Aspects of Household Heterogeneity in New Keynesian Economics

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This book is chiefly addressed to my fellow economists
– J.M. Keynes

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Preface

This dissertation is the result of my PhD studies at the School of Economics and Management at the University of Aarhus. The first equations were derived sometime during the summer of 2004, and it has taken form during the time I have spent at the University, at Danmarks Nationalbank in Copenhagen, and visiting the Universitat Pompeu Fabra in Barcelona. I am very grateful to Danmarks Nationalbank for financing these studies.

I would like to thank a number of people who have helped me along the way. First and foremost, I would like to thank my main supervisor Torben M. Andersen for his guidance and encouragement. Also, I would like to thank my secondary supervisor Niels Haldrup for standing by in case of econometric emergencies. And I dare not forget the PhD students at the School: You are great! A million thanks to Malene Kallestrup for always being there, and to Søren Tang Sørensen for always pointing me to Hayek (and for living in SW7). I wish I could also thank Henrik Filskov Simonsen; I think of late nights spent working on proofs for PhD Maths assignments and I am reminded of what really matters in life. At the bank, I would particularly like to thank Peter Storgaard and Kim Abildgren for many helpful discussions. Many thanks also to Jordi Galí for kindly sponsoring my stay at the UPF. I have greatly benefited from the eight months I was part of the stimulating intellectual environment there. Special thanks go to my co-author Francesco Furlanetto for our fruitful collaboration, his friendship, and not least for invaluable encouragement when it was badly needed. Finally, I am grateful to the assessment committee, John Driffill, Svend E. Hougaard Jensen and Bo Sandemann Rasmussen, for their thorough and constructive work, and for their many helpful comments and suggestions at the pre-defence. There are much better places to thank family and friends than the preface to a PhD dissertation. I will find such a place soon.

Martin Seneca
Aarhus, July 2008

Introduction and summary

I began my PhD studies shortly after Michael Woodford published his monumental book 'Interest and Prices, Foundations of a Theory of Monetary Policy' (Woodford, 2003). This book has been a companion throughout my PhD studies, and I have spent many hours reading it, reproducing its main equations, and pondering its arguments. It is a book that brings frustration as well as enlightenment – much like everything else a PhD student has to go through – but it is a book that has had an immense influence on the framing of my thinking about macroeconomics and economic policy. Indeed, the models in this thesis are closely related to the class of models considered by Woodford (2003), namely the dynamic stochastic general equilibrium (DSGE) models with imperfect competition, nominal rigidities and endogenous monetary policy that have come to dominate much of macroeconomics.[1]

Woodford (2003) refers to this class of models as neo-Wickellian stressing that monetary policy is conducted by setting a short-term interest rate in response to macroeconomic developments as suggested by Knut Wicksell in *his* 'Interest and Prices' more than a century ago (Wicksell, 1898). Goodfriend and King (1997) suggest instead that the models represent a new neo-classical synthesis because they accommodate different assumptions about the frictions that may potentially obstruct the efficiency of markets (as they do in Keynesian economics), hence replacing past methodological disagreements between the ever feuding schools of macroeconomic thought by the near consensus view that economic dynamics should be derived from well-defined decision problems of economic agents as in the neo-classical real business cycle (RBC) literature. The synthesis is 'new' to distinguish it from the 'old' neo-classical synthesis in which the long-run is classical and the short run Keynesian. In the new synthesis, the classical long run is to be

[1] The seminal contribution is Yun (1996).

thought of as an important dynamic benchmark or, in the words of Woodford (2003, p. 9), a '*virtual* equilibrium of the economy at each point in time – the equilibrium that one would have if wages and prices were not in fact sticky.'

Other economists – less afraid, perhaps, of the possible stigma related to the ideas exposed by John Maynard Keynes in his 'General Theory' (Keynes, 1936), and not least to the later interpretations thereof – refer to the models as New Keynesian. One example is Galí (2008), whose admirably clear exposition of the material I have much to thank for appreciating the underlying ideas. Hereby, they emphasise that labour and goods markets may be imperfectly competitive in these models, and that wages or prices may fail to adjust fully and instantaneously to shocks. But importantly, the DSGE framework applied means that it is a new generation of New Keynesian economics in comparison with the New Keynesian literature of the 1980s as defined and exemplified by the contributions in the volumes edited by Mankiw and Romer (1991). This early generation of New Keynesian economics developed Keynesian mechanics without the ad hoc assumption of the old Keynesian approach, but abstracted from issues related to risk, dynamics and/or general equilibrium effects. As 'Neo-New Keynesian Economics' is as much a tongue twister as 'The New Neo-Classical Synthesis', or perhaps more because of habit than anything else, I follow Galí (2008) and others in referring to the current generation of DSGE models with imperfect competition and nominal rigidities simply as 'New Keynesian', and it is in this sense that the inclusion of that term in the title of this thesis should be understood.

Despite the rigorous DSGE approach, the equilibrium conditions of the basic New Keynesian models can be summarised in a very simply form that is reassuringly similar to the equations of the old ad hoc models. In particular, the dynamics of the simplest models can be expressed in log-linear terms as an expectation-augmented Phillips curve, an IS equation and a monetary policy rule. But there are important differences, in particular because explicit attention to dynamic effects makes the models forward-looking. The expectations that matter for inflation dynamics are first and foremost current expectations of future inflation, and the IS equation gives the relation between interest rates and output in a forward-looking manner. Equally important, the basic model can be extended in numerous ways without compromising the ambitious methodological standards set by the DSGE approach. The models in this thesis are examples of such extensions.

Because of this appeal, DSGE modelling has become increasingly popular, and notably so even in the time I have spent as a PhD student, not

only as the comme-il-faut approach in academic macroeconomics, but also in policy making institutions. Central banks, in particular, have built and in many instances estimated medium-sized New Keynesian models often using recently developed Bayesian methods in macroeconometrics. These models are increasingly the driving forces behind economic policy deliberations at these central banks, and in many instances they have replaced existing structural macroeconometric models for both policy experiment and forecasting purposes. This is the case in particular in central banks that have adopted inflation targeting.

The models in this thesis are probably too stylised to be taken to the data in a way that can be expected to be meaningful, at least in their current forms, and no attempt has been made to estimate them formally. But that is not to say that they are not inspired by the data or influenced by the empirical evidence. Observations about structural economic characteristics and evidence from empirical studies in the literature have served as important inputs to the motivations for the chapters that follow.

The models in this thesis differ from most of the New Keynesian literature by emphasising household heterogeneity, that is, by the assumption that households in the economy have different characteristics. As obvious as this may appear to be to the uninitiated, macroeconomists usually assume that households are all alike, or more subtly that the economic decisions by households in the economy can be summarised by the decisions of a representative household without much consideration for the conditions under which this assumption provides a meaningful approximation to reality. In this thesis, no such representative household exits, though the deviations from the representative household assumption are admittedly small as the representative household approach to aggregation is often maintained.

Hence, I by no means claim to provide anything close to an exhaustive analysis of household heterogeneity, but instead I provide two examples of a basic kind of household heterogeneity with implications for the propagation of shocks to the economy, and I analyse some of these implications in the following four chapters. Indeed, in most (though not all) of the analysis, households are inherently identical in the sense that they have identical preferences, but importantly they face different constraints in their immediate economic environments, i.e. in the environments in which they have to make economic decisions, with non-trivial effects on their behaviour. Such an analysis is a first step – or rather a supplement – to the analysis of the implications of deeper household heterogeneity.

* * *

The thesis consists of two parts each divided into two self-containing chapters.

In part I consisting of chapters 1 and 2, households supply their labour services in a heterogeneous labour market. In particular, it is assumed that households in different countries in a monetary union (or alternatively in different sectors of a closed economy) are allowed to adjust their wages with different frequencies and that the market power with which they can set wages differ.

Chapter 1 is called 'Labour market asymmetries in a monetary union' and is joint work with Torben M. Andersen. This paper takes a first step in analysing how a monetary union performs in the presence of labour market asymmetries. Differences in wage flexibility, market power and country sizes are allowed for in a setting with both country-specific and aggregate shocks. The implications of asymmetries for both the overall performance of the monetary union and the country-specific situation are analysed. It is shown that asymmetries can have important effects, and that there are substantial spillover effects. Aggregate output volatility is not strictly increasing in nominal rigidity but hump-shaped. A disproportionate share of the consequences of wage inflexibility may fall on small countries. In the case of country-specific shocks a country unambiguously benefits in terms of macroeconomic stability by becoming more flexible, but in general an inflexible country does not necessarily achieve more output stability by becoming more flexible.

The title of Chapter 2 is 'Monetary policy and welfare in a monetary union with labour market heterogeneity'. The paper addresses the question of how monetary policy should be conducted in a monetary union when labour market structures are different in member countries. As this is the case in the European Monetary Union, this question should be of particular interest to European policy makers. The paper develops a dynamic stochastic general equilibrium model of a two-country monetary union with labour market heterogeneity. Asymmetries in labour market structures are proxied for by different degrees of nominal flexibility. A welfare loss function derived as a second-order approximation to household utility is evaluated. The results suggest that price inflation targeting may lead to non-negligible welfare losses compared to monetary policy alternatives when the important nominal rigidity is in the labour market. Welfare may be noticeably improved

in such a case by targeting wage inflation especially when shocks are highly correlated across member countries and when labour markets are very heterogeneous. Further welfare improvements can be obtained by putting more weight to fighting wage inflation in the more rigid labour market if shocks are less than perfectly correlated.

In part II consisting of chapters 3 and 4, households differ with respect to access to capital and financial markets. Some households, call them rule-of-thumb consumers, are barred from access to these markets and have no choice but to consume their current income. Other households, however, have full access to capital and financial markets, which means that they substitute consumption intertemporally by investing in capital goods or by buying and selling financial assets. As an alternative interpretation, rule-of-thumb consumers may have access to financial and capital markets, but nonetheless choose not to take part in them for some reason or other, maybe because they are myopic, extremely impatient, ignorant or afraid. Under this interpretation, rule-of-thumb consumers are inherently different from their optimising neighbours.

Chapter 3, 'Rule of thumb consumers, productivity and hours', is joint work with Francesco Furlanetto. In this chapter we study the transmission mechanisms of productivity shocks in a model with rule-of-thumb consumers. In the literature, this financial friction has been studied only with reference to fiscal shocks. We show that the presence of rule-of-thumb consumers is also very helpful in accounting for recent empirical evidence on productivity shocks. Rule-of-thumb agents, together with nominal and real rigidities, play an important role in reproducing the negative response of hours and the delayed responses of output and consumption after a productivity shock.

In Chapter 4, 'Fiscal shocks, rule-of-thumb consumers and real rigidities', which is joint work with Francesco Furlanetto, we show that empirically plausible results on the effects of fiscal shocks in Galí et al. (2007) rely on an excessive degree of price stickiness and an implausibly large percentage of financially constrained agents. We show that it is possible to obtain an empirically appealing consumption multiplier for plausible values of these parameters if real rigidities are added to the model. Real rigidities in the form of habit persistence, fixed firm-specific capital and Kimball demand curves interact in interesting ways with nominal and financial rigidities, and we show that they are useful in the study of fiscal shocks in addition to monetary and productivity shocks as has been shown in the previous literature.

The thesis has two main messages – or perhaps, given the risk of succumb-

ing to the temptation of drawing too strong conclusions from the evidence presented, the results provide two main indications. The first is that consumer heterogeneity – either in the form of inherent differences in household characteristics or in the form of asymmetries in the immediate economic environment in which individual households have to act – matters for the aggregate economy and for economic policy, and that more research should be devoted to understanding the implications of such heterogeneity both for social welfare and for the design of policy. The second message is that the interaction of nominal, real and financial rigidities is important in macroeconomic analysis even when a frictionless, perfectly competitive *virtual* general equilibrium is taken to be an important benchmark. The extent to which this latter indication must be modified by advances in the understanding of the potential limits to – or deviations from standard assumptions on – economic agents' decision making capabilities is, I believe, a particularly interesting avenue for further research in macroeconomics.

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Dansk resumé

Denne afhandling består af fire selvstændige kapitler. Afhandlingen er inddelt i to dele bestående af henholdsvis kapital 1-2 og kapitel 3-4.

I afhandlingens første del udbyder husholdningerne deres arbejdskraft i et heterogent arbejdsmarked. Mere specifikt antages det, at husholdninger i forskellige lande i en møntunion (eller alternativt i forskellige sektorer i en lukket økonomi) justerer deres lønninger med forskellig frekvens og har forskellige grader af markedsmagt i lønfastsættelsen.

Kapitel 1 hedder "Labour market asymmetries in a monetary union" og er skrevet sammen med Torben M. Andersen. Kapitel tager et første skridt i analysen af, hvordan en møntunion fungerer, når arbejdsmarkedsstrukturerne er forskellige på tværs af landegrænser. Forskelle i lønfleksibilitet, markedsmagt og landestørrelse tillades i en model med både landespecifikke og fælles stød. Implikationerne af asymmetrier for både den overordnede makroøkonomiske stabilitet i møntunionen og den landespecifikke situation analyseres. Det vises, at asymmetrier kan have vigtige effekter, og at der er betydelige spill-over-effekter. Blandt andet viser analysen, at den aggregerede outputvariabilitet ikke er strengt stigende i graden af nominel rigiditet, men snarere pukkelformet. En disproportional andel af konsekvenserne af manglende lønfleksibilitet kan tilfalde små lande, og et ufleksibel land opnår ikke nødvendigvis større stabilitet ved at blive mere fleksibelt.

Titlen på kapital 2 er "Monetary policy and welfare in a monetary union with labour market heterogeneity". Kapitlet adresserer spørgsmålet om, hvordan pengepolitikken skal indrettes i en møntunion, når arbejdsmarkedsstrukturerne er forskellige i unionens medlemslande. Da dette er tilfældet i Den Europæiske Møntunion, bør dette spørgsmål have særlig interesse for europæiske policymakere. I kapitlet udvikles en dynamisk stokastisk generel ligevægtsmodel for at besvare dette spørgsmål. Asymmetrier i arbejdsmarkedets strukturer er repræsenteret af forskellige grader af nominelle rigiditeter. En velfærdstabsfunktion udledes som en andenordensapproksi-

mation til husholdningernes nytte. Resultaterne indicerer, at en prisinflationsmålsætning kan føre til ikke-ubetydelige velfærdstab, når den vigtigste nominelle rigiditet er at finde på arbejdsmarkedet og ikke på varemarkedet. Velfærden i møntunion kan mærkbart forbedres i et sådant tilfælde ved at anvende løninflation som mål, især når stød er stærkt korrelerede på tværs af landegrænser, og når arbejdsmarkederne er meget heterogene. Velfærden kan yderligere forbedres ved, at centralbanken lægger mere vægt på løninflationen i landet med det mest rigide arbejdsmarked, når stød er mindre end perfekt korrelerede.

I afhandlingens anden del bestående af kapitel 3-4, er husholdningerne forskellige med hensyn til deres adgang til finans- og kapitalmarkederne. Nogle husholdninger, kald dem tommelfingerregelforbrugere, har ikke adgang til disse markeder og har dermed ingen anden mulighed end at forbruge deres disponible indkomst i hver periode. Men andre husholdninger har fuld adgang til finans- og kapitalmarkederne, hvilket betyder, at de kan substituere forbrug intertemporalt ved at investere i kapital eller ved at købe og sælge finansielle aktiver. Som en alternativ fortolkningsmulighed kan tommelfingerregelforbrugerne tænkes af have adgang til finans- og kapitalmarkederne, men ikke desto mindre vælge ikke at tage del i dem af en eller anden grund, måske forbi de er myopiske, særdeles utålmodige, ignorante eller bange for at tage del i disse markeder. Under denne alternative fortolkning er tommelfingerregelforbrugerne forskellige fra optimerende forbrugere i deres personlige karakteristika.

Kapitel 3, "Rule-of-thumb consumers, productivity and hours", er skrevet med Francesco Furlanetto. I dette kapitel undersøger vi transmissionsmekanismen af produktivitetsstød i en model med tommelfingerregelforbrugere. I litteraturen er denne finansielle friktion kun blevet undersøgt i relation til finanspolitiske stød. Vi viser, at tilstedeværelsen af tommelfingerregelforbrugere også hjælper med at forklare nylige empiriske resultater om produktivitetsstød. Tommelfingerregelforbrugere, sammen med nominelle og reale rigiditeter, er vigtige for at reproducere den negative reaktion af arbejdstimer og de forsinkede reaktioner af produktion og forbrug efter et produktivitetsstød.

I kapital 4, "Fiscal shocks, rule-of-thumb consumers and real rigidities", som er skrevet med Francesco Furlanetto, viser vi, at empirisk plausible resultater i J. Galí, J.D. López-Salido og J. Vallés, "Understanding the effects of governmetn spending on consumption", Journal of the European Economics Association, 5, pp. 227-270, er baseret på en overdrevet grad af

prisrigiditet og en uplausibel stor procentdel af finansielt begrænsede forbrugere. Vi viser, at det er muligt at opnå en empirisk plausibel forbrugsmultiplikator for plausible værdier for disse parametre, hvis reale rigiditeter introduceres i modellen. Reale rigiditeter i form af vanedannelse i forbruget, fast virksomhedsspecifik kapital og Kimball-efterspørgselskurver interagerer på interessant vis med nominelle og finansielle rigiditeter, og vi viser, at de er nyttige i analysen af fiskale stød i tillæg til monetære og teknologiske stød, som det tidligere er demonstreret i litteraturen.

Afhandlingen har to hovedbudskaber - eller måske snarere, givet risikoen for at falde for fristelsen til at drage for stærke konklusioner på baggrund af de fremlagte beviser, giver resultaterne to hovedantydninger. Den første er, at forbrugerheterogenitet – enten i form af forskelle i forbrugernes personlige karakteristika eller i form af asymmetrier i den umiddelbare økonomiske sfære, i hvilken husholdningerne skal agere – er af betydning for samfundsøkonomien og for den økonomiske politik, og at mere forskning skulle dedikeres til at forstå implikationerne af en sådan heterogenitet både for den samlede velfærd og for udformningen af den økonomiske politik. Det andet budskab er, at samspillet mellem nominelle, reale og finansielle rigiditeter er vigtigt i makroøkonomisk analyse, selv når en friktionsløs, fuldkommen kompetitiv, *virtuel* generel ligevægt med rimelighed anses for at være et vigtigt benchmark. Graden med hvilken denne sidste implikation skal modificeres som professionen gør fremskridt i forståelsen af de potentielle begrænsninger i – eller afviger fra standardantagelser om – økonomiske agenters evner til at træffe økonomiske beslutninger er efter min mening et særligt interessant emne for den fremtidige makroøkonomiske forskning.

Part I

Chapter 1

Labour market asymmetries in a monetary union*

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Abstract

This paper takes a first step in analysing how a monetary union performs in the presence of labour market asymmetries. Differences in wage flexibility, market power and country sizes are allowed for in a setting with both country-specific and aggregate shocks. The implications of asymmetries for both the overall performance of the monetary union and the country-specific situation are analysed. It is shown that asymmetries can have important effects, and that there are substantial spill-over effects. Aggregate output volatility is not strictly increasing in nominal rigidity but hump-shaped. A disproportionate share of the consequences of wage inflexibility may fall on small countries. In the case of country-specific shocks a country unambiguously benefits in terms of macroeconomic stability by becoming more flexible, but in general an inflexible country does not necessarily achieve more output stability by becoming more flexible.

JEL classification: E30, E52, F41.

Key words: wage formation, nominal wage rigidity, staggered contracts, monetary policy, monetary union, business cycles, shocks

*For useful comments, we thank seminar participants at Danmarks Nationalbank, Universitat Pompeu Fabra, Kiel Institute for the World Economy, SAE 2007 in Granada, and the Royal Economic Society's 2008 Conference at Warwick. The paper has appeared as CEPR Discussion Paper No. 6800 and Danmarks Nationalbank Working Paper No. 53.

1 Introduction

In the run-up to the establishment of the European Monetary Union, much focus was on whether the potential member countries fulfilled the conditions for an optimal currency area. According to the traditional theory on optimal currency areas, participation in a currency union and the implied loss of autonomy in monetary policy require that labour markets are flexible in adjusting to country-specific shocks (see e.g. de Grauwe, 2005). The flexibility can either be in terms of wage adjustment or labour mobility. In a second wave of the literature, it was stressed that the conditions for an optimal currency area are endogenous since participation in a currency union affects market fundamentals (via further integration), incentives in wage formation and possibly the incentive to undertake structural reforms (see e.g. Rose and Frankel, 1998, and Calmfors, 2001). Whether the countries constitute an optimal currency union is an ex-post rather than an ex-ante question. In either case, it is presumed that a monetary union eventually will be characterized by symmetries.

However, perceiving a currency union as a homogeneous area is in most cases misleading, and for the European Monetary Union (EMU) in particular this assumption does not seem appropriate. European countries are fairly heterogeneous, reflecting different institutional, political and historical developments. The homogeneity assumption did not hold neither in an ex ante and nor in an ex post sense so far since the incidence and nature of reforms undertaken in recent years do not seem to indicate that these differences are about to be eliminated. Differences in business cycle situations and the tensions these have created in the assessment of the common monetary policy of the ECB reveal that countries are affected by different shocks and/or structures.

This paper takes a first step in analysing the role of asymmetries for business cycle fluctuations within a currency union, focussing on the role of asymmetric sizes, structures and shocks. One reason why such asymmetries may be important and why the homogeneity assumption underlying standard approaches is potentially misleading arises from the observation that the common monetary policy tends to react to common or aggregate shocks within the area. Hence, asymmetries are potentially more important since they cannot easily be countered via the common monetary policy. Asymmetries across countries may arise directly from country-specific shocks, but even aggregate or common shocks may create asymmetric effects when they

interact with differences in size and structures across the member countries. This, in turn, implies that aggregate measures of the performance of the union, like inflation and output (gaps) to which the common monetary policy reacts, may critically depend on the asymmetries. To further complicate matters, the "asymmetries" are transmitted across member countries via trade links. It is an implication that the performance of the monetary union is not in general well described by a "representative" country approach.

The framework used in this paper is based on recent intertemporal approaches in both open economy macroeconomics and in the closed-economy New Keynesian literature on monetary policy.[1] To focus on the interactions within the monetary union, it is considered to be a closed area where the member countries share a common monetary policy and engage in trade with each other. The focus is positive in the sense of exploring the consequences of asymmetries (size, structure and shocks) for the business cycle performance at the country and aggregate level for a given monetary policy setting. The specific structural asymmetries analyzed apply to the labour market, and differences in nominal adjustment and the degree of competition in the labour market are allowed for. These two types of structural dimensions can be seen as examples of structural differences in the real and the nominal dimension. They are further motivated by the fact that it is well-established that there are such differences across European countries (member or potential member countries of EMU), see e.g. the OECD (various issues), European Commission (2006) and Arpaia and Pichelmann (2007).[2] The specific shocks considered are supply (productivity) shocks. Modern business cycle theory has studied such shocks extensively and therefore constitutes a natural benchmark for the analysis of the implications of asymmetries. However, the main mechanisms are not dependent on whether shocks are originating on the supply or the demand side. Finally, we model monetary policy by a simple Taylor-rule in accordance with much recent research on monetary policy.[3]

Whether participation in a monetary union strengthens the incentive to undertake reforms can not really be addressed without having analysed how

[1] Seminal contributions are Obstfeld and Rogoff (1995) and Yun (1996), respectively. For monographic expositions, cf. Obstfeld and Rogoff (1996) and Woodford (2003).

[2] Recent work has also pointed to the importance of downward nominal wage rigidities and cross-country differences in this form of rigidity, see Holden and Wulfsberg (2008).

[3] See, for instance, Smets and Wouters (2003) for an estimated Taylor rule for the euro area, and Galí and Gertler (2007) for a recent discussion.

the monetary union works in the presence of given asymmetries. The debate seems to take it for granted that more flexibility is good both from a country and an overall monetary union perspective. Is it necessarily the case that less flexible countries suffer from a disproportionate share of economic problems and therefore have the largest incentives to undertake reforms? Is it only asymmetries across large member countries that matter, or are the structural characteristics of smaller member countries also important? Surprisingly, these questions have not been much researched. We shed some light on these questions by considering the implications of asymmetric structures both for the aggregate performance of a monetary union and country-specific performance. Interestingly, we find that there may be a conflict between what may seem best from a country perspective and from an overall monetary union perspective. Moreover, changes in structural parameters may release spill-over effects between member countries, suggesting that non-cooperative structural policy making does not necessarily imply structures which are optimal seen from both an overall monetary union and a country-specific perspective.

Other forms of asymmetries or heterogeneities in a monetary union have been addressed in the literature. One important issue is the interdependency between monetary and fiscal policy when the former is centralised and the latter decentralised. This naturally leads to potential asymmetries in fiscal policy, which raises questions concerning interdependencies between national fiscal policies as well as between the aggregate fiscal stance and the monetary policy (see, e.g., Lombardo and Sutherland, 2004, Beetsma and Jensen, 2005, and Andersen, 2005). Also, Benigno (2002) analyses, from a normative point of view, how monetary policy should be designed when member countries have different degrees of nominal price rigidities, and it is shown that the central bank should attach more weight to inflation in countries characterised by more nominal inertia. Beetsma and Jensen (2004, 2005) allow for labour market asymmetries in their analysis of the interactions between monetary and fiscal policy in a monetary union. Dellas and Tavlas (2004) present a three-country model allowing for asymmetries in nominal wage flexibility, and find that countries with a high degree of nominal wage rigidity are better off in a monetary union. Similarly, Hallett and Jensen (2001) find that countries wish to join a monetary union only when its markets are relatively more flexible. Andersen (2008) also analyses the implications of labour market asymmetries in a monetary union in an intertemporal model, but in a setting with one-period contracts.

The paper is organised as follows. The model structure is laid out in section 2, and the equilibrium processes for output and inflation are determined in section 3. Section 4 considers the shock transmission in a symmetric baseline example. The implications of various forms of asymmetries are explored in section 5 both from a unionwide and country-specific perspective. Section 6 offers a few concluding comments.

2 A monetary union with heterogeneous labour markets

Consider a monetary union where the central bank has the monetary authority over I separate and otherwise independent countries (or more generally regions) indexed by i. In particular, the central bank sets the nominal interest rate R_t earned on risk-free nominal bonds throughout this monetary union between periods t and $t+1$. The union is closed to the outside world.[4] Each country i is populated by a continuum of households $h \in [0,1]$ and has a continuum of firms $f \in [0,1]$. Countries may be of different sizes, where the relative size of country i is given by $v_i \in [0,1]$ such that $\sum_{i=1}^{I} v_i = 1$. All firms in a given country produce the same internationally traded consumption good, different from those produced in other countries (a specialised production structure). For simplicity, product markets are assumed to be perfectly competitive and prices to be flexible. Labour markets, in contrast, are imperfectly competitive and have nominal rigidities in the form of nominal wage contracts. Labour market structures are allowed to differ across countries in terms of degrees of market power in wage setting, and degrees of nominal rigidity are different across countries. There is no mobility of labour across borders.

2.1 Firms

2.1.1 Labour demand

In each period t, each household h in country i supplies a differentiated labour service $N_{it}(h)$. The labour used in production in country i, L_{it}, is

[4] This means that the model could also be interpreted as a closed-economy model of a single country with I sectors, which are potentially asymmetric in terms of structures and shocks.

assumed to be an aggregate of the continuum of labour services supplied by the households:

$$L_{it} = \left[\int_0^1 N_{it}(h)^{\frac{\xi_i - 1}{\xi_i}} \, dh \right]^{\frac{\xi_i}{\xi_i - 1}} \tag{1}$$

where $\xi_i > 1$ is the elasticity of substitution between labour services.

Each household determines its wage rate, $W_{it}(h)$, taking into account how firms' labour demand depends on the wage (a right-to-manage structure). That is, given wages, actual employment is determined by labour demand. The demand for household h's labour service is determined by the cost minimization problems of the country's firms, which minimise costs, taking households' wage rates, $W_{it}(h)$, as given. The representative firm minimizes

$$\int_0^1 W_{it}(h) N_{it}(h) \, dh \tag{2}$$

with respect to $N_{it}(h)$ subject to (1). This leads to a demand for household h's labour service given by

$$N_{it}(h) = \left(\frac{W_{it}(h)}{W_{it}} \right)^{-\xi_i} L_{it} \tag{3}$$

where W_{it} is the wage index defined by

$$W_{it} = \left[\int_0^1 W_{it}(h)^{1-\xi_i} \, dh \right]^{\frac{1}{1-\xi_i}} \tag{4}$$

This wage index has the property that the minimum cost of acquiring L_{it} units of aggregate labour is given by $W_{it} L_{it}$. It follows that the labour demand elasticity is ξ_i, and hence the market power of wage setters is inversely related to ξ_i. The demand for household h's labour service is a decreasing function of the household's relative wage.

2.1.2 Profit maximisation

The representative firm in country i produces output Y_{it} according to the production function

$$Y_{it} = \frac{1}{\gamma} L_{it}^{\gamma} U_{it}^{1-\gamma} \tag{5}$$

where U_{it} is the stochastic period-t productivity of firms in country i, and $0 < \gamma < 1$ is the degree of returns to scale. Real capital is disregarded to

simplify, but decreasing returns can be interpreted as arising from a second factor of production in fixed supply.

Product markets are perfectly competitive, and the representative firm in country i maximizes profits, which it distributes to households. There are no nominal price rigidities, and the firm takes the price of its product, P_{it}, as given. The profit maximization problem yields a demand for aggregated labour services given by

$$L_{it} = \left(\frac{W_{it}}{P_{it}}\right)^{-\frac{1}{1-\gamma}} U_{it} \tag{6}$$

Inserting in (5) gives the supply relation

$$Y_{it} = \frac{1}{\gamma}\left(\frac{W_{it}}{P_{it}}\right)^{\frac{\gamma}{\gamma-1}} U_{it} \tag{7}$$

In logs and measuring variables as deviations from steady state[5], we have

$$y_{it} = \beta\left(p_{it} - w_{it}\right) + u_{it} \tag{8}$$

where $\beta \equiv \gamma/\left(1-\gamma\right)$. The supply shock is assumed to be generated by the process

$$u_{it} = \rho_{uu} u_{it-1} + \varepsilon_{it} \tag{9}$$

where $-1 < \rho_{uu} < 1$ and $\varepsilon_{it} \tilde{} iid\ N\left(0, \sigma_i^2\right)$. The correlation of the innovations $Corr(\varepsilon_i, \varepsilon_j)$ is denoted ρ_{ε}. Hence, different situations can easily be characterized by varying this correlation coefficient. If $\rho_{\varepsilon} = 1$, it follows that all innovations are identical across regions, i.e., $\varepsilon_{it} = \varepsilon_t\ \forall i$, which corresponds to an aggregate shock. If $\rho_{\varepsilon} = 0$, the shocks are idiosyncratic or country-specific shocks.

[5]In order to solve the model, it is written in log-deviations from the non-stochastic steady state. Steady-state values are indicated by omission of time subscripts, and lower-case letters denote (log-)deviations from steady-state values of corresponding upper-case variables ($x_t \equiv dX_t/X \approx \ln\left(X_t/X\right)$). Throughout, aggregate log-variables are defined as weighted averages of country-specific log-variables, i.e., for any variable x, we have $x_t = \sum_{i=1}^{I} v_i X_{it}$. In general, a log-linearization around a steady-state of $X_t = \sum_{i=1}^{I} V_i X_{it}$ gives this average where $v_i = \frac{V_j X_j}{X}$. Symmetry of the steady state implies $v_i = V_i$.

2.2 Households

2.2.1 Consumption and bond holdings

Household h in country i has the utility function

$$E_t \sum_{\tau=0}^{\infty} \delta^{\tau} \left[\frac{\sigma}{\sigma - 1} C_{it+\tau}(h)^{\frac{\sigma-1}{\sigma}} - \frac{1}{1+\mu} N_{it+\tau}(h)^{1+\mu} \right] \tag{10}$$

where E_t is an operator representing expectations over all states of the economy conditional on period-t information, $\delta \in (0,1)$ is the subjective discount factor, and $C_{it+\tau}(h)$ is a real consumption index for period $t+\tau$, $N_{it+\tau}$ is labour supply in period $t+\tau$. $\sigma > 0$ is the elasticity of intertemporal substitution of consumption and $\mu > 0$.[6]

The consumption index is defined over the differentiated commodities produced in the union's member countries. Specifically,

$$C_{it}(h) = \left[\sum_{j=1}^{I} v_j C_{ijt}(h)^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}} \tag{11}$$

where $\theta > 0$, v_j is the relative size of country j (as noted above), and $C_{ijt}(h)$ represents consumption of country j's commodity by household h in country i. In every period t, this household chooses $C_{ijt}(h)$ for a given level of real consumption by minimizing

$$\sum_{j=1}^{I} P_{jt} C_{ijt}(h)$$

subject to (11). This yields a demand for country j's product by household h in country i given by

$$C_{ijt}(h) = \left(\frac{P_{jt}}{v_j P_t} \right)^{-\theta} C_{it}(h) \tag{12}$$

[6]Real money balances could be included in the utility function in order to analyse money demand. However, the central bank's policy instrument is the interest rate, while it passively supplies the money demanded by households. Thus, as long as money enters additively separably in the utility function, nothing will change in what follows since the inclusion of money will only add a money demand relation recursively determining money demand as a function of the variables of interest. See, e.g., Woodford (2003) for a discussion.

when P_t is the price index defined by

$$P_t = \left[\sum_{j=1}^{I} v_j \left(\frac{P_{jt}}{v_j}\right)^{1-\theta}\right]^{\frac{1}{1-\theta}} \tag{13}$$

This price index has the property that the minimum cost of acquiring C_{it} units of real consumption is given by $P_t C_{it}$. From (12) it follows that θ is the price elasticity of demand for C_{ijt}.

Asset markets are assumed to be complete (see discussion below), i.e., available financial assets completely span the possible states of the economy. This assumption leads to the following period-t flow budget constraint for a household in country i:

$$E_t\left[Q_{t,t+1} B_{it}(h)\right] + P_t C_{it}(h) = B_{it-1}(h) + W_{it}(h) N_{it}(h) + \Pi_{it}(h) \tag{14}$$

The right-hand side gives available resources as the sum of initial financial wealth, $B_{it-1}(h)$, labour income, $W_{it}(h) N_{it}(h)$, and nominal profit income, $\Pi_{it}(h)$. The left-hand side represents the allocation of resources to consumption, $P_t C_{it}(h)$, and bond-holdings, $E_t\left[Q_{t,t+1} B_{it}(h)\right]$, where $Q_{t,t+1}$ is the asset pricing kernel.[7]

Given existing wage contracts, the household maximises expected utility (10) subject to the sequence of budget constraints (14) and (implicitly) a solvency condition. Defining the net risk-free nominal interest rate R_t by the relation $(1+R_t)^{-1} = E\left[Q_{t,t+1}\right]$, the first-order conditions determining the optimal choice of consumption and bond-holdings can be combined to yield the Euler equation

$$C_{it}(h)^{-\frac{1}{\sigma}} = \delta(1+R_t) E_t\left(C_{it+1}(h)^{-\frac{1}{\sigma}} \frac{P_t}{P_{t+1}}\right) \tag{15}$$

summarising the household's intertemporal consumption decisions.[8]

[7] The asset-pricing kernel is the period-t price of a claim to one unit of currency in state s^{t+1} in period $t+1$ divided by the probability of that state occurring conditional on period-t information, $\Pr_t\left(s^{t+1}\right)$. The bond B_{it} is a random variable paying $B_{it}\left(s^{t+1}\right)$ units of currency in state s^{t+1} in period $t+1$. At time t, the household chooses the complete specification of this random variable in all states s^{t+1}. It follows that $E_t\left[Q_{t,t+1} B_{it}\right]$ is the allocation of resources to a portfolio of bonds.

[8] Note that it is an implication of (15) that monetary policy affects aggregate demand in all countries symmetrically.

2.2.2 Wage setting

To model nominal rigidities, it is assumed that wages are set by households in a staggered fashion with random duration of wage contracts analogous to the mechanism in Calvo (1983). In particular, in every period each household in country i is allowed to reset the wage rate it demands for its labour service with a fixed probability $(1-\alpha_i)$. Hence, the wage rate set by household h at time t, $W_{it}^{*}(h)$, is the prevailing wage rate for the household at time $t+\tau$, i.e., $W_{it+\tau}(h) = W_{it}^{*}(h)$, with probability α_i^{τ}, and the expected duration of a contract given by $(1-\alpha_i)^{-1}$. For given wages, employment is determined from the demand side.

When a household resets its wage, it does so to maximise expected utility (10) subject to the demand for its labour (3), its budget constraint (14) and the price setting mechanism just described. For a household changing its wage rate at time t, this is equivalent to maximizing the following function with respect to $W_{it}^{*}(h)$ subject to (3) and (14):[9]

$$E_t \sum_{\tau=0}^{\infty} (\alpha_i \delta)^{\tau} \left[\frac{\sigma}{\sigma-1} C_{it+\tau}(h)^{\frac{\sigma-1}{\sigma}} - \frac{1}{1+\mu} N_{it+\tau}(h)^{1+\mu} \right] \tag{16}$$

The first-order condition becomes

$$E_t \sum_{\tau=0}^{\infty} (\alpha_i \delta)^{\tau} \left[\left(\frac{\xi_i}{1-\xi_i} N_{it+\tau}(h)^{\mu} + C_{it+\tau}(h)^{-\frac{1}{\sigma}} \frac{W_{it}^{*}(h)}{P_{t+\tau}} \right) N_{it+\tau}(h) \right] = 0 \tag{17}$$

It follows that the monopolistically competitive household sets its wage rate so that the marginal utility of income from an extra unit of labour effort is a constant mark-up over the marginal disutility in discounted expected value terms. This captures the standard result in wage bargaining models that the market power of wage setters depends on the elasticity of labour demand (see, e.g., Blanchard and Fisher (1989)).

In the special case with flexible wages where households are allowed to reset the wage each period, the first-order condition collapses to

$$\frac{W(h)_{it}^{*}}{P_t} = \frac{\xi_i}{\xi_i - 1} \frac{N(h)_{it}^{\mu}}{C(h)_{it}^{-\frac{1}{\sigma}}} = \frac{\xi_i}{\xi_i - 1} MRS(h)_{it} \tag{18}$$

[9] This differs from (10) in that implicit terms representing states where the wage to be set is not the prevailing wage are excluded.

where MRS_{it} is the marginal rate of substitution between consumption and leisure.

As shown in appendix A, the first-order condition for the representative household's wage-setting problem (17), the household labour demand relation (3) and the law of motion of the wage index (25) can be used to derive the following wage-setting equation for country i (in logs):

$$\omega_{it} = \Lambda_i \left[mrs_{it} - (w_{it} - p_t) \right] + \delta E_t \omega_{it+1} \tag{19}$$

where $\omega_{it} = w_{it} - w_{it-1}$ is wage inflation, $mrs_{it} = \mu l_{it} + \sigma^{-1} c_t$ is the marginal rate of substitution, and Λ_i is a decreasing function of the Calvo parameter α_i and of the elasticity of substitution between labour services ξ_i:

$$\Lambda_i = \frac{(1-\alpha_i)(1-\alpha_i\delta)}{\alpha_i (1+\mu\xi_i)} \tag{20}$$

For later reference, note that Λ_i depends on both the parameter characterising wage adjustment, α_i, and the parameter determining the mark-up or market power in wage formation, ξ_i.

2.2.3 Risk-sharing

Staggered wage setting implies that households in a given country are not identical, and there is no representative household. In the general case, individual decisions will depend on initial wealth, which implies that decisions will be path dependent, cf. Obstfeld and Rogoff (1995) and Corsetti and Pesenti (2001). This causes both substantial technical problems and problems with multiplicity of equilibria. This has been overcome in the literature either by imposing assumptions precluding wealth transfers or by assuming that risk-sharing arrangements are in place. We choose the latter approach here. By assuming that all households in the monetary union have entered the world with the same level of wealth, the complete-markets assumption implies that they will choose the same consumption levels, i.e., risk-sharing is complete both within and between the member countries of the monetary union.[10] While overcoming technical problems, this assumption also serves the purpose of focusing on the implications of supply side asymmetries across member countries in a monetary union without mixing them with demand

[10] This follows from the first-order conditions of the utility-maximisation problems as all households face the same asset-pricing kernel.

and wealth effects. Risk-sharing implies that $C_{it}(h) = C_{it}$ for all h, and therefore the h index can be dropped in what follows.

Aggregate demand for good j can now be defined as the weighted sum over countries i of (12):

$$D_{jt} = \sum_{i=1}^{I} v_i C_{ijt} = \left(\frac{P_{jt}}{v_j P_t}\right)^{-\theta} C_t \tag{21}$$

where

$$C_t = \sum_{i=1}^{I} v_i C_{it} \tag{22}$$

is defined as aggregate unionwide consumption. International risk-sharing implies that $C_{it} = C_{jt}$ for all i, j. Hence, $C_{it} = C_t$ for all i.

In logs, the Euler equation (15) becomes

$$E_t c_{t+1} = c_t + \sigma\left(r_t - E_t \pi_{t+1}\right) \tag{23}$$

where $r_t \equiv \log(1+R_t)$ and $\pi_{t+1} = p_{t+1} - p_t$ is inflation, and aggregate demand for commodity j (21)

$$d_{jt} = -\theta\left(p_{jt} - p_t\right) + c_t \tag{24}$$

In addition, the complete-markets assumption implies that the fraction $(1-\alpha_i)$ of households in country i changing their wage rates at time t choose the same rate W_{it}^*. The remaining fraction α_i of households continue with the wage rate prevailing at time $t-1$ where the distribution of wage rates is unchanged. Hence, the law of motion of the aggregate wage index in country i is given by

$$W_{it} = \left[\int_0^1 W_{it}(h)^{1-\xi_i}\, dh\right]^{\frac{1}{1-\xi_i}} = \left[\alpha_i W_{it-1}^{1-\xi_i} + (1-\alpha_i)\left(W_{it}^*\right)^{1-\xi_i}\right]^{\frac{1}{1-\xi_i}} \tag{25}$$

2.3 Monetary policy

The aim of this paper is to consider the implications of labour market asymmetries for a given monetary policy. Therefore, a standard monetary policy reaction function is specified, namely a so-called Taylor rule, cf. Taylor

(1993). Specifically, it is assumed that the interest rate is determined by (in logs)[11]

$$r_t = k_\pi \pi_t + k_y \hat{y}_t \tag{26}$$

where $\hat{y}_t = y_t - \bar{y}_t$ is the output gap. The level of 'potential' output, $\bar{y}_t$, used in this definition is the level of output under flexible wages. As shown in appendix B, it is given by

$$\bar{y}_t = \Xi u_t \tag{27}$$

where

$$\Xi = \frac{1+\mu}{1+\mu(1+\beta)+\beta\sigma^{-1}} \tag{28}$$

Note that the level of output under flexible wages is independent of monetary policy.

3 Equilibrium inflation and output

Market clearing requires that demand for each good equals its supply. That is, for all commodities $i = 1, 2, \ldots, I$ the equilibrium conditions read (in logs)

$$d_{it} = y_{it} \tag{29}$$

This implies that there is no aggregate net wealth accumulation or decumulation:

$$c_t = y_t \tag{30}$$

Wages are determined according to (25), and for these wages employment is demand determined from (6). Finally, the interest rate is given by (26).

The steady state is symmetric so that $B_i = 0$, $C_i = C = Y_i = Y$, $R = \delta^{-1} - 1$, $v_i^{-1} P_i = P$ and $W_i = W$. The equilibrium process for the endogenous variables can be explicitly characterized by solving the model by the undetermined coefficients method (see Appendix C).[12]

[11] Empirically interest rate smoothing is important, we disregard it here to simplify the exposition and focus on the role of asymmetries.

[12] The minimal state representation of the equilibrium is followed, cf. McCallum (1983,1999).

Output and inflation in country i are given by

$$y_{it} = \sum_{j=1}^{I} v_j b_0^{ij} u_{jt} + \sum_{j=1}^{I} v_j b_1^{ij} u_{jt-1} + \sum_{j=1}^{I} v_j b_2^{ij} y_{jt-1} \tag{31}$$

and

$$\pi_{it} = \sum_{j=1}^{J} v_j c_0^{ij} u_{jt} + \sum_{j=1}^{J} v_j c_1^{ij} u_{jt-1} + \sum_{j=1}^{J} v_j c_2^{ij} y_{jt-1} \tag{32}$$

Note that the system is in $ARMA\,(1,1)$ form. Aggregate output and inflation follows straightforwardly by aggregation of (31) and (32).

3.1 Numerical illustrations

To illustrate the model's properties, we shall later present numerical simulations. They are made for a two-country version of the model, i.e., the case where $I = 2$ and $i \in \{1, 2\}$. Numerical results are particularly useful since the complexity of the model makes it difficult to extract analytical results. Restricting attention to the two-country case avoids unnecessary complications while illustrating the main properties of the model.

The log-linear version of the model is solved numerically using DYNARE,[13] and the solution is used to calculate moments of the variables of interest. These moments are taken as measures of the performance of the macroeconomic variables. Simulations are performed for country-specific shocks, imperfectly correlated shocks and unionwide shocks, i.e., for the cases $\rho_{\varepsilon} = 0$, $\rho_{\varepsilon} = 0.5$ and $\rho_{\varepsilon} = 1$.[14]

4 Symmetry and shock transmission

To set the scene for the subsequent discussion of heterogeneities, it is useful to consider the symmetric case, i.e., the case where all countries are of equal sizes ($v_i = v$ for all i) and have the same structural parameters ($\alpha_i = \alpha$ and $\xi_i = \xi$ for all i). However, shocks are allowed to differ. This structurally symmetric

[13] For documentation, see www.cepremap.cnrs.fr/dynare/.

[14] The choice of the values for the remaining parameters is inspired by estimated parameters for the euro area in Smets and Wouters (2003). The baseline values are $\alpha_i = 0.75$, $\xi_i = 4$, $\sigma_i^2 = 1$, $\delta = 1$, $\sigma = \frac{2}{3}$, $\mu = 1$, $\theta = 4$, $\gamma = 0.7$, $\rho = 0.95$, $k_{\pi} = 1.5$, $k_y = 0.5$.

case serves the purpose of clarifying the basic mechanisms operating in the model.

4.1 Shock transmission

Consider the transmission of a productivity shock specific to country i. On impact, since prices are fully flexible, this shock will tend to both increase the output of commodity i, y_i, and to decrease it's price, p_i, cf. (8). Hence, the terms of trade change to the disfavour of country i in the sense that the relative price of its export good decreases. However, this works to shift demand from goods produced in other parts of the monetary union towards country i's commodity, cf. (24). In turn, this shift in demand causes the price of foreign products ($p_j,\ j \neq i$) to fall, but not by as much as the price of commodity i.[15] Consequently, the output level in the other country decreases, i.e., a country-specific shock induces a negative correlation in country-specific outputs via the terms of trade effect. The terms of trade effect plays a crucial role in the difference between the volatility of country-specific and aggregate output since from (24) we have

$$Var(y_i) = Var(y) + \theta^2 Var(p_i - p) - 2\theta Cov(p_i - p, y)$$

Note that in the limiting case of perfectly correlated shocks (aggregate shocks), there are no terms of trade changes, and hence $Var(y_i) = Var(y)$ and country-specific outputs are perfectly correlated. Similar reasoning applies to inflation; that is, country-specific prices (inflation) are positively correlated, and more so the larger the correlation in the shocks.

4.2 Output and inflation volatility

As mentioned above, the implications of various structural factors are assessed in terms of the standard deviations and correlations of the two key variables - output and inflation. Figure 1 shows the standard deviation for country-specific as well as aggregate output and inflation as a function of $\alpha = \alpha_1 = \alpha_2$ for various levels of the correlation of shocks.

[15] Note that the decrease in prices triggers a monetary expansion which, in turn, increases activity in both countries. If the response is sufficiently strong, it is possible that output increases in both countries. In the simulations reported, the parameter values ensure that this does not happen; i.e. the direct effects of the shocks described in the text dominate.

First, note a basic smoothing effect in the sense that aggregate output is less volatile than country-specific outputs (except in the limiting case of perfectly correlated shocks where they are equal). This is a direct consequence of smoothing by aggregation since

$$\sigma^2(y) = (v_1)^2\sigma^2(y_1) + (v_2)^2\sigma^2(y_2) + 2v_1v_2\sigma(y_1, y_2) \tag{33}$$

and hence in the symmetric case ($\sigma^2(y_1) = \sigma^2(y_2) = \sigma^2(y_i)$ and $v_1 = v_2 = 1/2$), we have

$$\sigma^2(y) = \frac{1}{2}\sigma^2(y_i)\left[1 + \rho(y_1, y_2)\right] \leq \sigma^2(y)$$

This expression also shows that the negative correlation in output for the member countries (for $\rho_\varepsilon < 1$) described above contributes to lower aggregate volatility. Since country-specific inflation rates are positively correlated, the smoothing effect for aggregate inflation is less strong.

Second, considering the effects of changing the degree of nominal rigidity, α, it is found that country-specific output volatility is strictly increasing in the degree of nominal rigidity if shocks are not too highly correlated. However, aggregate output variability is hump-shaped for all three types of shocks. Consequently, there is a critical level of nominal flexibility, α^*, where aggregate output variability is increasing in α for $\alpha < \alpha^*$, and decreasing in α for $\alpha > \alpha^*$. In the numerical illustration, the critical value α^* is close to one half, corresponding to expected contract lengths of two periods.

To understand the mechanism generating this hump, note that since output is generated by an $ARMA(1,1)$ process, the unconditional variance (in the case of common shocks) is given as

$$VAR(y_t) = \frac{b_0 + b_1^2 + 2b_0b_1b_2}{1 - b_2^2}\frac{1}{1 - \rho_u^2}\sigma_\varepsilon^2 \tag{34}$$

where the coefficients are all functions of the nominal rigidity parameter, α, i.e., $b_0 = b_0(\alpha)$, $b_1 = b_1(\alpha)$ and $b_2 = b_2(\alpha)$. It is seen that the variance depends on the properties of the shock, but also on the endogenous responses captured by the impact effect (b_0) and the persistence-generating mechanisms (b_1,b_2). Note that b_0 is increasing in α, while b_1 and b_2 are decreasing in α. It follows that stronger nominal rigidities (higher α) tend to increase output variability by increasing the impact effects of shocks, and to lower variability by reducing the persistence in the response to shocks. These two counteracting effects create the hump-shaped relation, where output variability is at first increasing and then decreasing in the nominal rigidity (α).

Figure 1 also shows the volatility of country-specific and unionwide inflation. In both cases, the variability of the inflation rate increases when α is low, and at some level it remains almost invariant to changes in α. Hence, while inflation volatility is quite sensitive to changes in the degree of nominal wage rigidity when wages are flexible (low α), it is relatively insensitive to changes in the degree of nominal wage rigidity when the starting point is one with rigid wages (high α).

Finally, considering the importance of imperfect competition (the ξ parameter), we also find (figures not shown) that aggregate output is less volatile than country-specific output (and identical for perfect correlation of shocks). In this case, more competition (higher ξ) implies less volatility of both aggregate output and inflation, but the effect levels off when converging to perfect competition (high ξ). However, country-specific output is more volatile, the higher ξ, unless the shocks are highly correlated. This is due to the implied reduction in the weight, Λ_i, put on the current relation between the marginal rate of substitution and the real wage in wage setting, cf. (20). This reduction in Λ_i leads to a weaker instantaneous reaction of wages and hence, trough its effect on marginal costs, of output. This implies a larger adjustment burden on prices and thus the terms of trade.

5 Asymmetries in size and structure

We now turn to an analysis of the implications of labour market asymmetries between the member countries of the monetary union. To clarify the issues, this proceeds in three steps. First, we consider asymmetries arising from different country sizes ($v_1 \neq v_2$), maintaining symmetric labour market structures across countries. Second, we consider structural asymmetries, while maintaining identical country sizes, with respect to the degree of nominal wage rigidity ($\alpha_1 \neq \alpha_2$) and the degree of competition ($\xi_1 \neq \xi_2$). Finally, we intersect the two dimensions of asymmetry, size and structure.

5.1 Different country sizes

To consider the effects of asymmetric country sizes, note first that it follows from (33) that (using $v_1 + v_2 = 1$)

$$\begin{aligned}\frac{\partial \sigma^2(y)}{\partial v_1} &= 2v_1\sigma^2(y_1) - 2(1-v_1)\sigma^2(y_2) + 2\left[1-2v_1\right]\sigma\left(y_1, y_2\right) \\ &= \sigma^2(y_i)(1-2v_1)2\left[\rho\left(y_1, y_2\right) - 1\right]\end{aligned}$$

If all structural parameters for the two countries are the same (implying $\sigma^2(y_1) = \sigma^2(y_2) = \sigma^2(y_i)$), we have

$$\frac{\partial \sigma^2(y)}{\partial v_1} = \sigma^2(y_i)(1-2v_1)2\left[\rho\left(y_1, y_2\right) - 1\right]$$

and hence

$$\begin{aligned}\frac{\partial \sigma^2(y)}{\partial v_1} &= 0 \qquad for \;\; \rho = 1 \\ \frac{\partial \sigma^2(y)}{\partial v_1} &> 0 \qquad for \; v_1 > \frac{1}{2} \;\; and \;\; \rho < 1 \\ \frac{\partial \sigma^2(y)}{\partial v_1} &< 0 \qquad for \; v_1 < \frac{1}{2} \;\; and \;\; \rho < 1\end{aligned}$$

i.e. asymmetric size exaberates aggregate output volatility unless the correlation of shocks is one. Similar reasoning applies to inflation volatility.

Figure 2 shows the standard deviations of output and inflation as a function of $\alpha = \alpha_1 = \alpha_2$ for different sizes of country 1 (the pattern is symmetric for country 2). This confirms that the level of aggregate output volatility is generally higher when countries are of asymmetric sizes. Moreover, aggregate output volatility is hump-shaped in nominal rigidity (α), as in the case with symmetric country sizes. However, the volatilities of the country-specific output levels differ. Output volatility is generally higher the smaller the country, and an increase in nominal wage rigidity (an increase in α) leads to larger increases in output volatility in the small country than in the large country (unless shocks are highly correlated). This suggests that nominal rigidities may be more problematic for small countries than for large countries.

For inflation, we find also that the level of volatility is generally higher due to asymmetric country sizes. Both aggregate and country-specific inflation tend to be increasing in nominal wage rigidity, but the effect levels off when

nominal rigidities reach a certain level. Country size matters less for country-specific inflation than for country-specific output.

Qualitatively, the effects of variations in the degree of imperfect competition are the same when countries have asymmetric sizes as when they are of equal sizes, but the level of volatility is generally larger with asymmetrically sized countries.

5.2 Asymmetric labour market structures

Next, we turn to the role of asymmetry with respect to structural factors. This issue is complicated by the fact that changing the structural parameters for one country has implications not only for the specific country but also at the aggregate level, and hence there is a risk of mixing up effects arising from asymmetries with effects arising from changing the aggregate properties of the currency union. To overcome this problem and to focus on the role of asymmetries for given aggregate structural characteristics of the monetary union, we keep aggregate variables unchanged in this subsection in the sense that the weighted average of the coefficients across countries is kept constant.[16]

Consider first the degree of wage flexibility. Figure 3 shows the behaviour of the standard deviations of output and inflation for different values of α_1 when the average degree of nominal wage rigidity is restricted to be 0.5, i.e.,

$$\bar{\alpha} = v_1\alpha_1 + v_2\alpha_2 = 0.5$$

For equally sized countries, i.e., for $v_1 = v_2 = 0.5$, this implies

$$\alpha_2 = 1 - \alpha_1$$

Hence, α_2 goes from 1 to 0 as α_1 goes from 0 to 1. In other words, the labour market in country 1 becomes relatively less flexible and the labour market in country 2 more flexible when moving from left to right on the x-axis.

We find that the volatility in aggregate output is somewhat lower with asymmetries in nominal rigidities than with symmetric labour market structures, i.e., asymmetries in nominal wage flexibility contribute to lowering

[16] In the next subsection, we fix the structural parameters of country 2 while allowing those of country 1 to vary in order to analyze the incentives for unilateral reform in country 1.

aggregate output volatility.[17] Similarly, for inflation we find that its volatility is reduced due to asymmetries. Interestingly, the form of the volatility of country-specific output as a function of the degree of nominal rigidity now changes and becomes hump-shaped. This suggests interesting spill-over effects in the structural parameters between the two countries. For country-specific output, we find that small asymmetries may imply a larger volatility in domestic output, while larger asymmetries may cause output volatility to be lower. This suggests that if countries have fairly asymmetric structures, the direction in which there is an incentive to change structural characteristics via reforms may be ambiguous.

Similarly, we consider the role of asymmetries in the parameters characterising the degree of monopolistic competition in labour markets, ξ_i for $i = 1, 2$. The average elasticity of substitution of labour services is restricted to be 4, i.e., $\overline{\xi} = 4$, and hence

$$\xi_2 = 8 - \xi_1$$

It turns out (figures not shown) that this form of asymmetry has essentially no effect on the volatility of domestic and aggregate output or on inflation.

5.3 Unilateral changes in structural characteristics

While the above sheds light on the implications of structural asymmetries, it does not directly clarify the incentives for structural reforms. Such reforms are unilateral and thus lead to changes in structural parameters in one country, leaving structural parameters in other countries unchanged. Assuming that the model's labour market parameters (α_i, ξ_i) can be affected through various structural policies, it is therefore of interest to determine the direction in which a country has an incentive to lead its reforms when the country aims at stabilizing its national output and inflation.

In figure 4, we present standard deviations as a function of α_1 when α_2 is fixed at 0.75. If shocks are uncorrelated, a lower value of α_1 decreases both the output and inflation volatility of country 1. Therefore, in this case a country has an incentive to implement reforms that make wages more flexible. Such

[17] At the unionwide level, the standard deviations are symmetric in the degree of nominal rigidity attaining a maximum at $\alpha_1 = \alpha_2 = 0.5$. This is the point where the two countries are identical. The symmetry, of course, arises because the countries have the same size so that the restriction $\bar{\alpha} = 0.5$ implies that $\alpha_2 = 1 - \alpha_1$.

reforms also tend to stabilize output in country 2, but not necessarily at the aggregate level. In addition, if α_1 is initially very high, the stabilization of country-specific output may be at the expense of a higher volatility of the country-specific inflation.

The picture is less clear if shocks are correlated. In this case, the volatilities of output and inflation in country 1 are hump-shaped in nominal rigidity in country 1. Hence, it is not necessarily in the interest of country 1 to implement labour market reforms inducing more flexibility. Moreover, an intermediary level of nominal flexibility in country 1 may bring about the highest output volatility for country 1, but the lowest level in country 2.

As suggested by the results in the previous subsection, we find that unilateral reforms aimed at changing the degree of monopolistic competition in labour markets have only very limited effect on the volatility of output and inflation. It follows that no incentives to change the degree of competition in labour markets follow from a concern over stabilization of output and inflation.

5.4 Different country-sizes and labour market structures

Finally, we turn to the interaction between the various forms of asymmetries. The preceding analysis has suggested that asymmetries in size and nominal flexibility are the more important both with respect to the aggregate and country-specific performance. In the following, a case is considered in which different country sizes (one country is small, the other large) interact with different degrees of nominal rigidities. The interaction between country sizes and degrees of imperfect competition, and the interaction between degrees of nominal rigidities and degrees of competition have also been considered. However, results are not reported as no new insights are gained by investigating these combinations of asymmetries. The former combination is close to the case with different country sizes only, and the latter to the case with asymmetries in nominal wage rigidity being the only deviation from the symmetric baseline example.

Consider the case where the weight of the small country is $v_1 = 0.3$ and aggregate nominal rigidity is $\overline{\alpha} = 0.75$. Figure 5 shows volatility of output and inflation. We find that more nominal rigidity in the small country – in combination with less nominal rigidity in the large country – has a hump-

shaped effect on aggregate output. This is the same type of result as found above. However, while the volatility of country 2 output is strictly increasing in the nominal rigidity in country 1, the volatility of country 1's output is hump-shaped if shocks are sufficiently correlated. This suggests that there may be an important negative externality from the small to the large country in the sense that more nominal rigidity in the small country may increase output volatility in the large country and decrease it in the small country. For inflation – both country-specific and aggregate – the volatility is hump-shaped. Hence, asymmetries lower inflation variability in this case.

6 Concluding remarks

In this paper, we have analyzed the consequences of labour market asymmetries in a monetary union with focus on degrees of nominal wage rigidity and of monopolistic competition in wage setting. We have considered both aggregate and country-specific shocks and how they are propagated across member countries that may not be equal in size. Moments of country-specific as well as unionwide output and inflation fluctuations have been calculated. These moments are taken as measures of the macroeconomic performance in the monetary union.

First, our results indicate that asymmetry in the sizes of member countries may in itself be an impediment to macroeconomic stability in a monetary union. In particular, the level of output and inflation volatility is generally higher when countries are of different sizes than when they are equal in size. In a monetary union consisting of a large core and a small periphery, the output in the periphery is generally more volatile than in the core. In addition, the periphery is more sensitive to changes in the degree of nominal rigidities.

Second, asymmetry in the degrees of nominal rigidity may smooth aggregate output and inflation volatility, while asymmetry in the degree of monopolistic competition has essentially no effect on the volatility of the macroeconomic variables of interest. Thus, the present analysis suggests that structural asymmetries alone are no hindrance to macroeconomic stability at the unionwide level. At the country level, however, the picture is less clear; our results indicate that there are non-trivial spill-over effects from asymmetries in nominal rigidities.

Third, when shocks are country-specific, i.e., when there is no correla-

tion between shocks hitting the countries in the monetary union, a country unambiguously benefits in terms of macroeconomic stability by pursuing unilaterally structural labour market reforms that reduce wage rigidities. For aggregate shocks hitting the whole monetary union with the same force, however, results are ambiguous.

Forth, we find that structural labour market reforms have different effects on macroeconomic stability at the country level than at the aggregate level. Hence, there is risk of a 'reform deficit' from the unionwide perspective. An individual member country may not have an incentive to reform its labour market unilaterally, while such reforms may be beneficial for the monetary union as a whole.

Given that only a few unambiguous results can be established, the incentives to undertake reform from the point of view of individual member countries, and the desirability of such reform from the point of view of the monetary union as a whole, depend crucially on the structural characteristics of national labour markets. An interesting topic for future research, then, is to estimate this model on data for a monetary union such as the EMU to identify and quantify the important asymmetries. As a final remark, we emphasize that, though we implicitly assume that macroeconomic stability is desirable, our statements about the volatility of macroeconomic variables cannot literally be interpreted as statements about welfare. Hence, in future research, we hope to address this issue – along with normative issues concerning monetary policy responses to structural asymmetries – in more explicit terms.

A Log-linearization

Write the first-order condition (17) as

$$W_{it}^{*}E_t\sum_{\tau=0}^{\infty}\left(\alpha_i\delta\right)^{\tau}\frac{C_{it+\tau}^{-\frac{1}{\sigma}}}{P_{t+\tau}}N_{it+\tau}=E_t\sum_{\tau=0}^{\infty}\left(\alpha_i\delta\right)^{\tau}\frac{\xi}{\left(\xi-1\right)}N_{it+\tau}^{1+\mu} \tag{35}$$

Taking the differential with respect to $W_{it+\tau}^{*}$, $C_{it+\tau}$, $P_{t+\tau}$ and $N_{it+\tau}$, evaluating at the steady-state values – W, C, P and N respectively – dividing through by W and rearranging gives the following log-linear approximation around the steady state:

$$\begin{aligned} v_{it}^{*} &= \left(1-\alpha_i\delta\right)E_t\sum_{\tau=0}^{\infty}\left(\alpha_i\delta\right)^{\tau}\left(\mu n_{it+\tau}+\sigma^{-1}c_{it+\tau}+p_{t+\tau}\right) \qquad (36)\\ &= \left(1-\alpha_i\delta\right)E_t\sum_{\tau=0}^{\infty}\left(\alpha_i\delta\right)^{\tau}\left[\mu\left(l_{it+\tau}-\xi\left(v_{it}^{*}-w_{it+\tau}\right)\right)+\sigma^{-1}c_{it+\tau}+p_{t+\tau}\right] \end{aligned}$$

where the second equality follows by using a log-linear version of (3) to replace $n_{it+\tau}$. Rearranging gives

$$v_{it}^{*}=\frac{\left(1-\alpha_i\delta\right)}{1+\mu\xi}\left(\mu l_{it}-\mu\xi w_{it}+\sigma^{-1}c_{it}+p_t\right)+\left(\alpha_i\delta\right)E_tv_{t+1}^{*} \tag{37}$$

Similarly, a log-linear approximation to (25) is given by

$$w_{it}=\alpha_iw_{it-1}+\left(1-\alpha_i\right)v_{it}^{*} \tag{38}$$

Subtracting w_{it} from both sides of (37) and using (38) to eliminate v_{it}^{*} gives

$$\omega_{it}=\frac{\left(1-\alpha_i\right)\left(1-\alpha_i\delta\right)}{\alpha_i\left(1+\mu\xi\right)}\left[\mu l_{it}+\sigma^{-1}c_{it}-\left(w_{it}-p_t\right)\right]+\delta E_t\omega_{it+1} \tag{39}$$

where $\omega_{it}=w_{it}-w_{it-1}$.

B Flexible wage equilibrium

Suppose wages are flexible as well as prices. In this case, the wage equation becomes

$$w_{it}=p_t+\mu l_{it}+\sigma^{-1}c_t \tag{40}$$

Substituting out l_{it} by a linear version of the production function (5) gives

$$w_{it} = p_t + \mu \left(\frac{1}{\gamma} y_{it} - \frac{1-\gamma}{\gamma} u_{it} \right) + \sigma^{-1} c_t \tag{41}$$

implying

$$w_t = p_t + \mu \left(\frac{1}{\gamma} y_t - \frac{1-\gamma}{\gamma} u_t \right) + \sigma^{-1} y_t \tag{42}$$

Inserting this in aggregated supply

$$y_t = \beta \left(p_t - w_t \right) + u_t \tag{43}$$

gives

$$\bar{y}_t = \beta \left(-\mu \left(\frac{1}{\gamma} \bar{y}_t - \frac{1-\gamma}{\gamma} u_t \right) - \sigma_t^{-1} \bar{y} \right) + u_t \tag{44}$$

or

$$\bar{y}_t = \frac{1+\mu}{1+\mu\left(1+\beta\right)+\beta\sigma^{-1}} u_t \tag{45}$$

C Sticky wage equilibrium

Imposing the equilibrium condition means $c_{it} = c_t = y_t$. Using this, the wage equation (19) and the aggregate supply relation (8) can be combined to give the 'AS' relation

$$\begin{aligned} & \pi_{it} + \beta^{-1} \left[\left(u_{it} - u_{it-1} \right) - \left(y_{it} - y_{it-1} \right) \right] \\ = \; & \Lambda_i \left[\left(\mu + \left(1+\mu \right) \beta^{-1} + \theta^{-1} \right) y_{it} + \left(\sigma - \theta^{-1} \right) y_t - \left(1+\mu \right) \beta^{-1} u_{it} \right] \\ & + \delta \left(E_t \pi_{it+1} + \beta^{-1} \left[\left(\rho_u - 1 \right) u_{it} - \left(E_t y_{it+1} - y_{it} \right) \right] \right) \end{aligned} \tag{46}$$

where a log-linear version of (5) has been used to substitute out l_{it}. Similarly, combining the Euler equation (23), the Taylor rule (26) and the intratemporal demand function (24) gives the 'IS' relation

$$\begin{aligned} & y_{it} - y_{it-1} \\ = \; & -\theta \left(\pi_{it} - \pi_t \right) + E_t y_{t+1} \\ & -\sigma \left(k_\pi \pi_t + k_y \left(y_t - \Xi u_t \right) - E_t \pi_{t+1} \right) - y_{t-1} \end{aligned} \tag{47}$$

Hence, two equations summarise the dynamics of output and inflation for each country. Disturbances to the system follow from the stochastic process

$$u_{it} = \rho_u u_{it-1} + \varepsilon_{it} \tag{48}$$

We guess that output and inflation in country i take the forms:

$$y_{it} = \sum_j v_j b_0^{ij} u_{jt} + \sum_j v_j b_1^{ij} u_{jt-1} + \sum_j v_j b_2^{ij} y_{jt-1} \tag{49}$$

and

$$\pi_{it} = \sum_j v_j c_0^{ij} u_{jt} + \sum_j v_j c_1^{ij} u_{jt-1} + \sum_j v_j c_2^{ij} y_{jt-1} \tag{50}$$

These conjectures imply the following expressions for aggregate output and inflation:

$$y_t = \sum_i \sum_j v_i v_j b_0^{ij} u_{jt} + \sum_i \sum_j v_i v_j b_1^{ij} u_{jt-1} + \sum_i \sum_j v_i v_j b_2^{ij} y_{jt-1} \tag{51}$$

$$\pi_t = \sum_i \sum_j v_i v_j c_0^{ij} u_{jt} + \sum_i \sum_j v_i v_j c_1^{ij} u_{jt-1} + \sum_i \sum_j v_i v_j c_2^{ij} y_{jt-1} \tag{52}$$

In addition, expectations become

$$E_t y_{it+1} = \sum_j v_j \left(b_0^{ij} \rho_u + b_1^{ij} \right) u_{jt} + \sum_j v_j b_2^{ij} y_{jt} \tag{53}$$

$$E_t \pi_{it+1} = \sum_j v_j \left(c_0^{ij} \rho_u + c_1^{ij} \right) u_{jt} + \sum_j v_j c_2^{ij} y_{jt} \tag{54}$$

$$E_t y_{t+1} = \sum_i \sum_j v_i v_j \left(b_0^{ij} \rho_u + b_1^{ij} \right) u_{jt} + \sum_i \sum_j v_i v_j b_2^{ij} y_{jt} \tag{55}$$

$$E_t \pi_{t+1} = \sum_i \sum_j v_i v_j \left(c_0^{ij} \rho_u + c_1^{ij} \right) u_{jt} + \sum_i \sum_j v_i v_j c_2^{ij} y_{jt} \tag{56}$$

To verify our conjectures, we find values of the coefficients $\left(b_0^{ij}, b_1^{ij}, b_2^{ij}, c_0^{ij}, c_1^{ij}, c_2^{ij} \right)$ that satisfy the restrictions imposed by the log-linear model. Inserting the conjectures in (46) gives

$$
\begin{aligned}
&\sum_j v_j c_0^{ij} u_{jt} + \sum_j v_j c_1^{ij} u_{jt-1} + \sum_j v_j c_2^{ij} y_{jt-1} \\
&+\beta^{-1}\left(u_{it} - u_{it-1}\right) - \beta^{-1}\left(\sum_j v_j b_0^{ij} u_{jt} + \sum_j v_j b_1^{ij} u_{jt-1} + \sum_j v_j b_2^{ij} y_{jt-1} - y_{it-1}\right) \\
= \;& \Lambda_i\left(\mu + (1+\mu)\beta^{-1} + \theta^{-1}\right)\left(\sum_j v_j b_0^{ij} u_{jt} + \sum_j v_j b_1^{ij} u_{jt-1} + \sum_j v_j b_2^{ij} y_{jt-1}\right) \\
&+\Lambda_i\left(\sigma - \theta^{-1}\right)\left(\sum_n \sum_j v_n v_j b_0^{nj} u_{jt} + \sum_n \sum_j v_n v_j b_1^{nj} u_{jt-1} + \sum_n \sum_j v_n v_j b_2^{nj} y_{jt-1}\right) \\
&-(1+\mu)\beta^{-1}\Lambda_i u_{it} + \delta \sum_j v_j\left(c_0^{ij}\rho_u + c_1^{ij}\right) u_{jt} \\
&+\delta \sum_n v_n c_2^{in}\left(\sum_j v_j b_0^{nj} u_{jt} + \sum_j v_j b_1^{nj} u_{jt-1} + \sum_j v_j b_2^{nj} y_{jt-1}\right) \\
&+\delta\beta^{-1}\left(\rho_u - 1\right) u_{it} - \delta\beta^{-1}\sum_j v_j\left(b_0^{ij}\rho_u + b_1^{ij}\right) u_{jt} \\
&-\delta\beta^{-1}\sum_j v_j b_2^{ij}\left(\sum_n v_n b_0^{jn} u_{nt} + \sum_n v_n b_1^{jn} u_{nt-1} + \sum_n v_n b_2^{jn} y_{nt-1}\right) \\
&+\delta\beta^{-1}\left(\sum_j v_j b_0^{ij} u_{jt} + \sum_j v_j b_1^{ij} u_{jt-1} + \sum_j v_j b_2^{ij} y_{jt-1}\right) \qquad (57)
\end{aligned}
$$

Equating coefficients on u_{it} gives the restriction

$$
\begin{aligned}
& v_i c_0^{ii} + \beta^{-1} - \beta^{-1} v_i b_0^{ii} \\
= \;& \Lambda_i\left(\mu + (1+\mu)\beta^{-1} + \theta^{-1}\right) v_i b_0^{ii} + \Lambda_i\left(\sigma - \theta^{-1}\right)\sum_j v_j v_i b_0^{ji} \\
& -(1+\mu)\beta^{-1}\Lambda_i + \delta v_i\left(c_0^{ii}\rho_u + c_1^{ii}\right) + \delta \sum_j v_j v_i c_2^{ij} b_0^{ji} \\
& +\delta\beta^{-1}\left(\rho_u - 1\right) - \delta\beta^{-1} v_i\left(b_0^{ii}\rho_u + b_1^{ii}\right) \\
& -\delta\beta^{-1}\sum_j v_j v_i b_2^{ij} b_0^{ji} + \delta\beta^{-1} v_i b_0^{ii} \qquad (58)
\end{aligned}
$$

Equating coefficients on u_{jt} where $j \neq i$ gives

$$\begin{aligned} & c_0^{ij} - \beta^{-1} b_0^{ij} \\ = \; & \Lambda_i \left(\mu + (1+\mu)\beta^{-1} + \theta^{-1}\right) b_0^{ij} + \Lambda_i \left(\sigma - \theta^{-1}\right) \sum_n v_n b_0^{nj} \\ & + \delta \left(c_0^{ij} \rho_u + c_1^{ij}\right) + \delta \sum_n v_n c_2^{in} b_0^{nj} \\ & - \delta\beta^{-1} \left(b_0^{ij} \rho_u + b_1^{ij}\right) - \delta\beta^{-1} \sum_n v_n b_2^{in} b_0^{nj} + \delta\beta^{-1} b_0^{ij} \end{aligned} \tag{59}$$

Equating coefficients on u_{it-1}:

$$\begin{aligned} & v_i c_1^{ii} - \beta^{-1} - \beta^{-1} v_i b_1^{ii} \\ = \; & \Lambda_i \left(\mu + (1+\mu)\beta^{-1} + \theta^{-1}\right) v_i b_1^{ii} \\ & + \Lambda_i \left(\sigma - \theta^{-1}\right) \sum_n v_n v_i b_1^{ni} + \delta \sum_n v_n v_i c_2^{in} b_1^{ni} \\ & - \delta\beta^{-1} \sum_n v_n v_i b_2^{in} b_1^{ni} + \delta\beta^{-1} v_i b_1^{ii} \end{aligned} \tag{60}$$

Equating coefficients on $u_{jt-1}$where $j \neq i$:

$$\begin{aligned} & c_1^{ij} - \beta^{-1} b_1^{ij} \\ = \; & \Lambda_i \left(\mu + (1+\mu)\beta^{-1} + \theta^{-1}\right) b_1^{ij} \\ & + \Lambda_i \left(\sigma - \theta^{-1}\right) \sum_n v_n b_1^{nj} + \delta \sum_n v_n c_2^{in} b_1^{nj} \\ & - \delta\beta^{-1} \sum_n v_n b_2^{in} b_1^{nj} + \delta\beta^{-1} b_1^{ij} \end{aligned} \tag{61}$$

Equating coefficients on y_{it-1} gives

$$\begin{aligned} & v_i c_2^{ii} - \beta^{-1} \left(v_i b_2^{ii} - 1\right) \\ = \; & \Lambda_i \left(\mu + (1+\mu)\beta^{-1} + \theta^{-1}\right) v_i b_2^{ii} + \Lambda_i \left(\sigma - \theta^{-1}\right) \sum_n v_n v_i b_2^{ni} \\ & + \delta \sum_n v_n v_i c_2^{in} b_2^{ni} - \delta\beta^{-1} \sum_n v_n v_i b_2^{in} b_2^{ni} + \delta\beta^{-1} v_i b_2^{ii} \end{aligned} \tag{62}$$

and on $y_{jt-1}$where $j \neq i$:

$$\begin{aligned} & v_j c_2^{ij} - \beta^{-1} v_j b_2^{ij} \\ = \; & \Lambda_i \left(\mu + (1+\mu)\beta^{-1} + \theta^{-1}\right) v_j b_2^{ij} + \Lambda_i \left(\sigma - \theta^{-1}\right) \sum_n v_n v_j b_2^{nj} \\ & + \delta \sum_n v_n v_j c_2^{in} b_2^{nj} - \delta\beta^{-1} \sum_n v_n v_j b_2^{in} b_2^{nj} + \delta\beta^{-1} v_j b_2^{ij} \end{aligned} \tag{63}$$

Inserting conjectures in (47) gives

$$\begin{aligned} & \sum_j v_j b_0^{ij} u_{jt} + \sum_j v_j b_1^{ij} u_{jt-1} + \sum_j v_j b_2^{ij} y_{jt-1} - y_{it-1} \\ = \; & -\theta \left(\sum_j v_j c_0^{ij} u_{jt} + \sum_j v_j c_1^{ij} u_{jt-1} + \sum_j v_j c_2^{ij} y_{jt-1} \right) \\ & + (\theta - \sigma k_\pi) \left(\sum_n \sum_j v_n v_j c_0^{nj} u_{jt} + \sum_n \sum_j v_n v_j c_1^{nj} u_{jt-1} + \sum_n \sum_j v_n v_j c_2^{nj} y_{jt-1} \right) \\ & + \sum_n \sum_j v_n v_j \left(b_0^{nj} \rho_u + b_1^{nj} \right) u_{jt} \\ & + \sum_m \sum_n v_m v_n b_2^{mn} \left(\sum_j v_j b_0^{nj} u_{jt} + \sum_j v_j b_1^{nj} u_{jt-1} + \sum_j v_j b_2^{nj} y_{jt-1} \right) \\ & - \sigma k_y \left(\sum_n \sum_j v_n v_j b_0^{nj} u_{jt} + \sum_n \sum_j v_n v_j b_1^{nj} u_{jt-1} + \sum_n \sum_j v_n v_j b_2^{nj} y_{jt-1} \right) + \sigma k_y \Xi \sum_j v_j u_{jt} \\ & + \sigma \sum_n \sum_j v_n v_j \left(c_0^{nj} \rho_u + c_1^{nj} \right) u_{jt} \\ & + \sigma \sum_m \sum_n v_m v_n c_2^{mn} \left(\sum_j v_j b_0^{nj} u_{jt} + \sum_j v_j b_1^{nj} u_{jt-1} + \sum_j v_j b_2^{nj} y_{jt-1} \right) - \sum_j v_j y_{jt-1} \end{aligned} \tag{64}$$

Equating coefficients on u_{jt} gives

$$
\begin{aligned}
& b_0^{ij} \\
= \; & -\theta c_0^{ij} + (\theta - \sigma k_\pi) \sum_n v_n c_0^{nj} + \sum_n v_n \left(b_0^{nj} \rho_u + b_1^{nj} \right) \\
& + \sum_m \sum_n v_m v_n b_2^{mn} b_0^{nj} - \sigma k_y \sum_n v_n b_0^{nj} + \sigma k_y \Xi v_j + \sigma \sum_n v_n \left(c_0^{nj} \rho_u + c_1^{nj} \right) \\
& + \sigma \sum_m \sum_n v_m v_n c_2^{mn} b_0^{nj} \qquad (65)
\end{aligned}
$$

On u_{jt-1}:

$$
\begin{aligned}
& b_1^{ij} \\
= \; & -\theta c_1^{ij} + (\theta - \sigma k_\pi) \sum_n v_n c_1^{nj} + \sum_m \sum_n v_m v_n b_2^{mn} b_1^{nj} \\
& - \sigma k_y \sum_n v_n b_1^{nj} + \sigma \sum_m \sum_n v_m v_n c_2^{mn} b_1^{nj} \qquad (66)
\end{aligned}
$$

y_{it-1}:

$$
\begin{aligned}
& v_i b_2^{ii} - 1 \\
= \; & -\theta v_i c_2^{ii} + (\theta - \sigma k_\pi) \sum_n v_n v_i c_2^{ni} + \sum_m \sum_n v_m v_n b_2^{mn} v_i b_2^{ni} \\
& - \sigma k_y \sum_n v_n v_i b_2^{ni} + \sigma \sum_m \sum_n v_m v_n c_2^{mn} v_i b_2^{ni} - v_i \qquad (67)
\end{aligned}
$$

and finally, y_{jt-1} where $j \neq i$:

$$
\begin{aligned}
& b_2^{ij} \\
= \; & -\theta c_2^{ij} + (\theta - \sigma k_\pi) \sum_n v_n c_2^{nj} + \sum_m \sum_n v_m v_n b_2^{mn} b_2^{nj} \\
& - \sigma k_y \sum_n v_n b_2^{nj} + \sigma \sum_m \sum_n v_m v_n c_2^{mn} b_2^{nj} - 1 \qquad (68)
\end{aligned}
$$

The restrictions (58)-(63) and (65)-(68) constitute a system of $6I^2$ equations determining the $6I^2$ coefficients in the conjectures. Indeed, this system is recursive. The $2I^2$ restrictions from equating coefficients on y_{it-1} may be combined to solve for $\{b_2^{ij}, c_2^{ij}\}_{ij}$, which may then be used in the $2I^2$ restrictions from u_{it-1} to solve for $\{b_1^{ij}, c_1^{ij}\}_{ij}$. Finally, these coefficients may be used in the restrictions from equation coefficients on u_{it} to find the remaining $2I^2$ coefficients $\{b_0^{ij}, c_0^{ij}\}_{ij}$.

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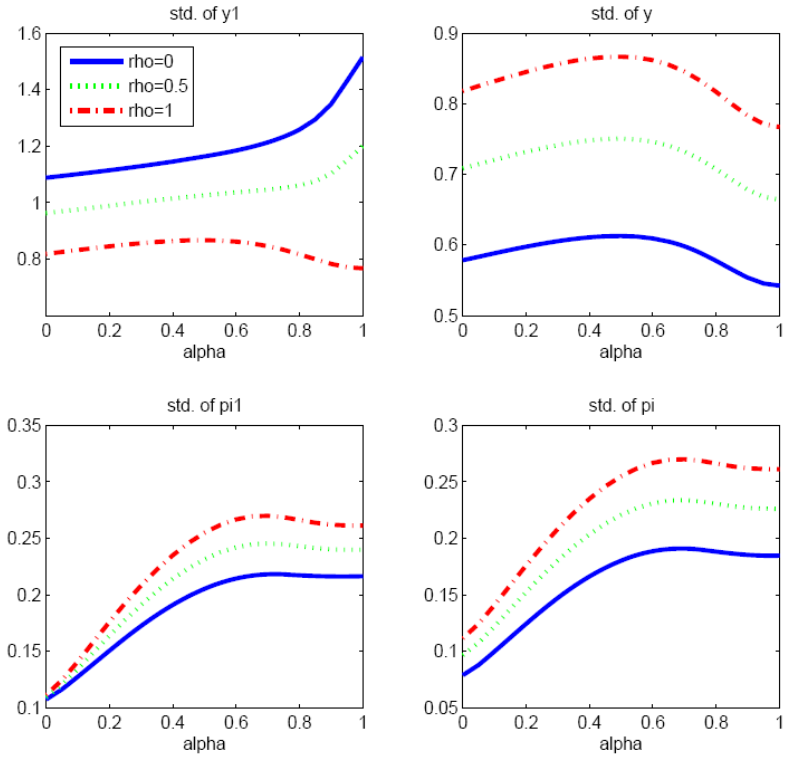

Figure 1: Standard deviations of country-specific and aggregate output and inflation as functions of nominal rigidity (α); symmetric structures ($\alpha_1 = \alpha_2 = \alpha; \xi_1 = \xi_2 = \xi; v_1 = v_2 = 0.5$) and different levels of shock correlations (ρ_ε).

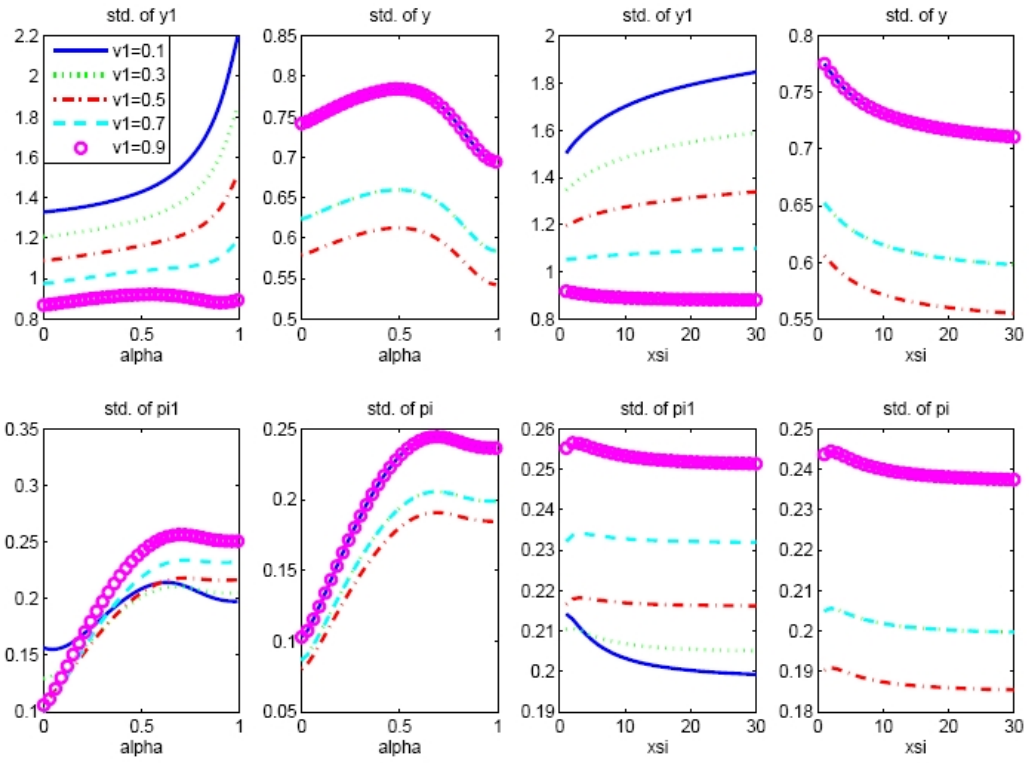

Figure 2: Standard deviations of country 1 and aggregate output and inflation as functions of country size (v_1); symmetric structures ($\alpha_1 = \alpha_2 = \alpha; \xi_1 = \xi_2 = \xi;$) and country-specific shocks ($\rho_\varepsilon = 0$).

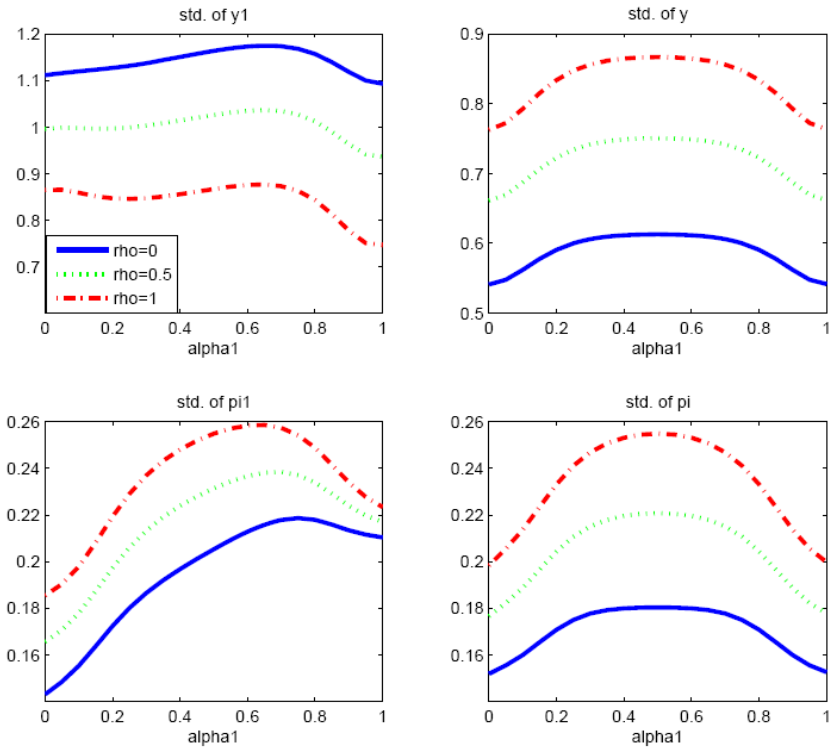

Figure 3: Standard deviations of country 1 and aggregate output and inflation as functions of country 1 nominal rigidity (α_1); average nominal rigidity $\overline{\alpha} = 0.5$, symmetry wrt. to competition and size ($\xi_1 = \xi_2 = \xi; v_1 = v_2 = 0.5$) for different levels of shock correlations (ρ_ε).

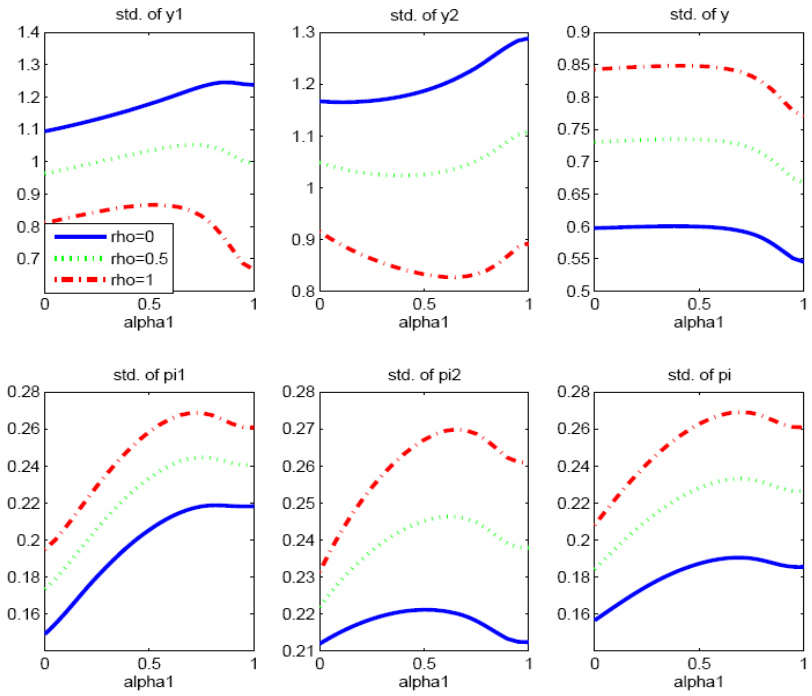

Figure 4: Standard deviations of country-specific and aggregate output and inflation as functions of a unilateral change in country 1 nominal rigidity (α_1); symmetry wrt. to competition and size ($\xi_1 = \xi_2 = \xi; v_1 = v_2 = 0.5; \alpha_2 = 0.75$) for different levels of shock correlations (ρ_ε)

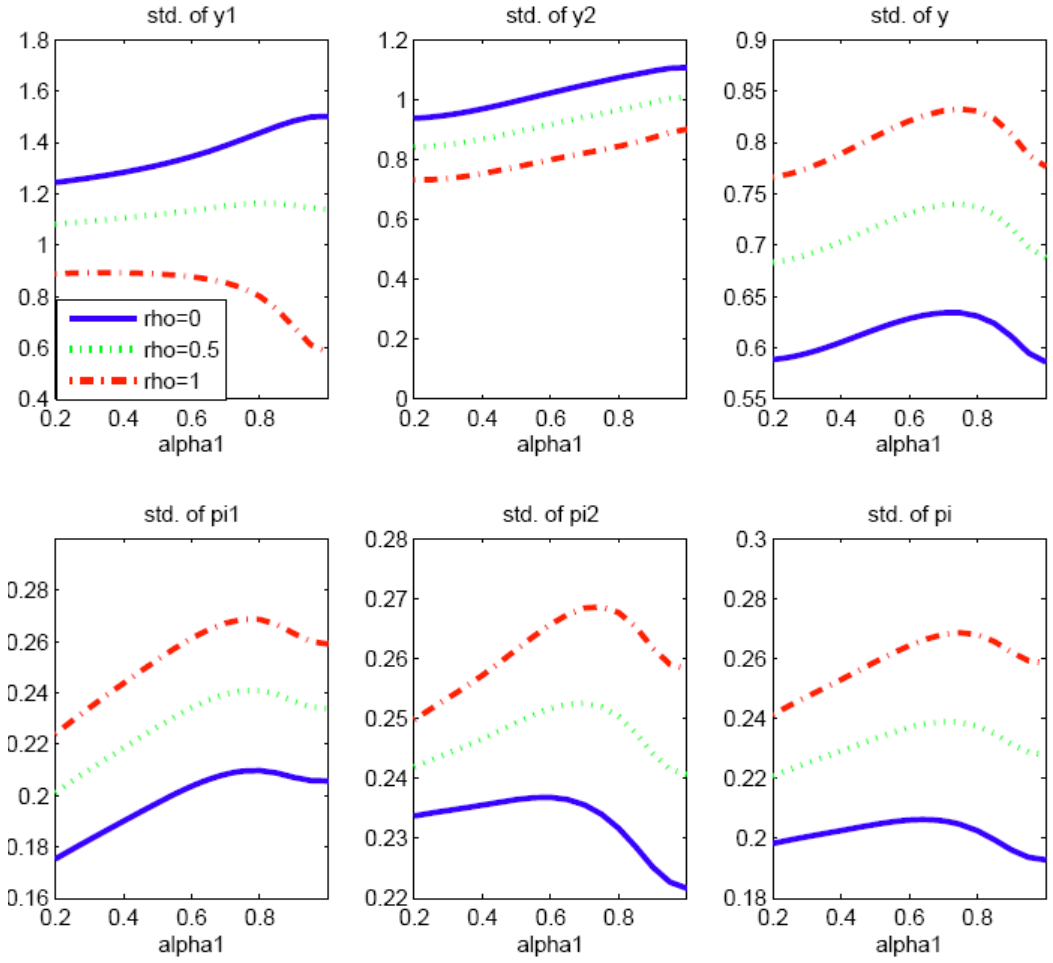

Figure 5: Standard deviations of country 1 and 2, and aggregate output and inflation as functions of country 1 nominal rigidity (α_1); average nominal rigidity $\overline{\alpha} = 0.75$, asymmetric sizes ($v_1 = 0.3, v_2 = 0.7$), symmetry wrt. to competition ($\xi_1 = \xi_2 = \xi$) for different levels of shock correlations (ρ_ε).

Chapter 2

Monetary policy and welfare in a monetary union with labour market heterogeneity*

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Abstract

How should monetary policy be conducted in a monetary union when labour market structures differ across member countries as is arguably the case in the euro area? This paper develops a dynamic stochastic general equilibrium model of a two-country monetary union with labour market heterogeneity to answer this question. Asymmetries in labour market structures are proxied for by different degrees of nominal flexibility. A welfare loss function derived as a second-order approximation to household utility is evaluated. Results suggest that price inflation targeting may lead to non-negligible welfare losses compared to monetary policy alternatives when the important nominal rigidity is in the labour market. Welfare may be noticeably improved in such a case by targeting wage inflation, especially when shocks are highly correlated across member countries and labour markets are very heterogeneous. Noticeable welfare improvements can be obtained by putting more weight on fighting wage inflation in the more rigid labour market if shocks are less than perfectly correlated.

JEL classification: E32

Key words: Inflation targeting, monetary policy, nominal wage rigidity, monetary union, business cycles, shocks

*I thank, without implicating, Torben M. Andersen for very useful comments on a very early draft of this paper.

1 Introduction

This paper addresses the question of how monetary policy should be conducted in a monetary union in which labour market structures are different across member countries. It it well known that such labour market asymmetries are characteristic for the European Monetary Union (EMU), not least in terms of labour market flexibility, cf. for instance Arpaia and Pichelmann (2007) or Holden and Wulfsberg (2008) for recent documentation. While these labour market asymmetries are frequently mentioned in discussions of labour market policies, surprisingly little research effort has been devoted to understanding the implications of labour market asymmetries for monetary policy. In particular, little is known about how monetary policy should be designed in a monetary union with labour market asymmetries.[1]

This paper takes a first step in providing answers to this question. It develops a dynamic stochastic general equilibrium (DSGE) model of a two-country monetary union with monopolistic competition in labour markets and nominal wage rigidities. Labour market asymmetries are modelled by allowing the degrees of monopolistic competition and of nominal rigidity to differ across the monetary union's member countries. Disturbances to the union may be common across the member countries or idiosyncratic. The paper proceeds by evaluating the welfare of alternative monetary policy rules that the central bank may contemplate, emphasising the potential need to deal with labour market asymmetries across the union. Following the approach of Rotemberg and Woodford (1999), welfare is evaluated using a quadratic loss function derived from second-order Taylor approximations to the levels of expected household utility. This exercise is a first important step in understanding how monetary policy is best designed when monetary policy is common across structurally diverse, interdependent economic regions.

The current practice in most central banks pursuing an independent monetary policy is to emphasise the stabilisation of goods price inflation based on the development of a consumer price index. This includes the European Central Bank (ECB) responsible for monetary policy in the EMU. This practice finds theoretical justification in the New Keynesian literature in which nomi-

[1] Beetma and Jensen (2004) allow for labour market asymmetries in a model of a monetary union but focus on monetary and fiscal policy interaction. Dellas and Tavlas (2005) present a three-country model allowing for asymmetries in nominal wage flexibility, and find that countries with a high degree of nominal wage rigidity are better off in a monetary union.

nal goods price rigidity is a distinguishing feature, see for instance Woodford (2003). In the basic New Keynesian model, nominal price rigidities lead to an inefficient allocation of resources in the economy unless the constraints represented by nominal rigidities do not bind. This happens in the model when inflation is stabilised.

As argued by Woodford (2003), allowing for nominal wage rigidities is probably not crucial if the objective of the analysis is to construct a positive model of the co-movements of output and inflation. But it is by no means obvious that nominal price rigidities are more important than nominal wage rigidities from an empirical perspective. Christiano et al. (2005), for instance, provide evidence of the contrary using US data. Nor does it appear to be empirically realistic to abstract from nominal wage rigidities. Though Smets and Wouters (2003) estimate a higher degree of nominal price rigidity than nominal wage rigidity for the euro area, their estimates suggest that nominal wage rigidities are substantial; the expected duration of wage contracts is estimated to be about one year.

Furthermore, nominal wage rigidities matter in important ways for a welfare-theoretic assessment of the proper goals of monetary policy as shown by Erceg et al. (2000). In a closed-economy with both nominal price and wage rigidities, they show that the optimal monetary policy places greater weight on stabilising inflation in the more rigid variable, prices or wages. That is, from a welfare-theoretic perspective, central banks should stabilise the nominal variables that fail to adjust so as to make the adjustment constraint non-binding, preventing misallocation in the markets characterised by rigidities. Indeed, they find that inflation targeting of the sort that is often considered to be a good approximation to actual central bank behaviour may induce substantial welfare losses when the important nominal rigidity is in the labour market as opposed to the goods market. A central bank operating with a seemingly empirically successful sticky price model may therefore seriously misjudge the welfare implications of its actions if sizeable nominal wage rigidities are present.

In a monetary union, the assessment of the welfare implications of alternative monetary policy prescriptions is further complicated by the fact that member countries often have very different characteristics along a number of dimensions of importance for the economic decision making of agents in the economy. Benigno (2004) addresses this question for the case of structural differences in goods markets. Specifically, he shows that optimal policy in a two-country monetary union in which nominal price rigidities differ across

the two member countries is such that a higher weight is given to fighting goods price inflation in the country with the highest degree of nominal price rigidity. Moreover, Lombardo (2006) assesses the welfare losses of simple monetary policy rules, including goods price inflation targeting allowing for asymmetries also in the degree of monopolistic competition in goods markets. He shows that the union's central bank should put a higher weight on the more competitive country. In fact, if the two countries differ sufficiently in terms of market power in goods markets, Benigno's (2004) result may be overturned as the central bank might optimally assign a higher weight to the country with more flexible prices if competition is sufficiently fierce in this country. This indicates that asymmetries in structural characteristics have important implications for the design of monetary policy.

As shown by Andersen and Seneca (2008), differences in structural features in labour markets across a monetary union have non-trivial implications for the propagation of shocks across the union and for the incentives for structural labour market reforms. This indicates that labour market asymmetries may be important for monetary policy, and providing a better understanding of these implications therefore seems an urgent task that should be of particular interest to policy makers in Europe, where such labour market asymmetries are present. To do so, this paper abstracts from other asymmetries that may characterise the monetary union so as to isolate the effects stemming from labour market asymmetries. The interesting question of how these asymmetries interact with other potential asymmetries is left for future research. Hence, structural characteristics in goods markets, preferences etc. will be symmetric across the union. In addition, goods prices will be assumed to be perfectly flexible. While this is a strong assumption, it serves the purpose of keeping a strict labour market perspective. Thus, what is allowed to differ are characteristics of the labour markets in which households offer their labour services. That is, in an important market for household welfare, households face different immediate economic environments as both the degree of market power in their wage setting and the expected duration of their wage contracts - or more generally employment contracts - may differ across the monetary union, though the emphasis will be on asymmetries in wage rigidities.

In this paper, the relative welfare under alternative simple monetary policy rules is investigated under the assumption that taxes are designed to off-set the distortions from monopolistic competition in the economy's steady state. This serves to focus attention on distortions induced by dif-

fering degrees of nominal wage rigidity. The monetary policies considered are, first, flexible monetary policy rules according to which the central bank may respond to price inflation, union-wide or national wage inflation, and the welfare-relevant output gap. Second, strict targeting rules are considered that lead to successful stabilisation of either aggregate price inflation, aggregate or country-specific wage inflation, or the output gap.

An important question for this paper to answer is whether inflation targeting by a monetary union's central bank similar to the one used by the ECB leads to substantial welfare losses when the important nominal rigidities are in the labour market, and, most importantly, when labour markets are characterised by differing degrees of nominal rigidity.

The results suggest that this may well be the case. In particular, when prices are flexible and wages rigid, flexible as well as strict price inflation targeting regimes lead to non-negligible welfare losses. Welfare may be noticeably improved by targeting wage inflation. The mistake made by targeting price inflation rather than wage inflation is found to be larger as shocks become more highly correlated across the monetary union's member countries, and as the degree of heterogeneity in the labour market structure increases. Finally, further welfare improvements can be obtained by putting more weight on fighting wage inflation in the country with a more rigid labour market, especially if shocks are idiosyncratic and labour market structures highly asymmetric.

The paper is organised as follows. Section 2 presents the model and derives its log-linear representation. Section 3 presents the welfare function used to evaluate alternative monetary policy rules. Section 4 presents the welfare analysis for a calibrated version of the model. Section 5 concludes.

2 The model

The model economy consists of two countries in a monetary union. Each country has a large number of households and a large number of firms. There is one central bank responsible for monetary policy throughout the union. In particular, the central bank sets the risk-free interest rate R_t earned on one-period risk-free bonds in the union's single financial market. There is no active fiscal policy for stabilisation purposes, and the union is closed to the outside world.

Labour is immobile across borders, and national labour markets are char-

acterised by monopolistic competition and nominal rigidities in the form of Calvo (1983) wage contracts of random duration. With monopolistic competition, each household supplies a differentiated labour service and has a certain degree of market power in setting the wage it demands for this service. Given the wage chosen, the household stands ready to supply the work hours demanded by firms. These assumptions lead to a downward-sloping demand curve for each household's labour service. Although both the degree of market power in wage setting and the degree of the nominal rigidities are allowed to differ between the two countries, emphasis is put on implications of differences in the expected duration of wage contracts. Such differences are taken to represent heterogeneity in labour market structure across the monetary union.

All firms in a given country are assumed to produce the same internationally traded good in a competitive market, but this good is differentiated from the goods produced by firms abroad. This leads to a downward-sloping demand curve for each country's product. To focus on the labour market, prices of goods are assumed to be perfectly flexible. It is assumed that the weight of each product in the consumption bundle is the same in both countries. Hence, there is no home bias in consumption. The weight assigned to each product is interpreted as the relative size of the country producing it.

2.1 Firms

The representative firm in country $i \in \{0, 1\}$ produces output Y_{it} according to the production function

$$Y_{it} = A_{it} N_{it}^{\gamma} \tag{1}$$

where A_{it} is the stochastic period-t productivity of firms in country i, and $0 < \gamma < 1$ is the degree of returns to scale. W_{it} represents aggregate wages in country i paid for the aggregate labour input into production, N_{it}, as described below. Real capital is disregarded to simplify, but decreasing returns can be interpreted as arising from a second factor of production in fixed supply.

The representative firm in country i maximises profits, which it distributes to households. There are no nominal price rigidities, and the firm takes the price of its product, P_{it}, as given. The profit maximisation problem yields

a demand for aggregated labour services defined by the relation

$$\frac{(1-t_i^e)\,W_{it}}{P_{it}} = \gamma A_{it} N_{it}^{\gamma-1} \tag{2}$$

when t_i^e is a fixed-rate employment subsidy paid to firms (and financed by lump-sum taxes, cf. below).[2] The labour demand relation equates the real wage (as perceived by firms) to the marginal product of labour.

2.2 Households

In each period t, each household h in country i supplies a differentiated labour service, $N_{it}(h)$. The labour used in production in country i, N_{it}, is assumed to be an aggregate of the continuum of labour services supplied by the households:

$$N_{it} = \left[\int_0^1 N_{it}(h)^{\frac{\xi_i-1}{\xi_i}}\,dh\right]^{\frac{\xi_i}{\xi_i-1}} \tag{3}$$

where $\xi_i > 1$ is the elasticity of substitution between labour services.

Each household sets the wage rate it demands for its labour service as described below and satisfies firms' labour demand at the chosen wage. That is, given existing wage contracts, household h's labour effort is determined by demand. This demand for household h's labour service is determined by the cost minimisation problems of the country's firms, which minimise costs taking households' wage rates, $W_{it}(h)$, as given. This leads to a demand for household h's labour service given by

$$N_{it}(h) = \left(\frac{W_{it}(h)}{W_{it}}\right)^{-\xi_i} N_{it} \tag{4}$$

when W_{it} is the wage index with the property that the minimum cost of N_{it} units of aggregate labour is given by $W_{it}N_{it}$. It follows that the demand for household h's labour service is a decreasing function of the household's relative wage with elasticity ξ_i. Hence, ξ_i is inversely related to the degree of market power in wage setting.

[2] The employment subsidy is used to neutralise the distorting effect from monopolistic competition in the steady state around which the model is log-linearised. This facilitates the welfare analysis of monetary policy alternatives emphasising implications of nominal wage rigidities.

Household h in country $i \in \{1,2\}$ has the utility function

$$E_t \sum_{\tau=0}^{\infty} \delta^{\tau} \left[\frac{\sigma}{\sigma-1} C_{it+\tau}(h)^{\frac{\sigma-1}{\sigma}} - \frac{1}{1+\mu} N_{it+\tau}(h)^{1+\mu} \right] \tag{5}$$

where E_t is an operator representing expectations over all states of the economy conditional on period-t information, $\delta \in (0,1)$ is the subjective discount factor, and $C_{it}(h)$ is a real consumption index. $\sigma > 0$ is the elasticity of intertemporal substitution of consumption, and $\mu > 0$ is the inverse of the Frisch labour elasticity.

The consumption index is defined over the differentiated commodities produced in the union's member countries. Specifically,

$$C_{it} = \left[v_1^{\frac{1}{\theta}} C_{i1t}^{\frac{\theta-1}{\theta}} + v_2^{\frac{1}{\theta}} C_{i2t}^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}} \tag{6}$$

where $\theta > 0$, v_j is the relative size of country $j \in \{1,2\}$, and C_{ijt} represents consumption of country j's commodity by households in country i. In every period t, households choose C_{ijt} for a given level of real consumption to minimise consumption expenditures. This yields a demand for country j's product in country i given by

$$C_{ijt} = v_j \left(\frac{P_{jt}}{P_t} \right)^{-\theta} C_{it} \tag{7}$$

when P_t is the price index defined by

$$P_t = \left[v_1 P_{1t}^{1-\theta} + v_2 P_{2t}^{1-\theta} \right]^{\frac{1}{1-\theta}} \tag{8}$$

This price index has the property that the minimum cost of C_{it} units of real consumption is given by $P_{it}C_{it}$.

Asset markets are assumed to be complete, i.e., available financial assets completely span the possible states of the economy.[3] This assumption leads to the following period-t flow budget constraint for a household in country i:

$$E_t \left[Q_{t,t+1} B_{it}(h) \right] + P_t C_{it}(h) + T_{it} = B_{it-1}(h) + W_{it}(h) N_{it}(h) + \Pi_{it} \tag{9}$$

[3]Note that this is likely to decrease the potential welfare improvements through stabilisation policy as it provides an insurance mechanism for households. In particular, households share consumption risk through these complete markets so that consumption levels are equalised across the monetary union.

The right-hand side gives available resources as the sum of initial financial wealth, $B_{it-1}(h)$, labour income, $W_{it}(h) N_{it}(h)$, and nominal profit income, Π_{it}. The left-hand side represents the allocation of resources to consumption, $P_t C_{it}(h)$, bond-holdings, $E_t[Q_{t,t+1} B_{it}(h)]$, where $Q_{t,t+1}$ is the asset pricing kernel, and to a lump-sum tax, T_{it}, used by the government to finance an employment subsidy paid to firms.

Households choose real consumption and wages to optimise expected utility (5) subject to the sequence of budget constraints (9) and labour demand (4).[4] Defining the net risk-free nominal interest rate R_t by the relation

$$(1 + R_t)^{-1} = E[Q_{t,t+1}] \tag{10}$$

the first-order conditions determining the optimal choice of consumption and bond holdings can be combined to yield the Euler equation

$$C_t^{-\frac{1}{\sigma}} = \delta (1 + R_t) E_t \left(C_{t+1}^{-\frac{1}{\sigma}} \frac{P_t}{P_{t+1}} \right) \tag{11}$$

where the international risk-sharing property that consumption is equalised across households and countries has been used.[5]

Wages are set by households in a staggered fashion. Following Calvo (1983), each household in country i is allowed to reset the wage rate it demands for its labour service with a fixed probability $(1 - \alpha_i)$. Hence, the wage rate set by household h at time t, $W_{it}^*(h)$, is the prevailing wage rate for the household at time $t + \tau$, i.e., $W_{it+\tau}(h) = W_{it}^*(h)$, with probability α_i^τ, and the expected duration of a contract is given by $(1 - \alpha_i)^{-1}$. The complete-markets assumption implies that the fraction $(1 - \alpha_i)$ of households in country i changing their wage rates at time t choose the same rate W_{it}^*. The remaining fraction α_i of households continue with the wage rate prevailing at time $t - 1$ where the distribution of wage rates is unchanged. Hence, the law of motion of the aggregate wage index in country i is given

[4] Implicitly, optimisation is also subject to a solvency condition that may be used to transform the sequence of flow budget constraints into a single life-time budget constraint. This has no effect on the first-order conditions, and since a log-linearised version of the model around its steady state is analysed here, the bounded stochastic shock processes specified below will ensure solvency at all times and in all states.

[5] Throughout, aggregate variables are indicated by the omission of country subindices and are given as averages of national variables with relative country sizes as weights.

by

$$W_{it} = \left[\int_0^1 W_{it}(h)^{1-\xi_i}\, dh\right]^{\frac{1}{1-\xi_i}} = \left[\alpha_i W_{it-1}^{1-\xi_i} + (1-\alpha_i)\left(W_{it}^*\right)^{1-\xi_i}\right]^{\frac{1}{1-\xi_i}} \quad (12)$$

When a household resets its wage, it does so to maximise expected utility (5) subject to the demand for its labour (4), its budget constraint (9) and the price setting mechanism just described. For a household changing its wage rate at time t, this is equivalent to maximizing the following function with respect to $W_{it}^*(h)$ subject to (4) and (9):[6]

$$E_t \sum_{\tau=0}^{\infty} (\alpha_i \delta)^\tau \left[\frac{\sigma}{\sigma-1} C_{it+\tau}(h)^{\frac{\sigma-1}{\sigma}} - \frac{1}{1+\mu} N_{it+\tau}(h)^{1+\mu}\right] \quad (13)$$

The first-order condition can be written as

$$E_t \sum_{\tau=0}^{\infty} (\alpha_i \delta)^\tau \left[\left(\frac{\xi_i}{1-\xi_i} N_{it+\tau}(h)^{\mu} + C_{t+\tau}^{-\frac{1}{\sigma}} \frac{W_{it}^*(h)}{P_{t+\tau}}\right) N_{it+\tau}(h)\right] = 0 \quad (14)$$

It follows that the monopolistically competitive household sets its wage rate so that the marginal utility of income from an extra unit of labour effort is a constant mark-up over the marginal disutility in discounted expected value terms.

Note that in the special case with flexible wages in which all households are allowed to reset the wage each period, the first-order condition collapses to

$$\frac{W_{it}}{P_t} = \frac{\xi_i}{\xi_i - 1} N_{it}^{\mu} C_t^{\frac{1}{\sigma}} \quad (15)$$

where $N_{it} = N_{it}(h)$ and $W_{it} = W_{it}^*(h)$ for all h. That is, wages are set so as to equalise the real wage (as perceived by the household) to a mark-up over the marginal rate of substitution.

2.3 Log-linear representation

Welfare will be evaluated using solutions to the model in log-linear form in which variables are expressed in log-deviations from the steady state with

[6] This differs from (5) in that implicit terms representing states where the wage to be set is not the prevailing wage are excluded.

stable wages.[7]

A log-linear version of the production function (1) is given by

$$y_{it} = a_{it} + \gamma n_{it} \tag{16}$$

Technology, a_{it}, is assumed follow a first-order autoregressive process

$$a_{it} = \rho_a a_{it-1} + \varepsilon_{it} \tag{17}$$

where the innovations, ε_{it}, are $N(0, \sigma_i^2)$ and may be correlated across countries as governed by the correlation parameter $\rho_\varepsilon \in [0, 1]$ .These innovations are assumed to be the only shocks to the monetary union.

A log-linearisation of the labour demand relation gives

$$w_{it} - p_{it} = a_{it} - (1 - \gamma) n_{it} \tag{18}$$

Note that the employment subsidy drops out of the log-linear labour demand relation. Hence, it has no effect on the dynamic responses to shocks in the log-linearised economy.

Log-linearisations of the first-order condition from the households' wage setting problem (14) and the law of aggregate wages (12) can be combined to yield a New Keynesian Phillips curve for wage inflation:[8]

$$\omega_{it} = \Lambda_i \left[\sigma^{-1} c_t + \mu n_{it} - (w_{it} - p_t)\right] + \delta E_t \omega_{it+1} \tag{19}$$

where $\omega_{it} = w_{it} - w_{it-1}$ is wage inflation in country i, and Λ_i is a decreasing function of the Calvo parameter α_i and of the elasticity of substitution between labour services ξ_i:

$$\Lambda_i = \frac{(1 - \alpha_i)(1 - \alpha_i \delta)}{\alpha_i (1 + \mu \xi_i)} \tag{20}$$

The supply side of the model is thus summarised by equations (16)-(19) for $i \in \{0, 1\}$. Note that (19) collapses to the labour supply relation

$$w_{it} - p_t = \sigma^{-1} c_t + \mu n_{it} \tag{21}$$

[7] Steady state variables are indicated by the omission of time subscripts, and lower-case letters denote log-deviations from steady-state values of corresponding upper-case variables.

[8] See Andersen and Seneca (2008, app. A) for details on the derivation.

in the virtual equilibrium with full wage flexibility, i.e. for $\alpha_i = 0$.

The demand side is represented by a log-linear version of the Euler equation (11) given as

$$c_t = E_t c_{t+1} - \sigma \left(r_t - E_t \pi_{t+1} \right) \tag{22}$$

where $\pi_t = p_t - p_{t-1}$ is price inflation, and a demand relation for the product produced in each country i

$$y_{it} = -\theta \left(p_{it} - p_t \right) + y_t \tag{23}$$

The latter is found by summing (7) over countries i, log-linearising, and imposing the equilibrium condition

$$y_{it}^d = y_{it} \tag{24}$$

where y_{it}^d is the aggregate demand in the monetary union for products produced in country i. Note that monetary policy affects the economy through the aggregate demand relation (22) only. In this sense, its effect on the two countries is symmetric.

2.4 Monetary policy

The model is closed by specifying the monetary policy reaction function. Two types of monetary policy rules are considered: Explicit flexible targeting rules and implicit strict targeting rules. Both types are within the class of simple monetary policy rules in the sense that they are operational rules that may be considered realistic in an actual monetary policy regime rather than the outcome of a Ramsey problem that the central bank may attempt to solve.[9]

With a flexible targeting rule, the central bank sets the risk-free interest rate in response to a vector of endogenous variables. Attention is restricted to rules that may be stated in log-linear form as

$$r_t = k_\pi \pi_t + k_1 \omega_{1t} + k_2 \omega_{2t} + k_y \hat{y}_t \tag{25}$$

where $\hat{y}_t = y_t - y_t^n$ is the output gap given as output in excess of the level of output in the virtual flexible wage equilibrium. As shown in appendix A, this "natural" level of output can be found by combining equations (16), (18), (21) and (23). It reads

$$y_t^n = \frac{1 + \mu}{(1 - \gamma) + \mu + \gamma \sigma^{-1}} a_t$$

[9] For a thorough discussion of optimal vs. simple policy rules, see Woodford (2003).

For $k_1 = k_2 = 0$, (25) reduces to the monetary policy rule specified by Taylor (1999), at least up to the definition of the output gap. Similarly, for $k_1 = k_2 = k_y = 0$, the rule reduces to a very simple rule according to which the central bank is only concerned about price inflation. More generally, however, (25) allows the central bank to respond not only to average wage inflation ($k_1 = k_2 > 0$), but also to place different weights on wage inflation in the two countries ($0 < k_1 \neq k_2 > 0$). In the welfare analysis below, it is of particular interest to investigate if welfare is improved by allowing the central bank to do so.

With a strict targeting rule, the central bank is assumed to successfully stabilise union-wide price inflation, national wage inflation in country 1, national wage inflation in country 2, aggregate wage inflation, or the output gap. Given these restrictions imposed on the equilibrium dynamics, the paths of the central bank's policy instrument will be complicated functions of the shocks to the economy. The evaluations of these rules are important benchmarks in an analysis of the relative importance of alternative target variables for monetary policy.

3 The welfare function

Following the approach of Rotemberg and Woodford (1999), the social welfare measure used to assess alternative monetary policy rules is derived from the utility functions of households. The welfare function gives the welfare losses experienced by households when the economy deviates from its efficient path expressed as a percentage of steady-state consumption. As shown in appendix B, under the assumption that the distortions associated with monopolistic competition have been eliminated in the steady-state by appropriate choices for the employment subsidies in the two countries (see further discussion below), the average period welfare loss can be expressed as

$$\mathcal{L}_t = \lambda_y VAR\left(\hat{y}_t\right) + \lambda_s E\left(\hat{s}_t^2\right) + \sum_{i=1}^{2} \lambda_{\omega i} VAR\left(\omega_{jt}^2\right) + t.i.p. \tag{26}$$

where $\hat{y}_t = y_t - y_t^n$ is the output gab, $\hat{s}_t = s_t - \tilde{s}_t^n$ is a terms-of-trade gap for the terms of trade defined as $s_t = p_{2t} - p_{1t}$, and $t.i.p.$ represents terms independent of policy that may safely be ignored when comparing monetary policy alternatives. The function parameters are given as

$$\begin{aligned}
\lambda_y &= \frac{1}{2}\left[\sigma^{-1} + \frac{(1-\gamma)+\mu}{\gamma}\right] \\
\lambda_s &= \frac{1}{2}\frac{(1+\mu)\,\theta^2}{\gamma} v_1 v_2 \\
\lambda_{\omega i} &= \frac{1}{2}\frac{v_i \xi_i \gamma \alpha_i \,(1+\mu\xi_i)}{(1-\alpha_i)\,(1-\alpha_i\delta)} = \frac{1}{2}\frac{v_i \xi_i \gamma}{\Lambda_i}
\end{aligned}$$

Welfare will be evaluated by the solutions to the model in log-linear form as described above. Given the loss function derived here, this gives a valid welfare ranking of policy alternatives only when the steady state is efficient; i.e. when the distortion from monopolistic competition that leads to an inefficiently low output level (regardless of whether nominal rigidities might be present) has been eliminated, cf. Kim and Kim (2003) and Woodford (2003, ch. 6). Otherwise, fluctuations around the steady state would have an asymmetric impact on welfare as positive shocks would push the economy closer towards the efficient level and so actually improve welfare, and negative shocks would push the economy a long way away from the efficient level and so reduce welfare considerably. To take account of such effects, a second-order approximation to the model's structural relations is required.

The efficient allocation is characterised by the efficiency conditions that the marginal rate of substitution equals the marginal product of labour in each country.[10] Combining (2) and (15) gives

$$\gamma A_{it} N_{it}^{\gamma-1} \frac{P_{it}}{P_t} = \frac{(1-t_i^e)\,\xi_i}{\xi_i - 1} C_t^{\frac{1}{\sigma}} N_{it}^{\mu} \tag{27}$$

which reduces to

$$\gamma A_i N_i^{\gamma-1} = \frac{(1-t_i^e)\,\xi_i}{\xi_i - 1} C^{\frac{1}{\sigma}} N_i^{\mu} \tag{28}$$

in the steady state. Hence, any remaining gap between the marginal rate of substitution and the marginal product of labour as a consequence of market power in wage setting can be eliminated in the steady state by setting $t_i^e = \xi_i^{-1}$. This assumption will be maintained in the welfare analysis.

[10] Formally, this can be verified as the outcome of a sequence of static social planner's problems maximising a weighted average of the instantaneous utility functions of a generic household in each country subject to the economy's resource constraints.

From (26), welfare losses can be seen to be driven by variation in four variables, namely the union-wide output gap, the terms-of-trade gap, and the two national wage inflation levels. Variation in these variables are related to the remaining distortions in the economy. The sources of these remaining inefficiencies are the nominal rigidities represented by staggered wage contracts of random duration, and an international externality similar to the one identified by Corsetti and Pesenti (2001).

The wage inflation and aggregate output gap fluctuations appear in (26) for the same reasons that they appear in loss functions for closed economies with nominal wage rigidity, cf. for instance Erceg et al. (2000) and Galí (2008, ch. 6). With nominal wage rigidities, wages are not fully adjusted in response to shocks to the economy, and the average mark-up of the real wage (as perceived by the household) over the marginal rate of substitution will generally differ from the desired one prevailing in the flexible wage equilibrium. In addition, the staggered wage setting implied by the random duration of contracts under the Calvo wage-setting scheme leads to a relative wage distortion and so to a suboptimal allocation of hours across households. Only when the economy is stabilised at a point where households have no incentives to change their wages, given the prevailing wages and current shocks to the economy, will the constraint represented by the nominal rigidity become unbinding. In this case, wages are stabilised and the flexible wage allocation is attained. The only difference to the closed economy is that the loss function weighs the contribution from wage inflation in each of the two countries according to its size and structural characteristics.

In the limiting case where $v_1 = 1$ and $v_2 = 0$, (26) reduces to the loss function for a closed economy with sticky wages and flexible prices (see for instance Galí, 2008, ch. 6):

$$\mathcal{L}_t = \lambda_y VAR\left(\hat{y}_t\right) + \lambda_{\omega 1} VAR\left(\omega_{1t}^2\right) \tag{29}$$

Similarly, when the two countries are identical and shocks are common to both countries, there are no terms-of-trade movements and the loss function becomes

$$\mathcal{L}_t = \lambda_y VAR\left(\hat{y}_t\right) + \lambda_{\omega} VAR\left(\omega_t^2\right) \tag{30}$$

where $\lambda_\omega = \lambda_{\omega 1} = \lambda_{\omega 2}$ and $\omega_t = \omega_{1t} = \omega_{2t}$. In this particular case, the monetary union is isomorphic to a standard New Keynesian economy with flexible prices and sticky wages, and the optimal policy takes a very simple form. The central bank should simply keep wages stable, and it follows

from (19) that the flexible wage equilibrium will automatically be attained. Moreover, given that an employment subsidy ensures efficiency in the steady state, (27) implies that this flexible wage allocation will also be efficient.

This result can be generalised further to the case with structural heterogeneity. This is because wage changes are the only mechanism through which common shocks can be propagated differently in the two countries. The structural features that are allowed to differ across countries are all contained in the composite parameter Λ_i in (19). Hence, if wage inflation is somehow successfully stabilised in each of the two countries, common shocks will induce identical behaviour of other national variables. This means that there can be no terms-of-trade changes, and so the only remaining potential source of welfare loss would be the output gap. By a similar argument as above, however, zero wage inflation is associated with a zero output gap. Hence, in the case of common shocks and structural heterogeneity, optimal monetary policy would be one that successfully stabilises wage inflation across the monetary union.[11]

In the general case with less than perfectly correlated shocks, the central bank faces a trade-off between stabilisation of all the variables entering the social welfare function. This is because the central bank needs to consider an international externality in addition to the distortions created by the nominal rigidities themselves. This extra distortion is reflected in the terms-of-trade gap term in (26). Notice also from (27) that, as a consequence of movements in the terms of trade, the "natural" flexible wage equilibrium is generally inefficient even in the case of an employment subsidy ensuring the efficiency of the steady state around which the model's structural relations are approximated. Consequently, the terms-of-trade gap entering (26) is defined not simply as the deviation of the terms of trade from its "natural" level under flexible wages, but rather as the deviation from a multiple of this "natural" level. Only in the special case where the intertemporal elasticity of substitution is one will the flexible wage equilibrium be efficient in the general case (see the appendix for details).

The international externality arises because movements in the terms of trade works to shift demand between goods produced in each of the two countries. For instance, an increase in the terms of trade defined as $s_t = p_{2t} - p_{1t}$

[11] This leaves open the question of whether the central bank is able to do so given that it has one instrument to its disposal, the effect of which goes through aggregate demand, cf. (22). See the welfare analysis below for further discussion.

works to shift demand from goods produced in country 2 to goods produced in country 1. This decreases the disutility from work in country 2 without reducing the utility from consumption, which stay constant as a consequence of international risk sharing. Hence, a positive technology shock in country 1, which increases output in country 1 and reduces the relative price of its product, effectively allows country 2 to act as a monopolist increasing its relative price and reducing its output. This means that more hours can be devoted to leisure without reducing the opportunity for consumption of other goods.[12] The flip-side of this, of course, is that households in country 1 work more without increasing their consumption. The presence of nominal rigidities works to amplify this effect on impact. This follows since wage adjustment costs will prevent workers in country 1 from increasing wages as much as they would have done without the restrictions of the Calvo wage-setting mechanism. As a consequence, firms demand more labour and the increase in output is larger than in the case of flexible wages. This, in turn, amplifies the terms-of-trade effect through the equilibrium effect on prices.

As noted above, the weights with which the output gap, the terms-of-trade gap and the wage inflation fluctuations enter the loss function depend on the model's structural parameters. Considering the weight on the output gap first, note that

$$\frac{\partial \lambda_y}{\sigma} < 0, \frac{\partial \lambda_y}{\gamma} < 0, \frac{\partial \lambda_y}{\mu} > 0 \tag{31}$$

That is, the weight on the output gap is decreasing in σ and γ, the elasticity of intertemporal substitution and the degree of returns to scale, respectively, and increasing in μ, the inverse of the Frisch labour elasticity. This is because a reduction in σ or γ, or an increase in μ, increases the inefficiency gap between the marginal product of labour and the marginal rate of substitution for any given deviation from the flexible wage equilibrium. To see this, consider (27) augmented with an inefficiency gap, gap_{it}, defined residually so

[12] In this economy with perfect risk sharing, consumption stays constant. In a more general setting, consumption of other goods may fall depending on the degree of substitutability, but as long as goods produced in the two countries are substitutes ($\theta > 1$), the reduction in utility from a fall in consumption will be smaller than the reduction in disutility from the change in workload. For $\theta = 1$, the two effects would cancel out. See De Paoli (2007) for a discussion in relation to a small open economy with sticky prices and flexible wages.

as to make the relation hold in the sticky wage equilibrium

$$\gamma A_{it} N_{it}^{\gamma-1} \frac{P_{it}}{P_t} = gap_{it} C_t^{\frac{1}{\sigma}} N_{it}^{\mu} \tag{32}$$

and notice that a given deviation of consumption and hours from their "natural" values would increase the value of gap_{it} needed to make this relation hold if σ or γ are decreased, or μ increased.

Similarly,

$$\frac{\partial \lambda_s}{\mu} > 0, \frac{\partial \lambda_s}{\theta} > 0, \frac{\partial \lambda_s}{\gamma} < 0 \tag{33}$$

which means that the weight on the terms-of-trade gap is increasing in μ and θ, and decreasing in γ. Intuitively, an increase in θ increases the effect of terms-of-trade movements on demand, a reduction in γ increases the change in hours needed to adjust to the change in demand, and an increase in μ amplifies the utility effect of such changes. Moreover, this weight is decreasing in the degree of asymmetry in size as reflected in $v_1 v_2$ (which takes its maximum for $v_1 = v_2 = 0.5$).

Finally, note that

$$\frac{\partial \lambda_{\omega i}}{v_i} > 0, \frac{\partial \lambda_{\omega i}}{\alpha_i} > 0, \frac{\partial \lambda_{\omega i}}{\xi_i} > 0, \frac{\partial \lambda_{\omega i}}{\mu} > 0, \frac{\partial \lambda_{\omega i}}{\gamma} > 0 \tag{34}$$

The first of these derivatives reflects the effect of relative size on the aggregate welfare function as described above, effectively because the central bank weighs welfare according to the mass of economic activity in the two countries. The second of the derivatives illustrates that an increase in the degree of nominal wage rigidity in a country increases the country's contribution to the monetary union's welfare loss. This indicates that the central bank should be more concerned about wage inflation in the country with the most rigid labour market as reflected in the expected duration of wage contracts. The third of the derivates suggests that the central bank should be more concerned about wage inflation in the country with the most competitive labour markets. In other words, the welfare implications of given nominal wage rigidities may be larger if the labour market is very competitive.[13] Finally, an increase in μ increases the effect of any given nominal

[13] This effect is similar to the finding of Lombardo (2006) that a central bank in a two-country monetary union with nominal price rigidities should be more concerned about price inflation arising in the member country with the most competitive goods market.

rigidity by reducing the slope of the Phillips curve for wage inflation, while an increase in γ amplifies the effect of a given suboptimal allocation of hours across households through the aggregate labour input.

4 Welfare analysis

This section evaluates a number of monetary policy alternatives using the loss function derived in the previous section. The loss function is evaluated using a solution to the log-linearised model in section 2 when the model's parameters are set at values commonly used in the literature. The use of the log-linear structural relations means that the welfare analysis is validly ranking monetary policy alternatives according to their ability to counter distortionary effects from nominal rigidities. The analysis abstracts from welfare improvements that could be made by an appropriately designed monetary policy to undo the steady state distortions from monopolistic competition. Instead, it is assumed that fiscal policy neutralises this distortion in the steady state around which the model is log-linearised.

4.1 Calibration

The model's parameters are set to values that are within the ranges considered in the literature and not too far from those estimated for the euro area, for instance by Smets and Wouters (2003). It is important to note, however, that the objective here is to illustrate particular features of the economy, rather than to build a full empirical model.

Considering the parameters governing preferences first, the value for the subjective discount factor is set to $\delta = 0.99$, corresponding to a steady-state interest rate of four per cent with the interpretation that a period corresponds to a quarter. In the baseline calibration, utility is assumed to be logarithmic in consumption ($\sigma = 1$), and the Frisch elasticity of labour is set to unity ($\mu = 1$). The elasticity of substitution between goods produced in the two countries is set to $\theta = 4$ in the baseline calibration. This is probably in the high end of empirical estimates, but it serves to highlight the effects in play in the model. A lower value is considered in the sensitivity analysis, and though this variable has a substantial influence on the level of welfare losses, the qualitative implications for the choice between monetary policy alternatives are robust to the changes in θ considered. In the baseline

calibration, the weights to the two countries' goods in the real consumption index are equalised to one half, i.e. $v_1 = v_2 = 0.5$.

The baseline calibration of the supply-side parameters is given by $\gamma = 0.7$, $\xi_1 = \xi_2 = 4, \rho_u = 0.9$ and $\sigma_1 = \sigma_2 = 0.15$. This corresponds to an economy with a labour share of 70 per cent and a wage mark-up of approximately 33 per cent. The technology shocks considered are temporary but very persistent. In the welfare analysis below, the size of these shocks as determined by the standard deviation are crucial in determining the levels of the welfare losses incurred under alternative monetary policy rules. For a quantitative assessment of the level of welfare losses it is therefore important to consider shocks of a size that is empirically plausible. However, this is not absolutely crucial for the purposes of this paper because the emphasis is on the contribution of labour market heterogeneity to the relative levels of welfare losses under alternative monetary policy rules. A more important qualification regards the absence of other shocks that may affect resource allocations under nominal rigidities. There is considerable controversy in the macroeconomic literature concerning the relative importance of different shocks in driving the business cycle (see for example Galí and Rabanal, 2005), and a quantitative assessment of the level of welfare losses would need to take the contribution of other shocks (e.g. demand shocks) to the fluctuations in endogenous variables into account. Concerning the correlation of shocks across countries, three cases are considered. In the first, $\rho_\varepsilon = 0$ and shocks are uncorrelated. This corresponds to purely country-specific shocks. In the second, $\rho_\varepsilon = 0.5$ and shocks are correlated put imperfectly so. And finally in the third, $\rho_\varepsilon = 1$ and shocks are perfectly correlated across countries. This corresponds to common shocks to the monetary union.

Concerning the degree of nominal rigidity, nine cases are considered that differ with respect to the average degree of nominal rigidity in the monetary union and the degree of heterogeneity across countries. The average degree of nominal rigidity is measured by the average expected duration of wage contracts under the Calvo wage setting scheme:

$$AED = \frac{v_1}{1-\alpha_1} + \frac{v_2}{1-\alpha_2} \tag{35}$$

Similarly, the degree of heterogeneity in nominal rigidities is measured by the relative expected duration defined as

$$RED = \frac{1-\alpha_1}{1-\alpha_2} \tag{36}$$

Welfare is evaluated for combinations of $AED \in \{3,4,5\}$ and $RED \in \{1,2,3\}$.

Finally, six combinations of values for the parameters in the flexible targeting rules are considered. First, a price inflation rule with $(k_\pi, k_1, k_2, k_y) = (1.5, 0, 0, 0)$, a symmetric wage inflation rule with $(k_\pi, k_1, k_2, k_y) = (0, 1.5, 1.5, 0)$, and an asymmetric wage inflation targeting rule with $(k_\pi, k_1, k_2, k_y) = (0, 1.1, 2, 0)$ so that the central bank puts more weight on the member country with the more rigid labour market (which is always country 2). Second, each of these rules are combined with a positive parameter on the output gap; i.e. three additional rules are considered with $(k_\pi, k_1, k_2, k_y) = (1, 5, 0, 0, 0.5/4)$, $(k_\pi, k_1, k_2, k_y) = (0, 1.5, 1.5, 0.5/4)$, and $(k_\pi, k_1, k_2, k_y) = (0, 1.1, 2, 0.5/4)$. The values of the parameters are of the size suggested by Taylor (1993).

4.2 Main results

Results from the baseline calibration are presented in tables 1-3 for RED given by 1, 2 and 3 quarters, respectively. That is, table 1 presents results for a symmetric monetary union, table 2 presents results for an asymmetric union where wage contracts are expected to be twice as long in country 2 as in country 1, and table 3 presents results for a highly heterogeneous monetary union with wage contracts thrice as long in country 2 as in country 1. In each table, the left panel gives results for $AED = 3$ corresponding to an average expected duration of wage contracts of three quarters, the middle panel gives results for $AED = 4$ equivalent to an average expected duration of one year, and the right panel for $AED = 5$. In each panel, the first column shows results for country-specific shocks with $\rho_\varepsilon = 0$, the second column for imperfectly correlated shocks with $\rho_\varepsilon = 0.5$, and the third column for common shocks with $\rho_\varepsilon = 1$.

Consider the symmetric case in table 1 first. When shocks are common, welfare losses can be eliminated if the central bank avoids targeting price inflation – at least up to the fourth decimal place of a percentage of steady state consumption. In this flexible-price economy, when the central bank does not interfere with the price adjustment process, movements in wages that would lead to an inefficient allocation can be avoided. All the central bank needs to do is to stand ready to respond to deviations from zero wage inflation so as to ensure determinacy; i.e. to rule out the possibility of ending up in a situation with inherent instability. The welfare losses resulting from both a flexible and a strict inflation rule are high compared to the other cases

considered, and they are increasing in the average degree of nominal rigidity. This indicates that if shocks are common to the monetary union and the important nominal rigidity is in the labour market, a central bank responsible for monetary policy in this union may inflict non-negligible welfare losses on the economy if its key target variable is price inflation.

When shocks are idiosyncratic, the welfare losses from inflation targeting policies are smaller than in the case of common shocks. Again, welfare losses are increasing the average expected duration of contracts. The efficient allocation can no longer be achieved, but welfare may be improved by letting the central bank respond to wage inflation instead of price inflation. In this case, the lowest welfare loss is obtained by targeting aggregate wage inflation, and a flexible wage inflation rule results in the same welfare loss as a strict aggregate wage inflation rule. In addition, the same welfare loss is achieved by allowing the central bank to respond to the output gap in addition to aggregate wage inflation in the flexible rule and to strictly target the output gap. This suggests that these alternatives may be close to achieving the best monetary policy can do within the class of monetary policy rules considered. Not surprisingly, given the symmetry of labour market structures in this case, welfare losses increase if the central bank is more concerned about wage inflation in one of the two member countries. Note also that strictly targeting wage inflation in one country increases welfare losses in comparison with targeting aggregate wage inflation. This is because stabilisation in one country comes at the expense of greater fluctuations in the other country.

Results for imperfectly correlated shocks are in between the two cases with idiosyncratic and common shocks, respectively. Welfare losses are higher under price inflation targeting than when shocks are uncorrelated, but they are noticeably smaller under wage inflation targeting regimes. Again, the results suggest that the central bank should target wages either by following a flexible wage inflation rule responding to aggregate wage inflation, or by strictly targeting aggregate wage inflation or the output gap. Consequently, as the gap between welfare losses under price and wage inflation targeting increases in the correlation of shocks, it appears that the mistake made by targeting price inflation rather than wage inflation when the important nominal rigidity relates to wages and not goods prices is more serious when shocks are highly correlated. The reason for this is the international externality working through the terms of trade, which is absent when shocks are common to the two countries. By dampening price inflation, the central bank also dampens the terms-of-trade movements. This in itself has a beneficial effect

on welfare off-setting some of the welfare losses resulting from the interference with the price adjustment process that would otherwise facilitate wage stability.

Table 2 presents results for the case with labour market heterogeneity in the sense that wage contracts can be expected to last twice as long in country 2 as in country 1. As discussed above, the optimal monetary policy in case of common shocks and labour market heterogeneity is one that successfully stabilises inflation in each of the two member countries. From table 2, it is seen that the optimal policy can be implemented as long as the central bank does not target price inflation but rather stands ready to react to deviations from the zero wage inflation target. Note also that the mistake made by targeting price inflation is larger in the case with labour market heterogeneity than in a symmetric monetary union. As before, welfare losses under price inflation targeting are increasing in the average degree of nominal rigidity.

When shocks are uncorrelated, welfare losses are all positive. Price inflation targeting either through a flexible rule or through strict targeting leads to considerably higher welfare losses than the other policy alternatives considered. Thus, the symmetric flexible wage inflation rule performs better than the flexible price inflation rule. Moreover, welfare may be noticeably improved by putting more weight on wage inflation in the more rigid country and less on the more flexible one. In addition, responding to the output gap as well further improves welfare. Notice also that strictly targeting wage inflation in the more rigid country comes close to achieving the same welfare loss as the asymmetric flexible rules, while the best policy response is one that stabilises the output gap completely. This indicates that further improvements in welfare may be achieved by carefully selecting the values of parameters in the flexible targeting rule. It this connection, it is important to note that it may not be possible for an actual central bank to respond to the output gap as it is defined here. The output gap entering the loss function is the welfare relevant output gap defined as output in deviations from the virtual flexible wage output. This generally makes the output gap unobservable, and targeting an output gap derived, say, by filtering an output series will be a very poor approximation to targeting $\hat{y}_t$, cf. for instance Sbordone (2002).

Results for shocks with $\rho_\varepsilon = 0.5$ fall in between results for idiosyncratic and common shocks with heterogeneous labour markets as well. Again, the best performing monetary policy under the baseline calibration is strict output gap targeting, and wage inflation targeting performs noticeably better

than price inflation targeting. But although welfare is improved by taking the differences in wage rigidity into account, the benefit from doing so is smaller in this case than in the case with idiosyncratic shocks.

Table 3 presents results for a highly asymmetric monetary union with a relative expected duration of wage contracts set to three. The main results discussed above go through to this case. For common shocks, welfare losses can be eliminated by abstaining from price inflation targeting. For idiosyncratic shocks, wage inflation targeting is better than price inflation targeting, and welfare can be further improved by assigning a higher weight to the country in which labour markets are more rigid. In addition, also targeting the output gap improves welfare, and the lowest welfare loss is obtained by strictly targeting the output gap. Again, welfare losses are increasing in the degree of nominal rigidity. The main difference is that the welfare improvement obtained by following an asymmetric wage inflation rule is higher in this case, where labour market structures are very asymmetric. Similarly, strictly targeting wage inflation in the more rigid country becomes more appealing as labour markets are more heterogeneous, and this is second only to the strict output targeting rule in this case.

4.3 Sensitivity analysis

Table 4 presents a sensitivity analysis for the case with $AED = 4$ and $RED = 2$ (cf. the middle panel in table 2). Alternative values are considered for σ, μ and θ, in particular $\sigma = 2/3$, $\mu = 0.2$ and $\theta = 1.5$.[14]

The left panel of table 4 presents results for $\sigma = 2/3$, keeping the remaining parameters at the baseline values. This change in the value of the intertemporal elasticity of substitution has no effect on the main results presented above. The main difference is that welfare losses are larger under the price inflation targeting regimes. A contributing factor to this result is the curvature effect from changing this parameter discussed in relation to the loss function. A lower σ increases the inefficiency gap and hence welfare losses for a given deviation from the flexible wage equilibrium. Hence, a monetary policy that fails to close this inefficiency gap will result in higher welfare losses when σ is low. Notice, however, that for $\sigma = 2/3$, the flexible wage targeting rules are unable to stabilise wages enough to completely eliminate

[14] Other values of these parameters have been considered, but the signs of derivatives of changes are the same as those discussed here.

welfare losses up to the fourth decimal place when shocks are common.

The middle panel of table 4 gives results for $\mu = 0.2$, keeping the remaining parameters at the baseline values (so that now again $\sigma = 1$). In this case, welfare losses are generally lower. A higher labour elasticity decreases the curvature effect contributing to this result, now along with the labour response effects working through the slope of the Phillips curve and with the direct utility effects of inefficiency as discussed in section 3.1. The main story that wage inflation targeting is better than price inflation targeting, and that welfare may be improved by following an asymmetric policy rule, is confirmed in this case. Moreover, the best monetary policy alternative considered is now the one that leads the central bank to stabilise wage inflation in the more rigid country.

The right panel of table 4 gives results for $\theta = 1.5$ keeping the remaining parameters at the baseline values. The main effect of this change in the international elasticity of substitution is to lower the welfare losses by reducing the international externality. As the benefit from targeting price inflation goes through the terms of trade, it follows that the largest relative reduction in welfare losses occurs under the wage targeting policy regimes. Again, an asymmetric wage targeting rule is preferable when shocks are idiosyncratic, and welfare losses can be eliminated by targeting wage inflation when shocks are common. It is clear, however, that once wages are targeted rather than prices, differences across the monetary policy alternatives are small.

5 Conclusion

This paper has addressed the question of how monetary policy should be conducted in a monetary union in which labour market structures are different across member countries. The analysis has maintained the assumption that the important nominal rigidities are in the labour market rather than in the goods market, and alternative simple monetary policy rules have been evaluated under this assumption that prices are flexible and wages sticky.

The results indicate that price inflation targeting may result in non-negligible welfare losses if nominal wage rigidities are important and nominal price rigidities are not. It is found that the mistake made by targeting price inflation under these assumptions is larger when shocks are common to the monetary union than when they are specific to the member countries due to an international externality. Also, the mistake is larger if the monetary

union's member countries are characterised by different degrees of nominal wage rigidity; i.e. when labour market structures are asymmetric.

Finally, the results suggest that the central bank can improve the welfare of the monetary union's citizens by putting more weight on wage inflation in the country with the more rigid labour market. The welfare improvements from doing so are larger when labour markets are very heterogeneous, and when shocks are idiosyncratic rather than correlated.

An interesting topic for further research is to further characterise the optimal monetary policy responses in a monetary union of the kind described in this paper.

A Natural equilibrium

In the virtual or "natural" equilibrium with flexible wages, the supply side is summarised by the production function

$$\begin{aligned} & y_{it} = a_{it} + \gamma n_{it} \\ \Leftrightarrow \quad & n_{it} = \gamma^{-1}\left(y_{it} - a_{it}\right) \end{aligned} \tag{37}$$

and the labour demand relation

$$w_{it} - p_{it} = a_{it} - (1-\gamma)\, n_{it} \tag{38}$$

while the labour supply relation is

$$w_{it} - p_t = \sigma^{-1} y_t + \mu n_{it} \tag{39}$$

and the relative demand

$$\begin{aligned} & y_{it} = -\theta\left(p_{it} - p_t\right) + y_t \\ \Leftrightarrow \quad & p_{it} - p_t = -\theta^{-1}\left(y_{it} - y_t\right) \end{aligned} \tag{40}$$

all for $i \in \{1, 2\}$.

Combining (37) and (38) gives

$$\begin{aligned} w_{it} - p_{it} &= a_{it} - (1-\gamma)\, n_{it} \\ &= a_{it} - (1-\gamma)\,\gamma^{-1}\left(y_{it} - a_{it}\right) \\ &= \frac{1}{\gamma} a_{it} - \frac{1-\gamma}{\gamma} y_{it} \end{aligned}$$

and after rearranging

$$y_{it} = \frac{1}{1-\gamma} a_{it} - \frac{\gamma}{1-\gamma}\left(w_{it} - p_{it}\right)$$

Substituting (39) and (40) into this relation gives

$$\begin{aligned} y_{it} &= \frac{1}{1-\gamma} a_{it} - \frac{\gamma}{1-\gamma}\left(\left(\sigma^{-1} y_t + \mu\gamma^{-1}\left(y_{it} - a_{it}\right)\right) + \theta^{-1}\left(y_{it} - y_t\right)\right) \\ &= \frac{1}{1-\gamma}\left[\left(1+\mu\right) a_{it} + \left(\theta^{-1} - \sigma^{-1}\right)\gamma y_t - \left(\mu + \gamma\theta^{-1}\right) y_{it}\right] \end{aligned}$$

Rearranging this expression gives the equilibrium value of country-specific output:

$$\begin{aligned} & y_{it}\left(1+\frac{\mu+\gamma\theta^{-1}}{1-\gamma}\right)=\frac{1}{1-\gamma}\left(\left(1+\mu\right)a_{it}+\left(\theta^{-1}-\sigma^{-1}\right)\gamma y_t\right) \\ \Leftrightarrow \quad & y_{it}\frac{\left(1-\gamma\right)+\mu+\gamma\theta^{-1}}{1-\gamma}=\frac{1}{1-\gamma}\left(\left(1+\mu\right)a_{it}+\left(\theta^{-1}-\sigma^{-1}\right)\gamma y_t\right) \\ \Leftrightarrow \quad & y_{it}^{n}=\frac{1+\mu}{\left(1-\gamma\right)+\mu+\gamma\theta^{-1}}a_{it}+\frac{\left(\theta^{-1}-\sigma^{-1}\right)\gamma}{\left(1-\gamma\right)+\mu+\gamma\theta^{-1}}y_t^{n} \end{aligned} \tag{41}$$

where a superscript n has been added to indicate equilibrium values in this "natural" equilibrium with flexible wages.

Aggregate output in the flexible wage equilibrium is found by aggregating (41) over countries and rearranging:

$$\begin{aligned} & y_t^{n}=\frac{1+\mu}{\left(1-\gamma\right)+\mu+\gamma\theta^{-1}}a_t+\frac{\left(\theta^{-1}-\sigma^{-1}\right)\gamma}{\left(1-\gamma\right)+\mu+\gamma\theta^{-1}}y_t^{n} \\ \Leftrightarrow \quad & \frac{\left(1-\gamma\right)+\mu+\gamma\theta^{-1}-\left(\theta^{-1}-\sigma^{-1}\right)\gamma}{\left(1-\gamma\right)+\mu+\gamma\theta^{-1}}y_t^{n}=\frac{1+\mu}{\left(1-\gamma\right)+\mu+\gamma\theta^{-1}}a_t \\ \Leftrightarrow \quad & y_t^{n}=\frac{1+\mu}{\left(1-\gamma\right)+\mu+\gamma\sigma^{-1}}a_t \end{aligned} \tag{42}$$

The equilibrium terms of trade under flexible wages can be found by combining (40) with (41):

$$\begin{aligned} & s_t^{n}=p_{2t}-p_{1t}=-\theta^{-1}\left(y_{2t}-y_{1t}\right) \\ \Leftrightarrow \quad & s_t^{n}=-\frac{1+\mu}{\left(1-\gamma\right)\theta+\mu\theta+\gamma}\left(a_{2t}-a_{1t}\right) \end{aligned} \tag{43}$$

B Welfare function

This section derives a second order approximation to the average welfare losses incurred by a generic household. The derivation follows the approach of Galí (2008).

In each country $i\in\{1,2\}$, there is a continuum of households indexed by $h\in[0,1]$ with an instantaneous utility function

$$U_{it}\left(h\right)=U\left(C_{it}\left(h\right),N_{it}\left(h\right)\right) \tag{44}$$

where $C_{it}(h) = C_t$ owing to perfect risk-sharing.

A second-order Taylor expansion of $U_t(h)$ around the steady state $U \equiv U(C,N)$ is given as

$$\begin{aligned} U_{it}(h) &= U + U_c(C_t - C) + U_n(N_{it}(h) - N) \\ &\quad + \frac{1}{2}U_{cc}(C_t - C)^2 + \frac{1}{2}U_{nn}(N_{it}(h) - N)^2 \\ &\quad + U_{cn}(C_t - C)(N_{it}(h) - N) + \mathcal{R}\left[(C_t, N_{it}(h)), (C,N)\right] \end{aligned}$$

where $\mathcal{R}(.,.)$ is a remainder term satisfying

$$\frac{\mathcal{R}\left[(C_t, N_{it}(h)), (C,N)\right]}{\|(C_t - C, N_{it}(h) - N)\|^3} \to 0$$

as $(C_t - C, N_{it}(h) - N) \to (0,0)$. Noting that $U_{cn} = 0$ by assumption, it follows that a second-order Taylor approximation expressed in terms of log-deviations from the steady state is given as

$$\begin{aligned} U_{it}(h) - U &\approx U_c C\left(\frac{C_t - C}{C}\right) + U_n N\left(\frac{N_{it}(h) - N}{N}\right) \\ &\quad + \frac{1}{2}U_{cc}C^2\left(\frac{C_t - C}{C}\right)^2 + \frac{1}{2}U_{nn}N^2\left(\frac{N_{it}(h) - N}{N}\right)^2 \\ &\approx U_c C\left(c_t + \frac{1}{2}c_t^2\right) + U_n N\left(n_{it}(h) + \frac{1}{2}n_{it}(h)^2\right) \\ &\quad + \frac{1}{2}U_{cc}C^2c_t^2 + \frac{1}{2}U_{nn}N^2 n_{it}(h)^2 \\ &= U_c C\left(y_t + \frac{1-\sigma^{-1}}{2}y_t^2\right) + U_n N\left(n_{it}(h) + \frac{1+\mu}{2}n_{it}(h)^2\right) \end{aligned}$$

where the second (approximate) equality makes use of the second-order approximations $C_t \approx C\left(1 + c_t + \frac{1}{2}c_t^2\right)$ and $N_{jt}(h) \approx N\left(1 + n_{jt}(h) + \frac{1}{2}n_{jt}(h)^2\right)$, and the third equality of the definitions $\sigma^{-1} = -U_{cc}C/U_c$ and $\mu = U_{nn}N/U_n$ as well as of the equilibrium condition $c_t = y_t$. Integrating over households gives

$$\begin{aligned} \int_0^1 (U_{it}(h) - U)\,dh &\approx U_c C\left(y_t + \frac{1-\sigma^{-1}}{2}y_t^2\right) \\ &\quad + U_n N\left(E_h n_{it}(h) + \frac{1+\mu}{2}E_h n_{it}(h)^2\right) \end{aligned}$$

Noting that

$$n_{it} + \frac{1}{2} n_{it}^2 \approx E_h n_{it}(h) + \frac{1}{2} E_h n_{it}(h)^2$$

and

$$E_h n_{it}(h)^2 \approx n_{it}^2 + \xi_i^2 VAR_h(w_{it}(h))$$

up to a second-order approximation, we can write this as

$$\begin{aligned} \int_0^1 (U_{it}(h) - U)\, dh &\approx U_c C \left(y_t + \frac{1-\sigma^{-1}}{2} y_t^2 \right) \\ &\quad + U_n N \left(n_{it} + \frac{1+\mu}{2} n_{it}^2 + \frac{\xi_i^2 \mu}{2} VAR_h(w_{it}(h)) \right) \end{aligned}$$

From the definition of aggregate employment, it follows that $\gamma n_{it} = y_{it} - a_{it} + d_{it}$ where

$$d_{it} \approx \frac{\xi_i}{2} VAR_h(w_{it}(h))$$

Using this gives

$$\begin{aligned} \int_0^1 (U_{it}(h) - U)\, dh &\approx U_c C \left(y_t + \frac{1-\sigma^{-1}}{2} y_t^2 \right) \\ &\quad + \frac{U_n N}{\gamma} \left(y_{it} + \frac{1+\mu}{2\gamma} (y_{it} - a_{it})^2 + \frac{(1+\xi_i \mu)\gamma \xi_i}{2} VAR_h(w_{it}(h)) \right) \\ &\quad + t.i.p. \end{aligned}$$

where $t.i.p.$ represents terms that are independent of policy (here terms in a_{it}) and so irrelevant for the evaluation of monetary policy alternatives.

Under the assumption of an efficient steady state (so that $-U_n/U_c = \gamma Y/N$), we can express the deviations of utility in percentage terms of steady-state consumption as follows:

$$\begin{aligned} \frac{U_{it} - U}{U_c C} &\approx y_t + \frac{1-\sigma^{-1}}{2} y_t^2 - y_{it} - \frac{1+\mu}{2\gamma} (y_{it} - a_{it})^2 \\ &\quad - \frac{(1+\xi_i \mu)\gamma \xi_i}{2} VAR_h(w_{it}(h)) \end{aligned}$$

where terms independent of policy have been dropped for convenience. Ag-

gregating over countries gives

$$\begin{aligned}\frac{U_t - U}{U_c C} &= \frac{1-\sigma^{-1}}{2} y_t^2 - \frac{1+\mu}{2\gamma} \sum_{i=1}^{2} v_i \left(y_{it} - a_{it}\right)^2 \\ &\quad - \sum_{i=1}^{2} v_i \frac{\left(1+\xi_i \mu\right) \gamma \xi_i}{2} VAR_h \left(w_{it}\left(h\right)\right)\end{aligned} \tag{45}$$

By inserting the expression (41) for country-specific output, the first two terms of this relation become[15]

$$\begin{aligned}&\frac{1-\sigma^{-1}}{2} y_t^2 - \frac{1+\mu}{2\gamma} \sum_{i=1}^{2} v_i \left(-\theta\left(p_{it} - p_t\right) + y_t - a_{it}\right)^2 \\ = \;& \frac{1-\sigma^{-1}}{2} y_t^2 - \frac{1+\mu}{2\gamma} \sum_{i=1}^{2} v_i \left(\left(\theta\left(p_{it} - p_t\right) + a_{it}\right)^2 + y_t^2 - 2\left(\theta\left(p_{it} - p_t\right) + a_{it}\right) y_t\right) \\ = \;& -\frac{1}{2}\left(\sigma^{-1} + \frac{\left(1-\gamma\right)+\mu}{\gamma}\right) y_t^2 + \frac{1+\mu}{\gamma} y_t a_t \\ & -\frac{1+\mu}{2\gamma} \sum_{i=1}^{2} v_i \left(\left(\theta\left(p_{it} - p_t\right) + a_{it}\right)^2\right) + t.i.p.\end{aligned}$$

After using (42) to replace a_t (and dropping $t.i.p.$), this expression can be written as

$$\begin{aligned}&-\frac{1}{2}\left(\sigma^{-1} + \frac{\left(1-\gamma\right)+\mu}{\gamma}\right) y_t^2 + \frac{\left(1-\gamma\right)+\mu+\gamma\sigma^{-1}}{\gamma} y_t y_t^n \\ &-\frac{1+\mu}{2\gamma} \sum_{j=1}^{2} v_j \left(\theta\left(p_{it} - p_t\right) + a_{it}\right)^2 \\ = \;& -\frac{1}{2}\left[\left(\sigma^{-1} + \frac{\left(1-\gamma\right)+\mu}{\gamma}\right)\left(y_t - y_t^n\right)^2 + \frac{1+\mu}{\gamma} \sum_{j=1}^{2} v_j \left(\theta\left(p_{it} - p_t\right) + a_{it}\right)^2\right]\end{aligned}$$

[15] Second-order terms from a second-order equivalent to (41) would only add third-order term to (45) due to the quadratic terms in this relation.

Since

$$\begin{aligned}
&\sum_{i=1}^{2} v_i \left(\theta \left(p_{it} - p_t\right) + a_{it}\right)^2 \\
= &\left(\sum_{i=1}^{2} v_i \left(\theta \left(p_{it} - p_t\right) + a_{it}\right)\right)^2 \\
&+v_1 v_2 \left[\left(\theta \left(p_{2t} - p_t\right) + a_{2t}\right) - \left(\theta \left(p_{1t} - p_t\right) + a_{1t}\right)\right]^2 \\
= &\ v_1 v_2 \left[\theta \left(p_{2t} - p_{1t}\right) + \left(a_{2t} - a_{1t}\right)\right]^2 + t.i.p. \\
= &\ v_1 v_2 \theta^2 \left(s_t - \frac{(1-\gamma) + \mu + \gamma\theta^{-1}}{1+\mu} s_t^n\right)^2 + t.i.p.
\end{aligned}$$

where $s_t \equiv p_{2t} - p_{1t}$ represents the terms-of-trade, we can write (45) as

$$\begin{aligned}
\frac{U_t - U}{U_c C} = &-\frac{1}{2}\left[\left(\sigma^{-1} + \frac{(1-\gamma)+\mu}{\gamma}\right)\left(y_t - y_t^n\right)^2 + \frac{(1+\mu)\theta^2}{\gamma} v_1 v_2 \left(s_t - \tilde{s}_t^n\right)^2\right] \\
&-\sum_{i=1}^{2} v_i \frac{(1+\xi_i\mu)\gamma\xi_i}{2} VAR_h\left(w_{it}\left(h\right)\right) \qquad (46)
\end{aligned}$$

where

$$\tilde{s}_t^n = \frac{(1-\gamma) + \mu + \gamma\theta^{-1}}{1+\mu} s_t^n$$

Taking the expected value then gives the following second-order approximation to the consumer's lifetime utility:

$$\begin{aligned}
\mathcal{W}_t = &\ E_t \sum_{\tau=t}^{\infty} \delta^\tau \frac{U_t - U}{U_c C} \\
= &-\frac{1}{2} E_t \sum_{\tau=t}^{\infty} \delta^\tau \left\{\left(\sigma^{-1} + \frac{(1-\gamma)+\mu}{\gamma}\right)\left(y_t - y_t^n\right)^2 + \frac{(1+\mu)\theta^2}{\gamma} v_1 v_2 \left(s_t - \tilde{s}_t^n\right)^2\right. \\
&\left.+\sum_{i=1}^{2} v_i \left(1+\xi_i\mu\right)\gamma\xi_i VAR_h\left(w_{it}\left(h\right)\right)\right\} \qquad (47)
\end{aligned}$$

By applying proposition 6.3 in Woodford (2003), we may write

$$VAR_h\left(w_{it}\left(h\right)\right) \approx \alpha_i VAR_h\left(w_{it-1}\left(h\right)\right) + \frac{\alpha_i}{1-\alpha_i}\omega_t^2$$

which is accurate up to a remainder term of third order. Integrating this expression forward and taking the discounted value yields the second-order accurate relation

$$\sum_{\tau=t}^{\infty}\delta^{\tau}VAR_{h}\left(w_{it}\left(h\right)\right)\approx\frac{\alpha_{i}}{\left(1-\alpha_{i}\right)\left(1-\alpha_{i}\delta\right)}\sum_{\tau=t}^{\infty}\delta^{\tau}\omega_{t}^{2}+t.i.p.$$

Consequently, $\mathcal{W}_t$ may be written as

$$\mathcal{W}_{t}=-E_{t}\sum_{\tau=t}^{\infty}\delta^{\tau}\left[\lambda_{y}\left(y_{t}-y_{t}^{n}\right)^{2}+\lambda_{s}\left(s_{t}-\tilde{s}_{t}^{n}\right)^{2}+\sum_{i=1}^{2}\lambda_{\omega i}\omega_{jt}^{2}\right] \tag{48}$$

where

$$\begin{aligned}\lambda_{y} &= \frac{1}{2}\left[\sigma^{-1}+\frac{\left(1-\gamma\right)+\mu}{\gamma}\right]\\ \lambda_{s} &= \frac{1}{2}\frac{\left(1+\mu\right)\theta^{2}}{\gamma}v_{1}v_{2}\\ \lambda_{\omega i} &= \frac{1}{2}\frac{v_{i}\xi_{i}\gamma\alpha_{i}\left(1+\mu\xi_{i}\right)}{\left(1-\alpha_{i}\right)\left(1-\alpha_{i}\delta\right)}=\frac{1}{2}\frac{v_{i}\xi_{i}\gamma}{\Lambda_{i}}\end{aligned}$$

Finally it follows that the *average period* welfare *loss* is

$$\mathcal{L}_{t}=\lambda_{y}VAR\left(\hat{y}_{t}\right)+\lambda_{s}E\left(\hat{s}_{t}^{2}\right)+\sum_{i=1}^{2}\lambda_{\omega i}VAR\left(\omega_{jt}^{2}\right) \tag{49}$$

where $\hat{y}_t = y_t - y_t^n$ is the welfare relevant output gap and $\hat{s}_t = s_t - \tilde{s}_t^n$ is a terms-of-trade gap.

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Table 1: Welfare losses for RED=1

$\mathcal{L}$	$AED=3$			$AED=4$			$AED=5$		
Flexible rules	$\rho_\varepsilon=0$	$\rho_\varepsilon=0.5$	$\rho_\varepsilon=0$	$\rho_\varepsilon=0$	$\rho_\varepsilon=0.5$	$\rho_\varepsilon=1$	$\rho_\varepsilon=0$	$\rho_\varepsilon=0.5$	$\rho_\varepsilon=1$
$(1.5;0;0;0)$	0.1659	0.2008	0.2358	0.2565	0.3266	0.3967	0.3346	0.4362	0.5378
$(0;1.5;1.5;0)$	0.0480	0.0240	0.0000	0.0581	0.0291	0.0000	0.0656	0.0328	0.0000
$(0;1.1;2;0)$	0.0488	0.0244	0.0000	0.0590	0.0295	0.0000	0.0665	0.0332	0.0000
$(1.5;0;0;0.125)$	0.1318	0.1497	0.1676	0.1678	0.1936	0.2194	0.1835	0.2096	0.2357
$(0;1.5;1.5;0.125)$	0.0480	0.0240	0.0000	0.0581	0.0291	0.0000	0.0656	0.0328	0.0000
$(0;1.1;2;0.125)$	0.0486	0.0243	0.0000	0.0588	0.0294	0.0000	0.0662	0.0331	0.0000
$\mathcal{L}$									
Strict rules									
$\pi_t=0$	0.1816	0.2243	0.2671	0.2375	0.2981	0.3587	0.2815	0.3567	0.4318
$\omega_t=0$	0.0480	0.0240	0.0000	0.0581	0.0291	0.0000	0.0656	0.0328	0.0000
$\omega_{1t}=0$	0.0637	0.0319	0.0000	0.0770	0.0385	0.0000	0.0866	0.0433	0.0000
$\omega_{2t}=0$	0.0637	0.0319	0.0000	0.0770	0.0385	0.0000	0.0866	0.0433	0.0000
$\hat{y}_t=0$	0.0480	0.0240	0.0000	0.0581	0.0291	0.0000	0.0656	0.0328	0.0000

Note: Benchmark calibration and $RED=1$. Flexible rules refer to monetary policy rules of the form (25) and are indicated by (k_π, k_1, k_2, k_y).

Table 2: Welfare losses for RED=2

$\mathcal{L}$	$AED=3$			$AED=4$			$AED=5$		
Flexible rules	$\rho_\varepsilon=0$	$\rho_\varepsilon=0.5$	$\rho_\varepsilon=0$	$\rho_\varepsilon=0$	$\rho_\varepsilon=0.5$	$\rho_\varepsilon=1$	$\rho_\varepsilon=0$	$\rho_\varepsilon=0.5$	$\rho_\varepsilon=1$
$(1.5;0;0;0)$	0.1745	0.2184	0.2623	0.2764	0.3612	0.4460	0.3667	0.4892	0.6116
$(0;1.5;1.5;0)$	0.0459	0.0229	0.0000	0.0556	0.0278	0.0000	0.0629	0.0315	0.0000
$(0;1.1;2;0)$	0.0423	0.0211	0.0000	0.0524	0.0262	0.0000	0.0601	0.0301	0.0000
$(1.5;0;0;0.125)$	0.1329	0.1570	0.1810	0.1697	0.2021	0.2345	0.1855	0.2181	0.2508
$(0;1.5;1.5;0.125)$	0.0449	0.0224	0.0000	0.0545	0.0273	0.0000	0.0618	0.0309	0.0000
$(0;1.1;2;0.125)$	0.0419	0.0210	0.0000	0.0521	0.0260	0.0000	0.0598	0.0299	0.0000
$\mathcal{L}$									
Strict rules									
$\pi_t=0$	0.1673	0.2035	0.2397	0.2209	0.2741	0.3273	0.2634	0.3306	0.3978
$\omega_t=0$	0.0533	0.0266	0.0000	0.0640	0.0320	0.0000	0.0720	0.0360	0.0000
$\omega_{1t}=0$	0.0770	0.0385	0.0000	0.0892	0.0446	0.0000	0.0976	0.0488	0.0000
$\omega_{2t}=0$	0.0443	0.0221	0.0000	0.0581	0.0290	0.0000	0.0687	0.0343	0.0000
$\hat{y}_t=0$	0.0403	0.0202	0.0000	0.0511	0.0256	0.0000	0.0592	0.0296	0.0000

Note: Benchmark calibration and $RED=2$. Flexible rules refer to monetary policy rules of the form (25) and are indicated by (k_π, k_1, k_2, k_y).

Table 3: Welfare losses for RED=3

$\mathcal{L}$	$AED=3$			$AED=4$			$AED=5$		
Flexible rules	$\rho_\varepsilon=0$	$\rho_\varepsilon=0.5$	$\rho_\varepsilon=0$	$\rho_\varepsilon=0$	$\rho_\varepsilon=0.5$	$\rho_\varepsilon=1$	$\rho_\varepsilon=0$	$\rho_\varepsilon=0.5$	$\rho_\varepsilon=1$
$(1.5;0;0;0)$	0.1829	0.2370	0.2912	0.3000	0.4029	0.5057	0.4092	0.5590	0.7088
$(0;1.5;1.5;0)$	0.0437	0.0218	0.0000	0.0528	0.0264	0.0000	0.0598	0.0299	0.0000
$(0;1.1;2;0)$	0.0369	0.0185	0.0000	0.0464	0.0232	0.0000	0.0539	0.0269	0.0000
$(1.5;0;0;0.125)$	0.1323	0.1633	0.1943	0.1698	0.2100	0.2502	0.1859	0.2266	0.2673
$(0;1.5;1.5;0.125)$	0.0414	0.0207	0.0000	0.0501	0.0251	0.0000	0.0569	0.0285	0.0000
$(0;1.1;2;0.125)$	0.0357	0.0179	0.0000	0.0452	0.0226	0.0000	0.0527	0.0264	0.0000
$\mathcal{L}$									
Strict rules									
$\pi_t=0$	0.1487	0.1763	0.2039	0.1989	0.2423	0.2857	0.2390	0.2956	0.3521
$\omega_t=0$	0.0580	0.0290	0.0000	0.0693	0.0347	0.0000	0.0776	0.0388	0.0000
$\omega_{1t}=0$	0.0822	0.0411	0.0000	0.0938	0.0469	0.0000	0.1015	0.0507	0.0000
$\omega_{2t}=0$	0.0306	0.0153	0.0000	0.0443	0.0221	0.0000	0.0550	0.0275	0.0000
$\hat{y}_t=0$	0.0301	0.0150	0.0000	0.0415	0.0208	0.0000	0.0502	0.0251	0.0000

Note: Benchmark calibration and $RED=3$. Flexible rules refer to monetary policy rules of the form (25) and are indicated by (k_π, k_1, k_2, k_y).

Table 4: Sensitivity analysis

$\mathcal{L}$	$\sigma = 2/3$			$\mu = 0.2$			$\theta = 1.5$		
Flexible rules	$\rho_\varepsilon = 0$	$\rho_\varepsilon = 0.5$	$\rho_\varepsilon = 0$	$\rho_\varepsilon = 0$	$\rho_\varepsilon = 0.5$	$\rho_\varepsilon = 1$	$\rho_\varepsilon = 0$	$\rho_\varepsilon = 0.5$	$\rho_\varepsilon = 1$
$(1.5; 0; 0; 0)$	0.3382	0.4545	,05708	0.1335	0.1568	0.1802	0.2254	0.3329	0.4404
$(0; 1.5; 1.5; 0)$	0.0554	0.0281	0.0009	0.0440	0.0220	0.0000	0.0054	0.0027	0.0000
$(0; 1.1; 2; 0)$	0.0526	0.0267	0.0008	0.0430	0.0215	0.0000	0.0051	0.0025	0.0000
$(1.5; 0; 0; 0.125)$	0.2173	0.2738	0.3302	0.1020	0.1101	0.1181	0.1216	0.1773	0.2330
$(0; 1.5; 1.5; 0.125)$	0.0545	0.0276	0.0007	0.0437	0.0218	0.0000	0.0053	0.0026	0.0000
$(0; 1.1; 2; 0.125)$	0.0523	0.0265	0.0007	0.0429	0.0214	0.0000	0.0051	0.0025	0.0000
$\mathcal{L}$									
Strict rules									
$\pi_t = 0$	0.2597	0.3320	0.4044	0.1235	0.1408	0.1582	0.1637	0.2400	0.3163
$\omega_t = 0$	0.0634	0.0317	0.0000	0.0454	0.0227	0.0000	0.0060	0.0030	0.0000
$\omega_{1t} = 0$	0.0876	0.0438	0.0000	0.0494	0.0247	0.0000	0.0081	0.0041	0.0000
$\omega_{2t} = 0$	0.0574	0.0287	0.0000	0.0410	0.0205	0.0000	0.0059	0.0030	0.0000
$\hat{y}_t = 0$	0.0511	0.0256	0.0000	0.0423	0.0211	0.0000	0.0050	0.0025	0.0000

Note: Parameters other than those mentioned are at their baseline values. Flexible rules refer to monetary policy rules of the form (25) and are indicated by (k_π, k_1, k_2, k_y).

Part II

Chapter 3

Rule-of-thumb consumers, productivity and hours*

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Abstract

In this paper we study the transmission mechanisms of productivity shocks in a model with rule-of-thumb consumers. In the literature, this financial friction has been studied only with reference to fiscal shocks. We show that the presence of rule-of-thumb consumers is also very helpful in accounting for recent empirical evidence on productivity shocks. Rule-of-thumb agents, together with nominal and real rigidities, play an important role in reproducing the negative response of hours and the delayed responses of output and consumption after a productivity shock.

JEL Classification: E32.

Keywords: rule-of-thumb consumers, productivity shocks, nominal rigidities, real rigidities

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1 Introduction

Recent research on fiscal policy in dynamic stochastic general equilibrium (DSGE) models has shown that deviations from Ricardian equivalence are instrumental in generating empirically plausible responses to government spending shocks. In particular, Galí et al. (2007) show that private consumption may rise after a positive shock to government spending if so-called rule-of-thumb consumers, who simply consume their current disposable income each period, are allowed to co-exist with intertemporally optimising consumers.[1] In the model, optimising consumers decrease their consumption following a government spending shock because they correctly anticipate a decline in life-time income as a consequence of taxation. But rule-of-thumb consumers increase their consumption if current disposable income increases. This happens in the model when the government finances the increase in its spending at least partially through the issuance of bonds, under assumptions of sticky prices and an imperfectly competitive labour market. In this case, if a sufficiently large fraction of households follow a rule of thumb, aggregate consumption rises.

A number of papers have further studied the implications of rule-of-thumb behaviour for fiscal policy in DSGE models, and rule-of-thumb consumers have become a standard ingredient in DSGE models at policy-making institutions, in particular at central banks.[2] But as far as we know, the implications of rule-of-thumb behaviour have not been investigated beyond the fiscal policy dimension so far. This is potentially important since rule-of-thumb consumers represent a substantial deviation from the standard optimising framework of DSGE models. In the baseline calibration in Galí et al. (2007), 50 per cent of households have no access to financial and capital markets and so cannot smooth consumption intertemporally. The market incompleteness introduced by this assumption may be suspected to have potentially sizeable effects on the model's propagation of shocks to variables other than

[1] To our knowledge, the idea that a fraction of consumers consume their current incomes each period, while the remaining fraction optimise intertemporally, was first put forward by Hall (1978) as an alternative to the permanent income hypothesis. Campbell and Mankiw (1989, 1991) reject the permant income hypothesis against this alternative, and Mankiw (2000) suggests that rule-of-thumb consumers should be included in models built for the analysis of fiscal policy issues.

[2] Papers include Andersen (2005), Bilbiie (2005), Coenen and Straub (2005), Colciago (2007), Erceg et al. (2006), Forni et al. (2007), Furlanetto (2007), Furlanetto and Seneca (2007), Galí et al. (2004) and Natvik (2006).

government spending. This is an important objection as counterfactual responses to other kinds of shocks may question the plausibility of introducing rule-of-thumb consumers even for analysing fiscal policy issues.

The purpose of this paper is to test this conjecture for the case of shocks to productivity. Hence, we analyse the impact of rule-of-thumb consumption behaviour on the propagation of technology shocks in the framework developed by Galí et al. (2007).[3] Considering the recent debate in macroeconomics on the importance of technology shocks for business cycle fluctuations, it seems particularly important to study the performance of the class of DSGE models with rule-of-thumb consumers in response to these shocks. On one hand, beginning with the seminal papers by Kydland and Prescott (1982) and Prescott (1986), real business cycle (RBC) theory suggests that technology shocks are the most important driving force behind business cycle fluctuations. On the other hand, a number of later papers, particularly Galí (1999), have challenged this claim based on empirical evidence on the impulse responses of macroeconomic variables. In this paper, we contribute to this debate by shedding light on how financial frictions in the form of rule-of-thumb behaviour may affect the transmission of technology shocks in the economy.

Galí (1999), and more recently Francis and Ramey (2005), provide evidence on responses to technology shocks in the US by identifying such shocks in an estimated vector autoregression (VAR) through long-run restrictions. In both studies, a positive technology shock has a significant negative effect on hours worked - in stark contrast with the predictions of the RBC literature. Furthermore, in both studies output does not respond to the shock on impact, but it increases with a lag. Since output and hours are strongly positively correlated in the data, it follows that technology shocks cannot be the main driving force behind business cycle fluctuations.

Of course, these claims have not stood unchallenged. Christiano et al. (2004) and McGrattan (2004) argue that Galí's (1999) results are sensitive to small changes in the specification of the empirical model. When hours are introduced in levels, and not in first differences as in Galí (1999), these authors obtain a positive response of hours. However, in recent papers Fernald

[3]We have also considered the effects of monetary, preference and cost-push shocks. The model with rule-of-thumb consumers delivers results very similar to the model without them as long as wages are sticky. Wage rigidity effectively shuts down the mechanisms through which rule-of-thumb behaviour may change the propagation of these shocks. Results are available upon request.

(2007) and Canova et al. (2007) show that once low-frequency movements in hours are taken into account, the negative response of hours is robust; see also Galí and Rabanal (2005) for a discussion. In addition, Gambetti (2005) confirms that hours fall using a bayesian VAR with time-varying coefficients. Consequently, we consider the evidence from the VAR literature to favour the view that hours decrease on impact of a technology shock.

An alternative empirical approach is taken by Basu et al. (2006). They use a sophisticated growth accounting framework to correct Solow residuals for the influences of increasing returns, imperfect competition, variable factor utilisation and sector compositional effects. Somewhat surprisingly, perhaps, this approach leads to results that are very similar to those of the VAR literature. In particular, Basu et al. (2006) estimate a significant decline in hours on impact of a technology shock, while they find a zero impact response of output.[4]

How can a theoretical model deliver a decline in hours after a technology shock? Galí (1999) shows how nominal rigidities, a key feature of New Keynesian models, can lead to such a response. However, Dotsey (2002) shows that this is true only if monetary policy is modelled as an exogenous money growth rule; when monetary policy follows a Taylor (1993) rule, hours increase as in the baseline RBC model. Francis and Ramey (2005) show that an RBC model augmented with real rigidities (habit persistence in consumption and capital adjustment costs) can generate a negative response of hours without relying on nominal rigidities.[5] In this paper, we show that a financial friction represented by rule-of-thumb behaviour affects the model's transmission mechanism in a way that makes it easier to obtain a decline in hours on impact of a productivity shock. In addition, we show how rule-of-thumb behaviour interacts with the nominal and real rigidities that have previously been considered in literature as potential explanations of the negative response of hours.

The Galí et al. (2007) model is characterised by three rigidities, namely

[4] The evidence in Basu et al. (2006) is based on macrodata for the US. Interestingly, using firm-level data for Italy and Sweden, respectively, Marchetti and Nucci (2005) and Carlsson and Smedsaas (2006) find that firms reduce the input of labour on impact of a positive technology shock. See also Carlsson (2003).

[5] Galí and Rabanal (2005) estimate a New Keynesian model using Bayesian techniques, and they find that both nominal and real rigidities are important, while Galí et al. (2003) detect significant differences across periods in the Federal Reserve's responses to technology shocks, reconciling the results of Galí (1999) and Dotsey (2002).

price stickiness, capital stickiness (due to capital adjustment costs) and the financial rigidity barring a fraction of households, the rule-of-thumb consumers, from access to financial and capital markets. We extend this framework in two steps.

In the first step, we extend the model with nominal wage rigidity as in Furlanetto (2007). Sticky wages have been shown to be important in order to generate plausible dynamics in macroeconomic variables in response to a wide variety of shocks, cf. Christiano et al. (2005). Moreover, Liu and Phaneuf (2005) show that sticky wages, in combination with sticky prices, are important in order to explain the dynamics of hours and wages following a productivity shock. The Galí et al. (2007) model extended with nominal wage rigidity is our baseline model.

In the second step, we introduce a fifth rigidity, namely consumption stickiness in the form of habit persistence in consumption. Habit formation has recently received a lot of attention in the literature, e.g. by Francis and Ramey (2005), Galí and Rabanal (2005) and Fève (2004). As we shall see, this extension allows us to explain empirical evidence on key macroeconomic variables besides hours and wages. In addition, it allows us to analyse the role played by many of the frictions studied in the literature and their interaction with rule-of-thumb consumers. Building on the terminology of McGrattan (2004), we refer to the Galí et al. (2007) model extended with both nominal wage rigidity and habit persistence as the *quintuple*-sticky model.

This paper's first key result is that the model with rule-of-thumb consumers is able to reproduce a sizeable decline in hours in keeping with the empirical evidence. Like Galí and Rabanal (2005), we find that a model with three types of rigidities (sticky prices, sticky wages and capital adjustment costs) can reproduce a negative response of hours - even under endogenous monetary policy in the form of a Taylor rule. But we show that this response is very small. As shown in figure 1, a one per cent increase in technology leads to a 0.2 per cent decline in hours worked on impact under our preferred calibration. In our baseline model with rule-of-thumb consumers, hours decline more: -0.6 per cent. Thus, the model's response coincides with the estimates in Basu et al. (2006) and Francis and Ramey (2005).

The intuition is the following. A positive shock to technology means that firms can produce a given level of output with fewer hours. Because prices are sticky, the level of output is determined by demand. This means that hours will go down if demand does not increase sufficiently after the shock. As to government spending shocks, rule-of-thumb and optimising households

react to technology shocks in different ways. Optimising consumers correctly anticipate an increase in life-time income and so they increase their consumption. This works to offset the decline in hours through aggregate demand. Rule-of-thumb consumers, in contrast, see current income go down because of combined effects of sticky prices and wages, and this makes them consume less. This curbs the aggregate demand effect, and hours decline more as a result when some households consume according to a rule of thumb. In a nutshell, our financial rigidity amplifies the impact of nominal and real rigidities, making the transmission more contractionary.

While the literature has studied the response of hours to technology shock in great detail, less attention has been devoted to the responses of other macroeconomic variables. Both Basu et al. (2006) and Francis and Ramey (2005) find not only a decline in hours on impact of a technology shock, but also a zero response of both output and consumption. Surprisingly, this additional evidence suggesting a delayed expansion in output and consumption following a technology improvement is yet to be explained in the theoretical literature.

The second key result of this paper is that the *quintuple*-sticky model can reproduce the zero impact-responses of output and consumption found in Basu et al. (2006) and Francis and Ramey (2005) in addition to a decline in hours worked. In the model, habit persistence works to smooth consumption, in effect delaying the full response of consumption to shocks. We stress, however, that the presence of all the five rigidities considered is crucial to obtain this result.

Consequently, rule-of-thumb agents are instrumental not only in obtaining a large negative response of hours, but also in reproducing delayed responses of output and consumption as in the empirical evidence. Thus, rule-of-thumb consumers, representing a substantial deviation from the standard optimising DSGE framework, do not worsen the performance of the model. In contrast, they can be very helpful in replicating important empirical regularities. This implies that researchers may safely rely on rule-of-thumb consumers in fiscal policy analyses in the sense that rule-of-thumb consumption behaviour generates reasonable responses to other shocks.

The paper has the following structure. In section 2 we briefly present the baseline model, and in section 3 we present impulse responses to technology shocks from this version of the model. In section 4 we discuss the *quintuple*-sticky model, we compare our results to other papers in the literature, and we present a sensitivity analysis. Section 5 concludes.

2 A DSGE model with rule-of-thumb consumers

The model is a standard New Keynesian model augmented with capital and rule-of-thumb consumers as in Galí et al. (2007), and with sticky wages as in Furlanetto (2007).[6] The economy consists of a continuum of firms, a continuum of households, a continuum of labour unions, a central bank responsible for monetary policy, and a government collecting lump-sum taxes.[7]

There is monopolistic competition in both goods and labour markets. In particular, there is a continuum of differentiated intermediate goods and a continuum of differentiated labour services. In the goods market, this leads to a downward-sloping demand curve for each intermediate good, and in the labour market it leads to a downward-sloping demand curve for each labour type.

A fraction λ of households are rule-of-thumb consumers - or 'spenders' in the terminology of Mankiw (2000). These consumers simply consume their respective disposable incomes each period. The remaining fraction $(1-\lambda)$ of households are optimisers - or 'savers' - who have access to both financial and capital markets. Hence, they choose plans for consumption, investment and bond holdings to maximise life-time utility. Wages are set by unions that each represent a differentiated type of labour service supplied by households. Wage rigidity is introduced by assuming adjustment costs as in Rotemberg (1982).

Each firm produces one of the differentiated intermediate goods. It does so by combining rented capital with a homogenous labour input constructed as a Dixit and Stiglitz (1977) aggregate of the differentiated labour services supplied by households. The firm sets its price according to a Calvo (1983) price-setting mechanism and stands ready to satisfy demand at the chosen price.

Each period begins by the realisation of shocks to the economy. We concentrate on technology shocks and abstract from other types of shocks that may affect the economy.

[6] In the appendix we further extend the model with habit formation in consumption.

[7] We abstract from fiscal policy as the model's propagation of government spending shocks has been thoroughly analysed in the literature, cf. references above.

2.1 Households

Households have identical instantaneous utility functions

$$U_t^i = \frac{\left(C_t^i\right)^{1-\sigma} - 1}{1-\sigma} - \frac{\left(N_t^i\right)^{1+\varphi}}{1+\varphi} \tag{1}$$

where $i \in \{o, r\}$ denotes the household's type, i.e. optimising or rule-of-thumb. C_t^i is the household's real consumption at time t (implicitly a Dixit and Stiglitz, 1977, index of intermediate goods), N_t^i is the hours worked by the household in period t, $\varphi > 0$ is the inverse of the Frisch labour elasticity, and $\sigma > 0$ is the coefficient of relative risk aversion and, at the same time, the inverse of the elasticity of intertemporal substitution.

An optimising household maximises expected life-time utility given by

$$E_t \sum_{k=0}^{\infty} \beta^k U_{t+k}^o$$

where E_t is an operator representing expectations over all states of the economy conditional on period-t information, and $\beta \in (0,1)$ is the subjective discount factor. Maximisation is subject to a sequence of flow budget constraints (and implicitly a no-Ponzi game condition):

$$P_t\left(C_t^o + I_t\right) + E_t\left(\Lambda_{t,t+1} B_{t+1}\right) = W_t N_t^o + R_t^k K_t + B_t - P_t T_t^o - F_t$$

where I_t is real investment, W_t is the nominal wage, R_t^k is the nominal rental rate on the stock of capital owned by the household at the beginning of period t, K_t, and T_t^o is the real lump-sum tax paid by optimising consumers. The right-hand side gives available resources as the sum of labour income, $W_t N_t^o$, income from renting capital to firms, $R_t^k K_t$, initial financial wealth, B_t, less nominal lump-sum taxes paid to the government, $P_t T_t^o$, and less a nominal union membership fee, F_t. On the left-hand side, resources are allocated to consumption, investment and a portfolio of bonds, $E_t\left(\Lambda_{t,t+1} B_{t+1}\right)$. $\Lambda_{t,t+1}$ is the stochastic discount factor. Hence, the gross risk-free interest rate is given by the relation $1 + R_t = \left(E_t \Lambda_{t,t+1}\right)^{-1}$.

The household's capital evolves according to

$$K_{t+1} = (1-\delta) K_t + \phi\left(\frac{I_t}{K_t}\right) K_t$$

where δ is the rate of depreciation, and $\phi(.)$ is an adjustment cost function satisfying $\phi(\delta) = \delta$, $\phi' > 0$, $\phi'(\delta) = 1$ and $\phi'' \leq 0$.

The optimisation problem, according to which the household chooses plans for consumption, bond holdings and investment, gives rise to the following first-order conditions that we state in log-linear form:[8]

$$c_t^o = E_t c_{t+1}^o - \frac{1}{\sigma}(r_t - E_t \pi_{t+1}) \tag{2}$$

$$k_{t+1} = (1-\delta) k_t + \delta i_t \tag{3}$$

$$q_t = -(r_t - E_t[\pi_{t+1}]) + [1 - \beta(1-\delta)] E_t\left[r_{t+1}^k - p_t\right] + \beta E_t[q_{t+1}] \tag{4}$$

$$i_t - k_t = \eta q_t \tag{5}$$

where $\eta \equiv -1/(\phi''(\delta)\delta)$. Here, (2) is the Euler equation, (3) is the capital accumulation equation, while (4) and (5) represent the dynamics of Tobin's q, denoted q_t, and its relation to investment, respectively.

A rule-of-thumb household faces the simple budget constraint

$$P_t C_t^r = W_t N_t^r - P_t T^r - F_t$$

where C_t^r is the household's real consumption at time t, N_t^r is the hours worked by the household in period t, and F_t is a nominal union membership fee. As a rule-of-thumb household simply consumes its current income, consumption follows directly from the budget constraint. A first-order log-linear approximation around the steady state with constant consumption equalised across households gives

$$c_t^r = \frac{WN}{PC}(w_t + n_t) \tag{6}$$

where omission of time subscripts indicate steady-state variables. Note that taxes drop out of the first-order approximation because we abstract from government spending shocks. Also, the union membership fee drops out because the fee is assumed to be a quadratic function of wage inflation, which is zero in the steady state, cf. below.

Aggregate variables are given as simple weighted averages:

$$c_t = \lambda c_t^r + (1-\lambda) c_t^o \tag{7}$$

$$n_t = \lambda n_t^r + (1-\lambda) n_t^o \tag{8}$$

[8]For details on the derivation we refer the reader to GLV (2007). Lowercase variables denote log-deviations from the steady state of the corresponding uppercase variables.

2.2 Firms

Each firm produces according to the technology

$$Y_t = A_t K_t^{\psi} N_t^{1-\psi}$$

where Y_t is output, A_t is a technology shock, and $0 \leq \psi \leq 1$. Each period, a firm is allowed to set a new price, P_t^*, with a fixed probability $(1-\theta_p)$ as in Calvo (1983). It does so to maximise the value of the firm to its owners, the optimising households,

$$\sum_{k=0}^{\infty} \theta^k E_t \left[\Lambda_{t,t+k} \left(P_t^* Y_{t+k|t} - W_{t+k} N_{t+k|t} - R_{t+k}^k K_{t+k|t} \right) \right]$$

where subscript $t+k|t$ indicates the value of the variable at time $t+k$ for a firm that has last reset its price in period t. Maximisation is subject to the downward-sloping demand curve it faces as a consequence of monopolistic competition.

As is wellknown, the optimality conditions from this problem imply the New Keynesian Phillips curve

$$\pi_t^p = \beta E_t \left(\pi_{t+1}^p \right) + \kappa_p mc_t \tag{9}$$

where $\kappa = (1-\beta\theta_p)(1-\theta_p)\theta_p^{-1}$, $\pi_t^p = p_t - p_{t-1}$ is price inflation, and where mc_t is real marginal costs given by

$$mc_t = (w_t - p_t) - (y_t - n_t) \tag{10}$$

In addition, cost minimisation implies that relative factor inputs satisfy the condition

$$k_t - n_t = (w_t - p_t) - \left(r_t^k - p_t \right) \tag{11}$$

Up to a first-order approximation, production is given by

$$y_t = a_t + \psi k_t + (1-\psi) n_t \tag{12}$$

2.3 Labour Unions

The economy has a continuum of unions $z \in [0,1]$ each representing a continuum of workers. A fraction $(1-\lambda)$ are optimising, and fraction λ are rule-of-thumb consumers. Each union sets the wage rate for its members,

who stand ready to satisfy firms' demand for their labour services at the chosen wage. The workers in a union provide the same type of labour (irrespective of their consumption behaviour) differentiated from the type of labour services provided by members of other unions. The labour service supplied by each union, $N(z)$, is a simple aggregate of its members' labour services. In turn, the labour entering the production function of any firm is a Dixit and Stiglitz (1977) aggregate of the labour services provided by the unions in the economy.

Each period, a representative union chooses $W_t(z)$ to maximise the present value of an average of its members' current and future period utility functions, that is,

$$\max_{W_t(z)} E_t \sum_{k=0}^{\infty} \beta^{t+k} \left[\lambda U_{t+k}^r + (1-\lambda) U_{t+k}^o \right]$$

subject to the labour demand functions and the budget constraints of its members, thus taking the effect of the wage decision on the income of its members into account. Wage adjustments are assumed to be costly. In particular, it is assumed that the wage adjustment cost is a quadratic function of the increase in the wage demanded by the union as modelled in Rotemberg (1982) for prices demanded by firms. For simplicity, the adjustment cost is proportional to the aggregate wage bill in the economy (this parallels the specification of price adjustment costs in Ireland, 2003). Though the wage bargaining process is not explicitly modelled, one way of thinking of this cost is that unions have to negotiate wages each period and that this activity demands economic resources; the larger the increase in wages obtained, the more effort unions would have needed to put into the negotiation process. Each member of the union covers an equal share of the wage adjustment cost by paying a union membership fee. Hence, the nominal fee paid by a member of union z at time t is given by

$$F_t(z) = \frac{\phi_w}{2} \left(\frac{W_t(z)}{W_{t-1}(z)} - 1 \right)^2 W_t N_t$$

where the size of the adjustment costs is governed by the parameter ϕ_w.

The optimality conditions imply a New Keynesian Phillips curve for wage inflation given by

$$\pi_t^w = \beta E_t \left(\pi_{t+1}^w \right) + \kappa_w \left(mrs_t - (w_t - p_t) \right) \tag{13}$$

where mrs_t is the average marginal rate of substitution given by

$$mrs_t = \sigma c_t + \varphi n_t \tag{14}$$

and the slope coefficient κ_w is

$$\kappa_w = \frac{\varepsilon_w - 1}{\phi_w}$$

The derivation is given in the appendix.[9]

In the special case where $\phi_w = 0$, the model effectively collapses to the model in Galí et al. (2007). Firms do not discriminate between consumer types in their labour demand, and so it follows from the unions' problems that $n_t^r = n_t^o = n_t$.

2.4 Monetary policy

The central bank controls the risk-free interest rate, which it sets according to a simple Taylor rule

$$r_t = r + \phi_\pi \pi_t \tag{15}$$

This specification implies that monetary policy is endogenous. The central bank responds to inflation, which is endogenously determined in the economy.

2.5 Equilibrium

Market clearing requires that

$$Y_t = C_t + I_t + G + F_t$$

where $G = T$ is government spending. In log-linear form, this becomes

$$y_t = \frac{C}{Y} c_t + \frac{I}{Y} i_t \tag{16}$$

[9]Instead of wage adjustment costs, we may assume that a union is allowed to reset its wage rate each period with a fixed probability $(1 - \theta_w)$ as in Calvo (1983). But to undo the implications of the implied heterogeneity across unions, each household must be assumed to provide all types of labour simultaneously in this case, or alternatively a risk-sharing arrangement between unions must be in place. This follows since rule-of-thumb consumers are barred from sharing risk through financial markets. Results, however, are very similar. In particular we would get a Phillips curve with $\kappa_w = (1 - \beta\theta_w)(1 - \theta_w)\theta_w^{-1}(1 + \varphi\varepsilon_w)^{-1}$ where ε_w is the wage elasticity of labour demand.

The only shocks to the economy that we consider are technology shocks. They evolve according to an autoregressive process of order one:

$$a_t = \rho_a a_{t-1} + e_{a,t} \tag{17}$$

It follows that the equilibrium dynamics are summarised by (2)-(17).

3 Impulse response analysis

In this section, we present impulse responses of key variables in a calibrated version of the model. To facilitate comparison, our calibration follows Galí et al. (2007). Hence, we consider a time period to be a quarter, and we set $\lambda = 0.5$, $\sigma = \eta = 1$, $\theta_p = 0.75$, $\psi = 0.3$, $\delta = 0.025$ and $\beta = 0.99$. In addition, we set $G/Y = 0.20$ with the implication that $I/Y = \alpha\delta\left(1/\beta + \delta\right)^{-1}\mu_p^{-1} = 0.18$, $C/Y = 0.62$ and $WN/PC = (1-\alpha)\,Y/C\mu_p$, under the assumption that steady-state price mark-ups $\left(\mu_p - 1\right)$ are 20 per cent, cf. Galí et al. (2007). However, we deviate from the calibration in Galí et al. (2007) in setting $\varphi = 1$ instead of $\varphi = 0.2$, a value we consider to be unrealistically low. Galí et al. (2007) need to set a high value for the labour elasticity to ensure determinacy of the equilibrium. But the introduction of wage rigidities increases the range of values of φ for which the equilibrium is determinate, cf. Colciago (2007). This allows us to set a more realistic value. Finally we set $\varepsilon_w = 4$ and $\phi_w = 174.7$. This corresponds to a steady-state wage mark-up of approximately 33 per cent, and a degree of price rigidity corresponding to $\theta_w = 0.75$ under the alternative Calvo (1983) wage-setting scheme, i.e. an average duration of wage contracts of four quarters.

We are interested in the implications of introducing rule-of-thumb consumers into the New Keynesian model, and so we compare the responses under the baseline calibration above with a calibration in which $\lambda = 0$, corresponding to a version of the model without rule-of-thumb consumers.

Figure 2 presents responses to a one standard deviation technology shock $\rho_a = 0.9$. Dashed lines are responses from the baseline model presented in the previous section, whereas solid lines are responses from the model without rule-of-thumb consumers.

Comparing the dashed and the solid lines, it is clear that the introduction of rule-of-thumb consumers is not without consequence for the responses to a technology shock. In particular, hours decline more following a positive productivity shock in the economy with rule-of-thumb behaviour in keeping

with the empirical evidence in Basu et al. (2006) and Francis and Ramey (2005). Indeed, with our baseline calibration, hours go down by -0.6 per cent in the period when the technology shock hits the economy. This coincides with the estimate in Basu et al. (2006) and Francis and Ramey (2005).

The transmission is as follows. The increase in productivity lowers firms' marginal costs. If prices were flexible, firms would lower their prices and increase supply. But since prices are sticky, some firms cannot do so and the reduction in the overall price level is limited. This means that output increases less than it would had prices been flexible. In addition, hours decline because the improvement in technology allows firms to produce the same output as before with less labour. The monetary policy authority reacts to the reduction in prices by a measured reduction of the nominal interest rate.

The fall in the interest rate makes it optimal for consumers to consume more in the current period. Optimising consumers realise this, and they also correctly anticipate that the productivity shock leads to an increase in permanent income. These two forces make optimising consumers increase their consumption.

Rule-of-thumb consumers behave differently, however. As their horizon is static, neither the increase in permanent income nor the reduction in real interest rates affects their consumption decisions. Instead, they choose consumption on the basis of current income, which is determined by current hours in production and the real wage. As noted above, hours decline because prices are sticky, but real wages respond little as a consequence of sticky wages. Hence, the decline in hours is larger than the increase in real wages, and current income declines. This makes rule-of-thumb agents consume less.

The effect on aggregate consumption depends on the relative importance of optimising and rule-of-thumb consumers in the economy, and on the size of their responses to the shocks. Aggregate consumption may still rise despite rule-of-thumb behaviour, but the presence of households that do not optimise intertemporally has an important contractionary effect. The aggregate demand effect that could potentially offset the initial reduction in hours is smaller because rule-of-thumb consumers decrease their consumption. From figure 2 we see this effect clearly: The model with rule-of-thumb consumers exhibits a smaller increase in aggregate consumption and output than the model without rule-of-thumb behaviour, and it exhibits a larger decline in hours.

This leads us to the first key result of this paper. A model with rule-

of-thumb consumers, interacting with nominal and real rigidities, can better explain the empirical evidence provided by Basu et al. (2006) and Francis and Ramey (2005). This is so even though the shock is too expansionary compared to the data.

We note that sticky wages is an essential assumption needed to obtain this result. In a model with flexible wages the increase in the real wage would be larger than the decrease in hours and rule-of-thumb agents would increase their level of consumption. There would be no contractionary effect from rule-of-thumb behaviour in this case.

Indeed, it is important to stress that all four frictions - sticky prices, sticky wages, rule-of-thumb behaviour and capital adjustment costs - are essential to subdue the expansionary effect of the shock. Sticky prices are needed for a decline in hours, and sticky wages are needed for this to lead to a reduction in the current income of rule-of-thumb consumers. A sufficiently high fraction of rule-of-thumb consumers, then, is needed for this reduction in current income to have an effect on the aggregate economy. And finally, capital adjustment costs are needed to dampen investment, an increase in which would otherwise offset the contractionary effect from the response of rule-of-thumb consumers.

Finally, we note that the real wage increase in the model with sticky wages perfectly fits the empirical evidence on the real wage response provided in Liu and Phaneuf (2005), whereas the response of the real wage in the model with flexible wages would be excessively procyclical. In our opinion, this fact is further confirmation that sticky wages is a sensible assumption.

From the analysis in this section, we conclude that the introduction of rule-of-thumb consumers, a considerable change to the standard DSGE set-up, does not lead to counterfactual responses to productivity shocks. On the contrary, (to the extent that a productivity shock leads to a decline in hours on impact) we find that the model's transmission mechanism is *improved*.

It is important to stress, however, that we perform a conditional analysis in the spirit of Galí (1999) and not an unconditional exercise as is typical in the RBC literature. That is, it is not our goal to reproduce the unconditional moments found in the data. Indeed, given the response of hours, our one-shock model would perform very badly in such an exercise. As in Galí (1999), Galí and Rabanal (2005) and Francis and Ramey (2005), productivity shocks are not the main driving force of aggregate fluctuations in our

model.[10] Nevertheless, we believe that our conditional analysis is relevant in order to evaluate the effects of rule-of-thumb behaviour. Even if productivity shocks are not the *main* driving force behind the business cycle, they still represent a source of fluctuations in the economy that needs to be considered in detail, especially because of the prominent role played by these shocks in the RBC literature.

4 Quintuple stickiness

In this section we present results from the model in section 2 extended with habit formation in consumption. That is, we let utility today depend not on consumption today by itself, but on consumption today relative to consumption in the previous period. This makes optimising households look back as well as forward when making consumption decisions. In addition, unions take the effect of habit on the utility of its members into account when setting wages. Thus, the introduction of habit formation in consumption changes the Euler equation of optimising consumers and the wage-setting equation. Details are given in the appendix.

Our model's *quintuple* stickiness makes our analysis more comprehensive than previous studies of technology shocks. In particular, we model the capital accumulation process explicitly, and we introduce endogenous monetary policy by letting the model's central bank respond to inflation developments. In comparison, Galí and Rabanal (2005) ignore investment dynamics in their model, while Francis and Ramey (2005) let monetary policy be exogenous. Finally, we consider the implication of credit constraints by allowing for rule-of-thumb behaviour.

The analysis of the *quintuple*-sticky model serves two purposes. First, the model helps us explain the empirical evidence on key macroeconomic variables besides hours worked. Second, it allows us to analyse the roles played by many of the frictions studied in the literature on technology shocks and the interaction of these frictions with rule-of-thumb consumers. We consider each of these issues in turn.

[10] To improve the unconditional performance of the model, we should include shocks to other variables such as demand shocks or possibly investment-specific technology shocks as in Fisher (2006).

4.1 Output, consumption and investment

The model presented in the previous section can easily reproduce the decline in hours after a technology shock found in the empirical literature. In addition, the response of the real wage seems plausible given the empirical evidence. But the model fares less well when considering other key macroeconomic variables for which we have empirical evidence. In particular, output, consumption and investment all increase on impact of a shock to technology in the model. This is in contrast to the evidence in Basu et al. (2006), who find that output and consumption change little on impact of a technology shock before increasing in the periods following the shock, whereas non-residential investment falls sharply on impact before rising. Francis and Ramey (2005) find similar responses for output and consumption, whereas the response of investment is statistically insignificant in their analysis.

Figure 3 presents the second key result in this paper. The *quintuple*-sticky model with rule-of-thumb consumers (dashed lines) can reproduce the zero impact responses of output and consumption. This is because habit persistence slows down the response of optimising consumers. With habit formation in consumption, optimising consumers need time to appreciate the increased scope for consumption given to them by the positive shock to technology. This leads to a hump-shaped response of optimising household's consumption, further restraining the expansion in the economy.

Now, perhaps the contractionary effects are even too strong in our *quintuple*-sticky model with rule-of-thumb consumers; hours go down more than one per cent, and aggregate consumption actually declines. However, as we show below, we may undo this excess contraction by modifying the baseline calibration, e.g. by lowering the percentage of rule-of-thumb consumers or the degree of price stickiness.[11] Here we keep the parameter values chosen by Galí et al. (2007) to facilitate comparison. An estimated model could deliver more precise guidance on the parameter values needed to replicate the empirical results. Our objective here is simply to show that the *quintuple*-sticky model with rule-of-thumb consumers delivers a very contractionary propagation of technology shocks, and that a financial friction in this form provides an additional explanation, along with nominal and real rigidities, of why a productivity shock may lead to a decline in hours on impact.

[11] Indeed, given the empirical evidence, both these parameters may appear to be uncomfortably high in our baseline calibration. For a discussion, see Furlanetto and Seneca (2007).

Figure 3 also shows responses to a technology shock for the model without rule-of-thumb consumers (solid lines). We see that rule-of-thumb behaviour is crucial in order to replicate the zero impact responses found in empirical studies. Without this friction, both output and consumption increase on impact of the shock.

Turning to investment, Basu et al. (2006) find a significantly negative response of this variable after a productivity shock. Given our analysis, this is puzzling. Indeed, investment increases after a positive technology shock in all versions of the model. In particular, the positive response of investment is not related to the presence of rule-of-thumb consumers. Basu et al. (2006) argue that their evidence on investment is compatible with a sticky price model in the case where monetary policy is exogenous. Once we allow for an endogenous reaction from the monetary authority, the response of investment is always both positive and fairly large. We believe that our assumption about monetary policy is the more reasonable one, however, and the evidence provided by Galí et al. (2003) supports this claim.

The identifying assumption often used in the empirical VAR literature is that a technology shock has a permanent effect on labour productivity. Therefore, we check whether a permanent technology shock delivers the same results as the temporary (but highly persistent) shock considered above. This is confirmed in figure 4. In particular, the impact responses of hours, the real wage, consumption and output are in line with the estimated responses in Basu et al. (2006) and Francis and Ramey (2005).

4.2 Interacting frictions and sensitivity analysis

It is important to note that all the frictions in the *quintuple*-sticky model are needed to obtain the results just considered. In figure 5, we show impulse responses to a (temporary, highly persistent) technology shock for the model with nominal rigidities only (sticky wages and sticky prices) and for the model with real rigidities only (capital adjustment costs and habit persistence), in both cases without rule-of-thumb consumers. We see that the model with nominal rigidities only (solid lines) performs poorly. Consumption, output, hours and investment all increase sharply following the technology shock. This is because, without capital adjustment costs, a technology shock leads to an investment boom that more than offsets the contractionary effect from other frictions in the model. In keeping with the theoretical results in Francis and Ramey (2005), the model with real rigidities only (dashed lines) is able

to reproduce a decline in hours, but output, consumption and investment responses are too expansionary compared to those estimated by Basu et al. (2006) and Francis and Ramey (2005).

Figure 5 confirms that real, nominal and financial frictions are all important to obtain a sizeable negative response of hours and a zero impact response of output and consumption. The same message comes from the more careful sensitivity analysis for the *quintuple*-sticky model that we present in figure 6. There, we plot the impact responses of output, consumption and hours for a large spectrum of parameter values to our temporary, but highly persistent technology shock. If lines are flat, it means that impact responses are not affected by the specific parameter considered.[12]

From figure 6 we see that the labour supply elasticity and the habit persistence parameter for constrained agents do not influence impact responses. The other panels confirm that impact responses are declining in the percentage of rule-of-thumb consumers, in the degree of wage and price rigidity, and in the degree of habit persistence in optimising consumption. Notice that the excess contraction in consumption disappears if λ is close to 0.25 instead of 0.5 as in the baseline calibration, or if we let θ_p be in the region of 0.6 instead of 0.75. The results for the coefficient in the Taylor rule suggest that the central bank will have to be very aggressive to overturn the results. On a similar note, if σ is very low, optimising consumers respond strongly to monetary policy and the contractionary effect from rule-of-thumb behaviour carries less weight at the aggregate level. Finally, when η is high it means that capital adjustment costs are low, in which case all the impact responses become positive. Again, this is because investment overreacts when prices are sticky and capital adjustment costs are low. Thus, figure 6 confirms that all five frictions in the model are needed to curb the expansionary effects of positive shocks to technology.

5 Conclusion

The introduction of rule-of-thumb consumers into the New Keynesian DSGE model has proven to be a useful way to explain responses to fiscal shocks. The purpose of this paper is to check whether the introduction of this substantial financial friction affects the transmission mechanism of productivity shocks.

[12] The analysis is partial in the sense that we vary one parameter at a time, while the remaining parameters are fixed at the values chosen for the baseline calibration.

We find that rule-of-thumb consumers, in combination with real and nominal rigidities, can explain a sizeable decline in hours worked after a positive productivity shock as suggested by the empirical evidence in Galí (1999), Francis and Ramey (2005) and Basu et al. (2006).

Moreover, we show that within our *quintuple*-sticky business cycle framework, only a combination of nominal rigidities, real rigidities and limited access to financial markets can reproduce a sizeable negative effect on hours *and* a zero impact response of output and consumption.

In addition, our *quintuple*-sticky model is a useful laboratory in which to compare results from many other papers in the literature. A model with real rigidities alone can explain a negative effect on hours, but the inclusion of a financial friction interacting with nominal rigidities is essential in order to reproduce a zero impact effect on output and consumption. A model with nominal rigidities alone cannot explain a decline in hours.

We conclude that the transmission mechanism for technology shocks is improved by including rule-of-thumb consumers in the model. Thus, our analysis suggests that researchers may safely build rule-of-thumb consumers into their models to reproduce empirically plausible responses to fiscal policy shocks without having to fear that the model becomes less realistic in other dimensions. Indeed, this financial friction may be an additional explanation, along with nominal and real rigidities, of why hours may decline following a productivity shock as the empirical literature suggests. An important topic for future empirical research is to investigate the relative importance of these frictions. In future work we therefore plan to estimate the *quintuple*-sticky model using Bayesian techniques.

A Appendix

Derivation of the wage schedule. The first-order condition to the union's problem becomes

$$\begin{aligned} 0 &= \lambda \frac{\partial U_t^r}{\partial C_t^r} \frac{\partial C_t^r}{\partial W_t(z)} + (1-\lambda) \frac{\partial U_t^o}{\partial C_t^o} \frac{\partial C_t^o}{\partial W_t(z)} - (N_t(z))^{\varphi} \frac{\partial N_t(z)}{\partial W_t(z)} \\ &\quad + \beta E_t \left[\lambda \frac{\partial U_{t+1}^r}{\partial C_{t+1}^r} \frac{\partial C_{t+1}^r}{\partial W_t(z)} + (1-\lambda) \frac{\partial U_{t+1}^o}{\partial C_{t+1}^o} \frac{\partial C_{t+1}^o}{\partial W_t(z)} \right] \end{aligned}$$

Since the demand for union z's type of labour service is given by the usual relation (implied by the Dixit-Stiglitz aggregator)

$$N_t(z) = \left(\frac{W_t(z)}{W_t} \right)^{-\varepsilon_w} N_t$$

we have

$$\frac{\partial N_t(z)}{\partial W_t(z)} = -\varepsilon_w \frac{N_t(z)}{W_t(z)}$$

and from the budget constraints we get

$$\frac{\partial C_t^i}{\partial W_t(z)} = \frac{1}{P_t} \left[(1-\varepsilon_w) N_t(z) - \phi_w \left(\frac{W_t(z)}{W_{t-1}(z)} - 1 \right) \frac{W_t N_t}{W_{t-1}(z)} \right]$$

and

$$\frac{\partial C_{t+1}^i}{\partial W_t(z)} = \frac{1}{P_{t+1}} \phi_w \left(\frac{W_{t+1}(z)}{W_t(z)} - 1 \right) \frac{W_{t+1}^2}{(W_t(z))^2} N_{t+1}$$

for $i \in \{o, r\}$.

Inserting these expressions in the first-order condition, imposing symmetry so that $W_t(z) = W_t$ and $N_t(z) = N_t$ for all z, and rearranging gives

$$\begin{aligned} 0 &= \left(\lambda \frac{\partial U_t^r}{\partial C_t^r} + (1-\lambda) \frac{\partial U_t^o}{\partial C_t^o} \right) \frac{W_t}{P_t} \left[(1-\varepsilon_w) - \phi_w (\Pi_t^w - 1) \Pi_t^w \right] + \varepsilon_w N_t^{\varphi} \\ &\quad + \beta E_t \left[\left(\lambda \frac{\partial U_{t+1}^r}{\partial C_{t+1}^r} + (1-\lambda) \frac{\partial U_{t+1}^o}{\partial C_{t+1}^o} \right) \phi_w \left(\Pi_{t+1}^w - 1 \right) \Pi_{t+1}^w \frac{W_{t+1}}{P_{t+1}} \frac{N_{t+1}}{N_t} \right] \end{aligned}$$

where $\Pi_t^w = W_t / W_{t-1}$ and

$$\frac{\partial U_t^i}{\partial C_t^i} = \left(C_t^i \right)^{-\sigma}$$

when the instantaneous utility is given by (1). Log-linearising gives (13) in the text.

Habit persistence. With habit persistence in consumption, the instantaneous utility function of a household is given by

$$U_t^i = \frac{\left(C_t^i - h_i \bar{C}_{t-1}^i\right)^{1-\sigma} - 1}{1-\sigma} - \frac{\left(N_t^i\right)^{1+\varphi}}{1+\varphi}$$

where $i \in \{o, r\}$ and $\bar{C}_{t-1}^i$ denotes aggregate consumption by households of type i at time t. The degree of habit in consumption is governed by the parameter h_i. With this specification, habit formation is external with respect to the household itself in the sense that the household ignores the effect of its current consumption choice on the lagged consumption term that enters the utility function next period. But habit formation is internal with respect to the type of household since the lagged consumption term is aggregate consumption by the class of households to which the household belongs as opposed to aggregate consumption by all households in the economy. In the limiting case where $h_i = 0$, there is no habit formation for a household of type i.

With habit formation, the marginal utility of consumption becomes

$$\frac{\partial U_t^i}{\partial C_t^i} = \left(C_t^i - h_i \bar{C}_{t-1}^i\right)^{-\sigma} = \left(C_t^i - h_i C_{t-1}^i\right)^{-\sigma}$$

where the last equality follows from the fact that all households of a given type are identical so that $C_t^i = \bar{C}_t^i$ for all t. Using this expression in the union's first-order condition and log-linearising gives (13) in the text only that now (14) must be replaced by

$$mrs_t = \chi_r \left(c_t^r - h_r c_{t-1}^r\right) + \chi_o \left(c_t^o - h_o c_{t-1}^o\right) + \varphi n_t \tag{18}$$

where

$$\chi_r = \sigma \frac{\lambda}{1-h_r} \frac{(1-h_o)^\sigma}{\lambda (1-h_o)^\sigma + (1-\lambda)(1-h_r)^\sigma}$$

and

$$\chi_o = \sigma \frac{(1-\lambda)}{1-h_o} \frac{(1-h_r)^\sigma}{\lambda (1-h_o)^\sigma + (1-\lambda)(1-h_r)^\sigma}$$

Note that for $h_r = h_o = 0$ this is identical to (14) in the text. For $h_r = h_o = h > 0$, we get $\chi_r = \lambda/(1-h)$ and $\chi_o = (1-\lambda)/(1-h)$. We generally assume that $h_r = 0$ and $h_o > 0$.

Habit persistence also changes the optimising household's stochastic discount factor, which is derived from its first-order conditions with respect to consumption and bond holdings. That is,

$$\Lambda_{t,t+k} = \beta \left(\frac{C^o_t - h_o C^o_{t-1}}{C^o_{t+k} - h_o C^o_{t+k-1}} \right)^{\sigma} \frac{P_t}{P_{t+k}}$$

Taking expectations of this equation with $k = 1$ gives the Euler equation for optimising consumption with habit persistence. The log-linear representation is given by

$$c^o_t = \frac{h_o}{1+h_o} c^o_{t-1} + \frac{1}{1+h_o} c^o_{t+1} - \frac{1-h_o}{1+h_o} \frac{1}{\sigma} \left(r_t - E_t \pi_{t+1} \right) \tag{19}$$

With habit formation, this equation replaces (2) in the text. Note that they are identical when $h_o = 0$.

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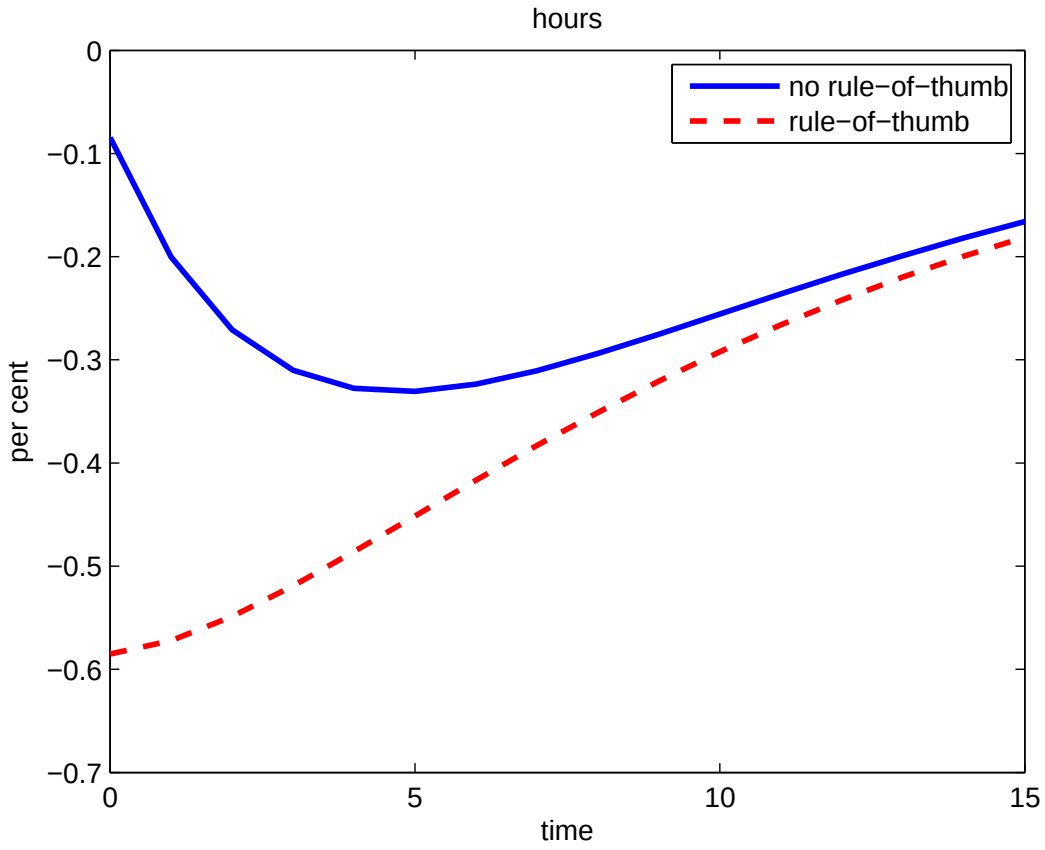

Figure 1: The response of hours to a technology shocks ($\rho_a = 0.9$) in the baseline model (with and without rule-of-thumb consumers).

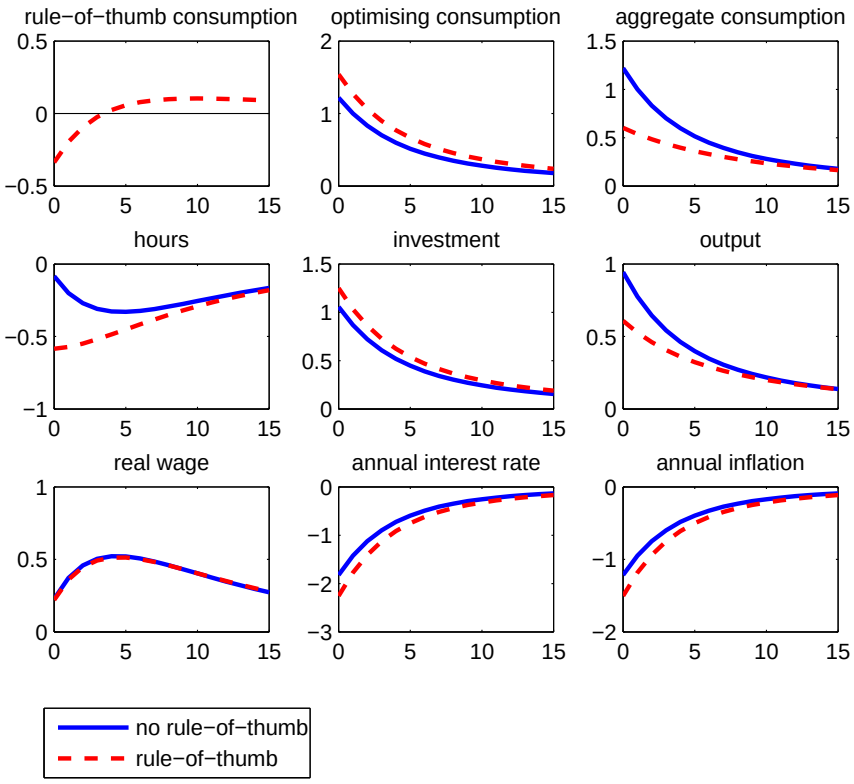

Figure 2: The responses of key variables to a technology shock ($\rho_a = 0.9$) in the baseline model (with and without rule-of-thumb consumers).

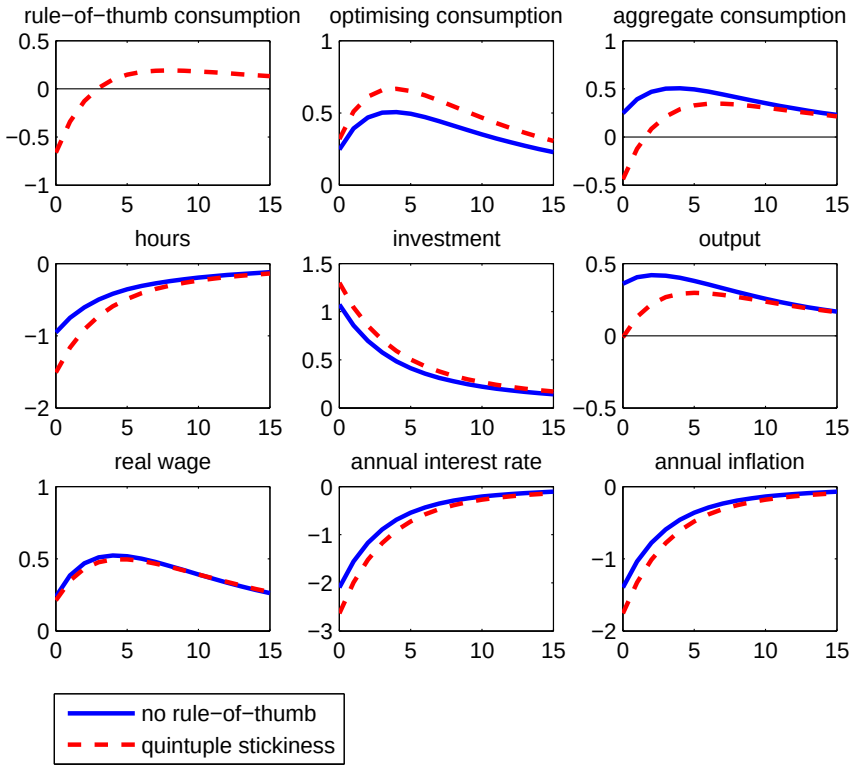

Figure 3: The response of key variables to a technology shock ($\rho_a = 0.9$) in the *quintuple*-sticky model.

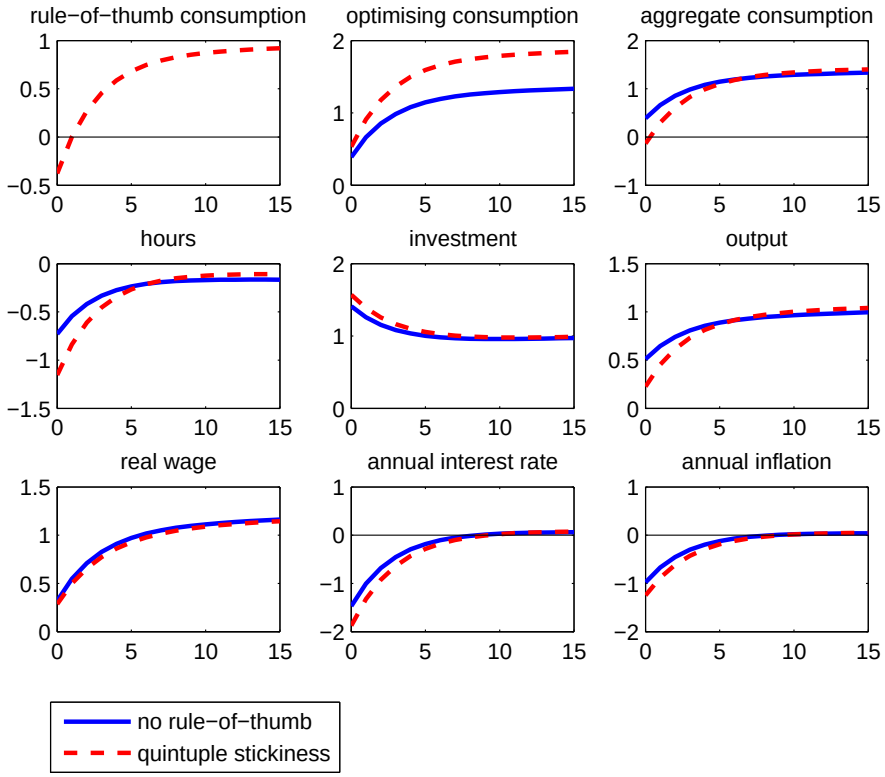

Figure 4: The responses of key variables to a technology shock ($\rho_a = 1$) in the *quintuple*-sticky model.

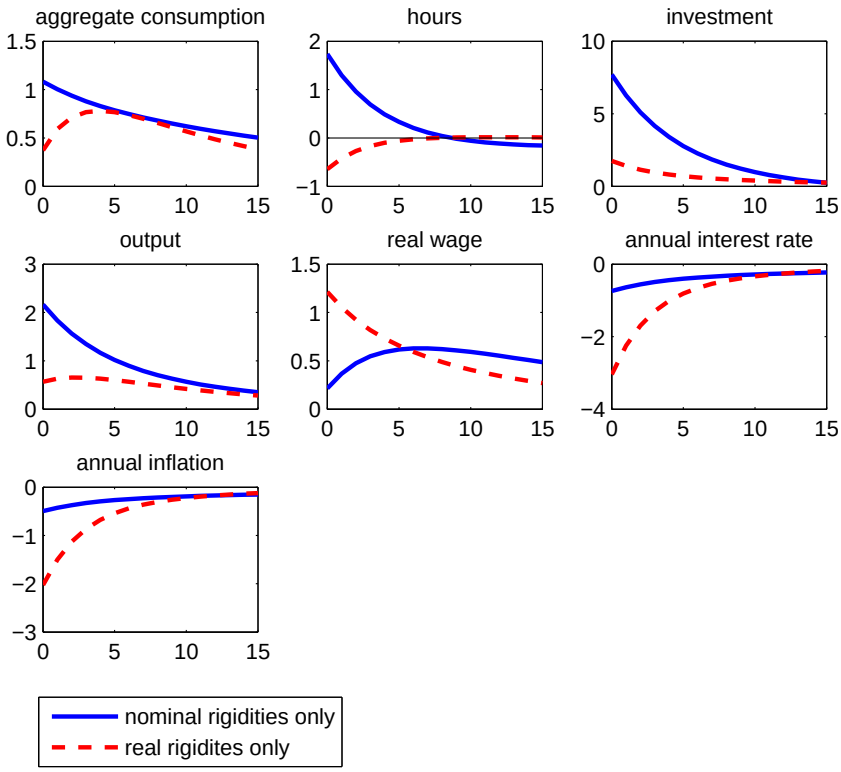

Figure 5: The responses of key variables to a technology shock ($\rho_a = 0.9$) in a version with nominal rigidities only (sticky prices and sticky wages) and in a version with real rigidities only (habit formation and capital adjustment costs).

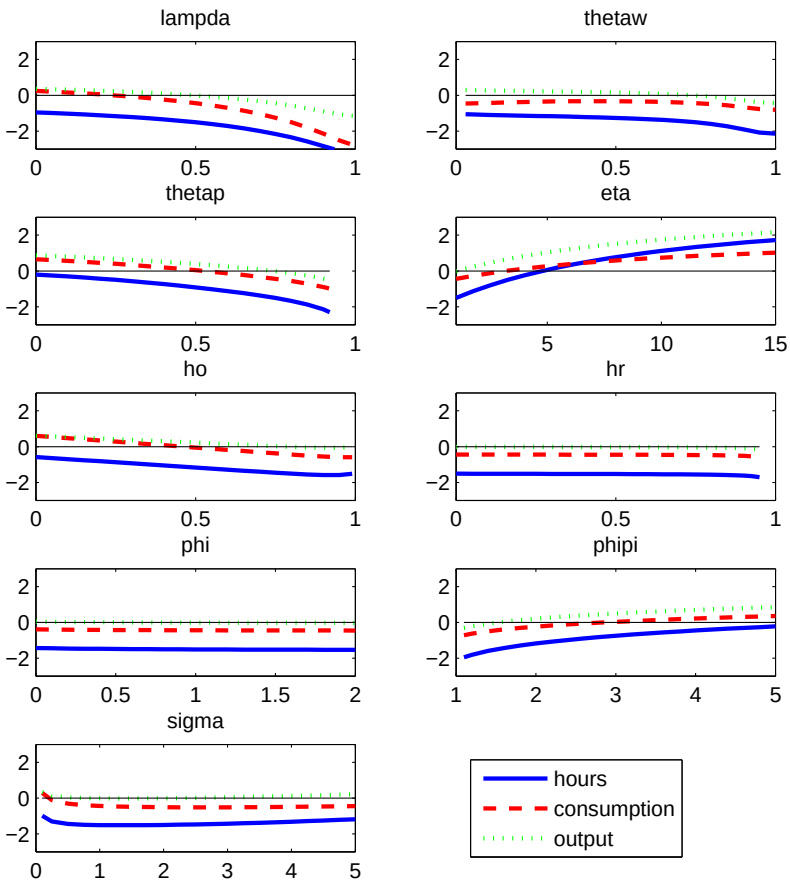

Figure 6: Sensitivity analysis: Impact-responses of hours, consumption and output as a function of parameter values.

Chapter 4

Fiscal shocks, rule-of-thumb consumers and real rigidities*

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Abstract

In this paper we show that empirically plausible results on the effects of fiscal shocks in Galí, López-Salido and Vallés (2007) rely on a high degree of price stickiness and a large percentage of financially constrained agents. Real rigidities in the form of habit persistence, fixed firm-specific capital and Kimball demand curves interact in interesting ways with nominal and financial rigidities and allow us to reproduce the same consumption multiplier as Galí et al. (2007) under only two and a half quarters of price stickiness, instead of four, and only 30 per cent of constrained agents instead of 50 per cent. Therefore, real rigidities are useful in the study of fiscal shocks in addition to monetary and productivity shocks as has been shown in the previous literature.

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1 Introduction

One of the most important developments in modern macroeconomics has been the replacement of traditional ad hoc models with dynamic stochastic general equilibrium (DSGE) models in economic policy analysis. In the New Keynesian DSGE literature, the bulk of the contributions have focused on monetary policy issues. But recently, a number of authors have begun to investigate the responses of key macroeconomic variables to fiscal shocks in the class of DSGE models with imperfect competition and nominal rigidities.

In a noteworthy example, Galí et al. (2007) show that nominal rigidities in combination with deviations from Ricardian equivalence can explain empirically observed responses to government spending shocks, while responses in the baseline real business cycle (RBC) model are in contrast with the empirical evidence. In particular, a number of recent empirical papers suggest that private consumption increases following a positive shock to government consumption.[1] While the RBC model predicts a decline in private consumption following such a shock, cf. Baxter and King (1993), private consumption may rise after a positive shock to government spending in the sticky-price model of Galí et al. (2007) if so-called rule-of-thumb consumers, who simply consume their current disposable income each period, are allowed to co-exist with intertemporally optimising consumers.[2] In the model, intertemporally optimising consumers decrease their consumption following a government spending shock because they correctly anticipate a decline in future income as a consequence of taxation. But rule-of-thumb consumers increase their consumption because their current income increases. Under the necessary auxiliary assumptions of sticky prices, monopolistic competition in the labour market and deficit financing, if a sufficiently large fraction of households behave according to a rule of thumb, aggregate consumption rises.

A potential weakness of the rule-of-thumb theory of consumption is that both the degree of nominal rigidity and the fraction of rule-of-thumb consumers needed to generate a positive response of consumption is uncomfort-

[1] See, for example, Blanchard and Perotti (2002), Caldara and Kamps (2007), Fatas and Mihov (2002), Galí et al. (2007) and Perotti (2005, 2008).

[2] Alternative approaches with similar objectives can be found in Bouakez and Rebei (2007), Linnemann and Shabert (2005), Linnemann (2006), López-Salido and Rabanal (2006) and Ravn, Schmitt-Grohé and Uribe (2007).

ably high given the recent empirical literature. In the baseline calibration in Galí et al. (2007), the expected duration of prices is set at one year, and half the consumers in the economy choose how much to consume by following a simple rule of thumb. Recent microeconomic evidence, however, points to two or three quarters of expected price duration, e.g. Bils and Klenow (2004) and Nakamura and Steinsson (2007), and several studies arrive at estimates of the percentage of rule-of-thumb consumers that are much lower than the 50 per cent originally suggested by Mankiw (2000). For instance, Campbell and Mankiw (1991) obtain 35 per cent for the US and 20 per cent for the UK, while Banerjee and Batini (2003) find 26 per cent for the US and 15 per cent for the UK.

The values of these parameters are crucial in the Galí et al. (2007) model. Once they are lowered to more realistic values (say, 2.5 quarters of price stickiness and 30 per cent of constrained agents as in our benchmark), the main result in Galí et al. (2007), i.e. that a model with rule-of-thumb consumers can generate a positive response of consumption following a government spending shock, is lost.

The main objective of this paper is to reconcile the evidence on these structural characteristics of the economy with the empirical responses of private consumption to a government spending shock. We show that this can be done by adding a number of what we consider to be realistic features to the model developed by Galí et al. (2007) to lower its dependence on price stickiness and households that do not take part in financial markets so as to smooth consumption.[3] The features we consider are real rigidities in the form of habit persistence in consumption, non-constant elasticities of demand, and fixed firm-specific capital.[4] Each of these rigidities has proven

[3]The idea that a fraction of households follow the simple rule of thumb that they consume their current disposable income each period, while the remaining fraction solve an intertemporal optimisation problem, was first put forward in the empirical consumption literature as an alternative to the permanent income hypothesis, see in particular Hall (1978) and Campbell and Mankiw (1989). We emphasise the interpretation that some households follow a rule of thumb because a financial friction bars them from participating in financial and capital markets. Alternatively, rule-of-thumb consumers may choose not to do so because of myopia or extreme impatience.

[4]We refer to all these three features as real rigidies to separate them conceptionally from the nominal rigidities that act as direct impediments to the adjustment of nominal variables, and from the financial constraint represented by rule-of-thumb consumers. Hence, our definition includes both the rigidities that work as direct impediments to the adjustment of real variables, and the 'real rigidities' of Ball and Romer (1990), the pres-

to be very useful in DSGE analyses in explaining empirical regularities of the transmission of other shocks, especially monetary shocks, see e.g. Christiano et al. (2005), Smets and Wouters (2003) and Woodford (2003), and productivity shocks, e.g. Francis and Ramey (2005) and Furlanetto and Seneca (2007). But their implications for the propagation of fiscal shocks have not been thoroughly analysed so far. This, in itself, provides a second motivation for this paper. Before giving a preview of the results, we briefly discuss each of these rigidities in turn.

The idea that habits may influence households' consumption behaviour grew out of the attempts in the mid-20th century empirical demand theory to explain the importance of lagged dependent variables in estimated demand functions, see e.g. Brown (1952), or the discussions of this literature in Deaton and Muellbauer (1980) and Deaton (1992). More recently, habit formation has been introduced into policy-oriented general equilibrium models following the specification in the asset pricing model by Abel (1990), in which utility today depends on consumption today relative to consumption in previous periods. For an example, see Christiano et al. (2005). In our model, habit persistence is important because it smooths the negative response of optimising households to a government spending shock. Hence, a smaller fraction of rule-of-thumb consumers is needed to generate a plausible response of aggregate consumption.

The second source of real rigidity that we introduce into the model is demand functions with non-constant elasticity of demand of the sort suggested by Kimball (1995). This represents a modification of the formalisation of monopolistic competition by Dixit and Stiglitz (1977) that has become standard in macroeconomics following the seminal paper by Blanchard and Kiyotaki (1987). The relative demand for an individual good is still decreasing in the relative price, but the elasticity – and hence the desired mark-up over marginal costs of the price-setting firm that produces it – now depend on its relative output. This induces a potential source of strategic complementarity in price setting in the model as discussed by Kimball (1995) and Woodford (2003, ch. 3). If the elasticity of demand falls with relative output, for instance, a firm that reduces its price will moderate its price reduction because the increase in demand it induces increases the desired mark-up. In this case, the firm is more reluctant to change prices away from the level charged

ence of which characterises an economy with strategic complementarity in price setting, cf. the discussion in Woodford (2003, ch. 3).

by other firms in the economy that may not be changing their prices in any given period. In this way, the Kimball demand specification amplifies the effect of any nominal price rigidity that prevents some firms from adjusting prices. This makes it possible to obtain realistic dynamics of key macroeconomic variables with lower degrees of nominal price stickiness as emphasised by Eichenbaum and Fisher (2007) and Levin et al. (2007).

Firm-specific capital is a relatively recent addition to the DSGE literature pioneered by Christiano (2005), Sveen and Weinke (2005) and Woodford (2005). The standard assumption in the literature is that firms rent perfectly mobile capital from households in a rental market. With firm-specific capital, in contrast, the economy's capital stock is owned by firms, and capital cannot be instantaneously reallocated across firms to equalise marginal costs. As argued, for instance, by Danthine and Donaldson (2002), the firm-specific capital assumption is probably the more appealing one in terms of realism.

For our purposes, the important implication of firm-specific capital is that it increases the strategic complementarity in price setting as described by Sveen and Weinke (2005) and Woodford (2005). For simplicity, we follow Coenen et al. (2007) by abstracting from the endogenous accumulation of firm-specific capital.[5] Instead, we assume that each firm is endowed with a fixed level of the capital good as in Sbordone (2002), resulting in a production process with decreasing returns to labour. With this specification, we retain the important implication of firm-specific capital that firms cannot reallocate capital instantaneously across firms to equalise marginal costs.

As already mentioned, non-constant elasticities of demand and fixed firm-specific capital help us to reduce the degree of price stickiness in the model, according to the mechanism explained in Sveen and Weinke (2005), Eichenbaum and Fisher (2007) and Woodford (2005). However, these frictions are also useful to lower the percentage of rule-of-thumb consumers in the model and this effect is new in the literature. In fact, they imply a lower inflation response to a given change in the marginal cost which translates into a lower response by the monetary policy authority. A lower increase in the interest rate pushes-up optimising consumption, making the model less dependent on rule-of-thumb consumers.

Introducing the rigidities just described in the model developed by Galí et al. (2007) gives us this paper's main result: Real rigidities are useful

[5] Similarly, some authors abstract from the endogenous capital accumulation process under the rental market assumption, e.g. Erceg, Henderson and Levin (2000).

not only in accounting for the economy's responses to monetary policy and productivity developments as has been emphasised in the existing literature, but also in accounting for responses to fiscal policy shocks. In particular, we arrive at an empirically plausible increase in private consumption following a government spending shock for a much lower degree of price rigidity and a much lower fraction of rule-of-thumb consumers than in Galí et al. (2007). With habit formation in consumption, fixed firm-specific capital and Kimball demand, we obtain the same consumption multiplier as in Galí et al. (2007) with two and a half quarters of expected price duration (as opposed to four) and 30 per cent of rule-of-thumb consumers (as opposed to 50). Thus, as in Furlanetto and Seneca (2007), we find an important role for the *interaction* of nominal, real and financial rigidities in realistically accounting for the empirical evidence on the response of a key macroeconomic variable to empirically important disturbances to the economy. Importantly, rule-of-thumb consumers remain essential to generate this result. An alternative perspective, then, is that the rule-of-thumb theory becomes more appealing in a setting that is probably more realistic than the one in which it was originally introduced.

The paper is organised as follows. In section 2, we present the model, and in section 3 the results. Section 5 gives a few concluding remarks.

2 The model

The model is a standard New Keynesian dynamic stochastic general equilibrium model augmented with habit persistence in consumption, Kimball (1995) demand curves and rule-of-thumb consumers. Except for the presence of real rigidities, the model is identical to Galí et al. (2007). The economy consists of a continuum of firms, a continuum of households, a continuum of labour unions, a central bank responsible for monetary policy, and a government collecting lump-sum taxes and issuing bonds to finance its expenditures. There is monopolistic competition in both goods and labour markets. In particular, there is a continuum of differentiated intermediate goods and a continuum of differentiated labour services. In the goods market, this leads to a downward-sloping demand curve for each intermediate good, and in the labour market it leads to a downward-sloping demand curve for each labour type.

A fraction λ of households are rule-of-thumb consumers - or 'spenders' in

the terminology of Mankiw (2000). These consumers simply consume their respective disposable income each period. The remaining fraction $1 - \lambda$ of households are optimisers - or 'savers' - that have access to financial markets. Hence, they choose plans for consumption and bond holdings to maximise lifetime utility. Consumers are assumed to form habits in consumption. That is, the utility a household obtains from a given level of consumption in a given period depends on the level of consumption in that period relative to the level of consumption in the previous period. Wages are set by unions each representing a differentiated type of labour service supplied by households. Wages are assumed to be flexible. That is, each union sets a new wage for its members each period to maximise an average of their utilities taking the effect of this wage on the members' budget constraints into account.

Each firm produces one of the differentiated intermediate goods. It does so by combining capital with a homogenous labour input constructed as a Dixit-Stiglitz aggregate of the differentiated labour services supplied by households. The firm sets its price according to a Calvo (1983) price-setting mechanism and stands ready to satisfy demand at the chosen price. The elasticity of the demand it faces depends on the level of output produced as in Kimball (1995). In particular, the elasticity of demand falls with the level of output. This is known to increase the degree of strategic complementarity in price-setting, cf. Woodford (2003, ch 3).

We consider two alternative assumptions concerning the structure of the capital market. Under the first assumption, the economy's capital stock is owned by the optimising households. In this case, firms rent the capital they employ in production in a common rental market, and capital can be reallocated across firms instantaneously. We allow for endogenous accumulation of capital under this assumption by letting households choose how much to invest in new capital each period. But we also assume that it is costly to adjust the capital stock. Consequently, the aggregate stock of capital is fixed in the limiting case where the capital adjustment cost goes to infinity. Rule-of-thumb consumers do not take part in the capital market. Under the second assumption, the capital stock is owned by firms, and capital cannot be reallocated across them. That is, capital is specific to individual firms. For simplicity, we abstract from endogenous capital accumulation and assume that the capital stock is fixed under this assumption. To encompass these two alternative assumptions on the structure of the capital market in the model, we define a dummy variable ι taking the value 1 under the rental

market assumption and 0 when capital is firm-specific[6], i.e.

$$\iota = \begin{cases} 1 & \text{if capital is owned by households} \\ 0 & \text{if capital is owned by firms} \end{cases}$$

Each period begins by the realisation of shocks to the economy. We concentrate on fiscal spending shocks and abstract from other shocks that may affect the economy.

2.1 Households

The instantaneous utility function of a household is given by

$$U_t^i = \frac{\left(C_t^i - h_i \bar{C}_{t-1}^i\right)^{1-\sigma} - 1}{1-\sigma} - \frac{\left(N_t^i\right)^{1+\varphi}}{1+\varphi} \tag{1}$$

where $i \in \{o, r\}$ denotes the type of household – optimising or rule-of-thumb – and $\bar{C}_{t-1}^i$ denotes aggregate consumption by households of type i at time t. The degree of habit in consumption is governed by the parameter h_i. With this specification, habit formation is external with respect to the household itself in the sense that the household ignores the effect of its current consumption choice on the lagged consumption term that enters the utility function in the next period. But habit formation is internal with respect to the type of household since the lagged consumption term is aggregate consumption by the class of households to which the household belongs as opposed to aggregate consumption by all households in the economy. In the limiting case where $h_i = 0$, there is no habit formation for a household of type i.

An optimising household maximises expected lifetime utility given by

$$E_0 \sum_{t=0}^{\infty} \beta^t U_t^o \tag{2}$$

where E_o is an operator representing expectations over all states of the economy conditional on period-0 information, and $\beta \in (0, 1)$ is the subjective

[6]Nothing, in principle, prevents this variable from taking intermediate values. This would correspond to an economy in which a share of the capital stock is owned by households and rented to firms, while the remaining share is firm-specific. We do not pursue this possibility here, though few things would change in the specification of the model.

discount factor. Maximisation is subject to a sequence of flow budget constraints (and implicitly a no-Ponzi game condition):

$$P_t C_t^o + E_t \left(\Lambda_{t,t+1} B_{t+1}\right) = W_t N_t^o + B_t - P_t T_t^o + \iota \left(R_t^k K_t^o - P_t I_t^o\right) \quad (3)$$

where W_t is the nominal wage, P_t is the aggregate price index and T_t^o is the real lump-sum tax paid by optimising consumers. The left-hand side gives the allocation of resourses to consumption and a portfolio of bonds, $E_t \left(\Lambda_{t,t+1} B_{t+1}\right)$, where $\Lambda_{t,t+1}$ is the stochastic discount factor so that the risk-free interest rate is given by the relation $1 + R_t = \left(E_t \Lambda_{t,t+1}\right)^{-1}$. The right-hand side gives available resources as the sum of labour income, $W_t N_t^o$, initial financial wealth, B_t, less nominal lump-sum taxes paid to the government, $P_t T_t^o$. Finally, under the assumption that the economy's capital stock is owned by households, the household receives rent for its capital, $R_t^k K_t^o$, where R_t^k is the rental rate of the capital it owns, K_t^o, and allocates resources to investment, $P_t I_t^o$. Under this assumption, the household's capital evolves according to

$$K_{t+1}^o = (1 - \delta) K_t^o + \phi \left(\frac{I_t^o}{K_t^o}\right) K_t^o \quad (4)$$

where δ is the rate of depreciation, and $\phi(.)$ is an adjustment cost function satisfying $\phi(\delta) = \delta$, $\phi' > 0$, $\phi'(\delta) = 1$ and $\phi'' \leq 0$.

The optimisation problem, according to which the household chooses plans for consumption and bond holdings, gives rise to a modified version of the well-known Euler equation which we state in log-linear form[7]:

$$c_t^o = \frac{1}{1 + h_o} c_{t-1}^o + \frac{1}{1 + h_o} E_t c_{t+1}^o - \frac{1 - h_o}{1 + h_o} \frac{1}{\sigma} \left(r_t - E_t \pi_{t+1}\right) \quad (5)$$

Because of habit formation in consumption, the Euler equation now contains a term in lagged consumption. Note that this equation reduces to the standard Euler equation for $h_o = 0$. For $\iota = 0$, i.e. under the assumption that firms own the capital stock, this is the only first-order condition for optimising consumers. For $\iota = 1$, i.e. with a rental market for capital, the optimising household also chooses investment. As shown by Galí et al. (2007), the first-order conditions to this problem represent the dynamics of Tobin's q and its relation to investment, and their log-linear forms are given by

$$q_t = -\left(r_t - E_t \left[\pi_{t+1}\right]\right) + \left[1 - \beta (1 - \delta)\right] E_t \left[r_{t+1}^k - p_t\right] + \beta E_t \left[q_{t+1}\right] \quad (6)$$

[7] In general, lower case variables denote log-deviations from corresponding uppercase variables. Omission of time subscripts indicates steady-state variables.

$$i_t - k_t = \eta q_t \tag{7}$$

where $\eta = -1/\left(\phi''\left(\delta\right)\delta\right)$.[8]

A rule-of-thumb household does not take part in financial or capital markets, and thus faces the following simple budget constraint regardless of the assumption on the ownership of capital:

$$P_t C_t^r = W_t N_t^r - P_t T_t^r \tag{8}$$

Here, C_t^r is the household's real consumption at time t, and N_t^r is the hours worked by the household in period t. As a rule-of-thumb household simply consumes its current income, consumption follows directly from the budget constraint. A first-order log-linear approximation around the steady state with constant consumption equalised across households gives

$$c_t^r = \frac{WN}{PC}\left(w_t + n_t\right) - \frac{Y}{C} t_t^r \tag{9}$$

where omission of time subscripts indicates steady-state variables.[9]

Aggregate variables are given as simple weighted averages:

$$c_t = \lambda c_t^r + \left(1-\lambda\right) c_t^o \tag{10}$$

$$n_t = \lambda n_t^r + \left(1-\lambda\right) n_t^o \tag{11}$$

and

$$t_t = \lambda t_t^r + \left(1-\lambda\right) t_t^o \tag{12}$$

2.2 Labour unions

The economy has a continuum of unions $z \in [0,1]$ each representing a continuum of workers, a fraction $(1-\lambda)$ are optimising, and a fraction λ are rule-of-thumb consumers. Each union sets the wage rate for its members, who stand ready to satisfy firms' demand for their labour services at the

[8] Note that i_t and k_t are the log-deviations from corresponding steady-state values of aggregate investment and capital, respectively, defined as $K_t = \left(1-\lambda\right) K_t^o$ and $I_t = \left(1-\lambda\right) I_t^o$.

[9] We maintain the assumption that consumption is equalised across agents in the steady state to facilitate comparability with Galí et al. (2007). For an alternative approach, see Natvik (2008).

chosen wage. The workers in a union provide the same type of labour (irrespective of their consumption behaviour) differentiated from the type of labour services provided by members of other unions. The labour service supplied by each union, $N(z)$, is a simple aggregate of its members' labour services. In turn, the labour entering the production function of any firm is a Dixit-Stiglitz aggregate of the labour services provided by the unions in the economy. Hence, the labour demand for a union's labour services is given by

$$N_t(z) = \left(\frac{W_t(z)}{W_t}\right)^{-\varepsilon_w} N_t \tag{13}$$

where $W_t(z)$ is the wage set by the union, and ε_w is the elasticity of labour demand.

Each period, a representative union chooses $W_t(z)$ to maximise the present value of an average of its members' current and future period utility functions, that is,

$$\max_{W_t(z)} E_t \sum_{k=0}^{\infty} \beta^{t+k} \left[\lambda U^r_{t+k} + (1-\lambda) U^o_{t+k}\right] \tag{14}$$

subject to the labour demand functions and the budget constraints of its members, thus taking the effect of the wage decision on the income of its members into account.

The first-order condition can be expressed in the form of Galí et al. (2007):

$$\left[\frac{\lambda}{MRS^r_t} + \frac{1-\lambda}{MRS^o_t}\right] = \frac{\varepsilon_w}{\varepsilon_{w-1}} \frac{W_t}{P_t} \tag{15}$$

where, now, the marginal rate of substitution is given by $MRS^i_t = \left(C^i_t - h_i C^i_t\right)^{\sigma} N^{\varphi}_t$ for $i \in \{o, r\}$ because of habit formation in consumption. As shown by Furlanetto and Seneca (2007), log-linearising this expression gives

$$w_t - p_t = \chi_r \left(c^r_t - h_r c^r_{t-1}\right) + \chi_o \left(c^o_t - h_o c^o_{t-1}\right) + \varphi n_t \tag{16}$$

where

$$\chi_r = \sigma \frac{\lambda}{1-h_r} \frac{(1-h_o)^{\sigma}}{\lambda (1-h_o)^{\sigma} + (1-\lambda)(1-h_r)^{\sigma}}$$

and

$$\chi_o = \sigma \frac{(1-\lambda)}{1-h_o} \frac{(1-h_r)^{\sigma}}{\lambda (1-h_o)^{\sigma} + (1-\lambda)(1-h_r)^{\sigma}}$$

2.3 Goods demand

The economy has a continuum of firms $j \in [0,1]$, each of which produces a differentiated product, $Y_t(j)$. The final good used in private and public consumption is an index of this continuum of intermediate goods. Following Kimball (1995) it is defined implicitly by the relationship

$$\int_0^1 \mathcal{G}\left(X_t(j)\right) dj = 1 \tag{17}$$

where $X_t(j) = Y_t(j)/Y_t$ is relative demand, and $\mathcal{G}(.)$ is a function satisfying $\mathcal{G}(1) = 1$, $\mathcal{G}' > 0$ and $\mathcal{G}'' < 0$.

For a given level of consumption and investment, and for given prices, $P_t(j)$, expenditure minimisation leads to the following demand for firm j's product

$$X_t(j) = \tilde{\mathcal{G}}\left(\frac{P_t(j) Y_t}{\upsilon_t}\right) \tag{18}$$

where $\tilde{\mathcal{G}}(.)$ is the inverse function of $\mathcal{G}'(.)$ and υ_t is the Lagrange multiplier from the minimisation problem. If we define the price deflator P_t implicitly by

$$P_t Y_t = \int_0^1 P_t(j) Y_t(j) dj \tag{19}$$

we have

$$\upsilon_t = P_t Y_t \left(\int_0^1 \mathcal{G}'\left(X_t(j)\right) X_t(j) dj\right)^{-1} \tag{20}$$

Note that the assumption that $\mathcal{G}'' < 0$ implies that this demand function is downward-sloping. It follows that the price elasticity of demand is given by

$$\xi\left(X_t(j)\right) = -\frac{\mathcal{G}'\left(X_t(j)\right)}{\mathcal{G}''\left(X_t(j)\right) X_t(j)} \tag{21}$$

In log-linear terms, the demand function becomes

$$y_t(j) = -\bar{\xi}\left(p_t(j) - p_t\right) + y_t \tag{22}$$

where $\bar{\xi} = \xi(1)$.

In the special case where

$$\mathcal{G}\left(X_t(j)\right) = \left(X_t(j)\right)^{\frac{\epsilon-1}{\epsilon}} \tag{23}$$

(17) reduces to the more common Dixit-Stiglitz aggregator, which leads to a constant elasticity of substitution since, in this case, $\xi\left(X_t\left(j\right)\right) = \bar{\xi}$ for all $X_t\left(j\right)$. As is well-known, this leads to a constant desired mark-up of price-setting firms given by $\mu_p = \bar{\xi}/\left(\bar{\xi} - 1\right)$. In the general Kimball specification, we allow the demand elasticity and hence the desired mark-up to vary with the level of output. For future reference define

$$\epsilon\left(X_t\left(j\right)\right) = \frac{\partial \xi\left(X_t\left(j\right)\right)}{\partial P_t\left(j\right)} \frac{P_t\left(j\right)}{\xi\left(X_t\left(j\right)\right)} \tag{24}$$

This is the own price elasticity of the elasticity of demand. In the steady state we have $\epsilon\left(1\right) = \bar{\epsilon}$. In the analysis, we employ the case where $\bar{\epsilon} > 0$, i.e., the case where the elasticity of demand is increasing in the price set by the firm, or equivalently decreasing in its relative output. This is known to increase the strategic complementarity in price setting as discussed in section 1.

2.4 Firms

Firm j produces according to the technology

$$Y_t\left(j\right) = \tilde{K}_t\left(j\right)^{\alpha} N_t\left(j\right)^{1-\alpha} \tag{25}$$

where $\tilde{K}\left(j\right)$ the capital used as input by firm j, $N_t\left(j\right)$ is the labour employed by the firm, and $0 < \alpha < 1$. When the capital is owned by the firms, we assume that all firms have identical endowments of capital and we normalise this level to 1. Denoting the household-owned capital employed in production by firm j by $K_t\left(j\right)$, we have in general that $\tilde{K}_t\left(j\right) = \left(K_t\left(j\right)\right)^{\iota}$. Note that real marginal costs are given by

$$MC_t\left(j\right) = \frac{W_t/P_t}{\left(1-\alpha\right)\left(\tilde{K}_t\left(j\right)/N_t\left(j\right)\right)^{\alpha}} \tag{26}$$

When firms rent capital from households, i.e. when $\iota = 1$, cost minimisation implies that firm j will choose factor inputs such that

$$\frac{W_t}{R_t^k} = \frac{1-\alpha}{\alpha} \frac{K_t\left(j\right)}{N_t\left(j\right)} \tag{27}$$

Since all firms have to pay the same wage for the labour they employ, and the same rental rate for the capital they rent, it follows that marginal costs

are equalised across firms under this assumption. In contrast, when $\iota = 0$ and capital is firm-specific, marginal costs will generally be different across firms.

We now turn to the firms' price-setting decisions. Each firm is allowed to set a new price, P_t^*, with a fixed probability $(1-\theta)$ as in Calvo (1983). This implies that the expected duration of prices is given by $(1-\theta)^{-1}$.The firm's decision is made to maximise the value of the firm to its owners, the optimising households, given by

$$\sum_{k=0}^{\infty} E_t \left\{ \Lambda_{t,t+k} \left[P_t^* Y_{t+k}(j) - \Psi\left(Y_{t+k}(j)\right) \right] \right\} \tag{28}$$

where $\Psi(.)$ is the cost function, subject to its production function (25) and to the demand for its product given by (18).[10]

The following first-order condition represents the price-setting equation:

$$\begin{aligned} &\sum_{k=0}^{\infty} \theta_p^k E_t \left\{ \Lambda_{t,t+k} Y_{t+k}(j) \left[P_t^* \left(1 - \xi\left(X_{t+k}(j)\right)\right) \right] \right\} \\ &= \sum_{k=0}^{\infty} \theta_p^k E_t \left\{ \Lambda_{t,t+k} Y_{t+k}(j) \left[\xi\left(X_{t+k}(j)\right) P_{t+k} MC_{t+k}(j) \right] \right\} \end{aligned} \tag{29}$$

where $MC_t(j)$ is firm j's real marginal cost given by (26).

From the log-linearisation of (29) we may derive the following New Keynesian Phillips curve for price inflation

$$\pi_t = \beta E_t \pi_{t+1} + \kappa m c_t \tag{30}$$

where the slope parameter κ is given by

$$\kappa = \frac{(1-\beta\theta)(1-\theta)}{\theta} \left(1 + \frac{\bar{\epsilon}}{\bar{\xi} - 1} + (1-\iota)\frac{\alpha}{1-\alpha}\bar{\xi} \right)^{-1} \tag{31}$$

The derivation is sketched in appendix A. Note that κ is declining in both θ (the degree of nominal rigidity) and $\bar{\epsilon}$ (the curvature of the demand parameter). Also $\kappa|_{\iota=0} < \kappa|_{\iota=1}$. That is, the New Keynesian Phillips curve is flatter with fixed firm-specific capital than with rental capital.

[10] With rental capital, the cost function is the value function from the cost minimisation problem. With fixed firm-specific capital, the cost function is simply $W_{t+k} N_{t+k}(j)$ where the production function is used to substitute for $N_{t+k}(j)$.

2.5 Economic policy

The specification of economic policy follows Galí et al. (2007). The central bank controls the risk-free interest rate, which it sets according to a simple Taylor rule

$$r_t = r + \phi_\pi \pi_t \tag{32}$$

The government budget constraint is

$$P_t T_t + R_t^{-1} B_{t+1} = B_t + P_t G_t \tag{33}$$

the linearisation of which becomes

$$b_{t+1} = \beta \left(b_t + g_t - t_t \right) \tag{34}$$

where $b_t = \left(B_t / P_{t-1} - B/P \right) Y$, $g_t = (G_t - G)/Y$ and $t_t = (T_t - T)/Y$. Fiscal policy is given by the rule

$$t_t = \phi_b b_t + \phi_g g_t \tag{35}$$

Government spending (normalised by steady-state output and expressed in deviations from steady state) evolves exogenously according to the following first-order autoregressive process

$$g_t = \rho_g g_{t-1} + \varepsilon_t \tag{36}$$

where $0 < \rho_g < 1$ and ε_t is white noise with variance σ_ε^2. With this specification, the government finances the exogenous disturbances to its spending in any given period partly through taxes, partly through the issuance of bonds.

2.6 Equilibrium

Market clearing requires that

$$Y_t = C_t + I_t + G_t \tag{37}$$

In log-linear form, this becomes

$$y_t = \frac{C}{Y} c_t + \frac{I}{Y} i_t + g_t \tag{38}$$

3 The consumption multiplier

As in Galí et al. (2007), we analyse the effects of government spending shocks emphasising the response of private consumption. Specifically, we focus on the impact response of aggregate private consumption following a shock to government spending normalised to one per cent of the level of output in the steady state. We refer to this impact response as the consumption multiplier. As shown by Galí et al. (2007), this impact multiplier is significantly above zero in the data.

3.1 The model without real rigidities

To set the scene, figure 1 shows the consumption multiplier as a function of the fraction of rule-of-thumb consumers, λ, and as a function of the degree of price rigidity, θ, in the model analysed by Galí et al. (2007). This is equivalent to the model in section 2 when $h_o = h_r = \bar{\varepsilon} = 0$ and $\iota = 1$. That is, it is a version of the model with a rental market for capital, without habit formation in consumption, and with a constant elasticity of demand. The calibration of the remaining parameters follows the baseline calibration in Galí et al. (2007). Hence, we consider a time period to be one quarter, and we set $\delta = 0.025$, $\alpha = 0.33$, $\sigma = \eta = 1$, $\beta = 0.99$, $\lambda = 0.5$, $\gamma_g = 0.2$, $\phi_\pi = 1.5$, $\phi_b = 0.33$, $\phi_g = 0.1$, $\bar{\xi} = 6$, $\rho_g = 0.9$ and $\varphi = 0.2$. Finally, in the baseline calibration $\lambda = 0.5$ and $\theta = 0.75$. Note for future reference that this baseline calibration gives a value of the consumption multiplier of approximately 1.2.

Consider the solid lines first. These lines show the consumption multiplier in the Galí et al. (2007) model as a function of λ (left panel) and θ (right panel) with the other parameters remaining as under the baseline calibration. We see that, keeping θ fixed at 0.75, the consumption multiplier is positive only for values of λ larger than 0.3. Similarly, keeping λ fixed at 0.5, the multiplier is positive only for values of θ above a critical value between 0.5 and 0.6 corresponding to between two and three quarters of expected price stickiness. Hence, if we lower one of these two key parameters from the value chosen under the baseline calibration to one that is more realistic given the empirical evidence described in section 1, the consumption multiplier is no longer positive.

Considering the dashed lines, we see that by lowering one of the two parameters to a more plausible value – $\theta = 0.6$ and $\lambda = 0.3$ respectively – we make it harder to obtain a positive consumption multiplier for all values

of the other parameter. For $\theta = 0.6$, the fraction of rule-of-thumb consumers needs to be close to 0.5 to drive the consumption multiplier above zero, and for $\lambda = 0.3$, the expected duration of prices must be longer than a year. Moreover, under our preferred calibration in which $\theta = 0.6$ and $\lambda = 0.3$ at the same time, the consumption multiplier is seen to be negative.

In sum, these pictures show that the positive response of consumption is a fragile result in two crucial dimensions. It relies on implausibly high values for the degree of nominal rigidity and the percentage of constrained agents. Our contribution is to provide a solution to this problem by reconciling a sizeable increase in consumption as in Gali et al. (2007) with reasonable values for the degree of nominal rigidity and the financial friction. We do this by adding real rigidities to the model.

3.2 Adding real rigidities

Motivated by the previous sensitivity analysis of the model in Galí et al. (2007), we now present responses from the model augmented with habit persistence, Kimball demand and fixed firm-specific capital. We set the fraction of rule-of-thumb consumers, λ, to 0.3 inspired by the empirical evidence discussed in section 1, and we set the degree of habit persistence of optimising households, h_o, equal to 0.85, a value which is within the range of values considered in the literature.[11] However, we let the degree of habit persistence of rule-of-thumb households be zero, that is, $h_r = 0$. This is to facilitate the interpretation that rule-of-thumb households are inherently different from optimising households by having an entirely static horizon.

The calibration of the curvature of the Kimball demand function, represented by $\bar{\varepsilon}$, is more difficult. As noted by Dossche et al. (2006), there is no agreement on what a plausible value might be for this parameter in the literature; estimates range from 1.3 (Bergin and Feenstra, 2000) to 471 (Kimball, 1995). In this section we therefore calibrate $\bar{\varepsilon}$ by fixing values for the slope of the New Keynesian Phillips curve, κ, and the degree of nominal rigidity, θ. This allows us to recover a value of $\bar{\varepsilon}$ implied by the expression for κ given in (31). We set θ at 0.6, cf. section 1, while we fix κ at 0.03 based on the reduced-form evidence on the slope of the New Keynesian Phillips curve in Galí et al. (2005) and Levin et al. (2007). The implied value of $\bar{\varepsilon}$ is 25.

[11] It falls between the value estimated by Christiano et al. (2005) and the one considered by Woodford (2003, ch. 5).

It is possible that 25 is still too high a value for $\bar{\epsilon}$, at least according to the evidence provided by Dossche et al. (2006). They suggest that a value around 4 is more reasonable, though they find evidence of considerable variation across sectors. We note that we would need a higher value of $\bar{\epsilon}$ (around 40) if we had kept the rental capital assumption. This illustrates that different real rigidities may interact in the economy in a way that allows us to consider reasonable values for other parameters representing real and financial rigidities.[12] Similarly, if we are slightly less ambitious in bringing down the expected duration of prices, we may obtain a value of $\kappa = 0.03$ with $\bar{\epsilon} = 4$ in the version of our model with firm-specific capital. This requires us to accept an expected duration of prices of slightly more than 3 quarters instead of our benchmark $2\frac{1}{2}$, but still in the range of the plausible values according to Nakamura and Steinsson (2007).

Note that our calibration of κ implies a much flatter New Keynesian Phillips curve than in Galí et al. (2007), where $\kappa = 0.0858$. In the model without real rigidities, we would need a Calvo parameter of 0.85 to generate a slope of 0.03, clearly an unrealistic value given the empirical evidence available.

Figure 2 presents impulse responses to key macroeconomic variables under this calibration along with responses from the model by Galí et al. (2007).[13] The main result of our paper is that the responses of consumption are nearly identical in the two models. In both cases, we obtain a consumption multiplier of approximately 1.2. Hence, the introduction of real rigidities in the form of habit persistence in consumption, Kimball demand and fixed firm-specific capital allows us to generate the same consumption multiplier as in Galí et al. (2007) with an expected price duration of two and a half quarters (instead of four) and with only 30 per cent of financially constrained agents (instead of 50). The crucial difference between the two models is that, in the model with real rigidities, both the fraction of rule-of-thumb consumers and the degree of price rigidity are more in line with the empirical evidence.

Part of the explanation for our result is that, in the model with real

[12] The model's equilibrium dynamics for variables other than investment is not affected by the choice of assumption concerning the structure of the capital market. We therefore omit reporting of the impulse responses for the rental capital case with Kimball demand and habit formation.

[13] The responses reported here are in percentage deviations from steady state and so they differ slightly from the ones reported in Galí et al. (2007), which are normalised by steady-state output.

rigidities, habit persistence works to mitigate the contractionary effect from Ricardian households by smoothing their response to the shock. Rule-of-thumb households still respond by increasing their consumption since the partial bond financing of the government spending shock makes current income go up. But with habit formation in consumption, optimising households need time to adjust to the lower level of consumption called for by the reduction in lifetime income that results from current *and* future taxation. This makes them reduce consumption less on impact of the shock. Though rule-of-thumb consumers now weigh less in the aggregate, the net effect on aggregate consumption is therefore unchanged.

This is not the only effect in play, however. With a relatively flat New Keynesian Phillips curve, a positive shock to government spending that increases firms' marginal costs by increasing aggregate demand in the economy has a smaller effect on inflation through the price-setting process. This makes the central bank respond by increasing interest rates less than in an economy with a steeper Phillips curve. This further moderates the negative consumption response of optimising consumers. It is the combination of habit formation in consumption and a less responsive demand effect through monetary policy that allows us to generate the same consumption multiplier as in Galí et al. (2007) for a lower percentage of rule-of-thumb consumers.

Importantly, the introduction of real rigidities that are known to increase the strategic complementarities in price setting, cf. Woodford (2003), allows us to reduce the slope of the Phillips curve without increasing the degree of nominal rigidity. In contrast, our analysis is consistent with fixing θ at 0.6 in keeping with microeconomic evidence on the frequency of price changes. Note also from figure 2 that the responses of the other aggregate variables are also nearly identical in the models. The only exception, of course, is investment, which is constant by assumption in the model with firm-specific capital.[14]

The importance of habit formation for the consumption response can be seen from figure 3, in which we report the consumption multiplier as a function of h_o keeping $\kappa = 0.03$ (left panel), and κ keeping $h_o = 0.85$

[14] As argued by Furlanetto (2007), the model in Galí et al. (2007) exhibits a counterfactually large response of the real wage. However, once he introduces a nominal wage rigidity that smoothes the wage response, the increase in consumption is confirmed. We have also considered a version of the model with real rigidities augmented with nominal wage rigidities. Results are qualitatively similar to the ones reported here. For sake of completeness, they are reported in appendix B.

(right panel) when $\lambda = 0.3$ (in contrast to the baseline $\lambda = 0.5$). Remaining parameters are at their baseline values. On the left panel it is seen that reducing the degree of habit persistence lowers the impact response of private consumption following a shock to government spending. In the extreme case without habit persistence, even if we allow for curvature in the demand curves by setting $\bar{\epsilon} = 25$ so that $\kappa = 0.03$ when $\theta = 0.6$, the consumption multiplier is small (albeit positive).

The right panel in figure 3 shows the consumption multiplier as a function of κ, the slope of the New Keynesian Phillips curve. As noted in section 2, this slope is inversely related to $\bar{\epsilon}$, meaning that κ goes from 0 to 0.1 as $\bar{\epsilon}$ goes from infinity to 0.1. That is, $\bar{\epsilon}$ declines as we move from left to right on the graph. When $\lambda = 0.3$ in the model with habit formation, we see that κ has to be close to 0.03 to generate a consumption multiplier close to 1.2. In particular, increasing the slope of the New Keynesian Phillips curve reduces the multiplier. For $\kappa = 0.0858$ as in the baseline calibration of Galí et al. (2007), we see that the multiplier falls to approximately 0.8 even when habit persistence curbs the contractionary effect from the 70 per cent of households that optimise intertemporally.

To summarise, we have shown that the empirically realistic consumption multiplier obtained by Galí et al. (2007) with 50 per cent of rule-of-thumb consumers and 4 quarters of expected price stickiness, can be obtained for considerably lower values of these parameters once real rigidities are added to the model. Habit formation, which directly smoothes the adjustment of private consumption of intertemporally optimising households, reduces the negative response of optimising consumers for a given monetary policy response. When combined with real rigidities that amplify the implications of nominal rigidities, the contractionary response of monetary policy to the fiscal expansion is reduced even for considerably lower degrees of nominal rigidities. This further reduces the negative consumption response of optimising households. The combination of these effects allows us to generate the same positive consumption multiplier as in Galí et al. (2007) with a percentage of rule-of-thumb consumers given by 30 and an expected duration of price rigidities given by two and a half periods.

4 Concluding remarks

This paper shows that the rule-of-thumb theory of consumption does not rely on a high degree of nominal rigidity or a large financial friction when accounting for the conditional responses to government spending shocks. When empirically plausible real rigidities are added to the model, they interact with nominal and financial rigidities in ways that allow us to specify more reasonable parameter values for all the rigidities at work in the model. Hence, we believe that this paper complements the analysis in Galí et al. (2007) by showing how the rule-of-thumb consumption theory becomes more appealing once realistic features are added to the model.

Interestingly, the same combination of real rigidities that we apply has been used in the previous literature to replicate conditional responses to other shocks, especially monetary shocks and technology shocks. Habit persistence has been used to reproduce the hump-shaped response of output and consumption on the impact of a monetary shock, while Kimball demand curves and firm-specific capital have been used to reconcile the microeconomic evidence on the degree of price rigidity with the macroeconomic evidence on the slope of the New Keynesian Phillips curve, cf. references in section 1. In a companion paper to this one, Furlanetto and Seneca (2007) show that the interaction of nominal, real and financial rigidities is also very helpful in accounting for the responses of hours worked following a productivity shock.

Thus, at a more general level, this paper contributes to this literature by showing how nominal and real rigidities may interact with a financial friction in ways that generate plausible dynamics following empirically important disturbances to the economy. We believe this is a further indication that, while the simple basic real business cycle framework is an important benchmark both conceptually and methodologically, a realistic model of the economy is likely to be one in which many frictions and rigidities interact. Providing further evidence on how this may occur – and not least further empirical evidence on the relative importance of these rigidities and frictions along the lines of Coenen and Straub (2005) and Forni, Monteforte and Sessa (2007) – is, we believe, an important topic for further research in macroeconomics.

A Appendix

The first-order condition to the price-setting problem is:

$$\sum_{k=0}^{\infty} \theta_p^k E_t \left\{ \Lambda_{t,t+k} Y_{t+k}(j) \left[P_t^* \left(1 - \xi\left(X_{t+k}(j)\right)\right) - \xi\left(X_{t+k}(j)\right) P_{t+k} MC_{t+k}(j) \right] \right\} = 0$$

We log-linearise this first-order condition to get

$$0 = E_t \sum_{k=0}^{\infty} \left(\theta_p \beta\right)^k \left[\left(1-\bar{\xi}\right) p_t^* - \left(1-\bar{\xi}\right) mc_{t+k}(j) - \left(1-\bar{\xi}\right) p_{t+k} - \bar{\epsilon}\left(p_t^* - p_{t+k}\right) \right]$$

where we have substituted in log-linearisations of (21) and (18). Since

$$mc_{t+k}(j) = mc_{t+k} - (1-\iota)\frac{\alpha}{1-\alpha}\bar{\xi}\left(p_t^* - p_{t+k}\right)$$

where mc_{t+k} is the average marginal cost in log-linear terms, we get

$$\begin{aligned}
&\frac{1}{1-\theta_p\beta}\left(1 + \frac{\bar{\epsilon}}{\bar{\xi}-1} + (1-\iota)\frac{\alpha\bar{\xi}}{1-\alpha}\right)\left(p_t^* - p_{t-1}\right) \\
&= E_t \sum_{k=0}^{\infty} \left(\theta_p\beta\right)^k \left[\left(1 + \frac{\bar{\epsilon}}{\bar{\xi}-1}\right)\left(p_{t+k} - p_{t-1}\right) + mc_{t+k} - (1-\iota)\frac{\alpha\bar{\xi}}{1-\alpha}\left(p_{t-1} - p_{t+k}\right) \right] \\
&= \left(1 + \frac{\bar{\epsilon}}{\bar{\xi}-1} + (1-\iota)\frac{\alpha\bar{\xi}}{1-\alpha}\right)\pi_t + mc_t \\
&+ \frac{1}{1-\theta\beta}\left(1 + \frac{\bar{\epsilon}}{\bar{\xi}-1} + (1-\iota)\frac{\alpha\bar{\xi}}{1-\alpha}\right) E_t\left(p_{t+1}^* - p_t\right) \\
&+ \frac{\theta\beta}{1-\theta\beta}\left(1 + \frac{\bar{\epsilon}}{\bar{\theta}-1} + (1-\iota)\frac{\alpha\bar{\theta}}{1-\alpha}\right)\pi_t
\end{aligned}$$

As shown by Eichenbaum and Fisher (2007), the price index implies that

$$p_t^* - p_{t-1} = \frac{\pi_t}{1-\theta}$$

Using this gives

$$\begin{aligned}
\frac{\pi_t}{1-\theta} &= (1-\theta\beta)\pi_t + (1-\theta\beta)\left(1 + \frac{\bar{\epsilon}}{\bar{\xi}-1} + (1-\iota)\frac{\alpha\bar{\xi}}{1-\alpha}\right)^{-1} mc_t \\
&\quad + \frac{\theta\beta}{1-\theta} E_t\pi_{t+1} + \theta\beta\pi_t
\end{aligned}$$

Rearranging gives the New Keynesian Phillips curve in the text:

$$\pi_t = \frac{(1-\theta\beta)(1-\theta)}{\theta_p}\left(1+\frac{\bar{\epsilon}}{\bar{\xi}-1}+(1-\iota)\frac{\alpha\bar{\xi}}{1-\alpha}\right)^{-1} mc_t + \beta E_t \pi_{t+1}$$

B Appendix

An unpleasant feature of the model presented in the previous section is that, independently of the presence of real rigidities, it implies a large increase in the real wage which is counterfactual. Many empirical studies – Blanchard and Perotti (2002), Perotti (2005), Fatas and Mihov (2002) among many others – find a zero response or at most a tiny positive response, in general not statistically significant. Furlanetto (2007) shows that by introducing sticky wages in the model, it is possible to reconcile a plausible conditional response of real wages and a positive and sizeable response of private consumption on the impact of a government spending shock. In other words, the Galí et al. (2007) result does not rely on the large counterfactual response of real wages, as one might intuitively think, but is confirmed in a more general setting with wage rigidities. For sake of completeness, we want to show that real rigidities can substitute for nominal and financial rigidities, also in a framework with sticky wages. As shown in Furlanetto and Seneca (2007), with sticky wages and habit formation in consumption, equation (15) is substituted by the following equation for wages

$$\pi_t^w = \beta E_t\left(\pi_{t+1}^w\right) + \kappa_w\left(mrs_t - (w_t - p_t)\right) \tag{39}$$

where mrs_t is the average marginal rate of substitution given by

$$mrs_t = \chi_r\left(c_t^r - h_r c_{t-1}^r\right) + \chi_o\left(c_t^o - h_o c_{t-1}^o\right) + \varphi n_t \tag{40}$$

and the slope coefficient κ_w is

$$\kappa_w = \frac{\varepsilon_w - 1}{\phi_w}$$

Here, ϕ_w governs the size of wage adjustment costs à la Rotemberg (1982).[15] We calibrate ε_w equal to 4 and ϕ_w equal to 454.5. This choice yields the

[15] Instead of wage adjustment costs, we may assume that a union is allowed to reset its wage rate each period with a fixed probability $1-\theta_w$ as in Calvo (1983). But to undo the implications of the implied heterogeneity across unions, a risk-sharing arrangement

same New Keynesian Phillips curve for wages as in a Calvo setting à la Erceg, Henderson and Levin (2000) with four quarters of wage stickiness.

A second criticism that can be raised to the Galí et al. (2007) model concerns the calibration of the inverse of the labor supply elasticity φ. Galí et al. (2007) are forced to set it at 0.2 to make the model determinate. However, the determinacy region is larger under sticky wages and therefore we can raise φ to more plausible values. We set φ equal to 3, consistent with a labor supply elasticity of 1/3, as in Galí and Monacelli (2005) and consistent with a considerable microeconomic evidence. In figure 4 we plot the impulse responses for the model in Galí et al. (2007) augmented with sticky wages along with a model further extended with real rigidities as in section 2 (Kimball demand and habit consumption, while keeping the rental capital assumption).

We see that the model with real rigidities can reproduce approximately the same multiplier as the model without real rigidities under only 30 percent of constrained agents. Thus, once again, real rigidities can substitute for nominal rigidities and financial frictions. Note also that real wages respond very little in both cases due to wage adjustment costs.

between unions must be in place. This follows since rule-of-thumb consumers are barred from sharing risk through financial markets. Results, however, are very similar. In particular we would get a Phillips curve with $\kappa_w = (1-\beta\theta_w)(1-\theta_w)\theta_w^{-1}(1+\varphi\varepsilon_w)^{-1}$ where θ_w is the Calvo parameter for wage setting.

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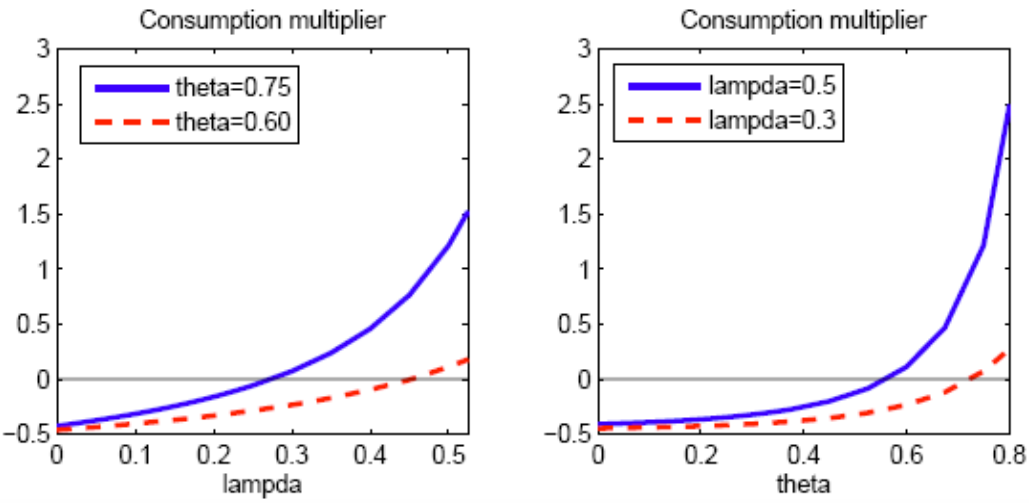

Figure 1: Impact consumption multiplier in the model by Galí et al. (2007) as function of λ, the fraction of rule-of-thumb consumers (left panel), and θ, the degree of price rigidity (right panel). Remaining parameters at baseline values.

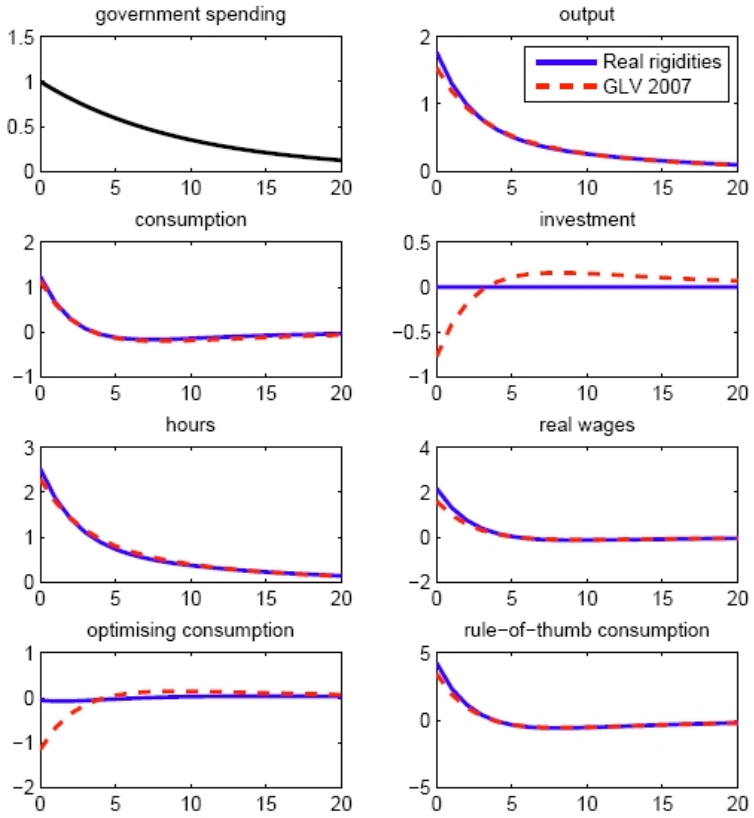

Figure 2: Impulse responses to a government spending shock normalised to one per cent of steady-state output for $\lambda = 0.5$ and $\theta = 0.75$ in the Galí et al. (2007) model (dashed lines), and for $\lambda = 0.3$ and $\theta = 0.6$ in an extended version of the model with real rigidities (solid lines).

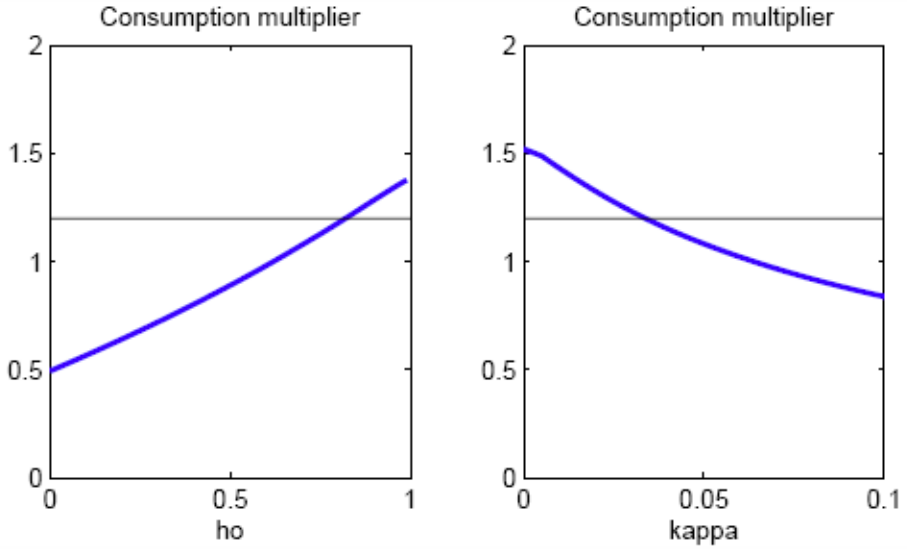

Figure 3: Impact consumption multiplier as a function of h_o, the degree of habit persistence of optimising households (left panel), and κ, the slope of the New Keynesian Phillips curve, for $\lambda = 0.3$ in the Galí et al. (2007) model augmented with real rigidities.

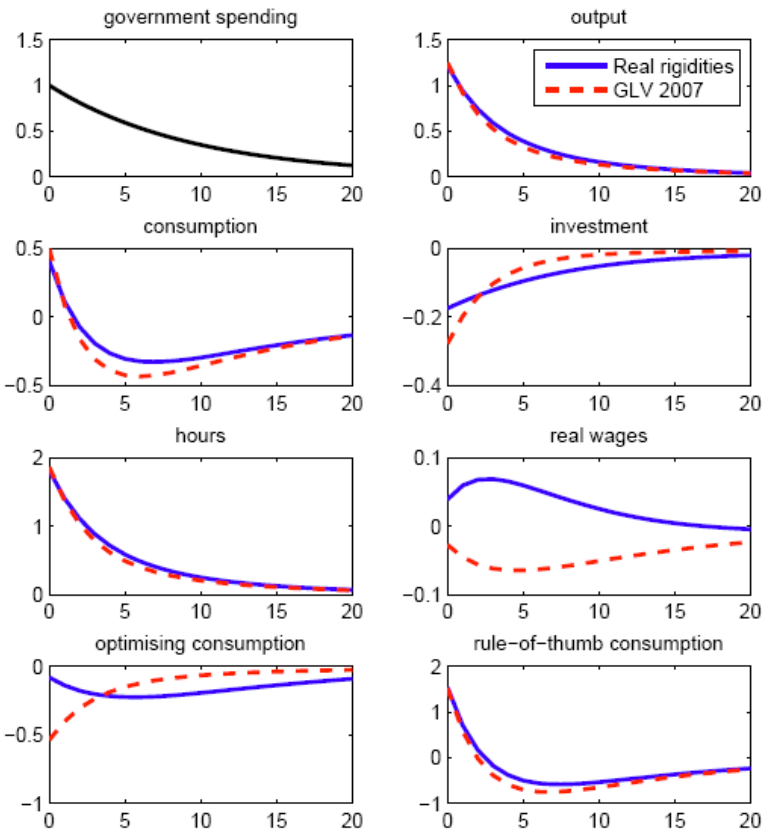

Figure 4: Impulse responses to a government spending shock normalised to one per cent of steady-state output for $\lambda = 0.5$ and $\theta = 0.75$ as in the Galí et al. (2007) model augmented with sticky wages (dashed lines), and for $\lambda = 0.3$ and $\theta = 0.6$ in an extended version with real rigidities in addition to sticky wages (solid lines).