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REVISITING POTENTIAL OUTPUT IN DENMARK

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RESUME

Dette arbejdspapir giver en detaljeret beskrivelse af Danmarks Nationalbanks nuværende metode til at estimere potentiel produktion og produktionsgabet for dansk økonomi. Det overordnede metodeapparat svarer til den tidligere produktionsfunktionstilgang i Andersen og Rasmussen (2011), men de enkelte delmodeller for erhvervsfrekvens, ledighed og totalfaktorproduktivitet er revideret. For det første så indarbejder den nye modellering af den strukturelle erhvervsfrekvens eksplit demografi på tværs af aldersgrupper, studietilbøjelighed og en "discouraged worker" effekt. For det andet så anvendes der nu en løn-Phillips-kurve i estimationen af den strukturelle ledighed i stedet for lønkvoten, da sidstnævnte lejlighedsvis revideres betydeligt. Endelig opstilles en unobserved components model for TFP-gabet, som erstatter det tidligere udvidede HP-filter. I modellen anvendes den ledige kapacitet i industrien fortsat som indikator for cykliske udsving i TFP. Resultaterne peger på, at danske økonomi opererer meget tæt på sit konjunkturneutrale niveau i 2016. Den overordnede opfattelse af produktionsgabet er således uændret.

ABSTRACT

This paper provides a detailed description of Danmarks Nationalbank's current methodology for estimating potential output and output gaps in the Danish economy. Although the general framework is similar to the previous production function approach in Andersen and Rasmussen (2011), each of the individual sub-models for the participation rate, unemployment and total factor productivity have been revised. First, the new approach to modelling the structural participation rate explicitly incorporates demographic variation across age groups, student propensity and a discouraged worker effect. Second, the estimation of structural unemployment now uses a wage Phillips curve rather than the wage share, since the latter is prone to substantial data revisions. Finally, an unobserved components model based on the degree of spare capacity in manufacturing is formulated for the total factor productivity (TFP) gap, thereby replacing the former extended HP filter. The results suggest that the Danish economy is operating very close to its cyclically neutral capacity in 2016. The overall view on the output gap is thus unchanged.

REVISITING POTENTIAL OUTPUT IN DENMARK

In economic theory, the output gap generally refers to the difference between actual output and potential output, i.e. the maximum production of goods and services that could be achieved while maintaining price stability over the medium term.[1] It is a notion constructed in part to evaluate current production capacity utilization and formulate policy prescriptions accordingly. This is important in a fixed exchange rate regime where only fiscal and structural policies can curb the build-up of macroeconomic imbalances. The estimation of the output gap thus plays a central role in Danmarks Nationalbank's assessment of the cyclical stance of the Danish economy.

Potential output is by nature an unobserved concept which is determined by the aggregate production capability of the economy. It generally depends on the number of people available to work, the number of hours they put in, the amount of buildings and machinery as well as the efficiency and intensity with which these production factors are used. When the economy is affected by demand shocks, the presence of rigidities may cause actual production to temporarily deviate from its potential due to the sluggish adjustment of wages and prices.

POTENTIAL OUTPUT AND OUTPUT GAPS ARE ESTIMATED USING THE PRODUCTION FUNCTION APPROACH

To quantify the size of the output gap, we revisit Danmarks Nationalbank's existing production function methodology for estimating potential GDP, cf. Andersen and Rasmussen (2011). In the production function approach, the output gap is determined by separate gaps for each of the input factors used in the production of goods and services (capital stock and labour) and their (combined) productivity, total factor productivity (TFP). The production function follows a standard Cobb-Douglas specification where the population size and the capital stock are both assumed to be structural. The output gap can thus be calculated as the sum of the total factor productivity gap and the employment gap multiplied by a constant wage share, cf. chart 1. Moreover, the employment gap can be further decomposed into contributions from the participation rate gap and the unemployment rate gap.[2]

The production function approach makes it possible to decompose the growth rate of potential output into the growth of the input factors. It can thus provide an economic interpretation of the developments in potential output as well as the output gap. This insight cannot be inferred by purely statistical methods.

A SHORT OVERVIEW OF THE METHODOLOGICAL CHANGES

While the overall methodological framework is similar to Danmarks Nationalbank's existing production function approach, the individual sub-models for TFP, unemployment and labour market participation have been revised. Most importantly, a new approach to estimating the structural participation rate is implemented which accounts for the impact of demographics as well as an increased propensity to study among younger generations. In order to capture a discouraged worker effect, i.e. the tendency for eligible workers to suspend their job search during periods with inadequate employment options, the unemployment gap is also included in the model for the participation rate gap. Moreover, the model for the unemployment gap has been made more robust to data revisions by relying on signals from real wages rather than the wage share in the non-primary private sector. Finally, a full model is now formulated for structural total factor productivity, thus replacing the previous extended HP filter approach.

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[1] The concept of potential output is not unambiguous. In New Keynesian DSGE models, it generally refers to the level of output that would prevail in the absence of nominal price and wage rigidities, cf. Vetlov et. al. (2011).

[2] See chapter 3 in Andersen and Rasmussen (2011) for a more detailed introduction to the production function method.

Illustration of the production function methodology Chart 1

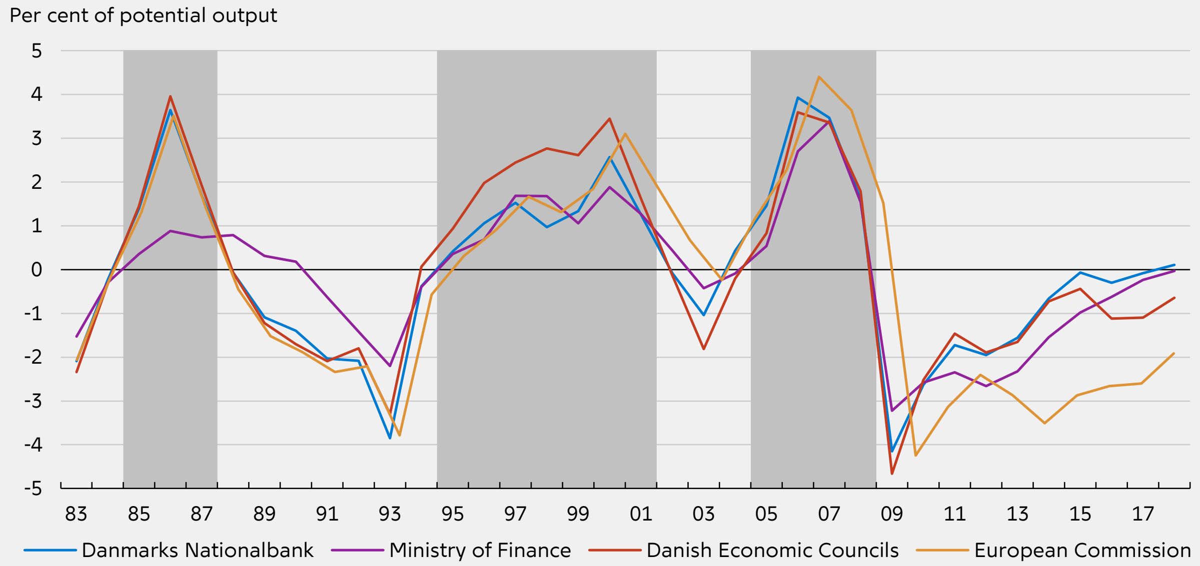
Note: The output gap is the sum of the total factor productivity gap and the employment gap multiplied by a wage share parameter which is set equal to the average in the private non-farm sector of 61 per cent since 1980. The employment gap can be approximated as the difference between the participation rate gap and the unemployment rate gap. Hours worked enter TFP, and the population as well as the capital stock are assumed to be structural.
Source: Danmarks Nationalbank.

OUTPUT GAPS ARE SUBJECT TO CONSIDERABLE UNCERTAINTY

The estimation of output gaps is associated with considerable uncertainty, particularly in terms of their real-time reliability and robustness to subsequent revisions. This implies that caution should be exercised when drawing conclusions about the exact size of the cyclical gap. An assessment of the cyclical stance of the economy should thus be based on a full set of business cycle indicators.

Several other economic institutions, e.g. the Danish Economic Councils, the Ministry of Finance, and the European Commission, also calculate output gaps for Denmark based on the production function method. These estimates broadly agree that the economic boom periods in the Danish economy peaked around 1986, 2000 and 2006, cf. chart 2. At the same time, the comtemporary output gap estimates exhibit considerable heterogeneity across institutions, thus illustrating the real-time reliability issue. Following a significant setback after the financial crisis, the output gap now appears to be closing, notably after the national accounts revision in November 2016 which implied a sizeable upward revision to GDP.

Output gaps for the Danish economy 1983-2018 Chart 2



Note: Shaded areas indicate periods where all output gap estimates are positive. The output gap from the European Commission is estimated before the revision of the national accounts in November 2016 which implied a substantial upwards revision of actual GDP.
Source: Danmarks Nationalbank, Ministry of Finance, Danish Economic Councils and the European Commission.

STRUCTURAL UNEMPLOYMENT

Structural (or natural) unemployment and the related unemployment gap are key components in the determination of potential output and the output gap. The unemployment gap itself serves as a central indicator for the labour market, and it enters as an indicator in the estimation of the trend participation rate.

STRUCTURAL UNEMPLOYMENT REFERS TO THE LEVEL OF UNEMPLOYMENT THAT IS CONSISTENT WITH STABLE WAGE AND PRICE GROWTH

The theory of a structural level of unemployment is a central element in understanding the labour market as well as the relationship between unemployment, wages and prices. This reflects the predominant view that while a short run trade-off between unemployment and inflation (when unemployment is low, inflation is high and vice versa) may exist, no such trade-off exits over a longer horizon. A low level of unemployment will, over time, not only lead to high but also rising inflation. Such persistent increases in inflation are not sustainable and will ultimately bring about higher unemployment. This is why structural unemployment is often defined as the level of unemployment consistent with stable inflation or the Non-Accelerating Inflation Rate of Unemployment (NAIRU), see for example Ball and Mankiw (2002). Structural unemployment arises for several reasons and reflects, inter alia, institutional features of the economy and the labour market, cf. box 1.

What is structural unemployment? Box 1

Structural unemployment arises for several reasons and reflects structural and institutional factors of the economy and the labour market. This can for instance be the presence of minimum wages, the level and accessibility of unemployment benefits, and employment legislation.

Part of the structural unemployment is frictional unemployment, i.e. unemployment arising from rigidities in searching for jobs and matching the supply and demand of labour. Frictional unemployment reflects the frequent but short unemployment spells workers have in the transition between jobs but also when entering and exiting the labour force. Factors such as the availability of job information and the degree of geographical mobility of labour determine the level of frictional unemployment.

Another part of structural unemployment is unemployment arising from a current mismatch between the demand and the supply of labour. Often this type of unemployment reflects a mismatch between the type of skills and education of the current unemployed persons and the skills companies are looking for. This type of structural unemployment is not of permanent character and can increase especially in relation to technological shocks that disrupt existing industries and create new ones.

The structural level is not fixed and may change over time in line with changes to the abovementioned factors. Many factors may potentially affect structural unemployment, and there is considerable uncertainty as to the precise causes that determine the level and development.

ESTIMATION OF STRUCTURAL UNEMPLOYMENT IS BASED ON PRICE AND WAGE DEVELOPMENTS

In this paper, structural unemployment is estimated on the basis of a Phillips curve that states a negative relation between the unemployment gap and the change in inflation in the short run: Rising inflation indicates, ceteris paribus, that unemployment is below its structural level, i.e. a negative unemployment gap, whereas falling inflation indicates a positive unemployment gap. Over a longer horizon, no such trade-off exits and structural unemployment can thus be defined as the level of unemployment at which inflation is constant.

Headline inflation in Denmark will, however, not necessarily yield a reliable signal of structural unemployment. As Denmark has a small open economy, Danish price developments are very much affected by price developments abroad and fluctuations in volatile energy and food prices.

Therefore, we use the development in wages to signal the magnitude of the pressure stemming from unemployment in the estimation of structural unemployment. Situations with labour market pressures and unemployment below the structural level tend to push up wage growth and vice versa when unemployment is above the structural level. Nevertheless, wage growth is not necessarily a signal of labour market pressure either. In a long-run perspecive, increases in real wages reflect labour productivity growth. Consequently, high growth in real wages does not necessarily indicate strong imbalances, provided that it is matched by productivity growth. We therefore use real wages controlling for productivity and terms of trade as well as short term fluctuations in commodity prices and the effective exchange rate, as formulated in the following relation:

$$\Delta w_t - \Delta p_t = \alpha_0 + \sum_{i=1}^{l} \alpha_i (\Delta w_{t-i} - \Delta p_{t-i}) + x_t' \beta + \gamma u_t^c + \varepsilon_t \quad (1)$$

where w_t and p_t is the logarithm of manufacturing wages and consumer prices, respectively, u_t^c is the unemployment gap, i.e. the deviation between actual and structural unemployment, and x_t is the vector of control variables (consisting of growth in average labour productivity in the private non-agricultural sector, changes in terms of trade, commodity price inflation, and changes in the effective exchange rate). The control variables are expressed as deviations from their respective sample means and are included both contemporaneously and with one and two lags. The error term ε_t is assumed to be normally distributed with a mean of zero. Formally, the model is estimated as an unobserved components model, cf. box 2.

Structural levels and cyclical gaps estimated using an unobserved components model BoksBox 2

The actual series for unemployment, the participation rate and total factor productivity are decomposed into structural and cyclical components using three seperate unobserved components models, UCM. In the UCM framework, the paths of unobservable variables are pinned down by imposing relations between them and the observable series. These relations are typically founded in economic theory. In the case of unemployment, for example, a Phillips curve is often used to decompose the observed series for unemployment into a structural and cyclical component by extracting signals from price or wage growth. Furthermore, the dynamics of the unobservable variables follow stochastic processes which in combination with the specified relationships between the observed and unobserved variables form the full system. Formally, the model can be described by a linear Gaussian state-space model of the following form:[A]

$$\xi_{t+1} = F\xi_t + v_{t+1} \quad \text{(A)}$$

$$Y_t = A'X_t + H'\xi_t + w_t \quad \text{(B)}$$

where equation (A) is known as the state equation which specifies the dynamics of the $rx1$ unobservable variables in the vector ξ_t. Let Y_t denote a $nx1$ vector observed at date t=1,…T, equation (B) is then the observation equation that contains the relation between the observable and unobservable variables. X_t is a k$x1$ vector of exogenous variables which may also influence the observable series. The error terms v_t and w_t are uncorrelated white noise components with fixed covariance matrices. The econometric task is to estimate the parameters in the matrices F, A and H, while simultanouesly finding values for the unobservable series of interest. In praxis, this is done by applying the Kalman filter to evaluate the likelihood function of the model. The Kalman filter works on the principle that an initial guess about the unobservable states is updated based on the difference between the model's prediction of the observable variables and their actual realizations. The state space models for unemployment, the participation rate and total factor productivity can be found in the appendix.

A. See chapter 13 in Hamilton (1994) for a detailed treatment of state space models and Kalman filtering.

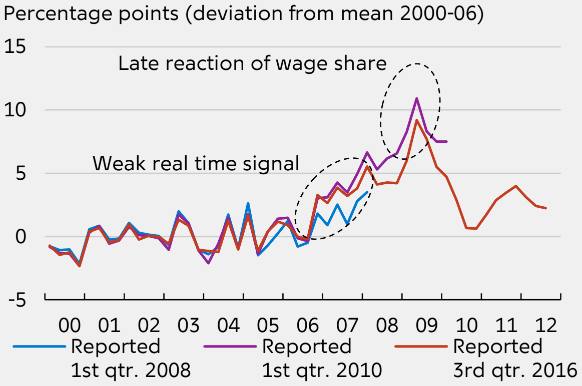
Equation (1) is closely related to the method in Andersen and Rasmussen (2011) who estimate structural unemployment on the basis of the wage share in the private non-agricultural sector, i.e. the ratio of total labour compensation to nominal gross value added in nominal terms. The wage share is, however, prone to frequent and sometimes substantial revisions of the national accounts, which may affect the estimation significantly, cf. box 3.

Revisions of the wage share can be substantial

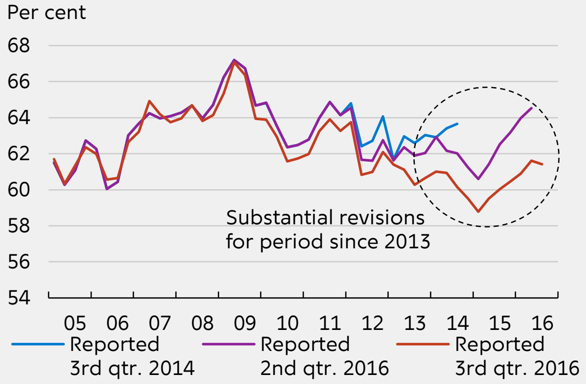
Box 3

The 2008 vintage of the national accounts originally showed that the wage share had been increasing since the beginning of 2005, although not neccesarily in a way that would suggest significant labour market pressure, cf. chart (left). After the outbreak of the financial crisis, however, the revised 2010 vintage pointed to a more pronounced increase in the wage share, indicating greater labour market tightness than previously observed. Today, cyclical movements in the wage share are now evident for the period 2005-10, even if the real time signal was less clear. Such revisions are quite frequent and the wage share has also been subject to substantial revisions in recent years, cf. chart (right). Moreover, the wage share tends to respond late to a tightening of labour market conditions, as firms often adjust their employment decisions sluggishly in response to changing demand conditions.

Wage share
Boom bust period 2006-09



Wage share
Period since 2013



Note: For comparison due to a level revision the wage shares are shown as deviation from the mean 2000 to 2006 in left chart. The charts show the wage share in the private non-agricultural sector from the flash estimate of national accounts.
Source: Statistics Denmark.

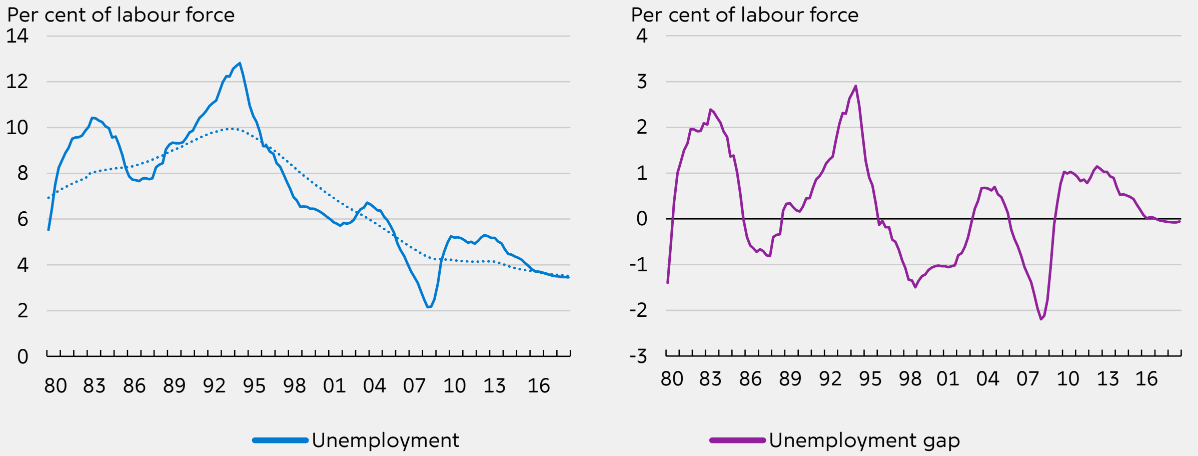
ESTIMATION RESULTS

Current gross unemployment is estimated to be close to its structural level, which is around 3.7 per cent of the labour force[3], corresponding to approximately 112,000 persons, cf. chart 3 (left).[4] The unemployment gap is virtually closed at the current stage, cf. chart 3 (right). Over the past five years structural unemployment has declined somewhat and the estimation finds no evidence of a significant increase in structural unemployment following the crisis in 2008-09. Hence, there is no clear sign of a hysteresis effect.

The marked decline in structural unemployment in the mid-1990s is driven by a series of labour market reforms which, inter alia, reduced the entitlement period for unemployment benefits and implemented more active labour market policies. The results are broadly in line with Danmarks Nationalbank's previous methodology for computing structural unemployment, cf. box 4.

3 Our definition of the labour force diverges from the one used by Statistics Denmark and the unemployment rate is thus not identical to the one reported by Statistics Denmark in their press releases.
4 The estimation is based on quarterly data from 1975 to 3rd quarter 2016.

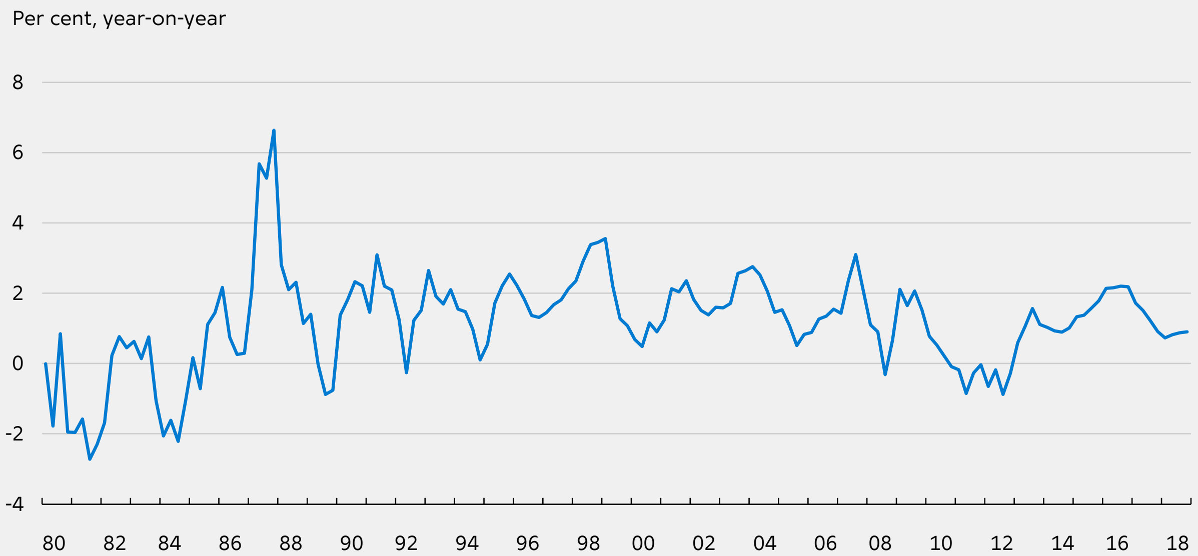
Actual and structural unemployment rate and the unemployment gap Chart 3



Note: The dotted line denotes structural unemployment. The estimation is based on quarterly data since 1975 till 3rd quarter 2016. From 4th quarter 2016 and forward the structural participation rates are determined by filtration.
Source: Statistics Denmark and own calculations.

The unemployment gap is smaller in 1986-87 than the increase in real wages would otherwise suggest, cf. chart 4. This may be related to the quite strong increase in commodity prices around that period. Additionally, chart 4 reveals a rather limited response in the real wage growth in the mid-2000s despite an overheating of the labour market, thus illustrating that the wage signal can be quite weak in real time. More generally, a flattening of the Phillips curve is seen across many countries, and it may, to some extent, be explained by increased global competition in goods and labour markets, cf. Borio and Filardo (2007). A firmer anchoring of inflation expectations also provides a part of the explanation.

Changes in real wages Chart 4

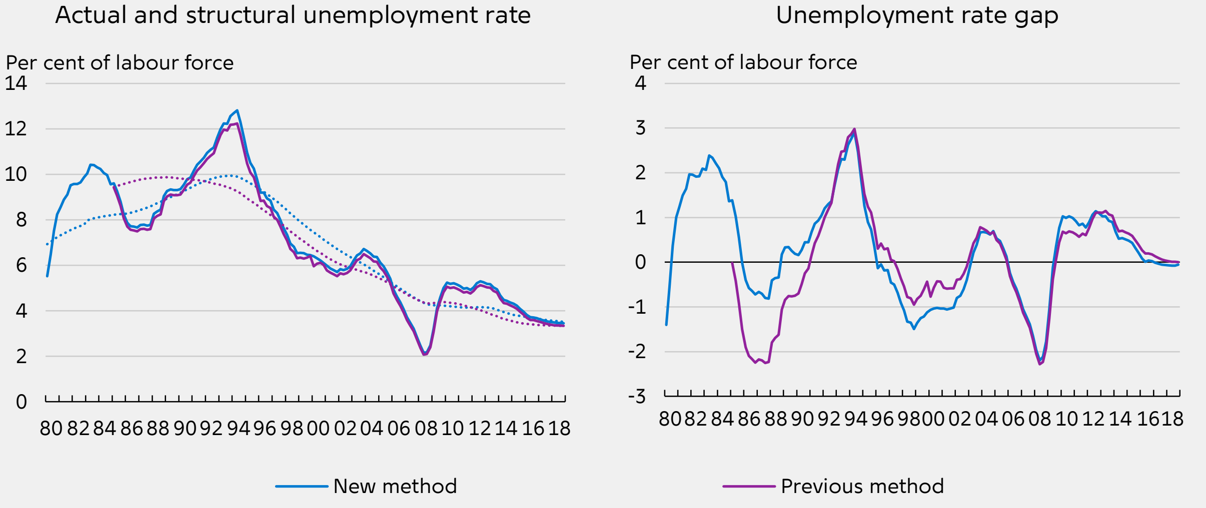


Source: Statistics Denmark.

Comparing the unemployment gap to the previous method Box 4

Our results generally identify the same cyclical movements in the unemployment rate as Andersen and Rasmussen (2011), and the two unemployment gaps basically coincide after 2002. In the late eighties, however, they differ substantially. This is partly related to the different starting points for the estimation periods, and, to a lesser extent, also the combined effect from volatile commodity prices and a reversed sign in the estimated coefficient for this control variable. Such differences illustrate the uncertainty with respect to modelling assumptions that are embedded in the estimations of unobservable variables.

Actual and structural unemployment rate and cyclical component of the unemployment rate



Note: Previous method refers to the model described in Andersen and Rasmussen 2011. Differences in actual unemployment in the chart to the left reflect a new definition of labour force (excluding employed persons on leave). Dotted lines indicate structural series, whereas the full lines refer to actual series.

Source: Statistics Denmark and own calculations.

The estimation finds a significant negative relation between the change in the real wage and the estimated unemployment gap, cf. table 1. An increase in the unemployment gap by 1 percentage point is, all other things being equal, associated with a drop in annual real wage growth of a little over 0.2 percentage point. For productivity, a 1 percentage point increase in average labour productivity is associated with an average rise in the real wage of close to 0.2 percentage point. The impact of an improvement in the terms of trade is of the same magnitude. These two effects reflect a limited initial reaction in the growth rate of real wages, which increases less than one-for-one with the growth rate of productivity and terms of trade. Higher commodity price inflation causes real wage growth to decline, but with a limited impact. This might reflect that a part of the commodity price development is captured in the import prices and thus the terms of trade, but also that fluctuations in commodity prices can be quite large. The change in real wage growth can be interpreted as inertia in wage setting, and the coefficient also shows some persistency.

Estimated coefficients					Table 1
Variable	Coefficient	Standard error	Variable	Coefficient	Standard error
Unemployment gap (t)	-0.22	(0.11)	Δ Effective krone rate (t)	0.01	(0.05)
Productivity growth (t)	0.17	(0.05)	Δ Effective krone rate (t-1)	0.17	(0.05)
Productivity growth (t-1)	-0.01	(0.06)	Terms of trade (t)	0.17	(0.06)
Productivity growth (t-2)	-0.03	(0.05)	Terms of trade (t-1)	-0.09	(0.07)
Commodity price inflation (t)	-0.02	(0.01)	Terms of trade (t-2)	-0.04	(0.05)
Commodity price inflation (t-1)	0.01	(0.01)	Δ Real wage (t-2)	0.47	(0.07)
			Constant	0.01	(0.00)

THE PARTICIPATION RATE

The employment potential is not only made up of people registered as unemployed. People outside the labour force also constitute a labour reserve for hiring firms.[5] Hence, as demand for labour rises during an economic expansion, it will not only result in a decreasing number of unemployed persons, also self-supporting unemployed, job seeking students and people living abroad will find work and enter the labour force. As a result, the participation rate fluctuates with the business cycle.

Cyclical movements in the participation rate are particularly pronounced for the age groups below 50 years of age, cf. chart 5 (left). This may reflect several factors. First, a high proportion of foreign labour employed in Denmark is below 50 years of age.[6] Second, for the 16-29-year-olds it also reflects that a higher proportion of the age group is in education compared with other age groups. Hence, economic fluctuations will to a greater extent be reflected in movements in and out of the workforce for these age groups.

Conversely, the participation rate of the older age groups is more stable over the business cycle, cf. chart 5 (right). Their labour market participation is to a larger degree determined by changes in the legislation of retirement over time. For example the introduction of the transitional allowance (overgangsydelse) in the mid-1990s and the subsequent abolition contributed to significant movements in the participation rate for the 50-59-year-olds. Another example is the number of retirement reforms since 1998 that have raised participation rates of the 60-66-year-olds.[7] As a result of health disparities and the possibility of retirement, the labour market attachment is considerably lower for the older age groups. Due to the different levels and cyclical developments of the participation rate across age groups, changes in demographics affect the structures of the labour market and the business cycle sensitivity of the aggregate participation rate.

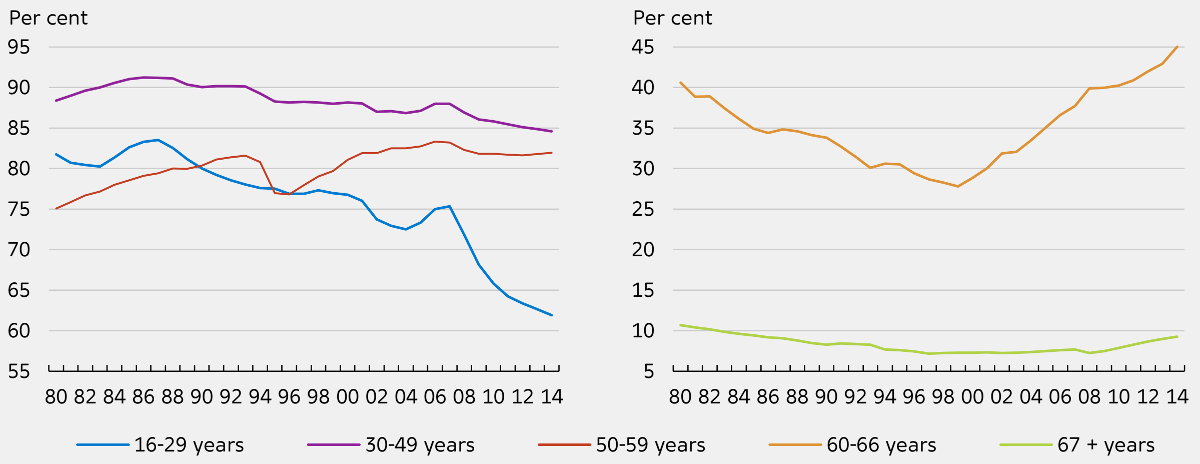
5 The labour force is defined as the sum of employed persons (excl. persons on leave, etc.) and register-based gross unemployment deducted people in subsidized employment schemes. People in subsidized employment are part of both employment in the national accounts and gross unemployment, hence they are deducted from gross unemployment in order to avoid double counting. Employment is measured by the number of persons, while unemployment is measured in full-time equivalents.

6 85 per cent of foreign payroll employment is between 16 and 49 years of age, cf. www.jobindsats.dk.

7 Examples are the 1998 early retirement reform, which, inter alia, increased the share of self-financing and tightened the offsetting effect of private pension savings, and the 2006 welfare agreement and the 2011 retirement reform, which gradually raises the early retirement age from 60 to 62 years over the period 2014-17.

Labour force participation rate across age groups

Chart 5



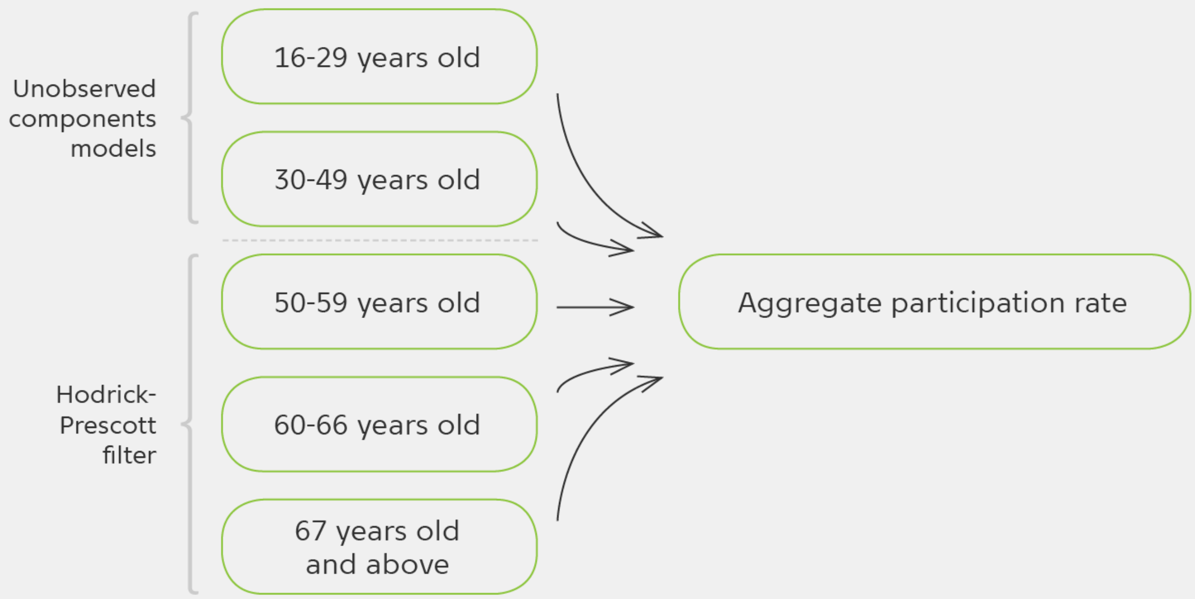
Note: Register-based labour force statistics (RAS).
Source: Statistics Denmark.

ESTIMATING THE CYCLICAL COMPONENT OF THE PARTICIPATION RATE

In order to capture demographic developments in the estimate of the participation rate gap, the labour force is divided into five age groups that are treated separately, cf. chart 6.[8] For the 16-29-year-olds and the 30-49-year-olds, the structural participation rate and the cyclical component are determined by means of unobserved components models, while the trend components of the groups above 49 years of age are determined using HP filters as there is little evidence of cyclical movements.

Illustration of estimation approach

Chart 6



8 The labour force is divided into age groups using register-based labour force statistics (RAS).

The models for the 16-29- and the 30-49-year-olds are largely similar. They are, however, specified individually in order to address age-specific conditions. Both models consist of two observation equations (equation (2) and (5)) and two state equations (equation (3) and (4)) presented below. Equation (2) denotes that the participation rate, E_t, can be decomposed into a structural component, E_t^*, and a cyclical component, E_t^c.

$$E_t = E_t^* + E_t^c \tag{2}$$

The development in the structural component is described by equation (3.1) for the 16-29-year-olds and (3.2) for the 30-49-year-olds. The structural participation rate is specified as a random walk. The trend is, however, also affected by the development in the observable variables $\Delta stud$ and $\Delta leave1629$ for the 16-29-year-olds and $\Delta leave3049$ for the 30-49-year-olds. The $\Delta leave$-variables denotes the change in the number of participants in leave schemes etc.[9] and are included in order to explicitly take into account the impact of a number of labour market policy measures. The variable $\Delta stud1629$ captures the changes in young people's propensity to pursue an education. The coefficient on $\Delta stud1629$ is set at -0.4, reflecting that on average around 60 per cent of students are employed.

16-29-year-olds: $$E_t^* = E_{t-1}^* + \mu_1 \Delta leave1629_t + \mu_2 \Delta stud_t + \varepsilon_t^* \tag{3.1}$$
30-49-year-olds: $$E_t^* = E_{t-1}^* + \mu_1 \Delta leave3049_t + \varepsilon_t^* \tag{3.2}$$

The cyclical component of the participation rate is described by equation (4.1) and (4.2) for the 16-29-year-olds and 30-49-year-olds, respectively. As persons who leave the labour force temporarily due to lack of job opportunities may become engaged in other activities, e.g. studying, there might be some persistence in the cyclical component of the participation rate. This persistence is captured by the lagged values of E^c on the right-hand side of the equation. In order to capture a "discouraged worker" effect, the unemployment gap effect is also included on the right-hand side of the equation for the 30-49-year-olds. It was not possible to identify a similar effect for the 16-29-year-olds, hence the variable is left out.

16-29-year-olds: $$E_t^c = \psi_1 E_{t-1}^c + \psi_2 E_{t-2}^c + \varepsilon_t^c \tag{4.1}$$
30-49-year-olds: $$E_t^c = \psi_1 E_{t-1}^c + \psi_2 E_{t-2}^c + \theta u_{t-1}^c + \varepsilon_t^c \tag{4.2}$$

Finally, equation (5.1) and (5.2) are the central observation equations for the 16-29-year-olds and 30-49-year-olds respectively. They link the share of manufacturing firms reporting a shortage of labour from Statistics Denmark's business sentiment surveys to the participation rate gap. It is expected that when the participation rate is above its structural level, the spare capacity in the labour market will be limited, and more firms will report a higher degree of labour shortage. For the 16-29-year-olds the indicator is lagged by 2 quarters reflecting that the influx of labour from outside the labour force may react slowly to the increased pressure on the labour market. The error terms ε_t^*, ε_t^c and $\varepsilon_t^{shortage}$ are all assumed to be normally, independently distributed with a mean of zero.

16-29-year-olds: $$shortage_{t-2} = \eta_0 + \eta_1 E_t^c + \varepsilon_t^{shortage} \tag{5.1}$$
30-49-year-olds: $$shortage_t = \eta_0 + \eta_1 E_t^c + \varepsilon_t^{shortage} \tag{5.2}$$

9 The variable leave etc. is taken from the national accounts. It consists of people with maternity benefits, sickness benefits as well the leave schemes for childminding, education and sabbaticals which were implemented during the mid-1990s and subsequently phased out.

ESTIMATION RESULTS

The models are estimated using quarterly data for the period 1st quarter 1983 to 3rd quarter 2016 and the Kalman filter. The estimated parameters all have the expected signs, cf. table 2. The $\Delta leave3049$ coefficient is estimated at -0.47, reflecting that 47 per cent of the change in the variable is counted as structural changes in the labour force. For the 16-29-year–olds the coefficient is smaller and statistically insignificant. In accordance with a discouraged worker effect, the coefficient of the unemployment gap has a negative sign. The coefficient is, however, also statistically insignificant. This might reflect collinearity between the shortage of labour indicator and the unemployment gap. The positive relationship between the shortage of labour in manufacturing and the estimated cyclical component of the participation rate is reflected in positive and statistically significant coefficient on the cyclical component of the participation rate.

Parameter estimates in the models for structural participation rate Tabele 2

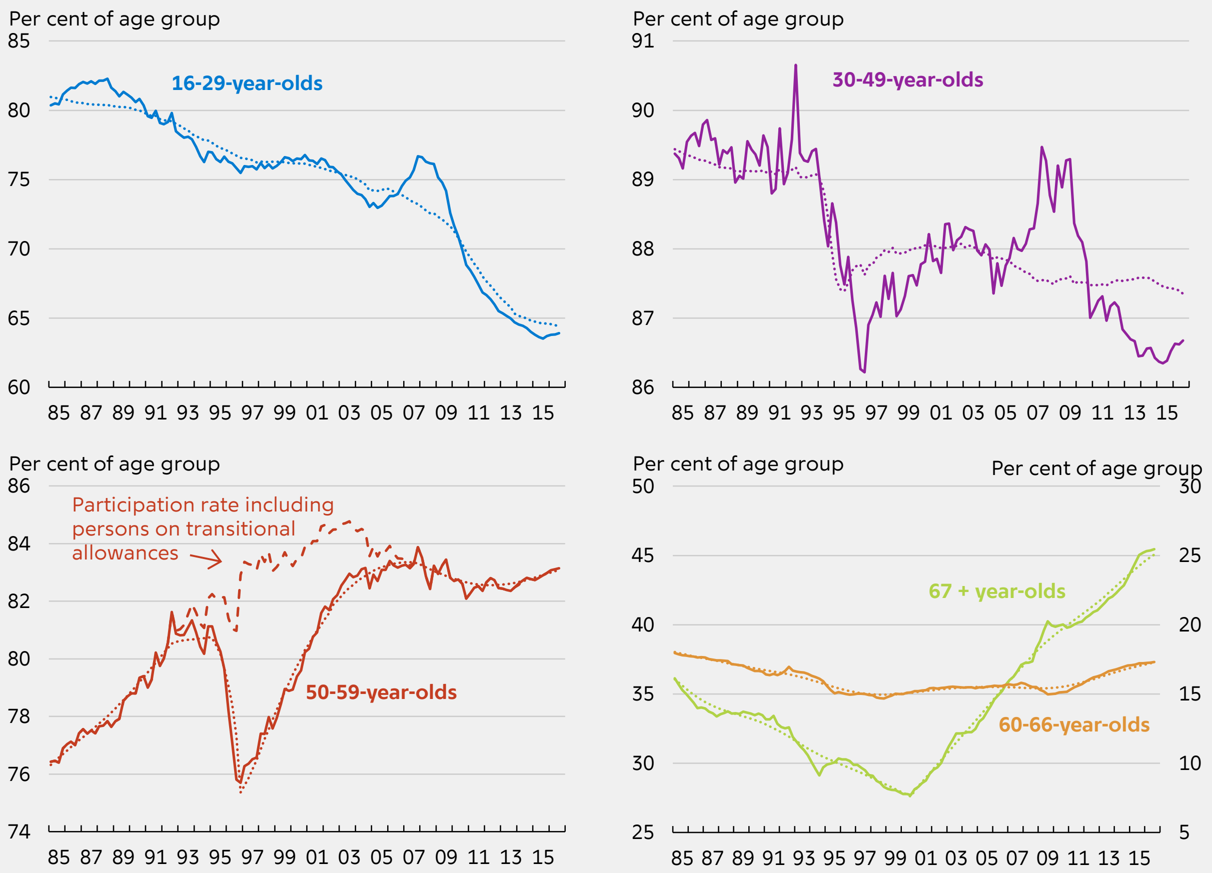
	16-29-year-olds		30-49-year-olds	
Parameter	Estimate	(Standard error)	Estimate	(Standard error)
Coefficients:				
μ_1: $\Delta leave$	-0.11	(0.34)	-0.48	(0.16)
μ_2: $\Delta stud$	-0.40			
ψ_1: $Lagged\ participation\ rate\ gap$	1.18	(0.12)	0.82	(0.09)
ψ_2: $Lagged\ participation\ rate\ gap$	-0.24	(0.12)	0.06	(0.09)
θ: $Unemployment\ gap$			-2.32	(2.85)
η_0: $Constant$	0.02	(0.01)	0.02	(0.00)
η_1: $Labor\ shortage$	0.01	(0.00)	0.01	(0.00)
Variance parameters:				
σ^*	0.05		0.01	
σ^c	0.42	(0.04)	0.34	(0.02)
$\sigma^{shortage}$	0.02	(0,00)	0.02	(0.00)

Source: Own calculations.

The structural participation rate for the 16-29-year-olds has dropped considerably since the mid-1980s, particularly in the wake of the financial crisis in 2008-09, cf. chart 7. This reflects an increased propensity to study among the younger age group. For the 30-49-year–olds an increased number of persons admitted to the labour market leave schemes in the mid-1990s reduced the structural participation rate. During the upswing in the mid-2000s and the subsequent recession both age groups shows a significant cyclical contribution from persons outside the labour force. As noted above, the trend component of the participation rate is determined using HP filters for persons above 49 years of age.[10] Overall the cyclical fluctuations in participation for the older age groups are limited.

[10] The smoothing parameter is set to 1600. In order to capture structural movements due to the introduction and the subsequent abolition of the transitional allowance scheme in the mid-1990s, a participation rate including persons receiving transitional allowance is HP filtered for the 50-59-year-olds. The structural participation rate of the 50-59-year-olds is then determined by subtracting persons on transitional allowance from the smoothed series, reflecting the assumption that each person admitted to the labour market scheme reduces the structural labour force 1:1 and vice versa.

Actual and structural participation rate for five age groups Chart 7



Note: The dotted line denotes the structural participation rate. The models are estimated for the period 1st quarter 1983 to 3rd quarter 2016.
Source: Own calculations.

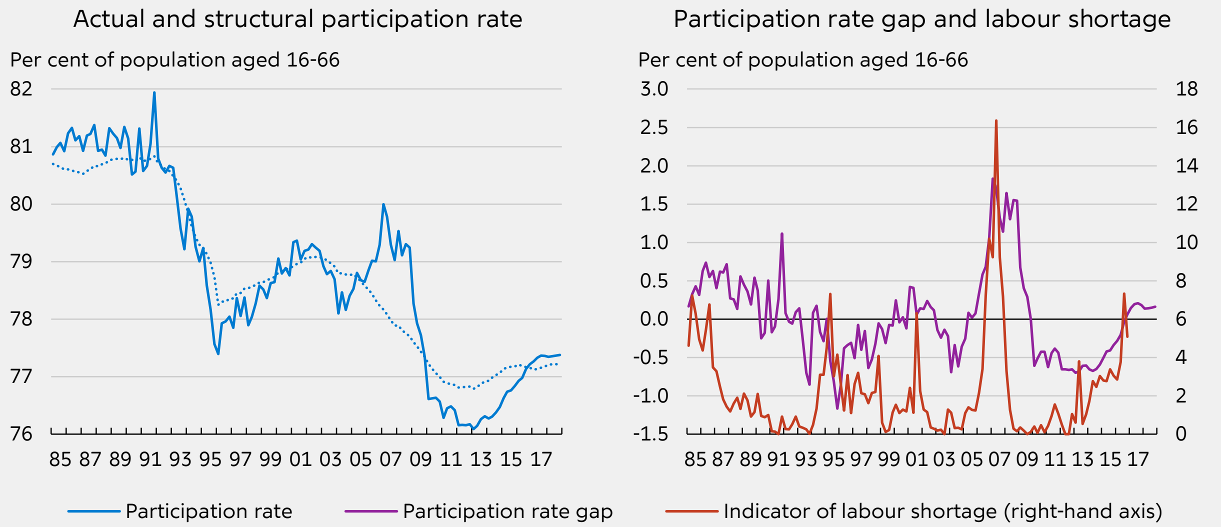
The aggregate structural participation rate has declined substantially since the beginning of the 1990s, cf. chart 8 (left). The decline is mainly attributable to two periods. Firstly, in the mid-1990s the introduction of a number of labour market policy measures resulted in a significant drop in the structural participation rate. Secondly, in the mid-2000s a combination of an aging population and an increasing propensity to study among the younger age group contributed to a declining structural participation rate[11].

In accordance with the labour shortage indicator, the participation rate shows a significant cyclical contribution during the boom in the mid-2000s and the subsequent downturn, cf. Chart 8 (right). The indicator is capped in the sense that it measures only the strength of the labour shortage and not the opposite situation with no shortage of labour. Hence there might be a risk of underestimating the strength of downturns and vice versa the strength of upswings. However, over the period from 1985 to today, the participation rate gap averages around zero. The difference between the actual and structural participation rate is assessed at -0.1 percentage points in 2016, which corresponds to around 5,000 persons outside the labour force. The labour force gap is primarily attributable to people in the group of 16-49–year-olds. The results are overall in line with the previous method, cf. box 5.

[11] The age composition of the work force is one of many demographic factors that may influence the structural participation rate. A large inflow of of immigrants and asylum seekers may for example also impact the average participation rate if their attachment to the labour market deviates from the existing population.

Actual and structural participation rate, cyclical component of the participation rate and labour shortage in manufacturing industry

Chart 8



Note: The participation rate is calculated as the number of persons in the labour force divided by the number of persons in the population in the group of 16-66-year-olds. The indicator of labour shortage indicates the share of firms in manufacturing industry, weighted by number of employees, reporting labour shortage as a production constraint. The indicator is seasonally adjusted.

Source: Own calculations.

LABOUR MARKET RESERVES ABROAD

The potential inflow of foreign workers makes up an important labour reserve for the Danish economy. The foreign reserve increased considerably with the EU enlargements in 2004 and 2007. During the boom in the mid-2000s, foreign labour proved to have a high degree of mobility and complemented the Danish workforce as demand for labour increased. The inflow of foreign labour may have contributed to reducing bottlenecks, dampen wage pressures and thereby support the upswing in the Danish economy.

The high mobility of international labour seems to continue during the current upswing, cf. Danielsen and Jørgensen (2015A). In recent years foreign labour has contributed to higher employment in a period where the number of Danish citizens of working age has declined. Without the influx of foreign labour in the mid-2000s and today, the shortage of labour would presumably be more pronounced.

However, determining the available reserves of labour abroad is complicated as it depends on a large number of factors that are difficult to model, cf. Danielsen and Jørgensen (2015B). These factors include the wage and working conditions on foreign labour markets as well as institutions, cultural and language barriers, etc. The estimation may capture some of the effects of the EU enlargement, given that the increased amount of available foreign labour is captured by the labour shortage indicator.

The general picture of the developments in the participation rate gap has not changed compared to the previous method

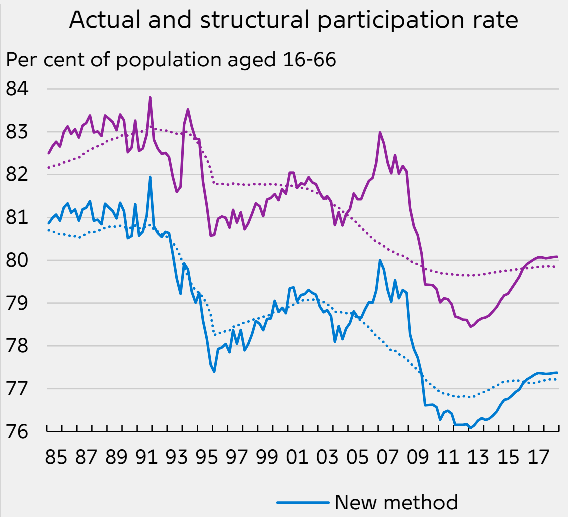
Box 5

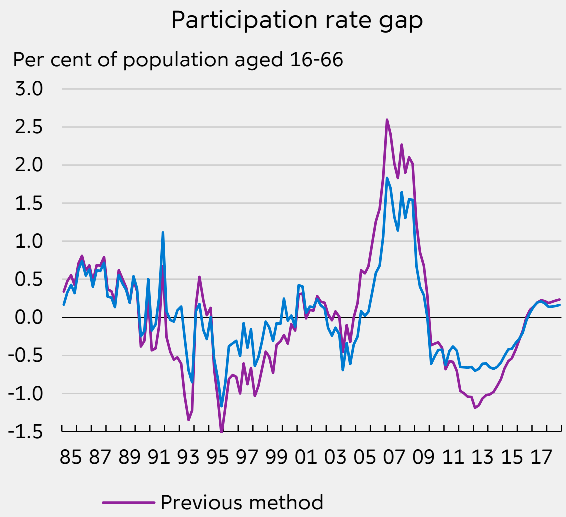
The estimate of the aggregated structural participation rate is generally in line with the estimated series of the previous method from the mid-1980s and forward, cf. chart (left).[A] Given the similarities between the methods, this is expected. However, both the actual and the structural participation rate are subject to a larger decline in the mid-1990s with the new method. This partly reflects that the workforce no longer includes people on leave etc. Hence, the inflows to labour market leave schemes in the mid-1990s drag down the potential workforce to a larger extent than previously. Subsequently, the gradual abolition of the leave schemes, including the transitional allowance, increases the structural participation rate with the new method of estimation.

In the early 2000s, the structural participation rate starts to decline. In the previous method, this is partly driven by an increasing proportion of 60-66-year-olds in the total working age population.[B] However, the method does not take into account that the participation rate of this age group increased during the period. The new method captures such changes in demographics. Here the decline of the structural participation rate is to a larger degree driven by an increased propensity to study among 16-29-year-olds, while the increasing labour force participation among the elderly all else equal lifts the structural participation rate.

The cyclical gap in the participation rate over the boom in the mid-2000s and during the subsequent downturn is narrowed slightly with the new method compared to the previous, cf. Chart (right). Part of this reflects that the cyclicality of persons on leave etc. is not included in the gap, as opposed to the previous method. The current assessment of the capacity available outside the labour force, however, is unchanged.

Actual and structural participation rate and cyclical component of the participation rate





Note: The participation rate is calculated as the number of persons in the labour force divided by the number of persons in the population in the group of 16-66-year-olds. The previous method included persons on leave etc. in the labour force.

Source: Own calculations.

A. See Andersen and Rasmussen (2011) for a review of the previous method. Some adjustments have been made after the publication of the article. Among other things, the concept of the labor force has changed.

B. In the previous method, shifts in demographics were handled by including the proportion of 60-66-year-olds as an explanatory variable in the estimation of the participation rate gap. The variable was intended to capture the effects of the lower labour market attachment of 60-66-year-olds. Hence, the increasing share of 60-66-year-olds among people aged 16-66 in the 2000s contributed to a reduction in the structural participation rate in the estimation.

TOTAL FACTOR PRODUCTIVITY

Total factor productivity (TFP) is the part of output not explained by the amount of labour and capital used in the production of goods and services. As such, it represents a measure of how efficiently and intensively input factors are utilized or, more generally, a proxy for the level of technology prevalent in the economy. Similarly, any measurement error in the calculations of capital and labour, including shifts in average hours worked, will also impact the TFP measure. Following the standard approach, we calculate TFP as the Solow residual of a Cobb-Douglas production function, cf. equation (6).

$$TFP_t = Y_t - \alpha L_t - (1-\alpha)K_t \tag{6}$$

Y_t denotes GDP in constant prices, L_t is the number of employees and K_t denotes the net capital stock. The wage share parameter α is calibrated to take a time-invariant value of 0.61. All variables are measured in logarithms and cover the total economy.

A stylized macroeconomic feature is that productivity tends to fluctuate procyclically over the business cycle, cf. Basu and Fernald (2000). During the initial phases of an economic expansion, companies typically respond to increasing demand by adjusting how intensively existing machines are used and how many hours their employees work. Capacity utilization – and hence productivity – will consequently accelerate at the beginning of an economic upturn and stall during a recession. This reflects the tendency for companies to adjust their input of capital and labour sluggishly due to the costs associated with hiring new staff or installing new machinery.

MODELLING TFP FLUCTUATIONS OVER THE BUSINESS CYCLE

To evaluate the relative importance of the cyclical and structural variation in productivity, we develop an unobserved components model according to equations (7)-(11). The approach imposes structure on the dynamics of the observable time series for actual TFP in order to decompose it into a structural and cyclical component, denoted below by TFP_t^* and TFP_t^c respectively.

$$TFP_t = TFP_t^* + TFP_t^c \tag{7}$$

Structural TFP is assumed to follow a random walk process with a time-varying drift element γ_t which by assumption evolves according to a separate random walk, cf. equation (8) and (9). The specification has the desirable feature that past productivity innovations accumulate into the economy's stock of technology, while also allowing for time variations in its underlying growth rate.

$$TFP_t^* = \gamma_t + TFP_{t-1}^* + \sigma_t^{f^*} \tag{8}$$

$$\gamma_t = \gamma_{t-1} + \sigma_t^{\gamma} \tag{9}$$

The cyclical component of TFP evolves in accordance with an autoregressive process that is designed to generate oscillations akin to the fluctuations observed for productivity over the business cycle, see equation (10). More fundamentally, the estimation of cyclical TFP uses information from Statistics Denmark's business sentiment survey on the degree of spare capacity in manufacturing, denoted CU_t in equation (11). This is motivated by several factors. First, the measure of spare capacity exhibits a relatively strong negative correlation with detrended TFP, suggesting that it may covariate with an unobservable cyclical TFP component.[12] Moreover, the indicator is measured with adequate precision and without subsequent revisions which, ceteris paribus, reduces periodic revisions to structural TFP estimates.[13]

$$TFP_t^c = \psi_1 TFP_{t-1}^c + \psi_2 TFP_{t-2}^c + \sigma_t^{f^c} \quad (10)$$

$$CU_t = \theta_1 CU_{t-1} + \theta_2 TFP_t^c + \sigma_t^{CU} \quad (11)$$

Finally, a series of error terms σ_t^i are included to capture irregular noise in the time series[14]. The model structure described above is written in state space form and estimated using the Kalman filter and maximum likelihood techniques.[15] The results, presented in table 3, suggest that the estimated series for cyclical TFP responds strongly to changes in spare capacity.

Model for TFP

Tabele 3

Parameter	Estimate	Standard error
Coefficients:		
ψ_1: *Lagged TFP gap*	0.73	(0.09)
ψ_2: *Lagged TFP gap*	0.12	(0.08)
θ_1: *Lagged spare capacity*	0.77	(0.05)
θ_2: *TFP gap*	-1.54	(0.38)
Variance parameters:		
$\sigma_t^{f^c}$	0.00	(0.00)
$\sigma_t^{f^*}$	0.01	(0.00)
σ_t^{Y}	0.00	(0.00)
σ_t^{CU}	0.05	(0.00)

Note: Rounding may result in a zero variance.
Source: Own calculations.

The results indicate that structural TFP growth began to slow in the mid-1990s and further decelerated at the turn of the millennium, cf. chart 9 (left). More concretely, the estimation suggests that structural TFP growth peaked at 1.5 per cent in 1995 and dropped to 0.3 per cent in 2007, after which it has remained broadly constant at this level. Looking at structural and cyclical productivity combined, the model for all practical purposes points to a closed TFP gap at the current juncture, following a significant cyclical setback in the aftermath of the 2008 financial crisis. Technically, this represents the fact that spare capacity has reached a level not too far from its

12 The correlation between (demeaned) spare capacity in manufacturing and HP filter detrended TFP is -0.57 in our data sample.

13 One potential drawback of an indicator which focuses exclusively on manufacturing is that it is not able to capture cyclical productivity fluctuations in services and construction. We have also experimented with the lagged employment gap to capture the idea that labour market pressure is often associated with a cyclical pick-up in productivity. The econometric results did, however, not suggest a statistical significant relationship.

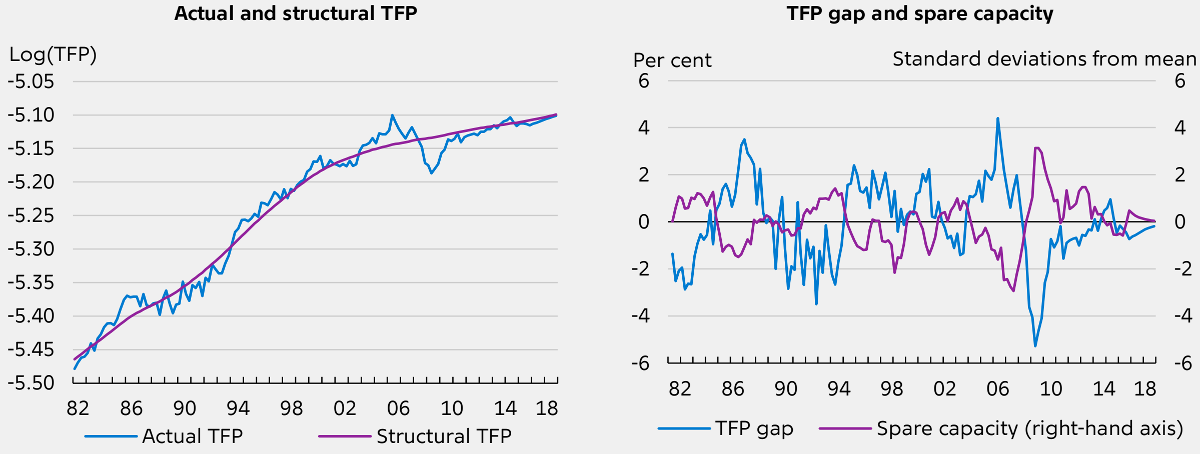
14 We decided not to include an error term in equation (1.2) in order not to break the identity saying that actual GDP equals the sum of cyclical and structural GDP. In other words, this decision ensures that the output gap is closing if actual GDP growth exceeds estimated potential GDP growth and vice versa.

15 The model needs a forecast for spare capacity which is assumed to revert back to a neutral position using a speed of convergence parameter of 0.25, i.e. 25 per cent of the difference between the contemporaneous spare capacity and its mean of zero will be neutralized each quarter.

long run average, as illustrated in chart 9 (right). This highlights the modelling assumption where cyclical TFP is essentially determined by fluctuations in the degree of spare capacity.

Danmarks Nationalbank has produced estimates of the Danish TFP gap based on the extended HP filter method developed by Andersen and Rasmussen (2011) since 2011. Comparing the two methodologies, both of which use the same spare capacity indicator, they arrive at broadly similar TFP gap estimates. There is, however, a tendency for the new estimate to give rise to less extreme values of the TFP gap.

Structural TFP estimate Chart 9



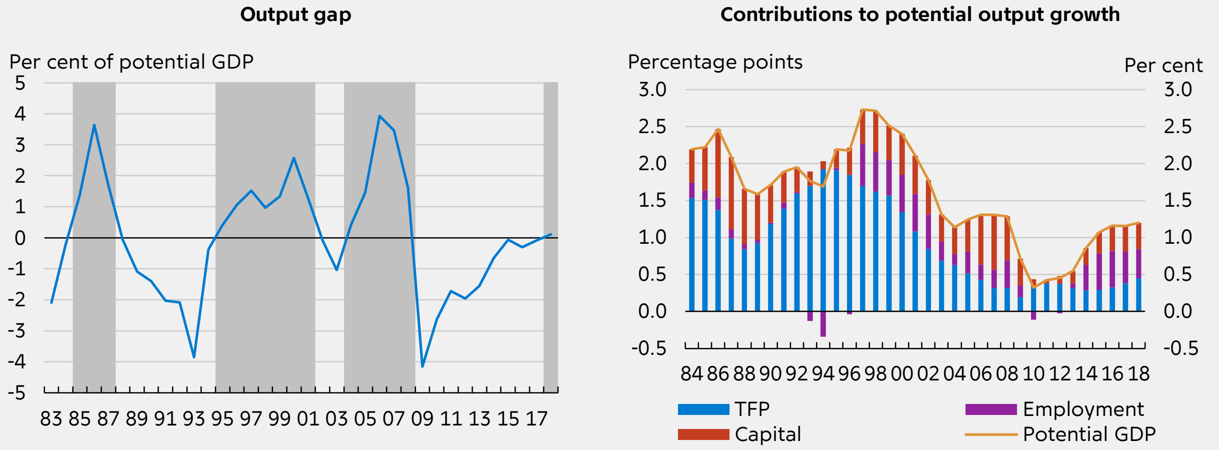
Source: Danmarks Nationalbank.

POTENTIAL OUTPUT AND THE OUTPUT GAP

Our measure of potential output is calculated by combining the estimated structural levels for unemployment, the participation rate and TFP from the preceding sections. The resulting series indicates that potential GDP growth has fallen since the late-1990s, mostly due to a setback in structural TFP growth, cf. chart 10. Another kink can be observed in the wake of the 2008 financial crisis, where subdued growth in structural employment and lower capital deepening related to low investment activity placed a drag on potential output growth. It has subsequently rebounded to 1.2 per cent in 2016 on account of capital accumulation and higher structural employment. This partly reflects the gradual phasing out of the voluntary early retirement scheme from 2014 and onwards.

Potential output

Chart 10



Note: Left-hand chart: Shaded areas indicate periods with a positive output gap. Right-hand chart: TFP, employment and capital denote the respective growth contribution in percentage points from these input factors to potential GDP growth.

Source: Danmarks Nationalbank.

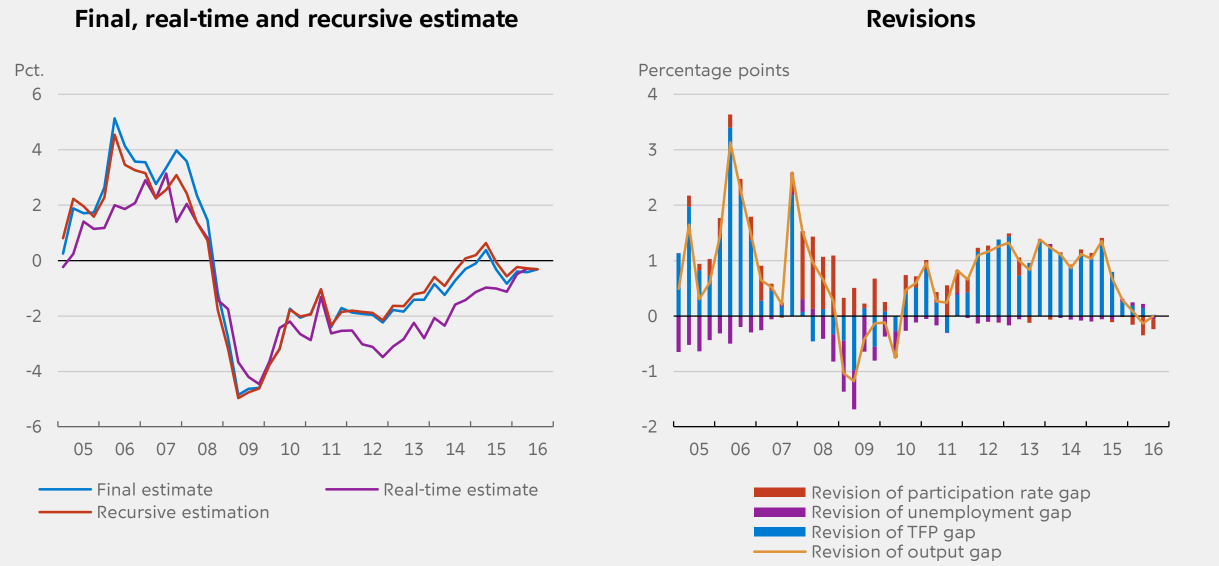
According to our estimates, the output gap was large and positive in the period leading up to 1986 when the implementation of policy measures in the form of a tax reform reducing the tax value of interest deductibility and the "Potato Cure", an austerity package, marked the beginning of a slowdown lasting into the early 1990s. Hansen et. al. (2003) find that the unwinding of the most important "Potato Cure" measures, together with a further decline in interest rates and moderate fiscal easing subsequently triggered an economic recovery. The decade ended with a boom and a positive output gap of 2.6 per cent, which was later followed by a relatively mild downturn in the beginning of the 2000s. In the period 2005-07, the Danish economy faced a substantial overheating with a positive output gap close to 4 per cent and significant pressure on the labour market. The outbreak of the global financial crisis caused a severe recession, which reached its trough in 2009, when actual GDP was more than 4 per cent below its potential. At the current juncture, following several years of a narrowing output gap, our estimates suggest that the Danish economy is operating very close to its cyclically neutral capacity in 2016.

REAL-TIME RELIABILITY AND ROBUSTNESS

Output gap estimates play a central role in Danmarks Nationalbank's assessment of the cyclical stance of the economy and serve to inform its policy recommendations. In praxis, it is thus important that they are available in a timely manner and with sufficient precision in order to have real policy applicability. Output gap estimates are, however, subject to uncertainty regarding their real-time reliability, robustness and ability to capture turning points. This is often reflected in sizable output gap revisions over time, either because new information emerges about the state of the economy or due to revised data.

To formally evaluate the robustness of the output gap methodology, a comparison between the most recent output gap vintage and its realtime estimate is shown in chart 11.[16] Overall, the method captures the cyclical turning points reasonably well in realtime, cf. chart 11 (left). However, the magnitude of the gap is subject to larger uncertainty. This is particularly the case during the upswing in 2006-08, when the real-time estimates were smaller than the final estimates, as was the case in the period 2011-15. A breakdown of the revisions to the output gap reveals that the revisions are primarily caused by revisions to the TFP gap, cf. chart 11 (right). This may reflect that GDP over time can be subject to substantial revisions. Conversely, the unemployment gap and participation rate gap are more robust as their data input is revised to a lesser degree.

Reliability and robustness Chart 11



Anm.: In the left-hand chart each point in the real-time estimate series reflects the estimate one would have obtained from the amount of information that was available after the first preliminary release of national accounts for that quarter. The final estimate series reflects the results discussed in the previous sections. The right-hand chart illustrates the difference between the final estimate and the real time estimate of the output gap as well as the contributions to the difference from the TFP gap, the unemployment gap and participation rate gap.

Source: Own calculations.

To illustrate the uncertainty related to data revisions, one can compare the real-time estimates with the results of a recursive estimation of the output gap. In general, the estimates in the

[16] In order to accomplish the real-time analysis, it has been necessary to construct a real-time dataset for each quarter back to the first quarter of 2005. That includes the construction of series that did not exist in the early years of the analysis, fx gross unemployment, persons on leave of absence etc. Each real-time estimate reflects the estimate one would obtain from the amount of information that is available immediately after the first preliminary release of the national accounts for that quarter combined with Denmarks Nationalbanks forecast at that time. Hence, the real-time estimate for the 1st quarter of 2016 reflects the estimate of the output gap one would obtain from the data that was available when Statistics Denmark published the first preliminary national accounts for the 1st quarter.

recursive estimation are much closer to the final estimates than the real-time estimates, reflecting that the uncertainty in the real-time estimates of the output gap are largely due to uncertainty about actual data. Hence, revisions of existing data represent a significant source of revisions to the estimates of the output gap. The uncertainty of the output gap illustrates that they should be interpreted with caution when assessing the economic situation.

APPENDIX

UNOBSERVED COMPONENTS MODEL FOR STRUCTURAL UNEMPLOYMENT

The model consists of the following equations:

$$u_t = u_t^* + u_t^c \tag{A.1}$$

$$\Delta u_t^* = \rho \, \Delta u_{t-1}^* + \zeta_t, \, \zeta_t \sim N(0, \sigma_\zeta^2) \tag{A.2}$$

$$u_t^c = \alpha_1 u_{t-1}^c + \alpha_2 u_{t-2}^c + \kappa_t, \kappa_t \sim N(0, \sigma_\kappa^2) \tag{A.3}$$

$$\Delta w_t - \Delta p_t = \alpha_0 + \alpha_1(\Delta w_{t-2} - \Delta p_{t-2}) + x_t'\beta + \gamma u_t^c + \varepsilon_t, \varepsilon_t \sim N(0, \sigma_\varepsilon^2) \tag{A.4}$$

where u_t is gross registered unemployment which covers both unemployed persons and persons in activation who either receive unemployment benefit or social assistance (and are classified as prepared for employment). A part of the activation is persons in subsidized employment. These persons are excluded from the estimation as they – while employed – to a lesser extent than the rest of the unemployed are considered to generate wage pressure. In equation (A.1), gross unemployment is decomposed into a structural component u_t^* and a cyclical component u_t^c.

Equation (A.2) models the change in structural unemployment as an autoregressive process where the parameter ρ controls its persistence. In equation (A.3), it is assumed that the cyclical component of unemployment can described by an AR(2) process. Equation (A.4) specifies the Phillips curve relationship between the unobservable unemployment gap and the year-on-year increase in real wages, $\Delta w_t - \Delta p_t$. The nominal wages refer to hourly wages in manufacturing from Danmarks Nationalbank's MONA database and they are deflated using the consumer price index. The control variables, x_t', include commodity price inflation, the change in the effective krone rate, the change in the terms of trade and the hourly productivity growth in the non-agricultural private sector. The noise terms ζ_t, κ_t and ε_t are all assumed to be normally, identically and independently distributed over time and mutually uncorrelated.

UNOBSERVED COMPONENTS MODEL FOR PARTICIPATION RATE

The aggregate structural participation rate is based on five separate submodels, each of which represents a specific age group. For the 16-29-year-olds and the 30-49-year-olds, the structural participation rate is determined by unobserved components models, whereas HP-filters are applied for the age groups above 49 years. The aggregate structural participation rate is computed as a weighted average of the structural participation rates across age groups.

Submodel for the group of 16-29–year-olds

The unobserved components model for the 16-29-year-olds consists of the following equations:

$$E_t = E_t^* + E_t^c \tag{A.5}$$

$$E_t^* = E_{t-1}^* + \mu_1 \Delta leave1629_t + \mu_2 \Delta stud_t + \varepsilon_t^*, \varepsilon_t^* \sim N(0, \sigma_{\varepsilon_t^*}^2) \tag{A.6}$$

$$E_t^c = \psi_1 E_{t-1}^c + \psi_2 E_{t-2}^c + \varepsilon_t^c, \varepsilon_t^c \sim N(0, \sigma_{\varepsilon_t^c}^2) \tag{A.7}$$

$$shortage_{t-2} = \eta_0 + \eta_1 E_t^c + \varepsilon_t^{shortage}, \varepsilon_t^{shortage} \sim N(0, \sigma_{\varepsilon_t^{shortage}}^2) \tag{A.8}$$

Where E_t is the participation rate for the 16-29-year-olds based on the register-based labour force statistics (RAS) which can be decomposed into a structural and cyclical component, denoted by E_t^* and E_t^c, respectively. Equation (A.6) states that the structural participation rate follows a random walk. It is further affected by the change in the persons enrolled in leave schemes, $\Delta leave1629_t$, and the number of students, $\Delta stud_t$. The number of people in leave schemes is from the national accounts. It consists of persons with maternity benefits, sickness benefits as well as leave schemes for childminding, education and sabbaticals which were implemented during the mid-1990s and subsequently phased out. The coefficient for the change in student propensity is calibrated to 0.4, reflecting that around 60 per cent of students are employed on average. In equation (A.7), the cyclical component of the participation rate is assumed to follow an AR(2) process. Equation (A.8) specifies the relationship between the cyclical component of the participation rate and the share of manufacturing firms reporting labour shortage from Statistics Denmarks business sentiment survey, $shortage_t$. The noise terms ε_t^*, ε_t^c, and $\varepsilon_t^{shortage}$ are all assumed to be normally, identically and independently distributed over time and mutually uncorrelated.

Submodel for the group of 30-49–year-olds

The unobserved components model for the 30-49-year-olds consists of the following equations:

$$E_t = E_t^* + E_t^c \tag{A.9}$$

$$E_t^* = E_{t-1}^* + \mu_1 \Delta leave3049_t + \varepsilon_t^*, \varepsilon_t^* \sim N(0, \sigma_{\varepsilon_t^*}^2) \tag{A.10}$$

$$E_t^c = \psi_1 E_{t-1}^c + \psi_2 E_{t-2}^c + \theta u_{t-1}^c + \varepsilon_t^c, \varepsilon_t^c \sim N(0, \sigma_{\varepsilon_t^c}^2) \tag{A.11}$$

$$shortage_t = \eta_0 + \eta_1 E_t^c + \varepsilon_t^{shortage}, \varepsilon_t^{shortage} \sim N(0, \sigma_{\varepsilon_t^{shortage}}^2) \tag{A.12}$$

Where the notation is analogous to the model for the 16-29-year-olds. In addition, a discouraged worker effect is included in equation (A.11), where the lagged unemployment gap, u_{t-1}^c, impacts the cyclical component of the participation rate.

UNOBSERVED COMPONENTS MODEL FOR TFP

The model consists of the following equations:

$$TFP_t = TFP_t^* + TFP_t^c \tag{A.9}$$

$$TFP_t^* = \gamma_t + TFP_{t-1}^* + \sigma_t^{f^*}, \sigma_t^{f^*} \sim N(0, \sigma_{f^*}^2) \tag{A.10}$$

$$\gamma_t = \gamma_{t-1} + \sigma_t^{\gamma}, \sigma_t^{\gamma} \sim N(0, \sigma_{\gamma}^2) \tag{A.11}$$

$$TFP_t^c = \psi_1 TFP_{t-1}^c + \psi_2 TFP_{t-2}^c + \sigma_t^{f^c}, \sigma_t^{f^c} \sim N(0, \sigma_{f^c}^2) \tag{A.12}$$

$$CU_t = \theta_1 CU_{t-1} + \theta_2 TFP_t^c + \sigma_t^{CU}, \sigma_t^{CU} \sim N(0, \sigma_{CU}^2) \tag{A.13}$$

Where TFP_t is the Solow residual from a constant returns Cobb Douglas production function which can be decomposed into a structural component TFP_t^* and a cyclical component TFP_t^c according to equation (A.9). Fluctuations in hours worked enter TFP because labour inputs is defined as the number of employees. In equation (A.10), structural TFP follows a random walk with a stochastic drift component, γ_t. The stochastic drift component is an unobservable state variable that follows a random walk, as specified in equation (A.11). In equation (A.12), it is assumed that the cyclical component of TFP can be described by an AR(2) process. Equation (A.13) specifies that the relationship between the unobservable cyclical TFP and the degree of spare capacity in manufacturing from Statistics Denmarks business sentiment survey, CU_t.The noise terms $\sigma_t^{f^*}$, σ_t^{γ}, $\sigma_t^{f^c}$ and σ_t^{CU} are all assumed to be normally, identically and independently distributed over time and mutually uncorrelated.

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