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RESUME

Jeg analyserer den naturlige realrente, hvorfor og hvordan den er medbestemmende for pengepolitikkens virkning på realøkonomien, og hvad der kan ske, hvis den bliver negativ. Det kan gøre pengepolitikken virkningsløs, hvilket i en situation med et negativt produktionsgab kan føre til en lang periode med lav vækst – *langvarig stagnation*. Danmark fører fastkurspolitik over for euroen. Derfor kan Nationalbanken ikke bruge pengepolitikken til at stabilisere økonomien. Men derfor kan det stadig være interessant at analysere påvirkningen fra pengepolitikken på realøkonomien og bruge analysen som et redskab til anbefalinger om økonomisk politik. Jeg bruger dette som motivation for at estimere den naturlige realrente på danske data ved hjælp af en model for en lille, åben økonomi. Resultaterne peger på en lav og måske negativ naturlig realrente for Danmark og på en negativ trend. Der argumenteres for, at det negative niveau må forventes at være midlertidigt, og at pengepolitikken, som Danmark importerer fra ECB, allerede stimulerer dansk økonomi nu eller vil gøre det i den nære fremtid, når den naturlige realrente forventes at stige. Det lave niveau for den naturlige realrente forventes dog at være af mere langvarig karakter. Det medfører en forhøjet sandsynlighed for, at cykliske stød kan skubbe den naturlige realrente tilbage i negativ, og derfor en forhøjet sandsynlighed for at ramme den effektive nedre grænse for den pengepolitiske rente.

ABSTRACT

This paper discusses the natural real rate, why and how it reflects the stance of monetary policy, and what can happen if it turns negative; make monetary policy ineffective, which in a situation with a negative output gap can lead to a long period of low growth - *secular stagnation*. Denmark as a fixed exchange rate regime vís-a-vís the euro and consequently has tied its policy rate to the policy rate in the euro zone. But it can still be interesting to analyse the stance of monetary policy and use it as input in policy recommendations for fiscal policy and other economic policies. Using this as motivation, the natural real rate is estimated using Danish data applying a model for the small-open economy. I find evidence for very low and perhaps negative levels of the natural real rate for Denmark and stronger evidence for a negative trend. It is argued that the negative levels can be expected to be temporary and that imported monetary policy from the ECB already is or will be stimulative in the future as the natural real rate increases. Low levels of the natural rate can remain in the near future leaving an elevated probability of cyclical factors pushing the natural real rate down and hence ending up in the lower bound for monetary policy rate again.

The Danish Natural Real Rate of Interest and Secular Stagnation

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March 13, 2015

Abstract

This paper discusses the natural real rate, why and how it reflects the stance of monetary policy, and what can happen if it turns negative; make monetary policy ineffective, which in a situation with a negative output gap can lead to a long period of low growth - *secular stagnation*. Denmark has a fixed exchange rate regime vís-a-vís the euro and consequently has tied its policy rate to the policy rate in the euro zone. But it can still be interesting to analyse the stance of monetary policy and use it as input in policy recommendations for fiscal policy and other economic policies. Using this as a motivation, the natural real rate is estimated using Danish data applying a model for the small-open economy. I find evidence for very low and perhaps negative levels of the natural real rate for Denmark and stronger evidence for a negative trend. It is argued that the negative levels can be expected to be temporary and that imported monetary policy from the ECB already is or will be stimulative in the future as the natural rate increases. Low levels of the natural rate can remain in the near future leaving an elevated probability of cyclical factors pushing the natural real rate down and hence ending up in the lower bound for monetary policy rate again.

*Address: Nationalbanken, Havnegade 5, 1093 Copenhagen K. The author thanks Mark Strøm Kristoffersen and Kim Abildgren for valuable comments and suggestions, and seminar participants at Danmarks Nationalbank for useful discussions. The viewpoints and conclusions stated are the responsibility of the author, and do not necessarily reflect the views of Danmarks Nationalbank.

1. Introduction

At the time of writing, the financial crisis has lasted around 7 years. In Denmark and many other countries growth is still sluggish, estimated economic slack is significant and deflation is a possibility in the near future. This long recession risks turning into *secular stagnation* introduced by Summers (2014): When in the absence of equilibrating forces low demand undermines potential growth, which may drive the natural rate of real interest - the real rate the economy demands to equate savings and investments and to close the output gap - below zero. Monetary policy might not be able to deliver these levels of real rates due to an effective lower bound for nominal interest rates and low inflation.[1] At the time of writing, there is even a risk that actual real rates can increase due to the risk of deflation, hardly what an economy needs in a situation with low demand and a negative output gap.

This paper analyses secular stagnation within a Danish context. Denmark has a fixed exchange rate regime vís-a-vís the euro and the policy rate is consequently tied to the policy rate in the euro zone. But it can still be interesting to analyse the stance of monetary policy imported from the euro-zone and using it as input in policy recommendations: If monetary policy imported from the euro area is too stimulative for the Danish economy this calls for contractionary discretionary *fiscal* policy; the remaining traditional policy instrument. Using this as motivation, I ask the following questions. What does secular stagnation means within a theoretical framework; how does an economy end up in a situation in which the output gap is negative and the lower bound for the monetary policy rate together with a negative natural rate rate makes monetary policy ineffective? The key word is the natural real rate of interest, NRR, which leads me to ask: Why is the NRR important, what is it and how is it determined in a small open economy?

Lastly, this paper evaluate the case of Denmark using an empirical model for the NRR. The seminal paper on the estimation of NRRs is Laubach and Williams (2003). However, their method is problematic in the case of Denmark for at least three reasons. Firstly, their solution to the so called pile-up problem estimating variances of zero, see Stock (1994), does not work well on Danish data. Secondly, the NRR is weakly identified using their method on Danish data essentially due to problems of unit-root. Thirdly, their model is a model for the closed economy.[2]

[1] Recent events have shown that there is no such thing as a *zero* lower bound on short term policy rates, but term premia and inflation and credit risk premia can prevent yields on bonds with longer maturities to fall to zero, and what matters for economic activity is not only the short term quarter to quarter real rate but the path of short term real rates or long terms rates.

[2] A similar model has however been estimated on Swedish, Canadian and Swiss data, see Bouis et al. (2013).

This paper instead estimates a model for a small open economy, addressing the last problem, using Bayesian techniques following Berger and Kempa (2014), addressing the first and the second problem. Specifically, the use of priors in the estimation can solve the pile-up problem, as the prior forces a part of the posterior distribution to be positive and hence address identification of the standard errors of transitory and permanent shocks on the NRR. Also, the Bayesian techniques make unit-root less of a problem and can possibly solve the identification problem of the natural rate of interest.

The result of the estimation can be summarised as follows. There is evidence of a slow downward movement in the NRR for Denmark. However, it turn negative only during the onset of the financial crisis around 2008 which points to a lack of demand/excess savings as an explanation behind the movement into negative territory. Thus, when both public and private balance sheets have been rebuild and the elevated levels of uncertainty have fallen, the NRR is likely to move back into positive territory and monetary policy can regain its role as a potential stabilising force. But the level can be expected to remain low in the near future globally as well as in Denmark. This has increased the probability of ending up in secular stagnation and in the lower bound, a probability which can be expected to remain elevated in the years to come. Economic policy needs to address this through structural reforms that can lift the natural rate of output and hence the NRR, flexible labour markets and active policies that reduces risk of hysteresis, and implementation of a symmetric fiscal policy framework, which reduces public debt in upturn leaving room for stimulating the economy in downturns. Finally, low interest rates can pose risk to financial stability.

One advantage of the empirical model is that it also provides an estimate of the output gap. The story the filtered output gap tells fairs well with actual business cycle and the economic history for Denmark during this period. One additional advantage of the empirical model is that it applies data for inflation and interest rates, which can be aligned to financial data. Given the forward looking behaviour of financial data it could be possible to utilise these data to provide forecasts of the business cycle. This is left for future research.

The remainder of this paper is organised as follows. Section (2) defines the NRR discuss its determinants, and shows why the NRR is an important determinant for monetary policy and what happens when the NRR is negative and monetary policy is bounded by the lower bound; secular stagnation. Section (3) presents the empirical model, section (4) presents the econometric techniques and section (5) presents data, while the results of the estimation is discussed in section (6). Policy and secular stagnation are discussed in section (7) and (8). Section (9) concludes.

2. What is the Natural Real Rate of Interest?

In what follows, I will discuss the NRR. The question about secular stagnation to a large extend hinges on the level of the NRR. I will in the sections to come analyse and derive a NRR from a small economic model or framework, the motivation being that it provides a coherent framework through which I can discuss both policy and use for a discussion of the underlying determinants of the NRR and consequently secular stagnation.

The terminology ”natural rate” is widely contributed to the work of Wicksell and later ”reinvented” by the DSGE literature and especially Woodford, see Knut (1936) and Woodford (2003).[3] Wicksell characterizes the NRR as the real interest rate that yields price stability and would equate real saving and investment in an otherwise equivalent non-monetary Walrasian economy.

> *There is a certain rate of interest on loans which is neutral in respect to commodity prices, and tends neither to raise nor to lower them. This is necessarily the same as the rate of interest which would be determined by supply and demand if no use were made of money and all lending were effected in the form of real capital goods.*
> *Knut Wickesell in Interest and Prices, 1898 p. 102*

The natural real rate defined by Wicksell around 100 years ago comes close to the natural real rate which, in a certain class of DSGE models, closes the output- and inflation gap.[4] The literature neither is clear on the definition of the NRR nor the terminology. Sometimes the natural rate is called the equilibrium real rate, the natural real rate or the neutral real rate. In what follows the time-varying real rate will be named the *natural real rate of interest* and its constant part, or steady-state, will be called the *neutral real rate of interest*. The difference between the natural and the neutral real rate is that the natural rate settles to the neutral rate when temporary shocks have died out.

Within these definitions, the neutral rate is a medium-to long-run concept which depends on lower frequency shocks. It can consequently be thought off as being determined by the economy in the long-run and by long-run relationships like trends in productivity, demographic trends etc, and it is possible to think of the NRR as moving around the level for the neutral rate. An appropriate framework for thinking about the neutral rate is

[3] See Woodford (2003) and the discussion in Amato (2005) for a analysis of both the NRR and its role in monetary policy

[4] That is, in DSGE-models in which the so called ”divine coincidence” holds, see as an example Galí (2009). If it does not hold, then it is an approximation as whenever there is a policy trade-off between closing the output gap and the inflation gap, the central bank has too few instruments, the monetary policy rate, to close two gaps.

therefore a growth model like the Ramsey-Cass-Koopmans model. The NRR in contrast is affected by all sorts of real, not nominal, shocks like government spending shocks or preference shocks. As an example, permanent shocks to productivity will in a DSGE-model lead to falling prices and increasing output, but once these business-cycle movements have died out, that is when prices and wages have been reset optimally, the output and inflation gaps will close and the neutral rate will settle on a new higher level. A temporary productivity shock will in contrast only affect the NRR.

I derive the NRR from first principles from a framework with an infinite living representative household. This is the framework used in many DSGE-models for the analysis of monetary policy. Importantly for the focus in this paper, the framework provides direct insight to the role of NRR for monetary policy and what and how it can happen that monetary policy can be impotent and the economy can end up in secular stagnation. Noteworthy, the setup provides a direct link between the Taylor-rule, the NRR and activity, provides the main economic explanation of the empirical model, and the theory plays can play the role as a framework upon which policy can be addressed and explained. There are however some disadvantages with this setup. Firstly, the setup implies that the NRR is solely a function of the consumption-savings decision of the household and investment demand plays no role though the framework can be expanded to include investments. The costs is a less appropriate model for investments and the advantage is a more simple setup. Secondly, the fact that the household lives infinitely makes the model a less suitable framework for the study of life-cycle behaviour.

With this in mind in what follows I will firstly derive the real rate in a closed economy setup, introduce the natural level of production, analyse an open economy setup, and then move on to discuss what determines the NRR. I will finally discuss the role of the NRR in the conduct of monetary policy and secular stagnation.

2.1. The Real Rate of Interest in a theoretical framework

I start from the standard utility maximisation problem for the representative consumer maximising discounted expected utility

$$\max_{(C_t,B_t)} V_t = \sum_{j=0}^{\infty} \beta^j E_t \left[U(C_{t+j}) \right], \tag{1}$$

with respect to consumption, C_t, and saving, B_t, such that the budget constraint holds,

$$B_{t+1} + P_t C_t = P_t X_t + I_t B_t. \tag{2}$$

Here I_t is the (gross) nominal interest rate on financial assets, and in what follows Π_t denotes (gross) inflation, π_t inflation, X_t denotes income, and P_t denotes the price level. β is the subjective discount factor. The first order condition with respect to savings can be written as

$$U_C(C_t) = \beta E_t\Big[U_C(C_{t+1})\frac{I_t}{\Pi_{t+1}}\Big], \tag{3}$$

in which $U_C(C_t)$ denotes marginal utility of extra consumption assumed to given by $C_t^{-\gamma}$, under the assumption of a CRRA utility function, $\frac{C_t^{1-\gamma}}{1-\gamma}$. These assumptions implies that (3) can be written as

$$C_t^{-\gamma} = \beta E_t\Big[C_{t+1}^{-\gamma}\frac{I_t}{\Pi_{t+1}}\Big] \tag{4}$$

Under an assumption of joint conditional normality with an approximate constant conditional variance-covariance matrix, relation (4) can be approximated as follows where lower case letters denote the log of the respective variable:

$$i_t = \delta + \gamma E_t[\Delta c_{t+1}] + E_t[\pi_{t+1}] - \frac{\gamma^2}{2}V_t[\Delta c_{t+1}] - \frac{1}{2}V_t[\pi_{t+1}] + \gamma Cov_t(\Delta c_{t+1}, \pi_{t+1}) \Leftrightarrow$$
$$r_t \equiv \delta + \gamma E_t[\Delta c_{t+1}] - \frac{\gamma^2}{2}V_t[\Delta c_{t+1}] = i_t - E_t[\pi_{t+1}] - \frac{1}{2}V_t[\pi_{t+1}] - \gamma Cov_t(\Delta c_{t+1}, \pi_{t+1}), \tag{5}$$

where $\delta \equiv -log(\beta)$. I lastly impose equilibrium in this simple closed economy model without investments that (log) output must equal (log) consumption, $y_t = c_t$,

$$r_t \equiv \delta + \gamma E_t[\Delta y_{t+1}] - \frac{\gamma^2}{2}V_t[\Delta y_{t+1}] = i_t - E_t[\pi_{t+1}] - \frac{1}{2}V_t[\pi_{t+1}] - \gamma Cov_t(\Delta y_{t+1}, \pi_{t+1}). \tag{6}$$

The real interest rate is high, when the consumers are relatively impatient; that is when δ is high. The intuition is that if the household wants to increase consumption now then equilibrium demands a higher real rate to induce them not to do so. The real rate is also increasing in expected future growth, because in the case of higher expected future growth it is advantageous to postpone consumption from today to tomorrow. Thus high real rates lower the level of consumption today, while raising its growth rate from today to tomorrow. The parameter γ determines the elasticity of expected growth on the real rate.

2.2. The Natural level of production, the Natural and the Neutral real rate

There is a close relationship between the NRR and natural output, y_t^N. Natural output can be defined as the level of production the economy would have produced if all prices and wages were fully flexible. The definition resembles the more traditional definition of potential production, c.f. the quote from Wicksell above, as deviations of natural output from actual output, the output gap, is the main determinant of price pressures in the economy. As an example, if the output gap is positive then production would have been lower if firms could have changed their prices instantly. In what follows, the output gap will be denoted as $x_t \equiv y_t - y_t^N$.

By inserting the natural level of output into the relation for the real rate, relation (6), the NRR can be found as the equilibrium real rate of return under fully flexible prices; that is, the real rate of return on savings necessary to ensure that aggregate demand is equal to natural output.

$$r_t^N \equiv \delta + \gamma E_t[\Delta y_{t+1}^N] - \frac{\gamma^2}{2} V_t[\Delta y_{t+1}^N] \tag{7}$$

For the purpose of this paper it is not necessary to make specific assumptions regarding the natural output and provide a complete specification of all the types of shocks that are likely to hit it. Instead it is sufficient to keep an over all view and divide these shocks into temporary and permanent shocks.[5] I consequently assume the economy grows by a constant growth rate denoted by g, which can be assumed to follow a simple stochastic process, $\Delta y_{t+1}^N = g + \varepsilon_{t+1}^g$. Here g captures the trend rate in productivity increases in the labour force etc. Further, the natural rate of output are hit by temporary shocks, Ω_t, which can be shocks to the labour supply, government spending shocks etc., times the respective shocks' elasticities, ψ reflecting the households' and firms' response to these shocks.

Under these simplifying assumptions relation (7) can be written as

$$r_t^N = (\delta + \gamma g) + \gamma E_t[\psi \Omega_{t+1}] - \frac{\gamma^2 \psi^2}{2} V_t[\Omega_{t+1}] \tag{8}$$

According to this paper's definition of the NRR, the *neutral real rate* is equal to $\delta + \gamma g$, which in this model is constant. But it is possible to think of this constant as depending on shocks, as an example shocks from international savings or investments from abroad, or shocks to trend growth rate in the economy, which in turn can affect the neutral rate. The NRR in turn, depends on the neutral rate and temporary shocks reflected in $\psi\Omega$. As an example, shocks to government spending will push natural output up and hence the NRR.

This analysis has left out the question of investments, which some uses as the framework to think about the NRR, see e.g. Lo and Rogoff (2015). This is also what Wicksell had in mind in the quotation above. As written in the introduction to this section, I have in the discussion in this paper chosen to focus on a more simple setup in terms of modeling, and consequently left out the investment decision of the firms and households. However,

[5] In Woodford (2003) a more complete specification of the determinants of these shocks. Here these are assumed to be government spending shocks, consumption shocks, technology shocks and labour supply shocks, see chapter 3.

investments in standard macro model of course do depend on the real rate of interest through cost of capital, and hence, relation (7) can be interpreted as also determining investment demand, see also Woodford (2003). I will, however, use an investment-savings framework for the determination of the global NRR presented next, as Denmark is a small open economy in which savings does not need to equal investments.

2.3. Open economy considerations

In an open economy the difference between savings and investments are net exports or changes in the current account. I will in this section talk about the global real rate and the global NRR and how it is likely to affect the domestic NRR. To the first end, I will use a savings-investment model framework. This is due to a wish for simplicity and for a wish to show how longer term trends in global investment demand and globals savings determines the global NRR, as these two are possible explanations to the observed behaviour of NRR explained in the sections to come. Using this definition, I have implicitly ignored the implication of a global business-cycle in the determination of the global NRR in contrast to the framework for the domestic economy above. This is due to a wish for simplicity as I avoid taking a stand on global monetary policy, output gap and inflation, and due to a wish to show how longer term trends in global investment demand and globals savings determines the global NRR, as these two are possible explanations to the observed behaviour of NRR explained in the sections to come. To the second end, I link the global real rate to the domestic through arbitrage; that is, through the real rate parity.

According to the uncovered real rate parity, the real rate in the home country through arbitrage can be expected to equal the real rate in the foreign economy, $r_t^{\star}$, minus the expected future change in the real exchange rate, q_t, and risk premia, rp_t^r.

$$r_t = r_t^{\star} - E_t[\Delta q_{t+1}] + rp_t^r, \tag{9}$$

Relation (7) is the uncovered interest rate parity written in real terms and establishes a relationship between the home real rate, the foreign real rate and the real exchange rate. In a situation in which the output gap and inflation gap are closed it is reasonable to assume that the real exchange rate is constant. Hence, the home NRR can to a large extend be expected to depend on the world NRR:

$$r_t^N = r_t^{\star,N} + rp_t^r. \tag{10}$$

The global NRR, $r_t^{\star,N}$, can be defined as the real rate of interest which equalizes global saving with global investments. Hence, global savings, investments and growth are important determinants of the home NRR.

As an example, higher savings from China and East-Asia are likely to depress the global NRR. This will all else being equal lead to capital inflows to Denmark through arbitrage putting upward pressure, that is appreciation, of the Danish exchange rate. The response of Danmarks Nationalbank is to decrease the Danish monetary policy rate, as Danmarks Nationalbank defends the fixed exchange rate regime. Lower nominal interest rates are likely to stimulate aggregate demand and result in upward inflationary pressures and ultimately a lower real rate. The equilibrium mechanism is the real exchange rate, the foreign price level in terms of Danish kroner divided by the Danish price level, through its effect on net exports: The increase in inflation causes a deterioration in the competitiveness of the Danish producers leading to a fall in exports. Through time, Danish producers need to face slower price increases than their foreign counterparts, relative disinflation, to rebuild competitiveness. If not, then the exchange rate peg will be under threat, see also Pedersen and Ravn (2014) for a discussion of this within a fixed exchange rate regime. The relationship between exports, the real exchange rate and the NRR provides a link to global capital markets, which can be exploited in the estimation of the NRR for Denmark without the need to model the foreign economy.

Denmark being a small economy can not influence the global NRR. But relation (10) shows that the Danish NRR does not necessarily need to be equal to the global NRR. Instead, risk premia can change. This opens up for a role for domestic factors to influence the domestic NRR. Knowledge of the domestic economy and its role for the domestic NRR is, however, of interest, as they determine the real exchange rate, the current account, net exports and the composition of demand. As an example, in the case of a relatively weak real exchange rate, Danish aggregate demand would consist of relative more exports and relative less consumption. Furthermore, in a fixed exchange rate regime, the difference between the actual real rate and the NRR can guide policy makers about the appropriate stance of fiscal policy and/or other economic instruments, as the policy rate in such a regime is fixed by the exchange rate. This will become clearer in the sections to come. Here I note that the discussion in this section motivates the use of the real effective exchange rate as a determinant of the domestic NRR in an empirical model with the advantage that it does not need the modeling of the foreign or global economy.

2.4. The determinants of the NRR

Relation (7) and relation (10) can be used as a framework for a discussion of the determinants of the NRR. Although the first term in that relation, minus the log of the subjective discount factor and the growth rate, in this framework is constant, it can be fruitful for the discussion of the NRR to think of it being hit by shocks. As an example, shocks to the discount factor can be interpreted as shocks to the savings in the economy: A lower value implies higher savings all else being equal. This term consequently can be interpreted as a term which reflects savings in the economy or equivalently, aggregate demand.[6] Further, the growth rate can be thought of as being hit by numerous permanent shocks like productivity discussed above.

The second term, $\gamma E_t[\Delta y^N_{t+1}]$, is the expected future growth rate of the natural level of output. Hence, everything that affects expectations of future growth increases the NRR through this term, for example increases in input factors in the production function, like investment booms, labour supply shocks, increases in government spending etc.

The third term, $\frac{\gamma^2}{2} V_t[\Delta y^N_{t+1}]$, reflects uncertainty or risk in the economy. Higher uncertainty puts downward pressure on the NRR due to as an example an increase in precautionary savings, or increased attractiveness of safe assets. Finally, the real rate parity, as discussed above, puts upward or downward pressure on the NRR through the global capital markets: Higher savings decreases the global NRR and puts downward pressure on the domestic NRR, while increasing demand for investments globally puts upward pressure on the global NRR and hence on the domestic NRR. Also, given the real rate parity, extra savings from abroad can be thought of as hitting the domestic economy through this term if risk premia in the parity conditions does not change.

I observe that the actual real rates and NRR can be expected to be correlated as they as an example both depends on economic growth, but they are not expected to co move perfectly. As an example, actual real rates depends on monetary policy, while the NRR is a real variable determined by the real side of the economy. This difference can thus be thought of as stemming from nominal factors especially nominal rigidities. The difference between actual real rates and NRR is indeed of great interest, as this difference determines the stand of monetary policy, as will be shown next.

[6]When the implications of the zero lower bound for the macro economy are studied in DSGE-models, a common method to force the economy towards the bound is through shocks to δ, see as an example Eggertsson and Woodford (2003). This is also the parameter which is changed when looking at changes in savings rates in growth models like the Solow or Ramsey models.

2.5. The NRR and monetary policy and secular stagnation

In what follows, I will show that within a simple DSGE-model as discussed above for monetary policy NRR is a key variable, and I will show that if the NRR is very negative, then monetary policy can become ineffective. This analysis provides the motivation for estimating the NRR and the theoretical background for secular stagnation.

Monetary policy in a closed economy, or in an open economy with e.g. inflation targeting, is often assumed to be conducted through a Taylor-rule for the monetary policy rate as follows, Taylor (1993)

$$i_t = \widetilde{i_t} + \bar{\pi} + \phi_\pi\Big(\pi_t - \bar{\pi}\Big) + \phi_y\Big(y_t - y_t^N\Big). \tag{11}$$

Here i_t denotes the short (one-period) nominal risk free rate and $\widetilde{i_t}$ denotes an exogenous given possible time-varying constant or level for the (long-run) real monetary policy rate, which I will come back to below.[7] In Taylor (1993) it was assumed that the target inflation rate, $\bar{\pi}$, and $\widetilde{i_t}$ were equal to 2 pct. per year respectively, and the coefficients, ϕ_π, ϕ_y were equal to a $1\frac{1}{2}$ and $\frac{1}{2}$ respectively. A key assumption in Taylor (1993) for this paper is that $\widetilde{i_t}$ is constant and equal to 2 pct. per year. If as an example inflation is zero, as it is at the time of writing, and the output gap is -$1\frac{1}{2}$, then the policy rate needs to be around $\frac{1}{4}$ pct. This is greater than but close to the actual Danish policy rate at the time of writing of around 0 even though monetary policy is aiming at stabilising the exchange rate against the euro.

I next introduce production into the model and I will talk about the real rate gap. In a standard DSGE-model production typically gives rise to a New-Keynesian Phillips curve, see as an example Woodford (2003)

$$\pi_t = \beta E_t[\pi_{t+1}] + \kappa x_t, \tag{12}$$

in which κ is a given constant parameter. I also rewrite (6) in terms of the output gap, $x_t \equiv y_t - y_t^N$:

$$x_t = E_t[x_{t+1}] - \frac{1}{\gamma}(i_t - E_t[\pi_{t+1}] - \delta - \gamma \Delta E_t[y_{t+1}^N] - \frac{\gamma^2}{2} V_t[\Delta y_{t+1}^N]) - rp_t,$$

in which I have defined the risk premia:

$$rp_t \equiv \frac{1}{2\gamma} V_t[\pi_{t+1}] + Cov_t(\Delta y_{t+1}, \pi_{t+1}) - \frac{\gamma}{2}\Big(V_t[\Delta y_{t+1}^N] - V_t[\Delta y_{t+1}]\Big).$$

[7] Here, I have added a constant rate of inflation, $\bar{\pi}$, and thus allowed for non-zero inflation target, the empirical relevant case, while in the model derivations, I have assumed a steady state rate of inflation of zero.

In this relation, the NRR can be recognized as being $r_t^N \equiv \delta + \gamma E_t[\Delta y_{t+1}^N] - \frac{\gamma^2}{2} V_t[\Delta y_{t+1}^N]$, which can be introduced above:

$$x_t = E_t[x_{t+1}] - \frac{1}{\gamma}(i_t - E_t[\pi_{t+1}] - r_t^N) - rp_t. \tag{13}$$

According to (13) it is the *real interest rate gap* - the difference between the actual real rate and the NRR, $\tilde{r}_{t+1} \equiv i_t - E_t[\pi_{t+1}] - r_t^N$ - which is important for aggregate demand. If the central bank wish to stimulate the economy and close the output gap, not only does the real rate need to be lower. It needs to be lower than the NRR. Intuitively and simplifying, the natural rate of output can be thought of as growing by the NRR, while actual output grows by the actual real rate. Hence, if actual output grows by a slower rate than natural output, then the real rate gap needs to be negative such that actual output is allowed to grow by a faster rate than natural output and hence close the discrepancy between the two; that is, to close the output gap.

Relation (13) visualises the problem of an effective lower bound on the monetary policy rate, a low if not negative NRR, and low inflation: When the NRR is zero, the real rate needs to be negative to stimulate aggregate demand and hence output. This is on average not a problem, as inflation seldom drops below zero. But if inflation is very low or even negative, and/or the NRR is very negative, the lower bound prevents the actual real rate to be sufficiently negative to drive the *real rate gap* into negative and hence stimulate the economy. As an example, if the NRR is 2 per cent and inflation is 2 per cent then from (11) the nominal policy rate stimulates the economy if the monetary policy rate is 2 per cent. But if the NRR is -2 and inflation is zero, then the nominal policy rate needs to be below -2 to be stimulative and close the output gap. Here a state in which output is depressed and monetary policy is impotent has occurred. This is what Larry Summers following Alvin Hansen have named *secular stagnation*, see Summers (2014).

From (12) and (13) it can be seen that if inflation has to be zero, then the output gap needs to be zero. That is not surprising as natural production is defined as the level of production in the economy in which no firm has the incentive to change its price; it is already at its optimum. If I substitute $y_t = y_t^N$ and $\pi_t = 0$ into (13), then I get the implied path of the monetary policy rate

$$i_t = r^N + \bar{\pi} - \gamma rp_t, \tag{14}$$

in which I have moved the time subscript on NRR to reflect that in a situation where the gaps are closed, the NRR must equal to the neutral real rate. If I do the same in the monetary policy rule, equation (11), I get

$$\begin{aligned} i_t &= \widetilde{i_t} + \bar{\pi} \Rightarrow \\ r^N - \gamma r p_t &= \widetilde{i_t} \end{aligned} \tag{15}$$

If the output gap is zero and inflation is on target, then the monetary policy rate is equal to the neutral real rate, plus the inflation target; from the discussion in section (2.2), and given that (15) was derived under constant inflation and an output gap of zero, the NRR must be the part which refers to the neutral rate. For monetary policy that implies that even in a situation in which the output- and inflation gap are zero, then the central bank needs to follow movements in the neutral real rate. And in a framework in which the neutral rate is time dependent, the natural level for the policy rate, here denoted by $\widetilde{i_t}$ will move.

Consequently, for monetary policy and the question of secular stagnation, a key variable is the NRR. In the following sections I will therefore estimate the NRR on Danish data to be able to discuss empirically the question of secular stagnation in a Danish context.

3. An empirical model of the Natural Real Rate for Denmark

The big challenge for the estimation of the NRR is that it is unobservable, but the theoretical framework from the previous sections can be used as a help. Specifically, economic theory points to possible dependence of e.g, on the growth rate in the economy, the output gap and inflation, which can be exploited in an empirical model for the NRR.

One approach is to apply a fully structural DSGE model. However, the estimates can be sensitive to the model assumptions of that particular model, and by using a less structural setting, data is allowed to speak more freely and the advantages of using a less structural setup can perhaps outweigh the disadvantage of not being able to provide a fully structural economic interpretation of the estimated time-series. This approach is chosen as the focus here to a greater extend is an estimation of the movements in the NRR, and to a lesser extend an economic interpretation of these movements.

The approach taken in this study is to apply the theory from small standard dynamic macroeconomic models with a dynamic IS-relation and a Phillips-curve and infer the NRR from movements in GDP. The seminal paper on the estimation of the NRR is Laubach and Williams (2003). Their method, however, turn out to be problematic in a Danish perspective due to at least three issues. Firstly, the model in Laubach and Williams (2003) is a model for a closed economy, while the Danish economy by any measure is open and small. Secondly, to identify the NRR it is important that the sum of the coefficients in the IS-relation, the relationship between the output gap, lags of the output gap and the real interest rate gap, are smaller than 1. Initial results form the estimation of the model in Laubach and Williams (2003) showed that this is not the case for Danish data. Thirdly, Laubach and Williams (2003) uses econometric techniques to resolve the so called ”pile-up” problem which works poorly on Danish data. The pile-problem is that some parameters can not be estimated efficiently. In the current model, the problem implies that the variance of the shock to the unobserved variables tends to be biased towards zero, see also Stock (1994). Laubach and Williams (2003) uses the theory discussed in Stock and Watson (1998), which, as noted before, does not provide reliable result in the case of Denmark.

The obvious way to solve the first problem is to apply insight from macroeconomic theory for small-open economies from the previous sections and use it in an expansion of the reduced-form model in Laubach and Williams (2003). One way to solve the two last problems is to use Bayesian techniques. Here prior information of the unknown parameter vector through a prior density can be exploited. Specifically, the prior distributions penalise likelihood-function in regions where the maximisation procedure wants zero-variances of the shocks and/or sum of coefficients above 1. Hence, the priors help to identify the unknown parameters. The empirical model is discussed next, while the econometric methodology is presented in section (4).

3.1. The estimated model

In this section I will present the empirical model following Berger and Kempa (2014). For convenience, the model is summarized in the appendix. The model consists of in total 13 equations in inflation, π_t, output, y_t, the real interest rate, r_t, and the real effective exchange rate, q_t. The real effective exchange rate is defined as the foreign price level, $P_t^\star$ divided by the effective nominal domestic exchange rate times the relative price level of the domestic economy, P_t, $q_t \equiv \frac{P_t^\star}{s_t^{Eff} P_t}$. An upward movement in q_t is consequently equal to an appreciation of the Danish real exchange rate. The advantage of the use of the real effective exchange rate is that there are no need to model the foreign economy, as hopefully will become clear below.

In what follows, I will let variables with superscript ”N” denote natural levels and a bar denotes the gap; the difference between actual levels and natural levels:

$$y_t = y_t^N + x_t^y \tag{16}$$

$$r_t = r_t^N + x_t^r \tag{17}$$

$$q_t = q_t^N + x_t^q \tag{18}$$

Following Laubach and Williams (2003), the NRR and natural level of output are modeled as follows:

$$y_t^N = y_{t-1}^N + g_{t-1} + \varepsilon_{t+1}^{y^N} \tag{19}$$

$$g_t = g_{t-1} + \varepsilon_{t+1}^{g} \tag{20}$$

$$r_t^N = \gamma g_{t-1} + z_{t-1} \tag{21}$$

$$z_t = z_{t-1} + \varepsilon_{t+1}^{z} \tag{22}$$

Movements in the natural level of output and the NRR are linked through the trend growth rate, g_t, which can be interpreted as long-run productivity growth or potential output, as in relation (8) in the theoretical framework. The NRR is also assumed to be affected by transitory shocks through the process z_t, which can be thought of as capturing the other terms in relation (7) besides expectations of the future growth rate: The subjective discount factor on savings, δ, and uncertainty or risk, $\frac{\gamma^2}{2} V_t[\Delta y_{t+1}^N]$. The coefficient γ plays the same role as its equivalent, the preference parameter, which governs intertemporal elasticity of substitution in relation (8).

The output gap is related to the real rate gap, motivated by the theory from section (2.5), and the real exchange rate gap allowing for the channels discussed in section (2.3).

$$x_t^y = \alpha_0^{x^y} x_{t-1}^y + \alpha_1^{x^y} x_{t-2}^y + \alpha_{x^r}^{x^y} x_t^r + \alpha_0^{x^q} x_t^q + \alpha_1^{x^q} x_{t-1}^q + \varepsilon_{t+1}^{x} \tag{23}$$

That is, the real exchange rate gap captures effects on the output gap from exports and the current account in a small-open economy framework: An appreciation of the Danish exchange rate makes Danish goods relatively more expensive in the world economy and hence exports are expected to fall depressing activity, and hence is a short-cut to capture effects from foreign demand on the Danish output gap. The expected signs of $\alpha_{0,1}^{x^q}$ are therefore negative.[8]

Inflation is assumed to be affected by the output gap based upon a Phillips-curve framework as in relation (12) as well as changes in the real exchange rate

$$\pi_t = \bar{\pi} + \beta_\pi^\pi \pi_{t-1} + \kappa_x^\pi x_t^y + \beta_q^\pi \Delta q_{t-1} + \varepsilon_{t+1}^{\pi} \tag{24}$$

The first terms are the (time-dependent) mean value of inflation which is thought of as capturing structural changes in the mean inflation in the Danish economy, see also figure (1). The real exchange rate affects inflation both through its first difference and indirectly through its effect on the output gap in equation (23). The first effect can be thought of as capturing the impact on foreign activity and inflation on the Danish CPI inflation. Hence,

[8] However, due to the Marshall-Lerner effect positive signs are not contrary to theory.

the expected sign of β_q^{π} is negative: An appreciation of the exchange rate, $\Delta q_t > 0$, implies that foreign goods are cheaper and hence, the rate of Danish CPI inflation should fall.[9]

The equilibrium effective real exchange rate is assumed to follow a random walk, while the real exchange rate gap is assumed to follow a AR(p) process

$$q_t^N = q_{t-1}^N + \varepsilon_{t+1}^q \tag{25}$$
$$x_t^q = \delta_0^{x^q} x_{t-1}^q + \delta_1^{x^q} x_{t-2}^q + \varepsilon_{t+1}^{x_q} \tag{26}$$

The theory behind relation (25) is basically PPP; that the natural level of the exchange rate is 1, and that deviations from it will be corrected, relation (26). Finally, the real rate gap is linked to the real exchange rate gap plus a stochastic term

$$x_t^r = c x_{t-1}^q + \kappa_{t-1} \tag{27}$$
$$\kappa_t = \rho_\kappa \kappa_{t-1} + \varepsilon_{t+1}^\kappa \tag{28}$$

Relation (27) can be interpreted as a real interest rate parity condition in gaps plus a stochastic term, κ_t, capturing risk premia. The expected sign of c is positive: If the real effective exchange is above its natural level, or simply in this model, its long-run or equilibrium level, the effective exchange rate is relatively strong and investors will expect a future depreciation. Hence, there will an outflow of capital from the domestic economy unless the real rate increases to compensate investors.

4. Estimation techniques

The estimation technique largely follows the techniques explained in Berger and Kempa (2014). The model summarized above can be cast into a linear Gaussian state-space model of the following form.[10]

$$Y_t = A' X_t + H' \xi_t + w_{t+1} \tag{29}$$
$$\xi_{t+1} = F \xi_t + K v_{t+1} \tag{30}$$

Equation (29) is the observation equation and equation (30) is the state equation. The vectors w_{t+1} and v_{t+1} are vector white noise with $E[v_t v_\tau'] = Q \ \ for \ \ t = \tau$ and 0 otherwise, and

[9]The model in Gali and Monacelli (2005), which can to some extend be regarded as a work-horse model for the small-open economy, provides direct insight in the economic shock which drives movements in the natural rate of output and the NRR in a full structural setup. As shown in Gali and Monacelli (2005), the NRR and natural output should depend on foreign demand. The inclusion of the real effective exchange rate can be thought of a capturing these terms in the model but without the need to actually model the foreign economies.

[10]See as an example Hamilton (1994), ch. 13 for a treatment of state-space models and Kalman filtering. The notation in that book is used in this paper.

$E[w_t w'_\tau] = R \ \ for \ \ t = \tau$ and 0 otherwise. The actual state-space form of the model estimated in this paper is shown in appendix. The task is to estimate the unknown parameters hidden in the matrices in equation (29) and (30) and to find a value for the unobserved state variables most importantly the NRR.

The likelihood-function and the unobserved states can be derived through the Kalman-filter. This filter uses the economic restrictions in the empirical model, the relationships between the real interest gap, the output gap, inflation and the effective exchange rate gap, to filter the unobserved states in the model. The Kalman filter works on the principle that the estimate of the unobserved state is adjusted based on how far away the model's prediction of GDP is from actual GDP given the behaviour of other variables. If the prediction is true, the filter does not adjust the estimate of NRR. If on the other hand, actual GDP is lower than predicted by the model, then the policy rate was likely to be less stimulative than the model had predicted and hence that the real rate gap was more positive than previously thought.

As noted previously, I analyse the model from a Bayesian point of view. That necessitates a choice of priors for the parameters in the model. I follow Berger and Kempa (2014) and assume a Gaussian prior distribution for all parameters except for the variance parameters which are assumed to be gamma distributed. The details can be found in table (1). Where appropriate, the mean and variance of the prior distributions are kept at their values in Berger and Kempa (2014). This implies among other things that the shock process are allowed to follow white-noise, their prior includes 0, that there are not imposed any strong beliefs upon the parameters which governs the effect of the real exchange rate gap on the real rate gap, c, and the real exchange rate gap on the output gap, α_q, and that where estimates for parameters are taken from other results in the literature, as an example Laubach and Williams (2003), the confidence intervals are chosen to be rather wide. Importantly, the variance of the long-run growth, σ^g, is ensured to be non-zero and the real rate gap is identified through priors on the sum of the coefficients in equation (23).

4.1. Parameter estimation

The task it to calculate the posterior mean, $\mathcal{P}^E(\Theta, Y)$, given by

$$\mathcal{P}^E(\Theta, Y) \equiv E[g(\Theta|Y)] = \int g(\Theta)p(\Theta|Y)d\Theta, \tag{31}$$

in which $g(\Theta)$ is a function which expresses the moments of the posterior densities, $p(\Theta|Y)$, in terms of the parameter vector. This function could be the parameter vector, impulse response function etc. The problem is that the density $p(\Theta|Y)$ is not a density with known analytical properties, which makes as an example direct sampling infeasible. Following Berger and Kempa (2014), I therefore use importance sampling, which is a posterior simulation method that tackles exactly this problem. The idea is to find a distribution that is similar to the posterior distribution and from which i.i.d. drawings can be made. It is possible to correct for the difference in the simulation and posterior distributions, in such a way that posterior moments can be approximated well.[11]

For the purpose of this study it suffices to provide an overview and intuition behind the technique. I refer to Geweke (2005) or DeJong and Dave (2007) for the details and the proofs. The idea is to use an importance density, $g(\Theta|Y)$, as a proxy for $p(\Theta|Y)$, where $g(\Theta|Y)$ must be chosen such that it is possible to sample from it directly and is as close as possible to the actual function, $p(\Theta|Y)$. The idea is to weight the draws from $g(\Theta|Y)$ where the weights reflect how important the sampled values are relative to other sampled values. Geweke (2005), cp. 4, shows consistency of the estimates for the number of draws going to infinity. In the current study I use a approximation for $p(\Theta|Y)$ by using a normal distribution with mean and variance-covariance matrix from the estimation of the mode.

In the simulation two changes to this normal distribution are made following Berger and Kempa (2014). Firstly, the variance-covariance matrix is scaled by 1.1 such the probability that the actual posterior density has thicker tails than importance density is reduced. Secondly, the importance density is sequentially updated such that starting with the estimated mode, the moments of the normal distribution are updated using estimated posterior moments for the actual density, which are then used in the simulation. I use 100.000 draws.

An estimate of the posterior mean of the parameter vector, Θ, is obtained by setting $g(\Theta|Y)$ equal to Θ, and taking $\widehat{\Theta}$ in the importance sampling. Estimates of the smoothed states, $\widehat{\xi_t}$, are calculated in a similar way using $\widehat{\Theta}$ in the Kalman smoother.

To calculate percentiles of the posterior densities, I follow Berger and Kempa (2014). I set $g(\Theta|Y)$ equal to an indicator function which is 1 if the j^{th} element of Θ is smaller than a certain value for that element and zero otherwise. Specifically, an estimate of the 10^{th} percentile of the posterior density is chosen such that the j^{th} element of Θ is smaller than 0.1. In the calculation of the uncertainty surrounding the smoothed states, $\widehat{\xi_t}$, both filter uncertainty and uncertainty about the estimated parameters are taken into account. An estimate of the 10^{th} percentile of the smoothed states is obtained by setting $g(\Theta|Y)$ equal to

[11] The Matlab toolbox Dynare used to estimated DSGE-models applies a more general approach than importance sampling namely MCMC and more specific, the Metropolis-Hastings algorithm. The importance sampling and the MCMC can be regarded as being similar as both classes of simulation methods aim to solve the same problem, namely to characterise an unknown distribution.

the j^{th} element in $\widehat{\xi_t}$ for the i^{th} draw subtracted by 1.645 times the respective elements of the smoothed state variance matrix obtained using $\widehat{\Theta}$.

5. Data

The data is shown in figure (1). I use seasonally adjusted quarterly data for Denmark from 1972 Q1 to 2014 Q4 for inflation (annualised first difference of the log CPI excluding food and energy), GDP (logs multiplied by 100) taken from quarterly national accounts and MONAs database, the effective real exchange rate calculated by Danmarks Nationalbank, and the ex post real rate. For the period 1997 Q1 to 2014 Q4, the nominal interest rate is calculated as the average over the quarter of the tomorrow-next day-to-day interest rate, while from 1972 Q1 to 1996 Q4 the Danmarks Nationalbank policy rate is used. Inflation expectations are calculated from the expectation of average inflation over the four quarters ahead from a univariate AR(3) of inflation estimated over the 36 quarters prior to the date at which expectations are being formed. The use of a rolling window for the estimation of the AR(3) process takes care of unstable parameters.

6. Results: Parameters and smoothed states

In the sections to come, I will discuss parameter estimates and smoothed states. I will in section (6.1) comment on the posterior distribution of the parameters and in section (6.2) interpret the smoothed states, that is the model's estimate of the NRR and the output gap. I will compare the estimated smoothed states with other measure in section (6.3) and look at estimates for other economies in section (6.4).

6.1. Posterior distribution of the parameters

The estimates of the parameters are shown in table (1), while the smoothed states are shown in figure (2). The estimated prior-posterior distributions are shown in figure (3). Starting with the prior and posterior distributions in figure (3), for the standard deviations all posterior distributions turn out to be narrower than the prior, which is an indication of substantial information in data. The posterior distribution of the standard deviation of the shock to the growth rate, g_t is small. This is the pile-up problem. However, the use of the prior ensures that the permanent shocks stemming from g_t are not estimated to be zero.

Turning to the IS-relation, the sum of the coefficients are smaller than one and shows a relatively small degree of persistence. The posterior mean of the coefficient on the real rate gap is around -0.35 and quite large relative to e.g. the estimates found in Laubach and Williams (2003) of around -0.1. Only the contemporaneous effect from movements in the the real exchange rate in the IS-relation is significant. Results for the real exchange rate equation shows a high degree of persistence and hence low degree of mean reversion. This is not surprising and confirms other results in the literature, see e.g. Rogoff (1996). The posterior mean estimate of the coefficient relating the trend growth rate to the NRR, γ, equals 3.72.

An interesting finding is that the output gap plays little role for inflation reflected in a low estimate of κ_x^π of 0.02 in the Phillips-curve, equation (24). The insensitivity of inflation to "slack" is a common finding. It is bad news for the Phillips-curve, but good news for question about secular stagnation as it prevents the large degree of slack to spill over to disinflation and consequently to increasing real rates that together with the lower bound in the monetary policy rate would have put further downward pressure on demand. There are many explanations for this in the literature, and this is not the place to discuss them all. Among the many explanations one is greater anchoring of inflation expectations due to greater central bank credibility and another is globalisation which have reduced the sensitivity of inflation to domestic conditions. The empirical model does support the latter explanations, as the coefficient on the change in the real effective exchange rate in the Phillips-curve, β_q^π, is larger than the coefficient on the output gap, κ_x^π, by a factor 10.

6.2. Posterior distribution of the smoothed states

Starting with the NRR shown in figure (2), it is estimated to follow the downward trend in the actual real rate starting from the the beginning of the 90s though starting from a lower starting point. As expected its movements are more smooth than the movements in the actual real rate. From around the onset of the financial crisis in 2007-08, the NRR turns sharply negative from a value of around 2 pct. per year to -1$\frac{1}{2}$ pct. per year and hits a value of around -2 around 2012. As expected, the 90 pct. confidence bounds are wide. This is a common finding in the literature. The bounds reflect both large filter uncertainty and parameter uncertainty, as noted above. According to the model, the NRR was only significantly positive from 1990 until 2002 and significantly negative only in 2013. The real rate gap, the difference between the actual real rate and the NRR was relatively high during the period 1987-1993, as can be seen from figure (2). During the build up to the financial crisis is was again positive reaching almost 2 pct., but it did not drop by a lot after the onset of the crisis. Hence, monetary policy imported from the ECB according to these results contracted Danish output during the boom, but was not very stimulative during the bust.

Moving on to the output gap, according to the model, there was a (significantly) positive output gap during the beginning of the 80s followed by a deep (significantly) recession. This squares quite well with the economic history of Denmark for that period. The subsequent period squares equally well: A long-lasting (insignificantly) expansion ending with a mild recession at the beginning of the 00's were followed by a (insignificant) boom. That boom ended with the bankruptcy of Lehmann Brothers in the autumn of 2008. As for the NRR, the confidence intervals around the mean estimate are quite wide.

According to the model, there is a downward trend in the growth rate of Danish GDP. It fell from a high of around 3 pct. in the beginning of the 90s to around 2 pct. per year in the mid-00s. During the onset of the financial crisis, the growth rate fell to around 0. Though I can not talk about causality in this non-structural setup, these developments feed into the estimated series for the NRR explained above. As an example, the drop in the NRR coincided with a large drop in natural output around the same period. Hence, according to the model, part of the downward movement in the NRR and the negative estimate for the NRR coincided with similar movements in natural output.

The filtered real effective exchange rate gap and the natural real effective exchange rate together with actual data are shown in the last row in figure (2). According to the model the effective real exchange rate was overvalued in the beginning of the 70s and again from 2008 to 2011 though only marginally significant. The natural real effective exchange rate shows quite modest fluctuations relative to the actual rate. The recent over valuation might be due to large depreciations of the dollar, the pound and the swedish kronar after the crisis, as these economies were able to lower their interest rate aggressively to combat the crisis.

6.3. Comparison with other measures of the NRR and the output gap

As a robustness check, I will in this section compare the smoothed states of the NRR and the output gap with other estimates. I will compare the output gap with Danmarks Nationalbank's estimates for the output gap using a similar model and technique, see Andersen and Rasmussen (2011), and with standard HP filters as well as a Band-Pass filter as alternatives to identifying both the output gap and the NRR. This is shown in figures (4) and (5). I use a standard smoothing parameter in the HP-filter of 1600 and I use parameters 4 and 32 in the Band-Pass filter.

As expected, the HP-filtered data follows the actual data well. For the output gap the model generates somewhat larger swings that the two filtered series. The output gap as measured by the methods in Andersen and Rasmussen (2011) is a lot bigger during the build-up to the financial crisis, but the two time-series are highly correlated and peak at the same data points within the sample.[12] In fact, the only big divergence between these two measures are during the period 2004-08, where the difference between the two measures are of order 2 percentage points. The method used in Andersen and Rasmussen (2011) resembles in some aspects the model used in this paper except for the Bayesian techniques used in the estimation and especially the use of data for the labour market; in the estimation, data for the labour market and not interest rates are used to filter the output gap. During the period 2004-08, unemployment in Denmark fell to historical low levels of around 1.4 per cent of the labour force. But the real rate gap was positive, see figure (2), as the ECB attempted to reign in the boom before the financial crisis. Danmarks Nationalbank followed the ECB in the defense of the fixed exchange rate. Based on this, the conjecture is that the 2 percentage points difference can be explained by the use of different data sets that showed different behaviour exactly during this period.[13]

Figure (5) shows the estimated NRR together with the actual (ex post) real rate and the two filtered series. As can be seen from the figure, the NRR from the model displays some notable differences with respect to the filtered series. The estimated NRR is clearly higher than the filtered series throughout the 1990s up until 2008. Hereafter, the series to a larger degree shows similar behaviour. However, neither do the filtered series follow the estimated NRR up during the 1990s nor do they fall during the end of the sample. This can be explained by the movements in the natural rate of output with respect to the trend, see section (6.2).

6.4. International developments and the estimated NRR for Denmark

In the figures in figur (6), the estimated series for the NRR for Denmark is compared to estimates for the U.S, estimated in Laubach and Williams (2003) and in Bouis et al. (2013), Canada estimated in Berger and Kempa (2014) and in Bouis et al. (2013), U.K., the euro-area and Japan all estimated in Bouis et al. (2013).

Perhaps with the exception of Japan, all estimates point to a downward trend in the NRR starting already from the 1970's for the U.S., and then a marked drop after the bankruptcy of Lehmann brothers in the third quarter of 2008. Secondly, the NRR went negative only with the onset of the financial crisis again with the exception of Japan. In the lower left figure in figure (6) a measure of the global long-term real rate is shown. As can

[12] The output gap in Andersen and Rasmussen (2011) is only calculated from the beginning of the 1980s.

[13] Including HP-filtered unemployment in the IS-relation did provide somewhat larger output gaps during the period 2006-08, but was left out of the model in this paper.

be seen, there is evidence of a decline in global real interest rates since the 80's. Further, the estimates for the NRR are model-dependent, which can be seen as an example for the estimates for the NRR for the U.S., middle figure to the left. For a Danish perspective it is reassuring that the Danish NRR follows the euro area NRR, and that the real rate gaps show similar behaviour. This implies that on average the Danish economy and the euro area share the same underlying characteristics and hence on average, the Danish economy imports optimal monetary policy from the ECB.

It was not the case that the Danish NRR followed global and euro area NRRs prior to the late 90s. The behaviour of the Danish NRR is markedly different from the other countries' NRR from the 1970's until at least 1995, which can be explained by the first oil-shock and especially the deep crisis which hit the Danish economy after the second oil shock at the beginning of the 1980's. This period was characterised by very high nominal interest rates, to a large extend explained by increases in the risk premia due to repeated devaluations, and high inflation hitting 16 percent per year. The fixed exchange rate regime vis-a-vis the D-mark came in place in 1982, which led to falling real rates and a boom in the Danish economy followed by a bust after contractionary fiscal policy aimed at constraining the boom slowed the economy down. As can be seen, after the mid-1990s convergence with the euro area was achieved and the Danish NRR has to a greater extend followed the international developments in the NRR.

Hence, both the downward movement in the NRR and the sharp fall into negative territory during the financial crisis is a common finding in many advanced economies and not specific for the Danish economy. This is not surprising. The theory presented in section (2.3) pointed to an effect on the domestic NRR from the global NRR through arbitrage on (increasingly) liberalised global capital markets. Hence, even though domestic factors can and do play a role, the figures in figure (6) show that developments on global capital markets play an important role and perhaps more and more so for the determination of the domestic NRR.

7. Policy and Secular Stagnation

In sections (6.1) to (6.4) I identified a downward movement in the NRR for Denmark and I found evidence for a negative NRR around the outbreak of the financial crisis. These findings can also be found in data for other developed economies. According to these results, there are some evidence that the Danish economy faces a risk of ending up in secular stagnation. In the following two sections, I will discuss appropriate policy in the light of the findings. To do so, I find it beneficial first to provide a further discussion outside the empirical model of the causes behind these observations first taking a global and then a domestic perspective.

7.1. Evidence of downward movement in the NRR and secular stagnation

There are many proposed explanations for the downward movement in the NRR, see e.g. the collection of papers in VoxEU.org (2014). The proposed explanations are both global and local. The IMF points to common global factors affecting to greater and greater extend, admittedly, actual real rates. Specifically, there is evidence of a common factor which accounts for a greater part of the movements in real rates and a greater role for global savings and investments, see IMF (2014).

Besides the global downward pressure on real interest rates, there are at least three other channels through which domestic economic developments can affect the domestic NRR, as explained in section (2), relation (7): Expected future growth rate of output, savings and uncertainty/risk. I will in what follows present some of the many proposed explanations.

Starting with the supply side of the economy, the literature has identified the following possible explanations behind the observed behaviour of the NRR. These points can be seen as shocks that is affecting the economy on frequencies longer than business-cycle frequency:

1. Productivity slow-down, see Gordon (2012).

2. Hysteresis on the labour market, see as an example the article by Edward L Glaeser in VoxEU.org (2014).

3. Quality of labour input, see as an example the article by Barry Eichengreen in VoxEU.org (2014); the innovations are there, contradicting item (1), but the quality of the labour force grows slower and hence the economy is not able to implement new technology as fast.

4. Demographics; the population is stagnant and life expectancy is increasing rapidly leading to lower potential growth and higher savings.

5. Bernankes international savings-glut, see also IMF (2014). This point can be seen together with item (4): Demographic pressures on world savings have depressed real rates, see also Barclays (2015) or GoldmanSachs (2015).

6. Low demand for investments from advanced economies, IMF (2014)

7. Change in portfolio away from equities towards bonds, see IMF (2014), and falling risk premia.

All these factors put downward pressure on the NRR. The first six items can be seen as affecting the NRR through their impact on the natural growth rate in the economy, while the last affects the NRR through risk.[14]

On the demand side, the following points have been raised, where these points can be seen as shocks that is affecting the economy on business-cycle frequency:

1. Balance-sheet recession and excess savings, see OECD (2014) and Koo (2014).

2. Fiscal austerity leading to extra savings from governments and less demand; a long periode with low public demand and/or higher taxes, which in both cases hurt demand.

3. Increasing inequality both between countries and within countries, see as an example Gordon (2012) or Edward L Glaeser in VoxEU.org (2014).

4. Low demand for investments from advanced economies, IMF (2014).[15]

Again, all items point in the same directions for what concerns the NRR: Down. Specifically, item (1) and (2) point to higher savings than usual. Item (3) works through aggregate demand and the marginal propensity to consume. That is, inequality within countries affect demand through moving purchasing power from some with a high propensity to consume to people with a relatively low propensity to consume. And fast growing economies especially in East-Asia, with relatively less developed financial markets save on the international financial markets. Also booming energy prices move purchasing power from countries with a relatively high propensity to consume to energy exporting economies which have a relatively low propensity to consume.

It is hard to evaluate and quantify each proposed explanation. The studies in IMF (2014) explains the global decline in actual real rates by higher savings from emerging markets and lower investments from advanced economies; even though investments are high in these countries the relative decline in the relative price of investments price means that the value of investments as a share of GDP has fallen. Lastly, the IMF identifies a portfolio shift from equity to bonds, which partly reflects higher demand for safe and liquid

[14] That is, items (1)-(6) can be seen as affecting the term $\gamma E_t[\Delta y^N_{t+1}]$ in relation (7), while (4)-(5) affects δ as well. Item (7) affects risk premia/uncertainty; $\frac{\gamma^2}{2}V_t[\Delta y^N_{t+1}]$.

[15] I have included the last items both under demand and supply, as investments are demand today and supply tomorrow

assets from emerging market economies to increase official foreign reserve accumulation and partly reflects an increase in the perceived riskiness of equity. This higher demand for safe assets puts downward pressure on real rates, see also the study in Caballero and Farhi (2014).

The downward movement into negative from the onset of the financial crisis points to lack of aggregate demand. Historically, financial crises are followed by deep slumps, see Rogoff and Reinhart (2008), which compares the subsequent macroeconomic developments after a financial crisis. Especially, financial crises are often followed by low level of investments which are not compensated by lower savings rates, see also IMF (2014). The restoration of balance sheets, both private and public, follows naturally after the fall of elevated asset prices.

There is however a caveat: The policy rate is prevented by the effective lower bound to stimulate the economy, the output gap is negative creating deflationary pressures, a negative NRR can be an absorbing state. That is, by the very nature of secular stagnation, a force is prevented from creating equilibrium. The result can thus be a truly secular - long lasting - recession. However, and as pointed out above, though all the items above pointed only in one direction with respect to the NRR, these forces can also be expected to be temporary: At some point in time, East-asians will start spending and not saving their extra income, investments depreciate and investors will one day be tired of not getting high returns on their bond portfolios; the studies in Barclays (2015) and GoldmanSachs (2015) both concludes that the global savings glut is over due to changes in demographics. The danger of a true long-lasting recession, however, motivates the implementation of policies that mitigates this risk, which I turn to next.

8. Policy within a Danish context

I will lastly discuss policy within a Danish context in a situation in which an economy faces the risk off secular stagnation. I will address three points: The natural rate of output, the labour market, and fiscal policy. By the very nature of secular stagnation, monetary policy is ineffective as the effective lower bound prevents nominal policy rates to be lowered sufficiently to stimulate aggregate demand.

For the Danish economy, there are some evidence that lower natural growth rates, see figure (2), which together with the international movements in the NRR, has pushed the Danish NRR down. If this is the case, then the obvious policy recommendation would be to raise the natural growth rate in the economy through structural reforms. Here there is a double dividend from higher retirement age, as this both increases the supply of labour and leads to lower savings, which both are expected to push the NRR up.[16] There are some evidence of a downward trend in productivity growth or a slowdown, see among many studies Produktivitetskommissionen (2012). Not only because productivity is the source of higher welfare but in this context also, higher productivity growth is important.

One major concern for the U.S. is hysteresis on the labour market with a decline in the employment-to-labour participation rate, see as an example VoxEU.org (2014): The participation rate does increase after a recession, but it does not reach previous peak before the onset of the next resulting in a downward trend, which lowers natural output. The Danish labour market is widely understood to be flexible and active labour market policies help to allocate labour from declining sectors to expanding sectors. Not only for growth and for the workers hit by recession, it is paramount to keep the labour market flexible for the risk of secular stagnation.

Finally, as monetary policy is ineffective in a period with secular stagnation, discretionary fiscal policy is the only traditional stabilising instrument left. This points to the importance of conducting symmetrical fiscal policy which reduces public debt during booms such that there are room to the combat downturns. This is especially so within the EU Stability and growth pact, which puts limits on the public deficit. Also, high level of public debt often coincide with low growth, as seen during this crisis, see also Rogoff and Reinhart (2008). Consequently, fiscal policy must aim to reduce the debt-to-GDP ratio when it is possible, because in recessions history has shown that it is difficult to stabilise it and almost impossible to reduce it.

9. Conclusion

This paper started out with a theoretical discussion of the NRR and why it is an important bench mark for monetary policy. It showed that if the NRR is sufficiently low, then monetary policy might be ineffective in stimulating aggregate demand, which in a situation with a negative output gap can lead to a long period of low growth; secular stagnation. Using this as motivation, the NRR was estimated on Danish data the main finding being that there are evidence of very low levels of the NRR and stronger evidence for a negative trend. These observations are also present for other advanced economies pointing to the importance of global factors as main determinants of domestic NRRs.

The discussion of the causes of these events pointed to cyclical factors as main cause of the negative NRR after the financial crisis. Cyclical factors are by nature temporary and

[16] While the retirement age in some developed countries are fairly low measured by the increase in longevity, the Danish economy has already increased it and it is questionable whether it can be increased by a lot more.

hence it can be expected that the NRR will move towards positive territory in the near future. Though secular stagnation is about negative NRR as an absorbing state for the economy, some of the explanations behind the cyclical downturn can not be expected to be long-lived and at some point must be expected to contribute positively to the NRR and demand within the coming future; fiscal austerity, debt overhang, rebuilding of balance sheets and excess uncertainty.

However, the observed low levels of the NRR might remain in the near future, say 5-10 years. Hence, in the future there might be an elevated probability of ending up in the lower bound again. This is so, as during a period with low NRR cyclical shocks all else equal needs to be smaller to do so. This points to policies that on a domestic level help to lift natural output - structural reforms - flexible labour markets that can mitigate hysteresis, and symmetrical fiscal policy which reduces public debt during booms such that there are room to combat to downturns. Finally, low levels of interest rates create search for yields, which can lead to asset price inflation and excess risk taking, see as an example Nationalbank (2014) section (4). These risks to financial stability must also be addressed in future policies.

10. Tables and figures

	Prior distribution			Posterior distribution		
	Type	Mean	s.d.	Mean	10 pct.	90 pct.
Phillipscurve, eq. (24): $\pi_t = \bar{\pi} + \beta_\pi^\pi \pi_{t-1} + \kappa_x^\pi x_t^y + \beta_q^\pi \Delta q_{t-1} + \varepsilon_{t+1}^\pi$						
$\overline{\pi_1}$	Normal	10	0.1	9.71	9.62	9.8
$\overline{\pi_2}$	Normal	5	5	4.12	3.97	4.28
$\overline{\pi_3}$	Normal	1.75	5	1.76	1.66	1.86
β_π^π	Normal	0.5	0.05	0.12	0.09	0.15
β_q^π	Normal	-0.25	0.01	-0.25	-0.27	-0.23
κ_x^π	Normal	0.5	0.25	0.02	0.01	0.04
$\sigma_{\varepsilon^\pi}^2$	Gamma	3	5	4.97	4.81	5.13
NRR, eq. (21), (22), (27), (28): $r_t^N = \gamma g_{t-1} + z_{t-1}$; $z_t = z_{t-1} + \varepsilon_{t+1}^z$; $x_t^r = c x_{t-1}^q + \kappa_{t-1}$; $\kappa_t = \rho_\kappa \kappa_{t-1} + \varepsilon_{t+1}^\kappa$						
γ	Normal	4	1	3.72	3.42	4.02
c	Normal	0	0.5	0.03	0.01	0.05
ρ_κ	Normal	0.5	0.25	0.92	0.91	0.94
$\sigma_{\varepsilon^z}^2$	Gamma	0.5	5	0.16	0.14	0.19
$\sigma_{\varepsilon^\kappa}^2$	Gamma	0.5	5	0.27	0.24	0.29
Exchange rate, eq. (25), (26): $q_t^N = q_{t-1}^N + \varepsilon_{t+1}^q$; $x_t^q = \delta_0^{x^q} x_{t-1}^q + \delta_1^{x^q} x_{t-2}^q + \varepsilon_{t+1}^{x_q}$						
$\delta_0^{x^q}$	Normal	1.5	0.25	1.48	1.44	1.53
$\delta_1^{x^q}$	Normal	-0.7	0.25	-0.52	-0.57	-0.48
$\sigma_{\varepsilon^{q^N}}^2$	Gamma	3	5	1.25	1.12	1.39
$\sigma_{\varepsilon^{x^q}}^2$	Gamma	3	5	1.01	0.87	1.14
IS-relation/Output, eq. (19), (20), (23): $y_t^N = y_{t-1}^N + g_{t-1} + \varepsilon_{t+1}^{y^N}$; $g_t = g_{t-1} + \varepsilon_{t+1}^g$; $x_t^y = \alpha_0^{x^y} x_{t-1}^y + \alpha_1^{x^y} x_{t-2}^y + \alpha_{x^r}^{x^y} x_t^r + \alpha_0^{x^q} x_t^q + \alpha_1^{x^q} x_{t-1}^q + \varepsilon_{t+1}^x$						
$\alpha_0^{x^Y}$	Normal	1.5	0.25	0.69	0.66	0.72
$\alpha_1^{x^Y}$	Normal	-0.7	0.25	0.13	0.1	0.15
$\alpha_{x^r,0}^{x^Y}$	Normal	-0.1	0.1	-0.35	-0.4	-0.31
$\alpha_{x^q,0}^{x^Y}$	Normal	0	0.15	-0.04	-0.07	-0.01
$\alpha_{x^q,1}^{x^Y}$	Normal	0	0.15	-0.02	-0.05	0.02
$\sigma_{\varepsilon^{y^N}}^2$	Gamma	0.25	5	0.21	0.16	0.26
$\sigma_{\varepsilon^g}^2$	Gamma	0.1	5	0.0028	0.0023	0.0034
$\sigma_{\varepsilon^{x^y}}^2$	Gamma	0.5	5	1.02	0.96	1.08

Table 1: The table reports prior and posterior distributions of the structural parameters of the model.

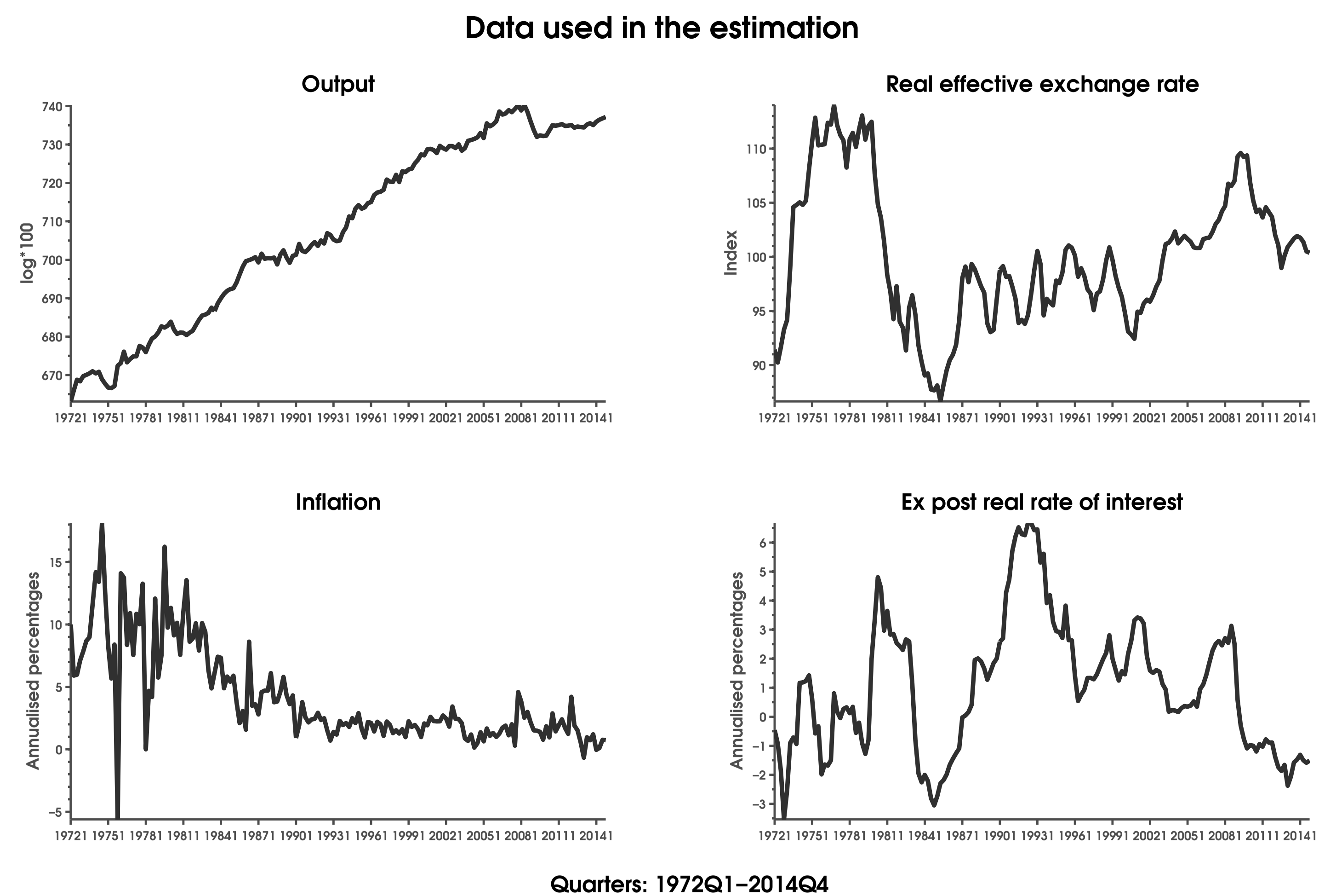

Figure 1: This figure shows the data used in the estimation of the model. The data sources are the MONA-database for output and inflation, while inflation figures used in the estimation of inflation expectations are taken from FRED before 1971. The real effective exchange rate is calculated by staff members of Danmarks Nationalbank. The model is estimated using data from 1972 Q1 to 2014 Q4.

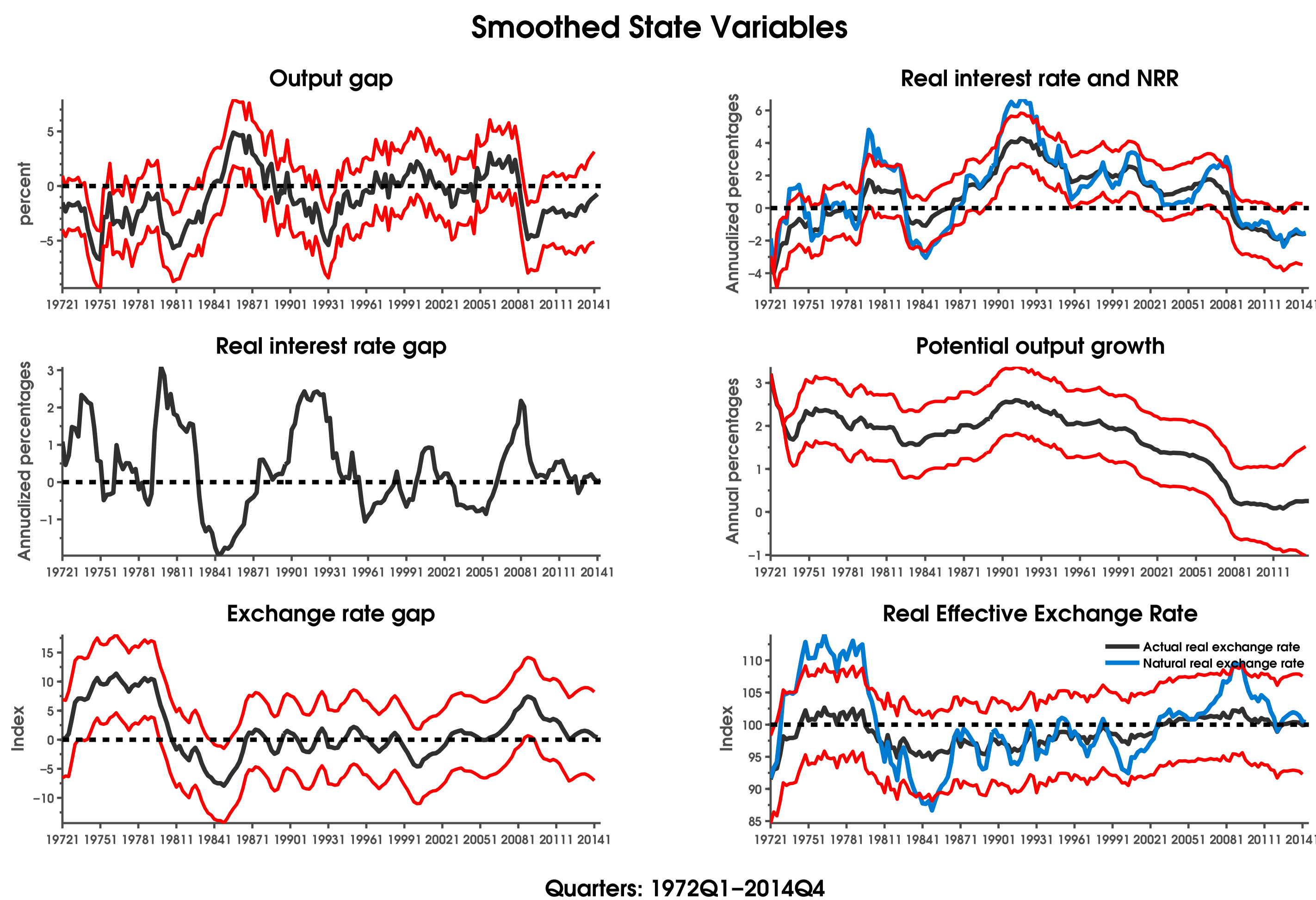

Figure 2: These figures show the trend and cyclical components for a selection of variables in the estimated model together with 90 pct. confidence bounds except for the real rate gap.

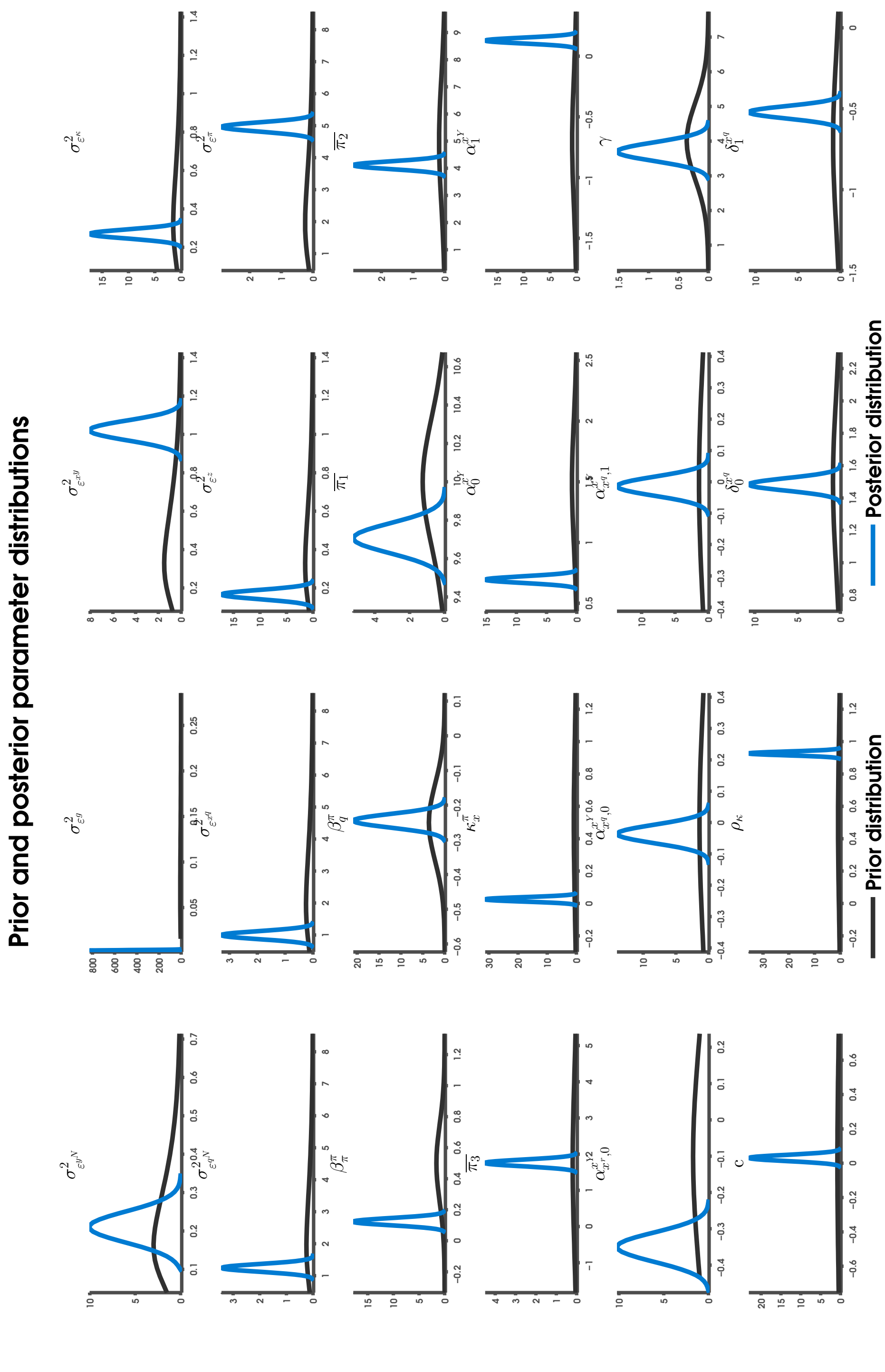

Figure 3: These figures show the prior and posterior distributions of the estimated parameters in the model

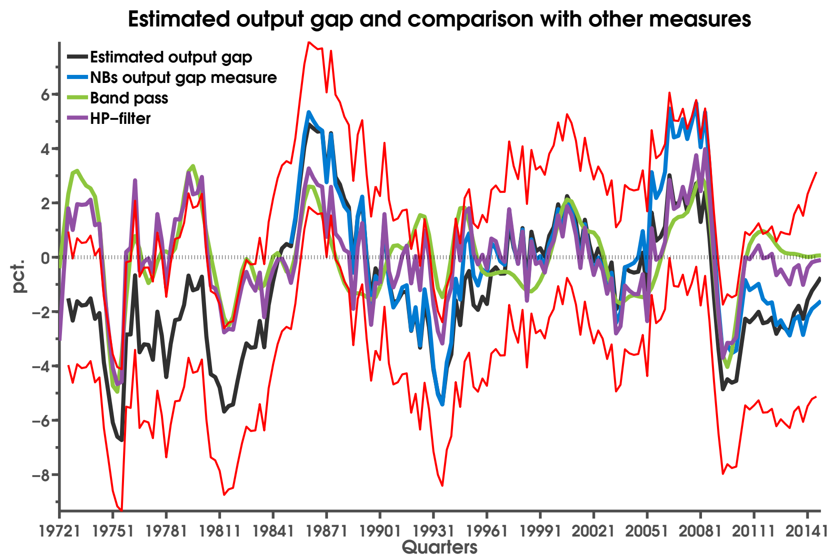

Figure 4: This figure shows the estimated output gap and 90 percentage confidence bounds together with different measures of the output gap: The output gap presented in Andersen and Rasmussen (2011) calculated using an unobserved component model using similar techniques as in this model, but applying data for the labour market, a Band-pass filter and a HP-filter.

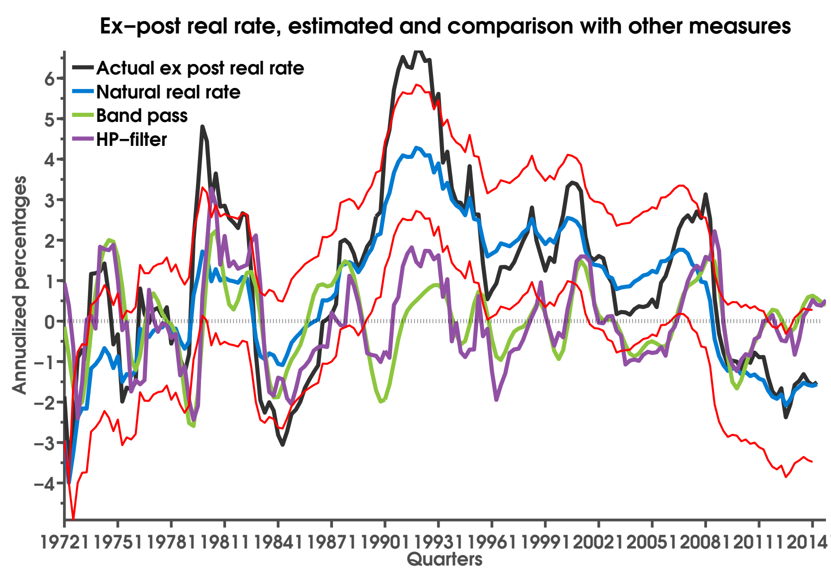

Figure 5: This figure shows the estimated NRR for Denmark and 90 percentage confidence bounds together with different measures of the NRR: The actual ex post real rate, a Band-pass filter and an HP-filter.

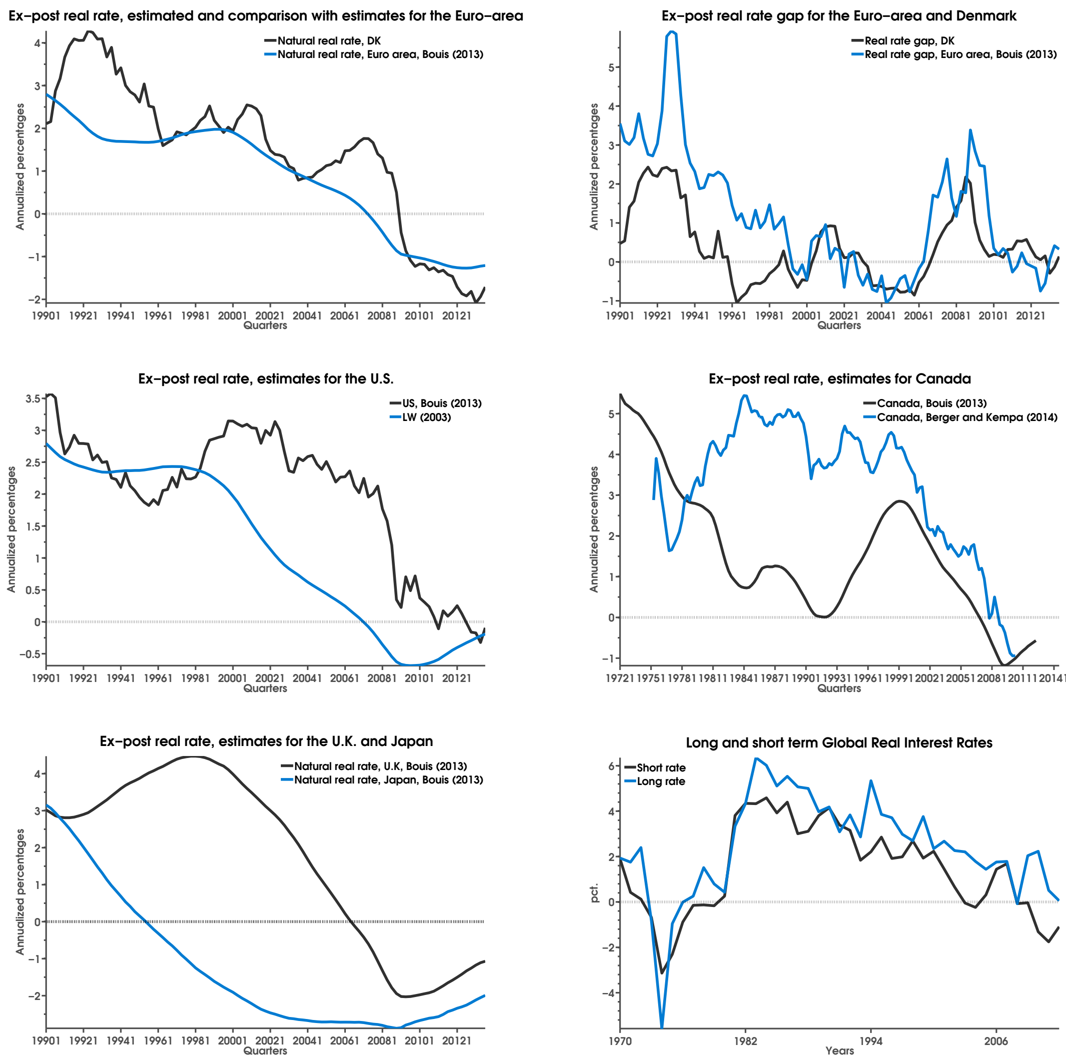

Figure 6: These figures compare the estimated NRR for Denmark with estimates for Japan, UK, the euro-area from Bouis et al. (2013), the US from Bouis et al. (2013) and Laubach and Williams (2003), and Canada from Bouis et al. (2013) and Berger and Kempa (2014). In the figure to the lower-right is shown IMF measure of the global actual short-term and long-term real rate, see IMF (2014).

A. The empirical model and the state-space representation

For convenience, the empirical model is summarized in the following equations:

$$
\begin{aligned}
y_t &= y_t^N + x_t^y \\
r_t &= r_t^N + x_t^r \\
q_t &= q_t^N + x_t^q \\
y_t^N &= y_{t-1}^N + g_{t-1} + \varepsilon_{t+1}^{y^N} \\
g_t &= g_{t-1} + \varepsilon_{t+1}^{g} \\
r_t^N &= \gamma g_{t-1} + z_{t-1} \\
z_t &= z_{t-1} + \varepsilon_{t+1}^{z} \\
x_t^y &= \alpha_0^{x^y} x_{t-1}^y + \alpha_1^{x^y} x_{t-2}^y + \alpha_{x^r}^{x^y} x_t^r + \alpha_0^{x^q} x_t^q + \alpha_1^{x^q} x_{t-1}^q + \varepsilon_{t+1}^{x} \\
\pi_t &= \bar{\pi} + \beta_\pi^\pi \pi_{t-1} + \kappa_x^\pi x_t^y + \beta_q^\pi \Delta q_{t-1} + \varepsilon_{t+1}^{\pi} \\
q_t^N &= q_{t-1}^N + \varepsilon_{t+1}^{q} \\
x_t^q &= \delta_0^{x^q} x_{t-1}^q + \delta_1^{x^q} x_{t-2}^q + \varepsilon_{t+1}^{x_q} \\
x_t^r &= c x_{t-1}^q + \kappa_{t-1} \\
\kappa_t &= \rho_\kappa \kappa_{t-1} + \varepsilon_{t+1}^{\kappa}
\end{aligned}
$$

The state-space representation is of the following form:

$$\begin{aligned} Y_t &= A' x_t + H' \xi_t + w_{t+1} \\ \xi_{t+1} &= F \xi_t + K v_t, \end{aligned}$$

with matrices given as follows:

$$Y_t = \begin{bmatrix} y_t & r_t & q_t & \pi_t \end{bmatrix}' \qquad X_t = \begin{bmatrix} DU_1 & DU_2 & DU_3 & \pi_{t-1} & \Delta q_t \end{bmatrix}'$$

$$\xi_t = \begin{bmatrix} y_t^N & g_t & x_t^y & x_{t-1}^y & r_t^N & x_t^r & \kappa_t & q_t^N & x_t^q & x_{t-1}^q & z_t \end{bmatrix}'$$

$$H = \begin{bmatrix} 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & \kappa_{x^y}^{\pi} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \qquad diag(Q) = \left[\sigma^2_{\varepsilon^{y^N}}, \sigma^2_{\varepsilon^{g}}, \sigma^2_{\varepsilon^{x^y}}, \sigma^2_{\varepsilon^{\kappa}}, \sigma^2_{\varepsilon^{q^N}}, \sigma^2_{\varepsilon^{x^q}}, \sigma^2_{\varepsilon^{z}}\right]'$$

$$A = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ \overline{\pi_1} & \overline{\pi_2} & \overline{\pi_3} & \beta_{\pi}^{\pi} & \beta_{q}^{\pi} \end{bmatrix}' \quad R = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \sigma^2_{\varepsilon^{\pi}} \end{bmatrix} \qquad K = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$F = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \alpha_0^{x^Y} & \alpha_1^{x^Y} & 0 & \alpha_{x^r,0}^{x^Y} & 0 & 0 & \alpha_{x^q,0}^{x^Y} & \alpha_{x^q,1}^{x^Y} & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \gamma & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & c & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \rho_{\kappa} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \delta_0^{x^q} & \delta_1^{x^q} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

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