WORKING PAPER

DANMARKS NATIONALBANK

27 FEBRUARY 2019 — NO. 136

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Abstract

Within a cointegrated VAR framework I show that the traditional money-demand relation, determined by a transaction effect and the opportunity cost of holding money, can no longer explain the recent development of monetary aggregates in Denmark. Instead, I argue that the introduction of housing wealth and the role of precautionary demand for liquidity improves both the explanatory power of money demand and stability of the long-run estimates. Identification of the long-run structure still suggests homogeneity between money and GDP together with a positive effect from the opportunity cost. Housing wealth enters the money-demand equation positively which is consistent with previous findings for the euro area and the US. To verify the implications of negative interest rates, I perform several forward-recursive tests and rolling-window estimations. In general, these tests confirm that the estimated money-demand relation behaves stably over time, reflecting that the negative interest rate environment has not contributed to any permanent effect on the determination of money demand. Instead, the analysis suggests that the introduction of negative policy rates has presumably provoked a temporary shock to the coefficient on the opportunity cost.

Resume

I en kointegreret VAR model viser jeg, at den traditionelle pengeefterspørgselsrelation, henholdsvis bestemt af en transaktionseffekt og alternativomkostningen ved at holde penge, ikke længere kan forklare udviklingen i pengemængden i Danmark. I stedet viser jeg, at introduktionen af boligformue og et forsigtighedsmotiv forbedrer forklaringsgraden og stabiliteten af pengeefterspørgselsrelationen. Identifikation af den langsigtede struktur indebærer stadig homogenitet mellem penge og BNP sammen med en positiv effekt fra alternativomkostningen. Boligformuen påvirker pengefterspørgslen positivt, hvilket er konsistent med lignende studier fra euroområdet og USA. For at verificere implikationerne af negative pengepolitiske renter, udfører jeg adskillige rekursive test. Disse test bekræfter generelt, at den estimerede pengeefterspørgselsfunktion opfører sig stabilt over tid, hvilket afspejler at det negative rentemiljø ikke har bidraget med nogen permanent effekt på bestemmelsen af pengeefterspørgslen. I stedet viser analysen dog, at indførelsen af negative renter formentlig har fremkaldt et midlertidigt stød til estimatet af alternativomkostningen.

Key words

Money demand; The cointegrated VAR model; Housing wealth; Long-run stability; Negative interest rates; Precautionary motive

JEL classification

C32; D15; D81; E40; E41; E50

Acknowledgements

I would like to thank Morten Spange, Søren Lejsgaard Autrup, Kim Abildgren and Jesper Pedersen for useful comments and suggestions.

The author alone is responsible for any remaining errors.

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Within a cointegrated VAR framework I show that the traditional money-demand relation, determined by a transaction effect and the opportunity cost of holding money, can no longer explain the recent development of monetary aggregates in Denmark. Instead, I argue that the introduction of housing wealth and the role of precautionary demand for liquidity improves both the explanatory power of money demand and stability of the long-run estimates. Identification of the long-run structure still suggests homogeneity between money and GDP together with a positive effect from the opportunity cost. Housing wealth enters the money-demand equation positively which is consistent with previous findings for the euro area and the US. To verify the implications of negative interest rates, I perform several forward-recursive tests and rolling-window estimations. In general, these tests confirm that the estimated money-demand relation behaves stably over time, reflecting that the negative interest rate environment has not contributed to any permanent effect on the determination of money demand. Instead, the analysis suggests that the introduction of negative policy rates has presumably provoked a temporary shock to the coefficient on the opportunity cost.

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The views expressed in this paper are those of the author, and do not necessarily correspond to those of Danmarks Nationalbank. I would like to thank Morten Spange, Søren Lejsgaard Autrup, Kim Abildgren and Jesper Pedersen for useful comments and suggestions. The author alone is responsible for any remaining errors.

1 Introduction

In order to foster economic growth in wake of the great recession, key monetary policy rates in several advanced economies have been ultra low or even negative for a number of years. Before the introduction of this unprecedented interest rate environment, negative policy rates were considered as a peculiarity that would be difficult and challenging to implement in practice; see e.g. Blomquist et al. (2011) and McAndrews (2015) who discuss and evalute alternative monetary policies at the zero lower bound. Others have expressed concerns that a prolonged period with negative interest rates could lead to asset price bubbles, affect the willingness to lend and cause financial market distortions, see e.g. Arteta et al. (2016), Carney (2016) and Aizenman et al. (2017).

Another implication of negative interest rates is that banks have been reluctant to pass on negative policy rates to firms' and especially to households' deposit accounts, creating incentives to reallocate the portfolio composition towards more liquid assets. For instance, Danmarks Nationalbank's imposition of negative monetary policy rates during 2012 has contributed to boosting the demand for deposits, as the return on deposit comparing to what is obtainable from other placements of funds has become more propitious, see Hensch and Pedersen (2018). Consequently, it is thus conceivable that such realloaction towards money balances has been a contributory factor in structurally shaping the way money demand is determined.

This paper examines whether the very expansionary monetary policy conducted in Denmark from 2012 and onwards has fundamentally affected the demand for money balances. Based on cointegration analysis, I show that the introduction of negative policy rates has not led to instability of the estimated coefficients to the long-run determinants of money demand. I interpret the result as if the impaired interest rate pass-through to deposit rates has not altered the underlying determination of money demand. The paper also establishes that money demand in Denmark can no longer be fully explained by the general macroeconomic determinants previously found in the literature, i.e. the needs for transactions and the opportunity cost of holding money.[3] Instead, I argue that recent developments in monetary aggregates indicate that an extension of the original empirical model is needed in order to ensure an empirically stable model. In particular, I show that the introduction of wealth/prices of financial aggregates and the role of precautionary demand for liquidity improves the explanatory power of money and sustain stability of the long-run parameters.

Theoretically, the link between wealth/prices of financial aggregates and the demand for money balances is motivated by Friedman (1988), who classified that the aggregate effect is composed of a wealth, a substitution and a transaction effect. Inspired by previous money-demand studies on data for the euro area and the US, I use housing wealth as an indicator of wealth/prices of financial aggregates, see for instance Greiber and Setzer (2007), de Santis et al. (2008) and Beyer (2009). Furthermore, the choice of housing wealth introduces an additional housing money channel - the collateral effect - saying that households' ability to borrow is influenced on their stock of collateral, see Iacoviello (2004) and (2005).

[3] See e.g. Johansen and Juselius (1990), Juselius (1998) and (2006), Coenen and Vega (2001), and Brand and Cassola (2004).

The idea that precautionary motives matter for money demand was introduced by Keynes (1936). He argued that precautionary motives exist as people need to meet unexpected outlays, basically related to future transaction needs. Within the empirical literature of money demand, the channel of precautionary savings has generally been examined together with the standard transaction motive as one aggregate effect. The reason is that a decompostion of which transactions reflecting current day-to-day needs or alternatively unexpected future outlays is difficult due to uncertainty, in general, can affect the economic environment in all conceivable configurations. Nevertheless, I follow the quite limited litterature on the cohesion between economic risk and money aggregates by applying labor market risk as a proxy for precautionary savings. In line with this litterature, larger labor market risk can be expected to raise the demand for money.

The empirical analysis shows that the introduction of housing wealth and the change in the unemployment rate improves the statistical model in terms of stability and stationarity of the long-run relations. Identification of the long-run structure suggests that the estimated impact from housing wealth is positive and very significant, reflecting that the collateral and the housing transaction effect exceed the ambigious portfolio effect. Furthermore, error-correction from housing wealth turns out to be unsubstantial, capturing that the variable, by itself, constitutes an underlying stochastic trend. Unexpectedly, the estimated coefficient on the change in the unemployment rate enters significantly the long-run money-demand function with negative sign, implying that people tend to dishoard money when labor market weakens. A conceivable explanation can be that the construction of the precautionary variable is vitiated with sizable measurement errors. In line with previous studies, the estimated effect of the opporturnity cost remains very significant and economically sizable. Moreover, the estimate is very stable over time, also in the period from 2012 and onwards. However, recursive analysis demonstrates that the estimate of the opportunity cost of holding money was hit by a temporary shock few quarters after the negative monetary policy rates were imposed. This may reflect that the introduction of negative policy rates has solely contributed to temporary effects on long-run money-demand determination.

Several robustness checks show that the results are robust to changing the scale variable, the measure of the opportunity cost and introducing a forward-looking measure of economic risk. The result reflects that the statistical model is quite robust to measurement changes, possibly indicating that the extent of the attenuation biases is tolerable.

The paper relates to several litteratures. First, the empirical analysis are related to previous studies on money demand in Denmark: Christensen & Jensen (1987), Juselius & Johansen (1990) and Hansen (1996) argued that long-run money demand was determined by the volume of transactions, the opportunity cost of money and foreign return variables. Andersen (2004) argued that the Danish money-demand relation could be explained slightly simpler, in that foreign return variables had no explanatory impact on money balances. In line with Andersen (2004), Juselius (1998) and (2006) showed in an expanded framework that money demand can also be explained by this simple relationship. More recently, Bang-Andersen et al. (2014) and Hensch and Pedersen (2018) demonstrate that housing wealth also has to be taken into consideration in money-demand determination.

On the international strand of empirical studies on money demand, the paper relates to studies that extend the conventional money-demand relation by incorporating housing or financial wealth variables, see Brand et al. (2002), Boone and van den Noord (2008), Greiber and Setzer (2007) and Beyer (2009). In general, they argue within the framework of the cointegrated VAR model that monetary expansion in the euro area at the beginning of the 2000s was driven by a sharp increase in financial prices, and especially in house prices. Consequently, they conclude that demand indicators and the opportunity cost are insufficient to replicate all the variation in monetary developments in case wealth variables are not included. Greiber and Setzer (2007) deliver the same evidence on US data, but they do perform identification on the long-run structure and recursive tests of the individual long-run estimates. Beyer (2009) shows in contrast to the remaining studies that other wealth aggregates, like financial wealth, do not have the same explanatory impact on money developments.

Finally, the paper also relates to the litterature that investigates the interactions between economic uncertainty, and more specifically labor market risk, and the demand for money holdings. Generally, the studies argue that unemployment risk plays a crucial role in households' decision on portfolio allocation between money and more risky assets. This channel has particularly been amplified by the great recession which has raised labor income uncertainty and thereby strengthened the precautionary demand for liquidity, see for instance Mody et al. (2012). Bondt (2009) shows that money is hoarded when unemployment risk rises, cohering with the general view that larger uncertainty on future income is channelized into a larger demand for riskless assets. However, Atta-Mensah (2004) and Seitz and von Landesberger (2014) find that a deterioration of labor market risk curbs the demand for money, possibly reflecting that a reallocation towards real assets occurs when uncertainty gets larger.

The rest of the paper is organized as follows. Section 2 presents the empirical framework and sheds light on how the several channels affecting money demand can be motivated theoretically. Then section 3 presents the statistical model, while section 4 outlines the data measurements. Subsequently, section 5 presents the empirical analysis, section 6 performs robustness checks and section 7 evaluates the main findings.

2 Empirical Framework

From a theoretical perpective, it is well-known that the standard money-demand equation can be derived from a classic money-in-the-utility function (MIU) model, see Petursson (2000) or Walsh (2010, chapter 2). To motivate precuationary demand for liquidity and the effects of housing aggregates on money demand, I introduce both the stochastic feature of households' preference suggested by Kim (2000) and the framework of Iacoviello (2004) into the general MIU model.[4] The combination of these single frameworks allows me to obtain a unified theory that incorporates the standard transaction motive and the opportunity cost, but also

[4]One of the advantages of this strategy is that I am capable of introducing a precautionary motive for money without necessarily being enclosed by numerical solution methods.

the effects of housing aggregates and uncertainty.

Inspired by Iacoviello (2004), I assume that the household sector can be divided into a fraction $1-\zeta$ of lenders and a fraction ζ of borrowers; lenders are able to finance their homes with own savings, while borrowers finance their houses with mortgage loans. Assuming isoelastic preferences, consistent with the instantanous utility function being parametrized within the standard constant relative risk aversion (CRRA) form, a representative consumer' lifetime welfare is:

$$\mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^{it}\left(\tilde{z}_t\frac{\left(\frac{M_t^i}{P_t}\right)^{1-\varphi}}{1-\varphi}+(1-\tilde{z}_t)\frac{C_t^{i\,1-\eta}}{1-\eta}+\delta^H\frac{H_t^{i\,1-\kappa}}{1-\kappa}\right)\right],\quad 1>\beta^i>0,\quad \varphi,\eta,\kappa,\delta^H>0 \tag{1}$$

where β^i is the discount factor, C_t^i is consumption, M_t^i is money balances, P_t is the price level of consumption and H_t^i is the volume of housing services. The notation $i\in\{l,b\}$ refers to lenders and borrowers, respectively, and $\mathbb{E}_t$ denotes expectations conditional on information at time t. In line with Iacoviello (2004), borrowers and lenders have identical preferences, but differ in the way they discount future utility; borrowers are assumed myopic, i.e. $\beta^b=0$.[5]

The stochastic variable z_t captures households' desire for consumption relative to real money balances. Following Kim (2000), I assume that the time-dependent preference shocks evolve according to the autoregressive process:

$$\tilde{z}_t=\log(z_t)=\rho\log(z_{t-1})+(1-\rho)\log(z)+\omega_t,\quad \rho\in(-1,1) \tag{2}$$

where ω_t is a serially uncorrelated shock which is normally distributed with zero mean and variance one. To clarify why the introduction of z_t can be interpreted as a precautionary channel, suppose for instance that the realized value of z_t is large. In such situation, households' desire to store a large part of their assets on highly liquid assets compared to using resources on current expenditures is large, reflecting their willingness to hold money for unexpected events. As a result, movements in z_t can be interpreted as shocks to labor market risk. However, it is important to emphasize that innovations in z_t can also reflect alternative money-demand shocks which are not necessarily related to uncertainty on future earnings.

Subsequently, both borrowers and lenders face the following flow of funds constraint:

$$C_t^i+B_t^i+\frac{M_t^i}{P_t}+\frac{Q_t}{P_t}(H_t^i-H_{t-1}^i)=Y_t^i+(1+r_{t-1})B_{t-1}^i+(1+d_{t-1})\frac{M_{t-1}^i}{P_t}, \tag{3}$$

where B_t^i is the stock of real bonds, Q_t is the house price level, Y_t^i is real income, r_t is the real return on bonds and d_t is the return on money. As borrowers can exclusively finance housing activities through mortgage loans, they also face the collateral constraint:

$$(1+i_t)B_t^b\leq\varsigma\frac{\mathbb{E}_tQ_{t+1}}{\mathbb{E}_tP_{t+1}}H_t^b, \tag{4}$$

[5] This entails that β^{bt} equals one for $t=0$, and zero otherwise.

saying that the amount they are capable of borrowing at time t cannot exceed a fraction $\varsigma \leq 1$ of the expected value of real estate holdings in the next period, appropriately discounted by the cost of borrowing. In case the debt constraint is not fulfilled, households have an incentive to accumulate infinite negative housing wealth. Iacoviello (2004) shows formally that the collateral constraint will always be binding. As a result, this will also be assumed here.

Maximization of households' lifetime utility faced by lenders and borrowers, respectively, in which they take (2), (3) and (4) into account, leads to the following decompressed expression for money demand:[6]

$$m_t - p_t = \underbrace{\phi_1(y_t - p_t)}_{\text{Transaction effect}} + \underbrace{\phi_2 \tilde{z}_t}_{\text{Precautionary effect}} + \underbrace{\phi_3(R_t^d - R_t^b)}_{\text{Speculative effect}} + \underbrace{\phi_4(wh_t - p_t)}_{\substack{\text{Housing wealth} \\ \text{effect}}}, \quad (5)$$

where small letters denote the variables in log values, R_t^d is the return on deposit, R_t^b is the nominal interest rate of bonds and wh_t is nominal housing wealth in logs. In general, the equation can be interpreted as a steady-state value of money demand in which all previous shocks (with or without persistence) have been neutralized.

Equation (5) reveals that the introduction of the money-demand shocks ensures that aggregate the transaction effect can be divided into demand for regular transaction today and the demand of handling unexpected situations that require cash outlay in future. This is an outcome of uncertainty that affects households' marginal utility of consumption relative to their marginal utility of money balances. The speculative effect represents the standard opportunity cost of holding money; a larger yield on bank deposits compared to the yield on bonds makes money holdings more attractive, thereby fostering the demand for money.

The housing wealth effect is shaped by four underlying factors (for a formal decomposition of these factors, see appendix A.1): (i) *The wealth effect*, saying that an increase in housing wealth typically leads to a reallocation of the desired portfolio composition, entailing that the demand for real balances may adjust to the altered portfolio composition; (ii) *the substitution effect*, capturing that an expected future rise in house prices makes house investment more attractive today compared to holding money balances, which automatically induces a reallocaton of the portfolio composition into housing activities in favor of money holdings.[7]; (iii) *the transaction effect*, saying that larger trade activity in the housing market typically leads to larger demand for money due to a simple need for transactions; (iv) *the collateral effect*, reflecting that a rise in housing wealth will typically increase the value of the collateral pledged by households and thereby foster the access to borrowing and money growth.

To argue empirically that the specification in (5) constitutes a long-run relationship, I will in the following introduce the statistical model to verify whether a deviation of money from its long-run equilibrium follows a stationary process. The money-demand relation holds several empirical predictions, which will be tested:

[6]See appendix A for a formal proof.

[7]Due to the construction of the underlying model, expected capital gains on housing also depend on the user cost of housing. Consequently, the substitution effect is also influenced by the cost of borrowing; a larger interest rate increases the user cost of housing which via the substitution effect boosts money demand.

$\phi_1 = 1$, $\phi_2 > 0$, $\phi_3 > 0$ and $\phi_4 \gtreqless 0$.[8]

3 The Cointegrated VAR Model

This section will briefly present the statistical model and the general notation that will be used throughout.

Consider the p-dimensional cointegrated VAR model:

$$H(r) : \Delta x_t = \alpha\beta' x_{t-1} + \sum_{i=1}^{k-1} \Gamma_i \Delta x_{t-i} + \alpha\beta_1' D_{s1994_t} + \mu_0 + \phi d_t + \varepsilon_t, \tag{6}$$

$$t = 1, 2, ..., T,$$

where x_t is of dimension $p \times 1$, α and β are of dimension $p \times r$, $r \leq p$, the short-run parameters $\Gamma_1, ..., \Gamma_{k-1}$ are $p \times p$ matrices, and ε_t is a sequence of independent Gaussian innovations with zero mean and the covariance matrix $\Omega > 0$. If the levels of x_t are cointegrated with r long-run relations then $\Pi = \alpha\beta'$ must have reduced rank, see Johansen (1996).

To match the data, that will be considered in the next section, I include a restricted level-shift dummy in the second quarter of 1994, D_{s1994_t}, in the deterministic specification. Finally, I also include an unrestricted constant, μ_0, and a set of dummy variables, d_t, which enter the model with unrestricted coefficients. The unrestricted dummy variables can be interpreted as large shocks to the multivariate system.

4 Data Measurements

I consider the data vector $x_t = (m_t : y_t : R_t^d : R_t^b : wh_t : \Delta U_t)'$, where m_t is the log of nominal money holdings, y_t is the log of nominal income, R_t^d is the deposit rate, R_t^b is the yield of a 10-year government bond, wh_t is the log of housing wealth and ΔU_t reflects the measure of precautionary savings, captured by the annual change in the unemployment rate. The effective sample is based on quarterly data covering the period 1988:1-2018:1, and all series, except interest rates, are seasonally adjusted.

The considered sample is selected to comparatively match the period in which the central rate of the Danish krone has been unchanged to the D-mark and subsequently to the euro. The advantage of such sampling is that the data do not cover several exchange rate regimes that might have changed the long-run coefficients over time. The choice of sample is also based on data limitation on alternative measures of precautionary demand for liquidity that will be relevant for potential robustness checks.

It is well-known that monetary economists in general prefer the monetary aggegate, $M3$, as the empirical measure of money, see Sriram (2001) for a discussion. Nevertheless, $M3$ is quite volatile for the Danish case, primarily reflecting technical issues, like maturities of mortgage bonds that cannot be considered as

[8] The potentially ambigious effect from housing wealth captures that the negative substitution effect could in theory exceed the remaining positive effects of housing wealth. However, from a empirical point of view, $\phi_4 > 0$ seems more reliable.

underlying determinants of money demand.[9] Based on this consideration, I will follow previous money-demand studies on Danish data by using the monetary aggregate, $M2$, as the empirical measure of the nominal money stock.[10]

Data on banks' average deposit rate, constructed as a weighted average of households' and non-financial corporations' deposit rates, are taken from Abildgren (2016) and Danmarks Nationalbank. Inspired by exisiting litterature on Danish money demand, I measure the alternative return to placing funds by the average yield of a 10-year government bond. This is also consistent with the idea that government bonds in general constitute benchmark bonds whose prices are transmitted into other bonds.

Current transaction needs and housing wealth are measured by GDP and households' stock of housing wealth, respectively, both stated in nominal terms. Both data series are taken from MONA data bank.[11] To capture precautionary demand for liquidity, I use the annual change in the unemployment rate, as labor market risk can in a broad sense be characterized as an inevitable risk which potentially leads to additional saving or a reallocation towards less risky assets in case of occurences of an unavoidable risk shock.[12]

In contrast to several money demand studies, data on money holdings, GDP and housing wealth are stated in nominal terms and inflation are excluded from the analysis. The reason is that simple Dickey-Fuller tests reveal that money holdings and GDP in nominal and real terms, respectively, can be characterized as $I(1)$ processes, reflecting that prices do not contain more than one unit root. Intuitively, this is an outgrowth of the very stable price developments observed over the past three decades, which basically coheres with the fixed exchange rate regime used as an intermediate target to transmit ECB's goal of price stability to the Danish economy. Another argument in favor of considering a nominal specification is that VAR modelling easily can become prohibitive in case of having too many autoregressive parameters. However, in section 6 I show that a nominal-to-real transformation does not affect the empirical results.

The variables are illustrated in figure 1. Graph (A) depicts GDP, money holdings and housing wealth, transformed into log values. Over the period, y_t tends to grow less rapidly than m_t, reflecting that the scale effect is not by itself capable of explaining the variation in money demand. This is consistent with the pattern in graph (B); the ratio seems to exhibit a well-fitted stationary behavior during the period 1988-2001, whereupon the stationary process is replaced by an increasing trending behavior, clearly indicating that y_t is insuffiicient to explain the evolution in m_t after 2001. Alternatively focusing on wh_t and the ratio between m_t and wh_t, depicted in graph (A) and (C), respectively, it is clear that the two variables have more or less been composed of the same common stochastic trends from the beginning of the 2000s and onwards. From an empirical point of view, this evidence shows that wh_t potentially constitutes a decisive component in terms

[9] The composition of $M3$ is constructed such that mortgage bonds with a longer original maturity are outside $M3$, while mortgage bonds with a shorter original maturity are inside $M3$. As mortgage institutions in general are issuing new bonds to replace bonds that mature, it is likely that mortgage institutions issue bonds with, let say, a longer original maturity to replace bonds having a shorter original maturity. Consequently, $M3$ will drop considerably although the structural money stock is completely unaffected.

[10] Data on $M2$ are taken from Abildgren (2016) and Danmarks Nationalbank.

[11] In Appendix A.2 I will for robustness considerations verify the empirical implications of applying alternative measures of the transaction variable.

[12] Data on the unemployment rate is from Statistics Denmark.

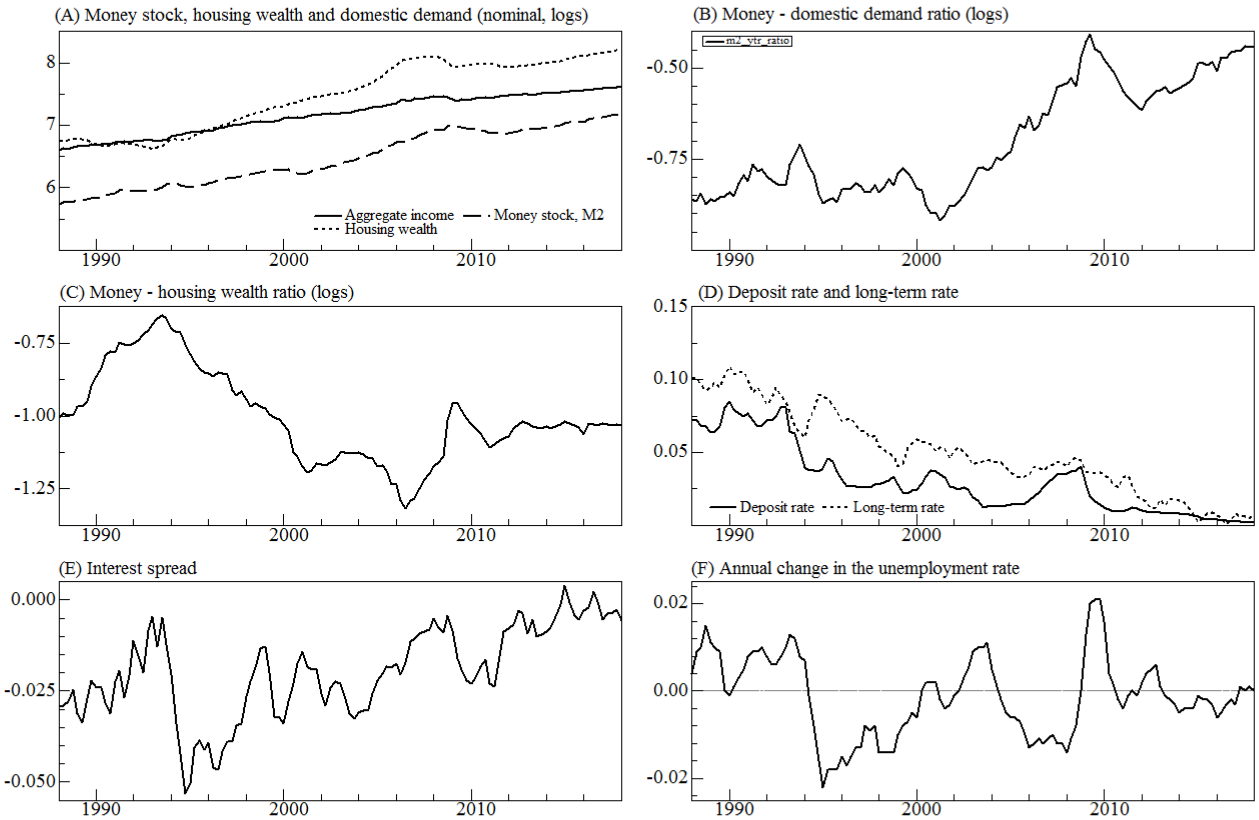

Figure 1: *Data and certain linear combinations.*

of describing the behavior of money demand in the first two decades of this century.

The deposit rate and the 10-year government bond rate are depicted in graph (D). During the sample period both interest rates have been trending downwards; R_t^b has been declining more rapidly than R_t^d, which is consistent with the gradual narrowing in the interest rate spread illustrated in graph (E). The graph also reveals that deposit rates have been decreasing more slowly than bond yields in the wake of the imposition of negative policy rates. The large drop in the interest rate differential in 1993-1994 can generally be attributed to the currency crisis in 1992-1993. As a result, it is relevant to include a restricted mean-shift dummy, *a priori*, in order to capture the statistical implications of the structural break in the interest rate spread.

Graph (F) shows the annual change in the unemployment rate. In general, ΔU_t is fluctuating around zero and exhibits, to some extent, degrees of persistency. This may reflect that the current economic situation plays a major role in households' perception of current labor market risk as well as the construction of the measure.

Generally, the majority of variables are clearly trending over time. Hence, a deterministic linear trend cannot be rejected *a priori*. However, as a deterministic trend does not seem to be a very intuitive determinant of money demand and the forthcoming empirical analysis at the same time suggests that a restricted linear trend does not enter the preferred long-run identified structure, I will for convenience not include a restricted trend prior to the cointegration analysis.

Finally, all variables are $I(1)$ processes which can easily be verified graphically by considering the first differences of the variables, see figure $A.1$ in appendix $A.3$. Simple Dickey-Fuller tests confirm this view.

5 Empirical Analysis

Following the proceduce in Juselius (2006), I observe several innovational outliers that can be removed by introducing well-specified dummy variables. I observe a (temporary) large shock to the deposit rate in 1993:2, possibly related to the currency crisis in 1992-1993. Such delayed dynamic effects in the data can effectively be alleviated by incorporating a transitory dummy in the period in which the innovational outlier is observed. I also include permanent blip dummies in 2000:3 and 2008:4 to take account of extraordinarily large shocks to the money stock, presumably caused by the dot-com bubble in 2000 and the financial crisis in 2008, respectively. The presence of the mean-shift dummy, restricted to be in the potential cointegration vectors, implies that one should at least incorporate its difference in the short-run structure of the model. Consequently, I include an unrestricted permanent intervention dummy in 1994:2.

Finally, I observe an additive transitory outlier in 1989:4. In general, additive outliers do not contribute to the nature of the autoregressive dynamics, as they lead to spuriously delayed effects which may bias the estimates. Thus, I follow Nielsen (2004) who argues that the additive outlier can effectively be removed prior to cointegration analysis by using linear interpolation in the data.

5.1 Lag Length Determination and Misspecifications

The stochastic variation in the data is assessed by using a combination of the general-to-specific procedure and the information criteria (SC, HQ and AIC). The advantage of this procedure is that it combines the influence of data and the benefits in terms of having the incomplexity of the autoregressive structure. The test statistics indicate that two lags are satisfactory in terms of incorporating the autoregressive nature of the variables. Thus, the error-correction form can be written as:

$$H(r) : \Delta x_t = \alpha[\beta' \beta_1'] \begin{pmatrix} x_{t-1} \\ D_{s1994_t} \end{pmatrix} + \Gamma_1 \Delta x_{t-1} + \mu_0 + \phi d_t + \varepsilon_t, \quad t = 1, 2, ..., T, \tag{7}$$

In general, the choice of lag length is only valid under the assumption of a correctly specified model. Table 1 reports the results of the misspecification tests of the single equations and the multivariate system for the unrestricted VAR model. Except for ΔU_t, the null hypothesis of no autocorrelation is accepted in the single equations. The autocorrelated errrors in ΔU_t are presumably caused by large degrees of persistency in the construction of the variable. However, in the robustness analysis, I alternatively show that the transformation to quarterly data in ΔU_t removes the autocorrelated errors, while the major empirical results still remain unchanged in the long-run part of the model.

The null hypothesis of no ARCH-effects is accepted for all variables. The residuals of R^d_t and ΔU_t seem to behave non-Gaussian according to the reports in table 1, while the null-hypothesis of normally distributed errors is accepted in the remaining equations. The two rejections of the normality assumption are essentially due to excees kurtosis caused by remaining moderate outliers. Even though non-Gaussian behaving residuals may lead to inefficient estimates, simulation studies have, however, shown that statistical inference is quite robust to excess kurtosis, see Juselius (2006). Consequently, the misspecification due to rejected normality may not be a serious problem.

	AR(1-2)		ARCH(1-2)		Normality	
Δm_t	1.20	[0.31]	2.36	[0.10]	4.56	[0.10]
Δy_t	0.81	[0.45]	0.38	[0.69]	0.92	[0.63]
ΔR^d_t	0.05	[0.96]	1.45	[0.23]	17.9	[0.01]
ΔR^b_t	3.18	[0.05]	0.17	[0.84]	1.29	[0.53]
Δwh_t	0.78	[0.46]	1.55	[0.21]	0.44	[0.80]
$\Delta^2 U_t$	9.05	[0.01]	0.56	[0.57]	11.1	[0.01]
Multivariate tests:	1.65	[0.01]	...		37.5	[0.01]

Table 1: *Test statistics for misspecification of the unrestricted VAR(2) system.* AR (1-2) are F-tests for autocorrelated residuals up to second order. The single equations and the multivariate tests are distributed as F(2,97) and F(72,451), respectively. ARCH (1-2) test for ARCH effects up to second order. The single equations are distributed as F(2,115). Finally, the tests for normality are distributed as $\chi^2(2)$ and $\chi^2(12)$ in the single equations and the multivariate tests, respectively.

5.2 The Cointegration Rank

To determine the cointegration rank, I start by considering the Johansen trace test. The test is based on (7) expressed in terms of its concentrated model:[13] $R_{0t} = \alpha\tilde{\beta}' R_{1t} + \varepsilon_t$, where R_{0t} is the short-run adjusted vector of endogenous variables and R_{1t} is the lagged short-run adjusted vector of the endogenous variables and the restricted mean-shift dummy, respectively. $\tilde{\beta}$ denotes coefficients to the cointegration relationships for the endogenous variables and the restriced level shift, respectively. The transformation of the cointegrated VAR model ensures that the long-run part of the model can be economically interpreted, as all short-run dynamics, unrestrited dummies and deterministic components have been concentrated out. As a result, I am left with a statistical model that solely captures the adjustment that takes place towards the long-run equilibrium relations.

The ML estimator can then be derived by using the general two step approch, see Johansen (1996). Thus,

[13]Formally, the concentrated model can be derived by transforming the CVAR(2) into compact form:

$$C_{0t} = \alpha\tilde{\beta}' \underbrace{\begin{pmatrix} x_{t-1} \\ D_{s1994_t} \end{pmatrix}}_{C_{1t}} + \Upsilon \underbrace{\begin{pmatrix} \Delta x_{t-1} \\ \mu_0 \\ d_t \end{pmatrix}}_{C_{2t}} + \varepsilon_t,$$

where Υ is a vector capturing the coefficients to the short-run parameters, the unrestricted mean and the unrestricted dummy variables, respectively. In order to estimate $\alpha\tilde{\beta}'$, I concentrate out the effect of C_{2t} on C_{0t} and C_{1t}, respectively, and then regress the cleaned C_{0t} (i.e. the residual called R_{0t}) on the cleaned C_{1t} (i.e. the residual called R_{1t}). For more details, see Johansen (1996).

the LR (trace) test statistic for two nested models, let's say $H(p)$ and $H(r)$, is:

$$\tau_{p-r} = LR(H(r)|H(p)) = -T \sum_{i=r+1}^{p} log(1 - \hat{\lambda}_i),$$

where the models meet the nested sequence $H(0) \subset \cdots \subset H(r) \subset \cdots \subset H(p)$, and $\hat{\lambda}_i$ captures the eigenvalues, explcitly linked to the cointegration vector i. The asymptotic distribtion of the rank tests converges in probability to some kind of a Dickey-Fuller distribution containing functionals of Brownian motions. As the distribution generally depends on deterministic specifications, such as an unrestricted constant and a mean-shift dummy restricted to the cointegration space, the distribution of the asymptotic trace test needs to be simulated, see Nielsen (2004*a*).[14]

		Simulated LR test						Bootstrap test	
p-r	r	Eig. Value	Trace	Trace*	P-value	P-value*	Largest non-unit eig.value	P-value	P-value*
6	0	0.37	147.6	127.8	[0.000]	[0.002]	0.717	[0.000]	[0.038]
5	1	0.22	91.9	80.7	[0.005]	[0.049]	0.722	[0.027]	[0.135]
4	2	0.20	62.7	55.9	[0.017]	[0.068]	0.810	[0.058]	[0.098]
3	3	0.14	35.6	30.8	[0.062]	[0.176]	0.771	[0.208]	[0.218]
2	4	0.10	17.8	14.0	[0.107]	[0.282]	0.952	[0.334]	[0.203]
1	5	0.04	4.7	2.7	[0.095]	[0.347]	0.988	[0.589]	[0.521]

Table 2: *Rank determination based on a simulated asymptotic distribution of Johansen trace test and bootstrap testing.* Asymptotic tables have been simulated based on the program developed by Nielsen (2004*b*). P-values are based on 5% critical values. The test statistics marked with an asterisk is the Bartleet-corrected trace test. In contrast to the general trace test, the Bartleet-corrected trace test corrects for small sample bias which in general leads to over-sized tests, see Johansen (2000, 2002*a* and 2002*b*).

The LR test statistics based on the top-bottom producere are reported in table 2. The null hypothesis of no cointegration is rejected for $r \leq 2$, while $r = 3$ is accepted with a p-value of 0.18. In the right panel, I have performed bootstrap likelihood ratio tests that tries to deal with the poor approximation of asymptotic inference in finite samples (especially in small samples).[15] Based on Cavaliere et al. (2012), the bootstrap version of the LR test cannot reject that the cointegration rank of three maximizes the explanatory power of the model in terms of stationarity.[16]

[14]In contrast, dummies for additive and innovational outliers do not influence the shape of the asymptotic distribution, as they correct for single observation shocks, see Johansen (1996, chapter 11).

[15]The substantial asymptotic distortion is a result of the complexity of the VAR model's dynamics, and the construction of the trace test that does not allow asymptotically for short-run effects, which turn out to be influential for the trace test in small samples, see Johansen (2002*b*). Since Bootstrap testing, that is based on a well-defined data generating process, has the advantage that it converges much faster to the true distribution, inference will automatically be much more reliable in small samples.

[16]The bootstrap Johansen trace test is implemented by estimating the restricted model $H(r)$, so that one obtains the estimates in (7) and the estimated residuals, $\{\hat{\varepsilon}_t\}$. Hence, the bootstrap test is then consistent with the generation of the artificial samples:

$$\Delta x_t^* = \hat{\alpha}\hat{\tilde{\beta}}' \begin{pmatrix} x_{t-1}^* \\ D_{s1994_t} \end{pmatrix} + \hat{\Gamma}_1 \Delta x_{t-1}^* + \hat{\mu}_0 + \hat{\phi} d_t + \varepsilon_t^*,$$

where $x_{-1}^* = x_{-1}$, $x_0^* = x_0$, and ε_t^* is drawn with replacement from the estimated residuals $\{\hat{\varepsilon}_t\}$. On each sample, one can calculate the test statistic for $H(r)$ and $H(p)$, respectively. Based on this sampling scheme, one can schematically re-estimate the restricted model and then simulate the distribution under the null hypothesis by using the generated bootstrap data. The bootsrapped p-values are generated from 400 replications and the sampling scheme is based on wild bootstrap.

Table 2 also documents that the largest unrestricted root is indisputably smallest for $r = 1$ and $r = 3$, while the choice of $r \geq 4$ could potentially lead to models containing I(2) trends. The graph of the recursive trace test (not depicted here) indicates moreover that linear growth is more or less present in the first two long-run relationships and partially for $r = 3$, whereas it does not seem to occur for the last two cointegration vectors. Indeed, the trace test component for the two smallest eigenvalues does not cross the 5% critical test value at any point in time, reconfirming that they may be equivalent to near unit roots.

In summary, the majority of tests pointed to a cointegration rank of three, while few others prefered one or two cointegration relationships. As the cointegration rank divides data into r relations in which the adjustment to equilibrium takes place and $p - r$ common stochastic trends, the choice of $H(r)$ will be very influential on whether the subsequent econometric analysis coincides with the expected economic hypothesis. A wrong choice of r can lead to wrong economic interpretation, as it leaves out additional information about of long-run equilibrium properties. Despite the analysis solely focuses on one theoretical equilibirum relationship, it is important to have in mind that the cointegration rank is not necessarily equivalent to the true number of theoretical relations, although it has been applied in several empirical applications on money demand. So, in order to ensure economically meaningful long-run properties as well as a well-specified cointegration space, I continue with a cointegration rank of $r = 3$, and let the identification process determine whether it is consistent with the theoretical specification of money demand.

5.3 Identification of the Long-run Structure

A natural first step in the long-run identification strategy is to examine whether the theoretical specification of money demand constitutes a stationary relationship. Such an analysis will contribute to spot whether the money-demand relation is empirically relevant for the identified long-run structure.

Formally, I test based on the test procedure in Johansen (1996) and Juselius (2006) whether one restricted relation lies in the cointegration space, while leaving the remaining relations unrestricted. Based on the concentrated model, I then have to restrict the cointegration space to the following null hypothesis: $\beta^c = (H_1\varphi_1, \xi)$, where φ_1 is a $s_1 \times 1$ matrix of unrestricted coefficients, s_1 denotes the numbers of free parameters in the restricted cointegration vector, H_1 is a known design matrix of dimension $7 \times s_1$, reflecting testable linear hypotheses, and ξ is a 7×1 matrix capturing the space of known coefficients.

	m_t	y_t	R_t^d	R_t^b	wh_t	ΔU_t	D_{s1994_t}	$\chi^2(v)$	p-value
$\mathbb{H}_1$	1	-1	17.7	-17.7	0	0	0	10.5(3)	0.015
$\mathbb{H}_2$	1	-1	15.0	-15.0	0	0	0.445	5.5(2)	0.059
$\mathbb{H}_3$	1	-1	11.9	-11.9	0.154	0	0	9.7(2)	0.077
$\mathbb{H}_4$	1	-1	61.2	-61.2	2.32	0	-2.95	0.9(1)	0.32
$\mathbb{H}_5$	1	-1	9.86	-9.86	0.156	-14.8	0	0.1(1)	0.75
$\mathbb{H}_6$	1	-1	5.34	-5.34	0.294	-13.8	-0.185	$\cdots$	$\cdots$

Table 3: *Testing money demand relations.*

Inspired by previous studies on money demand in Denmark, I consider the hypotheses related to the well-known money-demand relationship, suggesting that money demand, as a function of a scale variable and the opportunity cost, describes an irreducible stationary proces. The hypotheses with and without the restricted mean shift are shown under $\mathbb{H}_1$ and $\mathbb{H}_2$ in table 3. The test statistics indicate that money demand can no longer be described by this simple relation; even when I allow for a shift in the equilibrium mean, stationary is only borderline accepted with a p-value of 0.06. As argued, this is due to other determinants which have gradually become more influential during the 2000s and around the financial crisis.

The hypotheses $\mathbb{H}_3$ and $\mathbb{H}_4$ add housing wealth into the general money-demand relation. In the presence of the mean-shift component, stationarity can easily be accepted. In the next group of hypotheses (i.e. $\mathbb{H}_5$ and $\mathbb{H}_6$), I try to investigate the impact of incorporating precautionary motives into the relation. In contrast to hypotheses $\mathbb{H}_1 - \mathbb{H}_3$, stationarity cannot be rejected in $\mathbb{H}_5$, while $\mathbb{H}_6$ cannot formally be tested due to the lack of free parameters.

Generally, the results show that the standard relationship, where money demand is solely driven by a scale variable and the opportunity cost, is not even close to be empirically acceptable, whereas the data in contrast support the idea that the introduction of housing wealth and the change in the unemployment rate resuscitate the empirical validity of the money-demand relation.

Before imposing identifying restrictions, it is appropriate to examine whether some of the variables can be characterized as common stochastic trends. Inspired by Johansen (1996) the null hypothesis of weak exogeneity is $\alpha = \tilde{H}\alpha_1$, where $\tilde{H}$ is a $6 \times s$ matrix and α_1 is of dimension $s \times r$ of non-zero α-coefficients. As the model contains 3 common stochastic trends, I must have that $s \geq 3$, such that the number of variables which are adjusting to the long-run relation i have to be larger or equal to the number of long-run relations. The individual tests indicate that the cumulated residuals of wh_t constitute a common stochastic trend, while R_t^b can only be borderline accepted as weakly exogenous. Weak exogeneity of housing wealth is also found for the euro area, see Beyer (2009).

5.3.1 Identifying Restrictions on the Long-Run Structure

Motivated by the results in the previous subsection, this section will go a step deeper in terms of identifying the long-run structure and discussing whether the identified long-run relations are economical meaningful.

Identification of the long-run structure requires restrictions on each cointegration vector: Specifying s_i free parameters in each cointegration vector, the concentrated model can then be restricted to: $R_{0t} = \sum_{i=1}^{3} \alpha_i \varphi_i' W_i' R_{1t} + \varepsilon_t$, where φ are $s_i \times 1$ matrices of unrestricted coefficients and W_i are known design matrices of dimension $7 \times s_i$, reflecting testable linear hypotheses. The principle of identification is to choose W_i such that $\tilde{\beta}_i$ cannot be composed by linear combinations of the remaining cointegration vectors.

As the aim of the paper is to identify a stable long-run relationship for money demand, the identification approch will be based on a combination between statistics and the theory motivated in section 2. Consequently, I impose an adequate set of over-identifying restrictions on the cointegration space that automatically allow for a stationary money-demand relation found in the previous section (i.e. hypotheses $\mathbb{H}_5$ and $\mathbb{H}_6$). Subsequently, I impose restrictions on insignificant β estimates, while still allowing for economic interpretation.

As a final step, I impose the weak exogeneity restriction on housing wealth. As the error-correction dynamics are very weak for some of the other variables, I also impose additional linear restrictions on the columns in α. Inspired by Juselius (2006) the null hypothesis of identifying restrictions is $\alpha = (A_1\gamma_1 : A_2\gamma_2 : A_3\gamma_3)$, where A_i is a $6 \times s_i$ matrix, γ_i is of dimension $s_i \times 1$, and s_i captures the numbers of non-zero α-coefficient in column i.[17] The over-identifying restrictions imposed on the individual α and β estimates produce the preferred long-run structure reported under $\tilde{\mathbb{H}}_1$ in table 4. The over-identifying restrictions can easily be accepted jointly with a p-value 0.59. The statistical model is thus empirically identified.

	$\tilde{\mathbb{H}}_1$					
	α_1	α_2	α_3	β_1	β_2	β_3
m_t	0.061 (4.7)	−0.912 (−4.7)	0	−1	0	0
y_t	0	0	−0.055 (−4.9)	1	0	1
R_t^m	0.008 (3.5)	−0.12 (−3.3)	0	7.07 (3.8)	1	0
R_t^b	0	0.163 (4.8)	−0.034 (−4.7)	−7.07 (−3.8)	−0.74 (−8.9)	4.0 (5.2)
wh_t	0	0	0	0.44 (8.7)	0	−0.27 (−6.8)
ΔU_t	0.011 (4.4)	−0.137 (−3.5)	0	−13.3 (−8.2)	0	0
D_{s1994_t}	...	...	...	0	0.04 (8.8)	0
LR statistic	12.17					
P-value	0.59					
Distribution	$\chi^2(14)$					

Table 4: *Identification of the long-run structure.* t-values are based on asymptotic standard errors and shown in parentheses.

Graph (A) in figure 2 illustrates the actual movements in the money stock and its model-based long-run target, while graph (B) depicts deviations from its long-run target. The graphs clearly demonstrate that the underlying macroeconomic determinants have been guiding the money stock when substantial changes in demand of the latter have occured. Besides the clear evidence of endogeneity of money, the deviations in graph (B) show, however, that money demand in the wake of the financial crisis has been marginally lower than predicted by the empirical analysis.

5.4 Economic Interpretation

Long-run identification discloses that the theoretical representation of money demand can be motivated empirically. In light of the theoretical specification, the magnitude of the estimated coefficients are economically sizable and have the expected signs except for the coefficient to ΔU_t, that in contrast has a negative impact on money demand.

In line with previous findings, the empirical evidence supports moreover that the interest rates have a symmetric effect on money demand, and long-run homogeneity between GDP and money are still empirically relevant. The estimated impact from housing wealth is positive and very significant, probably reflecting that the magnitude of the collateral, the transaction and the wealth effect easily exceeds the substitution effect.

[17] Conditional on the identified structure of $\tilde{\beta}$, the concentrated model under the null hypothesis is thus: $R_{0t} = \sum_{i=1}^{3} A_i\gamma_i\varphi_i' W_i' R_{1t} + \varepsilon_t$, where the LR test procedure can then be derived by partitioning the system and subsequently solving the standard eigenvalue problem, see Johansen (1996).

The positive effect from housing wealth is also found for the euro area and for US with almost identical magnitudes of the estimates, see Greiber and Setzer (2007).

In contrast to the theory, the estimated effect from ΔU_t is negative. It means that households tend to demand less money when labor market risk is deteriorating. The results are, however, consistent with the findings in Atta-Mensah (2004) and Seitz and von Landesberger (2014) who also document that larger unemployment uncertainty reduces the demand for money holdings. Although the result does not seem very intuitive, their interpretation is that households' find real assets more advantageous than nominal assets when economic uncertainty is present. Another explanation that seems more dependable is that changes in unemployment rates may capture other movements than economic uncertainty, reflecting the difficulty of measuring precautionary savings.[18]

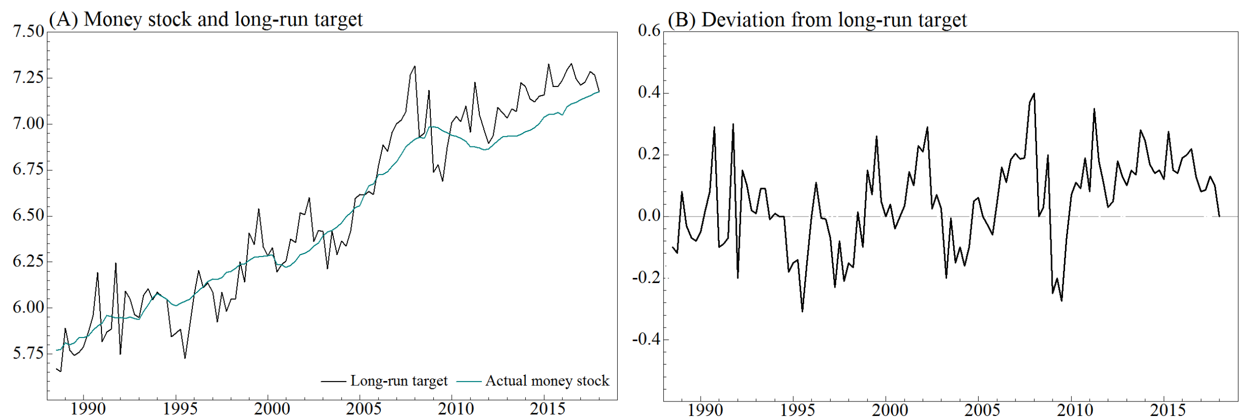

Figure 2: *Long-run target and deviations from money-demand relation.* Both graphs are based on the concentrated model.

Deviations from equilibrium are mainly corrected by money holdings that without presence of short-run effects eliminate 6.1% of a misalignment each quarter, while error-correction from the deposit rate and labor risk are more or less neglible. Intuitively, it coheres with the well-known idea that households gradually adjust their deposit account towards their desired level in case the economy is hit by shocks.

The remaining cointegration relations are spanned by the interest rate spread including the mean-shift dummy, and an aggregate income relationship saying that aggregate demand is positively related to housing wealth and negatively related to the long-term interest rate.[19]

5.5 Long-run stability and implications of negative interest rates

Here, I present forward-recursive tests and a rolling-windows estimation of the full model and the individual coefficients to the identified long-run structure. In contrast to full-sample estimation that provides estimates

[18]In section 6 I will return to this issue.

[19]In general, stationary of the interest rate spread is consistent with economic theory, but a cointegration coefficient of 0.74 is, however, not very meaningful. A reasonable explanation might be that the deposit rate is composed by an average of several bank deposit accounts with large variation in the yields. Juselius (1998) and (2006) find a similar identified long-run relation for the interest rate differential with cointegration coefficients of 0.5 and 0.8, respectively.

based on the maximum number of observations, the idea of recursive analysis is to spot potential changes and structural breaks in estimated coefficients.

As a starting point, I consider a recursive test of the full long-run model, here clarified by the recursively calculated log likelihood test, see Hansen og Johansen (1999). Intuitively, the test compared the influence of data of the sub-samples and the baseline sample, adjusted for the relative length of the baseline sample and the number of parameters. The test statistic is shown in graph (A) in figure 3, where the 95% quantile is characterized by the horizontal dashed line. The graph clearly indicates that constancy of the long-run structure can jointly be accepted, recalling that variability in the beginning of the sub-sample is quite prevalent when the baseline sample is relatively small compared to the number of parameters.

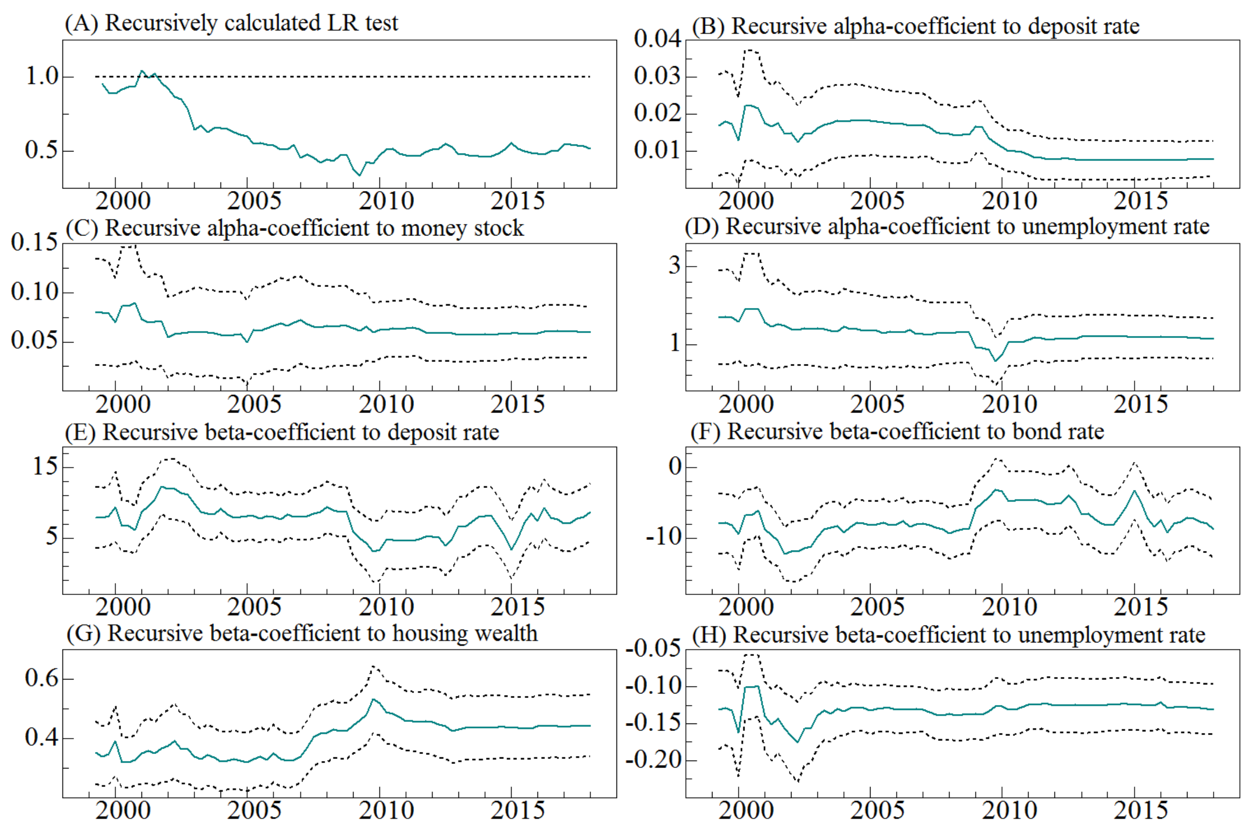

Figure 3: *Forward-recursive tests.* The recursively calculated LR test is based on the concentrated model and the sub-samples: $t_1 = 1999.4, ..., 2018.1$. The dashed lines indicate the 95% confidence bands. The recursively calculated coefficients of $\alpha(t_1)$ and $\beta(t_1)$ for the money-demand relation are also based on the sub-samples $t_1 = 1999.4, ..., 2018.1$. In each baseline sample, all short-run parameters are fixed at their full-sample estimates.

The remaining graphs in figure 3 depict forward-recursive estimation of the individual α and β vectors under $\tilde{\mathbb{H}}_1$.[20] The recursive graphs are produced by comparing the space of the individual full-sample estimate with the accompanying spanned estimates of the respective sub-samples, see Hansen and Johansen (1999) for details. Generally, the graphs illustrate that the estimates have behaved remarkably stably over the considered

[20] The individual α and β estimates to the remaining two cointegration relationships are shown in figure A.2 under additional results in appendix A.3.

sample, and the narrowing of the confidence bands in several of the recursively calculated estimates reflect an increasing information on the long-run parameters.

However, as forward-recursive estimation adheres to the starting-point observations, it inherently has a tendency to misjudge potential changes in the parameters towards the end of the full sample. As a result, the assessment of the timing of changes in the coefficients, specifically related to the introduction of negative policy rates, is probably much more deceptive when forward-recursive analysis is applied. To deal with these considerations, figure 4 shows a rolling-window estimation of the long-run money-demand parameters.[21] In line with the forward-recursive tests, the graphs indicate, to a satisfactory extent, that money-demand estimates behave remarkably stably. This reflects that the inclusion of wh_t and ΔU_t have improved stability and explanatory power of long-run money demand. Moreover, all estimates remain significant for all sub-sample points except for the estimates to the opportunity cost that turn insignificant for the short period 2011:2-2012:4.

In contrast to the forward-recursive estimation, the rolling-window approach additionally reveals information about the cohesion between negative policy rates and money demand. More specifically, graph (A) and (B) in figure 4 show that Danmarks Nationalbank's imposition of negative policy rates on the 24th May 2012 (see grey bars) induced a delayed temporary shock to the estimated coefficient to the opportunity cost. The time-lag impact coincides with the general idea that the pass-through to deposit rates is typically characterized by some degree of delay, see e.g. Autrup et al. (2016). In spite of that, it is, however, also crucial to emphasize that from a maximum statistical perspective, the temporary effect solely raises the estimate to its average level, arguably capturing that the estimate of the opportunity cost has gradually become less influential over a longer time frame. Nevertheless, I interpret the result as if the introduction of negative policy rates has not affected the structural determination of money demand permanently, but only temporarily.

A final note is that the forward-recursive tests reveal that the β-coefficient to housing wealth starts increasing slightly at the beginning of the 2000s. These movements contribute to the idea that housing wealth has been accountable for a larger part of the build-up in money holdings in the first two decades of this century. Cohesively, this interpretation is consistent with the rolling-window estimation, suggesting that the estimate tends to remain at that level at the end of the sample period.

6 Robustness Checks

So far, the statistical model has not been accountable for measurement errors, sample selectivity, or omitted variable considerations, like for instance the implications of a nominal-to-real transformation. In the next section, I will go through some of these issues and evaluate their statistical implications for the empirical results. The robustness tests for alternative measures of some of the endogenous variables are shown in

[21] Based on a 22-year rolling window. The start and end years of the recursive sample are not quite decisive for the results. However, it is important to notice that the size of the rolling window is chosen adequately large in order to prevent a too large drop in the power of the estimates. Nevertheless, the rolling-window analysis can still be used to assess the overall development in the estimated coefficients and the impact of a negative interest rate environment.

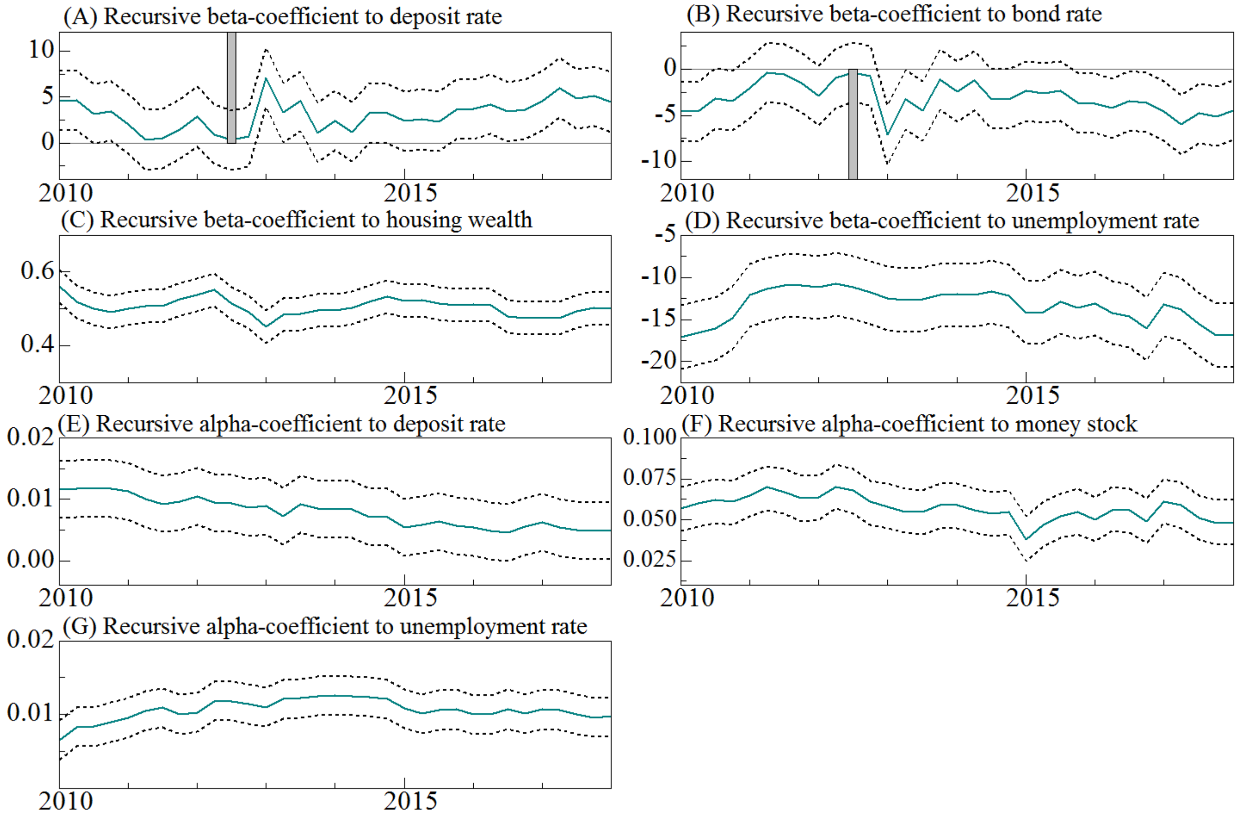

Figure 4: *Rolling-window estimation based on the concentrated model for 22-year samples: (1988:3-2010:1)-(1996:3-2018:1).* The dashed lines indicate the 95% confidence bands. The grey bars captures the time period in which negative monetary policy rates were introduced.

appendix A.2. In general, the results show that scale variable and the opportunity cost are robust to several measurement changes, indicating that the attenuation bias of these measures are tolerable.

6.1 Measuring Precautionary Demand for Money

The misspecification tests in section 5 stated that the annual change in the unemployment rate exhibited strong signs of autocorrelated errors. Since simulation studies in general indicate that valid statistical inference might be quite sensitive to violation of autocorrelated errors, it is decisive to examine whether the empirical results are driven by these autocorrelated errors in ΔU_t. Consequently, I consider quarterly data on the changes in the unemployment rate, whose variation is quite less persistent, but still follows the same underlying stochastic pattern, see figure 5. The null hypothesis of no autocorrelation is now accepted both in the single equation and for the full multivariate system. The simulated and the bootstrapped Johansen trace test suggest now a cointegration rank of four, while the remaining rank tests primarily argue for $r = 3$. Nevertheless, I choose $r = 3$, such that the framework remains consistent with previous findings. Identification of the long-run structure suggests a model which is equivalent to the baseline model.

The results are shown under $\tilde{\mathbb{H}}_1^*$ in table 5. Stationarity of the individual relations can be jointly accepted

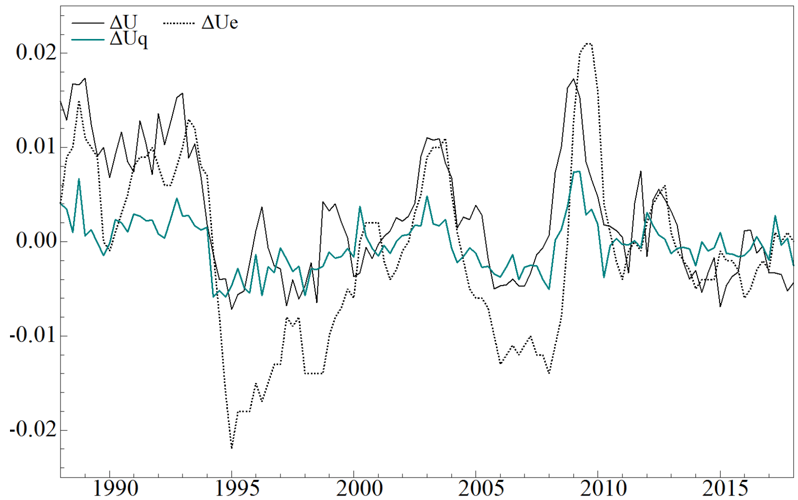

Figure 5: *Measures of precautionary demand for liquidity.* The notation ΔUe and ΔUq denote households' expectations on the general unemployment rate in a year comparing with today and the quarterly change in the unemployment rate, respectively.

with a p-value of 0.42.[22] Furthermore, the estimated money-demand equation suggests that the coefficients to the interest rate differential and housing wealth are more or less unaffected, while the impact from ΔU_t has increased quite dramatically due to the transformation from yearly to quarterly data.

	$\check{\mathbb{H}}_1^*$						$\check{\mathbb{H}}_2^*$	
	α_1	α_2	α_3	β_1	β_2	β_3	α_1	β_1
m_t	0.038 (3.3)	-0.534 (-3.2)	0	-1	0	0	0.039 (2.8)	-1
y_t	0	0	-0.052 (-4.6)	1	0	1	0	1
R_t^d	0.006 (3.2)	-0.087 (-2.9)	0	7.24 (3.6)	1	0	0.009 (3.7)	4.59 (2.3)
R_t^b	0	0.154 (4.6)	-0.032 (-4.6)	$-$ 7.24 (-3.6)	- 0.79 (-8.1)	4 (4.8)	0	$-$ 4.59 (-2.3)
wh_t	0	0	0	0.411 (7.4)	0	-0.263 (-6.1)	0	0.422 (6.8)
ΔU_t	0.008 (5.7)	-0.095 (-4.5)	0	$-$ 56.7 (-8.2)	0	0	0.011 (5.5)	$-$ 19.5 (-5.7)
D_{s1994_t}	$\cdots$	$\cdots$	$\cdots$	0	0.039 (7.2)	0	$\cdots$	-0.368 (4.5)
LR statistics	14.48						5.60	
P-value	0.42						0.35	
Distribution	$\chi^2(14)$						$\chi^2(5)$	

Table 5: *Identification of the long-run structure.* t-values are based on asymptotic standard errors and shown in parentheses.

Generally, the inadequacy of using ΔU_t, both measured in yearly and quartely data, to reflect precautionary demand for liquidity is that the variables do not capture forward looking expectations on households' labor market risk. To deal with that, I introduce a specific indicator, ΔU_t^e, which tries to measure households' expectations on the general unemployment rate in a year compared to the rate today.[23] Graphically, figure 5

[22] Forward-recursive estimation also documents that the null-hypothesis of non-constancy can easily be rejected, both for the full system and for the individual estimates, see Figure A.3 under additional results in appendix A.3.

[23] Data are taken from Statistics Denmark's consumer confidence survey. The indicator is constructed by sample surveys, in which a representative sample of persons 16-74 years are asked about: How do you think your employment situation will be in a year compared with today. Since the data are based on a sample survey, they are subject to a certain degree of statistical uncertainty.

reveals that ΔU_t^e evolves to varying degrees according to the movements in ΔU_t, presumably reflecting that consumers' expectations are in general based on their current employment situation.

In contrast to ΔU_t, the errors do not show any signs of non-normality or autocorrelation. Miscellaneous rank tests suggest a cointegration rank of one, generally due to near unit roots of the largest non-unit eigenvalue for $r = 2$ and $r = 3$, respectively. Nevertheless, identification of the single cointegration relationship turns out to be almost equivalent to the money-demand equation in $\tilde{\mathbb{H}}_1$, even in spite the restricted mean-shift being now entered into the relation. This can be seen by considering the long-run estimates of the money-demand equation depicted under hypothesis $\tilde{\mathbb{H}}_2^*$ in table 5. Stationarity can moreover be accepted with a p-value of 0.35 and forward-recursive tests suggest that the estimated coefficients evolve fairly constantly over the effective sample, see figure A.4 under additional results in appendix A.3.

The attempt to introduce forward-looking labor market risks has alleviated the effect of the opportunity cost slightly, while the impact from housing wealth has increased a little. Even though the estimate of ΔU_t^e is larger, it is still negative and comparatively close to the baseline estimate, probably reconfirming that future labor market events are mainly influenced by unemployment risks observed today.

6.2 Testing nominal-to-real transformation

As argued in section 4, a nominal-to-real transformation should not affect the estimates of the money-demand equation substantially in case long-run homogeneity between m_t and y_t is present. To investigate whether the intuition is consistent with the data, I consider the adjusted data vector $\tilde{x}_t = (\tilde{m}_t : \tilde{y}_t : R_t^d : R_t^b : \tilde{wh}_t : \Delta U_t : \Delta p_t)'$, where $\tilde{m}_t$ is the real money stock, $\tilde{y}_t$ is real GDP, $\tilde{wh}_t$ is real housing wealth and Δp_t is annual inflation.

	$\tilde{\mathbb{H}}_3^*$					
	α_1	α_2	α_3	β_1	β_2	β_3
$\tilde{m}_t$	0.057 (4.8)	−0.422 (−1.6)	0	−1	0	0
$\tilde{y}_t$	0	0	0	1	0	1
R_t^d	0.007 (3.5)	− 0.21 (−4.5)	0	4.33 (2.5)	1	0
R_t^b	0	0.112 (1.8)	−0.017 (−3.9)	− 4.33 (−2.5)	-0.447 (−9.6)	6.36 (12.9)
$\tilde{wh}_t$	0	0	0	0.531 (6.6)	0	0
ΔU_t	0.009 (4.6)	0	0.006 (1.7)	− 14.7 (−7.3)	0	0
Δp_t	0	0	0.036 (3.8)	0	0	− 6.36 (−12.9)
D_{s1994_t}	...	...	...	−0.314 (4.1)	0.028 (7.7)	0
LR statistics	23.38					
P-value	0.27					
Distribution	$\chi^2(20)$					

Table 6: *Identification of the long-run structure.* t-values are based on asymptotic standard errors and shown in parentheses.

The over-identified long-run structure is shown in table 6 and can be accepted with a p-value of 0.27. As expected, the nominal-to-real transformation has implied that the estimate of housing wealth has increased somewhat, but it is still positive and very significant, while the estimate of ΔU_t is more or less unaffected. The estimate of the opportunity cost has dropped noticeably, reflecting that the inclusion of the mean-shift dummy may have absorbed a sizable part of the exogenous variation in the interest differential. Consistent with previous findings (e.g. Juselius (1998) and (2006)), the empirical evidence also supports that money does not seem to be inflationary.

Weak exogeneity of housing wealth can still be accepted and error-correction of the money-demand equation does still take place primarily through money holdings. Finally, forward recursive tests are shown in figure 6. In line with the baseline model, the transformation to real terms has not substantially changed that non-constancy of the parameters can be rejected.

Generally, the decrease in the explanatory power of the system does not neccesarily reflect that the empirical model performs less favourably than the baseline model. Instead, it reflects that cointegrated VAR modelling typically becomes prohibitive in case of having too many autoregressive parameters. Consequently, the overall results indicate that the empirical framework is robust to a nominal-to-real transformation.

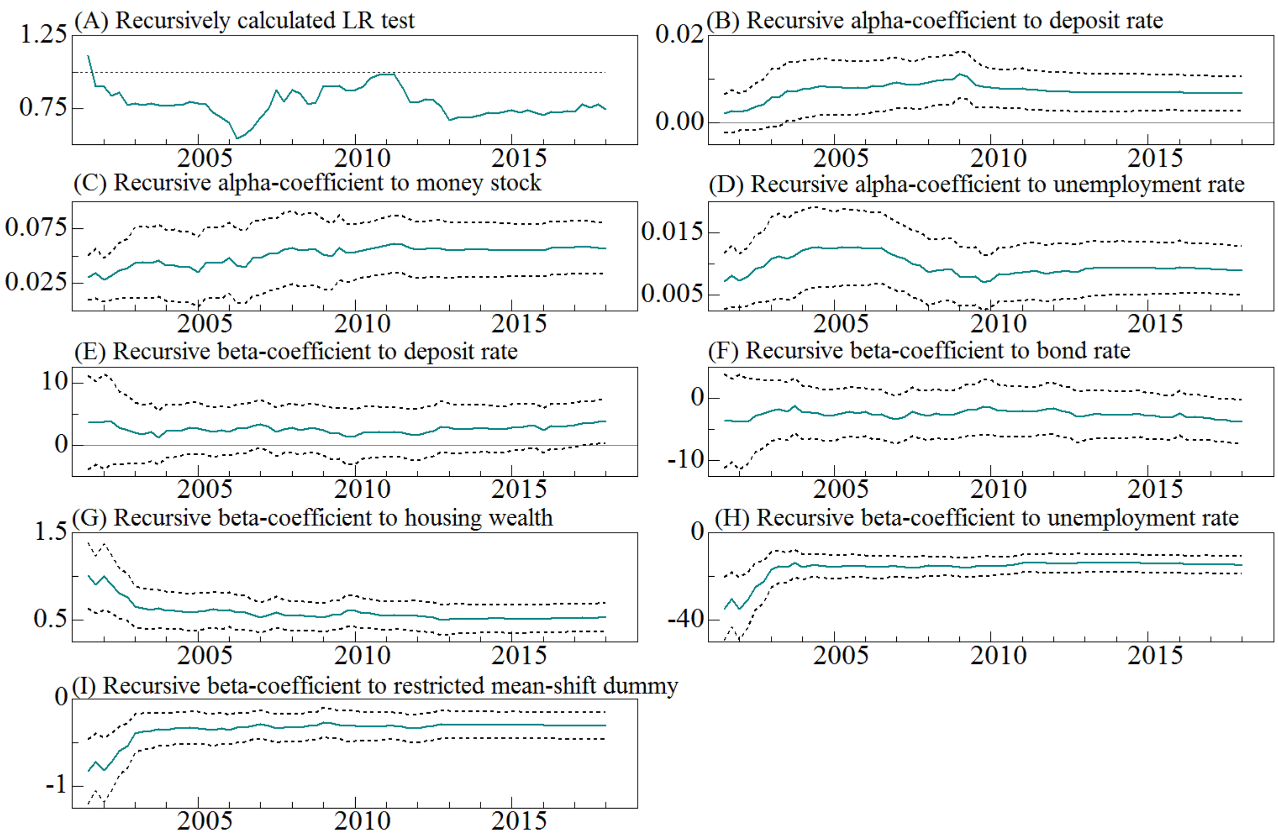

Figure 6: *Forward-recursive tests.* The recursively calculated LR test is based on the concentrated model and the sub-samples: $t_1 = 1999.4, ..., 2018.1$. The dashed lines indicate the 95% confidence bands. The recursively calculated coefficients of $\alpha(t_1)$ and $\beta(t_1)$ for the money-demand relation are also based on the sub-samples $t_1 = 1999.4, ..., 2018.1$. In each baseline sample, all short-run parameters are fixed at their full-sample estimates.

7 Conclusion

One of the implications of negative policy rates in Denmark has been the reduced pass-through from monetary policy to deposit rates, which has automatically contributed to boosting the incentives to hold deposits relative to alternative placements of funds. The paper documents that this channel has not permanently affected the underlying determination of money demand and the stability of the long-run determinants. Instead, the analysis shows that the introduction of negative monetary policy rates has solely contributed to a temporary shock to the long-run coefficient to the opportunity cost.

In terms of the empirical framework the paper also contributes to the litterature on money-demand estimation in Denmark. Specifically, the cointegration analysis demonstrates that the general macroeconomic determinants are no longer sufficient to describe the variation in monetary aggregates in Denmark during the period 1988-2018. Instead, the paper argues that a stable long-run relationship for money demand can alternatively be established by introducing housing wealth and the annual change in the unemployment rate to the empirical framework. Error-correction dynamics suggest that deviations from the long-run relation are mainly corrected via changes in money holdings, whereas housing wealth is not determined by other variables in the system. I interpret this as evidence that the gradual expansion of money from the beginning of the 2000s has not contributed to movements in property prices in this period.

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Appendix

A.1. Derivation of Eq. (5)

First, I solve the decision problem for the lenders. To do so, I define the state variable:

$$\omega_t \equiv \frac{Q_t}{P_t} H_{t-1}^l + (1 + r_{t-1}) B_{t-1}^l + (1 + d_{t-1}) \frac{P_{t-1}}{P_t} \frac{M_{t-1}^l}{P_{t-1}} \tag{A.1}$$

Hence, the transition equation for the state variable is:

$$\begin{aligned} \omega_{t+1} &= \frac{\mathbb{E}_t Q_{t+1}}{\mathbb{E}_t P_{t+1}} H_t^l + (1 + d_t) \frac{P_t}{\mathbb{E}_t P_{t+1}} \frac{M_t^l}{P_t} + (1 + r_t) B_t^l \\ &= \frac{\mathbb{E}_t Q_{t+1}}{\mathbb{E}_t P_{t+1}} H_t^l + (1 + d_t) \frac{P_t}{\mathbb{E}_t P_{t+1}} \frac{M_t^l}{P_t} + (1 + r_t) \left(Y_t^l - C_t^l - \frac{Q_t}{P_t} H_t^l - \frac{M_t^l}{P_t} + \omega_t \right) \end{aligned} \tag{A.2}$$

Based on the household's initial level of resources, the value function can be defined as follows:

$$V(\omega_t) = \max_{C_t^l, \frac{M_t^l}{P_t}, H_t^l} \left\{ \mathbb{E}_0 \left[\tilde{z}_t \frac{\left(\frac{M_t^l}{P_t} \right)^{1-\varphi}}{1 - \varphi} + (1 - \tilde{z}_t) \frac{{C_t^l}^{1-\eta}}{1 - \eta} + \delta^H \frac{{H_t^l}^{1-\kappa}}{1 - \kappa} + \beta V(\omega_{t+1}) \right] \right\} \tag{A.3}$$

The necessary first-order conditions then state:

$$(1 - \tilde{z}_t) {C_t^l}^{-\eta} - \beta V'(\omega_{t+1})(1 + r_t) = 0 \tag{A.4}$$

$$\tilde{z}_t {\frac{M_t^l}{P_t}}^{-\varphi} - \beta V'(\omega_{t+1}) \left[(1 + r_t) - (1 + d_t) \frac{P_t}{\mathbb{E}_t P_{t+1}} \right] = 0 \tag{A.5}$$

$$\delta_t^H {H_t^l}^{-\kappa} - \beta V'(\omega_{t+1}) \left[(1 + r_t) \frac{Q_t}{P_t} - \frac{\mathbb{E}_t Q_{t+1}}{\mathbb{E}_t P_{t+1}} \right] = 0 \tag{A.6}$$

By using the Fisher parity, (A.5) and (A.6), I obtain:

$$\frac{\tilde{z}_t {\frac{M_t^l}{P_t}}^{-\varphi}}{\delta_t^H {H_t^l}^{-\kappa}} = \frac{i_t - d_t}{\frac{Q_t}{P_t} \left[1 + i_t - \frac{\mathbb{E}_t Q_{t+1}}{Q_t} \right]}, \tag{A.7}$$

where i_t is the nominal yield on bonds. Equation (A.7) says that the marginal rate of substitution between real money and housing services reflects the return on money, represented by the opportunity cost of holding money, relative to the user cost of housing.

Subsequently, the money-demand equation for the borrowers can be solved by considering a standard

Lagrange optimization problem. By substituting (3) into (4), the optimization problem reads:

$$\max_{C_t^b, \frac{M_t^b}{P_t}, H_t^b} \tilde{z}_t \frac{\left(\frac{M_t^b}{P_t}\right)^{1-\varphi}}{1-\varphi} + (1-\tilde{z}_t)\frac{C_t^{b\,1-\eta}}{1-\eta} + \delta^H \frac{H_t^{b\,1-\kappa}}{1-\kappa} -$$
$$\lambda_t \left(C_t^b - \varsigma \frac{\mathbb{E}_t Q_{t+1}}{\mathbb{E}_t P_{t+1}(1+i_t)} H_t^b + \frac{M_t^b}{P_t} + \frac{Q_t}{P_t}(H_t^b - H_{t-1}^b) - Y_t^b - (1+r_{t-1})B_{t-1}^b - (1+d_{t-1})\frac{M_{t-1}^b}{P_t} \right) \quad \text{(A.8)}$$

where λ_t denotes the Lagrange multiplier. Thus, the first-order conditions for the borrowers are:

$$(1-\tilde{z}_t)C_t^{b\,-\eta} = \tilde{z}_t \frac{M_t^b}{P_t}^{-\varphi} = -\lambda_t \quad \text{(A.9)}$$
$$\delta_t^H H_t^{b\,-\kappa} = \lambda_t \frac{Q_t}{P_t}\left[1 - \varsigma \frac{P_t}{\mathbb{E}_t P_{t+1}} \frac{\mathbb{E}_t Q_{t+1}}{(1+i_t)Q_t}\right] \quad \text{(A.10)}$$

Taking the collateral constraint, (A.9) and (A.10) into account, I get:

$$\frac{\tilde{z}_t \frac{M_t^b}{P_t}^{-\varphi}}{\delta_t^H H_t^{b\,-\kappa}} = \frac{1}{\frac{Q_t}{P_t}\left[1 - \frac{B_t^b}{Q_t H_t^b}\right]}, \quad \text{(A.11)}$$

Due to the lenders and borrowers constitute a fixed fraction of the total household sector, aggegate money must satify the equation:

$$\log M_t = (1-\zeta)\log M_t^l + \zeta \log M_t^b \quad \text{(A.12)}$$

Finally, the aggregate money demand equation can be derived by combining (A.7), (A.11) and (A.12):

$$m_t - p_t = -\frac{1}{\varphi}\log\delta^H + \underbrace{\frac{1}{\varphi}\log C_t}_{\text{Transaction effect}} + \underbrace{\frac{1}{\varphi}\tilde{z}_t}_{\text{Precautionary effect}} \underbrace{-\left(\frac{1-\zeta}{\varphi}\right)\log(i_t - d_t)}_{\text{Speculative effect}} + \underbrace{\frac{\kappa}{\varphi}\log\left(\frac{Q_t H_t}{C_t P_t}\right)}_{\text{Housing transaction effect}}$$
$$+ \underbrace{\frac{1-\kappa}{\varphi}\log\left(\frac{Q_t}{C_t P_t}\right)}_{\text{Wealth effect}} + \underbrace{\frac{\zeta}{\varphi}\log\left(1 - \frac{P_t B_t^b}{Q_t H_t^b}\right)}_{\text{Collateral effect}} + \underbrace{\left(\frac{1-\zeta}{\varphi}\right)\log\left(i_t - \frac{\mathbb{E}_t Q_{t+1} - Q_t}{Q_t}\right)}_{\text{Substitution effect}} \quad \text{(A.13)}$$

To transform the theorectical specification into an empirically testable cointegration framework, I assume that the individual housing effects are decomposed to one aggregate housing factor. Based on previous litterature, the most appropriate measure in terms of capturing all the individual effects in one single variable is housing wealth. The reason is that changes in housing wealth can both involve shifts in quantities and prices. At the same time, an increase in housing wealth also contributes to larger borrowing access due to the enhancement of the collateral pledged.

As monetary aggregates both include money holdings from households and non-financial corporations, it seems more prevailing to use aggregate income as a measure of transaction instead of private consumption before (A.13) is taking to the data. To ensure empirical tractability I express the opportunity cost (i.e. $-\log(i_t - d_t)$) as $R_t^d - R_t^b$. Based on these considarations, the theoretical specification in (A.13) can be rewritten into the empirically testable long-run relation for money demand given by equation (5) in the main text.

A.2. Alternative measures of endogenous variables

Generally, mortgage credit is quite prevalent in Denmark. As a result, purchase of mortgage bonds or servicing of mortgages could be regarded as a better alternative to placing funds in deposit accounts comparing to government bonds. For a robustness check, It would therefore be intriguing to examine the empirical implications of alternatively using the effective rate of a mortgage bond. As 30-year mortgage bonds are the most widely diffused mortgages among households, it seems appropriate to use the effective average yield of these.[24]

The change in the measure of the opportunity cost does not convey to any additional outliers or misspecification issues. Furthermore, the statistical model still suggests a lag length of two and a cointegration rank of three. By identifying the long-run structure according to the procedure in section 5.3, I obtain the same over-identifying space of $\tilde{\beta}$ presented under $\tilde{\mathbb{H}}_1$, while the restrictions on the error-correction parameters differ slightly. The estimates of the full identified long-run structure are given by the hypothesis $\tilde{\mathbb{H}}_4^*$ in table A.1, and can easily be accepted with a p-value of 0.28. In contrast to the baseline model, $\tilde{\mathbb{H}}_1$, the impact of the opportunity cost has increased. This can presumably be explained by a larger interest differential between the deposit rate and the mortgage rate, as well as the currency crise had a larger negative impact on the effective return on government bonds compared to yields of mortgage bonds. The estimates of housing wealth and ΔU_t are more or less unchanged. Forward-recursive tests still indicate that the null hypothesis of non-constancy of the broad system and the individual estimates can be rejected.

I have also run the model with an average yield of mortgage bonds in order to take variation in duration across mortgages into account. The result is shown under $\tilde{\mathbb{H}}_5^*$. The long-run identified structure can still be accepted and estimated coefficients in the money-demand relation do not differ much from the estimates under $\tilde{\mathbb{H}}_4^*$.

It is well known that several studies apply different scale variables. For instance, it might be a problem that GDP incorporates foreign earnings received by domestic employers and the return on domestic wealth to foreigners. Consequently, the models $\tilde{\mathbb{H}}_6^*$ and $\tilde{\mathbb{H}}_7^*$ consider alternative measures of y_t that try to take some of these issues into account.

The model under $\tilde{\mathbb{H}}_6^*$ applies GDI as the measure of the transaction effect. As expected, the long-run estimates are more or less identical to baseline ones, corresponding to the modest data difference between GDI and GDP in Denmark. The joint over-identified system is accepted with a p-value of 0.19.

Alternatively, one could also argue that the choice of transaction variable has to reflect the volume of payments in domestic currency. Hence, an appropriate variable which in a broad sense satisfies this requirement is domestic demand. The model under $\tilde{\mathbb{H}}_7^*$ reveals that the introduction of domestic demand

[24]Data are from Danmarks Nationalbank.

	$\tilde{\mathbb{H}}_4^*$						$\tilde{\mathbb{H}}_5^*$		$\tilde{\mathbb{H}}_6^*$		$\tilde{\mathbb{H}}_7^*$	
	α_1	α_2	α_3	β_1	β_2	β_3	α_1	β_1	α_1	β_1	α_1	β_1
m_t	0.055 (4.7)	− 0.495 (−1.8)	− 0.068 (−3.1)	−1	0	0	0.058 (4.5)	-1	− 0.058 (−4.6)	−1	0.066 (4.7)	−1
y_t	0	0	− 0.036 (−4.4)	1	0	1	0	1	0	1	0	1
R_t^m	0.008 (3.5)	− 0.217 (−4.0)	0.007 (1.6)	10.8 (6.3)	1	0	0.008 (3.3)	10.6 (6.3)	0.007 (3.0)	6.08 (3.1)	0.01 (3.8)	4.86 (3.0)
R_t^b	0	0.179 (4.4)	− 0.015 (−3.2)	− 10.8 (−6.3)	- 0.812 (−13.1)	5.45 (5.3)	0	− 10.6 (−6.3)	0	− 6.08 (−3.1)	0	− 4.86 (−3.0)
wh_t	0	0	0	0.455 (9.0)	0	− 0.169 (−3.7)	0	0.403 (9.0)	0	0.428 (7.6)	0	0.384 (8.5)
ΔU_t	0.011 (4.5)	− 0.198 (−3.8)	0	− 14.0 (−8.4)	0	0	0.011 (4.5)	− 13.5 (−8.2)	0.012 (4.7)	− 13.4 (−7.7)	0.013 (4.6)	− 11.5 (−8.1)
D_{s1994_t}	...	...	...	0	0.037 (8.5)	0	...	0	...	0	...	0
LR statistics	14.34						16.33		18.47		19.48	
P-value	0.28						0.18		0.19		0.11	
Distribution	$\chi^2(12)$						$\chi^2(12)$		$\chi^2(14)$		$\chi^2(14)$	

Table A.1: *Identification of the long-run structure.* t-values are based on asymptotic standard errors and shown in parentheses. In line with hypothesis $\tilde{\mathbb{H}}_1$, the statistical model under hypotheses $\tilde{\mathbb{H}}_5^*$, $\tilde{\mathbb{H}}_6^*$ and $\tilde{\mathbb{H}}_7^*$ also suggest a lag length of two and a cointegration rank of three. For convenience, the estimates of the remaining two cointegration relationships for these hypotheses are not depicted. However, the relations are identical to the respective ones in $\tilde{\mathbb{H}}_1$ and the estimates are also more or less alike.

delivers more or less the same long-run estimates and the system can still be accepted jointly. Despite the over-identified system is accepted with a noticeably smaller p-value than the baseline model, the value is, however, still within the acceptable range.

A.3. Additional results

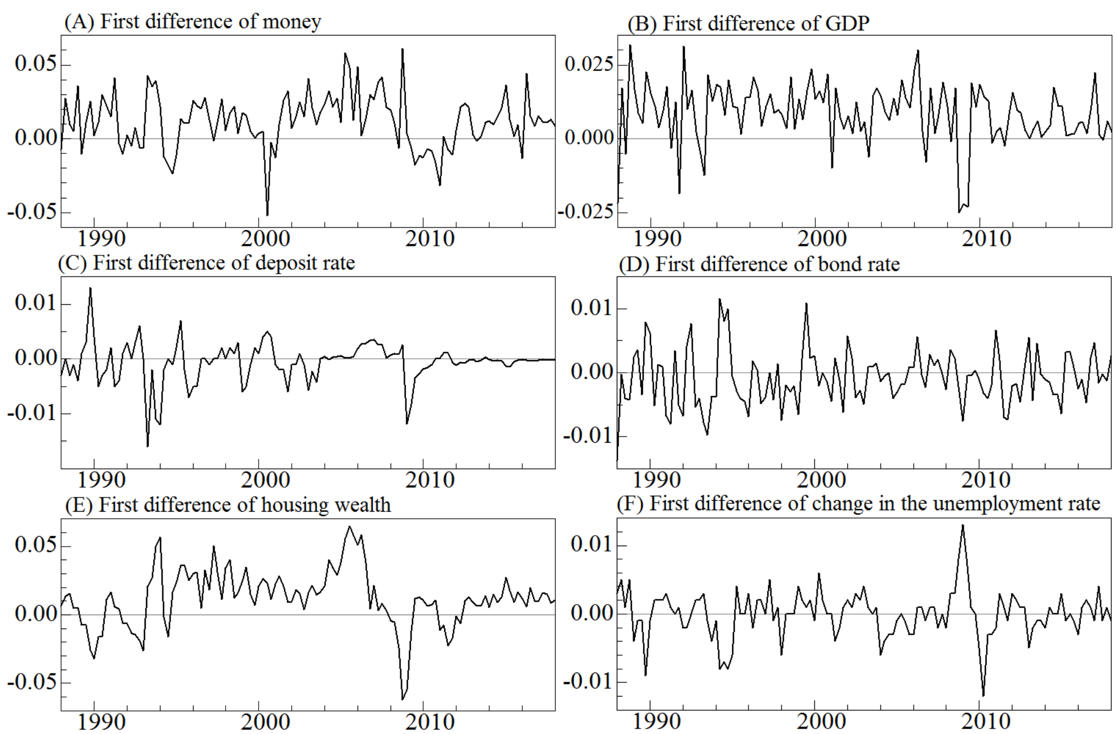

Figure A.1: *Data in first differences.*

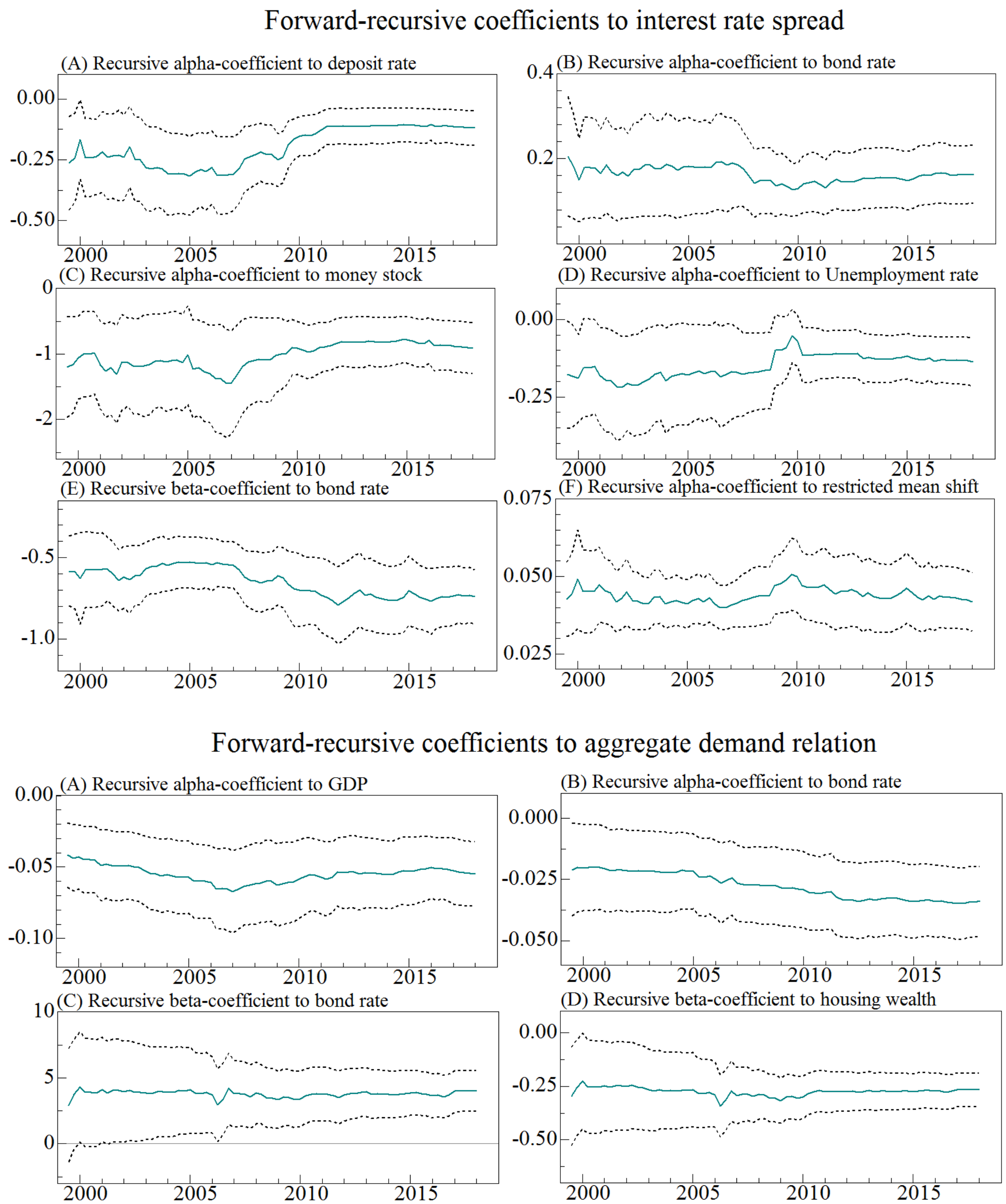

Figure A.2: *Forward-recursive tests of the two remaining cointegration relations.* The recursively calculated LR test is based on the concentrated model and the sub-samples: $t_1 = 1999.4, ..., 2018.1$. The dashed lines indicate the 95% confidence bands. The recursively calculated coefficients of $\alpha(t_1)$ and $\beta(t_1)$ for the money-demand relation are also based on the sub-samples $t_1 = 1999.4, ..., 2018.1$. In each baseline sample, all short-run parameters are fixed at their full-sample estimates.

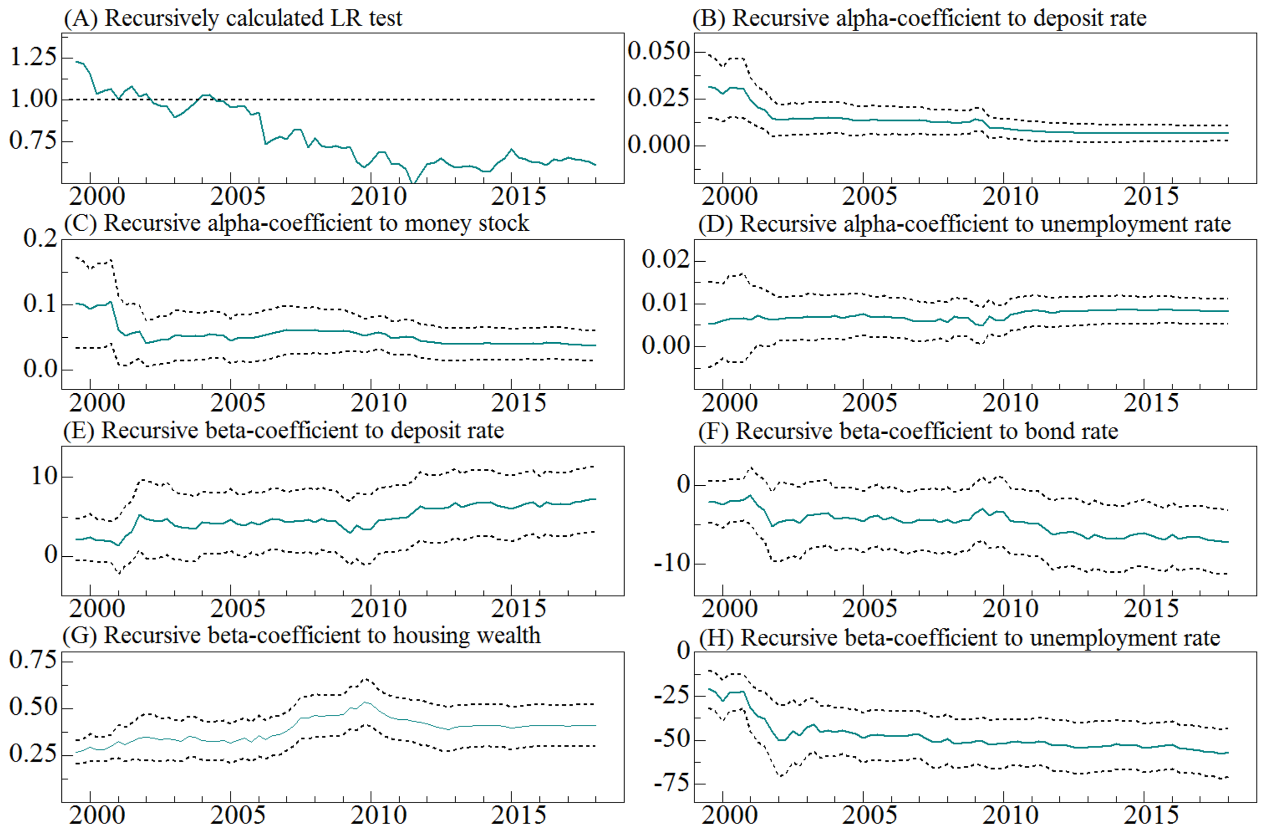

Figure A.3: *Forward-recursive tests.* The recursively calculated LR test is based on the concentrated model and the sub-samples: $t_1 = 1999.4, \ldots, 2018.1$. The dashed lines indicate the 95% confidence bands. The recursively calculated coefficients of $\alpha(t_1)$ and $\beta(t_1)$ for the money-demand relation are also based on the sub-samples $t_1 = 1999.4, \ldots, 2018.1$. In each baseline sample, all short-run parameters are fixed at their full-sample estimates.

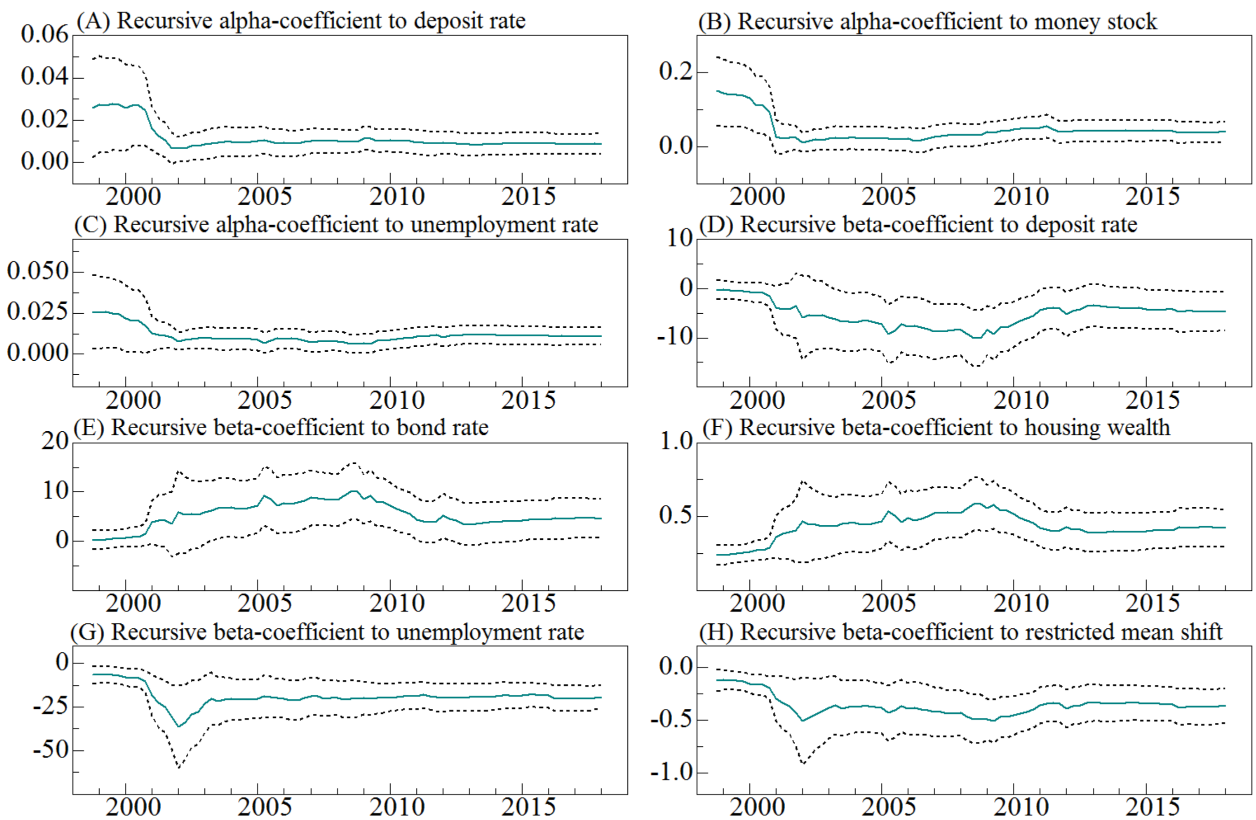

Figure A.4: *Forward-recursive estimates of the money-demand relation.* The recursively calculated LR test is based on the concentrated model and the sub-samples: $t_1 = 1999.4, ..., 2018.1$. The dashed lines indicate the 95% confidence bands. The recursively calculated coefficients of $\alpha(t_1)$ and $\beta(t_1)$ for the money-demand relation are also based on the sub-samples $t_1 = 1999.4, ..., 2018.1$. In each baseline sample, all short-run parameters are fixed at their full-sample estimates.

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