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A regional model of the Danish housing market

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Abstract
We estimate a regional model of the prices of Danish single-family houses and show that submarkets are interconnected via relative prices, giving rise to a ripple effect. We find strong evidence of a ripple effect in the short run of the model, but less so in the long run. We extend the model to allow for heterogeneity in fundamental elasticities based on factors such as local supply constraints and demographic compositions. We find that house prices are more sensitive to developments in fundamental factors, such as the housing stock, income and user costs in e.g. the Copenhagen area. Additionally, we document that the ripple effect is stronger from Copenhagen to other parts of Denmark than it is in the opposite direction.

Resume
Vi estimerer en regional model for priser på enfamiliehuse og viser, at delmarkederne er forbundet via de relative priser, hvilket giver anledning til en bølgeeffekt. Vi finder stærke tegn på en bølgeeffekt på kort sigt og i mindre grad på langt sigt. Vi udvider modellen til at tillade heterogenitet i grundlæggende elasticiteter baseret på fx lokale udbudsbegrænsninger og den demografiske sammensætning, og vi konstaterer, at boligpriserne er mere følsomme over for udviklingen i fundamentale faktorer, såsom boligmassen, indkomst og brugeromkostninger fx i og omkring København. Derudover dokumenterer vi, at bølgeeffekten er stærkere fra København til de øvrige dele af Danmark end i modsat retning.

Key words
House-price dynamics, regional house prices, global VAR models.

JEL classification
C22; G12; R31.

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A regional model of the Danish housing market*

Simon Juul Hviid†

November, 2017

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We estimate a regional model of the prices of Danish single-family houses and show that sub-markets are interconnected via relative prices, giving rise to a ripple effect. We find strong evidence of a ripple effect in the short run of the model, but less so in the long run. We extend the model to allow for heterogeneity in fundamental elasticities based on factors such as local supply constraints and demographic compositions. We find that house prices are more sensitive to developments in fundamental factors, such as the housing stock, income and user costs in e.g. the Copenhagen area. Additionally, we document that the ripple effect is stronger from Copenhagen to other parts of Denmark than it is in the opposite direction.

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1 Introduction

The substantial divergence in regional house prices seen over the past decades is not only a feature of the Danish housing market, but also of housing markets in most other countries where prices tend to increase beyond the national average in and around the largest cities, see e.g. Claeys et al. (2017). Our focus will be on single-family houses.[1] Figure 1 illustrates the price levels in Danish municipalities in 2016. The aim of this paper is to investigate the evolution of regional house prices to see if developments can be explained by the evolution of fundamental economic factors.

It is important for policy makers to have a solid understanding of the driving forces in house-price formation in order to implement directed and appropriate policy measures nationally as well as regionally, e.g. directed macroprudential policy such as loan-to-value or loan-to-income requirements, and local regulations on land use including parcelling new land. Additionally, as Grinderslev et al. (2017) find a strong connection between the house price cycle and the business cycle and Leamer (2007) goes further and finds that housing *is* the business cycle, developments in the housing market could also be taken into account in fiscal policy.

Our starting point is to develop a simple theoretical model that comprises the ideas of Meen (1999) and Oikarinen (2006) that increasing house prices in one geographical area will shift demand to other areas ceteris paribus. We do this in a framework of residential sorting where individual choices are modelled and can be aggregated to a measure of regional demand. From this model, we can derive inverted demand functions for all regions, which are clearly interconnected, as the prices in one geographical area are not only a function of variables in this particular region, but of variables across all regions in the housing market. In this framework, a shift the the relative prices between regions will give rise to a ripple effect as marginal house buyers will change their choice of location and thereby shift housing demand. The link from individual choices to the inverted demand function is a contribution to the theoretical literature on regional house-price formation.

Motivated by the theoretical model, our empirical approach follows the literature on global VAR models, which are most commonly applied to models of international trade. This approach comprises the ideas of Cameron et al. (2006). Additionally, we let the regional observables be weakly exogenous, as suggested in Pesaran et al. (2000). Heebøll (2014) estimates a somewhat similar model for the Danish housing market.[2] Our global VAR model in error-correction form with exogenous regressors (GVECMX) has several advantages. The specification of a dynamic panel of inverted demand functions can be estimated relatively sparsely in a reduced-rank panel-regression framework where prices in other regions are the driving force of the ripple

[1] We disregard prices of owner-occupied flats, leaving us with 82 per cent of the total housing market and 29 per cent in Copenhagen City. See Hviid et al. (2016) for a model of prices of owner-occupied flats in Copenhagen.

[2] The model differs to a non-trivial extent in specification and estimation. For instance, an embedded endogeneity issue exists in the lag structure, which does not apply to our model specification.

effect. In this way we allow regional price equilibria to be modelled locally, but include the price of the outside option, as the theoretical model suggests.

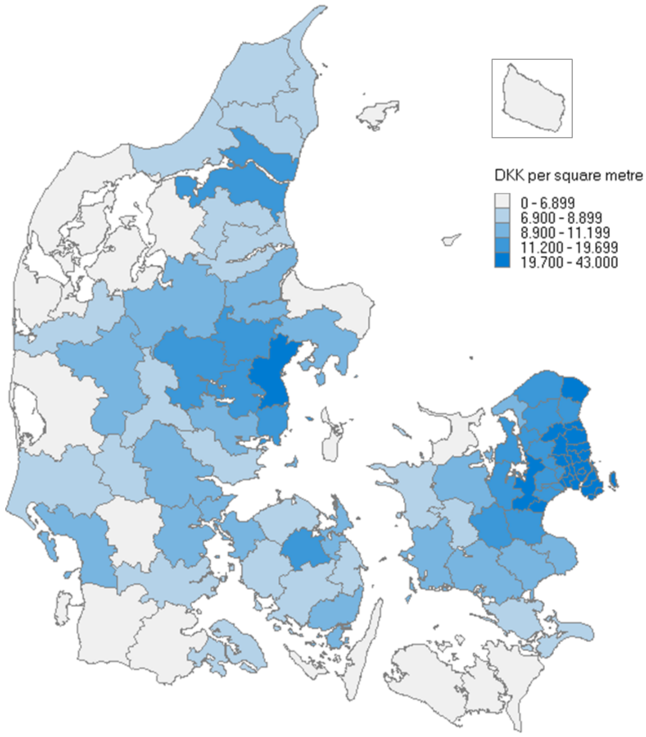


Figure 1: Prices of single-family houses per square metre in 2016Q4 across Danish municipalities

We estimate the model on a panel of regional prices of single-family houses along with regional measures of income, housing stock, and user costs from 1987-2016 at a quarterly frequency. We use a revealed-preference approach to construct a weighting matrix, which we use to measure the relative importance of prices in the work locations to the residential location. In particular, we let the relative amount of interregional commutes be the measure of connectedness between regions. This weighting matrix is an essential part of the global component in the modelling approach. In the empirical results we find strong evidence of a ripple effect in the short run of the model, but less so in the long run. We interpret this finding as the ripple effect being essential in understanding price developments over a horizon of some years, but in the long-run equilibrium prices are predominantly determined by regional fundamental factors. However, we also find that adjustment to this long-run equilibrium takes a substantial amount of time.

As a final exercise, we extend the model and allow for cross-regional variation in long-run elasticities by specifying the GVECMX model with random coefficients in the long-run relation. Specifically, we let regional differences in supply elasticities and demographic compositions identify the cross-regional differences in the estimated coefficients. This approach is inspired by e.g. Saiz (2010), Chen et al. (2016), and Hilber and Vermeulen (2016). We find that these factors explain some of the cross-sectional variation across regions and overall find that Copenhagen is more sensitive to changes in the housing stock, income, and costs. Additionally, we find that Copenhagen is less impacted by the price developments in other regions, which is in line with Oikarinen (2006), i.e. that the economic centres are also the epicentre of the ripple effect.

The paper proceeds as follows: Section 2 sets up the theoretical model. Section 3 derives the empirical model from a standard VAR framework. The results are outlined in section 4, and section 5 concludes.

2 The economic model

2.1 Residential sorting and price formation

To motivate the empirical part of the paper, we set up a simple framework that follows the literature on residential sorting, see e.g. Bayer et al. (2016). We follow and extend the model of Hilber and Vermeulen (2016) to provide an inverse demand function, which spans several geographical locations.

The formation of house prices effectively starts at the household level. Households observe prices and amenities across different locations and optimize their location thereafter in the standard Tiebout (1956)-sorting fashion. Assume there are H households of which an individual household is indexed h, and I locations indexed i. Let P_i be the price in a given location and let Z_i be a vector of local, observable and unobservable, amenities, including labour market conditions, user costs, etc. Let there further be an idiosyncratic taste shifter, φ_{hi}, for each household-location combination. The taste shifter captures idiosyncratic features such as the location of relatives, workplace, workplace of spouses, etc. In this case, the household would value living in location i by $V_h(P_i, Z_i, i) + \varphi_{hi}$. Here we assume additive separability of the idiosyncratic taste shifter for simplicity. Otherwise, there are no assumptions on the structural form of the value function. The household location can be expressed as the following rule,

$$i^* = \text{argmax}_{i \in \{1,\ldots,I\}} V_h(P_i, Z_i, i) + \varphi_{hi},$$

where i^* denotes the optimal location of household h. Let F denote the conditional cumulative distribution of the idiosyncratic taste shifter. Often, these are assumed to follow a type 1 extreme-value distribution, as it provides convenient closed-form solutions to discrete-choice models. For any given F, the expected demand in location i can be expressed as follows,

$$Q^D(P, Z, i) = H\left(1 - F(P, Z, i)\right), \tag{1}$$

i.e. the fraction of the total population that is expected to prefer to reside in the given location. Here P and Z are a vector and a matrix that collect prices and amenities in all regions. The important feature of this particular expression of expected demand is that it not only takes prices and amenities in the particular location as an argument in F, but prices and amenities in all locations. Hence, a change in prices or amenities in one location effectively spills over to other locations as it affects expected demand in all locations. In the short run we expect the supply of housing in location i, Q_i^S, to be constant, and by imposing the market-clearing condition $Q_i^S = Q_i^D \ \forall i \in \{1, \ldots, I\}$, we obtain the inverted demand functions for each location from (1)

$$P_i = G\left(P_{-i}, Q^S, Z, H, i\right).$$

To be agnostic about the specific relations, we let G characterize this inverted relationship and note that as for the local demand, local prices are affected by prices in other regions, which we denote P_{-i}, as well as demand and amenities across all regions.

As such, local house prices are exogenous to the individual. However, at the aggregate level, individual decisions accumulate and determine the relative price levels across locations.

This simple framework abstracts from many important features of the housing market, such as e.g. the distinction between renting and owning, and the inherent heterogeneity of houses and households over and above what can reasonably be expected to be captured by the idiosyncratic taste shifter. However, it serves to illustrate that housing sub-markets are inherently interconnected, and that this connection is expected to affect the formation of local house prices.

3 The econometric model

Our empirical approach departs from a standard VAR model. Let X_t be a vector of $k+1$ local variables in I regions, including house prices, and let T denote the length of the sample in the time dimension. According to Granger's representation theorem, the VAR model has a corresponding error correction form,

$$\Delta X_t = a + \Pi X_{t-1} + \sum_{j=1}^{J} \Gamma_j \Delta X_{t-j} + e_t, \tag{2}$$

where we would expect Π to have reduced rank. Here e_t is a vector of idiosyncratic error terms where $e_t \sim N(0, \Omega)$. An unrestricted estimation of this model would imply estimation of $(k+1)I + (k+1)^2 I^2 (J+1)$ parameters using $T(k+1)I$ observations, which quickly becomes infeasible as k or I increase. Hence, we use two advances in the literature. First, we follow

Pesaran et al. (2000), assuming that the k local variables are weakly exogenous to the price vector, P_t. We partition the error term into a vector of errors related to prices and one related to the other regressors, i.e. $e_t = (e'_{pt}, e'_{xt})'$, with a corresponding partition of the variance matrix

$$\Omega = \begin{pmatrix} \Omega_{pp} & \Omega_{px} \\ \Omega_{xp} & \Omega_{xx} \end{pmatrix}.$$

In this case, the price-error vector can conditionally be expressed in terms of e_{xt} as $e_{pt} = \Omega_{px}\Omega_{xx}^{-1}e_{xt} + \epsilon_t$, where $\epsilon_t \sim N(0, \Omega_{pp} - \Omega_{px}\Omega_{xx}^{-1}\Omega_{xp})$. Now, making a similar partition of the parameters in (2) as for e_t, such that $a = (a'_p, a'_x)'$, $\Pi = (\Pi'_p, \Pi'_x)'$, and $\Gamma_j = (\Gamma'_{pj}, \Gamma'_{xj})'$ $\forall j \in \{1, \ldots, J\}$, we insert e_{pt} in (2), to get the following model,

$$\Delta P_t = c + \Pi_{px} X_{t-1} + \sum_{j=1}^{J} \Upsilon_j \Delta P_{t-j} + \sum_{j=0}^{J} \Psi_j \Delta \tilde{X}_{t-j} + \epsilon_t,$$

where $\tilde{X}_t$ is the $k \times I$ vector of all regressors except house prices. Here, we have that $\Pi_{px} = \Pi_p - \Omega_{px}\Omega_{xx}^{-1}\Pi_x$, and $c = a_p - \Omega_{px}\Omega_{xx}^{-1}a_x$, $\Upsilon_j = \Upsilon_{pj} - \Omega_{px}\Omega_{xx}^{-1}\Upsilon_{xj}$, $\Psi_j = \Upsilon_{pj} - \Omega_{px}\Omega_{xx}^{-1}\Upsilon_{xj}$ $\forall j \in \{1, \ldots, J\}$, and lastly $\Psi_0 = \Omega_{px}\Omega_{xx}^{-1}$.

Second, we use the global error-correction model (GVECM) proposed by Pesaran et al. (2004). Let W be an $I \times I$ weighting matrix with $\sum_{i=1}^{I} w_{ij} = 1$ and $w_{ij} \geq 0$ $\forall i, j \in \{1, \ldots, I\}$. If we assume that the interconnectedness of all regions is restricted to be reflected in the relative prices, i.e. prices in other regions weighted by the matrix W, we effectively restrict all coefficients on cross-regional parameters to be zero. Denoting the measure of prices in other regions as P^*_{it} for region i, it can be expressed as

$$P^*_{it} = \sum_{j=1}^{I} w_{ij} P_{jt}, \text{ where } w_{ii} = 0.$$

This approach implies that the only channel by which the individual regions are interconnected is through prices. Let us assume that Π_{px} has reduced rank. This provides us with a panel of region-specific empirical models of the inverted demand function. If we let i denote regions, the model takes the form

$$\Delta P_{it} = a_i + \alpha_i \beta'_i X_{it} + \sum_{j=1}^{J} \gamma_{ij} \Delta P_{i,t-j} + \sum_{j=1}^{J} \Upsilon^*_{ij} \Delta P^*_{i,t-j} + \sum_{j=0}^{J} \Psi_{ij} \Delta \tilde{X}_{i,t-j} + \varepsilon_{it}.$$

In the baseline model, we assume constant coefficients across all regions but allow for regional fixed effects, i.e.

$$\Delta P_{it} = a_i + \alpha\beta' X_{it} + \sum_{j=1}^{J} \gamma_j \Delta P_{i,t-j} + \sum_{j=1}^{J} \Upsilon_j^* \Delta P_{i,t-j}^* + \sum_{j=0}^{J} \Psi_j \Delta \tilde{X}_{i,t-j} + \varepsilon_{it}. \quad (3)$$

This assumption will later be relaxed.

4 Empirical results

4.1 The data

Our sample spans 30 years, from 1987 to 2016, and covers 10 Danish regions.[3] The main part of the data is readily available from Statistics Denmark, but a few other sources have been drawn upon. The empirical model is specified at quarterly frequency. However, as many of the data sources are only available at annual frequency, we use Lagrange polynomials to interpolate the data using aggregate series. If no aggregate series are available at the quarterly frequency, we interpolate linearly. All relevant series are deflated using the private consumption deflator from the national accounts. Below is a description of the individual data series.

House prices From 1992 to 2016, Statistics Denmark provides quarterly price indices of single-family houses. Prior to 1992, Told & Skat made a biannual publication in which house prices are covered regionally. We merge the two sources and adjust for seasonality.

Income At the annual frequency, the Income Statistics provides the disposable income aggregated at the regional level for all individuals above the age of 14. The statistics are available with a lag of approximately one year; hence, the data is projected using household disposable income from the national accounts.

Housing stock From the Danish BBR registry we have access to detailed information of each housing unit in Denmark, including the gross size of each unit in square metres and the amount of square metres that are approved for residence. We aggregate the latter at the regional level for privately-owned housing units but exclude holiday homes, single-room units, and public institutions. There is a break in data from 2004 to 2005, a period in which there was substantial construction of new housing units.[4] We smooth the data by the average growth rate of 2003-04 and 2005-06, except for Copenhagen, where there is no evidence of such a break.

[3] The region of Bornholm is excluded due to limited availability of data back in time.
[4] See Statistics Denmark (2008) for details on the data break.

User costs User costs consist of several elements. Most of these do not differ regionally or are at least expected to do so to a very limited extent. We use the rate of a 30-year fixed-rate mortgage, which is the most popular mortgage in Denmark. Mortgages are subject to a time-varying degree of interest-rate deductibility, which we account for. We include contributions in the user cost. As the relation explains real house prices, we include a term in the user cost, that is thought to capture inflation expectations, π^e, modelling the inflation expectation by a Muth-Pischke smoothing (See Muth (1960) and Pischke (1995)), i.e. letting $\pi_t^e = (1-\gamma)\pi_{t-1}^e + \gamma\pi_t$, where π is measured over a one-year horizon. Here $\gamma = 0.125$ is chosen a priori as in Dam et al. (2011). One of the important region-specific variables to include in the user cost is housing taxes. Danish housing taxes consist of two, the local land tax paid to municipalities (and counties in the past) and property taxes collected by the government. From a separate registry, we can observe public valuations of properties and plots which we then can use to calculate the paid taxes in each year. We aggregate this number and divide it by the regional housing stock, as described above, multiplied by square metre prices at the regional level from House Price Statistics.[5] Hereby, we get a precise estimate of the effective housing-tax rates at the regional level at an annual frequency.

Minimum first-year payments We measure the minimum first-year payments as the mortgage rate of a flexible-rate mortgage including contributions. Repayments are included, but after the introduction of interest-only loans these fall substantially; however, still including repayments on housing debt to banks. Housing taxes, as described above, are included.

4.2 The measure of prices in other regions

We construct a measure of prices in other regions by choosing a parametrization of W. As argued above, the ripple effect is a consequence of a shift in demand from one region to another. Following Cameron and Muellbauer (1998), we assume that households prefer to minimize commuting costs and apply a revealed-preference approach by using the fraction of commutes between regions in order to determine the relative interconnectedness of individual regions.[6] Specifically, we calculate the fraction of households that commute from one home-location region to another work-location region of total commutes out of the home-location region. As an illustration, table 1 shows the implied weights in 2016, measured in per cent, where the rows sum to unity.

We see, that in general most commutes occur between neighbouring regions and that there is some degree of clustering within Zealand and Jutland in particular. However, we also see that a relatively large share of commuters from all regions work in Copenhagen whereas few

[5]These statistics are published by Finance Denmark.

[6]Data stems form Danish registers in which work location is observed as of 1 November, but residential location is observed as of 31 December. This implies that a small fraction of the measured commutes reflect actual moves.

Table 1: The weighting matrix in 2016Q4 measured in per cent

	KBH	KBO	NSJ	QSJ	VSJ	FYN	SJY	QJY	VJY	NJY
Copenhagen City, KBH	0	67	16	7	5	1	1	2	0	1
CPH surroundings, KBO	71	0	16	8	3	0	1	1	0	0
North Zealand, NSJ	42	49	0	4	2	0	1	1	0	0
East Zealand, QSJ	35	45	8	0	10	1	1	1	0	0
W. & S. Zealand, VSJ	25	32	6	30	0	2	2	2	1	1
Funen, FYN	12	9	2	2	8	0	50	11	3	2
South Jutland, SJY	7	8	2	1	2	21	0	35	21	4
East Jutland, QJY	9	6	2	1	1	4	31	0	34	13
West Jutland, VJY	4	4	1	1	1	2	22	45	0	20
North Jutland, NJY	9	9	3	1	2	2	9	36	29	0
Average	21	23	6	5	3	3	12	13	9	4

individuals commute to the island Funen. We allow the weighting matrix to be time-varying, as we would expect improving infrastructure to enable commuting over larger distances, but these improvements might not be uniform across regions. One example is that the building of the bridge across the Great Belt improved commuting between the two major islands, Funen and Zealand. An alternative revealed-preference measure could be to construct a weighting matrix by actual moves. However, as these are more volatile over time and influenced by where most new construction is located, which should point in the opposite direction in prices, we use commutes. In the third column of table 4 in appendix D, we do some robustness checks with respect to this alternative measure.

4.3 The baseline model

The empirical estimation of the models departs from the VECM model, for which the results are reported in column 1 of table 2. We estimate the model by reduced-rank panel regression and include regional fixed effects in order to capture some of the location-specific unobservables, e.g. large trends such as urbanization. Stepwise we introduce the specifications with exogenous regressors and the global component by including the outside prices to the model in column 2 and 3 respectively. The fourth column reports the results of the GVECMX specification.

Overall we see that the various specifications do not change point estimates of the coefficients on the long-run relation dramatically, while the R-squared increase from .36 to .51 as we move from the VECM to the GVECMX specification. One important finding is, however, that when the prices in other regions are included, the coefficient on the lagged price growth turns insignificant. At the same time, there is not much change to the loading coefficient on the lagged price level, and the coefficient on the lagged level of outside prices, i.e. the weighted prices in other regions, is not significant at conventional significance levels. This leads to the main take away from comparing these specifications; the ripple effect from prices in one region

Table 2: Estimation results of baseline models

Variables	VECM	VECMX	GVECM	GVECMX
p_{t-1}	-0.032***	-0.034***	-0.029***	-0.034***
	(0.002)	(0.002)	(0.007)	(0.008)
yd_{t-1}	0.090***	0.106***	0.063***	0.069***
	(0.012)	(0.017)	(0.013)	(0.009)
s_{t-1}	-0.165***	-0.202***	-0.129***	-0.166***
	(0.021)	(0.030)	(0.021)	(0.017)
c_{t-1}	-0.250***	-0.308***	-0.161***	-0.280***
	(0.034)	(0.032)	(0.029)	(0.026)
Δp_{t-1}	0.505***	0.514***	0.064	0.065
	(0.037)	(0.038)	(0.049)	(0.054)
Δyd_{t-1}	-0.045**		-0.050**	
	(0.016)		(0.018)	
Δs_{t-1}	-2.766***		-1.749**	
	(0.513)		(0.665)	
Δc_{t-1}	0.271***		0.235***	
	(0.041)		(0.034)	
Δyd_t		0.173***		0.157***
		(0.023)		(0.020)
Δs_t		-2.514***		-1.367*
		(0.562)		(0.629)
Δc_t		-0.384***		-0.417***
		(0.038)		(0.026)
p^*_{t-1}			0.008	0.013
			(0.006)	(0.008)
Δp^*_{t-1}			0.567***	0.579***
			(0.053)	(0.055)
Constant	0.977***	1.146***	0.695***	0.975***
	(0.086)	(0.061)	(0.070)	(0.048)
Observations	1,170	1,180	1,170	1,180
R-squared	0.362	0.414	0.453	0.508
Fixed effects	10	10	10	10

Notes: Robust standard errors in parentheses. *** p<0.01, ** p<0.05, * p<0.1. The table shows estimation results of models of real price growth. The estimation sample covers 1987Q1-2016Q4 for 10 Danish regions. All models include unreported regional fixed effects.

to others is a far more important component to account for than the momentum effect, or adaptive expectations, captured by the lagged endogenous variable in the short run, while, in the long run, prices are predominantly determined by local fundamental factors, which are here captured by disposable income, the housing stock, user costs, and the minimum first-year payment.

Figure 2 plots the actual real growth rate of prices of single-family homes in Copenhagen City along with the fitted value from the GVECMX model and the implied residuals. We see that the model fits the data relatively well. Only in the years preceding the financial crisis, we see slight bunching in standard errors. It is well documented that this period was subject to a speculative house-price bubble in the Danish housing market, see e.g. Hviid (2017). It is interesting to note that the model fits data well despite that there is only regional fixed effects included in the model to capture cross-regional differences in the dynamics of the model. This leads us to believe that despite large geographical differences in the developments on the housing market, the same fundamental forces drive the prices – it is mainly a matter of differences in fundamentals. Figure 6 includes similar plots for the other 9 regions in the model.

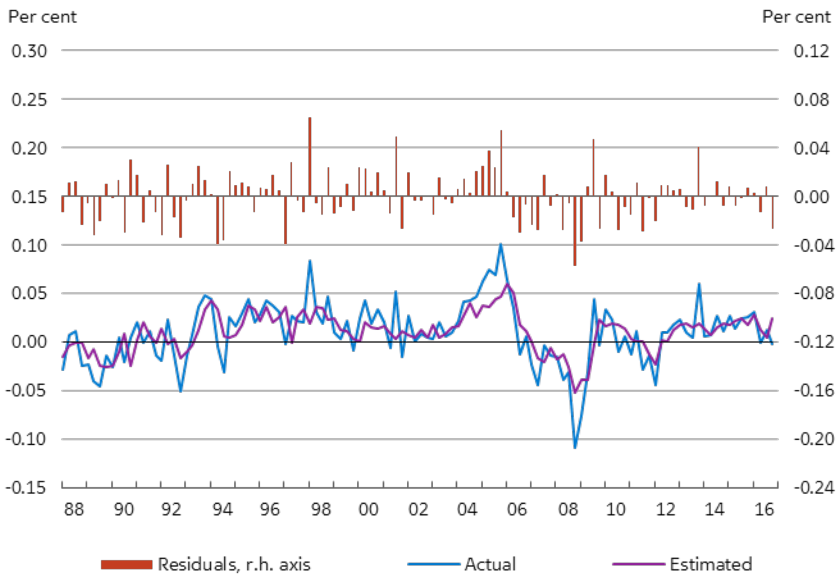


Figure 2: Fit of the baseline GVECMX model for Copenhagen City

As a robustness check, we show estimations of the model, where regions are weighted by the relative size of the housing stock in each year, which is reported in table 4 in appendix D. Likewise, we estimate the model excluding Copenhagen City, in case significantly different forces should be at play in this region. In both cases we find very limited sensitivity of all parameters.

4.4 Impulse-response functions

An illustrative way of assessing the dynamics of the model is to calculate impulse responses to different shocks to the estimated model. A standard way of doing this would be to present the model in (3) on its VAR representation, in which case the impulse-response function has a simple analytical form. However, a simpler way to do this for a GVECMX would be to calculate the one-step-ahead impulse-response function (IRF). Let ξ be a $(k+1)I$ length vector of shocks and let $\tilde{\xi}$ be the same vector, but excluding price shocks, which we abstract from here. Then the one-step-ahead IRF for the panel of regions takes the form

$$\begin{aligned}\Delta P_{t+1} =& \hat{a} + \text{diag}\left((\mathbb{I}_I \otimes \hat{\alpha}\hat{\beta}')\left(X_t + \xi\right)\right) + \text{diag}\left((\mathbb{I}_I \otimes \hat{\gamma})\Delta P_t\right) + \text{diag}\left((\mathbb{I}_I \otimes \hat{\Upsilon}^*)\Delta P_t^*\right) \\ &+ \text{diag}\left((\mathbb{I}_I \otimes \hat{\Psi})\left(\Delta\tilde{X}_t + \tilde{\xi}\right)\right),\end{aligned}$$

where $\otimes$ is the Kronecker product. Accumulating changes and calculating the IRFs in a recursive fashion, we can obtain h-step-ahead IRFs.

Assuming that the model is initially in long-run equilibrium, and restricting the constant to zero, we can interpret the impulses as deviations from the long-run equilibrium as if no shocks occurred, i.e. the partial effect of the shock.

Suppose that we impose a permanent shock to the income level in Copenhagen City of 1 per cent. Figure 3 (a) plots the impulse responses for a selection of regions from this particular shock. We see that prices in Copenhagen City increase, but also that the shock ripples to other regions in which there are no other changes than the change in Copenhagen City house prices.[7] Another important feature is that there is a substantial amount of persistence in the price level in all regions. From the coefficient on p_{t-1} in table 2, which for the GVECMX specification is 0.034, we see that the loading from the long-run relation, α, in (3) is fairly small, i.e. only 3.4 per cent of the deviation from the long-run equilibrium is corrected in each quarter.

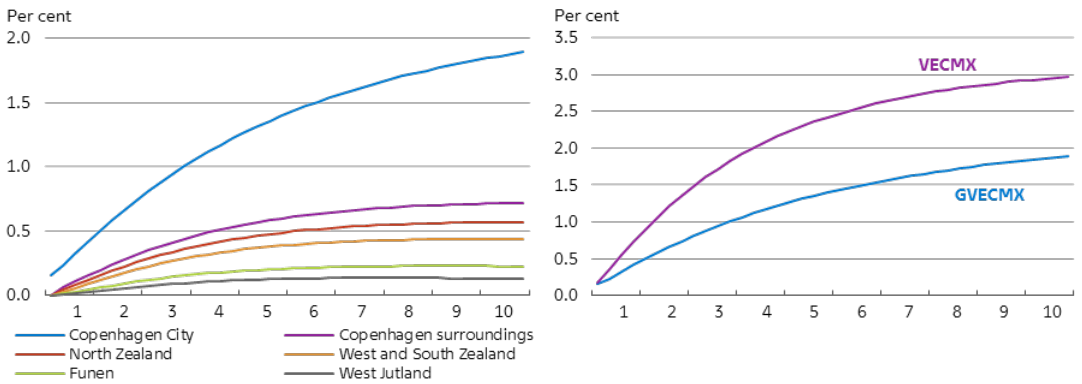
Additionally, we can compare the impulse responses of the same shock to income in Copenhagen City for different model specifications. In particular, we wish to illustrate the importance of allowing for the ripple effect across regions. Figure 3 (b) illustrates the IRFs for the VECMX and the GVECMX specifications. It is interesting to see that excluding the interconnectedness from the regional model tends to overestimate the sensitivity towards income shocks. This finding alludes to the observation that the ripple effect stabilizes local prices, as some of the demand shock is transferred to other regions.

In figure 7 in appendix C, we provide similar impulse responses of a permanent increase in the costs and the housing stock in Copenhagen City.

4.5 Housing-supply constraints

As argued above, local housing-supply constraints, due to e.g. limited availability of undeveloped land or local regulation of the housing market, can impact price formation. In regions where housing supply is constrained, a fraction of housing demand shocks are expected to

[7] Note that we do not restrict the coefficient on p^* to be zero despite that it statistically is not significantly different from zero.



(a) IRF for selected regions (b) VECMX vs. GVECMX for Copenhagen City

Figure 3: Impulse response of a 1 per cent permanent increase in disposable income in Copenhagen City for selected regions (left) and for two different model specifications for Copenhagen City alone (right).

impact prices even over the longer horizons. For each of the regions, we estimate the local price elasticity of housing supply following Wheaton et al. (2014). Let S_t denote the local housing supply, then the model takes the form

$$\Delta S_t = \alpha_1(S_{t-1} - (\beta_1 + \beta_2 P_{t-1}) + \sum_{l=1}^{L} \gamma_{sl} \Delta P_{t-l} + \sum_{l=1}^{L} \omega_{sl} \Delta S_{t-l}$$

$$\Delta P_t = \alpha_2(S_{t-1} - (\beta_1 + \beta_2 P_{t-1}) + \sum_{l=1}^{L} \gamma_{pl} \Delta P_{t-l} + \sum_{l=1}^{L} \omega_{pl} \Delta S_{t-l}$$

Here, it is the β_2 coefficient that is of particular interest. It shows the long-run price elasticity of housing supply. In practice, this model could be estimated in one step or in a two-step procedure where the first step estimates the long-run relation, which provides super-consistent estimates of the coefficients in the long-run relation, β, as first suggested by Engle and Granger (1987). We apply both estimation approaches and find only small differences in the point estimates of β across regions. Lutkepohl (2007) has argued that the first step in the two-step approach is a simple way of getting consistent estimates of long-run relations.

This estimation procedure provides one single measure of the elasticity over the sample period. However, local supply constraints are likely time varying, as available land is often available further away from city centres as population increases and cities evolve. As in this case, the inverse of the population density serves as an alternative measure of local supply constraints. Figure 4 plots the super-consistent estimates of the price elasticity of housing supply, estimated over the entire sample, against the population density of each of the regions in 2016. It is clear that there is a strong negative relation between the two measures. Therefore, we proceed by using the local population density as a time-varying proxy for the degree of local supply constraints.

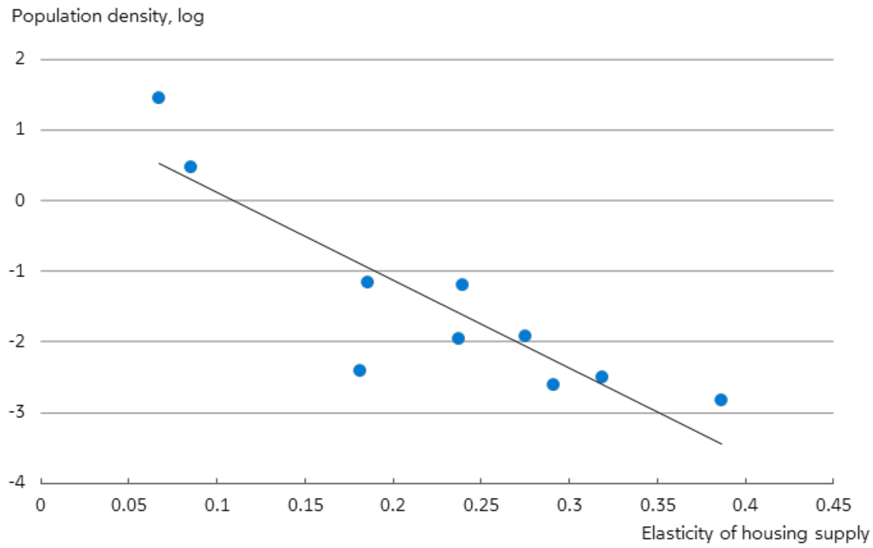


Figure 4: Population density in 2016Q4 in 10 Danish regions versus estimated local supply elasticities.

One way of thinking about the effect of local supply constraints, would be to consider the market tightness. In the labour market, market tightness is measured as the relative amount of demand and supply of labour, such that wages tend to increase when aggregate search is low and there are many vacancies. A similar way of thinking can be applied to the housing market, see e.g. Wheaton (1990). For obvious reasons we do not observe the search intensity of buyers, nor do we have long and reliable series for listed houses. As an alternative we consider the amount of families in each region relative to the the number of housing units. We can think of this as a measure of the housing-market tightness in the sense that supplied and demanded housing differ, which, according to the model outlined above, should affect prices.

4.6 Demographics

In the empirical model, the measure of income is aggregate disposable income in each of the regions. To this extent, some of the movements in income over time are a result of movements in demographics at the level of overall population. However, household demand for housing changes over the life cycle, which implies that the demographic composition should also affect housing demand. Over the life cycle, households enter the housing market at some point, typically in their younger years, then up-size when e.g. children arrive, and subsequently down-size or leave the housing market as the children leave the nest or household earners reach the age of retirement. We try to allow for such variation by constructing a measure of demographic composition. In particular, we consider two groups. First, we count the population in each region aged 25-39, as this group covers households, which we would expect to enter the housing market or be the most likely to up-size. Second, we count the population

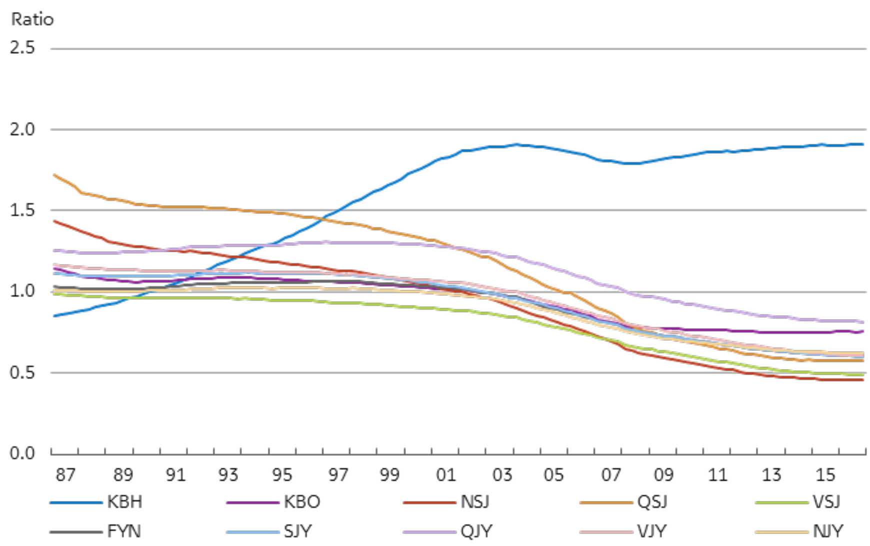


Figure 5: The ratio of the regional population aged 25-39 to those at ages above 60.

aged above 60, i.e. those in retirement the children of which are likely to have left home. By considering the ratio between the size of the two groups, we get a measure of the composition of households tending to increase local demand, and households tending to decrease local demand.

Figure 5 illustrates the ratio of young and old households in the 10 regions across time. We see that the general ageing of the population drags down the ratio for most regions; however, in Copenhagen, there has been a large inflow of younger individuals, which could explain some of the relative increase in the Copenhagen house prices.

4.7 Random-coefficients estimation

As is evident from the plots in figure 2 and 6, the model fits the data in all regions relatively well, and the model does explain about half of the variation in house prices across all regions, which is reasonable. However, there might still be some region specifics, such as those mentioned above, that might improve the model if included. Therefore, we extend the model by a random-coefficients modelling approach, where the pulling forces from the long-run relation is allowed to vary across regions in a systematic way.

In particular, we interact the components of the long-run relation with the variables described in sections 4.5 and 4.6 and report the results in table 3, where we include the random coefficients one at a time, but focus on the full random-coefficients specification, that includes all terms.

The demographic composition is interacted with disposable income and the costs of owning, as we expect that younger households entering the housing market have lower wealth accumulated, and are thus more sensitive to variations in income and cost of owning a house. We find that it is particularly the minimum first-year payments that impacts the price formation, and see that areas with a larger fraction of young versus elderly individuals react more strongly to changes in these key demand-side variables.

We interact the housing stock with the measure of tightness in the regional housing market, i.e. the proxy for demand and supply of local housing. In places where there is a shortage of supply, we should expect house prices to be more sensitive to changes in the housing stock than elsewhere. We do not find such an effect, when it is the only included random coefficient; however, when all measures are taken into account in the full random-coefficients model, there is a significant effect. This suggests that the housing demand in especially Copenhagen is more sensitive to a change in the housing supply than other regions.

The housing-supply elasticity, measured by the population density, is found to have a strong impact on the spillover from outside prices. In particular we see that regions with a high population density is less influenced in the long run by prices in other regions which suggests that prices mainly ripple from densely populated areas to the rest of the country, whereas the return ripple is less important. The implication of this finding is that areas such as Copenhagen have a larger impact on nation-wide house prices than the size in itself justifies. This goes beyond the feature that Copenhagen does already have a higher weight than other regions in the weighting matrix. Hereby we confirm the findings of Meen (1999) from the UK and Oikarinen (2006) from Finland, that economic centres are the point of departure of the ripple effect.

Overall, when we include all of the random coefficients, we gain about 2.5 per cent in explanatory power of the model, which is a relatively small gain from including seven additional regressors on a model with 20 coefficients (10 of those are fixed effects). The main insight obtained from this exercise is that Copenhagen is more sensitive to changes in the fundamental variables than other parts of the country, and that Copenhagen plays a predominant role in the evolution in prices across the country by being the strongest source of the ripple effect in house prices.

5 Conclusions

In this paper we have investigated price formation in regional sub-markets of the Danish housing market for single-family houses and the inherent interconnectedness of these sub-markets.

We develop a simple theoretical model from which we can derive an inverted demand function for each region. The model implies that prices in one geographical area depend on

Table 3: Estimation results of random-coefficients models

Variables	GVECMX Baseline	$\times bm$	$\times p^*$	$\times yd$	$\times mfy$	Full
p_{t-1}	-0.034***	-0.037***	-0.030***	-0.051***	-0.060***	-0.060***
	(0.008)	(0.010)	(0.006)	(0.014)	(0.012)	(0.011)
yd_{t-1}	0.069***	0.054***	0.108***	0.050***	0.054***	0.114***
	(0.009)	(0.007)	(0.021)	(0.012)	(0.013)	(0.019)
s_{t-1}	-0.166***	-0.127***	-0.243***	-0.092***	-0.102***	-0.186***
	(0.017)	(0.009)	(0.037)	(0.025)	(0.027)	(0.044)
c_{t-1}	-0.280***	-0.290***	-0.299***	-0.278***	-0.289***	-0.341***
	(0.026)	(0.028)	(0.028)	(0.038)	(0.035)	(0.040)
p^*_{t-1}	0.013	0.019	0.008	0.030**	0.037**	0.041***
	(0.008)	(0.010)	(0.006)	(0.013)	(0.012)	(0.012)
Δp_{t-1}	0.065	0.060	0.061	0.063	0.066	0.047
	(0.054)	(0.055)	(0.053)	(0.053)	(0.053)	(0.055)
Δyd_t	0.157***	0.149***	0.177***	0.147***	0.149***	0.178***
	(0.020)	(0.019)	(0.025)	(0.023)	(0.023)	(0.028)
Δs_t	-1.367*	-1.171*	-0.952	-1.047	-1.029	0.379
	(0.629)	(0.622)	(0.733)	(0.572)	(0.585)	(0.744)
Δc_t	-0.417***	-0.420***	-0.423***	-0.410***	-0.415***	-0.430***
	(0.026)	(0.028)	(0.025)	(0.030)	(0.029)	(0.031)
Δp^*_{t-1}	0.579***	0.579***	0.566***	0.556***	0.547***	0.515***
	(0.055)	(0.055)	(0.056)	(0.056)	(0.055)	(0.050)
$tight_{t-1}$		-0.161				-0.609***
		(0.194)				(0.172)
$tight_{t-1} \times s_{t-1}$		0.019				0.124**
		(0.056)				(0.043)
$dens_{t-1}$			0.524***			1.009***
			(0.149)			(0.207)
$dens_{t-1} \times p^*_{t-1}$			-0.038***			-0.074***
			(0.011)			(0.014)
yo_{t-1}				-0.017	0.031***	-0.031*
				(0.017)	(0.003)	(0.017)
$yo_{t-1} \times yd_{t-1}$				0.008*		0.014***
				(0.004)		(0.003)
$yo_{t-1} \times mfy_{t-1}$					-0.306**	-0.282**
					(0.101)	(0.117)
Constant	0.975***	0.873***	1.089***	0.801***	0.860***	0.887***
	(0.048)	(0.078)	(0.069)	(0.085)	(0.086)	(0.138)
Observations	1,180	1,180	1,180	1,180	1,180	1,180
R-squared	0.508	0.510	0.512	0.516	0.518	0.532
Fixed effects	10	10	10	10	10	10

Notes: Robust standard errors in parentheses. *** $p<0.01$, ** $p<0.05$, * $p<0.1$. The table shows estimation results of models of real price growth. The estimation sample covers 1987Q1-2016Q4 for 10 Danish regions. All models include unreported regional fixed effects.

prices in other regions, motivating our empirical model. The empirical specification is a global error-correction model with exogenous regressors where we let the ripple from one region to the other be driven by the relative prices across regions. The model is estimated using a panel of prices of single-family houses along with regional measures of income, housing stock, and user costs from 1987-2016 at a quarterly frequency. Prices in other regions are measured by weighting them by revealed preferences, using information on commutes between regions.

We find strong evidence of a ripple effect in the short run of the model, but less so in the long run, which is interpreted as the ripple effect being essential in understanding price developments as a pushing effect, but prices are predominantly determined by regional fundamental factors in the long run. The model is extended to allow for cross-regional variation in long-run elasticities in a random coefficients version of the GVECMX model's long-run relation. We use regional differences in supply elasticities and demographic compositions to identify these cross-regional differences in the long-run relations. This exercise confirms that some of the variation across regions can be explained by these factors, and that e.g. Copenhagen is more sensitive to changes in the fundamental factors. Lastly, we find that Copenhagen is less impacted by price developments in other regions. Rather, it is the source of the ripple effect.

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A Data overview

Variables		Measurement
House prices	p_t	Real, log
Disposable income	yd_t	Real, log
Housing stock	s_t	Square meters, log
Costs	c_t	Real, annualized
Minimum first-year payments	mfy_t	Property taxes, minimum mortgage rate and minimum repayments
Outside prices	p_t^*	House prices in other regions weighted by commuting patterns. Real, log
Market tightness	$tight_t$	Families relative to houses
Density	$dens_t$	Population relative to area
Demographic composition	yo_t	Population of age 25-39 years relative to 60 years and above

B Model fit

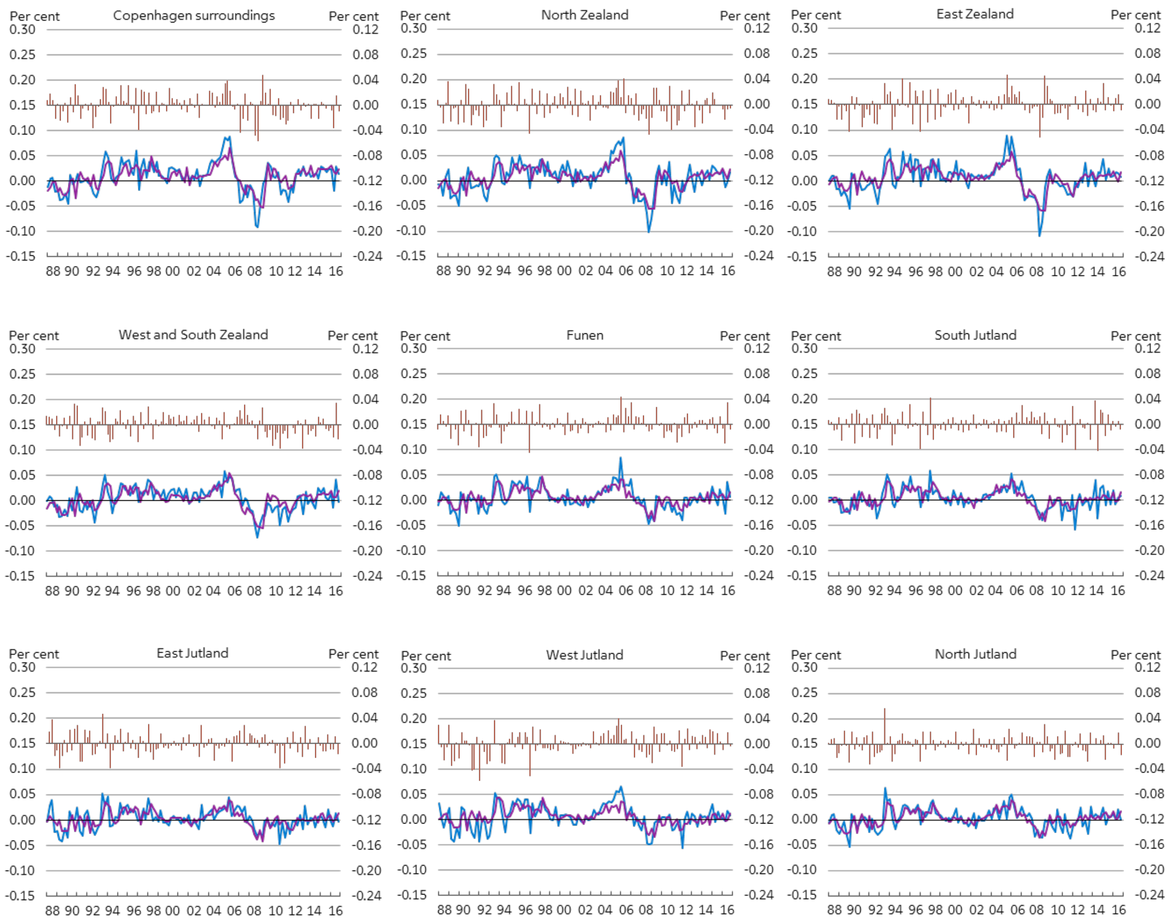

Figure 6: Model fit for all regions except for Copenhagen City (see main text)

C Impulse-response functions

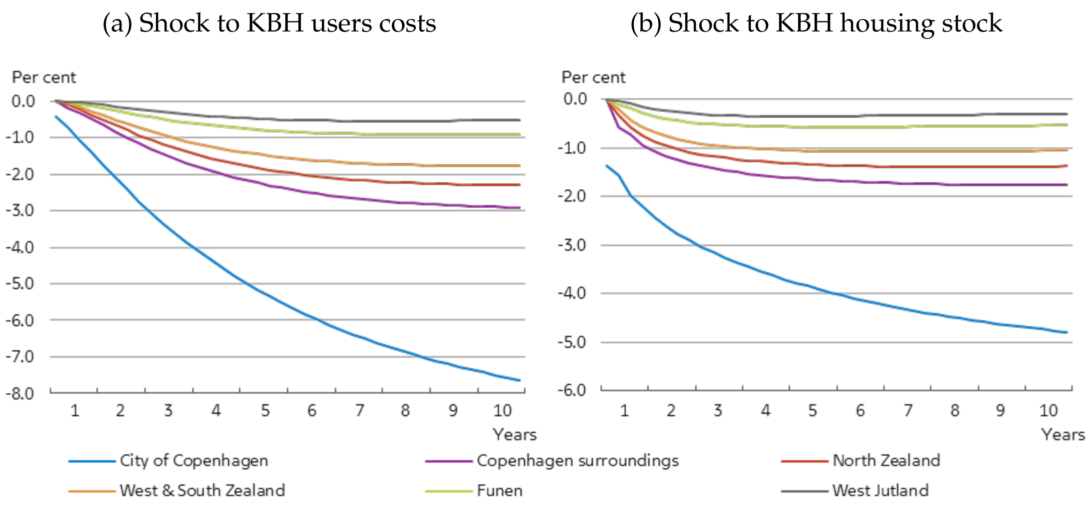

Figure 7: Impulse response of a 1 percentage point permanent increase in user costs (left) and a 1 per cent increase in the housing stock (right) in Copenhagen City for selected regions.

D Robustness

Table 4: Robustness checks of the model specification

Variables	GVECMX OLS	WLS	alt. W	No KBH
p_{t-1}	-0.034***	-0.034***	-0.039***	-0.031***
	(0.009)	(0.010)	(0.009)	(0.010)
yd_{t-1}	0.069***	0.074***	0.077***	0.066***
	(0.018)	(0.019)	(0.019)	(0.021)
s_{t-1}	-0.166***	-0.171***	-0.184***	-0.161***
	(0.031)	(0.032)	(0.034)	(0.035)
c_{t-1}	-0.280***	-0.275***	-0.291***	-0.279***
	(0.065)	(0.068)	(0.066)	(0.068)
Δp_{t-1}	0.065	0.060	0.121***	0.078*
	(0.043)	(0.046)	(0.044)	(0.046)
Δyd_t	0.157***	0.157***	0.155***	0.151***
	(0.026)	(0.027)	(0.027)	(0.027)
Δs_t	-1.367**	-1.359**	-1.374**	-1.399**
	(0.610)	(0.626)	(0.615)	(0.649)
Δc_t	-0.417***	-0.408***	-0.415***	-0.412***
	(0.048)	(0.051)	(0.048)	(0.051)
p^*_{t-1}	0.013*	0.010		0.011
	(0.008)	(0.008)		(0.008)
Δp^*_{t-1}	0.579***	0.577***		0.553***
	(0.046)	(0.049)		(0.046)
$W_t^{move} P_t$			0.017**	
			(0.008)	
$\Delta W_t^{move} P_t$			0.534***	
			(0.048)	
Constant	0.999***	1.006***	0.612***	0.941***
	(0.149)	(0.157)	(0.087)	(0.153)
Observations	1,180	1,180	1,180	1,062
R-squared	0.508	0.502	0.496	0.504
Fixed effects	10	10	10	10

Notes: Robust standard errors in parentheses. *** p<0.01, ** p<0.05, * p<0.1.
The table shows estimation results of models of real price growth.
The estimation sample covers 1987Q1-2016Q4 for 10 Danish regions.

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