DANMARKS
NATIONALBANK

WORKING PAPERS

October 2016 | No. 108

AN ESTIMATED DSGE-MODEL FOR DENMARK WITH HOUSING, BANKING, AND FINANCIAL FRICTIONS

Jesper Pedersen
Danmarks Nationalbank
Jpe@nationalbanken.dk

DANMARKS
NATIONALBANK

ISSN (online) 1602-1193

DANMARKS
NATIONALBANK
WORKING PAPERS

AN ESTIMATED DSGE MODEL FOR DENMARK WITH HOUSING, BANKING, AND FINANCIAL FRICTIONS

Contact for this working paper:

Jesper Pedersen
Danmarks Nationalbank
Jpe@nationalbanken.dk

ABSTRACT

An Estimated DSGE model for Denmark with Housing, Banking, and Financial Friktions
The financial crisis has moved attention to the modeling of financial frictions and banks in DSGE models. The preceding housing boom put focus on the need to incorporate developments in the residential sector, including house prices. This paper documents an estimated DSGE model of the Danish economy with financial frictions, banking and a construction sector.

RESUME

En Estimeret DSGE model for Danmark med boliger, finansiel sector og finansielle friktioner
Den finansielle krise satte fokus på nødvendigheden af at modellere finansielle friktioner og en finansiel sektor i DSGE modeller. De forudgående stigninger i boligpriserne satte fokus på nødvendigheden af at kunne analysere boligmarkedet og mulige samspil mellem denne og den øvrige realøkonomi. Dette arbejdspapir dokumenterer en estimeret DSGE model for dansk økonomi med finansielle friktioner, finansiel sektor og byggesektor.

JEL CLASSIFICATION

E17, E32, E62, E65, F41

KEYWORDS

DSGE Models, Estimation, Financial frictions, Banking.

An Estimated DSGE model for Denmark with Housing, Banking, and Financial Frictions*

Jesper Pedersen†
Danmarks Nationalbank

October 2016

Abstract

The financial crisis has moved attention to the modeling of financial frictions and banks in DSGE models. The preceding housing boom put focus on the need to incorporate developments in the residential sector, including house prices. This paper documents an estimated DSGE model of the Danish economy with financial frictions, banking and a construction sector.

JEL classification: E17, E32, E62, E65, F41.

Keywords: DSGE Models, Estimation, Financial frictions, Banking.

*The views expressed in this paper are those of the author, and do not necessarily correspond to those of Danmarks Nationalbank. The author especially thanks Søren Hove Ravn for his contribution to the developing of the model at its initial stage when he was employed at Danmarks Nationalbank, and for comments and suggestions to the final draft. The author also thanks colleagues at Danmarks Nationalbank and especially Mark Strøm Kristoffersen for useful comments . The author lastly thanks colleagues at the Banco de España for their hospitality and help during the authors stay in 2016, in particular Eva Ortega, Samuel Hurtado, and Carlos Thomas.

†Address: Danmarks Nationalbank, Havnegade 5, 1093 Copenhagen, Denmark. E-mail: jpe@nationalbanken.dk.

1. Introduction

The Danish economy experienced a boom in real house prices starting from 1993 peaking in 2007, like so many other developed economies, see figure (1). In this period, the real house price increased by around 250 percent. While it might be discussed how large the discrepancy between fundamentals and the real house price was during this period, the subsequent fall of around 30 percent points to some degree of overvaluation. The large increase in house prices coincided with an increase in residential investments as a percentage of GDP from 4 percent to almost 7 percent just before the outbreak of the financial crisis, and a rise in private consumption and real GDP. The fact that residential investments to GDP ratio fell back to around 4 pct. lends additional support to the hypothesis of an overvaluation of house prices. The subsequent fall coincided with a deep recession and a large fall in private consumption and investment. Behind this cycle the financial sector played a large role pushing up the credit-to-GDP ratio during the boom.

The DSGE-model estimated in Pedersen and Ravn (2013) neither had a residential sector nor financial frictions. Consequently, the movements explained before could not be analysed within that model. The current paper describes changes made to the model in Pedersen and Ravn (2013) in order to introduce a housing market, a banking sector and financial frictions. The goal is to have a structural, dynamic, general equilibrium model with forward looking agents, which can be used to interpret the current state of the economy, to analyse policy changes, and perhaps to generate predictions for key macroeconomic variables.

The model builds on a sequence of influential papers. The financial accelerator of Bernanke et al. (1999) demonstrated the potential importance of financial factors for macroeconomic fluctuations in a general equilibrium setup. Building on the work of Kiyotaki and Moore (1997), Iacoviello (2005) highlighted the important role played by asset prices (in particular, house prices) in a world with collateralized household debt. Iacoviello and Neri (2010) demonstrated that in order to gain a full understanding of the macroeconomic effects of fluctuations in the housing market, an explicit modeling of the construction sector is needed. Studies by Davis and Heathcote (2007) and Liu et al. (2013) point to the importance of a fixed supply of land in order to account for, respectively, movements in the house price and comovement between house prices and business investment. Finally, Gerali et al. (2010) were among the first to combine a profit-maximizing but less than perfectly competitive banking sector with collateralized household and corporate debt in a general equilibrium setup. The model presented in this paper features all these aspects.

The introduction of a housing market involves the following steps in line with Iacoviello and Neri (2010): First, the need to distinguish between patient and impatient households, the latter of which will have a lower discount factor than the former such that the model includes borrowing and saving in steady state. This also implies that the production function must be modified to incorporate two different types of labor. Second, demand for housing services are included in the model through the household's utility function giving rise to an additional first-order condition related to the choice of housing 'consumption'. Third, the impatient household will be subject to a collateral constraint, which must be taken into account in his utility maximization. The introduction of demand for housing services together with the collateral constraint have the potential to generate wealth effects on consumption from increases in asset prices as well as crowding-in of consumption in response to fiscal policy shocks. Forth, in addition to the original production function, a housing construction sector is introduced, which constructs housing according to a production function with inputs of land and intermediate input from the goods sector. Finally, to also introduce a role for credit frictions on the firm side, I introduce a third type of agent, the entrepreneur, who rents capital and land to the firms in the two sectors. The entrepreneurs are also assumed to be impatient and are subject to a collateral constraint as in Liu et al. (2013).

The introduction of housing into a DSGE-model gives the model the potential to explain house prices developments in terms of structural shocks. Further, the model can analyse whether and how house prices affect consumption and investments through wealth and income effects working through the financial frictions. Construction has the potential to address the effects on demand on the goods market from house price developments. Here the question is to what extend spillovers from developments in the construction sector affected prices, wages and employment in the rest of the economy. These questions will be addressed within the model in a historical shock decomposition.

The introduction of the banking sector broadly follows the setup of Gerali et al. (2010). There is a continuum of profit-maximizing banks, which receive deposits from patient households and use those funds to make loans to impatient households and entrepreneurs. As such, the balance sheet of each bank simply consists of bank loans on the asset side and deposits plus bank capital on the liabilities side. Bank lending is collateralized because of agency problems between borrower and lender. In line with microeconomic theories of banking competition, see, e.g., Freixas et al. (1997), it is assumed that competition in the banking sector is less than perfect. In particular, I assume that each bank operates under monopolistic competition in the market for bank loans. Furthermore, I assume that lending rates set by the banks are sticky in a way similar to prices and wages set by firms and workers in the economy. These assumptions give rise to endogenous spreads between the interest rate set by the central bank and the interest rates on bank loans and imperfect pass-through from changes in policy rates to market interest rates. Finally, banks are subject to a regulatory capital requirement, from which it is costly to deviate.

During and after the financial crisis, the macroeconomic profession in general came under scrutiny as very few economists had foreseen the depth of the crisis. DSGE-models were in particular subject to that criticism as they neither had a financial sector nor financial frictions, which possibly could have guided and warned policy makers and forecasts. The collateral constraint and the capital requirements introduced above are the financial frictions in this model and the banking sector is the financial sector. This is a relatively simple model, but it does have the potential to translate the type of shocks seen during as an example the failure of Lehmann-brothers.

As will become clearer during the presentation of the model, the model will be able to address both supply of credit and interest spreads between policy rates and lending rates endogenously. Further, even though the model cannot per see say much about funding liquidity and credit risk between the continuum of identical banks, the model is able to give a picture of the effects of such shocks and identify them in a historical shock decomposition. That is, even though there is no frictions on the funding market and therefore no liquidity stop etc., the result of a breakdown of the money market, namely higher interest spreads, and the effects on the real economy is modeled. Such a shock has a lot in common with risk shocks depressing both consumption and investment as it affects the funding costs for households and firms. On this background, shocks to the funding costs of banks, whether it originates from the foreign economy through higher ECB policy rates or through spreads, have the same consequences as uncertainty or risk shocks, see Christiano et al. (2014).

In what follows, the model is set up with focus on the changes included with respect to the model in Pedersen and Ravn (2013). After presenting the model, I evaluate the ability of the model to fulfill the goals setup in the beginning of the introduction: To have a structural, dynamic, general equilibrium model with forward looking agents, which can be used to interpret the current state of the economy, to analyse policy changes, and generate predictions for key macroeconomic variables. I do so through impulse response functions, in which I chose to focus on fiscal policy only for simplicity, and I next use the estimation to decompose the development of Danish GDP, the real house price, and inflation through the 2000's. I lastly look at forecasts produced by the model.

As will be shown, the model can be used to analyse the effects of various changes to fiscal policy variables, while the explicit modeling of financial frictions and a banking sector open up the possibility to study, as an example, counter-cyclical capital buffers. Further, the shock decomposition is a useful instrument to study counter-factuals – what would have happen if the LTV-ratio was changed in response to changes in the house price. In turn, the explicit modeling of forward-looking behaviour and expectations make the model a useful instrument for the study of changes to taxation on housing and/or changes in tax-subsidies on interest rate payments. I leave all this for future research.

2. The Model

2.1. Patient Households

The problem of the representative patient household is to choose consumption, C_t^P, housing services, H_t^P, and real deposits, D_t^P, placed in the banking sector paying the gross interest rate R_t^D, so as to maximize its stream of discounted future utility. The household also chooses its labour supply, but I treat this decision separately in section (2.5). The utility function is given by:

$$E_0 \sum_{t=0}^{\infty} \left(\beta^P \mathcal{C}_t\right)^t \left(\log\left(C_t^P - h^C C_{t-1}^P\right) + \varsigma^{HP} \mathcal{H}_t \log\left(H_t^P\right) - \eta_N \chi_t O_t^P \int_0^1 N_t^P(h)\, dh\right), \tag{1}$$

$0 < \beta^P < 1$ is the discount factor. I assume that the discount factor for the patient household, β^P, is greater than the discount factor for the impatient household, β^I, presented in the section to come. This implies that the impatient household will be a borrower in equilibrium and the patient household will be a lender. $h^C > 0$ measures the degree of (external) habit formation in private consumption. $\mathcal{C}_t$ is a shock to the household's preference for consumption today versus tomorrow, an intertemporal preference shock, which is given by:

$$\frac{\mathcal{C}_t}{\bar{\mathcal{C}}} = \left(\frac{\mathcal{C}_{t-1}}{\bar{\mathcal{C}}}\right)^{\rho_{\mathcal{C}}} \exp \varepsilon_{t+1}^{\mathcal{C}}, \tag{2}$$

where $\bar{\mathcal{C}} > 0$, $0 < \rho_{\mathcal{C}} < 1$, and where $\varepsilon_{t+1}^{\mathcal{C}}$ is an i.i.d. stochastic process with mean zero and variance $\sigma^{\mathcal{C}}$. The second term measures the utility arising from the consumption of housing goods, where the parameter $\varsigma^{HP} > 0$ is used to calibrate the steady state level of housing, and $\mathcal{H}_t$ can be interpreted as a housing demand (or 'taste') shock, and follows:

$$\frac{\mathcal{H}_t}{\bar{\mathcal{H}}} = \left(\frac{\mathcal{H}_{t-1}}{\bar{\mathcal{H}}}\right)^{\rho_{\mathcal{H}}} \exp \varepsilon_{t+1}^{\mathcal{H}}, \tag{3}$$

where $\bar{\mathcal{H}} > 0$, $0 < \rho_{\mathcal{H}} < 1$, and where $\varepsilon_{t+1}^{\mathcal{H}}$ is an i.i.d. stochastic process with mean zero and variance $\sigma^{\mathcal{H}}$.The third term in the utility function denotes disutility of labor. I will define the variables and parameters associated with this term in subsection (2.5).

Utility maximization is subject to the following budget constraint:

$$\begin{aligned}
&\left(1+\tau_t^{VAT}\right)\frac{P_t^C}{P_t}C_t^P + Q_t^H\left(H_t^P - \left(1-\delta^H\right)H_{t-1}^P\right) + \omega T_t + D_t^P - \frac{R_{t-1}^D D_{t-1}^P}{\pi_t^{DK}} + \\
&B_t^{DK} - \frac{R_{t-1}^{DK}}{\pi_t^{DK}}B_{t-1}^{DK} + \tau_t^H Q_t^H H_t^P \qquad (4)\\
= \;& \left(1-\tau_t^N\right)W_t^P N_t^P - \frac{\tau_t^B D_{t-1}^P\left(R_{t-1}^D - 1\right)}{\pi_t^{DK}} + \frac{\delta^B K_{t-1}^B}{\pi_t^{DK}} + \Pi_t^{Retail} + \Pi_t^X,
\end{aligned}$$

where P_t is the overall price level (producer price) to be defined below, Q_t^H denotes the real house price, and $\delta^H > 0$ is the depreciation rate of the housing stock. W_t^P is the real wage rate. I let τ_t^{VAT}, τ_t^B, τ_t^N and τ_t^H be the tax rates on consumption (i.e., a Value added tax, VAT), interest income, labor, and housing property. T_t denotes real lump-sum taxes, and $\delta^B K_t^B$ denotes dividends paid out by banks. Finally, the term $B_t^{DK} - \frac{R_{t-1}^{DK}}{\pi_t^{DK}}B_{t-1}^{DK}$ reflects that patient households are enrolled in a pension scheme, under which they hold outstanding real government debt, B_t^{DK}, which is then repaid with interest R_t^{DK} in the next period. Hence, the household holds government debt and can be thought of as the Danish ATP pension system. π_t^{DK} is the change in the Danish PPI-deflator. Π_t^{Retail}, Π_t^X denotes real profits received from firms and construction firms respectively, as the patient households are the owners of intermediate firms and construction firms. Finally, the parameter, $\omega > 0$, is the relative size of the patient household compared to the impatient household.

Observe that, unlike Pedersen and Ravn (2013), I do not allow (patient) households to directly access both a domestic and a foreign bond market. In the present model, bank deposits make domestic bonds redundant, as both are assets that pay a risk-free rate of interest one period ahead. Moreover, I assume that households can only access foreign bonds through the banking sector. In practice, as shown later, this implies that households will still be able to save at the international risk-free interest rate.[1]

The first-order conditions related to the utility maximization problem of the patient household are as follows:

$$C_t^P : \frac{P_t^C}{P_t}\lambda_t^P\left(1+\tau_t^{VAT}\right) = \frac{1}{C_t^P - h^C C_{t-1}^P}, \qquad (5)$$

$$H_t^P : \varsigma^{HP}\mathcal{H}_t\frac{1}{H_t^P} + \left(\beta^P C_t\right)\left(1-\delta^H\right)E_t\left(\lambda_{t+1}^P Q_{t+1}^H\right) = \left(1+\tau_t^H\right)\lambda_t^P Q_t^H, \qquad (6)$$

$$D_t^P : \lambda_t^P = \left(\beta^P C_t\right)R_t^D E_t\left(\frac{\lambda_{t+1}^P}{\pi_{t+1}^{DK}}\right) - \left(\beta^P C_t\right)\left(R_t^D - 1\right)E_t\frac{\tau_{t+1}^B\lambda_{t+1}^P}{\pi_{t+1}^{DK}}. \qquad (7)$$

[1]The assumption that households can only access international bonds via the banking sector is needed to ensure that funds are channeled from domestic savers to domestic borrowers *via* the banking sector and not outside it. Also notice that it is assumed that patient households passively hold domestic government bonds.

Here, λ_t^P denotes the Lagrange multiplier associated with the budget constraint. The first and third equation may be combined to obtain a standard Euler equation and consequently a relation between consumption and the real rate of interest. This is standard and are explained further in e.g. Pedersen (2012) or Galí (2009). The second equation may be interpreted as an Euler equation for housing: The patient household trades off the cost (in terms of foregone non-durable consumption) of purchasing a unit of housing today against the utility gain from that extra unit of housing plus the expected resale value in the next period. This will be analysed in greater detail in box C and in section (2.3.7).

Box A: A note on housing property tax in Denmark

The nominal amount Danish households have to pay in taxes on the value of the property is currently frozen to its 2002 level. This means that when house prices rose during the mid-00's, the effective tax rate on housing fell. In the estimation of the model explained below, I will use the effective tax rate on housing, τ_t^H, and freeze the tax burden in simulations. This will be done by a slight change to both household's budget constraints in the following manner:

$$\begin{aligned} & Q_t^H\left(H_t^j - \left(1-\delta^H\right)H_{t-1}^j\right) - \tau_t^H Q_t^H H_t^j \\ \Rightarrow\quad & Q_t^H\left(H_t^j - \left(1-\delta^H\right)H_{t-1}^j\right) - \left(\frac{\tau_t^H}{Q_t^H H_t^j}\overline{Q^H H^j}\right)Q_t^H H_t^j, \; j = P, I, \end{aligned}$$

such that $Q_t^H\left(H_t^j - (1-\delta^H)H_{t-1}^j\right) - \tau_t^H\overline{Q^H H^j}$ for $j = P, I$. Hence, tax payments will in simulations be paid from the steady state value of housing using the statutory tax rate.[a]

[a]Similar considerations apply to the land tax, τ_t^L. Here the main difference is that the tax on land is only allowed to increase by a certain amount each year. This is for simplicity not addressed in the model.

2.2. Impatient Households

Like his patient counterpart, the representative impatient household maximizes a utility function of the form:

$$E_0 \sum_{t=0}^{\infty}\left(\beta^I \mathcal{C}_t\right)^t\left(\log\left(C_t^I - h^C C_{t-1}^I\right) + \varsigma^{HI}\mathcal{H}_t \log\left(H_t^I\right) - \eta_N \chi_t O_t^I \int_0^1 N_t^I(h)\,di\right),$$

where the interpretation of all variables is the same as for the patient households. An important assumption is that the discount factor is assumed to be strictly lower than that of patient households, $\beta^I < \beta^P$; in equilibrium the impatient household will wish to borrow while the patient household lends.[2] As will become clear later, lending from patient households to impatient households is done through the financial sector. Observe that the taste shocks $\mathcal{C}_t$ and $\mathcal{H}_t$ are assumed to be common for the two types of agent. Once again, I treat the labor supply decisions in a separate section.

[2]Notice, that this is implicitly implied in the way the consumers' problems are stated. But this simply reflects that impatient household will not lend in equilibrium and therefore for simplicity this decision is left out.

The choice variables for the impatient household are the level of consumption, C_t^I, housing services, H_t^I, and domestic real borrowing, B_t^I. I assume that the impatient household does not have access to international financial markets, and therefore is not allowed to borrow abroad. Furthermore, the level of credit he can obtain domestically is limited by the following collateral constraint, which ties his borrowing capacity to the expected discounted real value of his home:

$$B_t^I \leq \Theta_t^I \frac{E_t\left(Q_{t+1}^H H_t^I \pi_{t+1}^{DK}\right)}{R_t^{L,I}}. \tag{8}$$

Here, the parameter $0 < \Theta^I < 1$ measures the loan-to-value ratio allowed by the lender. I assume that the loan-to-value ratio follows the process

$$\frac{\Theta_t^I}{\overline{\Theta^I}} = \left(\frac{\Theta_{t-1}^I}{\overline{\Theta^I}}\right)^{\rho_\Theta} \exp \varepsilon_t^\Theta,$$

where $\overline{\Theta_t^I} > 0$, $0 < \rho_\Theta < 1$, and where ε_t^Θ is an i.i.d. stochastic process with mean zero and variance σ^Θ.

As described later, $R_t^{L,I}$ denotes the gross lending rate faced by impatient households, which represents the relevant discount factor for the lender. Following Kiyotaki and Moore (1997), constraint (8) may be motivated by a limited enforcement problem: As the lender cannot force the borrower to repay the loan, the borrower must promise to hand over his home to the lender in the case of no repayment in the next period. As a result, the lender wants to tie the size of the loan to the expected value of the borrower's assets in the next period when he would be able to repossess the assets in case of default; it is the expected future house price that determines the loan-to-value constraint. The presence of a loan-to-value ratio strictly smaller than one reflects that assets have a higher value outside delinquency.

The impatient household is also faced with the following budget constraint:

$$\begin{aligned} & \left(1+\tau_t^{VAT}\right)\frac{P_t^C}{P_t}C_t^I + Q_t^H\left(H_t^I - \left(1-\delta^H\right)H_{t-1}^I\right) - B_t^I + \frac{R_{t-1}^{L,I}}{\pi_t^{DK}}B_{t-1}^I + (1-\omega)\,T_t \\ = \; & \kappa_t^{RI}\frac{\left(R_{t-1}^{L,I}-1\right)}{\pi_t^{DK}}B_{t-1}^I + \left(1-\tau_t^N\right)W_t^I N_t^I - \tau_t^H Q_t^H H_t^I, \end{aligned} \tag{9}$$

where κ_t^{RI} is the tax deduction allowance on mortgage interest expenses. I assume that the tax deduction follows the process

$$\frac{\kappa_t^{RI}}{\overline{\kappa^{RI}}} = \left(\frac{\kappa_{t-1}^{RI}}{\overline{\kappa^{RI}}}\right)^{\rho_{\kappa^R}} \exp \varepsilon_t^{\kappa^{RI}}$$

, where $\overline{\kappa^{RI}} > 0$, $0 < \rho_{\kappa^R} < 1$, and where $\varepsilon_t^{\kappa^{RI}}$ is an i.i.d. stochastic process with mean zero and variance $\sigma^{\kappa^{RI}}$.

The first-order conditions of the impatient household are given by:

$$C_t^I : \frac{P_t^C}{P_t}\lambda_t^I\left(1+\tau_t^{VAT}\right) = \frac{1}{C_t^I - h^C C_{t-1}^I}, \tag{10}$$

$$H_t^I : \varsigma^{HI}\mathcal{H}_t\frac{1}{H_t^I} + \left(\beta^I C_t\right)\left(1-\delta^H\right)E_t\left(\lambda_{t+1}^I Q_{t+1}^H\right) + \Theta_t^I\mu_t^I\frac{E_t[Q_{t+1}^H\pi_{t+1}^{DK}]}{R_t^{L,I}} = \left(1+\tau_t^H\right)\lambda_t^I Q_t^H, \tag{11}$$

$$B_t^I : \lambda_t^I - \mu_t^I = \left(\beta^I C_t\right)R_t^{L,I}E_t\left(\frac{\lambda_{t+1}^I}{\pi_{t+1}^{DK}}\right) - \left(\beta^I C_t\right)\left(R_t^{L,I}-1\right)\kappa_{t+1}^{RI}E_t\left(\frac{\lambda_{t+1}^I}{\pi_{t+1}^{DK}}\right), \tag{12}$$

where μ_t^I denotes the shadow value associated with the collateral constraint, (8). The presence of the collateral constraint gives rise to an additional term in the first-order condition for housing services, equation (11): On top of the utility gain from owning a house and the expected resale value, the impatient household must look at the gains from a marginal loosening of its collateral constraint when deciding whether or not to buy an additional unit of housing.

Likewise, the collateral constraint affects the optimal borrowing decision, equation (12). That is, it limits consumption smoothing. In the absence of the collateral constraint, or if the constraint becomes non-binding ($\mu_t^I = 0$), the impatient household would behave exactly like the patient household.[3] But if the multiplier all else equal increases, for the sake of argument from 0 to a positive value, $\mu_t^I \geq 0$, then, from equation (12), $\lambda_t \geq \beta C_t E_t\left(\lambda_{t+1}\frac{R_t^{L,I}}{\pi_{t+1}^{DK}}\right)$. Consequently, the marginal utility of current consumption exceeds the marginal gain of shifting consumption intertemporally. The higher the value of the multiplier, the higher is the marginal benefit of doing so and hence of acquiring one more durable good today, post it as collateral, borrow, and consume non-durable goods. A positive multiplier thus implies that the collateral constraint is tighter.

I finally notice that given that the borrowing constraint, equation (8), holds with equality, consumption for impatient households are given by

$$\left(1+\tau_t^{VAT}\right)\frac{P_t^C}{P_t}C_t^I =$$

$$\left[Q_t^H\left(\left(1-\delta^H\right)H_{t-1}^I - H_t^I\right) + B_t^I - \frac{R_{t-1}^{L,I}}{\pi_t^{DK}}B_{t-1}^I + \kappa_t^{RI}\frac{\left(R_{t-1}^{L,I}-1\right)}{\pi_t^{DK}}B_{t-1}^I + \left(1-\tau_t^N\right)W_t^I N_t^I - \tau_t^H Q_t^H H_t^I\right]$$

That is, to some extend impatient households act as rule-of-thumb consumers consuming out of current income and what they can borrow. But it is important to note that credit

[3] I note that impatient households do not hold domestic government bonds. That reflects that impatient household do not save not even in mandatory pension schemes.

constrained households do optimise and form expectations of future variables, but the credit constrain restrain them for implementing their optimal choices. As an example, in the case of an expansionary fiscal policy shock impatient households can see that they have become poorer due to expected future tax payments. But they cannot smooth their consumption in response to this shock. Rule-of-thumb consumers, however, do not form expectations and optimise consumption; they simple consume what they can. Specifically, and as shown in Andrés et al. (2015) in a different model, consumption of impatient household can as an approximation be thought of as being determined by after-tax labour income and after-tax net-wealth defined as $NW_t \equiv Q_t^H H_{t-1}^I - \frac{R_t^{L,I}}{\pi_{DK}^t} B_t^I$.

2.3. Production, entrepreneurs, capital goods producers, land and residential investments

While the household side of the model is fairly standard the production side is not. I have chosen a decentralised setup. This involves that the various sectors are split up between various producers. The motivation behind this strategy is that it makes the rather large model easier to understand.

There are two main production sectors, an intermediate goods sector and a final goods sector. The role of the intermediate goods producer is to cost minimize with respect to its inputs. This leads to optimal factor demand for capital goods and labour. The intermediates operate under monopolistic competition and can set prices thus providing a way for introducing price rigidities. The intermediate good can be exported or sold domestically to the final goods sector. Here a continuum of final goods producers combine the intermediate goods and sell them under perfect competition. These final goods producers enter in the model for two reasons. Firstly, to provide a demand function for each intermediate good, and, secondly, to derive a price index. The final good can be used for consumption, capital production, housing production or used by the government. The households, residential investors and capital goods producers in turn combine these final goods with imported goods. This setup is quite standard except for the introduction of residential investments.

I introduce capital goods producers. Their role is to provide a market price for capital. This is a modeling device, which is equivalent to a centralised setup in which capital formation is done internally within the entrepreneurs problem, see also Christiano et al. (2005). I introduce a residential investment, or housing sector. The motivation for this is twofold, and was explained in section (1). Firstly, the housing sector played a key role during the bust and preceding boom in the Danish and in many other economies. Secondly, a housing sector allows me to study interactions between housing taxes and LTV-ratios on one side and the real economy on the other side. The housing sector consists of a continuum of identical producers which operate under perfect competition. Their job is to collect housing units from the final goods producers with land to produce a residential housing unit. The idea behind this modeling strategy is inspired from Justiniano et al. (2013), but they do not take into account movements in land prices in their setup. The advantage of this simple setup, in comparison with as an example Iacoviello and Neri (2010) is that their strategy needs to take into account in- and outflow of labour between a residential sector and a goods sector. This is straightforward to model, but it complicates, among many things, the calculation of the steady state in large-scale models and involves multiple labour markets. The current setup is simple, and, as an example, a boom in house prices and residential investments leads to intuitive simple spillovers to the rest of the economy.

I lastly introduce an entrepreneur. I want to study financial frictions not only for households, but also for firms. In the literature, there exists at least two widespread approaches. One is the so-called financial accelerator mechanism of Bernanke et al. (1999), where the external financing cost faced by each firm depends negatively on that firm's equity or net worth. The higher the firm's net worth, the lower is the external finance premium. The second approach follows the tradition of Kiyotaki and Moore (1997) and Iacoviello (2005) outlined above by subjecting firms' demand for credit to a collateral constraint. Each firm's borrowing capacity is tied to the value of that firm's total assets (or net worth). In other words, this mechanism works through the *quantity* of credit available to firms, while the mechanism of Bernanke et al. (1999) works via the *price* of credit. Effectively, however, the quantitative impact of each of the two modeling approaches is very similar in general equilibrium. I therefore, and for internal consistency, opt for the approach of Iacoviello (2005) and apply a collateral constraint also on the firm side.

To make way for a collateral constraint on the supply side of the goods market, I therefore introduce a representative entrepreneur. The entrepreneur maximises consumption by renting out capital and land to firms, either intermediates or residential investors, buying land and capital goods from the capital goods producers and patient households. Land in this model is in fixed supply. For the entrepreneur capital and land serve the dual role as sources of income and collateralizable assets. Like the impatient household, the entrepreneur has a lower discount factor than the savers in the economy, ensuring that the collateral constraint is binding in steady state. The entrepreneur also resembles the impatient household in that he has access only to domestic financial markets. The entrepreneur can consequently borrow from the banks up to a certain percentage of the value of capital goods and land bought from the patient households and capital goods producer. It gets income from by renting capital goods and land to producers, and hence increases consumption, if prices and interest rates make it profitable to do so. In this sense, the rather abstract household, the entrepreneur, can be thought of as a financial investor, mutual fund, or hedge fund, maximising consumption for savers through the rental- and financial markets. Notice, that the entrepreneur does not own the firms; it only rents inputs to them.

In what follows, I will in greater detail describe the various agents on the production side starting with the intermediate goods producers, continuing with final goods producers, capital goods producers, and residential investment goods producers, and in the end describe the entrepreneur's problem, which collects the individual pieces together.

2.3.1 Intermediate Goods Producers

There is a continuum (of unit length) of firms in the intermediate goods sector, each of which operates under monopolistic competition. These firms are owned by the patient household. Each firm j uses private and public capital as well as labor to produce a firm-specific output according to the following production function:

$$Y_t D_t = A_t^Y \left(\left(\overline{K}_{t-1}^Y \right)^{1-\eta} \left(K_{t-1}^G \right)^{\eta} \right)^{\alpha^Y} \left(N_t^{total} \right)^{1-\alpha^Y}, \tag{13}$$

where $\alpha^Y, \eta > 0$ are parameters and $\overline{K}_{t-1}^Y = u_t^Y \mathcal{K}_t K_{t-1}^Y$ is the effective capital stock being utilised in a given period. Note that while the capital stock per se is predetermined, the rate of utilisation may be adjusted within-period, and is subject to shocks in that period. D_t is a measure of price dispersion, and A_t^Y measures total factor productivity (TFP) in the goods sector. It is assumed that A_t^Y consists of two terms; a transitory component $A_t^{Y,T}$, and a permanent component $A_t^{Y,P}$, so that $A_t^Y = A_t^{Y,T} A_t^{Y,P}$. The transitory component evolves according to:

$$\frac{A_t^{Y,T}}{\overline{A}^{Y,T}} = \left(\frac{A_{t-1}^{Y,T}}{\overline{A}^{Y,T}} \right)^{\rho_A} \exp \varepsilon_t^{A_{Y,T}}, \tag{14}$$

with $\overline{A}^{Y,T} > 0$, $0 < \rho_A < 1$, and where $\varepsilon_t^{A_{Y,T}}$ is an i.i.d. stochastic process with mean zero and variance σ^A. The permanent component follows the process:

$$dA_t \equiv \frac{A_t^{Y,P}}{A_{t-1}^{Y,P}} = \lambda_{At}^Y, \tag{15}$$

where, in turn,

$$\frac{\lambda_{At}^Y}{\overline{\lambda}_A^Y} = \left(\frac{\lambda_{At-1}^Y}{\overline{\lambda}_A^Y} \right)^{\rho_{\lambda_A}} \exp \varepsilon_t^{A_{Y,P}}, \tag{16}$$

with λ_{At}^Y measuring the growth rate in technology or TFP in the goods sector, while λ_A^Y is the steady state growth rate, $0 < \rho_{\lambda_A} < 1$, and $\varepsilon_t^{A_{Y,P}}$ is an i.i.d. stochastic process with mean zero and variance σ^{λ_A}.[4]

The problem of each firm is to maximize its profits subject to the production function. This problem gives rise to the following first-order conditions, where I have dropped the j's for simplicity:

$$r_t^{K,Y} = \frac{(1-\eta)\,\alpha^Y Y_t mc_t}{u_t^Y \mathcal{K}_t K_{t-1}^Y}, \tag{17}$$

[4] I point out for clarity that the total productivity shock, A_t^Y, affects the economy through equation (13). I have however written the model in detrended form and consequently only the part of A_t^Y which is related to the transitory part, $A_t^{Y,T}$, shows up in (13).

$$W_t^{tot} = \frac{\left(1 - \alpha^Y\right) Y_t mc_t}{N_t^{tot}}, \tag{18}$$

where mc_t is the marginal cost of production, which is identical to the Lagrange multiplier associated with the production function in the cost minimisation problem. That is, the cost in money terms of producing one more good.

Capital is hired from the entrepreneur at the rental rate $r_t^{K,Y}$, while labour input is assumed to be a Cobb-Douglas aggregate of employment from the two types of household,

$$N_t^{tot} = \left(N_t^P\right)^{\omega} \left(N_t^I\right)^{1-\omega}.$$

As explained in section (2.5), the respective employment rates are indices of a continuum of different labour types, i, with varying degrees of disutility of work, j. The intermediates minimize expenditures on the two types of labour from the respective households, which gives rise to two labour demand functions. The optimal use of the two labour aggregates, $N_t^{Y,P}, N_t^{Y,I}$, are in turn found by minimizing total wages given these demand functions, which yields an expression for the overall wage level

$$W_t^{tot} = \left(\omega^{-\omega} \left(1 - \omega\right)^{-(1-\omega)}\right) \left(W_t^P\right)^{\omega} \left(W_t^I\right)^{1-\omega}$$

The total wage, W_t^{tot}, can be thought off as the marginal cost in money terms of using one more unit of labour input; that is, the lagrange multiplier on the wage minimisation problem.

The intermediates also need to decide upon the utilization of their capital. The degree of capital utilization is measured by the variable u_t^Y, and is subject to the capital utilization shock $\mathcal{K}_t$. The function $z^{u^Y}\left(u_t^Y \mathcal{K}_t\right)$ measures the cost of changing the degree of capital utilization, which I assume takes on the following functional form:

$$z^{u^Y}\left(u_t^Y \mathcal{K}_t\right) = c_1 \left(u_t^Y \frac{\mathcal{K}_t}{\bar{\mathcal{K}}} - \overline{u^Y}\right) + \frac{c_2}{2} \left(u_t^Y \frac{\mathcal{K}_t}{\bar{\mathcal{K}}} - \overline{u^Y}\right)^2, \tag{19}$$

where $c_1, c_2 > 0$ are parameters, and $\overline{u^Y}$ is the steady state level of capital utilization, which I set to 1.[5] The utilization shock $\mathcal{K}_t$ evolves according to:

$$\frac{\mathcal{K}_t}{\bar{\mathcal{K}}} = \left(\frac{\mathcal{K}_{t-1}}{\bar{\mathcal{K}}}\right)^{\rho_{\mathcal{K}}} \varepsilon_{t+1}^{\mathcal{K}}, \tag{20}$$

where $0 < \rho_{\mathcal{K}} < 1$, $\bar{\mathcal{K}} = 1$ is the steady state value of the shock process, and $\varepsilon_{t+1}^{\mathcal{K}}$ is an i.i.d. normal shock. This problem involves the following first order condition:

[5]When I solve the model, I then need to scale the tax deduction from capital depreciation and the utilization cost in the budget constraint with the trend growth of investment-specific technology so as to ensure that these are not eroded over time.

$$r_t^{K,Y} = \frac{z^{u^Y\prime}\left(u_t^Y \mathcal{K}_t\right)}{\left(1-\tau_t^K\right)}, \tag{21}$$

where $z^{u^Y\prime}\left(u_t^Y \mathcal{K}_t\right)$ is the derivative of the utilization cost function with respect to u_t^Y,

$$z^{u^{Y\prime}}\left(u_t^Y \mathcal{K}_t\right) = c_1 + c_2\left(u_t^Y \frac{\mathcal{K}_t}{\bar{\mathcal{K}}} - \overline{u^Y}\right).$$

I introduce sticky prices into the model by assuming that intermediate goods firms are subject to staggered price setting. In particular, following Calvo (1983) each firm is only allowed to change its price in any given period with probability $(1-\theta_P) < 1$. Since all firms are identical ex ante, this implies that only a fraction $(1-\theta_P)$ of firms will reset their price each period. Of the remaining θ_p firms, I allow a fraction Γ_t to index their price to the rate of inflation in the CPI-index, π_t^C, while the remaining fraction of firms keep their price unchanged. When a given firm is allowed to re-optimize its price, it solves a dynamic optimization problem, taking into account that the price it sets is likely to prevail for $\frac{1}{1-\theta_p}$ periods. I can write the resulting first-order condition as:

$$\widetilde{P}_t(j) = \frac{\epsilon_t^P}{\epsilon_t^P - 1} E_t \sum_{s=0}^{\infty} \left(\beta^P C_t \theta_P\right)^s \frac{\lambda_{t+s}^P}{\lambda_t^P} \frac{Y_{t+s}(j)\, mc_{t+s} P_{t+k}}{Y_{t+s}(j)}, \tag{22}$$

where $\widetilde{P}_t(j)$ is the price set by intermediate firm j if it is allowed to change its price in period t. As all firms are identical, this price will be the same for all firms. Note also that the relevant stochastic discount factor is the one for patient households, $\beta^P C_t \frac{\lambda_{t+s}^P}{\lambda_t^P}$, as these are the owners of the firms. Finally, ϵ_t^P is the elasticity with which final goods producers substitute between different varieties of the intermediate good, and is given by:

$$\left(\frac{\epsilon_t^P}{\epsilon^P}\right) = \left(\frac{\epsilon_{t-1}^P}{\epsilon^P}\right)^{\rho_{\epsilon^P}} \exp \varepsilon_t^{\epsilon^P}, \tag{23}$$

where $\varepsilon_t^{\epsilon^P}$ is an i.i.d. stochastic process with mean zero and variance σ^{ϵ^P}, and where $0 < \rho_{\epsilon^P} < 1$. $\epsilon^P > 0$ measures the steady state elasticity of substitution. I can write the evolution of the aggregate price index as

$$P_t = \left[\theta_P \left(\pi_{t-1}^C\right)^{(1-\epsilon_t^P)\Gamma_t} P_{t-1}{}^{1-\epsilon_t^P} + (1-\theta_P)\left(\widetilde{P}_t\right)^{1-\epsilon_t^P}\right]^{\frac{1}{1-\epsilon_t^P}}, \tag{24}$$

highlighting that the share $(1-\theta_P)$ of prices are reset in each period. Finally, D_t measures the loss associated with price dispersion, and is given by

$$D_t = (1-\theta_P)\left(\widetilde{P}_t\right)^{-\epsilon_t^P} + \theta_P \left(\pi_{t-1}^C\right)^{-\epsilon_t^P \Gamma_t} \left(\pi_t^{DK}\right)^{\epsilon_t^P} D_{t-1}, \tag{25}$$

where π_t^{DK} is the domestic inflation rate of the Danish producer price index, π_t^C is the inflation rate in the Danish CPI-index, the parameters Γ_t denote price indexation to CPI inflation.

2.3.2 Final Goods Producers

Firms in the final goods sector operate under perfect competition. They collect a variety of intermediate goods and repackage these into a final good to be used for consumption, either by the government or by private households, investment. In doing so, they solve a cost minimization problem by choosing intermediate input goods so as to produce the final output, Y_t, at the lowest possible price. Final goods producers aggregate intermediate goods according to:

$$Y_t = \left(\int_0^1 Y_t(j)^{\frac{\epsilon_t^P - 1}{\epsilon_t^P}} dj \right)^{\frac{\epsilon_t^P}{\epsilon_t^P - 1}}. \tag{26}$$

I can write the price index of domestically produced final goods as:

$$P_t = \left(\int_0^1 P_t(j)^{1-\epsilon_t^P} dj \right)^{\frac{1}{1-\epsilon_t^P}}, \tag{27}$$

where $P_t(j)$ is the price set by intermediate goods firm j.

2.3.3 Final Consumption, Investment, Residential investments, and Capital goods

I assume that each type of household $j = P, I, E$ combine domestically, $C_t^{j,DK}$, and foreign, $C_t^{j,F}$, produced goods into a final composite consumption good, C_t^j, according to a constant elasticity of substitution (CES) technology:

$$C_t^j = \left(\vartheta_c^{\frac{1}{\upsilon_c}} \left(C_t^{j,DK} \right)^{1-\frac{1}{\upsilon_c}} + (1-\vartheta_c)^{\frac{1}{\upsilon_c}} \left(\left(1-\chi_t^C\right) C_t^{j,F} \right)^{1-\frac{1}{\upsilon_c}} \right)^{\frac{1}{1-\frac{1}{\upsilon_c}}}, \tag{28}$$

where $\upsilon_c > 1$ measures the elasticity of substitution between foreign and domestic goods, and $\vartheta_c > 0$ measures the steady state share of foreign and domestic goods in the consumption basket, and thus also the degree of home bias in consumption assumed to be equal for all household types. I follow Erceg et al. (2000) and Christoffel et al. (2008) and assume that there is a cost to adjusting the share of imported consumption goods, represented by the function χ_t^C, which is given by:

$$\chi_t^C = \frac{\chi_C}{2} \left(\frac{\frac{C_t^F}{C_t} \omega_t^I}{\frac{C_{t-1}^F}{C_{t-1}}} - 1 \right)^2, \tag{29}$$

with $\chi_C > 0$ measuring the adjustment cost, and where ω_t^I is an import shock, which follows the process:

$$\left(\frac{\omega_t^I}{\overline{\omega}^I}\right) = \left(\frac{\omega_{t-1}^I}{\overline{\omega}^I}\right)^{\rho_{\omega^I}} \exp\left(\varepsilon_t^{\omega^I}\right), \tag{30}$$

with $\overline{\omega}^I > 0$, $0 < \rho_{\omega^I} < 1$, and where $\varepsilon_t^{\omega^I}$ is an i.i.d. stochastic process with mean zero and variance σ^{ω^I}. Notice, that it is costly to change import relative to domestic produced goods in the index. It is not costly to change both imported and domestically produced goods by the same amount.

Likewise, capital good producers combine foreign and domestic investment goods into a final sector-specific investment good using a similar CES technology:

$$I_t^Y = \left(\vartheta_{Y,I}^{\frac{1}{v_I}} \left(I_t^{Y,DK}\right)^{1-\frac{1}{v_I}} + (1-\vartheta_{Y,I})^{\frac{1}{v_I}} \left(\left(1-\chi_t^I\right) I_t^{Y,F}\right)^{1-\frac{1}{v_I}}\right)^{\frac{1}{1-\frac{1}{v_I}}}, \tag{31}$$

where the parameters are defined as above. The adjustment cost function χ_t^I is defined similar to that for consumption goods. Observe that I allow the home bias parameter, $\vartheta_{Y,I}$ and ϑ_C, to differ between sectors. This reflects e.g. that the share of imported investment goods is much lower in the consumption goods sector than in the investment goods sector.

I assume that housing goods, I_t^H, also consist of both foreign and domestically produced goods with its own home bias parameter, ϑ_X, and an adjustment cost function χ_t^H defined similar to that for consumption and investment goods:

$$I_t^H = \left(\vartheta_{X,I}^{\frac{1}{v_I}} \left(I_t^{H,DK}\right)^{1-\frac{1}{v_I}} + (1-\vartheta_{X,I})^{\frac{1}{v_I}} \left(\left(1-\chi_t^{IX}\right) I_t^{H,F}\right)^{1-\frac{1}{v_I}}\right)^{\frac{1}{1-\frac{1}{v_I}}}. \tag{32}$$

I note that the government is assumed to have a home-bias of 1 and hence does not need to collect imported goods into its final consumption unit.

As in Erceg et al. (2000), and taken consumption as an example, the optimal composition of final consumption is found by choosing the values of C_t^{DK} and C_t^F that solve a cost-minimization problem subject to (28). The two resulting first-order conditions are:

$$\frac{P_t^{DK}}{P_t^C} = \left(\frac{\vartheta_c}{C_t^{DK}}\right)^{\frac{1}{v_c}} \left(\vartheta_c^{\frac{1}{v_c}} \left(C_t^{DK}\right)^{1-\frac{1}{v_c}} + (1-\vartheta_c)^{\frac{1}{v_c}} \left(\left(1-\chi_t^C\right) C_t^F\right)^{1-\frac{1}{v_c}}\right)^{\frac{1}{v_c-1}},$$

$$\frac{P_t^F}{P_t^C} = \left(\frac{(1-\vartheta_c)}{(1-\chi_t^C)C_t^F}\right)^{\frac{1}{v_c}} (1-\chi_t^C - (\chi_t^C)' C_t^F) *$$
$$(\vartheta_c^{\frac{1}{v_c}} (C_t^{DK})^{1-\frac{1}{v_c}} + (1-\vartheta_c)^{\frac{1}{v_c}} (C_t^F(1-\chi_t^C))^{1-\frac{1}{v_c}})^{\frac{1}{v_c-1}}, \tag{33}$$

which can be combined to yield:

$$\frac{C_t^{DK}}{C_t^F} = \frac{\vartheta_c}{1-\vartheta_c}\left(\frac{P_t^F}{P_t^{DK}}\right)^{\upsilon_c}\left(1-\chi_t^C\right)\left[1-\chi_t^C-\left(\chi_t^C\right)' C_t^F\right]^{-\upsilon_c}, \tag{34}$$

where P_t^{DK} and P_t^F denote the price of domestic and foreign goods, respectively. I can write the relative prices of consumption and investment goods as follows:

$$\frac{P_t^C}{P_t} = \left(\vartheta_c + (1-\vartheta_c)\left(\frac{\frac{P_t^F}{P_t^{DK}}}{1-\chi_t^C-\left(\chi_t^C\right)' C_t^F}\right)^{1-\upsilon_c}\right)^{\frac{1}{1-\upsilon_c}}, \tag{35}$$

Note that in the absence of adjustment costs, the optimal composition would depend only on the relative price, the elasticity of substitution and the steady state consumption shares. Also, for consumption the optimal consumption needs to be found for each agent, the patient and impatient household as well as for the entrepreneur. Similar results hold for investment goods and housing goods.

PPP does not hold in this model due to the presence of home bias. That is, for equal cost of buying a basket consisting of goods produced at home versus a basket consisting of goods produced abroad, the Danish household would prefer the home basket.

2.3.4 Capital goods producers

As noted previously, introducing capital good producers is a device to derive a market price for capital. As shown later, this is necessary to determine the value of entrepreneurs' collateral. I assume that the patient household owns the capital goods producers.

The capital goods producers do as follows. At the beginning of each period each capital producer among the continuum of producers buys an amount, I_t^Y, of the final good from the final goods producers, and the stock of depreciated capital from the previous period, $\left(1-\delta^{K,Y}\right)K_{t-1}^Y$, from the entrepreneur, which rents capital to the intermediate goods producers to the rental rate r_t^{KY}. New capital stock is sold back to entrepreneurs at the end of the period at the price P_t^k. Capital can be converted one-to-one into new capital, while the transformation of the final good is subject to adjustment costs. Hence, the capital producers technology is:

$$K_t^Y = \left(1-\delta^{K,Y}\right)K_{t-1}^Y + \left(1-S_t^Y\right)\mathcal{I}_t I_t^Y, \tag{36}$$

where the function $S_t^Y = \frac{\kappa_I}{2}\left(\frac{I_t^Y}{I_{t-1}^Y}-\gamma^I\right)^2$ is the investment adjustment cost function, with the parameter $\kappa_I > 0$ measuring the cost of changing the investment level. Following Christiano et al. (2005), the consequence of introducing an investment adjustment cost function is that it gives rise to a wedge between the price of new investment and the price of installed capital outside steady state. $\gamma^I > 0$ denotes the steady state growth rate of investment. $\mathcal{I}_t$ is a transitory investment-specific technology shock, which evolves according to:

$$\frac{\mathcal{I}_t}{\bar{\mathcal{I}}} = \left(\frac{\mathcal{I}_{t-1}}{\bar{\mathcal{I}}}\right)^{\rho_{\mathcal{I}}} \varepsilon^{\mathcal{I}}_{t+1}, \tag{37}$$

with $\bar{\mathcal{I}} > 0$, $0 < \rho_{\mathcal{I}} < 1$, and where $\varepsilon^{\mathcal{I}}_{t+1}$ is an i.i.d. stochastic process with mean zero and variance $\varepsilon^{\mathcal{I}}_{t+1}$. The permanent (non-stationary) investment-specific technology shock is given by Z^P_t, so that $Z_t = \mathcal{I}_t Z^P_t$.[6] The permanent component follows the process:

$$dZ_t \equiv \frac{Z^P_t}{Z^P_{t-1}} = \lambda_{zt}, \tag{38}$$

where, in turn

$$\frac{\lambda_{zt}}{\overline{\lambda_z}} = \left(\frac{\lambda_{zt-1}}{\overline{\lambda_z}}\right)^{\rho_{\lambda_Z}} \exp \varepsilon^{Z_P}_t.$$

Thus, λ_{zt} denotes the time t growth rate of investment-specific technology, while $\overline{\lambda_z} > 0$ is the steady state growth rate. $\varepsilon^{\lambda_Z}_t$ is an i.i.d. stochastic process with mean zero and variance σ^{λ_Z}, while $0 < \rho_{\lambda_Z} < 1$. I thus assume a negative trend in the relative price of investment goods in the form of a non-stationary investment- specific technology shock. The drop in the relative price of investment goods is typically ascribed to IT-related investment components, such as computers.

The maximisation problem for the capital goods producers,

$$\max_{I^Y_t} E_t \left[\sum_{j=0}^{\infty} \left(\beta^P\right)^j \lambda^P_{t+j} \left\{P^K_{t+j} \left[\left(1-\delta^{K,Y}\right) K^Y_{t+j-1} + \left(1 - S^Y_{t+j}\right) \mathcal{I}_{t+j} I^Y_{t+j}\right] - P^Y_{t+j} I^Y_{t+j}\right\}\right],$$

leads to the following optimality condition:

$$\frac{P^{I,Y}_t}{P_t} = Q^Y_t \mathcal{I}_t \left[1 - S^Y_t - S^{Y,\prime}_t I^Y_t\right] + \left(\beta^P C_t\right) E_t \left[Q^Y_{t+1} \mathcal{I}_{t+1} \frac{\lambda^P_{t+1}}{\lambda^P_t} S^{Y,\prime}_{t+1} I^Y_t \left(\frac{I^Y_{t+1}}{I^Y_t}\right)^2\right], \tag{39}$$

in which $Q^Y_t \equiv \frac{P^K_t}{P_t}$ is the real price of capital or Tobin's Q. To get some intuition about how investments work in this model, it can be useful to rewrite equation (39) as follows

$$\lambda^P_t \frac{P^{I,Y}_t}{P_t} = Q^Y_t \lambda^P_t \mathcal{I}_t \left[1 - S^Y_t - S^{Y,\prime}_t I^Y_t\right] + \left(\beta^P C_t\right) E_t \left[Q^Y_{t+1} \mathcal{I}_{t+1} \lambda^P_{t+1} S^{Y,\prime}_{t+1} I^Y_t \left(\frac{I^Y_{t+1}}{I^Y_t}\right)^2\right].$$

The left hand side of this expression is the marginal costs, in terms of utility, of a unit of investment. This must be equal to the marginal benefit, which consists of the value in terms

[6] I point out for clarity that the total investment shock, Z_t, affects the economy through equation (36). I have however written the model in detrended form and consequently only the part of Z_t, which is related to the transitory part, $\mathcal{I}_t$, shows up in (36).

of utility of the extra capital the extra investment produces, $Q_t^Y \lambda_t^P \mathcal{I}_t \left[1 - S_t^Y - S_t^{Y,\prime} I_t^Y\right]$, plus the fact that the extra investment also saves potential adjustment costs the next period discounted to the present, $\left(\beta^P C_t\right) E_t \left[Q_{t+1}^Y \mathcal{I}_{t+1} \lambda_{t+1}^P S_{t+1}^{Y,\prime} I_t^Y \left(\frac{I_{t+1}^Y}{I_t^Y}\right)^2\right]$.

2.3.5 Residential investment

I assume the existence of a continuum of construction firms, which rent land from the entrepreneurs and buy housing investment goods, I_t^H, from the final goods producers. The construction firms produce final residential unit, X_t, under perfect competition. The idea is that the construction firm sort to say, finds land and puts a house on it. I assume that the patient households are the owners of the construction firms. Notice that the production of housing does include the use of labour and capital, which enter implicitly through the production of the housing investment goods.[7]

The technology used by the construction firms is a standard Cobb-Douglas production function in land and housing investments goods

$$X_t = A_t^X \left(l_{t-1}\right)^{\alpha^X} \left(I_t^{\tilde{H}}\right)^{1-\alpha^X}.$$

The firms pay installation costs in the same way as capital goods producers face investment adjustment costs. Intuitively, the goods purchased need to be converted into housing goods and that is costly. Hence, the production function includes $I_t^{\tilde{H}}$, which is given by

$$I_t^{\tilde{H}} = I_t^H \left(1 - S_t^X\right),$$

in which $S_t^X = \frac{\kappa_{IX}}{2}\left(I_t^H / I_{t-1}^H - 1\right)^2$, with the parameter $\kappa_{IX} > 0$ measuring the cost of changing the investment level.

The residential investors face a similar problem as the capital goods producers. This leads to the following relation:

$$\frac{P_t^{I,X}}{P_t} = Q_t^H \left(1 - \alpha^X\right)\left[1 - S_t^X - S_t^{X,\prime} \frac{I_t^H}{I_{t-1}^H}\right] \frac{X_t}{I_t^{\tilde{H}}} + \beta^P C_t E_t \left[\frac{\lambda_{t+1}^P}{\lambda_t^P} Q_{t+1}^H \left(1 - \alpha^X\right) \frac{X_{t+1}}{I_{t+1}^{\tilde{H}}} S_{t+1}^{X,\prime} \left(\frac{I_{t+1}^H}{I_t^H}\right)^2\right] \quad (40)$$

Consequently, the real house price becomes the shadow value of the investment cost constraint, as this would be the value to investors of a marginal relaxation of this constraint.

Finally, total factor productivity in the construction sector, A_t^X, consists of a permanent and a temporary component similarly to the goods sector. However, in line with data for

[7] A different modeling strategy would have been to set up a construction producing sector with its own capital and labour input. However, that would have involved different wages for the same type of workers, unless the labour input choice would be interdependent. That would in turn have involved a less tractable model analytically, that would create a complex interplay between borrowing constraints and labour supply. That in turn would create problems for a large-scale model like the current model. See also the comments made in Iacoviello and Neri (2010). The chosen setup in this model is analytically much simpler.

Denmark, I assume that the trend growth rate of productivity in the construction sector is lower than in the rest of the economy; $\lambda_A^Y > \lambda_A^X$. This assumption, together with the fact that land is in fixed supply, also makes way for an upward trend in real house prices in the model.

2.3.6 The Entrepreneur's Problem

The entrepreneur rents capital and land to the firms, either intermediates or residential investors, to maximise its consumption. I assume that the entrepreneur obtains utility from non-durable consumption only:

$$E_0 \sum_{t=0}^{\infty} \left(\beta^E C_t\right)^t \log\left(C_t^E - h^C C_{t-1}^E\right), \tag{41}$$

and where, as already mentioned, $\beta^E < \beta^P$. The entrepreneur faces the following budget constraint:

$$\begin{aligned} & \left(1+\tau_t^{VAT}\right)\frac{P_t^C}{P_t}C_t^E - B_t^E + \frac{R_{t-1}^{L,E}B_{t-1}^E}{\pi_t^{DK}} \\ & +Q_t^L l_t\left(1+\tau_t^L\right) + \frac{P_t^{IY}}{P_t}I_t^Y \\ = \quad & Q_t^L l_{t-1} + r_t^L l_{t-1} + r_t^{K,Y} u_t^Y \mathcal{K}_t K_{t-1}^Y \\ & -\left(\tau_t^K r_t^{K,Y} u_t^Y \mathcal{K}_t - \tau_t^K \delta^{K,Y} + z^{u^Y}\left(u_t^Y \mathcal{K}_t\right)\right) K_{t-1}^Y \\ & + \frac{\kappa_t^{RE} B_{t-1}^E \left(R_{t-1}^{L,E} - 1\right)}{\pi_t^{DK}} \end{aligned} \tag{42}$$

The representative entrepreneur must pay for his non-durable consumption, new investment in the capital stock to be rented to intermediates, and new additions to his stock of land, l_t.[8] His income derives from renting his stock of capital (net of capital taxes and tax deductions on debt, κ_t^{RE}, and utilization costs) and land to the firms to the rental rate. r_t^L. I assume that the tax deduction for the entrepreneur follows the process

$$\frac{\kappa_t^{RE}}{\overline{\kappa^{RE}}} = \left(\frac{\kappa_{t-1}^{RE}}{\overline{\kappa^{RE}}}\right)^{\rho_{\kappa^R}} \exp \varepsilon_t^{\kappa^R E}$$

, where $\overline{\kappa^{RE}} > 0$, $0 < \rho_{\kappa^R} < 1$, and where $\varepsilon_t^{\kappa^R E}$ is an i.i.d. stochastic process with mean zero and variance $\sigma^{\kappa^{RE}}$. Finally, the entrepreneur must pay a property tax, τ_t^L, on his land holdings. I assume that the entrepreneur needs to use his accumulated stock of capital and land as

[8] It is assumed that the stock of land does not depreciate.

collateral:

$$B_t^E \leq \Theta_t^E \frac{E_t\left[Q_{t+1}^Y K_t^Y \pi_{t+1}^{DK} + Q_{t+1}^L l_t \pi_{t+1}^{DK}\right]}{R_t^{L,E}}, \tag{43}$$

where, as above, $0 < \Theta^E < 1$ is the loan-to-value (LTV) ratio faced by entrepreneurs, and where Q_t^Y denotes the price of installed capital coming from the maximisation problem for the capital goods producers. The need to post collateral is an assumption and does not arise endogenously. It can however, be motivated by absence of contract enforcement like for the impatient households.

2.3.7 The Entrepreneur's Optimal Behaviour

The problem of the entrepreneur is to maximize (41) subject to (42). He does so by choosing his consumption, C_t^E, borrowing, B_t^E, stock of land, l_t, and capital, K_t^Y. The first-order conditions arising from this problem are:

$$C_t^E : \frac{P_t^C}{P_t}\lambda_t^E\left(1+\tau_t^{VAT}\right) = \frac{1}{C_t^E - h^C C_{t-1}^E} \tag{44}$$

$$B_t^E : \lambda_t^E - \mu_t^E = \left(\beta^E \mathcal{C}_t\right) R_t^{L,E} E_t\left(\frac{\lambda_{t+1}^E}{\pi_{t+1}^{DK}}\right) - \left(\beta^E \mathcal{C}_t\right)\left(R_t^{L,E}-1\right)\kappa_{t+1}^{RE} E_t\left(\frac{\lambda_{t+1}^E}{\pi_{t+1}^{DK}}\right) \tag{45}$$

$$l_t : \left(1+\tau_t^L\right)\lambda_t^E Q_t^L = \left(\beta^E \mathcal{C}_t\right) E_t\left[\lambda_{t+1}^E\left(r_{t+1}^L + Q_{t+1}^L\right)\right] + \mu_t^E \Theta_t^E \frac{E_t\left(Q_{t+1}^L \pi_{t+1}^{DK}\right)}{R_t^{L,E}} \tag{46}$$

$$\begin{aligned} K_t^Y : Q_t^Y = {} & \beta^E \mathcal{C}_t E_t\left[\frac{\lambda_{t+1}^E}{\lambda_t^E}\left(r_{t+1}^{K,Y}(1-\tau_{t+1}^K)u_{t+1}^Y \mathcal{K}_{t+1} + \delta^{K,Y}\tau_{t+1}^K - z^{u^Y} u_{t+1}^Y \mathcal{K}_{t+1}\right) + (1-\delta^{K,Y})Q_{t+1}^Y)\right] \\ & + \Theta_t^E \frac{\mu_t^E}{\lambda_t^E}\frac{E_t[Q_{t+1}^Y \pi_{t+1}^{DK}]}{R_t^{L,E}}. \end{aligned} \tag{47}$$

Here, I let λ_t^E and μ_t^E denote the Lagrange multipliers associated with respectively the budget constraint and the collateral constraint. The multiplier on the collateral constraint, μ_t^E, equals the increase in lifetime utility from borrowing an additional amount to a cost equal to the interest rate charged by the banks, $R_t^{L,E}$, consuming or investing the proceeds and reducing consumption the following period; it measures the shadow price of a marginal slack in that constraint.

Relation (46) says that the entrepreneur buys land until the marginal cost of doing so, the after tax costs in terms of utility, $\left(1+\tau_t^L\right)\lambda_t^E Q_t^L$, equals the discounted marginal benefit which consists of the rental income from housing producers and the resale value of housing, $r_{t+1}^L + Q_{t+1}^L$, and finally the collateral value of buying an extra unit of land, $\mu_t^E \Theta_t^E \frac{E_t\left(Q_{t+1}^L \pi_{t+1}^{DK}\right)}{R_t^{L,E}}$.

The intuition behind the first order condition with respect to capital can perhaps be explained easier by rewriting the condition in the following manner:

$$\lambda_t^E Q_t^Y = \beta^E C_t E_t[\lambda_{t+1}^E (r_{t+1}^{K,Y}(1-\tau_{t+1}^K)u_{t+1}^Y \mathcal{K}_{t+1} + \delta^{K,Y}\tau_{t+1}^K - z^{u^Y}(u_{t+1}^Y \mathcal{K}_{t+1}) + (1-\delta^{K,Y})Q_{t+1}^Y)] + \Theta_t^E \mu_t^E \frac{E_t[Q_{t+1}^Y \pi_{t+1}^{DK}]}{R_t^{L,E}}. \quad (48)$$

Hence, when the entrepreneur decide how much capital he wants, he equates the marginal costs in terms of utility of an extra unit of capital, $Q_t^Y \lambda_t^E$, to the marginal benefit in terms of utility of expanding capital by one extra unit. The marginal benefit consists of the after tax return from the extra capital plus the resale value in the next period minus the compensation for utilisation rate plus the value of being able to expand the collateral constraint. This last term, $\mu_t^E \Theta_t^E \frac{E_t(Q_{t+1}^Y \pi_{t+1}^{DK})}{R_t^{L,E}}$, is new relatively to a standard DSGE model with capital and investments.

Box C: More on the determination of house prices

The previous sections have documented two important new features in comparison with a more standard DSGE model like the one in Pedersen and Ravn (2013) namely residential investments and financial frictions. Here the aim is to provide more insight to how these features work. I start with residential investments and the determination of the house price.

Defining the stochastic discount factor for the patient household as $M_{t+1}^P \equiv (\beta^P C_t)\frac{\lambda_{t+1}^P}{\lambda_t^P}$, the marginal utility of housing as $U_{H^P,t} \equiv \varsigma^{HP}\mathcal{H}_t \frac{1}{H_t^P}$, and the marginal rate of substitution between consumption of goods and housing as $MRS_t^{C^P,H^P} \equiv \frac{U_{H^P,t}}{U_{C^P,t}}$, equation (6), the first order condition for housing for the patient household, can be rearranged as

$$\left(1+\tau_t^H\right)Q_t^H = MRS_t^{C^P,H^P} + \left(1-\delta^H\right)E_t\left(M_{t+1}^P Q_{t+1}^H\right).$$

This relation says that the (after tax) house price can be written as the relative housing services or flow of benefits the consumer gets in terms of consumption, $MRS_t^{C^P,H^P}$, plus the expected discounted house price tomorrow.

The relation can also be understood in terms of user cost of housing, $usc_{t+1} \equiv \left(1+\tau_t^H\right)Q_t^H - (1-\delta^H)E_t\left(M_{t+1}^P Q_{t+1}^H\right)$, which says that the consumer consumes housing until the marginal benefit in terms of marginal utility, $MRS_t^{C^P,H^P}$, equals the costs of doing so, the user cost of housing. The user cost depends positively on the house price today and negatively on the future house price, which constitutes the expected discounted marginal utility of the gains from expanding future consumption through the non-depreciated resale value of the extra unit of housing. This can perhaps be seen more clearly from a log-linearisation of the user-cost:

$$\widetilde{mrs}_t \approx \widetilde{\tau_t^H} + \overline{MRS}\widetilde{q_t^H} - \left(1-\delta^H\right)\beta\widetilde{\Delta q_{t+1}^H} + \left(r_t^{LI} - \pi_{t+1}^{DK}\right),$$

in which a " ~ " above a lower case variable denotes deviations from steady state in percentages.

As above, the user cost of purchasing an extra housing good consists of tax payments on the value of the house, $\widetilde{\tau_t^H}$, the real price of housing, $\overline{MRS}\widetilde{q_t^H}$, and the real interest rate the household pays on mortgage debt, $\left(r_t^{L,I} - \pi_{t+1}^{DK}\right)$. Lower real interest rates, given the supply of housing, increases the price of housing as investments in other assets yield lower returns, so investors will purchase more housing, which pushes up the house price until the rate of return on all assets in the economy are equalised. From the user cost must be deducted the expected discounted resale value of the extra housing unit after depreciation, $(1-\delta^H)\beta\widetilde{\Delta q_{t+1}^H}$, and the part of the debt that a household can deduct part of the interest payments in the household's tax bill reflected in the term, $\kappa_t^{RI}\frac{\left(R_{t-1}^{L,I}-1\right)}{\pi_t^{DK}}$ left out in the calculations above for brevity.

Both expressions show that expectations play a key role for current house prices. Specifically, the current house price can be written as the discounted sum of future housing services:

$$Q_t^H\left(1+\tau_t^H\right) = E_t\left[\sum_{j=0}^{\infty} M_{t,t+j}^P \frac{(1-\delta^H)^j}{\left(1+\tau_{t+j}^H\right)} MRS_{t+j}^{C^P,H^P}\right]. \tag{49}$$

This relation is an asset price condition: The price of housing today is the expected, discounted after tax value of housing dividends. This implies as an example, that changes in tax rates in the future have consequences for the current house price now and not only when the changes are implemented.

Everything said above relates to the patient household and only partly to the impatient household. The impatient household differs from the patient by its collateral constraint. This constraint affects the first order condition with respect to housing, equation (11), through the term $\Theta_t^I\mu_t^I\frac{E_t\left(Q_{t+1}^H\pi_{t+1}^{DK}\right)}{R_t^{L,I}}$. The insight is however the same: Expectations are key to the development of house prices. As an example, the user cost for the impatient household can be written as

$$MRS_t^{C^I,H^I} = \left(1+\tau_t^H\right)Q_t^H - \left(1-\delta^H\right)E_t\left(M_{t+1}^I Q_{t+1}^H\right) - \Theta_t^I\frac{\mu_t^I}{\lambda_t^I}\frac{E_t\left(Q_{t+1}^H\pi_{t+1}^{DK}\right)}{R_t^{L,I}}, \tag{50}$$

or the house price can be written as

$$\left(1+\tau_t^H\right)Q_t^H = MRS_t^{C^I,H^I} + \left(1-\delta^H\right)E_t\left(M_{t+1}^I Q_{t+1}^H\right) + \Theta_t^I\frac{\mu_t^I}{\lambda_t^I}\frac{E_t\left(Q_{t+1}^H\pi_{t+1}^{DK}\right)}{R_t^{L,I}}, \tag{51}$$

and consequently includes a further term, which lowers the user cost of housing compared to the patient household. As noted previously, this term reflects the utility gain the impatient household gets from a marginal loosening of its collateral constraint. That is, by purchasing more housing units the impatient household also expands its opportunities to increase its borrowing. Again, higher expected house prices in the future lower the user cost not only through the second term but also through the third terms as higher house prices loosen the collateral constraint. From expression (51), it can also be seen that the LTV-ratio, Θ_t^I, is a determinant of the development of house prices, as it determines how much the impatient household can use housing as collateral and hence the marginal valuation of extra housing.

All the above relates to the *demand* for housing. The supply of housing is given by the construction sector and more specifically through the lagrange-multiplier on the construction firms cost minimisation problem. The output price equals their marginal costs as these firms operate under perfect competition.[a]

[a]For Danish readers familiar with the large-scale macroeconometric models, like ADAM, MONA and/or SMEC there is a quite close analogy between how construction and housing is set up in this model and the previous mentioned models. Demand for housing stems from the household's first order conditions. Construction firm supplies what is demanded to the given price. The real house price can be interpreted as a Tobins Q relationship. As an example, in SMEC Tobins' Q equals $\frac{p_t^h}{(p_t^{land})^{0.2}(p_t^{INV})^{0.8}}$. However, one major difference between the two modeling strategies is the formation of expectations and forward-looking behaviour, which can play a key role for the determination of house prices as shown above.

Box D: More on the financial accelerator mechanism(s)

This model has two collateral constraints and, as I will show below, two financial accelerator mechanism: One on the household side and one on the production side through the entrepreneur. From equation (8), borrowing of impatient households is tied to the collateral constraint. This consists of the expected future value of its house. If the house price increases due to a shock, the collateral value increases which increase the borrowing capacity allowing the impatient household to spend more. Given that impatient households have a higher consumption propensity than patient households, the increase in borrowing increases demand further and leads to an amplification or accelerator mechanism. As written in the introduction to this paper the housing boom in the Danish economy coincided with quite high consumption growth. Clearly, in a representative agent framework, this cannot be due to increases in housing wealth. Data, however, points to a positive increase in consumption in response to a shock to the house price, see e.g. Iacoviello (2005) or Carroll et al. (2011). This model with the two household setup, a collateral constraint, and different discount factors has the potential to generate such comovements.

Overall, the same mechanism plays out on the production side. Here the entrepreneur on top of its budget constraint faces a collateral constraint, equation (43), which depends on expected future prices of capital and land. The borrowing is thus tied to asset prices. While the impatient households' collateral constraint has the ability to generate an amplification mechanism through consumption, the collateral constraint the entrepreneur faces works through investments. When, as an example, the land price or the price of capital increase, the entrepreneur can post more collateral to the banks, increase its borrowing, buy more land and capital, which increases demand in the economy and an amplification mechanism plays out. This is the financial accelerator of Bernanke et al. (1999) and Kiyotaki and Moore (1997). Again, this mechanism has the potential to amplify responses to shocks in comparison with the model in Pedersen and Ravn (2013). It also has the potential to explain the comovements between output and credit, as noted in the introduction to this paper.

2.4. The Banking Sector

As described in the introduction, banks are subject to monopolistic competition in the markets for loans. To describe banks' profit maximization, it is convenient to assume that each bank consists of three separate branches: A deposit branch, whose job it is to set the deposit rate, R_t^D, and receive deposits from patient households, a loan branch, which similarly extends loans to impatient households and entrepreneurs and sets the corresponding lending rates, $R_t^{L,I}$ and $R_t^{L,E}$, and finally a wholesale branch which acts as the link between the two other branches, and makes the overall decisions of how much to borrow and lend, how much capital to hold etc. In other words, the wholesale branch manages the bank's balance sheet, which simply equals loans to deposits plus bank capital. Bank capital is accumulated via retained earnings.

I deviate from the setup of Gerali et al. (2010) in some aspects. While Gerali et al. (2010) assume monopolistic competition also in the market for deposits, I assume that the deposit market is characterized by perfect competition, and that, correspondingly, deposit rates are fully flexible. The reason for this is twofold: First, I believe that agency problems in banking are likely to be much more important for bank lending than for deposits partly because of deposit insurance mechanisms. Second, and more importantly, Denmark's fixed exchange rate requires me to assume that (patient) Danish households and foreign households have access to the same savings rate in order to ensure that equilibrium real interest rates are equalized across countries. Yet, if I were to assume monopolistic competition in the deposit market, this would drive a wedge between the interest rate in the eurozone and the deposit rate in Denmark. One potential solution would be to introduce monopolistic competition in the deposit market in the eurozone, but this would greatly enhance the number of equations in the model. Instead, I abstract from monopolistic competition in the European and in the Danish deposit markets. In the following, I describe each of the three branches in turn.

2.4.1 The Wholesale Branch

It may be helpful to think of the wholesale branch as the headquarters of each bank, with the loan and deposit branches operating according to its instructions. Unlike the two retail branches, the wholesale branch operates under perfect competition.

I first present the balance sheet of each wholesale bank j^b:

$$B_t\left(j^b\right) + D_t^*\left(j^b\right) = D_t^P\left(j^b\right) + K_t^B\left(j^b\right), \tag{52}$$

where $K_t^B\left(j^b\right)$ denotes bank j^b's accumulated stock of bank capital, and where, as described in the next two subsections, $B_t\left(j^b\right)$ and $D_t^P\left(j^b\right)$ denote total lending and total domestic deposits of bank j^b. $D_t^*\left(j^b\right)$ denotes bank j^b's borrowing/lending on international financial markets, where it has access to unlimited funds at the gross interest rate R_t^{INT}, which equals the gross policy rate set by the European Central Bank, R_t^{ECB}, plus a risk premium that depends on the ratio of foreign Danish private-sector debt to GDP.[9] Formally, the effective interest rate is given by:

$$R_t^{INT} = R_t^{ECB} \exp\left(-\psi_d^{D^*}\left(\frac{D_t^*}{Y_t} - \frac{\overline{D}^*}{\overline{Y}}\right)\right)\left(\frac{\mathcal{R}_t}{\bar{\mathcal{R}}}\right). \tag{53}$$

If the ratio of Denmark's aggregate net foreign asset position to GDP falls short of its steady state level, Danish banks will have to pay a risk premium on top of the interest rate set by the ECB. This reflects that foreign investors will be less willing to lend additional funds to Danish debtors. In turn, the higher interest rate will make it less attractive for Danish banks to borrow abroad, so that eventually the asset-to-output ratio will return to its steady state level. In this respect, $\psi_d^{D^*} > 0$ measures the sensitivity of the risk premium with respect to the net level of holdings of foreign bonds, or equivalently, Denmark's net foreign asset position.[10] I assume that each bank does not internalize the effects on Denmark's net foreign asset position, and thus on the risk premium, of its individual international debt. Consequently, R_t^{INT} is the de facto policy rate for the Danish economy. Moreover, I let $\mathcal{R}_t$ denote a shock to the risk premium. This shock evolves as:

$$\frac{\mathcal{R}_t}{\bar{\mathcal{R}}} = \left(\frac{\mathcal{R}_{t-1}}{\bar{\mathcal{R}}}\right)^{\rho_{\mathcal{R}}} \exp \varepsilon_{t+1}^{\mathcal{R}}, \tag{54}$$

where $\bar{\mathcal{R}} = 1$, $0 < \rho_{\mathcal{R}} < 1$, and where $\varepsilon_{t+1}^{\mathcal{R}}$ is an i.i.d. stochastic process with mean zero and variance $\sigma^{\mathcal{R}}$.

I point out that R_t^{INT} can differ from the gross borrowing rate for the Danish government, R_t^{DK}, both due to different debt elasticities and due to the fact that the risk premia differs between the two types of debt contracts. One depends on public sector debt while the other depends on total Danish debt. I stress that this way of modeling only reflects a wish to have more flexibilities in conducting policy experiments, as an example shocks to the perceived ability of the government to pay off its debt and/or stress on the financial markets.

As already mentioned, banks accumulate capital out of retained earnings according to the following law of motion:

$$\pi_t^{DK} K_t^B\left(j^b\right) = \left(1 - \delta^B\right) K_{t-1}^B\left(j^b\right)\frac{\kappa_t^B}{\bar{\kappa}^B} + \Xi_{t-1}\left(j^b\right), \tag{55}$$

where $\Xi_{t-1}\left(j^b\right)$ measures overall bank profits, i.e. from all three branches, and $\delta^B > 0$ may be interpreted as dividend payments to the owners of the bank, i.e. the patient households. The implication of equation (55) is that increases in bank capital can only come by through

[9] Note that I have placed $D_t^*\left(j^b\right)$ on the asset side of the bank's balance sheet. The intuition is, that it can be thought of as Denmark's net foreign asset position.

[10] The assumption of a risk premium on foreign bonds is only made to ensure a stationary model as in Schmitt-Grohé and Uribe (2003). Without such an assumption it would be possible for the consumers to borrow indefinitely in the international bond market and consume the proceeds.

retained earnings and consequently rules out all other options for recapitalisation like raising capital on the stock market. Remedying shortage of capital thus necessarily takes time. However, during the crisis, banks which were exposed to large write-downs necessarily also had difficulties raising new capital. Hence, during deep financial crisis the assumption behind equation (55) are perhaps not that unrealistic and during ”normal” times, banks are able to build up their capital through retained earnings and can avoid the stock market.

κ_t^B is a shock to bank capital and can be interpreted as a sudden, unexpected write down shock as an example due to fire sales in a financial crisis or counterparty risk and/or default. This shock serves the purpose of providing insight into the functioning of the model. I assume that the shock evolves as:

$$\frac{\kappa_t^B}{\bar{\kappa}^B} = \left(\frac{\kappa_{t-1}^B}{\bar{\kappa}^B}\right)^{\rho_{\kappa^B}} \exp\left(\epsilon_t^{\kappa^B}\right),$$

where $\bar{\kappa}^B = 1,\ 0 < \rho_{\kappa^B} < 1$, and where $\epsilon_t^{\kappa^B}$ is an i.i.d stochastic process with mean zero and variance σ_{κ^B}.

The wholesale branch of the bank chooses the overall amounts of lending and deposits of the bank so as to maximize profits. In addition, the wholesale branch takes into account the capital requirement imposed on banks: In any given period, it is costly for a bank to deviate from a target value $\kappa^B > 0$ for the bank's capital-to-assets ratio, $K_t^B\left(j^b\right)/B_t\left(j^b\right)$. I follow Gerali et al. (2010) in assuming that the costs of deviating take on a quadratic form and are proportional to bank capital. I may then write the problem of the wholesale branch as:

$$\max_{B_t(j^b),D_t^P(j^b),D_t^*(j^b)} \left(R_t^L - 1\right) B_t\left(j^b\right) + \left(R_t^{INT} - 1\right) D_t^*\left(j^b\right) - \left(R_t^D - 1\right) D_t^P\left(j^b\right)$$
$$-\frac{\Phi^B}{2}\left(\frac{K_t^B\left(j^b\right)}{B_t\left(j^b\right)} - \kappa^B\right)^2 K_t^B\left(j^b\right),$$

subject to (52), which I have inserted before maximizing. Here the parameter $\Phi^B > 0$ is the cost of deviating from the capital-ratio, κ^B. In the expression above, R_t^L denotes the gross interest rate charged by the wholesale branch on the loans it makes to the loan branch, and R_t^D is the gross interest rate paid by the wholesale branch on the funds it receives from the deposit branch. I assume that each individual bank does not internalize the effects on the Danish risk premium from its choice of international borrowing. The first-order conditions for $D_t^P\left(j^b\right)$ and $B_t\left(j^b\right)$ may be combined and rewritten to yield:

$$R_t^L = R_t^D - \Phi^B\left(\frac{K_t^B}{B_t} - \kappa^B\right)\left(\frac{K_t^B}{B_t}\right)^2, \tag{56}$$

where I consider a symmetric equilibrium and therefore have excluded the index j^b's. This condition shows that if the capital-to-asset ratio (or inverse leverage ratio) of the bank falls short of its target ratio, the wholesale branch will charge a lending rate that exceeds the deposit rate, R_t^D, at which it remunerates its deposit branch, so as to increase its capital ratio. The further away the actual capital-ratio is from the target ratio and the more costly it is to do so, governed by $\Phi^B > 0$, the higher is the spread. Equation (56) can therefore be interpreted as a loan supply schedule: When loans, B_t, increase, the capital-asset ratio falls below target, κ^B, and the bank is induced to raise the lending rate to balance the extra income from giving the loan with the cost of deviating from the optimal capital ratio.

The first-order condition for $D_t^*\left(j^b\right)$ may be combined with the other first-order conditions to yield:

$$R_t^D = R_t^{INT}. \tag{57}$$

This condition ties the domestic deposit rate to the international funding rate faced by banks, which in turn is a function of the ECB's policy rate, and thus is exogenous to the Danish economy.

Furthermore, I assume that Danish banks also have access to unlimited funds from Danmarks Nationalbank at the gross Danish policy rate R_t. By no-arbitrage arguments, the policy rate and the international interest rate (i.e., the ECB's interest rate plus risk premium) will always be equalized in equilibrium, reflecting Denmark's fixed exchange rate towards the euro. In addition, also by no-arbitrage, the deposit rate paid by the wholesale to the deposit branch will also be equal to the policy rate, $R_t^D = R_t$.

Finally, I can define the wholesale interest rate spread as the difference between its lending rate and its deposit rate. This can be written as:

$$S_t^W \equiv R_t^L - R_t^D = -\Phi^B\left(\frac{K_t^B}{B_t} - \kappa^B\right)\left(\frac{K_t^B}{B_t}\right)^2. \tag{58}$$

2.4.2 The Loan Branch

Monopolistic competition in the market for bank loans allows banks to charge a lending rate that exceeds the marginal cost of lending an extra unit of funds. In particular, I assume that the market for bank loans is characterized by Dixit and Stiglitz (1977) competition, where a continuum of banks supply slightly differentiated loan products at different lending rates, and where borrowers combine these loans into a composite loan basket under a constant elasticity of substitution so as to minimize their interest rate expenses. In other words, unlike under perfect competition, if an individual bank decides to raise its lending rate, it will not lose all (but only some) of its customers. Microeconomic theory of banking competition points to a number of reasons for such a market, such as switching costs, relationship banking, asymmetric information, or local monopolies.[11]

[11]Following Andrés and Arce (2012) and Andrés et al. (2013), an alternative way of capturing imperfect competition in the banking sector would be to apply Salop (1979) circular city model.

Consider first the demand for bank loans from impatient households. Given his total loan demand $B_t^I\left(i^b\right)$, each impatient household, i^b, seeks to minimize total loan repayment in the next period by choosing how much to borrow, $B_t^I\left(i^b, j^b\right)$, from each bank j^b:

$$\min_{B_t^I\left(i^b,j^b\right)} \int_0^1 \left(R_t^{L,I}\left(j^b\right)-1\right) B_t^I\left(i^b, j^b\right) dj^b,$$

subject to $\left[\int_0^1 B_t^I\left(i^b, j^b\right)^{\frac{\varepsilon_t^{bI}-1}{\varepsilon_t^{bI}}} dj^b\right]^{\frac{\varepsilon_t^{bI}}{\varepsilon_t^{bI}-1}} = B_t^I\left(i^b\right)$, i.e. that the weighted sum of his loans equals his total loan demand, where $\varepsilon_t^{bI} > 1$ is the elasticity of substitution with which impatient households substitute between loans from different banks. $R_t^{L,I}$ is gross the lending rate on loans to impatient households. Solving this problem, I arrive at an expression for impatient household i^b's demand for loans from bank j^b, which in a symmetric equilibrium may be written as the representative impatient household's demand:

$$B_t^I\left(j^b\right) = \left(\frac{R_t^{L,I}\left(j^b\right)-1}{R_t^{L,I}-1}\right)^{-\varepsilon_t^{bI}} B_t^I. \tag{59}$$

According to this expression, the demand for loans from bank j^b is a decreasing function of that bank's lending rate to impatient households relative to the aggregate lending rate in the economy, and a positive function of the impatient household's total loan demand. The elasticity of substitution ε_t^{bI} determines the size of the drop in the demand for loans from each bank following from a marginal increase in that bank's (relative) lending rate.

Bank lending to entrepreneurs is determined in a similar fashion. This implies that I can write the representative entrepreneur's optimal demand for loans from bank j^b as:

$$B_t^E\left(j^b\right) = \left(\frac{R_t^{L,E}\left(j^b\right)-1}{R_t^{L,E}-1}\right)^{-\varepsilon_t^{bE}} B_t^E, \tag{60}$$

where $R_t^{L,E}$ is the lending rate on loans to entrepreneurs, and $\varepsilon_t^{bE} > 1$ is the entrepreneur's elasticity of substitution between loans from individual banks.

Consider next the problem of the loan branch of the bank itself, which takes demand for its loan products, (59) and (60), as given when maximizing profits by setting its lending rates. The loan branch borrows funds from the wholesale branch of the bank at the rate R_t^L, and transforms these into loans made available to impatient households and entrepreneurs at lending rates $R_t^{L,I}$ and $R_t^{L,E}$ set as markups over R_t^L. Profits of the loan branch of the bank are then given by interest income from loans to households and entrepreneurs minus interest expenses paid to the wholesale branch, and net of the adjustment costs the bank must pay when changing its lending rates, which take on a quadratic form as in Rotemberg (1982) and are assumed to be proportional to aggregate loan returns so as not to become unimportant as bank lending increases. The problem of loan branch j^b may be written as:

$$\max_{R_t^{L,I}(j^b), R_t^{L,E}(j^b)} E_0 \sum_{t=0}^{\infty} M_{0,t}^P \left[\begin{array}{c} \left(R_t^{L,I}\left(j^b\right)-1\right) B_t^I\left(j^b\right)+ \\ \left(R_t^{L,E}\left(j^b\right)-1\right) B_t^E\left(j^b\right)-\left(R_t^L-1\right) B_t\left(j^b\right) \\ -\frac{\Phi^{B,I}}{2}\left(\frac{R_t^{L,I}(j^b)-1}{R_{t-1}^{L,I}(j^b)-1}-1\right)^2\left(R_t^{L,I}-1\right) B_t^I \\ -\frac{\Phi^{B,E}}{2}\left(\frac{R_t^{L,E}(j^b)-1}{R_{t-1}^{L,E}(j^b)-1}-1\right)^2\left(R_t^{L,E}-1\right) B_t^E \end{array}\right],$$

where $B_t\left(j^b\right)=B_t^I\left(j^b\right)+B_t^E\left(j^b\right)$ may be inserted, and where in turn I may insert for $B_t^I\left(j^b\right)$ and $B_t^E\left(j^b\right)$ from (59) and (60). $M_{t,t}^P=\left(\beta^P C_t\right)\frac{\lambda_{t+1}^P}{\lambda_t^P \pi_{t+1}^{DK}}$ is the stochastic discount factor of the patient household, who owns the banks. I may write the first-order conditions for lending to each type of customer $j=I,E$ as:

$$\begin{aligned} 0 \;=\;& \left(\frac{R_t^{L,j}\left(j^b\right)-1}{R_t^{L,j}-1}\right)^{-\varepsilon_t^{bj}} B_t^j-\varepsilon_t^{bj}\left(R_t^{L,j}\left(j^b\right)-1\right)\left(\frac{R_t^{L,j}\left(j^b\right)-1}{R_t^{L,j}-1}\right)^{-\varepsilon_t^{bj}-1} B_t^j \frac{1}{R_t^{L,j}-1} \\ & +\varepsilon_t^{bj}\left(R_t^L-1\right)\left(\frac{R_t^{L,j}\left(j^b\right)-1}{R_t^{L,j}-1}\right)^{-\varepsilon_t^{bj}-1} B_t^j \frac{1}{R_t^{L,j}-1} \\ & -\Phi^{B,j}\left(\frac{R_t^{L,j}\left(j^b\right)-1}{R_{t-1}^{L,j}\left(j^b\right)-1}-1\right)\left(R_t^{L,j}-1\right) B_t^j \frac{1}{R_{t-1}^{L,j}\left(j^b\right)-1} \\ & +\Phi^{B,j} E_t\left[M_{t,t+1}^P\left(\frac{R_{t+1}^{L,j}\left(j^b\right)-1}{R_t^{L,j}\left(j^b\right)-1}-1\right)\left(R_{t+1}^{L,j}-1\right) B_{t+1}^j\left(\frac{R_{t+1}^{L,j}\left(j^b\right)-1}{\left(R_t^{L,j}\left(j^b\right)-1\right)^2}\right)\right]. \end{aligned}$$

In a symmetric equilibrium, all banks will be charging the same interest rate, and all customers (of each group) will borrow the same amount from each bank. Imposing this, I may re-write the first-order condition as:

$$\begin{aligned} 0 \;=\;& 1-\varepsilon_t^{bj}+\varepsilon_t^{bj}\frac{R_t^L-1}{R_t^{L,j}-1}-\Phi^{B,j}\left(\frac{R_t^{L,j}-1}{R_{t-1}^{L,j}-1}-1\right)\frac{R_t^{L,j}-1}{R_{t-1}^{L,j}-1} \\ & +\Phi^{B,j} E_t\left[M_{t,t+1}^P\left(\frac{R_{t+1}^{L,j}-1}{R_t^{L,j}-1}-1\right)\left(\frac{R_{t+1}^{L,j}-1}{R_t^{L,j}-1}\right)^2\frac{B_{t+1}^j}{B_t^j}\right], \end{aligned} \tag{61}$$

where I still distinguish between the lending rates offered by the bank to its customers, $R_t^{L,j}$, and the rate at which the loan branch borrows funds from the wholesale branch, R_t^L. (61) shows that the lending rate offered to customers is an increasing function of R_t^L, which in turn is a function of the policy rate, as described above. This may be seen more clearly if I abstract from interest rate adjustment costs, $\Phi^{B,j} = 0$, in which case (61) boils down to:

$$\left(R_t^{L,j} - 1\right) = \frac{\varepsilon_t^{bj}}{\varepsilon_t^{bj} - 1}\left(R_t^L - 1\right)\ ,\ j = I, E.$$

From this expression, it can be seen directly that the net lending rate is set as a markup, $\frac{\varepsilon_t^{bj}}{\varepsilon_t^{bj}-1}$, over the marginal cost of funds faced by the loan branch, R_t^L, recalling that $\varepsilon_t^{bj} > 1$.[12] I let the market power be time-varying. Specifically, I assume that the elasticities of substitution between different loans for households and the entrepreneur vary according to the following processes:

$$\frac{\varepsilon_t^{bj}}{\overline{\varepsilon^{bj}}} = \left(\frac{\varepsilon_{t-1}^{bj}}{\overline{\varepsilon^{bj}}}\right)^{\rho_{\varepsilon^{bj}}} \exp\left(\varepsilon_t^{b,j}\right),\ j = I, E,$$

where $\overline{\varepsilon^{bj}} = 1$, $0 < \rho_{\varepsilon^{bj}} < 1$, and where $\varepsilon_t^{b,j}$ is an i.i.d. stochastic process with mean zero and variance $\sigma^{b,j}$.

In the case of fully flexible loan rates, I may write the spread between the lending rates and the policy rate as:

$$S^{L,j} \equiv R_t^{L,j} - R_t = \frac{\varepsilon_t^{bj}}{\varepsilon_t^{bj} - 1} S_t^W + \frac{1}{\varepsilon_t^{bj} - 1}\left(R_t - 1\right),$$

using the expression for $R_t^{L,j}$ just derived, the fact that $R_t^D = R_t$, and the definition of S_t^W from (58).

2.4.3 The Deposit Branch

The deposit branch collects deposits from each patient household in the economy and hands the funds over to the wholesale branch. As described previously, patient households can only invest in (foreign and domestic) bonds via the banks, i.e. by handing over their savings to a bank as deposits and have the bank buy the bonds. Therefore, patient households place all of their savings as deposits.

As described above, I let the deposit branch of the bank operate under perfect competition. As a result, its task is fairly simple: It receives deposits from patient households at the economy's deposit rate, R_t^D, and channels these funds on to the wholesale branch at the same rate. In other words, its profit maximization problem is trivial. The setup of the deposit branch also implies that the deposit rate is equalized to the international funding rate faced by banks, i.e. that $R_t^D = R_t^{INT}$, as described above.

[12] If I increase the elasticity of substitution, the banks lose market power, and eventually, as $\varepsilon_t^{bj} \to \infty$, the markup will converge to 1, as in the case of perfect competition.

2.4.4 Bank Profits and the Model's Interest Rates

I can add together the profits of each of the three branches of the bank to obtain the following expression for bank profits, where I have abstracted from index j^b's:

$$\begin{aligned}\Xi_t \;=\;& \left(R_t^{L,I}-1\right)B_t^I+\left(R_t^{L,E}-1\right)B_t^E+\left(R_t^{INT}-1\right)D_t^*-\left(R_t^D-1\right)D_t^P\\ &-\frac{\Phi^B}{2}\left(\frac{K_t^B}{B_t}-\kappa^B\right)^2K_t^B\\ &-\frac{\Phi^{B,I}}{2}\left(\frac{R_t^{L,I}-1}{R_{t-1}^{L,I}-1}-1\right)^2\left(R_t^{L,I}-1\right)B_t^I\\ &-\frac{\Phi^{B,E}}{2}\left(\frac{R_t^{L,E}-1}{R_{t-1}^{L,E}-1}-1\right)^2\left(R_t^{L,E}-1\right)B_t^E,\end{aligned}\qquad(62)$$

reflecting that bank profits are a function of the spread between its lending rates and its deposit rates or funding costs, minus the costs of deviating from its optimal capital level and the costs of adjusting its interest rates.

At the end of the description of the banking sector, it may be useful to summarize how the different interest rates in the banking sector are related to each other. These include the economy's deposit rate, R_t^D, the lending rate set by the wholesale branch on loans to the loan branch, R_t^L, the lending rates charged by the loan branch on loans to impatient households, $R_t^{L,I}$, and entrepreneurs, $R_t^{L,E}$, and finally the Danish policy rate, R_t, and the international interest rate, R_t^{INT}, which in turn is given by the policy rate set by the ECB, R_t^{ECB}, plus a risk premium. These interest rates are related as follows:

$$R_t^D \underset{No-arbitrage}{=} R_t^{INT} \underset{No-arbitrage}{=} R_t \underset{\Pi}{<} R_t^L \underset{Markup}{<} R_t^{L,I}, R_t^{L,E},$$

where Π denotes profit maximization by the wholesale branch.

2.5. The Labor Market

As in Pedersen and Ravn (2013), I model the labor market following Galí et al. (2011). In the present setup, however, I need to treat the two types of households (patient and impatient) separately. The main building blocks of the labour market are wage-setting households and sticky wages. Just as for the pricing behaviour of the firms in the model, sticky wages are achieved by assumption using the theory of Calvo (1983). In this framework, unemployment is due solely to a non-competitive labour market in which heterogeneous types of labour can set a wage above the market clearing wage. Unemployment varies due to changes in the average wage markup in the economy due to wage rigidities.

I assume the existence of two representative households each with a continuum of members indexed by

$$(h,k) \in [0,1] \times [0,1]$$

Index h refers to differentiated labour services. Hence, I assume the existence of heterogeneous types of labour specialized in various fields. This implies that each labour supplier has some market power to set its wage. I assume the existence of a continuum of labour unions each representing the different labour types. Index k refers to the household member's disutility from work. Hence, the household consists of many labour types who each have a certain degree of disutility from work.[13] I assume full consumption risk sharing across the household, implying that I do not need to take care of different consumption levels and hence marginal utilities, and that the individual members of the household have the household in mind when maximizing utility. The employment level is determined on the firm side - the household simply supplies the given number of workers at the going real wage.

When household j chooses how much labor to supply in sector i $\left(N_t^i, i = P, I\right)$, it equalises the marginal rate of substitution between leisure and consumption to the real wage:

$$MRS_t^i \equiv \frac{\chi_t O_t^j \left(N_t^i\right)^{\phi}}{\lambda_t^j} = \left(1 - \tau_t^N\right) W_t^i, \tag{63}$$

where $MRS_t^i \equiv -\frac{U_{N,t}^{i,}}{U_{C,t}^i}$ is the household's marginal rate of substitution between consumption and leisure, and $\phi > 0$ is the inverse of the Frisch elasticity. That is, $\frac{1}{\phi}$ measures by how much the households' labor supply changes in percent when the real wage increases by one percent holding consumption constant. Intuitively, at the optimum the disutility of working more must be compensated by what the real wage can buy in utility terms. If not, the household would be able to reshuffle between consumption and labour and achieve a higher utility. The variable O_t^j is defined as:

$$O_t^j = \frac{z_t^j}{\left(C_t^j - h^C C_{t-1}^j\right)\left(1 + \tau_t^{VAT}\right)\left(P_t^C / P_t\right)} = z_t^j \lambda_t^j, \tag{64}$$

with z_t^j evolving according to:

$$z_t^j = \left(z_{t-1}^j\right)^{1-\nu} \left[\left(C_t^j - h^C C_{t-1}^j\right)\left(1 + \tau_t^{VAT}\right)\left(P_t^C / P_t\right)\right]^{\nu}, \tag{65}$$

where $\nu \in [0,1]$. Following Galí et al. (2011), I may interpret z_t^j as a smooth trend for (habit-adjusted) aggregate consumption. In other words, O_t^j is smaller than one when consumption grows faster than this smooth trend, and vice versa. As seen from (63), this

[13] The parameter η_N in the utility function will help to determine the steady-state employment, but is left out in what follows.

implies a drop in the marginal disutility of labor, so that each individual will be willing to work at a lower wage rate, *ceteris paribus*. The parameter ν determines the strength of the wealth effect on labor supply. That is, by how much labor supply is affected by changes in wealth: If ν is close to 1, the wealth effect is quite strong, while the wealth effect disappears when ν tends to 0. [14] I assume the same preferences across households and across sectors except for the subjective discount factors. I also assume an equal degree of price stickiness across sectors. This does not imply that the wages are equalised across sectors and households due to different consumption levels.

Finally, in (63), the term χ_t represents an exogenous shock to labor supply, which evolves according to:

$$\frac{\chi_t}{\overline{\chi}} = \left(\frac{\chi_{t-1}}{\overline{\chi}}\right)^{\rho_\chi} \exp\left(\varepsilon_t^\chi\right), \tag{66}$$

where ε_t^χ is an i.i.d. stochastic process with mean zero and variance σ^χ, while $0 < \rho_\chi < 1$. I assume that the labor supply shock hits the labor supply of both households.

In equilibrium, a given individual will participate in the labor market if and only if the net benefits from doing so exceed that individual's total disutility of labor. I can write this participation condition as:

$$\lambda_t^j \left(1 - \tau_t^N\right) W_t^i \geq \Upsilon_t\left(k\right), \tag{67}$$

where the left-hand side measures the after-tax real wage rate as measured in utility units, and where $\Upsilon_t\left(k\right) \equiv \chi_t O_t^j k^\phi$ represents disutility from working. Here it is important that the individuals of each type of labour h are ordered by their disutility of labor and that the condition is related to the household's marginal disutility of work. Disutility from working thus consists of the exogenous shock to labor supply χ_t, the endogenous process O_t^j as described above, and individual-specific labor disutility.

This implies that the labor force will consist of all individuals for which the above condition is satisfied. In a symmetric equilibrium, I can write the labor force of type j households in sector i, $L_t^{i,j}$, as:

$$L_t^i = \left(\frac{\left(1 - \tau_t^N\right) W_t^i}{\chi_t z_t^j}\right)^{\frac{1}{\phi}}. \tag{68}$$

That is, the labour force consists of the k individuals for which condition (67) is satisfied. The sum of these participation rates across labour and household types gives the model's aggregate labour force. Notice that the labour force is time-varying. It may increase, for example, due to labour supply shocks which decrease the marginal disutility of working.

Next I define a notion of unemployment as $U_t^i \equiv \frac{L_t^{i,}}{N_t^i}$, i.e. the ratio between the labour force and employment of a given household type. I define the (log) average wage markup as the difference between the real wage and the marginal rate of substitution between

[14]As discussed by Galí et al. (2011), a low value of ν is necessary to ensure that not only employment, but also the labor force moves in a procyclical fashion in response to shocks originating from the demand side.

consumption and work:

$$\mu_{w,t}^{i} \equiv \log(\left(1-\tau_{t}^{N}\right)W_{t}^{i}) - \phi\left(z_{t}^{j} + N_{t}^{i} + \log\left(\chi_{t}\right)\right).$$

The wage markup for each household type in each sector varies as long as wages are not fully flexible, and it is non-zero as long as the labour market is not fully competitive. I can use this expression together with the participation condition, (68), to write

$$\mu_{w,t}^{i} = \phi u_{t}^{i},$$

where $u_t^i \equiv \log\left(U_t^i\right)$. Notice that the natural rate of unemployment $u_{n,t}^i$ is determined as $\mu_w^i = \phi u_{n,t}^i$. Hence, unemployment in this model is due solely to a non-competitive labour market in which heterogeneous types of labour can set a wage above the market clearing wage, and unemployment varies due to changes in the average wage markup in the economy. That is, due to wage rigidities. The natural rate of unemployment is higher the higher is the degree of monopolistic competition and the higher is the Frisch elasticity of labour supply. When this elasticity is high, the members of the household are more willing to substitute in and out of employment.

I introduce a variable that for estimation purposes aggregates the two unemployment rates in the economy

$$U_t^{tot} \equiv \left(U_t^P\right)^{\omega}\left(U_t^I\right)^{1-\omega}$$

2.5.1 The wage decision

The wage formation follows the description in Pedersen and Ravn (2013). Recall that households supply differentiated types of labour services, giving rise to monopolistic competition for labour. Furthermore, I assume that the household's face Calvo-style wage stickiness. The nature of the problem implies that all households who can reoptimize the wage rate in a given period choose the same wage $\widetilde{w}_{t,j}^{P}$ according to the following first-order condition:

$$0 = \sum_{s=0}^{\infty}\left(\beta^{j}C_{t}\theta_{W}\right)^{s}E_{t}\left[N_{t+s}^{j}\left(i\right)U_{C}^{j}\frac{\widetilde{w}_{t}^{j}\left(i\right)}{P_{t+s}} + \frac{\epsilon_{t}^{W}}{\epsilon_{t}^{W}-1}U_{N}^{j}\right],\ j = P, I, \tag{69}$$

where U_C^j and U_N^j denotes the marginal utility of consumption and labour, and where $0 < \theta_W < 1$ is the wage stickiness parameter, and ϵ_t^W is the elasticity of substitution between labor types, which evolves according to:

$$\left(\frac{\epsilon_t^W}{\epsilon^W}\right) = \left(\frac{\epsilon_{t-1}^W}{\epsilon^W}\right)^{\rho_{\epsilon^W}} \exp \varepsilon_t^{\epsilon^W}, \tag{70}$$

where $\varepsilon_t^{\epsilon^W}$ is an i.i.d. stochastic process with mean zero and variance σ^{ϵ^W}, and where $0 < \rho_{\epsilon^W} < 1$. Of the remaining types of workers θ_W, I allow a fraction Γ_W to index their wage to the steady state rate of inflation in the CPI-index, π_t^C, while the remaining fraction keep their wage unchanged. We can consequently write the evolution of the wage level in the private sector as:

$$W_t^j = \left[\theta_W \left(\left(\pi_{t-1}^C\right)^{\Gamma_W} W_{t-1}^j\right)^{1-\epsilon_t^W} + (1-\theta_W)\left(\widetilde{w}_t^j\right)^{1-\epsilon_t^W}\right]^{\frac{1}{1-\epsilon_t^W}} \quad j = P, I. \tag{71}$$

2.6. Trade and the two foreign economies

Denmark's fixed exchange rate towards the euro implies that I need to include two foreign economies in the model: the eurozone, EA, towards which Denmark has a fixed exchange rate, and Rest of the World, RoW, towards which the exchange rate is fully flexible and exogenous for Denmark due to the small-economy assumption and with monetary policy given from the eurozone. The two foreign economies are otherwise completely identical, and are taken as completely exogenous, so that movements in the Danish economy do not affect the foreign economies. I also do not model trade or other interactions between the eurozone and the rest of the world. The models for the two economies and the estimation can be found in appendix (13).

2.7. Exports

The role of the export sector is to buy final domestic goods, differentiate them, set a price and sell them to final goods producers in the eurozone or the rest of the world. The motivation behind the introduction of the import and export sectors is to be able to model an imperfect pass-through from changes in prices and the exchange rate to the Danish economy through estimation of the parameters in the export- and import relations. Hence, I can let the data determine the degree of the pass-through. This modeling strategy is the same as in Pedersen and Ravn (2013) and Burriel et al. (2010).

I can write the world demand for Danish exports, Ex_t, as:

$$Ex_t = X_t^{Ex} Y_t^W \left(\frac{P_t^X}{Px_t^W}\right)^{-\varepsilon^{World}}, \tag{72}$$

where the parameter ε^{World} denotes the elasticity with which world consumers substitute between Danish and foreign goods. The demand for Danish exports is thus increasing in world output and decreasing in the ratio between the relative price of Danish exports, P_t^X, and the relative world market price, Px_t^W. I define the latter as:

$$Px_t^W = Px_{t-1}^W \frac{\pi_t^W}{\pi_t^{DK}}, \tag{73}$$

where π_t^W is the world inflation rate, as described above in appendix (13). The relative price of Danish exports, P_t^X, is defined as:

$$P_t^X = P_{t-1}^X \frac{\pi_t^X}{\pi_t^{DK}}, \tag{74}$$

where π_t^X is the inflation rate in Danish exports prices, as described below. Finally, the export demand shock X_t^{Ex} evolves according to:

$$\frac{X_t^{Ex}}{\bar{X}^{Ex}} = \left(\frac{X_{t-1}^{Ex}}{\bar{X}^{Ex}}\right)^{\rho_{Ex}} \exp\left(\varepsilon_t^{Ex}\right), \tag{75}$$

where ε_t^{Ex} is an i.i.d. stochastic process with mean zero and variance σ^{EX}, and where $0 < \rho_{Ex} < 1$.

Firms in the export sector are faced with price rigidities of the same form as in the domestic production sector. I can therefore write the optimal export price $\widetilde{P}_t^X$ set by a given firm j in the export sector that is allowed to change its price in period t as:

$$\widetilde{P}_t^X(j) = \frac{\epsilon_t^X}{\epsilon_t^X - 1} E_t \sum_{s=0}^{\infty} (\beta\theta_X)^s \frac{\lambda_{t+s}}{\lambda_t} \frac{Y_{t+s}^W(j)\, mc_{t+s}^X P_{t+k}^X}{Y_{t+s}^W(j)}, \tag{76}$$

where θ_X is the Calvo stickiness parameter in the export sector, and mc_t^X is the real marginal cost for the export firms in foreign currency terms equal to $mc_t^X = \frac{P_t}{P_t^X}$. Finally, ϵ_t^X is the elasticity of substitution between the goods produced by each individual firm in the export sector, which follows the process:

$$\left(\frac{\epsilon_t^X}{\epsilon^X}\right) = \left(\frac{\epsilon_{t-1}^X}{\epsilon^X}\right)^{\rho_{\epsilon^X}} \exp \varepsilon_t^{\epsilon^X}, \tag{77}$$

where $\varepsilon_t^{\epsilon^X}$ is an i.i.d. stochastic process with mean zero and variance σ^{ϵ^X}, and where $0 < \rho_{\epsilon^X} < 1$. Of the remaining θ_X firms, I allow a fraction Γ^{X_t} to index their price to the rate of CPI inflation, while the remaining fraction of firms keep their price unchanged. The inflation rate in Danish export prices will then satisfy:

$$1 = \theta_X \left(\pi_{t-1}^C\right)^{\Gamma_{X_t}\left(1-\epsilon_t^X\right)} \left(\pi_t^X\right)^{\epsilon_t^X - 1} + (1-\theta_X)\left(\frac{\widetilde{P}_t^X}{P_t^X}\right)^{1-\epsilon_t^X}. \tag{78}$$

2.8. Imports

The structure of the importing sector can be described as follows: A continuum of import differentiators import a homogenous final good from foreign exporters, differentiate the good (say, by adding brand names), and sell the differentiated products to Danish households and firms, who, as described above, solve a cost minimization problem when they choose between imported and domestically produced goods. The world market price of import goods, which in turn determines the marginal cost of Danish import differentiators, is computed as a weighted average of prices in the eurozone and the rest of the world.[15] I can write the marginal cost for an import differentiator as:

$$mc_t^M = \frac{Px_t^W}{P_t^M}, \tag{79}$$

where, as described in the previous subsection, Px_t^W is the relative world market price, and P_t^M is the price of imported goods relative to Danish goods,

$$P_t^M = P_{t-1}^M \frac{\pi_t^M}{\pi_t^{DK}} \tag{80}$$

I define the inflation rate of import prices in Denmark, π_t^M, below. Just like domestic and exporting firms, the firms in the import sector face sticky prices as in Calvo (1983). I can therefore write the optimal price $\widetilde{P}_t^M$ chosen by a given import differentiator j that is allowed to change its price in period t as:

$$\widetilde{P}_t^M(j) = \frac{\epsilon_t^M}{\epsilon_t^M - 1} E_t \sum_{s=0}^{\infty} (\beta\theta_M)^s \frac{\lambda_{t+s}}{\lambda_t} \frac{Im_{t+s}(j)\, mc_{t+s}^M P_{t+k}^M}{Im_{t+s}(j)}, \tag{81}$$

where θ_M is the Calvo stickiness parameter in the import sector. Im_t denotes total Danish demand for imported goods. In the expression for the optimal price, ϵ_t^M is the elasticity of substitution between the goods of each import differentiator, which follows the process:

$$\left(\frac{\epsilon_t^M}{\epsilon^M}\right) = \left(\frac{\epsilon_{t-1}^M}{\epsilon^M}\right)^{\rho_{\epsilon^M}} \exp \varepsilon_t^{\epsilon^X}, \tag{82}$$

where $\varepsilon_t^{\epsilon^M}$ is an i.i.d. stochastic process with mean zero and variance σ^{ϵ^M}, and where $0 < \rho_{\epsilon^M} < 1$. Of the remaining θ_M firms, I allow a fraction Γ_{M_t} to index their price to the steady state rate of inflation, while the remaining fraction of firms keep their price unchanged. Finally, analogous to the previous subsection, the inflation rate in Danish import

[15] The modeling of the import sector involves one important drawback. Consider for example a situation where the US dollar appreciates against the Danish krone. This drives up the aggregate import price faced by Danish households and firms, who in turn choose to buy fewer imported goods from RoW *and* from EA, even though the exchange rate towards the Euro is unaffected.

prices satisfy:

$$1 = \theta_M \left(\pi^C_{t-1}\right)^{\Gamma_{M_t}\left(1-\epsilon^M_t\right)} \left(\pi^M_t\right)^{\epsilon^M_t - 1} + (1-\theta_M)\left(\frac{\widetilde{P}^M_t}{P^M_t}\right)^{1-\epsilon^M_t} \tag{83}$$

I finally point out that the presence of staggered import- and export prices imply that the model in the short run allows for deviation from the law of one price. That is, the same good can be sold at different exchange-rate adjusted prices in different countries.

2.9. Fiscal and monetary policy

The fiscal sector follows closely the setup in Pedersen and Ravn (2013). In addition to that setup, this model has three new fiscal instruments: Tax on the value of a residential unit, τ^H_t, a land tax, τ^L_t, and subsidy to interest rate payments on debt, κ^{RI}_t and κ^{RE}_t. The role of the public sector in the model is to raise taxes to be used for public consumption, public investment, and transfers. Public consumption, G_t, evolves according to:

$$\frac{G_t}{\overline{G}} = \left(\frac{G_{t-1}}{\overline{G}}\right)^{\rho_G} \exp\left(\varepsilon^G_t\right), \tag{84}$$

where ε^G_t is an i.i.d. stochastic process with mean zero and variance σ^G, $0 < \rho_G < 1$, and where $\overline{G}$ is given by:

$$\overline{G} = G^Y Y \tag{85}$$

where Y denotes total steady state output, and G^Y is the steady state share of government spending of goods and services produced by the intermediate goods producers and public production.

As for government investments, I assume that these are implemented with a lag. Specifically, I assume that an investment that is decided on in period t can only be initiated in period $t+M$ and is finalized in period $t+N$. In other words, I allow for *time to build* as well as *time to plan* as in Leeper et al. (2010). To this end, I need to distinguish between planned public investment, denoted by $I^{G,B}_t$, and implemented public investment denoted by I^G_t. Planned public investment evolves according to:

$$\frac{I^{G,B}_t}{\overline{I}^G} = \left(\frac{I^{G,B}_{t-1}}{\overline{I}^G}\right)^{\rho_{IG}} \exp\left(\varepsilon^{IG}_t\right), \tag{86}$$

where ε^{IG}_t is an i.i.d. stochastic process with mean zero and variance σ^{IG}, $\overline{I}^G$ is the steady state level of government investment, and $0 < \rho_{IG} < 1$. Due to the assumption of time to build, implemented investment only adds to the stock of public capital with a lag:

$$K^G_t = \left(1-\delta^G\right) K^G_{t-1} + I^{G,B}_{t-N}, \tag{87}$$

where $\delta^G > 0$ is the depreciation rate of public capital, and N is the number of periods it takes from an investment project is decided upon and until the investment is finalized. Note that investment-specific technology shocks also affect the accumulation of the public capital stock. This ensures a stable long-run relationship between the size of the public and the private capital stock along the balanced growth path.[16] Moreover, to take into account that planned investments affect the actual investment level (and hence, economic activity) with a lag, I let actual public investment be given by:

$$I_t^G = \sum_{i=M}^{N-1} \phi_i^I I_{t-i}^{G,B}, \tag{88}$$

with $\phi_i^I > 0$ determines how much is built in the respective period out of the total stock planned, and where M is the number of periods that pass from a project is decided on until it is initiated. I_t^G is thus a measure of all ongoing government investment projects at time t.

On the revenue side, the government raises seven different types of taxes and pays one subsidy: A labor income tax, τ_t^N, a capital income tax, τ_t^K, a value added tax, τ_t^{VAT}, a tax on domestic bond returns, τ_t^B, a tax on the value of the housing unit, τ_t^H, a land tax, τ_t^L, a subsidy on interest rate debt payment for the entrepreneur and the impatient household respectively, κ_t^{RE} and κ_t^{RI}, and a lump-sum tax T_t. By adjusting the tax rates, the government ensures that its intertemporal budget constraint, to be presented below, is always satisfied. This is done via the following type of tax rule:

$$\frac{X_t}{\overline{X}} = \left(\frac{X_{t-1}}{\overline{X}}\right)^{\rho_X} \varepsilon_t^X \left(\frac{B_{t-1}/Y_{t-1}}{\omega^D}\right)^{(1-\rho_X)e_X^{aux}\zeta_X},$$

for $X = \left\{\tau_t^N, \tau_t^K, \tau_t^{VAT}, \tau_t^B, T_t, \tau_t^H, \tau_t^L, \kappa_t^{R,E}, \kappa_t^{R,I}\right\}$. Here, $\overline{X}$ is the steady state value of X, while $0 < \rho_X < 1$. ε_t^X is a white noise shock associated with shocks to each tax rate X. Moreover, $\zeta_X > 0$ measures how strongly each fiscal instrument reacts to deviations of the debt-to-GDP ratio from its long-run target value, ω^D, reflecting that if the debt-to-GDP ratio overshoots its long-run target, one or more of the tax rates will eventually have to be raised. Finally, the dummy variable e_X^{aux} essentially switches the adjustment term on or off. I can set this to zero in order to undertake simulation experiments in which the government only starts raising taxes after a certain number of periods. Only lump-sum taxes react to deviations from the steady-state debt-level unless otherwise stated.

[16]The growth in investment-specific technology is related to the negative trend in the relative price of investment goods such as high-tech products, IT, software etc. Since many public investments also comprise such products, it seems reasonable to assume that public investments are also affected by the negative trend in the relative price of these.

I am now ready to present the government's intertemporal budget constraint, which takes the following form:

$$B_t^{DK} + TR_t = \frac{R_{t-1}^{DK}}{\pi_t^{DK}} B_{t-1}^{DK} + G_t + I_t^G, \tag{89}$$

where I have defined tax revenues TR_t as:

$$\begin{aligned} TR_t \equiv & -B_{t-1}^E \frac{\kappa_t^{R,E}\left(R_{t-1}^{L,E}-1\right)}{\pi_t^{DK}} - - + B_{t-1}^I \frac{\kappa_t^{R,I}\left(R_{t-1}^{L,I}-1\right)}{\pi_t^{DK}} + \tau_t^B \frac{\left(R_{t-1}^D-1\right)}{\pi_t^{DK}} D_{t-1}^P \\ & + T_t + \tau_t^{VAT} \frac{P_t^C}{P_t}\left(C_t^E + C_t^P + C_t^I\right) + \tau_t^N W_t^{tot} N_t^{tot} \\ & + \tau_t^H Q_t^H H_t^{tot} + \tau_t^L Q_t^L l_t + \tau_t^K\left(r_t^{K,Y} u_t^Y \mathcal{K}_t - \delta^{K,Y}\right) K_{t-1}^Y, \end{aligned} \tag{90}$$

The interest rate on Danish public debt is assumed to follow the process

$$R_t^{DK} = \left(R_{t-1}^{DK}\right)^{\rho_{DK}} \left(R_t^{ECB} \exp\left(\eta_{R^{DK}}\left(\frac{B_{t-1}^{DK}}{Y_{t-1}} - \omega^d\right)\right)\right)^{\rho_{DK}},$$

in which ρ_{DK} is a parameter calibrated to match the duration of total Danish public debt, $\eta_{R^{DK}} > 0$ is another parameter that determines by how much R_t^{DK} increases when the Danish public debt to GDP ratio deviates from its steady state, ω^d. This formulation captures that all else being equal higher debt levels increase the risk premium on the debt payments.

2.10. Market Clearing

I can write the aggregate resource constraint of the Danish economy as:

$$\begin{aligned} Y_t = & \left(C_t^{P,DK} + C_t^{I,DK} + C_t^{E,DK}\right) + I_t^{Y,DK} + I_t^{H,DK} + G_t + I_t^G \\ & + z^u\left(u_t^Y \mathcal{K}_t\right) K_{t-1}^Y + \frac{Adj_{t-1}^B}{\pi_t^{DK}} + Ex_t, \end{aligned} \tag{91}$$

, in which Adj_t^B denotes all adjustment costs in the banking sector, and where $z^u\left(u_t^Y \mathcal{K}_t\right) K_{t-1}^Y$ are the capital utilisation costs. Moreover, equilibrium in the housing market requires that

$$H_t^{tot} = H_t^P + H_t^I, \tag{92}$$

where the aggregate stock of housing evolves according to:

$$H_t^{tot} = \left(1-\delta^H\right) H_{t-1}^{tot} + X_t, \tag{93}$$

where $\delta^H > 0$ is the depreciation rate of the housing stock. The stock of land l_t is constant and may be normalized to 1.

Finally, Denmark's net foreign asset position is given by:

$$D_t^* = \frac{R_{t-1}^{ECB} \exp(-\psi_d^{D^*} \left(\frac{D_{t-1}^*}{Y_{t-1}} - \frac{\overline{D}^*}{\overline{Y}}\right))}{\pi_t^{DK}} D_{t-1}^* + Ex_t - Im_t, \tag{94}$$

so that net foreign asset holdings increase if Danish exports exceed imports in a given period, and imports must necessarily be given by

$$\begin{aligned} Im_t \quad \equiv \quad & \frac{P_t^C}{P_t}\left(C_t^E + C_t^P + C_t^I\right) - \left(C_t^{E,DK} + C_t^{P,DK} + C_t^{I,DK}\right) + \\ & \left(\frac{P_t^{IX}}{P_t} I_t^H - I_t^{H,DK}\right) + \left(\frac{P_t^{IY}}{P_t} I_t^Y - I_t^{Y,DK}\right), \end{aligned}$$

for market clearing to hold.

2.11. Trends and stationary equilibrium

As explained in the text above, I follow Pedersen and Ravn (2013) and assume that there is positive growth in total factor productivity, and a negative trend in the relative price of investment goods. In addition, I follow Iacoviello and Neri (2010) and assume that the trend growth rate in TFP in the construction sector is lower than in the goods production sector. The assumption of a lower productivity growth in the construction sector in combination with the presence of an input factor in construction which is in fixed supply, i.e. land, gives rise to an upward trend in the relative price of housing services, in line with the data for the sample period.

The model thus features three deterministic trends: Growth in total factor productivity in the production, A_t^Y, and in the construction sector, A_t^X, and in investment-specific technology, Z_t. This implies that aggregate macroeconomic variables, such as output and consumption, fluctuate around a balanced growth path. In order to solve the model, I therefore need to rewrite the equations in terms of detrended stationary variables and find the steady state of the stationary model. The explicit modeling of trends allows me to use trending data. An alternative is to specify the model without trends and use detrended data. However, the introduction of trends in the model firstly leave out questions of which method to use for detrending, HP-filtered data, linear trends etc., and provide insight into the relative trends among the variables. That is why a trending real house price can be observed for Danish data in the sample period. Also, detrended data necessarily implies loss of information for the same reason.

As explained in appendix (9), I can write the compounded trend growth of output as

$$d\Gamma_t \equiv \left(dA_t dZ_t^{\alpha^Y}\right)^{\frac{1}{1-\alpha^Y}},$$

where I have taken into account that both public and private capital in the production sector (but not in the construction sector) are affected by investment-specific technological progress. Here, dA_t is the rate of total factor productivity growth in the goods sector, and dZ_t is the rate of investment-specific technology growth. Variables that trend with output will in the model thus have a trend growth rate of $d\Gamma_t$.

I can likewise write the compounded trend growth rate of construction as

$$d\Gamma_t^X \equiv dA_t^X \left(d\Gamma_t\right)^{1-\alpha^X}.$$

The relative growth rate between residential investments and consumption goods can be derived as follows:

$$\frac{dX_t}{dY_t} = \frac{dA_t^X \left(d\Gamma_t\right)^{1-\alpha^X}}{d\Gamma_t} = dA_t^X d\Gamma_t^{-\alpha^X},$$

Hence, the real house price in the model grows by $\frac{d\Gamma_t^{\alpha^X}}{dA_t^X}$, and there is a positive growth rate in the real house price due to an input, which is in fixed supply namely land. This is in line with data, see figure (1). Further, as residential investment during the sample period has experienced practically zero productivity growth, the relative long-run growth rate can be written as

$$\frac{dX_t}{dY_t} = d\Gamma_t^{-\alpha^X}.$$

The real house price will thus grow faster than output due to land and due to lower productivity growth in the construction sector.

To obtain a stationary equilibrium, I then make the following transformations of the endogenous variables: I define $\widetilde{Y}_t = \frac{Y_t}{\Gamma_t}$ as the stationary counterpart of Y_t. Similarly, I define $\widetilde{C}_t^j = \frac{C_t^j}{\Gamma_t}$, $\widetilde{G}_t = \frac{G_t}{\Gamma_t}$, $\widetilde{T}_t = \frac{T_t}{\Gamma_t}$, $\widetilde{B}_t^j = \frac{B_t^j}{\Gamma_t}$, $\widetilde{D}_t^P = \frac{D_t^P}{\Gamma_t}$, and so forth, and I define $\widetilde{K}_t^Y = \frac{K_t^Y}{Z_t\Gamma_t}$, $\widetilde{I}_t^Y = \frac{I_t^Y}{Z_t\Gamma_t}$, and $\widetilde{K}_t^G = \frac{K_t^G}{Z_t\Gamma_t}$, where I have taken into account that capital and investment grow at a faster rate than output in the non-stationary model. I also define $\widetilde{\lambda}_t = \lambda_t\Gamma_t$ so as to ensure that the shadow price of consumption remains stable as the level of consumption grows, and I let $\widetilde{Q}_t^Y = Q_t^Y Z_t$, so that the relative price of investment goods changes over time along with investment-specific technological progress.

For the variables related to the construction sector, I use the growth rate of housing investment as explained above. I define: $\widetilde{X}_t = \frac{X_t}{\Gamma_t^X}$, $\widetilde{Q}_t^H = \frac{Q_t^H\Gamma_t^X}{\Gamma_t}$ (so that $\widetilde{Q_t^H X_t} = \frac{Q_t^H X_t}{\Gamma_t}$ grows at the same rate as output), $\widetilde{H}_t^P = \frac{H_t^P}{\Gamma_t^X}$, $\widetilde{H}_t^I = \frac{H_t^I}{\Gamma_t^X}$, $\widetilde{I}_t^H = \frac{I_t^H}{\Gamma_t}$. Appendix (9) describes in greater detail how the model is detrended and how the various variables are affected by the trends in the model.

3. Estimation

I will now estimate the parameters in the model primarily the structural shocks. This motivation is that this allows me to do counterfactual analysis' - change one subset of historical shocks with new shocks - and to do a historical shock decomposition of the macroeconomic variables.

3.1. Methodology

The econometric methodology is the same as described in Pedersen and Ravn (2013), and I refer to that publication and as an example Herbst and Schorfheide (2015) or Fernández-Villaverde (2010) for details. Here I note that I will apply Bayesian techniques to estimate parameters and shocks of interest. I however only partially estimate the model. That is, the objective in this paper is to estimate only the shock processes, autoregressive parameters and standard deviations of the structural shocks, and only a subset of the parameters. I will instead fix the rest of the (structural) parameters to the values estimated in Pedersen and Ravn (2013) and/or used in the analysis above. The parameters I have calibrated in the estimation are shown in table (3). In appendix (10) the measurement equations measuring the data to the model variables and the shocks I estimate are discussed. Appendix (11) describes data used in the estimation and how data is transformed before it is fed into the model. Appendix (12) analyses sensitivity and identification showing that the model is identified. Estimation and identification analysis is done using Dynare version 4.4.3 and Matlab 2015b.

3.2. Priors

I will start with some comments on the choice of priors. Following Del Negro and Schorfheide (2008), it is possible to divide the parameters into three different groups. The first group consists of parameters that can be identified from the steady-state relationships for instance the long-run growth rate, $d\Gamma$. Priors for as an example $d\Gamma$ are often based on sample averages, and I follow this approach to limit the amount of parameters I need to estimate. That is, I calibrate these parameters. The second group consists of parameters which control the endogenous propagation mechanisms without affecting the steady-state. These parameters belongs to as an example frictions, like the calvo-parameters for prices, θ_p. I set priors based on microeconometric evidence and previous results from other models estimated on data for other countries, following the literature. The choice of priors for this group of parameters can be found in tables (4)-(6).

The last group of parameters characterizes the law of motion of the exogenous shocks; the autoregressive parameters and the standard deviation of the innovations to the shocks. These parameters determine the volatility and persistence of the macroeconomic variables conditional on the other two groups of parameters. The challenge is that the shocks are unobserved.[17] Del Negro and Schorfheide (2008) suggests forming priors based on pre-sample observations about the dynamics of the observables and then map these into beliefs about the persistence and volatility of the exogenous shocks. I will follow this idea but not the approach. I will instead change some of the priors such that not only are moments and autocorrelations approximately fitted to data, but also such that the historical shock decomposition provides answers comparable to previous findings. This explain why not all the priors for the autoregressive parameters and the standard deviation for the innovations are the same.

3.3. Parameter estimates

The posterior mode of the estimated parameters are shown in tables (4)-(6). The right hand side of the tables shows the posterior distribution of the parameters. I highlight the following in observations. I find a negligible value for the preference parameter which governs the wealth effect, ν. This implies a low degree of wealth effect on labour supply. This contrasts the findings in Pedersen and Ravn (2013), who found a value of around $\frac{1}{2}$. However, this model is widely different and among many differences includes liquidity constrained households, which may explain the widely different finding.

On the nominal side of the economy, the estimate of the Calvo parameter for domestic prices is quite standard in the literature. Similar values are found for the Calvo-parameter for import-, export-prices and for wages. Price indexations are very low for domestic-, export, and import prices, which might reflect that Danish producers cannot rely on nominal exchange rates as an adjustment mechanism.

With regard to parameters on the financial side of the model, the estimation point to quite high costs of deviating from the optimal capital ratio, Φ^B, around 25, while the cost of changing the interest rates are somewhat lower compared to findings on data for the Euro Area, see Gerali et al. (2010). As will become clearer below, the relatively high adjustment costs can be due to the fact that the parameter Φ^B is identified, but poorly.

A way to check the quality of the estimation is by comparing the prior and posterior distributions of each parameter. This is also a method to evaluate the choices made with regards to the priors. This is done in figures (8) to (10). In general the figures show that the data is informative about the posterior distribution. That is, the posterior distribution is not equal to the prior, and hence the estimated parameters are not defined by their prior. As revealed by the figures, some priors are set quite tight. This reflect to some degree a necessity and the spirit of Bayesian estimation; without these tight priors the model would not work well in some important dimensions like impulse response functions or the historical shocks decomposition. See also the discussion in the previous section.

[17] However, it is possible to approximate some of these unobserved states to observed macroeconomic time-series, as I will show in section (6).

Convergence is addressed in appendix (14), and I refer to that appendix for further details.

3.4. Variance decompositions

I next analyse the model through a variance decomposition, VDC. The aim of the exercise is to provide a quantitative insight as to which structural shocks on average in the estimated model during the sample period give rise to variability in the endogenous variables in the model.

In tables (7) to (10) I show the contribution of the structural shocks in the model to the forecast error variances of a selected set of observed variables. For simplicity I only look at the variance decomposition for the 1$^{\text{st}}$, 4$^{\text{th}}$, 12$^{\text{th}}$ and infinite quarter horizon; the unconditional variance decomposition. In what follows I will mainly focus on the VDC for real GDP, PPI-inflation, the real house price, and construction. In figure (11(a)) to (11(d) the 39 shocks in the model are grouped into 6 groups: Demand, supply, fiscal policy, foreign shocks, markup shocks and shock in banking sector, and the combined contribution to the variance decomposition of the sub-group of shocks on the respective variables are shown for the different horizons.

On an overall level, foreign shocks are very important drivers of the Danish business cycle. The group of structural shocks originating in the two foreign economies (inflation, output, interest rates, and the effective exchange rate) account for between 65-75 percent of the variations in real GDP at all horizons. In particular, the shock to output in the euro-area is by far the largest contributor to movements in Danish GDP at all horizons. This should not be surprising, as Denmark is a very small and open economy with a fixed exchange rate towards the Euro.

While the finding that variations in Danish GDP and inflation are to a large extent driven by shocks from abroad may not seem very surprising, it does stand in contrast to the results of Justiniano and Preston (2010). This was also a finding in Pedersen and Ravn (2013), and I refer to that paper for a further discussion. The result in this paper is that including financial frictions and residential investments do not change this finding. I also highlight, that even though the transmission of shocks works through the interest rate, the variance decomposition shows that interest rate shocks in the eurozone are much less important than output shocks. The explanation is that movements in the euro zone interest rate, which is set according to a Taylor rule, are primarily driven by shocks to eurozone output and inflation, whereas monetary policy shocks are less important.

Regarding the real house price and construction, the picture is somewhat different. For the real house price demand plays are much larger role. This can be explained by the housing preference shock. As will become clearer in the sections to come, this shock has also been a major driver historically of the movements in the house price and to some extent captures features of the housing market not captured by the model like financial liberalisation, non-rational expectations of future house prices and possibly bubbles. Also, for the house price, fiscal policy shocks also plays are relatively large role. This is primarily due to the tax on housing. As explained in the model-section, the effective rate varies with the movements in the house price such that when the house price increases, the effective tax rate falls contributing further to the increase in the house price. This is what the variance decomposition captures. Further, as explained in box C in and (2.3.7) in the sections to come, the shocks that have the potential to move the real house price a lot are exactly the housing preference shock and taxes on housing. That is because they affect the housing demand symmetrically for both types of households. This is not so for as an example shocks to interest rates, as one household lends and the other borrows and consequently, and hence there are counteracting effects on total demand for housing. Construction largely moves in response to its own shock; the productivity shock in the residential investment sector.

3.5. Autocorrelations and correlation with GDP

I will in this section evaluate the estimated model through a comparison between the model implied autocorrelations and correlation with GDP and their empirical equivalents. It is not trivial to calculate the empirical moments as the time series are trending, see as an example DeJong and Dave (2007) for a text-book treatment. In what follows, I will not take a stand on the best way to detrend data and I will consequently compare the model to both HP-filtered data, Band-pass filtered data and data detrended with a constant trend.

I compare the correlations between GDP and other variables in the model at different horizons. These are shown in figure (12). In general, the instantaneous correlation between output and real variables, output, private consumption, exports, imports and labour market variables, are fitted quite well in the model. Depending on the empirical measure of inflation and real wages, the cross-correlations are not matched well to data. The model fits quite well the cross-correlation between output and both construction and the real house price, but not with respect to loans. I will come back to this point later. Here it suffices to note that all debt in the model is one-period debt. Hence, intuitively the model can have difficulties in matching the credit-cycles observed in data. But the model does seem to provide a reasonable fit to the time-series behaviour of real variables.

4. Analysing the model: Impulse response functions

In what follows, I will provide insight to the functioning of the model through impulse response functions, IRFs. Among the many shocks in the model, I will focus on the effects on the model economy of a temporary increase in public consumption of goods equivalent to 1 pct. of GDP. That is, the fiscal multiplier. The motivation is that discretionary fiscal policy is the main traditional economic policy instrument in an economy with a fixed exchange regime as monetary policy is devoted solely to maintaining the exchange rate policy. For a discussion of this point, see i.e. Pedersen and Ravn (2014). Together with the presentation of the model above, the transmission mechanism explained below can be seen as providing the main economic intuition behind some of the non-standard features in the model. I will start by presenting some of these non-standard features.

4.1. New fundamental channels in the model

The model has some new channels compared to a more standard DSGE model like the one in Pedersen and Ravn (2013) without housing, banking and financial frictions, which I from now on will denote the standard DSGE-model. I will start by providing some intuition to what can be expected from this model compared to smaller DSGE models without these features. Overall the model has the following additional channels: The collateral channel, the nominal debt channel, the financial friction channel, and the banking channel. Here I briefly explain how these channels can alter the impulse responses in the economy in response to the standard model. The channels are of course interrelated in general equilibrium but for simplicity it can be helpful to address them separately. Also, I will later try to recalibrate some parameters, both estimated and calibrated, to isolate the functioning of the various channels in the case of the analysis of a shock to government consumption.

The collateral channel or asset price channel works through the collateral constraint(s). Everything that affects the value of the collateral will affect the consumption of the impatient household and the entrepreneurs as well as investments. Intuitively, a looser collateral constraint or, equivalently, an increase in the value of the collateral the consumer can pledge increases the ability of the agents to borrow, which they will do, and consume and/or invest more. This channel was discussed in section (2.3.7). I will try to illustrate how the collateral channel works on the household side by comparing the fiscal multiplier with and without the collateral constraint.

The nominal debt effect also called the debt-deflation channel, cf. Fisher (1933), works through the collateral constraint(s) and *inflation*. A (surprise) increase in inflation lowers the real value of debt and hence enhances the ability of impatient consumers to pay back their debt. That is, a (positive) surprise inflation transfers wealth from lenders towards borrowers, who have a higher propensity to consume, which therefore increases total private consumption in the economy. I will try to illustrate how this channel works on the household

Model Name	Explanation	Calibration
Full Model (FM)	All features included and standard calibration	-
− Banking frictions (FMB)	Flexible rates, no capital ratio, small markups	Φ^B=0,$\Phi^{B,E}$=0
		$\Phi^{B,I}$=0, κ^B=0.0050,
		$\overline{\epsilon^{b,I}}$=10, $\overline{\epsilon^{b,E}}$=10,
− Debt deflation channel (FMD)	Leave out inflation in collateral constraints	-
− Housing sector (FMH)	No housing demand. No construction	$\overline{X} \approx 0$
		$\delta^H \approx 0, \beta^I = \beta^P$
		$\varsigma^{HI} \approx 0$
− Asset price channel (FMA)	Collateral constraints constants	$B^E = \overline{\Theta^E}$
		$B^I = \overline{\Theta^I}$
, with Rule-of-Thumb HH (FMRoT)	Impatient HH act as Rule-of-Thumb consumers	$\overline{\Theta^I} \approx 0$
		$\varsigma^{HI} \approx 0$

Table 1: This table shows various model calibrations used to derive and show key aspects of the model. The third column shows the changes in the model calibration vis-a-vis the standard model shown in table (3).

side by comparing the fiscal multiplier with and without inflation in the collateral constraint.

The financial friction or banking channel effect goes through the banking system. The banks in the economy determine the supply of credit through changes in interest rates and quantities. All shocks that increase bank leverage make the banks try to rebalance assets and liabilities by reducing loans and increasing deposits. I will try to illustrate how this channel works by recalibrating the parameters in the banking sector which determines how large these effects are.

4.2. A shock to discretionary fiscal policy: The Fiscal Multiplier – An analysis of policy changes

I will spend some time on the effects on the model economy of a temporary increase in public consumption of goods equivalent to 1 pct. of GDP. That is, the fiscal multiplier. The motivation is that discretionary fiscal policy is the main traditional economic policy instrument in an economy with a fixed exchange regime, as monetary policy is devoted solely to maintaining the exchange rate policy. For a discussion of this point, see i.e. Pedersen and Ravn (2014). This shock, the economic intuition behind the effects and the model presentation above, can be seen as providing the main economic intuition behind some of the non-standard features in the model.

I will consider the impact from the various new channels on the fiscal multiplier through a sequence of different calibrations. This is done to try as best as possible to identify the main drivers behind the economic effects and to quantify the relative strengths of the new features in the model compared to the model in Pedersen and Ravn (2013). Each calibration attempts to isolate the channels explained above and/or include some new features in the case of the Rule-of-Thumb consumers. It is however not possible to completely shut off all the new properties in the model as the models can not be fully nested. The multiplier in the full model includes all possible channels and the estimated parameters. I will compare the multiplier with in all 6 model versions presented in table (1).

For an in-depth theoretic treatment of the fiscal multiplier and a comparison between VARs, macroeconometric models and DSGE-models on Danish data, see Pedersen (2012). Ravn and Spange (2014) studies the empirical effect of discretionary fiscal policy in a structural vector autoregression, SVAR, and I set the persistence of the shock equal to the estimated persistence in that study, 0.8. The results are presented in figure (13). In the figures is plotted the average effect over a year.[18]

4.2.1 The fiscal multiplier: The complete model

I start by commenting on the full model, or the bench-mark model, which includes all the features explained above. On impact, GDP increases by around 0.70 pct. in the year following the 1 pct. increase in public expenditures. The effect on GDP dies off relatively quickly in the full model and is negligible after 2 years and zero after 3 years. After the third year, the effect on GDP turns negative all through to the end of the simulation period of 10 years approaching around -0.1 pct. around 5 years.

There are a host of explanations for this result and the underlying mechanisms. I will in what follows go through these. Here the different model calibrations can be helpful in disentangling the main drivers. Of the variables on the supply-balance, I will not go through imports and exports as their behaviour in response to fiscal policy shocks are quite standard, see also box E. I will instead focus on the labour market, investments, the real house price and residential investments and consumption for both patient and impatient households.

4.2.2 The labour market

In the New-Keynesian framework, output is in the short-run demand determined due to nominal rigidities. One underlying assumption is that firms produce all output demanded at given prices. If the firms could they would keep their price equal to a markup over marginal costs, see equation (22). Naturally, marginal costs are increasing in output due to decreasing returns to scale and increasing disutility of working at the labour supply-side. Hence, demand shocks will partly result in inflationary pressures through increasing marginal costs and partly in increases in output, as firms which can not increase prices will instead increase production and hence lower their markup; it is still profitable to produce as they set prices above marginal costs.

To be able to produce the output, they need more workers and hence employment increases and unemployment decreases. To be able to attract workers, who were willing to supply the given amount of work before the shock, to the given real wage, the firms must offer higher wages. This is the demand effect on the real wage, but not the only effect. The households also reacts to the fiscal policy shock as explained next.

[18]It is argued in Mountford and Uhlig (2009) and in Uhlig (2010) that short-run multipliers can be misleading. For ease of comparison with traditional macroeconometric models, I have however chosen to analyse the short-run multipliers and not the present value multipliers

4.2.3 Consumption and Exports

Patient households rationally see that they are poorer as the government has spent some of their money, which they need to pay back at a later stage.[19] As both non-durable consumption, housing and leisure are normal goods, they consequently lower their demand for these goods and work harder. This is the neo-classical *wealth-effect*. The increase in the labour supply is the supply-effect on the labour market, which puts downward pressure on the real wage. In equilibrium, as discussed above, the real wage increases.

Through the fixed exchange rate regime, the real rate decreases as inflation is not combated through higher policy rates. This all else being equal stimulates demand for interest rate sensitive goods today relative to the future, an *intertemporal-effect*. Further, higher prices all else being equal makes domestic consumption goods more expensive; a *substitution-effect*. Exports are crowded out due to loss of competitiveness, see box (E).[20]

Box E: Exports and model determinacy

Multiple solutions and determinacy, that is, that the model is not driven by pure ”sun-spot” shocks, is an issue in rational expectations model. for a textbook treatment, see Woodford (2003). For this box, it is sufficient to say, that in an economy with a floating exchange rate or a closed economy with inflation targeting, the monetary policy rule ensures is the mechanism which makes the model determinate. As an example, if the households suddenly expects for some reason – sunspots – that inflation will be higher in the future, then real rates fall, which makes it cheaper to tilt consumption from the future to the present increasing demand now. But rational households know that if monetary policy follows a Taylor-rule, then the policy rate will be increased by more than inflation, which increases real rates and hence depresses consumption and demand, and hence, the expectations will not be fulfilled, and hence, in a rational expectations equilibrium, the agents can not expect inflation in the first place to be higher in the future.

In contrast, in a small open economy with fixed exchange rate and consequently no independent monetary policy, foreign trade works as the stabilising mechanism. As an example, in this model the nominal exchange rate vis-a-vis the euro is constant and equal to one. Furthermore, all goods are traded and the terms of trade is therefore equal to the real exchange rate. Also, in the model the real exchange rate is calculated using producer prices, which determines cost of production and hence competitiveness. Disregarding home-bias, the law of one price says that a good in Denmark must costs same as the abroad through goods arbitrage, and (relative) purchasing power parity says then the real exchange rate needs to be 1 in the long-run.

This means that the price level in Denmark relative to that in the euro area is pinned down in the long run: Any temporary, relative increases in the Danish price level must be undone by a

[19]I recall that government debt is stabilised through lump-sum taxes

[20]This is another difference between a model for the fixed exchange rate regime and a floating rate regime with independent monetary policy. In such a regime, through the UIP condition it is not obvious that inflationary pressures lead to real exchange rate depreciation, as the interest rate also moves; it depends on the persistence of the shocks and expectations of the interest rate differential. However, in a fixed exchange rate regime, the policy rate stays constant and hence only the relative prices move.

period of relative deflation, i.e. a period where the price level in Denmark grows at a slower rate than in the euro area.

This implies that foreign trade is what stabilises inflation in the model and makes the model determinate, closes the model and ensure long-run stability. Temporary increase in the domestic countrys rate of inflation vs-a-vs the foreign country must be followed by a period of lower inflation; or relative deflation - bygones cannot be bygones in a fixed exchange rate regime, see also a similar discussion in Pedersen and Ravn (2014). This is also what is shown in figure (13): Inflation initially spikes upward, but it is already negative during the second year and henceforth gradually approaches zero from below - a deflationary period.

The wealth effect discussed above for the patient household also have implications for the consumption for the impatient households.The impatient household consumption is determined not only by current labour income, as for rule-of-thumb households, but also by their net-wealth defined as asset holdings net of debt. Their net-wealth decreases as the real house price falls. However, as can be seen from figure (13), consumption for impatient households does increase because of the extra labour income received as both the real wage and employment increase. That it, the impatient household is credit constraint but it does not need to borrow to finance all its purchases. It can also consume out of current income and both the real wage and employment increase initially and thus push up the impatient household consumption.

In the case of a fiscal policy shock, what makes consumption of the impatient household is the income it receives and not so much the value of its collateral constraint. When the collateral constraint is loosened, impatient households increase their consumption of non-durables, housing and leisure: A looser borrowing constraint allows the impatient households to get closer to satisfying the desire for early consumption dictated by their (relative) impatience. However, the collateral constraint tightens after the initial periods, which can be seen from behaviour of borrowing for the impatient household in figure (13). As noted above, this is because the real house price falls. This is in line with empirical evidence, see as an example Andrés et al. (2015).

4.2.4 The real house price and construction

The determination of the real house price in this model was explained in box C. To explain the relationship between housing demand, the house price and consumption on non-housing goods in the specific case of a fiscal policy shock, it can be useful to rewrite the patient households' Euler-equation for housing in the following way:

$$V_t^P \equiv Q_t^H U_{C,t}^P = U_{H,t}^P + \left(1-\delta^H\right)\beta^P E_t\left(V_{t+1}^P\right) \Leftrightarrow \tag{95}$$

$$V_t^P = E_t\left[\sum_{k=0}^{\infty}\left[\left(1-\delta^H\right)\beta^P\right]^k U_{Ht+k}^P\right], \tag{96}$$

where I for simplicity have left out taxes on housing, as they are constant in the case of a public expenditure shock given debt is stabilised by lump-sum taxes.

The right hand side of relation (96) is almost constant in case of temporary shocks, like a temporary fiscal policy shock: Housing is a slow-moving stock variable. Hence, whatever moves marginal utility of consumption, $U^{P}_{C,t}$, must to a first approximation imply a movement of opposite sign in the real house price, Q^{H}_{t}, see also Barsky et al. (2007).

In case of a fiscal policy shock, the wealth effect all else equal causes consumption of the patient households to fall and hence marginal utility to rise. The discounted sum of future marginal utilities of housing tend to increase, as patient households also decrease their demand for housing due to the wealth effect, but, as discussed above, this sum can be thought of a being constant. Goods inflation initially increase. Hence, the real house price is likely to decrease on impact though this depend on the strength of the wealth effect and the degree of nominal rigidities. In this model, the real house price falls, and as marginal utility of consumption stays elevated, the real house price does stays depressed as well. In short, the real house price follows the path of consumption of patient households. In the estimated model, the real house price falls on impact, continues falling until around 4 years, though the fall is relatively small.

Hence, in this model, the real house price is to a first approximation determined by the patient household though the impatient household also demands housing. The difference between the two households are of course that the impatient households are exposed to the collateral requirement. Specifically, the impatient household derives utility from having extra housing as it can be used to loosen the collateral constraint and hence to finance consumption. The multiplier on the collateral constraint consequently breaks the insight above as the multiplier falls on impact – a looser collateral constraint – reflecting an income effect and expectations of higher housing value, and a debt-deflation effect. The effect on the marginal utility and the house price can be deducted from the relationship below

$$
\begin{aligned}
V^{I}_{t} &\equiv Q^{H}_{t}U^{I}_{C,t} = U^{I}_{Ht} + \left(1-\delta^{H}\right)\beta^{I}E_{t}\left(V^{I}_{t+1}\right) + \gamma^{I}_{t} \Leftrightarrow \\
V^{I}_{t} &= E_{t}\left[\sum_{k=0}^{\infty}\left[\left(1-\delta^{H}\right)\beta^{I}\right]^{k}\left(U^{I}_{Ht+k} + \gamma^{I}_{t+k}\right)\right] \\
,\ \gamma^{I}_{t} &\equiv \Theta^{I}_{t}\mu^{I}_{t}\frac{E_{t}\left(Q^{H}_{t+1}\pi^{DK}_{t+1}\right)}{R^{L,I}_{t}}.
\end{aligned}
\tag{97}
$$

Again, marginal utility of housing is quasi-constant, and low frequency movements in the shadow value of housing, V^{I}_{t}, is consequently dominated by changes in γ^{I}_{t}, which falls as income increases in the case of an increase in public consumption, as discussed above. The impatient consumers use part of their extra income to buy more housing (not shown in the figure), both newly constructed and housing stock from patient households. Due to the extra housing the collateral constraint initially gets looser, borrowing increases and this fuels demand.

Patient households need to provide the extra resources consumed by the other households, which they do in response to the wealth effect discussed above. That is, the fall in wealth induces them to consume less, sell part of the impatient households' housing stock, reduce their accumulation of capital and work harder. The decrease in demand for houses is thus met by the patient households, who do not use their homes as collateral. But in equilibrium, the valuation of housing for the two households must be the same since housing is a homogeneous good traded on the same market to the same price. Hence, the marginal utility of housing needs to be equalised across households and this is achieved by reallocation of houses between the two households.[21] This reallocation increases the marginal utility of the housing stock in the hand of the patient households and decreases it for the impatient households, thus compensating for the higher collateral value enjoyed by the latter. Hence, there is a great degree of reallocation of housing between agents in response to shocks in the economy.

Construction falls together with the house price; a fiscal policy shock is also in the construction sector a demand shock but a negative demand shock in contrast to its effect on the goods sector.[22]

4.2.5 Investment

I next turn to investments and the response to the fiscal policy shock for the entrepreneurs. In a neo-classical model the response of investments depends among many things on the Frisch-elasticity and the persistence of the shock, see Baxter and King (1993). Intuitively, the more persistent is the shock, the bigger is the wealth-effect on consumption and labour supply, and the more the households are willing to supply labour the bigger the effect on output. Consequently, the more likely is it that investments are not crowded-out and the more likely is it that investment increases. The intuition goes through in this model which involves many new features including nominal rigidities, and the transmission mechanism from fiscal policy shocks to investments is more complicated.

Recalling that employment increases, unemployment decreases and the real wages are higher in response to the fiscal policy shock. Higher labour input and production in turn

[21] This can perhaps be seen most clearly from the standard optimality condition that the marginal rate of substitution between two goods must equal the relative prices. Now assume that utility is linear in consumption, such that marginal utility is constant, that the depreciation rate on housing is one, such that housing becomes a non-durable good leaving out the resale value in the consumers' first order condition. Then the patient household's marginal utility of housing equals the PCP deflated house price and the impatient household's marginal utility of housing equals the PCP deflated house price corrected for the lagrange multiplier on the collateral constraint.

[22] Notice also the existence of a third accelerator mechanism in the model besides the other two on the household- and firm side respectively. If a shock increases the house price and consequently construction, if the shock is not very short-lived, then the increase in construction leads to higher output though the indirect effect from demand for goods to the construction sector. This in turn increases the demand for housing through the same mechanism as described above and the circle continues. Notice, that the increase in construction puts even higher upward pressure on the labour market and wages and consequently on marginal costs and prices. This illustrates an interplay between housing demand, house prices and the goods sector.

implies higher marginal product of capital, which all else being equal induce firms to invest. [23] This effect is initially amplified by the fall in the real interest rate. Investments slowly returns to its steady-state as production and employment returns to their steady-state, while the real rate starts to increase.

4.2.6 Relative strength of various new channels

In figure (13) is shown the fiscal multiplier for the 5 calibrations presented in table (1). The response of GDP is not economically different across the different calibrations; it depends on other standard features of the model, like the degrees of crowding-out, the degree of nominal rigidities etc. The first year effects on GDP are within the range of 0.6 for the model without housing, and 0.85 for the model without the accelerator mechanism.

The main explanation of the relatively low multiplier in the model without housing is that consumption of impatient households is not crowded-in. The model without the asset price channel generates a higher multiplier, as without the financial accelerator mechanism, borrowing of firms does not fall. In the other models, and as discussed in previous sections, investments increases in response to the extra demand partly induced by a fall in the real interest rate. Specifically, the price of capital and land, Q_t^Y and Q_t^L, increases for one-period in the full model induced by the fall in the real rate. This is extra income for the entrepreneur, which consumes part of it, and investments increases. However, in the following periods, the price of capital and land falls depressing borrowing through the collateral constraint, which depends on expected future prices. This makes it more difficult to finance investments. Consequently, investments is higher in the model without the accelerator mechanism.

The accelerator mechanism on the household side of the economy makes the effect of rule-of-thumb households to consume relatively less compared with liquidity constrained households. That is because the rule-of-thumb households cannot use the extra purchases of housing to finance consumption. This limits the crowding-in of consumption.

I notice, that borrowing for the entrepreneur in the full model falls and stays below zero for many quarters. As borrowing is zero or negative in all the model versions, income for the entrepreneur must be the main determinant of the consumption profile.

In general the debt-deflation channel plays a minor role in the model. That can be due to the fact that all debt is one-period debt and hence, households and firms refinance every period. Obviously, the exclusion of the housing sector from the model changes the effect of construction and the behaviour of the real house price. It also affects consumption of both households through the channels discussed above.

[23] Recall from expression (48) that when the entrepreneur decide how much capital to invest in, the marginal costs in terms of utility of an extra unit of capital, $Q_t^Y \lambda_t^E$ is equated to the marginal benefit in terms of utility of expanding capital by one extra unit. The marginal benefit in turn consists of the after tax return from the extra capital plus the resale value in the next period minus compensation for utilisation rate plus the value of being able to expand the collateral constraint.

4.2.7 Loans and interest rates

In the discussion above, I have talked little about the banks. This is because the way banks operate in the model have little implications for shocks to the real economy, as can be seen in figure (13). That is, in "peace" time, the banks simply takes deposits, put a markup on the interest rates and lend out money. They do have implications for monetary policy shocks due to the presence of sticky interest rates, but the monetary transmission mechanism is not of main interest to a country with a fixed exchange rate regime. I will in the historical shock decomposition explain what happens when the banking sector stops working smoothly.

In the case of a fiscal policy shock, loans to impatient households increases due to the income effect. Impatient households purchase more housing both because they receive extra income from labour income, but also because they gain extra utility from housing purchases due to their collateral value. This allows them to increase the stock of loans. The lending is met by patient households which have decreased their consumption and consequently increased their savings, which they put into banks.

On the firm side, the firms take up less loans, as their collateral has fallen. This is so as even though the price of capital, or Tobin's Q, initially increases, what matters in the collateral constraint for the firms is the future expected value, see also the discussion in the model setup. Further, the price of land falls as construction falls. In total, borrowing by firms fall.[24]

5. In-sample forecasting – Predictions for key macroeconomic variables

The purpose of building a DSGE-model for Denmark has not been to develop a tool for forecasting. It is nonetheless interesting to analyse the forecast behaviour of the model. I leave a thorough analysis of the models' ability to forecast key macroeconomic variables for the future as that would require a sequential reestimation of the model using real-time data. I instead in the current work simply address forecasting using the estimated parameters and smoothed shocks and variables based on the full sample.

Specifically, I forecast the output gap 12 quarters ahead at each data point in time. The illustrative results can be seen in figure (14). As can be seen, the model has a tendency to forecast a closing of the output gap; when the output gap is positive, the model want GDP to fall back to its trend and vice-versa. This is natural: The model predicts that the economy will revert to its steady state. This is especially so during the very large negative output gap after the crisis. But the model does forecast a wider output gap in the beginning of the crisis during the period 2001-2004.

Looking at yearly growth rates in figure (15), the picture is, of course, the same: The

[24] I notice that the banks' lending rate does move no matter the calibration of the model. That is due to the endogenous risk premia on net foreign asset position needed to determine foreign debt in the model.

model wants GDP to return to its steady state. But interesting, the model does forecast a recession from around 2007-08, and when GDP starts to drop in mid-2008, the model seems to capture it, see figure (16). It, however, forecast the recession to be shorter lived, than it really was, and it cannot capture the depth of it.

As stated above, it remains to be seen whether the model can produce good forecasts. However, the benefit of the model in comparison with pure statistical models or models with less structure, is that the DSGE-models can give a structural interpretation of the forecast. That is, it can decompose the forecast, produced by the DSGE-model or by satellite models, into the models' structural shocks and hence give an economic interpretation of the forecasts. This has not been done in this paper. Instead, historical shock decompositions are produced and analysed in the following sections.

6. Historical decomposition of the real house price, inflation and output – An interpretation of the state of the economy

One advantage of an estimated, structural DSGE-model is the ability to decompose macroeconomic variables into the structural shocks in the model. A structural shock can be defined as uncorrelated shock with a clear economic interpretation. A historical shock decomposition can be a powerful storytelling device and can guide policy. As an example in an economy with inflation targeting, a positive output gab due to productivity shocks must be accommodated, while positive demand shocks must be combated, see Galí (2009). But this policy advice hinges on the ability to know which shocks that have hit the economy.

Though it is possible to decompose all the macroeconomic variables in the model into the structural shocks, I will focus on GDP, Y_t, the real house price, Q_t^H, and CPI-inflation, π_t^{CPI}. I will focus on the period mid-00s until today. The decompositions are shown in figures (29(a)) to (40(c)). For output and the real house price I have chosen to show both the decomposition of the output gap and the real house price gap together with the decomposition of the year-on-year growth rates. Here the gap is defined as deviations around a stochastic time trend. I have shown year-on-year growth rates to be able to compare the estimation result and decompositions directly to widely used definition of growth rates for the respective variables. But I have kept the figures, which are showing the respectively variables gaps, as the decompositions made through data, model, and the econometric techniques sees the gap, and I wish to compare the filtered shock's effects on the variables with empirical, observed counterparts. Before I turn to the actual decompositions, I will however first look at the smoothed innovations to the structural shocks in the model, as these provide the building block of the decomposition.

6.1. Smoothed shocks - The Financial crisis seen through the lenses of structural shocks

In figures (17) to (22), the smoothed Kalman filtered innovations, as an example ϵ^G, for the estimated shocks in the model are shown using the posterior mode parameters, while the process are shown in figures (23) to (28), as an example G_t. Productivity shocks, both temporary and permanent shocks in both sectors in the economy, all show a large fall during the outbreak of the financial crisis in 2008Q3. Output fell rapidly during that quarter, but employment fell by less. The model consequently filter out a large fall in productivity. Labour supply fell rapidly (seen as a positive shock), but not by enough to overturn this result. I will have more to say about this in the historical shock decomposition. Here it suffices to say, that the model filter out large negative supply shocks during the financial crisis, and large positive supply shocks before.

On the demand side, the model filter out a large spike in risk during 2008Q3. This leads to a fall in both consumption and investments. This spike coincide with negative preference shocks for housing and consumption, which further depressed consumption through wealth- and financial accelerator effects. Investment shocks contributed to this large fall in aggregate demand. Later, export shocks also clearly fell reflecting the marked fall in world trade. Hence, demand also clearly played role for the financial crisis.

While the Danish price markup shock did not react stronger than expected to the negative shocks coming from demand, the crisis is easy to detect in the rest of the markup shocks. Some of the negative demand came from the foreign sectors. These effects, lower demand for Danish exports, are however standard, and will consequently not be address further here.

While the outbreak of the crisis is clearly detectable in the smoothed innovations, the crises does not seem to be a non-linear event at least when domestic shocks only are considered. In table (2) moments for the innovations are shown together with test for normality. Overall, the Gaussian assumption for the structural shocks is supported by data though some shocks, notable the shock to preferences for housing, show signs of non-normality.

This conclusion, and the fact that productivity fell during the crisis is not supported by similar analysis on US-data, see as an example Lindé et al. (2015). As the authors point out, that the density of a linear Gaussian DSGE model can model the probability of the large events, as shown in table (2), means that the model considers the crisis as a likely event. Consequently, the model can be an instrument for analysing risk scenarios and/or stress test.

Distribution of structural shocks

Shock	Standard deviation (*100)	Skewness	Kurtosis	JB-test
ϵ^{G}	0.717	0.192⋆	2.75⋆	0
$\epsilon^{A_{Y,P}}$	0.0327	-0.139⋆	3.29⋆	0
$\epsilon^{A_{Y,T}}$	0.439	-0.0112⋆	3.19⋆	0
$\epsilon^{\epsilon^{W}}$	0.0555	0.0293⋆	3.05⋆	0
$\epsilon^{\epsilon^{X}}$	0.589	-0.172⋆	3.21⋆	0
$\epsilon^{\omega^{I}}$	1.18	-0.0752⋆	2.84⋆	0
ϵ^{C}	0.458	0.631	5.51	1
$\epsilon^{\epsilon^{P}}$	2.32	-0.182⋆	3.11⋆	0
$\epsilon^{\epsilon^{M}}$	0.58	-0.0888⋆	2.96⋆	0
ϵ^{Ex}	0.297	0.109⋆	3.67⋆	0
$\epsilon^{\mathcal{I}}$	1.01	-0.593	5.25	1
ϵ^{χ}	3.21	0.153⋆	3.13⋆	0
$\epsilon^{A_{X,T}}$	3.06	1.23	7.05	1
$\epsilon^{\mathcal{R}}$	0.184	-0.121⋆	3.1⋆	0
ϵ^{H}	2.7	0.109⋆	6.32	1

Table 2: **Distribution of estimated structural shocks:** This table reports empirical moments of a subset of the structural shocks. Shocks to fiscal policy instruments and the foreign economy are not considered. A ⋆ indicate significance at 5 pct. probability. That is, the skewness and/or kurtosis are not significantly different from 0 and 3 respectively. The JB-test is an indicator function that is 1 if null can be rejected and 0 if accepted. The null is that the respective shock is normally distributed.

6.2. Historical decomposition of output gap

In figure (29(a)) is shown the historical shock decomposition of real GDP. I will focus on the period 2000 to the present as this period compromise the build up to the financial crisis, the collapse of Lehman and the subsequent slump. The thick line in the figure shows the difference between actual GDP and an estimated stochastic trend. I will in what follows denote this difference the output gap. But as noted above, the distance between trend and actual GDP cannot be interpreted as the amount of "overheating" in the economy or inflationary pressure; as an example productivity shocks increase the output gap but tend to be deflationary when the output gap is defined as above. To talk about output gap and inflationary pressures, the gap needs to be defined relative to *natural* output, or the production in the economy that would prevail under flexible prices and wages. This is left for future work.

In the beginning of the sample, GDP is below trend but on an upward path after the low-growth period during the beginning of the 90's. This cycle ends at the beginning of the 00's with a mild economic downturn, which was succeeded by a large upturn culminating at the outbreak of the financial crisis. That crisis is clearly detectable in figure (29(b)) showing large negative year-on-year growth rates. The historical shock decomposition in figure (29(a)) is a device that can help to understand these movements. As the model has many shocks, the shocks are divided into subgroups, as shown in the figure: Financial shocks, markup shocks, demand, supply, foreign shocks and fiscal policy shocks. I will structure the text which follows according to this division of the shocks.

6.2.1 Output gap: Demand

One driver behind the Danish business-cycle during the period 2004 to the present has been demand. The subgroup labeled demand consists of the intertemporal consumption shock, investment shock, risk premia shock, import and export shocks and the housing preference shock.

Demand contributed positively to the output gap from the beginning of the 00's all the way to the outbreak of the financial crisis. When looking at the individual shocks in the group of shocks called demand in figure (29(c)) before the crisis, both export-, consumption-, risk premia-, and housing preference shocks contributed positively to output. I point to two observations from this period. Firstly, the housing preference shock, reflecting increases in the real house price, had spill-overs to the real economy. Secondly, the consumption shock only contributed negatively in 2009. The latter observation implies that although consumption fell during the initial phase of the crises, the recession that followed was so deep that consumption should have fallen by more. Likewise, the economy has in the late stage, 2013-15, recovered sufficiently such that consumption should in fact be higher that it is. Also, the temporary investment shock has contributed negatively to the output gap after all through the period after the outbreak of the crisis. This, according to the model, reflects that investments have been very depressed when taking into account the state of the economy.

Here I will emphasize the role played by the risk premia shock, as this is new compared to the analysis in Pedersen and Ravn (2013) and to standard DSGE-models without banking and financial frictions. I recall that a risk premia shock affects the loan rates the branches have to pay to the wholesale branch. Hence, a negative risk premia shock implies all else being equal, that the marginal cost of providing funds to firms and households falls, or, equivalently, that the markup the bank charges households and firms falls. This makes it cheaper to consume and invest and demand increases in the economy.

Does the picture in figure (29(c)) make sense with regard to the risk premia shock and the economic history for Denmark during that period? The historical shocks decomposition and the way the smoothed shocks are calculated can perhaps seem like a black box. I have therefore tried to compare the series for the smoothed structural shocks to observed equivalents when they exist to provide a sanity check on the decompositions of the variables. Natural equivalents to the risk premia shock are interest rate spreads. In the estimation of the model, I have used data for the loan rate to households. I compare in figure (32) the difference between this rate and the short term policy rate with the risk premia shock. A more obvious candidate than this spread could be a corporate bond spread to capture credit risk. This spread is however not used in the estimation, but would all else being equal have given a larger effect on the macroeconomic variables if they were included. I have also included the effect from the risk premia shock on the output gap in the figure.

In figure (32) is shown the standardised spread, the standardised risk premia shock, and the standardised effect on the output gap in the decomposition. It can be seen that these series are correlated. This is confirmed by the scatter plot and the simple regression on the observed spread. As an example, the observed risk premia spiked after Lehman in 2008 Q3, and this shows up in the shocks and its effect on GDP as a positive risk shock depressing output. The empirical spread captures credit risk. Hence, a risk premia shock in the model can be thought of as capturing perceptions of credit risk and/or the willingness to supply credit.

The effect of a consumption shock is standard: It is a shock to the desire to consume now and save less. That is, given the real rate of interest and expectations of future income and wealth, a positive consumption shocks all else equal implies that the consumer wants to consume now and save less. The consumption shock, however, only affects consumption, while the risk premia shock both affect consumption and investment in the same direction. In can therefore play a larger role for the business-cycle than consumption shock alone. Intuitively, the preference shock for consumption captures developments in consumption in excess of what the economic state or/and expectations of income in the future commands. A rough approximation could following this line of thought be the consumption-to-GDP ratio. As for the risk shock, I plot the effect of the consumption shock in the historical shock decomposition together with observed consumption-to-GDP ratio in figure (33). As can be seen from this figure, there is a close correlation between these two series. I do the same for investments in figure (34), and I reach the same conclusion.

6.2.2 Output gap: Supply

The group of shocks named supply includes technology shocks, both temporary and permanent in the goods sector, the labour supply shock, and temporary technology shocks in the housing sector. As can be seen from figure (29(a)) and (30(b)), this group has had a large impact on the output gap both in the upturn and in the downturn.

The interpretation of productivity shock and in general, capacity shocks, as a driver of increasingly unbalanced growth of the Danish economy during the build up to the financial crisis is, however, misleading. As also discussed in Pedersen and Ravn (2013), from a DSGE perspective the economy's response to productivity shocks is efficient and hence does not call for economic stabilization policy. Rather, it suggest that the actual overheating of the Danish economy during these years may have been smaller than previously thought as the economy seems to have been able to expand potential production and the level of capacity utilisation.

This points to the importance of identifying the fundamental drivers of the business cycle especially if policy makers wish to react to output gaps. I again point to the discussion in Pedersen and Ravn (2013) for the point of supply as a main driver of the Danish business-cycle during this period. In this paper, I will instead point to two new observations: The role of the labour supply shock, which for identification purposes was not included in the previously mentioned paper, and a closer comparison between technology shocks and observed productivity

The subgroup named ”supply” affect on the output gap is depicted in figure (30(a)). Within that subgroup, the labour supply shock, ϵ^X, has been a main driver pushing GDP up. This is not surprising. As can be seen in the left-hand figure in figure (35), which is based on the same idea as for the risk premia shock discussed in the previous section, there is a close correspondence between the labour supply shock and observed employment. And as can be seen in the right-hand figure, that relationship is statistically significant.

I next compare the group of technology shocks within the subgroup named supply, with hourly productivity in the private sector in the same way as I did for the risk premia shock discussed in the previous section.[25] This is shown in (36) together with the measure of productivity used in the estimation, employment in persons divided by GDP. This is, so to say, what the model sees in the estimation. For comparison, I have also included a measure of hourly productivity in private non-agricultural sector defined as net factor income divided by hours. This series is a more common measure of productivity.

The relationship is weaker than the relationship between the labour supply shock and employment, but still significant statistically and economically. Based on these observations, it is not surprising that the model assign a large role to technology shocks in the historical decomposition; the model sees that employment rose by less than GDP during the upturn pointing to positive productivity growth, and it saw that employment fell by less than GDP during the downturn.

6.2.3 Output gap: Markup shocks

I next look at the group which consists of shocks to the economys markup shocks. This subgroup consists of shocks to wage and price markups (domestic prices, import prices and export prices). In the build-up to the financial crisis, the markup shocks taken together affected the output gap negatively but then positively, see figure (30(c)). The first effect is primarily due to the export price markup and domestic price markup shock which were relatively high before the crisis. The economic intuition behind this observation is that domestic producers in those years utilized the extraordinary high domestic and foreign demand to increase their margins more than usually. At the later stage, domestic producers and exporters should in fact have increased their prices by even more than they did due to the extraordinary high demand during this period. They did not and that affected the output gap positively in 2007.

As for demand and supply, I next look at observed measures of markup shocks and compare them to the effect from the models’ equivalent on the output gap. This is done in figure (37) for the price markup. If I divide IMI, a measure of domestic determined inflation

[25]In the MONA database from Danmarks Nationalbank, this is the variable named *probx* defined as added value in the private non-agricultural sector divided by hours.

reflecting developments in profit and wages, by GDP, I can obtain a quite close correlation with this empirical measure of the price markup and the affect from the price markup on GDP. The same is true for the price of oil. I divide by GDP to account for the fact the markup shocks' effect in the decomposition in the model are relative to activity. The close correlation between the oil price and the domestic markup shocks reflects that the model neither has an oil price nor is the oil price used in the estimation. Domestic price markup shock consequently to some extend captures movements not captured by the model.

The wage markup shock plays little role in the shock decomposition according to the model. Further, I have not been able to find a relationship between the import price markup shocks and an observed equivalent. This can perhaps be due to its close correspondence to the import shock, see the identification and sensitivity analysis in appendix (12), or due to the fact that if the model captures well domestic prices, then import price markup can be thought of as residuals.

6.2.4 Output gap: Foreign sector

In figure (31(a)) is shown the contribution from the subgroup named foreign shocks. These shocks are shocks to GDP, inflation, monetary policy shock for both foreign economies and the UIP-shock. This group of shocks contributed slightly negatively to the Danish output gap just before the crisis and then negatively from around 2008. The latter can be explained by inflation shocks from the euro area and tighter monetary policy from Rest-of-World, which mitigated the increase in GDP from both foreign economies. Inflation shocks from the euro area implies that Danish goods have become relatively more expensive and hence less competitive pushing exports, and thus GDP, down.

This picture changes from 2007 from where inflation shocks from the euro area depresses the Danish output gap, while GDP-shocks from both economies mitigate this. Later in the crisis, Danish GDP have been held down by weak demand for Danish exports from the euro area, while expansive monetary policy have stimulated it.

6.2.5 Output gap: Fiscal Policy shocks

In figure (31(c)) is shown the contribution from the subgroup named fiscal policy to the output gap. This subgroup consists of public consumption and investments, labour-, housing-, and land tax as well as shocks to the interest rate deductions. According to the model, the fiscal policy shock with the largest impact on the output gap through the sample has been public consumption. Its impact has been asymmetrical: Though at some points during the boom it was negative - and hence contractionary -, if was a lot more positive during the downturn – that is positive and expansionary. The tax on the value of housing has only contributed a little to the output gap, but as shown later its impact on the real house price has been economically important.

Comparing the public consumption shock's effect on the historical shock decomposition to the public consumption to GDP ratio (standardised), there can be observed a quite close correspondence between the shocks the model finds have affected the business-cycle and fiscal policy, see figure (38).

6.3. Historical decomposition of the real house price

As presented in the introduction, the Danish economy experienced, like many other advanced economies, a large upswing in house prices during the period leading up to the financial crises and a large correction afterwards. In what follows I will through the lenses of the historical shock decomposition of the real house price analyse these developments. I will in what follows concentrate on the period 2000-2015.

In figure (39(a)), the quarter-to-quarter changes in the real house price around a stochastic trend are decomposed into the structural shocks in the model grouped as above for output. The model attributes a lot of the movements in the house price to the supply side; the supply side contributed negatively to the movements in the real house price. That is, supply did react to the increase in the real house price putting downward pressure on the real house price - the specific economic details were explained in the text; A Tobin's Q framework. Looking at the residential investments-to-GDP ratio, there is a close correspondence between data and housing supply shock, see figure (41). That is, increases in residential investments above its steady state translate into positive output gaps and negative real house price gaps - as expected. This implies that the boom in residential investments during the 00's put downward pressure on the real house price and upward pressure on real activity.

Looking at the demand factors, a big part of the housing boom can be explained by preference shock for housing, see figure (39(c)). The consumption shock also played a role for the house price during the boom. But the housing preference shock has been the dominating factor on the demand side for the development in house prices. Perhaps surprising, the housing preference shock has help to stem the fall in the house price during the slump after the crisis in 2011-2014. The explanation is, that even though the real house prices were flat during this period, the developments in the economy called for an even less positive development in the house price.

In figure (42), I compare the effect from the housing preference shock on both the output gap and the real house price in the historical shock decomposition with observed real house price. From this figure it can be seen that the housing preference shock drives both movements in the output gap and the real house price gap in very similar matters. Naturally, the effects are not of similar magnitude with the shock affecting the house price approximately 5 times more than the output gap.

What is the housing preference shock? Preference shocks are partly a measure of our ignorance; they capture what is not in the model. And in the case of Denmark during the 00's, there occurred, besides the changes to the taxation of housing, see next, some profound changes to the housing market namely the introduction and adaption of new type of mortgage contracts. Before the 00's almost 100 pct. of Danish household financed their purchases of housing using fixed-rate 30 year contracts with amortisation. That changed markedly during the 00's. In 2006, as an example, only 40 percent of the outstanding stock of loans were of the previous mentioned type. Instead 35 percent were interest-only loans, while 60 percent were variable rate loans. This changes is not captured in the model, and this is perhaps what the housing preference shock captures, see also Liu et al. (2013).

Looking at the markup shocks in figure (40(a)), the export price markup shock and the domestic price markup have been the most important drivers within this subgroup of shocks. In figure (40(c)), the contribution to the real house price from fiscal policy shocks are plotted. According to the model, fiscal policy contributed positively to the increase in the real house price before the outbreak of the financial crisis and put downward pressure after the crisis. The main contributor has been the tax on the value of the house, and is linked to the discussion in section (2.1): The effective tax on housing is kept constant at its 2002 level meaning that increasing prices does not imply increasing tax payments and vice versa. Hence, the tax on housing becomes pro-cyclical contributing to swings in house prices. According to the model around 1.5-2 percentage points of the real house price gap could in 2006 be attributed to this (unfortunate) way of taxing housing.

6.4. Historical decomposition of inflation

The year-to-year change in the CPI-deflator and its decomposition is shown in figure (43). The steady-state value of the CPI-deflator in the model is 1 meaning that the steady-state inflation is 0. To get actual observed inflation, around 1.85 pct. needs to be added to the corresponding two series. As I have not included measurement errors on the CPI-deflator, the smoothed series in the figure is equivalent to actual inflation plus the average inflation through the sample, 1.85 pct.

Inflation was relatively high during build up to the financial crisis and peaked around 2009. Surprisingly, it did not fall a lot during the outset of the financial crisis, and picked up again quite rapidly. It has however lately fallen to around -1 pct. pr. year, which, again, can be a counter-intuitive when the output gap is negative. The model attributes a large part of the movements in inflation to markup shocks and shocks from the trading partners. When looking at the group of shocks named foreign shocks in figure (44), it can be seen that the source of this imported inflation is less clear. In cases when prices in the Rest-of-World and the euro area increase faster than Danish prices, thus leading to improvements in the competitiveness of Danish goods and inflationary pressures, output, and hence demand form Danish exports, are low leading to negative pressures on Danish prices. But during the crisis period with falling demand for Danish exports from both economies, this lead to deflationary pressures for Danish goods prices, which only abated during 2012, when stimulative monetary policy both from the ECB and Rest-of-World provided stimulus to

Danish prices. The crisis in the euro area, low demand for Danish exports, has put downward pressures on Danish prices in the last part of the sample, 2014-2015.

Another contributor to low inflation in the latter part of the sample has been markup-shocks, which consists both of shocks to domestic-, import-, and export price markup as well as the wage markup, see figure (45). In the model, prices are set as a markup above marginal costs and prices are sticky. This implies that the markup, the difference between price and marginal costs, is endogenous and captures, as an example, that increases in costs due to higher wage demands, cannot fully and immediately be passed on to higher prices thus leading to lower markups. This is not what markup shocks capture. They capture movements in the relationship between prices and markups not captured in the endogenous movements in prices and marginal costs, and can be compared to residuals the difference being that the shock is identified and can be given an economic interpretation.

From figure (45) the main markup shocks within that group of shocks has been the markup shocks to Danish prices. I recall from section (2.7) that domestic prices are sticky and set as a markup above marginal costs. Hence, positive markup shocks for domestic prices means that domestic prices has increased faster than marginal costs would have called for. This is a negative shock for the output gap, as discussed above, as it implies that Danish firms have become less competitive. This means less inflationary pressures in the economy and hence that inflation is likely to fall in response to positive export markup shocks.

Lastly, in figure (46) is shown the impact from supply shocks on inflation. As expected, the correlation between inflation and supply is negative. As an example, the positive labour supply shocks during the boom mitigated inflationary pressures, while contributed to them, when the shock turn sign. The same is true for the productivity shocks. This emphasizes the need to be able to identify the background behind positive output gaps for a central bank, which job either is to target inflation or make sure that inflation is not higher than the inflation in the economy to which an exchange rate is pegged: If the shock which lead to a positive output gap is a supply shock, then it is most likely not inflationary and hence, the output gap reveals a false sign of an overheated economy.

7. Conclusion

I have in this document presented a structural, dynamic, general equilibrium model with forward looking agents with financial frictions, banking, and residential investments. The model was taken to data and its empirical fit was evaluated. I next used to the model to analyse policy changes trough the analysis of the effect in the model of the fiscal multiplier, I used a structural shock decomposition to interpret movements in key macroeconomic variables, and I briefly looked at the model's ability to generate predictions for GDP.

8. Tables and figures

	Value	Parameter explanation	Explanation
Private sector, preferences			
ς^{HI}	0.6	Pref. param. for housing, impatient households	Determines relative consumption of housing
ϑ_C	0.5	Degree of home-bias, consumption	Import content in total private consumption
$\vartheta_{Y,I}$	0.2	Degree of home-bias, investment	Import content in investments
$\vartheta_{X,I}$	0.8	Degree of home-bias, investment	Import content in investments, housing
β^P	0.9975	Subjective discount factor, patient households	SS real rate of 3.75 pct. pr. year
β^I	0.97	Subjective discount factor, impatient households	Lower than patient hh
β^E	0.97	Subjective discount factor, entrepreneur	Lower than patient hh
υ_C	1.5	Sub. ela in consumption	Estimated WP 88
υ_I	1.5	Sub. ela in investment	Estimated WP 88
χC	1.9998	Adj. costs, consumption	Estimated WP 88
χI	2	Adj. costs, investment	Estimated WP 88
Private sector, production			
$\delta^{K,Y}$	0.025	Private capital depreciation	Standard value and det. SS investments
ϵ^P	6	Sub. between goods	Markup of 20 pct., standard
ϵ^W	5.5	Sub. between labour types	Natural rate of unemployment of 4 pct.
ϵ_X	6	Sub. between goods, export	Markup of 20 pct., standard
ϵ^M	6	Sub. between goods, import	Markup of 20 pct., standard
α^Y	0.35	Capital share	SS labour share in production
α^X	0.25	Lands share in production of housing	Follows Grinderslev and Smidt (2006)
δ^H	0.025	Depreciation of housing	Standard and helps to determine SS housing
c_1	0.039035	Set st. $\bar{u}$=1	Simplification
c_2	2	Adj. cost. capital utilisation	From WP 88, sat as difficult to identify
γ^I	1.0045	SS inv. growth	Ensures SS inv. adj. costs are zero in SS
Private sector, others			
ω	0.5	Share of patient households	Iacoviello (2005), Quint and Rabanal (2013) mfl.
$\Psi_d^{D\star}$	0.005	Risk premia for inv. in foreign bonds	Small as possible to ensure stationarity of NFA
$\eta_{R^{DK}}$	0.02	Risk premia for inv. in Danish debt	Corsetti et al. (2013)
ω_X	0.5	Relativ size of foreign economies	Average trading weights
Public sector			
$\tau^{\bar{H}}$	0.0263	Tax on value of housing and land, see text.	Own calculations
$\tau^{\bar{L}}$	0.00263	Land tax	Mona-database
τ^N	0.31	Tax on labour income	Average
τ^K	0.25	Tax on capital income	Mona-database
τ^{VAT}	0.25	Consumption tax	Mona-database
τ^B	0.25	Tax on income from bonds	Mona-database
$\bar{B}$	1.8	Public Debt	Average over last 10 years. Pct. of GDP/4
$\bar{Ig}$	0.022886	Public investments	Average over sample. Pct. of GDP
$\bar{G}$	0.26	Public consumption	Average (consumption and services). Pct. of GDP
$\kappa^{\bar{R}E}$	0.3179	Tax deduction on debt, firms	Mona-database
$\kappa^{\bar{R}I}$	0.3179	Tax deduction on debt, households	Mona-database
$\eta_{R^{DK}}$	0.02	Duration Danish public debt	-
$\rho_{R^{DK}}$	0.8	Duration Danish public debt	-
δ^G	0.025	Depreciering of public capital	Standard
η	0.015	Public capital in production	Micro-data, Kamps (2004)
I_0	0	Immediate implementation of public inv.	Time-to-build, Leeper et al. (2010)
I_1	0.33333	q.=1,2,3 implementation of publlic inv.	Time-to-build, Leeper et al. (2010)
ζ_T	0.05	Ela. of lump-sum taxes wrt. public debt	Ensures non-explosive public debt
Banks			
δ^B	0.037026	Depreciering of bank capital	Set in calculations of SS
κ^B	0.09	Leverage ratio/Capital requirements	Basel III

Table 3: **Calibrated parameters:** This table shows that parameters which are calibrated in the estimation. WP 88 refers to Pedersen and Ravn (2013).

		Prior distribution				Posterior distribution					
		Type	Source	Mean	s.d.	Mean	Mode	s.d.	Median	5 pct.	95 pct.
Preferences and production											
Habit formation	h^C	Beta	SW07	0.5	0.1	0.77	0.72	0.042	0.78	0.73	0.82
Wage indexation	Γ_W	Beta	Medea	0.1	0.1	0.53	0.58	0.15	0.55	0.28	0.8
Price indexation	Γ_t	Beta	Medea	0.4	0.2	0.11	0.068	0.061	0.1	0.01	0.21
Import price indexation	Γ_{Mt}	Beta	Medea	0.5	0.2	0.2	0.12	0.086	0.18	0.036	0.36
Export price indexation	Γ_{Xt}	Beta	Medea	0.35	0.2	0.25	0.1	0.088	0.22	0.011	0.45
Calvo, wages	θ_w	Beta	SW03	0.7	0.05	0.67	0.68	0.049	0.67	0.58	0.75
Calvo, import price	θ_M	Beta	SW03	0.75	0.05	0.82	0.87	0.055	0.83	0.74	0.91
Calvo, export price	θ_X	Beta	SW03	0.75	0.05	0.74	0.78	0.037	0.76	0.62	0.84
Calvo, price	θ_p	Beta	SW03	0.75	0.013	0.73	0.73	0.013	0.73	0.71	0.76
Pref. for degree of wealth-effect	ν	Beta	Gali11	0.5	0.2	0.083	0.039	0.019	0.049	0.0026	0.23
Costs of deviating from cap-ratio.	Φ^B	Gamma	Gerali et al (2010)	25	5	23	25	5	23	15	30
Costs of changing interest rates, HH.	$\Phi^{B,I}$	Normal	Gerali et al (2010)	2	2	5.8	5.5	0.94	5.8	4.2	7.4
Costs of changing interest rates, E.	$\Phi^{B,E}$	Gamma	Gerali et al (2010)	5	2.5	9.8	8.8	1.6	9.7	6.9	13
Inv. adj. costs	κ_I	Gamma	-	5	1	4.4	3.7	0.82	4.3	2.8	5.9
Inverse Frisch Labour elasticity	ϕ	Gamma	-	5.5	2	6.5	5.9	1.4	6.1	3.2	9.8
Residential inv. adj. costs	κ_{IX}	Gamma	-	2	1	1.8	1.2	0.69	1.7	0.51	2.9
Elasticity of export	ϵ^{World}	Normal	-	5	0.5	1.9	3	0.6	2.1	-0.12	3.2

Table 4: **Estimated parameters:** This table reports the prior distribution and the posterior mode estimates of the structural parameters for the model. ADLS07 refers to Adolfson (2007), SW03 refers to Smets and Wouters (2003), Medea refers to Fernandez-Villaverde et al. (2009), Gali refers to Gali et al. (2011), and Gerali refers to Gerali et al. (2010).

		Prior distribution				Posterior distribution					
		Type	Source	Mean	s.d.	Mean	Mode	s.d.	Median	5 pct.	95 pct.
Persistence of shocks											
Persistence of temp. tech. shock	$\rho_{A_{Y,T}}$	Beta	SW07	0.75	0.05	0.81	0.82	0.042	0.81	0.74	0.88
Persistence of C. shock	ρ_{C}	Beta	SW07	0.85	0.1	0.77	0.72	0.073	0.79	0.63	0.9
Persistence of inv. shock	$\rho_{\mathcal{I}}$	Beta	SW07	0.7	0.05	0.69	0.6	0.059	0.69	0.61	0.78
Persistence of export shock	ρ_{Ex}	Beta	SW07	0.85	0.1	0.96	0.99	0.012	0.98	0.89	1
Persistence of import shock	ρ_{ω^I}	Beta	SW07	0.85	0.1	0.27	0.25	0.073	0.26	0.15	0.39
Persistence of riskpremia shock	$\rho_{\mathcal{R}}$	Beta	SW07	0.8	0.1	0.46	0.51	0.1	0.46	0.32	0.61
Persistence of labour income tax shock	ρ_{N}	Beta	SW07	0.5	0.1	0.26	0.25	0.063	0.25	0.16	0.36
Persistence of labour supply shock	ρ_{χ}	Beta	-	0.75	0.05	0.89	0.89	0.021	0.89	0.86	0.93
Persistence of pref. shock to housing	$\rho_{\mathcal{H}}$	Beta	SW07	0.85	0.1	0.9	0.89	0.047	0.9	0.83	0.97
Persistence of temp. tech. shock, housing	$\rho_{A_{T,X}}$	Beta	SW07	0.85	0.1	0.79	0.8	0.035	0.79	0.73	0.84
Persistence of shock to interest rate markup, HH.	$\rho_{\epsilon^{b,I}}$	Beta	SW07	0.85	0.1	0.84	0.91	0.086	0.85	0.69	0.98
Persistence of shock to interest rate markup, E.	$\rho_{\epsilon^{b,E}}$	Beta	SW07	0.85	0.1	0.27	0.25	0.074	0.26	0.15	0.38
Std. of shocks											
Tax on housing shock	ϵ^{τ^H}	Inv. gamma		0.012	2	0.03	0.029	0.0021	0.03	0.026	0.033
Tax on land shock	ϵ^{τ^L}	Inv. gamma		0.01	2	0.016	0.015	0.0011	0.015	0.014	0.017
Subsidy to interest rate, HH.	$\epsilon^{\kappa^{RI}}$	Inv. gamma		0.02	2	0.0045	0.0044	0.00031	0.0045	0.004	0.005
Subsidy to interest rate, E.	$\epsilon^{\kappa^{RE}}$	Inv. gamma		0.02	2	0.016	0.0092	0.0038	0.012	0.0045	0.03
Temp. tech shock, housing sector	$\epsilon^{A_{X,T}}$	Inv. gamma		0.02	2	0.031	0.029	0.0042	0.031	0.025	0.037
Shock to pref. for housing	ϵ^{H}	Inv. gamma		0.02	2	0.035	0.037	0.0095	0.033	0.018	0.051
Labour supply shock	ϵ^{χ}	Inv. gamma		0.02	2	0.031	0.027	0.0065	0.029	0.015	0.047
Markup shock, interest rate, HH.	$\epsilon^{b,I}$	Inv. gamma		0.02	2	0.014	0.0089	0.0034	0.011	0.0049	0.025
Markup shock, interest rate, E.	$\epsilon^{b,E}$	Inv. gamma		0.02	2	0.52	0.48	0.074	0.52	0.39	0.66
Perm tech shock	$\epsilon^{A_{Y,P}}$	Inv. gamma		0.001	0.1	0.00071	0.0007	0.00015	0.0007	0.00045	0.00097
Wage markup shock	ϵ^{ϵ^W}	Inv. gamma		0.02	2	0.17	0.0093	0.0038	0.12	0.0048	0.38
Temp. tech shock	$\epsilon^{A_{Y,T}}$	Inv. gamma		0.01	2	0.0052	0.0059	0.00075	0.0051	0.0039	0.0064
Consumption shock	ϵ^{C}	Inv. gamma		0.02	0.1	0.008	0.0073	0.0022	0.0074	0.0041	0.012
Price markup shock	ϵ^{ϵ^P}	Inv. gamma		0.02	2	0.024	0.023	0.0021	0.024	0.02	0.028
Investment shock	$\epsilon^{\mathcal{I}}$	Inv. gamma		0.02	2	0.024	0.09	0.027	0.018	0.0045	0.048
Export shock	ϵ^{Ex}	Inv. gamma		0.02	2	0.0076	0.0078	0.0023	0.0074	0.0046	0.011
Import shock	ϵ^{ω^I}	Inv. gamma		0.02	2	0.012	0.012	0.00097	0.012	0.011	0.014
Import price markup shock	ϵ^{ϵ^M}	Inv. gamma		0.02	2	0.016	0.016	0.0047	0.015	0.008	0.022
Export price markup shock	ϵ^{ϵ^X}	Inv. gamma		0.02	2	0.013	0.011	0.0027	0.011	0.0047	0.025
Riskpremia shock	$\epsilon^{\mathcal{R}}$	Inv. gamma		0.02	2	0.0027	0.0028	0.00026	0.0027	0.0023	0.003
Public consumption shock	ϵ^{G}	Inv. gamma		0.01	2	0.0072	0.0072	0.0006	0.0072	0.0062	0.0082
Public investment shock	ϵ^{IG}	Inv. gamma		0.075	2	0.065	0.064	0.0047	0.065	0.057	0.072
Tax on labour income shock	ϵ^{N}	Inv. gamma		0.06	2	0.013	0.013	0.00093	0.013	0.011	0.014
EA price shock	$\epsilon^{\Pi,EA}$	Inv. gamma		0.01	2	0.0044	0.0043	0.00039	0.0043	0.0037	0.005
EA output shock	$\epsilon^{y,EA}$	Inv. gamma		0.01	2	0.0011	0.0011	8e-05	0.0011	0.00097	0.0012
ECB policy rate shock	$\epsilon^{r,EA}$	Inv. gamma		0.01	2	0.0031	0.0029	0.00037	0.003	0.0025	0.0036
RoW output shock	$\epsilon^{y,ROW}$	Inv. gamma		0.01	2	0.0016	0.0015	0.00014	0.0016	0.0013	0.0018
RoW price shock	$\epsilon^{\Pi,ROW}$	Inv. gamma		0.012	2	0.0022	0.0022	0.00019	0.0022	0.0019	0.0025
RoW interest rate shock	$\epsilon^{r,ROW}$	Inv. gamma		0.01	2	0.0019	0.0019	0.00017	0.0019	0.0016	0.0022
UIP shock	ϵ^{UIP}	Inv. gamma		0.01	2	0.0035	0.0031	0.00065	0.0034	0.0023	0.0047

Table 5: **Estimated parameters:** This table reports the prior distribution and the posterior mode estimates of the structural parameters for the model. ADLS07 refers to Adolfson (2007), SW03 refers to Smets and Wouters (2003), Medea refers to Fernandez-Villaverde et al. (2009), Gali refers to Gali et al. (2011), and Gerali refers to Gerali et al. (2010).

		Prior distribution				Posterior distribution					
		Type	Source	Mean	s.d.	Mean	Mode	s.d.	Median	5 pct.	95 pct.
Std. of measurement errors											
Inflation, RoW	*pieRoWt*	Inv. gamma		0.01	2	0.0022	0.0021	0.00031	0.0021	0.0017	0.0027
Output, RoW	*YRoWt*	Inv. gamma		0.02	2	0.0036	0.0035	0.00042	0.0036	0.0029	0.0043
Interest rate, RoW	*RRoWt*	Inv. gamma		0.015	2	0.0025	0.0026	0.00029	0.0025	0.0021	0.003
Inflation, EA	*pieEAt*	Inv. gamma		0.02	2	0.0045	0.0047	0.0007	0.0045	0.0035	0.0056
Output, EA	*YEAt*	Inv. gamma		0.02	2	0.0033	0.0032	0.00035	0.0033	0.0027	0.0039
Interest rate, EA	*RECBt*	Inv. gamma		0.02	2	0.0076	0.0073	0.00055	0.0075	0.0066	0.0085
Unemploytment	*Ut*	Inv. gamma		0.005	2	0.0015	0.0014	0.00026	0.0015	0.001	0.002
Employment	*Npt*	Inv. gamma		0.005	2	0.0017	0.0015	0.0003	0.0016	0.0011	0.0022
Wages	*Wpt*	Inv. gamma		0.005	2	0.0047	0.0055	0.00056	0.0048	0.0029	0.0064
Output	*Yt*	Inv. gamma		0.005	2	0.0051	0.0047	0.00057	0.0051	0.004	0.0061
Consumption	*Ct*	Inv. gamma		0.0025	2	0.0077	0.0012	0.00048	0.0077	0.0054	0.011
Investments	*It*	Inv. gamma		0.005	2	0.065	0.058	0.0061	0.065	0.057	0.074
Import	*Mt*	Inv. gamma		0.005	2	0.019	0.017	0.002	0.019	0.016	0.022
Export	*EXt*	Inv. gamma		0.005	2	0.017	0.016	0.0014	0.017	0.015	0.02
Import prices	*pm*	Inv. gamma		0.005	2	0.016	0.016	0.0012	0.016	0.014	0.018
Export prices	*pe*	Inv. gamma		0.005	2	0.015	0.016	0.0012	0.015	0.0092	0.019
Exchange rate	*fx*	Inv. gamma		0.01	2	0.0087	0.0087	0.00079	0.0087	0.0074	0.01
Investment deflator	*pi*	Inv. gamma		0.005	2	0.028	0.027	0.002	0.027	0.024	0.031
Deflator for residential investment	*PXt*	Inv. gamma		0.005	2	0.024	0.023	0.0017	0.024	0.021	0.027
Loans to entrepreneurs	*BEt*	Inv. gamma		0.05	2	0.017	0.016	0.0012	0.017	0.015	0.019
Loans to households	*BIt*	Inv. gamma		0.05	2	0.032	0.05	0.013	0.029	0.014	0.05

Table 6: **Estimated parameters:** This table reports the prior distribution and the posterior mode estimates of the measurement errors in the estimation of the model. See appendix (11) for a description of the data used in the estimation.

Horizon: 1^{st} **q.**

	GDP	Consumption	Investment	Export	Import	Inflation	Wages	Employ.	Unemp.	Construction	Houseprice	Loans	Rate, HH.
Public consumption	2.03	0.09	0.05	0.01	0.07	0.11	0.12	1.73	1.69	0.00	0.01	0.03	0.00
Temp. tech.	0.00	0.57	0.24	0.48	0.31	5.10	0.00	14.49	14.31	0.13	0.46	0.15	0.01
Tax, capital	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Tax, consumption	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Public inv.	0.00	0.00	0.01	0.00	0.01	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00
Intertemporal con.	1.03	5.48	0.14	0.05	0.96	0.24	0.28	0.87	0.94	0.03	1.93	0.09	0.00
Tax, bonds	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Tax, labour	0.01	0.05	0.00	0.00	0.01	0.00	0.00	0.01	0.08	0.00	0.02	0.93	0.00
Risk	5.04	13.77	8.73	0.01	12.83	0.22	0.41	4.31	4.41	0.20	7.52	3.17	58.74
Investment	0.10	6.20	19.53	0.35	1.55	0.40	0.11	0.09	0.09	0.01	0.00	0.00	0.01
Import	1.08	0.40	1.21	0.12	0.20	1.58	0.62	0.92	0.94	0.15	0.15	0.75	0.00
Labour supply	0.00	0.00	0.00	0.02	0.00	0.05	0.86	0.00	3.41	0.00	0.02	0.02	0.00
Price markup	8.31	6.28	7.76	2.95	10.16	44.65	42.40	7.04	6.14	1.06	8.62	47.23	0.01
Wage markup	0.00	0.00	0.00	0.00	0.00	0.00	0.07	0.00	0.00	0.00	0.00	0.00	0.00
Export	7.78	2.63	2.22	16.60	2.69	1.03	1.51	6.64	6.46	0.02	0.58	1.40	0.03
RoW, inflation	0.86	0.26	0.19	1.95	0.26	0.08	0.07	0.74	0.72	0.00	0.02	0.12	0.00
RoW, GDP	2.10	0.00	0.05	11.57	0.02	0.07	0.07	1.79	1.75	0.01	0.06	0.01	0.00
RoW, Interest rate	15.11	12.23	11.66	14.54	13.72	2.79	2.94	12.89	12.57	0.07	2.03	5.40	0.10
EA, inflation	7.51	4.63	4.99	9.72	5.49	3.89	3.68	6.40	6.13	0.03	0.25	0.94	8.50
EA, GDP	28.46	29.26	28.26	20.16	32.60	35.72	31.06	24.22	22.83	0.07	2.51	1.81	6.03
EA, Interest rate	6.09	7.31	6.72	3.06	8.07	1.04	1.12	5.20	5.10	0.03	1.70	2.40	24.24
Export price markup	8.33	4.72	4.53	12.59	5.14	2.12	3.06	7.10	6.85	0.00	0.55	1.38	0.05
Import price markup	0.47	1.35	0.66	0.07	1.36	0.15	0.32	0.40	0.42	0.01	0.00	0.31	0.00
Perm. tech.	0.28	0.38	0.47	0.04	0.53	0.12	10.63	0.54	0.58	0.07	1.01	0.01	0.01
Perm. investment	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
UIP	3.84	2.08	1.89	5.68	2.28	0.49	0.50	3.27	3.20	0.01	0.33	1.06	0.02
Capital util.	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Housing preference	0.67	1.14	0.15	0.00	0.70	0.04	0.05	0.57	0.61	3.79	48.08	20.17	0.01
Bank markup, HH.	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Bank markup, E.	0.02	0.05	0.05	0.00	0.06	0.00	0.00	0.02	0.02	0.00	0.01	0.00	2.22
Tax, housing	0.53	1.08	0.32	0.01	0.81	0.04	0.06	0.45	0.48	1.63	16.82	10.43	0.01
Tax, land	0.01	0.02	0.02	0.00	0.02	0.00	0.00	0.01	0.01	0.00	0.00	0.00	0.00
Tax, deduction, E.	0.00	0.00	0.01	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Tax, deduction, HH.	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Loan-to-value	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Perm. tech, housing	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Temp. tech, housing	0.34	0.01	0.14	0.01	0.16	0.05	0.06	0.29	0.26	92.68	7.34	2.18	0.00

Table 7: **Variance decompositions:** This table reports the forecast-error-variance decomposition of selected variables at the 1st quarter horizon. The decomposition is conducted only for the structural shocks part of the forecast errors.

Horizon: 4^{th} q.

	GDP	Consumption	Investment	Export	Import	Inflation	Wages	Employ.	Unemp.	Construction	Houseprice	Loans	Rate, HH.
Public consumption	0.93	0.07	0.02	0.04	0.04	0.08	0.01	0.96	0.97	0.02	0.08	0.05	0.01
Temp. tech.	0.87	0.31	0.06	1.57	0.14	3.37	2.06	6.85	7.23	0.54	2.52	3.12	0.01
Tax, capital	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Tax, consumption	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Public inv.	0.13	0.01	0.00	0.01	0.00	0.01	0.00	0.13	0.13	0.00	0.01	0.00	0.00
Intertemporal con.	1.19	12.38	0.16	0.20	1.70	0.23	0.03	1.28	1.48	0.02	0.84	2.05	0.01
Tax, bonds	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Tax, labour	0.01	0.05	0.00	0.00	0.01	0.00	0.00	0.01	0.02	0.00	0.01	0.20	0.00
Risk	3.40	15.64	3.94	0.02	9.86	0.16	0.06	3.14	3.55	0.10	3.54	8.79	49.78
Investment	2.53	3.07	50.26	1.68	19.41	0.81	1.20	0.15	0.11	0.13	0.27	7.90	0.12
Import	0.56	1.43	1.56	0.76	0.84	1.44	0.31	0.49	0.49	0.32	0.53	1.57	0.02
Labour supply	0.06	0.01	0.00	0.08	0.01	0.06	0.41	0.06	2.35	0.02	0.07	0.08	0.00
Price markup	17.87	10.99	6.37	8.59	14.63	27.99	84.15	16.89	13.12	2.70	17.85	62.01	0.09
Wage markup	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00
Export	4.55	2.37	1.44	6.75	2.02	0.92	0.25	4.47	4.50	0.01	0.24	0.34	0.03
RoW, inflation	0.51	0.21	0.08	0.82	0.16	0.06	0.00	0.51	0.52	0.00	0.01	0.03	0.00
RoW, GDP	1.58	0.01	0.07	5.74	0.03	0.07	0.01	1.67	1.70	0.03	0.16	0.14	0.02
RoW, Interest rate	11.52	9.98	6.19	11.06	9.41	2.72	0.32	10.99	11.19	0.04	0.88	1.45	0.10
EA, inflation	7.37	4.63	3.47	8.86	4.74	4.88	0.42	7.06	7.11	0.09	0.20	0.21	11.31
EA, GDP	29.05	25.49	17.98	30.42	24.84	53.16	3.54	27.36	27.27	0.08	1.00	3.06	14.62
EA, Interest rate	4.64	5.20	3.34	3.66	5.02	0.99	0.13	4.36	4.44	0.01	0.70	0.65	20.98
Export price markup	9.60	4.31	2.75	16.88	3.71	2.15	0.52	9.31	9.37	0.02	0.37	0.38	0.04
Import price markup	0.17	0.64	0.75	0.43	0.56	0.20	0.59	0.17	0.17	0.10	0.06	0.10	0.00
Perm. tech.	0.25	0.18	0.28	0.18	0.32	0.16	5.91	1.02	1.06	0.18	1.10	1.05	0.02
Perm. investment	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
UIP	2.14	1.67	1.00	2.17	1.55	0.44	0.05	2.07	2.11	0.01	0.14	0.27	0.02
Capital util.	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Housing preference	0.36	0.61	0.07	0.01	0.36	0.03	0.01	0.36	0.40	8.00	41.13	3.83	0.01
Bank markup, HH.	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Bank markup, E.	0.01	0.03	0.01	0.00	0.02	0.00	0.00	0.01	0.01	0.00	0.01	0.05	2.81
Tax, housing	0.30	0.68	0.14	0.02	0.46	0.03	0.01	0.29	0.32	4.23	17.05	2.25	0.01
Tax, land	0.00	0.01	0.01	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.00
Tax, deduction, E.	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Tax, deduction, HH.	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Loan-to-value	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Perm. tech, housing	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Temp. tech, housing	0.40	0.03	0.06	0.02	0.15	0.04	0.01	0.39	0.38	83.33	11.22	0.41	0.00

Table 8: **Variance decompositions:** This table reports the forecast-error-variance decomposition of selected variables at the 4th quarter horizon. The decomposition is conducted only for the structural shocks part of the forecast errors.

Horizon: 12^{th} **q.**

	GDP	Consumption	Investment	Export	Import	Inflation	Wages	Employ.	Unemp.	Construction	Houseprice	Loans	Rate, HH.
Public consumption	0.50	0.06	0.01	0.06	0.02	0.07	0.01	0.67	0.69	0.03	0.09	0.04	0.01
Temp. tech.	1.91	0.54	0.09	1.96	0.42	2.96	1.76	4.68	5.03	0.79	2.58	2.10	0.00
Tax, capital	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Tax, consumption	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Public inv.	0.09	0.01	0.00	0.01	0.00	0.02	0.00	0.12	0.12	0.00	0.01	0.01	0.00
Intertemporal con.	0.72	12.60	0.31	0.41	1.19	0.20	0.01	1.01	1.23	0.28	1.13	1.75	0.01
Tax, bonds	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Tax, labour	0.00	0.03	0.00	0.00	0.01	0.00	0.00	0.01	0.02	0.00	0.01	0.06	0.00
Risk	1.81	11.50	2.18	0.04	5.85	0.18	0.15	2.21	2.52	0.17	2.50	2.61	36.73
Investment	10.32	3.34	42.61	8.59	23.05	0.84	6.26	0.14	0.11	1.36	3.01	46.24	0.34
Import	0.53	1.57	1.09	0.43	0.96	1.27	0.45	0.46	0.45	0.27	0.35	0.81	0.01
Labour supply	0.27	0.07	0.01	0.27	0.06	0.05	0.50	0.36	1.85	0.09	0.19	0.17	0.00
Price markup	16.65	12.45	5.10	7.77	13.21	24.96	73.41	18.45	14.22	2.49	12.83	27.39	0.07
Wage markup	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Export	3.11	2.18	1.64	4.51	1.73	0.70	0.82	3.53	3.59	0.02	0.17	0.15	0.04
RoW, inflation	0.27	0.15	0.04	0.37	0.09	0.05	0.00	0.35	0.37	0.00	0.01	0.01	0.00
RoW, GDP	0.90	0.01	0.10	2.71	0.07	0.18	0.00	1.21	1.25	0.03	0.13	0.18	0.02
RoW, Interest rate	7.45	7.77	4.10	7.10	6.24	2.09	0.46	8.36	8.70	0.03	0.56	1.15	0.18
EA, inflation	6.61	5.08	4.02	7.16	4.99	4.21	0.58	6.91	7.16	0.34	0.40	1.64	11.22
EA, GDP	34.67	29.53	26.51	38.06	30.37	58.93	4.16	33.77	34.74	1.88	2.33	9.20	34.34
EA, Interest rate	2.99	3.86	2.04	2.67	3.13	0.76	0.18	3.28	3.41	0.02	0.48	0.47	14.65
Export price markup	7.77	3.90	2.44	14.46	2.76	1.80	1.15	8.89	9.09	0.05	0.59	0.23	0.04
Import price markup	1.20	2.59	6.53	1.61	4.01	0.16	2.75	0.72	0.58	0.47	0.12	0.70	0.02
Perm. tech.	0.28	0.44	0.34	0.43	0.22	0.14	7.24	2.58	2.46	0.33	1.15	2.67	0.04
Perm. investment	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
UIP	1.33	1.30	0.66	1.29	1.02	0.34	0.07	1.53	1.59	0.00	0.09	0.20	0.03
Capital util.	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Housing preference	0.20	0.49	0.04	0.02	0.20	0.03	0.01	0.25	0.28	11.28	33.29	1.14	0.01
Bank markup, HH.	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Bank markup, E.	0.00	0.02	0.01	0.00	0.02	0.00	0.00	0.00	0.00	0.00	0.01	0.04	2.23
Tax, housing	0.17	0.50	0.08	0.04	0.26	0.02	0.02	0.20	0.23	9.66	19.84	0.87	0.01
Tax, land	0.00	0.01	0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.02	0.00
Tax, deduction, E.	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Tax, deduction, HH.	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Loan-to-value	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Perm. tech, housing	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Temp. tech, housing	0.25	0.03	0.03	0.02	0.10	0.04	0.01	0.32	0.31	70.38	18.12	0.14	0.00

Table 9: **Variance decompositions:** This table reports the forecast-error-variance decomposition of selected variables at the 12th quarter horizon. The decomposition is conducted only for the structural shocks part of the forecast errors.

Horizon: ∞ q.

	GDP	Consumption	Investment	Export	Import	Inflation	Wages	Employ.	Unemp.	Construction	Houseprice	Loans	Rate, HH.
Public consumption	0.34	0.04	0.01	0.05	0.03	0.06	0.01	0.63	0.65	0.03	0.07	0.01	0.01
Temp. tech.	1.44	0.49	0.11	1.46	0.45	2.84	1.10	4.41	4.76	0.68	2.04	0.69	0.00
Tax, capital	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Tax, consumption	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Public inv.	0.06	0.00	0.00	0.00	0.00	0.01	0.00	0.11	0.12	0.00	0.01	0.00	0.00
Intertemporal con.	0.74	8.36	0.33	0.40	1.17	0.19	0.10	1.01	1.18	0.41	1.13	1.04	0.01
Tax, bonds	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Tax, labour	0.00	0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.02	0.00	0.01	0.02	0.00
Risk	1.24	7.33	2.00	0.07	4.82	0.18	0.12	2.10	2.39	0.15	1.97	0.76	35.88
Investment	13.82	7.32	38.45	13.40	18.65	1.04	9.24	0.18	0.13	3.21	6.27	34.96	0.50
Import	0.38	1.01	0.94	0.33	0.77	1.21	0.31	0.44	0.43	0.23	0.27	0.34	0.01
Labour supply	0.34	0.12	0.02	0.32	0.11	0.05	0.36	0.54	1.76	0.10	0.20	0.13	0.00
Price markup	11.35	7.90	4.50	5.48	10.68	23.82	45.28	17.80	13.82	2.25	10.24	7.91	0.10
Wage markup	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Export	4.42	5.34	2.79	6.38	3.27	0.68	8.63	3.35	3.41	0.49	0.24	3.06	0.04
RoW, inflation	0.18	0.10	0.04	0.25	0.07	0.05	0.00	0.33	0.35	0.00	0.01	0.01	0.00
RoW, GDP	0.66	0.04	0.11	2.07	0.09	0.19	0.01	1.16	1.21	0.05	0.13	0.07	0.03
RoW, Interest rate	5.27	4.98	3.49	5.37	5.00	2.00	0.58	7.85	8.21	0.04	0.46	0.91	0.19
EA, inflation	5.37	4.18	3.61	5.78	4.44	4.03	1.40	6.58	6.85	0.48	0.52	2.60	11.03
EA, GDP	42.56	40.36	30.83	41.82	38.76	60.30	18.83	35.69	36.99	7.07	7.87	38.48	35.50
EA, Interest rate	2.10	2.44	1.78	2.08	2.54	0.73	0.19	3.08	3.22	0.02	0.38	0.31	14.31
Export price markup	5.51	3.46	2.12	10.17	2.50	1.86	2.15	8.47	8.72	0.38	0.77	1.26	0.05
Import price markup	2.60	3.71	7.69	3.14	5.09	0.18	5.42	0.79	0.63	0.66	0.25	4.39	0.06
Perm. tech.	0.24	1.23	0.44	0.37	0.23	0.14	6.15	3.30	2.85	0.30	0.93	2.20	0.04
Perm. investment	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
UIP	0.94	0.83	0.56	0.96	0.82	0.32	0.09	1.44	1.51	0.01	0.07	0.15	0.03
Capital util.	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Housing preference	0.14	0.32	0.04	0.02	0.17	0.03	0.01	0.24	0.27	9.99	26.59	0.31	0.01
Bank markup, HH.	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Bank markup, E.	0.00	0.01	0.01	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.01	2.18
Tax, housing	0.12	0.38	0.08	0.06	0.23	0.02	0.02	0.19	0.22	12.17	20.05	0.34	0.01
Tax, land	0.00	0.00	0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.00
Tax, deduction, E.	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Tax, deduction, HH.	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Loan-to-value	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Perm. tech, housing	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Temp. tech, housing	0.18	0.02	0.04	0.03	0.09	0.04	0.01	0.31	0.31	61.28	19.49	0.04	0.00

Table 10: **Variance decompositions:** This table reports the forecast-error-variance decomposition of selected variables at the infinite horizon; the unconditional variance decomposition. The decomposition is conducted only for the structural shocks part of the forecast errors.

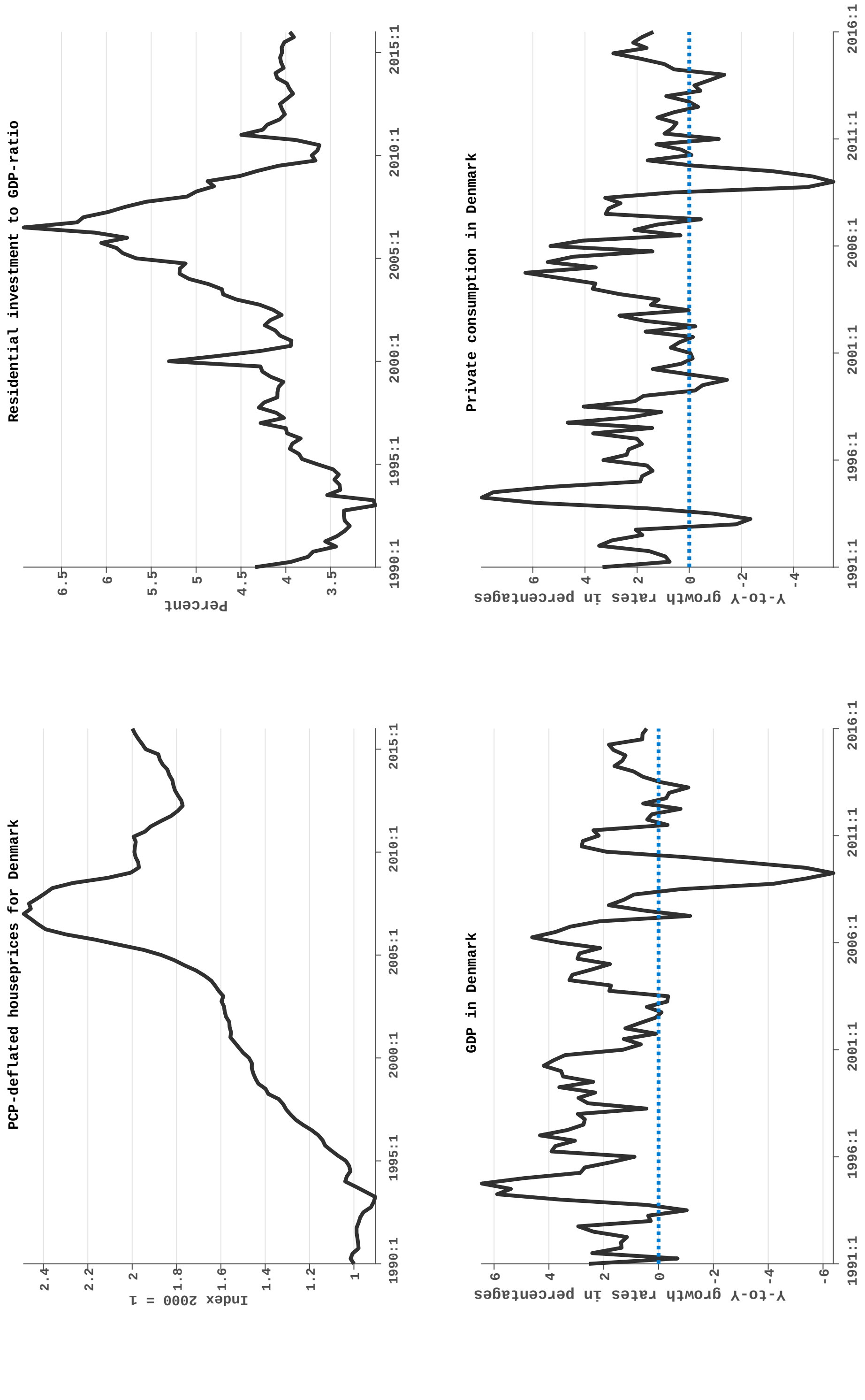

Figure 1: **Some facts about the Danish economy**
All data are taken from Danmarks Nationalbank, MONA database. Data manipulations are done with IRIS.

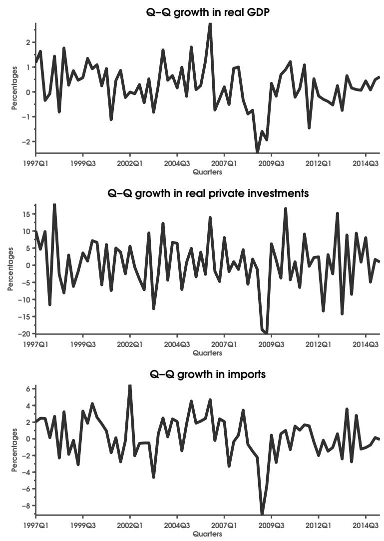

Figure 2: **Transformed observed time series** This figure shows the time series of the observed variables used in the estimation of the model.

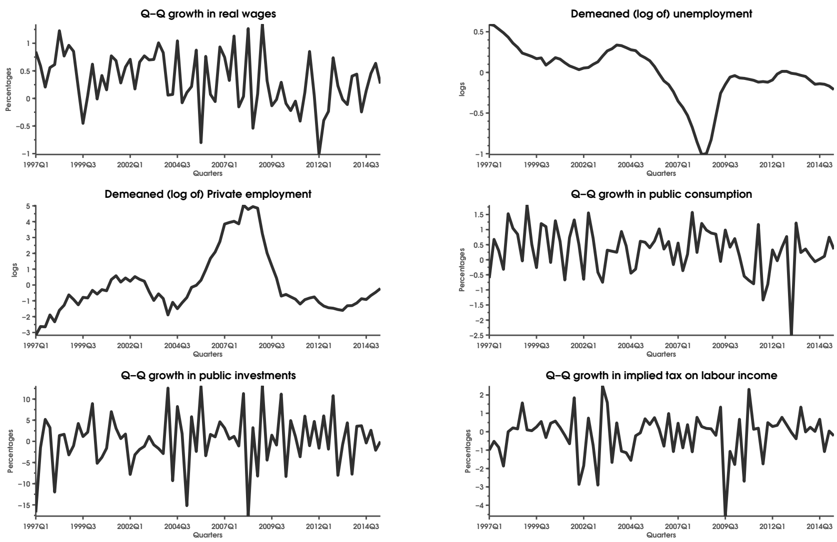

Figure 3: **Transformed observed time series** This figure shows the time series of the observed variables used in the estimation of the model.

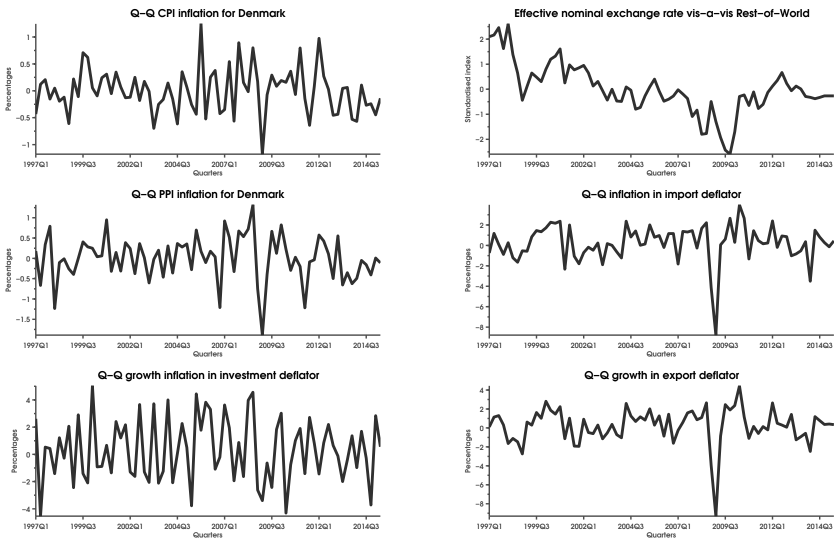

Figure 4: **Transformed observed time series** This figure shows the time series of the observed variables used in the estimation of the model.

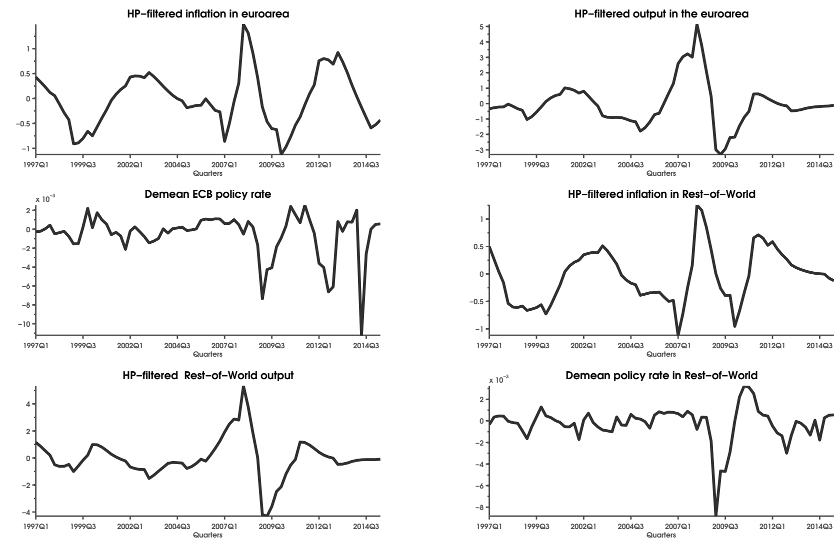

Figure 5: **Transformed observed time series** This figure shows the time series of the observed variables used in the estimation of the model.

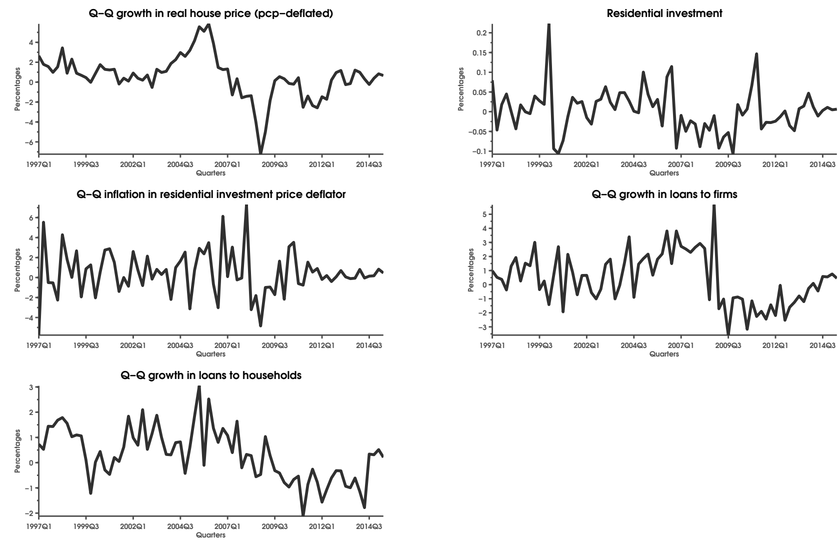

Figure 6: **Transformed observed time series** This figure shows the time series of the observed variables used in the estimation of the model.

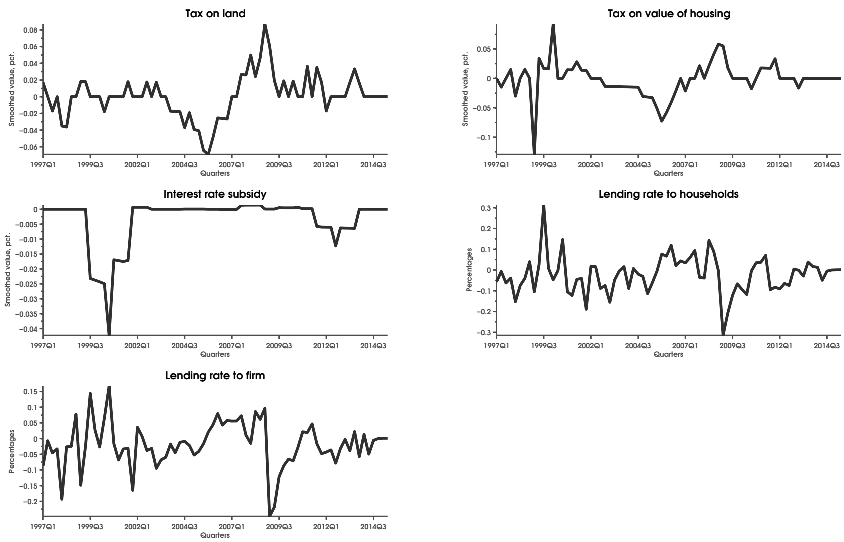

Figure 7: **Transformed observed time series** This figure shows the time series of the observed variables used in the estimation of the model.

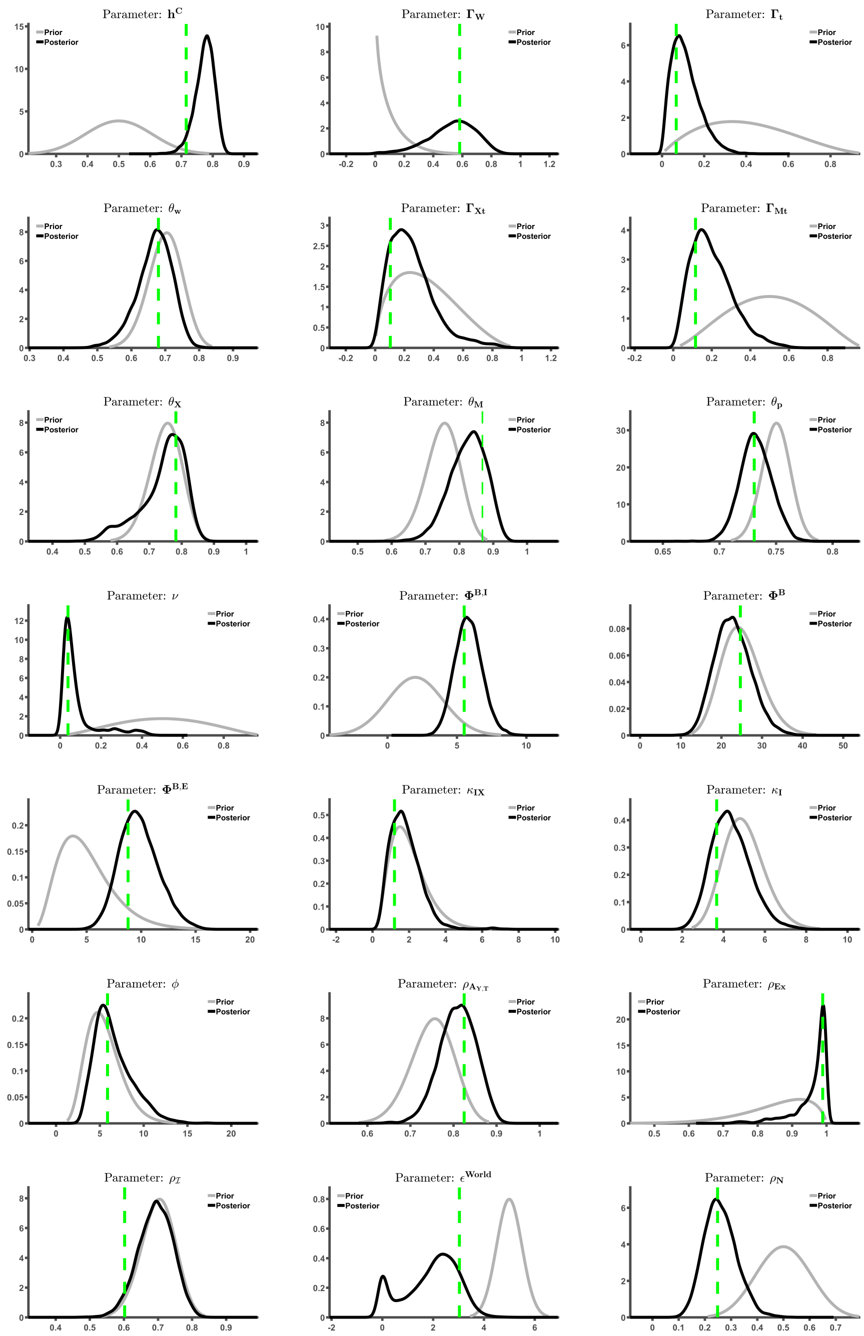

Figure 8: **Prior and posterior** Prior and posterior distributions of the estimated parameters.

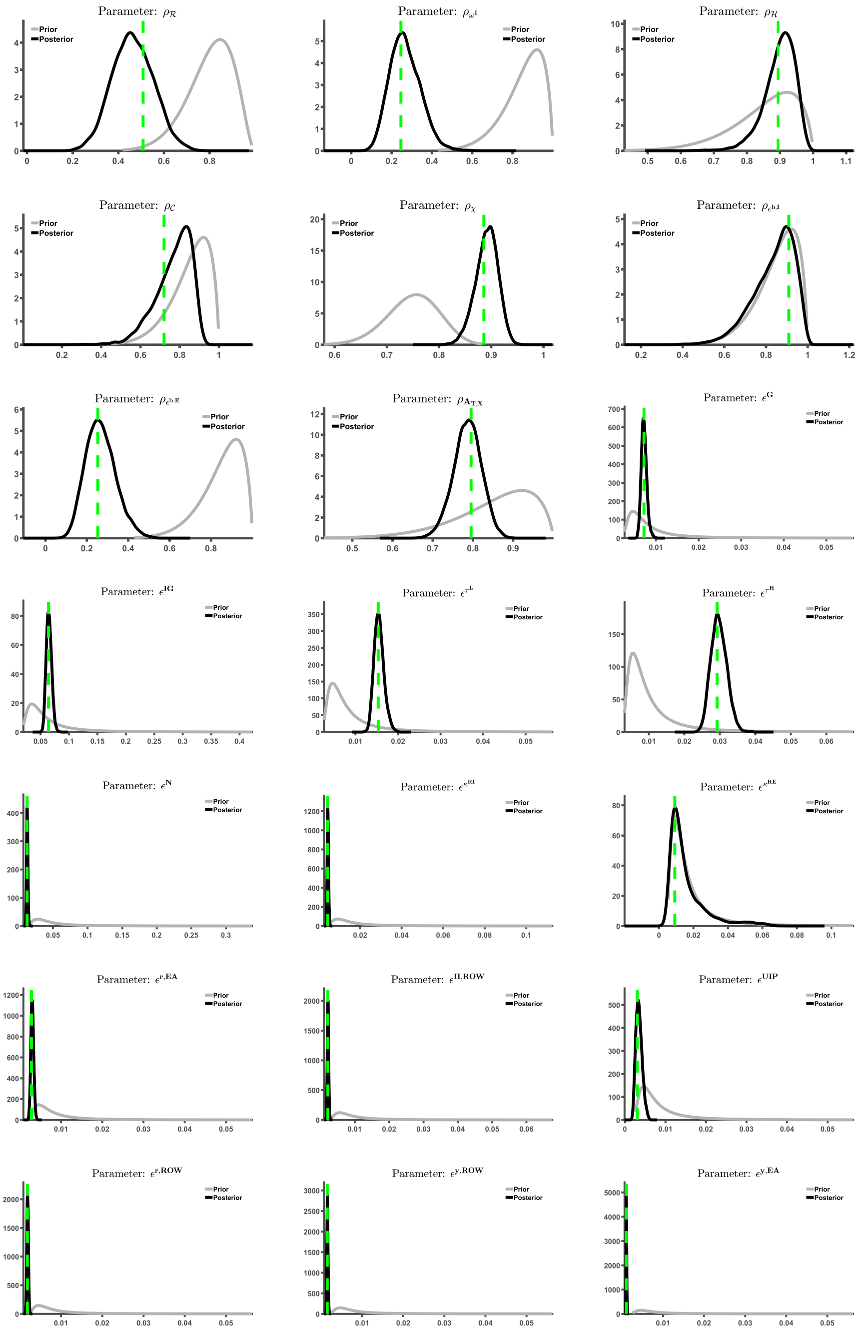

Figure 9: **Prior and posterior** Prior and posterior distributions of the estimated parameters.

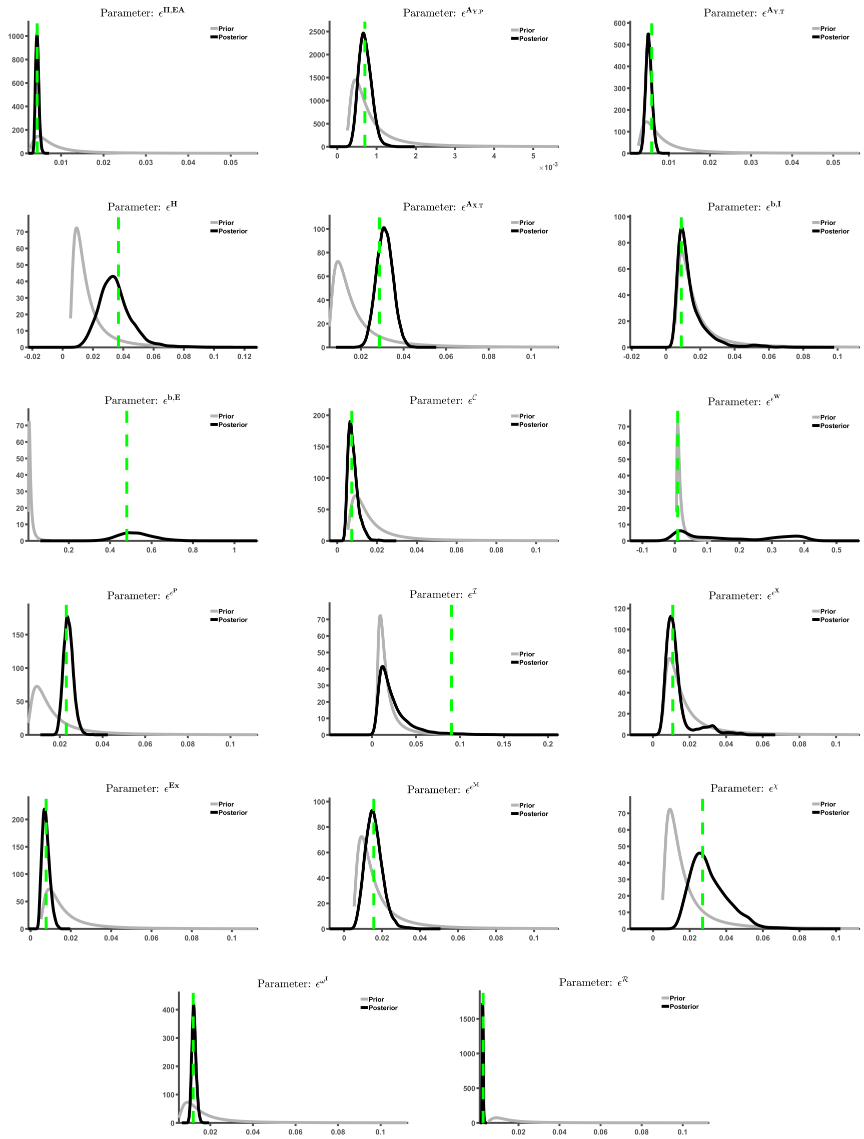

Figure 10: **Prior and posterior** Prior and posterior distributions of the estimated parameters.

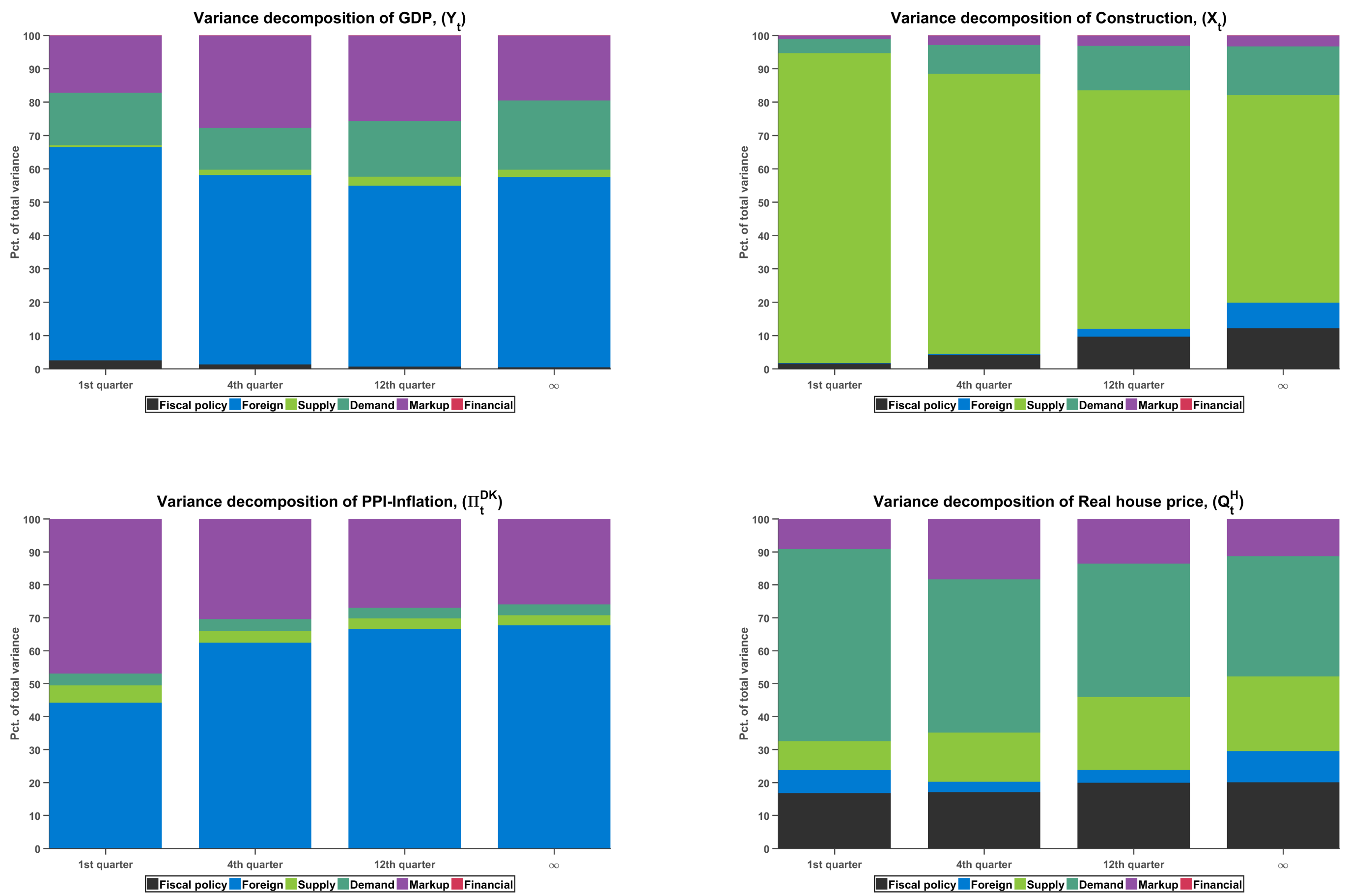

Figure 11: **Variance decomposition of shocks.**
This figure shows the variance decomposition of real GDP for various horizons. The shocks are grouped into groups as explained in the text. The individual contributions of each shock to each variable can be seen in tables (7) to (10).

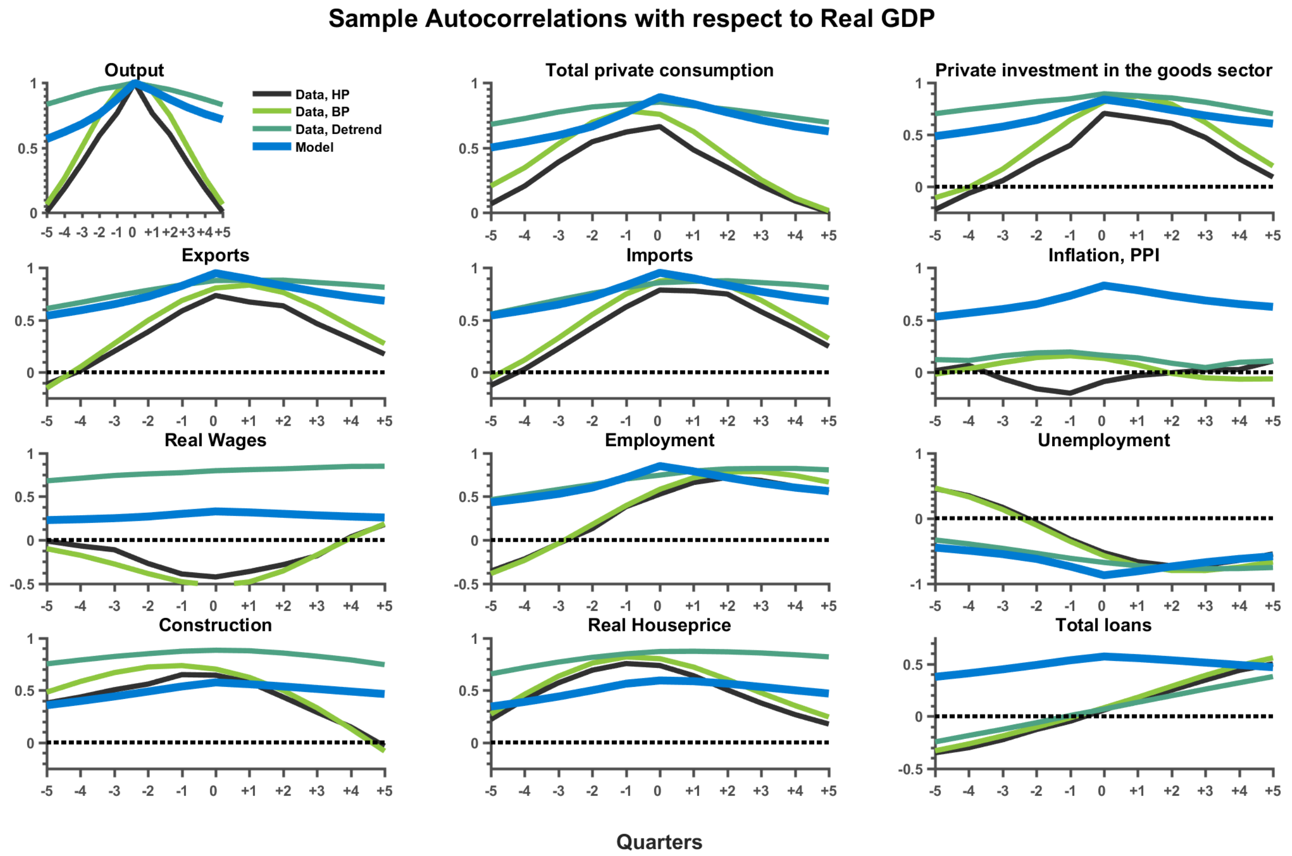

Figure 12: **Sample and model-based autocorrelations between Real GDP and selected variables**
This figure shows the model-based autocorrelations between real GDP and selected variables along with the sample counterparts. The sample cross-correlations are calculated on HP-filtered data using a λ-parameter equal to 1600, on Band-pass filtered data for cycles between 6 and 40 quarters, and detrended data with a constant trend.

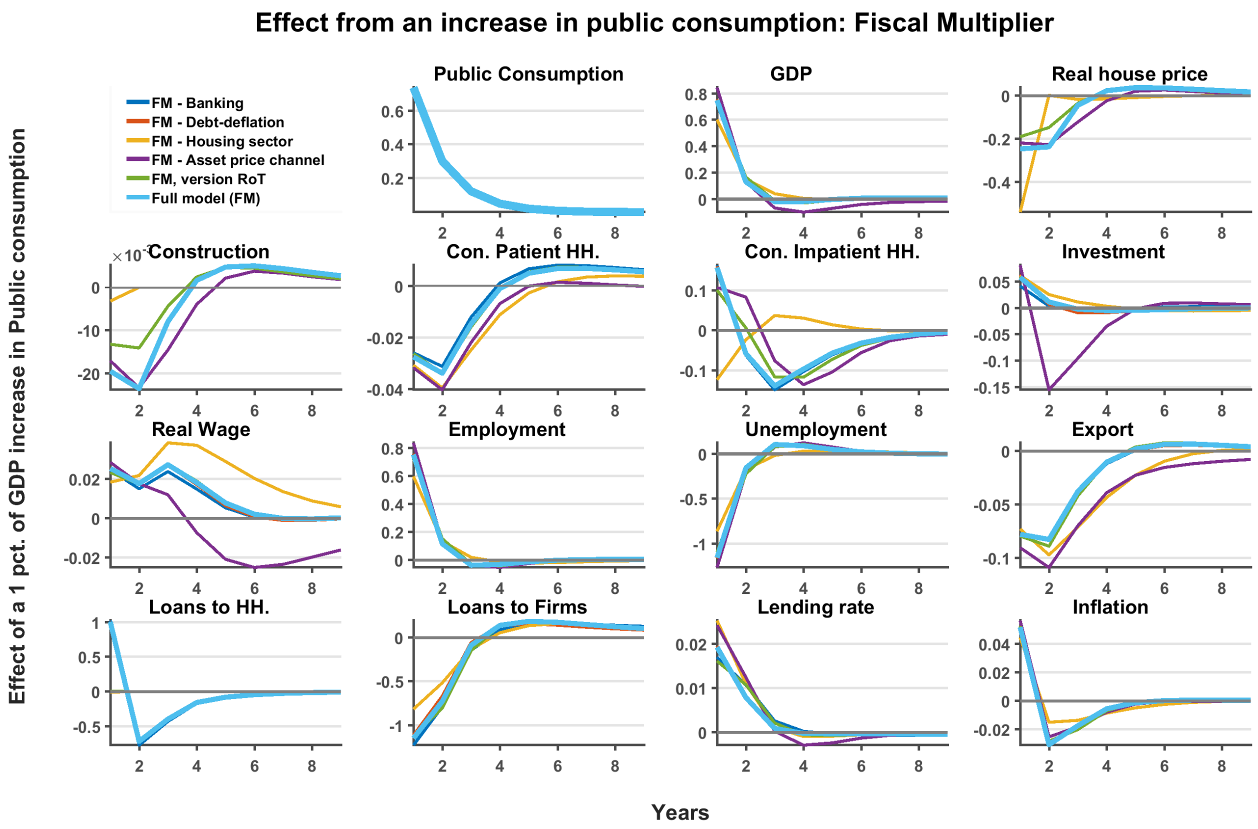

Figure 13: **Fiscal multiplier, $\frac{dY_t}{dG_t}$ - model comparison**
This figure shows impulse response function for a shock to public expenditures, G_t, equivalent to 1 pct. of steady state GDP with an autoregressive parameter equal to 0.8 or a half-life of 3 quarters. The impulses are yearly responses defined as the mean response in four quarters. The model is calibrated in different ways to highlight model properties, se also table (1). The responses are also compared to the impulse response function in the full model.

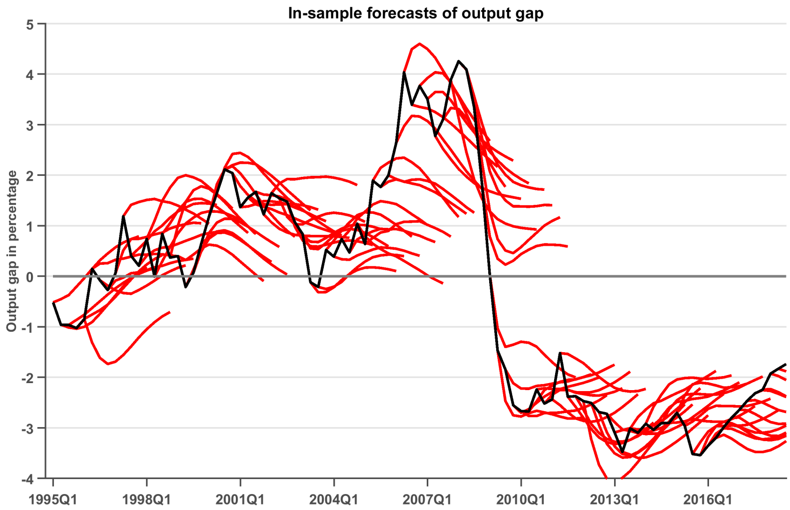

Figure 14: **In-sample forecasting of the output gap**
The figure shows the output gap produced by the model defined as the deviation in Danish real GDP (black line) from a linear stochastic trend , see also definition in text, together with in-sample forecasts 12 quarters ahead (red lines) for each quarter.

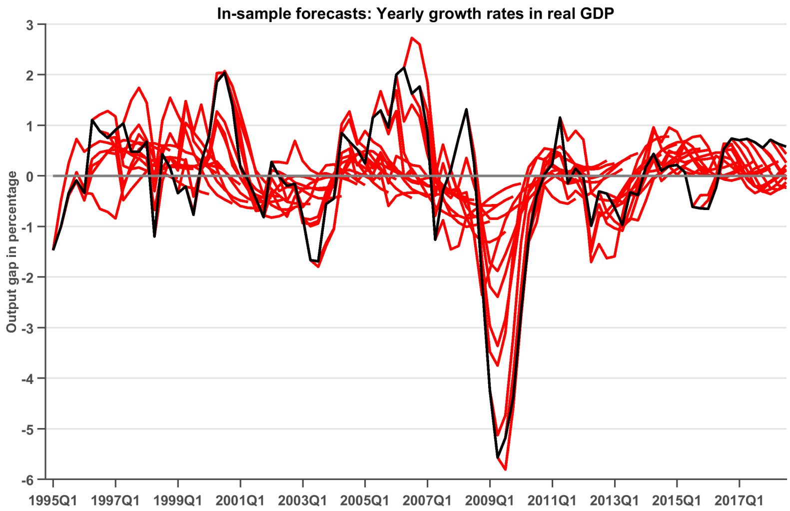

Figure 15: **In-sample forecasting of yearly growth rates in real GDP**
The figure shows yearly growth rates in GDP produced by the model together with in-sample forecasts 12 quarters ahead (red lines) for each quarter.

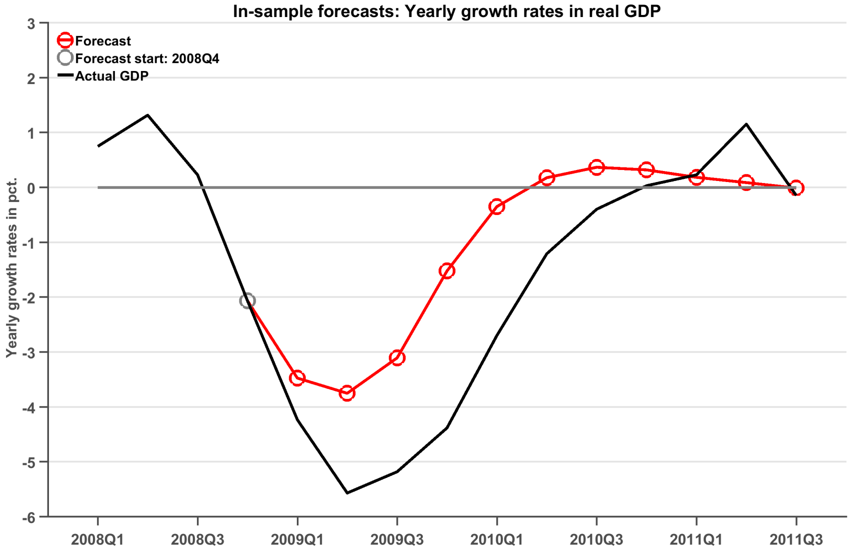

Figure 16: **In-sample forecasting of yearly growth rates in real GDP**
The figure shows yearly growth rates in GDP produced by the model together with in-sample forecasts 12 quarters ahead (red lines) starting from 2008, 4^{th} quarter.

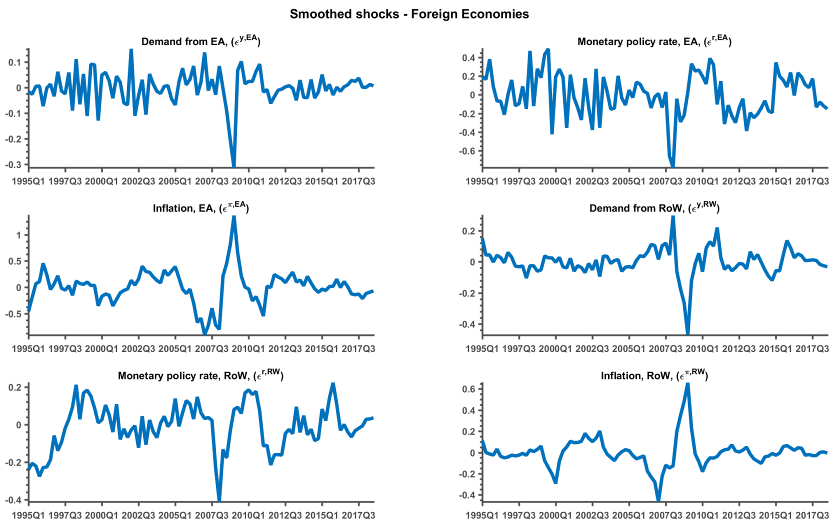

Figure 17: **Smoothed shocks**
This figure shows the smoothed estimates of the models structural shocks used in the estimation based on the posterior mode estimates of the model's structural parameters.

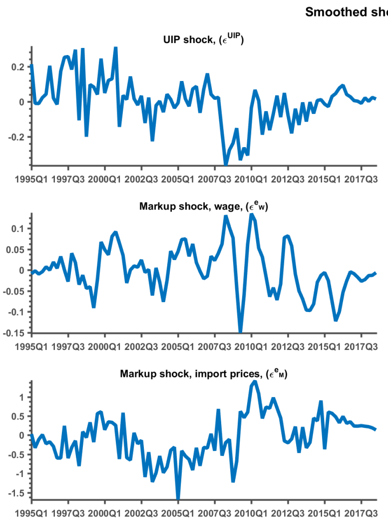

Figure 18: **Smoothed shocks**
This figure shows the smoothed estimates of the models structural shocks used in the estimation based on the posterior mode estimates of the model's structural parameters.

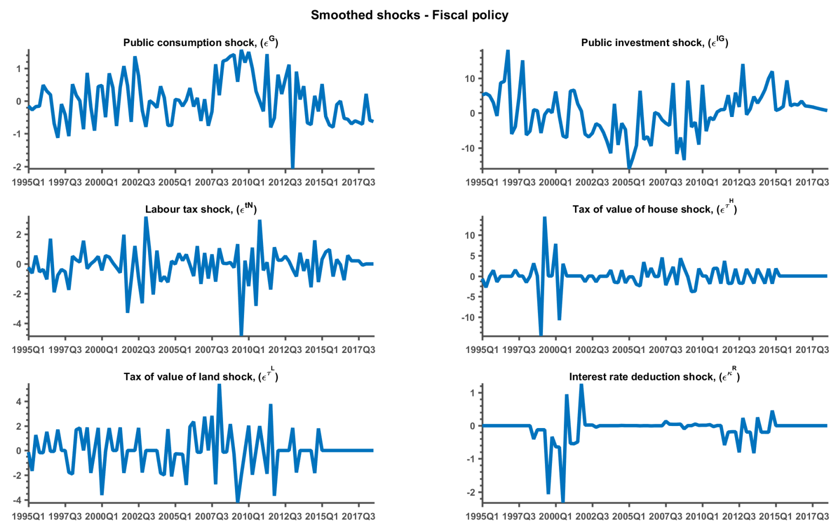

Figure 19: **Smoothed shocks**
This figure shows the smoothed estimates of the models structural shocks used in the estimation based on the posterior mode estimates of the model's structural parameters.

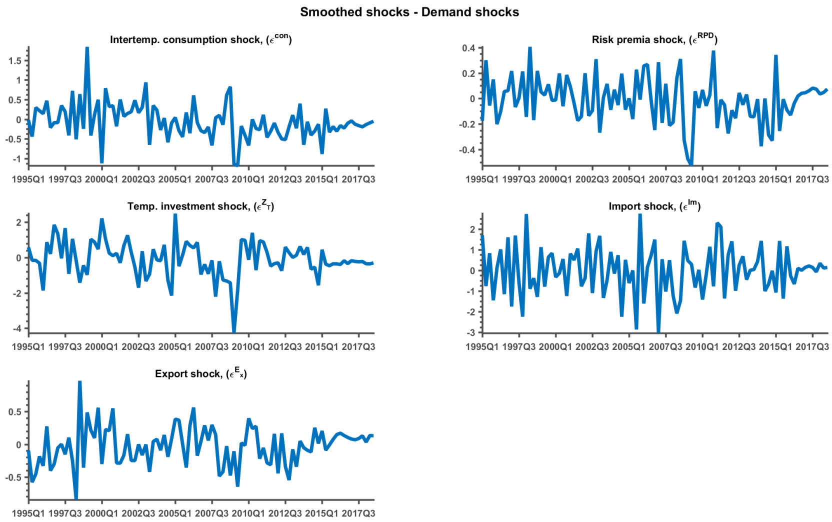

Figure 20: **Smoothed shocks**
This figure shows the smoothed estimates of the models structural shocks used in the estimation based on the posterior mode estimates of the model's structural parameters.

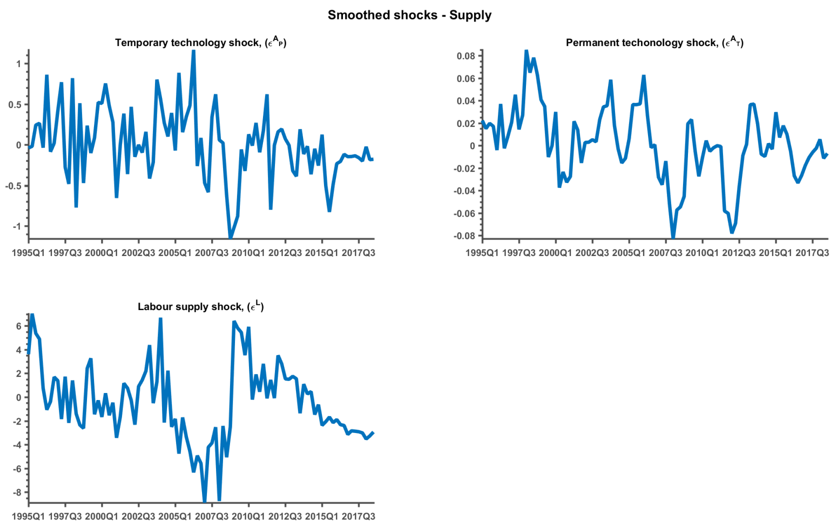

Figure 21: **Smoothed shocks**
This figure shows the smoothed estimates of the models structural shocks used in the estimation based on the posterior mode estimates of the model's structural parameters.

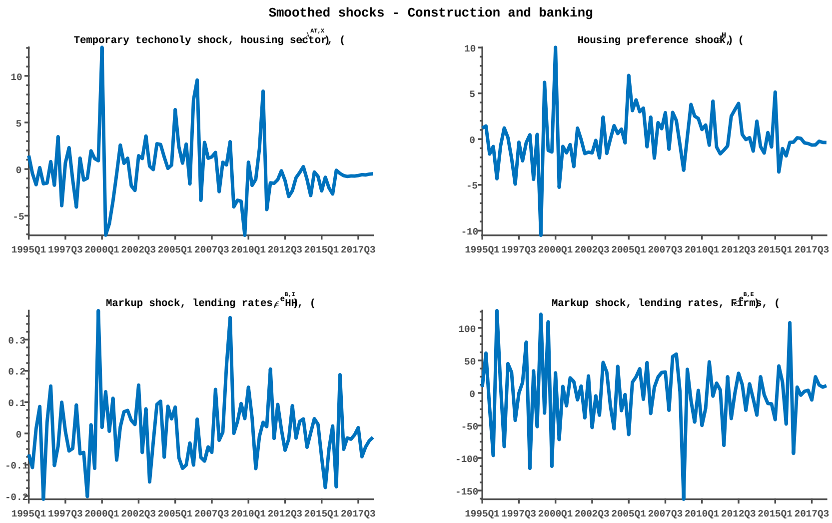

Figure 22: **Smoothed shocks**
This figure shows the smoothed estimates of the models structural shocks used in the estimation based on the posterior mode estimates of the model's structural parameters.

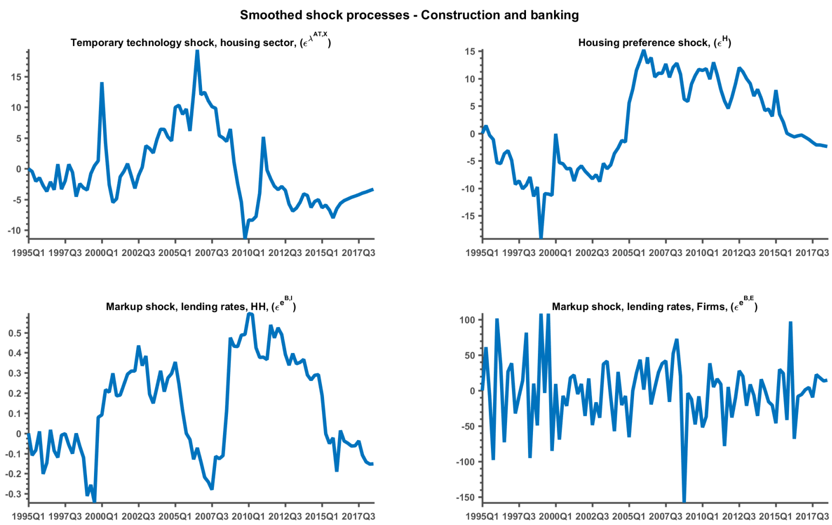

Figure 23: **Smoothed processes for the structural shocks**
This figure shows the smoothed estimates of the models structural shocks processes used in the estimation based on the posterior mode estimates of the model's structural parameters.

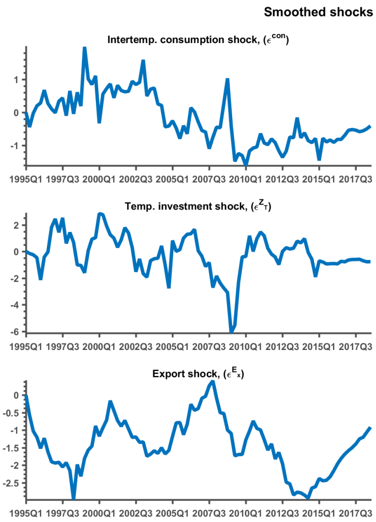

Figure 24: **Smoothed processes for the structural shocks**
This figure shows the smoothed estimates of the models structural shocks processes used in the estimation based on the posterior mode estimates of the model's structural parameters.

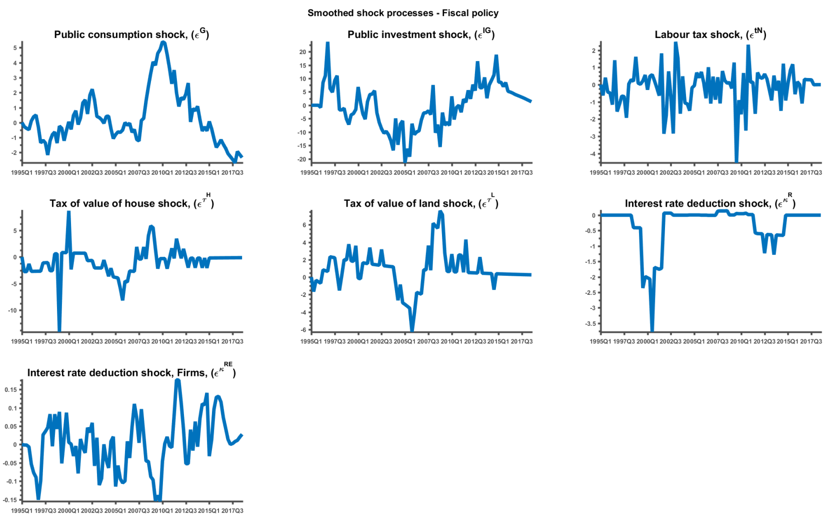

Figure 25: **Smoothed processes for the structural shocks**
This figure shows the smoothed estimates of the models structural shocks processes used in the estimation based on the posterior mode estimates of the model's structural parameters.

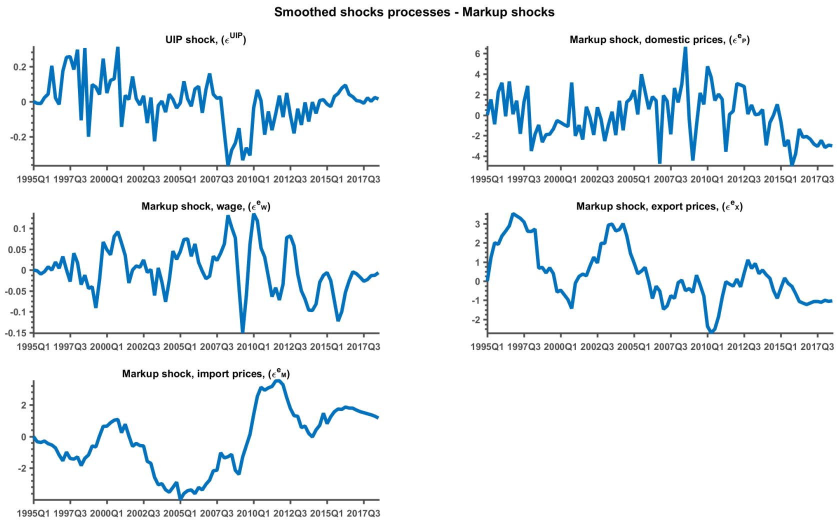

Figure 26: **Smoothed processes for the structural shocks**
This figure shows the smoothed estimates of the models structural shocks processes used in the estimation based on the posterior mode estimates of the model's structural parameters.

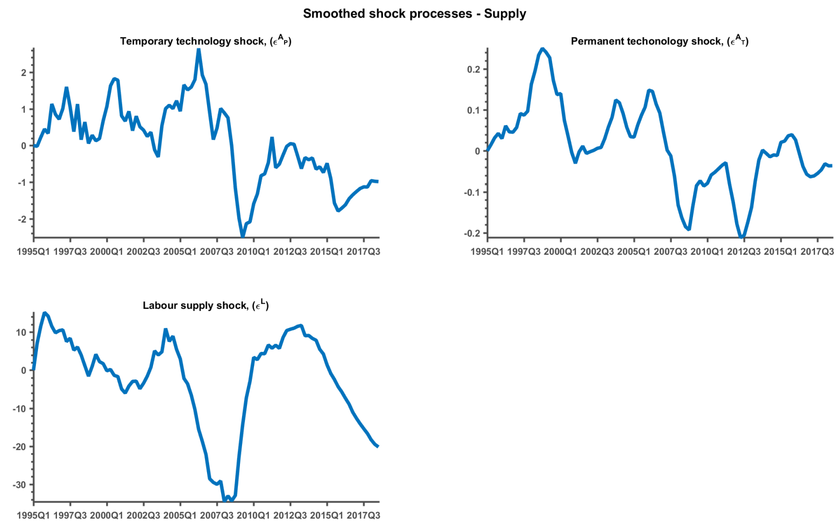

Figure 27: **Smoothed processes for the structural shocks**
This figure shows the smoothed estimates of the models structural shocks processes used in the estimation based on the posterior mode estimates of the model's structural parameters.

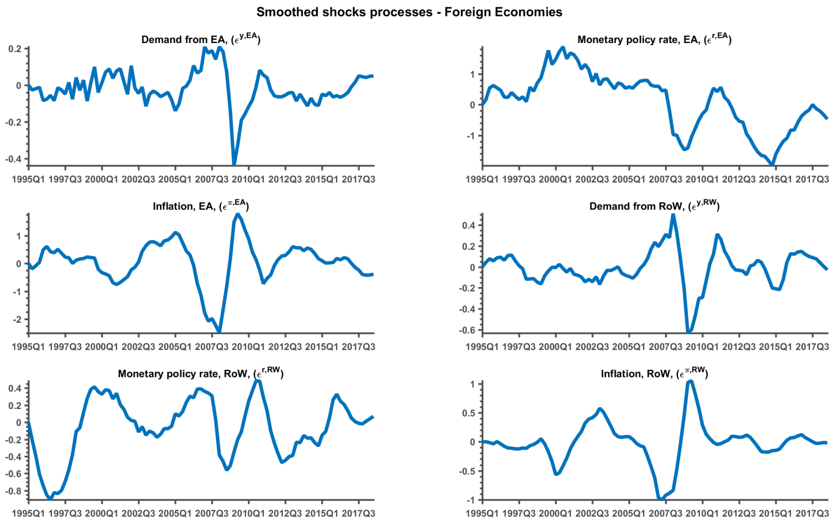

Figure 28: **Smoothed processes for the structural shocks**
This figure shows the smoothed estimates of the models structural shocks processes used in the estimation based on the posterior mode estimates of the model's structural parameters.

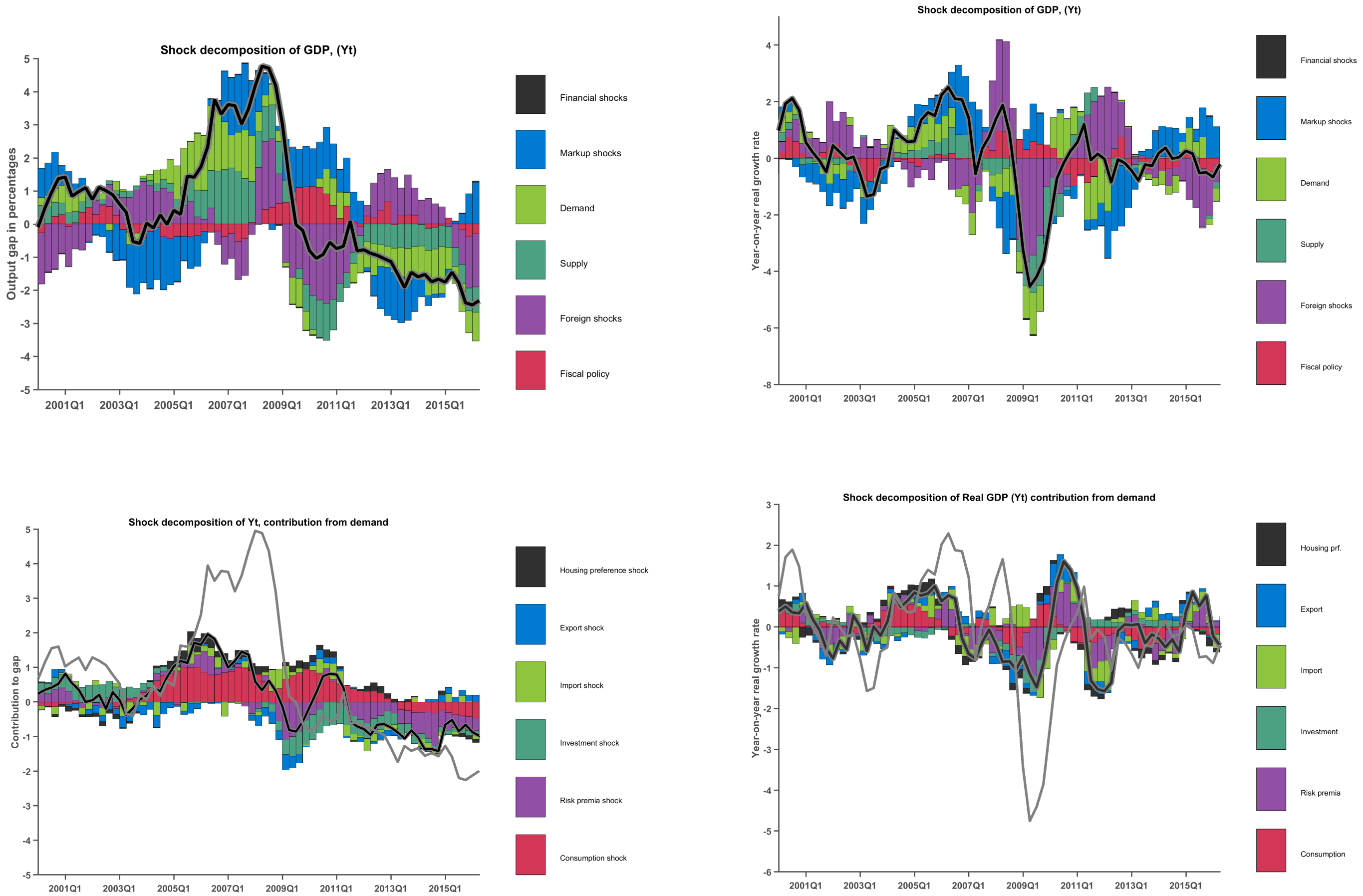

Figure 29: **Historical Shock Decomposition of real GDP and the contribution from the subgroup named demand shock.**

- Top left figure: The historical quarter to quarter deviation in Danish real GDP (solid lines) from a linear stochastic trend decomposed into the structural shocks in the model. The shocks are grouped into categories as explained in the text. Residual contributions, which capture the influence of the initial state of the economy and measurement errors, are not shown. The decomposition have been computed using the posterior mode estimates of the model's structural parameters.
- Top right figure: The change from one year ago in real GDP decomposed into the structural shocks in the model. The shocks are grouped into categories as explained in the text.
- Bottom left figure: The contribution from the subgroup named demand to the quarter to quarter deviation in real GDP from a linear stochastic trend. The thick black line is equal to the columns for the subgroup "Demand" in the figure above.
- Bottom right figure: The contribution from the subgroup named demand to the change one year ago in real GDP. The thick black line is equal to the columns for the subgroup "Demand" in the figure above.

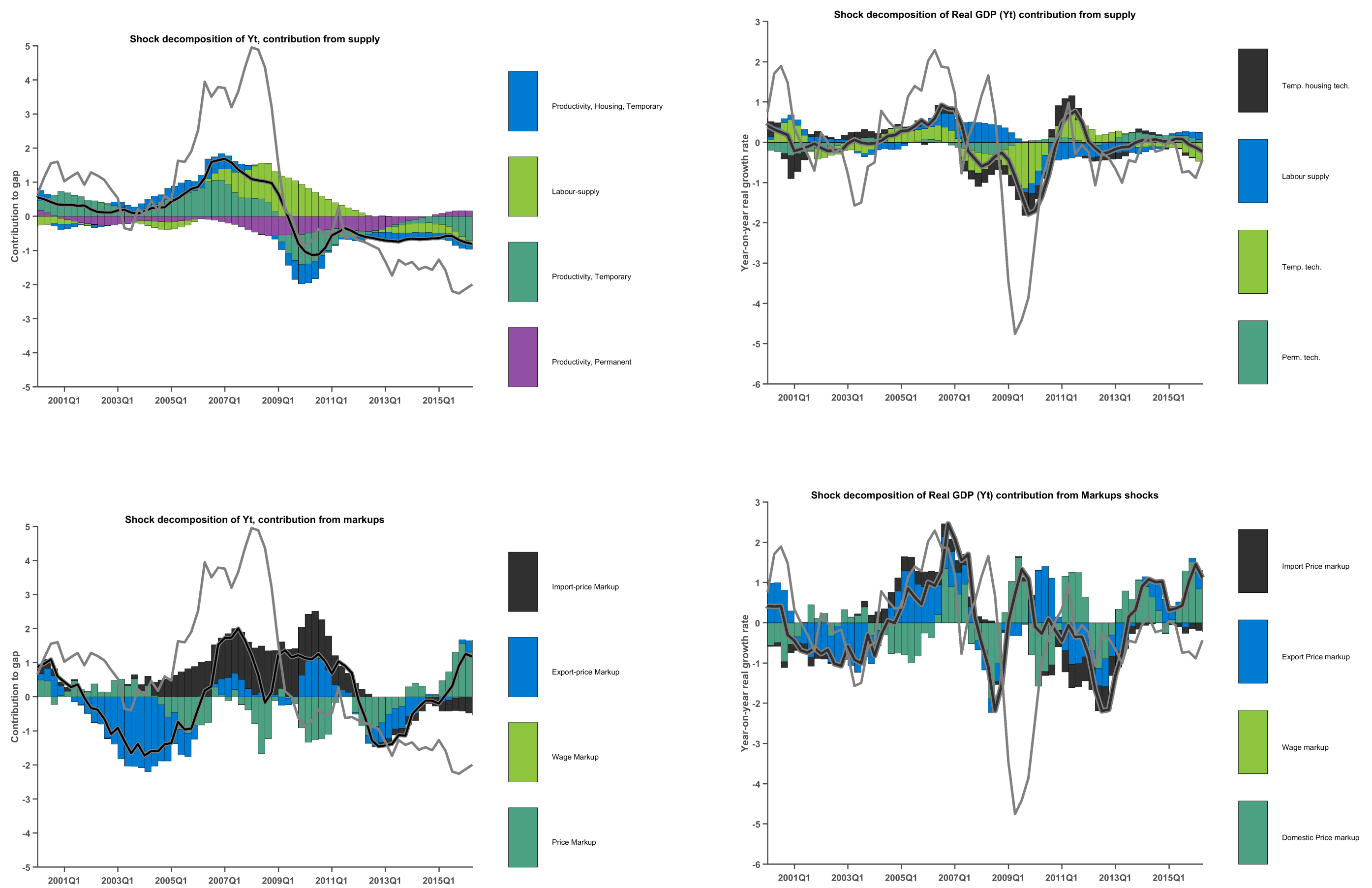

Figure 30: **Historical Shock Decomposition of real GDP: Contribution from supply and markup shocks.**

- Top left figure: The contribution from the subgroup named supply to the quarter to quarter deviation in real GDP from a linear stochastic trend. Residual contributions, which capture the influence of the initial state of the economy and measurement errors, are not shown. The decomposition have been computed using the posterior mode estimates of the model's structural parameters. The thick black line is equal to the columns for the subgroup "supply" in figure (29(a)).
- Top right figure: The contribution from the subgroup named supply to the change one year ago in real GDP. The thick black line is equal to the columns for the subgroup "supply" in figure (29(b)).
- Bottom left figure: The contribution from the subgroup named markup shocks to the quarter to quarter deviation in real GDP from a linear stochastic trend. The thick black line is equal to the columns for the subgroup "markup" in figure (29(a)).
- Bottom right figure: The contribution from the subgroup named markup shocks to the change one year ago in real GDP. The thick black line is equal to the columns for the subgroup "markup" in figure (29(b)).

Figure 31: **Historical Shock Decomposition of real GDP: Contribution from foreign and fiscal policy shocks.**

- Top left figure: The contribution from the subgroup named foreign shocks to the quarter to quarter deviation in real GDP from a linear stochastic trend. Residual contributions, which capture the influence of the initial state of the economy and measurement errors, are not shown. The decomposition have been computed using the posterior mode estimates of the model's structural parameters. The thick black line is equal to the columns for the subgroup ”foreign” in figure (29(a)).
- Top right figure: The contribution from the subgroup named foreign shocks to the change one year ago in real GDP. The thick black line is equal to the columns for the subgroup ”foreign” in figure (29(b)).
- Bottom left figure: The contribution from the subgroup named fiscal policy shocks to the quarter to quarter deviation in real GDP from a linear stochastic trend. The thick black line is equal to the columns for the subgroup ”Fiscal policy” in figure (29(a)).
- Bottom right figure: The contribution from the subgroup named fiscal policy shocks to the change one year ago in real GDP. The thick black line is equal to the columns for the subgroup ”Fiscal policy” in figure (29(b)).

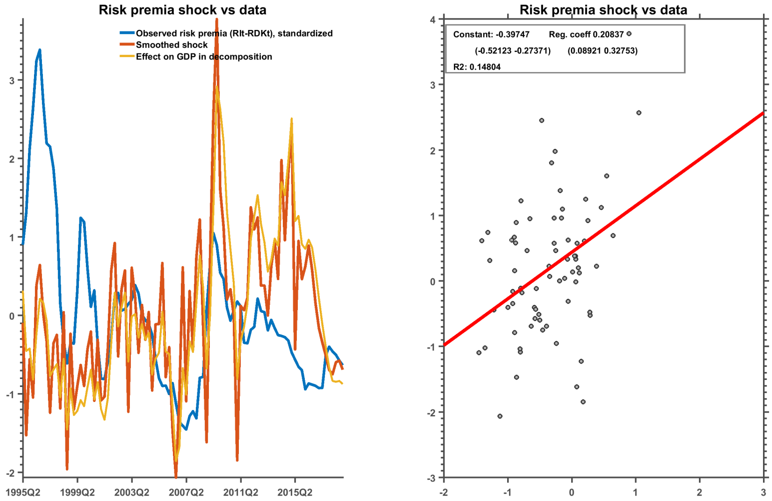

Figure 32: **Historical Shock Decomposition, Risk premia shock**
This figure shows the relationship between observed demeaned interest rate spread and the affect from the risk premia shock, $\epsilon^{\mathcal{R}}$, on the output gap in the historical shock decomposition. The spread is the difference between the monetary policy rate for Denmark and the lending rate to household for housing purchases. See also appendix (11) for a description of data. The decomposition have been computed using the posterior mode estimates of the model's structural parameters. Both series have been demeaned and divided by its standard deviation.

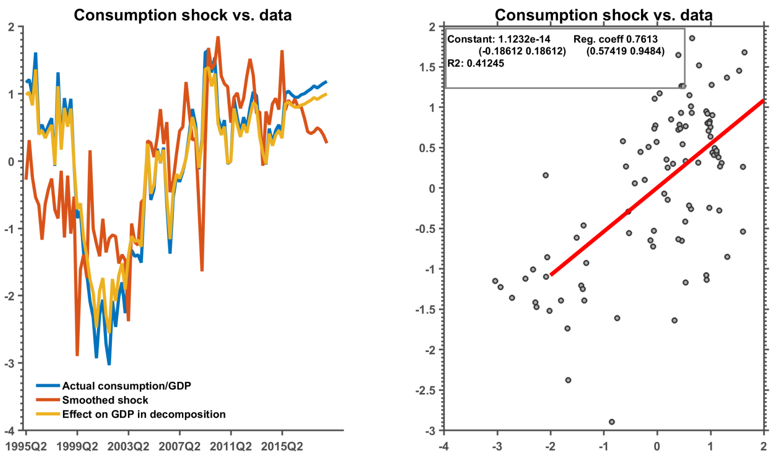

Figure 33: **Historical Shock Decomposition, Consumption shock**
This figure shows the relationship between observed consumption-to-GDP ratio and the affect from the preference shock for consumption today relative to the future, ϵ^{C}, on the output gap in the historical shock decomposition. The decomposition have been computed using the posterior mode estimates of the model's structural parameters. Both series have been demeaned and divided by its standard deviation.

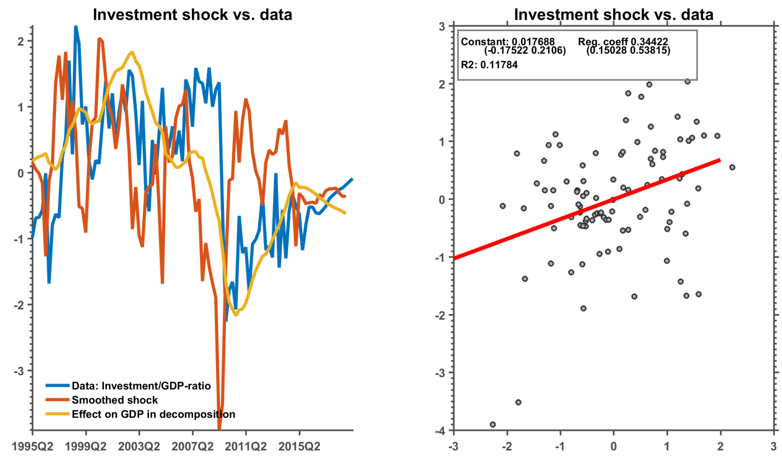

Figure 34: **Historical Shock Decomposition, Temporary Investment shock**
This figure shows the relationship between observed investment-to-GDP ratio and the affect from the temporary investment shock , ϵ^{I}, on the output gap in the historical shock decomposition. The decomposition have been computed using the posterior mode estimates of the model's structural parameters. Both series have been demeaned and divided by its standard deviation.

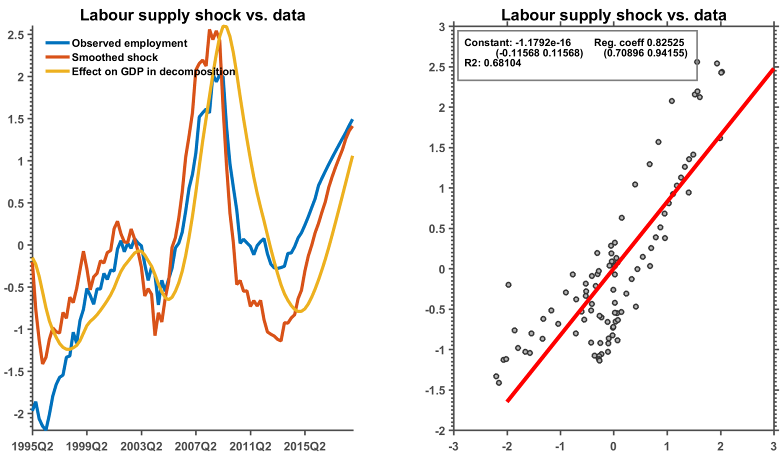

Figure 35: **Historical Shock Decomposition, Labour supply shock**
This figure shows the relationship between observed employment and the affect from the labour supply shock, ϵ^{X}, on the output gap in the historical shock decomposition. Employment consists of total employment in public- and private sector as well self-employed. Both series have been demeaned and divided by its standard deviation. The decomposition have been computed using the posterior mode estimates of the model's structural parameters.

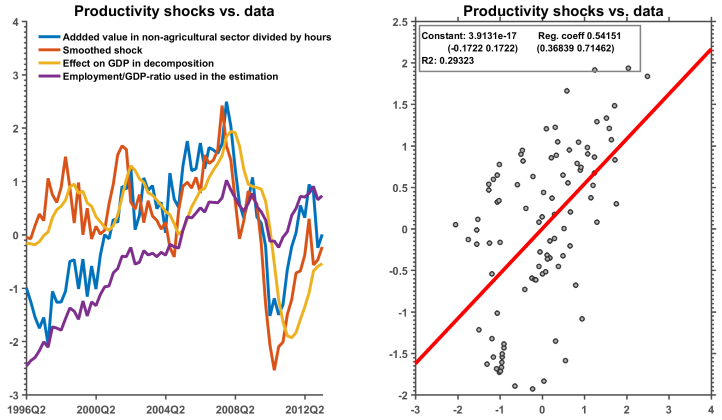

Figure 36: **Historical Shock Decomposition, Productivity shocks**
This figure shows the relationship between observed employment and the affect from the technology shocks,$\epsilon^{A_{Y,T}}$, on the output gap in the historical shock decomposition. Observed productivity is the detrended serie for added value in non-agricultural sector divided by hours, and employment divided by GDP respectively. The first series is taken from the Mona-data base, probx. Both series have been demeaned and divided by its standard deviation. The decomposition have been computed using the posterior mode estimates of the model's structural parameters.

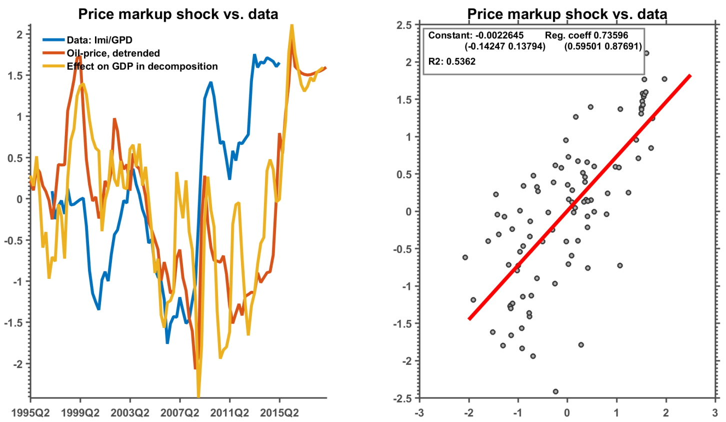

Figure 37: **Historical Shock Decomposition, Domestic Price Markup-shock**
This figure shows the relationship between observed prices (demeaned IMI divided by output) and the affect of the domestic price markup shock, ϵ^{ϵ^P}, on the output gap in the historical shock decomposition. Both series have been demeaned and divided by its standard deviation. The decomposition have been computed using the posterior mode estimates of the model's structural parameters.

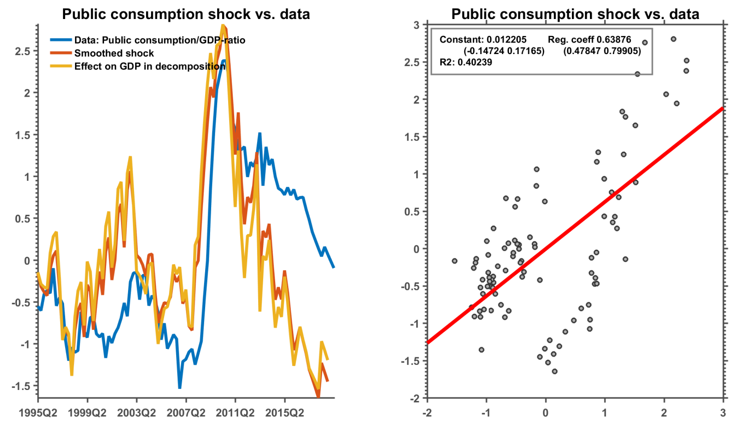

Figure 38: **Historical Shock Decomposition, Public consumption shock**
This figure shows the relationship between observed public consumption divided by output and the affect of the public consumption shock, ϵ^G, on the output gap in the historical shock decomposition. Both series have been demeaned and divided by its standard deviation. The decomposition have been computed using the posterior mode estimates of the model's structural parameters.

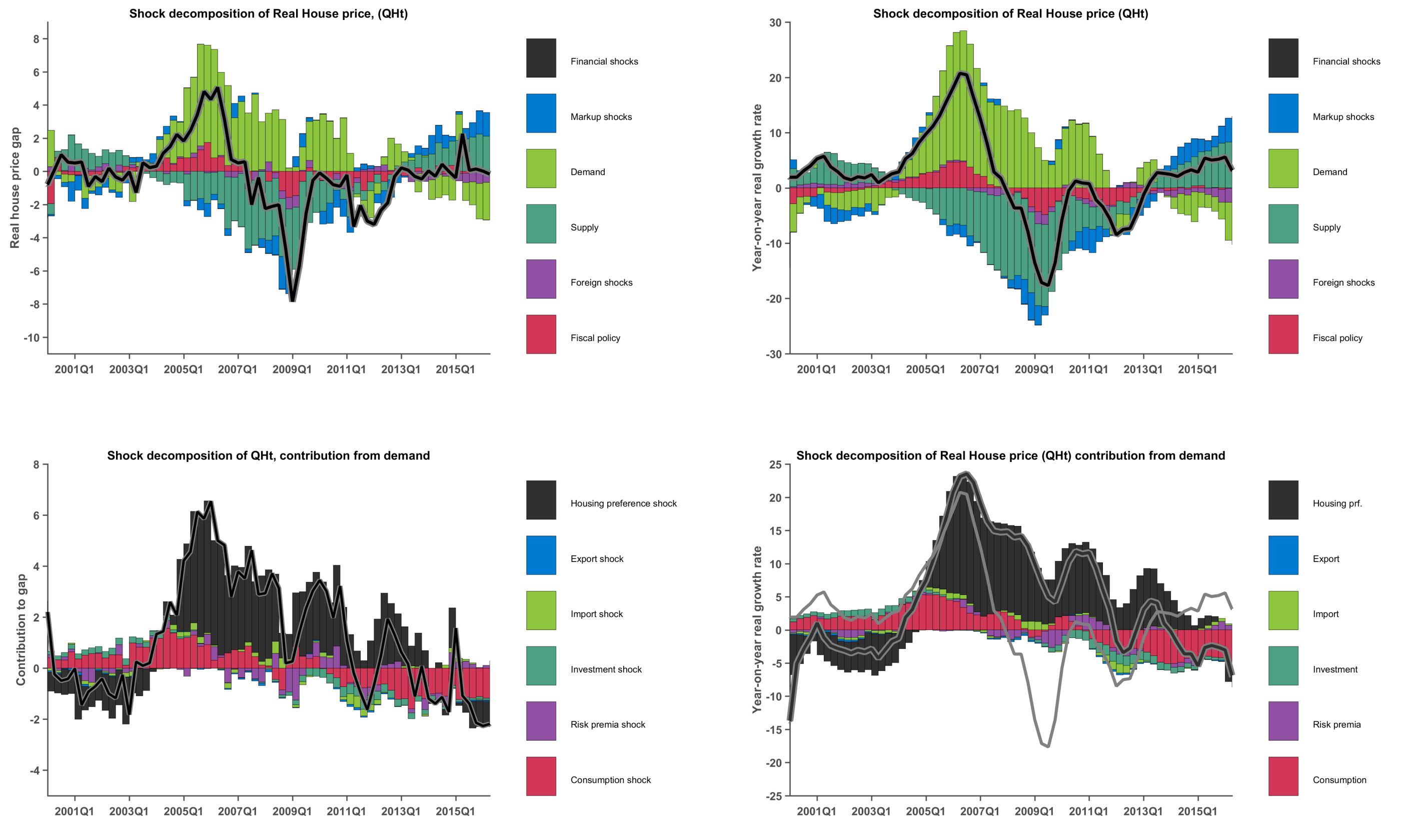

Figure 39: **Historical Shock Decomposition of the real house price, Q_t^H**

- Top left figure: The historical quarter to quarter deviation in Danish real house price (solid lines) from a linear stochastic trend decomposed into the structural shocks in the model. The shocks are grouped into categories as explained in the text. Residual contributions, which capture the influence of the initial state of the economy and measurement errors, are not shown. The decomposition have been computed using the posterior mode estimates of the model's structural parameters.

- Top right figure: The change from one year ago in real house price decomposed into the structural shocks in the model. The shocks are grouped into categories as explained in the text.

- Bottom left figure: The contribution from the subgroup named demand to the quarter to quarter deviation in real house price from a linear stochastic trend. The thick black line is equal to the columns for the subgroup "demand" in figure (39(a)).

- Bottom right figure: The contribution from the subgroup named demand to the change one year ago in the real house price. The thick black line is equal to the columns for the subgroup "demand" in figure (39(b)).

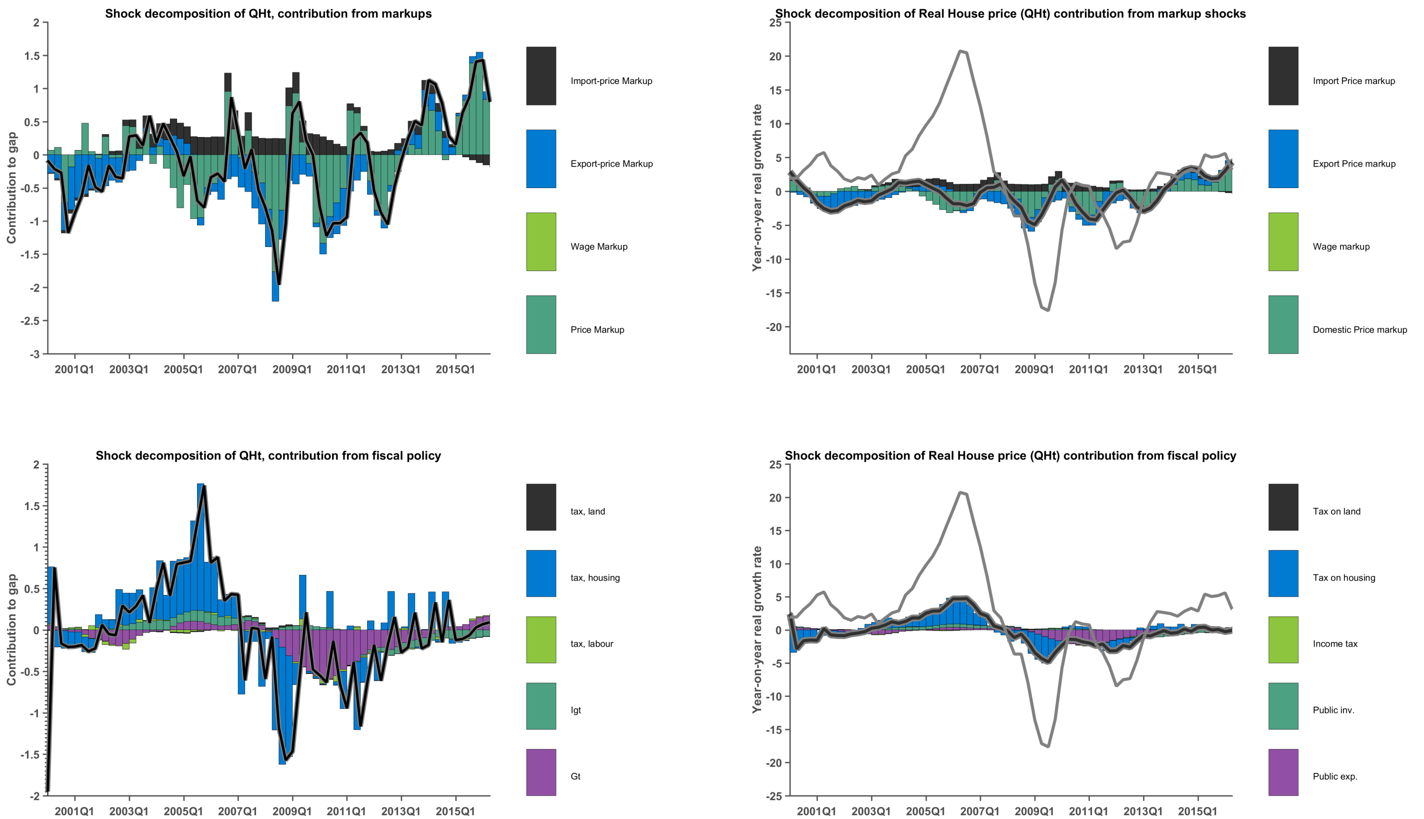

Figure 40: **Historical Shock Decomposition, real house price, Q_t^H**

- Top left figure: The contribution from the subgroup named markup shocks to the quarter to quarter deviation in real house price from a linear stochastic trend. The thick black line is equal to the columns for the subgroup ”markup” in figure (39(a)).
- Top right figure: The contribution from the subgroup named markup shocks to the change one year ago in the real house price. The thick black line is equal to the columns for the subgroup ”markup” in figure (39(b)).
- Bottom left figure: The contribution from the subgroup named fiscal policy to the quarter to quarter deviation in real house price from a linear stochastic trend. The thick black line is equal to the columns for the subgroup ”fiscal policy” in figure (39(a)).
- Bottom right figure: The contribution from the subgroup named fiscal policy to the change one year ago in the real house price. The thick black line is equal to the columns for the subgroup ”fiscal policy” in figure (39(b)).

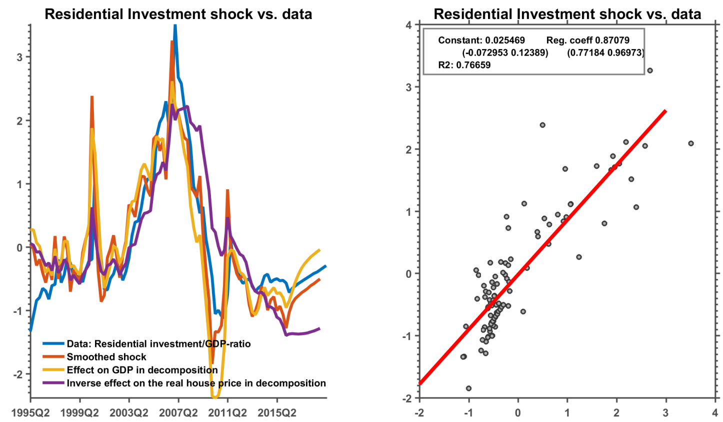

Figure 41: **Historical Shock Decomposition, Temporary productivity shock in the residential sector**
This figure shows the relationship between observed real residential investment-to-GDP ratio and the effect of the temporary productivity shock in the housing sector, $\epsilon^{A_{X,T}}$, on the output gap and the real house price gap in the historical shock decomposition. Both series have been demeaned and divided by its standard deviation. The decomposition have been computed using the posterior mode estimates of the model's structural parameters.

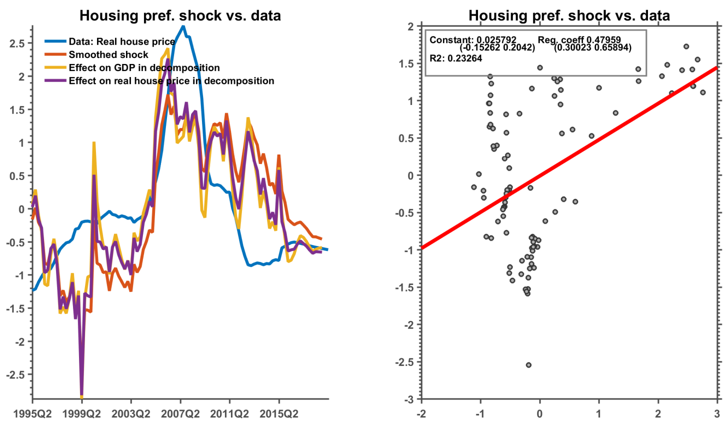

Figure 42: **Historical Shock Decomposition, Housing Preference Shock**
This figure shows the relationship between the observed real house price and the effect of the housing preference shock, ϵ^{H}, on the output gap and the real house price gap in the historical shock decompositions. Both series have been demeaned and divided by its standard deviation. The decomposition have been computed using the posterior mode estimates of the model's structural parameters.

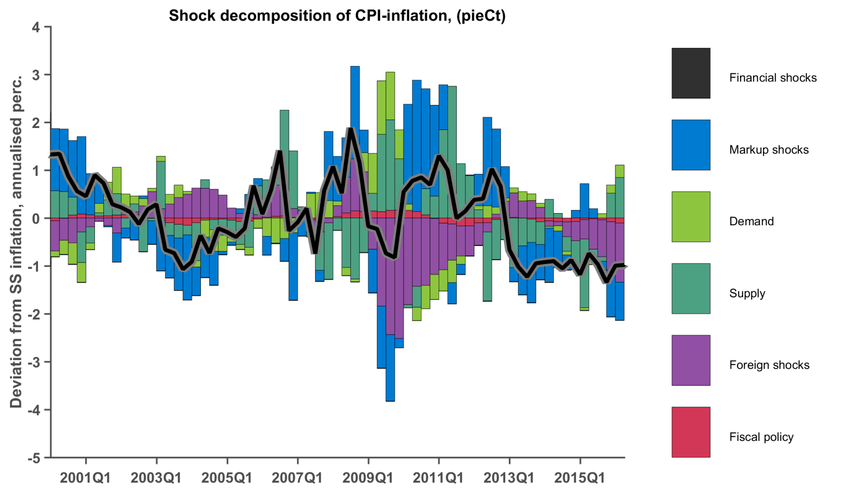

Figure 43: **Historical Shock Decomposition for CPI-inflation, π_t^C**
This figure shows the year-to-year change in the CPI-deflator. decomposed into the structural shocks in the model. The shocks are grouped into categories as explained in the text. Residual contributions, which capture the influence of the initial state of the economy and measurement errors, are not shown. The decomposition have been computed using the posterior mode estimates of the model's structural parameters. The steady state of the CPI-deflator in the model is 1.

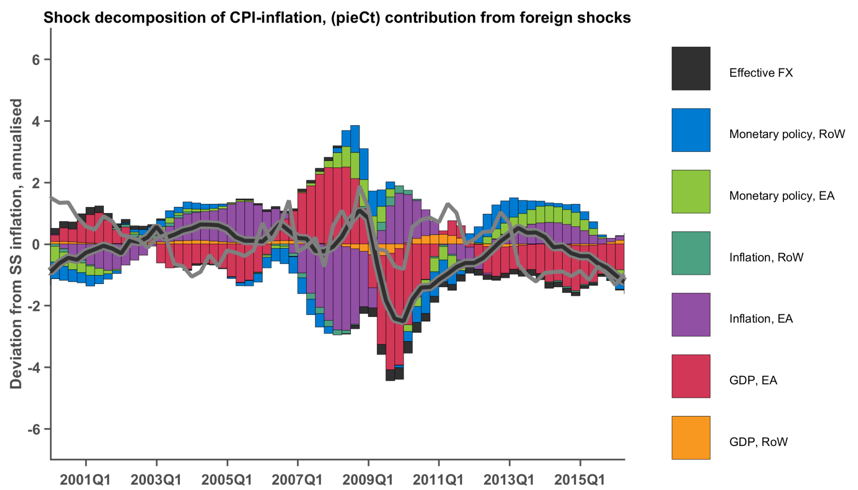

Figure 44: **Historical Shock Decomposition for CPI-inflation, π_t^C**
This figure shows the combined contribution of the category named foreign shocks (solid lines), as explained in the text. The light-gray line is the inflation in the PPI-deflator. Residual contributions, which capture the influence of the initial state of the economy and measurement errors, are not shown. The decomposition have been computed using the posterior mode estimates of the model's structural parameters. The steady state of the CPI-deflator in the model is 1.

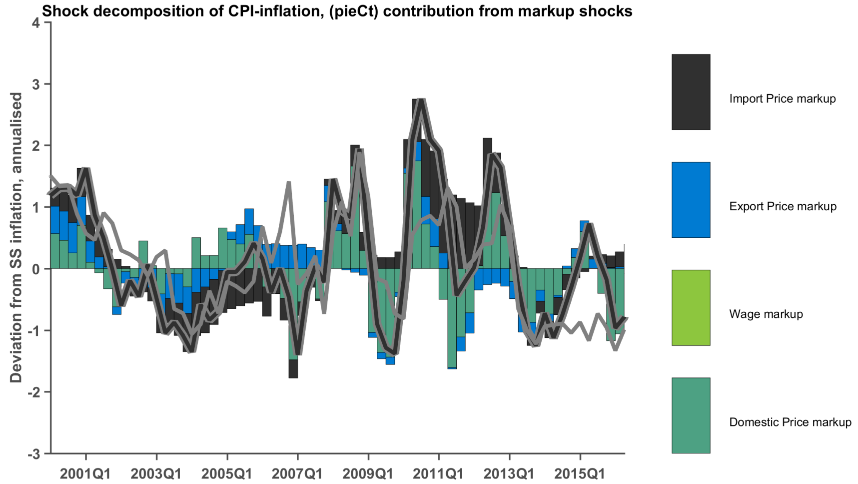

Figure 45: **Historical Shock Decomposition for CPI-inflation,** π_t^C
This figure shows the combined contribution of the category named markup (solid lines), as explained in the text. The light-gray line is the inflation in the PPI-deflator. Residual contributions, which capture the influence of the initial state of the economy and measurement errors, are not shown. The decomposition have been computed using the posterior mode estimates of the model's structural parameters. The steady state of the CPI-deflator in the model is 1.

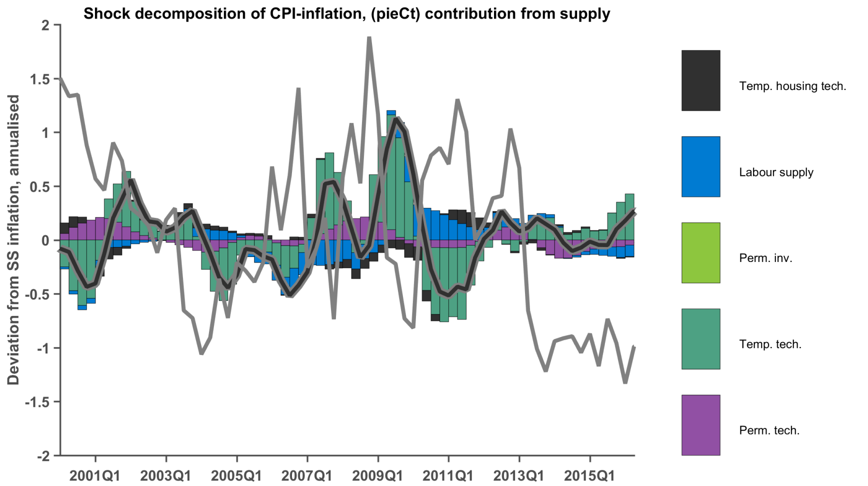

Figure 46: **Historical Shock Decomposition for CPI-inflation,** π_t^C
This figure shows the combined contribution of the category named supply (solid lines), as explained in the text. The light-gray line is the inflation in the PPI-deflator. Residual contributions, which capture the influence of the initial state of the economy and measurement errors, are not shown. The decomposition have been computed using the posterior mode estimates of the model's structural parameters. The steady state of the CPI-deflator in the model is 1.

9. Appendix A: Growth and detrending

There are three trends in the economy: Growth in total factor productivity (TFP) in the goods sector, A_t^Y, growth in investment-specific technology, Z_t, and growth in TFP in the construction sector, A_t^X. The first two trends are identical to those in Pedersen and Ravn (2013), while the latter is introduced along with the housing and construction sector.

I begin by computing the compounded growth rate of output. This will be important, as many other variables in the model will grow at the same rate as output along the economy's balanced growth path. The production function in the goods sector is given by:

$$Y_t D_t = A_t^Y \left(\left(\overline{K}_{t-1} \right)^{1-\eta} \left(K_{t-1}^G \right)^{\eta} \right)^{\alpha^Y} \left[\left(N_t^P \right)^{\omega} \left(N_t^I \right)^{1-\omega} \right]^{1-\alpha^Y}.$$

For the purpose here, I can for simplicity consider the following version, using the fact that the two types of capital (public and private) and of labour, N_t^P, N_t^I, each share the same growth rate:

$$Y_t D_t = A_t^Y \left(K_{t-1} \right)^{\alpha^Y} \left(N_t \right)^{1-\alpha^Y}$$

In this expression, I know that TFP grows at the rate dA_t^Y. Moreover, due to growth in investment-specific technology (IST), the capital stock grows at a faster rate than output. In particular, it grows with the growth rate of output *times* the growth rate in IST. Finally, price dispersion, D_t, and labor input are stationary variables that do not grow along the balanced growth path. I can use these insights to derive an expression for the growth rate of output by dividing the production function in period t with that in period $t-1$:

$$\frac{Y_t D_t}{Y_{t-1} D_{t-1}} = \frac{A_t^Y}{A_{t-1}^Y} \left(\frac{K_{t-1}}{K_{t-2}} \right)^{\alpha^Y} \left(\frac{N_t}{N_{t-1}} \right)^{1-\alpha^Y} \Leftrightarrow$$

$$dY_t dD_t = dA_t^Y \left(dK_{t-1} \right)^{\alpha^Y} \left(dN_t \right)^{1-\alpha^Y},$$

where $dX_t \equiv \frac{dX_t}{dX_{t-1}}$ denotes the growth in variable X_t. Since price dispersion and labor input are stationary, I have that $dD_t = dN_t = 1$, so that these terms cancel out,

$$dY_t = dA_t^Y \left(dK_{t-1} \right)^{\alpha^Y}.$$

I can now impose the relation between the growth rates of capital and output, which, as described above, tells us that:

$$dK_t = dY_t dZ_t.$$

Moreover, along the balanced growth path, where all shocks are zero and have died out, it will be the case that $dK_{t-1} = dK_t$. We can therefore rewrite the expression above as:

$$dY_t = dA_t^Y \left(dK_{t-1} \right)^{\alpha^Y} \Leftrightarrow$$

$$dY_t = dA_t^Y \left(dY_t dZ_t\right)^{\alpha^Y} \Leftrightarrow$$
$$\left(dY_t\right)^{1-\alpha^Y} = dA_t^Y \left(dZ_t\right)^{\alpha^Y} \Leftrightarrow$$
$$dY_t = \left(dA_t^Y dZ_t^{\alpha^Y}\right)^{\frac{1}{1-\alpha^Y}} \equiv d\Gamma_t.$$

In other words, output, consumption, and many other variables will grow with the economy's compounded growth rate denoted $\Gamma_t \equiv \left(A_t^Y Z_t^{\alpha^Y}\right)^{\frac{1}{1-\alpha^Y}}$.[26] Likewise, this term shows up in the measurement equations when the model is taken to data.

I can likewise derive an expression for the growth rate of housing construction, X_t. As for the goods sector, I may consider the following, simplified production function without loss of generality:

$$X_t = A_t^X \left(l_{t-1}\right)^{\alpha^X} \left(\widetilde{I_t^H}\right)^{1-\alpha^X}$$

Once again, I can divide by the lagged production function and rewrite the equation in terms of changes:

$$dX_t = dA_t^X dl_t^{\alpha^X} \left(d\widetilde{I_t^H}\right)^{1-\alpha^X}$$

Land is in fixed supply and hence also stationary; $dl_t = 1$. Intermediate inputs in the construction sector, I_t^H, grows at the same rate as output since this is produced from the same technology:

$$dX_t = dA_t^X \left(d\Gamma_t\right)^{1-\alpha^X} \equiv d\Gamma_t^X.$$

The relative growth rate between residential investments and goods can be derived as follows:

$$\frac{dX_t}{dY_t} = \frac{dA_t^X \left(d\Gamma_t\right)^{1-\alpha^X}}{d\Gamma_t} = dA_t^X d\Gamma_t^{-\alpha^X},$$

and as residential investment during the sample period has experienced practically zero productivity growth, the relative long-run growth rate is

$$\frac{dX_t}{dY_t} = d\Gamma_t^{-\alpha^X}$$

Hence, the real house price in the model grows by $d\Gamma_t^{\alpha^X}$; there is a positive growth rate in the real house price due to a factor which is in fixed supply, land, and due to lower productivity growth in the construction sector.

From a computational perspective, growth is introduced in the same way as in Pedersen and Ravn (2013). The idea is the following: In each equation, all non-stationary variables are multiplied by the level of that particular variable's growth. In the next step, I then seek to

[26] Note that the expression for the compounded growth rate in Iacoviello and Neri (2010) differs somewhat (eq. (12) in that paper). This is because they have the TFP term inside the expression for labor (see their eq. (6)), so that TFP growth affects output less than 1-for-1. In contrast, this model, as well as that of Liu et al. (2013), features an exponent of 1 on the TFP term.

transform those level variables into growth rates by manipulating the equations; the model does not want the *level* of the growth variables (e.g.,Z_t) to appear anywhere in the equations, but only the *rate* of growth (e.g., $dZ_t \equiv \frac{Z_t}{Z_{t-1}}$), for which I have meaningful steady state values (these are given by, respectively, λ_A^Y, λ_A^X, λ_z). The challenge is therefore to figure out what is the correct growth rate of each variable is. For many variables, the growth rate will be a combination of the different trends.

Consider as an example the first-order condition for consumption of patient households, which is given by:

$$\frac{P_t^C}{P_t}\lambda_t^P\left(1+\tau_t^{VAT}\right)=\frac{1}{C_t^P-h^C C_{t-1}^P}.$$

The tax rate, τ_t^{VAT}, is stationary. In addition, recall that $\frac{P_t^C}{P_t}$ measures the relative price of consumption and not a price level and therefore is this variable also stationary. This leaves consumption and the shadow price as the only trending variables. As discussed above, consumption grows at the same rate as output. Accordingly, the shadow price needs to take this growth into account; otherwise the ever-increasing levels of consumption would eventually drive the shadow price to zero. This can be seen from the simplest possible version of this equation: In the absence of shocks, taxes, nominal rigidities and habits, the equation would simply collapse to $\lambda_t^P = \frac{1}{C_t^P}$. Hence, λ_t^P grows at the inverse growth rate of consumption.

We can then manipulate the first-order condition as:

$$\frac{P_t^C}{P_t}\lambda_t^P\left(1+\tau_t^{VAT}\right)=\frac{1}{C_t^P-h^C C_{t-1}^P}\Leftrightarrow$$

$$\frac{P_t^C}{P_t}\frac{\lambda_t^P}{\Gamma_t}\left(1+\tau_t^{VAT}\right)=\frac{1}{C_t^P\Gamma_t-h^C C_{t-1}^P\Gamma_{t-1}}\Leftrightarrow$$

$$\frac{P_t^C}{P_t}\frac{\lambda_t^P}{\Gamma_t}\left(1+\tau_t^{VAT}\right)=\frac{\frac{1}{\Gamma_t}}{C_t^P-h^C C_{t-1}^P\frac{\Gamma_{t-1}}{\Gamma_t}}\Leftrightarrow$$

$$\frac{P_t^C}{P_t}\lambda_t^P\left(1+\tau_t^{VAT}\right)=\frac{1}{C_t^P-h^C C_{t-1}^P\frac{1}{d\Gamma_t}}.$$

This is the growth-adjusted version of the first-order condition. Note that only growth *rates* appear; no levels. Intuitively, the equation shows that in the habit formation of households, I need to account for the growth rate in consumption when computing the habit-adjusted consumption level from which the household derives utility.

I can proceed in the same way with all other equations of the model. In the following, I will only show the derivations for a representative subsample of the equations.

First order condition for the patient household with respect to housing:

$$\varsigma^H \mathcal{H}_t \frac{1}{H_t^P \Gamma_t^X} + \left(\beta^P C_t\right)\left(1-\delta^H\right) E_t\left(\frac{\lambda_{t+1}^P}{\Gamma_{t+1}} \frac{Q_{t+1}^H \Gamma_{t+1}}{\Gamma_{t+1}^X}\right) = \left(1+\tau_t^H\right) \frac{\lambda_t^P}{\Gamma_t} \frac{Q_t^H \Gamma_t}{\Gamma_t^X} \Leftrightarrow$$

$$\begin{aligned} \varsigma^H \mathcal{H}_t \frac{1}{H_t^P} + \left(\beta^P C_t\right)\left(1-\delta^H\right) E_t\left(\lambda_{t+1}^P \frac{Q_{t+1}^H \Gamma_t^X}{\Gamma_{t+1}^X}\right) &= \left(1+\tau_t^H\right) \lambda_t^P Q_t^H \Leftrightarrow \\ \varsigma^H \mathcal{H}_t \frac{1}{H_t^P} + \left(\beta^P C_t\right)\left(1-\delta^H\right) E_t\left(\lambda_{t+1}^P \frac{Q_{t+1}^H}{d\Gamma_{t+1}^X}\right) &= \left(1+\tau_t^H\right) \lambda_t^P Q_t^H, \end{aligned}$$

where it should be noted that the relative house price Q_t^H grows at the inverse rate of the growth rate of housing (i.e., $\frac{1}{\Gamma_t^X}$) *times* the growth rate of output, so that relative expenditures on land $Q_t^H H_t^P$ grow at the same rate as output and consumption. As explained by Iacoviello and Neri (2010), this helps reconcile the upward trend in relative house prices and the stationary nominal expenditure share on household investment goods.

FOC for patient deposits becomes:

$$\frac{\lambda_t^P}{\Gamma_t} = \beta^P R_t^D E_t\left(\frac{\lambda_{t+1}^P}{\Gamma_{t+1}\pi_{t+1}^{DK}}\right) - \beta^P\left(R_t^D - 1\right) E_t \frac{\tau_{t+1}^B \lambda_{t+1}^P}{\Gamma_{t+1}\pi_{t+1}^{DK}} \Leftrightarrow$$

$$\lambda_t^P = \beta^P R_t^D E_t\left(\frac{\lambda_{t+1}^P}{d\Gamma_{t+1}\pi_{t+1}^{DK}}\right) - \beta^P\left(R_t^D - 1\right) E_t \frac{\tau_{t+1}^B \lambda_{t+1}^P}{d\Gamma_{t+1}\pi_{t+1}^{DK}}.$$

FOC for the impatient households housing services:

$$\begin{aligned} & \frac{\varsigma^H \mathcal{H}_t}{H_t^I \Gamma_t^X} + \left(\beta^I C_t\right)\left(1-\delta^H\right) E_t\left(\frac{\lambda_{t+1}^I}{\Gamma_{t+1}} Q_{t+1}^H \frac{\Gamma_{t+1}}{\Gamma_{t+1}^X}\right) \\ + \frac{\Theta_t^I \mu_t^I}{\Gamma_t} \frac{E_t\left(Q_{t+1}^H \Gamma_{t+1} \pi_{t+1}^{DK}\right)}{\Gamma_{t+1}^X R_t^{L,I}} \quad = \quad & \left(1+\tau_t^H\right) \frac{\lambda_t^I}{\Gamma_t} \frac{Q_t^H \Gamma_t}{\Gamma_t^X} \\ \Leftrightarrow \end{aligned}$$

$$\begin{aligned} & \frac{\varsigma^H \mathcal{H}_t}{H_t^I} + \left(\beta^I C_t\right)\left(1-\delta^H\right) E_t\left(\lambda_{t+1}^I \frac{Q_{t+1}^H}{d\Gamma_{t+1}^X}\right) \\ + \frac{\Theta_t^I \mu_t^I d\Gamma_{t+1}}{d\Gamma_{t+1}^X} \frac{E_t\left(Q_{t+1}^H \pi_{t+1}^{DK}\right)}{R_t^{L,I}} \quad = \quad & \left(1+\tau_t^H\right) \lambda_t^I Q_t^H, \end{aligned}$$

since the shadow price associated with the collateral constraint, μ_t^I, follows the same trend as λ_t^I.

FOC for impatient borrowing:

$$\frac{\lambda_t^I}{\Gamma_t} - \frac{\mu_t^I}{\Gamma_t} = \left(\beta^I C_t\right) R_t^{L,I} E_t\left(\frac{\lambda_{t+1}^I}{\Gamma_{t+1}\pi_{t+1}^{DK}}\right) - \left(\beta^I C_t\right)\left(R_t^{L,I} - 1\right)\kappa_{t+1}^R \tau_{t+1}^B E_t\left(\frac{\lambda_{t+1}^I}{\Gamma_{t+1}\pi_{t+1}^{DK}}\right) \Leftrightarrow$$

$$\lambda_t^I - \mu_t^I = \beta^I R_t^{L,I} E_t\left(\frac{\lambda_{t+1}^I}{d\Gamma_{t+1}\pi_{t+1}^{DK}}\right) - \left(\beta^I C_t\right)\left(R_t^{L,I} - 1\right)\kappa_{t+1}^R \tau_{t+1}^B E_t\left(\frac{\lambda_{t+1}^I}{d\Gamma_{t+1}\pi_{t+1}^{DK}}\right).$$

Impatient collateral constraint treated as an equality:

$$B_t^{I,DK} = \Theta_t^I \frac{E_t\left(\frac{\Gamma_{t+1}}{\Gamma_t}\frac{\Gamma_t^X}{\Gamma_{t+1}^X} Q_{t+1}^H H_t^I \pi_{t+1}^{DK}\right)}{R_t^{L,I}} \Leftrightarrow$$

$$B_t^{I,DK} = \Theta_t^I \frac{E_t\left(\frac{d\Gamma_{t+1}}{d\Gamma_{t+1}^X} Q_{t+1}^H H_t^I \pi_{t+1}^{DK}\right)}{R_t^{L,I}}.$$

Impatient HH budget:

$$\begin{aligned}
&\left(1+\tau_t^{VAT}\right)\frac{P_t^C}{P_t} C_t^I \Gamma_t + Q_t^H H_t^I \Gamma_t - \left(1-\delta^H\right) Q_t^H \frac{\Gamma_t}{\Gamma_t^X} H_{t-1}^I \Gamma_{t-1}^X \\
&- B_t^{I,DK}\Gamma_t + \frac{R_{t-1}^{L,I} B_{t-1}^{I,DK}\Gamma_{t-1}}{\pi_t^{DK}} \\
= \quad & \frac{\kappa_t^{R,I} B_{t-1}^{I,DK}\Gamma_{t-1}\left(R_{t-1}^{L,I}-1\right)}{\pi_t^{DK}} + \left(1-\tau_t^n\right) w_t^I N_t^I \Gamma_t - \tau_t^H Q_t^H H_t^I \Gamma_t
\end{aligned}$$

$$\begin{aligned}
&\left(1+\tau_t^{VAT}\right)\frac{P_t^C}{P_t} C_t^I + Q_t^H\left(H_t^I - \left(1-\delta^H\right)\frac{H_{t-1}^I}{d\Gamma_t^X}\right) \\
&- B_t^{I,DK} + \frac{R_{t-1}^{L,I} B_{t-1}^{I,DK}}{\pi_t^{DK} d\Gamma_t} \\
= \quad & \frac{\kappa_t^{R,I} B_{t-1}^{I,DK}\left(R_{t-1}^{L,I}-1\right)}{d\Gamma_t \pi_t^{DK}} + \left(1-\tau_t^n\right) w_t^I N_t^I - \tau_t^H Q_t^H H_t^I.
\end{aligned}$$

Entrepreneur budget:

$$
\begin{aligned}
& \left(1+\tau_t^{VAT}\right)\frac{P_t^C}{P_t}C_t^E\Gamma_t+\frac{P_t^{I,Y}}{P_t}\frac{1}{Z_t}I_t^Y\Gamma_tZ_t+\frac{P_t^{IX}}{P_t}I_t^H\Gamma_t \\
& -B_t^{E,DK}\Gamma_t+\frac{R_{t-1}^{L,E}B_{t-1}^{E,DK}\Gamma_{t-1}}{\pi_t^{DK}}+Q_t^Ll_t\left(1+\tau_t^L\right)\Gamma_t \\
= \; & \Gamma_tQ_t^ll_{t-1}+\Gamma_tr_t^Ll_{t-1}-\left(\frac{\tau_t^Kr_t^{K,Y}u_t^Y\mathcal{K}_t}{Z_t}-\frac{\tau_t^K\delta^{K,Y}}{Z_t}+\frac{z^u\left(u_t^Y\mathcal{K}_t\right)}{Z_t}\right)K_{t-1}^Y\Gamma_{t-1}Z_{t-1} \\
& +\frac{\kappa_t^{R,E}B_{t-1}^{E,DK}\Gamma_{t-1}\left(R_{t-1}^{L,E}-1\right)}{\pi_t^{DK}}
\end{aligned}
$$

$$
\begin{aligned}
& \left(1+\tau_t^{VAT}\right)\frac{P_t^C}{P_t}C_t^E+\frac{P_t^{I,Y}}{P_t}I_t^Y+\frac{P_t^{I,X}}{P_t}I_t^H \\
& -B_t^{E,DK}+\frac{R_{t-1}^{L,E}B_{t-1}^{E,DK}}{\pi_t^{DK}d\Gamma_t}+Q_t^Ll_t\left(1+\tau_t^L\right) \\
= \; & \Gamma_tQ_t^ll_{t-1}+\Gamma_tr_t^Ll_{t-1} \\
& -\left(\tau_t^Kr_t^{K,Y}u_t^Y\mathcal{K}_t-\tau_t^K\delta^{K,Y}+z^u\left(u_t^Y\mathcal{K}_t\right)\right)K_{t-1}^Y\frac{1}{d\Gamma_tdZ_t} \\
& +\frac{\kappa_t^{R,E}B_{t-1}^{E,DK}\left(R_{t-1}^{L,E}-1\right)}{d\Gamma_t\pi_t^{DK}}
\end{aligned}
$$

where the capital tax payments and deductions and the utilisation costs of capital have been scaled by the IST growth in the goods production sector. As in Pedersen and Ravn (2013), this ensures that these costs are not eroded over time. Likewise, the relative price of investment goods in the goods sector needs to be adjusted for growth (in this case, negative growth), while the price of investment goods in the construction sector does not. Finally, the price of land grows at the same rate as output because the stock of land is constant.

Entrepreneur collateral:

$$
B_t^{E,DK}\Gamma_t=\Theta_t^E\frac{E_t\left[\frac{Q_{t+1}^Y}{Z_{t+1}}K_t^Y\Gamma_tZ_t\pi_{t+1}^{DK}+Q_{t+1}^L\Gamma_{t+1}l_t\pi_{t+1}^{DK}\right]}{R_t^{L,E}}\Leftrightarrow
$$

$$
B_t^{E,DK}=\Theta_t^E\frac{E_t\left[Q_{t+1}^YK_t^Y\frac{1}{dZ_{t+1}}\pi_{t+1}^{DK}+Q_{t+1}^Ld\Gamma_{t+1}l_t\pi_{t+1}^{DK}\right]}{R_t^{L,E}}.
$$

Entrepreneur FOC for land:

$$
\begin{aligned}
\left(1+\tau_t^L\right)\frac{\lambda_t^E}{\Gamma_t}Q_t^L\Gamma_t &= \left(\beta^E C_t\right)E_t\left[\frac{\lambda_{t+1}^E}{\Gamma_{t+1}}\left(\frac{\Gamma_{t+1}}{\Gamma_{t+1}^X}r_t^L\Gamma_t^X+Q_{t+1}^L\Gamma_{t+1}\right)\right] \\
+\frac{\mu_t^E}{\Gamma_t}\Theta_t^E\frac{E_t\left(Q_{t+1}^L\Gamma_{t+1}\pi_{t+1}^{DK}\right)}{R_t^{L,E}} &\Leftrightarrow
\end{aligned}
$$

$$
\begin{aligned}
\left(1+\tau_t^L\right)\lambda_t^E Q_t^L &= \left(\beta^E C_t\right)E_t\left[\lambda_{t+1}^E\left(r_t^L+Q_{t+1}^L\right)\right] \\
&\quad +\mu_t^E\Theta_t^E\frac{E_t\left(Q_{t+1}^L d\Gamma_{t+1}\pi_{t+1}^{DK}\right)}{R_t^{L,E}},
\end{aligned}
$$

where I have used the fact that the rental rate on land grows at the same rate as construction. This follows intuitively from the fact that land is in fixed supply. From a computational viewpoint, the FOC for profit maximization with respect to land can be manipulated as follows:

$$
\begin{aligned}
r_t^L\Gamma_t^X &= \frac{\alpha^X X_t\Gamma_t^X mc_t^X}{l_{t-1}} \Leftrightarrow \\
r_t^L &= \frac{\alpha^X X_t mc_t^X}{l_{t-1}}
\end{aligned}
$$

which clearly shows that the rental rate on land must grow at the rate Γ_t^X in order to obtain a sensible stationary equation.

Entrepreneur FOC for intermediates:

$$
\begin{aligned}
\frac{P_t^{IX}\Gamma_t^X}{P_t\Gamma_t} &= \frac{\left(1-\alpha^X\right)X_t mc_t^X}{I_t^H} \Leftrightarrow \\
\frac{P_t^{IX}}{P_t} &= \frac{\left(1-\alpha^X\right)X_t\Gamma_t^X mc_t^X}{I_t^H\Gamma_t}
\end{aligned}
$$

FOC for labor:

$$
w_t^j\Gamma_t = \frac{mc_t\left(1-\alpha^Y\right)\omega Y_t\Gamma_t}{N_t^j} \Leftrightarrow
$$

$$
w_t^{Y,P} = \frac{mc_t\left(1-\alpha^Y\right)\omega Y_t}{N_t^j},\ \mathit{for}\ j=P,I
$$

Rental rate, goods sector:

$$
\frac{r_t^{K,Y}}{Z_t}u_t^Y\mathcal{K}_t = \frac{(1-\eta)\,\alpha^Y Y_t\Gamma_t}{K_{t-1}^Y\Gamma_{t-1}Z_{t-1}} \Leftrightarrow
$$

$$r_t^{K,Y} u_t^Y \mathcal{K}_t = \frac{(1-\eta)\,\alpha^Y Y_t}{K_{t-1}^Y} \frac{\Gamma_t Z_t}{\Gamma_{t-1} Z_{t-1}},$$

where I then define $Invgrowth_t = \frac{\Gamma_t Z_t}{\Gamma_{t-1} Z_{t-1}}$ in the codes.

About the different rental rates of the model: The rental rate of land is increasing due to the scarcity of land, as we saw above. In contrast, the rental rate of capital in the goods sector is declining due to IST growth. In particular, it is declining at the same rate as the relative price of capital (or investment) in the goods sector *relative* to the output of the goods sector, i.e. consumption goods. We know that the relative price of investment goods declines by Z_t because the capital-output ratio grows by Z_t; hence, also r_t^{KY} declines by Z_t, as I used above. FOC for investment in goods sector is unchanged from Pedersen and Ravn (2013).

FOC for capital

$$\begin{aligned}
Q_t^Y &= \left(\beta^E C_t\right) E_t \frac{\lambda_{t+1}^E}{\lambda_t^E} * \\
&\left[r_{t+1}^{K,Y}\left(1-\tau_{t+1}^K\right) u_{t+1}^Y \mathcal{K}_t + \tau_{t+1}^K \delta^{K,Y} - z^u\left(u_{t+1}^Y \mathcal{K}_{t+1}\right) + \left(1-\delta^{K,Y}\right) Q_{t+1}^Y\right] \\
&+\Theta_t^E \frac{\mu_t^E}{\lambda_t^E} \frac{E_t Q_{t+1}^Y \pi_{t+1}^{DK}}{R_t^{L,E}}
\end{aligned}$$

$$\begin{aligned}
\frac{Q_t^Y}{Z_t} &= \left(\beta^E C_t\right) E_t \frac{\lambda_{t+1}^E \Gamma_t}{\lambda_t^E \Gamma_{t+1}} * \\
&\left[\frac{r_{t+1}^{K,Y}\left(1-\tau_{t+1}^K\right) u_{t+1}^Y \mathcal{K}_t}{Z_{t+1}} + \frac{\tau_{t+1}^K \delta^{K,Y}}{Z_{t+1}} - \frac{z^u\left(u_{t+1}^Y \mathcal{K}_{t+1}\right)}{Z_{t+1}}\right] \\
&+\beta^E E_t \frac{\lambda_{t+1}^E \Gamma_t}{\lambda_t^E \Gamma_{t+1}}\left(1-\delta^{K,Y}\right) \frac{Q_{t+1}^Y}{Z_{t+1}} + \Theta_t^E \frac{\mu_t^E \Gamma_t}{\lambda_t^E \Gamma_t} \frac{E_t \frac{Q_{t+1}^Y}{Z_{t+1}} \pi_{t+1}^{DK}}{R_t^{L,E}}
\end{aligned}$$

$$\begin{aligned}
Q_t^Y &= \left(\beta^E C_t\right) E_t \frac{\lambda_{t+1}^E}{\lambda_t^E} \frac{\Gamma_t Z_t}{\Gamma_{t+1} Z_{t+1}}\left[r_{t+1}^{K,Y}\left(1-\tau_{t+1}^K\right) u_{t+1}^Y cap_t + \tau_{t+1}^K \delta^{K,Y} - z^u\left(u_{t+1}^Y cap_{t+1}\right)\right] \\
&+\left(\beta^E C_t\right) E_t \frac{\lambda_{t+1}^E}{\lambda_t^E}\left(1-\delta^{K,Y}\right) Q_{t+1}^Y \frac{\Gamma_t Z_t}{\Gamma_{t+1} Z_{t+1}} + \Theta_t^E \frac{\mu_t^E}{\lambda_t^E} \frac{E_t Q_{t+1}^Y \pi_{t+1}^{DK}}{R_t^{L,E}} \frac{Z_t}{Z_{t+1}}
\end{aligned}$$

$$\begin{aligned}
Q_t^Y &= \left(\beta^E C_t\right) E_t \frac{\lambda_{t+1}^E}{\lambda_t^E} \frac{\Gamma_t Z_t}{\Gamma_{t+1} Z_{t+1}} * \\
&\left[r_{t+1}^{K,Y}\left(1-\tau_{t+1}^K\right) u_{t+1}^Y \mathcal{K}_t + \tau_{t+1}^K \delta^{K,Y} - z^u\left(u_{t+1}^Y \mathcal{K}_{t+1}\right) + \left(1-\delta^{K,Y}\right) Q_{t+1}^Y\right] \\
&+\Theta_t^E \frac{\mu_t^E}{\lambda_t^E} \frac{E_t Q_{t+1}^Y \pi_{t+1}^{DK}}{R_t^{L,E}} \frac{1}{dZ_{t+1}}.
\end{aligned}$$

Production function, goods sector:

$$Y_t D_t = A_t^Y \left(\left(\overline{K}_{t-1}^Y \right)^{1-\eta} \left(K_{t-1}^G \right)^{\eta} \right)^{\alpha^Y} \left(N_t^{Tot} \right)^{1-\alpha^Y} \Leftrightarrow$$

$$Y_t \Gamma_t D_t = A_t^Y \left(\left(\overline{K}_{t-1}^Y \Gamma_{t-1} Z_{t-1} \right)^{1-\eta} \left(K_{t-1}^G \Gamma_{t-1} Z_{t-1} \right)^{\eta} \right)^{\alpha^Y} \left(N_t^{Tot} \right)^{1-\alpha^Y} \Leftrightarrow$$

$$Y_t D_t = \frac{A_t^Y \left(\Gamma_{t-1} Z_{t-1} \right)^{\alpha^Y}}{\Gamma_t} \left(\left(\overline{K}_{t-1}^Y \right)^{1-\eta} \left(K_{t-1}^G \right)^{\eta} \right)^{\alpha^Y} \left(N_t^{Tot} \right)^{1-\alpha^Y} \Leftrightarrow$$

$$Y_t D_t = \frac{A_t^Y \left(\left(A_{t-1}^Y Z_{t-1}^{\alpha^Y} \right)^{\frac{1}{1-\alpha^Y}} Z_{t-1} \right)^{\alpha^Y}}{\left(A_t^Y Z_t^{\alpha^Y} \right)^{\frac{1}{1-\alpha^Y}}} \left(\left(\overline{K}_{t-1}^Y \right)^{1-\eta} \left(K_{t-1}^G \right)^{\eta} \right)^{\alpha^Y} \left(N_t^{Tot} \right)^{1-\alpha^Y} \Leftrightarrow$$

$$Y_t D_t = \frac{A_t^Y \left(A_{t-1}^Y \right)^{\frac{\alpha^Y}{1-\alpha^Y}} \left(Z_{t-1}^{\frac{\alpha^Y}{1-\alpha^Y}} Z_{t-1} \right)^{\alpha^Y}}{\left(A_t^Y \right)^{\frac{1}{1-\alpha^Y}} Z_t^{\frac{\alpha^Y}{1-\alpha^Y}}} \left(\left(\overline{K}_{t-1}^Y \right)^{1-\eta} \left(K_{t-1}^G \right)^{\eta} \right)^{\alpha^Y} \left(N_t^{Tot} \right)^{1-\alpha^Y} \Leftrightarrow$$

$$Y_t D_t = \left(A_t^Y \right)^{1-\frac{1}{1-\alpha^Y}} \left(A_{t-1}^Y \right)^{\frac{\alpha^Y}{1-\alpha^Y}} Z_{t-1}^{\frac{\alpha^Y}{1-\alpha^Y}} Z_t^{-\frac{\alpha^Y}{1-\alpha^Y}} \left(\left(\overline{K}_{t-1}^Y \right)^{1-\eta} \left(K_{t-1}^G \right)^{\eta} \right)^{\alpha^Y} \left(N_t^{Tot} \right)^{1-\alpha^Y} \Leftrightarrow$$

$$Y_t D_t = \left(\frac{A_{t-1}^Y}{A_t^Y} \right)^{\frac{\alpha^Y}{1-\alpha^Y}} \left(\frac{Z_{t-1}}{Z_t} \right)^{\frac{\alpha^Y}{1-\alpha^Y}} \left(\left(\overline{K}_{t-1}^Y \right)^{1-\eta} \left(K_{t-1}^G \right)^{\eta} \right)^{\alpha^Y} \left(N_t^{Tot} \right)^{1-\alpha^Y} \Leftrightarrow$$

$$Y_t D_t = \left(dA_t^Y dZ_t \right)^{-\frac{\alpha^Y}{1-\alpha^Y}} \left(\left(\overline{K}_{t-1}^Y \right)^{1-\eta} \left(K_{t-1}^G \right)^{\eta} \right)^{\alpha^Y} \left(N_t^{Tot} \right)^{1-\alpha^Y},$$

where I have used the definition of $\Gamma_t \equiv \left(A_t^Y Z_t^{\alpha^Y} \right)^{\frac{1}{1-\alpha^Y}}$.

Production function in the residential investment sector:

$$\begin{aligned}
X_t &= A_t^X \left(l_{t-1} \right)^{\alpha^X} \left(\widetilde{I}_t^H \right)^{1-\alpha^X} \Leftrightarrow \\
X_t \Gamma_t^X &= A_t^X \left(l_{t-1} \right)^{\alpha^X} \left(\widetilde{I}_t^H \Gamma_t \right)^{1-\alpha^X} \Leftrightarrow \\
X_t &= \frac{A_t^X \Gamma_t^{1-\alpha^X}}{\Gamma_t^X} \left(l_{t-1} \right)^{\alpha^X} \left(\widetilde{I}_t^H \right)^{1-\alpha^X} \Leftrightarrow \\
X_t &= \frac{\Gamma_t^X}{\Gamma_t^X} \left(l_{t-1} \right)^{\alpha^X} \left(\widetilde{I}_t^H \right)^{1-\alpha^X} \Leftrightarrow \\
X_t &= \left(l_{t-1} \right)^{\alpha^X} \left(\widetilde{I}_t^H \right)^{1-\alpha^X}
\end{aligned}$$

Capital accumulation:

$$K_t^Y = \left(1-\delta^{K,Y}\right) K_{t-1}^Y + \left(1-S_t\right) Z_t^T I_t^Y \Leftrightarrow$$

$$K_t^Y \Gamma_t Z_t = \left(1-\delta^{K,Y}\right) K_{t-1}^Y \Gamma_{t-1} Z_{t-1} + \left(1-S_t\right) Z_t^T I_t^Y \Gamma_t Z_t \Leftrightarrow$$

$$K_t^Y = \left(1-\delta^{K,Y}\right) K_{t-1}^Y \frac{\Gamma_{t-1} Z_{t-1}}{\Gamma_t Z_t} + \left(1-S_t\right) Z_t^T I_t^Y.$$

Goods market clearing:

$$Y_t = C_t^{P,DK} + C_t^{I,DK} + C_t^{E,DK} + I_t^{Y,DK} + I_t^{H,DK} + G_t + I_t^G + z^u\left(u_t^Y \mathcal{K}_t\right) K_{t-1}^Y + \frac{Adj_{t-1}}{\pi_t^{DK}} + Ex_t \Leftrightarrow$$

$$Y_t = C_t^{P,DK} + C_t^{I,DK} + C_t^{E,DK} + I_t^{Y,DK} + I_t^{H,DK} + G_t + I_t^G + \frac{z^u\left(u_t^Y \mathcal{K}_t\right) K_{t-1}^Y}{d\Gamma_t dZ_t} + \frac{Adj_{t-1}}{d\Gamma_t \pi_t^{DK}} + Ex_t.$$

FOC for loan branch of bank:

$$0 = 1-\varepsilon_t^{bj}+\varepsilon_t^{bj}\frac{R_t^L-1}{R_t^{L,j}-1}-\Phi^{B,j}\left(\frac{R_t^{L,j}-1}{R_{t-1}^{L,j}-1}-1\right)\frac{R_t^{L,j}-1}{R_{t-1}^{L,j}-1}+\Phi^{B,j}E_t q_{t,t+1}^P\left(\frac{R_{t+1}^{L,j}-1}{R_t^{L,j}-1}-1\right)\left(\frac{R_{t+1}^{L,j}-1}{R_t^{L,j}-1}\right)^2\frac{B_{t+1}^j}{B_t^j}.$$

Here, I first insert the expression for $q_{t,t+1}^P = \left(\beta^P C_t\right)\frac{E_t \lambda_{t+1}^P}{\lambda_t^P \pi_{t+1}^{DK}}$:

$$\begin{aligned} 0 \quad &= \quad 1-\varepsilon_t^{bj}+\varepsilon_t^{bj}\frac{R_t^L-1}{R_t^{L,j}-1}-\Phi^{B,j}\left(\frac{R_t^{L,j}-1}{R_{t-1}^{L,j}-1}-1\right)\left(\frac{R_t^{L,j}-1}{R_{t-1}^{L,j}-1}\right) \\ &\quad +\Phi^{B,j}E_t\left(\beta^P C_t\right)\frac{E_t\lambda_{t+1}^P}{\lambda_t^P\pi_{t+1}^{DK}}\left(\frac{R_{t+1}^{L,j}-1}{R_t^{L,j}-1}-1\right)\left(\frac{R_{t+1}^{L,j}-1}{R_t^{L,j}-1}\right)^2\frac{B_{t+1}^j}{B_t^j} \\ &\Leftrightarrow \end{aligned}$$

$$\begin{aligned} 0 \quad &= \quad 1-\varepsilon_t^{bj}+\varepsilon_t^{bj}\frac{R_t^L-1}{R_t^{L,j}-1}-\Phi^{B,j}\left(\frac{R_{t+1}^{L,j}-1}{R_t^{L,j}-1}-1\right)\left(\frac{R_t^{L,j}-1}{R_{t-1}^{L,j}-1}\right) \\ &\quad +\Phi^{B,j}E_t\left(\beta^P C_t\right)\frac{E_t\lambda_{t+1}^P\Gamma_t}{\lambda_t^P\pi_{t+1}^{DK}\Gamma_{t+1}}\left(\frac{R_{t+1}^{L,j}}{R_t^{L,j}}-1\right)\left(\frac{R_{t+1}^{L,j}-1}{R_t^{L,j}-1}\right)^2\frac{B_{t+1}^j\Gamma_{t+1}}{B_t^j\Gamma_t} \\ &\Leftrightarrow \end{aligned}$$

$$\begin{aligned} 0 \quad = \quad & 1-\varepsilon_t^{bj}+\varepsilon_t^{bj}\frac{R_t^L-1}{R_t^{L,j}-1}-\Phi^{B,j}\left(\frac{R_{t+1}^{L,j}-1}{R_t^{L,j}-1}-1\right)\left(\frac{R_t^{L,j}-1}{R_{t-1}^{L,j}-1}\right) \\ & +\Phi^{B,j}E_t\left(\beta^P C_t\right)\frac{E_t\lambda_{t+1}^P}{\lambda_t^P\pi_{t+1}^{DK}d\Gamma_{t+1}}\left(\frac{R_{t+1}^{L,j}}{R_t^{L,j}}-1\right)\left(\frac{R_{t+1}^{L,j}-1}{R_t^{L,j}-1}\right)^2\frac{B_{t+1}^j d\Gamma_{t+1}}{B_t^j} \\ \Leftrightarrow & \end{aligned}$$

$$\begin{aligned} 0 \quad = \quad & 1-\varepsilon_t^{bj}+\varepsilon_t^{bj}\frac{R_t^L-1}{R_t^{L,j}-1}-\Phi^{B,j}\left(\frac{R_{t+1}^{L,j}-1}{R_t^{L,j}-1}-1\right)\left(\frac{R_t^{L,j}-1}{R_{t-1}^{L,j}-1}\right) \\ & +\Phi^{B,j}E_t\left(\beta^P C_t\right)\frac{E_t\lambda_{t+1}^P}{\lambda_t^P\pi_{t+1}^{DK}}\left(\frac{R_{t+1}^{L,j}-1}{R_t^{L,j}-1}-1\right)\left(\frac{R_{t+1}^{L,j}-1}{R_t^{L,j}-1}\right)^2\frac{B_{t+1}^j}{B_t^j}, \end{aligned}$$

i.e. all the growth-related terms cancel out, and we are left with the original equation. This holds also for the equivalent FOC for the deposit branch.

Law of motion for bank capital:

$$\pi_t^{DK}K_t^B\Gamma_t=\left(1-\delta^B\right)K_{t-1}^B\Gamma_{t-1}+\Xi_{t-1}\Gamma_{t-1}\Leftrightarrow$$

$$\pi_t^{DK}K_t^B d\Gamma_t=\left(1-\delta^B\right)K_{t-1}^B+\Xi_{t-1}.$$

The expression for bank profits is unaffected, as the growth terms may simply be cancelled out.

Net foreign asset position:

$$D_t^*=\frac{R_{t-1}^{ECB}\exp(-\psi_d\left(\frac{D_{t-1}^*}{Y_{t-1}}-\frac{\overline{D}^*}{\overline{Y}}\right))}{\pi_t^{DK}}D_{t-1}^*+Ex_t-Im_t\Leftrightarrow$$

$$D_t^*\Gamma_t=\frac{R_{t-1}^{ECB}\exp(-\psi_d\left(\frac{D_{t-1}^*\Gamma_{t-1}}{Y_{t-1}\Gamma_{t-1}}-\frac{\overline{D}^*}{\overline{Y}}\right))}{\pi_t^{DK}}D_{t-1}^*\Gamma_{t-1}+Ex_t\Gamma_t-Im_t\Gamma_t\Leftrightarrow$$

$$D_t^*=\frac{R_{t-1}^{ECB}\exp(-\psi_d\left(\frac{D_{t-1}^*}{Y_{t-1}}-\frac{\overline{D}^*}{\overline{Y}}\right))}{\pi_t^{DK}}D_{t-1}^*\frac{1}{d\Gamma_t}+Ex_t-Im_t.$$

The process for z_t^P reads:

$$z_t^P=\left(z_{t-1}^P\right)^\nu\left(\frac{1}{\lambda_t^P}\right)^{1-\nu},$$

which becomes:

$$z_t^P\Gamma_t=\left(z_{t-1}^P\Gamma_{t-1}\right)^\nu\left(\frac{\Gamma_t}{\lambda_t^P}\right)^{1-\nu}\Leftrightarrow$$

$$z_t^P = \left(z_{t-1}^P \frac{1}{d\Gamma_t}\right)^{\nu} \left(\frac{1}{\lambda_t^P}\right)^{1-\nu},$$

where I recall that also the smooth process for habit-adjusted consumption, i.e. z_t^P, needs to be adjusted for growth. The explanation is as follows: In computing O_t^P, which feeds into the patient household's disutility of labor, I need to compare the growth rate of (habit-adjusted) consumption to its steady state growth rate: If consumption grows faster than this rate, O_t^P is smaller than 1, as explained in Pedersen and Ravn (2013). As a result, in a model where consumption is stationary, so is z_t^P; but in a model with growth in consumption in steady state, such as this model, the variable z_t^P needs to grow at the same rate in order for this comparison (and thus for the computation of O_t^P) to make sense at every point in time. In other words, z_t^P grows at the same rate as consumption. On the other hand, O_t^P is a stationary variable that needs not be transformed. The equation for O_t^P then becomes:

$$O_t^P = z_t^P \lambda_t^P \Leftrightarrow$$

$$O_t^P = z_t^P \Gamma_t \frac{\lambda_t^P}{\Gamma_t} \Leftrightarrow$$

$$O_t^P = z_t^P \lambda_t^P,$$

i.e. this equation remains unchanged.

The wage process for the wage rate for households of type $j = P, I$ is:

$$\left(w_t^j\right)^{1-\epsilon_t^W} = \theta_W \left(w_{t-1}^j\right)^{1-\epsilon_t^W} + (1-\theta_W)\left(\widetilde{w}_t^j\right)^{1-\epsilon_t^W} \Leftrightarrow$$

$$\left(w_t^j \Gamma_t\right)^{1-\epsilon_t^W} = \theta_W \left(w_{t-1}^j \Gamma_{t-1}\right)^{1-\epsilon_t^W} + (1-\theta_W)\left(\widetilde{w}_t^j \Gamma_t\right)^{1-\epsilon_t^W} \Leftrightarrow$$

$$\left(w_t^j\right)^{1-\epsilon_t^W} = \theta_W \left(w_{t-1}^j \frac{1}{d\Gamma_t}\right)^{1-\epsilon_t^W} + (1-\theta_W)\left(\widetilde{w}_t^j\right)^{1-\epsilon_t^W} \Leftrightarrow$$

$$\left(w_t^j\right)^{1-\epsilon_t^W} = \theta_W \left(w_{t-1}^j \frac{1}{d\Gamma_t}\right)^{1-\epsilon_t^W} + (1-\theta_W)\left(\widetilde{w}_t^j\right)^{1-\epsilon_t^W} \Leftrightarrow$$

$$w_t^j = \left[\theta_W \left(w_{t-1}^j \frac{1}{d\Gamma_t}\right)^{1-\epsilon_t^W} + (1-\theta_W)\left(\widetilde{w}_t^j\right)^{1-\epsilon_t^W}\right]^{\frac{1}{1-\epsilon_t^W}}.$$

10. Appendix B: Measurement equations, Shocks, and Calibration

10.1. Calibration

In what follows, I will describe the calibrations of the models' parameters. l aim to hit "big ratios"; residential construction to GDP ratio, private consumption to GDP ratio etc. I set the deep parameters which also existed in the model of Pedersen and Ravn (2013) to the estimated values in that model if they are not reestimated. I calibrate the parameters which are new in this model following Iacoviello and Neri (2010) and Gerali et al. (2010). I deviate from this strategy for some parameters to get plausible impulse responses. The calibrated parameters can be found in table (3). In these tables it is also briefly explained how the standard DSGE-parameters are calibrated. Consequently, in what follows not all the parameter calibrations are explained in detail.

I aim to get best possible fit of the average GDP-ratios in the supply balance. I use the steady state value of the export shock, x^Z, and the labour supply shock, $\bar{\chi}$, given all the structural parameters, to hit the consumption to GDP ratio of approximately 50 pct. In the model, a higher steady state private consumption to GDP ratio implies a lower export to GDP-ratio. Hence, I can not hit both ratios at the same time. In the model, the export to GDP ratio is around $37\frac{1}{2}$ pct., which is below the empirical ratio of around 50 pct. The part of the households which are credit constrained, the calibration of its labour supply etc. implies that patient household consumes around 3/4 of total private consumption, the entreprenuer around 5 pct. and the impatient household around 20 pct. The public consumption to GDP ratio is set to 26 pct. of GDP approximately equal to its long run empirical average. I calibrate the investment share to 18 pct. of GDP using depreciation of capital, $\delta^{K,Y}$. Imports in steady state follows from the steady state ratio of the rests of the variables.

I set the steady state level of construction to the long run average for the Danish economy, $4\frac{1}{2}$ pct. of GDP. I use the preference parameter on the impatient households' utility function, ς^{HI}, to calibrate the relative steady state ratios of housing wealth among the two types of households; $\omega Q_t^H H_t^{tot}$ for for the patient household and $(1-\omega) Q_t^H H_t^{tot}$ for the impatient household. I calibrate the total household wealth using depreciation of housing together with the preference parameters previously mentioned. I aim for a level of total housing wealth in percentage of GDP of around 1.75, which equals the empirical counterpart in the later part of the sample. As it is difficult to get good data for land, I have calibrated the value of land to 50 pct. of GDP.

I use the preference parameter, η_N, in the household's utility function for labour to hit a value of around 1/3 for labour for both type of households. I calibrate the substitution of different labour types, ϵ^W, to give a markup that gives a steady state unemployment of around 4 pct. of the labour force, which is believed to be the structural level for Denmark, see Andersen and Rasmussen (2011).

I set the production function parameter for construction, α^X, to 0.8, which implies that a house consists roughly of 20 pct. land and 80 pct. goods. This follows the strategy from macroeconometric model's for Denmark, see Grinderslev and Smidt (2006).

I calibrate the home-bias parameter for construction, ϑ_X, to 0.8 based on estimates from input-output tables, see og Byggestyrelsen (2008). I calibrate the home-bias to 1 for public consumption. Public consumption in this model includes both consumption of goods, which can have a large import component, and services, which have a much lower import component, as well as public production such as education and child care. This latter component of public consumption is the largest and only has an import share of around 10 pct. Based on input-output tables it could perhaps be justified to set it to 0.9, but the parameter is set to 1 for simplicity. All tax rates are calibrated to its equivalent counter part in the Danmarks Nationalbanks' macroeconometric model, MONA, see Nationalbanken (2003).

10.2. Measurement equations

In the estimation it is necessary to link observable data to variables in the model. I will in this section use GDP as an example. In the model GDP is growing due to the assumption on a trend in productivity and the price of investment goods. I need to map the difference in GDP stationarized, $\Gamma_t \frac{Y_t}{Y_{t-1}}$, in which Γ_t is the long-run quarter-to-quarter growth rate of output, to data. One time period in the model is 1 quarter. I consequently need to take log-differences of GDP but not demean it: $gy_t \equiv \frac{\log\left(Y_t^{obs}\right)}{\log\left(Y_{t-1}^{obs}\right)}$. Consequently, all variables which trend with the trend rate of GDP need to be link to data using in the following way:

$$gz_t^1 = \Gamma_t \frac{Z_t^1}{Z_{t-1}^1}, \; for \; Z_t^1 = Y_t, Ex_t, IM_t, G_t, I_t^g, W_t^{Tot}, \pi_t^M, \pi_t^{Ex}, \pi_t^X, B_t^E, B_t^I, W_t^{tot}$$

Likewise, all variables which grows with the trend rate of residential investments, Γ_t^X, need to be multiplied by that growth rate

$$gz_t^X = \Gamma_t \frac{Z_t^X}{Z_{t-1}^X}, \; for \; Z_t^X = X_t, \pi_t^X,$$

, while the real house price grows by the ratio of the growth rate in the goods sector, Γ_t, to the growth rate in the construction sector, Γ_t^X, such that the measurement equation becomes

$$gQ_t^H = \frac{\Gamma_t}{\Gamma_t^X} \frac{Q_t^H}{Q_{t-1}^H}$$

Investments and private consumption are in data measured including imports, $C_t = C_t^{DK} + C_t^W$, while the respective model variables is an CES aggregate of the two subcomponents. Further data measures total private consumption while in the model private consumption is divided into the three agents, patient and impatient households and the entreprenuer. Taken consumption for patient households as an example, I add the demand for consumption goods produced in Denmark, $C_t^{P,DK}$, and consumption goods imported, $C_t^{P,F}$,

$$\begin{aligned} C_t^{P,DK} &= \vartheta_c \left(\frac{P_t}{P_t^C}\right)^{-\upsilon_c} C_t^P, \\ C_t^{P,F} &= (1-\vartheta_c)\left(\frac{P_t^M}{P_t^C}\right)^{-\upsilon_c} \left[1-\chi_t^C - \left(\chi_t^C\right)' C_t^F\right]^{\upsilon_c} \frac{C_t^P}{\left(1-\chi_t^C\right)} \end{aligned}$$

$$C_t^{P,DK} + C_t^{P,F} = \left[\vartheta_c \left(\frac{P_t}{P_t^C}\right)^{-\upsilon_c} + (1-\vartheta_c)\left(\frac{P_t^M}{P_t^C}\right)^{-\upsilon_c} \left[1-\chi_t^{P,C} - \left(\chi_t^{P,C}\right)' C_t^{P,F}\right]^{\upsilon_c} \left(1-\chi_t^{P,C}\right)\right] C_t^P$$

I then add the consumption for the three types of consumers,

$$\begin{aligned} C_t^{Tot} &\equiv \left(C_t^{P,DK} + C_t^{P,F}\right) + \left(C_t^{I,DK} + C_t^{I,F}\right) + \left(C_t^{E,DK} + C_t^{E,F}\right) \\ &= \left[\vartheta_c \left(\frac{P_t}{P_t^C}\right)^{-\upsilon_c} + (1-\vartheta_c)\left(\frac{P_t^M}{P_t^C}\right)^{-\upsilon_c} \left[1-\chi_t^{P,C} - \left(\chi_t^{P,C}\right)' C_t^{P,F}\right]^{\upsilon_c} \left(1-\chi_t^{P,C}\right)\right] C_t^P + \\ &\quad \left[\vartheta_c \left(\frac{P_t}{P_t^C}\right)^{-\upsilon_c} + (1-\vartheta_c)\left(\frac{P_t^M}{P_t^C}\right)^{-\upsilon_c} \left[1-\chi_t^{I,C} - \left(\chi_t^{I,C}\right)' C_t^{I,F}\right]^{\upsilon_c} \left(1-\chi_t^{I,C}\right)\right] C_t^I + \\ &\quad \left[\vartheta_c \left(\frac{P_t}{P_t^C}\right)^{-\upsilon_c} + (1-\vartheta_c)\left(\frac{P_t^M}{P_t^C}\right)^{-\upsilon_c} \left[1-\chi_t^{E,C} - \left(\chi_t^{E,C}\right)' C_t^{E,F}\right]^{\upsilon_c} \left(1-\chi_t^{E,C}\right)\right] C_t^E, \end{aligned}$$

, which allows me to write

$$gc_t = \Gamma_t \log(\left(\frac{C_t^{Tot}}{C_{t-1}^{Tot}}\right)$$

The measurement equation for investments is similarly given by:

$$\begin{aligned} I_t^{Tot} &\equiv I_t^{Y,DK} + I_t^{Y,F} = \left[\vartheta_I \left(\frac{P_t}{P_t^{I,Y}}\right)^{-\upsilon_I} + (1-\vartheta_I)\left(\frac{P_t^M}{P_t^{I,Y}}\right)^{-\upsilon_I} \left[1-\chi_t^I - \left(\chi_t^I\right)' I_t^{I,Y}\right]^{\upsilon_I} \left(1-\chi_t^I\right)\right] I_t^Y \\ gI_t^y &= \Gamma_t Zt \log(\left(\frac{I_t^{Tot}}{I_{t-1}^{Tot}}\right), \end{aligned}$$

where the trend variable is corrected by the trend rate in investment prices.

Some variables are stationary in the model and are consequently matched as

$$gz_t^2 = \frac{Z_t^2}{\bar{Z}},\ for\ Z_t^2 = U_t^{Tot}, N_t^{Tot}, \pi_t^{DK}, \tau_t^n$$

The investment price are assumed to follow a different trend than output:

$$\pi_t^{IY} = \frac{\log\left(P_t^{IY}\right)}{\log\left(P_{t-1}^{IY}\right)} = \left(\frac{\Gamma_t}{Z_t}\right)\pi_t^{IY}$$

This is also so for public investments.

Data for the two foreign economies are HP-filtered. Hence, there is no need to stationarize in the measurement equations

$$gz_t^f = \frac{Z_t^f}{Z_{t-1}^f}, \; for \; Z_t^f = FX_t, Y_t^{EA}, Y_t^{RoW}, \pi_t^{EA}, \pi_t^{RoW}, fx_t$$

I recall that interest rates are simply demeaned and hence link one-to-one to the interest rates in the model. Finally, all taxes and subsidies are measured with respect to their steady states.

10.3. Shocks

The model includes all in all 38 structural shocks including public finance shocks, but I only use 30 in the estimation of the model. I also use the 7 shocks from the DSGE-model for the two foreign economies:

- Public consumption shock, ϵ^{G}
- Public investment shock, ϵ^{IG}
- Shock to tax on housing, ϵ^{τ^H}
- Shock to tax on land, ϵ^{τ^L}
- Shock to interest rate deductions, E., $\epsilon^{\kappa^{RE}}$
- Shock to interest rate deductions, HH., $\epsilon^{\kappa^{RI}}$
- Tax on labour income shock, ϵ^{N}
- EA price shock, $\epsilon^{\Pi,EA}$
- EA output shock, $\epsilon^{y,EA}$
- ECB interest rate shock, $\epsilon^{r,EA}$
- RoW price shock, $\epsilon^{\Pi,ROW}$
- RoW output shock, $\epsilon^{y,ROW}$
- RoW interest rate shock, $\epsilon^{r,ROW}$
- UIP shock, ϵ^{UIP}
- Perm. tech shock, $\epsilon^{A_{Y,P}}$
- Temp. tech shock, $\epsilon^{A_{Y,T}}$
- Wage markup shock, ϵ^{ϵ^W}
- Temp. tech shock, housing sector, $\epsilon^{A_{X,T}}$
- Export price markup shock, ϵ^{ϵ^X}
- Import shock, ϵ^{ω^I}
- Consumption shock, ϵ^{C}
- Price markup shock, ϵ^{ϵ^P}
- Import price markup shock, ϵ^{ϵ^M}
- Export shock,ϵ^{Ex}
- Temp. investment shock, $\epsilon^{\mathcal{I}}$
- Labour supply shock,ϵ^{χ}
- Riskpremia shock,$\epsilon^{\mathcal{R}}$

- Preference shock for housing, ϵ^{H}
- Markup shock, interest rate, E. $\epsilon^{b,I}$
- Markup shock, interest rate, HH. $\epsilon^{b,E}$

As already described, all shocks are assumed to feed into first-order autoregressive processes, except for the shock to the ECB policy rate, the shock to the policy rate for the rest of the world, and the UIP shock, and all price markup shocks, which are all white noise, and the shocks to public finance variables. I have not included shocks to the trend in the relative price of investment goods, ϵ^{Z_p}, as the fit of especially GDP deteriorated. In the estimation I assume that 21 variables are measured with measurement errors. This concerns GDP, wages, employment, unemployment, exports, imports, private consumption, private investments, debt for households and firms, import-, export- and residential investment deflators as well as all data for the two foreign economies.

11. Appendix C: Data

I estimate the model on data running from 1995Q1 to 2018Q4. I use the first years as 'training' sample for the Bayesian estimation which I afterwards discard in the analysis. This has the advantage that initial conditions in the historical shock decomposition are likely to have vanished in the sample period. I add the most recent forecast produced by Danmarks Nationalbank for the 3-year period after the sample to get better estimates of the long term trend in data. The latter must be seen as a consequence of the crisis period which might bias the long-run growth rate downwards leaving a worse fit at the end of the sample. Strictly speaking I should perhaps allow for a structural break but I leave that for future work. I use data from 1995 although the European currency union was not in place before 1999. I consequently weight the costs of having less data less than the costs of using data from a group of countries within a currency union, which was not in place at that point in time. I believe, however, that the initial euro zone countries to some extent shared business cycles already in 1995, as also suggested by Dam (2008).

In estimating the model, I use times series for 33 macroeconomic variables. I notice that unlike the estimation of a wide range of DSGE-models, I use employment rather than hours worked, and redefine the wage as the wage per worker rather than the wage per hour. This is so as the model focuses on variations in labor at the extensive margin, in a way consistent with the conventional definition of unemployment. The following time series for Danish variables describing the Danish real economy are taken from Statistics Denmark and Danmarks Nationalbank with source in parentheses:

- Real GDP: Danmarks Nationalbank, MONA (variable name: *fy*).
- GDP-deflator: Danmarks Nationalbank, MONA (variable name: $\frac{y}{fy}$).
- Private consumption: Danmarks Nationalbank, MONA (*fcp*).

- Government spending: Danmarks Nationalbank, MONA (*fco*).
- Government investment: Danmarks Nationalbank, MONA (*fio*).
- Exports: Danmarks Nationalbank, MONA (*fe*).
- Imports: Danmarks Nationalbank, MONA (*fm*).
- Total private investment: Danmarks Nationalbank, MONA (*fip*; chained sum of all types of private investment, including inventories but excluding construction).
- Labor income tax: Danmarks Nationalbank, MONA (*bsda*).
- Employment: Danmarks Nationalbank, MONA (*qp+qs+qo*).
- Unemployment: Danmarks Nationalbank, MONA (*ul* / (*qp+qo+qs*)).
- Industry nominal wages deflated by CPI: Danmarks Nationalbank, MONA (*lna* / *pcp*).
- Consumer price index: Danmarks Nationalbank, MONA (*pcp*).
- Investment deflator: Danmarks Nationalbank, MONA (*pip*; relative price of total private investment).
- Import price deflator: Danmarks Nationalbank, MONA (*pm*).
- Export price deflator: Danmarks Nationalbank, MONA (*pe*).
- Effective Danish exchange rate: Danmarks Nationalbank, MONA (*efkrks*).
- eurozone inflation: OECD (obtained from Ecowin).
- Rest of World inflation: OECD (Ecowin).
- eurozone real GDP: OECD (Ecowin).
- Rest of World real GDP: OECD (Ecowin).
- ECB policy (nominal) interest rate: OECD (Ecowin).
- Rest of World implied nominal interest rate: OECD (Ecowin).

The 'Rest of World'-variables are defined as the weighted sum of GDP, inflation or the policy rate of the Danish trading partners excluding trading partners within the eurozone. The eurozone and the Rest of the World are approximately of equal size. Data is taken from the Ecowin data base and OECD.

Finally, which is new in comparison to Pedersen and Ravn (2013), I use financial data to estimate the model; 4 in total. I follow Gerali et al. (2010) and use time series for nominal lending rates to firms and households, as well as the outstanding stock of loans. All data is collected and published by Danmarks Nationalbank:

- Loans to household: Outstanding amount of loans to households for house purchasing, total maturity, neither seasonally nor working day adjusted.
- Loans to firms: Outstanding amount of loans to non-financial corporations, total maturity, neither seasonally nor working day adjusted.
- Nominal interest rate on loans to households: Annualized agreed rate on loans for house purchases, total maturity.
- Nominal interest rate on loans to firms: Annualized agreed rate on loans other than bank overdrafts to non-financial corporations with maturity of over one year.

I also include the following variables on the construction side of the model:

- Residential investments: Danmarks Nationalbank, MONA (variable name:*fih*)
- Residential investment deflator: Danmarks Nationalbank, MONA (variable name:*pih*)
- Real house price: Danmarks Nationalbank, MONA (variable name: *kp/pcp*)
- Effective tax rate on value of housing, MONA and own calculations
- Effective tax rate on value of land, MONA and own calculations
- Tax deduction on mortgage loans, MONA and own calculations

Prior to estimation, I transform the time series into quarter-on-quarter growth rates, approximated by the first difference of their logarithm. As explained above, in the model, I include a trend in productivity and in the relative price of investments goods. The variables in the model therefore have trends and consequently, I do not demean data. I also divide by the working age population to take out possible effects on GDP from demographics, as in the model I assume the existence of representative households.

All interest rates are simply demeaned. Also, a number of additional transformations are made to ensure that variable measurement is consistent with the properties of the model's growth path. Firstly, I remove the sample growth rate differentials between the export and import variables and Danish GDP, as these variables in the sample have grown faster than GDP reflecting globalisation, see also Pedersen and Ravn (2013). I do the same for outstanding loans and I also divide by the working age population and I translate the series to real terms. Secondly, for the effective exchange rate I standardise the serie by its standard deviation and I deduct the mean. Thirdly, tax rates are smoothed using a simple moving average filter to avoid large spikes in data due to institutional changes. Lastly, I HP-filter data for the foreign economies as I do not model trends in the foreign economies.

Data is shown in figures (2) to (7).

12. Appendix D: Identification

Identification can be a big issue in the estimation of DSGE-models, see among many Canova and Sala (2009) for a discussion of the problems. In this section, I will analyse the strength of the identification in the estimated model using Dynare in-build toolbox documented in Ratto and Iskrev (2011). This is done through a number of figures produced by Dynare after the estimation. Specifically, after estimation Dynare performs local identification checks for the estimated parameters (and not the calibrated parameters). The program analyses the first two moments, calculate the Jacobien with respect to the estimated parameters and check its column rank. If the Jacobian has a full rank, then the parameters are locally identifiable. This condition is both necessary and sufficient when the shocks are normally distributed and the number of rows are equal to the number of columns in the Jacobian matrix, see Ratto and Iskrev (2011).

In figure (47) is shown a measure of the identification strength of the parameters based on the information matrix calculated in the estimation of the model. I stress that the in-build Dynare programs can not handle measurement errors, as I have used them in the estimation. Instead, the programs can only handle errors added directly to the observation equations. This does make a difference and as will be discussed later, it affects the identification of some of the shocks.

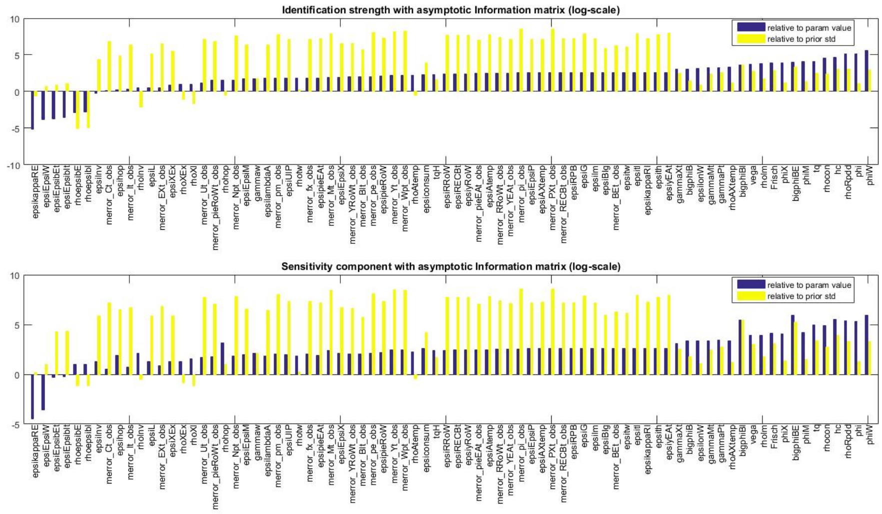

Figure 47: **Identification using information from observables**

Intuitively, the bars in the figure represent the normalized curvature of the log likelihood function at the prior mean in the direction of the parameter. If a bar is zero, the parameter is not identified - the likelihood function is flat in this direction. The larger the absolute value of the bars, the stronger is the identification. Admittedly difficult to see from the figure, all parameters are identified.

The lower panel in figure (47) decomposes the effect shown in the upper panel. Weak identification can be due to either other parameters linearly compensating/replacing the effect of a parameter such that two parameters have the same effect on the likelihood, or due to the fact that the likelihood does not change at all with the respective parameter; sensitivity. It is sensitivity which is shown in the bottom panel of figure (47). The bottom panel shows that none of the parameters give rise to a sensitivity of zero although, naturally, some parameters have a higher sensitivity than others.

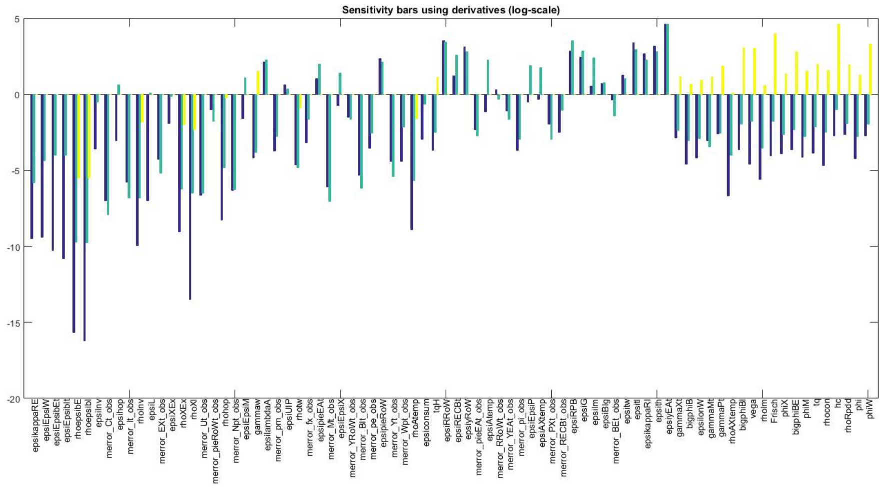

Figure 48: **Sensitivity plot**

In figure (48), an aggregate measure of how changes in the estimated parameters impact the model moments is shown. In the figure, three different measures of sensitivity are shown. For a presentation of why and how the three types of scaling are done, see Ratto and Iskrev (2011). The bars shown in the figure depict the length of three different standardized Jacobian matrices for the respective parameter shown on the x-axis. If the moment matrix indicates non-identifiability and the model solution matrix indicates identifiability, it means that the observables used in the estimation are not sufficient. Again, for no parameters are all the bars zero at the same time.

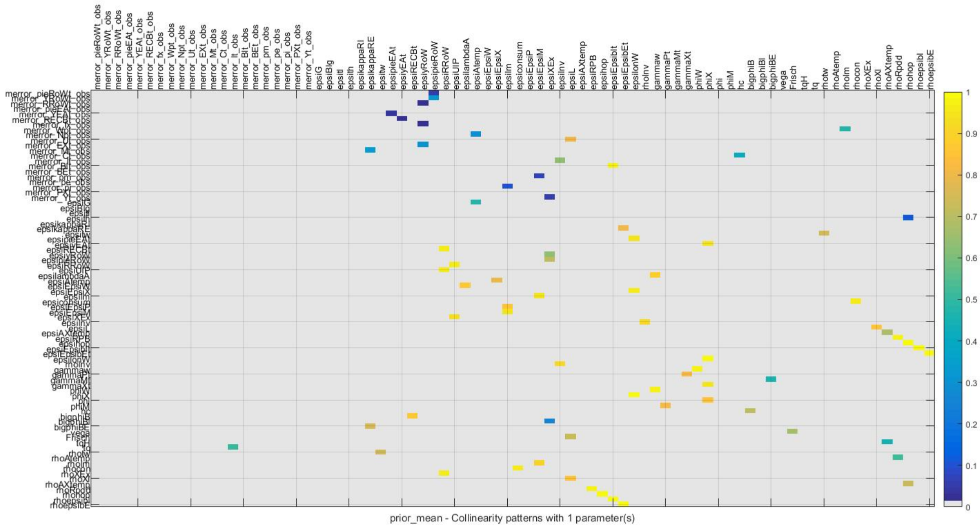

Figure 49: **Identification using information from observables**

Figure (49) helps to detect collinearities. That is, parameters which have the same affect on the likelihood function. Specifically, the figure shows which linear combination of parameters shown in the columns best replicates/replaces the effect of the parameter depicted in the row on the moments of the observables. High values imply the relative redundancy and thus weak or un-identifiability of the parameter under consideration. I again refer to Ratto and Iskrev (2011) for details. For each parameter regressions are run of the column of the Jacobian on all possible combinations of other Jacobian columns. The procedure looks for the combination with the highest R^2 in the regressions. The darker yellow the squares are, the more critical is the collinearity between parameters.

There are some dark yellow points in the rather large matrix. That is, in the estimation of the model, some parameters have the same affect as other parameters. The pair of parameters are however mostly the autoregressive parameter in a shock process with its own shock, and what intuitively can be expected; as an example import price markup shock and import shock.

Identification patterns can be analysed through a decomposition of the information matrix or of the Jacobian. In figure (50), is shown the eigenvectors of such a decomposition relative to the smallest and largest singular values. The smaller the eigenvectors, the more affected is the corresponding parameter by none- or weak identification. The larger it is, the stronger is the identification relative to the rest of the parameters. A singular value of 0, implies that the parameter is completely unidentified and the parameter thus have no effect on the likelihood. The parameter combinations associated with the smallest singular values are closest to being perfectly collinear and thus redundant.

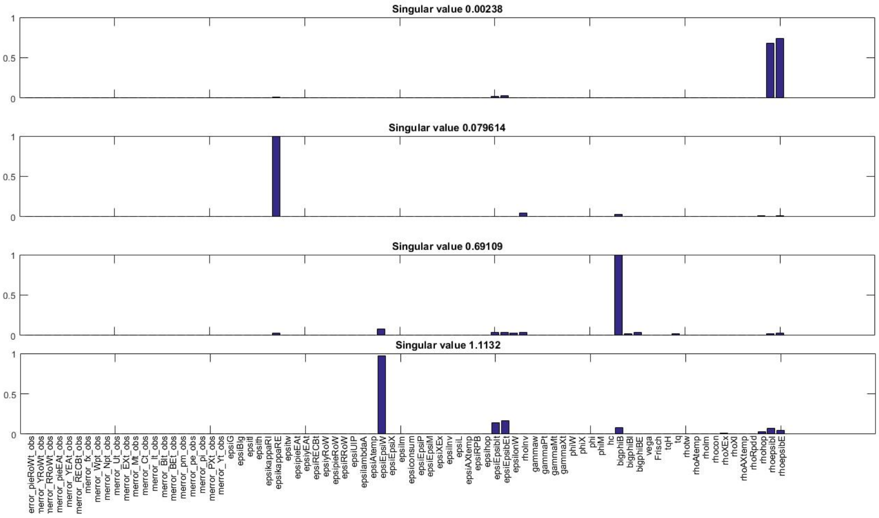

Figure 50: **Identification analysis: Strongest and weakest identification**

In figure (50) the smallest singular values are shown. It can be seen that no parameter has a value of zero and hence all parameters are identified, although of course some are better identified than others. Specifically, the persistence parameters for the markup shocks to lending rates in the banking sector are relatively weakly identified. The same goes for the interest rate deduction shock on interest payments on the firm side. This is confirmed in the historical shock decomposition for both set of parameters; the markup shocks on the lending rates and the interest rate payment subsidy play a small role. Looking at the posterior-prior plot for the cost parameter for deviating from the capital ratio for the banks, it is also less surprising that this parameter, Φ^B, is only weakly identified.

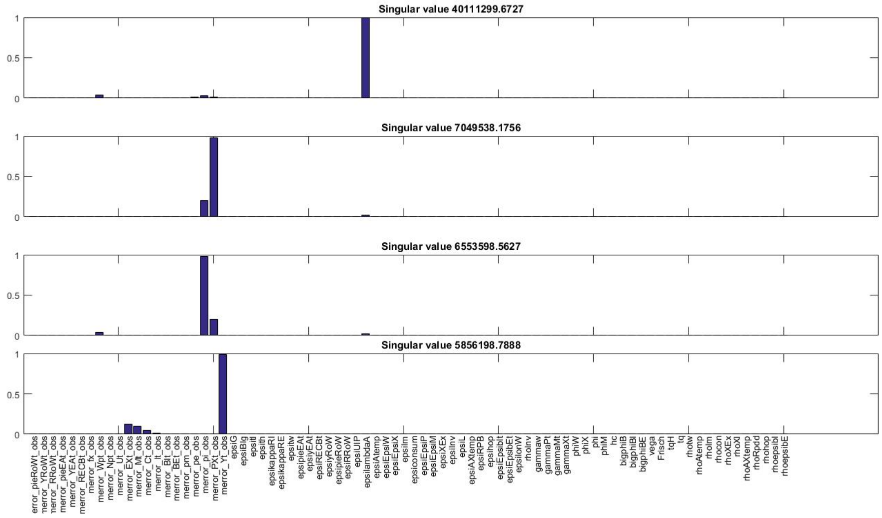

Figure 51: **Identification analysis: Strongest and weakest identification**

-

Figure (51) shows the highest singular values. The interpretation is the same as for figure (50) except for now depicting the parameters being most uncorrelated and thus best identified. Perhaps misleading, these tend to be the ”measurement” errors. I recall that the in-build Dynare programs for performing the identification and sensitivity analysis only works with either no measurement errors or ”measurement” errors added directly to the measurement equations. Of the structural shocks, the permanent technology shock is the best identified shock.

13. Appendix E: The eurozone and rest of world models

As mentioned in the text, the modeling of the two foreign economies follows the approach in Pedersen and Ravn (2013). The main impacts of the foreign countries on the Danish economy work through trade and interest rates. I consequently aim for the most flexible model for these two economies and downplay the microfoundations Further, I do not aim to estimate a common trend for all the three economies as the data points to different steady state growth rates in output. Denmark's role as a small open economy implies that I can model the foreign economies as being exogenous to the Danish economy. Also, I do not aim to model the interrelations between the euro zone and rest of the world, and I consequently do not model trade between these two economies. I do, however, include a UIP-relation between the policy rate in the two countries so that I can pin down the effective exchange rate between Denmark and rest of world.

The model for the foreign economies consequently are set as follows. Denmark's fixed exchange rate towards the euro implies that I need to include two foreign economies in the model: One (the euro zone, EA for short) towards which Denmark has a fixed exchange rate, and one (the Rest of the World; RoW for short) towards which the exchange rate is fully flexible and exogenous for Denmark due to the small-economy assumption and with monetary policy given from the euro zone. The two foreign economies are otherwise completely identical, and are taken as completely exogenous, so that movements in the Danish economy does not affect the foreign economies. I also do not model trade or other interactions between the euro zone and the rest of the world. Each of foreign economies is described by a basic 3-equation New Keynesian model, so that for $j = (EA, RoW)$ I have:

$$\frac{Y_t^j}{\overline{Y}^j} = \left(\frac{Y_{t+1}^j}{\overline{Y}^j}\right)^{\rho_Y^j} \left(\frac{Y_{t-1}^j}{\overline{Y}^j}\right)^{1-\rho_Y^j} \left(\frac{\frac{R_t^j}{\overline{R}^j}}{\frac{\pi_{t+1}^j}{\overline{\pi}^j}}\right)^{-\phi_Y^j} \left(\frac{\epsilon_{Y,t}^j}{\overline{\epsilon}_Y^j}\right), \tag{98}$$

$$\frac{\pi_t^j}{\overline{\pi}^j} = \left(\frac{\pi_{t+1}^j}{\overline{\pi}^j}\right)^{\rho_\pi^j} \left(\frac{\pi_{t-1}^j}{\overline{\pi}^j}\right)^{1-\rho_\pi^j} \left(\frac{Y_t^j}{\overline{Y}^j}\right)^{\phi_\pi^j} \left(\frac{\epsilon_{\pi,t}^j}{\overline{\epsilon}_\pi^j}\right), \tag{99}$$

$$\frac{R_t^j}{\overline{R}^j} = \left(\frac{R_{t-1}^j}{\overline{R}^j}\right)^{\rho_R^j} \left[\left(\frac{\pi_t^j}{\overline{\pi}^j}\right)^{\Gamma_\pi^j} \left(\frac{Y_t^j}{\overline{Y}^j}\right)^{\Gamma_Y^j}\right]^{1-\rho_R^j} \left(\frac{\epsilon_{R,t}^j}{\overline{\epsilon}_R^j}\right). \tag{100}$$

Here, (98) is a hybrid dynamic IS-type relation that links output to the real interest rate, (99) is a version of a hybrid New Keynesian Phillips Curve linking the rate of inflation to real activity, and (100) is a Taylor rule that determines monetary policy in each of the two regions as a function of inflation and economic activity. See Galí (2009) for a detailed exposition of the 3-equation New Keynesian model. In turn, the shock processes in each of these equations are given as AR(1)-processes:

$$\frac{\epsilon^j_{k,t}}{\bar{\epsilon}^j_k} = \left(\frac{\epsilon^j_{k,t-1}}{\bar{\epsilon}^j_k}\right)^{\rho^j_{\epsilon^k}} \varepsilon^{k,j}_t, \tag{101}$$

for $j = (EA, RoW)$ and $k = (Y, \pi, R)$, and where the $\varepsilon^{k,j}_t$'s are i.i.d. normal processes. The parameters in the IS curve $\left(\rho^j_Y, \phi^j_Y\right)$ and the New Keynesian Phillips Curve $\left(\rho^j_\pi, \phi^j_\pi\right)$, as well as the reaction parameters in the Taylor rule $\left(\Gamma^j_\pi, \Gamma^j_Y\right)$ are chosen in line with the literature.[27] The six shocks are included in the estimation to account for the contribution to the Danish business cycle of foreign shocks.

Finally, I can write world output and inflation as:

$$\frac{Y^W_t}{\overline{Y}^W} = \left(\frac{Y^{EA}_t}{\overline{Y}^{EA}}\right)^{\omega_X} \left(\frac{Y^{RoW}_t}{\overline{Y}^{RoW}}\right)^{1-\omega_X}, \tag{102}$$

$$\frac{\pi^W_t}{\overline{\pi}^W} = \left(\frac{\pi^{EA}_t}{\overline{\pi}^{EA}}\right)^{\omega_X} \left(\frac{FX_t}{FX}\frac{\pi^{RoW}_t}{\overline{\pi}^{RoW}}\right)^{1-\omega_X}, \tag{103}$$

where the parameter $\omega_X > 0$ measures the relative size of the eurozone, and where FX denotes the change in the *effective* exchange rate of the Danish krone. The effective exchange rate is given by:

$$\frac{FX_t}{FX} = \frac{\frac{R^{ECB}_t}{\overline{R}^{ECB}}}{\frac{R^{RoW}_t}{\overline{R}^{RoW}}} \varepsilon^{UIP}_t,$$

so that the effective exchange rate moves in response to interest rate differentials between the eurozone and the rest of the world. ε^{UIP}_t is an i.i.d. normal shock process.

I use the same sample as for the Danish economy. The estimation is done through a two-step procedure: In the first step, I estimate the two separate small-scale DSGE models of each of the foreign blocks shown above. In the second step, where I estimate the main model for Denmark, I include these estimated relations, and then estimate the shocks in the foreign models by including the data for the foreign economies. The parameter estimates are shown in the following table.

[27] It may be difficult to distinguish interest rate smoothing from persistence in the shocks hitting the interest rate rule. I therefore decide to eliminate the latter by fixing the parameter $\rho^j_{\epsilon^R} = 0$ for $j = (EA, RoW)$.

		Prior distribution				Posterior distribution					
		Type	Source	Mean	s.d.	Mean	Mode	s.d.	Median	5 pct.	95 pct.
Parameters for euroarea model											
Weight on expected output	ρ_Y^{EA}	Normal	-	0.75	0.1	0.64	0.63	0.061	0.64	0.54	0.74
Weight on real rate of interest	ϕ_Y^{EA}	Normal	-	0.4	0.05	0.35	0.35	0.052	0.35	0.26	0.44
Weight on expected inflation	ρ_π^{EA}	Normal	-	0.75	0.1	0.86	0.86	0.067	0.86	0.75	0.97
Weight on output	ϕ_π^{EA}	Normal	-	0.75	0.1	0.59	0.59	0.092	0.59	0.44	0.74
Policy interest rate smoothing parameter	ρ_R^{EA}	Normal	-	0.85	0.1	0.98	0.98	0.0027	0.98	0.98	0.99
Persistence of shock to output	$\rho_{\epsilon_Y}^{EA}$	Beta	-	0.85	0.1	0.9	0.91	0.02	0.9	0.86	0.93
Persistence of shock to inflation	$\rho_{\epsilon_\pi}^{EA}$	Beta	-	0.85	0.1	0.76	0.76	0.049	0.76	0.68	0.85
Shock to output	ϵ_Y^{EA}	Inv. gamma	-	0.01	2	0.003	0.003	0.001	0.003	0.002	0.004
Shock to inflation	ϵ_π^{EA}	Inv. gamma	-	0.01	2	0.026	0.026	0.004	0.026	0.019	0.032
Monetary policy shock	ϵ_R^{EA}	Inv. gamma	-	0.01	2	0.003	0.003	0.00	0.0025	0.002	0.0031
Policy interest rate, reaction to inflation	Γ_π^{EA}	Calibrated		1.5							
Policy interest rate, reaction to inflation	Γ_Y^{EA}	Calibrated		0.125							
Parameters for rest of world model											
Weight on expected output	ρ_Y^{RoW}	Normal	-	0.75	0.1	0.69	0.68	0.068	0.69	0.57	0.8
Weight on real rate of interest	ϕ_Y^{RoW}	Normal	-	0.5	0.05	0.52	0.51	0.051	0.52	0.43	0.6
Weight on expected inflation	ρ_π^{RoW}	Normal	-	0.75	0.1	0.88	0.88	0.084	0.88	0.73	1
Weight on output	ϕ_π^{RoW}	Normal	-	0.75	0.1	0.32	0.28	0.063	0.31	0.21	0.44
Policy interest rate smoothing parameter	ρ_R^{RoW}	Beta	-	0.85	0.1	0.84	0.85	0.021	0.85	0.81	0.88
Persistence of shock to output	$\rho_{\epsilon_Y}^{RoW}$	Beta	-	0.85	0.1	0.76	0.77	0.037	0.76	0.7	0.82
Persistence of shock to inflation	$\rho_{\epsilon_\pi}^{RoW}$	Beta	-	0.85	0.1	0.8	0.8	0.053	0.8	0.72	0.89
Shock to output	ϵ_Y^{RoW}	Inv. gamma	-	0.01	2	0.002	0.002	0.0003	0.002	0.002	0.003
Shock to inflation	ϵ_π^{RoW}	Inv. gamma	-	0.01	2	0.002	0.002	0.00032	0.002	0.002	0.003
Monetary policy shock	ϵ_R^{RoW}	Inv. gamma	-	0.01	2	0.002	0.002	0.00022	0.002	0.002	0.002
Policy interest rate, reaction to inflation	Γ_π^{RoW}	Calibrated		1.5							
Policy interest rate, reaction to inflation	Γ_Y^{RoW}	Calibrated		0.125							

Table 11: **Estimated and calibrated parameters for the foreign countries:** This table reports the prior distribution and the posterior mode estimates of the structural parameters for the model.

14. Appendix F: Convergence statistics

Convergence of the algorithm is assessed by looking at the plots of the draws following conventional theory in Brooks and Gelman (1998). In each of the following figures, the univariate convergence diagnostics are plotted. The figures in the first column show the appended (Interval) convergence diagnostics for the 80% interval. The blue line shows the 80% interval/quantile range based on the pooled draws from all sequences, while the red line shows the mean interval range based on the draws of the individual sequences. The second and third column with the appended (m2) and (m3) show an estimate of the same statistics for the second and third central moments, i.e. the squared and cubed absolute deviations from the pooled and the within-sample mean, respectively.

If the chains have converged, the two lines should stabilize horizontally and should be close to each other. In the figures, the number of parameter draws are increasing along the x-axis. The analysis is shown in figures (52(a))-(57).[28] Convergence is in general achieved though less so for some parameters.

The last figures show the multivariate convergence diagnostic. This diagnostics is the same as the univariate one depicted in the first figures, except for the statistics now being based on the range of the posterior likelihood function instead of the individual parameters. Thus, the posterior kernel is used to aggregate the parameters. Convergence is indicated by the two lines stabilizing and being close to each other.

[28] I have for brevity left out the convergence analysis for the standard deviation of the measurement errors.

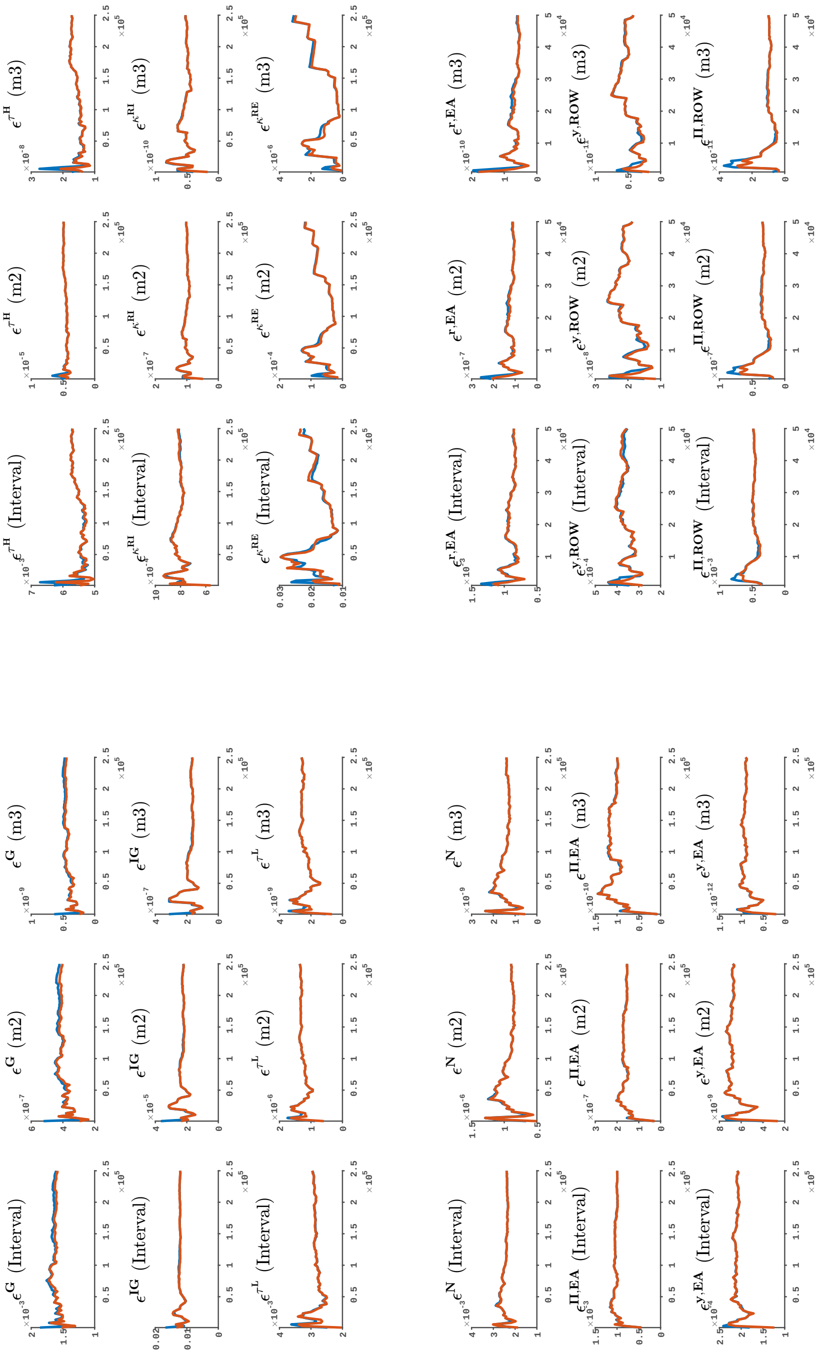

Figure 52: Univariate diagnostics as in Brooks and Gelman (1998)

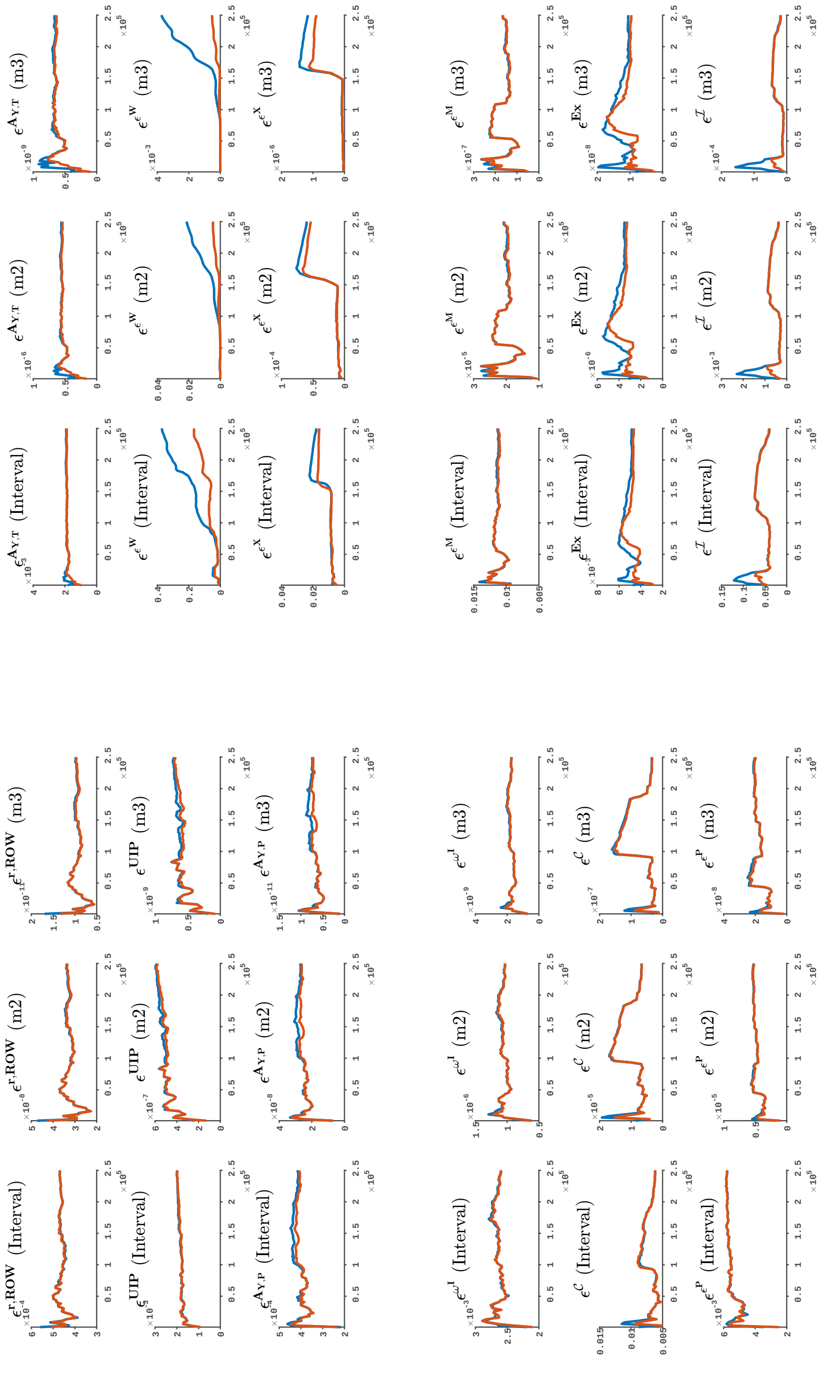

Figure 53: Univariate diagnostics as in Brooks and Gelman (1998)

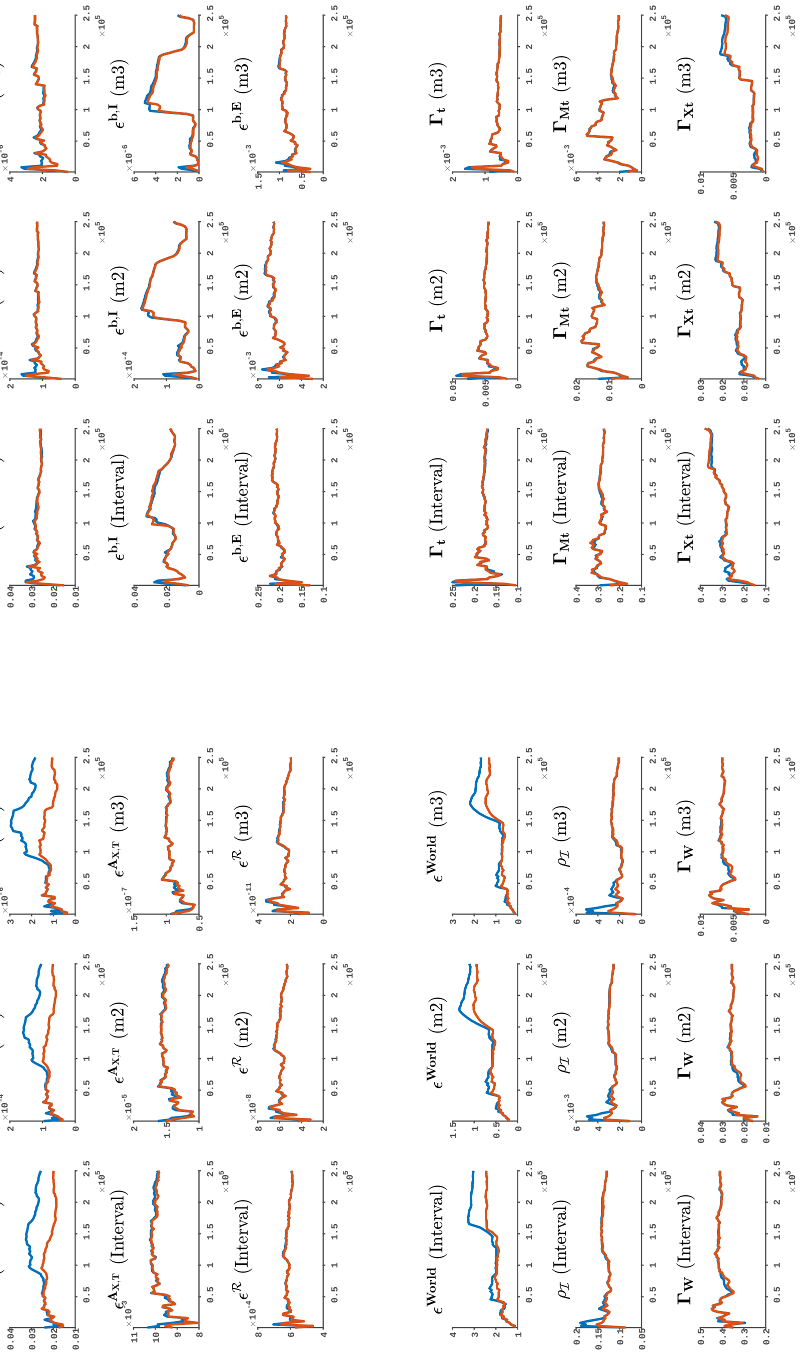

Figure 54: Univariate diagnostics as in Brooks and Gelman (1998)

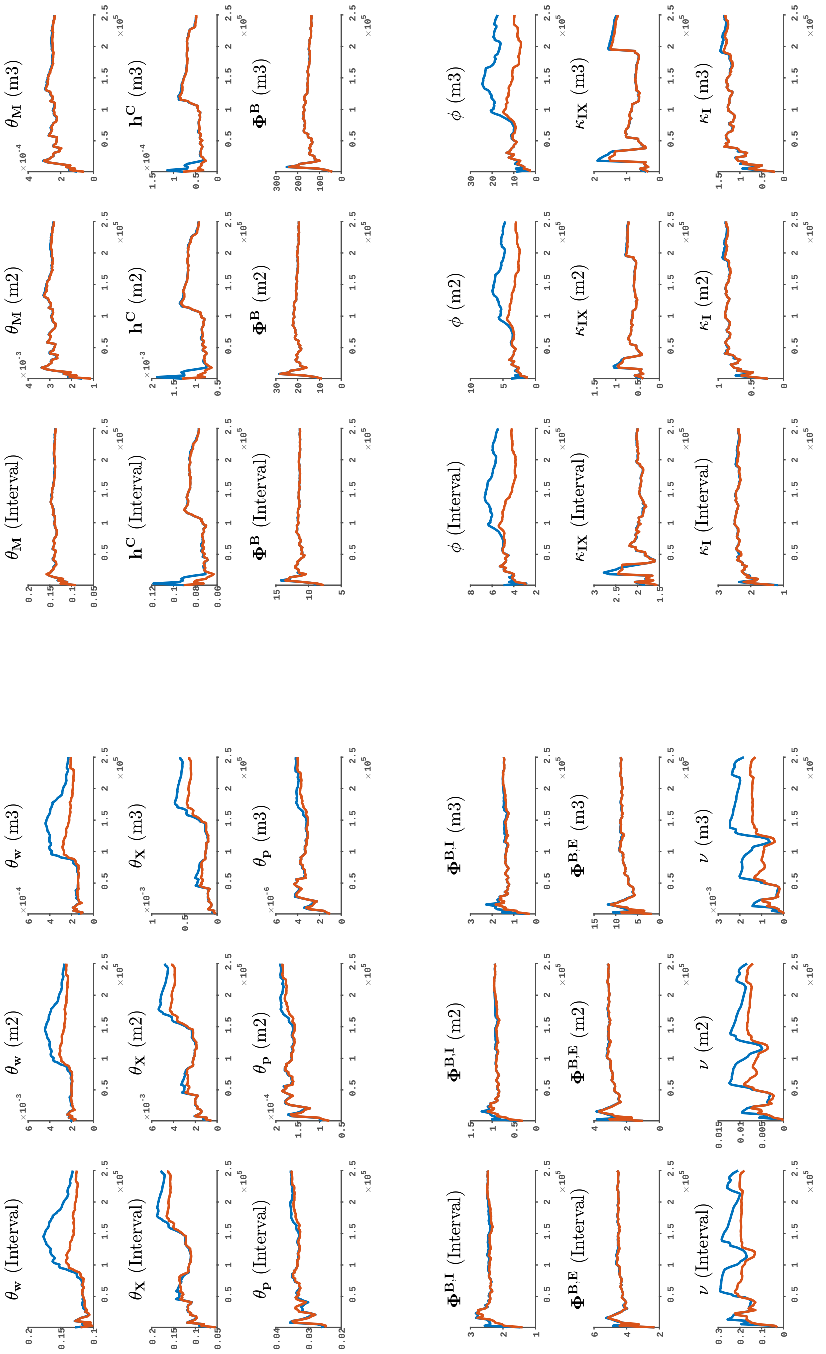

Figure 55: **Univariate diagnostics as in Brooks and Gelman (1998)**

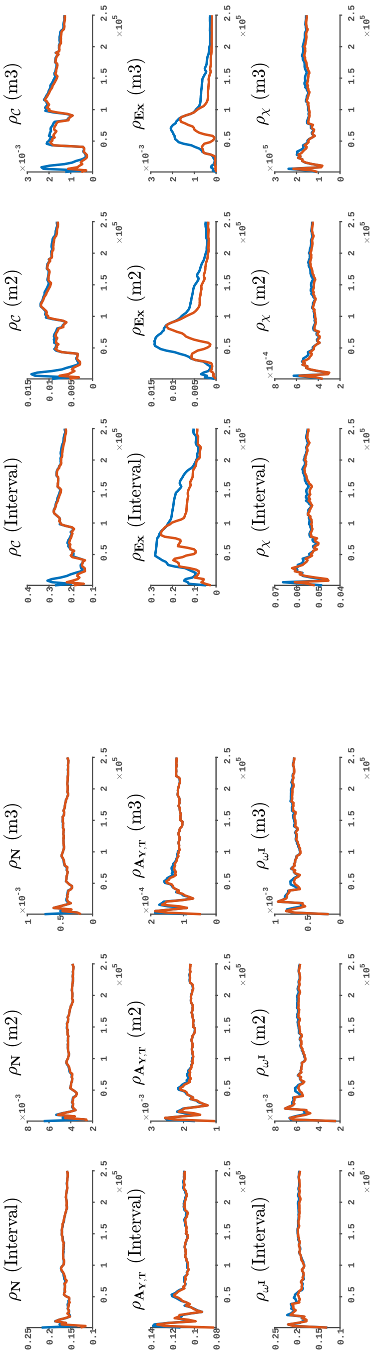

Figure 56: **Univariate diagnostics as in Brooks and Gelman (1998)**

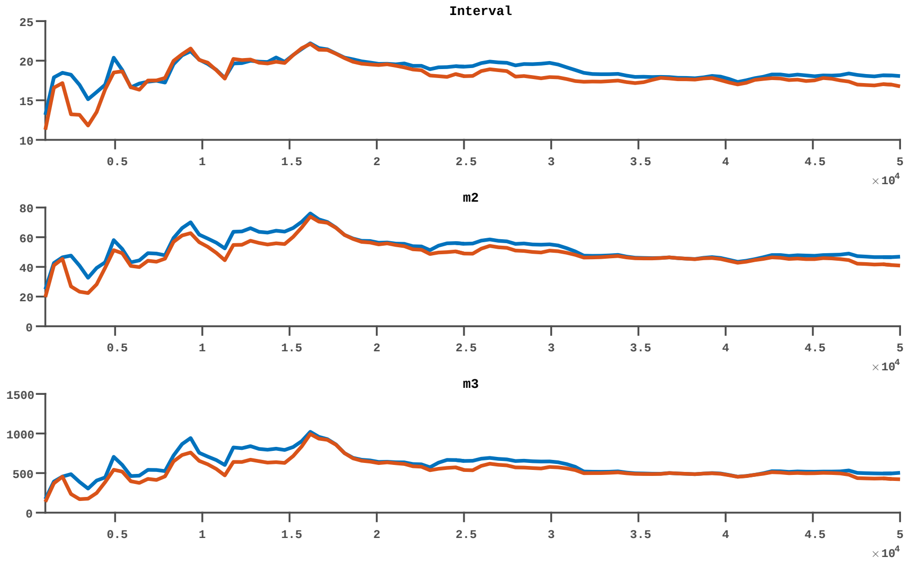

Figure 57: **Multivariate diagnostics as in Brooks and Gelman (1998)**

15. Appendix G: Model equations from Dynare

$$\lambda_t^P \, (\frac{P^C}{P})_t = \frac{1}{\left(C_t^P - h^C \, C_{t-1}^P \, \frac{1}{d\Gamma_t}\right) \left(1 + \tau_t^{VAT}\right)}$$

$$\lambda_t^P = \beta^P \, \frac{C_t}{\bar{C}} \, \lambda_{t+1}^P \, \frac{R_t^D}{\Pi^{DK}{}_{t+1} \, d\Gamma_{t+1}} - \beta^P \, \frac{C_t}{\bar{C}} \, \left(R_t^D - 1\right) \, \frac{\lambda_{t+1}^P \, \tau_{t+1}^B}{\Pi^{DK}{}_{t+1} \, d\Gamma_{t+1}}$$

$$(\frac{P^C}{P})_t \, \lambda_t^I = \frac{1}{\left(1 + \tau_t^{VAT}\right) \left(C_t^I - \frac{1}{d\Gamma_t} \, h^C \, C_{t-1}^I\right)}$$

$$\varsigma^{HI} \, \mathcal{H}_t \, \frac{1}{H^I{}_t} + \frac{C_t}{\bar{C}} \, \beta^I \, \left(1 - \delta^H\right) \, \lambda_{t+1}^I \, Q_{t+1}^H \, \frac{1}{d\Gamma_{t+1}^X} + \mu_t^I \, \Theta_t^I \, \frac{\Pi^{DK}{}_{t+1} \, Q_{t+1}^H}{R_t^{L,I}} \, \frac{d\Gamma_{t+1}}{d\Gamma_{t+1}^X} = \lambda_t^I \, \left(1 + \tau_t^H\right) \, Q_t^H$$

$$\lambda_t^I - \mu_t^I = \lambda_{t+1}^I \, \frac{C_t}{\bar{C}} \, \beta^I \, R_t^{L,I} \, \frac{1}{\Pi^{DK}{}_{t+1} \, d\Gamma_{t+1}} - \frac{C_t}{\bar{C}} \, \beta^I \, \left(R_t^{L,I} - 1\right) \, \frac{\lambda_{t+1}^I \, \kappa_{t+1}^{RI}}{\Pi^{DK}{}_{t+1} \, d\Gamma_{t+1}}$$

$$B_t^I = \Pi^{DK}{}_{t+1} \, \frac{d\Gamma_{t+1}}{d\Gamma_{t+1}^X} \, H^I{}_t \, Q_{t+1}^H \, \Theta_t^I \, \frac{1}{R_t^{L,I}}$$

$$\begin{aligned}(1-\omega) \, T_t + C_t^I \, (\frac{P^C}{P})_t \, \left(1 + \tau_t^{VAT}\right) + Q_t^H \, \left(H^I{}_t - \left(1 - \delta^H\right) \, \frac{H^I{}_{t-1}}{d\Gamma_t^X}\right) - B_t^I + R_{t-1}^{L,I} \, B_{t-1}^I \, \frac{1}{d\Gamma_t \, \Pi^{DK}{}_t} \\ = \frac{1}{d\Gamma_t \, \Pi^{DK}{}_t} \, B_{t-1}^I \, \kappa_t^{RI} \, \left(R_{t-1}^{L,I} - 1\right) - \tau_t^H \, QHs \, HIs + \left(1 - \tau_t^N\right) \, W^I{}_t \, N^I{}_t\end{aligned}$$

$$(\frac{P^C}{P})_t \, \lambda_t^E = \frac{1}{\left(1 + \tau_t^{VAT}\right) \left(C_t^E - \frac{1}{d\Gamma_t} \, h^C \, C_{t-1}^E\right)}$$

$$\lambda_t^E - \mu_t^E = \frac{1}{\Pi^{DK}{}_{t+1} \, d\Gamma_{t+1}} \, \frac{C_t}{\bar{C}} \, \beta^E \, R_t^{L,E} \, \lambda_{t+1}^E - \frac{C_t}{\bar{C}} \, \beta^E \, \left(R_t^{L,E} - 1\right) \, \frac{\lambda_{t+1}^E \, \kappa_{t+1}^{RE}}{\Pi^{DK}{}_{t+1} \, d\Gamma_{t+1}}$$

$$\lambda_t^E \, \left(1 + \tau_t^L\right) \, Q_t^L = \frac{C_t}{\bar{C}} \, \beta^E \, \lambda_{t+1}^E \, \left(r^L{}_{t+1} \, \frac{1}{d\Gamma_{t+1}} + Q_{t+1}^L\right) + \Pi^{DK}{}_{t+1} \, d\Gamma_{t+1} \, Q_{t+1}^L \, \mu_t^E \, \Theta_t^E \, \frac{1}{R_t^{L,E}}$$

$$Q_t^Y = \frac{C_t}{\bar{C}} \, \beta^E \, \frac{\lambda_{t+1}^E}{\lambda_t^E \, \Delta I_{t+1}} \, \left(\left(1 - \delta^{K,Y}\right) \, Q_{t+1}^Y - z^{u^Y} (u_t^Y \mathcal{K})_{t+1} + r_{t+1}^{K,Y} \, \left(1 - \tau_{t+1}^K\right) \, u_{t+1}^Y \, \frac{\mathcal{K}_t}{\bar{\mathcal{K}}} + \delta^{K,Y} \, \tau_{t+1}^K\right) + \Pi^{DK}{}_{t+1} \, Q_{t+1}^Y \, \mu_t^E \, \Theta_t^E \, \frac{1}{\lambda_t^E \, R_t^{L,E}} \, \frac{1}{\Delta Z_{t+1}}$$

$$B_t^E = \Theta_t^E \, \frac{1}{R_t^{L,E}} \, \left(\Pi^{DK}{}_{t+1} \, \frac{1}{\Delta Z_{t+1}} \, Q_{t+1}^Y \, K_t^Y + \Pi^{DK}{}_{t+1} \, d\Gamma_{t+1} \, Q_{t+1}^L \, l_t\right)$$

$$\begin{aligned}l_t \, \left(1 + \tau_t^L\right) \, Q_t^L + (\frac{P^C}{P})_t \, \left(1 + \tau_t^{VAT}\right) \, C_t^E + (\frac{P^{IY}}{P})_t \, I_t^Y - B_t^E + \frac{1}{d\Gamma_t \, \Pi^{DK}{}_t} \, R_{t-1}^{L,E} \, B_{t-1}^E \\ = Q_t^L \, l_{t-1} + l_{t-1} \, r^L{}_t + \frac{\mathcal{K}_t}{\bar{\mathcal{K}}} \, r_t^{K,Y} \, u_t^Y \, K_{t-1}^Y \, \frac{1}{\Delta I_t} + \frac{1}{d\Gamma_t \, \Pi^{DK}{}_t} \, B_{t-1}^E \, \kappa_t^{RE} \, \left(R_{t-1}^{L,E} - 1\right) - \frac{1}{\Delta I_t} \, K_{t-1}^Y \, \left(\frac{\mathcal{K}_t}{\bar{\mathcal{K}}} \, u_t^Y \, r_t^{K,Y} \, \tau_t^K - \delta^{K,Y} \, \tau_t^K + z^{u^Y} (u_t^Y \mathcal{K})_t\right)\end{aligned}$$

$$(\frac{P^{IY}}{P})_t = Q_t^Y \, \mathcal{I}_t \left(1 - S_t^Y - \Delta I_t \, \frac{I_t^Y}{I_{t-1}^Y} \, S_t^{1,Y}\right) + \frac{1}{\Delta Z_{t+1}} \, Q_{t+1}^Y \, \frac{1}{d\Gamma_{t+1}} \, \beta^P \, \frac{C_t}{\bar{C}} \, \frac{\lambda_{t+1}^P}{\lambda_t^P} \, \mathcal{I}_{t+1} \left(\frac{\Delta I_{t+1} \, I_{t+1}^Y}{I_t^Y}\right)^2 S_{t+1}^{1,Y}$$

$$K_t^Y = \frac{\left(1 - \delta^{K,Y}\right) K_{t-1}^Y}{\Delta I_t} + I_t^Y \, \mathcal{I}_t \left(1 - S_t^Y\right)$$

$$S_t^Y = \frac{\kappa_I}{2} \left(\frac{I_t^Y \, \Delta I_t}{I_{t-1}^Y} - \gamma^I\right)^2$$

$$S_t^{1,Y} = \kappa_I \left(\frac{I_t^Y \, \Delta I_t}{I_{t-1}^Y} - \gamma^I\right)$$

$$r_t^{K,Y} = \frac{z^{u^{Y1}} (u_t^Y \mathcal{K})_t}{1 - \tau_t^K}$$

$$r_t^{K,Y} = \frac{\Delta I_t \, mc_t \, \alpha^Y \, (1 - \eta) \, Y_t}{K_{t-1}^Y \, \frac{\mathcal{K}_t}{\bar{\mathcal{K}}} \, u_t^Y}$$

$$\frac{W^P{}_t}{W^I{}_t} = \frac{\omega}{1 - \omega} \, \frac{N^I{}_t}{N^P{}_t}$$

$$W_t^{tot} = \frac{Y_t \, mc_t \left(1 - \alpha^Y\right)}{N^{tot}{}_t}$$

$$W_t^{tot} = \left(\frac{W^P{}_t}{\omega}\right)^{\omega} \left(\frac{W^I{}_t}{1 - \omega}\right)^{1-\omega}$$

$$N^{tot}{}_t = N^{P\omega}_t \, N^{I1-\omega}_t$$

$$P^{\tilde{D}K}{}_t = M^P \, \frac{\epsilon^P{}_t}{\bar{\epsilon^P}} \, \frac{q1_t}{q2_t}$$

$$q1_t = mc_t \, \lambda_t^P \, Y_t + \beta^P \, \frac{C_t}{\bar{C}} \, \theta_p \, \Pi^{DK}{}^{\epsilon^P}_{t+1} \, q1_{t+1}$$

$$q2_t = \frac{\lambda_t^P \, Y_t}{(\frac{P^C}{P})_t} + \beta^P \, \frac{C_t}{\bar{C}} \, \theta_p \, \Pi^{DK}{}^{\epsilon^P - 1}_{t+1} \, q2_{t+1}$$

$$1 = \theta_p \, \Pi^{C}{}^{\Gamma_t \, (1-\epsilon^P)}_{t-1} \, \Pi^{DK}{}^{\epsilon^P - 1}_t + \left(1 - \theta_p\right) P^{\tilde{D}K}{}^{1-\epsilon^P}_t$$

$$D_t = \left(1 - \theta_p\right) P^{\tilde{D}K}{}^{(-\epsilon^P)}_t + \theta_p \, \Pi^{DK}{}^{\epsilon^P}_t \, \Pi^{C}{}^{\Gamma_t \, (-\epsilon^P)}_{t-1} \, D_{t-1}$$

$$\Pi^C{}_t = \frac{(\frac{P^C}{P})_t \, \Pi^{DK}{}_t}{(\frac{P^C}{P})_{t-1}}$$

$$X_t = A^{X,T}{}_t \, l_{t-1}^{\alpha^X} \, \tilde{IH}_t^{1-\alpha^X}$$

$$H_t^{tot} = X_t + \left(1 - \delta^H\right) H_{t-1}^{tot} \, \frac{1}{d\Gamma_t^X}$$

$$H_t^{tot} = H^I{}_t + H^P{}_t$$

$$l_t = 1$$

$$\tilde{I^H}_t = I^H{}_t \left(1 - S_t^X\right)$$

$$S_t^X = \frac{\kappa_{IX}}{2} \left(\frac{I^H{}_t \, d\Gamma_t^X}{I^H{}_{t-1}} - \gamma^{IX}\right)^2$$

$$S_t^{1,X} = \kappa_{IX} \left(\frac{I^H{}_t \, d\Gamma_t^X}{I^H{}_{t-1}} - \gamma^{IX}\right)$$

$$\left(\frac{p^{IX}}{p}\right)_t = Q_t^H \left(1 - \alpha^X\right) \frac{X_t}{\tilde{I^H}_t} \left(1 - S_t^X - S_t^{1,X} \frac{I^H{}_t}{I^H{}_{t-1}}\right) + \left(1 - \alpha^X\right) Q_{t+1}^H \frac{1}{d\Gamma_{t+1}} \beta^P \frac{C_t}{\bar{C}} \frac{\lambda_{t+1}^P}{\lambda_t^P} \frac{X_{t+1}}{\tilde{I^H}_{t+1}} S_{t+1}^{1,X} \left(\frac{I^H{}_{t+1}}{I^H{}_t}\right)^2$$

$$r^L{}_t = \frac{Q_t^H \, X_t \, \alpha^X}{l_{t-1}}$$

$$d\Gamma_t^X = d\Gamma_t$$

$$Y_t \, D_t = A_t^{Y,T} \left(\Delta A_t \, \Delta Z_t\right)^{\frac{(-\alpha^Y)}{1-\alpha^Y}} \left(K_{t-1}^Y \frac{\mathcal{K}_t}{\bar{\mathcal{K}}} u_t^Y\right)^{\alpha^Y (1-\eta)} {K^G}_{t-1}^{\alpha^Y \eta} {N^{tot}}_t^{1-\alpha^Y}$$

$$Y_t = C^{P,DK}{}_t + C^{I,DK}{}_t + C^{E,DK}{}_t + I^{Y,DK}{}_t + I_t^G + G_t + Ex_t + I^{HDK}{}_t - \frac{Adj_{\Pi t-1}^B}{d\Gamma_t \, \Pi^{DK}{}_t} + K_{t-1}^Y \frac{z^{u^Y}(u_t^Y \mathcal{K})_t}{\Delta I_t}$$

$$D_t^* = Ex_t + \frac{1}{d\Gamma_t} D_{t-1}^* \frac{R_{t-1}^{ECB} \, exp\left(\left(-\Psi_d^{D^\star}\right)\left(\frac{D_{t-1}^*}{Y_{t-1}} - \frac{\bar{D^*}}{\bar{Y}}\right)\right)}{\Pi^{DK}{}_t} - Im_t$$

$$z^{u^Y}(u_t^Y \mathcal{K})_t = c_1 \left(\frac{\mathcal{K}_t}{\bar{\mathcal{K}}} u_t^Y - 1\right) + \frac{c_2}{2} \left(\frac{\mathcal{K}_t}{\bar{\mathcal{K}}} u_t^Y - 1\right)^2$$

$$z^{u^{Y1}}(u_t^Y \mathcal{K})_t = c_1 + \left(\frac{\mathcal{K}_t}{\bar{\mathcal{K}}} u_t^Y - 1\right) c_2$$

$$\chi^{P,C}{}_t = \frac{\chi C}{2} \left(\frac{\omega_t^I \, \frac{C^{P,W}{}_t}{C_t^P}}{\frac{C^{P,W}{}_{t-1}}{C_{t-1}^P}} - 1\right)^2$$

$$(\chi^{P,C})'_t = \chi C \left(\frac{\omega^I_t \frac{C^{P,W}{}_t}{C^P_t}}{\frac{C^{P,W}{}_{t-1}}{C^P_{t-1}}} - 1 \right) \frac{\omega^I_t \frac{1}{C^P_t}}{\frac{C^{P,W}{}_{t-1}}{C^P_{t-1}}}$$

$$d\chi^{P,C}{}_t = 1 - \chi^{P,C}{}_t - C^{P,W}{}_t \, (\chi^{P,C})'_t$$

$$\chi^{I,C}{}_t = \frac{\chi C}{2} \left(\frac{\omega^I_t \frac{C^{I,W}{}_t}{C^I_t}}{\frac{C^{I,W}{}_{t-1}}{C^I_{t-1}}} - 1 \right)^2$$

$$(\chi^{I,C})'_t = \chi C \left(\frac{\omega^I_t \frac{C^{I,W}{}_t}{C^I_t}}{\frac{C^{I,W}{}_{t-1}}{C^I_{t-1}}} - 1 \right) \frac{\omega^I_t \frac{1}{C^I_t}}{\frac{C^{I,W}{}_{t-1}}{C^I_{t-1}}}$$

$$d\chi^{I,C}{}_t = 1 - \chi^{I,C}{}_t - C^{I,W}{}_t \, (\chi^{I,C})'_t$$

$$\chi^{E,C}{}_t = \frac{\chi C}{2} \left(\frac{\omega^I_t \frac{C^{E,W}{}_t}{C^E_t}}{\frac{C^{E,W}{}_{t-1}}{C^E_{t-1}}} - 1 \right)^2$$

$$(\chi^{E,C})'_t = \chi C \left(\frac{\omega^I_t \frac{C^{E,W}{}_t}{C^E_t}}{\frac{C^{E,W}{}_{t-1}}{C^E_{t-1}}} - 1 \right) \frac{\omega^I_t \frac{1}{C^E_t}}{\frac{C^{E,W}{}_{t-1}}{C^E_{t-1}}}$$

$$d\chi^{E,C}{}_t = 1 - \chi^{E,C}{}_t - C^{E,W}{}_t \, (\chi^{E,C})'_t$$

$$C^P_t = \left(\vartheta_C^{\frac{1}{v_C}} \, {C^{P,DK}_t}^{1-\frac{1}{v_C}} + (1 - \vartheta_C)^{\frac{1}{v_C}} \, \left(C^{P,W}{}_t \left(1 - \chi^{P,C}{}_t \right) \right)^{1-\frac{1}{v_C}} \right)^{\frac{1}{1-\frac{1}{v_C}}}$$

$$\frac{C^{P,DK}{}_t}{C^{P,W}{}_t} = \left(1 - \chi^{P,C}{}_t \right) \frac{\vartheta_C}{1 - \vartheta_C} \, (\frac{P^M}{P})^{v_C}_t \, d{\chi^{P,C}_t}^{(-v_C)}$$

$$C^I_t = \left(\vartheta_C^{\frac{1}{v_C}} \, {C^{I,DK}_t}^{1-\frac{1}{v_C}} + (1 - \vartheta_C)^{\frac{1}{v_C}} \, \left(C^{I,W}{}_t \left(1 - \chi^{I,C}{}_t \right) \right)^{1-\frac{1}{v_C}} \right)^{\frac{1}{1-\frac{1}{v_C}}}$$

$$\frac{C^{I,DK}{}_t}{C^{I,W}{}_t} = \left(1 - \chi^{I,C}{}_t \right) \frac{\vartheta_C}{1 - \vartheta_C} \, (\frac{P^M}{P})^{v_C}_t \, d{\chi^{I,C}_t}^{(-v_C)}$$

$$C^E_t = \left(\vartheta_C^{\frac{1}{v_C}} \, {C^{E,DK}_t}^{1-\frac{1}{v_C}} + (1 - \vartheta_C)^{\frac{1}{v_C}} \, \left(C^{E,W}{}_t \left(1 - \chi^{E,C}{}_t \right) \right)^{1-\frac{1}{v_C}} \right)^{\frac{1}{1-\frac{1}{v_C}}}$$

$$\frac{C^{E,DK}{}_t}{C^{E,W}{}_t} = \left(1 - \chi^{E,C}{}_t \right) \frac{\vartheta_C}{1 - \vartheta_C} \, (\frac{P^M}{P})^{v_C}_t \, d{\chi^{E,C}_t}^{(-v_C)}$$

$$(\frac{P^C}{P})_t = \left(\vartheta_C + (1-\vartheta_C)\left(\frac{(\frac{P^M}{P})_t}{d\chi^{P,C}{}_t}\right)^{1-\upsilon_C}\right)^{\frac{1}{1-\upsilon_C}}$$

$$\chi^{Y,I}{}_t = \frac{\chi I}{2}\left(\frac{\omega^I_t \frac{I^{Y,W}{}_t}{I^Y_t}}{\frac{I^{Y,W}{}_{t-1}}{I^Y_{t-1}}} - 1\right)^2$$

$$(\chi^{Y,I})'_t = \chi I\left(\frac{\omega^I_t \frac{I^{Y,W}{}_t}{I^Y_t}}{\frac{I^{Y,W}{}_{t-1}}{I^Y_{t-1}}} - 1\right)\frac{\omega^I_t \frac{1}{I^Y_t}}{\frac{I^{Y,W}{}_{t-1}}{I^Y_{t-1}}}$$

$$d\chi^{Y,I}{}_t = 1 - \chi^{Y,I}{}_t - I^{Y,W}{}_t\,(\chi^{Y,I})'_t$$

$$I^Y_t = \left(\vartheta_{Y,I}^{\frac{1}{\upsilon_I}}\, {I^{Y,DK}_t}^{1-\frac{1}{\upsilon_I}} + \left(1-\vartheta_{Y,I}\right)^{\frac{1}{\upsilon_I}}\left(I^{Y,W}{}_t\left(1-\chi^{Y,I}{}_t\right)\right)^{1-\frac{1}{\upsilon_I}}\right)^{\frac{1}{1-\frac{1}{\upsilon_I}}}$$

$$\frac{I^{Y,DK}{}_t}{I^{Y,W}{}_t} = \left(1-\chi^{Y,I}{}_t\right)\frac{\vartheta_{Y,I}}{1-\vartheta_{Y,I}}\,(\frac{P^M}{P})_t^{\upsilon_I}\, d{\chi^{Y,I}_t}^{(-\upsilon_I)}$$

$$(\frac{P^{IY}}{P})_t = \left(\vartheta_{Y,I} + \left(1-\vartheta_{Y,I}\right)\left(\frac{(\frac{P^M}{P})_t}{d\chi^{Y,I}{}_t}\right)^{1-\upsilon_I}\right)^{\frac{1}{1-\upsilon_I}}$$

$$\Pi I^Y{}_t = \frac{\Pi^{DK}{}_t\,(\frac{P^{IY}}{P})_t}{\Delta Z_t\,(\frac{P^{IY}}{P})_{t-1}}$$

$$\chi^{I^H}{}_t = \frac{\chi I}{2}\left(\frac{\omega^I_t \frac{I^{HW}{}_t}{I^H{}_t}}{\frac{I^{HW}{}_{t-1}}{I^H{}_{t-1}}} - 1\right)^2$$

$$(\chi^{I^H})'_t = \chi I\left(\frac{\omega^I_t \frac{I^{HW}{}_t}{I^H{}_t}}{\frac{I^{HW}{}_{t-1}}{I^H{}_{t-1}}} - 1\right)\frac{\omega^I_t \frac{1}{I^H{}_t}}{\frac{I^{HW}{}_{t-1}}{I^H{}_{t-1}}}$$

$$d\chi^{I^H}{}_t = 1 - \chi^{I^H}{}_t - I^{HW}{}_t\,(\chi^{I^H})'_t$$

$$I^H{}_t = \left(\vartheta_{X,I}^{\frac{1}{\upsilon_I}}\, {I^{HDK}_t}^{1-\frac{1}{\upsilon_I}} + (1-\vartheta_{X,I})^{\frac{1}{\upsilon_I}}\left(I^{HW}{}_t\left(1-\chi^{I^H}{}_t\right)\right)^{1-\frac{1}{\upsilon_I}}\right)^{\frac{1}{1-\frac{1}{\upsilon_I}}}$$

$$\frac{I^{HDK}{}_t}{I^{HW}{}_t} = \left(1-\chi^{I^H}{}_t\right)(\frac{P^M}{P})_t^{\upsilon_I}\,\frac{\vartheta_{X,I}}{1-\vartheta_{X,I}}\, d{\chi^{I^H}_t}^{(-\upsilon_I)}$$

$$(\frac{p^{IX}}{p})_t = \left(\vartheta_{X,I} + (1-\vartheta_{X,I})\left(\frac{(\frac{P^M}{P})_t}{d\chi^{I^H}{}_t}\right)^{1-\upsilon_I}\right)^{\frac{1}{1-\upsilon_I}}$$

$$\Pi I^X{}_t = \frac{\Pi^{DK}{}_t\, (\frac{p^{IX}}{p})_t}{(\frac{p^{IX}}{p})_{t-1}}$$

$$1 = \epsilon_t^{b,I} - \epsilon_t^{b,I}\,\frac{R_t^L - 1}{R_t^{L,I} - 1} + \Phi^{B,I}\,\frac{\left(R_t^{L,I} - 1\right)\left(\frac{R_t^{L,I}-1}{R_{t-1}^{L,I}-1} - 1\right)}{R_{t-1}^{L,I} - 1} - \beta^P\,\frac{C_t}{\bar{C}}\,\Phi^{B,I}\,\frac{\lambda_{t+1}^P}{\lambda_t^P\,\Pi^{DK}{}_{t+1}}\left(\frac{R_{t+1}^{L,I} - 1}{R_t^{L,I} - 1} - 1\right)\left(\frac{R_{t+1}^{L,I} - 1}{R_t^{L,I} - 1}\right)^2\,\frac{B_{t+1}^I}{B_t^I}$$

$$1 = \epsilon_t^{b,E} - \epsilon_t^{b,E}\,\frac{R_t^L - 1}{R_t^{L,E} - 1} + \Phi^{B,E}\,\frac{\left(R_t^{L,E} - 1\right)\left(\frac{R_t^{L,E}-1}{R_{t-1}^{L,E}-1} - 1\right)}{R_{t-1}^{L,E} - 1} - \frac{\lambda_{t+1}^P}{\lambda_t^P\,\Pi^{DK}{}_{t+1}}\,\beta^P\,\frac{C_t}{\bar{C}}\,\Phi^{B,E}\left(\frac{R_{t+1}^{L,E} - 1}{R_t^{L,E} - 1} - 1\right)\left(\frac{R_{t+1}^{L,E} - 1}{R_t^{L,E} - 1}\right)^2\,\frac{B_{t+1}^E}{B_t^E}$$

$$S_t^{L,I} = R_t^{L,I} - R_t^{INT}$$

$$S_t^{L,E} = R_t^{L,E} - R_t^{INT}$$

$$D_t^* + B_t = D_t^P + K_t^B$$

$$B_t = B_t^I + B_t^E$$

$$d\Gamma_t\,\Pi^{DK}{}_t\,K_t^B = K_{t-1}^B\,\left(1 - \delta^B\right)\,\frac{\bar{\kappa^B}}{\kappa_t^B} + \Xi_{t-1}$$

$$\Xi_t = \left(R_t^{L,I} - 1\right)B_t^I + \left(R_t^{L,E} - 1\right)B_t^E - \left(R_t^D - 1\right)D_t^P + D_t^*$$
$$\left(R_t^{INT} - 1\right) - K_t^B\,\frac{\Phi^B}{2}\left(\frac{K_t^B}{B_t} - \kappa^B\right)^2 - B_t^I\left(R_t^{L,I} - 1\right)\frac{\Phi^{B,I}}{2}\left(\frac{R_t^{L,I} - 1}{R_{t-1}^{L,I} - 1} - 1\right)^2 - B_t^E\left(R_t^{L,E} - 1\right)\frac{\Phi^{B,E}}{2}\left(\frac{R_t^{L,E} - 1}{R_{t-1}^{L,E} - 1} - 1\right)^2$$

$$R_t^L = R_t^D - \Phi^B\left(\frac{K_t^B}{B_t} - \kappa^B\right)\left(\frac{K_t^B}{B_t}\right)^2$$

$$R_t^D = R_t^{INT}$$

$$R_t^{INT} = exp\left(\left(-\Psi_d^{D\star}\right)\left(\frac{D_{t-1}^*}{Y_{t-1}} - \frac{\bar{D^*}}{\bar{Y}}\right)\right)R_t^{ECB}\,\frac{\mathcal{R}_t}{\bar{\mathcal{R}}}$$

$$S_t^W = R_t^L - R_t^D$$

$$\frac{R_t^{DK}}{R\bar{DK}} = \left(\frac{R_{t-1}^{DK}}{R\bar{DK}}\right)^{\rho_{RDK}}\left(\frac{R_t^{ECB}}{R\bar{ECB}}\,exp\left(\eta_{R^{DK}}\left(\frac{B_{t-1}^{DK}}{Y_{t-1}} - \omega^d\right)\right)\right)^{1-\rho_{RDK}}$$

$$z^P{}_t = \left(\frac{1}{d\Gamma_t}\,z^P{}_{t-1}\right)^{1-\nu}\left((\frac{P^C}{P})_t\left(C_t^P - h^C\,C_{t-1}^P\,\frac{1}{d\Gamma_t}\right)\left(1 + \tau_t^{VAT}\right)\right)^{\nu}$$

$$(\frac{P^C}{P})_t\left(C_t^P - h^C\,C_{t-1}^P\,\frac{1}{d\Gamma_t}\right)\left(1 + \tau_t^{VAT}\right)O^P{}_t = z^P{}_t$$

$$q1W^P_t = {O^P}_t \left(\frac{{W^P}_t}{{\tilde{W}^P}_t}\right)^{\epsilon^W\,(1+\phi)} \chi_t\, {N^P}^{1+\phi}_t + \beta^P\, \frac{C_t}{\bar{C}}\, \theta_w\, q1W^P_{t+1}\, {\Pi^C}^{(1+\phi)\,\epsilon^W\,(-\Gamma_W)}_{t-1}\, {\Pi^{DK}}^{\epsilon^W\,(1+\phi)}_{t+1} \left(\frac{{\tilde{W}^P}_{t+1}}{{\tilde{W}^P}_t}\right)^{\epsilon^W\,(1+\phi)}$$

$$q2W^P_t = \left(1-\tau^N_t\right)\, \lambda^P_t\, {N^P}_t \left(\frac{{W^P}_t}{{\tilde{W}^P}_t}\right)^{\epsilon^W} + \beta^P\, \frac{C_t}{\bar{C}}\, \theta_w\, q2W^P_{t+1}\, {\Pi^C}^{\Gamma_W\,(1-\epsilon^W)}_{t-1}\, {\Pi^{DK}}^{\epsilon^W-1}_{t+1} \left(\frac{{\tilde{W}^P}_{t+1}}{{\tilde{W}^P}_t}\right)^{\epsilon^W}$$

$${\tilde{W}^P}_t = M_W\, \frac{{\epsilon^W}_t}{\bar{\epsilon W}}\, \frac{q1W^P_t}{q2W^P_t}$$

$${W^P}_t = \left(\theta_w\, {\Pi^C}^{\Gamma_W\,(1-\epsilon^W)}_{t-1}\, {W^P}^{1-\epsilon^W}_{t-1} \left(d\Gamma_t\, {\Pi^{DK}}_{t-1}\right)^{\epsilon^W-1} + (1-\theta_w)\, {\tilde{W}^P}^{1-\epsilon^W}_t\right)^{\frac{1}{1-\epsilon^W}}$$

$${\mathcal{W}^P}_t = \frac{\left(1-\tau^N_t\right)\, {W^P}_t}{\frac{{O^P}_t\, \chi_t\, {N^P}^{\phi}_t}{\lambda^P_t}}$$

$${L^P}_t = \left(\frac{\left(1-\tau^N_t\right)\, {W^P}_t}{{z^P}_t\, \chi_t}\right)^{\frac{1}{\phi}}$$

$${U^P}_t = {\mathcal{W}^P}^{\frac{1}{\phi}}_t$$

$${z^I}_t = \left(\frac{1}{d\Gamma_t}\, {z^I}_{t-1}\right)^{1-\nu} \left((\frac{P^C}{P})_t\, \left(1+\tau^{VAT}_t\right) \left(C^I_t - \frac{1}{d\Gamma_t}\, h^C\, C^I_{t-1}\right)\right)^{\nu}$$

$$(\frac{P^C}{P})_t\, \left(1+\tau^{VAT}_t\right) \left(C^I_t - \frac{1}{d\Gamma_t}\, h^C\, C^I_{t-1}\right) {O^I}_t = {z^I}_t$$

$$q1W^I_t = {O^I}_t\, \chi_t \left(\frac{{W^I}_t}{{\tilde{W}^I}_t}\right)^{\epsilon^W\,(1+\phi)} {N^I}^{1+\phi}_t + {\Pi^{DK}}^{\epsilon^W\,(1+\phi)}_{t+1}\, {\Pi^C}^{(1+\phi)\,\epsilon^W\,(-\Gamma_W)}_{t-1}\, \frac{C_t}{\bar{C}}\, \beta^I\, \theta_w\, q1W^I_{t+1} \left(\frac{{\tilde{W}^I}_{t+1}}{{\tilde{W}^I}_t}\right)^{\epsilon^W\,(1+\phi)}$$

$$q2W^I_t = \left(1-\tau^N_t\right)\, \lambda^I_t\, {N^I}_t \left(\frac{{W^I}_t}{{\tilde{W}^I}_t}\right)^{\epsilon^W} + {\Pi^{DK}}^{\epsilon^W-1}_{t+1}\, {\Pi^C}^{\Gamma_W\,(1-\epsilon^W)}_{t-1}\, \frac{C_t}{\bar{C}}\, \beta^I\, \theta_w\, q2W^I_{t+1} \left(\frac{{\tilde{W}^I}_{t+1}}{{\tilde{W}^I}_t}\right)^{\epsilon^W}$$

$${\tilde{W}^I}_t = M_W\, \frac{{\epsilon^W}_t}{\bar{\epsilon W}}\, \frac{q1W^I_t}{q2W^I_t}$$

$${W^I}_t = \left(\left(d\Gamma_t\, {\Pi^{DK}}_{t-1}\right)^{\epsilon^W-1} \theta_w\, {\Pi^C}^{\Gamma_W\,(1-\epsilon^W)}_{t-1}\, {W^I}^{1-\epsilon^W}_{t-1} + (1-\theta_w)\, {\tilde{W}^I}^{1-\epsilon^W}_t\right)^{\frac{1}{1-\epsilon^W}}$$

$${\mathcal{W}^I}_t = \frac{\left(1-\tau^N_t\right)\, {W^I}_t}{\frac{\chi_t\, {O^I}_t\, {N^I}^{\phi}_t}{\lambda^I_t}}$$

$$L^{I}{}_{t} = \left(\frac{\left(1 - \tau_t^N\right) W^{I}{}_{t}}{\chi_t \, z^{I}{}_{t}} \right)^{\frac{1}{\phi}}$$

$$U^{I}{}_{t} = \mathcal{W}^{I}{}_{t}^{\frac{1}{\phi}}$$

$$Im_t = (\frac{P^C}{P})_t \left(C_t^E + C_t^P + C_t^I\right) - \left(C^{P,DK}{}_t + C^{I,DK}{}_t + C^{E,DK}{}_t\right) + I^{H}{}_t \, (\frac{p^{IX}}{p})_t - I^{HDK}{}_t + (\frac{P^{IY}}{P})_t \, I_t^Y - I^{Y,DK}{}_t$$

$$(\frac{Px^W}{P})_t = \frac{(\frac{Px^W}{P})_{t-1}}{\Pi^{DK}{}_t} \, \Pi_t^W$$

$$(\frac{P^M}{P})_t = \frac{(\frac{P^M}{P})_{t-1}}{\Pi^{DK}{}_t} \, \Pi^{M}{}_t$$

$$mc_t^M = \frac{(\frac{Px^W}{P})_t}{(\frac{P^M}{P})_t}$$

$$q1m_t = Im_t \, \lambda_t^P \, mc_t^M + \beta^P \, \frac{C_t}{\bar{C}} \, \theta_M \, q1m_{t+1} \, \Pi^{M}{}_{t+1}^{\epsilon^M}$$

$$q2m_t = \lambda_t^P \, Im_t + \beta^P \, \frac{C_t}{\bar{C}} \, \theta_M \, q2m_{t+1} \, \Pi^{M}{}_{t+1}^{\epsilon^M - 1}$$

$$\tilde{P^M}_t = \mathcal{M}_M \, \frac{\epsilon_t^{Mt}}{\epsilon^{\bar{M}t}} \, \frac{q1m_t}{q2m_t}$$

$$1 = \theta_M \, \Pi^{C}{}_{t-1}^{\Gamma_{Mt} \left(1 - \epsilon^M\right)} \, \Pi^{M}{}_t^{\epsilon^M - 1} + (1 - \theta_M) \, \tilde{P^M}_t^{1 - \epsilon^M}$$

$$Ex_t = X_t^{Ex} \, Y_t^W \left(\frac{(\frac{P^X}{P})_t}{(\frac{Px^W}{P})_t} \right)^{\left(-\epsilon^{World}\right)}$$

$$(\frac{P^X}{P})_t = \frac{(\frac{P^X}{P})_{t-1}}{\Pi^{DK}{}_t} \, \Pi^{X}{}_t$$

$$mc_t^X = (\frac{P^X}{P})_t^{(-1)}$$

$$q1x_t = Y_t^W \, \lambda_t^P \, mc_t^X + \beta^P \, \frac{C_t}{\bar{C}} \, \theta_X \, q1x_{t+1} \, \Pi^{X}{}_{t+1}^{\epsilon_X}$$

$$q2x_t = \lambda_t^P \, Y_t^W + \beta^P \, \frac{C_t}{\bar{C}} \, \theta_X \, q2x_{t+1} \, \Pi^{X}{}_{t+1}^{\epsilon_X - 1}$$

$$\tilde{P^X}_t = \mathcal{M}_X \, \frac{\epsilon_t^{Xt}}{\epsilon^{\bar{X}t}} \, \frac{q1x_t}{q2x_t}$$

$$1 = \theta_X \, {\Pi^C}_{t-1}^{\Gamma_{Xt}\,(1-\epsilon_X)} \, {\Pi^X}_t^{\epsilon_X - 1} + (1-\theta_X)\, \tilde{P^X}_t^{1-\epsilon_X}$$

$$B_t^{DK} + T_t = \frac{1}{d\Gamma_t \, {\Pi^{DK}}_t} B_{t-1}^E \, \kappa_t^{RE} \left(R_{t-1}^{L,E} - 1\right) + \frac{1}{d\Gamma_t \, {\Pi^{DK}}_t} B_{t-1}^I \, \kappa_t^{RI} \left(R_{t-1}^{L,I} - 1\right) + G_t + I_t^G + \frac{1}{d\Gamma_t} B_{t-1}^{DK} \frac{R_{t-1}^{DK}}{{\Pi^{DK}}_t}$$
$$-\left(C_t^E + C_t^P + C_t^I\right) \left(\frac{P^C}{P}\right)_t \tau_t^{VAT}$$
$$-\tau_t^N \left({W^I}_t \, {N^I}_t + {W^P}_t \, {N^P}_t\right) - \frac{1}{\Delta I_t} K_{t-1}^Y \, \tau_t^K \left(r_t^{K,Y} \frac{\mathcal{K}_t}{\bar{\mathcal{K}}} u_t^Y - \delta^{K,Y}\right) - \tau_t^H \, QHs \, Htots - l_t \, \tau_t^L \, Q_t^L - \frac{1}{d\Gamma_t \, {\Pi^{DK}}_t} D_{t-1}^P \, \tau_t^B \left(R_{t-1}^D - 1\right)$$

$${K^G}_t = \frac{{K^G}_{t-1} \left(1 - \delta^G\right)}{\Delta I_t} + \frac{AUX\_ENDO\_LAG\_18\_2_{t-1}}{\Delta I_t \, \Delta I_{t-1} \, AUX\_ENDO\_LAG\_66\_1_{t-1}}$$

$$\frac{\tau_t^N}{\tau^N} = \left(\frac{\tau_{t-1}^N}{\tau^N}\right)^{\rho_N} \left(\frac{\frac{B_{t-1}^{DK}}{Y_{t-1}}}{\omega^d}\right)^{(1-\rho_N)\,\zeta_N} exp\left(\epsilon_t^N\right)$$

$$\frac{\tau_t^K}{\tau^K} = \left(\frac{\tau_{t-1}^K}{\tau^K}\right)^{\rho_K} \left(\frac{\frac{B_{t-1}^{DK}}{Y_{t-1}}}{\omega^d}\right)^{(1-\rho_K)\, e\, \zeta_K} exp\left(\epsilon_t^K\right)$$

$$\frac{\tau_t^{VAT}}{\tau^{VAT}} = \left(\frac{\tau_{t-1}^{VAT}}{\tau^{VAT}}\right)^{\rho_C} \left(\frac{\frac{B_{t-1}^{DK}}{Y_{t-1}}}{\omega^d}\right)^{e\,(1-\rho_C)\,\zeta_{VAT}} exp\left(\epsilon_t^{VAT}\right)$$

$$\frac{\tau_t^B}{\tau^B} = \left(\frac{\tau_{t-1}^B}{\tau^B}\right)^{\rho_B} \left(\frac{\frac{B_{t-1}^{DK}}{Y_{t-1}}}{\omega^d}\right)^{e\,(1-\rho_B)\,\zeta_B} exp\left(\epsilon_t^B\right)$$

$$T_t - \bar{T} = \rho_T \, \left(T_{t-1} - \bar{T}\right) + \left(\frac{B_{t-1}^{DK}}{Y_{t-1}} - \omega^d\right) e \, (1-\rho_T) \, \zeta_T$$

$$T_t^{fon} = B_t^{DK} - \frac{1}{d\Gamma_t} B_{t-1}^{DK} \frac{R_{t-1}^{DK}}{{\Pi^{DK}}_t}$$

$$B_t^{DK,GDP} = \frac{B_t^{DK}}{Y_t \, 4}$$

$$\frac{I_t^{G,B}}{\bar{I^g}} = exp\left(\epsilon_t^{IG}\right) \left(\frac{I_{t-1}^{G,B}}{\bar{I^g}}\right)^{\rho_{Big}}$$
$$\left(\frac{AUX\_ENDO\_LAG\_18\_1_{t-1}}{\bar{I^g}}\right)^{rhoIBg\_2} \left(\frac{AUX\_ENDO\_LAG\_18\_2_{t-1}}{\bar{I^g}}\right)^{rhoIBg\_3}$$
$$\left(\frac{AUX\_ENDO\_LAG\_18\_3_{t-1}}{\bar{I^g}}\right)^{rhoIBg\_4} \left(\frac{AUX\_ENDO\_LAG\_18\_4_{t-1}}{\bar{I^g}}\right)^{rhoIBg\_5}$$

$$I_t^G = I_t^{G,B}\, I_0 + I_{t-1}^{G,B}\, I_1\, \frac{1}{d\Gamma_t\, \Delta Z_t} + \frac{1}{d\Gamma_t\, \Delta Z_t\, \Delta Z_{t-1}\, d\Gamma_{t-1}}\, I_2\, AUX\_ENDO\_LAG\_18\_1_{t-1} + I_3\, AUX\_ENDO\_LAG\_18\_2_{t-1}\, \frac{1}{d\Gamma_t\, \Delta Z_t\, \Delta Z_{t-1}\, d\Gamma_{t-1}\, AUX\_ENDO\_LAG\_65\_1_{t-1}\, AUX\_ENDO\_LAG\_63\_1_{t-1}}$$

$$\frac{G_t}{\bar{G}} = exp\left(\epsilon_t^G\right) \left(\frac{G_{t-1}}{\bar{G}}\right)^{\rho_G} \left(\frac{AUX\_ENDO\_LAG\_45\_1_{t-1}}{\bar{G}}\right)^{rhoG\_2} \left(\frac{AUX\_ENDO\_LAG\_45\_2_{t-1}}{\bar{G}}\right)^{rhoG\_3} \left(\frac{AUX\_ENDO\_LAG\_45\_3_{t-1}}{\bar{G}}\right)^{rhoG\_4} \left(\frac{AUX\_ENDO\_LAG\_45\_4_{t-1}}{\bar{G}}\right)^{rhoG\_5} \left(\frac{AUX\_ENDO\_LAG\_45\_5_{t-1}}{\bar{G}}\right)^{rhoG\_6} \left(\frac{AUX\_ENDO\_LAG\_45\_6_{t-1}}{\bar{G}}\right)^{rhoG\_7}$$

$$\frac{Y_t^{EA}}{\bar{Y^{EA}}} = \left(\frac{Y_{t+1}^{EA}}{\bar{Y^{EA}}}\right)^{ay} \left(\frac{Y_{t-1}^{EA}}{\bar{Y^{EA}}}\right)^{1-ay} \left(\frac{\frac{R_t^{ECB}}{\bar{R^{ECB}}}}{\frac{\Pi_{t+1}^{EA}}{\bar{\Pi^{EA}}}}\right)^{(-phiyEA)} \frac{e_t^{y,EA}}{\bar{e_{y,EA}}}$$

$$\frac{\Pi_t^{EA}}{\bar{\Pi^{EA}}} = \left(\frac{\Pi_{t+1}^{EA}}{\bar{\Pi^{EA}}}\right)^{apie} \left(\frac{\Pi_{t-1}^{EA}}{\bar{\Pi^{EA}}}\right)^{1-apie} \left(\frac{Y_t^{EA}}{\bar{Y^{EA}}}\right)^{phipieEA} \frac{e_t^{\Pi,EA}}{\bar{e_{\Pi,EA}}}$$

$$\frac{R_t^{ECB}}{\bar{R^{ECB}}} = \left(\frac{R_{t-1}^{ECB}}{\bar{R^{ECB}}}\right)^{0.75} \left(\left(\frac{\Pi_t^{EA}}{\bar{\Pi^{EA}}}\right)^{Ipie} \left(\frac{Y_t^{EA}}{\bar{Y^{EA}}}\right)^{Iy}\right)^{0.25} \frac{e_t^{RECB}}{\bar{e_{RECB}}}$$

$$\frac{e_t^{y,EA}}{\bar{e_{y,EA}}} = \left(\frac{e_{t-1}^{y,EA}}{\bar{e_{y,EA}}}\right)^{rho\_e\_y} exp\left(\epsilon_t^{y,EA}\right)$$

$$\frac{e_t^{\Pi,EA}}{\bar{e_{\Pi,EA}}} = \left(\frac{e_{t-1}^{\Pi,EA}}{\bar{e_{\Pi,EA}}}\right)^{rho\_e\_pie} exp\left(\epsilon_t^{\Pi,EA}\right)$$

$$\frac{e_t^{RECB}}{\bar{e_{RECB}}} = exp\left(\epsilon_t^{r,EA}\right)$$

$$\frac{Y_t^{RoW}}{\bar{Y^{RoW}}} = \left(\frac{Y_{t+1}^{RoW}}{\bar{Y^{RoW}}}\right)^{ayRoW} \left(\frac{Y_{t-1}^{RoW}}{\bar{Y^{RoW}}}\right)^{1-ayRoW} \left(\frac{\frac{R_t^{RoW}}{\bar{R^{RoW}}}}{\frac{\Pi_{t+1}^{RoW}}{\bar{\Pi^{RoW}}}}\right)^{(-phiyRoW)} \frac{e_t^{y,ROW}}{\bar{e_{y,RW}}}$$

$$\frac{\Pi_t^{RoW}}{\bar{\Pi^{RoW}}} = \left(\frac{\Pi_{t+1}^{RoW}}{\bar{\Pi^{RoW}}}\right)^{apieRoW} \left(\frac{\Pi_{t-1}^{RoW}}{\bar{\Pi^{RoW}}}\right)^{1-apieRoW} \left(\frac{Y_t^{RoW}}{\bar{Y^{RoW}}}\right)^{phipieRoW} \frac{e_t^{\Pi,ROW}}{\bar{e_{\Pi,RW}}}$$

$$\frac{R_t^{RoW}}{\bar{R^{RoW}}} = \left(\frac{R_{t-1}^{RoW}}{\bar{R^{RoW}}}\right)^{rho\_i\_RoW} \left(\left(\frac{\Pi_t^{RoW}}{\bar{\Pi^{RoW}}}\right)^{IpieRoW} \left(\frac{Y_t^{RoW}}{\bar{Y^{RoW}}}\right)^{IyRoW}\right)^{1-rho\_i\_RoW} \frac{e_t^{R,ROW}}{\bar{e_{R,RW}}}$$

$$\frac{e_t^{y,ROW}}{e_{y,\bar{R}W}} = \left(\frac{e_{t-1}^{y,ROW}}{e_{y,\bar{R}W}}\right)^{rho\_e\_yRoW} exp\left(\epsilon_t^{y,ROW}\right)$$

$$\frac{e_t^{\Pi,ROW}}{e_{\Pi,\bar{R}W}} = \left(\frac{e_{t-1}^{\Pi,ROW}}{e_{\Pi,\bar{R}W}}\right)^{rho\_e\_pieRoW} exp\left(\epsilon_t^{\Pi,ROW}\right)$$

$$\frac{e_t^{R,ROW}}{e_{R,\bar{R}W}} = exp\left(\epsilon_t^{r,ROW}\right)$$

$$\frac{Y_t^W}{Y^{\bar{W}}} = \left(\frac{Y_t^{EA}}{Y^{\bar{E}A}}\right)^{\omega_X} \left(\frac{Y_t^{RoW}}{Y^{\bar{R}oW}}\right)^{1-\omega_X}$$

$$\frac{\Pi_t^W}{\Pi^{\bar{W}}} = \left(\frac{\Pi_t^{EA}}{\Pi^{\bar{E}A}}\right)^{\omega_X} \left(\frac{\Pi_t^{RoW} \frac{FX_t}{\bar{FX}}}{\Pi^{\bar{R}oW}}\right)^{1-\omega_X}$$

$$\frac{FX_t}{\bar{FX}} = \frac{\frac{R_t^{ECB}}{R^{\bar{E}CB}}}{\frac{R_t^{RoW}}{R^{\bar{R}oW}}} exp\left(\epsilon_t^{UIP}\right)$$

$$d\Gamma_t = \left(\Delta A_t \, \Delta Z_t^{\alpha^Y}\right)^{\frac{1}{1-\alpha^Y}}$$

$$\Delta A_t = \lambda_{A\,t}^Y$$

$$\Delta Z_t = \lambda_{Z\,t}$$

$$\frac{\lambda_{A\,t}^Y}{\bar{\lambda}_A^Y} = \left(\frac{\lambda_{A\,t-1}^Y}{\bar{\lambda}_A^Y}\right)^{\rho_{\lambda_A}} exp\left(\epsilon_t^{A_{Y,P}}\right)$$

$$\frac{\lambda_{Z\,t}}{\bar{\lambda}_Z} = \left(\frac{\lambda_{Z\,t-1}}{\bar{\lambda}_Z}\right)^{\rho_{\lambda_Z}} exp\left(\epsilon_t^{Z_P}\right)$$

$$\Delta A_t^X = \lambda_t^{A,X}$$

$$\frac{\lambda_t^{A,X}}{\lambda^{\bar{A},X}} = \frac{\left(\frac{\lambda_{t-1}^{A,X}}{\lambda^{\bar{A},X}}\right)^{\rho_{A,X}}}{exp\left(\epsilon_t^{A_{X,P}}\right)}$$

$$d\Gamma_t^X = \Delta A_t^X \, d\Gamma_t^{1-\alpha^X}$$

$$\Delta I_t = d\Gamma_t \, \Delta Z_t$$

$$\frac{\tau_t^H}{\bar{\tau^H}} = \left(\frac{\tau_{t-1}^H}{\bar{\tau^H}}\right)^{\rho_{\tau_H}} \left(\frac{\frac{B_{t-1}^{DK}}{Y_{t-1}}}{\omega^d}\right)^{e\left(1-\rho_{\tau_H}\right)\zeta_H} exp\left(\epsilon_t^{\tau^H}\right)$$

$$\frac{\tau_t^L}{\bar{\tau^L}} = \left(\frac{\tau_{t-1}^L}{\bar{\tau^L}}\right)^{\rho_{\tau^L}} \left(\frac{\frac{B_{t-1}^{DK}}{Y_{t-1}}}{\omega^d}\right)^{e\left(1-\rho_{\tau^L}\right)\zeta_L} exp\left(\epsilon_t^{\tau^L}\right)$$

$$\frac{\Theta_t^I}{\bar{\Theta^I}} = \left(\frac{\Theta_{t-1}^I}{\bar{\Theta^I}}\right)^{\rho_\Theta} exp\left(\epsilon_t^{\Theta}\right)$$

$$\frac{\Theta_t^E}{\bar{\Theta^E}} = exp\left(\epsilon_t^{\Theta}\right) \left(\frac{\Theta_{t-1}^E}{\bar{\Theta^E}}\right)^{\rho_\Theta}$$

$$\frac{\kappa_t^{RI}}{\bar{\kappa^{RI}}} = \left(\frac{\kappa_{t-1}^{RI}}{\bar{\kappa^{RI}}}\right)^{\rho_{\kappa^R}} exp\left(\epsilon_t^{\kappa^{RI}}\right)$$

$$\frac{\kappa_t^{RE}}{\bar{\kappa^{RE}}} = \left(\frac{\kappa_{t-1}^{RE}}{\bar{\kappa^{RE}}}\right)^{\rho_{\kappa^R}} exp\left(\epsilon_t^{\kappa^{RE}}\right)$$

$$\frac{\mathcal{H}_t}{\bar{\mathcal{H}}} = \left(\frac{\mathcal{H}_{t-1}}{\bar{\mathcal{H}}}\right)^{\rho_{\mathcal{H}}} exp\left(\epsilon_t^{H}\right)$$

$$\frac{\epsilon_t^{b,I}}{\epsilon^{\bar{b},I}} = \left(\frac{\epsilon_{t-1}^{b,I}}{\epsilon^{\bar{b},I}}\right)^{\rho_{\epsilon^{b,I}}} exp\left(\epsilon_t^{b,I}\right)$$

$$\frac{\epsilon_t^{b,E}}{\epsilon^{\bar{b},E}} = \left(\frac{\epsilon_{t-1}^{b,E}}{\epsilon^{\bar{b},E}}\right)^{\rho_{\epsilon^{b,E}}} exp\left(\epsilon_t^{b,E}\right)$$

$$\frac{A^{X,T}{}_t}{A^{\bar{X},T}} = \left(\frac{A^{X,T}{}_{t-1}}{A^{\bar{X},T}}\right)^{\rho_{A_{T,X}}} exp\left(\epsilon_t^{A_{X,T}}\right)$$

$$\frac{C_t}{\bar{C}} = \left(\frac{C_{t-1}}{\bar{C}}\right)^{\rho_C} exp\left(\epsilon_t^{C}\right)$$

$$\frac{A_t^{Y,T}}{A^{\bar{Y},T}} = \left(\frac{A_{t-1}^{Y,T}}{A^{\bar{Y},T}}\right)^{\rho_{A_{Y,T}}} exp\left(\epsilon_t^{A_{Y,T}}\right)$$

$$\frac{\mathcal{R}_t}{\bar{\mathcal{R}}} = \frac{\left(\frac{\mathcal{R}_{t-1}}{\bar{\mathcal{R}}}\right)^{\rho_{\mathcal{R}}}}{exp\left(\epsilon_t^{\mathcal{R}}\right)}$$

$$\frac{\mathcal{I}_t}{\bar{\mathcal{I}}} = \left(\frac{\mathcal{I}_{t-1}}{\bar{\mathcal{I}}}\right)^{\rho_{\mathcal{I}}} exp\left(\epsilon_t^{\mathcal{I}}\right)$$

$$\frac{\omega^I_t}{\bar{\omega^I}} = \left(\frac{\omega^I_{t-1}}{\bar{\omega^I}}\right)^{\rho_{\omega^I}} exp\left(\epsilon^{\omega^I}_t\right)$$

$$\frac{\chi_t}{\bar{\chi}} = \left(\frac{\chi_{t-1}}{\bar{\chi}}\right)^{\rho_\chi} exp\left(\epsilon^\chi_t\right)$$

$$\frac{\epsilon^P{}_t}{\bar{\epsilon^P}} = \left(\frac{\epsilon^P{}_{t-1}}{\bar{\epsilon^P}}\right)^{\rho_{\epsilon^P}} exp\left(\epsilon^{\epsilon^P}_t\right)$$

$$\frac{\epsilon^W{}_t}{\epsilon\bar{W}} = \left(\frac{\epsilon^W{}_{t-1}}{\epsilon\bar{W}}\right)^{\rho_{\epsilon^W}} exp\left(\epsilon^{\epsilon^W}_t\right)$$

$$\frac{X^{Ex}_t}{X^{\bar{E}x}} = \left(\frac{X^{Ex}_{t-1}}{X^{\bar{E}x}}\right)^{\rho_{Ex}} exp\left(\epsilon^{Ex}_t\right)$$

$$\frac{\epsilon^{Mt}_t}{\epsilon^{\bar{M}t}} = \left(\frac{\epsilon^{Mt}_{t-1}}{\epsilon^{\bar{M}t}}\right)^{\rho_{\epsilon^M}} exp\left(\epsilon^{\epsilon^M}_t\right)$$

$$\frac{\epsilon^{Xt}_t}{\epsilon^{\bar{X}t}} = \left(\frac{\epsilon^{Xt}_{t-1}}{\epsilon^{\bar{X}t}}\right)^{\rho_{\epsilon^X}} exp\left(\epsilon^{\epsilon^X}_t\right)$$

$$\frac{\mathcal{K}_t}{\bar{\mathcal{K}}} = \left(\frac{\mathcal{K}_{t-1}}{\bar{\mathcal{K}}}\right)^{\rho_{\mathcal{K}}} exp\left(\epsilon^{\mathcal{K}}_t\right)$$

$$\frac{\kappa^B_t}{\bar{\kappa^B}} = \left(\frac{\kappa^B_{t-1}}{\bar{\kappa^B}}\right)^{\rho_{\kappa^B}} exp\left(\epsilon^{\kappa^B}_t\right)$$

$$Adj^B_{\Pi t} = K^B_t \left(\frac{K^B_t}{B_t} - \kappa^B\right)^2 \left(-\left(\frac{\Phi^B}{2}\right)\right) - B^I_t \left(R^{L,I}_t - 1\right) \frac{\Phi^{B,I}}{2} \left(\frac{R^{L,I}_t - 1}{R^{L,I}_{t-1} - 1} - 1\right)^2 - B^E_t \left(R^{L,E}_t - 1\right) \frac{\Phi^{B,E}}{2} \left(\frac{R^{L,E}_t - 1}{R^{L,E}_{t-1} - 1} - 1\right)^2$$

$$C^{tot}{}_t = C^E_t + C^P_t + C^I_t$$

$$\frac{K^B}{B^{tot}{}_t} = \frac{K^B_t}{B_t}$$

$$\varsigma^{HP} \frac{\mathcal{H}_t}{H^P{}_t} + \frac{1}{d\Gamma^X_{t+1}} Q^H_{t+1} \lambda^P_{t+1} \beta^P \frac{C_t}{\bar{C}} \left(1 - \delta^H\right) = Q^H_t \lambda^P_t \left(1 + \tau^H_t\right)$$

$$U^{tot}{}_t = U^{P\omega}_t \, U^{I}{}^{1-\omega}_t$$

$$\Pi^{Retail}{}_t = Y_t - \left(\frac{\mathcal{K}_t}{\bar{\mathcal{K}}} r^{K,Y}_t u^Y_t K^Y_{t-1} \frac{1}{\Delta I_t} + W^I{}_t N^I{}_t + W^P{}_t N^P{}_t\right)$$

$$\frac{R^{Taylor}_t}{\overline{R^{Taylor}}} = \left(\frac{R^{Taylor}_{t-1}}{\overline{R^{Taylor}}}\right)^{0.90} \left(\left(\frac{\Pi^{DK}{}_t}{\Pi\bar{d}k}\right)^{1.5} \left(\frac{Y_t}{\bar{Y}}\right)^{0.125}\right)^{0.1}$$

$$\frac{D^{\star}_t}{Y_t}_t = \frac{D^{*}_t}{Y_t}$$

$$\Pi^{Retail,X}{}_t = Q^H_t X_t - l_{t-1} r^L{}_t - I^H{}_t \, (\frac{p^{IX}}{p})_t$$

$$Yt\_obs_t = log\left(d\Gamma_t \frac{Y_t}{Y_{t-1}}\right)$$

$$\begin{aligned}
exp(Ct_obs_t)/d\Gamma_t = {} & C^I_t \left(\vartheta_C \, (\frac{P^C}{P})^{\upsilon_C}_t + \left(1-\chi^{I,C}{}_t\right)(1-\vartheta_C)\left(\frac{(\frac{P^M}{P})_t}{(\frac{P^C}{P})_t}\right)^{(-\upsilon_C)} d\chi^{I,C}{}^{\upsilon_C}_t\right) + \\
& C^E_t \left(\vartheta_C \, (\frac{P^C}{P})^{\upsilon_C}_t + \left(1-\chi^{E,C}{}_t\right)(1-\vartheta_C)\left(\frac{(\frac{P^M}{P})_t}{(\frac{P^C}{P})_t}\right)^{(-\upsilon_C)} d\chi^{E,C}{}^{\upsilon_C}_t\right) + \\
& C^P_t \left(\vartheta_C \, (\frac{P^C}{P})^{\upsilon_C}_t + \left(1-\chi^{P,C}{}_t\right)(1-\vartheta_C)\left(\frac{(\frac{P^M}{P})_t}{(\frac{P^C}{P})_t}\right)^{(-\upsilon_C)} d\chi^{P,C}{}^{\upsilon_C}_t\right) / \\
& C^I_{t-1} \left(\vartheta_C \, (\frac{P^C}{P})^{\upsilon_C}_{t-1} + (1-\vartheta_C)\left(\frac{(\frac{P^M}{P})_{t-1}}{(\frac{P^C}{P})_{t-1}}\right)^{(-\upsilon_C)} \left(1-\chi^{I,C}{}_{t-1}\right) d\chi^{I,C}{}^{\upsilon_C}_{t-1}\right) + \\
& C^E_{t-1} \left(\vartheta_C \, (\frac{P^C}{P})^{\upsilon_C}_{t-1} + (1-\vartheta_C)\left(\frac{(\frac{P^M}{P})_{t-1}}{(\frac{P^C}{P})_{t-1}}\right)^{(-\upsilon_C)} \left(1-\chi^{E,C}{}_{t-1}\right) d\chi^{E,C}{}^{\upsilon_C}_{t-1}\right) + \\
& C^P_{t-1} \left(\vartheta_C \, (\frac{P^C}{P})^{\upsilon_C}_{t-1} + (1-\vartheta_C)\left(\frac{(\frac{P^M}{P})_{t-1}}{(\frac{P^C}{P})_{t-1}}\right)^{(-\upsilon_C)} \left(1-\chi^{P,C}{}_{t-1}\right) d\chi^{P,C}{}^{\upsilon_C}_{t-1}\right)
\end{aligned}$$

$$It\_obs_t = log\left(\frac{\vartheta_{Y,I} \, (\frac{P^{IY}}{P})_t^{\left(-\left(1-\frac{1}{\upsilon_I}\right)\right)} + \left(1-\vartheta_{Y,I}\right)\left(\frac{(\frac{P^M}{P})_t}{(\frac{P^{IY}}{P})_t \, (1-\chi^{Y,I}{}_t)}\right)^{\left(-\left(1-\frac{1}{\upsilon_I}\right)\right)}}{\vartheta_{Y,I} \, (\frac{P^{IY}}{P})_{t-1}^{\left(-\left(1-\frac{1}{\upsilon_I}\right)\right)} + \left(1-\vartheta_{Y,I}\right)\left(\frac{(\frac{P^M}{P})_{t-1}}{(\frac{P^{IY}}{P})_{t-1} \, (1-\chi^{Y,I}{}_{t-1})}\right)^{\left(-\left(1-\frac{1}{\upsilon_I}\right)\right)}} \frac{I^Y_t}{I^Y_{t-1}} \, d\Gamma_t \, \Delta Z_t\right)$$

$$EXt\_obs_t = log\left(d\Gamma_t \frac{Ex_t}{Ex_{t-1}}\right)$$

$$Mt\_obs_t = log\left(d\Gamma_t \frac{Im_t}{Im_{t-1}}\right)$$

$$Wpt\_obs_t = log\left(d\Gamma_t \frac{W^{tot}_t}{W^{tots}}\right)$$

$$Ut\_obs_t = log\left(\frac{U^{tot}{}_t}{Utots}\right)$$

$$Npt\_obs_t = log\left(\frac{N^{tot}{}_t}{Ntots}\right)$$

$$RECBt\_obs_t = log\left(\frac{R_t^{ECB}}{R^{\bar{E}CB}}\right)$$

$$pieEAt\_obs_t = log\left(\frac{\Pi_t^{EA}}{\Pi^{\bar{E}A}}\right)$$

$$YEAt\_obs_t = log\left(\frac{Y_t^{EA}}{Y^{\bar{E}A}}\right)$$

$$RRoWt\_obs_t = log\left(\frac{R_t^{RoW}}{R^{\bar{R}oW}}\right)$$

$$pieRoWt\_obs_t = log\left(\frac{\Pi_t^{RoW}}{\Pi^{\bar{R}oW}}\right)$$

$$YRoWt\_obs_t = log\left(\frac{Y_t^{RoW}}{Y^{\bar{R}oW}}\right)$$

$$fx\_obs_t = log\left(\frac{FX_t}{\bar{FX}}\right)$$

$$Cgt\_obs_t = log\left(d\Gamma_t \, \frac{G_t}{G_{t-1}}\right)$$

$$Igt\_obs_t = log\left(\Delta Z_t \, d\Gamma_t \, \frac{I_t^{G,B}}{I_{t-1}^{G,B}}\right)$$

$$Twt\_obs_t = log\left(\frac{\tau_t^N}{\bar{\tau^N}}\right)$$

$$KappaRt\_obs_t = log\left(\frac{\kappa_t^{RI}}{\kappa^{\bar{R}I}}\right)$$

$$Tauht\_obs_t = log\left(\frac{\tau_t^H}{\tau^{\bar{H}}}\right)$$

$$Tault\_obs_t = log\left(\frac{\tau_t^L}{\tau^{\bar{L}}}\right)$$

$$pieDKt\_obs_t = log\left(\frac{\Pi^{DK}{}_t}{\Pi \bar{d}k}\right)$$

$$pm\_obs_t = log\left(d\Gamma_t \, \frac{\Pi^M{}_t}{\Pi \bar{M}}\right)$$

$$pe\_obs_t = log\left(d\Gamma_t \, \frac{\Pi^X{}_t}{\Pi \bar{X}}\right)$$

$$pi\_obs_t = log\left(\frac{\Pi I^Y{}_t}{pieIYs}\,\frac{d\Gamma_t}{\Delta Z_t}\right)$$

$$Xt\_obs_t = log\left(d\Gamma_t^X\,\frac{X_t}{X_{t-1}}\right)$$

$$QHt\_obs_t = log\left(Q_t^H\,\frac{d\Gamma_t}{d\Gamma_t^X}\right)$$

$$PXt\_obs_t = log\left(d\Gamma_t\,\frac{\Pi I^X{}_t}{\Pi I^X{}_{t-1}}\right)$$

$$BEt\_obs_t = log\left(d\Gamma_t\,\frac{B_t^E}{B_{t-1}^E}\right)$$

$$BIt\_obs_t = log\left(d\Gamma_t\,\frac{B_t^I}{B_{t-1}^I}\right)$$

$$RLEt\_obs_t = log\left(\frac{R_t^{L,E}}{RLEs}\right)$$

$$RLIt\_obs_t = log\left(\frac{R_t^{L,I}}{RLIs}\right)$$

$$pieCt\_obs_t = log\left(\frac{\Pi^C{}_t}{\bar{\Pi C}}\right)$$

References

Andersen, A. L. and M. H. Rasmussen (2011). Potential Output in Denmark. *Danmarks Nationalbank Monetary Review 3rd quarter, part 2*.

Andrés, J. and Ó. Arce (2012). Banking Competition, Housing Prices and Macroeconomic Stability. *The Economic Journal 122*(565), 1346–1372.

Andrés, J., Ó. Arce, and C. Thomas (2013). Banking Competition, Collateral Constraints, and Optimal Monetary Policy. *Journal of Money, Credit and Banking 45*(s2), 87–125.

Andrés, J., J. E. Boscá, and J. Ferri (2015). Household Debt and Fiscal Multipliers. *Economica 82*(s1), 1048–1081.

Barsky, R. B., C. L. House, and M. S. Kimball (2007). Sticky-price models and durable goods. *American Economic Review 97*(3), 984.

Baxter, M. and R. G. King (1993). Fiscal policy in general equilibrium. *The American Economic Review*, 315–334.

Bernanke, B. S., M. Gertler, and S. Gilchrist (1999). The financial accelerator in a quantitative business cycle framework. *Handbook of macroeconomics 1*, 1341–1393.

Brooks, S. P. and A. Gelman (1998). General methods for monitoring convergence of iterative simulations. *Journal of computational and graphical statistics 7*(4), 434–455.

Burriel, P., J. Fernández-Villaverde, and J. F. Rubio-Ramírez (2010). MEDEA: a DSGE model for the Spanish economy. *SERIEs 1*(1-2), 175–243.

Calvo, G. A. (1983). Staggered prices in a utility-maximizing framework. *Journal of Monetary Economics 12*(3), 383–398.

Canova, F. and L. Sala (2009). Back to square one: identification issues in DSGE models. *Journal of Monetary Economics 56*(4), 431–449.

Carroll, C. D., M. Otsuka, and J. Slacalek (2011). How large are housing and financial wealth effects? A new approach. *Journal of Money, Credit and Banking 43*(1), 55–79.

Christiano, L. J., M. Eichenbaum, and C. L. Evans (2005). Nominal rigidities and the dynamic effects of a shock to monetary policy. *Journal of political Economy 113*(1), 1–45.

Christiano, L. J., R. Motto, and M. Rostagno (2014). Risk Shocks. *American Economic Review 104*(1), 27–65.

Christoffel, K., G. Coenen, and A. Warne (2008). The new area-wide model of the euro area-a micro-founded open-economy model for forecasting and policy analysis. *ECB Working Papers 944*.

Corsetti, G., K. Kuester, A. Meier, and G. J. Müller (2013). Sovereign risk, fiscal policy, and macroeconomic stability. *The Economic Journal 123*(566), F99–F132.

Dam, N. A. (2008). Konjunkturcykler i Danmark og Europa. *Nationaløkonomisk Tidsskrift 146*.

Davis, M. A. and J. Heathcote (2007). The price and quantity of residential land in the United States. *Journal of Monetary Economics 54*(8), 2595–2620.

DeJong, D. and C. Dave (2007). *Structural macroeconomics*. Princeton University Press Princeton.

Del Negro, M. and F. Schorfheide (2008). Forming priors for DSGE models (and how it affects the assessment of nominal rigidities). *Journal of Monetary Economics 55*(7), 1191–1208.

Dixit, A. K. and J. E. Stiglitz (1977). Monopolistic competition and optimum product diversity. *The American Economic Review*, 297–308.

Erceg, C. J., D. W. Henderson, and A. T. Levin (2000). Optimal monetary policy with staggered wage and price contracts. *Journal of Monetary Economics 46*(2), 281–313.

Fernández-Villaverde, J. (2010). The econometrics of DSGE models. *SERIEs 1*(1-2), 3–49.

Fisher, I. (1933). The debt-deflation theory of great depressions. *Econometrica: Journal of the Econometric Society*, 337–357.

Freixas, X., J.-C. Rochet, et al. (1997). *Microeconomics of banking*, Volume 2. MIT press Cambridge, MA.

Galí, J. (2009). *Monetary Policy, inflation, and the Business Cycle: An introduction to the new Keynesian Framework*. Princeton University Press.

Galí, J., F. Smets, and R. Wouters (2011). Unemployment in an estimated new keynesian model. *NBER Macroeconomics Annual*, 329–360.

Gerali, A., S. Neri, L. Sessa, and F. M. Signoretti (2010). Credit and Banking in a DSGE Model of the Euro Area. *Journal of Money, Credit and Banking 42*(s1), 107–141.

Grinderslev, D. and J. Smidt (2006). SMEC - Description and Model Properties. Technical report, The Danish Economic Councils.

Herbst, E. P. and F. Schorfheide (2015). *Bayesian Estimation of DSGE Models*. Princeton University Press.

Iacoviello, M. (2005). House prices, borrowing constraints, and monetary policy in the business cycle. *American economic review*, 739–764.

Iacoviello, M. and S. Neri (2010). Housing market spillovers: Evidence from an estimated DSGE model. *American Economic Journal: Macroeconomics*, 125–164.

Justiniano, A. and B. Preston (2010). Can structural small open-economy models account for the influence of foreign disturbances? *Journal of International Economics 81*(1), 61–74.

Justiniano, A., G. E. Primiceri, and A. Tambalotti (2013). Household leveraging and deleveraging. Technical report, National Bureau of Economic Research.

Kamps, C. (2004). *The dynamic macroeconomic effects of public capital: theory and evidence for OECD countries*. Springer.

Kiyotaki, N. and J. Moore (1997). Credit chains. *Journal of Political Economy 105*(21), 211–248.

Leeper, E. M., T. B. Walker, and S.-C. S. Yang (2010). Government investment and fiscal stimulus. *Journal of Monetary Economics 57*(8), 1000–1012.

Lindé, J., F. Smets, and R. Wouters (2015). Macro Models Used at Central Banks and New Challenges. Technical report, Working Paper, Handbook of Macroeconomics II.

Liu, Z., P. Wang, and T. Zha (2013). Land-Price Dynamics and Macroeconomic Fluctuations. *Econometrica 81*(3), 1147–1184.

Mountford, A. and H. Uhlig (2009). What are the effects of fiscal policy shocks? *Journal of applied econometrics 24*(6), 960–992.

Nationalbanken (2003). *MONA - en kvartalsmodel af dansk økonomi*. Nationalbanken.

og Byggestyrelsen, E. (2008). Input-output analyser for byggeriet. Technical report, Erhvervs- og Byggestyrelsen.

Pedersen, J. (2012). Fiscal Policy in Macroeconomic models. *Danmarks Nationalbank Monetary Review 3rd quarter, part 2*.

Pedersen, J. and S. r. H. Ravn (2013). What Drives the Business Cycle in a Small Open Economy? Evidence from an Estimated DSGE Model of the Danish Economy. *Danmarks Nationalbank Working Paper* (88).

Pedersen, J. and S. r. H. Ravn (2014). A Taylor Rule for Fiscal Policy in a Fixed Exchange Rate Regime. *Danmarks Nationalbank Working Paper* (90).

Quint, D. and P. Rabanal (2013). Monetary and macroprudential policy in an estimated DSGE model of the euro area. *International Monetary Fund* (13-209).

Ratto, M. and N. I. Iskrev (2011). Identification analysis of DSGE models with DYNARE. *MONFISPOL 225149*, 26.

Ravn, S. H. and M. Spange (2014). The Effects of Fiscal Policy in a Small Open Economy with a Fixed Exchange Rate. *Open Economies Review 25*(3), 451–476.

Salop, S. C. (1979). Monopolistic competition with outside goods. *The Bell Journal of Economics*, 141–156.

Schmitt-Grohé, S. and M. Uribe (2003). Closing small open economy models. *Journal of International Economics 61*(1), 163–185.

Uhlig, H. (2010). Some fiscal calculus. *The American Economic Review 100*(2), 30–34.

Woodford, M. (2003). *Interest and Prices*. Princeton University Press.