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Large sigma events in the European FX markets

– Stylised facts from 273 years of quarterly data[1]

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[1] The author wishes to thank colleagues from Danmarks Nationalbank for useful comments on preliminary versions of this paper. The author alone is responsible for any remaining errors.

Abstract

We offer a closer look at the frequency distribution of nominal price changes in the foreign exchange markets for a sample of 10 European exchange-rate pairs on the basis of a unique quarterly data set spanning 273 years. Our analysis clearly illustrates the risk of seriously underestimating the probability and magnitude of tail events when frequency distributions of nominal exchange-rate changes are derived on the basis of fairly short data samples. We suggest that financial institutions and regulators should have an eye for the long-term historical perspective as a source of inspiration when designing "worst case scenarios" or "severe stress scenarios" in relation to risk assessments and stress tests.

Key words: Economic history; Realised exchange-rate volatility; Risk management; Fat tailed distributions; Kernel density estimation.

JEL Classification: C14; C58; F31; G32; N23; N24.

Resumé (Danish summary)

Vi belyser hyppighedsfordelingen af nominelle valutakursændringer for 10 europæiske valutakryds på basis af et unikt kvartalsvis datasæt, som dækker de seneste 273 år. Vores analyse illustrerer klart risikoen for alvorlig en undervurdering af sandsynligheden for og størrelsen af halebegivenheder, når hyppighedsfordelinger af nominelle valutakursændringer udledes på basis af relativt korte tidsrækker. Vi foreslår, at finansielle institutioner og myndigheder lader sig inspirere af den langsigtede økonomisk-historiske udvikling i forbindelse med design af "worst case scenarier" eller "hårde stress scenarier".

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1. Introduction

In recent papers, Cotter *et al*. (2008) and Daníelsson (2008) reviewed the probability of tail events under a normal distribution. The motivation was statements in the press suggesting that the daily losses in some financial institutions during the recent crisis represented events that were only supposed to happen once in every 100,000 years or events that represented so-called "25-sigma loss events" several days in a row. A 25-sigma loss event denotes a drop in daily asset returns of more than 25 standard deviations from the mean. Based on the assumption that the daily losses are normally distributed, a 25-sigma loss event is to be expected every 1.309E+135 years[2]. According to Cotter *et al*., *op. cit*., the probability of a 25-sigma loss event under the normal distribution can be compared to the probability of winning the UK National Lottery 21 or 22 times in a row or "... being on a par with Hell freezing...". It is thus very unlikely that financial price changes follow the "bell curve", which was already a well-established fact in the seminal papers by Mandelbrot (1963) and Fama (1965). The recent work by Reinhart and Rogoff (2009) – counting numerous incidences of financial crises during the past 800 years – also clearly suggests that periods with severe financial stress are not incidents that only occur once every 100,000 years.

Financial institutions rely heavily on a wide range of quantitative methodologies and tools to manage and stress test exchange-rate risks where the related frequency distributions of nominal exchange-rate changes are derived on the basis on historical data. However, often the historical data sets applied are fairly short covering at best the most recent decades or so which almost by definition contains relatively few very extreme observations. So, even if the risk management tools and stress test models do not rely on the normal distribution the use of relative short data samples implies a risk of underestimating the probability and magnitude of tail events such as large exchange-rate movements related to currency crises, changes of monetary regimes, banking crises, debt crises, severe stock-market collapses, wars, episodes of high inflation or hyperinflation *etc*.

In the paper at hand we take a closer look at the frequency distribution of nominal price changes in the foreign exchange (FX) markets for a sample of 10 European exchange-rate pairs on the basis of a unique quarterly data set spanning 273 years constructed by the authors. The large number of data points covering different crises as well as non-crises regimes have the potential to offer a better empirical description of the occurrence of large sigma events in the FX market than can be obtained in a short data sample spanning only a

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[2] Scientific notation, i.e. 1.309E+135 means 1.309 times ten raised to the power of 135 ($1.309\text{x}10^{135}$). In the cases with more than 7 standard deviations from the mean the number of years between expected occurrences under the normal distribution can not be compiled with standard software programs. In the paper at hand they have therefore been compiled via the tail-probability approximation method outlined in Annex A.

few decades. The analysis in the paper clearly illustrate the risk of seriously underestimating the probability and magnitude of tail events when frequency distributions of nominal exchange-rate changes are derived on the basis of fairly short data samples.

2. A brief review of related literature

Our paper relates most closely to the literature on the volatility and distribution of nominal exchange-rate changes and on long-run behaviour of exchange rates.

It has long been a well-documented fact that the observed distribution of nominal exchange-price changes tends to have fatter tails than the normal distribution (Westerfield, 1977; Rogalski and Vinso, 1978; Boothe and Glassman, 1987). This implies that a higher number of large exchange-rate changes are observed compared to what can be expected under a normal distribution. This stylised empirical fact holds across exchange-rate regimes (fixed or floating) and can even be observed for black-market exchange rates (Pond and Tucker, 1988; Koedijk *et al*., 1990; Koedijk and Kool, 1992). The observed heavy tails ("fat tails", "leptokurtosis" or "excess kurtosis") of the distribution of exchange-rate changes might reflect either that the changes come from fat tail distributions which are fixed over time or from distributions which vary over time, for instance in relation to change of exchange-rate regime (Hsieh, 1988; Koedijk *et al*., 1992).

A range studies have focused on the validity of Purchasing-Power-Parity (PPP) as a long-run parity condition and estimation of half-lives of real-exchange-rate shocks based on data sets covering a time span of at least a couple of centuries or so. Studies within this line of research include Lothian and Taylor (1996, 2000, 2008), Cuddington and Liang (2000), Peel and Venetis (2003), Calderón and Ducan (2003), Murray and Papell (2005) and Christou *et al*. (2009). However, all these studies are based on data on an annual frequency and none of the studies focus on nominal exchange-price changes from a risk-management perspective.

The main part of literature on the volatility and distribution of nominal exchange-rate changes is based on data set covering only the most recent decades, cf. e.g. the survey in de Vries (1994). Only a few papers have studied exchange-rate behaviour based on monthly or quarterly data sets spanning several centuries. De Vries (2001) has studies the volatility of the nominal NLG-GBP exchange rate on monthly data for the period 1766-2000. Bernholz *et al*. (1985) study nominal and real exchange rate behaviour under inflationary conditions in 17 historical cases from the period 1703-1981. Some of the cases are based on monthly exchange rates, but most are based on annual exchange-rate data. Craighead (2010) takes a brief view on the nominal USD-GBP exchange-rate volatility across time and monetary regimes based on monthly data over the period 1794–2005 but focuses on the behaviour of the real exchange rate. Ahmad and Craighead (2011) use the real exchange rate from the same data set to assess temporal aggregation biases in the half-lives of PPP deviations.

In the paper at hand we analyse the distribution of nominal exchange-rate fluctuations on the basis of a unique quarterly data set for 10 European exchange-rate pairs covering a time span of 273 years constructed by the authors. To the best of our knowledge this is the first study on nominal exchange-rate changes for a large number of exchange-rate pairs based on quarterly data for spanning almost three centuries. The unique long-span data set covers several low probability events such as currency crises, changes of monetary regimes, banking crises, debt crises, severe stock-market collapses, wars and episodes of high inflation or hyperinflation. It has therefore the potential to give a better empirical description of the occurrence of large price fluctuations in the FX markets than can be obtained on the basis of shorter data samples.

3. The data set

Recently Norges Bank and Sveriges Riksbank have published comprehensive collections of historical monetary statistics including long-span time series on monthly nominal exchange rates, cf. Eitrheim *et al*. (eds.) (2004) and Edvinsson *et al*. (eds.) (2010). Combined with Rubow (1918), Wilcke (1929), Friis and Glamann (1958) and Denzel (1999) as well as exchange-rate data from NBER's Macrohistory Database, the Danish Central Bureau of Statistics (Statistics Denmark) and the Danish central bank (Danmarks Nationalbank) we were able to construct quarterly average nominal exchange-rate series for 10 European exchange-rate pairs spanning the period 1740q1-2012q4.

The exchange-rate series are partly based on direct quotes and partly compiled as synthetic cross rates derived from an assumption of perfect international arbitrage. In the latter case the cross rates might have been adjusted to take into account differences in the levels of cross rates and direct rates. Furthermore, in some cases adjustments have been made in order to take into account differences between bid, mid and offer rates. A few missing quarterly observations have been interpolated or estimated from annual data.

Naturally, the assumption of perfect international arbitrage is debatable in relation to a study on FX markets going three centuries back. However, several studies have indicated that effective arbitrage in the FX market is not a phenomenon restricted to the late 20th and early 21st century.

Officer (1985, 1986) and Canjels *et al*. (2004) found a strong integration of the Anglo-American FX market in the last two decades of the 19th century and the first decade of the 20th century. Esteves *et al*. (2009) also found that the efficiency of the Lisbon/London FX market reached a level close to those of core currencies in the 1880s. Ugolini (2010) found a strong integration of the FX market of the five main international financial centres already during the 1840s.

The almost perfect gold point arbitrage found during the last decades of the 19th century was supported by lower freight rates on shipping specie, lower insurance costs, improved speed of oceanic transportation due to steamships and faster transatlantic communication due to the availability of cable communication. The opening of a permanent trans-Atlantic telegraph cable in 1866 played a crucial role in integrating the financial markets in New York and London, cf. Garbade and Silber (1978). Prior to the opening of the cable it took about three weeks to communicate price information by ship from New York to London. After the opening of the cable the time delay dropped to round 1 day. The study by Garbade and Silber, *op. cit*., indicates that the cable was used for financial-market arbitrage between New York and London immediately after its opening in 1866. In today's foreign exchange markets, the word "cable" is still used as slang for the exchange rate between US dollar and the British pound.

According to Flores (2007) the exchange rate vis-á-vis the British pound of South American countries were published in the newsmagazine, The Economist, with a time lag of 1 month in 1870 since the information had to be transmitted by steamships. The fasted route by ship from Buenos Aires to London took 27 days. At the end of the decade – after the introduction of the telegraph – the time lag of the South American exchange rates published in The Economists was reduced to two days.

Schubert (1989) illustrates that arbitrage on the FX markets of London and Amsterdam functioned well even during most of the 18th century measured by deviations between London-Paris, London-Lisbon and London-Hamburg cross rates and direct rates, taking into account that arbitrage took place under uncertainty due to the communication lag. Major unexploited arbitrage opportunities occurred mainly during periods of wars.

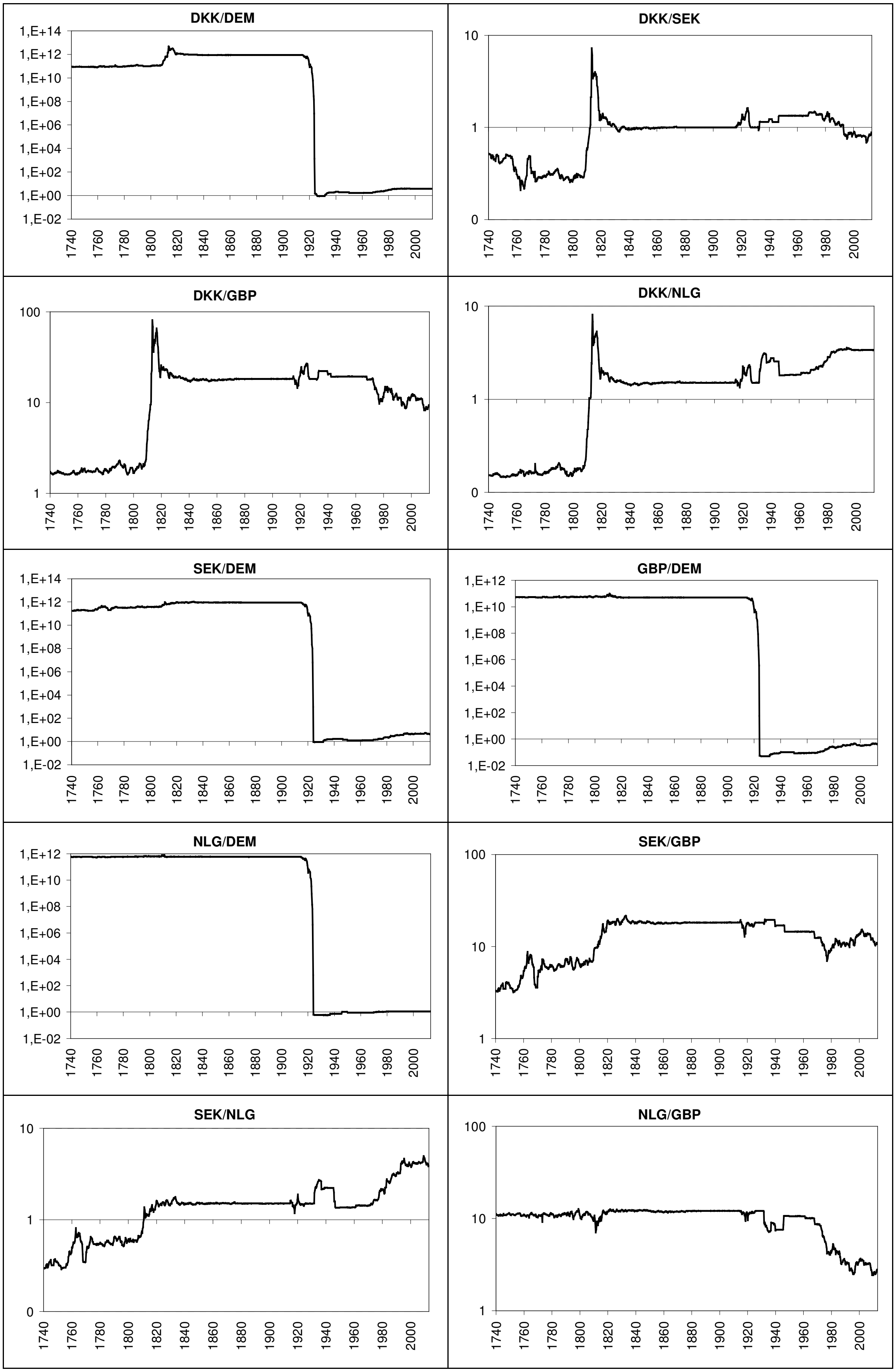
For several of the countries in our data set the currency unit of account has changed during the sample period. For presentational reasons and to ensure comparability across time we converted most of the time series to contemporary currency unit of accounts using the official conversion rates, cf. Table 1. However, for euro area countries we converted the time series to the currency unit of account that was in use just prior to the formation of the euro in 1999. The currencies covered are Danish kroner (DKK), Deutsche marks (DEM), Swedish kronor (SEK), British pounds (GBP) and Netherlands guilder (NLG). The exchange rate notation used in the paper at hand is the following: DKK/DEM denotes DKK per DEM, SEK/GBP denotes SEK per GBP, *etc*.

Table 1: Changes in currency unit of accounts since 1740

Currency	Unit of account
Danish kroner (DKK)	The Danish currency unit of account changed from "rigsdaler kurant" to "riksbankdaler" in 1813, from "riksbankdaler" to "rigsdaler" in 1854 and from "rigsdaler" to "kroner" (DKK) in 1875. Prior to 1875 the following official conversion rates has been used: 6 rigsdaler kurant = 1 rigsbankdaler = 1 rigsdaler = 2 DKK.
Deutsche marks (DEM)	Before unification of Germany in 1871 the Hamburger reichsthaler banco was among the most important currency units of the German states and has therefore been used in this study. The currency unit of account of the unified German changed from "old Reichsmark" (introduced in 1873) to "Rentenmark" in 1923, from "Rentenmark" to "new Reichsmarks" in 1924 and from "new Reichsmarks" to "Deutsche Mark" (DEM) in 1948. At the currency reform in 1948 establishing the Deutsche Mark, West Germans could exchange 60 Reichsmarks for 60 Deutsche Mark. Most other conversions were made at a ratio of 1 Deutsche Mark for 10 Reichsmarks. The exchange rate between the USD and the DEM was maintained equivalent to the old USD-DEM exchange rate. Prior to 1948 the following official conversion rates has been used: 1 Hamburger reichsthaler banco = 4.5 old Reichsmark and 1,000,000,000,000 old Reichsmark = 1 Rentenmark = 1 new Reichsmarks = 1 DEM. Since January 1999 calculated on the basis the DEM-to-EUR conversion rate fixed at 1 January 1999 (1 EUR = 1.95583 DEM).
Swedish kronor (SEK)	The Swedish currency unit of account changed from "daler kopparmynt" (=72 marks kopparmynt = 18 daler kopparmynt) to "riksdaler banco" (= 48 skilling banco) in 1777, from "riksdaler banco" to "riksdaler riksmynt" in 1858 and from "riksdaler riksmynt" to "krona" (SEK) in 1873. Prior to 1873 the following official conversion rates hs been used: 48 marks kopparmynt = 12 daler kopparmynt = 32 skilling banco = 48 skilling riksgälds = 1 riksdaler riksmynt = 1 SEK.
British pounds (GBP)	The British Pound (GBP) dates back to the 8th century, so no conversions have been necessary.
Netherlands guilder (NLG)	Netherlands Guilder was introduced in 1543. Since January 1999 calculated on the basis of the NLG-to-EUR conversion rate fixed at 1 January 1999 (1 EUR = 2.20371 NLG).

Figure 1 shows the 10 exchange-rate pairs for the full sample period. Some major economic events are clearly visible: The high Danish inflation and bankruptcy of the Danish state in the early 19th century, the end of the Classical Gold Standard around World War I, the German hyperinflation in the early 1920s and the break-down of the Bretton Woods system in the early 1970s. The data set is available on request in an electronic form.

Figure 1: Bilateral exchange rates 1740-2012, semi-logarithmic scale



4. Stylised facts on the empirical distribution of exchange-rate changes

For the full sample period 1740-2012 we can compile 1,091 quarter-on-quarter percentage changes in the nominal exchange rate for each currency pair. A range of summary statistics is shown in Table 2. Table 3 shows the results if the sample period is restricted to the post Bretton Woods period (1974-2012), whereas Table 4 covers the post EMS-crisis period (1996-2012) only.

Table 2: Summary statistics for 1,091 quarter-on-quarter percentage changes in bilateral nominal exchange rates 1740-2012

Currency pairs	DKK/DEM	DKK/SEK	DKK/GBP	DKK/NLG	SEK/DEM
Maximum	107.1	99.7	101.5	110.0	50.2
99% percentile	17.0	12.2	12.7	18.2	13.2
95% percentile	3.9	5.1	4.7	4.1	5.2
5% percentile	-4.6	-4.4	-4.4	-3.2	-4.8
1% percentile	-30.6	-13.1	-10.7	-13.2	-26.9
Minimum	-100.0	-43.9	-42.7	-44.2	-100.0
Mean	-0.3	0.2	0.3	0.4	-0.5
Standard deviation	9.8	6.0	6.1	6.1	8.4
Skewness	-1.6	8.7	10.1	9.8	-6.6
Kurtosis	63.9	141.6	165.6	159.1	72.7
Jacque-Bera	169039.9	887234.2	1220939.1	1125698.7	229120.4
Currency pairs	GBP/DEM	NLG/DEM	SEK/GBP	SEK/NLG	NLG/GBP
Maximum	49.8	67.1	32.0	29.7	24.8
99% percentile	12.2	9.2	11.4	13.4	8.3
95% percentile	4.7	2.2	5.2	4.9	3.4
5% percentile	-4.3	-2.6	-4.6	-3.9	-4.0
1% percentile	-28.7	-27.9	-8.0	-9.1	-9.5
Minimum	-100.0	-100.0	-22.5	-22.4	-24.5
Mean	-0.6	-0.7	0.2	0.3	-0.1
Standard deviation	8.2	8.0	3.4	3.4	2.9
Skewness	-7.1	-7.5	1.5	1.2	0.6
Kurtosis	77.4	88.4	17.9	17.8	20.5
Jacque-Bera	261153.0	341818.6	10489.6	10157.8	13999.3

Table 3: Summary statistics for 156 quarter-on-quarter percentage changes in bilateral nominal exchange rates 1974-2012

Currency pairs	DKK/DEM	DKK/SEK	DKK/GBP	DKK/NLG	SEK/DEM
Maximum	4.6	6.6	9.2	5.0	18.0
99% percentile	4.0	5.0	7.4	4.1	13.8
95% percentile	2.8	3.8	5.8	2.5	5.7
5% percentile	-0.8	-5.0	-6.6	-0.9	-3.8
1% percentile	-1.9	-10.9	-10.2	-1.5	-5.0
Minimum	-2.7	-14.5	-12.3	-2.6	-6.4
Mean	0.3	-0.2	-0.2	0.3	0.7
Standard deviation	1.1	3.0	3.7	1.0	3.4
Skewness	1.6	-1.5	-0.4	1.9	1.8
Kurtosis	4.3	5.0	0.6	6.1	6.5
Jacque-Bera	74.3	80.3	40.0	157.6	164.3
Currency pairs	GBP/DEM	NLG/DEM	SEK/GBP	SEK/NLG	NLG/GBP
Maximum	14.1	2.1	13.8	18.5	8.0
99% percentile	12.1	1.7	11.4	14.5	7.4
95% percentile	7.3	1.0	6.1	5.2	5.1
5% percentile	-4.6	-0.6	-4.9	-3.9	-6.8
1% percentile	-7.2	-1.0	-8.2	-5.0	-10.7
Minimum	-7.3	-3.0	-10.8	-6.4	-12.5
Mean	0.7	0.0	0.1	0.6	-0.5
Standard deviation	3.8	0.5	3.7	3.4	3.7
Skewness	0.8	-0.2	0.6	1.9	-0.5
Kurtosis	1.2	10.4	2.0	7.4	0.8
Jacque-Bera	36.9	358.7	16.2	222.5	38.6

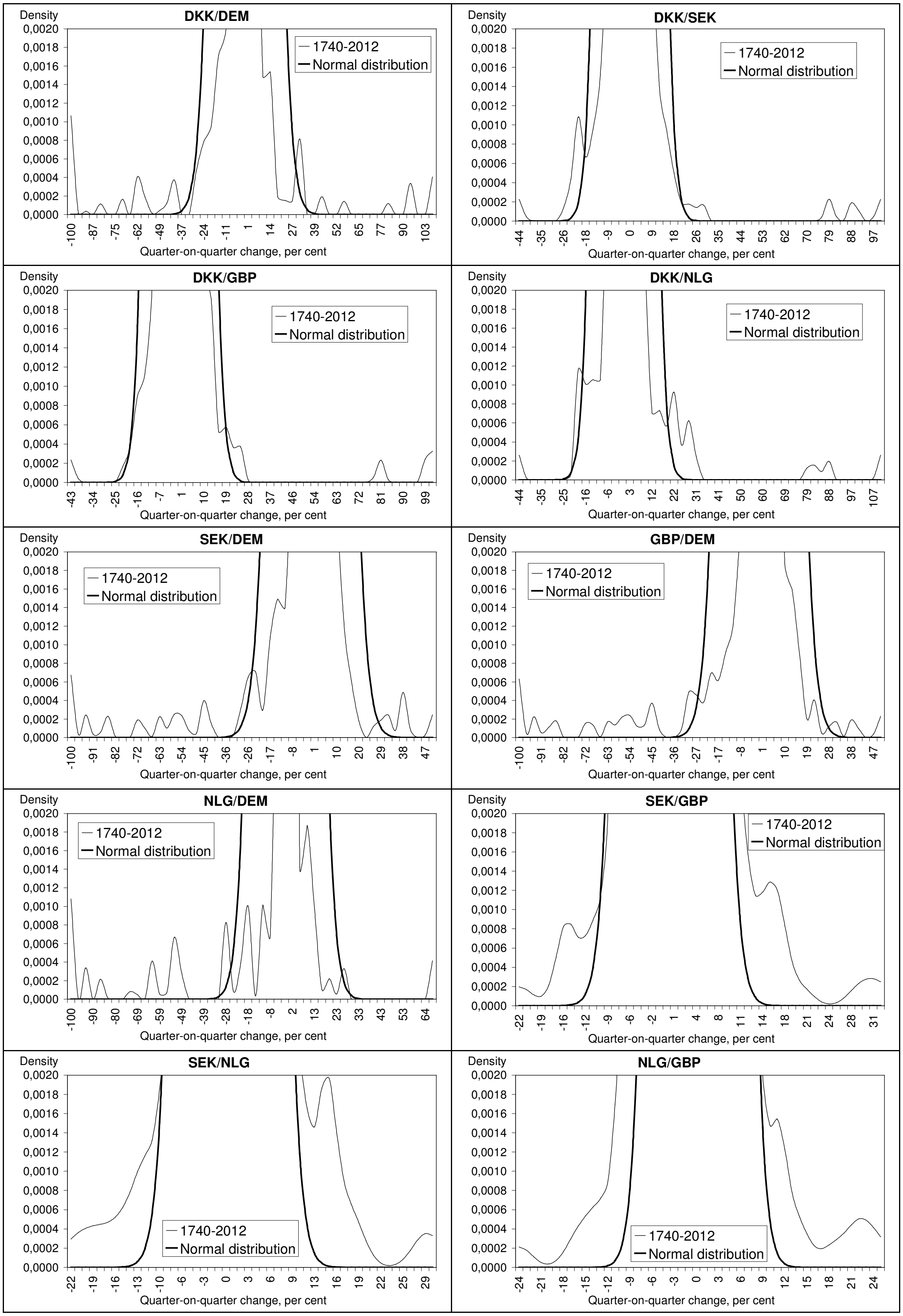
Table 4: Summary statistics for 68 quarter-on-quarter percentage changes in bilateral nominal exchange rates 1996-2012

Currency pairs	DKK/DEM	DKK/SEK	DKK/GBP	DKK/NLG	SEK/DEM
Maximum	0.2	5.7	7.3	0.2	7.9
99% percentile	0.2	4.8	7.1	0.2	7.3
95% percentile	0.1	4.0	4.3	0.1	4.5
5% percentile	-0.2	-4.2	-4.9	-0.2	-3.9
1% percentile	-0.5	-6.8	-7.0	-0.7	-4.6
Minimum	-0.6	-7.4	-7.9	-0.7	-5.3
Mean	0.0	0.1	0.1	0.0	-0.1
Standard deviation	0.1	2.5	3.0	0.1	2.5
Skewness	-2.0	-0.5	-0.3	-2.2	0.6
Kurtosis	6.8	0.9	0.2	7.8	1.2
Jacque-Bera	83.9	14.5	22.9	118.1	13.5
Currency pairs	GBP/DEM	NLG/DEM	SEK/GBP	SEK/NLG	NLG/GBP
Maximum	8.6	0.3	9.8	7.9	8.0
99% percentile	7.5	0.2	7.3	7.3	7.8
95% percentile	5.2	0.1	4.5	4.5	4.3
5% percentile	-4.2	0.0	-4.8	-3.9	-4.9
1% percentile	-7.2	-0.1	-5.1	-4.5	-7.0
Minimum	-7.3	-0.1	-5.5	-5.3	-7.9
Mean	-0.1	0.0	0.1	-0.1	0.2
Standard deviation	3.1	0.0	2.9	2.5	3.1
Skewness	0.4	3.6	0.5	0.7	-0.2
Kurtosis	0.4	15.7	0.8	1.3	0.4
Jacque-Bera	21.0	602.8	16.3	13.3	19.6

From Table 2-4 some noteworthy observations immediately leap to the eye. First, in all three sample periods the quarterly exchange-price changes are far from following a normal distribution. The quarterly changes exhibit a pronounced "fat tailed" property measured by the kurtosis, and the Jacque-Bera statistics clearly reject normality at all conventional significance levels. The fat tails are also clearly visible in Figure 2, which shows the tails of the probability density function of the normal distribution and the density functions estimated[3] on the basis of the observed quarter-on-quarter changes of the 10 exchange-rate pairs for the full sample period.

[3] Estimated by a Gaussian adoptive kernel density estimator, cf. Annex B.

Figure 2: Tails of the estimated density functions for the observed the quarter-on-quarter exchange-rate changes 1740-2012



Second, the maximum and minimum values as well as the tail percentiles are substantially larger in the full sample in Table 2 than in the two shorter samples in Table 3 and 4. This point is also illustrated in Figure 3 and 4. In Figure 3 we show the probability density functions estimated on the basis of the observed quarter-on-quarter changes in the full sample period 1740-2012 together with the minimum and maximum of the quarter-on-quarter exchange-rate changes observed in the post-Bretton Woods period. For each of the 10 exchange-rate pairs a large share of the probability mass is located outside the interval delimited by the minimum and maximum from the post-Bretton Woods period. In Figure 4 the minimum and maximum values are derived on the basis of the post EMS-crisis period, and here the share of the probability mass located outside the interval delimited by the minimum and maximum is even larger. This clearly illustrate the risk of seriously underestimating the significance and magnitude of low probability events when exchange-rate changes are derived on the basis of fairly short data samples.

Figure 3: Tails of the estimated density functions for the observed the quarter-on-quarter exchange-rate changes 1740-2012 and min/mix observed 1974-2012

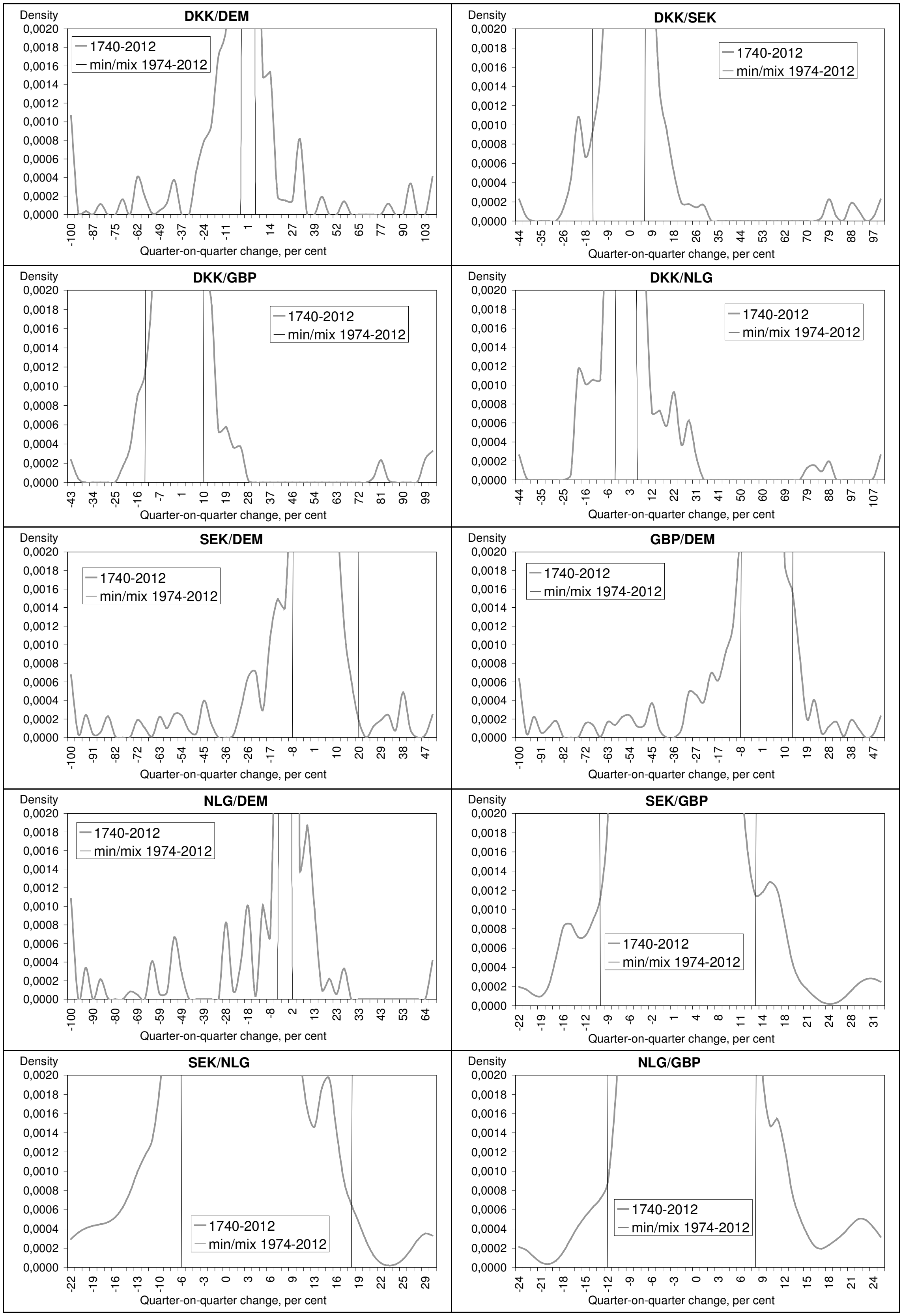


Figure 4: Tails of the estimated density functions for the observed the quarter-on-quarter exchange-rate changes 1740-2012 and min/mix observed 1996-2012

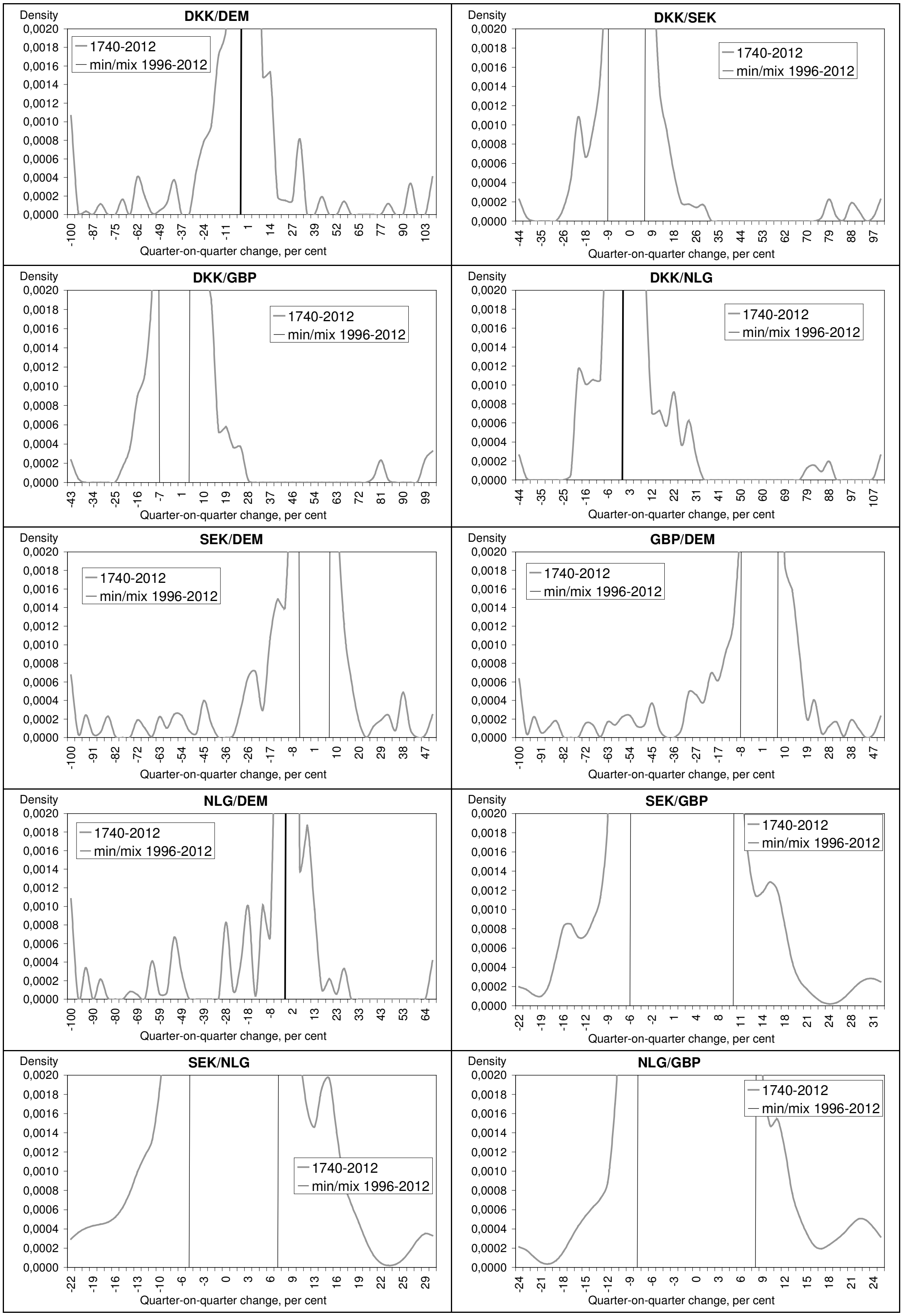


Table 5 reports the observed occurrences of large sigma events in our full sample of quarterly exchange-rate changes. We observe 3-10 occurrences of quarterly changes (drops or increases) in the exchange rates which are larger than six standard deviations from the mean during the past 273 years. Under the normal distribution a 6-sigma event can only be expected to occur once every 126 million years. Furthermore, we observe 2-7 occurrences of quarterly changes in the exchange rates, which are larger than eight standard deviations from the mean. An 8-sigma event can only be expected to occur once every 2.009E+14 years under the normal distribution. To put this figure in perspective it can be noted that the period that has elapsed since the "Big Bang" of the Universe according to European Space Agency (2013) is approximately 13.82 billion (1.382E+10) years!

Table 5: Number of occurrences of large sigma events in the quarterly-on-quarter changes in 10 nominal bilateral exchange rates 1740-2012

Quarter-on-quarter drop or increase in exchange rate of more than x standard deviations from the mean	Number of years between expected occurrences under a normal distribution	Observed number of occurrences									
		DKK/DEM	DKK/SEK	DKK/GBP	DKK/NLG	SEK/DEM	GBP/DEM	NLG/DEM	SEK/GBP	SEK/NLG	NLG/GBP
x = 1	0.79	53	81	74	55	59	55	41	180	167	162
x = 2	5.5	28	25	23	28	26	25	24	53	50	52
x = 3	93	18	12	8	14	18	16	18	21	26	24
x = 4	3945	16	6	5	7	13	12	11	12	16	12
x = 5	435381	12	4	4	4	11	11	11	4	7	7
x = 6	126247085	9	4	4	4	9	9	10	3	4	5
x = 7	9.704E+10	8	4	4	4	6	6	8	2	2	4
x = 8	2.009E+14	7	3	3	3	5	5	6	2	2	2
x = 9	1.108E+18	5	3	3	3	4	4	4	1	0	0
x = 10	1.640E+22	3	3	3	3	4	4	4	0	0	0
x = 11	6.542E+26	0	3	3	3	3	3	3	0	0	0
x = 12	7.036E+31	0	3	3	3	0	2	2	0	0	0
x = 13	2.043E+37	0	3	3	3	0	0	0	0	0	0
x = 14	1.604E+43	0	2	2	2	0	0	0	0	0	0
x = 15	3.405E+49	0	1	2	1	0	0	0	0	0	0
x = 16	1.957E+56	0	1	2	1	0	0	0	0	0	0
x = 17	3.044E+63	0	0	0	1	0	0	0	0	0	0
x = 18	1.283E+71	0	0	0	0	0	0	0	0	0	0

5. Finalising remarks

The most recent global financial crisis since 2007/2008 has once again reminded us of the critical importance of taking into account the infrequent occurrence of severe negative shocks in relation to risk assessments. The analysis in this paper has clearly illustrated the risk of seriously underestimating the probability and magnitude of tail events when frequency distributions of nominal exchange-rate changes are assessed on the basis of fairly short historical data samples.

A range of studies suggests that similar conclusions might be reached in other areas of relevance for financial risk management and stress tests. Barro (2006) lists 65 episodes of 15 percent or greater decline in real per capita GDP for 35 countries in the period 1900-2000. Reinhart and Rogoff (2009) count 250 sovereign external default episodes and 68 cases of default on domestic debt in the period 1800-2009. Reinhart and Rogoff, *op. cit*., also lists around 360 cases of banking crisis in 137 countries in the period from 1800 and until the recent global financial crisis, and Laeven and Valencia (2012) list as much as 147 systemic banking crises in more than 100 countries alone in the period 1970-2011. Mehl (2013) identify 43 global stock market volatility shocks over the period 1885-2011.

It might therefore be useful to have an eye for the long-term historical perspective as a source of inspiration when designing "worst case scenarios" or "severe stress scenarios" in relation to risk assessments and stress tests by financial institutions or regulators. As noted by Varotto (2012): "Among the main advantages of historical scenarios is the fact that they are plausible, if only because they have occurred before, and are not as sensitive to model risk as hypothetical scenarios".

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Annex A: Tail-probability approximation in the normal distribution

Let X~N(0,1) be a random variable following a standard normal distribution with a mean of zero and a standard deviation of 1. The cumulative distribution function for X is given by:

$$[A.1]\ P(X \le x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-\frac{u^2}{2}} du,$$

where x also can be seen as the number of standard deviations away from the mean under a normal distribution with a mean of μ and a standard deviation of σ.

The so-called error function (erf) used in mathematical physics is according to formula 7.1.1 in Abramowitz and Stegum (eds.) (1964) given by:

$$[A.2]\ \mathrm{erf}(y) = \frac{2}{\sqrt{\pi}} \int_{0}^{y} e^{-t^2} dt.$$

The complementary error function (erfc) is according to formula 7.1.2 in Abramowitz and Stegum, *op. cit*., given by:

$$[A.3]\ \mathrm{erfc}(y) = 1 - \mathrm{erf}(y).$$

The standard normal cumulative distribution function can then according to formula 7.1.22 in Abramowitz and Stegum, *op. cit*., be written as:

$$[A.4]\ P(X \le x) = \frac{1}{2}(1 + \mathrm{erf}(z)) \text{ where } z = \frac{x}{\sqrt{2}}.$$

In the normal distribution the one-sided tail probability p that X is more than x standard deviations away from the mean is then given by:

$$[A.5]\ p = 1 - P(X \le x) = \frac{1}{2}(1 - \mathrm{erf}(z)) = \frac{1}{2}\mathrm{erfc}(z) \text{ where } z = \frac{x}{\sqrt{2}}.$$

According to formula 7.12.1 in Boisvert *et al*. (eds.) (2010) the following asymptotic expansion of the complementary error function applies for $y \to \infty$:

$$[A.6]\ \mathrm{erfc}(y) \approx \frac{e^{-y^2}}{y\sqrt{\pi}} \sum_{m=0}^{\infty} (-1)^m \frac{1 \cdot 3 \cdot 5 \cdot \ldots \cdot (2m-1)}{(2y^2)^m}$$

Following Cotter *et al*. (2008) in a choice of m=3 in [A.6], the one-sided tail probability p that X is more than x standard deviations away from the mean under the normal distribution is then approximately given by:

$$[A.7]\ p \approx \frac{e^{-z^2}}{2z\sqrt{\pi}} \left(1 - \frac{1}{2z^2} + \frac{1 \cdot 3}{(2z^2)^2} + \frac{1 \cdot 3 \cdot 5}{(2z^2)^3}\right) \text{ where } z = \frac{x}{\sqrt{2}}.$$

In the main text we refer to a situation where the daily asset returns are assumed to be normally distributed. Table A.1 shows the probability of a drop in daily asset returns of more than x standard deviations from the mean under the normal distribution. Furthermore, the Table shows the number of days and years (consisting of 250 trading days) between expected occurrences under the normal distribution. In the cases with more than 7 standard deviations from the mean the figures in Table A.1 have been compiled via the tail-probability approximation formula [A.7].

Table A.1: Probability and occurrence of large sigma loss events when daily asset returns are assumed to follow a normal distribution

Drop in daily asset price of more than x standard deviations from the mean	Probability	Number of days between expected occurrences	Number of years (250 trading days) between expected occurrences
x = 1	15.866	6.3	0.0
x = 2	2.275	44.0	0.2
x = 3	0.135	740.8	3.0
x = 4	0.0032	31559.6	126
x = 5	0.000029	3483046.3	13932
x = 6	0.000000099	1009976678	4039907
x = 7	0.000000000129	7.76E+11	3105395365
x = 8	6.2216E-14	1.607E+15	6.429E+12
x = 9	1.1287E-17	8.860E+18	3.544E+16
x = 10	7.620E-22	1.312E+23	5.249E+20
x = 11	1.911E-26	5.234E+27	2.093E+25
x = 12	1.776E-31	5.629E+32	2.252E+30
x = 13	6.117E-37	1.635E+38	6.539E+35
x = 14	7.794E-43	1.283E+44	5.132E+41
x = 15	3.671E-49	2.724E+50	1.090E+48
x = 16	6.389E-56	1.565E+57	6.261E+54
x = 17	4.106E-63	2.435E+64	9.742E+61
x = 18	9.741E-71	1.027E+72	4.106E+69
x = 19	8.527E-79	1.173E+80	4.691E+77
x = 20	2.754E-87	3.632E+88	1.453E+86
x = 21	3.279E-96	3.049E+97	1.220E+95
x = 22	1.440E-105	6.945E+106	2.778E+104
x = 23	2.331E-115	4.291E+116	1.716E+114
x = 24	1.390E-125	7.192E+126	2.877E+124
x = 25	3.057E-136	3.272E+137	1.309E+135

In the main text we also compare the observed quarter-on-quarter percentage changes in the nominal exchange rate with a situation where the quarterly exchange-price changes are assumed to follow a normal distribution. Table A.2 shows the probability of a quarter-on-quarter drop or increase in the exchange rate of more than x standard deviations from the mean under the normal distribution. Furthermore, the Table shows the number of quarters and years between expected occurrences under the normal distribution. In the cases with more than 7 standard deviations from the mean the figures in Table A.2 have also been compiled via the tail-probability approximation formula [A.7].

Table A.2: Probability and occurrence of large sigma events when quarter-on-quarter exchange-rate changes are assumed to follow a normal distribution

Quarter-on-quarter drop or increase in exchange rate of more than x standard deviations from the mean	Probability	Number of quarters between expected occurrences	Number of years between expected occurrences
x = 1	31.731	3.15	0.79
x = 2	4.550	22	5.5
x = 3	0.270	370	93
x = 4	0.0063	15780	3945
x = 5	0.000057	1741523	435381
x = 6	0.000000198	504988339	126247085
x = 7	0.000000000258	3.882E+11	9.704E+10
x = 8	1.244E-13	8.036E+14	2.009E+14
x = 9	2.257E-17	4.430E+18	1.108E+18
x = 10	1.524E-21	6.562E+22	1.640E+22
x = 11	3.821E-26	2.617E+27	6.542E+26
x = 12	3.553E-31	2.815E+32	7.036E+31
x = 13	1.223E-36	8.174E+37	2.043E+37
x = 14	1.559E-42	6.416E+43	1.604E+43
x = 15	7.342E-49	1.362E+50	3.405E+49
x = 16	1.278E-55	7.826E+56	1.957E+56
x = 17	8.212E-63	1.218E+64	3.044E+63
x = 18	1.948E-70	5.133E+71	1.283E+71
x = 19	1.705E-78	5.864E+79	1.466E+79
x = 20	5.507E-87	1.816E+88	4.539E+87
x = 21	6.559E-96	1.525E+97	3.812E+96
x = 22	2.880E-105	3.472E+106	8.681E+105
x = 23	4.661E-115	2.145E+116	5.363E+115
x = 24	2.781E-125	3.596E+126	8.990E+125
x = 25	6.113E-136	1.636E+137	4.089E+136

Annex B: Non-parametric kernel density estimation

Kernel density estimation can be seen as a technique to construct smooth and differentiable histograms.

Let y_1, ..., y_n be n observations from a random variable Y with a unknown probability density function φ. Define a set of evenly spaced reference points over the range covered by the observed data. Following Ahamada and Flachaire (2010), the adoptive kernel density estimate f(y) at each reference point, y, can then be defined by:

$$[B.1]\ f(y) = \frac{1}{n}\sum_{i=1}^{n}\frac{1}{h\lambda_i}K\left(\frac{y-y_i}{h\lambda_i}\right),$$

where h is the global bandwidth parameter, λ_i is a parameter and K() is the kernel weight, which is a function that satisfies:

$$[B.2]\ \int_{-\infty}^{\infty} K(x)dx = 1.$$

The Gaussian kernel weight – which is applied in the paper at hand[4] – is given by the standard normal probability density function:

$$[B.3]\ K(x) = \frac{1}{\sqrt{2\pi}}e^{-\frac{x^2}{2}}.$$

The exits a range of other kernel weights than the Gaussian kernel. However, in practice the choice of kernel is of relative little importance for the results, cf. Ahamada and Flachaire, *op. cit*.

λ_i is a parameter that varies with the local concentration of data. The distributions of the quarter-on-quarter exchange-rate changes in section 5 are very long-tailed, and the concentration of data is very heterogeneous. In such cases the parameter λ_i is smaller in the middle of the distributions with high data concentrations. The parameter λ_i is larger in the tails of the distributions where the concentrations of data is low.

[4] The estimated densities shown in the paper at hand have been compiled by the use of R and the quantreg package.