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WHAT ARE THE EFFECTS OF CHANGES IN TAXATION AND NEW TYPES OF MORTGAGES ON THE REAL ECONOMY? – THE CASE OF DENMARK DURING THE 00'S

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What are the effects of changes in taxation and new types of mortgages on the real economy?
– The case of Denmark during the 00's

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ABSTRACT

What are the effects of changes in taxation and new types of mortgages on the real economy?
- The case of Denmark during the 00's
What are the effects of introducing interest-only, flexible-rate mortgage contracts and a tax-freeze on housing wealth for the real- and financial economy? I study this within a medium-size DSGE model with housing, banking and financial frictions, and the coexistence of flexible-, fixed-rate and interest-only mortgage contracts. I find that the introduction and the adoption of flexible-rate and interest-only mortgage contracts together with a freeze of taxation of the housing wealth can explain around 1/3 of the real house price gap in Denmark during the period 2004-06 and 15-40 per cent of the output gap. The household debt-to-GDP would have been almost constant instead of increasing by 20 percentage points. Finally, the analysis points to a more volatile Danish economy after the implementation of the structural changes to the economy.

RESUME

Hvad er effekterne på realøkonomien af ændret boligbeskatning og introduktionen af nye låneformer?
- Et case studie af dansk økonomi i 00'erne
Hvad er effekterne på aktivitet og boligpriserne af introduktionen af rentetilpasningslån og afdragsfrihed og skattestoppet på boligmarkedet? Jeg analyserer spørgsmålet inden for rammerne af en DSGE-model med et boligmarked, finansielle friktioner, og banker udvidet med rentetilpasningslån og afdragsfrihed. Jeg finder, at introduktionen af de nye låntyper og ændret boligbeskatning kan forklare ca. 1/3 af boligprisens afgivelse fra en langsigtet trend i perioden 2004-06 og ca. 15-40 pct. af produktionsgabet. Husholdningernes gæld i forhold til BNP ville uden ændringer have været 20 procentpoint lavere. Endelig peger analysen på, at dansk økonomi er blevet mere volatil efter implementering af de nævnte ændringer i rammebetingelser på boligmarkedet.

JEL CLASSIFICATION

E17, E32, E62, E65, F41

KEYWORDS

DSGE Models, House prices, Financial frictions, Banking.

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Abstract

What are the effects of introducing interest-only, flexible-rate mortgage contracts and a tax-freeze on housing wealth for the real- and financial economy? I study this within a medium-size DSGE model with housing, banking and financial frictions, and the coexistence of flexible, fixed-rate and interest-only mortgage contracts. I find that the introduction and the adoption of flexible-rate and interest-only mortgage contracts together with a freeze of taxation of the housing wealth can explain around $\frac{1}{3}$ of the gap in the real house price in Denmark during the period 2004-06 and 15-40 per cent of the output gap. The household debt-to-GDP would have been almost constant instead of increasing by 20 percentage points. Finally, the analysis points to a more volatile Danish economy after the implementation of the structural changes to the economy.

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1. Introduction

Danish house prices were booming in the 00's rising by more than 20 per cent in 2006. They subsequently experienced comparable negative growth rates after the onset of the financial crisis in 2008. This coincided with a boom-bust cycle in Danish economy with large increases in residential investments, private consumption and GDP. A similar development can be found in other developed economies.

In what follows I will study the role played by two structural changes for these observations, which to some extent only happened in Denmark: The introduction and adoption of new mortgage contracts and a freeze of the nominal amount payed on housing wealth. These two changes are shown in figure (1) and (2). In figure (1) is shown the outstanding stock of mortgage debt contracts distributed on type of contracts; flexible-rate, fixed-rate contracts with and without principal payments. From this figure it can be seen that pre-2002, almost all mortgage contracts were fixed rate with amortisation and 30-year maturity. In 2007 this was only so for around 40 per cent of the outstanding contracts. The rest was interest-rate only contracts and/or flexible-rate mortgage contracts. The introduction and adoption of new mortgage contracts coincided with the change in the taxation of housing wealth shown in figure (2). Here is shown a change in the taxation of housing wealth: Starting from January 1st 2002 the nominal amount Danish households have to pay in taxes on the value of the property has been frozen to its 2002 level. The effect of this way of taxing housing wealth is that when house prices rose during the mid-00s, the effective tax rate on housing fell and vice-versa during the bust.

Based on this, I ask: *What are the effects from these structural changes on the real house price, household debt and real GDP during the period 2002-08?* To answer this question, I use a medium-size small-open economy DSGE model with banks, financial frictions, and housing for Denmark, see Pedersen (2016). One contribution of this paper is the extensions made to this model. Specifically, I expand this model with long-term debt contracts with and without amortisation and the co-existence of flexible-rate and fixed-rate contracts. I do this through a reinterpretation of the Calvo-setup, usually used to model sticky prices and/or wages. I reinterpreted the probability of being able to reset interest rates as instead determining how often mortgage lending rates are set on the borrowers side thus effectively determining the average duration of the mortgage portfolio in the economy. This avoids a large number of state-variables, while still being able to model the co-existence of fixed- and flexible-rate mortgage contracts of varying maturity. Further, through the explicit modeling of the amortisation schedule of these contracts I allow for interest-only mortgage contracts (I.O. loans). I use this framework to analyse the permanent and transitory effects, and the effects on second-order moments from these changes.

The results from this paper point to a combined effect from changes in the taxation of housing wealth and the introduction and adoption of new types of mortgage contracts as follows. The real house price gap would without these changes have been around 8 percentage points lower during the period 2002-06, and the construction-to-GDP ratio would have been around 1 percentage point lower. The output gap would in 2005 have been around 0.75 percentage points lower. Equivalently, these two changes to the housing market explained around $\frac{1}{3}$ of the real house price gap and 15-40 per cent of the output gap. This leaves a large part unexplained. This is not too surprising. Here I only consider the two mentioned structural changes to the economy, while in data a lot of different shocks hit the Danish economy during the period under study.[1]

On the financial side, the debt-to-GDP ratio for households would have been almost constant without these two changes, while it in fact increased by 20 percentage points. Further, these changes have had implications for the business cycle – equivalent shocks propagate stronger after the changes to the housing market. This concerns both the real and the financial side of the economy. Specifically, using the coefficient of variation of the changes in the variables in the model as a measure of the business cycle, the real house price and construction can be expected to be respectively 25 and 15 per cent more volatile, while debt to households can be expected to be 50 per cent more volatile. Private consumption can in the future expected to move by 10 per cent more.

While I find that the main drivers for first-order moments have been taxation of housing wealth, the adoption of new types of mortgage contracts have had permanent effects on especially debt. That is, if the composition of the stock of outstanding mortgage contracts in 2008 had been constant and taxation kept frozen in all future, then in the new regime household debt would had been around 20 per cent higher compared to the old regime. Likewise, the real house price would have been 3 per cent higher and residential investments around 10 per cent higher. Also, while I find that the interest-only mortgage contracts are not likely to have had a large effect on either, this is not so for these permanent effects. I find that they contribute to around half of the build-up of private debt in the new regime relative to the situation in 2002.[2]

[1] In Pedersen (2016) the real house price is decomposed into structural shocks. There I find that both foreign shocks, like demand for Danish exports and monetary policy, productivity shocks etc. contributed to the house price gap during the period under study. Further, housing preference shocks played a large part. These shocks can be interpreted as the consumers getting more utility of a given level of housing or that given fundamentals, the household demanded more housing. This is thus a sign of a house price bubble.

[2] These finding are not a case against financial liberalization. In the current study I do not consider the effects on welfare. To this end I both need a heterogeneous agent model and endogenous defaults. A an example, differences in risk aversion could make it welfare improving to finance housing using flexible-rate contracts, while I.O loans provide an insurance against, as an example, uninsurable unemployment risk. This is not the focus of this paper.

In the case of Denmark, Dam et al. (2011) study similar questions. They find that the real house price would without I.O. mortgage contracts have been around 15 per cent lower in 2006-07. Likewise without flexible-rate loans the house price would have been around 15 per cent lower in 2006-07. Without changes to the taxation the real house price would have been around 7.5 per cent lower in 2006-07. In Dam et al. (2011) a traditional macroeconometric model is used. While a standard, well-tried tool with data consistent framework, the model lacks forward looking economic agents and simulating the model and calculating higher-order moments is not straightforward.

In this paper, I apply a structural general equilibrium model with forward-looking behaviour and expectations. I specifically extend the model in Pedersen (2016) in two directions. Firstly, I introduce long-term debt contracts with and without amortisation following Kydland et al. (2012). Secondly, I allow for the coexistence of fixed and flexible-rate mortgage contracts. The motivation behind using a structural DSGE model is that house prices are forward-looking variables and can be thought of as a stock price. Housing is a (very) durable good, and hence, not only the economic conditions today but also in the future determine the current real house price. It is thus necessary to model the formation of expectations. Further, with a structural model the calculation of moments and impulses is straightforward, and a structural model should be robust to the Lucas-critique and policy changes then become meaningful.

To be able to study the effects on the real economy of introducing flexible-rate mortgage contracts with and without amortisation obviously a model needs to include all types of mortgage contracts including long-term debt contracts. But also financial frictions are needed: In a model with perfect financial markets, amortisation is saving which can be undone through borrowing. Therefore amortisation should not in such a model affect the user cost of housing and thus financial markets and the real economy. Likewise, there should not be any effect from introducing flexible-rate mortgage contracts in a frictionless world, as in risk adjusted expectations terms, there should not be any difference between fixed and flexible-rate contracts. This is due to no-arbitrage and any risk would be priced in the bond.

The financial frictions in the model are on the financial side of the model banks with capital requirements as in Gerali et al. (2010). This was also used in Pedersen (2016), and in this paper the same structure with only minor changes is used. On the household and firm side, the financial frictions are collateral requirements, see Iacoviello (2005): The impatient household in the economy needs to post collateral to be able to borrow, and they can only borrow up to a certain amount of the value of their collateral being determined by the value of their housing stock. Likewise on the firm side, the firms can only borrow up to a certain value of their collateral, here in the form of the value of their stock of capital.

The model, however, excludes three channels which are likely to affect the results. Firstly, expectations are formed rationally. It could be that long periods of rising house prices could lead to overoptimism pushing the actual movements in the real house price away from fundamentals. The fact that Pedersen (2016) finds a large role for housing preference shocks in a structural shock decomposition partly confirms this intuition. Rational expectations naturally rule out this possibility, but non-rational expectations could increase the estimates of effects on the economy from the structural changes. Secondly, the model rules out defaults. All else being equal, the introduction of default would increase the impact of the two changes to the economy on expected volatility. That is, if the probability and cost of financial crises are increasing the debt level, then a higher level of debt as a consequence of I.O. loans and lower effective taxation on housing wealth would increase the amplitude of the business cycle. Lastly, it is assumed that the collateral constraint is everywhere and always binding. But if the house price is hit by a very negative shock, then it is implicitly assumed that households are forced to pay back the mortgage loan faster than is imposed in the contract. In an asymmetric collateral constraint such a shock would have let the household into a deleveraging cycle, see also Andrés et al. (2014). It is, however, unclear how this affects the results, as in this study the house price is almost always affected positively.

Previous papers have considered long-term debt contracts with fixed and flexible-rate, but not simultaneously. Calza et al. (2013) consider the effects of monetary policy under fixed or flexible-rate mortgage contracts. However, in their model, the two types of mortgages do not coexist. Rubio (2011) considers a model in which fixed-rate and floating-rate mortgage contracts coexist. Her model captures that, all else being equal, shocks which transmit to the economy through interest rates are mitigated under fixed-rate mortgages. But her model does not include amortisation and long-term debt contracts. Brzoza-Brzezina et al. (2014) expands on the framework in Calza et al. (2013) to long-term debt contracts and consider both flexible and variable rate mortgage contracts. Also, flexible-rate contracts are not one-period contracts, and they do not consider the coexistence of all types of contracts.

I introduce long-term debt with and without amortisation building upon the work of Gelain et al. (2015) and Kydland et al. (2012). I reinterpret the financial market structure introduced by Gerali et al. (2010). Specifically, the markup which arise due to monopolistic competition in the banking sector is in this model interpreted as risk premia inherent and observed in data for fixed-rate, long maturity mortgage contracts. The stickiness of the interest rates are in turn interpreted as a measure of the average duration of the mortgage contract and hence for how long these interest rates are fixed in the contracts.

This paper is organised as follows. In section (2), the changes to the model in Pedersen (2016) are explained, and their impact on the model is analysed together with the transmission mechanism for the instruments. The scenarios shown are explained in section (3) and the results are discussed in section (4). I discus the results outside the model in section (5). Section (6) concludes.

2. The Model

The questions I seek to answer in this paper will be analysed using the model in Pedersen (2016) with some extensions. These extensions are explained in the sections to come. The effect on the economy from these extensions is analysed through impulse response functions. In overall terms, the model in Pedersen (2016) is a small-open economy model with a fixed exchange-rate regime, housing, construction, a financial sector and financial frictions. The model is estimated on quarterly Danish data running from 1995 to 2016. The data set includes information on house prices, loans and loan rates. I will refer to Pedersen (2016) for detail in what follows, and here only analyse and present the changes made to that model.

The extensions made to the model in Pedersen (2016) are long-term debt with varying amortisation, and flexible- and fixed-rate mortgages. These are necessary extensions to the model for the questions addressed in this paper. On the household side, the idea is to setup a representative mortgage contract with an interest rate, an amortisation rate, and duration which reflects the average mortgage contract in Denmark. A set up with as an example four representative (impatient) households, which each finance their housing purchases using fixed-rate, flexible-rate with and without amortisation would a priori yield similar conclusions but with the cost of having a much larger model. On the banking side the idea is to use the standard Calvo-setup for interest rates normally applied to model sticky prices, see Galí (2009). The advantage of this approach is that it is memoryless and hence there is no need to follow all cohorts of fixed and flexible-rate mortgage contracts.

2.1. Model extension: Long-term debt contracts and amortisation

In the model in Pedersen (2016) all debt contracts are one-period. This is standard in the literature, but the questions addressed in this paper cannot be answered within such a framework. Specifically, there can be no meaningful distinction between interest-only loans and debt contracts with amortisations if all debt is rolled over each period. Similar, if the contract only runs for one period, then the distinction between fixed- and flexible-rate mortgages becomes less interesting. I will for these reasons introduce long-term debt contracts into the model in Pedersen (2016) with a process for the amorisation rate. In the next section, I will explain how I introduce fixed and flexible-rate mortgage contracts.

In what follows, I will only present the changes to the model. I will follow Gelain et al. (2015) for the introduction of long-term debt contracts. I will follow Kydland et al. (2012) for the specification of the amortisation of the long-term debt.

Let B_t^I be the stock of real debt outstanding at the of the period, $B_t^{I,New}$ new borrowing incurred in period t, and let $\delta_t^{\mathcal{A}}$ be the amortisation rate. The law of motion for debt is given by

$$B_t^I = \left(1 - \delta_{t-1}^{\mathcal{A}}\right) \frac{B_{t-1}^I}{\pi_t^{DK}} + B_t^{I,New}. \tag{1}$$

π_t^{DK} is the change in the Danish PPI-deflator. If $\delta_t^{\mathcal{A}} = 0 \ \forall t$ then no debt is amortised and all debt contracts are perpetuities. If $\delta_t^{\mathcal{A}} = 1 \ \forall t$ the model collapses into a model with one-period debt contracts only.

I follow Kydland et al. (2012) in the specification of $\delta_t^{\mathcal{A}}$. One approach is to simply let the process be a constant, $\delta^{\mathcal{A}}$. However, that would not take into account the actual amortisation schedule for Danish mortgage contracts, which to a large degree are annuities. Kydland et al. (2012) propose a process for $\delta_t^{\mathcal{A}}$ as follows

$$\delta_t^{\mathcal{A}} = \left(1 - \frac{B_t^{I,New}}{B_t^I}\right) \left(\delta_{t-1}^{\mathcal{A}}\right)^{\alpha^\delta} + \frac{B_t^{I,New}}{B_t^I} \left(1 - \alpha^\delta\right)^{\kappa^\delta}, \tag{2}$$

where $\alpha^\delta \in [0, 1)$ and $\kappa^\delta > 0$ are constants. By setting $\alpha^\delta = 0$, amortisation becomes equal to one, $\delta_t^{\mathcal{A}} = 1$, and the model collapses into a model in which all debt is rolled over each period. As shown in Kydland et al. (2012), the parameters $(\alpha^\delta, \kappa^\delta)$ can be calibrated to match approximately the amortisation of a 30-year mortgage contract, which is standard in Denmark.[3]

Equation (1) and (2) can be combined as follows

$$\delta_t^{\mathcal{A}} = \left(1 - \alpha^\delta\right)^{\kappa^\delta} + \frac{B_{t-1}^I}{\pi_t^{DK} B_t^I} \left(1 - \delta_{t-1}^{\mathcal{A}}\right) \left(\left(\delta_{t-1}^{\mathcal{A}}\right)^{\alpha^\delta} - \left(1 - \alpha^\delta\right)^{\kappa^\delta}\right), \tag{3}$$

which only depends on the stock of debt.[4] I treat the amortisation process as being taken as given for the households. Consequently, they do not take into account that debt-level affects the amortisation rate when determining their borrowing in equilibrium.

The budget constraint for the impatient households is given by, see also Pedersen (2016):[5]

$$\begin{aligned} & \left(1 + \tau_t^{VAT}\right) \frac{P_t^C}{P_t} C_t^I + Q_t^H \left(H_t^I - \left(1 - \delta^H\right) H_{t-1}^I\right) + \frac{R_{t-1}^{L,I}}{\pi_t^{DK}} B_{t-1}^I + (1 - \omega) T_t \\ = \ & B_t^I + \left(1 - \tau_t^N\right) W_t^I N_t^I - \tau_t^H Q_t^H H_t^I. \end{aligned} \tag{4}$$

[3] I refer to Kydland et al. (2012) for the details on how this can be done.

[4] The amortisation process depends on the change in the stock of debt, as, by the nature of an annuity, principal payments are lower and interest payments higher at the initiation of the debt contract.

[5] Compared to the model in Pedersen (2016), I disregard tax deduction on interest-rate payments.

Here P_t is the overall price level (producer price), P_t^C is the consumer price, Q_t^H denotes the real house price, and $\delta^H > 0$ is the depreciation rate of the housing stock. The parameter ω denotes the share of impatient households in the economy. W_t^I and N_t^I are respectively employment and the real wage rate. I let τ_t^{VAT}, τ_t^B, τ_t^N and τ_t^H be the tax rates on consumption (i.e., a Value added tax, VAT), interest income, labor, and housing property. T_t denotes real lump sum taxes. C_t^I denotes consumption. Using (1) the budget constraint can with long-term debt be written as

$$\begin{aligned} &\left(1+\tau_t^{VAT}\right)\frac{P_t^C}{P_t}C_t^I + Q_t^H\left(H_t^I - \left(1-\delta^H\right)H_{t-1}^I\right) - B_t^{I,New} + \frac{\left(R_{t-1}^{L,I} - 1 + \delta_{t-1}^{\mathcal{A}}\right)}{\pi_t^{DK}}B_{t-1}^I \\ = &\ -\left(1-\omega\right)T_t + \left(1-\tau_t^N\right)W_t^I N_t^I - \tau_t^H Q_t^H H_t^I. \end{aligned} \tag{5}$$

The terms in brackets in front of the stock of loans in the budget constraint, $\left(R_{t-1}^{L,I} - 1 + \delta_{t-1}^{\mathcal{A}}\right)$, are net interest payments and principal on the stock of loans.

The introduction of long-term debt contracts does not change the nature of the budget constraint per se, as can be seen from the expressions above. And long-term debt contracts only matter for the equilibrium dynamics if financial frictions are introduced into the model. Following Iacoviello (2005) and the setup in Pedersen (2016), I assume the existence of a collateral constraint. That is, the outstanding debt of the household cannot exceed some fraction, Θ_t^I, of the value of the housing stock, $H_t^I Q_t^H$,

$$B_t^{I,New} = \Theta_t^I\left(\frac{E_t\left[Q_{t+1}^H \pi_{t+1}^{DK}\right]H_t^I}{R_t^{L,I}} - B_t^I\right). \tag{6}$$

Hence, the collateral constraint is imposed on new debt; the household can only take on new debt, if the collateral value, the first term in (6), is higher than the existing stock of debt, B_t^I.[6]

Assuming that the collateral constraint binds, I can rewrite equation (6) in terms of the stock of debt only using equation (1)

$$B_t^I = \frac{\Theta_t^I}{1+\Theta_t^I}E_t\left[\pi_{t+1}^{DK}Q_{t+1}^H\right]H_t^I\frac{1}{R_t^{L,I}} + \frac{1-\delta_{t-1}^{\mathcal{A}}}{1+\Theta_t^I}\frac{B_{t-1}^I}{\pi_t^{DK}}. \tag{7}$$

I notice, that the parameter Θ_t^I is the loan-to-value on net-worth, and the steady-state LTV ratio is equal to $\frac{\Theta^I}{\Theta^I+\delta^{\mathcal{A}}} = \frac{Q^H H^I \pi^{DK}}{R^{L,I}}$. In the calibration of the model, Θ_t^I is set such that the steady state loan-to-value ratio equals 0.8. This implies that the household can borrow up to 4 times its net-worth in steady-state. I will comment more on the optimal behavoiur of the borrower in the section to come analysing how the introduction of long-term debt contracts changes propagation in the model. First, I point out one inconvenience of the setup: Households

[6]In what follows, I assume that the collateral constraint is always and everywhere binding. In case of one-period debt, this question has been studied in depth, see e.g. Iacoviello (2005).

can be forced to pay back their mortgage loans faster than the contractual agreements. That is, if a shock forces the value of the collateral constraint below the loan-to-value, then implicitly in the model, the households not only stop borrowing more, they must also pay back the difference between the value of the housing stock and the loan-to-value. This is assumed for simplicity. Here the implicit assumption is that the borrowers in the economy are a representative household covering the full Danish economy and it is unlikely that the average Danish household will be below water.[7]

2.2. Model extension: Flexible- and fixed-rate mortgage contracts

Having introduced long-term debt contracts, the next step for addressing the questions in this paper is to introduce flexible and fixed-rate mortgage contracts. Clearly these contracts need to coexist. To this end, I continue with the representative mortgage contract framework in Pedersen (2016), which is built upon the framework in Gerali et al. (2010). I reinterpret the markup in the competitive monopolistic framework for the banking sector as risk premia reflecting duration risk from financing housing through fixed-rate/long maturity contracts. This is a short cut in modeling risk and risk premia, but it is a tractable way forward that introduces a way to implement the change from fixed-rate contracts to flexible-rate contracts, which is the aim of this analysis. I next reinterpret sticky lending rates to households as reflecting the duration of the mortgage-rate contract instead: The longer the duration of the contract, the less often the bank can change the interest rate on the outstanding loan portfolio, and this translates into different stickyness of lending rates. Specifically, the Calvo parameter is interpreted as a measure of the average duration of the mortgage contract, and hence how often interest rates are fixed in the contracts.[8]

In detail, in the banking sector I assume that the banks set a markup over the lending rate, as in the original model in Pedersen (2016). In the steady state the markup is such that the average interest rate on the average contract is 5 per cent per year. The markup reflects both monopolistic competition in the banking sector, as in Pedersen (2016), and generally it reflects the existence of a interest-rate spread between interest rates on mortgage loans and the policy rate. It is hence a short cut to model term premia. It is also, as explained next, a short cut to model the empirical spread between fixed- and flexible-rate mortgages and hence introduces a way forward to implement scenarios in which the outstanding stock of loans are changed from fixed to flexible rate contracts.

Also, the banks can not change the interest rate on the mortgage-contracts every period. Specifically, a Calvo-style setup is applied to the lending rates from the banks to the

[7]In Andrés et al. (2014), this asymmetry is explicitly modelled such that shocks which make the value of the collateral households or firms' can post fall below the LTV ratio, force the economy into a deleveraging cycle. Here households can only consume out of income and firms can only invest out of earnings; they can not borrow. This is all left for future research.

[8]A draw-back of this approach is that strictly speaking, the individual household does not know the duration of its mortgage contract. In the aggregate, this is however not important and does not change the results in this paper.

households. As is well-known, one big computational advantage of this setup is that it is memoryless. This implies a reduction in the number of state variables as there is no need to follow all the mortgage contracts through time. The Calvo-parameter for the mortgage contract is denoted by θ_R, and is calibrated such that it reflects the average length of the representative mortgage contract; the expected duration of the contract in this framework is $\frac{1}{1-\theta_R}$.

The interpretation of the θ_R is therefore slightly different from the sticky-price setup, see Galí (2009). In that framework, the Calvo-parameter denotes the probability of being able to change prices for the firms. This is also so in this framework mathematically, but here the Calvo-parameter is interpreted as determining the duration of the average mortgage contract in the economy, and the implications are the same: The bank is not able to change the interest rate in the contract on existing loans, but must wait until the contract is refinanced and/or new loans come in. If as an example $\theta_R = 0$, then the model collapses into a model in which all loans are flecible- rate loans, and when $\theta_R \to 1$ once the interest-rate is set it cannot be changed reflecting a model with fixed-rate mortgage contracts.

Adding to the setup in Pedersen (2016), I will in this paper instead of Taylor-type adjustment costs assume that the banks can not change the interest rate set on its portfolio of mortgage contracts every period.[9] In the steady-state, the optimal lending rate, $R^{\tilde{L,I}}$, is equal to

$$\widetilde{R^{L,I}} = \frac{\epsilon_R}{\epsilon_R - 1} R_t^L, \tag{8}$$

in which the first term, $\frac{\epsilon_R}{\epsilon_R-1}$ is the markup, or spread, between the lending rate from the wholesale bank to the branch, R_t^L. The actual representative or average mortgage rate evolves as

$$R_t^{L,I} = \left(\theta_R \, R_{t-1}^{L,I\ 1-\epsilon_R} + (1-\theta_R) \; R^{\tilde{L,I}}{}_t^{1-\epsilon_R} \right)^{\frac{1}{1-\epsilon_R}}. \tag{9}$$

2.3. The role of long-term debt contracts and amortisation for house prices and the real economy

I next analyse how the introduction of fixed- and flexible-rate and long-term/short-term mortgage contracts affects the business cycle in the model. I will firstly look at the effect in the model of a shock to the amortisation rate, which is seen as the effect of increasing the part of the stock of outstanding mortgage contracts which are I.O contracts. Secondly, I will look at the effect in the model of a shock to the risk premium reflecting a change in the stock of outstanding mortgage contracts which are flexible-rate contracts.

I will calibrate the parameters to the values estimated in Pedersen (2016) and keep them fixed throughout the scenarios. I will then vary new parameters introduced in the model extensions. The idea is to calibrate the model under the structure of the housing

[9] In appendix are shown all the equations which have changed compared to the model in Pedersen (2016).

market before 2002, in which all mortgage contracts were fixed-rate with amortisation and 30-year maturity, and the situation around 2008, in which part of the debt contracts were flexible-rate without amortisation. In the 2002 calibration, I calibrate the parameters in the amortisation process, κ^{δ} and α^{δ}, such that the amortisation rate reflects a mortgage contract with a maturity of 30 years. For the 2008 calibration, these parameters are set such that the average loan contract is of one period maturity, basically collapsing the model into a standard DSGE model with one-period debt contracts. That was not the case in 2008, but this is done to highlight the mechanims played by long-term debt.[10] I change the degree of how often the banks can change the interest rate on their loans, θ_R, or, equivalently, the degree of flexible-rate contracts in the portfolio from approximately 0.95 to 2/3. Everything else is kept constant including the taxation of housing wealth.

2.3.1 The effect of a shock to the amortisation rate

In figure (3) is shown a temporary shock to the amortisation rate keeping the maturity of the contracts fixed. Here is considered a higher amortisation rate; i.e. $\delta_t^{\mathcal{A}}$ increases. Recalling that higher amortisation implies that a larger part of the outstanding debt is paid back each period, this, all else being equal, implies that the household is forced to pay more during the period of the shock before the household can borrow more. The increase in principal payments drags down borrowing, and the borrowing constraint tightens. One unit of housing is less valuable in the collateral constraint, and the house price falls. As can be seen in the figure, this pushes consumption and output down.

These effects can be explained through the collateral constraint, equation (7), and the first-order equation for the impatient households with respect to borrowing,

$$\lambda_t^I - \mu_t^I = \beta^I E_t\left[\lambda_{t+1}^I \frac{1}{\pi_{t+1}^{DK}}\right] R_t^{L,I} - \beta^I E_t\left[\frac{\mu_{t+1}^I}{\pi_{t+1}^{DK}\left(1+\Theta_{t+1}^I\right)}\right]\left(1-\delta_t^{\mathcal{A}}\right). \tag{10}$$

Introducing the collateral constraint in equation (10), the first-order condition with respect to borrowing for the impatient households can be written as

$$\begin{aligned}
\lambda_t^I - \mu_t^I &= \beta^I E_t\left[\lambda_{t+1}^I R_t^{L,I} \frac{1}{\pi_{t+1}^{DK}}\right] - \beta^I E_t\left[\frac{\mu_{t+1}^I}{\pi_{t+1}^{DK}\left(1+\Theta_{t+1}^I\right)}\right]\left(1-\delta_t^{\mathcal{A}}\right) \Leftrightarrow \\
\frac{\mu_t^I}{\lambda_t^I} \equiv \psi_t &= 1 - E_t\left[\frac{\lambda_{t+1}^I}{\lambda_t^I}\beta^I \frac{1}{\pi_{t+1}^{DK}} R_t^{L,I}\right] + E_t\left[\beta^I \frac{\mu_{t+1}^I \lambda_{t+1}^I}{\lambda_t^I \lambda_{t+1}^I \pi_{t+1}^{DK}\left(1+\Theta_{t+1}^I\right)}\right]\left(1-\delta_t^{\mathcal{A}}\right) \Leftrightarrow \\
\psi_t &= 1 - E_t\left[M_{t+1} R_t^{L,I}\right] + E_t\left[M_{t+1}\psi_{t+1}^I \frac{\left(1-\delta_t^{\mathcal{A}}\right)}{\left(1+\Theta_{t+1}^I\right)}\right]
\end{aligned} \tag{11}$$

[10] To see how the model collapses into a model with one-period debt contracts, see section (2.1).

Here I have introduced the stochastic discount factor, $E_t[M_{t+1}] \equiv E_t\left[\beta^I \frac{\lambda^I_{t+1}}{\lambda^I_t}\frac{1}{\pi^{DK}_{t+1}}\right]$. It can be seen from this expression that the marginal benefit in terms of utility of acquiring an extra unit of debt, ψ_t, depends on the utility gain from consuming the proceeds minus the cost of doing so, $1 - E_t\left[M_{t+1}R^{L,I}_t\right]$, plus the value of the stock of debt not amortised discounted to the present, the last term $E_t\left[M_{t+1}\psi^I_{t+1}\frac{(1-\delta^{\mathcal{A}}_t)}{(1+\Theta^I_{t+1})}\right]$. This term is decreasing in the amortisation rate and in the loan-to-value rate.

Long-term debt and the collateral constraint introduce two new channels relative to the model with one-period debt contracts only: Firstly, when choosing how much to borrow, the household takes into account that new borrowing today is long-term and hence is committed to pay interest rate and principal on the stock of debt not only today but also in the future. This is reflected in the last term in expression (11). And secondly, the stock of debt in the future, and not only in the present, determines the amount of new debt the household can take on; the impatient households smooth their borrowing due to this forward-looking behaviour.

This also implies that a tighter collateral constraint, a strictly positive value of the multiplier, μ^I_{t+j}, $j > 1$, is not reflected one-to-one in new borrowing; the problem for the households of borrowing has become more dynamic in the sense that when the households optimise their borrowing, they need to take into account the state of the economy today and in the future. This makes shocks which affect debt more persistent. It can also be seen that increases in the amortisation rate, $\delta^{\mathcal{A}}_t$, translates directly into a tightening of the collateral constraint, which will allow the household to borrow less. This will in turn move house prices and a financial accelerator mechanism kicks in from house prices to borrowing. Naturally, the quantitative effects depend in general equilibrium on the behaviour of savers and banks.

From equation (10) it can be seen that if the consumer does not have a limit on borrowing, that is when the collateral constraint does not bind and the multiplier always is zero, $\mu^I_t = 0 \forall t$, the first-order condition for borrowing collapses into the standard Euler-equation. Hence, it can explicitly be seen, and as suggested above, that it is the introduction of financial frictions which gives the potential to alter the equilibrium dynamics under long-term debt contracts. If all debt is one-period debt, $\forall t\ \delta^{\mathcal{A}}_t = 0$, then the model collapses into the standard collateral constraint as in Iacoviello (2005) or Pedersen (2016).

2.3.2 The effect of a shock to flexible-fixed-rate mortgage contracts

In figure (4) is shown the effect of a shock to the risk premium between fixed and flexible-rate mortgage contracts in the two calibrations, or states, of the economy. The shock is a temporary shock. Here is considered a lower risk premium; $\frac{\epsilon_R}{\epsilon_R - 1}$ decreases. This shock can be interpreted as a higher degree of flexible-rate mortgage contracts in the economy.

A decrease in the markup, here interpreted as borrowers facing a lower mortgage rate, increases borrowing for both firms and households as borrowing has become cheaper. This naturally fuels debt: The impatient household can get closer to satisfying its desire for consumption now by buying more housing, taking on more debt and consume the proceeds. Consumption increases in both calibrations. This is due to the financial accelerator effect on the household side: The real house price increases, which relaxes the collateral constraint further leading to more borrowing whose proceeds can be consumed.[11]

Borrowing increases by more in the 2008 calibration. This is due to debt being short-term and especially because the shock to the lending rate is transmitted much faster due to flexible rates. The extra housing is partly met by extra residential investments and partly by patient households, who sell part of their housing stock. The effect on the real house price is greatest in the 2008 calibration, which can be explained through the same mechanism working for the response of consumption: Loans increase by more in the 2008 calibration due to a relatively lower duration of the outstanding mortgage contracts. Hence, borrowing can respond faster to changes in mortgage rates and consequently the real house price can increase by more.

The major differences between the two calibrations are exactly the response of debt. Specifically in the 2008 calibration, debt responds much faster than in the 2002 calibration. And interestingly, the debt-to-GDP ratio actually falls in the 2002 calibration due to a denominator effect, while this ratio increases in the 2008 calibration. The response to leveraging from changes in interest rates has been debated recently in the literature. The motivation has been the question about what monetary policy should do in response to increases in debt and asset prices, see Svensson (2013).

[11] Here, the assumption that the collateral constraint always binds becomes crucial. This issue is discussed in depth in e.g. Iacoviello (2005).

2.3.3 How does long-term debt contracts and fixed/flexible-rate mortgage change the transmission of shocks?

How does long-term debt and fixed/flexible-rate mortgages change the transmission of shocks in the economy? That is, how does, say, a shock to the preferences for housing propagate through the economy under flex-rate, fixed-rate, long-term and short-term mortgage debt contracts?

To answer this, I look at a set of IRFs for a set of variables and for a set of shocks. In figure (5) to (8) are shown the effect of a shock to four representative shocks, a risk shock, $\epsilon^{\mathcal{R}}$, a temporary productivity shock, $\epsilon^{A_{Y,T}}$, a housing preference shock, ϵ^{H}, and an intertemporal consumption shock, ϵ^{C}, on GDP, the real house price, consumption, loans to households, debt-to-GDP ratio and inflation.

Pedersen (2016) provides a detailed discussion presentation of how the shocks considered here propagate through the model. I will consequently only be brief. The risk shock, $\epsilon^{\mathcal{R}}$, can be interpreted as a funding shock for banks increasing the marginal costs of providing loans. A positive risk shock will thus increase the interest rate on all newly issued loans, or refinancing of existing loans, and thus make credit more expensive for the borrower. A temporary productivity shock, $\epsilon^{A_{Y,T}}$, works as a standard temporary productivity shock, while a housing preference shock, ϵ^{H}, makes both household demand more housing for given expectations of the future and for given user costs. An intertemporal consumption shock, ϵ^{C}, makes households more impatient lifting consumption from the future to the present.

To isolate the effects of the various new channels in the model, I consider the following different calibrations of the model:

1. 2002 calibration: Long-term fixed-rate debt and no tax-freeze
2. 2002 calibration with tax-freeze
3. 2002 calibration with flexible-rate loans
4. 2002 calibration with I.O. loans
5. 2008 calibration: Short-term debt with flexible rate and tax freeze.

Overall shocks are transmitted in fairly similar ways across the models with regard to GDP. Long-term debt, as expected, introduces under-, and overshooting in the responses. The economy reacts quicker and by more in response to the shocks if debt is short-term debt. Clearly, flexible-rate mortgage contracts make the reaction even faster. On top of this, the tax freeze makes the transmission of shocks greater. That is the case both in the 2002 calibration with long-term, fixed-rate debt contracts and in the case of short-term debt with flexible-rates.

The response of debt differs across the four models. I will use the housing preference shock to explain why household debt reacts differently across the 5 calibrations. With regard to the housing preference shock the effects on GDP are largest in model (4). Short-term debt implies that the stock of household debt increases quickly; this was explained in the previous section. If debt is one-period debt, when the impatient households determine their borrowing, they do not need to take into account that debt in the next period can be posted as collateral; the impatient household can reoptimise all periods without the need to think about the stock of outstanding debt. It also implies that debt returns much quicker to the steady-state in comparison with a model with long-term debt contract.

A higher stock of debt implies more collateral, which the impatient household can borrow against and consume the proceeds. Consumption is consequently higher in model (3) and (4). On top of this, under a tax-freeze, the consumer has no need to take into account extra tax payments when purchasing more housing stock, and consequently, the response of borrowing and consumption is largest in model (4).

With long-term debt contracts, the response of debt is firstly muted and secondly more persistent. Again, this is because the households smoothen its borrowing over time and rationally sees that the boom does not last forever, and that the households need to carry over to the following period the stock of debt; he cannot reoptimise on the existing stock of debt. This naturally also has consequences for consumption, which inherits the cyclical response of debt.

3. Scenarios

Having presented a model which can be used to analyse the policy questions set out in the introduction to this paper, I next explain how I can include into the model the changes to the Danish housing market experienced from approximately 2002 until the outbreak of the financial crisis shown in figure (1) and (2). The scenarios are shown in figure (9). Specifically, in figure (9) are shown the shocks implemented in the model for each year. The 2002 scenario is thus the change in the outstanding mortgage stock in that year together with the change in taxation. The total scenario for the period 2002-06 is the lower envelope of the lines in the figure.

I analyse the transition path to the new steady-state from these changes focusing on the initial part from 2002 to 2007.[12] The effect from, say, taxation is the combined effect over the years, from the 2002 scenario to the 2005 scenario. I denote the effects from this analysis the *Temporary effects*. The effects are presented and shown in section (4.1).

I next look at the effects from these changes on the business cycle. That is, I analyse how these changes to the housing market have affected the transmission of shocks in the

[12] I thus simulate the model under certainty equivalence. Hence, households and firms know and can calculate their optimal response to the changes once these are revealed. I consider a sequence of changes of simulations each starting from the transition path from the previous change.

economy when the new steady-state has been reached. I do this through second-order moments. Here the IRFs analysed in the previous section provide intuition behind the results. This analysis is done in section (4.2), *Effects on the business cycle*.

Lastly, I will analyse the long run or permanent effects from the changes to taxation and the introduction of new mortgage contracts. This is done in section (4.3), *Permanent effects*. Here, I compare the level in the old steady state with the level of the variables in the new steady state. First, I will explain in greater detail how the actual changes in the economy are mapped into the model.

3.1. Mapping of actual changes on the housing market to the model

I need to map the changes on the Danish housing market shown in figure (1) and (2) to the model. The scenarios implemented in the model are shown in figure (9). Changes to the amortisation rate is implemented through the process for $\delta_t^{\mathcal{A}}$. As an example, starting from 2004, the amortisation rate is changed permanently such that the principal payments on the mortgage contract in the model reflect the average contract: In 2004, the amortisation rate in the scenario falls to approximately 15 per cent as in the data shown in figure (2). When the scenarios end in 2006, the rate has fallen by around 35 per cent. Starting from 2002, almost all mortgage contracts were amortized over a 30-year period. Therefore, the process for amortisation, equation (3), is set such that it resembles a 30-year mortgage annuity before 2002.

For the tax on housing, when I calculate second-order moments and IRFs, I assume that the tax rate, τ_t^H, is constant, but the base is fixed to the steady-state value of the housing stock and the real house price, which was also done in Pedersen (2016). This implies that tax payments do not change with changes to the value of the housing stock, and hence the effective tax rate falls. It is not straightforward to include the tax freeze in the scenarios; simply introducing a tax freeze does not in itself affect the equilibrium dynamics in the model without the realisation of shocks. I make permanent shocks to the tax rate each year with the size of the shock set such that the shock resembles as closely as possible the effective tax-rate on housing in figure (1). In this way, the agents in the economy face a declining tax rate. Naturally, this is in fact not how the tax rule works; it is only a permanent lower tax payment if the real house price increases above its level in 2002. In a model like the one used here with forward looking agents the permanent change in the tax rate will be capitalised into the real house price today, and hence, the effects shown later can be thought of as an upper bound given model uncertainty and its specification.

The change from fixed-rate mortgages to flexible-rate mortgages shown in figure (2) are implemented through changes in the markup, $\mathcal{M}_R$, on the lending rate for households, $R_t^{I,L}$, over the lending rate in the bank, $R_t^{I,L}$: $R_t^{I,L} = \mathcal{M}_R R_t^L$. In the steady state, the (annualized) lending rate to households is 5 per cent, and this is taken as the starting point in 2002; the average interest rate paid on fixed-rate mortgages. In 2003, as an example, around 30 per cent of the outstanding mortgage debt was already flexible-rate contracts the remaining being fixed-rate, see figure (2). Empirically, the spread between fixed and variable rate mortgages has been around $2\frac{1}{2}$ per cent. Hence, in the scenario for the year 2003, I consequently lower the markup such that the lending rate to households is $4\frac{1}{4}$ per cent.[13] I further change the parameter, θ_R, determining the average duration of the loan port folio for the banks such that it reflect how often interest rate can be changed.

To isolate effects from the sequence of changes made, I will in what follows present four scenarios:

1. The tax-rule for housing is changed from being paid on the actual value of the housing stock to the value on the steady-state value
2. The change from fixed to flexible-rate mortgage is introduced
3. The change from amortisation to interest-only loans is introduced
4. All changes are introduced (scenario (1) to scenario (3)).

The effect from these four scenarios are thus the sum of the changes to the housing market implemented each year. Changes to fiscal policy within a DSGE model also necessitate an instrument to stabilise public debt. I assume throughout that public debt is stabilised using lump sum taxation paid by patient and impatient households according to their relative size in the economy.

4. Results

I will in this section discuss the effects within the structural model of introducing the changes to the model. I will focus on the temporary effects in section (4.1). I will in section (4.2) look at the effects on the business cycle or volatility of key economic variables, while I will analyse long-term or permanent-term effects in section (4.3).

4.1. Temporary effects - the period 2002-2007

The immediate effects, defined as the effect during the period starting in the first quarter of 2002 and ending in the forth quarter 2006, are shown in figure (10) for the real house price, debt to households and construction-to-GDP ratio. Here I have also included the individual effects from the four scenarios to be better able to understand the main drivers of the results. I compare the results to actual data.

Overall, according to the model the changes in taxation of housing and the adoption of new mortgage contracts played a economically significant role both for the development in the real economy and on financial markets. Take the real house price first. From figure (10)

[13] $(70\% * 5\% + 30\% * \left(5\% - 2\frac{1}{2}\%\right)$ equals $4\frac{1}{4}\%)$.

it can be seen that the model in fact explains too much of the developments in the real house price through the period 2001-04. From around 2004-Q2 the real house price rose too much to be explained by the change in taxation and the adoption of new mortgage contracts.

On the financial side, the debt-to-GDP ratio for households would have been almost constant. The main economic mechanisms were explained in section (2): I.O loans and flexible-rate loans make borrowing cheaper and relaxes the collateral constraint fueling demand for both housing and consumption for impatient households. Taxation of housing alters relative after-tax prices of housing versus consumption goods.

In figure (11) is shown the effects as deviations from the initial steady-state. I interpret these deviations as gaps; differences between the movements in the variables away from their steady states. For the house price gap, the difference between the real house price and its steady-state, is 8 percentage points. Depending on the method to detrend the house price, the observed real house price gap was between 3-5 percentage points in 2004 and around 15 percentage points in 2005. Hence, the model can explain around half the real house price gap during this period. In 2006 the observed gap widens to close to 30 percentage points; the model can thus not explain the subsequent movements in the real house price.[14] Naturally, this leaves a large part of the gaps unexplained. To explain this I stress that I look at the impact of the two structural changes to the Danish housing market and nothing more; a lot of shocks hit the Danish economy during the period under study. In Pedersen (2016) the real house price is decomposed into structural shocks. There I find that both foreign shocks, like demand for Danish exports and monetary policy, productivity shocks etc., contributed to the house price gap during the period under study. Further, housing preference shocks played a large part. These shocks can be interpreted as the consumers getting more utility of a given level of housing, or that given fundamentals the households demanded more housing. This is thus a sign of a house price bubble.

As expected the increase in the real house price stimulates construction pushing the construction-to-GDP ratio up by 1 percentage point. The observed ratio was just above 6 per cent increasing from around 4. Hence, the scenarios can explain almost half the increase in the construction-to-GDP ratio. GDP is also stimulated pushing the output gap in the model up by around 0.6 percentage points in 2004, see figure (12). The actual output gap was as an example 0.25 percentage points in 2004 and 4.5 percentage points in 2006, see as an example Andersen and Rasmussen (2011). Hence, the changes in the scenarios can explain around 1/3 of the actual output gap. Total private consumption can explain part of this increase in the output gap: Total private consumption is 0.5 per cent above its steady state in 2004 in response to the changes considered in this study. The aggregate response of consumption covers the fact that consumption of patient households falls, while the consumption of impatient households increases due to the relaxation of the borrowing constraint.

Looking at the individual scenarios, the main driver has, according to the model, been

[14] In figure (10) is shown the detrended real house price using a linear trend and normalising the serie to 1 in 2001.

the changes to taxation. This is so both for the financial variables, the real house price and debt, as well as for the real variables, GDP, construction and consumption. The reason why taxation plays such a large role is that it affects the user cost of housing for *both* households in combination with the collateral constraint. Lower taxes on housing wealth cannot be smoothed for the impatient households. These would, if they could, try to smooth the extra lump sum tax payments needed to stabilise public debt.

The introduction of I.O loans contributed significantly to the build-up of debt and consumption in the late part of the period under study. The reason why I.O. loans affect debt relatively strongly is due to their direct effect on the user cost of borrowing, which falls, as discussed in section (2.1).

Summing up with regards to the business-cycle effects of introducing new mortgage contracts and a tax-freeze on housing wealth during 2002-08, the output gap would have been around 1/3 smaller, and the real house price gap would have been almost half the size. Further, the household debt-to-GDP ratio would have been almost constant during that period. The main driving force has been the tax freeze.

4.1.1 Reflections on the size of the temporary effects

It is perhaps surprising that the introduction of new mortgage contracts, according to this analysis, has not had larger effects on the real and financial economy, and smaller effects than found in Dam et al. (2011). As presented in section (2), the model extension which has the ability to make the effects large is the collateral constraint and thus the financial accelerator on the household side of the model. As shown in Iacoviello (2005), this mechanism has as an example the ability to match data for the wealth effect from housing on consumption for the U.S. But consumption is much lower for Denmark. Hence, the spill-overs from the housing market to the real economy is relatively smaller in the model.

Further, rational expectations, forward looking behaviour and general equilibrium makes it relatively more challenging to model large effects compared to the traditional macroeconometric models; as an example lower taxation of housing wealth needs to be funded in the model used in this study. This lowers the effects. And lower margins in the banking sector, due to the switch from fixed- to flexible-rate contracts, mean lower dividend payments to the owners of the banks, the patient households, which consequently lower their consumption.

With regard to the results found in Dam et al. (2011) it is finally worth mentioning that it their study it was assumed that without the changes made to the structure of the Danish housing market, the non-explained part of the developments in the real house price would not have happened.

4.2. Effects on the business-cycle

In this section, I analyse the effect of the policy changes on the business cycle defined as the volatility of the endogenous variables in the model. To this end, I calculate the coefficient of variation, the standard deviation divided by the mean, of a subset of the macroeconomic variables in the model before any changes were made to the market for housing in Denmark in 2002. I compare these with the coefficient of variation calculated at the new steady-state reached after implementation of the scenarios above. In what follows, I will calibrate the parameters to the values estimated in Pedersen (2016), which is especially so for the estimated standard deviations of the innovations to the structural shocks. The assumption is that the structural shocks estimated in Pedersen (2016) are truly structural and do not change in any economically significant way in the different regimes.

I consequently use coefficient of variation as a measure of volatility or "uncertainty". The standard deviations reflect the (log) changes of each variable from its respective steady state. The mean is the steady state. To calculate the standard deviations, I simulate the models' structural shocks using the standard deviations of the innovations as estimated in Pedersen (2016). I note that I keep these innovations fixed in all the scenarios such that it is not the size of the shocks that changes but the transmission mechanism in the model. In the case of taxation, I simply change the tax rule such that the tax rate and the base are kept constant at their steady state value in 2002.

I will focus on the case in which all changes are made to the housing market. For comparison and for analysing the effects from the different changes, I will also calculate the standard deviations in the other three scenarios presented in section (3). The results are shown in table (7).

Compared to the case before 2002, the real house price can be expected to vary by close to 20 per cent more at the new steady state. This change in the volatility can be attributed mostly to the change in taxation and the introduction of flexible-rate mortgage contracts. I will in the section to come explain in greater detail why flexible-rate mortgage contracts affect the volatility of house prices. With regard to taxation the reason why house prices vary by more under a tax freeze is that tax payments do not change with the real house price, and hence, an automatic stabiliser is taken away.

Naturally, the greater volatility of house prices also affects residential investments, which also have become around 20 per cent more volatile. This also affects the volatility of debt to households, which has become 25 per cent more volatile. Private consumption can in the future be expected to move by 5 per cent more, which is mainly driven by the consumption of the impatient households'; they are exposed to a collateral constraint, which has now become more volatile.[15]

[15] It is perhaps a bit surprising that the volatility of GDP does not change by more. Public consumption and exports are respectively 26 and 38 per cent of steady-state output, but the volatility of exports naturally does not change much in response to the changes in the economy, and public consumption is exogenous. Further, investments and consumption for the entrepreneur are small parts of output in steady state and their volatility

Finally, in table (7), I also calculate the relative standard deviation with respect to GDP. This is another measure of volatility in the economy. As can be seen, both the real house price, construction and especially household debt varies more compared with GDP after the changes to the housing market than before.

4.2.1 Effects on the business cycle – the role played by variable-rate contracts

While the change from fixed to flex-rate mortgage contracts has not had large immediate effects on the economy, it does contribute significantly to the change in the second-order moments. The explanation is that with a larger proportion of flexible-rate mortgage contracts in the economy, shocks which propagate through interest rates propagate stronger, and the flex-rate mortgage rate adjusts quicker to shocks.

This result can perhaps on the outset seem counter-intuitive. The shocks and parameters are estimated on Danish data for a period in which the Danish business cycle has followed closely the business cycle in the euro area, and therefore Denmark has on average imported a monetary policy stance, which has been close to what the Danish economy would call for in terms of stabilisation of inflation and output. Based on this insight, a higher degree of interest-rate sensitivity should, all else being equal, be stabilising and not destabilising for the Danish economy as shown here. However, this insight only concerns the imported changes in interest rates and not domestically generated shocks. Even though the monetary policy rate is imported from the euro area, the interest rate faced by borrowers on their mortgage contracts does not vary one-to-one with the imported monetary policy rate and it is not orthogonal to domestic shocks.

In detail, the lending rate faced by the households and firms is determined by the banks in the economy, and through the frictions in the model, specifically the capital ratio, all shocks which affects the borrowing decision for banks will effect lending rates. The wholesale branch of the bank chooses the overall amounts of lending and deposits of the bank so as to maximize profits. In addition, the wholesale branch takes into account a capital requirement imposed on banks: In any given period, it is costly for a bank to deviate from a target value $\kappa^B > 0$ for the bank's capital-to-assets ratio, $K_t^B\left(j^b\right)/B_t\left(j^b\right)$. The first-order condition yields:

$$R_t^L = R_t^D - \Phi^B\left(\frac{K_t^B}{B_t} - \kappa^B\right)\left(\frac{K_t^B}{B_t}\right)^2. \tag{12}$$

The parameter $\Phi^B > 0$ is the cost of deviating from the capital-ratio, κ^B, R_t^L denotes the gross interest rate charged by the wholesale branch on the loans it makes to the loan branch, and R_t^D is the gross interest rate paid by the wholesale branch on the funds it receives from the deposit branch.[16] In equilibrium that interest rate is equal to the ECB monetary policy rate.

does not change much; the changes on the market for housing do not spill-over to that part of the economy. Also, both construction and private consumption have a non-significant import content. Hence, although these variables become relatively more volatile, output is affected relatively little.

[16]Here is considered a symmetric equilibrium, such that I can continue with a representative bank in what

Condition (12) show that if the capital-to-asset ratio (or inverse leverage ratio) of the bank falls short of its target ratio, the wholesale branch will charge a lending rate that exceeds the deposit rate, R_t^D, at which it remunerates its deposit branch so as to increase its capital ratio. The further away the actual capital ratio is from the target ratio and the more costly it is to do so, determined by the parameter Φ^B, the higher the spread. Equation (12) can therefore be interpreted as a loan supply schedule: When loans, B_t, increase, the capital-to-asset ratio falls below target, κ^B, and the bank is induced to raise the lending rate to balance the extra income from giving the loan with the cost of deviating from the optimal capital ratio.

The monetary policy rate imported from the euro zone affects the first term one-for-one, R_t^D.[17] But all shocks which affect either desired borrowing or bank capital will affect the second term in expression (12), $\Phi^B\left(\frac{K_t^B}{B_t}-\kappa^B\right)\left(\frac{K_t^B}{B_t}\right)^2$, lending rates to bank-lending branches or mortgage institutions, and thus affect the interest rate faced by borrowers in the economy. And clearly, the more these lending rates respond to shocks, the greater are the effects on the cost of borrowing.

4.3. Permanent effects

I lastly look at the permanent effects from the changes to the structure of the housing market on both real and financial variables. I define permanent effects as the difference between the value of a variable in the model in the old steady state with respect to its value in the new steady state when reached. The new steady state must therefore be understood as the point in time in the future where all agents have made their optimal decisions and the economy has fully reacted to these changes. These values can be found in table (7).

I point to the following observations. The real house price is around 3 per cent higher in the new steady state and construction has increased by almost 10 per cent. Further, debt to households has increased by 20 per cent. This is due to the increasing demand for housing due to the relaxation of the collateral constraint, which has made the impatient household able to buy more housing financed by borrowing – the financial accelerator on the household side of the model, and due to taxation, which has made the patient household increase their purchases of housing. Equilibrium has been restored through an increase in the price which stimulates construction. In the end, consumption is almost constant in the new steady state for impatient households: They pay fewer taxes on their housing stock and more interest and amortisation on a now higher debt level. Consumption for patient households falls. This can be explained by crowding out: GDP increases by a little, while construction increases by a lot taking resources away from non-durable consumption to consumption of housing, and patient households are the providers of funds to the banking sector.

follows.

[17]Here for simplicity I disregard a small risk premium on the Danish net foreign asset position needed to ensure stationarity of that variable.

Investments do not change in the new steady state. Even though the entrepreneurs face slightly higher lending rates, which all else being equal push borrowing and investments down, they can also post more collateral due to higher land rents. In total, these two effects effectively cancel out.[18]

The main driver for first-order moments has been taxation of housing wealth, and this is also so for the permanent effects discussed here except for I.O. loans on especially debt. The adoption of interest-only contracts have pushed borrowing up by 7 per cent; in the model the impatient household used the lower borrowing costs to take on more debt and thus come close to satisfying its desire to consume more now relative to the future.

5. Reflections outside the model and policy

As emphasized previously, according to the model GDP is not affected greatly by the changes to the market structure for housing. However, this is likely to be a lower bound for the estimate of these effects. As shown in this study both the volatility in debt and the real house price can be expected to be higher in the future after the introduction of new mortgage contracts and the tax freeze. And large swings in asset prices a priori lead to increases in default rates as house prices and economic activity falls after negative shocks to the economy. In a model with household default and/or bank and/or default in the production sector, this can be expected to lead to larger swings in output.

Should the introduction of new loan types have been postponed either to after a bust or indefinitely? In practice, though costly, it was possible to get interest-only loans before 2002 in Denmark. That could be done by refinancing the mortgage contracts whenever the value of the debt was below the LTV ratio. As such, the introduction of interest-only loans can be regarded as "legalising" an already widely used practice with the added value for the households that is has become cheaper. What the model does not capture is that the amount not used to pay principal on an interest-only loan can be used to pay down other types of debt, which often are more expensive, as they are can not or only to a lesser extend be used as collateral, such as credit card debt, student loans, or car loans. And interest-only loans can further play the role as a buffer to unexpected temporary shocks to income like spells of unemployment. Neither of these effects are modelled here.

I have shown that within the model framework, the introduction of new mortgage contracts and a tax freeze have had profound consequences for the output gap and the way shocks propagate through the economy. I will not consider a fully-fledged welfare analysis, as the model leaves out the costs of higher debt; that is there are no defaults in the model. There are consequently no costs of having a "high" debt. Also, the results can not be used as a case against financial liberalisation, as the welfare implications of introducing the

[18]Notice that if I had assumed that patient households were the land owners, then investment would fall in the economy and there would have been a traditional crowding-out of real investments from higher residential investments. I refer again to the paper Pedersen (2016) for a full presentation of the model used here.

contracts are not analysed in the current study.[19] Hence, though the model is quite large and captures many aspects of the Danish economy, it is not at its current stage suited to discuss the normative implications of the introduction of these new loan contracts.

Should Danish policy makers have responded to the build-up of debt and rising house prices? The question about pricking asset bubbles through leaning against the wind, or clean it up after the bubble has burst, remains unanswered. In a model which shares some characteristics of the model in the current study including long-term debt contracts, Gelain et al. (2015) study the question about leaning against the credit cycle. They show that a policy in which the monetary policy rate responds to household debt induces equilibrium indeterminancy. I leave an analysis of macro-prudential tools within the current setup to future research.

6. Conclusion

In this paper I have asked: *What are the effects from the introduction and adoption of new types of mortgage contracts and a freeze on the nominal tax paid on housing wealth on the real house price, household debt, and real GDP during the period 2002-08?* I found that the two changes to the structure of the housing market in Denmark significantly affected both the financial side and the real side of the Danish economy during the period of study. Specifically, the combined effect of the changes can explain up to around $\frac{1}{3}$ of the real house price gap and 15-40 per cent of the output gap. I further showed that these changes have had implications for the business cycle – equivalent shocks propagate stronger after the changes to the housing market. Using the coefficient of variation of the changes in the variables in the model as a measure of the business cycle, the real house price and construction have become respectively 25 per cent and 15 per cent more volatile, while debt to households has become 50 per cent more volatile.

This paper has also pointed to a need for further research. Firstly, the term premium is not explicitly modelled; it is simply assumed to be the markup over the whole-sale lending rate. It therefore neither reflects credit nor duration or liquidity risk. This leads to the second point: There are no defaults in the model. Hence, the model can not be used to study the welfare effects of financial stability, e.g. high debt levels. Within the model used in this paper, I could only address the volatility of high leverage, but not e.g. risk of default and its dependence on the debt level. Thirdly, future research could analyse microfoundations for the choice of mortgage contracts and include such a framework within a DSGE model.

[19] As an example, it could be that some agents would like to carry more risk and finance housing purchases using, say, flexible-rate contracts due to differences in risk aversion.

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7. Tables and figures

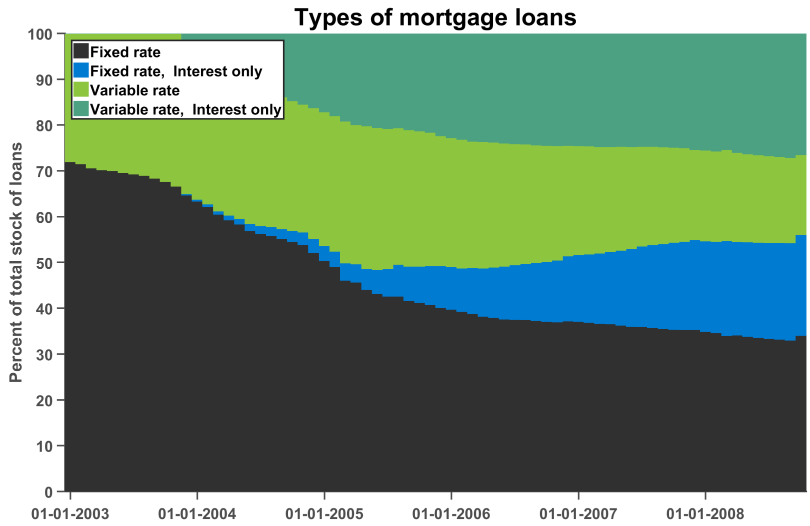

Figure 1: **Changes in mortgage loans**
In the figure is shown the effective tax rate on housing together with the real house price. Source: Danmarks Nationalbank.

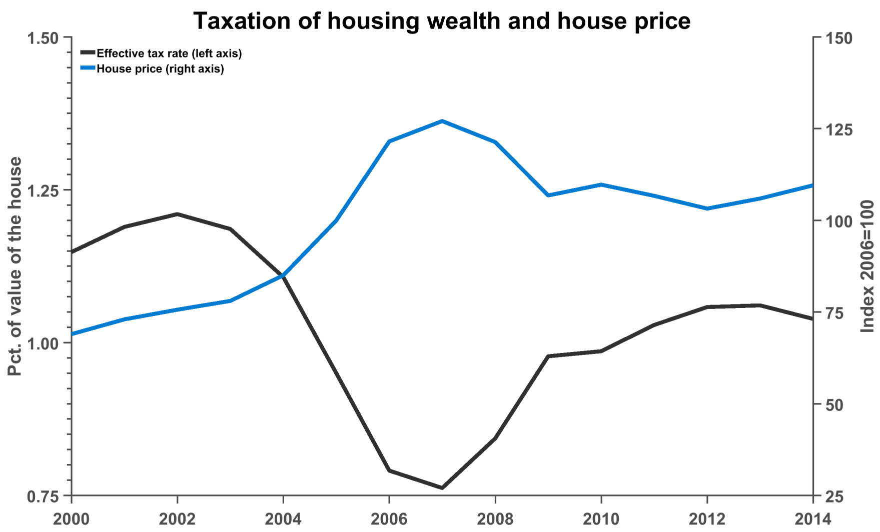

Figure 2: **Changes in mortgage loans**
In the figure is shown the development in the stock of outstanding mortgage contracts. Source: Danmarks Nationalbank.

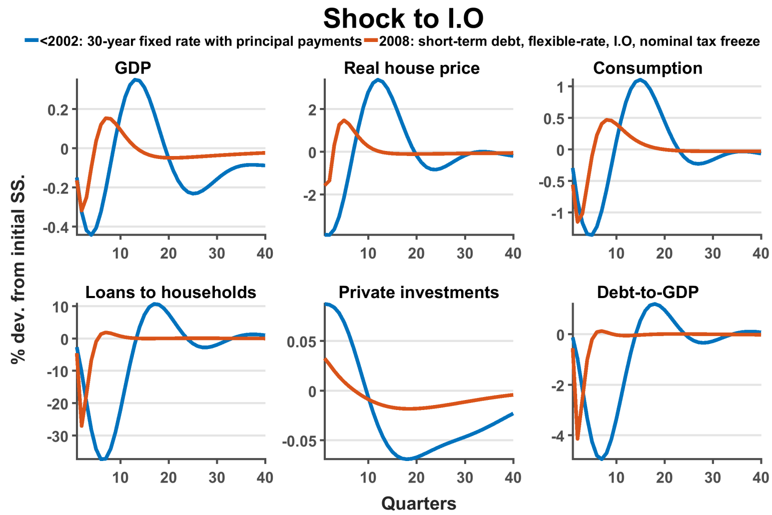

Figure 3: **Impulse response functions**
The figures show impulse response functions for a subset of the macroeconomic variables with respect to shock to amortisation rate. The size of the shock is held constant. The parameters are calibrated to the estimated and calibrated parameters in Pedersen (2016) except for a subset of parameters to be able to define a 2002 calibration and a 2008 calibration. These two calibrations reflect respectively the structure for the housing market in 2002, with a 30-year fixed-rate mortgage contract with principal payments, and in 2008 with a flexible-rate, short-term debt contract and a nominal tax freeze.

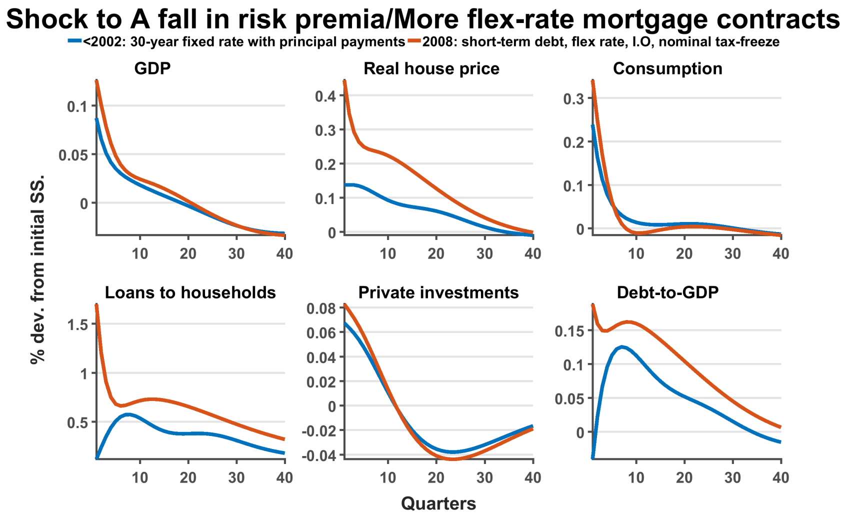

Figure 4: **Impulse response functions**
The figures show impulse response functions for a subset of the macroeconomic variables with respect to shock to the mortgage rate. The size of the shock is held constant. The parameters are calibrated to the estimated and calibrated parameters in Pedersen (2016) except for a subset of parameters to be able to define a 2002 calibration and a 2008 calibration. These two calibrations reflect respectively the structure for the housing market in 2002, with a 30-year fixed-rate mortgage contract with principal payments, and in 2008 with a flexible-rate, short-term debt contract and a nominal tax freeze.

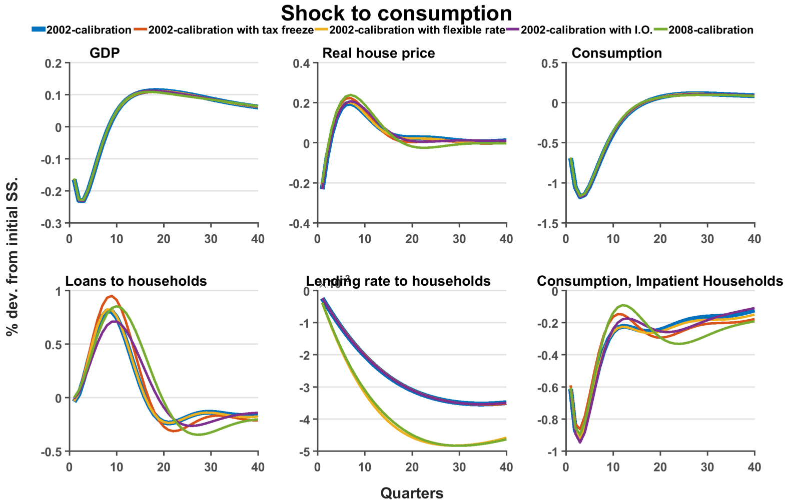

Figure 5: **Impulse response functions**
The figures show impulse response functions for a subset of the macroeconomic variables with respect to an intertemporal preference shock. I consider the following different calibrations of the model: (1) 2002 calibration: Long-term fixed-rate debt and no tax freeze. (2) 2002 calibration with tax freeze. (3) 2002 calibration with flexible-rate loans. (4) 2002 calibration with interest-only loans. (5) 2008 calibration: Short-term debt with flexible rate, tax freeze.

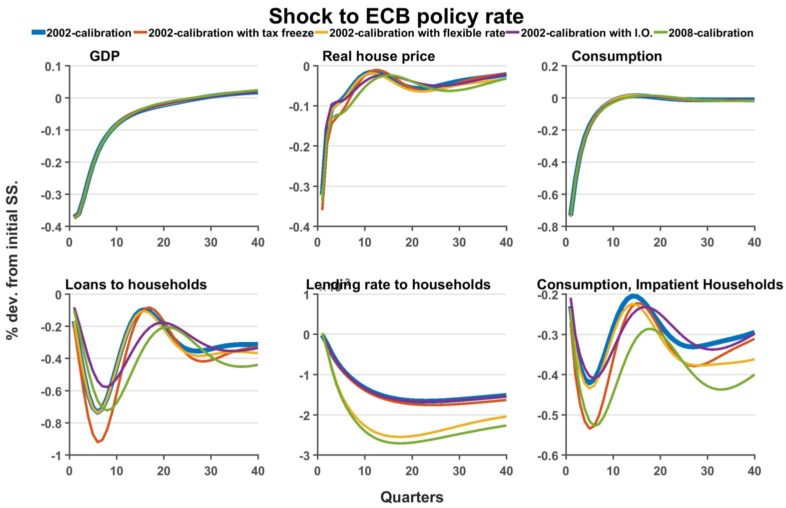

Figure 6: **Impulse response functions**
The figures show impulse response functions for a subset of the macroeconomic variables with respect to a shock to the ECB monetary policy rate. I consider the following different calibrations of the model: (1) 2002 calibration: Long-term fixed-rate debt and no tax freeze. (2) 2002 calibration with tax freeze. (3) 2002 calibration with flexible-rate loans. (4) 2002 calibration with interest-only loans. (5) 2008 calibration: Short-term debt with flexible rate, tax freeze.

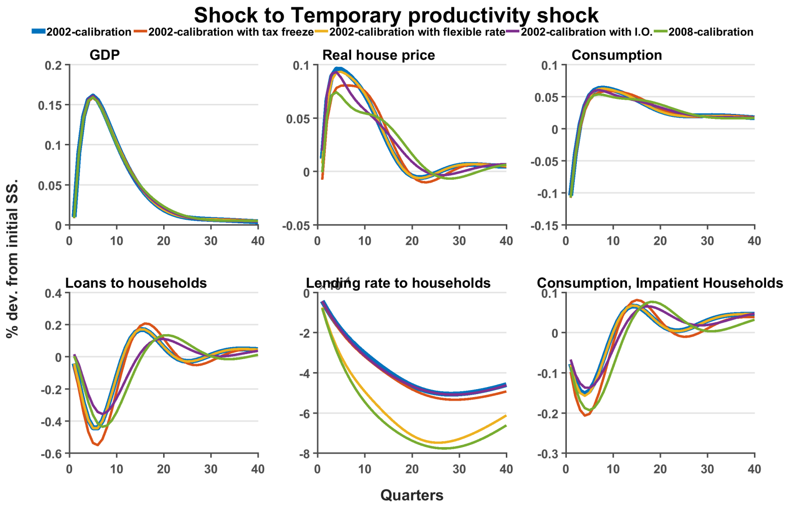

Figure 7: **Impulse response functions**
The figures show impulse response functions for a subset of the macroeconomic variables with respect to a temporary productivity shock. I consider the following different calibrations of the model: (1) 2002 calibration: Long-term fixed-rate debt and no tax freeze. (2) 2002 calibration with tax freeze. (3) 2002 calibration with flexible-rate loans. (4) 2002 calibration with interest-only loans. (5) 2008 calibration: Short-term debt with flexible rate, tax freeze.

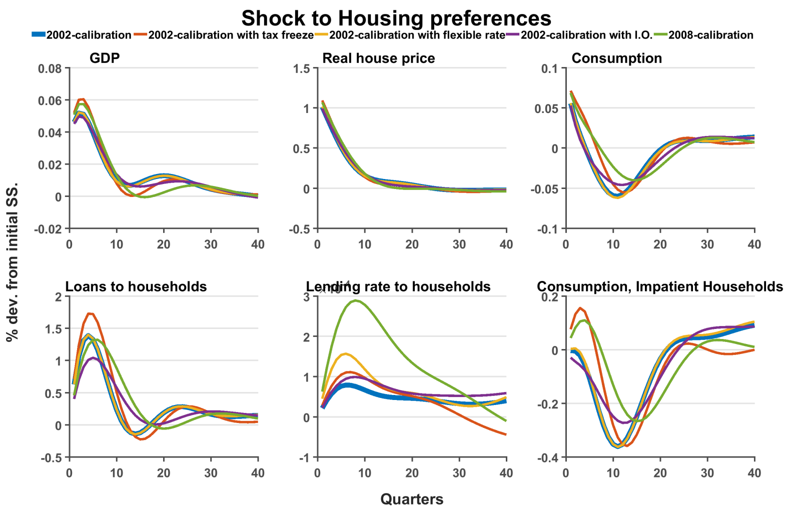

Figure 8: **Impulse response functions**
The figures show impulse response functions for a subset of the macroeconomic variables with respect to a housing preference shock. I consider the following different calibrations of the model: (1) 2002 calibration: Long-term fixed-rate debt and no tax freeze. (2) 2002 calibration with tax freeze. (3) 2002 calibration with flexible-rate loans. (4) 2002 calibration with interest-only loans. (5) 2008 calibration: Short-term debt with flexible rate, tax freeze.

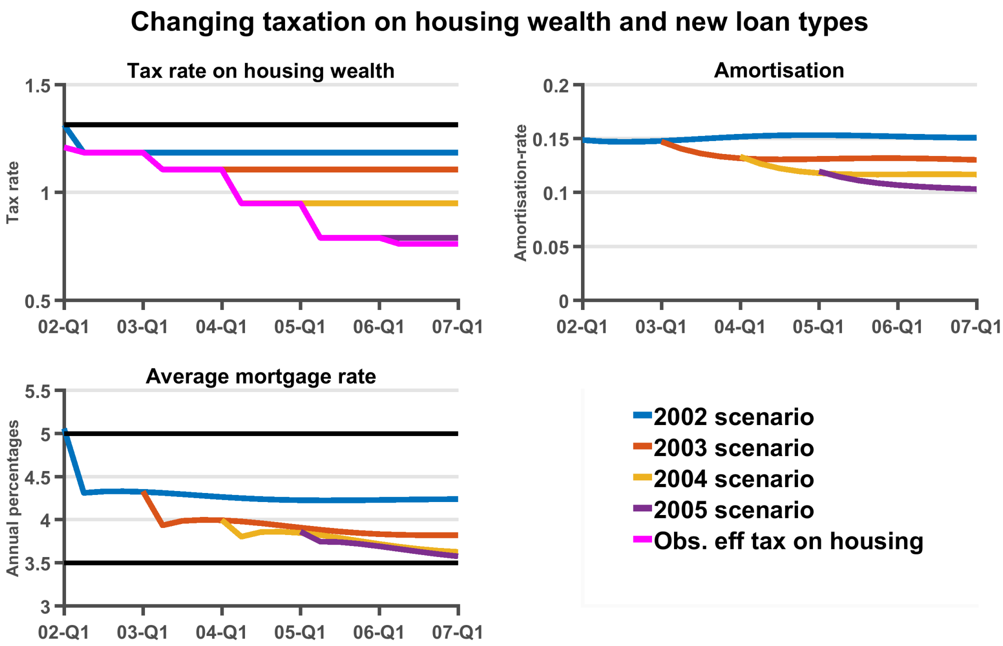

Figure 9: **Scenarios:** In this figure is shown the scenarios as presented in section (3).

Figure 10: **The effect on the real house price, construction and household debt**
In this figure is shown the effect from the scenarios presented in section (3) on the real house price, construction and household debt. Only the temporary effects are shown corresponding to the period under study – 2002-06. In the figures are also shown data. To be able to compare data with the model simulations the following calculations are done. For the real house price is shown detrended data normalised to 1 in 2000. The construction-to-GDP ratio is normalised in the model to 4 per cent as it was in data, while household debt-to-GDP ratio likewise is normalised to 50 per cent.

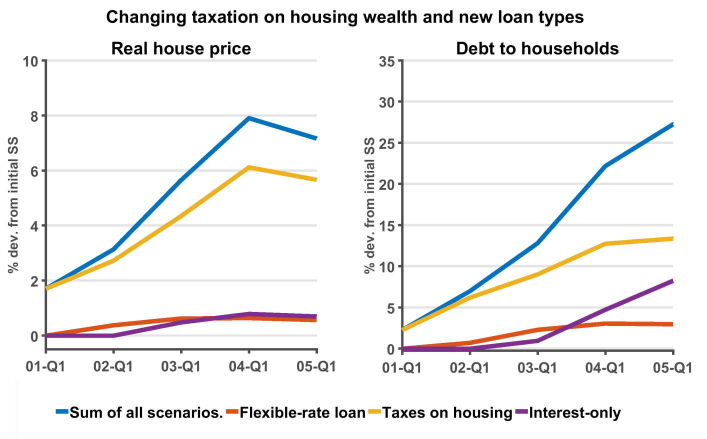

Figure 11: **The effect on the real house price and household debt**
In this figure is shown the effect from the scenarios presented in section (3) on the real house price and household debt. Only the temporary effects are shown corresponding to the period under study – 2002-06. In the figures are also shown data. To be able to compare data with the model simulations the following calculations are done. For the real house price is shown detrended data normalised to 1 in 2000. The construction-to-GDP ratio is normalised in the model to 4 per cent as it was in data, while household debt to GDP ratio similary is normalised to 50 per cent.

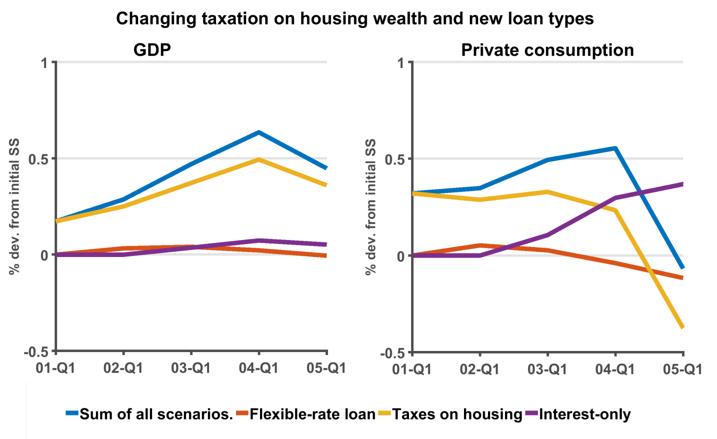

Figure 12: **The effect on the GDP and total private consumption**
In this figure is shown the effect from the scenarios presented in section (3) on GDP and total private consumption (sum of the consumption of patient- and impatient households and the entrepreneur). Only the temporary effects are shown corresponding to the period under study – 2002-06. In the figures are also shown data. To be able to compare data with the model simulations the following calculations are done. For the real house price is shown detrended data normalised to 1 in 2000. The construction-to-GDP ratio is normalised in the model to 4 per cent as it was in data, while household debt-to-GDP ratio is likewise normalised to 50 per cent.

Variable	$Co - Var(j)^{OLD}$	$Co - Var(j)^{NEW}$	Change (%)	$\frac{Co-Var(j)}{Co-Var(Y_t)}^{OLD}$	$\frac{Co-Var(j)}{Co-Var(Y_t)}^{NEW}$	$\Delta SS\%$
Scenario (1): Taxation						
GDP, (Y_t)	1.04	1.04	-0.34	1.00	1.00	0.15
Real house price, (Q_t^H)	1.15	1.29	12.35	1.10	1.24	3.00
Residential inv., (X_t)	3.01	3.28	8.98	0.13	0.14	9.25
Household debt, (B_t^I)	5.76	6.62	14.80	4.17	4.80	10.50
Total private cons., (C_t^{tot})	1.37	1.41	3.37	0.65	0.67	-0.50
Consumption, patient, (C_t^P)	0.81	0.82	1.65	0.29	0.30	-0.75
Consumption, impatient, (C_t^I)	4.80	5.29	10.26	0.39	0.43	0.25
Investment, (I_t^Y)	2.64	2.62	-0.89	0.35	0.35	0.10
Scenario (2): Fixed-to-floating						
GDP, (Y_t)	1.04	1.07	2.34	1.00	1.00	0.00
Real house price, (Q_t^H)	1.15	1.27	10.73	1.10	1.19	0.30
Residential inv., (X_t)	3.01	3.42	13.46	0.13	0.14	1.00
Household debt, (B_t^I)	5.76	6.54	18.61	4.17	4.73	3.00
Total private cons., (C_t^{tot})	1.37	1.38	1.30	0.65	0.64	-0.15
Consumption, patient, (C_t^P)	0.81	0.88	7.78	0.29	0.31	-0.50
Consumption, impatient, (C_t^I)	4.80	5.79	22.97	0.39	0.46	1.30
Investment, (I_t^Y)	2.64	2.77	5.02	0.35	0.36	-0.35
Scenario (3): I.O. loans						
GDP, (Y_t)	1.04	1.04	-0.41	1.00	1.00	0.00
Real house price, (Q_t^H)	1.15	1.13	-1.56	1.10	1.09	-0.10
Residential inv., (X_t)	3.01	3.03	0.59	0.13	0.13	-0.25
Household debt, (B_t^I)	5.76	5.45	-6.32	4.17	3.94	7.25
Total private cons., (C_t^{tot})	1.37	1.36	-0.03	0.65	0.65	0.00
Consumption, patient, (C_t^P)	0.81	0.81	-0.44	0.29	0.29	0.10
Consumption, impatient, (C_t^I)	4.80	4.86	2.71	0.39	0.40	-0.40
Investment, (I_t^Y)	2.64	2.63	-0.21	0.35	0.35	-0.10
Scenario (4): All changes						
GDP, (Y_t)	1.04	1.06	1.36	1.00	1.00	0.05
Real house price, (Q_t^H)	1.15	1.37	19.70	1.10	1.30	3.00
Residential inv., (X_t)	3.01	3.69	22.59	0.13	0.16	9.75
Household debt, (B_t^I)	5.76	6.92	23.55	4.17	5.01	21.50
Total private cons., (C_t^{tot})	1.37	1.43	4.53	0.65	0.67	-0.50
Consumption, patient, (C_t^P)	0.81	0.90	10.14	0.29	0.32	-1.25
Consumption, impatient, (C_t^I)	4.80	6.26	34.75	0.39	0.51	1.50
Investment, (I_t^Y)	2.64	2.75	4.07	0.35	0.36	0.00

Table 1: **Theoretical moments before and after changes to market structure for housing:** This table reports theoretical moments calculated in the model. "Co-Var" refers to the coefficient of variation (the ratio of the standard deviation to the mean). "Old" refers to the conditions for the housing market before the changes to the taxation of housing and the introduction of new mortgage loans. Public debt is stabilised using lump sum taxation in all the scenarios. In the calculation of the moments, the estimated standard deviation of the shocks are taken from Pedersen (2016). $\Delta SS\%$ refers to the difference between the steady state before the implementation of the scenario and after the new steady state has been reached. This is in the text referred to as the permanent effects of the policy changes. "New" refers to the model in which the changes in the scenarios are made, as described in text.

8. Appendix: Dynare equations

$$\varsigma^{HI}\,\mathcal{H}_t\,\frac{1}{H^I{}_t}+\frac{C_t}{\bar{C}}\,\beta^I\left(1-\delta^H\right)\lambda^I_{t+1}\,Q^H_{t+1}\,\frac{1}{d\Gamma^X_{t+1}}+\mu^I_t\,\Theta^I_t\,\frac{\Pi^{DK}{}_{t+1}\,Q^H_{t+1}}{R^{L,I}_t}\,\frac{d\Gamma_{t+1}}{d\Gamma^X_{t+1}}=\lambda^I_t\left(1+\tau^H_t\right)Q^H_t$$

$$\varsigma^{HI}\,\mathcal{H}_t\,\frac{1}{H^I{}_t}+\frac{C_t}{\bar{C}}\,\beta^I\left(1-\delta^H\right)\lambda^I_{t+1}\,\left(\left(1-WW\right)QHt\_hj_{t+1}+WW\,QHt\_A_t\right)\frac{1}{d\Gamma^X_{t+1}}$$

$$+\mu^I_t\,\frac{\Theta^I_t}{1+\Theta^I_t}\,\frac{\Pi^{DK}{}_{t+1}\,\left(\left(1-WW\right)QHt\_hj_{t+1}+WW\,QHt\_A_t\right)}{R^{L,I}_t}\,\frac{d\Gamma_{t+1}}{d\Gamma^X_{t+1}}=\lambda^I_t\left(1+\tau^H_t\right)Q^H_t$$

$$\lambda^I_t-\mu^I_t=\lambda^I_{t+1}\,\frac{C_t}{\bar{C}}\,\beta^I\,R^{L,I}_t\,\frac{1}{\Pi^{DK}{}_{t+1}\,d\Gamma_{t+1}}-\frac{C_t}{\bar{C}}\,\beta^I\left(R^{L,I}_t-1\right)\frac{\lambda^I_{t+1}\,\kappa^{RI}_{t+1}}{\Pi^{DK}{}_{t+1}\,d\Gamma_{t+1}}$$

$$\lambda^I_t-\mu^I_t=\lambda^I_{t+1}\,\frac{C_t}{\bar{C}}\,\beta^I\,R^{L,I}_t\,\frac{1}{\Pi^{DK}{}_{t+1}\,d\Gamma_{t+1}}-\frac{C_t}{\bar{C}}\,\beta^I\left(R^{L,I}_t-1\right)\frac{\lambda^I_{t+1}\,\kappa^{RI}_{t+1}}{\Pi^{DK}{}_{t+1}\,d\Gamma_{t+1}}-\frac{C_t}{\bar{C}}\,\beta^I\,\frac{\mu^I_{t+1}}{\Pi^{DK}{}_{t+1}\,d\Gamma_{t+1}\left(1+\Theta^I_t\right)}\left(1-\mathcal{A}_t\right)$$

$$B^I_t=\Pi^{DK}{}_{t+1}\,\frac{d\Gamma_{t+1}}{d\Gamma^X_{t+1}}\,H^I{}_t\,Q^H_{t+1}\,\Theta^I_t\,\frac{1}{R^{L,I}_t}$$

$$B^I_t=\frac{\Theta^I_t}{1+\Theta^I_t}\,\Pi^{DK}{}_{t+1}\,\frac{d\Gamma_{t+1}}{d\Gamma^X_{t+1}}\,H^I{}_t\,\left(\left(1-WW\right)QHt\_hj_{t+1}+WW\,QHt\_A_t\right)\frac{1}{R^{L,I}_t}+\frac{\frac{1-\mathcal{A}_{t-1}}{1+\Theta^I_t}\,B^I_{t-1}}{d\Gamma_t\,\Pi^{DK}{}_t}$$

The following four equations substitute for equation (61) in Pedersen (2016) for j=I:

$$q1R_t=B^I_t\,\lambda^P_t\,R^L_t+\beta^P\,\frac{C_t}{\bar{C}}\,\theta_p\,q1R_{t+1}$$

$$q2R_t=\lambda^P_t\,B^I_t+\beta^P\,\frac{C_t}{\bar{C}}\,\theta_p\,q2R_{t+1}$$

$$R^{\tilde{L},I}{}_t=\frac{epsiRt_t}{epsiRt_t-1}\,\frac{q1R_t}{q2R_t}$$

$$R^{L,I}_t=\left(\theta_p\,R^{L,I}_{t-1}{}^{1-epsiRt_t}+\left(1-\theta_p\right)R^{\tilde{L},I}{}_t^{1-epsiRt_t}\right)^{\frac{1}{1-epsiRt_t}}$$

$$Passtrought_t=B^I_t\,R^L_t\,\frac{1}{epsiRt_t-1}$$

$$\Xi_t=\left(R^{L,I}_t-1\right)B^I_t+\left(R^{L,E}_t-1\right)B^E_t-\left(R^D_t-1\right)D^P_t+D^*_t$$

$$\left(R^{INT}_t-1\right)-K^B_t\,\frac{\Phi^B}{2}\left(\frac{K^B_t}{B_t}-\kappa^B\right)^2-B^I_t\left(R^{L,I}_t-1\right)\frac{\Phi^{B,I}}{2}\left(\frac{R^{L,I}_t-1}{R^{L,I}_{t-1}-1}-1\right)^2-B^E_t\left(R^{L,E}_t-1\right)\frac{\Phi^{B,E}}{2}\left(\frac{R^{L,E}_t-1}{R^{L,E}_{t-1}-1}-1\right)^2$$

$$\Xi_t = \left(R_t^{L,I} - 1\right) B_t^I + \left(R_t^{L,E} - 1\right) B_t^E - \left(R_t^D - 1\right) D_t^P + D_t^* \left(R_t^{INT} - 1\right) - K_t^B \frac{\Phi^B}{2} \left(\frac{K_t^B}{B_t} - \kappa^B\right)^2 - Passtrought_t$$

The following four equations substitute for equation (61) in Pedersen (2016) for j=E:

$$q1RE_t = B_t^E \lambda_t^P R_t^L + \beta^P \frac{C_t}{\bar{C}} \theta_p \, q1RE_{t+1}$$

$$q2RE_t = \lambda_t^P B_t^E + \beta^P \frac{C_t}{\bar{C}} \theta_p \, q2RE_{t+1}$$

$$R^{\tilde{L},I}{}_t = MarkupER \frac{q1RE_t}{q2RE_t}$$

$$R_t^{L,E} = \left(\theta_p \, {R_{t-1}^{L,E}}^{1-\epsilon_R} + \left(1 - \theta_p\right) {R^{\tilde{L},I}{}_t}^{1-\epsilon_R}\right)^{\frac{1}{1-\epsilon_R}}$$

$$\varsigma^{HP} \frac{\mathcal{H}_t}{H^P{}_t} + \frac{1}{d\Gamma_{t+1}^X} Q_{t+1}^H \lambda_{t+1}^P \beta^P \frac{C_t}{\bar{C}} \left(1 - \delta^H\right) = Q_t^H \lambda_t^P \left(1 + \tau_t^H\right)$$

$$\varsigma^{HP} \frac{\mathcal{H}_t}{H^P{}_t} + \frac{1}{d\Gamma_{t+1}^X} \left(\left(1 - WW\right) QHt\_hj_{t+1} + WW \, QHt\_A_t\right) \lambda_{t+1}^P \beta^P \frac{C_t}{\bar{C}} \left(1 - \delta^H\right) = Q_t^H \lambda_t^P \left(1 + \tau_t^H\right)$$

$$\frac{\mathcal{A}_t}{\bar{\bar{\mathcal{A}}}} = (1 - AA)^{KA} + (1 - \mathcal{A}_{t-1}) \frac{B_{t-1}^I}{B_t^I \, d\Gamma_t \, \Pi^{DK}{}_t} \left(\mathcal{A}_{t-1}^{AA} - (1 - AA)^{KA}\right)$$

$$\frac{epsiRt_t}{epsiRs} = \left(\frac{epsiRt_{t-1}}{epsiRs}\right)^{rhoEpsiR} exp\left(epsiEpsiR_t\right)$$

$$QHt\_hj_t = Q_t^H$$

$$QHt\_A_t = QHt\_hj_t \, LL + (1 - LL) \, QHt\_A_{t-1}$$

$$\left(\frac{p^{IX}}{p}\right)_t = Q_t^H \left(1 - \alpha^X\right) \frac{X_t}{\tilde{I^H}_t} \left(1 - S_t^X - S_t^{1,X} \frac{I^H{}_t}{I^H{}_{t-1}}\right) + \left(1 - \alpha^X\right) \left(Q_{t+1}^H\right) \frac{1}{d\Gamma_{t+1}} \beta^P \frac{C_t}{\bar{C}} \frac{\lambda_{t+1}^P}{\lambda_t^P} \frac{X_{t+1}}{\tilde{I^H}_{t+1}} S_{t+1}^{1,X} \left(\frac{I^H{}_{t+1}}{I^H{}_t}\right)^2$$

$$\left(\frac{p^{IX}}{p}\right)_t = Q_t^H \left(1 - \alpha^X\right) \frac{X_t}{\tilde{I^H}_t} \left(1 - S_t^X - S_t^{1,X} \frac{I^H{}_t}{I^H{}_{t-1}}\right) + \left(1 - \alpha^X\right) \left(\left(1 - WW\right) QHt\_hj_{t+1} + WW \, QHt\_A_t\right) \frac{1}{d\Gamma_{t+1}} \beta^P \frac{C_t}{\bar{C}} \frac{\lambda_{t+1}^P}{\lambda_t^P} \frac{X_{t+1}}{\tilde{I^H}_{t+1}} S_{t+1}^{1,X} \left(\frac{I^H{}_{t+1}}{I^H{}_t}\right)^2$$