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Torben M. Andersen

University of Aarhus, CEPR, and EPRU

and

Niels C. Beier

Danmarks Nationalbank, Copenhagen

International transmission of transitory and persistent monetary shocks under imperfect information

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Resumé
Vi analyserer transmissionen af monetære stød i en *New Open-Economy Macroeconomics* model med en-periode nominelle lønkontrakter og imperfekt information. De monetære stød har transitoriske og persistente komponenter, der hver især kun bliver kendt gennem læring over tid. Tilpasningen til stødene ændres i forhold til tilfældet med fuldstændig information. Der er persistente effekter på internationale relative priser, og ved persistente stød kan den nominelle valutakurs udvise *delayed overshooting*. I nogle tilfælde er der (ex post) *excess returns*, da der både er et positivt rentespænd og en apprecierende valuta (eller omvendt). Til sidst vises, at asynkrone lønkontrakter forøger persistensen.

Abstract
We analyze the transmission of monetary shocks in a new open-economy macroeconomics model with one-period nominal contracts and imperfect information. Shocks may have transitory and persistent components, which only through accumulation of information over time becomes known. Responses to shocks are altered compared to the case of full information. There are persistent effects on international relative prices, and delayed exchange-rate overshooting is possible following a persistent shock. In some cases, there are (ex post) excess returns as a positive interest rate spread is accompanied by an appreciating currency (or vice versa). Lastly, it is demonstrated that staggering re-inforce persistence.

International transmission of transitory and persistent monetary shocks under imperfect information*

Torben M. Andersen†
University of Aarhus, CEPR, and EPRU

Niels C. Beier
Danmarks Nationalbank

May 2004

Abstract

We analyze the transmission of monetary shocks in a new open-economy macroeconomics model with one-period nominal contracts and imperfect information. Shocks may have transitory and persistent components which only through accumulation of information over time becomes known. Responses to shocks are altered compared to the case of full information. There are persistent effects on international relative prices, and delayed exchange-rate overshooting is possible following a persistent shock. In some cases, there are (ex post) excess returns as a positive interest rate spread is accompanied by an appreciating currency (or vice versa). Lastly, it is demonstrated that staggering reinforce persistence.

Keywords: exchange rates; imperfect information; new open economy macroeconomics; nominal shocks; transitory and persistent shocks; persistence.

JEL classification: E32; F41

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†Address: University of Aarhus, Department of Economics, DK-8000 Aarhus C, Denmark. E-mail: tandersen@econ.au.dk. Telephone: +45 8942 1609.

1 Introduction

Information flows are essential for financial markets, and a great deal of resources are absorbed by analyzing the developments in asset prices. Market participants and the media exert much effort to interpret current market signals in an attempt to infer information of relevance for the future to answer the pertinent question whether a given change is purely temporary (noise) or lasting. An obvious market where information plays a crucial role is the foreign-exchange market since exchange rates can change instantaneously to new information. A recent example is the development in the US-Euro exchange rate where the issue of distinguishing transitory from persistent changes has taken center-stage.

Despite considerable resources invested in information processing, market expectations of future exchange rates deviate systematically ex post from realized exchange rates. As is evident from Figure 1, where actual US-dollar exchange rates are depicted along the Consensus forecast based on information available 4 months ahead, market prediction errors are highly persistent, suggesting fundamental information problems.

Figure 1 about here

The purpose of this paper is twofold. First, we show that persistent deviations between expected and realized exchange rates do not necessarily reflect market anomalies, but may arise as a consequence of an inability to distinguish between *transitory* and *persistent* shocks. Second, we explore how these (rational) prediction errors affect the international transmission of monetary shocks with special focus on the response of exchange rates, terms of trade, and interest rates.

To address the international transmission of monetary shocks we need an explicit intertemporal general equilibrium model. The specific structure builds on the new open-economy macroeconomics launched by Obstfeld and Rogoff (1995), which has proved to be a useful framework (for a survey, see Lane, 2001). In the specific application here we introduce the minimum assumption needed to create non-neutralities by assuming one-period nominal (wage) contracts. Since the assumed nominal contracting setup does not in itself contribute to generate interesting dynamics, we are able to focus on the dynamic implications of imperfect information.[1] We present an explicit analytical solution of the model, with the advantage that we can identify the mechanisms through which information problems affect market reactions under rational expectations.

Although the problem of distinguishing between transitory and persistent influences can arise from a whole range of sources (see below), we focus on monetary shocks. It is a simple way to illustrate the gist of imperfect

[1] Absent imperfect information nominal shocks have real effects in one period.

information: current signals may later turn out to have conveyed irrelevant noise, whereas other pieces of information may turn out to be useful, but to have been given insufficient weight. Ex-ante there is a non-trivial problem in disentangling the two. Furthermore, it is well documented that nominal shocks play a significant role in explaining open-economy variables like exchange rates (e.g., Canova and De Nicoló, 2002), and focussing on nominal shocks allows us to address the issue why nominal exchange-rate changes can have persistent real effects even when nominal rigidities are short-lived (Rogoff, 1996). Specifically, we model shocks to the money stock as being either transitory or persistent, and while there is full information about the current money stock, there is imperfect information on the implication of current shocks for the future (all market participants observe the money stock, but do not directly observe the transitory and persistent component).

Several puzzles are unexplained in open-economy macroeconomics. We demonstrate that the informational problem of disentangling transitory and persistent changes combined with one-period contracts can possibly explain three of these puzzles: persistent effects on international relative prices, delayed overshooting, and the well-known observation that a depreciating (appreciating) currency is accompanied by a negative (positive) interest rate spread. First and foremost, learning gives an intuitive account of the persistent effects of nominal shocks on international relative prices (terms of trade and real exchange rate; Rogoff 1996). When a given shock hits the economy, the agents take time to pin down the nature of the shock, and during this time there will be real effects. Furthermore, we show that a persistent shock is capable of generating delayed nominal exchange-rate overshooting (Eichenbaum and Evans, 1995). Following a positive (persistent) monetary shock, which only gradually dies out, the nominal exchange rate depreciates on impact, but rather than settling on an appreciating path as the shock dies out, the nominal exchange rate keeps depreciating further for some time, before appreciating towards its pre-shock value. During the periods with depreciation, the interest rate spread is negative (Frankel and Rose, 1995), which shows that a depreciating (appreciating) currency can be accompanied by a negative (positive) interest rate spread. If market participants cannot readily distinguish noise (transitory shocks) from fundamental changes in market conditions (persistent shocks), they tend to react too strongly to the former and too little to the latter. As a consequence, sensitivity to noise and fundamentals is increased and decreased, respectively. The net effect – in this rational expectations setting – is that exchange-rate volatility is decreased. Thus, excess volatility in exchange rates is still most likely best explained by irrationalities (e.g., noise traders, see Devereux and Engel, 2002). Lastly, we show that if the informational problem of shock confusion is combined with overlapping wage contracts, the two propagation mechanisms interact nontrivially. Persistence is increased and in the cases with nominal delayed overshooting, hump-shaped responses of the real variables are possible as well.

This open-economy paper is related to a vast and growing literature documenting and exploring the macroeconomic implications of information problems primarily in closed economies.[2] Persistent errors in expectations are not isolated to foreign-exchange markets. Evans and Wachtel (1993) is a classic reference on inflation expectations, which also recently has been explored by Carroll (2003). In his model agents only occasionally update expectations from news wires, and this creates stickyness in expectations. Mankiw and Reis (2002) analyze the dynamic implications of inertia in information dissemination and show that a model with sticky information can account better for business cycle facts than the standard sticky-price model.

The information problem arising from errors in preliminary data on which agents react has been analyzed by Bomfim (2001) focussing on the implications for business cycle fluctuations, while Faust *et al.* (2003) show that exchange-rate models in general perform much better when evaluated on the basis of preliminary rather than final data.

The interaction between policy and expectations formation when policy responses are based on imperfect knowledge of the true state of the economy or when the private sector holds incomplete knowledge on policy objectives has been addressed by Erceg and Levin (2003), Lansing (2000), Orphanides (2001), Romer and Romer (2000), and Rudebusch (2001).

In a related paper Gourinchas and Tornell (2002) analyze the interest-rate spread and exchange-rate dynamics when agents misperceive the true underlying process generating shocks. They find that misperception can explain the forward premium puzzle as well as delayed overshooting.

Although our crude characterization of transitory and persistent shocks can be given a narrow motivation by errors in preliminary money stock data (Bomfim, 2001; Mankiw *et al.*, 1984), and builds on the monetary approach to exchange-rate determination, we think the insights provided go beyond this specific way of modelling foreign-exchange markets. The point that determinants of exchange rates and interest rates have both transitory and persistent components is generic to any model of exchange-rate determination. Thus, our model can be given a wider interpretation along the lines of those given in the literature, cf. above. There is also a parallel between this paper and the so-called micro-structure model of financial markets (e.g., O'Hara, 1995) in the sense of stressing the fact that market participants face non-trivial information problems, and that it is crucial for market behavior how new information enters the market. The present paper asks whether these information problems have any interesting macroeconomic implications, and focusses on the dynamic adjustment process addressing stylized

[2] The importance of distinguishing between transitory and persistent shocks goes back to Muth's (1960) discussion of the optimality of adaptive expectations, see also Sargent (1982). On the role of informational problems for macroeconomic adjustment, see also Andersen (1994).

facts, which is hard to reconcile with standard open macro models.

The paper is organized as follows: the two-country model with a flexible exchange rate is set up in section 2. The stochastic process for money and the information structure are defined in section 3. Section 4 describes the equilibrium. Section 5 considers the dynamics of nominal shocks and includes numerical illustrations of the main findings. Section 6 extends the model with staggering, and discussion and concluding remarks are presented in section 7.

2 A stochastic two-country model

Following Obstfeld and Rogoff (1995, 2000) we consider a symmetric two-country model with a flexible exchange rate and specialized production. There are two equally-sized countries and two goods, one produced by Home and one produced by Foreign firms. There are two assets in the economy: money and a real bond, where the latter is traded in a perfect international capital market. There is no real capital and no internationally mobile labor.

Workers are organized in (monopoly) unions, and each union represents a (small) subset of workers supplying labor to a given group of firms. Each union is utilitarian and chooses a wage for period t given all available information in period $t-1$ to maximize the expected utility of workers, which in turn depends on the wage income and the disutility of work. Employment is determined by firms given the wage set by the union (right-to-manage structure).[3] All other prices are determined in competitive markets.[4]

2.1 Firms, consumers, and the government

Home firms demand labor, produce the Home good, and are price takers in both product and labour markets. The good is produced subject to a decreasing returns technology linking labor input N and Home output Y^h. In the following superscript h (f) refers to traded variables originating in Home (Foreign), and Foreign variables are denoted by an asterisk.

$$Y_t^h = N_t^\gamma, \quad 0 < \gamma < 1.$$

Profits are distributed to households. Profit maximization yields the following labor demand and output supply (in logarithms)

$$n_t = \eta_{nw} \left(p_t^h - w_t \right), \quad \eta_{nw} = (1-\gamma)^{-1}, \tag{1}$$

$$y_t^h = \eta_{yw} \left(p_t^h - w_t \right), \quad \eta_{yw} = \gamma (1-\gamma)^{-1}. \tag{2}$$

[3] We assume that workers are willing to participate in the sense that for any labor demand, the marginal consumption value of the real wage is larger than the marginal disutility of effort (Corsetti and Pesenti, 2001).

[4] All proofs and derivations are relegated to the appendix.

The countries are inhabited by consumers who consume goods (C), supply labor (N), and hold money (M) as well as bonds. Let E_t be the expectations operator conditional on period-t information (the information structure is defined below), and P the consumer price index, then the representative consumer's objective function is

$$U_t = E_t \sum_{j=0}^{\infty} \delta^j \left[\frac{\sigma}{\sigma - 1} C_{t+j}^{\frac{\sigma-1}{\sigma}} + \frac{\lambda}{1-\beta} \left(\frac{M_{t+j}}{P_{t+j}} \right)^{1-\beta} - \frac{\kappa}{1+\mu} N_{t+j}^{1+\mu} \right], \qquad (3)$$

$$\sigma > 0, \quad \lambda > 0, \quad \beta > 0, \quad \kappa > 0, \quad \mu > 0, \quad 0 < \delta \leq 1.$$

$$C_t = (C_t^h)^{\frac{1}{2}} (C_t^f)^{\frac{1}{2}}.$$

Thus, the real consumption index aggregates across consumption of the Home good (C_t^h) and the Foreign good (C_t^f), where the elasticity of substitution between the two goods is assumed to be one. Our results apply to a more general CES specification, but the unitary elasticity of substitution simplifies the analytics significantly. The price index corresponding to composite consumption is also Cobb-Douglas

$$P_t = 2(P_t^h)^{\frac{1}{2}} (P_t^f)^{\frac{1}{2}},$$

where P_t^h (P_t^{*h}) is the price of the Home good in Home (Foreign) currency and P_t^f (P_t^{*f}) is the price of the Foreign good in Home (Foreign) currency. As our focus will be on nominal wage rigidity we assume that the law of one price holds for both goods, i.e. $P_t^h = S_t P_t^{*h}$ and $P_t^f = S_t P_t^{*f}$. S is the nominal exchange rate defined as the Home price of Foreign currency. A direct implication of the law of one price is that PPP holds as well, that is, $P_t = S_t P_t^*$. As a consequence, the subsequent analysis will focus on how nominal shocks affect the terms of trade. It can be shown in a setting including nontradables (Hau, 2000) that the movements in the real exchange rate are qualitatively equivalent to the movements in the terms of trade; hence, our results can be directly related to the PPP puzzle.[5]

We assume that there is one internationally traded real bond denoted in the composite consumption good C. Let r_t be the consumption-based real interest rate between dates t and $t+1$. The consumer's budget constraint for any period t is given by

$$P_t B_t + M_t + P_t C_t = (1 + r_{t-1}) P_t B_{t-1} + M_{t-1} + W_t N_t + \Pi_t + P_t \tau_t. \qquad (4)$$

The right-hand side gives available resources as the sum of the gross return on bond holdings $(1 + r_{t-1}) P_t B_{t-1}$, initial money holdings M_{t-1}, labor income $W_t N_t$, nominal profit income Π_t, and transfers from the government

[5] For alternative assumptions concerning price setting and discussion see, e.g., Betts and Devereux (2000), and Obstfeld (2001).

$P_t\tau_t$. Resources are allocated to consumption P_tC_t, nominal money holdings M_t, and bond holdings P_tB_t.

Given the constant elasticity consumption index, Home consumers' demands for the Home good and the Foreign good are

$$D_t^h = \frac{1}{2}\left(\frac{P_t^h}{P_t}\right)^{-1} C_t, \quad D_t^f = \frac{1}{2}\left(\frac{P_t^f}{P_t}\right)^{-1} C_t,$$

respectively, and mutatis mutandis for the demands by Foreign consumers. Aggregating we find total demand for the Home good to be (and similarly for the Foreign good)

$$D_t \equiv D_t^h + D_t^{*h} = \frac{1}{2}\left(\frac{P_t^h}{P_t}\right)^{-1} (C_t + C_t^*), \tag{5}$$

The household chooses consumption (C), money demand (M), bond demand (B) and wages (W) to maximize (3) subject to the sequence of budget constraints given in (4). In determining the wage it is taken into account that employment is (demand) determined. Written in log-linear form[6] the first-order conditions are given by the Euler equation, money demand and the wage rate:

$$E_t c_{t+1} = c_t + \sigma \log(1 + r_t), \tag{6}$$

$$m_t - p_t = \eta_{mc} c_t - \eta_{mc}^1 E_t c_{t+1} + \eta_{mp}(p_t - E_t p_{t+1}), \tag{7}$$

$$w_t = E_{t-1}\left[\eta_{wp} p_t^h + (1 - \eta_{wp})\left(s_t + p_t^{*f}\right) + \eta_{wc} c_t\right], \tag{8}$$

$$\eta_{mc} = [\sigma(1-\delta)\beta]^{-1}, \qquad \eta_{mc}^1 = \delta\eta_{mc}, \qquad \eta_{mp} = \delta[(1-\delta)\beta]^{-1},$$

$$\eta_{wp} = (1 + \mu\eta_{nw})^{-1}(0.5 + \mu\eta_{nw}), \qquad \eta_{wc} = [\sigma(1 + \mu\eta_{nw})]^{-1}.$$

Lower-case denotes the log-deviations from a symmetric steady state of the corresponding upper-case variables, and all constants are neglected, since our primary interest is the adjustment process to shocks.[7] The wage equation (8) satisfies the basic homogeneity property generic to any micro-founded wage-setting model, and implies that nominal wages and, thus, prices depend on expected exchange rates. This captures a channel through which exchange rates affect the real side of the economy. Note that the log-linearized version of the Home price index is $p_t = \frac{1}{2}(p_t^h + s_t + p_t^{*f})$, and the terms of trade are defined as $q_t \equiv p_t^h - p_t^f = p_t^h - p_t^{*f} - s_t$.

[6] The model is specified to yield a log-linear model. However, log-linearizations are needed for money demand and the budget constraint.

[7] These constant terms include conditional variance terms which are constant under the stochastic process considered.

We assume that the government balances its budget each period, i.e. $M_t - M_{t-1} = P_t \tau_t$. In other words, the only role of the government is to issue money. Money is transferred to Home consumers in a lump-sum fashion. The stochastic process governing money supply along with the assumptions on the information structure are described in detail in the next section. We end the description of the model by noting that Foreign is completely symmetric and that a (symmetric) equilibrium exists (see appendix) in which money is neutral absent nominal rigidities.

3 Information structure and money supply

In a flexible exchange-rate regime, changes in supply and demand translate immediately into changes in the exchange rate. It follows that changes in exchange rates may originate from various forms of shocks arising on either the demand or the supply side. These shocks could be real or monetary in nature and leave a non-trivial problem of separating transitory from persistent changes in the exchange rate. Building this problem into a fully specified general equilibrium model is by no means trivial, since it requires not only a specification of shocks which have transitory and persistent components, but also an account of how these shocks affect the agents (preferences, endowments, technology, etc.).

To simplify we focus on nominal shocks. Thereby we also address the more difficult problem of explaining persistent effects of nominal shocks. Since the information problem of interpreting changes is essential to our story, we exploit the model simplification which can be achieved by considering changes in the money stock, which are either transitory or persistent. It is assumed that all current information is freely available, but agents face the problem of making inferences about its implications for the future.

3.1 The money-supply process

A straightforward way to introduce the problem of distinguishing between transitory and persistent influences is to assume that the relative money-supply process is[8]

$$m_t - m_t^* = z_t + u_t, \tag{9}$$

$$z_t = \theta z_{t-1} + \varepsilon_t, \quad 0 < \theta < 1,$$

where u and ε are independent and normally distributed mean-zero shocks with variances σ_u^2 and σ_ε^2.

[8] Since information flows continuously and the wage contracts are assumed to be fixed for a given period of time, it follows that some aggregation of information has already implicitly taken place in transforming data to match the contract length.

The money-supply process captures that some changes are transitory (u) and some are persistent (z), and that agents cannot readily disentangle one type of shock from the other. They only observe the sum of the two components. Agents know current and past realizations of relative money supplies, but they cannot directly observe whether current changes are transitory or persistent. Hence, agents' information set, I_t, is given as $\left\{m_t - m_t^*, m_{t-1} - m_{t-1}^*, ...\right\} \subseteq I_t$. Accordingly, agents learn over time as they accumulate information.

The specification (9) can be given several interpretations. Empirically it can be motivated by noise in preliminary announcements of money stock data (Bomfim, 2001; Mankiw *et al.*, 1984), where u represents the measurement error or noise. Eq. (9) can also be interpreted literally as reflecting that money-supply (monetary policy) changes may be either transitory or persistent. This may arise if the policy maker reacts to changes in variables, which in turn are affected by shocks that may be either transitory or persistent in nature. Another reason may be that the policy maker operates under imperfect information or that market participants have different information than the policy maker, or that there may be imperfect knowledge about the objectives of the policy maker.[9,10] However, the specific formulation adopted here may also capture more general information problems. It is natural to interpret the transitory component (the u-part) as reflecting noise, and the persistent part (the z-part) as fundamentals, since the former is not helpful in predicting the future while the latter does affect future market conditions. In a money-supply setting, this would apply if the liquidity created in the financial system is not one-to-one related to the money base (which can be observed with high precision), that is, the money multiplier varies and there is imperfect information on the causes. The formulation can also reflect that the aggregate information set available in the market is not sufficiently detailed to allow a precise identification of market fundamentals of importance for future market developments (Figlewski, 1982).

3.2 Expectations formation

Given the assumptions made above on the money supply and the information structure we can turn to expectations formation. Predicting the future money supply is a question of predicting its persistent component, i.e.

$$E_t\left(m_{t+1} - m_{t+1}^*\right) = E_t\left(z_{t+1}\right),$$

where E_t is shorthand for the mathematical expectation conditional on the information set I_t. Information on future changes in the relative money

[9] Cukierman and Meltzer (1986) show why the policy maker for strategic reasons may disseminate imprecise information.

[10] The contemporaneous debate on transparency in monetary policy making can be interpreted as a way to minimize the noise component and thereby provide more information on the fundamentals underlying monetary policy.

supply arrives via observations of the relative money supply, and it can be shown by use of Kalman-filter techniques that the conditional expectation can be written as (Hamilton, 1994)

$$\begin{aligned} & E_t\left(m_{t+1}-m_{t+1}^*\right) \\ = \quad & \theta E_{t-1}\left(m_t-m_t^*\right)+\theta h\left[m_t-m_t^*-E_{t-1}\left(m_t-m_t^*\right)\right], \end{aligned} \tag{10}$$

$$h=\frac{1+\Delta-\left(1-\theta^2\right)\eta}{1+\Delta+\left(1+\theta^2\right)\eta}\in(0,1), \quad \Delta^2=\left[\left(1-\theta^2\right)\eta-1\right]^2+4\eta,$$

$$\eta=\frac{\sigma_u^2}{\sigma_\varepsilon^2}, \quad \frac{\partial h}{\partial \eta}<0, \quad \frac{\partial h}{\partial \sigma_u^2}<0, \quad \frac{\partial h}{\partial \sigma_\varepsilon^2}>0.$$

Expectations of tomorrow's relative money supply are given as a weighted sum of yesterday's expectations of today's relative money supply and the information obtained by observation of today's relative money supply. The latter is the difference between the actual period-t money supply and its expected value (conditional on period $t-1$ information) times θh, since a fraction h of this is perceived to be persistent of which a fraction θ carries forward to the next period. The coefficient h is crucial for the updating of expectations since it determines the weight given to new information. It is decreasing in the noise-to-signal ratio η. That is, if all shocks are transitory ($\eta \rightarrow \infty$) we have $h=0$, and the information content of the signal is nil, whereas if all shocks are persistent ($\eta \rightarrow 0$) we have $h=1$ reflecting that current signals contain all information of relevance for predicting future money supplies. Interestingly, when the noise-to-signal ratio is one, h is greater than one-half. This reflects the learning aspect involved in expectation formation. Not only does a given surprise reflect information about new shocks; it contains information about last period's surprise as well, i.e. learning. A positive surprise last period followed by another positive surprise this period indicates that a larger part of last period's surprise was due to a persistent shock than was originally expected.

If we interpret the transitory shock (u) as noise and the persistent part (z) as fundamentals, the updating formula has a very intuitive interpretation. The more noise (larger σ_u^2, smaller h), the less weight is put on the current observation of money supply since agents know that current movements tend to reflect noise; current signals have a low information content.

Since the updating of expectations to shocks is crucial to the results of this paper, it is useful to consider the learning process in some detail. The following tracks the adaptation of expectations to given shocks. The nature of shocks are unknown to the agents, but known to the analyst.

Figure 2 describes the adjustment path for the actual and (un)expected money stock to transitory and persistent shocks, respectively. In both cases we consider a 1 percent positive shock to the relative money stock.[11]

[11] We consider expectations under the assumption that they have been zero up to date

Figure 2 about here

For a transitory shock we see that, although the relative money supply is only affected in one period, it takes several periods for the agents to learn that the shock was transitory. Accordingly, a positive transitory shock will imply that the money stock is unanticipatedly low in subsequent periods until agents eventually learn the type of the shock.

In the case of a persistent shock it also takes several periods before expectations and de facto money converge; money is unexpectedly high for a number of periods. Furthermore, there may be delayed overshooting. Initially expectations rise, and only several periods later do they begin to fall. Delayed overshooting occurs when $E_t\left(m_{t+1} - m^*_{t+1}\right) < E_{t+1}\left(m_{t+2} - m^*_{t+2}\right)$, and this is ensured if $1 - 2\theta + \theta h < 0$. This condition will turn up later when we consider exchange-rate dynamics. Lastly, sensitivity analysis with respect to the noise-to-signal ratio, η, shows that the more noise (η large), the longer it takes for agents to learn that the shock in fact was persistent.

Figure 2 vividly illustrates that conditional on a particular shock the learning process implies systematic expectations errors. This is essential if a given business cycle (of a duration of some years) is interpreted as a realization of a particular string of shocks, i.e., a business cycle episode is represented by a small sample of observations from an underlying stochastic process with properties assumed known to the agents. Hence, when relating our analysis to actual observations like those reported in Figure 1, we take the perspective that they should be interpreted in terms of theoretical results which consider the adjustment process contingent on a particular string of shocks.[12] In our case, agents are rational and know the properties of the stochastic process (9), and to simplify, we consider monetary shocks only.

4 Equilibrium

Characterizing the equilibrium analytically is not only complicated by the presence of nominal contracts and the information problem, but also the intertemporal structure linking current and future decisions via expectations. We demonstrate in the appendix how to find an analytical solution so that we can explicitly characterize the processes for the endogenous variables.

Since Obstfeld and Rogoff (1995), the qualitative working of this type of model (with a one-period nominal contract) has been well understood;

1, where a one-time 1 percent positive monetary shock hits the economy. No shock occurs after that, which is unknown to the agents. For instance, in the case of a transitory shock, the agents believe part of the initial shock to be persistent, and part of the negative surprise the period after is seen as being due to a new shock.

[12] The reason is that in a large sample under the assumption of rational expectations there would obviously be no systematic expectations error. Agents would be right on average. In a short sample, however, this would not necessarily be the case, and this motivates the perspective taken here.

a positive monetary shock leads to an exchange-rate depreciation (terms-of-trade deterioration) leading to a switch in demand towards the Home good, and thus Home production increases. Wealth reallocations are ruled out by the assumption of a unitary demand elasticity, and, therefore, relative consumption between Home and Foreign is invariant to all types of shocks. These basic effects are not changed, but richer dynamics arise under imperfect information. The assumption on demand elasticity allows us to demonstrate the main points in the least technical way. The results carry over to the general case, where demand elasticity is different from one.

Consider as a benchmark for the subsequent analysis the case where agents have perfect information allowing them to identify transitory and persistent shocks to the relative money supply, that is, the underlying process generating relative money supplies (9) is unchanged, and there are still surprises. The information set is now $\{z_t, z_{t-1},...,u_t, u_{t-1},...\} \subseteq \widehat{I}_t$ (compare with I_t). In this case the nominal exchange rate can be written

$$s_t = \phi_{sz} z_t + \phi_{su} u_t, \tag{11}$$

$$0 < \phi_{su} = \left(1 + \eta_{mp}\right)^{-1} < \left(1 + \eta_{mp} - \theta\eta_{mp}\right)^{-1} = \phi_{sz} < 1.$$

Transitory shocks have a smaller effect on the exchange rate than persistent shocks, since the latter also affect future money supplies. The terms of trade are given as

$$q_t = \phi_{q\varepsilon} \varepsilon_t + \phi_{qu} u_t, \tag{12}$$

$$-1 < \phi_{q\varepsilon} = -\gamma\phi_{sz} < -\gamma\phi_{su} = \phi_{qu} < 0.$$

In the absence of information problems the real adjustment to unanticipated shocks is ended after a period of time equal to the contract length. The dynamics is trivial and the impulse-response functions are implausible.

4.1 Imperfect Information

With informational imperfections we have

$$s_t = \theta\left(1-h\right) s_{t-1} + \phi_{sm}\left(m_t - m_t^*\right) + \phi_{sm}^1 \left(m_{t-1} - m_{t-1}^*\right), \tag{13}$$

$$\phi_{sm} = \frac{1 + \eta_{mp} - \theta\left(1-h\right)\eta_{mp}}{\left(1+\eta_{mp}\right)\left(1 + \eta_{mp} - \theta\eta_{mp}\right)} > 0,$$

$$\phi_{sm}^1 = \theta\left(h-1\right)\phi_{sm} + \theta h \frac{\theta\left(1-h\right)\eta_{mp}}{\left(1+\eta_{mp}\right)\left(1+\eta_{mp}-\theta\eta_{mp}\right)}.$$

Similarly, the terms of trade can be written[13]

$$q_t = \theta\left(1-h\right) q_{t-1} + \phi_{qm}\left(m_t - m_t^*\right) + \phi_{qm}^1 \left(m_{t-1} - m_{t-1}^*\right), \tag{14}$$

[13] Or $q_t = \phi_{qm}\left[m_t - m_t^* - E_{t-1}\left(m_t - m_t^*\right)\right]$; only unanticipated nominal shocks have real effects.

$$\phi_{qm} = -\gamma\phi_{sm}, \quad \phi^1_{qm} = -\theta\phi_{qm}.$$

The implications for the adjustment of nominal interest rates (i_t) can easily be worked out by noting that the real asset available to households implies that it is possible to construct a nominal asset for which the return is given by uncovered interest rate parity[14] ($R_t \equiv \log(1+i_t)$)

$$R_t - R^*_t = E_t s_{t+1} - s_t. \tag{15}$$

Using the equilibrium value for the exchange rate, it follows that the interest rate spread can be written

$$R_t - R^*_t = \theta(1-h)(R_{t-1} - R^*_{t-1}) + \phi_{im}(m_t - m^*_t) + \phi^1_{im}\left(m_{t-1} - m^*_{t-1}\right), \tag{16}$$
$$\phi_{im} = \theta h\left(1 + \eta_{mp} - \theta\eta_{mp}\right)^{-1} - \phi_{sm} < 0, \quad \phi^1_{im} = -\phi^1_{sm}.$$

Relative production is inversely related to the terms of trade, $y^h_t - y^{*f}_t = -q_t$, hence we focus in the following on the process for the nominal exchange rate, the terms of trade, and the interest rate spread, noting that the real effects for production and thus employment can easily be inferred from the behavior of the terms of trade. It is clear that imperfect information generates a richer adjustment path for both the nominal exchange rate and the terms of trade. In particular, there are non-trivial dynamics running beyond the length of nominal contracts (one period).

5 Dynamics under imperfect information

We now turn to a detailed analysis of the dynamic adjustment path under imperfect information. In particular we lay out the economy's response following an increase in relative money supply. Our strategy is to see to which extent imperfect information alters the impulse-response functions to shocks, and whether the responses are consistent with the stylized facts in open-economy macroeconomics. In particular, can learning help explain some of the puzzles, for example, excess exchange-rate volatility (relative to fundamentals), persistent effects of nominal shocks on international relative prices, delayed nominal exchange-rate overshooting, and the fact that investors in countries with high interest rates at times also tend to reap the benefits of an appreciating currency?

The analytical results are supplemented by numerical illustrations[15] based

[14] Follows by use of PPP. Constants are disregarded.

[15] A thorough quantitative investigation warrants the model to be augmented with capital, price and wage staggering as well as calibration of the informational parameters, θ and $h(\eta)$. What should be clear from our simple exercise, though, is the potential for persistence in international relative prices.

on the following parameter values[16]: $\gamma = 0.67$, $\mu = 10$, $\sigma = 0.75$, $\beta = 9$, $\delta = 1/1.05$, $\theta = 0.9$, and $\eta = 1$. In Figure 3 we provide impulse-response functions to a 1 percent increase in Home (relative) money in period 1 which is either transitory or persistent under both full and imperfect information.

Figure 3 about here

As a benchmark Figure 3 provides the responses to the two types of shocks under full information. In both cases there are only one-period dynamics in the terms of trade, and the real effects are increasing in the persistence of the shock. For the interest rate spread the impact effect is decreasing. Since a fall in the interest rate is necessary to make agents hold the money over to the next period, it follows that the more persistent the shock, the less the interest rate is expected to rise in the future, and therefore the less it has to fall on impact. The nominal exchange rate displays one-period dynamics for a transitory shock and multi-period dynamics for a persistent shock.

Under imperfect information the dynamic responses reflect the learning process when agents over time acquire more information as illustrated in Figure 2 on updating of expectations. In particular, the terms of trade deteriorate on impact, but then improve given a transitory shock. This reflects that the monetary change disappears, but agents still expect the money shock to have increased, since it takes a while before sufficient information is accumulated to infer that the shock is transitory. The subsequent terms-of-trade improvement is gradually worked out of the system. While persistent shocks also have persistent effects on nominal variables (exchange rate, interest rate spread) under full information, the dynamic adjustment process is different and includes persistent real effects under imperfect information.

5.1 Excess sensitivity to noise and volatility

Under imperfect information the impact effect of an expansion in (relative) Home money supply (regardless of the type of shock) is a depreciation of the nominal exchange rate, a terms-of-trade deterioration, and a fall in the interest rate spread, i.e.

$$\frac{\partial s_t}{\partial (m_t - m_t^*)} = \phi_{sm} > 0, \ \frac{\partial q_t}{\partial (m_t - m_t^*)} = \phi_{qm} < 0, \frac{\partial (R_t - R_t^*)}{\partial (m_t - m_t^*)} = \phi_{im} < 0$$

Consider first how markets react to news. The impact effect under imperfect information is the same whether the shock is transitory or persistent,

[16] The productivity parameter γ is chosen to match the wage share of about 2/3, while μ is chosen to imply a labor-supply elasticity of 0.1. The next three coefficients correspond to those adopted in, for example, Sutherland (1996). The last coefficient value is arbitrarily set at 1.

since agents cannot immediately distinguish the two. It turns out that the impact response can be written as a weighted average of the impact effects of a transitory and a persistent shock under full information, respectively. More specifically, for the nominal exchange rate (similar reasoning follows straightforwardly for the other variables):

$$\phi_{sm} = (1-h)\,\phi_{su} + h\phi_{sz}. \tag{17}$$

The intuition for the weighting is that $(1-h)$ measures the weight attached to the shock being transitory, and h to it being persistent. Since

$$\phi_{su} < \phi_{sm} < \phi_{sz},$$

it follows that there is an overreaction, or excess sensitivity, to transitory shocks or noise ($\phi_{su} < \phi_{sm}$), while there is an underreaction to persistent changes or fundamentals ($\phi_{sm} < \phi_{sz}$). The intuition is straightforward; part of a transitory shock is taken to be persistent and vice versa. It is an implication that the less informed the market (high η, low h), the more cautious is the market adjustment (ϕ_{sm} lower). That is, if the information quality of observable signals is low, the response to persistent shocks tends to be muted, since a larger fraction of changes is taken to reflect noise.

Given the averaging in the response to shocks under imperfect information, it might be inferred that the average response, or more generally, volatility, is unaffected by the information problem. This is not correct, as it can be shown[17] that the volatility of the nominal exchange rate is lower under imperfect information. To see why, consider a case where $\sigma_u^2 = 0$ and $\sigma_\varepsilon^2 > 0$, i.e. there is no information problem. If σ_u^2 increases there will be two effects on volatility under imperfect information. There will be a direct positive effect as there will be more transitory shocks, but there will be an indirect effect as well working in the opposite direction since the information content of signals decreases. The impact effect on persistent shocks decreases, reducing volatility. Under perfect information only the direct effect will be present and, hence, volatility under perfect information is larger than under imperfect information case. It is an interesting corollary that an improvement in information does not reduce volatility (see also Bomfim, 2001). As $\sigma_u^2 \to \infty$ there will again de facto be no information problem, and volatility under both full and imperfect information will be the same.

The comparison made here is between a case where agents can and cannot ex-post distinguish between transitory and persistent shocks. In either case transitory shocks are present. It is trivial to show that transitory shocks or noise create more volatility in exchange rates compared to a situation

[17] We show this analytically in the appendix. The unconditional exchange-rate volatility can be found by writing eq. (13) in a Wold representation and applying standard techniques for finding the unconditional variance.

without noise. Although agents cannot observe the type of shock, we assume rational expectations, and under this assumption, while sensitivity to noise is increased, volatility is reduced. Devereux and Engel (2002) analyze a similar model with irrational expectations (among noise traders), and they demonstrate that this can generate exchange-rate volatility (see also Duarte and Stockman, 2001; Bacchetta and Wincoop, 2003).[18]

5.2 Persistent terms-of-trade effects

A standing puzzle in open-economy macroeconomics is persistence in international relative prices (real exchange rate, terms of trade), which Rogoff (1996) coined the PPP puzzle.[19] Although imperfect information under rational expectations does not account for volatility in exchange rates (and thus in international relative prices), it does give an intuitive explanation for the observed persistence; on impact agents cannot observe the type of shock, and it simply takes time to extract the relevant information.

It is well-known that intertemporal general equilibrium business-cycle models have difficulties matching the observed strong persistence in the adjustment process. In the new open-economy macroeconomics literature there has been focus on persistence generated by staggered contracts, and there is a growing consensus that the critical determinants of persistence are: marginal costs' sensitivity to output; and prices' sensitivity to marginal costs (Lane, 2001). Researchers try to find conditions under which the sensitivity is low in both cases. The present analysis suggests another source of persistence, and interestingly, it works independently of properties of marginal costs and prices over the business cycle.[20]

The equilibrium processes for the endogenous variables, given by (13), (14) and (16), and Figure 3, reveal that informational problems result in a more complicated dynamic adjustment path driven by accumulation of information over time, i.e. learning. The interim dynamic process for the variables is seen to follow an ARMA(1,2) process in the relative money supply, and the autoregressive part is the same for all three variables, not only the terms of trade. This indicates that persistence in the adjustment process spreads to all variables. Furthermore, the autoregressive coefficient ($\theta[1-h]$) depends *only* on the parameters characterizing the information structure (θ,h), which brings out that the information structure has a po-

[18] Devereux and Engel (2002) make other assumptions as well; e.g., local currency pricing and incomplete capital markets.

[19] Our stripped-down model can be extended to cases with a non-constant real exchange rate by including local currency pricing or non-tradables.

[20] This is basically a question of the type: is the glass half empty or half full? Learning in our model can be seen as driving a wedge between marginal costs and output, or in other words, making marginal costs less dependent on output. When learning is slow (much noise) there are persistent effects on output without (relative) wage adjustment. Imperfect information implicitly makes marginal cost less sensitive to output changes.

tentially important role for the dynamic adjustment process, even if nominal contracts only have a duration of one period. Specifically, we find that more persistence in the persistent part of the shocks (high θ) and more information confusion (high, η, low h) generate the strongest persistence in the response of the variables following nominal shocks. A high θ means that great emphasis is put on changes being persistent and, therefore, on last period's expectations, and a low h means that little new information of relevance for predicting future money supplies is obtained from the most recent observations, cf. (10). While the moving average part of the process differs across variables, the autoregressive part works similar to all types of shocks, and this shows that there will be persistence in the adjustment to shocks irrespective of whether they are transitory or persistent.

5.3 Delayed nominal exchange-rate overshooting

Eichenbaum and Evans (1995) present nominal exchange-rate impulse-response functions to monetary shocks, which are hump-shaped, or rather, display delayed overshooting. It turns out that persistent shocks can generate delayed overshooting (cf. Figure 3), which requires (for an expansionary shock)

$$\frac{\partial s_t}{\partial \varepsilon_t} < \frac{\partial s_{t+1}}{\partial \varepsilon_t}, \quad \frac{\partial s_{t+j+1}}{\partial \varepsilon_t} < \frac{\partial s_{t+j}}{\partial \varepsilon_t}, \quad j \geq 1.$$

A necessary condition for delayed overshooting is that $1 - 2\theta + \theta h < 0$, implying that a combination of a high degree of persistence in the shock (large θ) and much noise (low information content of signals, i.e. small h) tend to create this phenomenon. This condition on θ and h is sufficient for generating delayed overshooting in money expectations (see section 3). Intuitively, this has to be fulfilled for delayed exchange-rate overshooting to occur, but it is not sufficient.

Figure 4 illustrates the impulse-response functions for different values of the signal-to-noise ratio η, and hence the coefficient h. For large values of η delayed overshooting disappears as learning becomes too slow, and similarly for small values as learning becomes too fast. Moreover a larger η implies smaller impact effects (vice versa for the interest rate spread), and more persistence as agents take longer to learn the exact nature of the shock.

Figure 4 about here

It can be shown that unconditional delayed overshooting is not possible[21], that is, within the present rational expectations framework delayed overshooting is shock conditional in the sense that it arises for persistent

[21] See the appendix. Gourinchas and Tornell (2002) find that unconditional delayed overshooting can arise if agents misperceive the process underlying the shocks.

shocks if there is sufficient confusion about whether it is actually a transitory shock. This implies that empirical studies which find unconditional delayed overshooting may suffer from a small sample problem in the sense that the particular sample has an overrepresentation of persistent shocks relative to the distribution underlying expectations formation (for a similar argument see Faust *et al.* (2003) for model evaluations based on different information sets than those available to agents, or Lansing (2000) for evaluations of policy rules not based on the information set available to policy makers). Alternatively, it is a rejection of the rational expectations hypothesis. Either way, the shock dependence in dynamic adjustment paths found here points to the problems of interpreting empirical analyses based on small samples.[22]

5.4 Interest rate spread: excess returns

It is a well-established empirical fact that an appreciating (depreciating) exchange rate tends to be accompanied by a positive (negative) interest rate spread (Frankel and Rose, 1995), and this is sometimes interpreted as evidence against the joint hypothesis of uncovered interest rate parity and rational expectations (Eichenbaum and Evans, 1995). As seen in Figure 3, the present framework explains that (ex post) excess returns are persistent due to the interplay between nominal rigidities and imperfect information (see also Gourinchas and Tornell, 2002). To put it differently, we find that expectational errors – in a rational expectations setting – can account for systematic interest rate spreads.[23] In particular we find that conditional on a persistent shock (mistaken to be partially transitory) it is possible to observe a depreciating currency and a negative interest-rate spread. Interestingly, this arises under the same circumstance as delayed overshooting, both of which were found in, e.g., Eichenbaum and Evans' analysis (1995), which supports the point made above on shock contingencies and small samples. It is also worth pointing out that the spread may be time-varying reflecting the type of shock hitting the economy and the learning problem.

Persistence in the interest rate spread does not leave any *ex ante* risk-free arbitrage possibilities, since the real rate of return is the same in both countries, and the difference in nominal interest rates under uncovered interest rate parity reflects the changes in the nominal exchange rate expected by all market participants.

[22] Note that consistent with this view other papers do not find delayed overshooting to be a robust fact (e.g., Kim and Roubini, 2000).

[23] Recall that the "expectational errors" are rational errors in the sense that the expectations are rational given the information set of the agents, and this information set contains the sum of the transitory and persistent components only.

6 Staggered contracts and imperfect information

To clarify the role of learning as a propagation mechanism the model presented above did not include other propagation mechanisms, but an interesting question is how various propagation mechanisms may interact. It is obvious in this setting to include staggered or overlapping wage contracts, and we consider so-called Taylor-contracts (see also Beier, 2001).[24] With an overlapping contract structure, wage setters take into account that their wage decision will interact both with existing and future contracts. This forward-backward looking feature has the potential to interact with imperfect information since each group of wage-setters write contracts at different points in time. The asynchronized timing and the gradual accumulation of information over time imply that each group will write contracts with different informations sets and thus form different expectations.

To formalize this idea we let the workers be organized in monopoly unions, where each union represents a small subset of workers. There are two equally sized groups of unions setting nominal wages in an overlapping fashion; one half setting the wage for two periods in even periods, and the other half in odd periods. The unions are utilitarian and take into account that employment is demand-determined given the wage set. For the group of unions setting the wage as of period $t-1$ and applying to periods t and $t+1$, it is easy to show that the expression of one-period contracts (consult eq. [8]) can be generalized to (see Andersen and Beier, 2003, 2004)

$$\begin{aligned} w_t &= \frac{1}{1+\delta} E_{t-1}\left[\eta_{wp} p_t^h + \left(1-\eta_{wp}\right)\left(s_t + p_t^{*f}\right) + \eta_{wc} c_t\right] \\ &\quad + \frac{\delta}{1+\delta} E_{t-1}\left[\eta_{wp} p_{t+1}^h + \left(1-\eta_{wp}\right)\left(s_{t+1} + p_{t+1}^{*f}\right) + \eta_{wc} c_{t+1}\right], \end{aligned}$$

and similarly for the group setting the wage as of periods $t-2$, t, $t+2$.

Figure 5 about here

Figure 5 shows the economy's response to a persistent expansionary monetary shock with staggered contracts under both perfect and imperfect information. The two propagation mechanisms interact to make the impulse-response qualitatively (for the parameters chosen) and quantitatively different. The terms of trade follow a (conditional) hump-shaped response and the adjustment process displays more persistence.

The interaction is underlined by the fact that both imperfect information and staggering are needed to generate the smooth hump shape in figure 5, although a hump-shape does not arise for all parameter values. A necessary condition is nominal delayed overshooting, but to get a hump shape for

[24] Erceg and Levin (2003) assume staggered contracts, but their focus is not on the interaction of propagation mechanisms.

real variables the internal propagation mechanism generated by staggering alone must be sufficiently strong (for a detailed analysis, see Andersen and Beier, 2003). The intuition is that with strong propagation generated by staggering wages (and thus prices) adjust slowly, and this is needed for the subsequent further depreciation of the nominal exchange rate to be transmitted into increased real effects. This line of reasoning also stresses that neither imperfect information nor a strong internal propagation mechanism from staggering viewed in isolation can generate the hump shape. Learning is needed to generate nominal delayed overshooting, and a strong internal propagation mechanism is needed to have sufficient sluggishness in the adjustment of nominal wages (and prices).

7 Concluding remarks

The problem of distinguishing between transitory and persistent monetary changes has been considered in an explicit intertemporal general equilibrium model focussing on adjustment of exchange rates and interest rates. A major finding is that imperfect information can have significant implications for the dynamic adjustment path to shocks, and that the learning process involved in disentangling the nature of shocks adds persistence to the adjustment process. Our model has been stripped down to be able to present clear-cut analytical and intuitive results, and this opens for interesting further work.

We have modelled the information problem in a stylized way given that the information structure is invariant over time. In reality the relative variances of shocks varies over time as do the markets' perceptions of these. All business cycles are different (IMF, 2002), and even ex post it can be difficult to obtain consensus regarding the causes of particular cycles. In this paper we have explored a minimal deviation from perfect information, and shown that this has significant effects for the adjustment. On the other hand, agents have rational expectations. They might not observe a particular shock, but on average they are not wrong. Ultimately, it is an empirical question how shocks are distributed, and how agents perceive the distribution as well as how agents actually update expectations (Carroll, 2003).

Additionally we demonstrated that learning interacted with staggering in a non-trivial way. The two interact to strengthen persistence in the adjustment process, and to enrich the dynamics, e.g. by causing a persistent monetary shock to induce a hump-shaped response. This indicates that the interplay of transmission mechanisms can possibly strengthen the internal propagation. Another candidate in this respect is capital formation. Furthermore, other shocks might be subject to transitory-persistent confusion (technology shocks), and it would be interesting to quantify the effects of imperfect information in a full-fledged business-cycle model.

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A Log-linearization

Our analysis builds on a version of the model in logarithms. The first-order conditions are

$$C_t^{-\frac{1}{\sigma}} = \delta\left(1+r_t\right)E_t\left(C_{t+1}^{-\frac{1}{\sigma}}\right), \tag{18}$$

$$C_t^{-\frac{1}{\sigma}} = \lambda\left(\frac{M_t}{P_t}\right)^{-\beta} + E_t\left(\delta C_{t+1}^{-\frac{1}{\sigma}}\frac{P_t}{P_{t+1}}\right). \tag{19}$$

As is apparent the Euler equation for money demand is not linear in logs and subsequently we have to approximate around a non-stochastic steady state. The steady-state version of the model is similar to that analyzed in, for example, Obstfeld and Rogoff (1995, 1998, 2000). We focus on a symmetric non-stochastic steady state where $B = B^* = 0$, $C = C^* = Y^h = Y^{*f} = Y = Y^*$, $r = \delta^{-1} - 1$, $\frac{P^h}{P} = \frac{P^f}{P} = \frac{P^{*h}}{P^*} = \frac{P^{*f}}{P^*} = 1$, $\frac{W}{P} = \frac{W^*}{P^*}$, and where money is neutral and the price level is determined from (19). Real incomes are $Y = \frac{P^h Y^h}{P}$ and $Y^* = \frac{P^{*f}Y^{*f}}{P^*}$. Steady-state values are indicated by omission of time subscripts.

Next step is to log-linearize the first-order conditions arising from consumer optimization (18)-(19). The log-linearized consumption Euler equation (6) is obtained by using the convenient formula for lognormally distributed variables

$$\log E\left(X^f\right) = fE\left[\log\left(X\right)\right] + \frac{f^2}{2}Var\left[\log\left(X\right)\right],$$

where f is a scalar and X is lognormally distributed. Money demand warrants a comment. Taking logs on both sides of (19) yields the log of a sum and it is easy to show that around a steady state (disregarding constants)

$$\log(X_t + Z_t) \approx \frac{X}{X+Z}\log(X_t) + \frac{Z}{X+Z}\log(Z_t). \tag{20}$$

Using this we get that

$$\begin{aligned}\log\left[\lambda\left(\frac{M_t}{P_t}\right)^{-\beta} + E_t\left(\delta C_{t+1}^{-\frac{1}{\sigma}}\frac{P_t}{P_{t+1}}\right)\right] \approx\ & (1-\delta)\log\left[\lambda\left(\frac{M_t}{P_t}\right)^{-\beta}\right] \\ & +\delta\log\left[E_t\left(\delta C_{t+1}^{-\frac{1}{\sigma}}\frac{P_t}{P_{t+1}}\right)\right].\end{aligned}$$

Equation (7) follows immediately with

$$\eta_{mc} = \frac{1}{\sigma\left(1-\delta\right)\beta}, \qquad \eta_{mc}^1 = \frac{\delta}{\sigma\left(1-\delta\right)\beta}, \qquad \eta_{mp} = \frac{\delta}{\left(1-\delta\right)\beta}.$$

While the model is specified so as to yield a log-linear structure, we have that the budget constraint is linear in levels, i.e.

$$B_t = (1 + r_{t-1}) B_{t-1} + Y_t - C_t. \tag{21}$$

Subtracting the steady-state version of the budget constraint from (21) and dividing by Y $(= C)$ we get

$$\begin{aligned} \frac{B_t - B}{Y} &= (1+r) \frac{B_{t-1} - B}{Y} + \frac{Y_t - Y}{Y} - \frac{C_t - C}{C} \\ &\quad + [(1 + r_{t-1}) - (1+r)] \frac{B_{t-1} - B}{Y}. \end{aligned}$$

The last term on the right-hand side is negligible as we look at small deviations around steady state. We end up with

$$b_t \approx \delta^{-1} b_{t-1} + y_t - c_t, \tag{22}$$

as $1 + r = \delta^{-1}$, $\log\left(\frac{Y_t}{Y}\right) \approx \frac{Y_t - Y}{Y}$, $b_t = \frac{B_t}{Y}$ and $\log\left(\frac{C_t}{C}\right) \approx \frac{C_t - C}{C}$.

B Wage setting

The union solves

$$\begin{aligned} &\underset{W_t}{\text{Max}} E_{t-1} \left(\zeta_t \frac{W_t}{P_t} N_t - \frac{\kappa}{1+\mu} N_t^{1+\mu} \right) \\ &\text{s.t. } N_t = \gamma^{\frac{1}{1-\gamma}} \left(\frac{P_t^h}{W_t} \right)^{\eta_{nw}}. \end{aligned}$$

where $\zeta_t = C_t^{-\frac{1}{\sigma}}$. The resulting first-order condition is

$$\begin{aligned} &E_{t-1} \left[\zeta_t \frac{N_t}{P_t} + \zeta_t \frac{W_t}{P_t} \frac{\partial N_t}{\partial W_t} - \kappa N_t^{1+\mu} \frac{\partial N_t}{\partial W_t} \right] = 0 \\ &\Rightarrow W_t = \kappa \frac{\eta_{nw}}{\eta_{nw} - 1} \frac{E_{t-1}\left(N_t^{1+\mu} \right)}{E_{t-1}\left(C_t^{-\frac{1}{\sigma}} \frac{N_t}{P_t} \right)}. \end{aligned}$$

Disregarding constants, this can be shown to yield (8) by utilizing that the variables are lognormally distributed (recall that the shocks, ε and u, are normally distributed), and (20) as well as the following formula, where b_1, b_2, and b_3 are scalars:

$$\log\left(X_1^{b_1} X_2^{b_2} X_3^{b_3} \right) \sim \text{lognormal}\left(b' \mu_X, b' \Sigma_X b \right)$$

$$X_i \sim \mathit{lognormal}\left(\mu_i, \sigma_i^2 \right), \; b = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}, \; \mu_X = \begin{pmatrix} \mu_1 \\ \mu_2 \\ \mu_3 \end{pmatrix},$$

and Σ_X is the variance-covariance matrix.

C The Kalman filter

In this section we set up the Kalman filter and derive the updating formula of the expectations. We follow Hamilton (1994) very closely. The general system can in state space representation be written (Hamilton, 1994, pp. 372-373, eqs. [13.1.1]-[13.1.5])

$$\boldsymbol{\xi}_{t+1} = \mathbf{F}\boldsymbol{\xi}_t + \mathbf{v}_{t+1},\ \mathbf{y}_t = \mathbf{A}'\mathbf{x}_t + \mathbf{H}'\boldsymbol{\xi}_t + \mathbf{w}_t,\ E(\mathbf{v}_t\mathbf{w}'_\tau) = 0 \text{ for all } t \text{ and } \tau,$$

$$E\left(\mathbf{v}_t\mathbf{v}'_\tau\right) = \begin{cases} \mathbf{Q} & \text{for } t=\tau \\ \mathbf{0} & \text{otherwise} \end{cases},\ E\left(\mathbf{w}_t\mathbf{w}'_\tau\right) = \begin{cases} \mathbf{R} & \text{for } t=\tau \\ \mathbf{0} & \text{otherwise} \end{cases}.$$

In our notation this corresponds to

$$\boldsymbol{\xi}_{t+1} = z_{t+1},\ \mathbf{F} = \theta,\ \mathbf{v}_{t+1} = \varepsilon_{t+1},\ \mathbf{y}_t = m_t - m_t^*,\ \mathbf{A}'\mathbf{x}_t = 0,$$

$$\mathbf{H}' = 1,\ \mathbf{w}_t = u_t,\ \mathbf{Q} = \sigma_\varepsilon^2,\ \mathbf{R} = \sigma_u^2.$$

Hamilton's (1994) Proposition 13.1, which states that as $T \to \infty$ (the updating process has been going on forever) then

$$\mathbf{P} = \mathbf{F}\left[\mathbf{P} - \mathbf{PH}\left(\mathbf{H}'\mathbf{PH} + \mathbf{R}\right)^{-1}\mathbf{H}'\mathbf{P}\right]\mathbf{F}' + \mathbf{Q},$$

$$\mathbf{K} \equiv \mathbf{FPH}\left(\mathbf{H}'\mathbf{PH} + \mathbf{R}\right)^{-1},$$

where $\mathbf{K}$ in our notation is θh. Inserting yields the expression given in the text. Reproducing Hamilton's (1994) equations [13.2.20] and [13.2.24] where the left-hand sides are the forecasts at time t of the permanent part and the sum of the permanent and temporary parts at time $t+1$ ($\mathbf{A}'\mathbf{x}_t = 0$)

$$\widehat{\boldsymbol{\xi}}_{t+1|t} = \mathbf{F}\widehat{\boldsymbol{\xi}}_{t|t-1} + \mathbf{K}\left(\mathbf{y}_t - \mathbf{H}'\widehat{\boldsymbol{\xi}}_{t|t-1}\right),\quad \widehat{\mathbf{y}}_{t+1|t} = \mathbf{H}'\widehat{\boldsymbol{\xi}}_{t+1|t}$$

leading to

$$(\widehat{m - m^*})_{t+1|t} = \widehat{z}_{t+1|t},\quad \widehat{z}_{t+1|t} = \mathbf{F}\widehat{z}_{t|t-1} + \mathbf{K}\left[(m_t - m_t^*) - \widehat{z}_{t|t-1}\right],$$

or in our notation

$$E_t\left(m_{t+1} - m_{t+1}^*\right) = \theta E_{t-1}\left(m_t - m_t^*\right) + \theta h\left[\left(m_t - m_t^*\right) - E_{t-1}\left(m_t - m_t^*\right)\right].$$

D Imperfect information

We solve for three variables: the nominal exchange rate, the terms of trade, and the nominal interest rate spread. We use the method of undetermined coefficients and take each variable in turn. Our guesses are

$$s_t = \pi_{sm}\left(m_t - m_t^*\right) + \pi_{sm}^1 E_{t-1}\left(m_t - m_t^*\right), \tag{23}$$

$$q_t = \pi_{qm}\left(m_t - m_t^*\right) + \pi_{qm}^1 E_{t-1}\left(m_t - m_t^*\right), \tag{24}$$

Next we consider the endogenous variables in turn.

D.1 Nominal exchange rate

The nominal exchange rate follows from the money market equilibrium condition yielding (just use the first-order equation for money [7])

$$s_t = \frac{1}{1+\eta_{mp}}\left(\eta_{mp} E_t s_{t+1} + m_t - m_t^*\right),$$

where we have used $c_t = c_t^*$. Consistency with (23) requires

$$\pi_{sm} = \frac{\eta_{mp}\left(\pi_{sm} + \pi_{sm}^1\right)\theta h + 1}{1+\eta_{mp}}, \quad \pi_{sm}^1 = \frac{\eta_{mp}\left(\pi_{sm} + \pi_{sm}^1\right)\theta\left(1-h\right)}{1+\eta_{mp}}$$

implying that

$$\begin{aligned}
\pi_{sm} &= \frac{1+\eta_{mp} - \theta\left(1-h\right)\eta_{mp}}{\left(1+\eta_{mp}\right)\left(1+\eta_{mp} - \theta\eta_{mp}\right)} \\
\pi_{sm}^1 &= \frac{\theta\left(1-h\right)\eta_{mp}}{\left(1+\eta_{mp}\right)\left(1+\eta_{mp} - \theta\eta_{mp}\right)}
\end{aligned}$$

Equation (23) can by use of

$$E_{t-1}\left(m_t - m_t^*\right) = \theta\left(1-h\right)E_{t-2}\left(m_{t-1} - m_{t-1}^*\right) + \theta h\left(m_{t-1} - m_{t-1}^*\right), \quad (25)$$

be rewritten as (13).

For both shocks we have

$$\begin{aligned}
\frac{\partial s_t}{\partial\left(m_t - m_t^*\right)} &= \phi_{sm} = \pi_{sm} > 0, \\
\frac{\partial \pi_{sm}}{\partial h} &= \frac{\theta\eta_{mp}}{\left[\left(1+\eta_{mp}\right)\left(1+\eta_{mp} - \theta\eta_{mp}\right)\right]^2} > 0.
\end{aligned}$$

D.1.1 Conditional delayed overshooting

In the case of a persistent shock [$\varepsilon_t > 0$, $\theta \in (0,1)$] we cannot reject the possibility of delayed overshooting. We need

$$\frac{\partial s_t}{\partial\left(m_t - m_t^*\right)} = \phi_{sm} < \phi_{ss}^1\phi_{sm} + \theta\phi_{sm} + \phi_{sm}^1 = \frac{\partial s_{t+1}}{\partial\left(m_t - m_t^*\right)},$$

or inserting

$$\left(\theta - 1\right)\left(\pi_{sm} + \pi_{sm}^1\right) - \theta\pi_{sm}^1 - \left(1 - 2\theta + \theta h\right)\pi_{sm}^1 > 0.$$

Hence, a necessary condition for delayed overshooting is $1 - 2\theta + \theta h < 0$, since both π_{sm} and π_{sm}^1 are positive.

D.2 Terms of trade

Equalizing relative demands (using [5] and its Foreign counterpart)

$$d_t - d_t^* = \log\left[\left(\frac{P_t^h}{P_t}\right)^{-1} C_t^w\right] - \log\left[\left(\frac{P_t^f}{P_t^*}\right)^{-1} C_t^w\right] = -q_t,$$

and relative supply (see eq. [2])

$$y_t^h - y_t^{*f} = -\eta_{yw}\left[(w_t - w_t^*) - \left(p_t^h - p_t^{*f}\right)\right] = -\eta_{yw}\left[(w_t - w_t^*) - q_t - s_t\right],$$

we get product market equilibrium

$$q_t = \frac{\eta_{yw}\left(2\eta_{wp} - 1\right)}{1 + \eta_{yw}} E_{t-1} q_t - \frac{\eta_{yw}}{1 + \eta_{yw}}\left(s_t - E_{t-1} s_t\right), \tag{26}$$

where we have used the expression for the wage rate and $c_t = c_t^*$. Invoking the expression for the nominal exchange rate we obtain

$$q_t = \frac{\eta_{yw}\left(2\eta_{wp} - 1\right)}{1 + \eta_{yw}} E_{t-1} q_t - \frac{\eta_{yw}}{1 + \eta_{yw}} \pi_{sm}\left[m_t - m_t^* - E_{t-1}\left(m_t - m_t^*\right)\right].$$

Using our guess (24) to find $E_{t-1} q_t$ and inserting, the restrictions are

$$\pi_{qm} = -\frac{\eta_{yw}\pi_{sm}}{\rho + \eta_{yw}} = -\gamma\pi_{sm} = -\gamma\phi_{sm},$$

$$\pi_{qm}^1 = -\theta\pi_{qm}.$$

Using (24), and (25) we can rewrite the terms of trade as (14).

The impact effect is

$$\frac{\partial q_t}{\partial\left(m_t - m_t^*\right)} = \pi_{qm} = -\gamma\pi_{sm} < 0,$$

and since π_{sm} is increasing in h, the absolute value of the impact effect is increasing as well.

D.3 Nominal interest rate spread

If we instead of a real bond assume a nominal bond with gross return $1 + i_t$ the Euler equation reads (see also Obstfeld and Rogoff, 1998):

$$C_t^{-\frac{1}{\sigma}} = \delta E_t\left[\frac{(1 + i_t) P_t}{P_{t+1}} C_{t+1}^{-\frac{1}{\sigma}}\right].$$

Combining with the Euler equation when using a real bond we get

$$1 + i_t = \left[E_t\left(\frac{C_{t+1}^{-\frac{1}{\sigma}}}{P_{t+1}}\right) P_t\right]^{-1} (1 + r_t) E_t\left(C_{t+1}^{-\frac{1}{\sigma}}\right).$$

Using the joint lognormality of C and P and using the Foreign version, we get uncovered interest rate parity (eq. [15]; disregarding constants). Finally, inserting for the nominal exchange rate we obtain (16).

The impact effect is ($R = \log[1 + i_t]$)

$$\begin{aligned}
\frac{\partial (R - R^*)}{\partial (m_t - m_t^*)} &= \phi_{im} \\
&= \frac{\theta h}{1 + \eta_{mp} - \theta \eta_{mp}} - \phi_{sm} \\
&= \frac{\theta h \left(1 + \eta_{mp}\right) - 1 - \eta_{mp} + \theta (1 - h) \eta_{mp}}{\left(1 + \eta_{mp} - \theta \eta_{mp}\right)\left(1 + \eta_{mp}\right)} < 0,
\end{aligned}$$

and

$$\frac{\partial \phi_{im}}{\partial h} > 0.$$

The less noise, the less impact on the spread.

E Perfect information

We guess

$$\begin{aligned}
s_t &= \phi_{sz} z_t + \phi_{su} u_t \\
q_t &= \phi_{q\varepsilon} \varepsilon_t + \phi_{qu} u_t
\end{aligned}$$

E.1 Nominal exchange rate

We start out with

$$\begin{aligned}
s_t &= \frac{1}{1 + \eta_{mp}} \left(\eta_{mp} E_t s_{t+1} + m_t - m_t^*\right) \\
&= \frac{\eta_{mp}}{1 + \eta_{mp}} E_t s_{t+1} + \frac{1}{1 + \eta_{mp}} (z_t + u_t) \Longrightarrow \\
s_t &= \left(\frac{\eta_{mp}}{1 + \eta_{mp}} \phi_{sz} \theta + \frac{1}{1 + \eta_{mp}}\right) z_t + \frac{1}{1 + \eta_{mp}} u_t.
\end{aligned}$$

Equalizing coefficients yields the expressions given in the text.

E.2 Terms of trade

We start out with (26)

$$q_t = \frac{\eta_{yw} \left(2\eta_{wp} - 1\right)}{1 + \eta_{yw}} E_{t-1} q_t - \frac{\eta_{yw}}{1 + \eta_{yw}} (s_t - E_{t-1} s_t).$$

Inserting $E_{t-1}q_t = 0$ yields

$$\begin{aligned} q_t &= -\frac{\eta_{yw}}{1+\eta_{yw}}\left(s_t - E_{t-1}s_t\right) \\ &= -\gamma\left(\phi_{sz}\varepsilon_t + \phi_{su}u_t\right), \end{aligned}$$

which are the coefficients postulated in the section.

E.3 Nominal interest rate spread

The spread follows from uncovered interest rate parity

$$\begin{aligned} & E_t s_{t+1} - s_t \\ = \; & E_t\left(\phi_{sz}z_{t+1} + \phi_{su}u_{t+1}\right) - \phi_{sz}z_t - \phi_{su}u_t \\ = \; & \phi_{sz}\theta z_t - \phi_{sz}z_t - \phi_{su}u_t = (\theta-1)\,\phi_{sz}z_t - \phi_{su}u_t. \end{aligned}$$

F Unconditional relative variances

In the following we show that the variability under imperfect information is less that under perfect information. Again we utilize the apparatus from Hamilton (1994).

F.1 Perfect information

The unconditional variance of the nominal exchange rate is

$$\begin{aligned} E\left(s_t^2\right) &= E\left\{\left[\phi_{su}u_t + \phi_{sz}\left(\varepsilon_t + \theta\varepsilon_{t-1} + \theta^2\varepsilon_{t-2} + ...\right)\right]^2\right\} \\ &= \phi_{su}^2\sigma_u^2 + \phi_{sz}^2\left(1+\theta^2+\theta^4+\theta^6+...\right)\sigma_\varepsilon^2 \\ &= \phi_{su}^2\sigma_u^2 + \frac{\phi_{sz}^2}{1-\theta^2}\sigma_\varepsilon^2 \end{aligned}$$

F.2 Imperfect information

We have that

$$\begin{aligned} s_t &= \pi_{sm}\left(m_t - m_t^*\right) + \pi_{sm}^1 E_{t-1}\left(m_t - m_t^*\right) \\ &= \phi_{sz}\left(m_t - m_t^*\right) - \pi_{sm}^1\left[\left(m_t - m_t^*\right) - E_{t-1}\left(m_t - m_t^*\right)\right] \end{aligned}$$

and let

$$\boldsymbol{\omega}_t = \left(m_t - m_t^*\right) - E_{t-1}\left(m_t - m_t^*\right)$$

thus using Hamilton's (1994) eq. (13.5.18)

$$m_t - m_t^* = \left(1 + (1-\theta L)^{-1}\theta h L\right)\boldsymbol{\omega}_t = \boldsymbol{\omega}_t + \theta h\left(\boldsymbol{\omega}_{t-1} + \theta\boldsymbol{\omega}_{t-2} + \theta^2\boldsymbol{\omega}_{t-3} + ...\right).$$

where (Eq. [13.5.16])

$$E\left(\boldsymbol{\omega}_t \boldsymbol{\omega}_t\right) = \mathbf{H}'\mathbf{P}\mathbf{H} + \mathbf{R} = \mathbf{P} + \sigma_u^2 = \frac{1}{1-h}\sigma_u^2$$

Now inserting:

$$\begin{aligned} s_t &= \phi_{sz}\left[\boldsymbol{\omega}_t + \theta h\left(\boldsymbol{\omega}_{t-1} + \theta \boldsymbol{\omega}_{t-2} + \theta^2 \boldsymbol{\omega}_{t-3} + ...\right)\right] - \pi_{sm}^1 \boldsymbol{\omega}_t \\ &= \phi_{sm}\boldsymbol{\omega}_t + \phi_{sz}\theta h\left(\boldsymbol{\omega}_{t-1} + \theta \boldsymbol{\omega}_{t-2} + \theta^2 \boldsymbol{\omega}_{t-3} + ...\right) \\ &= \left(\phi_{sm} - \phi_{sz} h\right)\boldsymbol{\omega}_t + \phi_{sz} h\left(\boldsymbol{\omega}_t + \theta \boldsymbol{\omega}_{t-1} + \theta^2 \boldsymbol{\omega}_{t-2} + \theta^3 \boldsymbol{\omega}_{t-3} + ...\right) \\ &= (1-h)\,\phi_{su}\boldsymbol{\omega}_t + \phi_{sz} h\left(\boldsymbol{\omega}_t + \theta \boldsymbol{\omega}_{t-1} + \theta^2 \boldsymbol{\omega}_{t-2} + \theta^3 \boldsymbol{\omega}_{t-3} + ...\right) \end{aligned}$$

meaning that the variance is

$$\begin{aligned} E\left(s_t s_t\right) &= \left[(1-h)^2 \phi_{su}^2 + 2(1-h)\,\phi_{su}\phi_{sz} h + \frac{\phi_{sz}^2 h^2}{1-\theta^2}\right]\left(\mathbf{P} + \sigma_u^2\right) \\ &= \left[(1-h)^2 \phi_{su}^2 + 2(1-h)\,\phi_{su}\phi_{sz} h + \frac{\phi_{sz}^2 h^2}{1-\theta^2}\right]\frac{1}{1-h}\sigma_u^2 \end{aligned}$$

F.3 Relative variance

The variance under imperfect information relative to perfect information is thus

$$\begin{aligned} & \frac{\left[(1-h)^2 \phi_{su}^2 + 2(1-h)\,\phi_{su}\phi_{sz} h + \frac{\phi_{sz}^2 h^2}{1-\theta^2}\right]\frac{1}{1-h}\sigma_u^2}{\phi_{su}^2 \sigma_u^2 + \frac{\phi_{sz}^2}{1-\theta^2}\sigma_\varepsilon^2} \\ = & \frac{\left[(1-h)^2 \phi_{su}^2 + 2(1-h)\,\phi_{su}\phi_{sz} h + \frac{\phi_{sz}^2 h^2}{1-\theta^2}\right]\frac{1}{1-h}\eta}{\phi_{su}^2 \eta + \frac{\phi_{sz}^2}{1-\theta^2}} \end{aligned}$$

For $\eta \to \infty$ or $\eta \to 0$ this ratio is 1. Performing a grid search shows that the ratio is less than unity for intermediate values of the noise-to-signal ratio. Hence, the nominal exchange rate variability is less under imperfect information.

G Unconditional delayed overshooting

Here we show that there can be no unconditional delayed nominal exchange-rate overshooting. Thus, if we do not condition on the type of the shock, but consider a general innovation

$$\boldsymbol{\omega}_t = \left(m_t - m_t^*\right) - E_{t-1}\left(m_t - m_t^*\right)$$

there cannot be delayed overshooting. Showing this boils down to finding the Wold representation of the nominal exchange rate, which we found above:

$$\begin{aligned} s_t &= \phi_{sz}\left[\boldsymbol{\omega}_t + \theta h\left(\boldsymbol{\omega}_{t-1} + \theta\boldsymbol{\omega}_{t-2} + \theta^2\boldsymbol{\omega}_{t-3} + ...\right)\right] - \pi^1_{sm}\boldsymbol{\omega}_t \\ &= \phi_{sm}\boldsymbol{\omega}_t + \phi_{sz}\theta h\left(\boldsymbol{\omega}_{t-1} + \theta\boldsymbol{\omega}_{t-2} + \theta^2\boldsymbol{\omega}_{t-3} + ...\right). \end{aligned}$$

The impact effect to a given shock is

$$\frac{\partial s_t}{\partial \boldsymbol{\varepsilon}_t} = \phi_{sm}$$

and the effect on the nominal exchange rate the period after is

$$\frac{\partial s_{t+1}}{\partial \boldsymbol{\varepsilon}_t} = \phi_{sz}\theta h.$$

It is seen that

$$\frac{\partial s_{t+1}}{\partial \boldsymbol{\varepsilon}_t} - \frac{\partial s_t}{\partial \boldsymbol{\varepsilon}_t} = \phi_{sz}\theta h - \phi_{sm} < 0.$$

Hence there cannot be unconditional delayed overshooting in this model.

H Equilibrium with staggered wage setting under imperfect information

H.1 Union wage setting

Here we follow Andersen and Beier (2003), and Beier (2001). Workers are organized in monopoly unions, where each union represents a small subset of workers supplying labor to group of firms. The contract structure is exogenous, where half the unions sign contracts in even periods and the other in odd periods. The utilitarian union presets the wage for periods t and $t+1$ given all information available in period $t-1$ to maximize expected utility of its members, which in turn depends on wage income and disutility of work. We assume a right-to-manage structure in which employment is determined by firms given the wage. The representative union's problem is given by

$$\underset{W}{\text{Max}}\; E_{t-1}\left[\zeta_t\frac{W}{P_t}N_t - \frac{\kappa}{1+\mu}N_t^{1+\mu} + \delta\left(\zeta_{t+1}\frac{W}{P_{t+1}}N_{t+1} - \frac{\kappa}{1+\mu}N_{t+1}^{1+\mu}\right)\right]$$

$$\text{s.t. } N_t = \alpha_n\left(\frac{P_t^h}{W}\right)^{\eta_{nw}},\; N_{t+1} = \alpha_n\left(\frac{P_{t+1}^h}{W}\right)^{\eta_{nw}},\; \zeta_t = C_t^{-\frac{1}{\sigma}},\; \alpha_n = \gamma^{\frac{1}{1-\gamma}}.$$

In Andersen and Beier (2003) and Beier (2001) we demonstrate that the solution is (disregarding constants)

$$w_t = \frac{1}{1+\delta}E_{t-1}w_t^{flex} + \frac{\delta}{1+\delta}E_{t-1}w_{t+1}^{flex},$$

where w_t^{flex} is the flexible wage which would have prevailed had wages been flexible. The flexible wage can be shown to be

$$w_t^{flex} = \eta_{wp} p_t^h + \left(1 - \eta_{wp}\right) \left(s_t + p_t^{*f}\right) + \eta_{wc} c_t,$$

where the constants are defined in the text. Again see Andersen and Beier (2003) for interpretations. Note that the relative aggregate wage can be written as

$$\overline{w}_t - \overline{w}_t^* = (2 + 2\delta)^{-1} \sum_{i=1}^{2} \sum_{j=1}^{2} \delta^{j-1} E_{t-i} \left[\left(2\eta_{wp} - 1\right) q_{t+j-i} + s_{t+j-i}\right],$$

where we have used that the relative Walrasian wage is (and $c_t = c_t^*$)

$$w_t^{flex} - w_t^{*flex} = \left(2\eta_{wp} - 1\right) q_t + s_t.$$

H.2 Equilibrium

In general we conjecture a solution for the nominal exchange rate and the terms of trade as

$$s_t = \pi_{sm} \left(m_t - m_t^*\right) + \pi_{sm}^1 E_{t-1} \left(m_t - m_t^*\right)$$

$$\begin{aligned} q_t &= \pi_{qq} q_{t-1} + \pi_{qm} \left(m_t - m_t^*\right) \\ &\quad + \pi_{qm}^1 \left(m_{t-1} - m_{t-1}^*\right) + \pi_{qm}^2 E_{t-1} \left(m_t - m_t^*\right), \end{aligned}$$

H.2.1 The nominal exchange rate

The nominal exchange rate is unchanged by the introduction of staggering. Thus we get

$$\begin{aligned} \pi_{sm} &= \frac{1 + \eta_{mp} - \theta\eta_{mp} + \theta h \eta_{mp}}{\left(1 + \eta_{mp} - \theta\eta_{mp}\right)\left(1 + \eta_{mp}\right)} \\ \pi_{sm}^1 &= \frac{(1 - h)\,\theta\eta_{mp}}{\left(1 + \eta_{mp}\right)\left(1 + \eta_{mp} - \theta\eta_{mp}\right)}. \end{aligned}$$

H.2.2 The terms of trade

From product market equilibria we get

$$-q_t = -\eta_{yw} \left[\left(\overline{w}_t - \overline{w}_t^*\right) - q_t - s_t\right] \Rightarrow q_t = \frac{\eta_{yw}}{\eta_{yw} + 1} \left[\left(\overline{w}_t - \overline{w}_t^*\right) - s_t\right]$$

Thus using the expression for aggregate wages (see above) we can write the terms of trade as

$$\begin{aligned} q_t &= +\eta_{qq}' \left(E_{t-2} q_{t-1} + \delta E_{t-2} q_t + E_{t-1} q_t + \delta E_{t-1} q_{t+1}\right) \\ &\quad + \eta_{qs}' \left[\left(E_{t-2} s_{t-1} - s_t\right) + \delta\left(E_{t-2} s_t - s_t\right) + \left(E_{t-1} s_t - s_t\right) + \delta\left(E_{t-1} s_{t+1} - s_t\right)\right] \end{aligned}$$

$$\eta'_{qq}=\frac{\eta_{yw}}{\eta_{yw}+1}\frac{2\eta_{wp}-1}{2(1+\delta)},\ \eta'_{qs}=\frac{\eta_{yw}}{\eta_{yw}+1}\frac{1}{2(1+\delta)}$$

Next we have that

$$\begin{aligned}
E_{t-2}s_{t-1} &= \left(\pi_{sm}+\pi_{sm}^{1}\right)E_{t-2}\left(m_{t-1}-m_{t-1}^{*}\right)\\
E_{t-2}s_{t} &= \left(\pi_{sm}+\pi_{sm}^{1}\right)E_{t-2}\left(m_{t}-m_{t}^{*}\right)\\
E_{t-1}s_{t} &= \left(\pi_{sm}+\pi_{sm}^{1}\right)E_{t-1}\left(m_{t}-m_{t}^{*}\right)\\
E_{t-1}s_{t+1} &= \left(\pi_{sm}+\pi_{sm}^{1}\right)E_{t-1}\left(m_{t+1}-m_{t+1}^{*}\right),
\end{aligned}$$

and

$$s_t-E_{t-2}s_{t-1}=\pi_{sm}\left(m_t-m_t^*\right)+\pi_{sm}^1E_{t-1}\left(m_t-m_t^*\right)-\left(\pi_{sm}+\pi_{sm}^1\right)E_{t-2}\left(m_{t-1}-m_{t-1}^*\right)$$

$$s_t-E_{t-2}s_t=\pi_{sm}\left(m_t-m_t^*\right)+\pi_{sm}^1E_{t-1}\left(m_t-m_t^*\right)-\left(\pi_{sm}+\pi_{sm}^1\right)E_{t-2}\left(m_t-m_t^*\right)$$

$$s_t-E_{t-1}s_t=\pi_{sm}\left(m_t-m_t^*\right)+\pi_{sm}^1E_{t-1}\left(m_t-m_t^*\right)-\left(\pi_{sm}+\pi_{sm}^1\right)E_{t-1}\left(m_t-m_t^*\right)$$

$$s_t-E_{t-1}s_{t+1}=\pi_{sm}\left(m_t-m_t^*\right)+\pi_{sm}^1E_{t-1}\left(m_t-m_t^*\right)-\left(\pi_{sm}+\pi_{sm}^1\right)E_{t-1}\left(m_{t+1}-m_{t+1}^*\right).$$

Next we invoke that

$$E_{t-1}\left(m_t-m_t^*\right)=(1-h)\,\theta E_{t-2}\left(m_{t-1}-m_{t-1}^*\right)+h\theta\left(m_{t-1}-m_{t-1}^*\right)$$

and thus

$$\begin{aligned}
& E_{t-2}\left[E_{t-1}\left(m_t-m_t^*\right)\right]\\
=\ & (1-h)\,\theta E_{t-2}\left(m_{t-1}-m_{t-1}^*\right)+h\theta E_{t-2}\left(m_{t-1}-m_{t-1}^*\right)\\
=\ & \theta E_{t-2}\left(m_{t-1}-m_{t-1}^*\right)
\end{aligned}$$

i.e.

$$\begin{aligned}
E_{t-2}\left(m_{t-1}-m_{t-1}^*\right) &= -\frac{h}{(1-h)}\left(m_{t-1}-m_{t-1}^*\right)\\
&\quad+\frac{1}{(1-h)\,\theta}E_{t-1}\left(m_t-m_t^*\right)
\end{aligned}$$

$$\begin{aligned}
E_{t-2}\left(m_t-m_t^*\right) &= -\frac{h\theta}{(1-h)}\left(m_{t-1}-m_{t-1}^*\right)\\
&\quad+\frac{1}{(1-h)}E_{t-1}\left(m_t-m_t^*\right)
\end{aligned}$$

$$\begin{aligned}
& s_t-E_{t-2}s_{t-1}\\
=\ & \pi_{sm}\left(m_t-m_t^*\right)+\pi_{sm}^1E_{t-1}\left(m_t-m_t^*\right)\\
& -\frac{\left(\pi_{sm}+\pi_{sm}^1\right)}{(1-h)\,\theta}E_{t-1}\left(m_t-m_t^*\right)+\frac{\left(\pi_{sm}+\pi_{sm}^1\right)h}{1-h}\left(m_{t-1}-m_{t-1}^*\right)
\end{aligned}$$

$$
\begin{aligned}
& s_t - E_{t-2}s_t \\
= \;& \pi_{sm}\left(m_t - m_t^*\right) + \pi_{sm}^1 E_{t-1}\left(m_t - m_t^*\right) \\
& - \left(\pi_{sm} + \pi_{sm}^1\right) E_{t-2}\left(m_t - m_t^*\right) \\
= \;& \pi_{sm}\left(m_t - m_t^*\right) + \pi_{sm}^1 E_{t-1}\left(m_t - m_t^*\right) \\
& - \frac{\left(\pi_{sm} + \pi_{sm}^1\right)}{(1-h)} E_{t-1}\left(m_t - m_t^*\right) + \frac{\left(\pi_{sm} + \pi_{sm}^1\right)\theta h}{1-h}\left(m_{t-1} - m_{t-1}^*\right)
\end{aligned}
$$

$$
\begin{aligned}
s_t - E_{t-1}s_t &= \pi_{sm}\left(m_t - m_t^*\right) + \pi_{sm}^1 E_{t-1}\left(m_t - m_t^*\right) \\
& \quad - \left(\pi_{sm} + \pi_{sm}^1\right) E_{t-1}\left(m_t - m_t^*\right) \\
&= \pi_{sm}\left(m_t - m_t^*\right) - \pi_{sm} E_{t-1}\left(m_t - m_t^*\right)
\end{aligned}
$$

$$
\begin{aligned}
s_t - E_{t-1}s_{t+1} &= \pi_{sm}\left(m_t - m_t^*\right) + \pi_{sm}^1 E_{t-1}\left(m_t - m_t^*\right) \\
& \quad - \left(\pi_{sm} + \pi_{sm}^1\right)\theta E_{t-1}\left(m_t - m_t^*\right) \\
&= \pi_{sm}\left(m_t - m_t^*\right) + \left[\pi_{sm}^1(1-\theta) - \theta\pi_{sm}\right] E_{t-1}\left(m_t - m_t^*\right)
\end{aligned}
$$

Inserting yields

$$
\begin{aligned}
q_t &= \eta_{qq}\left(E_{t-2}q_{t-1} + \delta E_{t-2}q_t + E_{t-1}q_t + \delta E_{t-1}q_{t+1}\right) \\
& \quad + \eta_{qm}\left(m_t - m_t^*\right) + \eta_{qm}^1\left(m_{t-1} - m_{t-1}^*\right) + \eta_{qm}^2 E_{t-1}\left(m_t - m_t^*\right)
\end{aligned}
$$

where

$$
\eta_{qq} = \frac{\eta_{yw}}{\eta_{yw}+1}\frac{1}{2(1+\delta)}\left(2\eta_{wp} - 1\right),
$$

$$
\eta_{qm} = -\frac{\eta_{yw}}{\eta_{yw}+1}\pi_{sm}
$$

$$
\eta_{qm}^1 = -\frac{\eta_{yw}}{\eta_{yw}+1}\frac{1}{2(1+\delta)}\frac{h}{1-h}(1+\delta\theta)\left(\pi_{sm} + \pi_{sm}^1\right),
$$

$$
\begin{aligned}
\eta_{qm}^2 &= -\frac{\eta_{yw}}{\eta_{yw}+1}\frac{1}{2(1+\delta)} \cdot \\
& \quad \left((1+\delta)\pi_{sm}^1 - \pi_{sm} + \delta(1-\theta)\pi_{sm}^1 - \delta\theta\pi_{sm} - \frac{\pi_{sm} + \pi_{sm}^1}{(1-h)\theta}(1+\theta\delta)\right) \\
&= -\frac{\eta_{yw}}{\eta_{yw}+1}\frac{1}{2(1+\delta)} \cdot \\
& \quad \left(-\pi_{sm}(1+\delta\theta) - \pi_{sm}^1(1+\delta\theta) + 2(1+\delta)\pi_{sm}^1 - \frac{\pi_{sm} + \pi_{sm}^1}{(1-h)\theta}(1+\theta\delta)\right) \\
&= -\frac{\eta_{yw}}{\eta_{yw}+1}\frac{1}{2(1+\delta)} \cdot \\
& \quad \left(-\left(\pi_{sm} + \pi_{sm}^1\right)(1+\delta\theta)\left[\frac{1+(1-h)\theta}{(1-h)\theta}\right] + 2(1+\delta)\pi_{sm}^1\right)
\end{aligned}
$$

$$\theta < 1 \Rightarrow \eta_{qm} + \eta_{qm}^1 + \eta_{qm}^2 \neq 0.$$

Note that as $\theta \to 1$ the expression converges to the one given in Beier (2001) p.127. ($\pi_{sm} + \pi_{sm}^1 \to 1$). In that case we have homogeneity

$$\theta = 1 \Rightarrow \eta_{qm} + \eta_{qm}^1 + \eta_{qm}^2 = 0.$$

Using our guess

$$\begin{aligned} q_t &= \pi_{qq} q_{t-1} + \pi_{qm} (m_t - m_t^*) \\ &\quad + \pi_{qm}^1 \left(m_{t-1} - m_{t-1}^*\right) + \pi_{qm}^2 E_{t-1} (m_t - m_t^*), \end{aligned}$$

to find $E_{t-2} q_{t-1}$, $E_{t-2} q_t$, $E_{t-1} q_t$, and $E_{t-1} q_{t+1}$ (leaving out consumption terms and using the useful formula for finding the expectations) we find

$$\begin{aligned} E_{t-2} q_{t-1} &= \pi_{qq} q_{t-2} + \pi_{qm} E_{t-2} \left(m_{t-1} - m_{t-1}^*\right) \\ &\quad + \pi_{qm}^1 \left(m_{t-2} - m_{t-2}^*\right) + \pi_{qm}^2 E_{t-2} \left(m_{t-1} - m_{t-1}^*\right) \\ &= q_{t-1} - \pi_{qm} \left(m_{t-1} - m_{t-1}^*\right) + \pi_{qm} E_{t-2} \left(m_{t-1} - m_{t-1}^*\right) \\ &= q_{t-1} - \pi_{qm} \left(m_{t-1} - m_{t-1}^*\right) \\ &\quad + \pi_{qm} \left[-\frac{h}{(1-h)} \left(m_{t-1} - m_{t-1}^*\right) + \frac{1}{(1-h)\theta} E_{t-1} (m_t - m_t^*) \right] \\ &= q_{t-1} - \frac{\pi_{qm}}{1-h} \left(m_{t-1} - m_{t-1}^*\right) + \frac{\pi_{qm}}{(1-h)\theta} E_{t-1} (m_t - m_t^*), \end{aligned}$$

$$\begin{aligned} E_{t-2} q_t &= \pi_{qq} E_{t-2} q_{t-1} + \pi_{qm} E_{t-2} (m_t - m_t^*) \\ &\quad + \pi_{qm}^1 E_{t-2} \left(m_{t-1} - m_{t-1}^*\right) + \pi_{qm}^2 E_{t-2} E_{t-1} (m_t - m_t^*) \\ &= \pi_{qq} E_{t-2} q_{t-1} + \left(\theta \pi_{qm} + \pi_{qm}^1 + \theta \pi_{qm}^2\right) E_{t-2} \left(m_{t-1} - m_{t-1}^*\right) \\ &= \pi_{qq} q_{t-1} - \pi_{qq} \frac{\pi_{qm}}{1-h} \left(m_{t-1} - m_{t-1}^*\right) + \pi_{qq} \frac{\pi_{qm}}{(1-h)\theta} E_{t-1} (m_t - m_t^*) \\ &\quad - \frac{h}{1-h} \left(\theta \pi_{qm} + \pi_{qm}^1 + \theta \pi_{qm}^2\right) \left(m_{t-1} - m_{t-1}^*\right) \\ &\quad + \left(\theta \pi_{qm} + \pi_{qm}^1 + \theta \pi_{qm}^2\right) \frac{1}{(1-h)\theta} E_{t-1} (m_t - m_t^*), \end{aligned}$$

$$E_{t-1} q_t = \pi_{qq} q_{t-1} + \pi_{qm}^1 \left(m_{t-1} - m_{t-1}^*\right) + \left(\pi_{qm} + \pi_{qm}^2\right) E_{t-1} (m_t - m_t^*),$$

$$\begin{aligned} E_{t-1} q_{t+1} &= \left(\theta \pi_{qm} + \pi_{qm}^1 + \theta \pi_{qm}^2\right) E_{t-1} (m_t - m_t^*) \\ &\quad + \pi_{qq} \left[\pi_{qq} q_{t-1} + \pi_{qm}^1 \left(m_{t-1} - m_{t-1}^*\right) + \left(\pi_{qm} + \pi_{qm}^2\right) E_{t-1} (m_t - m_t^*)\right] \end{aligned}$$

we end up with the following restrictions (See Andersen and Beier (2003) for analysis on π_{qq})

$$\pi_{qq} = \eta_{qq} \left[1 + (1+\delta) \pi_{qq} + \delta \pi_{qq}^2\right]$$

$$\pi_{qm} = \eta_{qm}$$

$$\begin{aligned}
\pi_{qm}^{1} &= \eta_{qm}^{1} \\
&+\eta_{qq}\left[-\frac{\pi_{qm}}{1-h}\right] \\
&+\eta_{qq}\delta\left[-\pi_{qq}\frac{\pi_{qm}}{1-h}-\left(\theta\pi_{qm}+\pi_{qm}^{1}+\theta\pi_{qm}^{2}\right)\frac{h}{1-h}\right] \\
&+\eta_{qq}\pi_{qm}^{1} \\
&+\eta_{qq}\delta\pi_{qm}^{1}\pi_{qq}
\end{aligned}$$

$$\begin{aligned}
\pi_{qm}^{2} &= \eta_{qm}^{2} \\
&+\eta_{qq}\left[\frac{\pi_{qm}}{(1-h)\,\theta}\right] \\
&+\delta\eta_{qq}\left[\pi_{qq}\frac{\pi_{qm}}{(1-h)\,\theta}+\left(\theta\pi_{qm}+\pi_{qm}^{1}+\theta\pi_{qm}^{2}\right)\frac{1}{(1-h)\,\theta}\right] \\
&+\eta_{qq}\left[\pi_{qm}+\pi_{qm}^{2}\right] \\
&+\delta\eta_{qq}\left[\left(\theta\pi_{qm}+\pi_{qm}^{1}+\theta\pi_{qm}^{2}\right)+\pi_{qq}\left(\pi_{qm}+\pi_{qm}^{2}\right)\right]
\end{aligned}$$

I Equilibrium with perfect information and staggering

I.1 Equilibrium

In general we conjecture a solution for the nominal exchange rate and the terms of trade as

$$s_t = \pi_{sm}\left(m_t - m_t^*\right)$$

$$\begin{aligned}
q_t &= \pi_{qq}q_{t-1} + \pi_{qu}u_t + \pi_{qu}^{1}u_{t-1} + \pi_{qm}\left(m_t - m_t^*\right) \\
&+\pi_{qm}^{1}\left(m_{t-1} - m_{t-1}^*\right)
\end{aligned}$$

I.1.1 The nominal exchange rate

The nominal exchange rate is unchanged by the introduction of staggering. Thus we get

$$\pi_{sm} = \frac{1}{1+\eta_{mp}-\theta\eta_{mp}}$$

I.1.2 The terms of trade

From product market equilibria we get

$$-q_t = -\eta_{yw}\left[\left(\overline{w}_t - \overline{w}_t^*\right) - q_t - s_t\right] \Rightarrow q_t = \frac{\eta_{yw}}{\eta_{yw}+1}\left[\left(\overline{w}_t - \overline{w}_t^*\right) - s_t\right]$$

Thus we can write the terms of trade as

$$\begin{aligned} q_t \;=\;& +\eta'_{qq}\left(E_{t-2}q_{t-1} + \delta E_{t-2}q_t + E_{t-1}q_t + \delta E_{t-1}q_{t+1}\right) \\ & +\eta'_{qs}\left[\left(E_{t-2}s_{t-1} - s_t\right) + \delta\left(E_{t-2}s_t - s_t\right) + \left(E_{t-1}s_t - s_t\right) + \delta\left(E_{t-1}s_{t+1} - s_t\right)\right] \end{aligned}$$

$$\eta'_{qq} = \frac{\eta_{yw}}{\eta_{yw}+1}\frac{2\eta_{wp}-1}{2(1+\delta)},\ \eta'_{qs} = \frac{\eta_{yw}}{\eta_{yw}+1}\frac{1}{2(1+\delta)}$$

Next we have that

$$\begin{aligned} E_{t-2}s_{t-1} &= \pi_{sm}E_{t-2}\left(m_{t-1} - m_{t-1}^*\right) = \pi_{sm}\theta\left(m_{t-2} - m_{t-2}^*\right) \\ E_{t-2}s_t &= \pi_{sm}E_{t-2}\left(m_t - m_t^*\right) = \pi_{sm}\theta^2\left(m_{t-2} - m_{t-2}^*\right) \\ E_{t-1}s_t &= \pi_{sm}E_{t-1}\left(m_t - m_t^*\right) = \pi_{sm}\theta\left(m_{t-1} - m_{t-1}^*\right) \\ E_{t-1}s_{t+1} &= \pi_{sm}E_{t-1}\left(m_{t+1} - m_{t+1}^*\right) = \pi_{sm}\theta^2\left(m_{t-1} - m_{t-1}^*\right) \end{aligned}$$

and

$$\begin{aligned} s_t - E_{t-2}s_{t-1} &= \pi_{sm}\left(m_t - m_t^*\right) - \pi_{sm}E_{t-2}\left(m_{t-1} - m_{t-1}^*\right) \\ &= \pi_{sm}\left(m_t - m_t^*\right) - \pi_{sm}\theta\left(m_{t-2} - m_{t-2}^*\right) \\ &= \pi_{sm}\left(m_t - m_t^*\right) - \pi_{sm}\left(m_{t-1} - m_{t-1}^*\right) + \pi_{sm}u_{t-1} \\ &= \pi_{sm}\left(m_t - m_t^*\right) - \pi_{sm}\left(m_{t-1} - m_{t-1}^*\right) + \pi_{sm}u_{t-1} \end{aligned}$$

$$\begin{aligned} s_t - E_{t-2}s_t &= \pi_{sm}\left(m_t - m_t^*\right) - \pi_{sm}E_{t-2}\left(m_t - m_t^*\right) \\ &= \pi_{sm}\left(m_t - m_t^*\right) - \pi_{sm}\theta\theta\left(m_{t-2} - m_{t-2}^*\right) \\ &= \pi_{sm}\left(m_t - m_t^*\right) - \pi_{sm}\theta\left(m_{t-1} - m_{t-1}^*\right) + \pi_{sm}\theta u_{t-1} \end{aligned}$$

$$\begin{aligned} s_t - E_{t-1}s_t &= \pi_{sm}\left(m_t - m_t^*\right) - \pi_{sm}E_{t-1}\left(m_t - m_t^*\right) \\ &= \pi_{sm}\left(m_t - m_t^*\right) - \pi_{sm}\theta\left(m_{t-1} - m_{t-1}^*\right) \\ &= \pi_{sm}u_t \end{aligned}$$

$$\begin{aligned} s_t - E_{t-1}s_{t+1} &= \pi_{sm}\left(m_t - m_t^*\right) - \pi_{sm}E_{t-1}\left(m_{t+1} - m_{t+1}^*\right) \\ &= \pi_{sm}\left(m_t - m_t^*\right) - \pi_{sm}\theta\left[\theta\left(m_{t-1} - m_{t-1}^*\right)\right] \\ &= \pi_{sm}\left(m_t - m_t^*\right) - \pi_{sm}\theta\left[\left(m_t - m_t^*\right) - u_t\right] \\ &= \pi_{sm}(1-\theta)\left(m_t - m_t^*\right) + \pi_{sm}\theta u_t \end{aligned}$$

Inserting yields

$$
\begin{aligned}
q_t &= \eta_{qq}\left(E_{t-2}q_{t-1} + \delta E_{t-2}q_t + E_{t-1}q_t + \delta E_{t-1}q_{t+1}\right) \\
&\quad + \eta_{qm}\left(m_t - m_t^*\right) + \eta_{qm}^1\left(m_{t-1} - m_{t-1}^*\right) + \eta_{qu}u_t + \eta_{qu}^1 u_t
\end{aligned}
$$

where

$$
\eta_{qq} = \frac{\eta_{yw}}{\eta_{yw}+1}\frac{1}{2(1+\delta)}\left(2\eta_{wp}-1\right),
$$

$$
\eta_{qm} = -\frac{\eta_{yw}}{\eta_{yw}+1}\frac{1}{2(1+\delta)}\left[1+\delta+\delta(1-\theta)\right]\pi_{sm}
$$

$$
\eta_{qm}^1 = -\frac{\eta_{yw}}{\eta_{yw}+1}\frac{1}{2(1+\delta)}\left(-1-\theta\delta\right)\pi_{sm},
$$

$$
\eta_{qu} = -\frac{\eta_{yw}}{\eta_{yw}+1}\frac{1}{2(1+\delta)}\left(1+\theta\delta\right)\pi_{sm},
$$

$$
\eta_{qu}^1 = -\frac{\eta_{yw}}{\eta_{yw}+1}\frac{1}{2(1+\delta)}\left(1+\theta\delta\right)\pi_{sm},
$$

Our guess is

$$
\begin{aligned}
q_t &= \pi_{qq}q_{t-1} + \pi_{qu}u_t + \pi_{qu}^1 u_{t-1} + \pi_{qm}\left(m_t - m_t^*\right) \\
&\quad + \pi_{qm}^1\left(m_{t-1} - m_{t-1}^*\right)
\end{aligned}
$$

lagging it once

$$
\begin{aligned}
q_{t-1} &= \pi_{qq}q_{t-2} + \pi_{qu}u_{t-1} + \pi_{qu}^1 u_{t-2} + \pi_{qm}\left(m_{t-1} - m_{t-1}^*\right) \\
&\quad + \pi_{qm}^1\left(m_{t-2} - m_{t-2}^*\right)
\end{aligned}
$$

thus

$$
\begin{aligned}
E_{t-2}q_{t-1} &= \pi_{qq}q_{t-2} + \pi_{qu}^1 u_{t-2} + \pi_{qm}\theta\left(m_{t-2} - m_{t-2}^*\right) + \pi_{qm}^1\left(m_{t-2} - m_{t-2}^*\right) \\
&= q_{t-1} - \pi_{qu}u_{t-1} - \pi_{qm}\left(m_{t-1} - m_{t-1}^*\right) \\
&\quad + \pi_{qm}\theta\left(m_{t-2} - m_{t-2}^*\right) \\
&= q_{t-1} - \pi_{qu}u_{t-1} - \pi_{qm}u_{t-1}
\end{aligned}
$$

$$
\begin{aligned}
E_{t-2}q_t &= \pi_{qq}E_{t-2}q_{t-1} + \pi_{qm}\theta\theta\left(m_{t-2} - m_{t-2}^*\right) + \pi_{qm}^1\theta\left(m_{t-2} - m_{t-2}^*\right) \\
&= \pi_{qq}\left\{q_{t-1} - \pi_{qu}u_{t-1} - \pi_{qm}u_{t-1}\right\} \\
&\quad + \left(\pi_{qm}\theta + \pi_{qm}^1\right)\left(m_{t-1} - m_{t-1}^*\right) - \left(\pi_{qm}\theta + \pi_{qm}^1\right)u_{t-1}
\end{aligned}
$$

$$
E_{t-1}q_t = \pi_{qq}q_{t-1} + \pi_{qu}^1 u_{t-1} + \pi_{qm}\theta\left(m_{t-1} - m_{t-1}^*\right) + \pi_{qm}^1\left(m_{t-1} - m_{t-1}^*\right)
$$

$$
\begin{aligned}
E_{t-1}q_{t+1} &= \pi_{qq}E_{t-1}q_t + \pi_{qm}E_{t-1}\left(m_{t+1} - m_{t+1}^*\right) + \pi_{qm}^1 E_{t-1}\left(m_t - m_t^*\right) \\
&= \pi_{qq}\left\{\pi_{qq}q_{t-1} + \pi_{qu}^1 u_{t-1} + \pi_{qm}\theta\left(m_{t-1} - m_{t-1}^*\right) + \pi_{qm}^1\left(m_{t-1} - m_{t-1}^*\right)\right\} \\
&\quad + \pi_{qm}\theta\theta\left(m_{t-1} - m_{t-1}^*\right) + \pi_{qm}^1\theta\left(m_{t-1} - m_{t-1}^*\right)
\end{aligned}
$$

Thus we get the following restrictions

$$\pi_{qq} = \eta_{qq}\left(1 + \delta\pi_{qq} + \pi_{qq} + \delta\pi_{qq}^2\right)$$

$$\pi_{qu} = \eta_{qu}$$

$$\pi_{qu}^1 = \eta_{qu}^1 + \eta_{qq}\left[-\pi_{qu} - \pi_{qm} - \delta\pi_{qq}\left(\pi_{qu} + \pi_{qm}\right) - \delta\left(\pi_{qm}\theta + \pi_{qm}^1\right) + \left(1 + \delta\pi_{qq}\right)\pi_{qu}^1\right]$$

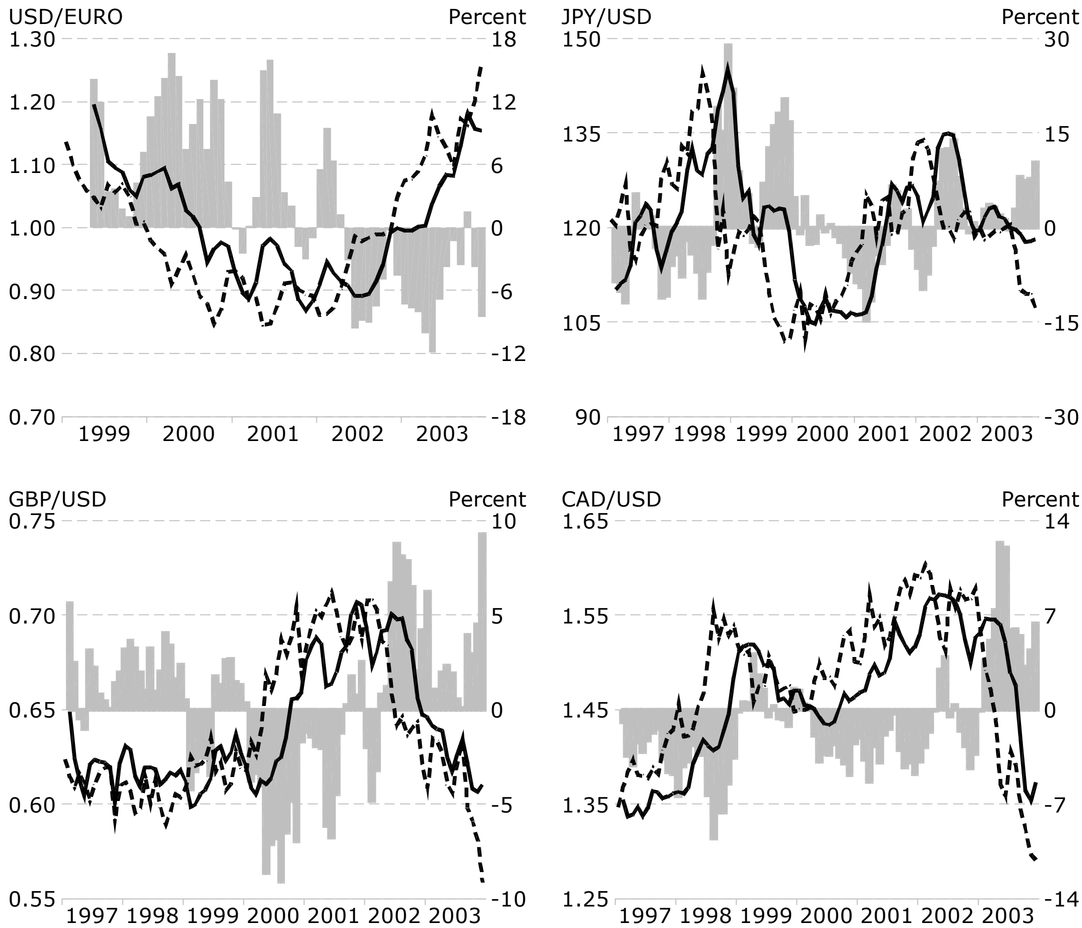
$$\begin{aligned} \pi_{qu}^1 &= \left[1 - \left(1 + \pi_{qq}\delta\right)\eta_{qq}\right]^{-1} \\ &\quad \left\{\eta_{qu}^1 + \eta_{qq}\left[-\pi_{qu} - \pi_{qm} - \delta\pi_{qq}\left(\pi_{qu} + \pi_{qm}\right) - \delta\left(\pi_{qm}\theta + \pi_{qm}^1\right)\right]\right\} \end{aligned}$$

$$\pi_{qm} = \eta_{qm}$$

$$\begin{aligned} \pi_{qm}^1 &= \eta_{qm}^1 + \eta_{qq}\left[\delta\left(\pi_{qm}\theta + \pi_{qm}^1\right) + \pi_{qm}\theta + \pi_{qm}^1\right] \\ &\quad + \eta_{qq}\delta\left[\pi_{qq}\left(\pi_{qm}\theta + \pi_{qm}^1\right) + \pi_{qm}\theta\theta + \theta\pi_{qm}^1\right] \end{aligned}$$

$$\begin{aligned} \pi_{qm}^1 &= \left[1 - \eta_{qq}\left(\delta + 1 + \delta\pi_{qq} + \delta\theta\right)\right]^{-1} \\ &\quad \left\{\eta_{qm}^1 + \theta\pi_{qm}\eta_{qq}\left[1 + \delta + \delta\left(\delta\pi_{qq} + \theta\right)\right]\right\} \end{aligned}$$

Figure 1: Expected (4-months horizon) and actual US-Dollar exchange rates



Note: The solid line (left-hand axis) is the US-dollar exchange rate against the euro, Japanese yen (JPY), British Pound (GBP), and Canadian dollar (CAD) end-of-month. The dashed line (left-hand axis) is the consensus forecast on a 4 month horizon. The bars (right-hand axis) are the forecast error in percentage terms

Sources: Actual exchange rates: EcoWin (code: 19005); Expected exchange rates: Consensus Economics Inc.

Figure 2: Updating of expectations

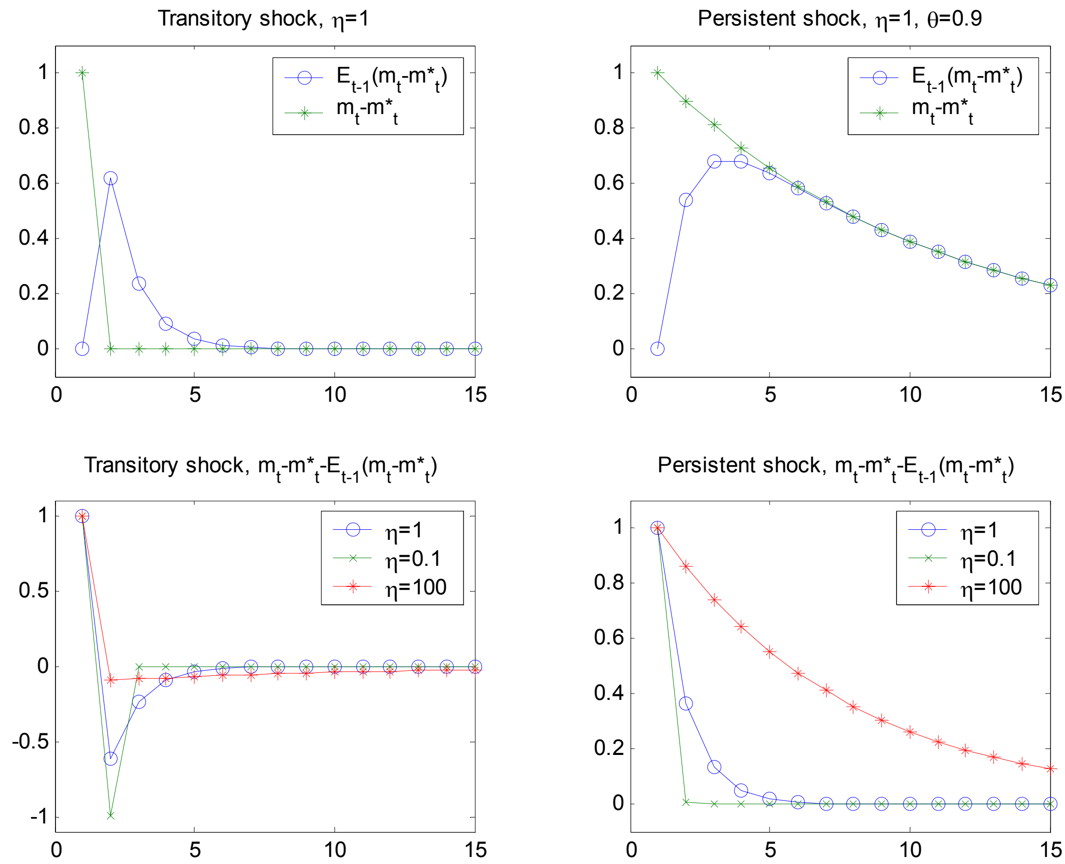
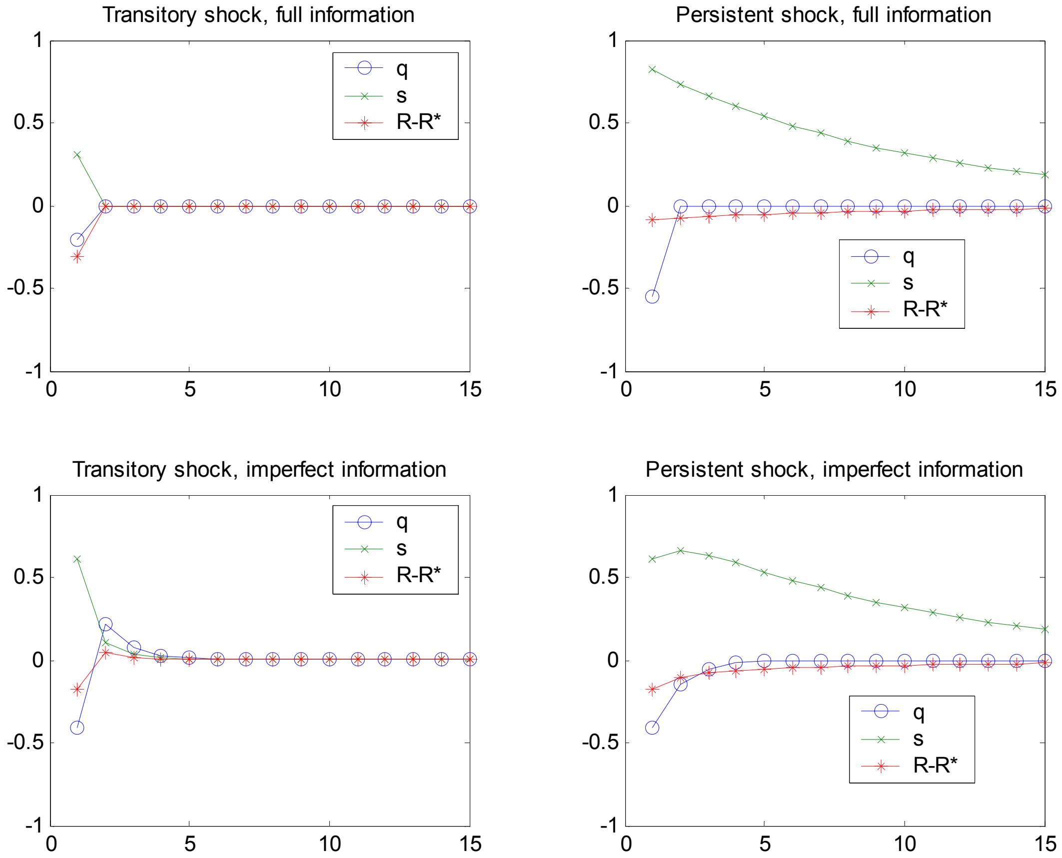
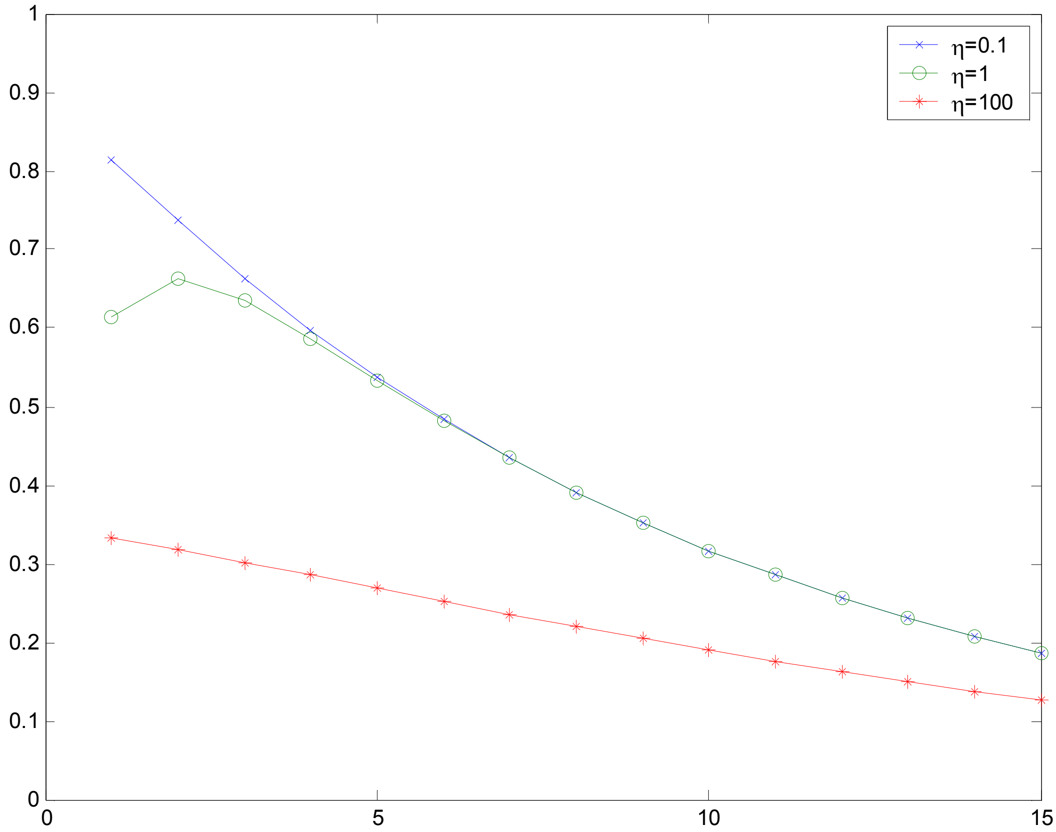


Figure 3: Impulse-response functions to a monetary shock



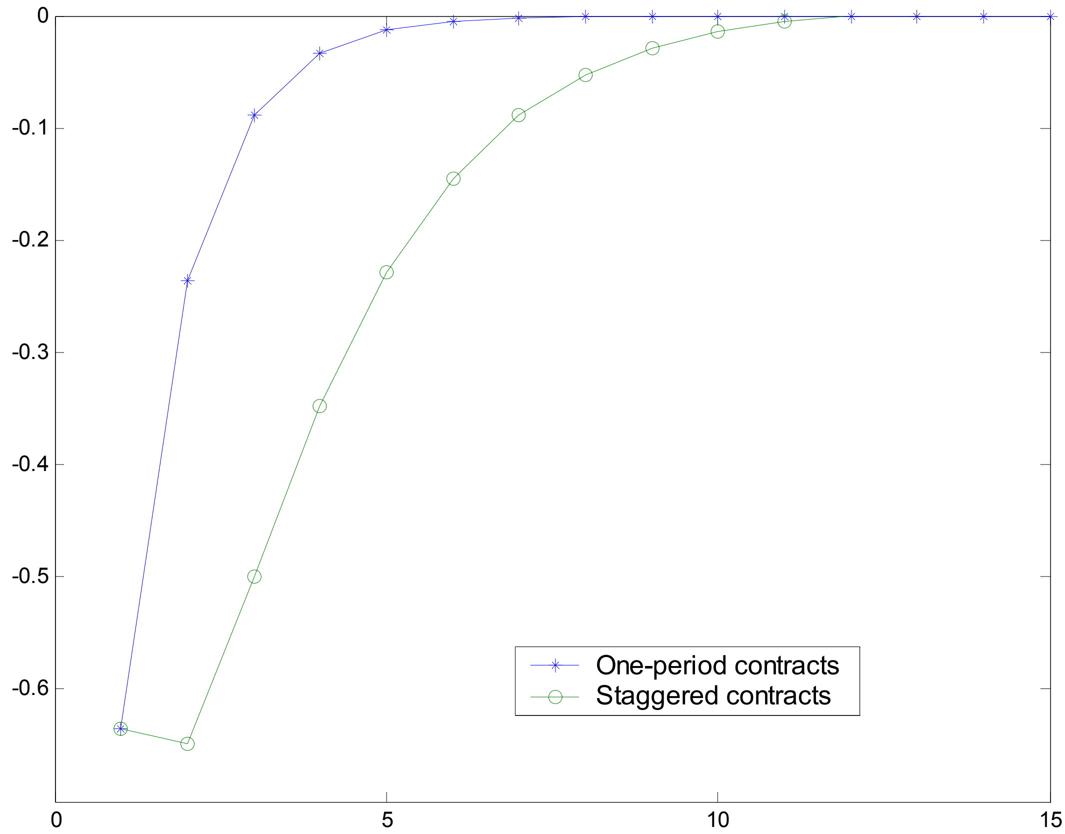
Note: The reaction of q, s, and R-R* to a one-unit increase in relative money in period 1. Drawn for θ=0.9 and η=1.

Figure 4: Exchange-rate response to a persistent shock



Note: The reaction of s to a one-unit increase in relative money in period 1. Drawn for θ=0.9.

Figure 5: Response to a persistent shock with imperfect information



Note: The reaction q to a one-unit increase in relative money in period 1. Drawn for θ=0.95, γ=0.95, and η=1.