Murcia \(2012\)](#) finds that SMM is already quite accurate when the simulated sample is five or ten times longer than the actual data.

values. Similar concerns apply to SMM as to GMM. The bias arises because the moments used to fit the model itself are correlated with the weighting matrix, and may thus be avoided by the use of fixed weights in the minimization. [Altonji and Segal \(1996\)](#) demonstrate that minimization schemes with fixed weights clearly dominate optimally weighted ones in such circumstances. [Ruge-Murcia \(2012\)](#) points out that parameter estimates remain consistent for any positive-definite weighting matrix, and finds that the accuracy and efficiency gains associated with an optimal weighting matrix are not overwhelming. Finally, we follow [Iacoviello \(2005\)](#) in attaching a weight of four to the moments of particular interest, which in our case are the skewness of output, consumption, and investment, while all other moments receive a unit weight. The empirical moments and their model counterparts upon estimation are reported in [Table F1](#).

When computing standard errors, we rely on a version of the delta method, as described, e.g., in [Hamilton \(1994\)](#). We approximate the numerical derivative of the moments with respect to the estimated parameters using the secant that can be computed by adding and subtracting  $\epsilon$  to/from the estimates, where  $\epsilon$  is a very small number. The covariance (or spectral density) matrix is estimated using the Newey-West estimator.

Table F1: Empirical and simulated moments.

	<i>Standard deviations (percent)</i>	
	Model simulations	US data, 1952:I–1984:II
Output	2.75	2.84
Consumption	2.00	2.23
Investment	6.25	6.91
House price	3.71	3.05
LTV ratio	7.46	6.96
	<i>Skewness</i>	
Output	−0.16	−0.38
Consumption	−0.17	−0.31
Investment	−0.07	−0.41
	<i>Autocorrelations</i>	
Output	0.78	0.82
Consumption	0.58	0.81
Investment	0.89	0.84
House price	0.61	0.79
LTV ratio	0.89	0.92
	<i>Correlations with output</i>	
Consumption	0.93	0.85
Investment	0.92	0.75
House price	0.66	0.38

## Appendix G: Skewness and the Great Moderation

In the main text, we demonstrated that our model was able to generate a more negatively skewed business cycle along with a drop in macroeconomic volatility when we raise the steady-state LTV ratios. However, the drop in the standard deviation of GDP growth documented in [Figure 10](#) falls short of the decline observed in US data during the Great Moderation period. In this respect, it is important to recognize that none of the factors to which the Great Moderation is typically ascribed are featured in our model. One widely cited explanation for the Great Moderation is the so-called “Good Luck” hypothesis, according to which the Great Moderation was simply a result of smaller shocks hitting the US economy (see, e.g., [Stock and Watson, 2003](#)).<sup>37</sup> The goal of this appendix is to demonstrate that our main finding of an increasingly negatively skewed business cycle holds up in an environment where increasing LTV ratios are combined with smaller macroeconomic shocks to obtain a drop in output volatility similar to that observed in the data. It is important to stress that this coexistence is not trivial: All else equal, reducing the size of the shocks hitting the economy lowers the probability that collateral constraints become non-binding, thus potentially weakening the key driver of business cycle skewness in our model.

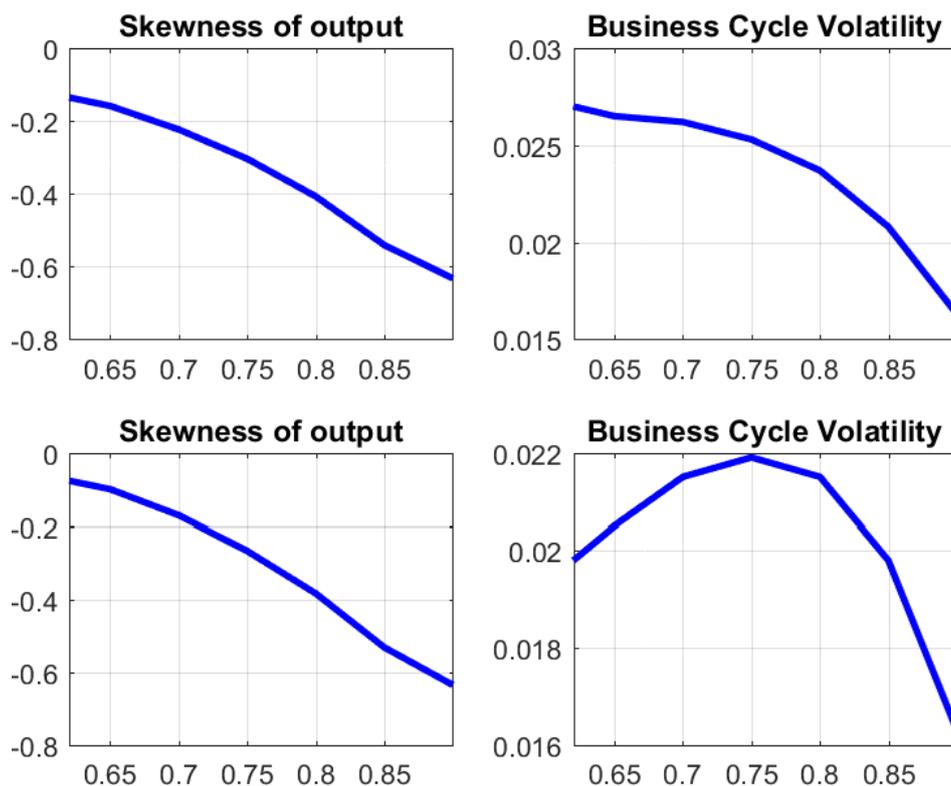
The Great Moderation entailed a decline in the volatility of GDP growth in the US economy of around 40%, cf. subsection 2.1. In the following, we engineer a similar decline in output volatility in our model simulations by reducing the standard deviations of all three shocks in the model, keeping their relative size fixed in accordance with the estimation in subsection 5.1.2. To obtain the desired drop in the volatility of GDP growth at  $s^I = 0.90$ , we need to reduce the standard deviation of each shock by 30%. We first assume that the decline in the size of the shocks occurred gradually along with the increase in LTV ratios. The top row of [Figure G1](#) shows the pattern of the standard deviation and skewness of GDP growth in this experiment. The right panel illustrates that in this case, the drop in macroeconomic volatility is similar to that observed in the data during the Great Moderation, as desired. In contrast to the results reported in subsection 6.3, the standard deviation of output growth now declines monotonically. The left panel shows that the decline in skewness of GDP growth survives in this environment. In fact, the magnitude of the drop in skewness is almost identical to the baseline results presented in subsection 6.2 of the main text. This demonstrates that the mechanism giving rise to business cycle skewness in our model is compatible with the “Good Luck” hypothesis of the decline in macroeconomic volatility observed during the Great Moderation.

Some may argue that the Great Moderation entailed a discrete, downward shift in the size of macroeconomic shocks hitting the US economy rather than the gradual decline assumed above. The bottom row of [Figure G1](#) shows the results from an experiment in which the standard deviation of each shock is reduced by 30% starting at  $s^I = 0.6239$ , and then kept at this new level as the LTV ratios are increased. By design, the standard deviation of GDP growth reaches the same level as in the top row of [Figure G1](#) when  $s^I = 0.90$ . The hump-shaped pattern of output volatility presented in subsection 6.3 is maintained in this experiment, though at a lower level. Importantly, skewness of GDP growth displays roughly the same pattern as in the top row, confirming again the robustness of our main finding.

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<sup>37</sup>Other popular explanations include better monetary policy ([Boivin and Giannoni, 2006](#)) and smaller dependence on oil ([Nakov and Pescatori, 2010](#)).

Figure G1: Skewness and the Great Moderation



Notes. The top row displays the skewness and standard deviation of output growth in the experiment where the size of shocks is reduced gradually, while the bottom row displays the same statistics in the case where the size of shocks is reduced at once. In each case, the numbers reported are median values from 501 stochastic model simulations of 2000 periods each.

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