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**Morten Lykke**

Financial Markets  
Danmarks Nationalbank, Copenhagen

and

**Kenneth Juhl Pedersen**

Financial Markets  
Danmarks Nationalbank, Copenhagen

and

**Heidi Mølgaard Vinther**

Financial Markets  
Danmarks Nationalbank, Copenhagen

**A Failure-Rate Model for the Danish  
Corporate Sector**

**5 May 2004**

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Please direct any enquiries to  
Danmarks Nationalbank, Information Desk, Havnegade 5, DK-1093 Copenhagen K  
Denmark  
Tel.: +45 33 63 70 00 (direct) or +45 33 63 63 63  
Fax : +45 33 63 71 03  
E-mail: [info@nationalbanken.dk](mailto:info@nationalbanken.dk)

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## **Abstract**

This paper presents an accounting-based model developed in Denmark's Nationalbank to predict failure rates in the Danish corporate sector. The model serves as a tool in analysing the Danish corporate sector in relation to financial stability. The main purpose is to assess the banks' credit risk on the corporate sector.

By using logistic regression the model is estimated on 300,000 accounts of the Danish corporate sector published in the period from 1995 to 1999. Nine indicators from the accounts, including more qualitative information about the company, are significant in predicting the failure rates of Danish companies. Besides the basic model, models at sector level and by number of employees in the company are estimated.

## **Resume**

Dette arbejdspapir præsenterer en regnskabsbaseret konkursmodel udviklet i Danmarks Nationalbank med henblik på at estimere konkurs sandsynligheder for danske virksomheder. Modellen anvendes i analysen af danske erhvervs virksomheder i relation til finansiel stabilitet. Formålet er at vurdere bankers kreditrisiko på erhvervs virksomheder.

På baggrund af 300.000 årsregnskaber fra danske aktie- og anpartselskaber aflagt i perioden 1995-1999 estimeres en logistisk regressionsmodel. Ni nøgletal fra regnskaberne, herunder flere kvalitative oplysninger, udgør de forklarende variabler, der definerer konkursmodellen. Foruden en overordnet model er konkursmodellen estimeret på brancheniveau og ud fra antal ansatte i virksomhederne.

## 1. Introduction

Corporate failures can lead to credit losses in the banking sector and under severe circumstances be a threat to financial stability. Analyses of financial stability include a description of financial health in the corporate sector. For that purpose a model has been developed in Danmarks Nationalbank to predict failure rates in the corporate sector based on published accounts. This paper presents the current state of this work<sup>1</sup>.

The model generates probabilities of failure for each individual company. The failure rates should be related to the actual debt position of the company to get a picture of the potential loss in the banking sector.

### 1.1 Pioneer work of failure-rate models

The early literature in the field of company failure-rate models comprises two different approaches<sup>2</sup>.

Beaver (1966) and Altman (1968) introduce the score-models based on discriminant analysis to distinguish between active and failed companies. Each company is assigned a score based on the linear discriminant function:

$$(1) \quad Z = \alpha_1 X_1 + \alpha_2 X_2 + \dots + \alpha_k X_k$$

The alphas are the discriminatory coefficients and the X's are the explanatory variables, which in this context are financial ratios from the published accounts.

The discriminatory coefficients are estimated on a sample of active and failed companies. The coefficients are estimated to provide the

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<sup>1</sup> Developing the failure-rate model is an on-going project in Danmarks Nationalbank. Consequently, the results presented in this paper may differ from the results presented in financial stability reports published by Danmarks Nationalbank. Similar work is carried out by central banks in UK, Belgium and Norway among others.

<sup>2</sup> Other approaches comprise non-parametric methods, cf. Frydman et al. (1985) and Back et al. (1996).

best discrimination between active and failed companies based on financial ratios in the sample, and each company is assigned a Z-value. The score-models entail an ordinal ranking of the companies, where a high Z-value indicates an active company, whereas a low Z-value indicates a failed company.

Altman (1968) examines published accounts from 66 companies and finds that financial ratios measuring the company's liquidity position, profitability and solvency predicts potential failure<sup>3</sup>. The explanatory variables enter the model with a negative effect on failure, i.e. an increase in liquidity position, profitability or solvency reduces the probability of failure.

Ohlson (1980) takes a different approach to model corporate failures using published accounts. Based on the maximum likelihood method the probability of failure for each company is estimated using logistic regression. According to this approach the probability of failure for a company is:

$$(2) P_i = \frac{1}{1 + \exp[-(\alpha_0 + \alpha_1 X_{1i} + \alpha_2 X_{2i} + \dots + \alpha_k X_{ki})]}$$

where P is the probability of failure. The  $\alpha$ -vector is the coefficients specifying the model and the X's are the explanatory variables, here financial ratios from companies' accounts.

Ohlson (1980) uses 2,163 accounts to specify the model. Ohlson (1980) finds a negative correlation between the probability of failure and the size, profitability and liquidity of the company. On the other hand the probability of failure is positively correlated with the company's gearing.

Platt & Platt (1991) finds that the probability of failure depends on which sector the company is operating in. Adjusting financial ratios to the sector average is found to be superior to a model without sector

<sup>3</sup> The model is refined further in Altman, Haldeman & Narayanan (1977) and Altman (2000).

adjustment. Similar conclusions are found in Sjøvoll (1999), Bernhardsen (2001) and Pedersen (2002).

Keasey & Watson (1987) find non-financial information from accounts to be significant in explaining the probability of failure. Among other factors their analyses show that a critical comment from the auditor and a delayed account significantly increases the probability of failure.

## 2. Econometric method and data

### 2.1 Econometric method

Danmarks Nationalbank's accounting-based model of failure rates<sup>4</sup> presented in *Financial stability 2003* is estimated using maximum likelihood method within the framework of logistic regression<sup>5</sup>.

The response variable is binary. If a company fails the last account of that company will have the response  $y_{i,t} = 1$ . As long as the company is active the response is  $y_{i,t} = 0$ . It is assumed that the values of the response, i.e. the failure rates, are observations of independent stochastic variables. The failure rate of company  $i$  is by definition continuous and written as:

$$(3) y_{i,t}^* = \beta' X_{i,t} + \varepsilon_{i,t}$$

where  $\varepsilon_{i,t}$  is an error term with  $E(\varepsilon_{i,t}) = 0$ .  $X_{i,t}$  is a vector of explanatory variables and  $\beta$  a vector of coefficients.

The actual failure rate  $y_{i,t}^*$  cannot be observed. At a specific point in time, it is observed whether the company is active or has failed. This discrete observation can be written as:

$$(4) y_{i,t} = \begin{cases} 1 & \text{if } y_{i,t}^* > 0 \\ 0 & \text{otherwise} \end{cases}$$

<sup>4</sup> The work is based on Pedersen (2002).

<sup>5</sup> cf. Johnston & DiNardo (1997) and Allison (1999)

Assuming a symmetric distribution the probability that a company has failed can be written as:

$$(5) \quad E(y_{i,t}) = \Pr(y_{i,t} = 0) \times 0 + \Pr(y_{i,t} = 1) \times 1 = \Pr(y_{i,t}^* > 0) = \\ \Pr(\beta' X_{i,t} + \varepsilon_{i,t} > 0) = \Pr(\varepsilon_{i,t} < \beta' X_{i,t}) = F(\beta' X_{i,t})$$

F is the cumulative distribution function of the error term,  $\varepsilon_{i,t}$ . Using the logit function, the expected probability that a company will fail,  $P_i$ , can be written as (leaving out the time indices):

$$(6) \quad F(X_i' \beta) = \frac{\exp(\beta_0 + \beta_1 x_{1i} + \beta_2 x_{2i} + \dots + \beta_k x_{ki})}{1 + \exp(\beta_0 + \beta_1 x_{1i} + \beta_2 x_{2i} + \dots + \beta_k x_{ki})} \\ = \frac{1}{1 + \exp[-(\beta_0 + \beta_1 x_{1i} + \beta_2 x_{2i} + \dots + \beta_k x_{ki})]} = P_i$$

with k explanatory variables. (6) can be rewritten as the odds ratio:

$$(7) \quad \log\left(\frac{P_i}{1 - P_i}\right) = \beta_0 + \beta_1 x_{1i} + \beta_2 x_{2i} + \dots + \beta_k x_{ki}$$

The model is non-linear so a marginal change in one variable depends on the level of the particular variable *ceteris paribus*. Thus, a change in a company's earnings will affect the failure rate differently depending on the initial earnings capacity. However, the compensation ratio between the explanatory variables is constant, cf. Bernhardsen (2001).

## 2.2 Data

In earlier studies such as Altman (1968), Altman et al. (1977), Dambolena and Khoury (1980), Hennawy and Morris (1983), Betts and Belhoul (1987), Platt and Platt (1991) failure-rate models are estimated on data not randomly selected and often consisting of less than 100 accounts. The companies in these studies are often "paired", so the active and failed companies have some of the same

characteristics (usually of the same size and from the same sector). This way of selecting data might cause selection bias.

As stated in Shumway (2001), most of the existing literature on estimating failure-rate models is based on a single account from each company. By choosing only one account per company the models ignore the fact that failure rates change over time. Why for instance do some companies stay active for a long period of time and then suddenly fail, whereas other companies manage to stay close to failure for a long period of time without failing? In order to incorporate these effects, a model using more than one account from each company is needed. Danmarks Nationalbank's model uses all accounts reported by each company. Following Shumway (2001), all the accounts reported up to a failure are defined as accounts from an active company. Only the last account of a failed company is characterized as a failure account.

Using all available accounts from the companies strengthen the estimation results. However, the technique applied assumes that the observations are independent which might be a strong assumption when considering accounts over time from the same company.

Danmarks Nationalbank's failure-rate model is estimated using accounting data on the Danish corporate sector from KOB A/S. KOB A/S collects annual accounts as they are registered with the Danish Commerce and Companies Agency, as well as public announcements of involuntary bankruptcy published in Statstidende (The Danish Official Gazette).

The database contains accounting data for Danish public limited liability companies (A/S) and private limited liability companies (ApS)<sup>6</sup>.

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<sup>6</sup> Instead of using published accounts the failure rate of a single company could be estimated on the basis of the financial markets valuation of the company, e.g. using the Merton approach, cf. Merton (1973,1974). As only a few hundred Danish companies are listed on the stock exchange, no distinction is made between listed and non-listed companies.

The estimation sample covers the period 1995-99 and contains approximately 300,000 annual accounts of which almost 8,000 from failed companies. Table 1 shows the number of active and failed companies from 1995 to 2002.

A company is categorized as failed if one of the following events has occurred:

- ◆ The company is being liquidated or is subject to compulsory liquidation
- ◆ The company has been dissolved, dissolved by the courts, or is subject to compulsory dissolution by the court
- ◆ The company is subject to a compulsory deed of arrangement with creditors or is subject to a compulsory scheme of arrangement with creditors.

It should be noted that this definition is broader than the juridical bankruptcy definition in order to capture a more precise picture of potential loss in financial institutions.

ACTIVE AND FAILED COMPANIES, 1995-2002								Table 1
	1995	1996	1997	1998	1999	2000	2001	2002
Active .....	53,963	54,758	55,889	57,899	60,671	63,668	67,237	71,632
Failed .....	1,633	1,472	1,350	1,555	1,858	2,272	2,505	1,125

Note: The number of failed companies in 2001 and 2002 can be subject to an increase due to the registration lag.  
Source: KOB A/S and own calculations.

The estimation period ends in 1999 due to a time lag between publication of the accounts and the official registration of failure. On average it takes 19 months from the accounting year of the last account until a failure is announced, cf. Table 2. Extending the estimation period to 2002 would imply an increased risk of accounts having a wrong response variable, as an apparently active company could in fact have failed. In addition, due to the time lag it is difficult to specify the exact timing of a failure.

In light of this the failure rate of a company in the model is interpreted as the probability of failure within the next few years.

TIME LAG FROM END OF LAST ACCOUNTING YEAR TO REGISTRATION OF FAILURE,  
NUMBER OF MONTHS

Table 2

	Months
10th percentile .....	8
50th percentile .....	18
90th percentile .....	34
Mean .....	19

Source: KOB A/S.

The following accounts have been excluded from the database:

- ◆ Holding companies
- ◆ Consolidated accounts
- ◆ Companies with total assets of less than DKK 50,000.

In addition, a few extreme observations have been identified in the data set. These outlier observations are not removed from the data set, but extreme values of financial ratios are truncated by the 1st and the 99th percentile of the variable in question. These adjustments have increased the performance of the model.

### 3. Danmarks Nationalbank's accounting-based failure-rate model

#### 3.1 The basic model

The estimated accounting-based failure-rate model contains a number of key financial ratios that are commonly used in strategic accounting analysis such as the company's liquidity position, profitability and solvency. Other variables are age and size of the company, form of ownership and critical auditor comments. The model has nine explanatory variables, i.e. five quantitative and four dummy variables ('D' is an abbreviation for a dummy variable). The sign in parenthesis indicates the expected effect on the failure rate of a change in the variable.

- ◆ *Capital base reduction*<sup>D</sup> (+). The dummy is equal to 1 if the company repeats the deficit for the year causing the company's capital base to fall below the initial level of capital required to set up a company. Otherwise the dummy is 0.
- ◆ *Debt ratio* (+). Short-term debt as a ratio of total assets. A company is more dependent on current earnings capacity if assets are financed by short-term debt.
- ◆ *Remark*<sup>D</sup> (+) is set to 1 if there is a critical auditor comment in the account. Otherwise the dummy is 0.
- ◆ *Size* (-) of a company is measured by total assets (in logarithms). A larger company is expected to have a lower failure rate.
- ◆ *Solvency* (-) is measured as equity as a ratio of total assets.
- ◆ *New company*<sup>D</sup> (+) measures the company's age. The dummy variable is equal to 0 if the company has been in business for more than five years, otherwise 1.
- ◆ *Corporate form*<sup>D</sup> (+). The dummy variable is 1 if the company is a private limited liability company (ApS) and equal to 0 if the company is a public limited liability company (A/S).
- ◆ *Adj. ROA* (-) measures the company's return on assets adjusted for the median return on assets in the sector where the company is operating.
- ◆ *Reduced liquidity* (+)<sup>7</sup> measures the short-term debt as a ratio of primary operating result to indicate the liquidity position of the company.

All variables in the estimated model are significant at a 5 per cent level and the coefficient estimates have the expected signs, cf. Table 3.

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<sup>7</sup> Caution must be made when the primary operating result is negative. Work is in progress to find an alternative measure of liquidity.

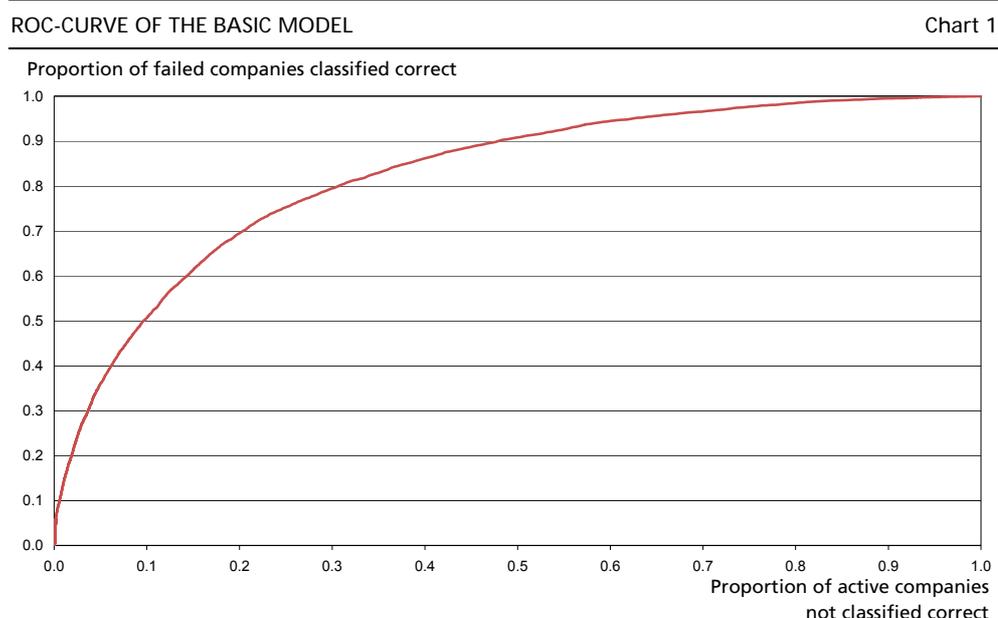
COEFFICIENT ESTIMATES AND STANDARD ERRORS IN THE BASIC MODEL		Table 3
	Coefficient	Odds ratio
Number of active companies .....	281,673	
Number of failed companies .....	7,787	
Intercept .....	-2.5464 (0.1018)	
Adj. ROA .....	-0.0036 (0.0004)	0.996
Solvency .....	-0.0062 (0.0004)	0.994
Size .....	-0.2794 (0.0102)	0.756
Reduced liquidity .....	0.0025 (0.0003)	1.003
New company .....	0.3502 (0.0256)	1.419
Remark .....	1.0987 (0.0286)	3.000
Corporate form .....	0.4557 (0.0343)	1.577
Capital base reduction .....	0.9438 (0.0297)	2.570
Debt ratio.....	0.2643 (0.0448)	1.302

Note: Standard errors are shown in brackets.  
Source: Own calculations.

The odds ratio measures the probability of a failure relative to a non-failure. The odds ratio of the dummy variable "remark" in the basic model is 3, and indicates that the probability of failure is three times higher when a company has a critical remark from the auditor compared to when the company does not have a critical remark *ceteris paribus*. Subtracting 1 from the odds ratio of a quantitative variable leads to the percentage change in the odds of failure for each unit increase in the quantitative variable *ceteris paribus*.

The model's ability to classify companies correct, i.e. as active or failed is evaluated using the c-value derived from the ROC-curve. The ROC-curve depicts the relation between the proportion of correctly classified failures (on the Y axis) and the proportion of active companies that are not correctly classified (on the X axis), i.e. a trade-off between classifying the failures correctly and at the same time misclassifying some active companies. The c-value can be seen as a continuum of threshold values of the failure rate. The curvature

of the ROC-curve indicates the classification ability of the model. The more the curve bends towards the 'north-west' the better is the classification ability of the model. The c-value measures the area under the ROC-curve and should be as close to 1 as possible. The c-value of the estimated failure-rate model is 0.825 cf. Chart 1.



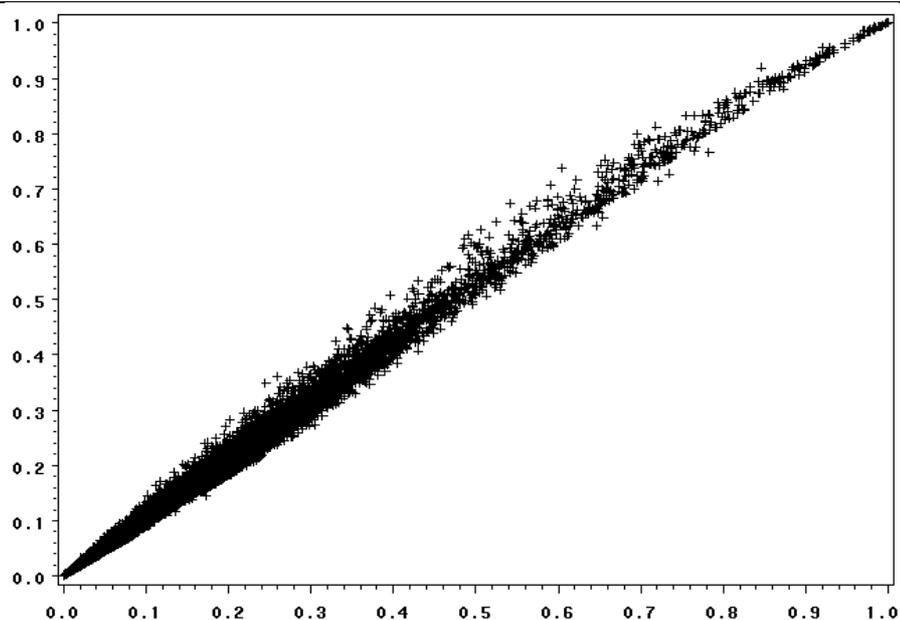
Source: Own calculations.

The model correctly classifies active and failed companies in approximately 80 per cent of the cases using a failure-rate threshold value of 2.5 per cent. Thus, a company is considered failed if the estimated failure rate is greater than 2.5 per cent. Testing the model out of sample and considering those companies that have failed since 2000, a total of almost 6,000 failures have been registered. The model predicts 67 per cent of the failures out of sample.

The model is robust to different estimation periods. The robustness is indicated in Chart 2, as the probabilities from two different estimation periods follow approximately a straight line.

THE MODEL ESTIMATED ON DIFFERENT TIME PERIODS

Chart 2



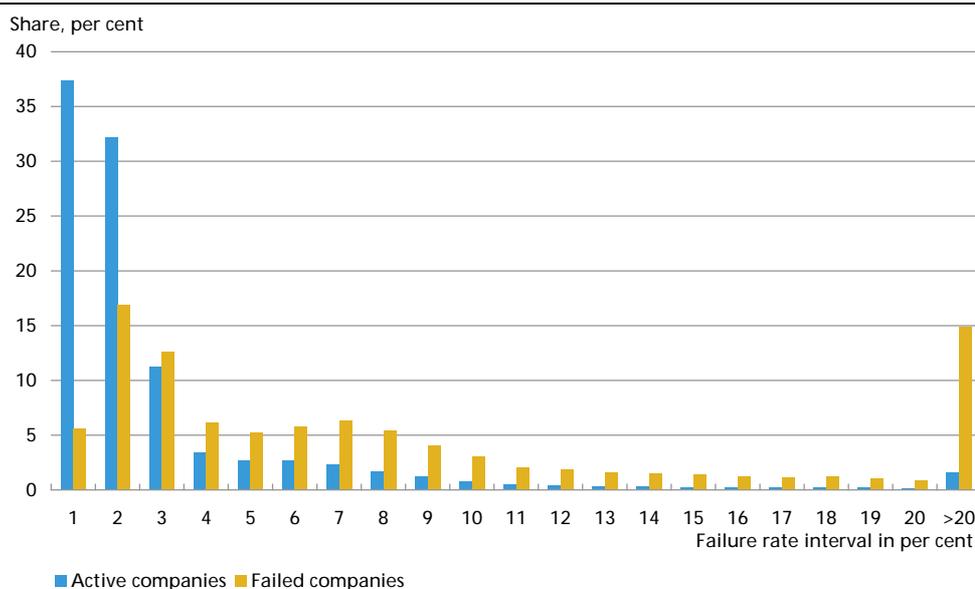
Note: X-Y plot of failure rates estimated on observations from 1995-1997 (X-axis) and a model estimated on observations from 1995-1999 (Y-axis).

Source: Own calculations.

50 per cent of the active companies have a failure rate lower than 1.33 per cent whereas 50 per cent of the failed companies have a failure rate lower than 5.59 per cent, cf. Chart 3.

DISTRIBUTION OF FAILURE RATES

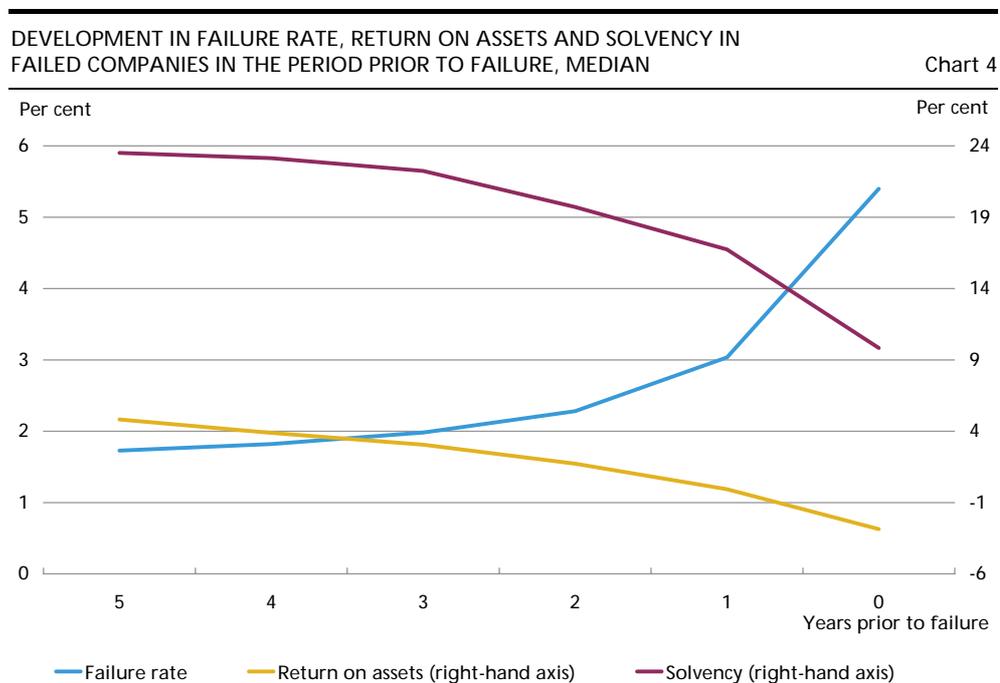
Chart 3



Note: Companies in the interval "1" have a failure rate greater than 0 and lower than or equal to 1 per cent etc. ">20" includes all the companies with a failure rate greater than 20 per cent.

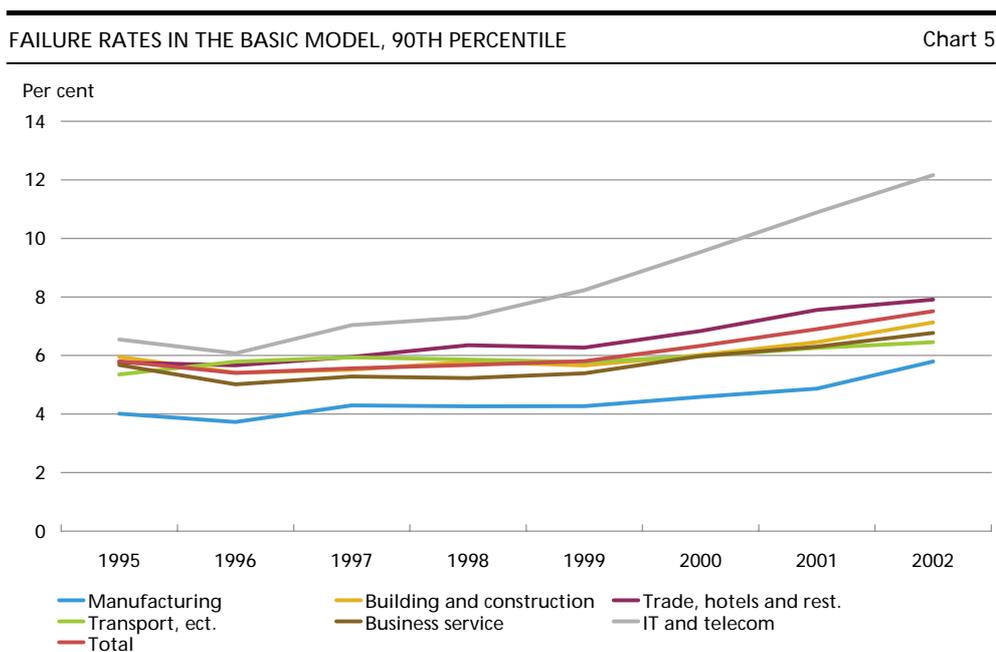
Source: Own calculations.

It is a characteristic feature that the failure rate only increases considerably in the last years prior to failure. The variables return on assets and solvency follow the same pattern, cf. Chart 4.



Note: Return on assets is defined as the primary operating result as a ratio of total assets. Solvency is defined as equity as a ratio of total assets.  
Source: Own calculation.

The failure-rate model enables analyses of the failure rate over time. Based on the basic model the failure rate has in general increased in the Danish corporate sector since 1999, cf. Chart 5.



### 3.2 Sector models

In order to take account of structural differences, by analysing whether the explanatory variables have different weight across sectors, a failure-rate model is estimated for each sector.

The sector models<sup>8</sup> are estimated using equation (2). The coefficients and standard errors are presented in Table 4 and the odds ratios in Annex 2. The signs of the coefficients in the sector models are as expected, cf. section 3.1. Comparing the basic model described in section 3.1 and the sector models reveals some differences. Overall, the coefficient estimates and the significance of the variables in the sector models deviate from the basic model, cf. Table 4.

<sup>8</sup> See Annex 1 for a definition of sectors.

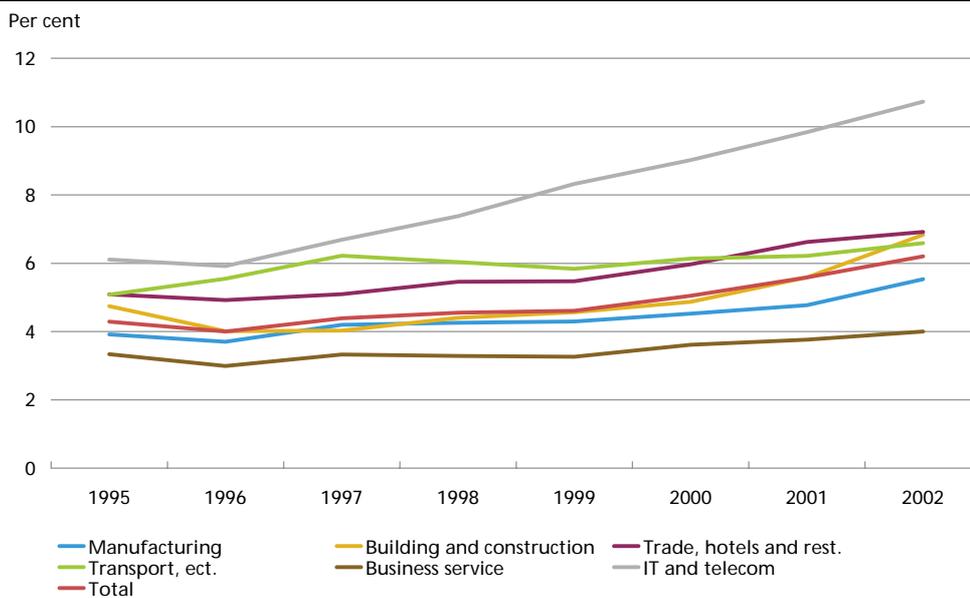
COEFFICIENTS ESTIMATES AND STANDARD ERRORS IN SECTOR MODELS						Table 4
	Manufacturing	Building and construction	Trade, hotels and rest.	Transport, etc.	Business service, etc.	IT and telecom
Number of active companies .....	46,746	35,030	83,028	14,224	76,188	14,075
Number of failed companies .....	979	880	2,081	440	1,289	498
Intercept .....	-3.6343 (0.3020)	-2.4567 (0.3790)	-2.6623 (0.2059)	-3.3205 (0.4503)	-2.6235 (0.2410)	-3.9035 (0.3962)
Adj. ROA.....	-0.0094 (0.1385)	-0.0084 (0.0017)	-0.0063 (0.0009)	-0.0076 (0.0019)	-0.0074 (0.0010)	<b>-0.0013</b> <b>(0.0011)</b>
Solvency.....	-0.0094 (0.1341)	-0.0140 (0.0019)	-0.0080 (0.0008)	-0.0056 (0.0019)	-0.0062 (0.0009)	<b>-0.0033</b> <b>(0.0018)</b>
Size .....	-0.1398 (0.0287)	-0.3095 (0.0368)	-0.2733 (0.0205)	-0.1800 (0.0429)	-0.3188 (0.0247)	-0.1043 (0.0385)
Reduced liquidity .....	0.0083 (0.0011)	0.0035 (0.0012)	<b>0.0015</b> <b>(0.0007)</b>	0.0052 (0.0009)	<b>-0.0001</b> <b>(0.0009)</b>	0.0039 (0.0014)
New company .....	0.2915 (0.0767)	0.8357 (0.0745)	0.3607 (0.0497)	0.4579 (0.1073)	<b>0.1067</b> <b>(0.0637)</b>	0.5049 (0.0974)
Remark .....	1.2476 (0.0797)	1.0557 (0.0826)	1.0927 (0.0540)	1.2072 (0.1174)	1.1356 (0.0705)	1.1501 (0.1117)
Corporate form.....	0.5053 (0.0877)	0.3695 (0.1144)	0.5610 (0.0699)	0.6342 (0.1446)	0.4934 (0.0865)	0.6332 (0.1287)
Capital base reduction .....	0.9832 (0.0868)	0.6830 (0.0982)	0.9806 (0.0597)	0.6435 (0.1278)	0.6882 (0.0731)	0.8387 (0.1146)
Debt ratio.....	<b>0.0182</b> <b>(0.1373)</b>	<b>0.3768</b> <b>(0.1963)</b>	<b>0.0036</b> <b>(0.0894)</b>	<b>0.2173</b> <b>(0.2036)</b>	0.3712 (0.1020)	<b>0.2878</b> <b>(0.1978)</b>

Note: The columns show coefficients while the standard errors are shown in brackets. Estimates in bold indicate insignificant variables at a 5 per cent level. Testing all sector models the t-test statistics reject the zero-hypothesis that the mean for active companies is identical to the mean for failed companies.

Source: Own calculations.

Chart 6 presents the results from the sector models. Business service has the lowest failure rate in the sector models. Although IT and telecom still have the highest failure rates, the ordering of the other sectors is different.

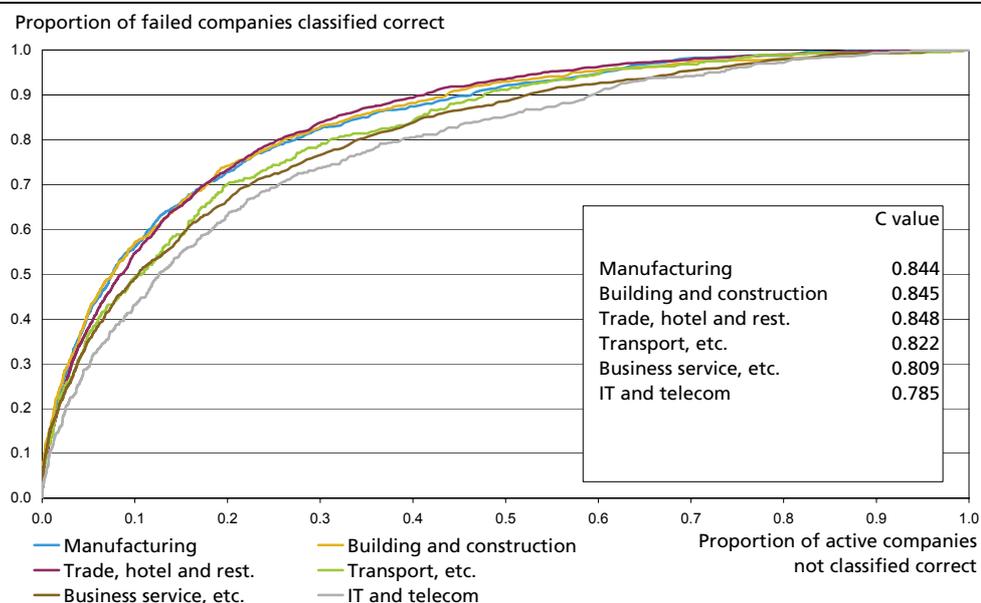
FAILURE RATES FROM THE SECTOR MODELS, 90TH PERCENTILE Chart 6



Note: Total contains the mentioned sectors.  
 Source: Own calculations.

The structural differences captured in the sector models are in general reflected in a better classification ability. Chart 7 presents the ROC-curves and c-values from the respective sector models.

ROC-CURVE OF THE SECTOR MODELS Chart 7



Source: Own calculation

### 3.3 Model based on number of employees in the company

In Denmark most of the companies are small and medium sized. More than 50 per cent of the companies have 2 to 10 employees<sup>9</sup>. A few per cent have more than 100 employees and these large companies are predominantly operating in manufacturing. Failures are rare in large companies. Close to one quarter of all the failures reported are in single proprietorship and more than 50 per cent of all the failures are in companies with 2 to 10 employees.

Estimating the failure-rate model on the basis of the number of employees in the company might disclose other structural differences in explaining the failure rate. One example to mention is the adjusted return on assets, which is only significant in the smallest companies. The coefficients estimated in the model based on the number of employees are presented in Table 5 and the odds ratios in Annex 3.

COEFFICIENT ESTIMATES AND STANDARD ERRORS IN THE MODEL BASED ON EMPLOYEES					
	Table 5				
Number of employees in the company	1	2-10	11-20	21-100	>100
Number of active companies .....	26,746	97,727	27,688	27,653	6,020
Number of failed companies .....	938	2,430	386	321	31
C-value .....	0.820	0.845	0.858	0.860	0.849
Intercept .....	-2.2100 (0.3365)	-4.0828 (0.2364)	-5.6061 (0.6842)	-2.8729 (0.7222)	1.4270 (2.4349)
Adj. ROA .....	-0.0036 (0.0010)	-0.0020 (0.0008)	<b>-0.0016</b> <b>(0.0025)</b>	<b>-0.0041</b> <b>(0.0021)</b>	<b>-0.0158</b> <b>(0.0101)</b>
Solvency .....	-0.0074 (0.0012)	-0.0071 (0.0009)	-0.0130 (0.0024)	-0.0114 (0.0024)	-0.0131 (0.0081)
Size .....	-0.3309 (0.0375)	-0.1430 (0.0249)	<b>0.0099</b> <b>(0.0684)</b>	-0.2312 (0.0671)	-0.5113 (0.1949)
Reduced liquidity .....	0.0044 (0.0012)	0.0039 (0.0006)	<b>0.0029</b> <b>(0.0016)</b>	0.0064 (0.0016)	0.0002 (0.0075)
New company .....	0.2920 (0.0731)	0.4573 (0.0454)	0.2233 (0.1240)	0.6420 (0.1343)	<b>0.5518</b> <b>(0.4980)</b>
Remark .....	0.9221 (0.0803)	1.1814 (0.0488)	1.0663 (0.1292)	1.6186 (0.1448)	1.2294 (0.6052)
Corporate form .....	0.4886 (0.1245)	0.4853 (0.0597)	0.8678 (0.1232)	0.4271 (0.1498)	<b>0.1011</b> <b>(1.0594)</b>
Capital base reduction .....	0.8038 (0.0879)	1.1598 (0.0542)	1.4455 (0.1392)	1.3818 (0.1442)	2.0215 (0.5006)
Debt ratio .....	<b>0.1305</b> <b>(0.1301)</b>	0.4703 (0.0952)	<b>0.4500</b> <b>(0.2333)</b>	<b>-0.0895</b> <b>(0.2437)</b>	-2.0737 (0.8215)

Note: The columns show coefficient estimates while the standard errors are shown in brackets. Estimates in bold indicate insignificant variables at a 5 per cent level.

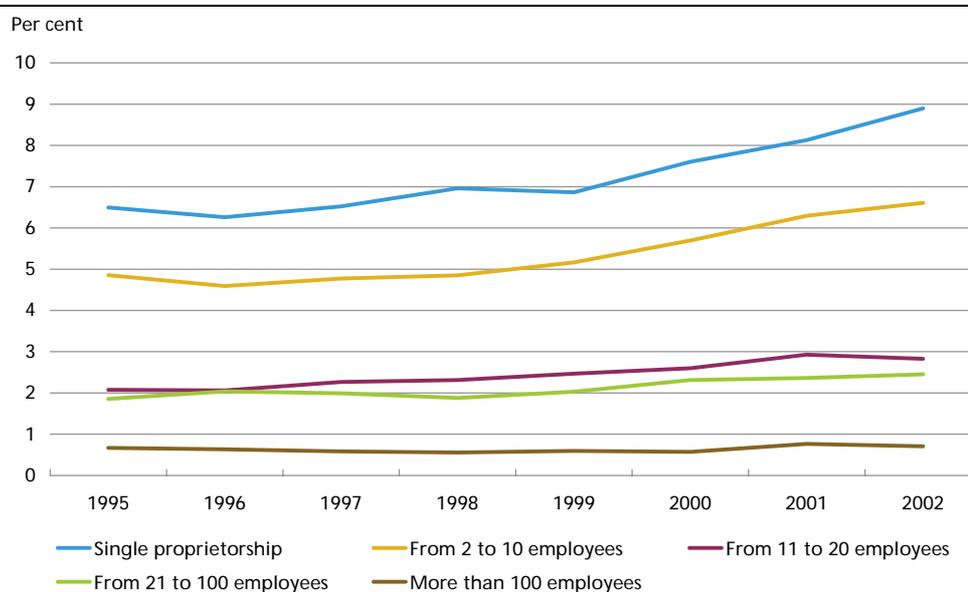
Source: Own calculations.

<sup>9</sup> Based on the database from KOB A/S. 65 per cent of the companies in the database report the number of employees in the company.

The failure rates are highly dispersed in the models based on the number of employees and the failure rates decrease significantly when companies have more than 10 employees, cf. Chart 8.

FAILURE RATES IN MODEL BASED ON THE NUMBER OF EMPLOYEES,  
90TH PERCENTILE

Chart 8



Source: Own calculation.

#### 4. Conclusion

Based on approximately 300,000 annual accounts of private and public limited liability companies this paper has presented a failure-rate model for the Danish corporate sector. The model is part of Danmarks Nationalbank's analysis of financial stability. Information on a company's return on assets, solvency, liquidity position, capital base, debt ratio, age, size, corporate structure, as well as critical remarks from auditors has been unified into a single measure, the failure rate. The failure rate is an estimate of the probability that the company will fail within the next few years (on average 19 months). The coefficients in the model are estimated using historical accounting information from active and failed companies from 1995 to 1999. This information enables the model to correctly discriminate active companies from failed companies eight out of ten times.

## Annex 1, Definition of sectors

The grouping of corporate sectors adheres to the common EU nomenclature (Nace Rev. 1.1). The corporate sectors used in the model are

- ◆ Manufacturing (12.00 <= NACE < 40.00)
- ◆ Building and construction (45.00 <= NACE < 50.00)
- ◆ Trade, hotel and restaurants (50.00 <= NACE < 60.00)
- ◆ Transport, etc. (60.00 <= NACE < 65.00)
- ◆ Business service, etc.<sup>10</sup> (70.00 <= NACE < 75.00  
and 93.00 <= NACE < 95.00)
- ◆ IT and telecom<sup>11</sup> (30.00 <= NACE < 31.00  
and NACE = 32.00  
and 32.20 <= NACE < 32.30  
and NACE = 51.64.10  
and NACE = 51.84.10  
and NACE = 51.84.20  
and NACE = 52.48.65  
and NACE = 52.48.66  
and NACE = 52.48.70  
and 64.20 <= NACE < 64.21  
and NACE = 71.33.10  
and 72.00 <= NACE < 73.00)

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<sup>10</sup> Business service, etc. includes e.g. business related to real property, car rental, machines, plant and equipment, etc., research and development, legal services, consultancy and cleaning services.

<sup>11</sup> IT and telecom comprises production of and trade in computer equipment, as well as telecommunication and data-processing equipment.

## Annex 2, Odds ratios in the basic model and the sector models

	Basic model	Manufacturing	Building and construction	Trade, hotels and rest.	Transport, etc.	Business service, etc.	IT and telecom
Active .....	281,673	46,746	35,030	83,028	14,224	76,188	14,075
Failures .....	7,787	979	880	2,081	440	1,289	498
Adj. ROA .....	0.996	0.390	0.431	0.534	0.464	0.478	<b>0.877</b>
Solvency .....	0.994	0.391	0.247	0.448	0.572	0.535	<b>0.720</b>
Size .....	0.756	0.870	0.734	0.761	0.835	0.727	0.901
Reduced liquidity .	1.003	1.008	1.004	<b>1.002</b>	1.005	<b>1.000</b>	1.004
New company .....	1.419	1.338	2.306	1.434	1.581	<b>1.113</b>	1.657
Remark .....	3.000	3.482	2.874	2.982	3.344	3.113	3.159
Corporate form ....	1.577	1.658	1.447	1.752	1.885	1.638	1.884
Capital base reduction .....	2.570	2.673	1.980	2.666	1.903	1.990	2.313
Debt ratio .....	1.302	<b>1.018</b>	<b>1.458</b>	<b>1.004</b>	<b>1.243</b>	1.450	<b>1.333</b>

Note: Odds ratios in bold indicate insignificant variables at a 5 per cent level.  
Source: Own calculations.

### Annex 3, Odds ratios in the models based on the number of employees

ODDS RATIOS IN THE MODELS BASED ON EMPLOYEES					
Number of employees in the company	1	2-10	11-20	21-100	>100
Number of active companies .....	26,746	97,727	27,688	27,653	6,020
Number of failed companies .....	938	2,430	386	321	31
Adj. ROA .....	0.996	0.998	<b>0.998</b>	<b>0.996</b>	<b>0.984</b>
Solvency .....	0.993	0.993	0.987	0.989	0.987
Size .....	0.718	0.867	<b>1.010</b>	0.794	0.600
Corporate form .....	1.630	1.625	2.382	1.533	<b>1.106</b>
Reduced liquidity .....	1.004	1.004	<b>1.003</b>	1.006	1.000
New company .....	1.339	1.580	1.250	1.900	<b>1.736</b>
Remark .....	2.514	3.259	2.905	5.046	3.419
Capital base reduction .....	2.234	3.189	4.242	3.982	7.550
Debt ratio .....	<b>1.139</b>	1.600	<b>1.568</b>	<b>0.914</b>	0.126

Note: Odds ratios in bold indicate insignificant variables at a 5 per cent level.  
Source: Own calculations.

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