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A DSGE Model with Occasionally Binding Constraints on Monetary Policy and Borrowing
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Resume

Key words
Zero lower bound; Fiscal policy; American Recovery and Reinvestment Act; Non-linear DSGE models; Housing friction; Occasionally binding constraints.

JEL classification
E12; E32; E44; E63; E65; R21; R31.

Acknowledgements
The author wishes to thank Søren Hove Ravn, Neil Webster, Martin Nygaard Jørgensen, Svend Greniman Andersen, Anders Larsen and other colleagues in Economics and Monetary Policy at Danmarks Nationalbank for useful comments and discussions of this paper.

The author alone is responsible for the viewpoints, conclusions and any remaining errors.
Fiscal Tools at the Zero Lower Bound:
A DSGE Model with Occasionally Binding Constraints on Monetary Policy and Borrowing

Jakob Feveile Adolfsen†

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The paper analyses the effectiveness of fiscal tools at the zero lower bound (ZLB). A non-linear New Keynesian DSGE model with occasionally binding constraints on monetary policy and borrowing is applied. When the ZLB binds in a liquidity trap, government spending becomes more effective in stimulating output whereas cuts in the income tax may lower output in the short run. Although the government spending multiplier increases at the ZLB, its size depends strongly on rational expectations to the liquidity trap length. In the beginning of 2009 market expectations reflected anticipations of a short stay at the ZLB. However, the American Recovery and Reinvestment Act was implemented on the basis of official reports and dominating literature that suggested the spending multiplier to be above 1 by assuming a long stay at the ZLB. Borrowing is based on the collateral value of the household’s stock of housing, and this credit channel has a negative implication on the effectiveness of government spending as a stabilising instrument due to a fall in the housing price. However, the channel might turn positive at the ZLB when allowing for public debt financing because of a wealth transfer to the borrowing household.

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1 Introduction

The bursting bubbles on housing and stock markets during the financial crisis created dramatic recessions in most advanced economies, and central banks eventually reached the zero lower bound (ZLB) on conventional monetary policy in many countries. In the

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*This working paper is based on a master thesis written with financial support from Danmarks Nationalbank. The author alone is responsible for the viewpoints, conclusions and any remaining errors.

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US specifically, the Effective Funds Rate of the Federal Reserve (Fed) was caught at the ZLB for 7 years from the end of 2008. Therefore, central banks and governments needed to look for alternative paths towards stabilising the business cycle. Unconventional monetary policy was conducted, while fiscal policy experts called for structural reforms along with public consolidation and/or expansions. The latter initiative marked an increased emphasis on Keynesian policies. Christiano et al. (2011) and Eggertsson (2011) argued that the ZLB had created a moment for extremely effective public spending in stabilising output with spending multipliers between 2-4 in their DSGE frameworks due to inflationary effects and a fixed nominal interest rate.1 In a similar type of model estimated on US data, Denes and Eggertsson (2009) found a spending multiplier around 3. In contrast, policies with deflationary implications in New Keynesian models such as income tax cuts were suggested to have negative implications on GDP at the ZLB in the short run (Eggertsson, 2011).

Along the findings in the macroeconomic literature, the Obama administration signed the American Recovery and Reinvestment Act (ARRA) in the beginning of 2009. It marked a historically large spending stimulus in the US. The policy was based on the widely cited report of Bernstein and Romer (2009) who founded their recommendation on a spending multiplier above 1 like the aforementioned papers. The ARRA therefore constitutes an interesting case for a discussion of public spending at the ZLB. The ARRA also included substantial elements of tax cuts, which are therefore addressed in this paper.

In a modified version of the New Keynesian DSGE model with housing from Guerrieri and Iacoviello (2017), I find that the government spending multiplier is higher at the ZLB than in normal times in accordance with Christiano et al. (2011) and Eggertsson (2011). In contrast, a cut in the income tax rate has a short-term negative effect on output at the ZLB. Despite these findings, my analysis finds government spending to be less expansionary at the ZLB as well as tax cuts to be less contractionary compared to Christiano et al. (2011) and Eggertsson (2011). While these papers find spending multipliers of 3.7 and 2.3 at the ZLB, I only find a multiplier of 0.89. Likewise, I find a tax multiplier of −0.05 at the ZLB while it is −1 in Eggertsson (2011). The expected length of the liquidity trap and rational expectations are found to be essential for the multiplier sizes. This is due to an immediate accumulated consumption response to future expected changes in the real interest rate. Christiano et al. (2011) and Eggertsson (2011)’s results rely on a long anticipated period of ZLB policies while the liquidity trap is short in

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1Mertens and Ravn (2014) show that, if the liquidity trap is a sunspot equilibrium entirely driven by pessimistic expectations without any changes in fundamentals, the government spending multiplier will be lower at the ZLB than in normal times. However, Aruoba et al. (2016) find that the liquidity trap in the US was more likely to have been caused by shifts in fundamentals than by a sunspot switch. I therefore stick to analysing a fundamentals-driven liquidity trap.
this paper. This crucial role of expectations in the New Keynesian model deserves more attention when evaluating public demand in liquidity traps.

A 7 year ZLB spell in the US might in this regard lead to very positive evaluations of the ARRA. However, from a New Keynesian view, what matters is the expected time at the ZLB when implementing the fiscal adjustment, and not the actual duration per se. Market expectations indicate that market participants did not expect the ZLB to bind for much more than a half year in the beginning of 2009. Hence, I question whether anticipations to the ARRA were too high, because they were based on an expectations assumption that was out of step with reality?

Due to the findings in Christiano et al. (2011) and Eggertsson (2011) as well as the recommendations in Bernstein and Romer (2009), the spending multiplier at the ZLB has been frequently discussed by others in different DSGE settings. In an estimated Smets and Wouters model\(^2\) with rule-of-thumb consumers, Cogan et al. (2010) also stress that Bernstein and Romer (2009) might have over-estimated the fiscal multiplier at the ZLB by relying on a permanent interest rate at zero as well as a permanent government expansion. However, Siemsen and Watzka (2013) criticise Cogan et al. (2010) for relying on an exogenously generated ZLB period. Like Cogan et al. (2010), I conclude that the recommendations in Bernstein and Romer (2009) might have been over-optimistic but in a model with an endogenously generated ZLB period. Drautzburg and Uhlig (2013) also estimate the ARRA to be less efficient in a richer model. They show that if a fiscal stimulus is financed by distortionary taxation, the ARRA multiplier would be only around 0.5 due to reduced work incentives and income.

Besides the ZLB, the declining housing market also played a vital role in the Great Recession by eroding the wealth of households (Guerrieri and Iacoviello, 2017). The bursting housing bubble led to eroded mortgage collateral and therefore tight constraints on household borrowing with severe consequences for the US economy. I therefore control for financial frictions in the housing market when evaluating fiscal policy during the crisis by using the model from Guerrieri and Iacoviello (2017).

Financial frictions have previously been implemented in DSGE models. Bernanke et al. (1999) emphasise a financial accelerator mechanism when firms’ borrowing is constrained by the value of their assets. In this framework, Carrillo and Poilly (2013) find that the borrowing constraint amplifies the government spending multiplier further at the ZLB. However, I suspect other forces to be at play when using the housing stock as collateral because government expansions tend to reduce house prices in New Keynesian models due to a negative correlation between the price on durable goods and the marginal utility of non-durable consumption (Barsky et al., 2007).

\(^2\)Smets and Wouters (2007).
Khan and Reza (2013) stress this negative effect as an almost inevitable implication of spending expansions in New Keynesian models with a housing market. However, the effect might vanish with a certain degree of monetary accommodation, which is interesting in the context of this paper as the ZLB de facto works as monetary accommodation to government spending. Bermperoglou (2015) finds that the tightening of a housing-based borrowing constraint during the crisis resulted in government spending becoming less effective on output due to its negative effect on the housing price. Likewise, Andrés et al. (2015) conclude that the spending multiplier decreases when house prices fall and households’ access to credit get restricted. However, both papers omit the ZLB on monetary policy.

Due to the negative effect from government spending on the housing price which erodes the collateral capacity of borrowing households, I find a negative contribution to the spending multiplier from a tight borrowing constraint. However, this effect is dampened by the ZLB, because the ZLB leads to crowding in of consumption and a smaller reduction in house prices. With debt-financed government spending, at the ZLB, the collateral channel might even contribute positively to the spending multiplier and become an accelerator. However, the overall impact from the collateral channel on the multiplier size is marginal in all cases.

The analysis is carried out in the following steps. First, I present the extended Guerrieri and Iacoviello (2017) model. Secondly, I simulate fiscal policies at the ZLB. The robustness of the results is checked with respect to present value multipliers replacing impact multipliers, higher impatience among borrowing households, and different degrees of fiscal persistence. Then, I discuss the implications of my results on the evaluation of the ARRA. In that regard, I focus on market expectations to the duration of the ZLB period. Lastly, I conclude and suggest that future research should add public investments to experiments similar to mine as well as focusing on a more realistic formation of expectations.

2 Model

I integrate fiscal policy through government spending, and later income taxation in the model from Guerrieri and Iacoviello (2017). This model builds on Iacoviello (2005) and implies a collateral channel based on the housing stock. In contrast to Iacoviello and Neri (2010), the supply of housing is assumed to be fixed at 1 for simplicity reasons.³

The ZLB is the main focus of the paper. Thus, the utility function of the two household types and the monetary policy rule have been simplified compared to Guerrieri and

³Khan and Reza (2013) stress that this does not change the dynamics of the model.
Iacoviello (2017) to make it more likely that the economy hits the ZLB. All exogenous variables follow an AR(1) process. Along with the first order conditions (FOCs), they can be found in the appendix.

2.1 Households

There are two types of households: A patient household that accumulates capital and own retail goods firms, and an impatient household that does not. The patient household lends money to the government and the impatient household based on collateral from the impatient household’s housing stock.

2.1.1 The patient household

The patient household gains utility from consumption, $c_t$, and housing, $h_t$, as well as disutility from working hours, $N_t$.\footnote{In contrast to Guerrieri and Iacoviello (2017), the utility function does not include superficial habits.}

$$
E_0 U_0 = E_0 \sum_{t=0}^{\infty} \beta^t \zeta_t \left( \log c_t + j_t \log h_t - \frac{N_t^{1+\varphi}}{1+\varphi} \right)
$$

(1)

$\beta$ is the discount factor on the future and $\varphi$ determines the Frisch elasticity. $\zeta_t$ is an intertemporal preference parameter. A shock that lowers $\zeta_t$ increases the taste for savings and therefore lowers consumption today. $j_t$ is an exogenous shock process for the utility weight on housing. The patient household maximises (1) subject to the following budget constraint:

$$
c_t + q_t h_t + \frac{R_{t-1}}{\pi_t} b_{t-1} + I_t = \frac{w_t}{X_{w,t}} N_t + q_t h_{t-1} + b_t + r_{k,t} K_{t-1} + \text{div}_t - (1-\sigma)T_t
$$

(2)

Total expenses go to consumption, housing, negative debt formation, $b_t < 0$, and investments, $I_t$. Income is generated through labour income, increases in the value of the housing stock, real interest service on loans, real interests on capital, and lump-sum dividends from retail firm ownership and trade unions, $\text{div}_t$. It is reduced by a lump-sum tax payment to the government, $T_t$, where $1-\sigma$ is the income share of patient households. $w_t$ is the real wage. $X_{w,t}$ is a wage markup between the wage accrued by the trade union and received by households.\footnote{Trade union profit is eventually transferred back to the household through $\text{div}_t$.} $q_t$ is the housing price. $R_{t-1}/\pi_t$ is the real interest rate on bonds. $r_{k,t}$ is the real rental rate of capital, and $K_t$ is the capital stock.

Capital is accumulated from investments under the exogenous technology, $a_t$, where $\phi$ allows for convex installation costs:
\[ K_t = a_t \left( I_t - \frac{\phi (I_t - I_{t-1})^2}{I} \right) + (1 - \delta_k) K_{t-1} \]  

(3)

2.1.2 The impatient household

The impatient household has the same type of utility function as the patient household but with a lower discount factor, \( \beta' < \beta \) (i.e. variables of the impatient household are denoted with a mark). The impatient household maximises:

\[ E_0 U'_0 = E_0 \sum_{t=0}^{\infty} (\beta')^t \zeta_t \left( \log c'_t + j_t \log h'_t - \frac{N_t^{1+\varphi}}{1+\varphi} \right) \]  

but faces the following budget constraint:

\[ c'_t + q_t h'_t + \frac{R_{t-1}}{\pi_t} b'_{t-1} = \frac{w'_t}{X'^t_{w,t}} N'_t + q_t h'_{t-1} + b'_t + div'_t - \sigma T_t \]  

(5)

Total expenses of the impatient household go to consumption and housing as well as to paying interest on debt accumulated in the previous period. Total income comes from labour supply, increases in the housing value, borrowing from the patient household, dividends from labour unions, and is reduced by lump-sum tax payments.

The impatient household also faces a constraint on borrowing, (6), which is linked to the value of its housing stock. The housing stock is put forward as collateral for the loan due to imperfect information between lender and borrower. Thus, housing serves two purposes for the impatient household: it is a utility-generating good and a facilitator of access to credit.

\[ b'_t \leq \gamma \frac{b'_{t-1}}{\pi_t} + (1 - \gamma) m q_t h'_t \]  

(6)

\( m \) is the loan to value (LTV) ratio. As stressed by Guerrieri and Iacoviello (2017), the borrowing constraint is only fully reset in every period for households who choose to refinance their loans, and empirically measures of aggregate debt tend to follow housing prices with a lag. The inclusion of lagged debt in (6) allows for this sluggishness in borrowing with \( \gamma \) being the degree of inertia in the borrowing limit. When shocks are large enough to push the housing price sufficiently upwards, the constraint does not bind, because the impatient household gets access to more funds than it wishes to consume. The latter illustrates the development in the borrowing market before the crisis. I allow for this possible scenario by letting the borrowing constraint be only occasionally binding.

Two FOCs are worth emphasising:

\[ \lambda_t \equiv \lambda'_t \equiv \lambda'_c \]  

(4)

\[ \lambda'_t / \lambda'_c \]  

is the Lagrange multiplier on the borrowing constraint normalised by the marginal utility of consumption.

---

6 The shock processes, \( \zeta_t \) and \( j_t \), are identical for both types of households.

7 \( \lambda_t \equiv \lambda'_t / \lambda'_c \) is the Lagrange multiplier on the borrowing constraint normalised by the marginal utility of consumption.
\[ U'_{c,t}(1 - \lambda_t) = \beta' \mathbb{E}_t \left( \frac{R_t - \lambda_{t+1} \gamma}{\pi_{t+1}} \right) \]  

(7)

\[ q_t U'_{c,t} = U'_{h,t} + \lambda_t U'_{c,t}(1 - \gamma)mq_t + \beta' \mathbb{E}_t U'_{c,t+1}q_{t+1} \]  

(8)

The distinct feature of the impatient household comes from the collateral concern. In the Euler condition, (7), a tightening of the borrowing constraint reduces the marginal utility from consumption (i.e. the left-hand side). In the expected marginal utility from consumption in the next period, the relative shadow price on borrowing is moreover deducted as inertia in the borrowing constraint means that higher debt in this period affects the borrowing limit positively in the next period. (8) shows the optimal housing decision. The term \( \lambda_t U'_{c,t}(1 - \gamma)mq_t \) reflects that, when calculating the expected gain from housing, the impatient household also needs to consider the relative utility gain stemming from the part of the borrowing constraint that gets loosened when expanding the collateral capacity.

2.2 Firms

Two types of firms exist in the model: wholesale firms and retail firms.

2.2.1 Wholesale firms

Wholesale firms operate under full competition. They sell real output, \( Y_t/P_t \), to retailers at \( P^w_t \). Hence, they maximise the following profit function:

\[ \max_{Y_t, N_t, N'_t, K_{t-1}} \Pi_t = \frac{Y_t}{X_{p,t}} - w_tN_t - w'_tN'_t - r_{k,t}K_{t-1} \]  

(9)

where \( X_{p,t} \equiv P_t/P^w_t \) is the markup charged by retailers over wholesale prices. They maximise the profit function under the following production technology:

\[ Y_t = N_t^{(1-\sigma)(1-\alpha)}N'_t^{\sigma(1-\alpha)}K_{t-1}^{\alpha} \]  

(10)

\( \alpha \) is the income share to capital, and \( 1 - \alpha \) is the income share to total labour input. The income share to labour is divided into two shares, \( \sigma \) to impatient households, and \( 1 - \sigma \) to patient households.

2.2.2 Retail firms

Retail firms operate under monopolistic competition. They buy wholesale goods, reassemble them into output goods at no costs and sell them at a price which is set at a markup
over the wholesale price.\textsuperscript{8} Retailers adjust prices imperfectly under a Calvo scheme. Thus, prices evolve according to the New Keynesian Phillips curve:

\[
\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} - \varepsilon \hat{x}_{p,t} + u_{P,t}, \quad \varepsilon = \frac{(1 - \theta \pi)(1 - \theta \pi \beta)}{\theta \pi} \tag{11}
\]

\[
\hat{\pi}_t \equiv \log (\pi_t/\pi) \quad \text{and} \quad \hat{x}_{p,t} \equiv \log (X_{p,t}/X_p) \quad \text{are log deviations from the steady state values of inflation and the price markup respectively, and} \quad u_{P,t} \quad \text{is an exogenous shock to the price markup.}
\]

### 2.3 Trade unions

Wage setting is modelled as price setting. Both types of households offer labour services to their trade union. The trade unions maximise profits when supplying the labour services of their members to labour packers by setting a markup, \(X_{w,t} \) and \(X'_{w,t} \), on the wage that they pay to households.\textsuperscript{9} Labour packers reassemble the labour services and sell them to wholesale firms under perfect competition. The New Keynesian Phillips curves of nominal wage inflation, \(\hat{\pi}^w_t \) and \(\hat{\pi}'^w_t \), are:

\[
\hat{\pi}^w_t = \beta E_t \hat{\pi}^w_{t+1} - \varepsilon_w \hat{x}_{w,t} + u_{W,t}, \quad \varepsilon_w = \frac{(1 - \theta_w)(1 - \beta \theta_w)}{\theta_w} \tag{12}
\]

\[
\hat{\pi}'^w_t = \beta E_t \hat{\pi}'^w_{t+1} - \varepsilon'_w \hat{x}'_{w,t} + u_{W,t}, \quad \varepsilon'_w = \frac{(1 - \theta_w)(1 - \beta' \theta_w)}{\theta_w} \tag{13}
\]

where \(\hat{\pi}^w_t \equiv \log(w_t \pi_t/w_{t-1} \pi) \). \(u_{W,t} \) is a shock to the wage markups.

### 2.4 The government

Monetary policy is determined from a Taylor rule subject to a ZLB constraint:

\[
R_t = \max [1, R_{TR,t}], \quad R_{TR,t} = R_{t-1}^R \left( \frac{\pi_t}{\pi} \right)^{(1-r_R)r_\pi} \left( \frac{Y_t}{Y} \right)^{(1-r_R)r_Y} \quad R^{1-r_R} \varepsilon_{R,t} \tag{14}
\]

where \(R_{TR,t} - 1 \) is the policy rate when the Taylor rule is active. Compared to Guerrieri and Iacoviello (2017), the Taylor rule is simplified so that the central bank only reacts on the last observation of quarterly inflation. \(r_R \) allows for interest rate smoothing, and \(\varepsilon_{R,t} \) is a policy-implied shock to the nominal interest rate.

Fiscal policy follows Galí et al. (2007). The government conducts fiscal policy through exogenous public spending, \(G_t \), prone to shocks, \(u_{G,t} \). It finances public spending through a mixture of government bonds, \(b_G^t \), and total lump-sum taxes paid by the patient and

\textsuperscript{8}This intuition follows Bernanke et al. (1999).

\textsuperscript{9}However, the profit from the wage markup is eventually transferred back to the households through \(div_t \) and \(div'_t \).
impatient households, \( T_t = (1 - \sigma)T_t + \sigma T_t \). So the budget constraint of the government is:

\[
T_t + b_t^G = \frac{R_{t-1}}{\pi_t} b_{t-1}^G + G_t
\]  

(15)

Generally, I consider a balanced budget meaning that \( b_t^G = 0 \) and \( T_t = G_t \).

In equilibrium all markets clear: consumption of housing sums to 1, and debt obligations sum to 0. The steady-state relations are provided in the appendix.

2.5 Calibration

The calibration of the model generally follows the estimated parameters from Guerrieri and Iacoviello (2017). They estimate their model with Bayesian methods on US time series from 1985-2011. All parameter values are shown in the appendix, but here I will stick to describing the changes to their calibration.

I have changed the target inflation from \( \pi = 1.005 \) to \( \pi = 1 \) (i.e. a zero inflation target) to ease the path towards the ZLB. A zero inflation target is widely applied in the literature for simplicity reasons although a 2 per cent target is more realistic.

I have taken fiscal parameters from Christiano et al. (2011), and \( b^G/Y \) is 0 with a balanced budget. The inclusion of government spending changes the steady-state value of output. In order to target a steady-state ratio of housing wealth to annual output of 1.5, I set \( j = 0.052 \).

Lastly, I have changed the estimated Calvo parameters from \( \theta_\pi = 0.9182 \) and \( \theta_w = 0.9163 \). Guerrieri and Iacoviello (2017)’s model is very rigid as wages and inflation react only modestly to exogenous shocks. To ease the path towards the ZLB I adopt the estimated values from the very similar model in Iacoviello and Neri (2010) (i.e. \( \theta_\pi = 0.83 \) and \( \theta_w = 0.79 \)).

2.6 Dynamics on the housing market

Before moving on with the simulation, I explain a key connection between consumption of the patient household and the housing price, an effect clarified by Barsky et al. (2007).

Housing demand of the patient household implies:

\[
q_t U_{c,t} = E_t \left( \sum_{s=0}^{\infty} \beta^s U_{h,t+s} \right)
\]  

(16)

(16) tells us that the shadow value of housing (i.e. the left-hand side) equals the discounted sum of utility from housing services (i.e. the right-hand side). Because housing is a stock that does not depreciate, the total housing stock of the household is close to
constant although shocks might cause the household to adjust the housing flow over time. In other words, since the marginal utility of housing depends on the housing stock, it is roughly constant over time (Sterk, 2010). Moreover, the household does not care much about the timing of durables due to a discount factor, $\beta$, close to 1. Therefore, the household is willing to substitute housing intertemporarily when the price changes, and the shadow value is only under very little influence from temporary shocks to the economy.

In contrast, the marginal utility (i.e. the shadow value) of consumption moves freely with the consumption level and the intertemporal preference parameter. Therefore, changes in the marginal utility of consumption must dictate the housing price. Specifically, it means that increases in the marginal utility of consumption of the patient household must be accompanied by equally large reductions in the housing price to obtain a near-constant shadow value of housing. The reason why house prices follow consumption of the patient and not the impatient household is because the demand of the latter for housing is less sensitive to expected changes in the marginal utility of consumption and house prices due to the collateral concern shown in (8).

3 Simulation

The two non-linearities from the occasionally binding ZLB and borrowing constraint complicate the simulation of the model. Like Guerrieri and Iacoviello (2017), I use the OccBin algorithm introduced in Guerrieri and Iacoviello (2015) to solve this problem. The algorithm is a piecewise linear perturbation method. It treats the 4 different cases where the constraints do and do not bind as different regimes which can be solved through linearisation around the non-stochastic steady state of a reference regime.\(^{10}\) The algorithm is explained fully in Guerrieri and Iacoviello (2015). As explained in Guerrieri and Iacoviello (2017) and shown in figure A.1 in the appendix, the borrowing constraint implies an asymmetry. Consumption responds more to changes in the housing price when the borrowing constraint is binding. The reason is that an increase (decrease) in the housing price loosens (tightens) the borrowing constraint of the impatient household through the collateral channel.

In the following, I present simulated effects of fiscal policy at the ZLB and in normal times. The liquidity trap has been simulated through a 7 standard deviations negative shock to the intertemporal preference parameter, $\zeta_t$, in period 2, which generates a ZLB spell of 4 quarters.\(^{11}\) The expected length of the liquidity trap is vital for the fiscal multiplier size as households adjust consumption immediately to expected changes in the

\(^{10}\)In the reference regime, the borrowing constraint binds and the ZLB does not bind.

\(^{11}\)See figure A.2 in the appendix.
future real interest rate. Therefore, since government spending reduces the real interest rate at the ZLB, I will show that the multiplier size explodes with the expected length of the ZLB spell. Christiano et al. (2011) and Eggertsson (2011) simulate fiscal policies in liquidity traps that last longer than 4 quarters. The large multipliers found in these papers depend on private expectations to long ZLB spells. However, I will later show that actual market expectations at the time around the ARRA implementation reflected that market participants expected the Fed to keep the Effective Funds Rate at the ZLB for a shorter period closer to the simulated liquidity trap length in this paper.

The shock size that generates the 4 quarter liquidity trap is relatively large and reflects that a single preference shock is an unrealistic way of modelling a liquidity trap. The ZLB spell in the US was rather generated from a sequence of negative wealth shocks followed by smaller shocks to intertemporal preferences and productivity (Cuba-Borda, 2014; Guerrieri and Iacoviello, 2017). However, modelling a liquidity trap through a large preference shock is a relatively standard way of analysing ZLB policy (Eggertsson, 2011; Christiano et al., 2011) due to its simplicity and tangibility when isolating the effects from the ZLB on fiscal multipliers.

3.1 Effect of government spending

I impose a positive government spending shock of 5 per cent in period 3 relative to the steady-state level, $G$. When raising government spending in period 3, the government conducts expansionary fiscal policy 1 quarter after the beginning of the recession due to an implementation lag. The implementation lag implies 3 quarters where both government spending is expansionary and the ZLB is binding. The impulse response functions at the ZLB have been calculated as the effects from a sequence with a preference shock in period 2 subtracted from a sequence with a preference shock in period 2 and a government spending shock in period 3. In the baseline scenario, I omit a preceding preference shock. The resulting impulse response functions are shown in figure 1.\footnote{The figure shows percentage deviations from the steady state except for the nominal interest rate and the borrowing multiplier where the deviations have been shown in percentage points and level instead. Additional impulse response functions of all simulations are shown in appendix F.}

The government lifts total demand. Moreover, retailers and trade unions lower their markups and raise prices and wages. The reaction of both types of households is to lower consumption and raise labour supply when they have to pay more taxes. In addition, lower income also forces the patient household to lower investment. The higher labour supply has a negative effect on nominal wages and inflation. These effects are standard in New Keynesian DSGE models. What is interesting are the effects from the collateral and ZLB constraints.
Figure 1: Effect of a 5 per cent increase in government spending

The collateral constraint contributes negatively to the effect from government spending. The housing price declines after a government shock due to lower consumption of the patient household. This reduces the collateral capacity of the impatient household who gradually reduces borrowing and consumption further. The financial accelerator (Bernanke et al., 1999) is therefore actually a decelerator, which has lowered the effective-
ness of government spending at the outset of the housing market crisis as credit constraints went from slack to tight.

The ZLB contributes positively to the effectiveness of government spending. Output increases more in the liquidity trap than in the baseline case. The reason is the absence of a stabilising reaction from the central bank on higher inflation and output that would crowd out consumption and investment. When the nominal interest rate is fixed at zero while inflation rises due to higher government spending, the real interest rate declines and crowds in consumption and investment.

The ZLB also interacts with the collateral channel. When consumption is crowded in by the real interest rate, the increase in the marginal utility of consumption of the patient household is counteracted. Hence, the ZLB induces a smaller reduction in the housing price than in the baseline case and therefore a less negative effect from the credit channel on consumption.

The high effectiveness of government spending in liquidity traps is a common result (Christiano et al., 2011; Eggertsson, 2011). However, it is useful to quantify the effects of the ZLB. The impact multiplier of government spending is defined as the absolute change in output relative to the absolute change in government spending within the same period. It is shown in table 1.

<table>
<thead>
<tr>
<th>Case</th>
<th>Impact multiplier of $G$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Baseline</td>
<td>0.70</td>
</tr>
<tr>
<td>ZLB</td>
<td>0.89</td>
</tr>
</tbody>
</table>

Source: Own simulation in Dynare.

We see that the standard multiplier is 0.70, which is reasonable compared to the findings of other papers.\footnote{Eggertsson (2011) finds a baseline multiplier of 0.48, and Christiano et al. (2011) find a multiplier of 1.05. A higher multiplier in the latter paper comes from the use of a non-separable utility function.} However, the difference between the ZLB and the baseline case is small in my experiment. Christiano et al. (2011) and Eggertsson (2011) find multipliers that are around 3-5 times larger at the ZLB than in their baseline cases, while the multiplier implied by my model only increases with around 0.2 at the ZLB.
The vital determinant of the ZLB multiplier size is the duration of the liquidity trap. Figure 2 shows different multipliers as a function of the time spent at the ZLB. It is clear that the multiplier increases almost exponentially with the duration of the liquidity trap. The longer agents expect the liquidity trap to last, the more an additional expected quarter at the ZLB will raise the multiplier. With an expected stay at the ZLB for 8 quarters, the impact multiplier will rise to 1.97, which is close to the result in Eggertsson (2011).

The duration of expansionary government spending at the ZLB is only 3 quarters in table 1. In comparison, Christiano et al. (2011) and Eggertsson (2011) work with a time interval in the range of 6-12 quarters without implementation lags. The short duration of the liquidity trap along with the implementation lag therefore accounts for most of the relatively small ZLB multiplier in my experiment. While this factor is mentioned in both papers, I here provide a more thorough discussion and explanation.

In a paper on forward guidance, McKay et al. (2016) explain how the effect originates from the nature of forward-looking agents in the New Keynesian model. Given a future expected decline in the real interest rate, the household increases consumption immediately. In order to satisfy the Euler equation in every period, the forward-looking household only changes consumption growth between the period of the interest rate reduction and the period after when consumption again becomes expensive. In other periods, consumption growth is unchanged. A higher income level generated through increased demand allows

14Due to persistence in government spending, it will stay above the steady-state value in subsequent periods where monetary policy is not at the ZLB.
the household to raise consumption immediately until the period after the interest rate reduction, when the consumption level falls back to the steady-state level. Thus, every intermediate period between the announcement and implementation of an interest rate cut adds to an accumulated effect on the immediate consumption level. These dynamics are identical to an expected government expansion at the ZLB as it de facto works as a real rate reduction due to inflationary effects. Moreover, at the ZLB, a feedback mechanism further increases immediate consumption as inflationary pressures from private crowding in reduces the real interest rate even more. These dynamics apply for every period of expansionary spending at the ZLB and cause an exploding multiplier size along the expectations to the liquidity trap length. The result is in accordance with Bletzinger and Lalik (2017). They show that large ZLB multipliers in the New Keynesian model rest on forward-looking agents who predict the length of the liquidity trap initially.

I have moreover suspected other features to influence the multiplier size. First, I have checked the effects from the collateral channel and wage rigidity. While both channels reduce the multiplier, the multiplier size does not come close to the ones in Christiano et al. (2011) and Eggertsson (2011) when omitting them.15

Secondly, I have run the experiment where the government more realistically finances spending through public debt. I have used the fiscal rule from Galí et al. (2007).16

\[
\frac{T_t - T}{Y} = \kappa_b \frac{b^G_t - b^G}{Y} + \kappa_G \frac{G_t - G}{Y}
\]  

(17)

The results are generally unchanged, and the impact multipliers are only marginally higher in both scenarios.17 However, the collateral channel now contributes positively to the ZLB multiplier. The postponed tax payment of the impatient household is paid by the patient household who purchases bonds from the government. Thus, a wealth transfer from the patient household occurs, and this effect is amplified by the ZLB where the real interest service on loans declines. The result is a reduced demand for housing of the patient household such that the housing stock and therefore collateral capacity and borrowing of the impatient household increases. This is shown in figure 3. However, as noted, the impact from debt financing on the multiplier size remains marginal.

---

15See figure A.4 and A.5 in the appendix.
16The sensitivity parameters are calibrated as in Galí et al. (2007) (i.e. \( \kappa_b = 0.33 \) and \( \kappa_G = 0.10 \)), and \( b^G/Y = 0.4732 \) to reflect the mean public debt to GDP ratio in the US in 1966-2008.
17The baseline multiplier is 0.72, and the ZLB multiplier is 0.94.
3.2 Effect of a cut in the income tax rate

Besides government spending, income taxation at the ZLB is widely discussed. I introduce a tax on labour income, \(\tau_t\), which is imposed equivalently on both household types. The income tax changes the budget constraints of the households and the government as well as the optimal labour choices. Details are shown in appendix E.

The income tax rate is lowered with 1 percentage point in period 3. The impulse response functions are shown in figure 4. A lower income tax rate raises the after-tax real wage. Thus, both household types increase their labour supply, and nominal wages gradually decline. Moreover, productivity of capital increases, which also contributes to a decline in marginal costs in the wholesale production and a lower inflation. At the ZLB, a lower inflation level translates into a higher real interest rate that dampens demand. Instead of raising output, the fiscal stimulation policy thereby lowers output in the first quarter before it starts having a positive impact.

The collateral channel affects the tax multiplier positively in the baseline case as a lower marginal utility of consumption translates into a higher housing price that expands the collateral capacity and lifts the consumption opportunities of the impatient household. This positive effect gets muted by the ZLB due to a lower housing price.
**Figure 4:** Effect of a 1 percentage point decrease in the income tax rate

The impact multiplier from a tax cut has been calculated as the absolute change in output relative to the negative change in the tax revenue, and is shown in table 2. The tax multiplier in the baseline case is 0.15. The reason why the tax multiplier is substantially lower than the spending multiplier is the choice of analysing the impact multiplier which only includes the effects in the first period of the policy change. Income tax cuts stimulate output over a longer time horizon, while government spending stimulates output immediately. Specifically, output peaks 2 quarters after the income tax cut.

**Table 2:** Effect of a cut in the income tax

<table>
<thead>
<tr>
<th>Case</th>
<th>Impact multiplier of $\tau$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Baseline</td>
<td>0.15</td>
</tr>
<tr>
<td>ZLB</td>
<td>-0.05</td>
</tr>
</tbody>
</table>

Source: Own simulation in Dynare.

At the ZLB, the tax multiplier is -0.05. Eggertsson (2011) finds a tax multiplier of -1 at the ZLB. My ZLB multiplier is less negative due the short length of the liquidity
trap, which can be seen in figure A.8 in the appendix. The intuition is similar to the one presented for the spending multiplier but now with an expected rise in the real interest rate.

3.3 Robustness

In this section I will check robustness of my results to factors that have been frequently discussed in the literature.

3.3.1 Present value multipliers

Uhlig (2010) argues that one needs to analyse the long-term multipliers in order to get a full picture of the economic dynamics when conducting fiscal policy. Present value multipliers sum all discounted, future effects on output relative to the changes in fiscal instruments.

The present value multiplier of government spending remains higher at the ZLB, while the tax multiplier remains lower.\(^{18}\) Moreover, the present value multipliers of taxation come closer to those of government spending. As mentioned in section 3.2, this reflects that government spending is a more short-lived policy than income tax cuts, because tax cuts also stimulate investments through higher post-tax income. Importantly, the large effects from expectations to the liquidity trap length remain. This can be seen in figure A.9 and A.10 in appendix F.

3.3.2 Higher impatience among credit-constrained households

The discount factor of the impatient household is estimated at a relatively high level in Guerrieri and Iacoviello (2017). It implies a more frequently slack borrowing constraint, and that the impatient household is almost as patient as the patient household. I run similar experiments with \(\beta' = 0.97\) replacing \(\beta' = 0.9922\) to check that my results do not rest on low impatience. I get similar effects\(^{19}\) and almost identical ZLB multipliers. The impact of the expected liquidity trap length is also unchanged as seen in figure A.13 and A.14 in the appendix.

3.3.3 Persistence of fiscal policy

In the last robustness check I change the persistence of fiscal policy, \(\rho_G\). This is related to Christiano et al. (2011) who stress that a spending stimulus becomes ineffective if it is permanent. Figure A.15 in the appendix shows that the impact multiplier remains

\(^{18}\)See table A.3 in the appendix.\(^{19}\)The impulse response functions can be seen in figure A.11 and A.12 in appendix F.
higher at the ZLB for plausible values of $\rho_G$. Moreover, the multiplier is higher for lower values of $\rho_G$. If the rise in government spending becomes less persistent, there will be fewer periods with a high level of government spending after the liquidity trap has ended. Initially, households will therefore reduce consumption less as they expect lower future real interest rates, and the spending multiplier will be higher.

Below a persistence parameter of 0.39, the ZLB multiplier is almost unchanged\(^{20}\). It shows that spending stimulates output exactly as long as it reduces the real interest rate without leading to higher expected real interest rates in the future. For government spending to be effective at the ZLB it needs to take place while and only while the ZLB is binding. Therefore, the effect of the expected duration of the liquidity trap is smaller if persistence of government spending is low. In that case, an extra expected quarter at the ZLB does not add much extra time with both a high level of government spending and a binding ZLB.\(^{21}\)

4 Discussion of the ARRA

In this section I apply my findings in a discussion of crisis management in the US. Specifically, I will discuss the ARRA.

The ARRA was a stimulus plan of 787 bn $ to be spent from 2009-2010. It consisted of both fiscal spending, tax cuts, and public investments. Denes and Eggertsson (2009) roughly estimate that 2/3 went to government spending and 1/3 to income tax cuts. The estimate is rough, but provides a good indication of the large weight on public spending in the ARRA.

By adopting these assumptions, a simplified calculation of the effect of the ARRA based on my multipliers show that it increased GDP by 1.6 per cent.\(^{22}\) A model-simulated ex-post estimate of the CEA\(^{23}\) was that the pact increased GDP by 2-2.5 per cent from late 2009 to mid-2011 alone (CEA, 2014). Denes and Eggertsson (2009) estimated a similarly large rise of 3.3 per cent in GDP as an implication of the ARRA. In contrast, Drautzburg and Uhlig (2013) found an ARRA multiplier below 1 in an estimated DSGE model as mentioned in the introduction. They come closer to my conclusions. However, both estimated DSGE models and model-simulated ARRA effects rely on ex-ante assumptions on multiplier sizes when identifying the counterfactual GDP growth. In other words, the effect of the ARRA could as well have been simulated before its implementation. What

\(^{20}\)It increases marginally.
\(^{21}\)See figure A.16 in appendix F.
\(^{22}\)The effect on GDP has been calculated as the multiplier of the ARRA, $2/3 \times 0.89 + 1/3 \times (-0.05) = 0.58$, times the expenses of the ARRA in per cent of GDP in 2008-2009, $787/29,249 \times 100 = 2.7$ per cent.
\(^{23}\)Council of Economic Advisers.
is needed is an identification strategy applied on historical data. However, due to rare occurrences of ZLB spells, the number of such studies is small.

In a study on historical US data, Ramey and Zubairy (2014) estimate multipliers in times with a binding and slack ZLB constraint. With multiple identification schemes they do not find any difference in multipliers in the different states.²⁴ Feyrer and Sacerdote (2011) exploit state-level variation in ARRA stimulus within the US and find lower ARRA multipliers than predicted by Bernstein and Romer (2009).²⁵ Wilson (2012) also exploits state-level variation in a different data set and estimates an ARRA multiplier closer to the projections of Bernstein and Romer (2009). The difference in estimates of the ARRA multiplier reflects the major uncertainty as to the actual effects of the ARRA due to the unknown counterfactual development of GDP. Further empirical research on ZLB multipliers is clearly needed.

However, what is certain is that the model-generated ZLB multipliers are low in this paper due to a short expected duration of the liquidity trap. Therefore, rather than observing the actual time that the Effective Federal Funds Rate remained at the ZLB, I turn to discuss the expectations to monetary policy at the time where the ARRA was implemented.

4.1 Expectations to the length of the liquidity trap

A standard measure of market expectations to monetary policy in the US is the Federal Funds 30 Days Future Rate. Figure 5 shows the market expectations at key dates of the ARRA implementation along with the actual Funds Rate and the upper limit of the ZLB margin.²⁶ The 26th of January 2009 was the date on which the ARRA was presented to the House of Representatives and on the 17th of February 2009 the ARRA was signed. In addition, the 1st of January 2009 is reported to show the expectations before the pact was officially announced.

The 3 expectation curves lead to the same conclusion. It is clear that the market expected the nominal interest rate to escape the ZLB after only 2 quarters in July 2009.²⁷ This is much shorter than assumed in papers such as Christiano et al. (2011) and Eg-

²⁴Only when excluding the sub-sample of World War II, they find some mixed evidence for a ZLB multiplier above 1.

²⁵However, Feyrer and Sacerdote (2011) underline that by emphasising state-level effects, they ignore positive nationwide effects of the ARRA. Moreover, the standard errors of their estimates do not allow them to reject the possibility that the ARRA might have been more effective than indicated by their point estimates.

²⁶The Fed defines a 25 basispoint wide interval for their target rate. The interval 0-0.25 represents the ZLB margin.

²⁷In contrast, the FOMC projected the ZLB to bind for 16 quarters in January 2009 (FOMC, 2009).
However we need to be careful when interpreting figure 5. When the interest rate is at the ZLB, the probability distributions of future interest rates become skewed to the right as there is zero probability of a lower future interest rate. In this case, market expectations to the interest rate are likely to suffer from a positive bias (FOMC, 2009). I have compared figure 5 to market expectations on the 9th of August 2011\textsuperscript{28} when the FOMC specified their forward guidance program to last for at least 2 years (Williams, 2013). The step was taken in order to improve transparency and strengthen the credibility of the forward guidance program. It represents the first date in data when the market actually expected the ZLB to last for at least 2 years. The strong effect from the Fed commitment on market expectations in August 2011 underlines that the low expectations to the liquidity trap duration in the beginning of 2009 were not a result driven purely by skewed probability distributions.

Rather, figure 5 reflects that agents underestimated the time at the ZLB due to imperfect forecasts on the future and a lack of commitment and transparency in Fed’s forward

\textsuperscript{28}See figure A.17 in the appendix.
guidance program. It is therefore fair to conclude that the optimistic anticipations to the ARRA rested on expectations to the liquidity trap length, which were inconsistent with actual expectations of private agents in the economy.

5 Conclusion

I find that the government spending multiplier is higher at the ZLB than under normal circumstances as in Christiano et al. (2011) and Eggertsson (2011). In contrast, cuts in the income tax rate might bear negative consequences on output in the short run. The main takeaway from the paper is the sensitivity of the multiplier size to expectations to the liquidity trap length. One should be aware of this effect when using a New Keynesian model for fiscal policy analysis at the ZLB, otherwise we risk having unrealistic and over-optimistic expectations. Market expectations to the Effective Federal Funds Rate around the ARRA announcement are more in line with the short ZLB spell in this paper than longer spells in Christiano et al. (2011), Eggertsson (2011), and Bernstein and Romer (2009). Hence, it is relevant to doubt whether the ARRA was based on realistic assumptions on expectations in the private economy.

Moreover, I find that the tightening of households’ borrowing constraint during the crisis might have contributed negatively to the spending multiplier, while this changes when introducing public debt financing at the ZLB. However, the overall effect remains marginal.

Further empirical research that identifies the ZLB effect on fiscal multipliers would be needed as the number of historical cases with liquidity traps increases. Moreover, the absence of public investments in my DSGE model is likely to cause a negative bias in the spending multiplier (Bouakez et al., 2014), which should be further emphasised. Another useful insight would come from replacing rational expectations with a more careful modelling of expectations.
Bibliography

Economica, 82, 1048–1081.


Appendices

A FOCs and the remaining model equations

\[ U_{c,t} = \lambda_{c,t} \]  
\[ U_{c,t} = \beta E_t \left( U_{c,t+1} \frac{R_t}{\pi_{t+1}} \right) \]  
\[ q_t U_{c,t} = U_{h,t} + \beta E_t(q_{t+1} U_{c,t+1}) \]  
\[ - U_{n,t} = U_{c,t} \frac{w_t}{X_{w,t}} \]  
\[ q_{k,t} U_{c,t} \left( 1 - \phi \frac{\Delta h}{I} \right) = U_{c,t} - \beta E_t \left( q_{k,t+1} U_{c,t+1} \frac{\Delta I_{t+1}}{I} \right) \]  
\[ q_{k,t} U_{c,t} \frac{1}{a_t} = \beta E_t \left( U_{c,t+1} \left( r_{k,t+1} + q_{k,t+1} \frac{1 - \delta_k}{a_{t+1}} \right) \right) \]  
\[ U'_{c,t} = \lambda'_{c,t} \]  
\[ U'_{c,t} (1 - \lambda_t) = \beta' E_t \left( U'_{c,t+1} \frac{R_t - \lambda_{t+1} \gamma}{\pi_{t+1}} \right) \]  
\[ q_t U'_{c,t} = U'_{h,t} + \lambda_t U'_{c,t} (1 - \gamma) mq_t + \beta' E_t U'_{c,t+1} q_{t+1} \]  
\[ - U'_{n,t} = U'_{c,t} \frac{w_t'}{X'_{w,t}} \]  
\[ (1 - \sigma)(1 - \alpha) \frac{Y_t}{X_{p,t}} = w_t N_t \]  
\[ \sigma(1 - \alpha) \frac{Y_t}{X_{p,t}} = w'_t N'_t \]  
\[ \frac{\alpha Y_t}{X_{p,t}} = r_{k,t} K_{t-1} \]  
\[ \hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} - \varepsilon_{\pi} \hat{x}_{p,t} + u_{F,t} \]  
\[ \hat{\pi}_{t}^{w} = \beta E_t \hat{\pi}_{t+1}^{w} - \varepsilon_{w} \hat{x}_{w,t} + u_{W,t} \]  
\[ \hat{\pi}_{t}^{'w} = \beta E_t \hat{\pi}_{t+1}^{'w} - \varepsilon_{w} \hat{x}_{w,t} + u_{W,t} \]  
\[ \log R_t = \max [0, \log R_{TR,t}] \]  
\[ \log R_{TR,t} = r_R \log R_{t-1} + (1 - r_R) r_x \log \left( \frac{\pi_t}{\pi} \right) + (1 - r_R) r_y \log \left( \frac{Y_t}{Y} \right) \]  
\[ + (1 - r_R) \log R + u_{R,t} \]  
\[ c_t + c_t' + K_t - (1 - \delta_k) K_{t-1} + G_t = Y_t \]  
\[ h_t + h_t' = 1 \]  
\[ b_t + b_t' + b_t'^G = 0 \]
B  Shocks

\[
\log \zeta_t = \rho_\zeta \log \zeta_{t-1} + u_{\zeta,t}, \quad u_{\zeta,t} \sim iidN(0, \sigma^2_\zeta) \tag{A.21}
\]

\[
\log j_t = \rho_j \log j_{t-1} + (1 - \rho_j) \log j + u_{j,t}, \quad u_{j,t} \sim iidN(0, \sigma^2_j) \tag{A.22}
\]

\[
\log a_t = \rho_K \log a_{t-1} + u_{K,t}, \quad u_{K,t} \sim iidN(0, \sigma^2_K) \tag{A.23}
\]

\[
u_{P,t} = \log e_{P,t} = \rho_P \log e_{P,t-1} + \epsilon_{P,t}, \quad \epsilon_{P,t} \sim iidN(0, \sigma^2_P) \tag{A.24}
\]

\[
\log e_{R,t} = \rho_R \log e_{R,t-1} + \epsilon_{R,t}, \quad \epsilon_{R,t} \sim iidN(0, \sigma^2_R) \tag{A.26}
\]

\[
\log G_t = \rho_G \log G_{t-1} + (1 - \rho_G) \log G + u_{G,t}, \quad u_{G,t} \sim iidN(0, \sigma^2_G) \tag{A.27}
\]
C Steady state

Table A.1: Steady-state relations

<table>
<thead>
<tr>
<th>Equation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R = \frac{\pi}{\beta}$</td>
<td>(SS.1)</td>
</tr>
<tr>
<td>$r_k = \frac{1}{\beta} - (1 - \delta_k)$</td>
<td>(SS.2)</td>
</tr>
<tr>
<td>$\lambda = \frac{1 - \beta'}{1 - 2\beta}$</td>
<td>(SS.3)</td>
</tr>
<tr>
<td>$q^{\prime} = \frac{j}{1 - \beta}$</td>
<td>(SS.4)</td>
</tr>
<tr>
<td>$\frac{q^{\prime h'}}{\psi} = \frac{j}{1 - \beta - \lambda(1 - \gamma)m}$</td>
<td>(SS.5)</td>
</tr>
<tr>
<td>$K = X_w r_k$</td>
<td>(SS.6)</td>
</tr>
<tr>
<td>$\frac{b'}{q^{\prime h'}} = \frac{m(1 - \gamma)}{1 - \gamma}$</td>
<td>(SS.7)</td>
</tr>
<tr>
<td>$T = \frac{(1 - \frac{1}{\beta} - 1) K^G}{Y} + G$</td>
<td>(SS.8)</td>
</tr>
<tr>
<td>$\psi' = \frac{(1 - \alpha)\sigma}{X_p X_w} - \frac{\sigma Y}{Y}$</td>
<td>(SS.9)</td>
</tr>
<tr>
<td>$\psi' = 1 - \frac{\psi}{Y} - \delta K + G$</td>
<td>(SS.10)</td>
</tr>
<tr>
<td>$N = \left( \frac{(1 - \alpha)(1 - \alpha)}{X_p X_w} \right)^{\frac{1}{1 + \phi}}$</td>
<td>(SS.13)</td>
</tr>
<tr>
<td>$N' = \left( \frac{\sigma(1 - \alpha)}{X_p X_w} \right)^{\frac{1}{1 + \phi}}$</td>
<td>(SS.14)</td>
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<tr>
<td>$Y = \left( \frac{K}{Y} \right)^{\frac{1}{1 - \alpha}} N^{1 - \sigma} N'^{\sigma}$</td>
<td>(SS.15)</td>
</tr>
<tr>
<td>$I = \delta K Y$</td>
<td>(SS.16)</td>
</tr>
<tr>
<td>$w = X_w \frac{K}{Y} N^\phi$</td>
<td>(SS.17)</td>
</tr>
<tr>
<td>$w' = X_w \frac{q^{\prime}}{Y} N'^{\phi}$</td>
<td>(SS.18)</td>
</tr>
<tr>
<td>$q = \frac{q^{\prime}}{Y} Y + \frac{q^{\prime h'}}{\psi} Y$</td>
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Table A.2: Calibrated parameters and steady-state values

<table>
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</tr>
<tr>
<td>$\beta'$</td>
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E Integrating an income tax

(2) changes to (2'), and the FOC with respect to labour hours, (A.4), changes to (A.4'):
\[ c_t + q_t h_t + \frac{R_{t-1}}{\tau_t} b_{t-1} + I_t = (1 - \tau_t) \frac{w_t}{X_{w,t}} N_t + q_t h_{t-1} + b_t + r_{kt} K_{t-1} + \text{div}_t - (1 - \sigma) T_t \]

\[ - U_{n,t} = U_{c,t}(1 - \tau_t) \frac{w_t}{X_{w,t}} \]

where \( \text{div}_t = \frac{X_{p,t}}{X_{p,t}} Y_t - \frac{X_{w,t}}{X_{w,t}} (1 - \tau_t) w_t N_t \). Similarly, the budget constraint and FOC of the impatient household change to (5') and (A.10'):

\[ c_t' + q_t h_t' + \frac{R_{t-1}}{\tau_t} b_{t-1}' = (1 - \tau_t) \frac{w_t'}{X_{w,t}'} N_t' + q_t h_{t-1}' + b_t' + \text{div}_t' - \sigma T_t \]

\[ - U'_{n,t} = U'_{c,t}(1 - \tau_t) \frac{w_t'}{X_{w,t}'} \]

where \( \text{div}_t' = (1 - \tau_t) \frac{X_{p,t}'}{X_{w,t}'} w_t' N_t' \). Moreover, the public budget constraint, (15), changes to (15'):

\[ T_t + \tau_t (w_t N_t + w_t' N_t') + b_t^G = \frac{R_{t-1}}{\tau_t} b_{t-1}^G + G_t \]

lastly, the income tax rate follows an AR(1) process, where \( \tau = 0.20 \) as in Eggertsson (2011) and \( \rho_\tau = 0.80 \) to match the persistence of government spending:

\[ \log \tau_t = \rho_\tau \log \tau_{t-1} + (1 - \rho_\tau) \log \tau + u_{\tau,t} \]

\[ u_{\tau,t} \sim iid N(0, \sigma_\tau^2) \]
F Figures and tables

Figure A.1: Asymmetric effects of an equally large increase and decrease in housing demand

Note: The housing price and consumption are multiplied by $-1$ for a negative housing price shock.
Source: Own simulation in Dynare.
Figure A.2: Effect of a 7 standard deviations decrease in the intertemporal preference parameter

Source: Own simulation in Dynare.
Figure A.3: Effect of a 5 per cent increase in government spending

Source: Own simulation in Dynare.
**Figure A.4:** Effect of a 5 per cent increase in government spending with $\sigma \to 0$

Note: The ZLB multiplier is 0.92.
Source: Own simulation in Dynare.

**Figure A.5:** Effect of a 5 per cent increase in government spending with $\theta_w \to 0$

Note: The ZLB multiplier is 1.04.
Source: Own simulation in Dynare.
**Figure A.6:** Effect of a 5 per cent increase in debt-financed government spending

Source: Own simulation in Dynare.
Figure A.7: Effect of a 1 percentage point decrease in the income tax rate

Source: Own simulation in Dynare.
Figure A.8: Impact tax multipliers and liquidity trap length

Source: Own simulation in Dynare.

Table A.3: Impact and present value multipliers

<table>
<thead>
<tr>
<th>Case</th>
<th>Impact multipliers</th>
<th>Present value multipliers</th>
</tr>
</thead>
<tbody>
<tr>
<td>Baseline</td>
<td>0.70</td>
<td>0.15</td>
</tr>
<tr>
<td>ZLB</td>
<td>0.89</td>
<td>−0.05</td>
</tr>
</tbody>
</table>

Source: Own simulation in Dynare.
**Figure A.9:** Spending present value multipliers

![Spending present value multipliers](image1)

Source: Own simulation in Dynare.

**Figure A.10:** Tax present value multipliers

![Tax present value multipliers](image2)

Source: Own simulation in Dynare.
Figure A.11: Effect of a 5 per cent increase in government spending with $\beta' = 0.97$

Source: Own simulation in Dynare.
Figure A.12: Effect of a 1 percentage point decrease in the income tax rate with $\beta' = 0.97$.

Source: Own simulation in Dynare.
Figure A.13: Spending impact multipliers with $\beta' = 0.97$

Source: Own simulation in Dynare.

Figure A.14: Tax impact multipliers with $\beta' = 0.97$

Source: Own simulation in Dynare.
**Figure A.15:** Impact spending multipliers and fiscal persistence

Note: $\rho_G$ is the persistence of shocks to government spending.
Source: Own simulation in Dynare.

**Figure A.16:** Spending impact multipliers with $\rho_G = 0.2$

Source: Own simulation in Dynare.
**Figure A.17:** Market expectations to the Effective Federal Funds Rate

![Graph showing market expectations to the Effective Federal Funds Rate from July 2011 to July 2013.](image)

- **Effective Federal Funds Rate**
- **Market Expectations, 09.08.11**
- **Upper Margin for ZLB**

Source: Federal Reserve Bank of St. Louis (FRED) and Macrobond.