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Corporate Debt Maturity and Investment over the Business Cycle

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Abstract
I study the business cycle dynamics of the maturity structure of the debt of U.S. non-financial firms. I document three facts: First, the aggregate share of long-term debt in total debt is pro-cyclical. Second, the long-term debt share of small firms has a higher standard deviation and correlation with output than the long-term debt share of large firms. Third, large firms tend to use a larger share of long-term debt in general. To account for these facts, I construct a quantitative dynamic equilibrium model in which firms optimally choose investment, leverage, debt maturity, dividends, and default, subject to idiosyncratic and aggregate risk. When they choose their debt maturity, firms trade off default premia and rollover costs. As a result, financially constrained firms endogenously prefer to issue short-term debt, because they face high default premia on long-term debt. Financially unconstrained firms issue long-term debt, because it has lower rollover costs. The model, which is parameterized to match cross-sectional moments, can match stylized facts about the level and dynamics of the maturity structure of debt, both in the aggregate and along the firm size distribution.

Resume
Jeg undersøger konjunkturdynamikken i løbetidsstrukturen for amerikanske ikke-finansielle virksomheders gæld. Jeg dokumenterer tre ting: For det første er den aggregerede langfristede gæld som andel af den samlede gæld procyklisk. For det andet har den langfristede gældsandel hos små virksomheder en højere standardafvigelse og korrelation med produktionen end den langfristede gældsandel hos store virksomheder. For det tredje har store virksomheder en tendens til generelt at anvende en større andel af langfristet gæld. For at gøre rede for dette konstruerer jeg en kvantitativ dynamisk ligevægtsmodel, hvor virksomheder optimalt vælger investering, gearing, gældens løbetid, udbytter og misligholdelse under hensyntagen til idiosynkratisk og aggregeret risiko. Når de vælger gældens løbetid, foretager virksomhederne en afvejning af risikopræmie og refinansieringsomkostninger. Derfor vil finansielt begrænsede virksomheder som udgangspunkt foretrække at udstede kortfristet gæld, fordi risikopræmien er høj for langfristet gæld. Virksomheder, der ikke er finansielt begrænsede, udsteder langfristet gæld, fordi fornyelsesomkostningerne er lavere. Modellen, der er parameteriseret, så den passer på tværsnitsmomenter, kan matche stiliserede fakta om niveauet for og dynamikken i gældens løbetidsstruktur, både samlet set og for de forskellige virksomhedsstørrelser.

Key words
Financial stability; Business enterprises and households; Models

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Abstract

I study the business cycle dynamics of the maturity structure of the debt of U.S. non-financial firms. I document three facts: First, the aggregate share of long-term debt in total debt is pro-cyclical. Second, the long-term debt share of small firms has a higher standard deviation and correlation with output than the long-term debt share of large firms. Third, large firms tend to use a larger share of long-term debt in general. To account for these facts, I construct a quantitative dynamic equilibrium model in which firms optimally choose investment, leverage, debt maturity, dividends, and default, subject to idiosyncratic and aggregate risk. When they choose their debt maturity, firms trade off default premia and rollover costs. As a result, financially constrained firms endogenously prefer to issue short-term debt, because they face high default premia on long-term debt. Financially unconstrained firms issue long-term debt, because it has lower rollover costs. The model, which is parameterized to match cross-sectional moments, can match stylized facts about the level and dynamics of the maturity structure of debt, both in the aggregate and along the firm size distribution.

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1 Introduction

Firms with a high level of short-term debt were much more negatively affected by the financial crisis of 2007-09 than firms with a high level of long-term debt. A high level of short-term debt exposed firms to the risk of an unexpected decrease in credit availability, which in turn forced them to cut down investment to repay outstanding debt.[1]

Despite these observations, the macroeconomic literature has so far not considered which factors determine the maturity structure of corporate debt. Typically, macroeconomic models with financial frictions treat all debt as short-term.[2] Yet for an average publicly traded U.S. firm between 1984 and 2012, only 36.8 percent of outstanding debt matured within the next year. In the aggregate, only 15.54 percent of corporate debt matured within the next year during the same time period. This discrepancy between the standard model assumption of short-term debt and the actual maturity structure observed in the data is economically important for how corporate debt affects corporate investment over the business cycle: Theories where firms rely exclusively on short-term debt emphasize liquidity constraints, such that firms reduce investment in a recession due to a high cost of re-financing.[3] Theories where firms will mostly use long-term debt emphasize other channels like the debt overhang problem first introduced by Myers (1977). According to this theory, firms reduce investment due to the failure of shareholders to internalize the benefits of investment to holders of outstanding debt.

In this paper, I study the determinants of the maturity structure of debt, both in the cross section and over the business cycle. Importantly, I also consider heterogeneity in the cyclical dynamics of the maturity structure for firms of different size. I document that in the aggregate, the share of long-term debt in total debt is pro-cyclical. This correlation varies substantially by firm size: For the largest 10 percent of firms as measured by assets, the share of long-term debt in total debt is *counter*-cyclical, while it is pro-cyclical for all other firms. Because the firm size distribution is very right-skewed, the behavior of large firms dominates the aggregate effect, such that it is easy to overlook the pro-cyclical debt maturity dynamics for the vast majority of small firms. In addition, large firms tend to use a larger share of long-term debt in general: The smallest 25 percent of firms have an average long-term debt share of 42 percent, the largest 10 percent of firms have an average long-term debt share of 82 percent.

To propose an explanation for these debt maturity dynamics, I construct a quantitative dynamic model that allows for a rich capital structure of firms: Firms can issue short-term debt, long-term debt and equity. They invest in productive capital, which is illiquid due to capital adjustment costs. Firms face both idiosyncratic and aggregate productivity risk. They also face aggregate

[1] Duchin, Ozbas, and Sensoy (2010) report that firms with high short-term debt restricted investment more during the financial crisis, while firms with high long-term debt did not significantly reduce investment. Almeida, Campello, Laranjeira, and Weisbrenner (2012) report that firms with a high fraction of debt maturing when the crisis hit decreased investment substantially more than firms without such a debt position.

[2] Examples are Jermann and Quadrini (2012), Khan and Thomas (2013), Gourio (2013) and Gilchrist, Sim, and Zakrajsek (2014).

[3] Seminal examples are Gomes (2001) and Hennessy and Whited (2007).

consumption risk, which affects asset prices in two ways: First, consumption risk creates a risk premium for assets whose returns co-move with consumption. This is true for both equity and debt in the model, since cash flows to equity are pro-cyclical and default rates are counter-cyclical, lowering the payouts to creditors in recessions. Second, it creates a time-varying term structure of risk-free interest rates: Short-term interest rates are lower than long-term interest rates in a recession and higher in an expansion. Due to limited liability, firms can default on outstanding debt.

The key ingredients of the model are these endogenous default premia, exogenous debt and equity issuance costs and a tax benefit of debt. Issuance costs reflect costs like underwriting fees, which the corporate finance literature considers to be important for corporate capital structure decisions, see e.g. Gomes (2001), Hennessy and Whited (2007), Titman and Tsyplakov (2007) and Eisfeldt and Muir (2016). When equity issuance costs are higher than debt issuance costs, as they are in my parametrization of the model, they create an incentive for low productivity firms to issue debt to avoid having to issue equity. The tax deductibility of interest expenses creates an incentive to issue debt for high productivity firms. This tax benefit of debt is also widely considered to be an important determinant of the corporate capital structure, see e.g. Modigliani and Miller (1963), Fischer, Heinkel, and Zechner (1989) and the empirical evidence in Heider and Ljungqvist (2015).

The main trade-off between short-term and long-term debt is the following: On the one hand, financially unconstrained firms that want to maintain a high leverage ratio for the tax benefit of debt want to keep the expected cost of rolling over debt low. They can do so by issuing long-term debt, since long-term debt only has to be rolled over infrequently. On the other hand, financially constrained firms that issue debt because they lack internal funds want cheap external liquidity and hence want to keep the default premium on newly issued debt low. They can do so by issuing short-term debt, since the default premium on short-term debt is endogenously lower than the default premium on long-term debt. There are two reasons for this: First, the holders of long-term debt price default risk over a longer time horizon and second, long-term debt creates an ex post incentive misalignment between the firm owners and the long-term creditors. This creates a commitment problem for the firm owners and increases the probability of default. A pecking order theory akin to Myers and Majluf (1984) arises: Very high productivity firms issue long-term debt. Medium productivity firms rely on internal funds, as long as they have sufficient internal liquidity. Low productivity firms use short-term debt, since they lack internal liquidity. Very low productivity firms issue equity, since they are effectively excluded from credit markets due to prohibitively high default premia. Since productivity is positively correlated with firm size, the model can match the stylized fact that small firms use a larger share of long-term debt. Importantly, the model generates an inverse u-shaped relationship between the long-term debt share and firm size, since very large firms and very small firms will not use short-term debt.

The model can match the stylized fact that the aggregate corporate debt maturity structure is pro-cyclical: If aggregate productivity decreases, the productivity distribution shifts to the left, such that firms will use more equity and short-term debt and less long-term debt. It can also match

the stylized fact that the debt maturity structure of small firms is more pro-cyclical than the debt maturity structure of large firms, because smaller firms tend to be in the region of the state space where a small change in (aggregate) productivity can lead firms to switch from not issuing debt to issuing short-term debt, which shortens their maturity structure.

To understand the incentive misalignment between shareholders and long-term creditors better, consider the case of a firm that has some outstanding long-term debt and a positive default probability in the next period. This firm has to decide how much to invest today. For simplicity, assume that the investment is financed with internal funds. If the firm will default in some states of the world in the next period, an investment today hence constitutes an intertemporal transfer from the owners of the firm to the creditors, because the firm owners carry the entire cost of the investment today, while the benefit of the investment in default states in the next period accrues to the creditors. The firm therefore has an incentive to underinvest, if it does not care about the value of debt. For short-term debt, this incentive to underinvest does not arise, because the effect of the investment on the value of short-term debt is internalized by the firm through the effect on current bond issuance revenue. But for long-term debt, this effect is not internalized, because newly issued long-term debt constitutes only a fraction of all outstanding long-term debt. Knowing that the firm will not act in their interest ex post, long-term creditors will hence demand a higher default premium ex ante.

To test the quantitative importance of this theory for aggregate and firm-level debt maturity structure dynamics, I calibrate the model to match several cross-sectional moments, among them the average share of long-term debt in the cross section and the default rate. The calibrated model captures that firms endogenously use mostly long-term debt and that larger firms use a larger share of long-term debt. While I choose the model parameters to match cross-sectional moments, the model can also match aggregate correlations, notably the pro-cyclicality of investment, the long-term debt share and long-term debt issuance as well as the counter-cyclicality of equity issuance, leverage and the default rate.

I show that issuance costs of debt and equity are important in terms of matching the level and dynamics of the debt maturity structure: Absent of equity issuance costs, there is no motive to issue short-term debt, because firms are never financially constrained: The cost of a unit of external equity is always one. Firms will then exclusively use long-term debt, independently of their size. Absent of debt issuance costs firms find it optimal to avoid using long-term debt, also independently of their size. As a consequence, a model without debt and equity issuance costs cannot explain the size heterogeneity in the level and dynamics of the debt maturity structure. This result complements Crouzet (2015), who shows in a similar quantitative model without debt issuance costs that the optimality of short-term debt is a result of the incentive problem between the firm and the creditors. Consumption risk helps to improve the model fit for the maturity dynamics substantially, because it leads to a counter-cyclical slope of the term structure of risk-free interest rates over the business cycle. Finally, removing the tax benefit of debt leads to much lower leverage and a lower long-term debt share both in the cross section and in the aggregate, and reduces both

the level and the counter-cyclicality of the default rate.

Related Literature My paper is related primarily to the literature on the cyclicality of the capital structure of non-financial firms. The closest paper to mine is Jungherr and Schott (2016). In independent and simultaneous work, they focus on the maturity dynamics of aggregate liabilities, using aggregate data from the financial accounts of the United States. They find the maturity dynamics of aggregate liabilities in the data to be *counter*-cyclical.[4] In terms of the theoretical analysis, my focus is more on firm heterogeneity over the business cycle, whereas they focus on aggregate dynamics.

Various other characteristics of the business cycle dynamics of the capital structure of non-financial firms have been studied in the literature, for example the choice of debt vs equity in Covas and Den Haan (2011) and Jermann and Quadrini (2012), of loans vs bonds in de Fiore and Uhlig (2011), Crouzet (2016) and Xiao (2017) or the choice between unsecured and secured debt in Azariadis, Kaas, and Wen (2016).

My paper is also related to the literature on how financial frictions amplify business cycle fluctuations and hence affect the real economy. Khan and Thomas (2013) develop a heterogeneous firm model in which firms issue secured short-term debt if they lack internal funds for investment. They do not consider default decisions. They propose a model which still focuses on short-term debt, but includes endogenous default in Khan, Senga, and Thomas (2014). Gilchrist, Sim, and Zakrajsek (2014) study uncertainty shocks in a heterogeneous agent model with financial frictions and defaultable short-term debt. Gomes, Jermann, and Schmid (2016) develop a heterogeneous agent model with nominal long-term debt, in which inflation risk affects investment and default through a debt overhang channel. However, none of these papers considers a debt maturity choice.

There is also a body of literature in corporate finance which studies the maturity structure of corporate debt, typically abstracting from aggregate dynamics. The closest paper to mine in this literature is Crouzet (2015), which discusses the determinants of the debt maturity structure in a stylized model with frictionless investment, no equity issuance and no aggregate uncertainty. He and Milbradt (2014) discuss the dynamics of debt maturity in a continuous time model. They solve their model in closed form and provide a theoretical discussion of the existence of various equilibria. My focus is different: I use a quantitative model to study the dynamics of debt maturity in a setting with rich cross-sectional heterogeneity of firms. Importantly, I discuss the role of aggregate uncertainty and investment for the maturity structure of debt. The focus on investment also distinguishes my paper from Chen, Xu, and Yang (2016), who investigate maturity choice in a

[4] There are two possible reasons why their data leads them to a different conclusion: First, their measure of short-term liabilities includes trade credit. This is important, since it is well known that during and after the financial crisis, there has been a collapse of trade and hence trade credit (Chor and Manova (2012)). This would show up as an increase in the maturity structure of debt in their data. In contrast, trade credit is a separate credit category in Compustat, which is the data I use. Second, their measure of short-term debt includes all loans except mortgages and excludes all bonds. In contrast, loans and bonds in Compustat are classified according to their actual maturity. This is also a potential an issue, because the fraction of loans in total debt financing is known to be pro-cyclical (de Fiore and Uhlig (2011)), which would show up as an increase in the maturity structure of debt during recessions in their data.

He and Milbradt (2014)-type model with illiquid bond markets and endogenous default. They show that a liquidity-default spiral may lead firms to shorten their maturity structure during recessions, despite the existence of rollover risk. However, they do not discuss the dynamics of maturity choice: Conditional on the aggregate state variable, debt maturity is static in their model.

Furthermore, there is a large body ofliterature in corporate finance and asset pricing that studies the role of macroeconomic risk for the investment and financing decisions of firms in dynamic models, starting with Gomes (2001). My paper builds on the model of Kuehn and Schmid (2014), who study the implications of endogenous investment for credit spreads in a model with only long-term debt. Hackbarth, Miao, and Morellec (2006) study leverage dynamics with long-term debt and aggregate uncertainty in a continuous-time framework and show that leverage is counter-cyclical. Bolton, Chen, and Wang (2013) study cash holdings in a single-factor model and find that firms issue external funds in times of low funding costs to build up precautionary cash buffers. The financing costs in their model are exogenous, since they do not consider risky debt. In a similar framework, Eisfeldt and Muir (2016) use an estimated model to provide evidence of a separate financial factor for the build-up of precautionary cash buffers through the issuance of external funds. Warusawitharana and Whited (2016) have a similar focus to Bolton et al. (2013) and Eisfeldt and Muir (2016), but provide a behavioral foundation for their financial shock in the form of an equity misvaluation shock. I contribute to this literature by studying the effects of aggregate risk on the debt maturity structure of firms.

Finally, my paper is related to the literature that studies how the maturity structure of debt affects investment, which starts with the seminal paper by Myers (1977). Moyen (2007) discusses the role of different maturity structures in relation to the quantitative importance of debt overhang. There are some differences between her framework and mine, most notably that in her model, firms either hold only short-term debt or only long-term debt and there is no endogenous debt maturity choice. Also, the long-term debt contract in her model is different. Diamond and He (2014) also discuss the effect of different maturity structures on debt overhang in a very different class of models for an exogenously given debt maturity structure.

I proceed as follows: In section 2, I show the main facts about the maturity structure of the debt of U.S. firms. In section 3, I outline the model of the decision problem of an equity-value maximizing firm and the bond pricing equations. Section 4 illustrates the determinants of the maturity structure of debt. Section 5 discusses my calibration strategy. In section 6, I present the numerical results. Section 7 concludes.

2 Stylized Facts about the Corporate Debt Maturity Structure

In this section, I describe the main empirical facts about the level and dynamics of the corporate debt maturity structure both at the firm level and in the aggregate. All data are from Compustat, which contains information about publicly traded U.S. firms. I include only non-financial firms. The time period is the first quarter of 1984 to the last quarter of 2012. Long-term debt is defined

as all debt with a residual maturity of at least one year. Debt includes notes, bonds, loans, credit lines and banker's acceptances. The main statistic I focus on is the long-term debt share (LT share) which is defined as long-term debt divided by total debt. I defer a detailed description of the data to section 5.1.

2.1 Heterogeneity in the Level of the Debt Maturity Structure

Firm Size Quantile	(1) Mean, LT Share	(2) St. Dev., LT Share	(3) Share of Agg. Assets	(4) Share of Agg. Debt
0% to 25%	0.419	0.368	0.006	0.006
25% to 50%	0.562	0.352	0.037	0.026
50% to 75%	0.706	0.327	0.151	0.131
75% to 90%	0.799	0.274	0.287	0.297
90% to 95%	0.822	0.252	0.195	0.207
95% to 99%	0.825	0.239	0.243	0.251
99% to 100%	0.823	0.232	0.081	0.082
0% to 90%	0.608	0.364	0.481	0.460
All Firms	0.632	0.359	1.000	1.000
Aggregate	0.845	0.012		
Observations				552987

Table 1: Summary statistics of the long-term debt share, by firm size.

In Table 1, I show two stylized facts: First, most outstanding debt of U.S. non-financial firms is long-term. Second, there is substantial heterogeneity in the level of the debt maturity structure across firms. Consider the row "Aggregate" of Table 1. The first column shows the share of long-term debt in total debt in the aggregate over the sample period. On average, 84 percent of the outstanding debt of U.S. non-financial firms has a residual maturity of at least one year. Assuming a geometric debt repayment rate, this corresponds to an aggregate debt maturity of about 6 years.[5] This is in stark contrast to the macroeconomic literature with financial frictions, which often assumes that the entire stock of corporate debt matures within the next quarter. It is also noteworthy that the *average* LT share is with 0.632 much lower than the *aggregate* LT share, corresponding to a maturity of about 2.3 years. This difference between the average and aggregate indicates already that firms which account for a large share of aggregate debt tend to hold long-term debt.

[5] If each quarter, a fraction μ of debt matures, the share of long-term debt is given by $l = (1-\mu)^4$. Then, the annual maturity $m = 1/(4\mu)$ can be calculated as

$$m = \frac{1}{4(1 - l^{0.25})}$$

To confirm this, consider the remaining rows of the first column of Table 1. These show the distribution of the long-term debt share conditional on the firm size distribution. I sort firms into quarter-specific size bins and compute the average long-term debt share within each size bin. A robust feature of the data is that larger firms tend to have a higher share of long-term debt: The average share of long-term debt of the smallest quartile of firms by assets is on average only 41.9 percent, corresponding to a debt maturity of about 1.3 years. The largest one percent of firms holds on average 82.3 percent of debt as long-term debt, corresponding to a debt maturity of about 6 years. The second column shows that the standard deviation of the maturity structure within a given size quantile is monotonically decreasing in size, which is also a robust feature of the data. Not only do larger firms have a larger share of long-term debt, but their maturity structure is also less volatile.

In the last two columns, I show the fraction of assets and debt that firms within a given size quantile account for, that is, the marginal distributions of assets and debt in Compustat. These make it clear that while the distributions of debt and assets are very right-skewed, with the largest 1 percent of firms accounting for roughly 8 percent of aggregate assets and debt. The smallest 90 percent of firms account nonetheless for a substantial fraction of 48 percent of assets and 46 percent of debt in the data.

2.2 Aggregate and Firm-Level Dynamics of the Debt Maturity Structure

Firm Size Quantile	(1) GDP(t-1)	(2) GDP(t)	(3) GDP(t+1)
0% to 25%	0.288**	0.335***	0.283**
25% to 50%	0.265**	0.408***	0.449***
50% to 75%	0.621***	0.722***	0.760***
75% to 90%	0.398***	0.547***	0.633***
90% to 95%	0.135	0.227*	0.286**
95% to 99%	−0.324***	−0.229*	−0.155
99% to 100%	−0.140	−0.0633	−0.006 32
0% to 90%	0.467***	0.613***	0.690***
All Firms	0.227*	0.396***	0.502***
Observations			116

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Table 2: Correlations of the detrended long-term debt share with detrended log real corporate sales, by firm size.

In terms of dynamics, the maturity structure of debt of non-financial U.S. firms varies substantially over the business cycle at the aggregate level. In Figure 1, I plot the cyclical component of the share of long-term debt in total debt for all firms in the Compustat sample, where I detrend the series using a simple linear-quadratic trend. It is evident that the share of long-term debt decreases

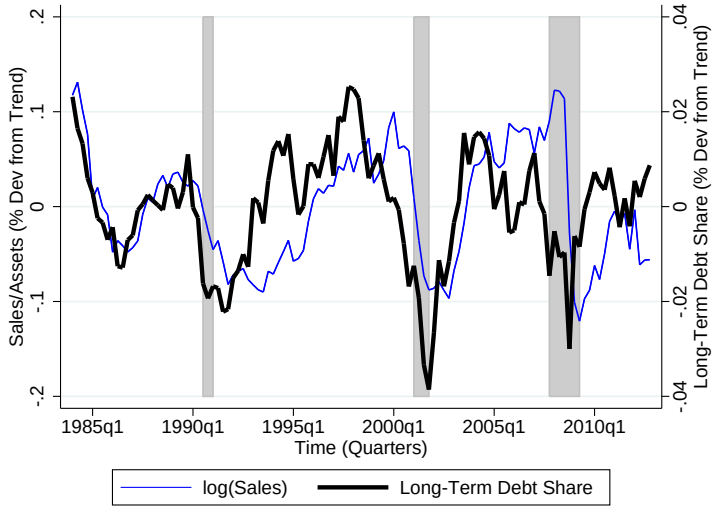

Figure 1: Figure 1 shows the cyclical component of the aggregate share of long-term debt in total debt for non-financial firms in the Compustat Quarterly database. The thin line is real aggregate sales. Both series are detrended with a linear-quadratic trend. The shaded areas indicate the NBER recession episodes.

during recessions and increases during expansions. For example, the long-term debt share during the financial crisis decreased by roughly 4 percent from the first quarter of 2007 to the last quarter of 2008.

In Table 2, I report the correlation coefficients between real corporate log sales and the share of long-term debt in total debt. Aggregate sales is my preferred measure of output, since it is the closest equivalent to output as measured in the model and since it reflects cyclical fluctuations in the corporate sector better than real GDP. I present the correlations computed using real GDP or real corporate log profits in Appendix A. In the aggregate, the debt maturity structure is pro-cylical, with a correlation of 0.396 for all firms, which increases to 0.613 for the smallest 90 percent of firms. This means that the fraction of due payments on outstanding debt increases exactly when internal funds are most valuable for firms, a seemingly puzzling observation.

The maturity structure also varies widely at the firm level. In Figure 2, I plot the time series for the long-term debt share of the smallest 50 percent of firms as well as the firms in the 50-75, 75-90 and 90-100 percent size quantiles. I follow Covas and Den Haan (2011) in choosing these cut-offs. The time series are de-trended using a linear quadratic trend. In the figure, they are furthermore smoothed using a moving average filter with two lags. In Table 2, the series are not smoothed. Figure 2 shows that the long-term debt share of small firms is pro-cyclical, while the long-term debt share of large firms is counter-cyclical.

This is confirmed by the correlations I report in Table 2: The long-term debt share of the firms up to the 90 percent size quantile is pro-cyclical, while for the largest 10 percent of firms, the long-term debt share is counter-cyclical. There is also an inverse u-shaped relationship between

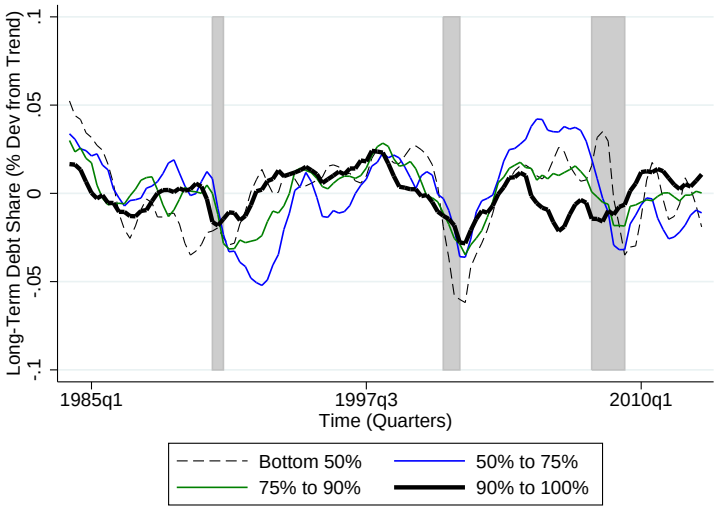

Figure 2: Figure 2 shows the cyclical component of the aggregate share of long-term debt for non-financial firms in the Compustat Quarterly database for different size bins. All series are detrended with a linear-quadratic trend. For better visibility, all series for this figure are smoothed using a moving average filter with two lags. The shaded areas indicate the NBER recession episodes.

firm size and the correlation of the long-term debt share with aggregate sales: For very small and very large firms, this correlation is lower than for medium-sized firms. These differences are large: For the 50 percent to 75 percent quantile, the contemporaneous correlation between the long-term debt share and real output is 0.722 and is significantly different from zero at the 0.1 percent level. For the 95 to 99 percent size quantile, the contemporaneous correlation is -0.229 and is significantly different from zero at the 1 percent level.

Explaining these patterns of the debt maturity structure is interesting for two reasons: First, they provide additional evidence about which financial frictions determine the capital structure decisions of non-financial firms both at the microeconomic and macroeconomic level. If firms predominantly rely on long-term debt, rollover costs are more important to the firms. If firms use mostly short-term debt, default premia and default incentives are more important. Second, the maturity structure itself is a factor that determines through which channels outstanding debt affects the investment decisions of firms. Specifically, an important question that arises is whether rollover costs or debt overhang are more important for the investment behavior of financially constrained firms. Answering this question requires a structural model of the optimal maturity choices of firms, because firms take the effect of their current debt maturity choices on future investment decisions into account in a forward-looking way. In the next section, I introduce such a model.

3 Model

The model consists of many firms i, a competitive bond market for short-term debt and long-term debt, which specifies the menu of bond prices which each firm faces and a representative household which owns all firms and holds all debt in the economy. Time is discrete: $t = 0, 1, \ldots, \infty$. The unit of time is a quarter.

Each firm i uses capital $K_{i,t}$ to produce output, subject to aggregate and idiosyncratic productivity risk. At each time t, firms decide how much to invest in capital, $K_{i,t+1}$, how much short-term debt $B^S_{i,t+1}$ and long-term debt $B^L_{i,t+1}$ to issue, how much dividend $D_{i,t}$ to pay and whether to default or not.

There are two types of debt contracts, short-term debt and long-term debt, which I model as bonds. Their prices are determined on a competitive market. More precisely, creditors take into account that firms might default at any point in time in the future and they form expectations over the future probability of default and the uncertain recovery value of a bond in default. Aggregate consumption risk yields a stochastic discount factor which generates an additional risk premium for both equity and risky debt.

In section 3.1, I present the firm problem. In section 3.2, I lay out how the bond prices are determined. I derive the stochastic discount factor in section 3.3.

3.1 Firm Problem

Objective Function The objective function of the firm is the present value of equity payouts, which consist of dividend payouts or equity issuance including equity issuance costs. The dividend payout of firm i in period t is denoted by $D_{i,t}$. $EIC(D)$ is an equity issuance cost which is zero when dividends are non-negative and positive when dividends are negative. Future cash flows are discounted with a stochastic discount factor $\Lambda(C_t, C_s)$, where C_t denotes aggregate consumption at time t. The present value of dividends at time t is given by

$$\mathbb{E}_t \left[\sum_{s=t}^{T} \Lambda(C_t, C_s) \left(D_{i,s} - EIC(D_{i,s}) \right) \right]. \tag{3.1}$$

T denotes the period in which it is optimal for the firm to default. In what follows, I will write the model in recursive form, using the notation that $X_t = X$ and $X_{t+1} = X'$ for any variable X.

Technology The profit function of the firm is given by

$$\Pi(K_i, \tilde{A}_i, Z) = \tilde{A}_i Z K_i^{\alpha} - \psi, \tag{3.2}$$

where K_i is the capital stock of the firm from the last period and $\alpha < 1$ describes returns to scale at the firm level. $\tilde{A}_i$ is the idiosyncratic productivity of the firm, which evolves according to

$$\begin{aligned} \ln \tilde{A}_i' &= \rho \ln \tilde{A}_i + \sigma^A \varepsilon_i^A, \\ \varepsilon_i^A &\sim \text{i.i.d.} N(0,1). \end{aligned} \tag{3.3}$$

The idiosyncratic productivity shocks ε_i^A are uncorrelated over time and across firms. Z is the aggregate productivity, common to all firms in the economy, which also follows a first-order autoregressive process:

$$\begin{aligned} \ln Z' &= \rho \ln Z + \sigma^Z \varepsilon^Z, \\ \varepsilon^Z &\sim \text{i.i.d.} N(0,1). \end{aligned} \tag{3.4}$$

In addition, firms have to pay a fixed cost of operation ψ. It can be interpreted for example as a maintenance cost or as administrative overheads. In the corporate finance literature, such a fixed production cost is used, for example, in Gomes (2001).

Since idiosyncratic and aggregate productivity have the same persistence, they can be collapsed into the state variable $A_i = \tilde{A}_i Z$, which evolves according to

$$\ln A_i' = \rho \ln A_i + \sigma^A \varepsilon_i^A + \sigma^Z \varepsilon^Z. \tag{3.5}$$

The conditional density function of A_i' is denoted by $f(A_i'|A_i)$.

Investment Capital follows the standard law of motion:

$$K_i' = (1-\delta)K_i + I_i, \tag{3.6}$$

where δ is the depreciation rate and I_i is investment. When installing new capital or selling old capital, the firm has to incur a quadratic capital adjustment cost with the functional form

$$AC(K_i, K_i') = \frac{\theta}{2}\left(\frac{K_i'}{K_i} - 1 + \delta\right)^2 K_i. \tag{3.7}$$

With these capital adjustment costs, I capture in a simple way that capital is illiquid. This form of capital adjustment costs is common in the investment literature, see for example Hayashi (1982). It is widely used in the corporate finance literature, for example in Bolton et al. (2013) and Eisfeldt and Muir (2016). Furthermore, Bloom (2009) reports that at the firm level, quadratic capital adjustment costs yield a good description of firm-level investment behavior, even if the capital adjustment costs are non-convex at the plant level.

Debt Financing The firm can issue short-term debt, $B_{S,i}$, and long-term debt, $B_{L,i}$. Short-term debt takes the form of a one-period contract. Long-term debt takes the form of a contract

with stochastic maturity μ.[6] This formulation is a common way to model long-term debt without introducing too many state variables in the model. It is for example used in the corporate finance literature in Leland (1998), Hackbarth et al. (2006) and Kuehn and Schmid (2014).[7] The level of long-term debt evolves according to

$$B'_{L,i} = (1-\mu)B_{L,i} + J_{L,i}, \tag{3.8}$$

where $J_{L,i}$ denotes long-term debt issuance. Long-term debt cannot be repaid early: $J_{L,i} \geq 0$. Leary and Roberts (2005) report that firms adjust their leverage only slowly towards a target leverage, which is consistent with a transaction cost for repurchases or, similarly, a repurchase constraint. Both short-term debt and long-term debt pay a coupon c.

Debt is risky, because firms can default. Issuance occurs at state-contingent prices Q_S and Q_L for short-term debt and long-term debt respectively. I will explain how these bond prices are determined in equilibrium in section 3.2.

There is a linear issuance cost ξ for debt. Debt issuance costs are equal for short-term debt issuance and long-term debt issuance. The functional form for debt issuance costs is given by

$$DIC(B'_{S,i}, J_{L,i}) = \xi \left(|B'_{S,i}| + |J_{L,i}| \right). \tag{3.9}$$

These issuance costs can be interpreted as flotation fees for new bond issues or bank fees for new loans. Such costs can arise in addition to the endogenous default premium. Typically, the literature considers either a combination of fixed and linear debt issuance costs or one of the two. An example for the former is Kuehn and Schmid (2014). A model which uses a purely linear issuance cost is Titman and Tsyplakov (2007).

Corporate Income Tax There is a proportional corporate income tax τ. Consistent with the U.S. tax code, taxable income is calculated as income less operating costs, depreciation and interest expense. This implies that there is a tax benefit for investment as well as debt issuance. As a consequence, from the perspective of the shareholder, debt issuance is cheaper than equity issuance, because a fraction of interest expense is implicitly rebated by the government. There is therefore an incentive for the firm to increase leverage up to the point where the marginal cost of debt in the form of issuance costs and the change in the default premium equal the marginal tax benefit of debt. This trade-off between the tax shield and the default premium is an important determinant of leverage, see for example Kraus and Litzenberger (1973) or Fischer et al. (1989). Effectively, it lowers the required return on equity relative to the required return on debt, making the creditors of the firm more patient than its shareholders. In this model, it is the reason why large, financially unconstrained firms issue debt.

[6] Each unit of debt is infinitely divisible, such that a fraction μ will mature every period, while the remaining fraction $1-\mu$ is rolled over into the next period.

[7] It is also a common assumption in the literature on sovereign default, for example in Hatchondo and Martinez (2009), Arellano and Ramanarayanan (2012) or Chatterjee and Eyigungor (2012).

Dividends and Equity Financing For convenience, I use the total amount of outstanding debt, $B_i = B_{S,i} + B_{L,i}$ and the fraction of long-term debt, $M_i = B_{L,i}/B_i$, as state variables. I collect the endogenous state variables of firm i in the tuple $\mathcal{S}_i = (K_i, B_i, M_i)$. K_i is the capital stock of the firm, B_i is the total amount of outstanding debt, and M_i is the fraction of outstanding debt that was issued in the form of long-term debt.

Dividends are given residually by the budget constraint of the firm,

$$\begin{aligned} D_i = (1-\tau)\underbrace{[\Pi(K_i, A_i) - \delta K_i - \psi - cB_i]}_{\text{Taxable Income}} \\ - \underbrace{((1-M_i) + \mu M_i)B_i}_{\text{Principal Repayment}} + \underbrace{K_i - K_i'}_{\text{Gross Investment}} - AC(K_i, K_i') \\ + \underbrace{Q_S(1-M_i')B_i'}_{\text{Revenue from ST Debt Issuance}} + \underbrace{Q_L(M_i'B_i' - (1-\mu)M_iB_i)}_{\text{Revenue from LT Debt Issuance}} \\ - \underbrace{DIC((1-M_i)B_i, M_i'B_i' - (1-\mu)M_iB_i)}_{\text{Debt Issuance Cost}}. \end{aligned} \tag{3.10}$$

Dividends can be negative. In this case, the firm issues seasoned equity and has to pay an equity issuance cost.[8] This cost is meant to capture monetary costs, such as underwriting fees, but also non-monetary costs like managerial effort and signalling costs conveyed through the issues.[9] The equity issuance cost consists of a fixed component ϕ_0 and a linear component ϕ_1, such that the average cost of issuing equity is decreasing in the size of the issue. The functional form is

$$EIC(D_i) = (\phi_0 + \phi_1|D_i|)\,\mathbb{1}_{(D_i<0)}. \tag{3.11}$$

This functional form is consistent for large firms with the empirical evidence in Altinkilic and Hansen (2000) and the structural estimation results in Hennessy and Whited (2007).[10] It is for example used in Gomes (2001), Cooper and Ejarque (2003), Gomes and Schmid (2010) and Kuehn and Schmid (2014).

Recursive Firm Problem and Default If the firm decides not to default, its problem is then to maximize the present value of dividends by choosing the capital stock K_i', debt B_i', the fraction of long-term debt M_i', and dividends D_i. The value function of a continuing firm i can be summarized

[8] Seasoned equity meaning that it is an issue by an already publicly traded firm.

[9] An example of a model in which equity issuance is associated with higher signalling costs than debt issuance is Myers and Majluf (1984).

[10] Although the main point of Altinkilic and Hansen (2000) is exactly that this formulation is not appropriate for smaller firms.

as

$$V^C(\mathcal{S}_i, A_i, C) = \max_{K_i', B_i', M_i', D_i} \{D_i - EIC(D_i) + \mathbb{E}\left[\Lambda(C, C') \int_0^\infty V(\mathcal{S}_i', A_i', C') f(A_i'|A_i) dA_i'|C\right]\}, \tag{3.12}$$

subject to the budget constraint 3.10, the constraints

$$K_i' \geq 0, \tag{3.13}$$

$$B_i' \geq (1-\mu) M_i B_i, \tag{3.14}$$

$$(1-\mu) M_i B_i / B_i' \leq M_i' \leq 1, \tag{3.15}$$

and bond pricing equations to be specified below. The constraints 3.14 and 3.15 arise due to the assumption that long-term debt cannot be repurchased.

Default occurs if the firm does not repay its debt, either for strategic reasons or because the firm cannot raise sufficient funds to repay outstanding liabilities. Since shareholders can simply walk away if the value of owning the firm becomes negative, the value of the firm to shareholders is bounded below by 0. The total value of equity is then

$$V(\mathcal{S}_i, A_i, C) = \max\left\{V^C(\mathcal{S}_i, A_i, C), 0\right\}. \tag{3.16}$$

3.2 Bond Markets

Payouts to Creditors The bond market is competitive. Short-term bonds and long-term bonds are discounted with the same discount factor as equity. Both bonds pay a fixed coupon c. Coupons are calculated according to

$$c = \frac{1}{\beta} - 1. \tag{3.17}$$

That is, coupons are chosen such that the values of risk-free bond prices in the absence of aggregate risk are both equal to 1.

If the firm does not default, the payment to the short-term creditors is $1 + c$. The payment to the long-term creditors is $\mu + c$. The outstanding fraction $(1 - \mu)$ of long-term debt is valued by creditors at the end-of-period bond price Q_L', such that the value of owning one unit of a long-term bond that is not in default is given by $\mu + c + (1 - \mu)Q_L'$.

If the firm decides to default on its outstanding debt, the firm is liquidated after production has taken place.[11] There is a cross-default clause: a default on short-term debt triggers a default on long-term debt and vice versa. In addition, there is a pari passu clause: bondholders have

[11] Corbae and D'Erasmo (2017) report that liquidations according to Chapter 7 of the U.S. bankruptcy code account for about 20 percent of all U.S. defaults. The rest are reorganizations following Chapter 11 of the U.S. bankruptcy code. While allowing for reorganizations would change the recovery value in default, it would not change the model mechanism substantially otherwise.

equal claims on the liquidation value of the firm, independently of the maturity of their bond. The liquidation value consists of the profits plus the depreciated capital stock. Consistent with the U.S. tax code, it is not possible to deduct interest expenses from taxable income in default.

A complication of the model with a quadratic capital adjustment cost is that capital is illiquid. I interpret the capital adjustment cost as a primitive of the model that also has to be paid if the firm is liquidated. Therefore, it is not optimal to uninstall the entire capital stock of the firm, because below some optimal disinvestment $I^* < 0$, the marginal adjustment cost from disinvesting an additional unit of the capital stock outweighs the marginal benefit. This optimal level of disinvestment is the solution to

$$\max_{I \leq 0} = -I - \frac{\theta}{2}\left(\frac{I}{K}\right)^2 K.$$

The solution is given by $I^* = -\frac{K}{\theta}$. With this disinvestment, the adjustment cost is given by $\frac{K}{2\theta}$, such that creditors receive $I^* - \frac{K}{2\theta}$ from the liquidation of the capital stock in the current period. The recovery value per unit of the bond is therefore

$$R(\mathcal{S}_i, A_i) = \chi \max\left(\frac{(1-\tau)(\Pi(K_i, A_i) - \psi - \delta K_i) - I^* - \frac{K}{2\theta}}{B_i}, 0\right). \tag{3.18}$$

Note that the model has an endogenous liquidation loss on capital due to the capital adjustment cost.

Bond Pricing Equations It is useful to define a default threshold set for productivity. The default threshold set is implicitly defined by

$$a^*(\mathcal{S}_i, C) = \left\{A \in \mathcal{A} : V^C(\mathcal{S}_i, A, C) = 0\right\}. \tag{3.19}$$

Suppose that the continuation value function V^C is strictly increasing in idiosyncratic productivity. Then, for each $(\mathcal{S}_i, C)$, the default threshold is unique and equation 3.19 defines a function for the default threshold productivity: For $A \leq a^*(\mathcal{S}_i, C)$, the firm will default, for $A > a^*(\mathcal{S}_i, C)$, the firm will continue.

The bond price functions are then

$$q_S(\mathcal{S}_i', A_i, C) = \mathbb{E}\left[\Lambda(C, C')\left(\int_{a^*(\mathcal{S}_i', C')}^{\infty}(1+c)f(A_i'|A_i)dA_i' + \int_0^{a^*(\mathcal{S}_i', C')} R(\mathcal{S}_i', A_i')f(A_i'|A_i)dA_i'\right)|C\right], \tag{3.20}$$

and

$$q_L(\mathcal{S}_i', A_i, C) = \mathbb{E}\left[\Lambda(C, C')\left(\int_{a^*(\mathcal{S}_i', C')}^{\infty} (\mu + c + (1-\mu)Q_L')f(A_i'|A_i)dA_i' + \int_0^{a^*(\mathcal{S}_i', C')} R(\mathcal{S}_i', A_i')f(A_i'|A_i)dA_i'\right)|C\right], \tag{3.21}$$

$$Q_L' = q_L(\mathcal{S}_i'', A_i', C').$$

That is, bond prices reflect the future default probabilities and the value of the firm in default. Future cash flows are discounted at the stochastic discount factor. Notably, while the short-term bond price only reflects the next-period default probability, the long-term bond price captures the entire future path of default probabilities through its recursive dependence on Q_L'. In what follows, I will call the next-period default probability short-run default risk and the default probability after the next period long-run default risk.

3.3 Stochastic Discount Factor

Equity and debt payouts are discounted with the stochastic discount factor

$$\Lambda(C, C') = \beta\left(\frac{C'}{C}\right)^{-\sigma}. \tag{3.22}$$

This discount factor is derived from a household whose consumption process co-moves with the aggregate productivity process: Household preferences are time-separable with discount factor β. The period felicity function has a constant relative risk aversion σ. The utility function therefore takes the recursive form

$$U(C) = \frac{C^{1-\sigma} - 1}{1-\sigma} + \beta\mathbb{E}_{C'}\left[U(C')|C\right] \tag{3.23}$$

The consumption process is driven by aggregate productivity and a process $\tilde{C}$ that is uncorrelated with aggregate productivity. $\tilde{C}$ represents cyclical movements in aggregate consumption that are unrelated to productivity, for example due to other economic shocks. It also follows a first-order autoregressive process:

$$\ln \tilde{C}' = \rho \ln \tilde{C} + \sigma^C \varepsilon^C. \tag{3.24}$$

Aggregate consumption depends on productivity Z with weight λ_1 and on $\tilde{C}$ with weight λ_2:

$$\ln C = \lambda_0 + \lambda_1 \ln Z + \lambda_2 \ln \tilde{C}. \tag{3.25}$$

Combining equations 3.22 and 3.25 yields the stochastic discount factor $\Lambda(C, C')$. This discount factor leads to a risk premium in the model, which has been established to be an important component of bond yields.[12] While the risk premium in the model is exogenous, default premia

[12] See for example Chen (2010) or Bhamra, Kuehn, and Strebulaev (2010).

do reflect the endogenous default decisions of firms in the model, and are therefore endogenously determined as well.

Aggregate consumption is exogenous, which by itself is not central for the main results. It implies that the term structure of risk-free interest rates is exogenous. Note, however, that I allow for correlation between aggregate productivity and aggregate consumption. The exogeneity of the term structure of risk-free interest rates is plausible, since I only model the markets for risky non-financial corporate debt and equity. The markets for government debt or household debt, for example, are outside the model. According to the Financial Accounts of the U.S., corporate business debt constituted only 17.9 percent of all outstanding debt in 2015. The market value of equity of non-financial domestic corporations constituted about 27.2 percent of the net wealth of the U.S. for the same year. In addition, the assumption of an exogenous aggregate consumption or an exogenous stochastic discount factor is common in the asset pricing and corporate finance literature. It is for example used in Campbell and Cochrane (1999).

3.4 Equilibrium

Define $\mathbb{S} \subseteq \mathbb{R}_2^+ \times [0,1]$, $\mathbb{A} \subseteq \mathbb{R}^+$ and $\mathbb{C} \subseteq \mathbb{R}^+$. The recursive competitive equilibrium for this economy is given by a set of policy functions $k : \mathbb{S} \times \mathbb{A} \times \mathbb{C} \to \mathbb{R}^+$, $b : \mathbb{S} \times \mathbb{A} \times \mathbb{C} \to \mathbb{R}^+$ and $m : \mathbb{S} \times \mathbb{A} \times \mathbb{C} \to [0,1]$ for capital, debt, and the share of long-term debt; a default policy function $d : \mathbb{S} \times \mathbb{A} \times \mathbb{C} \to \{0,1\}$, value functions $V_C : \mathbb{S} \times \mathbb{A} \times \mathbb{C} \to \mathbb{R}$ and $V : \mathbb{S} \times \mathbb{A} \times \mathbb{C} \to \mathbb{R}$; and bond price functions $q_S : \mathbb{S} \times \mathbb{A} \times \mathbb{C} \to \mathbb{R}$ and $q_L : \mathbb{S} \times \mathbb{A} \times \mathbb{C} \to \mathbb{R}$, such that for every firm i:

- for any $(\mathcal{S}_i, A_i, C) \in \mathbb{S} \times \mathbb{A} \times \mathbb{C}$, given q_S and q_L, $k(\mathcal{S}_i, A_i, C)$, $b(\mathcal{S}_i, A_i, C)$ and $m(\mathcal{S}_i, A_i, C)$ maximize the continuation problem 3.12, with the solution to the firm problem given by $V_C(\mathcal{S}_i, A_i, C)$.

- for any $(\mathcal{S}_i, A_i, C) \in \mathbb{S} \times \mathbb{A} \times \mathbb{C}$, given q_S and q_L, the firm chooses a default policy d such that

$$d(\mathcal{S}_i, A_i, C) = \begin{cases} 0 & \text{if } V^C(\mathcal{S}_i, A_i, C) > 0 \\ 1 & \text{if } V^C(\mathcal{S}_i, A_i, C) \leq 0 \end{cases}.$$

  The value function V is given by

$$V(\mathcal{S}_i, A_i, C) = V^C(\mathcal{S}_i, A_i, C)(1 - d(\mathcal{S}_i, A_i, C))$$

- for any $(\mathcal{S}_i', A_i, C) \in \mathbb{S} \times \mathbb{A} \times \mathbb{C}$, given k, b ,m, d, V_C and V, $q_S(\mathcal{S}_i', A_i, C)$ and $q_L(\mathcal{S}_i', A_i, C)$ are the solutions to the bond pricing equations 3.20 and 3.21.

The first equilibrium condition states that the firm makes optimal investment and debt issuance decisions, taking the bond prices as given, the second states that the firm makes an optimal default decision, taking the bond prices as given, and the third condition states that bond price schedules incorporate the true default probability of the firm, taking firm policies as given.

4 The Determinants of Debt Maturity

In this section, I will outline the determinants of the maturity structure of a single firm and how it varies with productivity. First, I will explain the different channels that determine the maturity structure of the firm. Then, I will discuss how aggregate consumption shocks affect the optimal maturity choice.

I will denote $Q_L = q_L(\mathcal{S}_i', A_i, C)$, $Q_S = q_S(\mathcal{S}_i', A_i, C)$, and, with some abuse of notation, $V = V(\mathcal{S}_i, A_i, C)$ and $V^C = V^C(\mathcal{S}_i, A_i, C)$ to increase readability.

Throughout this section, I assume that the value function V is once differentiable in K, B, M and A and the bond price functions Q_S and Q_L are differentiable in K', B', M' and A. I further assume that the short-term and long-term bond prices are weakly increasing in A, i.e. $\frac{\partial Q_S}{\partial A} \geq 0$ and $\frac{\partial Q_L}{\partial A} \geq 0$. I do not make these assumptions when I solve the model numerically later on.

4.1 The General Case

The optimal maturity choice is given by the first-order condition with respect to M' in the continuation problem of the firm presented in equation 3.12.

Consider for simplicity the case of a firm which cannot issue seasoned equity.[13] The full problem for a firm which faces a non-negativity constraint on dividends is

$$
\begin{aligned}
V^C_{D\geq 0} = \max_{K',B',M'} & \left\{ D + \mathbb{E}\left[\Lambda(C,C')V(\mathcal{S}_i', A_i', C')|A_i, C\right]\right\} \qquad (4.1)\\
\text{s.t.} & \\
D &\geq 0\\
M' &\leq 1\\
M' &\geq (1-\mu)\frac{MB}{B'}\\
B' &\geq 0
\end{aligned}
$$

and the budget constraint 3.10. I denote as λ_D the contemporaneous shadow cost of internal funds. Further, I denote as $\lambda_{M,0}$ and $\lambda_{M,1}$ the multipliers for the constraints $M' \geq (1-\mu)MB/B'$ and

[13] Due to the non-convex equity issuance cost, the continuation problem of the firm is the maximum of the problem of a firm which faces a non-negativity constraint on dividends and the problem of a firm which issues equity:

$$V^C = \max(V^C_{D\geq 0}, V^C_{D<0}).$$

$M' \leq 1$, respectively.[14] The Lagrangian for this problem is

$$\begin{aligned}\mathcal{L} = & D(1+\lambda_D) + \mathbb{E}\left[\Lambda(C,C')V(\mathcal{S}_i', A_i', C')|A_i, C\right] \\ & + \lambda_{M,1}(1-M') \\ & + \lambda_{M,0}\left(M' - (1-\mu)\frac{MB}{B'}\right) \\ & + \lambda_B B'.\end{aligned}$$

The first-order condition for M' in the Lagrangian to problem 4.1 is

$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial M'} = & \Bigg[(Q_L - Q_S)B' + \\ & \frac{\partial Q_S}{\partial M'}(1-M')B' + \frac{\partial Q_L}{\partial M'}\left(M'B' - (1-\mu)MB\right)\Bigg](1+\lambda_D) + \\ & \mathbb{E}\left[\Lambda(C,C')\frac{\partial V'}{\partial M'}|A, C\right] = \lambda_{M,1} - \lambda_{M,0}. \end{aligned} \tag{4.2}$$

Differentiating equation 3.10, the envelope condition yields

$$\begin{aligned}\frac{\partial V}{\partial M} = \frac{\partial D}{\partial M} &= (\mu + (1-\mu)Q_L - 1 + (1-\mu)\xi)B(1+\lambda_D) \\ &= (1-\mu)(Q_L - 1 + \xi)B(1+\lambda_D),\end{aligned} \tag{4.3}$$

if the firm is in a no-default state. Otherwise, the $\partial V/\partial M = 0$, that is, the value of equity in default is insensitive to the state of the firm. Combining 4.2 and 4.3, we get

$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial M'} = & \Bigg[(Q_L - Q_S)B' + \\ & \frac{\partial Q_S}{\partial M'}(1-M')B' + \frac{\partial Q_L}{\partial M'}\left(M'B' - (1-\mu)MB\right)\Bigg](1+\lambda_D) + \\ & \mathbb{E}\left[\Lambda(C,C')(1-\mu)(Q_L' - 1 + \xi)B'(1+\lambda_D')\mathbb{1}_{(V'>0)}|A, C\right] = \lambda_{M,1} - \lambda_{M,0}. \end{aligned} \tag{4.4}$$

The interpretation of this first-order condition is that the benefit and the cost of marginally increasing the share of long-term debt must be equal. This decision concerns only the *maturity structure* of debt in the next period, but not the *leverage*. The total amount of debt issuance and hence the leverage choice is given by the first-order condition for total debt, B'.

The choice of M' is a portfolio choice how to allocate borrowing among different types of debt for a given amount of total debt B'. The cost of issuing marginally more long-term debt is here the opportunity cost of issuing marginally less short-term debt.[15]

[14] The lower bound on M' arises from the assumption of no debt repurchases.

[15] Of course, B' and M' are in the end jointly determined.

4.2 Optimal Maturity Choice without Consumption Risk

First, I will focus on the main trade-off between short-term debt and long-term debt in a setup without consumption risk. That is, $\lambda_1 = \lambda_2 = 0$. In this case, the discount factor $\Lambda(1,1)$ equals β and the risk-free bond prices for short-term debt and long-term debt are both equal to 1.

The general first-order condition 4.4 contains many different terms, so I will consider three different types of firms: In the first case, I discuss the case of a firm which never defaults. This is the case of a low leverage, high productivity firm. For such a firm, neither short-term debt nor long-term debt are risky. In the second case, the firm may default only after the next period. The firm hence has no short-run default risk, but some long-run default risk. Then, short-term debt is risk-free while long-term debt is risky. In the third case, the firm also has some short-run default risk, such that both short-term debt and long-term debt are risky.

In this way, I can introduce the channels that affect the optimal maturity choice one by one. I will first focus on the main trade-off between rollover costs and default risk and then add other channels.

4.2.1 Case I: No Default Risk

In the case of no default risk and no consumption risk, bond prices do not include a default premium: $Q_S = Q_L = 1$. In addition, bond prices are insensitive to changes in the maturity structure of the firm: $\partial Q_S/\partial M' = \partial Q_L/\partial M' = 0$. The first-order condition 4.4 reduces to

$$\frac{\partial \mathcal{L}}{\partial M'} = \beta(1-\mu)\xi B' \mathbb{E}\left[(1+\lambda_D')|\mathcal{Y}\right] = \lambda_{M,1} > 0.$$

This optimality condition states that the benefit of increasing the long-term debt share is that the firm has to pay less rollover costs, ξ, in the next period if it uses relatively more long-term debt. Consequently, a firm that can issue debt without risk wants to set the long-term debt share as high as possible: $M' = 1$, which implies a positive multiplier $\lambda_{M,1} > 0$.

4.2.2 Case II: Risk-Free Short-Term Debt, Risky Long-Term Debt

If the firm has no short-run default risk, but some long-run default risk, the short-term bond price does not include a default premium, while the long-term bond price does: $Q_S = 1, Q_L < 1$. The short-term bond price remains insensitive to the maturity structure of the firm: $\partial Q_S/\partial M' = 0$.

The first-order condition for the long-term debt share becomes[16]

$$\frac{\partial \mathcal{L}}{\partial M'} = \left[\underbrace{(Q_L - 1)\, B'}_{\text{Change, Marginal Revenue}} + \underbrace{\frac{\partial Q_L}{\partial M'} \left(M'B' - (1-\mu)MB \right)}_{\text{Change, Intramarginal LT Revenue}} \right] (1 + \lambda_D) +$$

$$\beta \mathbb{E} \left[(1-\mu)(\underbrace{1 - Q'_L}_{\substack{\text{Change, Endogenous} \\ \text{Rollover Cost}}} + \underbrace{\xi}_{\substack{\text{Change, Exogenous} \\ \text{Rollover Cost}}}) B'(1 + \lambda'_D) | A_i \right]$$

$$= 0.$$

There are four important terms: The first two terms describe the change in the revenue from issuing new bonds if the firm decides to issue long-term bonds instead of short-term bonds. These are the costs of issuing a higher share of long-term debt. The last two terms describe the change in future rollover costs if the firm issues marginally more long-term debt. These are the benefits of issuing a higher share of long-term debt. Relative to case I in section 4.2.1, the first three terms are new, whereas the exogenous rollover cost also arises in a situation with risk-free short-term *and* long-term debt.

The first term is the *change in the marginal revenue of debt issuance*: If the firm issues marginally more debt as risky long-term debt instead of risk-free short-term debt, it has to incur a default premium, captured by the term $Q_L - 1$. If this default premium is high, the firm prefers to issue short-term debt by setting a low M'.

The second term captures how a marginally larger long-term debt share affects the *intramarginal revenue from long-term debt issuance*. This effect arises, since a higher share of long-term debt today adversely affects firm policies in the future: The derivative $\frac{\partial Q_L}{\partial M'}$ is given by differentiating equation 3.21 with respect to M':

$$\frac{\partial Q_L}{\partial M'} = \beta(1-\mu)\mathbb{E}\left[\frac{\partial Q'_L}{\partial K''}\frac{\partial K''}{\partial M'} + \frac{\partial Q'_L}{\partial B''}\frac{\partial B''}{\partial M'} + \frac{\partial Q'_L}{\partial M''}\frac{\partial M''}{\partial M'} | A_i \right].$$

Since the envelope theorem does not apply to the bond price, the effects of current choices on future choices enter the current bond price.[17] It is not possible to find analytic expressions for $\frac{\partial K'}{\partial M}$, $\frac{\partial B'}{\partial M}$ and $\frac{\partial M'}{\partial M}$. In the numerical solution to the model, the policy function for the next-period capital stock K'' is decreasing in M', while the policy function for the next-period level of debt B'' is increasing in M'. The former is the debt overhang, the latter the debt dilution channel. Both of these channels are discussed in detail in Jungherr and Schott (2017). These effects arise because the firm acts only in the interest of the shareholder and does therefore not internalize the effect of its decisions of the value of outstanding long-term debt in default states. Since the benefits of

[16] From this section onward, I focus on the case of an interior solution, so $\lambda_{M,1} = \lambda_{M,0} = 0$.

[17] The reason why the envelope theorem does not apply to the bond prices is that firms do maximize over the market value of equity, but not over the market value of debt.

investment and the costs of debt issuance arise in the future, the larger the share of firm value that accrues to long-term debt, the lower will be investment and the higher debt issuance. As a consequence, a higher share of outstanding long-term debt will in general increase long-run default risk, by adversely affecting future firm policies, which drives down the price of long-term debt today. In this case, $\frac{\partial Q_L}{\partial M'} < 0$.

The third term is the *endogenous rollover cost*. If the firm issues short-term debt, it has to repay the entire amount at the face value in the next period. If the firm instead issues long-term debt, it can roll over a fraction $1-\mu$ of debt at the market value. The market value of long-term debt is below its face value because of long-run default risk. Therefore, being able to roll over long-term debt at the market value leads to lower rollover costs for the firm.

In addition, since the market value of long-term debt is low whenever cash flows to equity are low, long-term debt creates a hedging benefit to shareholders. However, this hedging benefit is quantitatively not that important since in the region of the state space where firms issue long-term debt, the equity value is almost linear, since the firm is far away from being liquidity constrained. Therefore shareholders are almost risk-neutral and do not value the hedging benefit highly.

4.2.3 Case III: Risky Short-Term Debt and Long-Term Debt

If short-term debt is also risky, the first-order condition for M' is

$$\frac{\partial \mathcal{L}}{\partial M'} = \Bigg[\underbrace{(Q_L - Q_S)B'}_{\text{Change, Marginal Revenue}} + \underbrace{\frac{\partial Q_S}{\partial M'}(1-M')B'}_{\text{Change, Intramarginal ST Revenue}} + \underbrace{\frac{\partial Q_L}{\partial M'}\left(M'B' - (1-\mu)MB\right)}_{\text{Change, Intramarginal LT Revenue}} \Bigg](1+\lambda_D) + \beta\mathbb{E}\left[(1-\mu)(1-Q_L'+\xi)B'(1+\lambda_D')\mathbb{1}_{(V'>0)}|A_i\right] = 0.$$

In this case, there are two new terms relative to the case in which only long-term debt is risky: First, the short-term bond price and the long-term bond price also incorporate a premium for short-run default risk. The long-term bond price can be written as

$$Q_L = Q_S + (1-\mu)\beta\mathbb{E}\left[\left(Q_L' - 1\right)\mathbb{1}_{(V'>0)}|A_i\right].$$

Hence, $Q_L - Q_S = (1-\mu)\beta\mathbb{E}\left[(Q_L'-1)\,\mathbb{1}_{(A'>a^*)}|A_i\right]$. Note that lengthening the maturity structure of debt does not change the default premium that the firm has to pay for short-run default risk, since both long-term debt and short-term debt price short-run default risk. The premium long-run default risk is the only change in the marginal revenue that arises when the firm increases the long-term debt share.

Second, if short-term debt is risky, the short-term bond price is also sensitive to the maturity structure of the firm. What matters for the short-term bond price is how a change in the maturity

structure of debt affects short-run default risk of the firm: The derivatives of the short-term and long-term bond prices in the case of risky short-term debt and long-term debt are given by:

$$\frac{\partial Q_S}{\partial M'} = \underbrace{\left[1 + c - R(K, B, a^*, 1)\right]}_{\text{Change in Repayment}} \underbrace{\frac{\frac{\partial V'}{\partial M'}}{\frac{\partial V'}{\partial A'}} f(a^*|A)}_{\substack{\text{Change in Next-Period} \\ \text{Default Probability}}} > 0,$$

$$\frac{\partial Q_L}{\partial M'} = \underbrace{\left[\mu + c + (1-\mu)Q_L' - R(K, B, a^*, 1)\right]}_{\text{Change in Repayment}} \underbrace{\frac{\frac{\partial V'}{\partial M'}}{\frac{\partial V'}{\partial A'}} f(a^*|A)}_{\substack{\text{Change in Next-Period} \\ \text{Default Probability}}}$$

$$+ \beta(1-\mu) \underbrace{\int_{a^*}^{\infty} \left(\frac{\partial Q_L'}{\partial K''} \frac{\partial K''}{\partial M'} + \frac{\partial Q_L'}{\partial B''} \frac{\partial B''}{\partial M'} + \frac{\partial Q_L'}{\partial M''} \frac{\partial M''}{\partial M'} \right) f(A'|A) dA}_{\text{Change in Long-Term Repayment and Default Probability}}.$$

Interestingly, increasing the long-term debt share can have opposite effects on the bond prices: A higher long-term debt share increases the price of short-term debt, because it reduces the short-run default risk. However, a higher long-term share also reduces investment in the next period and increases debt issuance in the next period, such that long-run default risk increases. In the quantitative version of the model, the latter effect dominates for the long-term bond price, such that lengthening the maturity structure increases the short-term bond price and decreases the long-term bond price.

In Figure 3, I depict the bond price as a function of the long-term share for two different levels of debt. All other variables are chosen to be the same. The parameters are from my baseline calibration. In the left panel, the level of debt chosen is high and as a consequence, the next-period default probability is high. The long-term bond price and the short-term bond price both increase for the most part in response to an increase in the long-term debt share. This is because such an increase lowers the next-period default probability, which is here the dominant effect.

In the right panel, the level of debt is low and therefore the next-period default probability is low. The long-term bond price decreases mostly if the long-term debt share increases. This is because a higher long-term debt share in this case leads to a higher long-term probability of default. In contrast to that, the short-term bond price increases, as in the left panel, monotonically with a higher long-term debt share.

In summary, if short-term debt and long-term debt are risky, the trade-off is fundamentally the same as in the case of risk-free short-term debt and risky long-term debt. Relative to the case with risk-free short-term debt, there is an additional benefit of issuing long-term debt, since a higher long-term debt share reduces the short-run default probability. However, in the quantitative model, firms will still prefer to issue short-term debt if they are liquidity-constrained.

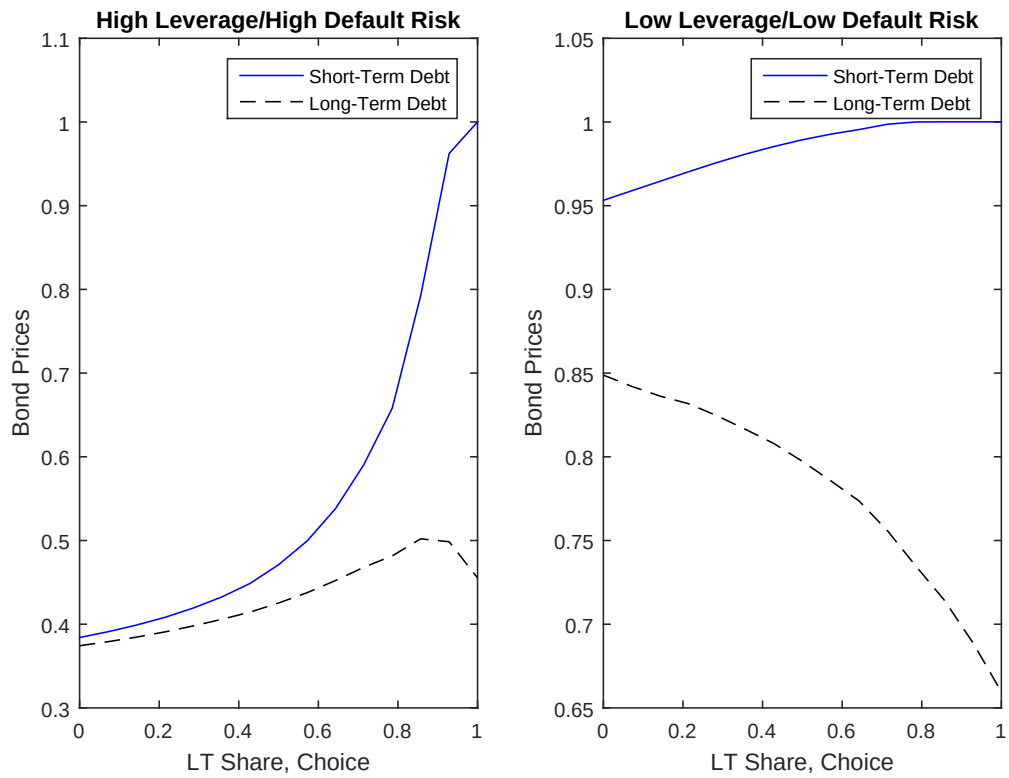

Figure 3: Bond prices as a function of the long-term debt share choice M'. The left panel depicts a situation with high debt and high next-period default risk, the right panel depicts a situation with low debt and low next period default risk. Capital, productivity and the aggregate state are the same in the left and the right panel.

4.2.4 Discussion and Implications for Macroeconomic Dynamics

The firm trades off rollover costs of short-term debt against the long-term default premium and the negative incentive effect of long-term debt issuance. This is the main trade-off I consider and therefore it deserves to be discussed in more detail. Consider the case of a firm with low productivity and a low capital stock. This firm issues debt due to a high value of internal funds, that is, since λ_D is high. It will have a low probability of long-term survival, and hence face a high long-term default premium on long-term debt. This is captured by the term $Q_L - 1 < 0$. Furthermore, by issuing long-term debt, such a firm would decrease the incentive for future investment, since a part of that investment would essentially be an intertemporal transfer of current shareholder funds to future bondholder funds. This is captured by the term $\frac{\partial Q_L}{\partial M'} < 0$. If these effects outweigh the rollover costs, such a liquidity-constrained firm will choose to mostly issue short-term debt.

Now consider a firm with a high capital stock and a high productivity. The motive for such a firm to issue debt is not a high value of λ_D, but the tax benefit of debt. Such a firm has a low

long-term default probability, and hence $Q_L - 1$ and $\frac{\partial Q_L}{\partial M'}$ will be close to 0. Therefore, such a firm will mostly be concerned about the rollover costs of debt and will issue long-term debt, as described in case I.

In this model, firms endogenously use different types of debt for different motives: Liquidity-constrained firms use short-term debt, while firms which care mostly about the tax benefit of debt use long-term debt. These two motives will later on give rise to the cyclical dynamics of debt maturity: Intuitively, the fraction of firms which issue short-term debt due to liquidity constraints increases in a recession, while the fraction of firms which issue long-term debt due to the tax benefit decreases. The motive to issue debt due to a liquidity shortfall is counter-cyclical, while the tax benefit of debt net of the default premium is pro-cyclical. As a consequence, the aggregate long-term debt share in the model will be pro-cyclical.

4.3 The Effect of Consumption Risk on the Optimal Maturity Structure

In the presence of consumption risk, the difference in bond prices can be decomposed into two terms:

$$Q_L - Q_S = \underbrace{Q_L^{RF} - Q_S^{RF}}_{\text{Risk-Free Term Spread}} + \underbrace{(Q_L - Q_L^{RF}) - (Q_S - Q_S^{RF})}_{\text{Long-Term Default Premium}}.$$

The new first term is the difference in risk-free bond prices. The second term is the long-term default premium. Bonds yield a fixed stream of income which is $1 + c$ for short-term bonds and $\mu + c$ per period for long-term bonds. In the presence of aggregate risk, a marginal unit of consumption more in a recession is more valuable than a marginal unit of consumption in an expansion. Hence, conditional on being in a recession, the fact that consumption is mean-reverting implies that consumption in the next period will be higher than in the current period and hence that risk-free bond prices in a recession are lower than in an expansion. Further, the positive autocorrelation of the shocks implies that the risk-free long-term bond price in the recession is lower than the risk-free short-term bond price, since the short-term bond will repay more in periods closer to the present, where consumption is lower.

With this decomposition, the first-order condition for the long-term debt share can be rewritten as

$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial M'} = \Bigg[&\left((Q_L^{RF} - Q_S^{RF}) + (Q_L - Q_L^{RF} - Q_S + Q_S^{RF})\right) B' + \\ &\frac{\partial Q_S}{\partial M'}(1 - M')B' + \frac{\partial Q_L}{\partial M'}\left(M'B' - (1-\mu)MB\right)\Bigg](1 + \lambda_D) + \\ &\mathbb{E}\left[\Lambda(C, C')\frac{\partial V'}{\partial M'}|A_i\right] = 0.\end{aligned}$$

Consumption risk is important for two reasons. First, the risk-free short-term bond price is higher than the risk-free long-term bond price in a recession and lower in expansions, as described

above. In other words, the term structure of risk-free bond *yields* is downward sloping in expansions and upward sloping in recessions. Without the shadow cost of internal funds, λ_D, this would not matter, since firms would then discount future cash flows at the same discount factor as creditors. However, a firm that values an additional unit of internal funds more highly today than tomorrow, i.e. with $\lambda_D > \lambda'_D$, will be myopic and hence prefer short-term debt relative to long-term debt *more* in a recession.

Second, as outlined in Chen (2010) and Bhamra et al. (2010), in the presence of aggregate risk, the bond yield contains a risk premium in addition to the risk-neutral default premium if the default probability is higher in recessions. This is because in that case, cash flows from the firms to creditors are low exactly when creditors value cash flows highly.

In summary, consumption risk introduces a new channel for the determination of the maturity structure through the time variation in the term structure of risk-free rates and emphasizes the importance of the default channel relative to the rollover channel by increasing default premia. The fact that the short-term bond price is higher than the long-term bond price in recessions makes short-term debt even more attractive if the firm issues debt due to liquidity constraints in a recession. This channel amplifies the counter-cyclicality of short-term debt issuance. In addition, the higher and more cyclical default premia reduce the net tax benefit of long-term debt, particularly in recessions. This effect should amplify the pro-cyclicality of long-term debt issuance.

5 Mapping the Model to the Data

In this section, I will describe the data and the selection of parameters. I divide the set of parameters into three subsets: I take the first set from the literature, estimate the second set of parameters directly from aggregate data, and choose the third set of parameters to match cross-sectional data moments in simulations of the model. I solve the model using value function iteration. The interested reader will find a detailed description of the solution algorithm in Appendix C.

5.1 Data

Cross-Sectional Data Firm data are from the merged CRSP Compustat Quarterly North America database. I focus on U.S. firms. The observation unit is a firm-quarter. I include data from the first quarter of 1984 to the last quarter of 2012.[18] I exclude regulated firms (SIC code 4900-4999), financial firms (SIC code 6000-6999) and non-profit firms (SIC code 9000-9999) from my sample, since the model is not appropriate for such firms. Furthermore, I exclude those observations which do not report total assets or those which report either negative assets or a negative net capital stock.

I calculate all flow variables from the cash flow statements of firms. Investment is capital expenditures minus sales of property, plant and equipment. Short-term debt issuance is defined as

[18] Data are in principle available since the first quarter of 1976, but before 1984, the sample composition in Compustat changed markedly from quarter to quarter.

change in current debt. Long-term debt issuance is long-term debt issuance minus long-term debt reduction. Equity issuance is sale of common and preferred stock minus purchase of common and preferred stock minus dividends. All flows are normalized by lagged total assets. I calculate the share of long-term debt to total debt as long-term debt divided by long-term debt plus current debt. I follow Whited (1992) to calculate market leverage: It is defined as the book value of short-term debt plus the market value of long-term debt divided by the sum of the book market of short-term debt, the market value of long-term debt and the market value of equity. To calculate the market value of long-term debt, I use the method by Bernanke, Campbell, Friedman, and Summers (1988). The market value of common stock is defined as the share price times the number of shares. The market value of preferred stock is defined by the current dividend for preferred stock divided by the current federal funds rate.

Aggregate Data I use the productivity time series from Fernald (2014) to compute productivity shocks and real personal consumption expenditure from the Bureau of Economic Analysis (BEA) to compute the stochastic progress for consumption. For real GDP, I either use real GDP from the BEA or aggregate real sales from Compustat. To compute real sales, I deflate nominal sales with a four quarter moving average of the consumer price index. All series are detrended with a quadratic trend, as are the other time series I aggregate from Compustat data.

The data for the default rate is from Ou, Chiu, and Metz (2011). I use the default rate for the largest sample, namely all firms from 1920 to 2011, which corresponds to 1.1 percent.

5.2 Simulation Procedure

I simulate a panel of 5000 firms for 2000 quarters. I use a burn-in period of 1000 quarters. Defaulted firms are replaced with new firms which draw a new productivity from the unconditional productivity distribution. Changing these values does not affect the results. These firms start out with zero debt and a very small capital stock.

In the data, short-term debt is defined as debt with a maturity of less than 1 year. This definition includes long-term debt with a residual maturity of less than 1 year. In the model, debt with a maturity of less than 1 year is given by

$$(1 - M_{i,t})B_{i,t} + (1 - (1 - \mu)^4)M_{i,t}B_{i,t}. \tag{5.1}$$

Therefore, the share of long-term debt in total debt at the firm level is given by

$$\frac{B_{i,t} - (1 - M_{i,t})B_{i,t} - (1 - (1 - \mu)^4)M_{i,t}B_{i,t}}{B_{i,t}} = (1 - \mu)^4 M_{i,t}. \tag{5.2}$$

Market leverage at the firm level is calculated as the market value of debt divided by the market

Name	Value	Role	Reference
Parameters from the Literature			
β	$1.04^{-1/4}$	Discount Factor	4% Annual Risk-Free Rate
σ	2	Utility Curvature	Campbell and Cochrane (1999)
μ	0.05	Long-Term Debt Repayment Rate	5 Year Maturity
τ	0.14	Corporate Income Tax Rate	Graham (2000)
χ	0.8	Recovery Rate	Kuehn and Schmid (2014)
θ	4	Capital Adjustment Cost	Warusawitharana and Whited (2016)
α	0.35	Production Function Curvature	Moyen (2007)
ρ_A	0.95	Persistence, Idiosyncratic Productivity	Katagiri (2014)
σ_A	0.1	Volatility, Idiosyncratic Productivity	Katagiri (2014)
Calibrated Parameters			
ξ	0.0025	Linear Debt Issuance Cost	Average Long-Term Debt Share
ϕ_0	0.1	Fixed Equity Issuance Cost	Size, Equity Issuance
ϕ_1	0.04	Linear Equity Issuance Cost	Frequency, Equity Issuance
ψ	1.8	Fixed Production Cost	Annual Default Rate
Estimated Parameters			
σ^Z	0.0071	Volatility, Aggregate Productivity	
σ^C	0.0058	Volatility, Consumption	
λ_1	0.113	Productivity Coefficient in Consumption	
λ_2	1	Consumption Coefficient	

Table 3: Parameter Choices. This table shows all model parameters, grouped into three categories: The first category shows parameters chosen from the literature, the second category shows parameters from the literature, the third category shows parameters taken from a production function estimation using a dynamic panel data estimator.

value of debt plus the ex dividend value of equity:

$$\frac{Q_{S,i,t}B_{S,i,t+1} + Q_{L,i,t}B_{L,i,t+1}}{V_{i,t} - D_{i,t} - EIC(D_{i,t}) + Q_{S,i,t}B_{S,i,t+1} + Q_{L,i,t}B_{L,i,t+1}}. \tag{5.3}$$

Book leverage is given by

$$\frac{B_{S,i,t+1} + B_{L,i,t+1}}{K_{i,t+1}}. \tag{5.4}$$

Finally, the market-to-book ratio is

$$\frac{V_{i,t} - D_{i,t} - EIC(D_{i,t}) + Q_{S,i,t}B_{S,i,t+1} + Q_{L,i,t}B_{L,i,t+1}}{K_{i,t+1}}. \tag{5.5}$$

5.3 Parameter Choices

Parameters from the Literature For the preferences of the representative household, I use a time preference rate, $\frac{1}{\beta} - 1$, of 4 percent per year and a risk aversion coefficient σ of 2, which is the value used in Campbell and Cochrane (1999). In the baseline calibration, I set the maturity

	(1) ln(Z)	(2) ln(C)
L.ln(Z)	0.9662***	-0.0263
	(34.14)	(-0.32)
ln(Z)		0.1128
		(1.44)
L.ln(C)		0.9172***
		(29.37)
Observations	115	115

t statistics in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Table 4: Estimation of aggregate processes.

of long-term debt to 5 years. This implies a quarterly repayment rate μ of 5 percent. Following Graham (2000), I set the corporate income tax rate τ to 14 percent. This is substantially lower than the true marginal U.S. corporate income tax rate, but corresponds to about the actual average tax rate of firms in the U.S. I set the recovery rate in default to 0.8 as in Kuehn and Schmid (2014). I set the autocorrelation of the productivity shock to 0.95 and the standard deviation of idiosyncratic productivity to 0.1, which is similar to Katagiri (2014). I use $\alpha = 0.35$ for the production function curvature. I set the capital adjustment cost θ to 4. Warusawitharana and Whited (2016) estimate an adjustment cost between 4 and 6 for large firms in Compustat. Bloom (2009) estimates an adjustment cost of 4.8 on Compustat data in his purely quadratic adjustment cost specification.

Estimated Parameters There are three parameters that govern aggregate uncertainty in the model: The volatility σ^Z of the aggregate productivity shock, the volatility σ^C of the consumption shock and the coefficient of productivity in consumption, λ_1. I set, without loss of generality, $\lambda_2 = 1$. The persistence of both aggregate shocks is equal to the persistence of the idiosyncratic shocks to keep the state space tractable. I estimate the following regression on detrended productivity and consumption data:

$$\ln Z_t = \rho^Z \ln Z_{t-1} + \eta_t^Z$$
$$\ln C_t = \lambda_1 \rho^Z \ln Z_{t-1} + \lambda_1(\ln Z_t - \rho^Z \ln Z_{t-1}) + \rho^C \ln C_{t-1} + \eta_t^C$$

I report the results in Table 4. The standard deviation of η_t^Z is 0.0071, the standard deviation of η_t^C 0.0058. Hence, I set $\lambda_1 = 0.113$, $\sigma^Z = 0.0071$, and $\sigma^C = 0.0058$. The regression further shows that setting $\rho^Z = \rho^C = 0.95$ is well within the range of plausible parameters.

Parameters Set to Match Cross-Sectional Moments I choose the debt issuance cost parameter ξ, the equity issuance cost parameters ϕ_0 and ϕ_1 and the fixed production cost ψ to match a set of cross-sectional moments. I choose to match the average share of long-term debt, the average

size and frequency of equity issuance and the cumulative one year default rate.

The average share of long-term debt is informative about the linear debt issuance cost ξ: the higher is the debt issuance cost ξ, the less attractive is short-term debt relative to long-term debt for the purpose of the tax benefit, and the higher is the share of long-term debt. The size and frequency of equity issuance are informative about the equity issuance costs ϕ_0 and ϕ_1: A higher ϕ_0 leads to a larger conditional size of equity issuance and a lower frequency of equity issuance. A higher ϕ_1 leads to a smaller conditional size of equity issuance, and a lower frequency of equity issuance. The default rate helps to identify the fixed cost ψ: A higher value for ψ implies a higher default rate.

Table 3 shows the parameters resulting from the moment matching exercise. The fixed cost ψ corresponds to about 10 percent of the steady state capital stock of the model. Kuehn and Schmid (2014) use a linear production cost that corresponds to 4 percent of the lagged capital stock.

	Model	Data
Long-Term Debt Share, Mean	64.044	63.200
Equity Issuance, Size	6.931	15.684
Equity Issuance, Frequency	18.552	10.100
Default Rate	1.628	1.146

Table 5: Model fit.

Table 5 reports targeted moments from a numerical simulation. The model matches the average long-term debt share well. The default rate in the model is close to the default rate in the data of about 1.1 percent. The model can also match the size and frequency of equity issuance. Overall, while the match between the model and the data is by no means perfect, it delivers plausible numbers for all targeted moments.

6 Quantitative Results

In this section, I report the implications of the model for the dynamics of the aggregate maturity structure. The aim of this section is to show that the model, which is parameterized to match firm-level moments, can also match aggregate dynamics. First, I will report aggregate means and correlations. Second, I consider how well the model fits the cross-sectional distributions of leverage and the debt maturity structure in the data and third how well it replicates the dynamics of the debt maturity structure across the size distribution of firms. Explaining the heterogeneity in the level and dynamics of the maturity structure for firms of different size classes is a key contribution of the paper relative to the existing literature.

6.1 Aggregate First Moments

In Table 6, I report aggregate summary statistics. I report moments for the aggregate long-term debt share, which is the key variable the model should explain, as well as book and market leverage,

	Data	Baseline	No DIC	No EIC	No Sigma	No Tax Benefit	No IC
Long-Term Debt Share, Mean	0.845	0.653	0.142	0.782	0.601	0.534	0.179
Long-Term Debt Share, StDev	0.012	0.012	0.009	0.004	0.007	0.017	0.008
Book Leverage, Mean	0.433	0.383	0.710	0.509	0.347	0.205	1.311
Book Leverage, StDev	0.033	0.013	0.017	0.008	0.008	0.010	0.035
Market Leverage, Mean	0.228	0.174	0.344	0.226	0.163	0.096	0.614
Market Leverage, StDev	0.040	0.006	0.009	0.005	0.004	0.005	0.006
Investment/Capital, Mean	0.032	0.037	0.037	0.037	0.037	0.037	0.037
Investment/Capital, StDev	0.005	0.004	0.004	0.004	0.002	0.004	0.004
Default Rate	1.146	1.628	1.062	1.292	2.260	1.052	1.158
Fraction, Liq. Constr. Firms	-	0.345	0.321	0	0.336	0.350	0
Equity Issuance, Frequency	0.101	0.185	0.211	0.285	0.180	0.201	0.470
ST Debt Issuance, Frequency	0.174	0.761	0.997	0.410	0.776	0.788	0.996
LT Debt Issuance, Frequency	0.161	0.662	0.287	0.833	0.576	0.341	0.583

Table 6: Aggregate summary statistics.

which together with the long-term debt share fully characterize the capital structure of the firms in the model. Since investment is a key reason why firms want to issue debt in this model, I also report moments for aggregate investment. Since they are key to the model mechanism, I further report the default rate and the fraction of liquidity-constrained firms. Finally, to give more insights into the financing choices of firms and how they vary with the model assumptions, I report the fraction of firms issuing equity, short-term debt and long-term debt, respectively.

I show the results for six different versions of the model: *Baseline* is the model with the parametrization as described in section 5. In *No DIC*, I set debt issuance costs to 0. In *No EIC*, I set equity issuance costs to 0. In *No Tax Benefit*, I remove the tax deductibility of coupon payments. Finally, in *No IC*, I remove both debt and equity issuance costs.

Removing debt issuance costs allows me to demonstrate the importance of rollover costs in the model. Without equity issuance costs, firms do no longer face liquidity constraints. A risk-neutral household does not charge a risk premium on debt and equity. Removing the tax benefit demonstrates the importance of the tax shield. The model version without issuance costs shows how debt issuance costs and equity issuance costs interact non-linearly.

The mean and standard deviation of the aggregate long-term debt share are well matched by the model. As in the data, the aggregate fraction of debt maturing within the next year is relatively low, though somewhat higher in the model. The standard deviation of the long-term debt share in the model is similar to the data. Overall, despite the fact that the model has has two contracts with different debt maturity, compared to the limitless contracting options in the data, it explains the aggregate maturity structure in the data well.

Aggregate book leverage in the model is similar in the model and in the data, but the volatility is lower. Book leverage is of separate interest, since it is much easier to measure in the data than market leverage. This is because it does not depend on prices and hence expectations. However it is

market leverage which is relevant for corporate decisions, since market leverage reflects expectations about the fraction of future cash flows accruing to the creditors of the firm. The model can also match the aggregate market leverage, but the volatility of market leverage is too low. This is because market leverage is calculated using stock prices in the data, and the model does not generate sufficient stock price volatility.

The model can also match the mean and standard deviation of the aggregate investment to capital ratio well. The mean of the investment to capital ratio is by construction equal to the deprecation rate, which corresponds surprisingly well to the average aggregate investment to capital ratio. The volatility of investment in the data is higher than in the model. Arguably, the model misses many important drivers of aggregate investment like uncertainty shocks (see for example Bloom (2009) and Bachmann and Bayer (2014)) or investment-specific technology shocks (see for example Fisher (2006) and Justiniano, Primiceri, and Tambalotti (2011)).

Comparing the different models, we can see that firms choose a substantially shorter debt maturity structure in the absence of exogenous rollover costs. Since short-term debt is not subject to debt overhang and debt dilution, this allows firms to massively expand leverage while maintaining a low default rate. Further, firms adjust their leverage every period by constantly issuing short-term debt.

In contrast, eliminating equity issuance costs substantially reduces the motive for liquidity-constrained firms to issue short-term debt: The long-term debt share increases markedly, while the frequency of short-term debt issuance decreases. Further, we can see that future liquidity constraints are an important determinant of the leverage choices of firms: If they are removed, firms choose a much higher long-term leverage.

Removing risk aversion of the representative household does not affect the levels of the aggregate ratios substantially, but it does reduce their volatilities.

Without the tax benefit of debt, firms use less long-term leverage, as seen by a lower leverage and a lower long-term debt share. This substantially reduces the default rate of firms.

Finally, removing both debt issuance costs and equity issuance costs allow firms to take on leverage which is more than 3 times higher than in the baseline model, while maintaining a very low default rate. Again, as in the case without debt issuance costs, firms can achieve this by issuing mostly short-term debt, as indicated by the low long-term debt share.

6.2 Aggregate Correlations

In Table 7, I report the correlations of the aggregate time series of the model with my preferred measure of output, aggregate sales. This is the better measure of corporate cash flows than real GDP and is also strongly pro-cyclical and hence a good proxy variable for the business cycle.

The model matches the signs of the correlations of the long-term debt share, book leverage and market leverage with output. In terms of flows, it matches the signs of investment, long-term debt issuance, equity issuance and the default rate with output.

Importantly, the long-term debt share in the model is pro-cyclical. The model matches the

	Data	Baseline	No DIC	No EIC	No Sigma	No Tax Benefit	No IC
Long-Term Debt Share	0.386	0.539	-0.294	0.737	0.406	0.326	-0.739
Book Leverage	-0.099	-0.627	0.801	0.008	-0.462	-0.410	0.756
Market Leverage	-0.277	-0.725	0.260	-0.186	-0.767	-0.548	0.823
ST Debt Issuance/Capital	0.179	-0.187	0.815	-0.524	-0.222	-0.020	0.828
LT Debt Issuance/Capital	0.510	0.450	0.437	0.739	0.286	0.259	0.506
Debt Issuance/Capital	0.383	-0.072	0.814	0.199	-0.135	0.013	0.828
Equity Issuance/Capital	-0.390	-0.614	-0.497	-0.321	-0.643	-0.523	-0.004
Investment/Capital	0.390	0.602	0.600	0.597	0.824	0.558	0.510
Default Rate	-0.553^*	-0.683	-0.578	-0.634	-0.520	-0.581	-0.597
Fraction, Liq. Constr. Firms	-	-0.639	-0.800	-	-0.835	-0.613	-
Equity Issuance, Frequency	0.255	-0.736	-0.861	-0.438	-0.774	-0.694	-0.225
ST Debt Issuance, Frequency	-0.073	-0.478	0.546	-0.905	-0.677	-0.698	0.640
LT Debt Issuance, Frequency	0.528	-0.224	0.149	0.199	-0.185	-0.173	-0.195

Table 7: Aggregate correlations with log(sales). The correlation reported in data for the default rate, noted by * is the correlation between the default rate and output growth reported in Kuehn and Schmid (2014).

correlation in the data quantitatively well.

Both book leverage and market leverage in the model are counter-cyclical. Market leverage has a more negative correlation with output than book leverage, because the market value of equity is more volatile than the market value of debt. This is also true in the data, although the model overstates the cyclicality of market leverage in the data. A negative correlation between market leverage and output implies that the fraction of cash flows accruing to creditors increases in a recession, which increases debt overhang problems in recessions and leads to a counter-cyclical default rate.

Short-term debt issuance is counter-cyclical in the baseline model, in contrast to the data. The main motive for short-term debt issuance is financial constraints in the form of a positive shadow cost of internal funds λ_D. As financial constraints are more severe in recessions, short-term debt issuance is higher during recessions. My measure of short-term debt issuance in the model is also not perfect, since I only observe the change in current debt, which also includes maturing long-term debt. Since I do not observe long-term debt maturing and repurchased separately, this problem cannot be easily remedied.

In line with the data, long-term debt issuance is pro-cyclical, since long-term debt is issued by unconstrained firms due to the tax benefit of debt. In a recession, default premia increase, while the tax benefit is constant. As a consequence, firms will issue less long-term debt.

Total debt issuance is counter-cyclical in the baseline model, also in contrast with the data. The reason for this is that counter-cyclical short-term debt issuance is too high relative to pro-cyclical long-term debt issuance. A model in which the maturity of long-term debt μ is also a choice variable could potentially better match both the dynamics of the long-term debt share and debt issuance.

Equity issuance is counter-cyclical in the baseline model as well as in the data. Financially constrained firms which cannot or do not want to issue debt will issue equity instead. This result is in line with the results in Jermann and Quadrini (2012), who also report counter-cyclical equity issuance. I interpret equity issuance strictly as liquidity injections due to a lack of internal funds, and measure it in the data accordingly. Other motives for equity issuance outlined in Fama and French (2005) exist, but those are not captured by this model. This is of relevance, since Covas and Den Haan (2011) document that other measures, which include for example stock compensation for employees or equity swaps during mergers, are actually pro-cyclical.

Finally, the investment-capital ratio in the model is pro-cyclical, as it is in the data. The relatively weak pro-cyclicality is surprising. This is due to the fact that I scale investment by the last period capital stock, which is also pro-cyclical. This mutes the pro-cyclicality of the investment to capital ratio relative to the level of investment.

Overall, while the model predicts a too high counter-cyclicality of short-term debt issuance and hence total debt issuance, it replicates aggregate dynamics in the data well. In particular, it matches the main fact that the aggregate maturity structure of debt is pro-cyclical.

Relative to the baseline model, removing debt issuance costs leads to a pro-cyclical aggregate leverage and a counter-cyclical long-term debt share. This is because the tax benefit of short-term debt becomes the dominant motive for debt issuance. Due to counter-cyclical default risk and hence credit spreads, this tax-shield motive is pro-cyclical.

The model without equity issuance costs has pro-cyclical total debt issuance, since firms issue less short-term debt due to counter-cyclical liquidity constraints and more long-term debt for the pro-cyclical tax benefit. Further, relative to the baseline model, both short-term debt issuance and long-term debt issuance become more cyclical.

As expected, removing the risk premia mutes the pro-cyclicality of long-term debt issuance and the default rate. The reason for the latter is a less volatile market value of equity. However it does not substantially affect the dynamics of short-term debt, which does not have a large risk premium.

Removing the tax benefit of debt reduces the pro-cyclicality of long-term debt issuance and the counter-cyclicality of short-term debt issuance markedly.

Finally, removing both debt and equity issuance costs leads to strongly pro-cyclical debt issuance and leverage dynamics. Equity issuance becomes basically a-cyclical. This is because firms use short-term debt as their main financing instrument, which has a tax benefit compared to equity and does not lead to debt overhang and debt dilution compared to long-term debt.

6.3 The Cross Section over the Cycle

To investigate how firms adjust their issuance in response to macroeconomic shocks, I group the firms from the simulated panel into 7 size quantiles and compute the correlation of the long-term debt share with output within each of these size quantiles.

As in the data, the cyclicality of the long-term debt share varies substantially by firm size. Both in the model and in the data, it is the medium-sized firms for which the long-term debt share

Firm Size Quantile	Data	Baseline	No DIC	No EIC	No Sigma	No Tax Benefit	No IC
0% to 25%	0.333	0.363	-0.233	0.008	-0.141	0.145	-0.527
25% to 50%	0.473	0.605	-0.449	0.879	0.170	0.194	-0.840
50% to 75%	0.658	0.570	-0.185	0.831	0.668	0.481	-0.103
75% to 90%	0.519	0.571	-0.205	0.723	0.729	0.647	-0.219
90% to 95%	0.269	0.803	-0.241	-0.134	0.574	0.270	-0.372
95% to 99%	-0.214	0.684	-0.179	-0.053	0.575	-0.193	-0.081
99% to 100%	-0.086	-0.043	-0.059	0.066	0.000	0.031	-0.156

Table 8: Correlations between the long-term debt share and log(sales) across the firm size distribution.

is the most sensitive to cyclical fluctuations. Small firms in the model are constrained and use predominantly short-term debt even during expansions, while medium-sized firms are unconstrained during expansions, but constrained during recessions. The larger the firms, the less likely it is that they are financially constrained during a recession, and the lower is their need to issue short-term debt. The baseline model can therefore account for the inverse u-shape in the dynamics of the long-term debt share across the firm size distribution.

These results support the hypothesis that the main driver of short-term debt issuance and hence a short maturity structure are liquidity constraints. In addition, the model also lends support to the theory that the fraction of liquidity-constrained firms increases during a recession.

Note also that for the largest firms, the maturity structure is basically a-cylical both in the model and in the data. The puzzling firms, from the perspective of the model, are the firms in the 95 to 99 percent size quantile for which the maturity structure of debt is *counter*-cyclical. Jungherr and Schott (2016) construct a model in which the share of long-term debt is counter-cyclical due to debt dilution: As firms in their model issue more debt, they shorten the maturity structure of debt. If newly issued short-term debt constitutes a very large share of newly issued debt, incentives are better aligned between firm owners and creditors. Creditors prefer short-term debt because long-term debt creates debt dilution issues. The distinction between their model and this model is that this model has a clear pecking order in which short-term and long-term debt are issued for different reasons, whereas in their model, short-term debt and long-term debt are to some extent substitutes.

One possible way to reconcile these results is if fixed costs constitute a large share of debt issuance costs, such that it is beneficial for large firms to also issue short-term debt due to the tax benefit. However, in such a model, large firms would choose a short debt maturity structure, which runs counter to the observation that the debt maturity structure is strictly more long-term for larger firms in the data. There is therefore scope for future research to understand and explain the debt maturity dynamics of the long-term debt share for very large firms.

Comparing the baseline model with the alternative models shows that none of the other models can replicate the dynamics in the data as well as the baseline model.

6.4 Untargeted Cross-Sectional Moments

	Data	Baseline	No DIC	No EIC	No Sigma	No Tax Benefit	No IC
				Long-Term Debt Share			
Mean	63.210	64.047	17.193	76.348	58.063	37.573	22.908
StDev	35.914	18.562	22.750	9.756	24.165	29.830	22.612
Correlation with Size	0.414	0.520	-0.290	0.609	0.489	-0.109	-0.437
Correlation with Market-To-Book	-0.190	-0.088	-0.278	0.300	-0.055	-0.057	-0.356
Correlation with Book Leverage	-0.030	-0.125	-0.125	0.151	-0.060	0.592	-0.391
				Market Leverage			
Mean	22.862	21.426	35.210	24.311	20.362	12.793	58.798
StDev	26.292	12.322	10.077	10.734	12.240	10.885	8.761
Correlation with Size	0.068	-0.666	0.044	-0.161	-0.660	-0.662	0.739
Correlation with Market-To-Book	-0.236	-0.564	-0.336	-0.577	-0.490	-0.465	0.298
Correlation with Book Leverage	0.413	0.864	0.656	0.829	0.865	0.903	0.686

Table 9: Cross-sectional summary statistics.

In the upper panel of Table 9, I show moments of the cross-sectional distribution of the long-term debt share of the model. I report the mean and standard deviation, as well as correlations with firm size and the market-to-book ratio, which are important determinants of leverage in the corporate finance literature, as well as the correlation with book leverage. Size is an important proxy variable for financial constraints, while the market-to-book ratio of the firm measures its growth opportunities and is highly correlated with its productivity. Note that I only target the mean for the long-term debt share. All numbers report percentages.

The model can accurately capture the mean for the long-term share, which is a targeted moment. The standard deviation of the long-term debt share is too low relative to the data. One reason is that in the data, there is a non-negligible fraction of firms which use exclusively short-term debt.

The model can also correctly account for the positive correlation between the share of long-term debt and firm size, which is crucial to match aggregate moments. The correlation with the market-to-book ratio is low, as in the data. However, the correlation with the market-to-book ratio is too weak in the model. The correlation with book leverage is too high. The reason is that the motive to use leverage in the model is too weak for large firms, such that small firms will simultaneously have a high leverage and a short debt maturity structure.

In the model without debt issuance costs, the correlation between the long-term debt share and firm size becomes negative, since large firms issue more short-term debt for the tax benefit. In contrast, in the model without equity issuance costs, this correlation becomes more positive, since larger firms can sustain a higher long-term leverage. Removing the tax benefit effectively eliminates any correlations between firm size and the long-term debt share, while, similar to the case without debt issuance costs, it is also strongly negative in the model without any issuance costs.

In the lower panel of Table 9, I show the distribution of market leverage. The model can match the average market leverage as well as the standard deviation of the market leverage well. However, the model predicts a strong negative correlation between market leverage and firm size, while this

correlation is weakly pro-cyclical in the data. The reason is that due to the fixed production cost, it is mostly small firms which issue debt because they are liquidity-constrained. Firms in which growth options constitute a large fraction of the firm value use less leverage, as shown by the negative correlation between the market-to-book ratio and market leverage. As expected, the correlation between book leverage and market leverage is positive both in the model and in the data.

Average leverage increases in the model without debt issuance costs. Also, the correlation between firm size and leverage increases. However the same is true for the model without equity issuance costs. Eliminating both debt and equity issuance costs leads to a threefold increase in average leverage, which strongly correlates with firm size and productivity.

Overall, these results show that debt issuance costs and equity issuance costs together are necessary to match cross-sectional moments for leverage and the debt maturity structure. The tax benefit of debt is necessary for the model to deliver a positive correlation between the long-term debt share and firm size. The importance of the tax benefit relative to the importance of liquidity constraints determines the sign of the correlation between firm size and leverage.

7 Conclusion

I study the determinants of aggregate and firm-level corporate debt maturity dynamics in a quantitative model with rich cross-sectional heterogeneity. In the model, firms prefer to issue long-term debt if they are financially unconstrained, because debt issuance costs imply that the tax advantage of long-term debt is much bigger than the tax advantage of short-term debt. Maturity dynamics are driven by liquidity-constrained firms issuing short-term debt to cover liquidity shortfalls.

The model can match the levels and dynamics of the debt maturity structure, both at the firm level and in the aggregate, and is consistent with other established facts about the dynamics of corporate financing and investment decisions.

The question of regulation naturally arises, given the incentive misalignment between shareholders and long-term creditors in the model. Can and should regulatory authorities develop rules such that the preferences of bondholders are better reflected in the decisions of firms? The results in this paper suggest that such rules can lead to higher investment rates.

Some interesting extensions of the model are left for future research. For example, I abstract from cash holdings and credit lines, which are additional sources of funds firms can use to reduce the incidence of liquidity shortfalls. Further, there are no labor market frictions in the model, which might be another important reason to issue short-term debt through a working capital requirement, as in Jermann and Quadrini (2012).

References

Almeida, H., M. Campello, B. Laranjeira, and S. Weisbrenner (2012). Corporate Debt Maturity Structure and the Real Effects of the 2007 Credit Crisis. *Critical Finance Review 1*, 3–58.

Altinkilic, O. and R. S. Hansen (2000). Are there economies of scale in underwriting fees? Evidence of rising external financing costs. *Review of Financial Studies 13*(1), 191–218.

Arellano, C. and A. Ramanarayanan (2012). Default and the Maturity Structure in Sovereign Bonds. *Journal of Political Economy 120*(2), 187–232.

Azariadis, C., L. Kaas, and Y. Wen (2016). Self-fulfilling credit cycles. *Review of Economic Studies 83*, 1364–1405.

Bachmann, R. and C. Bayer (2014). Investment dispersion and the business cycle. *American Economic Review 104*(4), 1392–1416.

Bernanke, B. S., J. Y. Campbell, B. M. Friedman, and L. H. Summers (1988). Is There a Corporate Debt Crisis? *Brookings Papers on Economic Activity 1988*(1), 83–139.

Bhamra, H. S., L. A. Kuehn, and I. A. Strebulaev (2010). The aggregate dynamics of capital structure and macroeconomic risk. *Review of Financial Studies 23*, 4187–4241.

Bloom, N. (2009). The Impact of Uncertainty Shocks. *Econometrica 77*(3), 623–685.

Bolton, P., H. Chen, and N. Wang (2013). Market timing, investment, and risk management. *Journal of Financial Economics 109*(1), 40–62.

Campbell, J. Y. and J. H. Cochrane (1999). By Force of Habit: A ConsumptionBased Explanation of Aggregate Stock Market Behavior. *Journal of Political Economy 107*(2), 205–251.

Chatterjee, S. and B. Eyigungor (2012). Maturity, indebtedness, and default risk. *American Economic Review 102*(10), 2674–2699.

Chen, H. (2010). Macroeconomic conditions and the puzzles of credit spreads and capital structure. *Journal of Finance 65*(6), 2171–2212.

Chen, H., Y. Xu, and J. Yang (2016). Systematic Risk, Debt Maturity, and the Term Structure of Credit Spreads. *Unpublished Manuscript*.

Chor, D. and K. Manova (2012). Off the cliff and back? Credit conditions and international trade during the global financial crisis. *Journal of International Economics 87*, 117–133.

Cooper, R. W. and J. Ejarque (2003). Financial frictions and investment: requiem in Q. *Review of Economic Dynamics 6*(4), 710–728.

Corbae, D. and P. D. D'Erasmo (2017). Reorganization or liquidation: bankruptcy choice and firm dynamics. *NBER Working Paper 23515*, 1–65.

Covas, F. and W. J. Den Haan (2011). The Cyclical Behavior of Debt and Equity Finance. *American Economic Review 101*(2), 877–899.

Crouzet, N. (2015). Default, Debt Maturity and Investment Dynamics. *Unpublished Manuscript*.

Crouzet, N. (2016). Aggregate Implications of Corporate Debt Choices. *Unpublished Manuscript*.

de Fiore, F. and H. Uhlig (2011). Bank finance versus bond finance. *Journal of Money, Credit and Banking 43*(7), 1399–1421.

Diamond, D. W. and Z. He (2014). A Theory of Debt Maturity: The Long and Short of Debt Overhang. *Journal of Finance 69*(2), 719–762.

Duchin, R., O. Ozbas, and B. A. Sensoy (2010). Costly external finance, corporate investment, and the subprime mortgage credit crisis. *Journal of Financial Economics 97*(3), 418–435.

Eisfeldt, A. L. and T. Muir (2016). Aggregate external financing and savings waves. *Journal of Monetary Economics 84*, 116–133.

Fama, E. F. and K. R. French (2005). Financing decisions: Who issues stock? *Journal of Financial Economics 76*(3), 549–582.

Fernald, J. (2014). A quarterly, utilization-adjusted series on total factor productivity. *Federal Reserve Bank of San Francisco Working Paper*.

Fischer, E. O., R. Heinkel, and J. Zechner (1989). Dynamic Capital Structure Choice: Theory and Tests. *Journal of Finance 44*(1), 19–40.

Fisher, J. D. M. (2006). The Dynamic Effects of Neutral and Investment-Specific Technology Shocks. *Journal of Political Economy 114*(3), 413–451.

Gilchrist, S., J. W. Sim, and E. Zakrajsek (2014). Uncertainty, Financial Frictions, and Investment Dynamics. *Unpublished Manuscript*.

Gomes, J., U. Jermann, and L. Schmid (2016). Sticky Leverage. *American Economic Review 106*(12), 3800–3828.

Gomes, J. F. (2001). Financing investment. *American Economic Review 91*(5), 1263–1285.

Gomes, J. F. and L. Schmid (2010). Levered Returns. *Journal of Finance 65*(2), 467–494.

Gourio, F. (2013). Credit risk and disaster risk. *American Economic Journal: Macroeconomics 5*(3), 1–34.

Graham, J. R. (2000). How Big Are the Tax Benefits of Debt? *Journal of Finance 55*(5), 1901–1941.

Hackbarth, D., J. Miao, and E. Morellec (2006). Capital structure, credit risk, and macroeconomic conditions. *Journal of Financial Economics 82*, 519–550.

Hatchondo, J. C. and L. Martinez (2009). Long-duration bonds and sovereign defaults. *Journal of International Economics 79*(1), 117–125.

Hatchondo, J. C., L. Martinez, and C. Sosa-Padilla (2016). Debt Dilution and Sovereign Default Risk. *Journal of Political Economy 124*(5), 1383–1422.

Hayashi, F. (1982). Tobin's Marginal q and Average q : A Neoclassical Interpretation. *Econometrica 50*(1), 213–224.

He, Z. and K. Milbradt (2014). Endogenous Liquidity and Defaultable Bonds. *Econometrica 82*(4), 1443–1508.

Heider, F. and A. Ljungqvist (2015). As Certain as Debt and Taxes: Estimating the Tax Sensitivity of Leverage from Exogenous State Tax Changes. *Journal of Financial Economics 118*(3), 684–712.

Hennessy, C. A. and T. M. Whited (2007). How Costly Is External Financing? Evidence from a Structural Estimation. *Journal of Finance 62*(4), 1705–1745.

Jermann, U. and V. Quadrini (2012). Macroeconomic Effects of Financial Shocks. *American Economic Review 102*(1), 238–271.

Jungherr, J. and I. Schott (2016). Optimal Debt Maturity and Firm Investment. *Unpublished Manuscript*.

Jungherr, J. and I. Schott (2017). Debt Dilution and Debt Overhang. *Unpublished Manuscript*, 1–44.

Justiniano, A., G. E. Primiceri, and A. Tambalotti (2011). Investment shocks and the relative price of investment. *Review of Economic Dynamics 14*(1), 102–121.

Katagiri, M. (2014). A macroeconomic approach to corporate capital structure. *Journal of Monetary Economics 66*, 79–94.

Khan, A., T. Senga, and J. K. Thomas (2014). Default Risk and Aggregate Fluctuations in an Economy with Production Heterogeneity. *Unpublished Manuscript*.

Khan, A. and J. K. Thomas (2013). Credit Shocks and Aggregate Fluctuations in an Economy with Production Heterogeneity. *Journal of Political Economy 121*(6), 1055–1107.

Kraus, A. and R. H. Litzenberger (1973). A State-Preference Model of Optimal Financial Leverage. *Journal of Finance 28*(4), 911–922.

Kuehn, L. A. and L. Schmid (2014). Investment-Based Corporate Bond Pricing. *Journal of Finance 69*(6), 2741–2776.

Leary, M. T. and M. R. Roberts (2005). Do firms rebalance their capital structures? *Journal of Finance 60*(6), 2575–2619.

Leland, H. E. (1998). Agency Costs, Risk Management, and Capital Structure. *Journal of Finance 53*(4), 1213–1243.

Modigliani, F. and M. H. Miller (1963). Corporate Income Taxes and the Cost of Capital: A Correction. *American Economic Review 53*(3), 433–443.

Moyen, N. (2007). How big is the debt overhang problem? *Journal of Economic Dynamics and Control 31*(2), 433–472.

Myers, S. C. (1977). Determinants of corporate borrowing. *Journal of Financial Economics 5*(2), 147–175.

Myers, S. C. and N. S. Majluf (1984). Corporate financing and investment decisions when firms have information that investors do not have. *Journal of Financial Economics 13*(2), 187–221.

Ou, S., D. Chiu, and A. Metz (2011). Corporate Default and Recovery Rates, 1920-2010. Technical report, Moody's Investor Service.

Titman, S. and S. Tsyplakov (2007). A dynamic model of optimal capital structure. *Review of Finance 11*(3), 401–451.

Warusawitharana, M. and T. M. Whited (2016). Equity Market Misvaluation, Financing, and Investment. *The Review of Financial Studies 29*(3), 603–654.

Whited, T. M. (1992). Debt, Liquidity Constraints and Corporate Investment: Evidence from Panel Data. *Journal of Finance 4*(4), 1425–1460.

Xiao, J. (2017). Corporate Debt Structure, Precautionary Savings, and Investment Dynamics. *Unpublished Manuscript*.

A Other Measures of Corporate Cash Flows

In this section, I describe how using other measures of corporate cash flows affects the main empirical facts.

	(1) GDP(t-1)	(2) GDP(t)	(3) GDP(t+1)
0% to 25%	0.176	0.202*	0.195*
25% to 50%	−0.0398	0.0518	0.101
50% to 75%	0.569***	0.651***	0.707***
75% to 90%	0.127	0.233*	0.311***
90% to 95%	−0.256**	−0.205*	−0.157
95% to 99%	−0.488***	−0.444***	−0.373***
99% to 100%	−0.302**	−0.234*	−0.170
0% to 90%	0.158	0.256**	0.328***
All Firms	−0.114	0.0001	0.101
Observations	116	116	116

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Table 10: Correlations of the Detrended Long-Term Debt Share with Detrended Logged real GDP, by Firm Size.

In Table 10, I report the correlation between the long-term debt share and real GDP. The upside of using real GDP is that it is conventionally used in the business cycle literature to compute the cyclicality of other variables. The downside is that it is only imperfectly related to corporate cash flows. This is problematic from a theoretical and empirical perspective: The mapping to the model counterpart, namely real aggregate corporate cash flow, is imperfect. For example, real GDP might include shocks to other sectors of the economy which are uncorrelated with cash flow in the corporate sector. For example, a sharp decrease in added value from the financial sector or the public sector would not be contemporaneously reflected in corporate cash flow.

Indeed, using real GDP instead of real aggregate sales changes the correlations substantially: The correlation between the long-term debt share of all firms and real GDP is zero, and the correlation between the long-term debt share and the bottom 90 percent of firms by size is only one third as high compared to the correlation when using sales.

In Table 11, I report the correlations of the long-term debt share with aggregate log real corporate profits. Using profits instead of sales is a better measure of corporate cash flows if there is significant cyclicality in the operating costs of firms, such that fluctuations in revenues do not present a full picture of the funds available to firms. However, the correlations are broadly similar to those in Table 2. The biggest difference is that with profits as measure of corporate cash flows, the maturity structure of the largest 5 percent is practically a-cyclical.

	(1) GDP(t-1)	(2) GDP(t)	(3) GDP(t+1)
0% to 25%	0.248**	0.274**	0.137
25% to 50%	0.311***	0.415***	0.344***
50% to 75%	0.589***	0.716***	0.705***
75% to 90%	0.405***	0.548***	0.559***
90% to 95%	0.231*	0.300**	0.297**
95% to 99%	−0.162	−0.0648	0.0110
99% to 100%	−0.176	−0.0592	0.0191
0% to 90%	0.530***	0.666***	0.659***
All Firms	0.322***	0.494***	0.539***
Observations	116	116	116

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Table 11: Correlations of the Detrended Long-Term Debt Share with Detrended Log Real Corporate Profits, by Firm Size.

B Derivation of the Derivatives in the Main Text

As in section 4, I assume that the bond prices are differentiable once in K', B', M' and A and value function is differentiable once in K, B, M and A. Further, I assume that the short-term and the long-term bond price are non-decreasing in A.

B.1 Value Function

The first-order condition for the long-term debt share is given by

$$\begin{aligned}\frac{\partial V}{\partial M'} = \Bigg[&(Q_L - Q_S)B' + \\ &\frac{\partial Q_S}{\partial M'}(1-M')B' + \frac{\partial Q_L}{\partial M'}\left(M'B' - (1-\mu)MB\right)\Bigg](1+\lambda_D) + \\ &\mathbb{E}\left[\Lambda(C,C')\frac{\partial V'}{\partial M'}|\mathcal{Y}\right] = \lambda_{M,1} - \lambda_{M,0}.\end{aligned}$$

The envelope condition yields

$$\begin{aligned}\frac{\partial V}{\partial M} &= (\mu + (1-\mu)Q_L - 1 + (1-\mu)\xi)B(1+\lambda_D) \\ &= (1-\mu)(Q_L - 1 + \xi)B(1+\lambda_D)\end{aligned}$$

if the firm is in a no-default state and

$$\frac{\partial V}{\partial M} = 0$$

if the firm is in a default state.

The derivative of the value function with respect to A is given by

$$\frac{\partial V}{\partial A_i} = (1-\tau)K^\alpha + \frac{\partial Q_S}{\partial A}(1-M')B' + \frac{\partial Q_L}{\partial A}(M'B' - (1-\mu)MB) + \mathbb{E}\left[\Lambda(C,C')\int_{-\infty}^{\infty}\frac{\partial V'}{\partial A_i'}\frac{\partial A_i'}{\partial A_i}f(A_i'|A_i)dA_i'|C\right].$$

Using $\frac{\partial A_i'}{\partial A_i} = \rho\frac{A_i'}{A_i}$ yields

$$\frac{\partial V}{\partial A_i} = (1-\tau)K^\alpha + \frac{\partial Q_S}{\partial A}(1-M')B' + \frac{\partial Q_L}{\partial A}(M'B' - (1-\mu)MB) + \mathbb{E}\left[\Lambda(C,C')\int_{-\infty}^{\infty}\frac{\partial V'}{\partial A_i'}\rho\frac{A_i'}{A_i}f(A_i'|A_i)dA_i'|C\right]$$

in no-default states and

$$\frac{\partial V}{\partial A_i} = 0$$

in default states.

Expanding the recursion, one can see that this value function derivative depends on the entire future path of the derivatives of the production function and the bond prices with respect to how the sequence of productivity changes with a current change in the productivity. The production function derivative is positive. As shown below, the short-term bond price derivative is non-negative. It is not possible to analytically determine the sign of these bond prices. If these derivatives are non-negative, which I assume and which is the case in simulations, the derivative of the value function with respect to idiosyncratic productivity is positive, i.e. $\frac{\partial V}{\partial A_i} > 0$, if the firm is in a no-default state.

B.2 Short-Term Bond Price

The sign of the short-term bond price derivative depends on how the default cut-off varies with the share of long-term debt. The default cut-off function $a^*(K_i, B_i, M_i, C)$, which exists if the value function derivative with respect to idiosyncratic is positive, i.e. $\frac{\partial V}{\partial A_i} > 0$, is implicitly defined by the equation

$$V(K_i, B_i, M_i, a^*(K_i, B_i, M_i, C), C) = 0.$$

Using the implicit function theorem, the derivative for a^* is given by:

$$\frac{\partial a^*}{\partial M_i} = -\frac{\frac{\partial V}{\partial M_i}}{\frac{\partial V}{\partial A_i}},$$

since $\frac{\partial V}{\partial M_i} > 0$ and $\frac{\partial V}{\partial A_i} > 0$, $\frac{\partial a^*}{\partial M_i} < 0$, i.e. since the default threshold in the next period is decreasing

in M.

With this information and using Leibniz' rule, the short-term bond price derivative with respect to the long-term debt share can be computed as

$$\frac{\partial Q_S}{\partial M_i'} = \mathbb{E}\left[\Lambda(C,C')\left(R(K_i',B_i',a^*,C') - (1+c)\right) f(a^*|A)\frac{\partial a^*}{\partial M_i'}|C\right].$$

Since $1+c \geq R(K,B,a^*,C)$, i.e. the creditor cannot recover more than his claim per unit of the bond in default, and $\frac{\partial a^*}{\partial M'} \leq 0$, i.e. the default cut-off for the next period is lower if the long-term share in the next period is higher, this derivative is positive.

B.3 Long-Term Bond Price

For the long-term bond price, the derivative with respect to the long-term debt share is given by

$$\begin{aligned}\frac{\partial Q_L}{\partial M'} = \mathbb{E}\Bigg[\Lambda(C,C')\Bigg[&\left[R(K_i',B_i',a^*,C') - (\mu + c + (1-\mu)Q_L')\right] f(a^*|A)\frac{\partial a^*}{\partial M_i'} \\ &+(1-\mu)\int_{a^*}^{\infty}\left(\frac{\partial Q_L'}{\partial K''}\frac{\partial K''}{\partial M'} + \frac{\partial Q_L'}{\partial B''}\frac{\partial B''}{\partial M'} + \frac{\partial Q_L'}{\partial M''}\frac{\partial M''}{\partial M'}\right) f(A'|A)dA\Bigg]|C\Bigg].\end{aligned}$$

It is not possible to determine the sign of this derivative, for two reasons: First, it is not necessarily the case that $(\mu + c + (1-\mu)Q_L') > R(K_i',B_i',a^*,C')$, since the continuation bond price $Q_L' = q_L(\mathcal{S}_i',a^*,C')$, which represents a part of the claim of the creditor on the firm, might be low.

Second, the future bond price also changes with future firm decisions, which depend on the policy for the long-term debt share today. Since the policy functions are unknown, it is not possible to determine these derivatives.

In general, the long-term bond price can therefore decrease in the long-term debt share. This can be the case in two situations: First, if defaulting would actually lead to a higher pay-off for creditors. Second, if a higher long-term share increases default risk after the next period through adversely affecting the firm policies in the next period.

C Numerical Algorithm

My solution algorithm is a value function iteration algorithm based on Hatchondo, Martinez, and Sosa-Padilla (2016). It works as follows:

1. Start with a guess for the expected value function and bond prices. The equilibrium for the infinite horizon model might not be unique. I therefore follow Hatchondo and Martinez (2009) and approximate the infinite horizon value functions by finite horizon value functions for the first period. Therefore, the initial guesses are the terminal value function and the terminal bond prices.

2. Compute the policy functions and value function. I approximate the value function between

grid points using linear interpolation. For capital and debt, I use grids with 25 points, respectively. For the share of long-term debt, I use 10 grid points. For the idiosyncratic productivity shock, I use 15 and for the aggregate productivity shock 3 grid points. I use a choice grid with 100 points for capital, 100 points for debt and 10 points for the long-term debt share.

3. Update the expected value function and bond prices. For the calculation of expectations, I approximate the expected value function $\mathbb{E}\left[V(\mathcal{S}_i', A_i, C)\right]$, $q_S(\mathcal{S}_i', A_i, C)$, and $q_L(\mathcal{S}_i', A_i, C)$ using linear interpolation on $(\mathcal{S}_i', A_i)$. I treat A_i as continuous, using Gauss-Legendre quadrature to calculate the expectation over A_i'. I compute these expectations on a grid with 50 points for K_i', 50 for B_i' and 10 for M_i'. I use 25 quadrature points to compute the expectations approximating the integrals piecewise in the default and no-default regions, using the exact default cut-offs for A_i. I use three Gauss-Hermite quadrature nodes for ϵ^Z and ε^C. By approximating the expectations, I only have to calculate expectations once outside the maximization step instead of many times within the maximization step.

4. Repeat until the updating errors in the expected value function and bond prices are smaller than 5e-2 for the value function and 1e-2 for the short-term bond price.

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