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A new approach to modelling banks' equity volatility: Adding time-to-maturity jumps

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Abstract
We add discrete jumps in the time-to-maturity of a firm's debt to the model of Engle and Siriwardane (2015), such that changes in equity volatility can be explained by the volatility of the firm's assets, its market leverage and investors' perception of the time-to-maturity of the firm's debt. For banks a shortening of the time-to-maturity can be interpreted as investor worries over whether the bank will experience funding or solvency problems. In line with this we find that calm periods generally coincide with a long perceived time-to-maturity and that crisis periods, where some banks experienced problems of different sorts, are characterized by a short perceived time-to-maturity. Changing perceptions of the time-to-maturity are broadly consistent with changes in the probability of default extracted for a structural Merton model, suggesting that the information in the perceived time-to-maturity clearly relates to the solvency of the firm, but that other factors, e.g. liquidity, also play a role. For financial researchers the model has several advantages as it allows one to disentangle which underlying factors affect changes in equity volatility as well as giving an indication of how the financial markets view the robustness of a bank when it comes to risks that affect both the solvency and liquidity situation of the bank.

Resume
Vi tilføjer spring i løbetiden for firmaers gæld i modellen foreslået af Engle og Siriwardane (2015). Ændringer i aktievolatiliteten forklares derfor med volatiliteten i firmaets aktiver, firmaets gearing og investorernes syn på gældens løbetid. En kort løbetid kan for banker fortolkes som bekymring fra investorerne for refinansieringen af bankens gæld og/eller bankens solvens. Konsistent med denne fortolkning finder vi, at rolige perioder generelt er sammenfaldende med lange løbetider, hvorimod kriseperioder generelt er karakteriseret ved korte løbetider. Ændringer i løbetiden er til en vis grad sammenfaldende med ændringer i *probability of default* udledt fra en Merton model, hvilket tyder på, at informationen i løbetiden er relateret til firmaets solvens, men at også andre ting, fx likviditet, er vigtige komponenter. Modellen har adskillige fordele, idet den dekomponerer aktievolatiliteten i de underliggende faktorer, og samtidig giver en indikation af de finansielle markeders syn på bankens robusthed, hvad angår solvens og likviditet.

Key words
Financial stability; Financial sector; Financial risks; Models

JEL classification
C32; G01; G12; G21; G32

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Abstract

We add discrete jumps in the time-to-maturity of a firm's debt to the model of Engle and Siriwardane (2015), such that changes in equity volatility can be explained by the volatility of the firm's assets, its market leverage and investors' perception of the time-to-maturity of the firm's debt. For banks a shortening of the time-to-maturity can be interpreted as investor worries over whether the bank will experience funding or solvency problems. In line with this we find that calm periods generally coincide with a long perceived time-to-maturity and that crisis periods, where some banks experienced problems of different sorts, are characterized by a short perceived time-to-maturity. Changing perceptions of the time-to-maturity are broadly consistent with changes in the probability of default extracted from a structural Merton model, suggesting that the information in the perceived time-to-maturity clearly relates to the solvency of the firm, but that other factors, e.g. liquidity, also play a role. For financial researchers the model has several advantages as it allows one to disentangle which underlying factors affect changes in equity volatility as well as giving an indication of how the financial markets view the robustness of a bank when it comes to risks that affect both the solvency and liquidity situation of the bank.

1 Introduction

The last ten years have been characterized by several periods of high equity volatility - especially for banks, see Figure 1.1. This paper attempts to explain these changes in equity volatility by changes in the following three factors: asset volatility, market leverage and time-to-maturity.[1] The first two factors are well-known and appear in popular models such as the Merton model, see e.g.

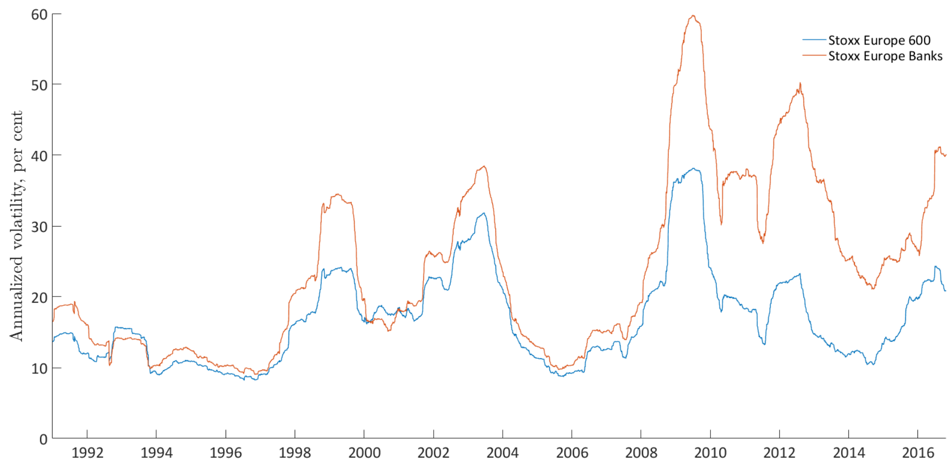
[1] Market leverage is here defined as D/E, where D is the book value of debt and E is the *market value* of equity, i.e. the value of the firm's outstanding shares.

Merton (1974). However, the last factor, time-to-maturity, is often assumed to take on a constant value in the literature even though changes in the time-to-maturity of a firm's debts also has an impact on equity volatility. To illustrate this, one can consider two firms that are similar in all aspects, with the exception of when they are expected to liquidate. In this case the firm with the shortest time-to-liquidation (time-to-maturity and time-to-liquidation will be used interchangeably throughout the paper) will have a higher equity volatility.[2] This is because the probability of being solvent when the firm is liquidated is highest for the firm with the shortest time-to-maturity. This implies that a shock to the current asset value will have a higher impact on the equity value for the firm with a short time-to-maturity. That is, equity volatility will be higher for a given level of asset volatility the shorter the time-to-liquidation.

Intuitively it makes particularly good sense to allow for changes in the time-to-maturity when modelling the equity volatility of banks. A unique feature of banks' business models is that they engage in both liquidity and maturity transformation. That is, they fund assets (typically illiquid loans with a medium- to long-term maturity) with liabilities that can be rather short-term and liquid, such as deposits and short-term wholesale funding (e.g. lending through the interbank market and commercial papers). As illustrated in Diamond and Dybvig (1983) this liquidity transformation can result in self-fulfilling panics and bank runs. This implies that banks have liabilities where the financial markets' perception of the time-to-maturity can change rapidly, even if the liability structure is unchanged. If for instance the deposits and short-term wholesale funding of a given bank are considered stable sources of funding (meaning that depositors are not believed to extract their deposits and it is possible to roll over short-term market-based funding) the liabilities of the bank can be considered as having a long time-to-maturity. On the other hand, if the bank is facing liquidity problems and is unable to roll over its short-term debt or is experiencing a withdrawal of deposits, the liabilities of a bank can be perceived by investors as having a short time-to-maturity. This "run" risk could in principle occur even if the bank is solvent, which makes it a highly relevant risk to model as it could determine whether policymakers should provide emergency liquidity or not, cf. e.g. Morris and Shin (2016). Besides funding problems sudden changes in the investors' perception of the time-to-maturity can also arise if a bank is forced to undergo a recapitalization by the authorities, which would take place if the authorities deem the bank to be undercapitalized, and will typically lead to a dilution or a complete write-down of existing shareholders. The bottom line is, that one can imagine scenarios where investors' perception of the time-to-maturity of a bank's liabilities can change quite rapidly and drastically. This can be driven by both pure

[2]Note that the firms must be solvent for this to be the case.

Figure 1.1: Equity volatility



Note: The volatility is calculated by taking the standard deviation on a 1-year rolling window of log returns.
Source: Bloomberg.

liquidity-related issues as well as solvency-related issues.[3] Given the prevalence of both liquidity- and solvency-related problems in banks during the financial crisis, this suggests that decreases in the perceived time-to-maturity can be part of the explanation of why the volatility on bank shares increased so much during this period, cf. Figure 1.1.

In order to test this hypothesis, one needs to construct a model that explicitly allows for changes in the perceived time-to-maturity of a bank's liabilities. This paper attempts to achieve this by expanding the Structural GARCH model introduced in Engle and Siriwardane (2015). To simplify, we only allow the perceived time-to-maturity to switch between two states. In one state the bank is considered safe, i.e. it is able to roll over its short-term debt and it is not subject to runs or solvency problems such that the perceived time-to-maturity on its liabilities is rather long. In the other state financial markets fear that the bank will be liquidated shortly, e.g. due to liquidity problems or a risk of a forced recapitalization by the authorities which will result in a short perceived time-to-maturity. The model estimates from the empirical applications are in line with what one would expect; namely that the rise in equity volatility during both the financial crisis and the European sovereign debt crisis was partly due to a decrease in investors' perception of the time-to-maturity on banks' liabilities.

[3] In general the two types of risk are difficult to consider in isolation and disentangle as e.g. liquidity risks are likely to arise when investors fear that the firm is becoming insolvent.

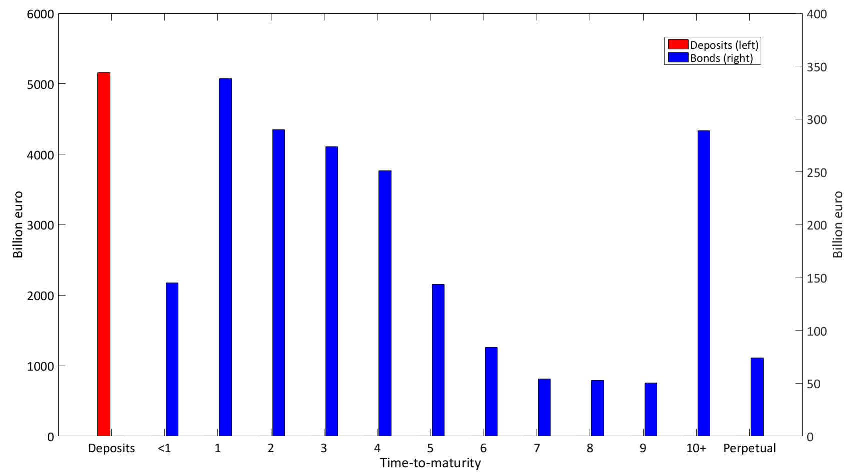
The paper proceeds with a literature review in section 2. Section 3 introduces the econometric model used to explain equity volatility. Subsequently section 4 and 5 introduces the data on which the model is estimated along with results. Section 6 concludes.

2 Literature and our idea

The literature offers a range of different explanations for the high equity volatility observed during the crisis. We focus on banks and present three different views of why banks' equity volatility can change. First, based on classic structural models, such as Merton (1974), where the unobserved asset volatility is backed out from observed equity volatility, one would conclude that the asset volatility exploded. The reason being that the only source of a higher equity volatility is higher asset volatility. Second, a range of econometric models, including Glosten et al. (1993), would attribute the rise in equity volatility to a series of negative returns resulting in a rise in market leverage, i.e. the leverage effect documented by Black (1976). Higher leverage implies that the equity becomes more risky and therefore also more volatile - since negative returns increase the leverage they also increase volatility. Third and last, the use of Markov switching regimes in GARCH models can introduce jumps in volatility consistent with the volatility clusters observed in practice, see e.g. Haas et al. (2004). Although the Markov switching property can be implemented in different ways, the main idea is that something fundamental has changed giving rise to a jump in the volatility process. Generally the literature is silent on which underlying factors can give rise to these regime shifts. To us, all three approaches capture relevant aspects of what can drive equity volatility. We therefore try to incorporate elements from all the models in one single econometric model.

Our model has its starting point in the classic Merton (1974) framework, which provides a simple relation between the assets and the equity of a firm. In this model the firm is believed to liquidate at a specific future point in time and afterwards the outstanding debt is repaid first and whatever is left accrue to equity holders. Therefore, the future payoff to equity holders resembles the payoff from a call option on the assets with strike and maturity determined by the size and maturity profile of the debt. That is, the equity investor could perceive the equity as a call option on the firm's assets and price it as such. The liquidation time, i.e. the expiry of the implicit call option, is of course an abstraction and therefore unknown. In the literature the expiry is often set to five years, which is meant to resemble the average maturity of a representative firm's outstanding debt. Figure 2.1 illustrates the aggregate liability structure of the banks in our sample. If deposits are considered

Figure 2.1: Aggregate debt profile for the banks in the sample, 2016



Note: The figure contains the amount of funding that stems from customers' deposits (retrieved from SNL) as well as the issuance of bonds (retrieved from Bloomberg).
Source: Bloomberg and SNL Financial.

a stable form of funding and therefore ignored, one could from the large amount of bonds with 10+ years to maturity conclude that the average bank has a long debt profile. However, if the bank experienced trouble, the deposits might become a less stable form of financing and the bank could furthermore experience difficulties in rolling over its short-term debt or risk being recapitalized.[4] The result of this would be a substantially shorter perception of the time-to-maturity of the bank's liabilities. Ideally one should be able to take these potential shifts in perceived time-to-maturity into account when modelling the value of equity, as changes in equity volatility would otherwise be ascribed to (unobservable) changes in asset volatility.

Due to its simplicity the Merton model is rather popular and widely used, but it has also been criticized. For example Nagel and Purnanandam (2015) note that the basic insight, that that the payoff to equity resembles a call, is not really appropriate for a bank, since a bank's assets are mainly loans with a limited upside. Further, the liquidation time is in practice not fixed, and therefore the running default model, Black and Cox (1976), would be more suitable. Despite their differences, these structural models are developed to price equity given the asset value, but can also be used to find the asset value when the equity value is observed. Using the models to back out asset values on different dates would lead to the conclusion that asset values change over time

[4] In many countries deposit insurance schemes ensure that deposits are protected if they are under a certain limit, thereby reducing the likelihood of runs for deposits that are covered by the schemes. In the EU deposits up to 100,000 euros are covered by such guarantee schemes.

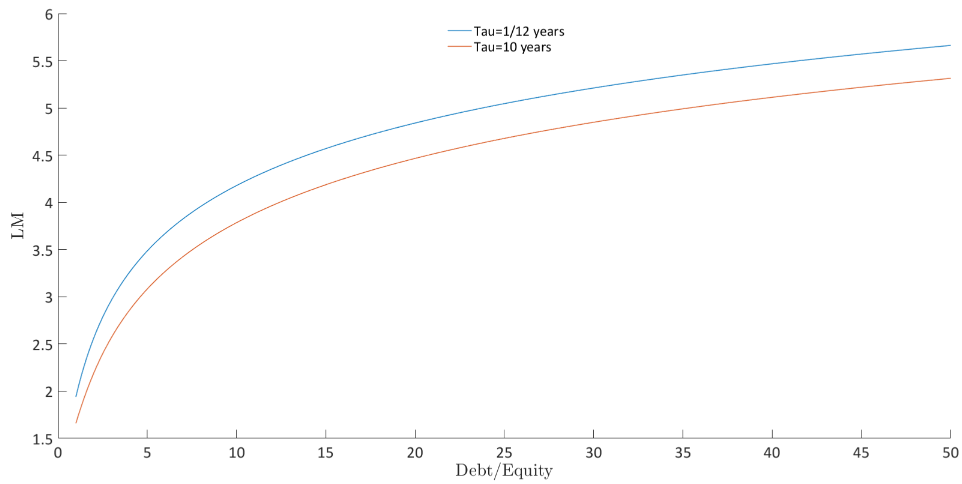
- and that they sometimes change *a lot*. This is because changes in the value of equity can only be driven by a change in the value of assets in these models, implying that changes in equity volatility can only arise due to changes in asset volatility.

In the econometric model of Engle and Siriwardane (2015) these findings are coupled with the earlier described leverage effect. Starting from the intuition of the Merton model, Engle and Siriwardane (2015) propose what they call a Structural GARCH, a model that explains equity returns as a function of a leverage multiplier and asset returns. Higher leverage implies that a shock to the assets of a certain size will have a higher impact on the value of equity. In that way daily returns interact through leverage; a negative return today leads to higher leverage, and therefore a higher impact on the equity return tomorrow for the same asset return. The fundamental idea is that investors' sensitivity to asset volatility increases as the firm becomes more leveraged. In their model a constant maturity is estimated as part of the model estimation. However, as we have argued earlier the maturity should in fact be the investors' perception of the maturity (or time-to-liquidation), which can change over time and thereby lead to changes in how asset returns affect equity returns - particularly for banks that are partly financed by funds of a short-term nature and can be subject to forced recapitalization by regulators. In order to capture this feature, we extend the model in Engle and Siriwardane (2015) in a simple and formal way that allows the perceived time-to-maturity of the call (the firm's equity) to switch between two states; one with a short time-to-maturity (the bank might collapse soon due to e.g. funding problems) and one with a long time-to-maturity (the bank is considered stable).

3 Model

The basic idea of the model in this paper is to price equity as a call option on assets and allow investors to change their perception of the maturity of the option. Although the idea is based on the classical Merton model, the implementation is made in the setup of the Structural GARCH introduced in Engle and Siriwardane (2015). This model uses the basic idea in the Merton model and develops an econometric model describing how equity returns, asset returns and leverage interact. For the deduction of the Structural GARCH and a more thorough explanation of the intuition of the model we refer to the original paper of Engle and Siriwardane (2015) or Grinderslev and

Figure 3.1: LM curves with two different times to maturity



Note: The leverage multiplier (LM) determines how much asset returns affect equity returns, cf. equations 1 and 4.

Kristiansen (2016). The essential equations are

$$r_{E,t} = LM_{t-1} r_{A,t} \tag{1}$$

$$r_{A,t} = \sqrt{h_{A,t}} \varepsilon_{A,t} \tag{2}$$

$$h_{A,t} = \omega + \alpha \left(\frac{r_{E,t-1}}{LM_{t-2}} \right)^2 + \gamma \left(\frac{r_{E,t-1}}{LM_{t-2}} \right)^2 \mathbb{1}_{r_{E,t-1}<0} + \beta h_{A,t-1} \tag{3}$$

$$LM_{t-1} = \left[\Delta_{t-1}^{BSM} \times g^{BSM} \left(E_{t-1}/D_{t-1}, 1, \sigma_{A,t-1}^{f}, \tau \right) \times \frac{D_{t-1}}{E_{t-1}} \right]^{\phi}, \tag{4}$$

where r_E is the return on equity. The return on assets, r_A, follows a GJR(1,1) process where $h_{A,t}$ defines the asset volatility. τ is the time-to-maturity, E the equity value and D the value of debt. The option delta, Δ^{BSM}, is based on the Black-Scholes-Merton (BSM) formula and g^{BSM} is the inverted call pricing formula. The BSM formula is used due to convenience, but the Structural GARCH is not heavily influenced by the flaws of the BSM model, since the final specification for the Leverage Multiplier, LM, in equation (4) is raised to the power of ϕ. Data is free to determine ϕ implying that a range of different option-pricing models can be obtained within this framework, see Engle and Siriwardane (2015) for more on this point. It also means that the specification takes a departure from the underlying idea that is based on the Merton model (which rests on the BSM model), but the Merton framework is included as a special case when $\phi = 1$. The model has five parameters, $\{\omega,\ \alpha,\ \gamma,\ \beta,\ \phi\}$, which are estimated by numerically maximizing a likelihood function.

Equations (1)-(4) give the relationship between asset returns, asset volatility and equity returns for a given bank. The basic idea is that the return on equity is the scaled return on assets. The scaling, LM, is primarily a function of the ratio between the value of debt and the value of equity, i.e. how leveraged the firm is. Higher leverage implies that asset returns have a greater impact on the option value because leverage scales asset returns and therefore impacts how much is left for equity at maturity. In Figure 3.1 the upwards sloping and concave LM curve is depicted. The positive slope implies that a series of negative asset (equity) returns increase leverage and thereby increase the leverage multiplier, this leads to a higher sensitivity to the next asset return, however with a diminishing effect due to the concavity of the LM curve. The scaling also depends on the time-to-maturity. Figure 3.1 illustrates two LM curves with different maturities, where the curve with the short maturity is above the one with the long maturity. This is in line with the intuition given earlier and illustrates how the sensitivity of equity to changes in assets (i.e. the leverage multiplier) is higher the shorter the time-to-maturity.

3.1 Modelling and estimating the changes in time-to-maturity

To implement changing perceptions of the maturity into the model in a formal and simple way, two states for the time-to-maturity are introduced, $\tau_t \in \{\tau_1, \tau_2\}$ with the changes in time-to-maturity being modelled with a Markov switching (MS) model (see e.g. Hamilton (1989)). Based on an initial run of the model on a sample of ten major European banks with two states where the time-to-maturity is restricted to lie in the interval of one month and 15 years, we find that there appear to be two distinct regimes, one with a short maturity of one month (1/12 years) and one with a long maturity around 10-12 years, cf. Table A.1. This clearly suggests that there is indeed one state in which a bank is considered rather safe (long time-to-maturity) and one where it is considered rather risky (short time-to-maturity). As the estimated time-to-maturities were more or less consistent across banks, we chose to simplify the model by fixing the maturities such that $\tau_1 = 1/12$ and $\tau_2 = 10$, which would comply with the data and the intuition of banks being subject to runs and forced recapitalizations, implying that they can be in either a "vulnerable" (short time-to-maturity) or a "normal" (long time-to-maturity) state. Fixing the maturities and only allowing the model to fit the probabilities of changing state leaves us with a total of seven parameters. With a Markov chain governing the switching between the two regimes, i.e. the time-to-maturity, the likelihood is split into two parts,

$$\mathcal{L}(t) = \mathcal{P}(\tau = \tau_1 | \mathcal{F}_t) \cdot \mathcal{L}(t, x, \tau_1) + \mathcal{P}(\tau = \tau_2 | \mathcal{F}_t) \cdot \mathcal{L}(t, x, \tau_2),$$

where $\mathcal{L}(t, x, \tau_i)$ is the original likelihood from Engle and Siriwardane (2015) with the time-to-maturity set to τ_i. The maximization of the likelihood is performed by a numerical optimization procedure. Due to the nature of the model, there is a high risk of finding a local optimum, and therefore we use simulated annealing to find the region of the optimum, followed by a local optimization procedure to find the exact optimum. Even though it is relatively easy to implement, the two-state Markov switching model has the disadvantage that it only allows for discrete shifts between the two regimes. Even if this fits with intuition of banks being either in a "vulnerable" or a "normal" state, it could be interesting to expand the model in order to allow investors to gradually change their perception of risk. However, as already described this would come at the cost of a higher complexity and estimation time of the model.

To summarize, the model incorporates two ways in which the equity sensitivity to asset returns can increase. The first way is an increase in the debt-to-equity ratio which results in a rightwards movement along the LM curve and a higher LM value. The firm becomes more levered and the evolution in the assets therefore "matters more" for the evolution in equity. The second way is a shift from the long to the short maturity, which results in an upwards shift of the LM curve. The interpretation is that the implicit call option expires sooner, and therefore changes in the value of the assets have a larger impact on the value of equity. In combination with asset volatility these two effects explain equity volatility.

4 Data

The model uses bank-specific data on equity returns together with the debt-to-equity ratio to disentangle asset volatility from equity volatility for a given bank. All data is from Bloomberg. More specifically, the model inputs are:

- Equity returns: daily returns of total return indices.
- Market capitalization: market value of the outstanding shares.
- Value of debt: book value of debt.
- Risk free rate: euro swap rates.

Liquid shares are a prerequisite for the model to be able to estimate reliable volatility estimates from daily returns. Therefore the focus is on major global banks as their shares are frequently traded and quite liquid. Further, the events of the financial crisis have a special interest since exactly this period was characterized by liquidity problems in many banks, which might translate into a short perception of maturity in the model. A long data period before and after the crisis is therefore essential. These two conditions lead to the selection of ten banks from the Stoxx Europe 600. These banks are Danske Bank, which is of special interest in Denmark, and the nine banks with the highest market capitalization in 2007 combined with the condition that data is available from 2000.[5] This ensures a sample of large banks with liquid shares and a data period covering a relatively long time span both before and after the financial crisis. Besides the financial crisis, the period also contains other crises such as the burst of the dot-com bubble and the sovereign debt crisis. Particularly during the financial crisis and the sovereign debt crisis, one would suspect investors to change their perception of the time-to-liquidation for some banks due to e.g. liquidity or solvency problems. While the intuition of the model is quite suited for banks, the model can of course also be used on non-financial firms. The caveat is that the intuition is less clear (non-financial firms e.g. rely less on "runnable" funding). In line with our expectations, a trial estimation on a small selection of big non-financial firms showed that they did not exhibit the same pattern as the banks in our sample; many of the non-financial firms had a stable long time-to-maturity, implying that allowing for regime shifts appears unnecessary. Our focus is therefore only on banks for the remainder of this paper.

5 Results

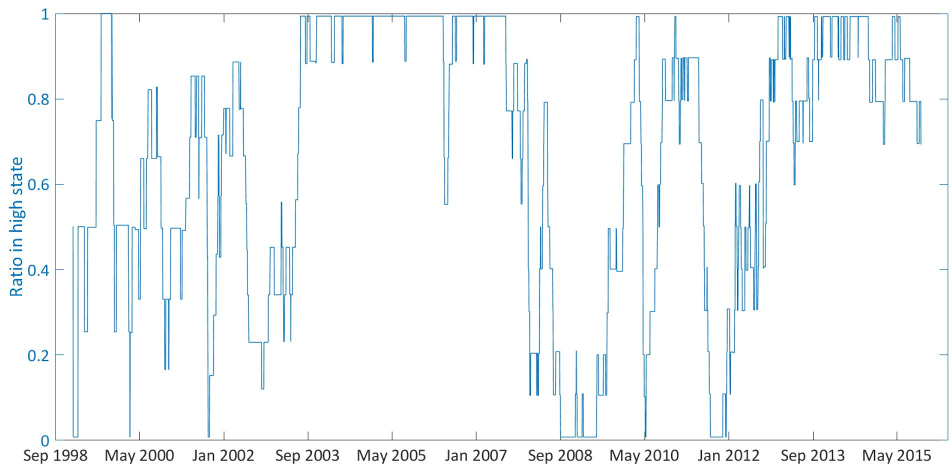
An issue with the Markov switching model is that its current state is unknown at any given point in time, one only knows the (filtered) probability of being in a certain state. To make a simple rule, we assume that the process at time t is in the state with the highest probability given the information set up to that time, i.e.

$$\tau_t = \begin{cases} 10 & , \ \mathcal{P}(\tau = 10|\mathcal{F}_t) > 0.5 \\ 1/12 & , \ otherwise \end{cases} \tag{5}$$

With this assumption it is easy to calculate the model-implied time-to-maturity τ given a filtered probability (which is available in real time). After obtaining a value for τ, one can calculate the LM curve, asset returns and volatility. Figure 5.1 illustrates the portion of the banks in the high time-to-maturity based on a sample from 2000-2016. It is worth noticing how there are periods where most banks are in the state with a high (low) time-to-maturity. As expected, the portion with a high

[5]Besides Danske Bank the sample includes (sorted here by start-2007 market capitalization): Banco Santander, BNP Paribas, ING Group, Banco Bilbao Vizcaya Argentaria, Deutsche Bank, NordeaBank AB, Erste Group Bank, Banco Popular Espanol and Skandinaviska Enskilda Banken (SEB). As we do not want to put too much emphasis on the results for specific banks we have decided to show aggregated results or, when referring to results for individual banks, results for Bank x.

Figure 5.1: Fraction of the banks in the high state, i.e. a long time-to-maturity



Note: The figure shows the fraction of the banks that are in the "normal" state with a long time-to-maturity. I.e. a ratio of one (zero) indicates that all banks are in the state with a long (short) maturity.
Source: Bloomberg and own calculations.

perceived time-to-maturity drops during the financial crisis and again during the sovereign debt crisis. This implies that the model indicates that the market perceived the time-to-maturity of the liabilities of the banks as being rather short during both crises. Similar observations can be made for the individual banks (a heat map is provided in Appendix B): The banks had a short perceived time-to-maturity during the financial crisis according to the model. The perceived time-to-maturity also decreased during the European sovereign debt crisis.

However, for several of the banks in our sample the perceived time-to-maturity also dropped during the burst of the dot-com bubble in the early 2000s. While one can rationalize the drop in the perceived time-to-maturity during both the financial crisis and the European sovereign debt crisis due to the liquidity and solvency problems faced by banks, it is not that obvious that the perceived time-to-maturity should decrease during the start of the 2000s. However, as illustrated in Figure B.2 in Appendix B, financial markets appear to have had some concerns about the solvency situation of banks when the dot-com bubble bursted. CDS spreads for several of the banks included in the sample rose to levels that were more than five times higher than what was observed in years with no market turmoil, such as the period leading up to the financial crisis. Although it should be emphasized that in comparison with the CDS spreads observed during the financial crisis and the European sovereign debt crisis (and even today), the levels were not particularly alarming. Hence, the change in perceived maturity might be due to the model only containing two regimes, implying that even minor shifts in the market's view on banks would appear quite substantial in the model. In addition to this, the shifts can possibly be an indication that the perceived time-to-maturity at times decreases due to e.g. increases in risk aversion among investors and not necessarily because they fear that the bank under consideration is facing problems. However, when comparing changes in the perceived time-to-maturity with changes in an estimated variance premium, there does not appear to be a clear link between the two, as one would expect if changes in the perceived time-to-maturity was primarily driven by changes in risk aversion among investors, cf. Figure C.1.[6]

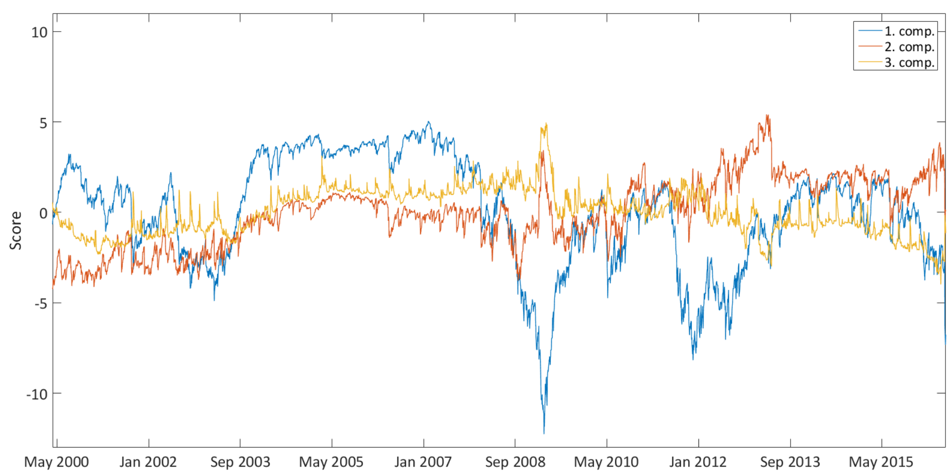
As a further investigation of whether or not the changes in τ are in fact just a reflection of changes in e.g. risk aversion among investors or the probability of default, we compare the filtered probabilities from the Markov switching (MS) model with the probability of default (PD). The PD is a concept often used to assess investors' view on the robustness of a bank. We use the Bloomberg estimates which are based on a Merton framework. We anticipate the Merton PD and the estimated probabilities from our MS model to capture some of the same information (the risk of default), but we do not expect the two series to be identical since the concepts are somewhat different[7] and because the Merton PD comes from a structural model whereas the probabilities from the MS model come from an econometric model, making the fundamental interpretations different. To investigate the extent to which the same information is captured by the two measures, their degree of co-movement is evaluated using a principal component analysis (PCA). A PCA converts series of data into linearly uncorrelated variables or principal components. The transformation ensures that the first component accounts for as much of the variability in the data as possible. The individual series under consideration load with a factor on this first component, and this loading helps to determine, firstly, the importance of this variable in explaining the movements in the component, and, secondly, whether it co-moves strongly with the other series under consideration. First, we perform the PCA between the filtered probabilities from the MS model and the Merton PD for each bank and find that the first component explains a substantial part of the variation for each bank, see Table D.1. In line with what one would expect, this implies that for each bank the PD and the filtered probabilities from the MS model contain some of the same information, but that there are differences illustrated with a non-negligible part of the variation being explained by the second principal component. Second, we perform the PCA on the filtered probabilities from the MS model for all banks, and likewise for the Merton PDs, see Table D.2. The results indicate

[6]The correlation between the inverse variance premium and an index of changes in the perceived time-to-maturity is only 0.12 for the entire sample and 0.31 for the period 1999-2002.

[7]The PD for instance depends on both the riskiness of assets (asset volatility) and the refinancing risk, while the time-to-liquidation in our framework is modelled separately from asset volatility.

that the filtered MS probabilities, respectively Merton PDs, are in part driven by a common factor which influences all the banks. In fact the first components from each of the two analyses seem to move together but the remaining components do not have this pattern indicating that some distinct drivers are affecting the filtered MS probabilities respectively the Merton PDs. Last, a PCA on both filtered MS probabilities and Merton PDs for all the banks again indicates that there is a relatively strong common factor across both series and banks, but that they are not entirely driven by the same underlying factors. In Figure 5.2 the first three components are illustrated. The first component, explaining 46 per cent of the total variation, follows the general pattern for the filtered MS probabilities: low during the dot-com bubble, high in the years leading to the financial crisis, very low during the financial crisis and the sovereign debt crisis, and finally a bit ambiguous in the last couple of years.[8] That is, there seems to be some common driving factor for the PD and the Markov chain implying that they contain some of the same information, but there also seems to be some uncommon information. This is in line with what we expect as the Merton PDs also capture the risk of being insolvent, which can occur independently of the firm being liquidated or restructured, although the two measures are closely related.

Figure 5.2: There is a common driver between the evolution in the perceived time-to-maturity and the Merton PDs



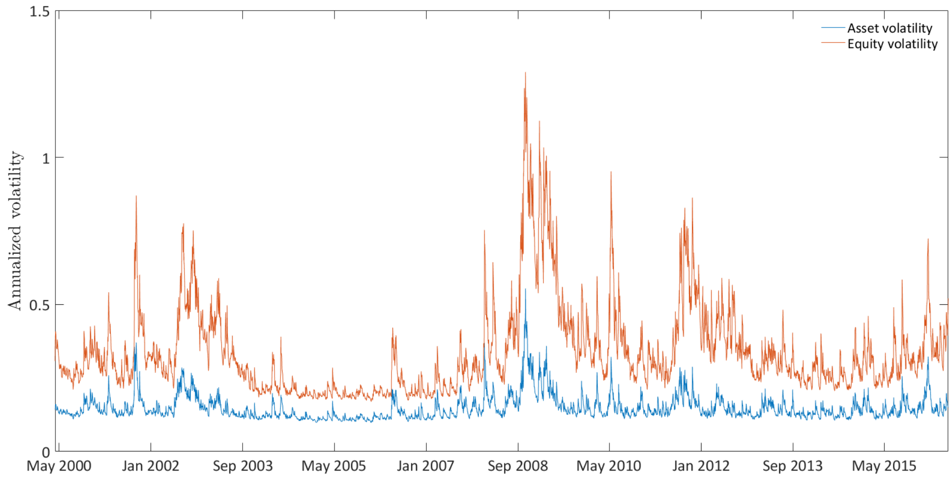
Note: The principal components are calculated by performing a principal components analysis when using the Merton PDs as well as the filtered probabilities from the Markov switching model for all banks in the sample as inputs.

[8] Recall that a low filtered probability of being in the state with a high time-to-maturity is equivalent to the bank in question being vulnerable.

6 Conclusion

The novelty of the model presented in this paper is the way in which equity volatility is modelled as a function of three factors: asset volatility, market leverage and perceived time-to-maturity. One advantage of this approach is that not all changes in equity volatility are ascribed to changes in the volatility of assets and market leverage, which is the case in Engle and Siriwardane (2015). Ideally, this should enable one to estimate the level and change in asset volatilities more accurately from an equity return series - thereby allowing one to obtain a better measure of the (evolution in the) riskiness of the firm's assets. This holds independently of whether τ changes due to changes in investors' attitude towards risk or because they fear that a bank will be subject to a run or a restructuring. However, since the market value of assets (and therefore asset volatility) is unobservable, it is difficult to test whether this is true in practice. With the assumption in equation (5) the equity and asset volatilities can be calculated from the model. Figure 6.1 shows the average model volatilities. Aggregate equity volatility rose considerably more than asset volatility during the financial crisis. That is, not all the equity volatility was due to an increase in asset volatility. Instead, our model suggests that equity volatility rose due to a combination of higher asset volatility, higher market leverage and a shorter perception of the time-to-maturity during 2008. The model thereby provides more guidance to the source of changes in equity volatility, and how market observers should interpret these changes.

Figure 6.1: Average annualized asset and equity volatility



Note: The numbers are average numbers for the banks considered in this paper. The asset volatility has been calculated using the model presented in Section 3.

Appendix A

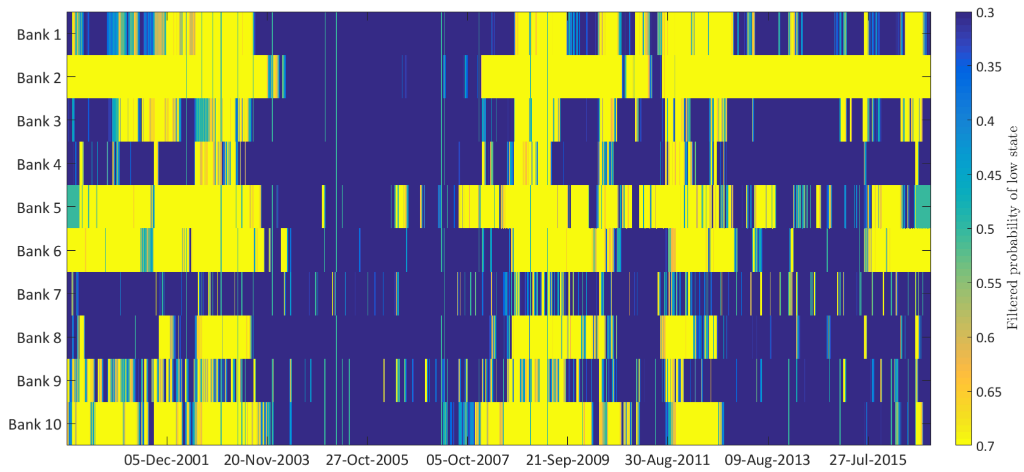
Table A.1: Estimated time-to-maturities from a two-state Markov switching model with freely varying parameters

Bank	τ_1	τ_2
Bank 1	1/12	10.4
Bank 2	1/12	11.5
Bank 3	1/12	10.7
Bank 4	1/12	10.9
Bank 5	1/12	10.6
Bank 6	1/12	10.9
Bank 7	1/12	10.9
Bank 8	1/12	10.2
Bank 9	1/12	10.7
Bank 10	1/12	11.3

Note: Banks included in the sample are Danske Bank and the nine largest (end 2007) members of Stoxx Europe 600 with data from 2000. For each bank the table shows the preferred maturities from a fit with free maturities. The short maturity is consistently hitting the lower bound, $\tau = 1/12$, and the long maturity is close to $\tau = 10$.

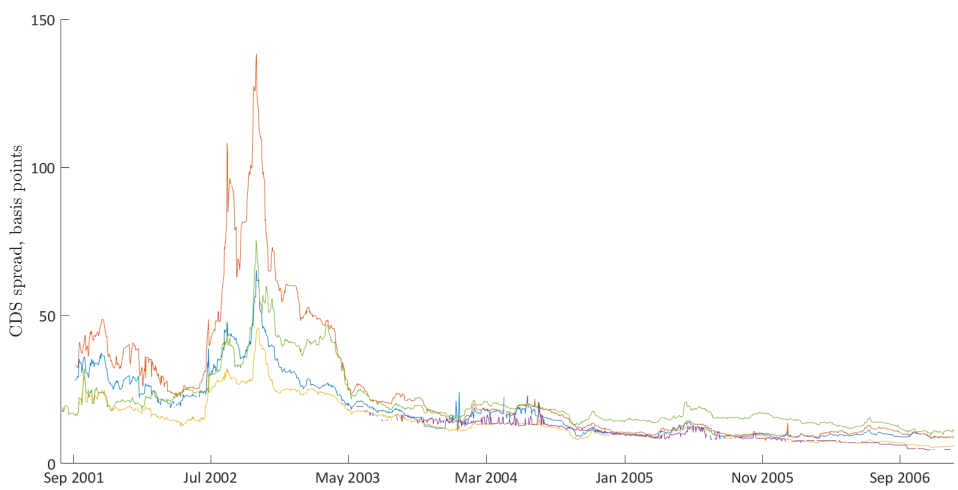
Appendix B

Figure B.1: Heat map for the filtered probabilities



Note: Yellow (blue) indicates a short (long) maturity.

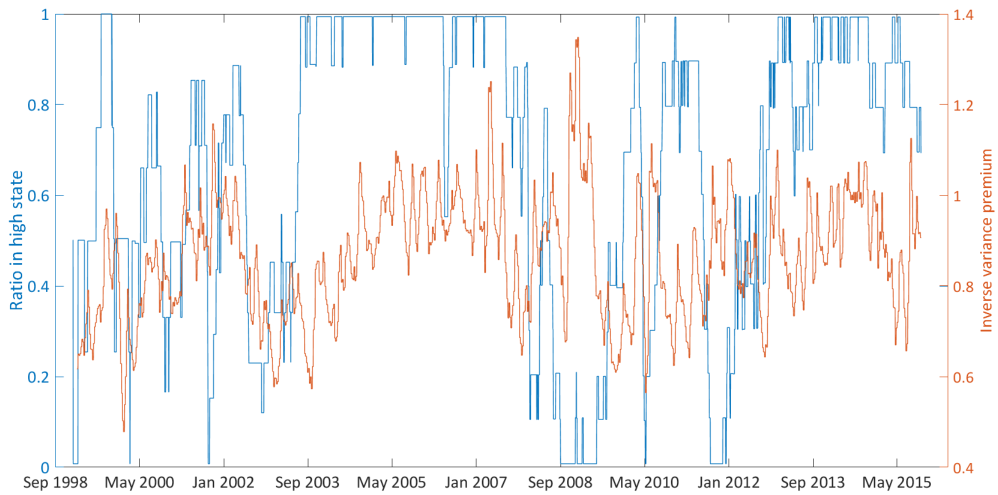
Figure B.2: CDS spreads had a peak at the time of the dot-com bubble burst



Note: The figure shows the evolution in Credit Default Swap (CDS) spreads for the five banks, where a long time series is available.

Appendix C

Figure C.1: The maturity index shows the ratio of the banks that have a long ($\tau = 10$) perceived maturity



Note: The variance premium is calculated as the ratio between the V2X-index, which depends on the implied volatility of equity options on the Stoxx Europe 50 index, and the conditional volatility on the Stoxx Europe 600 index calculated using a GJR-GARCH model. The inverse variance premium is high (low) when the implicit volatility on equity options is low (high) relative to the realized volatility.

Appendix D

Table D.1: Principal Components Analysis between the probability of high state and the Merton PD for each bank

Bank	*1st component*	*2nd component*
Bank 1	78.5	21.5
Bank 2	66.2	33.8
Bank 3	70.9	29.1
Bank 4	62.1	37.9
Bank 5	58.2	41.8
Bank 6	63.4	36.6
Bank 7	61.3	38.7
Bank 8	78.4	21.6
Bank 9	56.0	44.0
Bank 10	77.7	22.3

Table D.2: First ten principal components for all banks

Component	*Only Markov prob.*	*Only Merton PD*	*Both*
1	57.5	58.1	45.8
2	10.6	16.0	17.0
3	7.2	7.4	8.0
4	6.9	6.0	4.6
5	4.6	3.5	3.5
6	3.6	2.9	3.1
7	3.2	2.1	2.8
8	2.7	1.8	2.0
9	2.1	1.3	1.7
10	1.7	0.8	1.6

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