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The Macroeconomic Effects of Shadow Banking Panics

Johannes Poeschl
jpo@nationalbanken.dk

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Abstract
We study the effects of shadow banking panics in a macroeconomic model with a rich financial system, including deposit-financed retail banks and wholesale-financed shadow banks. Shadow banking panics occur when retail banks choose not to roll over their lending to shadow banks. Occasionally binding financial constraints of retail banks increase the likelihood and amplify the severity of such shadow banking panics. The model can quantitatively match the dynamics of key macroeconomic and financial variables around the US financial crisis. We quantify the impact of wholesale funding market interventions akin to those implemented by the Federal Reserve in 2008, finding that they reduced the fall in output by about half a percentage point. Unconditionally, central bank interventions reduce output volatility and the likelihood of banking panics.

Resume
Vi undersøger virkningerne af panik i skyggebanksektoren i en makroøkonomisk model med et rigt finansielt system, der omfatter indlånsfinansierede detailbanker og engrosfinansierede skyggebanker. Der opstår panik i skyggebanksektoren, når detailbanker vælger ikke at lægge deres udlån ud til skyggebanker. Lejlighedsvis bindende finansielle begrænsninger for detailbanker øger sandsynligheden for og forstærker alvoren af en sådan panik. Modellen matcher kvantitativt dynamikken ved de vigtigste makroøkonomiske og finansielle variabler i forbindelse med finanskrisen i USA. Vi kvantificerer virkningen af interventioner på engrosfinansieringsmarkedet svarende til dem, der blev gennemført af den amerikanske centralbank i 2008, og konstaterer, at de reducerede faldet i produktionen med omkring et halvt procentpoint. Centralbankinterventioner reducerer ubetinget volatiliteten i produktionen og sandsynligheden for panik i banksektoren.

Key words
Financial Institutions; Financial Stability; Interconnectedness; Monetary Policy

JEL classification
E44; G24; G28.

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The Macroeconomic Effects of Shadow Banking Panics *

Johannes Poeschl
Danmarks Nationalbank

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Abstract

We study the effects of shadow banking panics in a macroeconomic model with a rich financial system, including deposit-financed retail banks and wholesale-financed shadow banks. Shadow banking panics occur when retail banks choose not to roll over their lending to shadow banks. Occasionally binding financial constraints of retail banks increase the likelihood and amplify the severity of such shadow banking panics. The model can quantitatively match the dynamics of key macroeconomic and financial variables around the US financial crisis. We quantify the impact of wholesale funding market interventions akin to those implemented by the Federal Reserve in 2008, finding that they reduced the fall in output by about half a percentage point. Unconditionally, central bank interventions reduce output volatility and the likelihood of banking panics.

Keywords: Banking panics, shadow banks, wholesale funding market, liquidity backstops.

JEL Classification: E440; G240; G280.

*This paper supersedes an earlier paper called *Bank Capital Regulation and Endogenous Shadow Banking Panics*, which was joint work with Xue Zhang. Contact information: Research Department, Danmarks Nationalbank, Havnegade 5, 1093 Copenhagen, Denmark, jpo@nationalbanken.dk. I am thankful to my advisors Klaus Adam, Michèle Tertilt and Georg Duernecker for their constant support and advice. I thank Axel Gottfries for discussing the paper at the Nordic Macro Meeting 2018, Francesco Ferrante for discussing the paper at the Federal Reserve Day Ahead Conference and participants at various other conferences and workshops for their comments. I am thankful to Mark Gertler, Nobuhiro Kiyotaki and Andrea Prestipino for making the code for Gertler, Kiyotaki, and Prestipino (2016) available to us. Financial support from Karin-Islinger-Stiftung and the Stiftung Geld und Währung is gratefully acknowledged. This paper represents solely the view of the author and does not in any way reflect the opinions of Danmarks Nationalbank.

1 Introduction

At the heart of the 2007-2009 financial crisis was the collapse of the shadow banking system.[1] Before the crisis, shadow banks were an integral part of the broader banking system of the US, providing liquidity and maturity transformation for the US economy by securitizing assets like mortgages or small business loans. They funded these activities primarily using the securitized assets as collateral to borrow on the wholesale funding markets.[2] Important wholesale funding markets include the asset-backed commercial paper markets (Covitz, Liang, and Suarez (2013)) or the tri-party repo market (Martin, Skeie, and Thadden (2014)). During the crisis, the shadow banking system came under severe stress when this major source of funding collapsed due to a banking panic following the bankruptcy of Lehman Brothers in September 2008.

The lenders on the wholesale funding market were often traditional financial institutions like commercial banks or money market mutual funds, which in turn refinanced themselves with deposits or deposit-like liabilities like money market mutual fund shares.[3] Since the shadow banking system was so deeply integrated into the financial system, its collapse cannot be understood without explaining the interaction between the traditional (retail) and the shadow banking sector. While this interaction has been studied in more stylized models, a quantitative macroeconomic model exploring the feedback effects between the run on the shadow banking sector and the broader financial system through the wholesale funding market and its consequences for the macroeconomy is still missing from the expanding literature of macroeconomic models of financial crises.[4] With this study, we attempt to fill this gap.

In section 2, we present stylized facts about the retail and the shadow banking sectors in the US during the 2007-2009 financial crisis. Using data from Compustat, we show five facts: first, before the crisis, the shadow banking sector was more leveraged than the retail banking sector. Second, there was a spike in leverage both in the retail banking sector and in the shadow banking sector during the financial crisis. Third, the shadow banking sector was especially exposed to short-term debt. Fourth, the shadow banking sector witnessed a decline in short-term liabilities, which, fifth, coincided with an especially large decline in net worth in the shadow banking sector and an increase in credit spreads on the wholesale funding market and the funding market, even for non-financial firms with low default risk. Taken together, we interpret these facts as evidence of a run on short-term wholesale funding of the shadow banking sector, which resulted in a negative spill-over to the traditional banking sector and a disruption of lending to the non-financial sector.

To explain these facts and study the aggregate implications of a banking panic in the shadow banking sector, we use section 3 to outline an otherwise conventional business cycle model in which we embed a rich financial system with a retail and a shadow banking sector, which we

1. See e.g. Gorton and Metrick (2012) or Covitz, Liang, and Suarez (2013). For a recent overview of the literature supporting this view, see Gertler and Gilchrist (2018) and Bernanke (2018).

2. For a detailed description, see Pozsar et al. (2012).

3. MMMF shares promise to maintain a stable net asset value of one US dollar and are usually redeemable at any time. For an overview of money market mutual funds and their role during the financial crisis, see Schmidt, Timmermann, and Wermers (2016).

4. E.g. He and Krishnamurthy (2012), Brunnermeier and Sannikov (2014), Gertler and Kiyotaki (2015), Gertler, Kiyotaki, and Prestipino (2016) or Gertler, Kiyotaki, and Prestipino (2019a), Paul (2018), Ferrante (2018a), Faria-e-Castro (2019).

model as in Gertler, Kiyotaki, and Prestipino (2016). The financial sector reflects many of the attributes of the pre-crisis US financial system: Retail banks, best thought of as representing depository institutions and money market mutual funds, lend on the wholesale funding market to shadow banks. The latter is for example finance companies, broker-dealers and structured investment vehicles. Consistent with the empirical evidence, shadow banks are net borrowers on the wholesale funding markets, are highly leveraged and hence are especially exposed to banking panics, which arise occasionally and endogenously in the form of rollover crises in the spirit of Cole and Kehoe (2000) and Gertler and Kiyotaki (2015) on the wholesale funding market.

We calibrate the model using a variety of asset price, balance sheet and macroeconomic data in section 4 and show as our main experiment in section 5 that the banking panics that arise in the model resemble the one experienced by the US in 2008: The initial losses of the shadow banking sector from 2007 onward spread through the entire financial system through plummeting asset prices, tightening the financial constraints of retail banks. This reduced the supply of wholesale funding, leading to elevated credit spreads on the wholesale funding market. The higher credit spreads deteriorated the balance sheets of the shadow banking sector further, to the point where a rollover crisis became possible in September 2008. The result of such a materialized rollover crisis in the model is a drastic reduction in net worth of and lending by retail and shadow banks. The sharp non-linearity in the net worth of the shadow banking sector leads to strong endogenous amplification of the initial shock to the economy: Despite only moderately large exogenous shocks, output, investment, consumption and hours fall substantially, replicating the dynamics in the data.

The feedback between the shadow banking sector and the retail banking sector through the wholesale funding market plays a key role: Falling asset prices interact with the financial constraints of the retail banking sector to drive up the credit spread and reduce credit supply on the wholesale funding market. This resembles the period from August 2007 to September 2008 when for example the asset-backed commercial paper market started to experience a substantial reduction in credit supply (Covitz, Liang, and Suarez (2013)). The higher funding cost deteriorates the balance sheet of the shadow banking sector to the point where it becomes susceptible to a panic-driven run similar to the one experienced by the US shadow banking system in September 2008. This wholesale credit supply channel arises only if both retail and shadow banks are financially constrained.

We study the effect of occasionally binding financial constraints of retail banks on the probability of a banking panic in the shadow banking sector in more detail in section 6. We show that financial constraints of the retail banking sector amplify the effects of macroeconomic shocks when they start binding. Moreover, they are essential for a financial crisis to occur in our main crisis experiment: In the absence of financial constraints, retail banks will increase lending to the non-financial sector when the shadow banking sector fails. Moreover, they will not reduce credit supply on the wholesale funding market, leading to a much lower credit spread. As a consequence, even conditional on a run, investment and asset prices fall less. Unconditionally, the probability of a run is much lower if retail banks are not financially constrained: Given the shocks that replicate the financial crisis of 2008 in our baseline experiment, no shadow banking panic would have occurred in a counterfactual model without financial constraints on retail banks. Modeling the financial constraints of retail banks is therefore important, even though the crisis originates in the shadow banking sector.

Modeling explicitly the interaction of retail and shadow banks on the wholesale funding market allows us to analyze the effects of a wholesale funding market intervention by the central bank. As we show in section 2, the Federal Reserve intervened massively in the wholesale funding market after the collapse of Lehman Brothers, through for example the commercial paper funding facility.[5] At its peak, the Federal Reserve held more than 330 billion dollars of commercial paper through this facility, corresponding to about 20 percent of all outstanding commercial paper. The main argument for these interventions at the time was that they would serve as a liquidity backstop to avert a rollover crisis. A quantitative evaluation of the effects of these wholesale funding interventions on the likelihood of a rollover crisis, as well as its interaction with the occasionally binding financial constraints of the retail banking sector is the third contribution of this paper.

We introduce government policy in section 7. The central bank can intervene on the wholesale funding market by raising deposits from households and lending them on the wholesale funding market. The benefit of the central bank doing so is that it does not face the same leverage constraint as the retail banking sector and can therefore act as a lender of last resort, stepping in to provide lending to the shadow banking sector when credit supply by the retail banking sector falls. The cost is that the central bank is less efficient at providing liquidity on the wholesale funding market. This captures in a reduced-form way the Federal Reserve bank's worry that its intervention could disrupt the functioning of the commercial paper market if its lending terms were too favorable. We pin down the parameters governing the costs of the central bank intervention by matching the size and the timing of the intervention in the data.

First, we study the short-run effect of an unanticipated ex post intervention after a banking panic. When we compare the models with and without intervention, we find that the policy is effective at bringing down spreads on the wholesale funding markets fairly quickly. The lower credit spreads reallocate retail bank lending from the wholesale funding market to the retail lending market, and allow the shadow banking sector to build up net worth faster. As a consequence, the fall in output due to the banking panic is about half a percentage point lower, which is a substantial amount. This can be compared to for example the 2 percentage point reduction in the output fall due to direct mortgage-backed security purchases found by Ferrante (2018b), which in total amounted to about 1.2 trillion US dollars, or the 1.5 percentage point reduction in the fall in output due to all liquidity interventions, which amounted to about 1 trillion US dollars, found by Del Negro et al. (2017).

Second, we study the long-run effect of introducing a permanent liquidity facility that becomes active whenever credit spreads on the wholesale funding markets are abnormally high. This is no longer a theoretical exercise: The Federal Reserve reactivated the commercial paper

5. For a description of the commercial paper funding facility (CPFF), see Adrian, Kimbrough, and Marchioni (2011). Other liquidity programs were for example the Asset-Backed Commercial Paper Money Market Mutual Fund Liquidity Facility (AMLF), the Term Asset-Backed Securities Loan Facility (TALF), and the Money Market Investor Funding Facility (MMIFF). For a comprehensive overview of all the policy interventions by the Federal Reserve in the aftermath of the 2007-2009 financial crisis, see Bernanke (2018). Adrian, Kimbrough, and Marchioni (2011) argue that the CPFF was among the most successful programs, because it tackled the root cause of the disruptions in the financial markets in October 2008.

funding facility in the aftermath of the financial disruptions in the first quarter of 2020. We find that unconditionally, this policy can reduce the likelihood of a shadow banking panic by about half a percentage point and reduce output volatility by about 0.05 percentage points.

Related Literature This paper is closely related to three strands of literature. The first literature incorporates shadow banks into macroeconomic models. We build directly on the work of Gertler, Kiyotaki, and Prestipino (2016), from now on GKP2016, who ahve developed a canonical macroeconomic framework of financial crises in the form of shadow bank runs. They extend Gertler and Kiyotaki (2015) by including a wholesale or shadow banking sector, which played an important part in the onset of the 2007-2009 financial crisis. Our contribution relative to that paper is threefold: First, we embed their model into a conventional New Keynesian model to analyse the macroeconomic effects of a banking panic on the wholesale funding market. Second, we explicitly account for the possibility that financial constraints of retail banks can be occasionally binding. We can therefore study the interaction between the occasionally binding constraints of lenders with a rollover crisis. Third, we use the model to study the effects of central bank interventions on the wholesale funding market.

Another paper which studies shadow banking panics is Begenau and Landvoigt (2018). Their focus is different, as they study optimal regulation of traditional banks in a model with shadow banks. They abstract from modeling a wholesale funding market. Meeks, Nelson, and Alessandri (2017) study the effects of the existence of a shadow banking sector on the propagation of various macroeconomic shocks, including financial shocks, but do not consider endogenous financial risk in the form of banking crises. Ferrante (2018a) presents a new channel through which shadow banks endogenously affect the asset quality of the economy, which leads to business cycle and banking crisis amplification. In his model, there is no wholesale funding market. Durdu and Zhong (2019) investigate the drivers of bank and non-bank credit cycles through the lens of a structural model. Fève, Moura, and Pierrard (2019) as well as Gebauer and Mazelis (2019) study regulatory spillovers in an estimated model with shadow banks. None of these papers consider the effects of anticipated banking panics on the wholesale funding market.

Other papers provide microfoundations for the role of shadow banks in more stylized, theoretical models, e.g. Gennaioli, Shleifer, and Vishny (2013), Luck and Schempp (2016), Moreira and Savov (2017) and Chretien and Lyonnet (2017). Farhi and Tirole (2017) provide a theoretical model of optimal macroprudential regulation in the presence of shadow banks. Related, there is a theoretical literature that emphasizes the role of the shadow banking system for regulatory arbitrage, for example Plantin (2014), Huang (2018) and Ordoñez (2018). Relative to this literature, our contribution is a quantitative dynamic model with retail banks and shadow banks that can account for the macroeconomic effects of the financial crisis.

A second literature studies the causes and non-linear propagation of severe financial crises in models with financial intermediation. The paper closest to this paper is Gertler, Kiyotaki, and Prestipino (2019a), from now on GKP2019, which introduces banking panics into a macroeconomic model. Our contribution relative to that paper is to model the wholesale funding market and the financial constraints of the retail banking sector. This allows us to study the interaction between the retail banking sector and the shadow banking sector, match the model to additional data and study the effects of wholesale funding market interventions by the central bank.

Other papers which model banking crises as rollover crises driven by sunspots are Martin, Skeie, and Thadden (2014), Paul (2018), Faria-e-Castro (2019), Robatto (2019) and Rottner (2020). There is a large literature which introduces financial crises in a different way, e.g. due to occasionally binding borrowing constraints (Mendoza (2010), Bianchi (2011), He and Krishnamurthy (2012), Brunnermeier and Sannikov (2014) or Akinci and Queralto (2017)), market freezes (Uhlig (2010), Boissay, Collard, and Smets (2016)), or bank default (Mendicino et al. (2019)). A key distinction to that literature is that we include both a retail and a shadow banking sector.

The paper is also related to a third literature, which analyses the macroeconomic effects of the liquidity measures by the Federal Reserve in response to the financial crisis. A key paper is Del Negro et al. (2017), which uses a quantitative model building on Kiyotaki and Moore (2012) to evaluate the Federal Reserve's liquidity facilities. The model is different from ours in a number of ways. First, the authors do not explicitly model the banking sector, and therefore also not the wholesale funding market. Second, the authors do not model endogenous banking panics. Doing so allows us to discuss the effects of the liquidity measures on the likelihood of a financial crisis. There are other papers considering the macroeconomic effects of direct asset purchases by the central bank (e.g. Gertler and Karadi (2011), Gertler, Kiyotaki, and Prestipino (2016), Ferrante (2018b) and Ferrante (2018a)). They do not explicitly discuss the liquidity interventions by the Federal Reserve. Arce et al. (2019) study central bank balance sheet operations in a model with an explicit interbank market, but do not consider banking panics. Similarly, Gertler and Kiyotaki (2011) discuss liquidity injections, but they have a different model of the interbank market and do not model endogenous banking panics.

2 Retail Banks, Shadow Banks, and the Wholesale Funding Market during the Financial Crisis

2.1 The Crisis on the Wholesale Funding Market

Figure 1 shows that the wholesale funding market experienced a severe disruption during the financial crisis, which spilled over to the non-financial sector. The blue, solid line is the TED spread, i.e. the spread between the 3-month LIBOR and the yield on 3-month US treasuries. The TED spread is a measure of the funding costs on the wholesale funding market. The red, dotted line is the spread between AAA-rated corporate bonds and 10 year treasuries. This AAA-10Y spread is a measure of the funding costs of prime non-financial borrowers. We also show the NBER recession dates represented by the shaded area.

We see that, typically, the TED spread is much lower than the AAA-10Y spread. From the third quarter of 2007 onward, both the TED spread and the AAA-10Y spread increased markedly, peaking in the last quarter of 2008. At that time, the TED spread had surpassed the level of the AAA-10Y spread, reflecting a historically unprecedented rise in funding costs on the wholesale funding market. Moreover, there were many borrowers on the wholesale funding market which were unable to roll over their debt at all (Gorton and Metrick (2012), Covitz,

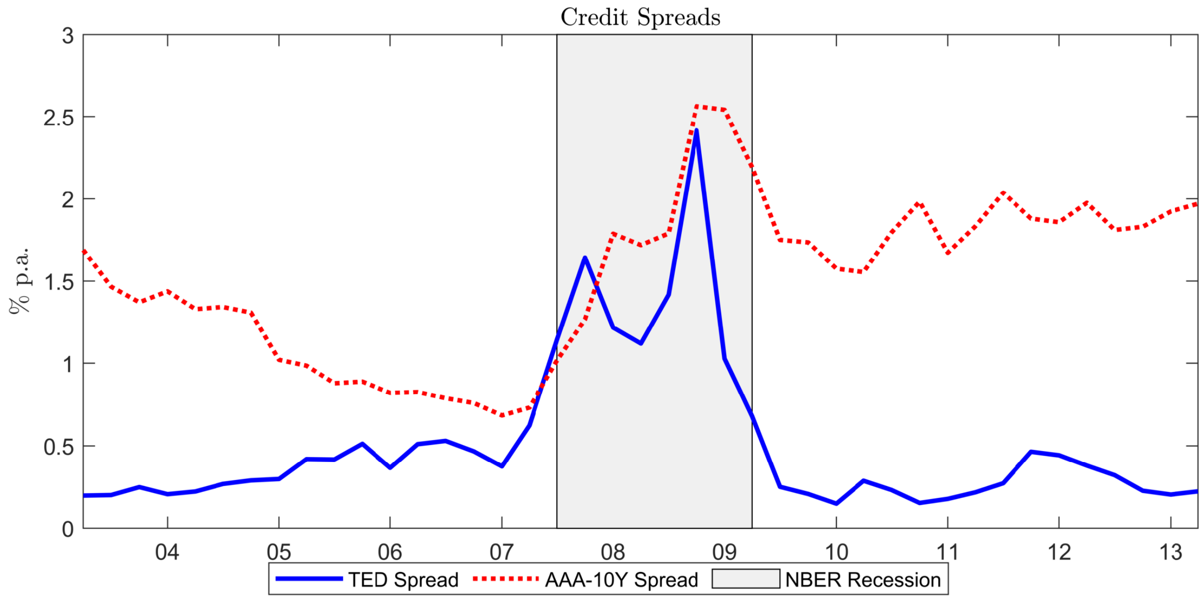

Figure 1: The TED spread during the financial crisis

Note: Data sources: Federal Reserve Bank of St. Louis and Moody's.

Liang, and Suarez (2013)).

2.2 Retail and Shadow Banks during the Crisis

The breakdown of the wholesale funding market had an especially severe impact on the shadow banking sector. In the first panel of Figure 2, we use data from Compustat to show that compared to the retail banking sector, the shadow banking sector was especially highly leveraged before the financial crisis. Our measure of leverage is the market leverage ratio, which we compute as the market value of equity plus the book value of current debt, long-term debt and deposits, divided by the market value of equity. We focus on market leverage and measure the net worth of the respective banking sectors as the market values of their equity, which is consistent with our model and in line with the literature, see e.g. Gertler and Kiyotaki (2015) and Gertler, Kiyotaki, and Prestipino (2019a).[6] This figure shows market leverage of retail banks as a blue, solid line and of shadow banks as a red, dotted line. In Appendix A.2, we explain which institutions we include in the retail and shadow banking sectors, respectively. We can see that market leverage of both retail banks and shadow banks increased during the financial crisis, peaking in the fourth quarter of 2008, and declining rapidly thereafter.

In the second panel of Figure 2, we furthermore show that the shadow banking sector was especially exposed to short-term debt, and experienced a much stronger outflow of short-term debt during the financial crisis than the retail banking sector. We show the share of short-term

6. There is some criticism in the literature regarding this choice: Adrian and Shin (2010), Adrian and Shin (2014) and Nuño and Thomas (2017) argue that book leverage is the relevant concept to look at when making statements about the effect of leverage and bank lending, and that book leverage is pro-cyclical, even when it is marked to market. He, Khang, and Krishnamurthy (2010) and He, Kelly, and Manela (2017) show that market leverage matters for asset prices, and that it is counter-cyclical.

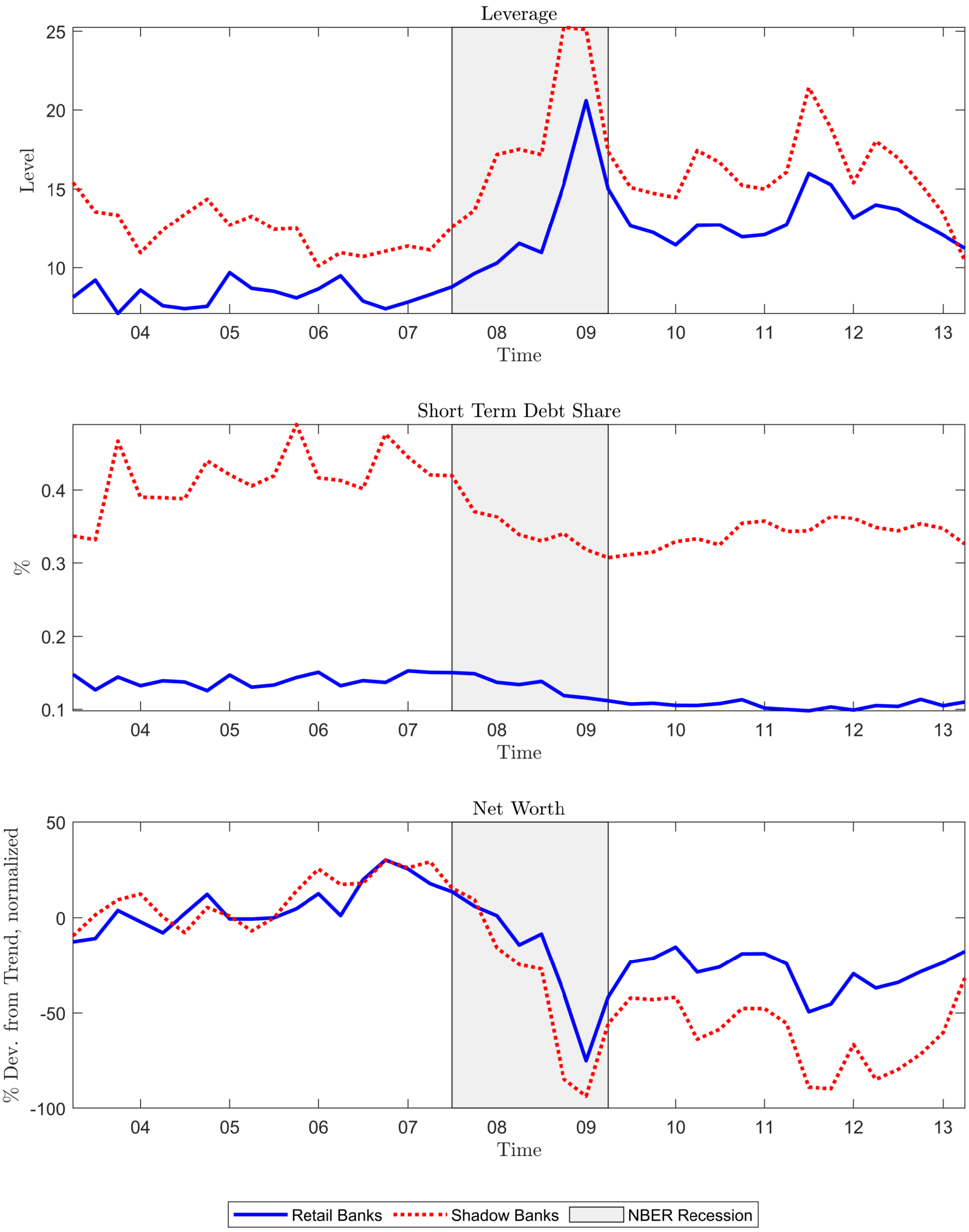

Figure 2: Capital Structure Dynamics of Retail and Shadow Banks

Note: Data source: Compustat. Retail banks are companies with SIC codes 602, 603 and 671, shadow banks are companies with SIC codes 614, 615, 616, 617, 620 and 621. Market leverage is the market value of equity plus the book value of short-term debt, long-term debt and accounts payable (i.e. deposits), divided by the market value of equity. The short-term debt share is short-term debt over short-term debt plus long-term debt plus accounts payable. Market capitalization is the market value of equity, computed as shares outstanding of common stock times the closing price for the quarter. For the last figure, we detrend market capitalization with a linear trend estimated using data from 1986Q1 to 2018Q4. We normalize it to zero in 2007Q3.

debt in total liabilities, which is short-term debt divided by short-term debt, long-term debt and deposits. Throughout, the short-term debt share of shadow banks is much higher than the short-term debt share of retail banks. Moreover, the short-term debt share of shadow banks starts decreasing in 2007 and continues to do so until the end of the financial crisis, decreasing overall by about a third. In comparison, the short-term debt share of retail banks is much more stable.

In the last panel of Figure 2, we show that the spike in leverage and the outflow of short-term debt of shadow banks coincided with a dramatic fall in net worth of the shadow banking sector. Relative to the beginning of the sample, net worth of shadow banks, which we again show as red, dotted line, fell by almost 100 percent. Net worth of the retail banking sector, however, also decreased dramatically by almost 60 percent.

2.3 Wholesale Funding Market Interventions by the Federal Reserve

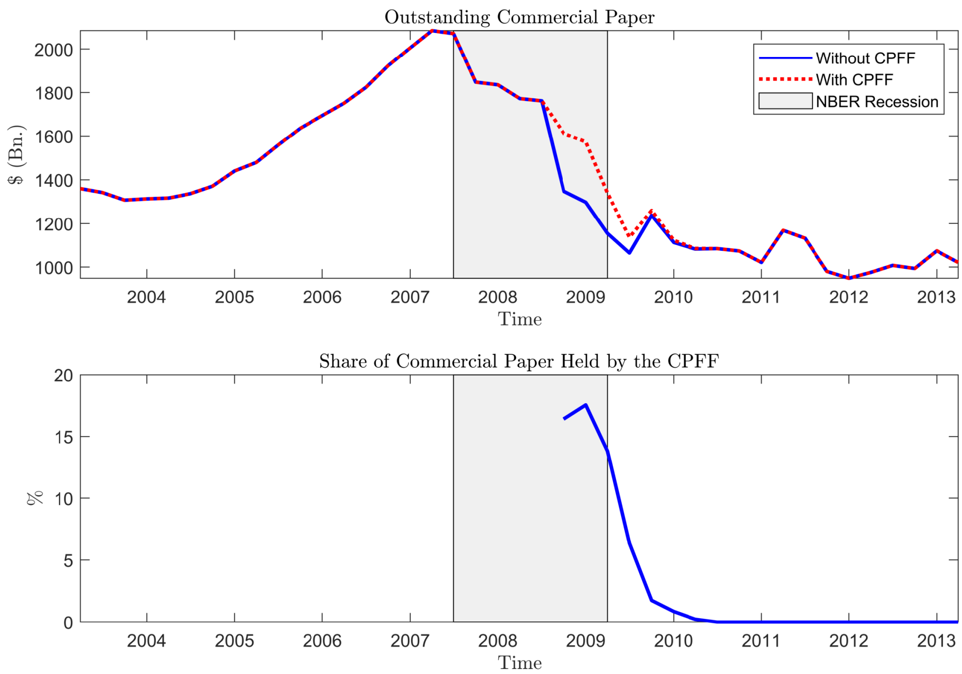

Figure 3: The Commercial Paper Funding Facility.

Note: Data source: Board of Governors of the Federal Reserve System

After the collapse of Lehman Brothers in September 2008, the Commercial Paper Funding Facility (CPFF) was introduced as a liquidity backstop in October 2008. This focus on the role as a liquidity backstop distinguished the CPFF from other programs, like the Asset-Backed Commercial Paper Money Market Mutual Fund Liquidity Facility (AMLF) and the Money Market Investor Funding Facility (MMIFF). In Figure 3, we show that the Federal Reserve intervention on the wholesale funding market was quantitatively large: At its peak, the central bank held more than 20 percent of all outstanding commercial paper, holding momentarily the fall in outstanding commercial paper since the beginning of the financial crisis.

We presented stylized facts that the financial crisis of 2007-2009 was associated with a severe disruption of the wholesale funding market, and that this disruption disproportionally affected shadow banks. Retail banks did, however, also experience a sharp fall in net worth and an increase in leverage. We also showed that in response to the contraction, the Federal Reserve intervened by providing a liquidity backstop on the wholesale funding market. In the next section, we are build a model that can both qualitatively and quantitatively account for these facts.

3 Model

In this section, we present a macroeconomic model with a detailed financial sector. The model is based on GKP2016 and GKP2019. The crucial feature of the model is a wholesale funding market, where retail and shadow banks can make loans to each other. As in reality, lending on the wholesale funding market is not covered by deposit insurance. Thus, a loss in lenders' confidence in the ability of borrowers to repay can lead to a roll-over crisis where the wholesale funding market dries up completely.

The rest of the model works as follows: There is a continuum of households, which each consist of a measure φ^R of retail bankers, a measure φ^S of shadow bankers and a measure $1 - \varphi^R - \varphi^S$ of workers. Workers consume, supply labor, invest into mutual funds and can make deposits at retail banks and shadow banks. Bankers make retail loans to consumption goods producers and wholesale loans to each other. Monopolistically competitive consumption goods producers produce intermediate goods using capital and labor, and set prices subject to price rigidities.[7] They finance capital purchases from capital goods producers with retail loans from banks and mutual funds. Intermediate goods are repackaged by a competitive final goods production sector. Capital goods producers transform final goods into capital goods subject to a capital adjustment cost.

Time is discrete, with $t = 0, 1, \dots, \infty$. We follow the convention that lower case letters for variables denote individual variables, while upper case letters denote aggregate variables.

3.1 Banks – Environment

We begin with an exposition of the problem of a bank. The rest of the model is quite standard.

3.1.1 Objective Function and Balance Sheet

Objective Function There are two types of bankers: retail bankers R and shadow bankers S. As we will make clear below, they differ in their cost function for making loans and in their ability to accumulate internal funds. Bankers of type J, $J \in \{R, S\}$ maximize expected discounted payouts to their household, which are given by

$$V_t^J = \mathbb{E}_t \Lambda_{t,t+1} (1 - p_{t+1}^{J,Default}) \left[\sigma^J n_{t+1}^J + (1 - \sigma^J) V_{t+1}^J \right], \tag{3.1}$$

7. We add price rigidities to the model, because, as Gertler, Kiyotaki, and Prestipino (2019a) show, they help the model to account for the depth and persistence of the financial crisis. Without price rigidities, it is difficult for the model to generate enough amplification to make banking panics possible.

where $\mathbb{E}_t$ denotes the expectation conditional on time t information. σ^J is an exogenous, type-specific exit probability and n^J_{t+1} is the net worth of the bank in period $t+1$. $\Lambda_{t,t+1}$ is the stochastic discount factor of the household to which the banker belongs between period t and period $t+1$. $p^{J,Default}_{t+1}$ is the probability that the bank defaults in period $t+1$. Intuitively, this payoff function implies that banks accumulate net worth until they exit, in which case they pay out their net worth to the households.[8]

Balance Sheet At time t, banks use deposit funding from households, d^J_{t+1}, and their equity, e^J_{t+1}, to finance retail loans to non-financial firms, a^J_{t+1}. For simplicity, we assume that there are no agency frictions between the banks and the ultimate borrowers. Therefore, retail loans can be interpreted as direct claims on the capital stock of the non-financial firms. Hence, these loans are valued at the market price of the non-financial firms' capital Q_t. Banks pay a bank-specific, linear loan-servicing fee f^J_t for outstanding retail loans at the end of period t to loan-servicing companies.[9] This loan-servicing fee can be interpreted as the cost of monitoring outstanding loans.

In addition to financing retail loans with deposits or equity, banks can borrow and lend on the wholesale funding market. We denote wholesale loans by b^J_{t+1}. $b^J_{t+1} > 0$ means that bank J lends on the wholesale funding market, while $b^J_{t+1} < 0$ denotes that bank J borrows on the wholesale funding market.

The balance sheet of a bank at the end of period t is given by

$$b_{t+1} + (Q_t + f^J_t)a^J_{t+1} = d^J_{t+1} + e^J_{t+1}. \tag{3.2}$$

Net Worth In period t, incumbent banks obtain a gross return on retail loans issued in period $t-1$, $R^A_t a^J_t$. Banks also pay a gross return from borrowing on the wholesale funding market $R^B_t b^J_t$. If they lend on the wholesale funding market, they receive a gross return from lending on the wholesale funding market, $\tilde{R}^B_t b^J_t$, with $\tilde{R}^B_t = \min\{x_t, 1\} R^B_t \leq R^B_t$ due to the default risk of wholesale borrowers.[10] x_t is the recovery rate on defaulted wholesale loans. Banks repay $R^D_t d^J_t$ to households for their deposits. A bank's net worth at the beginning of period t is thus given by

$$n^J_t = R^A_t a^J_t + \tilde{R}^B_t b^J_t - R^D_t d^J_t \tag{3.3}$$

if the bank lends on the wholesale funding market and

$$n^J_t = R^A_t a^J_t + R^B_t b^J_t - R^D_t d^J_t \tag{3.4}$$

if the bank borrows on the wholesale funding market. Since the banks cannot raise additional equity, their equity at the end of the period is equal to their net worth at the beginning of the

8. It is straightforward to show that whenever bankers face the risk of becoming financially constrained in the future, it is optimal for them not to pay out dividends.

9. Such companies are for example appraisal management companies, which determine the value of a property, or credit bureaus, which determine the credit-worthiness of a household.

10. The exact relationship between $\tilde{R}^B_t$ and R^B_t becomes clear once we introduce default in section 3.2.3.

period:

$$e_{t+1}^J = n_t^J. \tag{3.5}$$

Entry and Exit With probability σ^J, a banker of type J experiences an exit shock. In the case of such a shock, the banker sells the bank's assets, repays its liabilities and returns to being a worker, transferring the bank's net worth to the household. It is common to introduce such an exit probability in the literature (see e.g. Gertler and Kiyotaki (2011), Gertler and Karadi (2011)) to ensure that banks do not outsave their borrowing constraints. The payouts from banks to households can be interpreted as dividend payments. To keep the measure of bankers constant over time, a fraction of workers σ^J become bankers of type J, receiving an exogenous endowment $\tilde{n}_t^J = \upsilon K_t/\sigma^J$ from their household.[11]

We make the following assumption regarding the exit probability of retail and shadow banks:

Assumption 1. *The exit probability of retail banks is lower than the exit probability of shadow banks, i.e.* $\sigma^R < \sigma^S$.

This assumption is not essential for the qualitative results, but it allows us to match the relative size of the retail and the shadow banking sector quantitatively. An interpretation of this assumption is that retail banks pay lower dividends than shadow banks.

3.1.2 Financial Frictions

Moral Hazard Problem With the assumptions so far, banks would be financially unconstrained and the capital structure of banks would be indeterminate. To introduce a role for the capital structure of banks in the model, we follow Gertler and Kiyotaki (2011) in assuming the following moral hazard problem: Banks can divert a fraction of their assets after they have made their borrowing and lending decisions. How much they can divert depends on the type of assets. We make the following assumptions regarding the diversion of assets:

Assumption 2. *A fraction*

- ψ, $0 < \psi < 1$, *of retail loans,*
- $\gamma\psi$, $0 < \gamma < 1$, *of wholesale loans, and*
- $\omega\psi$, $0 < \omega < 1$, *of wholesale funded (securitized) retail loans*

can be diverted.

Assuming $\omega < 1$ is not essential for the results, but it helps us to match both the high leverage as well as the large holdings of unsecured short-term debt of shadow banks, which are two of the stylized facts we showed in section 2.

$\gamma < 1$ captures the benefits of holding wholesale loans relative to retail loans. It is a simple way to model that wholesale loans on the wholesale funding market trade at a lower premium

11. We scale the endowment of newly entering banks by the capital stock to ensure that the arguably stylized assumptions on entry do not affect the comparative statics through changes in the relative size of the endowment.

than even bonds issued by prime borrowers on the retail funding market, as we showed in section 2. In reality, there are a variety of reasons why wholesale loans are considered safer than retail loans: For example, there is a higher standardization of wholesale loan contracts compared to retail loan contracts, which means that there exists a market and therefore also a known market price for them. Hence, the potential for diversion is much higher for retail loans compared to wholesale loans.

Taken together, ω and γ capture the benefits of securitization. By securitizing loans first into asset-backed securities and then into collateralized debt obligations, wholesale borrowers created securities that could serve as collateral in wholesale funding markets like the asset-backed commercial paper markets. Those asset-backed securities were considered safe, and their market value was supposedly easy to verify for creditors (see e.g. Gorton and Metrick (2012)). The collateral underlying a retail loan can for example be commercial real estate or the physical capital stock of a firm, for which only a rough estimate of the market value exists.

3.2 Banks – Optimality

Incentive Constraint If a banker diverts assets, he will not repay his bank's liabilities. His creditors will subsequently force the bank to close down. Diversion occurs at the end of the period before the uncertainty about the next period is resolved. Therefore, creditors can ensure that diversion will never occur in equilibrium by imposing an incentive constraint on the banker. This incentive constraint states that the benefit of diversion to the banker must be smaller or equal to the franchise value of continuing to operate the bank. If the bank lends on the wholesale funding market, i.e. $b^J_{t+1} > 0$, the incentive constraint is

$$\psi\left[(Q_t + f^J_t)a^J_{t+1} + \gamma b^J_{t+1}\right] \leq V^J_t. \tag{3.6}$$

To create a^J_{t+1} units of retail loans, the bank must obtain financing $(Q_t + f^J_t)a^J_{t+1}$, from which it can divert a fraction ψ. To create b^J_{t+1} units of wholesale loans, the bank must obtain financing b^J_{t+1}, of which it can divert a fraction $\psi\gamma$.

If the bank borrows on the wholesale funding market, i.e. $b^J_{t+1} \leq 0$, the incentive constraint is instead

$$\psi\left[(Q_t + f^J_t)a^J_{t+1} + b^J_{t+1} - \omega b^J_{t+1}\right] \leq V^J_t. \tag{3.7}$$

To create a^J_{t+1} units of retail loans, the bank needs to obtain financing $(Q_t + f^J_t)a^J_{t+1}$. This financing can come from deposits and equity for the retail loans and from wholesale funding for the securitized retail loans. The bank can divert a fraction ψ of the non-securitized amount of the loans, but only a fraction $\psi\omega$ of the securitized amount.

The Leverage Ratio Define the leverage ratio of a bank as

$$\phi^J_t \equiv \frac{(Q_t + f^J_t)a^J_{t+1} + \gamma \max(b^J_{t+1}, 0)}{n^J_t}, \tag{3.8}$$

i.e. the fraction of bank assets that require some equity financing divided by the net worth, or equity, of the bank. Remember that a fraction $1 - \gamma$ of wholesale loans is non-divertable and hence does not require equity financing.

We guess (and verify) that the value function of the bank is linear in its net worth: $V_t^J = \Omega_t^J n_t^J$. Ω_t^J is the unit franchise value of the bank. With this in mind, and using equation 3.8, we can rewrite equations 3.6 and 3.7 as leverage constraints. If the bank is a wholesale lender, the incentive constraint is

$$\phi_t^J \leq \frac{\Omega_t^J}{\psi}. \tag{3.9}$$

If the bank is a wholesale borrower, the incentive constraint is instead

$$\phi_t^J \leq \frac{\Omega_t^J}{\psi\omega} - \frac{1-\omega}{\omega}. \tag{3.10}$$

3.2.1 Optimal Retail Banker Decisions

We consider an equilibrium where retail bankers issue deposits and lend on both the retail and the wholesale funding markets, whereas shadow banks do not issue deposits and borrow on the wholesale funding market. We also abstract from the default risk of retail banks: $p_{t+1}^{R,default} = 0$ at all times. We now characterize the optimal decision rules of bankers and their creditors in this equilibrium.

The maximization problem of the retail banker is to choose ϕ_t^R, b_{t+1}^R, and d_{t+1}^R to maximize their objective function 3.1 subject to the balance sheet constraint 3.2, the law of motion for net worth 3.3 and the occasionally binding incentive constraint 3.9. Denote as $\mu_t^{R,IC}$ the Lagrange multiplier on the incentive constraint of the retail bank. In the case of a non-binding incentive constraint, and given that it is optimal for retail banks to lend on both markets, the first order conditions are given by

$$\mu_t^{IC,R} = \frac{1-\psi}{\psi}\mathbb{E}_t\tilde{\Omega}_{t+1}^R\left[\frac{R_{t+1}^A}{Q_t + f_t^R} - R_{t+1}^D\right] = 0 \tag{3.11}$$

and

$$\mu_t^{IC,R} = \frac{1-\psi}{\psi\gamma}\mathbb{E}_t\tilde{\Omega}_{t+1}^R\left[\tilde{R}_{t+1}^B - R_{t+1}^D\right] = 0. \tag{3.12}$$

$\tilde{\Omega}_{t+1}^R \equiv \Lambda_{t,t+1}\left[\sigma^R + (1-\sigma^R)\Omega_{t+1}^R\right]$ is the stochastic discount factor of the retail banker. Equation 3.12 implies that retail banks pass through their own borrowing costs, given by R_{t+1}^D, to the wholesale funding markets. The credit spread between the wholesale funding rate and the deposit rate is a premium for the risk-adjusted expected loss in default in case of a shadow bank default. Using $\tilde{R}_{t+1}^B = x_{t+1}R_{t+1}^B$, we can rearrange equation 3.12 to write the spread as

$$\begin{aligned}\Delta_{t+1}^B &\equiv R_{t+1}^B - R_{t+1}^D \\ &= \left(\frac{1}{\mathbb{E}_t x_{t+1}} - 1\right)R_{t+1}^D - \frac{cov\left(\tilde{\Omega}_{t+1}^R, x_{t+1}\right)}{\mathbb{E}_t\tilde{\Omega}_{t+1}^R\mathbb{E}_t x_{t+1}},\end{aligned} \tag{3.13}$$

where x_{t+1} is the recovery rate on wholesale loans, which we define below. $cov(x, y)$ is the covariance between x and y. The first term is a premium for the expected loss in default, the second term a premium for the co-movement of the loss in default and the stochastic discount factor of the retail banking sector. Similarly, we can define a credit spread on loans made to non-financial firms by retail banks as

$$
\begin{aligned}
\Delta_{t+1}^{A,R} &\equiv \mathbb{E}_t \frac{R_{t+1}^A}{Q_t + f_t^R} - R_{t+1}^D \\
&= -\frac{cov\left(\tilde{\Omega}_{t+1}^R, \frac{R_{t+1}^A}{Q_t+f_t^R}\right)}{\mathbb{E}_t \tilde{\Omega}_{t+1}^R}.
\end{aligned} \tag{3.14}
$$

Figure 4 shows the leverage of the retail bank ϕ_t^R as well as the Lagrange multiplier $\mu_t^{R,IC}$ and the credit spreads Δ_{t+1}^B and $\Delta_{t+1}^{A,R}$ as a function of the net worth of the retail and the shadow banking sectors. In the upper left panel, we show the leverage implied by the incentive constraint 3.9 or the first order condition 3.11. We can see from the upper right panel that the incentive constraint does not bind if the net worth of the retail banking sector or the net worth of the shadow banking sector are high. The credit spreads are consequently low.

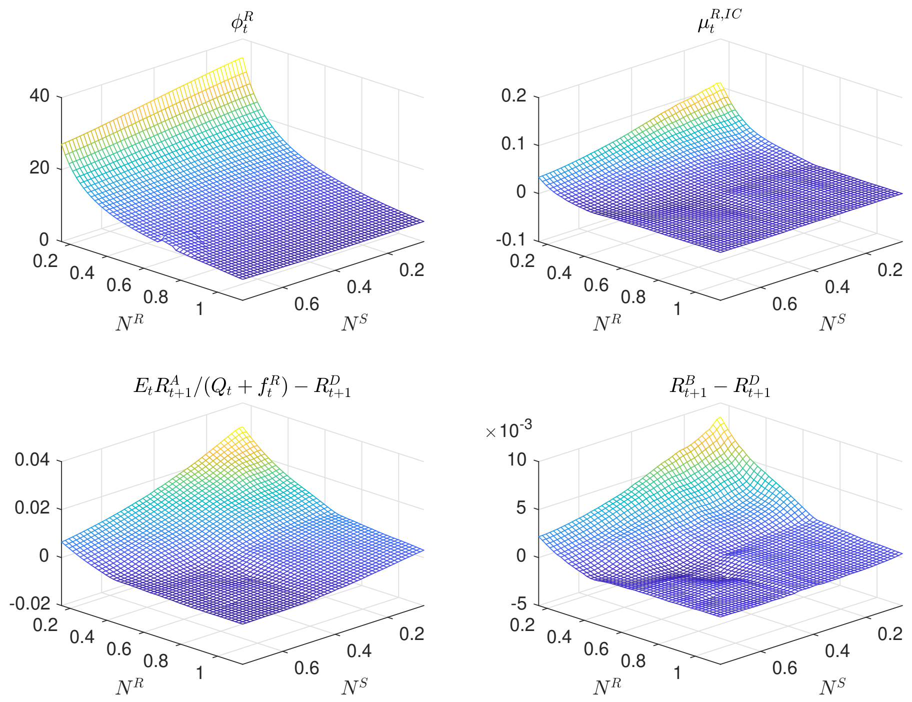

Figure 4: Policy Functions of the Retail Bank as a Function of its Net Worth

Consider next the case of a binding incentive constraint or capital requirement, such that leverage is given by

$$
\phi_t^R = \frac{\Omega_t^R}{\psi}. \tag{3.15}
$$

It is optimal for retail banks to lend on both the retail and the wholesale funding markets, if the following condition holds:

$$
\mathbb{E}_t \tilde{\Omega}_{t+1}^R \left[\tilde{R}_{t+1}^B - R_{t+1}^D\right] = \gamma \mathbb{E}_t \tilde{\Omega}_{t+1}^R \left[\frac{R_{t+1}^A}{Q_t + f_t^R} - R_{t+1}^D\right] \tag{3.16}
$$

Equation 3.16 states that the excess return for lending a unit of wholesale loans must equal the excess return of lending γ units of retail loans; To finance an additional unit of wholesale loans, the retail bank must give up γ units of retail loans, if the leverage constraint binds. Thus, the financial constraints of the retail banks, which are the lenders on the wholesale funding market, will be reflected in the wholesale credit spread. By rearranging equation 3.16, we can write the wholesale funding credit spread Δ^B_{t+1} as

$$\Delta^B_{t+1} = \underbrace{\left(\frac{1}{\mathbb{E}_t x_{t+1}} - 1\right) R^D_{t+1} - \frac{cov\left(\tilde{\Omega}^R_{t+1}, x_{t+1}\right)}{\mathbb{E}_t \tilde{\Omega}^R_{t+1} \mathbb{E}_t x_{t+1}}}_{\text{Default Premium}} + \gamma \underbrace{\frac{\mathbb{E}_t \tilde{\Omega}^R_{t+1}\left[\frac{R^A_{t+1}}{Q_t + f^R_t} - R^D_{t+1}\right]}{\mathbb{E}_t \tilde{\Omega}^R_{t+1} x_{t+1}}}_{\text{Liquidity Premium}}. \tag{3.17}$$

This spread has two components: As before, in the case of the non-binding incentive constraint, it contains a premium for the risk-adjusted loss in default. Moreover, it contains a liquidity premium which reflects the financial constraints of the retail banks in the wholesale funding market. The parameter γ determines to what extent the financial constraints of lenders are reflected in the wholesale funding credit spread: For $\gamma = 0$, they are not reflected in the credit spread at all, whereas for $\gamma = 1$, they are fully incorporated in the credit spread. As we see in Figure 4, the incentive constraint starts to bind if the net worth of retail banking sector or the net worth of the shadow banking sector is low, implying a positive Lagrange multiplier and a positive credit spread on the wholesale funding market.

3.2.2 Optimal Shadow Banker Decisions

Shadow bankers do not issue deposits and borrow on the wholesale funding market. Their maximization problem is to choose ϕ^S_t and b^S_{t+1} to maximize 3.1 subject to the incentive constraint 3.10, the law of motion for net worth 3.4 and the balance sheet constraint 3.2. We focus on the case where the incentive constraint is always binding, which will be the relevant case in our calibration below. Hence, leverage is determined by the incentive constraint:

$$\phi^S_t = \frac{1}{\psi\omega}\Omega^S_t - \frac{1-\omega}{\omega}. \tag{3.18}$$

Plugging 3.4, 3.2 and 3.10 into 3.1 and rearranging, equation 3.18 yields the following expression for shadow bank leverage:

$$\begin{aligned}
\phi^S_t &= \frac{\mathbf{E}_t \tilde{\Omega}^S_{t+1} R^B_{t+1} - \psi(1-\omega)}{\psi\omega - \mathbf{E}_t \tilde{\Omega}^S_{t+1}\left(\frac{R^A_{t+1}}{Q_t} - R^B_{t+1}\right)} \\
&= \frac{\mathbf{E}_t \tilde{\Omega}^S_{t+1} R^B_{t+1} - \psi(1-\omega)}{\psi\omega - \mathbf{E}_t \tilde{\Omega}^S_{t+1}\left(\Delta^{A,S}_{t+1} - \Delta^B_{t+1}\right)}.
\end{aligned} \tag{3.19}$$

$\tilde{\Omega}^S_{t+1} \equiv \Lambda_{t,t+1}(1-p^{S,Default}_{t+1})\left[\sigma^S + (1-\sigma^S)\Omega^S_{t+1}\right]$ is the stochastic discount factor of the shadow banker, adjusted for the default probability $p^{S,Default}_{t+1}$ and the value of an additional unit of net worth in the next period, $\left[\sigma^S + (1-\sigma^S)\Omega^S_{t+1}\right]$. $\Delta^{A,S}_{t+1} \equiv \frac{R^A_{t+1}}{Q_t} - R^D_{t+1}$ is the credit spread for loans made to the non-financial sector by the shadow banking sector. Equation 3.19 implies that shadow bank leverage is increasing in the credit spread for retail loans and decreasing in the credit spread for wholesale loans. An increase in the banking panic probability also lowers leverage by lowering $\tilde{\Omega}^S_{t+1}$. Leverage is furthermore increasing in the diversion parameter ω.

3.2.3 Optimal Rollover Decision of Shadow Bankers' Creditors

For ease of exposition, and since we abstract from the default risk of retail banks, we only characterize the rollover decision of shadow bankers' creditors. We focus on the case where a bank that exclusively borrows on the wholesale funding market defaults, since these were the more relevant runs in the financial crisis of 2007-2008.[12]

Illiquidity and Default The incentive constraint 3.10 implies that lenders are not willing to lend to a shadow banker with a negative net worth, because for any positive amount of lending, the incentive constraint would be violated. If a creditor would nevertheless lend to such a shadow banker, the shadow banker would choose to divert assets and default on the debt.

By the definition of bank net worth, a negative net worth also means that the assets of the bank are insufficient to cover its liabilities: $n^S_t \leq 0 \iff R^A_t a^S_t + R^B_t b^S_t \leq 0$. Hence, a bank with a negative net worth cannot access external funds and does not have enough internal funds to repay its liabilities. It is illiquid and will default on its liabilities.

If a bank defaults, it is liquidated. Creditors will recover the assets of the bank instead of their wholesale loan. The recovery rate on their lending is given by

$$x_t = \frac{R^A_t a^S_t}{|R^B_t b^S_t|}. \tag{3.20}$$

If the bank is illiquid, the creditors do not recover their claim in full: $n^S_t < 0 \Rightarrow x_t < 1$.

Equilibrium Multiplicity Since there are no idiosyncratic shocks to bank net worth, either all or no incumbent shadow banks default. Consider two returns on retail loans R^A_t and R^{A*}_t, where $R^A_t \geq R^{A*}_t$. R^A_t is the equilibrium return in normal times when shadow banks operate, R^{A*}_t the equilibrium return when shadow banks are in default and do not operate. The associated recovery rates implied by equation 3.20 are given by x_t and x^*_t, with $x_t \geq x^*_t$.

We can distinguish three situations: If $x_t \geq 1$ and $x^*_t \geq 1$, it is always optimal for creditors to roll over their debt. Using the terminology of Cole and Kehoe (2000), the economy is in the no-crisis zone. If $x_t \geq 1$ and $x^*_t < 1$, it is optimal for a creditor to roll over his debt if all other creditors do so, and to not roll over if no other creditor does. The economy is in the crisis zone. Finally, if $x_t < 1$ and $x^*_t < 1$, it is not optimal for the creditor to roll over his debt no matter what, and the economy is in the default zone. Hence, the rollover policy of a shadow

12. Moreover, abstracting from deposit panics, which can be ruled out due to deposit insurance, defaults on deposits do not occur in simulations of the calibrated model.

bank creditor in period t is given by

$$\begin{cases} \text{Do not roll over} & \text{if } x_t < 1 \text{ and } x_t^* < 1 \\ \text{Do not roll over} & \text{if } x_t \geq 1 \text{ and } x_t^* < 1 \text{ and all other creditors do not roll over.} \\ \text{Roll over} & \text{if } x_t \geq 1 \text{ and } x_t^* < 1 \text{ and all other creditors roll over.} \\ \text{Roll over} & \text{if } x_t \geq 1 \text{ and } x_t^* \geq 1. \end{cases} \tag{3.21}$$

Banking Panics and Sunspots A banking panic is a situation where all incumbent shadow banks default and all shadow bankers having entered the economy in the current period postpone entry until the next period.[13] A banking panic is a situation of coordination failure: By coordinating on not to roll over their debt, creditors drive the shadow banking system into default, even though the shadow banking sector would have been able to service its debt if creditors coordinated to roll it over. This situation may arise if the economy is in the crisis zone or the default zone.

The coordination mechanism of shadow bank creditors is a sunspot shock: If the economy is in the crisis zone, the optimal strategy of each shadow bank creditor is to roll over his debt when all others do so and to not roll over his debt when nobody else does. Agents will decide not to roll over their debt if they observe a sunspot shock, which occurs with probability π. Otherwise, they will roll over their debt. Thus, the probability of a banking panic is given by the probability of the economy being in the crisis zone times the probability of a sunspot:

$$\mathbb{E}_t p_{t+1}^{S,Panic} = \pi \mathbb{E}_t \mathbb{1}(x_{t+1}^* < 1). \tag{3.22}$$

This probability is endogenous and state-dependent. We can rewrite x_t^* as

$$x_t^* = \frac{\frac{R_t^A}{Q_t}}{R_t^D} \frac{R_t^D}{R_t^B} \frac{\phi_{t-1}^S}{\phi_{t-1}^S - 1}. \tag{3.23}$$

Hence, the probability of a banking panic is increasing in leverage, decreasing in the credit spread on retail loans and increasing in the credit spread on wholesale loans. Agents will correctly anticipate whether a banking panic can occur or not. The probability of a systemic bank default is the sum of the probability of the economy being in the default zone plus the probability of the economy being in the crisis zone and experiencing a banking panic.

$$\mathbb{E}_t p_{t+1}^{S,Default} = \mathbb{E}_t p_{t+1}^{S,Panic} + (1 - \pi) \mathbb{E}_t \mathbb{1}(x_{t+1} < 1). \tag{3.24}$$

Notice the subtle difference between a systemic bank default and a banking panic: A banking panic occurs only if there is both a systemic bank default and if newly entering shadow banks decide to postpone entry.

Figure 5 shows the probability of a banking panic $\mathbb{E}_t p_{t+1}^{S,Panic}$, as well as shadow bank leverage and the credit spreads on wholesale loans and non-financial loans as a function of the net worth

13. The latter assumption is crucial, since newly entering shadow banks would otherwise take over a large share of the assets of incumbent shadow banks, substantially reducing the scope for equilibrium multiplicity. It is common in the literature (Gertler and Kiyotaki (2015), Gertler, Kiyotaki, and Prestipino (2016), Gertler, Kiyotaki, and Prestipino (2019a), and Gertler, Kiyotaki, and Prestipino (2019b)).

of the retail banking sector and the net worth of the shadow banking sector. We see that

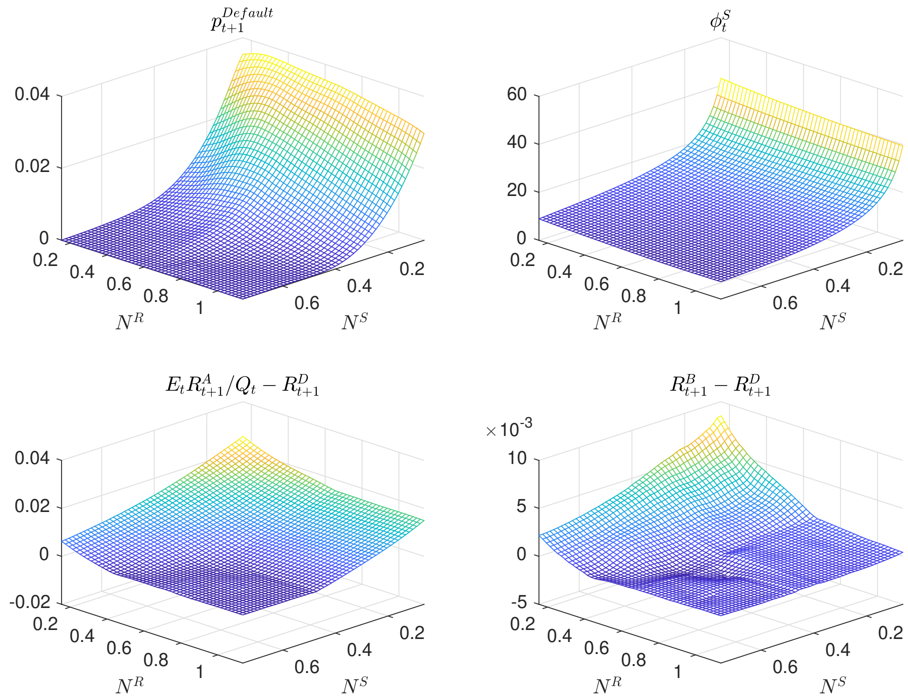

Figure 5: The Default Probability of Shadow Banks as a Function of Retail and Shadow Bank Net Worth.

the probability of a banking panic is monotonically decreasing in the net worth of the shadow banking sector. It is non-monotonic in the net worth of the retail banking sector: For a high net worth of the retail banking sector, when the incentive constraint of the retail banking sector is not binding, an increase in retail bank net worth lowers the probability of a banking panic. For low values of net worth of the retail banking sector, when the incentive constraint is binding, an increase in retail bank net worth increases the probability of a banking panic. This is, because with a binding incentive constraint, an increase in retail bank net worth relaxes the incentive constraint of the retail banking sector, which lowers the credit spread on the wholesale funding market as we showed in Figure 4. This increases the likelihood of a banking panic, as it reduces the excess return of shadow banks. As we showed in equation 3.23, there is a second, positive effect: A increase in retail bank net worth leads to a reduction of the wholesale funding spread, which increases the excess return of shadow banks and thereby reduces the likelihood of a banking panic. Taken together, these two effects lead to the non-monotonic relationship between the net worth of the retail banking sector and the shadow banking panic probability. Both of these effects arise only if both retail and shadow banks face financial constraint, and the second effect arises only if retail banks and shadow banks are linked through the wholesale funding market, highlighting the importance of these two assumptions.

3.3 Households

Preferences Households maximize utility from consumption. Their utility function is given by

$$\mathbb{E}_t\left[\sum_{s=t}^{\infty}\beta^{s-t}\left[U(c_s^H)-G(l_s^H)-\sum_{J\in H,R}\zeta^J(\tilde{A}_{t+1}^J,A_{t+1})\right]\right], \tag{3.25}$$

where β is the discount factor of the household. c_t^H denotes household consumption, l_t^H labor supply in period t. $U(c)$ is the current utility function of the household from consumption, $G(l)$ the disutility of labor. $\zeta^J(\tilde{A}^J, A)$ is a utility loss due to the effort of loan monitoring of sector J.

The stochastic discount factor of the household between period t and $t+s$ is given by

$$\Lambda_{t,t+s}=\beta^{s-t}\frac{U'(c_{t+s}^H)}{U'(c_t^H)}.$$

Household Budget Constraint Households consume and make deposits d_{t+1}^H at banks. They supply labor and receive W_t as labor income. They invest a_{t+1}^H into mutual funds at price Q_t and subject to a fee f_t^H, receiving a return R_t^A on past investments.[14] In addition, they own the banks and firms and receive their profits Π_t.[15] Deposits yield a safe gross return R_t^D. The budget constraint of the household is

$$c_t^H+d_{t+1}^H+(Q_t+f_t^H)a_{t+1}^H=R_t^Dd_t^H+W_tl_t^H+\Pi_t. \tag{3.26}$$

The maximization problem of the household is to choose c_t^H, l_t^H, d_{t+1}^H and a_{t+1}^H to maximize 3.25 subject to 3.26. The first order conditions of the household are given by

$$Q_t+f_t^H=\mathbb{E}_t\Lambda_{t,t+1}R_{t+1}^A \tag{3.27}$$

$$1=\mathbb{E}_t\Lambda_{t,t+1}R_{t+1}^D \tag{3.28}$$

$$G'(l_t^H)=U'(c_t^H)W_t. \tag{3.29}$$

Loan-Servicing Firms For the retail loans on their balance sheet at the end of the period, banks and households have to pay a loan-servicing cost. Households, retail banks and shadow banks have access to different loan-servicing technologies. Loan-servicing is provided by specialized firms, owned by households, which operate in a competitive industry. Servicing loans requires effort, which we model as a utility cost to the owner of the loan-servicing firm. The effort cost is quadratically increasing in the total amount of loans serviced, $\tilde{A}_{t+1}^J$.[16] It is given by

$$\zeta^J(\tilde{A}_{t+1}^J,A_{t+1})=\eta^J\left(\max\left\{\frac{\tilde{A}_{t+1}^J}{A_{t+1}}-\zeta^J,0\right\}\right)^2A_{t+1} \tag{3.30}$$

14. We introduce direct intermediation, since, in its absence, deposits would be the only asset that households have access to, which would complicate the equilibrium in the market for deposits.

15. Profits of capital producers are 0 in steady state, but may arise outside of the steady state due to the capital adjustment cost.

16. There is one unit of monitoring per unit of loans made: $\tilde{A}_{t+1}^J=A_{t+1}^J$.

Loan-servicing firms charge a linear fee f_t^J to the banks and households for their services. The objective function of these firms is given by:

$$V_t^{L,J} = f_t^J U'(c_t^H)\tilde{A}_{t+1}^J - \eta^J \left(\max\left\{ \frac{\tilde{A}_{t+1}^J}{A_{t+1}} - \zeta^J, 0 \right\} \right)^2 A_{t+1},$$

where f_t^J is the loan-servicing fee per unit of the loan. The fee f_t^J is taken as given by households and banks, and is determined in equilibrium such that the loan-servicing firms are willing to service all loans of the banks. It is given by

$$f_t^J = \frac{\eta^J}{U'(c_t^H)} \left(\max\left\{ \frac{\tilde{A}_{t+1}^J}{A_{t+1}} - \zeta^J, 0 \right\} \right). \tag{3.31}$$

Regarding the cost of screening, we make the following assumption:

Assumption 3. *Shadow banks have lower loan-servicing costs than retail banks. Households have the highest loan-servicing costs:* $\zeta^H < \zeta^R < \zeta^S$.

In the calibrated model, we choose a ζ^S high enough for $f_t^S = 0$ at all times. The result of this assumption is that no single sector is efficient enough to provide lending to the entire economy at zero cost. η is a crucial parameter, since it controls by how much asset prices need to fall for the retail banking sector and the household sector to be willing to absorb the liquidated assets of the shadow banking sector if the latter defaults.

3.4 Production

The production side of the economy is standard and follows GKP2019. Final goods producers repackage inputs from intermediate goods producers. Capital goods producers transform final goods into capital goods.

3.4.1 Final Goods Producers

Competitive final goods producers repackage intermediate goods $y_t(i)$, which they purchase from a continuum of intermediate goods producers i at price $p_t(i)$, to produce output Y_t according to the following production function:

$$Y_t = \int_0^1 \left(y_t(i)^{\frac{\varepsilon-1}{\varepsilon}} di \right)^{\frac{\varepsilon}{\varepsilon-1}}. \tag{3.32}$$

Cost minimization yields a demand for intermediate good i given by

$$y_t(i) = \left(\frac{p_t(i)}{P_t} \right)^{-\varepsilon} Y_t, \tag{3.33}$$

where P_t is a composite price index given by

$$P_t = \left[\int_0^1 p_t(i)^{1-\varepsilon} di \right]^{\frac{1}{1-\varepsilon}}. \tag{3.34}$$

3.4.2 Intermediate Goods Producers

Intermediate goods producers choose labor $l_t(i)$ and capital $k_t(i)$ to produce intermediate goods $y_t(i)$ at the minimal cost. They set a price $p_t(i)$, subject to a quadratic Rotemberg (1982) price adjustment cost with parameter ρ^R, taking the demand function from final goods producers as given. Their production function is

$$y_t(i) = k_t(i)^\alpha l_t(i)^{1-\alpha}. \tag{3.35}$$

Cost minimization yields a marginal cost function

$$mc = \frac{1}{\mathcal{M}_t} = \left(\frac{r_t^A}{\alpha}\right)^\alpha \left(\frac{W_t}{1-\alpha}\right)^{1-\alpha}, \tag{3.36}$$

where $\mathcal{M}_t$ is the markup over marginal cost, $r_t^A = R_t^A - (1-\delta)Q_t$ is the user cost of capital, and W_t is real wage. The intermediary goods producers choose prices $p_t(i)$ to maximize

$$\sum_{s=t}^{\infty} \Lambda_{t,s} \left[p_s(i) y_s(i) - \frac{1}{\mathcal{M}_s} y_s(i) - \frac{\rho^R}{2} \left(\frac{p_s(i)}{p_{s-1}(i)} - 1 \right) Y_s \right], \tag{3.37}$$

subject to the demand for the intermediate good by the final goods producer 3.33. All intermediate goods producers choose the same price, such that $p_t(i) = P_t$. The first order condition of the intermediate goods producers with respect to the optimal price is given by

$$(\pi_t - 1)\pi_t - \frac{\varepsilon}{\rho^R}\left(\frac{1}{\mathcal{M}_t} - \frac{\varepsilon - 1}{\varepsilon}\right) = \mathbb{E}_t \Lambda_{t,t+1} \frac{Y_{t+1}}{Y_t} (\pi_{t+1} - 1)\pi_{t+1}, \tag{3.38}$$

where $\pi_t \equiv \frac{P_t}{P_{t-1}}$ is the gross inflation rate.

The intermediate goods producers own the capital stock. Since they are all identical, they all hold the same amount of capital: $k_t(i) = K_t$. The aggregate capital stock follows the law of motion

$$S_{t+1} = (1-\delta)K_t + X_t, \tag{3.39}$$

$$K_t = Z_t S_t. \tag{3.40}$$

with depreciation rate δ and investment X_t. We distinguish between capital at the end of $t-1$, S_t, and capital at the beginning of t, K_t. The distinction arises because of Z_t, which is a capital quality shock that creates exogenous variation in the value of capital, following Merton (1973) and Gertler and Karadi (2011). It can be interpreted as a fraction of the capital stock losing its economic value. It follows the law of motion

$$\ln Z_t = \rho^Z \ln Z_{t-1} + \varepsilon_t^Z,$$

with $\varepsilon_t^Z \sim N(-(\sigma^Z)^2, \sigma^Z)$.

Capital purchases are financed using state-contingent retail loans. Hence, the balance sheet constraint of the intermediate goods producers is

$$A_{t+1}^{H}+A_{t+1}^{R}+A_{t+1}^{S}=S_{t+1}. \tag{3.41}$$

3.4.3 Capital Goods Producers

Capital goods producers use a technology which transforms I_t units of final goods into X_t units of capital goods. They face a concave production function, which we specify following Jermann (1998) and Boldrin, Christiano, and Fisher (2001):

$$X_t=\left(\frac{\theta_1}{1-\theta_0}\left(\frac{I_t}{K_t}\right)^{1-\theta_0}+\theta_2\right)K_t. \tag{3.42}$$

The production function is scaled by the aggregate capital stock K_t, which the capital goods producers take as given. The capital goods producers maximize profits with respect to I_t, which are given by

$$\Pi_t^{Q}=Q_tX_t-I_t, \tag{3.43}$$

subject to 3.42. The first order condition from the solution of the capital goods producers' problem yields the following expression for the price of capital:

$$Q_t=\frac{1}{\theta_1}\left(\frac{I_t}{K_t}\right)^{\theta_0}. \tag{3.44}$$

θ_0 is the elasticity of the capital price to the investment-capital ratio. Due to the concave production function, the capital goods producers may earn non-zero profits outside the steady state. They are owned by the households and any profits or losses are transferred to the households in each period.

3.5 Policy, Aggregation and Market Clearing

3.5.1 Monetary Policy

The central bank sets the nominal interest rate R_{t+1}^{N} according to a rule which responds to inflation π_t with elasticity κ^{π} and a measure of the output gap with elasticity κ^{y}:

$$R_{t+1}^{N}=\frac{1}{\beta}\pi_t^{\kappa^{\pi}}\left(\frac{\mathcal{M}_t}{\mathcal{M}_t^{n}}\right)^{\kappa^{y}}. \tag{3.45}$$

$\mathcal{M}_t^{n}=\frac{\varepsilon}{\varepsilon-1}$ is the optimal markup of the intermediate goods producers absent price rigidities. We use this measure instead of the actual output gap, since we have a closed form expression for optimal markup absent price rigidities, but not for the natural rate of output Y_t^{n}. This markup is a measure of the labor market wedge due to nominal rigidities.

3.5.2 Aggregation

There is no idiosyncratic uncertainty for households, such that we can consider the problem of a representative household. Moreover, since the policy functions of an individual bank are linear in net worth, we will characterize the equilibrium in terms of the aggregate decisions of the banking sectors. The aggregate net worth of the retail and shadow banking sectors is given by the sum of the net worth of incumbent and newly entering banks:

$$N_t^R = \max\left\{R_t^A A_t^R + R_t^B B_t^R - R_t^D D_t^R, 0\right\}(1-\sigma^R) + \upsilon K_t \tag{3.46}$$

$$N_t^S = \begin{cases} \max\left\{R_t^A A_t^S + R_t^B B_t^S, 0\right\}(1-\sigma^S) + \upsilon K_t & \text{if no panic occurs} \\ 0 & \text{if a panic occurs} \\ \upsilon K_t + (1-\sigma^S)\upsilon K_{t-1} & \text{in the period after a panic.} \end{cases} \tag{3.47}$$

Aggregate profits are given by the profits of screening firms $\Pi_t^{L,H} + \Pi_t^{L,R}$, intermediate goods producers Π_t^F, and capital goods producers Π_t^Q, plus the sum of the net worth of exiting retail banks and shadow banks minus the net worth of entering banks:

$$\Pi_t = \Pi_t^Q + \Pi_t^F + \sigma^R n_t^R + \sigma^S n_t^S + \Pi_t^{L,H} + \Pi_t^{L,R} - 2\upsilon K_t \tag{3.48}$$

Aggregate output is given by the production function:

$$Y_t = K_t^\alpha L_t^{1-\alpha}. \tag{3.49}$$

3.5.3 Market Clearing

The markets for retail loans,

$$S_{t+1} = A_{t+1}^H + A_{t+1}^R + A_{t+1}^S \tag{3.50}$$

labor,

$$L_t^H = L_t \tag{3.51}$$

deposits,

$$D_{t+1}^H = D_{t+1}^R \tag{3.52}$$

wholesale loans,

$$0 = B_{t+1}^R + B_{t+1}^S \tag{3.53}$$

investment

$$X_t = S_{t+1} - (1-\delta)K_t \tag{3.54}$$

and loan services

$$A_{t+1}^J = \tilde{A}_{t+1}^J, \quad J \in \{H, R\} \tag{3.55}$$

have to clear. Since there is a representative household, the individual consumption and aggregate consumption are equal, $c_t^H = C_t^H$. Household consumption can be inferred from the aggregate resource constraint:

$$C_t^H = Y_t - I_t - G - \frac{\rho^R}{2}(\pi_t - 1)^2 Y_t, \tag{3.56}$$

where G denotes government consumption.

4 Calibration

For our main results, we solve the model numerically. To do so, we calibrate the model to match key moments of the US economy during the 1986-2018 period at a quarterly frequency. Since we solve a complicated non-linear model, estimating it is infeasible. We therefore divide the parameters into three blocks. Parameters in the first block include the technology, preference and policy parameters. We use conventional values for those parameters. The second block of parameters are those for the financial sector. We set them to match the stochastic steady state of the model to long-run data averages. The data for these targets are credit spreads as well as data from the financial accounts of the US and Compustat. The third block of parameters are specific to bank runs or specify the exogenous stochastic processes. We internally calibrate those parameters to match the business cycle properties and dynamics of the 2008 financial crisis.

4.1 Parameters

Name	Value	Interpretation	Target or source
(a) Preferences			
σ	2	Utility of consumption	Risk aversion = 2
ϕ	0.5	Disutility of labor	Frisch elasticity of labor supply = 2
β	0.9902	Household discount factor	Real interest rate 4% p.a.
μ	1.3172	Disutility of labor	Labor supply = 1 in SS
(b) Technology			
α	0.36	Capital share in production	Capital income share = 36%
δ	0.025	Depreciation rate	Annual deprecation rate = 10%
θ_0	0.25	Capital adjustment cost	Elasticity of investment to capital price = 25%
θ_1	0.5302	Capital adjustment cost	Capital price in steady state = 1
θ_2	-0.0083	Capital adjustment cost	$X_t = I_t$ in steady state
ε	11	Elasticity of substitution btw varieties	Markup = 10%
ρ^R	1000	Price adjustment cost	Elasticity of inflation to marginal cost = 1%
ρ^Z	0.7	Autocorrelation, productivity	$\rho(Y_t, Y_{t-1}) = 0.9$
σ^Z	0.005	Standard deviation, productivity shock	$\sigma(Y_t) = 0.03$
(c) Government and monetary policy			
G	0.2473	Government consumption	Government consumption share in output = 20 %
κ^Y	0.125	Weight on output in Taylor rule	Standard value
κ^π	1.5	Weight on inflation in Taylor rule	Standard value
(d) Financial sector			
γ	0.4234	Diversion benefit of wholesale lending	Change in TED spread in crisis: = 4.1% p.a.
ζ^H	0.1777	Household bank capital holding cost	AAA-10Y treasury spread = 1.35% p.a.
ζ^R	0.4366	Retail bank capital holding cost	BAA-10Y treasury spread = 2.33% p.a.
σ^R	0.0488	Retail bank exit rate	$K^R/K = 0.45$
σ^S	0.0894	Shadow bank exit rate	$K^S/K = 0.35$
ψ	0.2778	Asset diversion share	$\phi^R = 10$
ω	0.7182	Diversion benefit of wholesale funding	$\phi^S = 15$
υ	0.0005	Banks' initial equity	Change in investment in crisis: -32.3% p.a.
η^H	0.25	Household capital holding cost	Change in AAA-10Y spread in crisis: 1.2% p.a.
η^R	0.175	Retail bank capital holding cost	Change in retail bank net worth in crisis: -68.5% p.a.
π	0.04	Sunspot probability	Crisis freq. of $\approx$ 1% per quarter

Table 1: Calibration.

Preferences. We list the preference parameters in panel (a) of Table 1. Regarding functional forms, we model the utility of consumption as

$$U(c) = \frac{c^{1-\sigma}}{1-\sigma} \tag{4.1}$$

and the disutility of labor as

$$G(l) = \mu \frac{l^{1+\phi}}{1+\phi}. \tag{4.2}$$

We choose the curvature of the utility of consumption to imply a risk aversion σ of the households of 2, and the curvature of the disutility of labor ϕ to imply a Frisch elasticity of 2. We set the discount rate of households β to target an annual risk-free real interest rate of 4 percent. We set the remaing labor disutility parameter μ to normalize labor to 1 in steady state.

Technology. The parameters in panel (b) of Table 1 describe the technology. The production function curvature α is set to match a capital share in GDP of 36 percent. The quarterly depreciation rate of capital δ is set to 0.025, implying an annual depreciation rate of 10 percent. We set the capital adjustment cost parameter θ_0 to 0.25, such that the elasticity of the capital price to investment is 25 percent. This is in line with estimates from, for example Eberly, Rebelo, and Vincent (2012). We choose the other two adjustment cost parameters θ_1 and θ_2 such that the price of capital in the stochastic steady state is 1 and the quarterly investment-to-capital ratio is 2.5 percent per quarter in steady state. We set the elasticity of substitution across varieties of intermediate goods to imply a markup of 10 percent, which follows Del Negro et al. (2017) and Gertler, Kiyotaki, and Prestipino (2019a). We set the price adjustment cost parameter ρ^R to get an elasticity of inflation to marginal cost of 0.1 percent, in line with Gertler, Kiyotaki, and Prestipino (2019a). We choose ρ^Z and σ^Z to match the conditional volatility and the autocorrelation of detrended GDP for the United States.

Government. Regarding government and monetary policy parameters, which we list in panel (c), we set G to match a share of government consumption in GDP in steady state of 20 percent, and the parameters of the Taylor rule to match an elasticity of the policy rate to inflation of 1.5 percent and to the markup of 0.125 percent, in line with Del Negro et al. (2017) and Gertler, Kiyotaki, and Prestipino (2019a).

Financial Sector. The parameters in panel (d) of Table 1 are specific to the financial sector. We target leverage ratios of 10 and 15 for retail banks and shadow banks, respectively, to calibrate the diversion parameters ψ and ω. This corresponds to the leverage ratios of retail banks and shadow banks in Compustat before the crisis. We choose the remaining diversion parameter γ to match the increase in the TED spread during the financial crisis. We set the exit shock probabilities σ^R and σ^S such that the share of assets intermediated by retail banks and shadow banks in steady state are 45 and 35 percent. The former corresponds to the share of retail bank assets of total assets of the financial sector in Compustat. The latter is higher than the share of shadow bank assets in Compustat, but corresponds to the size of the shadow banking sector used by Gertler, Kiyotaki, and Prestipino (2019a) and Begenau and Landvoigt

(2018). For the capital-holding cost parameters ζ^H and ζ^R, we target the spreads of the Moody's BAA yield and the Moody's AAA yield over a 10-year treasury bond.[17] We map the expected return on assets of retail banks to the return on AAA-rated bonds and the expected return on assets of shadow banks to the return on BAA-rated bonds to capture the fact that retail banks were lending mostly to prime borrowers, whereas shadow banks were also lending to subprime borrowers (Pozsar et al. (2012)). We set the banks' endowments υ to target the fall in investment during the financial crisis. We set the remaining holding cost parameters η^H and η^R to match the increase in the Moody's AAA spread and the fall in retail bank net worth during the financial crisis. Finally, we set the probability of the sunspot to target a frequency of financial crises of 4 percent per year, in line with Jordà, Schularick, and Taylor (2011).

4.2 Solution Method

We solve the model non-linearly with a projection method on a sparse grid. To construct the sparse grid, we use the toolbox of Judd et al. (2014). Solving the model using global methods has three key advantages: First, it allows us to accurately characterize the dynamics of the economy very far away from steady state. This is important, since a financial crisis will wipe out the net worth of the shadow banking sector and substantially reduce the net worth of the retail banking sector below its steady state level. Second, the non-linear solution allows us to accurately compute risk premiums in the model. This is crucial, since asset price dynamics are key for generating financial crises in the model. Third, and most importantly, banking panics introduce substantial non-linearity due to endogenous, time-varying uncertainty into the model, which perturbation methods cannot capture. Details of the solution algorithm are found in Appendix C.

4.3 Model Fit

Table 2 shows how well the baseline model matches the targeted moments. For comparison, we report in the last column the results of an alternative model in which retail banks do not face financial frictions. This latter model is qualitatively very similar to the model of Gertler, Kiyotaki, and Prestipino (2019a). Overall, we can see that the baseline model matches the targeted moments well, which is not obvious, given that it is a complicated non-linear model

17. We compute the yields on 10-year treasuries, AAA-rated and BAA-rated bonds in the model as the geometric average of short rates over a 10-year horizon to match the maturity of the bonds in the model more closely to the maturity of the bonds in the data. The corresponding formulas are given by

$$R^D_{t,t+40} = \left[\mathbb{E}_t \prod_{\tau=0}^{39} R^D_{t+\tau,t+1+\tau}\right]^{\frac{1}{40}}, \tag{4.3}$$

for 10 year treasuries, as well as by

$$R^{AAA}_{t,t+40} = \left[\mathbb{E}_t \prod_{\tau=0}^{39} \frac{R^A_{t+\tau,t+1+\tau}}{Q_{t+\tau} + f^R_{t+\tau}}\right]^{\frac{1}{40}}, \tag{4.4}$$

$$R^{BAA}_{t,t+40} = \left[\mathbb{E}_t \prod_{\tau=0}^{39} \frac{R^A_{t+\tau,t+1+\tau}}{Q_{t+\tau}}\right]^{\frac{1}{40}}, \tag{4.5}$$

for AAA-rated and BAA-rated bonds, respectively.

with multiple equilibria and occasionally binding constraints. The model without financial frictions of retail banks can by construction not match the AAA-10Y spread, the TED spread, and retail bank leverage. It also produces a much smaller increase in credit spreads as well as a much smaller fall in investment and retail bank net worth in a panic. Moreover, the frequency of banking panics is much lower, despite shadow bank leverage being higher. The BAA-10Y credit spread is also much lower than in the baseline model. These results give us a first indication that modelling retail banks explicitly is important for the propagation of banking panics.

	Data	Baseline	No R-IC
St. Dev., Output	2.450	2.350	1.872
Autocorrelation, Output	0.973	0.877	0.910
Mean, AAA-10Y Spread	1.354	1.658	0.031
Mean, BAA-10Y Spread	2.330	2.641	1.977
Mean, Retail Bank Asset Share	0.450	0.445	0.462
Mean, Shadow Bank Asset Share	0.350	0.357	0.342
Mean, Retail Bank Leverage	10.000	9.700	1.000
Mean, Shadow Bank Leverage	15.000	11.065	13.093
Frequency of Banking Panics	4.089	4.523	0.174
TED Spread Increase in Run	4.133	5.284	0
AAA-10Y Spread Increase in Run	1.208	0.697	0
Fall in Retail Net Worth in Run	-68.545	-76.516	-3.813
Fall in Investment in Run	-32.307	-30.542	-20.866

Table 2: Model Fit: Targeted Moments

Note: The model moments come from a simulation of the baseline model of 1000 economies for 2000 periods, discarding the first 1000 periods as burn-in. The data for output, consumption, investment, hours, and net worth are detrended with a quadratic trend estimated using data from 1986Q1 to 2018Q4. The model and the data for the AAA-10Y spread, the BAA-10Y spread and the TED spread are annualized.

Table 3 displays business cycle moments. Again, we compare the baseline model to the data and an alternative model in which retail banks do not face financial frictions. We can see that the model produces plausible volatilities and business cycle correlations and autocorrelations of macroeconomic aggregates, asset prices and financial sector variables. In particular, the model matches the fact that bank net worth is pro-cyclical and that credit spreads are counter-cyclical. It also matches the volatilities of these variables well.

The TED spread in the data is pro-cyclical, which is counter-intuitive. The reason is that it did not increase during the 1990 and 2001 recessions, which were not accompanied by financial crises. As we are primarily interested in providing a model of the 2008 financial crisis, this is not a big concern.

Hours in the model are weakly counter-cyclical, and even more so in the model without retail bank financial constraints. As we show below, they are pro-cyclical on impact, but then overshoot after a few periods. Introducing wage stickiness or using Greenwood et al. (1988)-preferences would help to overcome this issue, albeit at the price of complicating the model somewhat. It is, however, not central for the main results.

	Data	Baseline	No R-IC
St. Dev.			
Output (Y)	2.450	2.350	1.872
Consumption (C_H)	2.881	2.545	2.357
Investment (I)	10.858	7.827	4.602
Hours (L)	3.805	3.191	2.553
Retail Bank Net Worth (N_R)	22.827	24.059	4.696
Shadow Bank Net Worth (N_S)	42.939	40.788	31.930
AAA-10Y Spread ($\mathbb{E}\frac{R^A}{Q} - R^D$)	0.459	0.164	0.005
BAA-10Y Spread ($\mathbb{E}\frac{R^A}{Q+f_R} - R^D$)	0.712	0.254	0.118
TED Spread ($R^B - R^D$)	0.403	0.673	0.003
Corr. W. GDP			
Output (Y)	1.000	1.000	1.000
Consumption (C_H)	0.952	0.769	0.879
Investment (I)	0.820	0.775	0.639
Hours (L)	0.629	-0.026	-0.360
Retail Bank Net Worth (N_R)	0.475	0.984	0.687
Shadow Bank Net Worth (N_S)	0.778	0.967	0.956
AAA-10Y Spread ($\mathbb{E}\frac{R^A}{Q} - R^D$)	-0.206	-0.801	0.951
BAA-10Y Spread ($\mathbb{E}\frac{R^A}{Q+f_R} - R^D$)	-0.137	-0.772	-0.839
TED Spread ($R^B - R^D$)	0.328	-0.617	-0.244
Autocorr.			
Output (Y)	0.973	0.877	0.910
Consumption (C_H)	0.985	0.994	0.993
Investment (I)	0.962	0.740	0.736
Hours (L)	0.984	0.858	0.908
Retail Bank Net Worth (N_R)	0.779	0.843	0.933
Shadow Bank Net Worth (N_S)	0.918	0.869	0.870
AAA-10Y Spread ($\mathbb{E}\frac{R^A}{Q} - R^D$)	0.910	0.748	0.971
BAA-10Y Spread ($\mathbb{E}\frac{R^A}{Q+f_R} - R^D$)	0.891	0.765	0.823
TED Spread ($R^B - R^D$)	0.805	0.573	0.631

Table 3: Model Fit: Business Cycle Statistics

Note: The model moments come from a simulation of the baseline model of 1000 economies for 2000 periods, discarding the first 1000 periods as burn-in. The data for output, consumption, investment, hours, and net worth are detrended with a quadratic trend estimated using data from 1986Q1 to 2018Q4. The model and the data for the AAA-10Y spread, the BAA-10Y spread and the TED spread are annualized.

5 The Macroeconomic Effects of a Shadow Banking Panic

Figure 6 shows the ability of the model to quantitatively reproduce the dynamics of key macroeconomic aggregates, banking sector balance sheet variables and asset prices during the US financial crisis of 2008. We conduct an experiment similar to Gertler, Kiyotaki, and Prestipino (2019a). We start the model in the stochastic steady state in the second quarter of 2004, when the output gap in the data was close to zero. We then feed in a sequence of shocks to match aggregate investment from the third quarter of 2004 to the third quarter of 2008. In the fourth quarter of 2008, we hit the economy with an additional shock that is just large enough to push it into the crisis zone, where the recovery value on wholesale loans in a panic is below one, such that both the panic and the no panic equilibria are possible. We then compute the impulse response of the economy if a banking panic occurs (the blue line), and the impulse response if no panic occurs (the red line). The data are represented by the black, solid line.

We can see that the panic probability rises once the negative shocks start to hit the economy. Even in the fourth quarter of 2008, though, a banking panic is a tail event: the probability of it occurring is less than 5 percent. The multiplier of retail banks, which measures whether the incentive constraint of retail banks binds, falls to zero after the initial positive shocks, and increases rapidly thereafter. The resulting low credit spreads on the wholesale funding market produces an additional build-up of leverage, which increases the likelihood of the banking panic occurring later. Once the panic happens, we see that there is substantial amplification of the shock in the sense that output, investment, consumption, and hours in the model simulation with the banking panic fall more than in the model simulation without the banking panic. In that way, the model can reproduce the fall in these variables quantitatively without relying on exceptionally large shocks.

The model accounts well for the dynamics of the TED spread, the BAA-10Y spread and the AAA-10Y spread after the crisis. However, it has trouble matching the historically very low AAA-10Y and BAA-10Y spreads before the crisis. Credit spreads might have been low due to economic mechanisms that we abstract from, for example due to investor sentiment (López-Salido, Stein, and Zakrajšek (2017)) or expectational errors (Bordalo, Gennaioli, and Shleifer (2018), Gertler, Kiyotaki, and Prestipino (2019a)). The leverage and the net worth dynamics of both retail and shadow banks before the crisis are matched well. After the crisis, leverage of both retail and shadow banks increases to very high values that the model cannot match. This is because net worth of the shadow banking sector in the model falls to zero and recovers only very slowly afterwards, whereas it recovers much faster in the data. Adding equity issuance as in Akinci and Queralto (2017) or Gertler, Kiyotaki, and Prestipino (2019b) to the model would help to resolve this issue without substantially affecting the pre-crisis results. Another way to resolve this issue would be to use a different incentive constraint, e.g. the one proposed in Adrian and Shin (2014), which has been used in Nuño and Thomas (2017) and Rottner (2020).

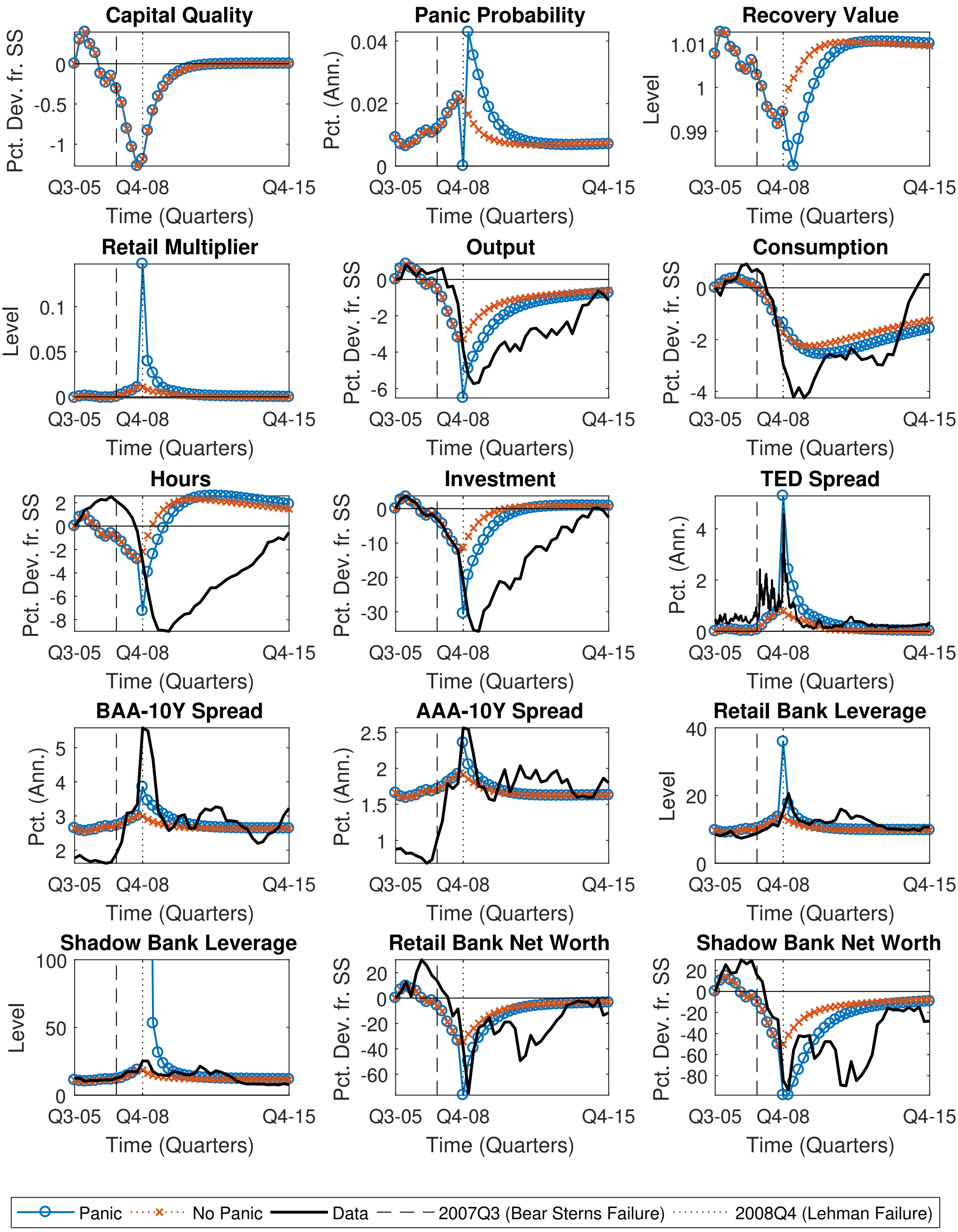

Figure 6: A banking panic in the model and in the data.

6 Financial Amplification Due To Retail Bank Constraints

6.1 Amplification during Normal Times

Figure 7 shows that financial constraints of retail banks amplify the endogenous response to exogenous shocks substantially. The blue line with circles shows the impulse responses to a one standard deviation negative capital quality shock in the baseline model, and the red line with crosses shows the alternative model in which retail banks are never financially constrained.

The response to a shock of the same size of all variables is substantially amplified in the baseline model compared to the alternative model. The mechanism is as follows: In the baseline model, the negative capital quality shock leads to a fall in the net worth of both retail and shadow banks. As a result, both sectors cut lending to the non-financial sector. Moreover, retail banks also cut lending on the wholesale funding market, which drives up the funding cost of shadow banks, forcing them to cut lending even further. In that sense, financial constraints of retail banks have a both a direct effect and an indirect wholesale credit supply effect working through the wholesale funding market on lending to the non-financial sector. Note also that the panic probability in the alternative model is on average much lower and is also substantially less responsive to the shock than in the baseline model.

6.2 The Role of Financial Constraints of Retail Banks During the Crisis

Figure 8 shows impulse responses of the exercise in Figure 6 for the baseline model and the alternative model without financial constraints of retail banks. We start both models in the respective stochastic steady state, feeding in the same sequence of capital quality shocks, and letting a run happen in the fourth quarter of 2008. In the model without financial constraints of retail banks, the recovery value does not fall enough to trigger a banking panic, so the panic we show for that model is an off-equilibrium event.

We can see that even conditional on a run, output, investment, consumption and hours fall much less in the alternative model without financial frictions of retail banks. This is because unconstrained retail banks increase lending substantially when the shadow banking sector fails. There is also less amplification in response to the positive shocks at the beginning of the sample. Note that this is despite the leverage of the shadow banking sector being higher in the alternative model. The banking panic probability is much lower throughout the sample.

7 The Macroeconomic Effects of Liquidity Interventions

As described in section 2, the Federal Reserve intervened on the wholesale funding markets shortly after the collapse of Lehman Brothers. In this section, we investigate the macroeconomic effects of such liquidity interventions.

We assume that the central bank can lend on the wholesale funding market, B_{t+1}^{CB}. The market-clearing condition on the wholesale funding market then becomes

$$B_{t+1}^{CB} + B_{t+1}^{R} + B_{t+1}^{S} = 0. \tag{7.1}$$

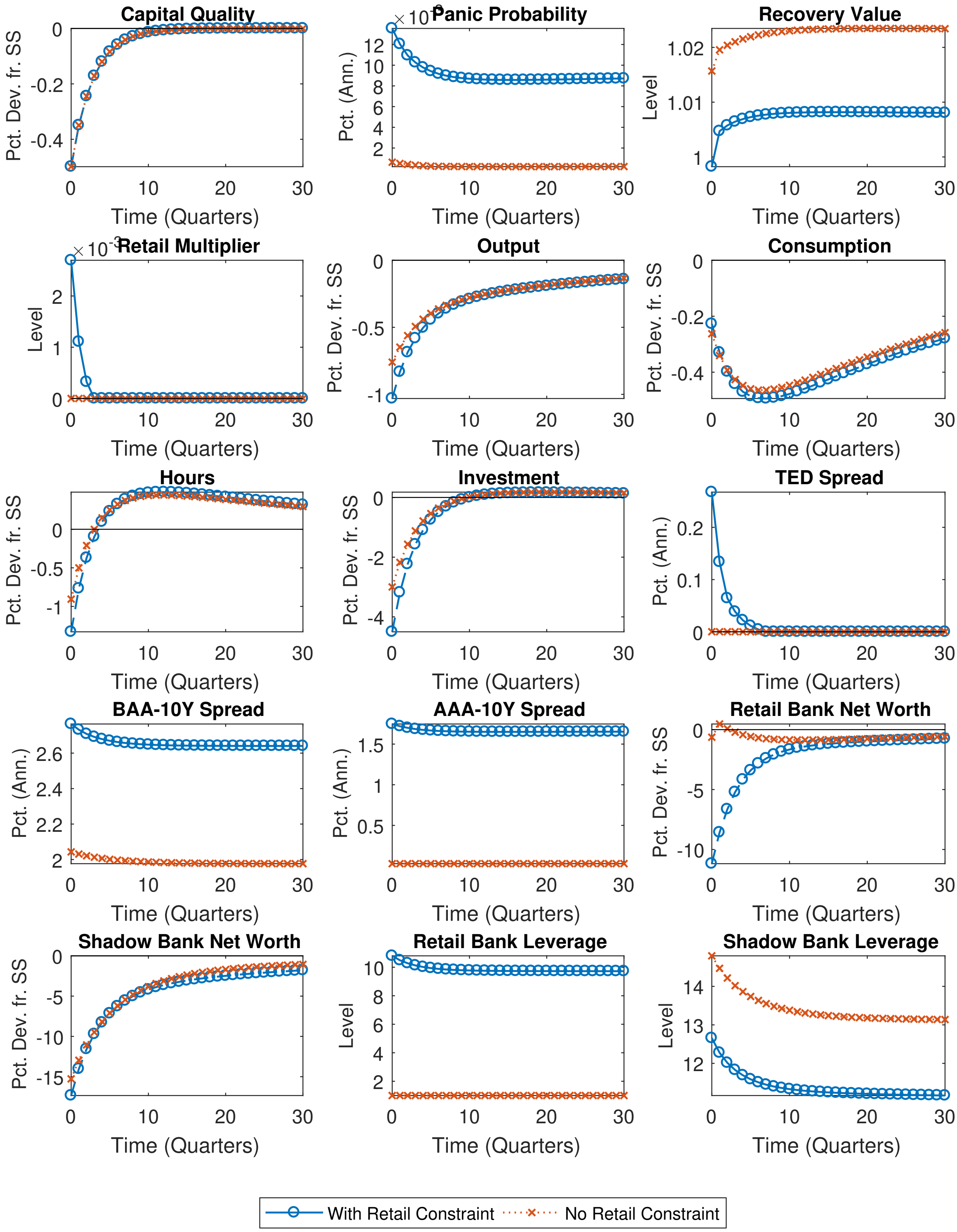

Figure 7: Amplification due to financial constraints of retail banks.

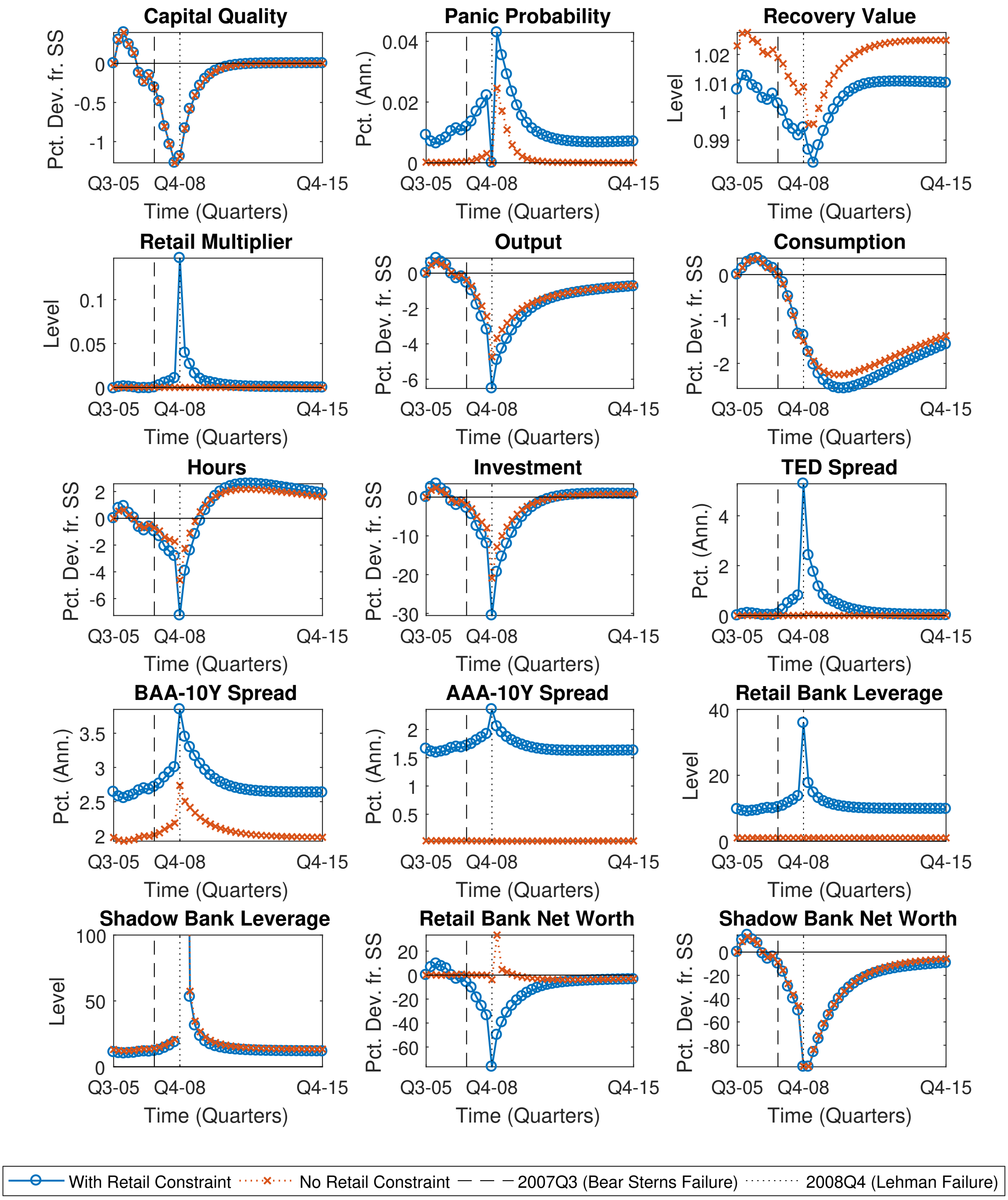

Figure 8: Amplification due to financial constraints of retail banks in the run experiment.

The central bank finances its loans on the wholesale funding market by issuing deposits. For simplicity, we assume that the central bank can raise deposits directly from households.[18] The balance sheet of the central bank is then given by

$$B_{t+1}^{CB} = D_{t+1}^{CB}. \tag{7.2}$$

In contrast to the retail and shadow banks, the central bank does not face a moral hazard problem. Thus, it does not need to finance a fraction of its lending with equity. The central bank can therefore provide lending at a lower rate than the retail banks: $R_{t+1}^{B,CB} < R_{t+1}^{B}$. However, the central bank faces a cost of conducting intermediation in the wholesale funding market. Otherwise, it would be optimal for the central bank to take over the entire wholesale funding market at all times. We model this cost as a utility cost for the household. We assume that the marginal cost of intervening on the wholesale funding market consists of a constant and a linear term:

$$mc(B_{t+1}^{CB}, B_{t+1}) = c_0 + c_1 \frac{B_{t+1}^{CB}}{B_{t+1}}. \tag{7.3}$$

The goal of the central bank is to choose $R_{t+1}^{B,CB}$ and B_{t+1}^{CB} to maximize household utility subject to its balance sheet constraint, equation 7.2. The central bank chooses the the terms of its intervention according to

$$\mathbb{E}_t \Lambda_{t,t+1} \left(x_{t+1} R_{t+1}^{B,CB} - R_{t+1}^{D} \right) = 0 \tag{7.4}$$

$$\mathbb{E}_t \Lambda_{t,t+1} \left(\tilde{R}_{t+1}^{B} - R_{t+1}^{D} \right) - c_0 - c_1 \frac{B_{t+1}^{CB}}{B_{t+1}} \geq 0, \tag{7.5}$$

$$B_{t+1}^{CB} \geq 0, \tag{7.6}$$

$$\left[\mathbb{E}_t \Lambda_{t,t+1} \left(\tilde{R}_{t+1}^{B} - R_{t+1}^{D} \right) - c_0 - c_1 \frac{B_{t+1}^{CB}}{B_{t+1}} \right] B_{t+1}^{CB} = 0. \tag{7.7}$$

That is, the central bank intervenes if the intervention implied by 7.5 is non-negative. It sets the interest rate to ensure that it does not make losses in expectation, and targets the size of the intervention to trade off the reduction of spreads on the wholesale funding market with the utility cost of lending. We pin down the parameters c_0 and c_1 to ensure that the central bank does not intervene in steady state and that the intervention in the period after a banking panic is around 17 percent of outstanding commercial paper, similar to the size of the commercial paper funding facility at its peak.

7.1 The Short-Run Effect of an Unanticipated Intervention

We first consider an unanticipated intervention by the central bank. For that purpose, we start a simulation of the economy in the fourth quarter of 2008, assuming that a banking panic has occurred in that period. We then simulate the model forward using the policy rules and laws of

18. Alternatively, we could assume that the central bank raises deposits from retail banks, who in turn raise deposits from households. If the retail banks cannot divert central bank deposits, they can finance them completely with household deposits, which is equivalent to assuming that the central bank can borrow directly from households.

motion of the model with the central bank intervention, and compare them to the baseline model without intervention. In Figure 9, we show the difference in the impulse responses between the baseline model, which are the same as in Figure 6, and the alternative model with the liquidity intervention.

The intervention starts in the period after the banking panic, when the wholesale funding market becomes active again, and stops when credit spreads on the wholesale funding markets return to normal levels. The intervention in the model matches the one in the data very well. The anticipation of the intervention does, however, already have an effect in the period of the banking panic.

As a consequence of the intervention, credit spreads on the wholesale funding market fall by about 1 percent. The AAA-10Y and the BAA-10Y spreads fall respectively by about 5 basis points. The small fall in these credit spreads is due to us computing 10-year spreads, which are not very volatile. The effects on the real economy are substantial: Due to the intervention, output and employment rise by about 0.3 percent, consumption by 0.1 percent and investment by about 1.5 percent.

Leverage of retail banks decreases temporarily, and then increases as the central bank intervention stops. Leverage of shadow banks decreases. The banking panic probability increases temporarily, which is explained by the lower credit spread, which lowers the profitability of shadow banks after a run.

7.2 The Long-Run Effects of a Permanent Liquidity Facility

Table 4 compares statistics from simulations of the baseline model to those from a model where the central bank intervenes whenever there is stress in the wholesale funding market.

	Baseline	Policy	% Change
Mean, Output	1.225	1.226	0.058
Mean, Consumption	0.734	0.734	0.043
Mean, Investment	0.242	0.243	0.171
St. Dev., Output	2.350	2.308	-1.784
St. Dev. Consumption	2.545	2.553	0.311
St. Dev. Investment	7.827	7.417	-5.228
Mean, Retail Bank Leverage	9.700	9.776	0.786
Mean, Shadow Bank Leverage	11.065	11.369	2.751
Frequency of Banking Panics	4.661	4.181	-10.295

Table 4: The Long-Run Effects of a Permanent Liquidity Facility.

Note: The moments come from a simulation of the baseline model of 1000 economies for 2000 periods, discarding the first 1000 periods as burn-in.

Unconditionally, switching to a regime with a permanent liquidity facility that becomes active whenever there is stress on the wholesale funding markets can reduce output volatility by about 1.8 percent or 0.04 percentage points. Consumption volatility increases slightly, but investment volatility decreases markedly by about 5 percent or 0.4 percentage points. The

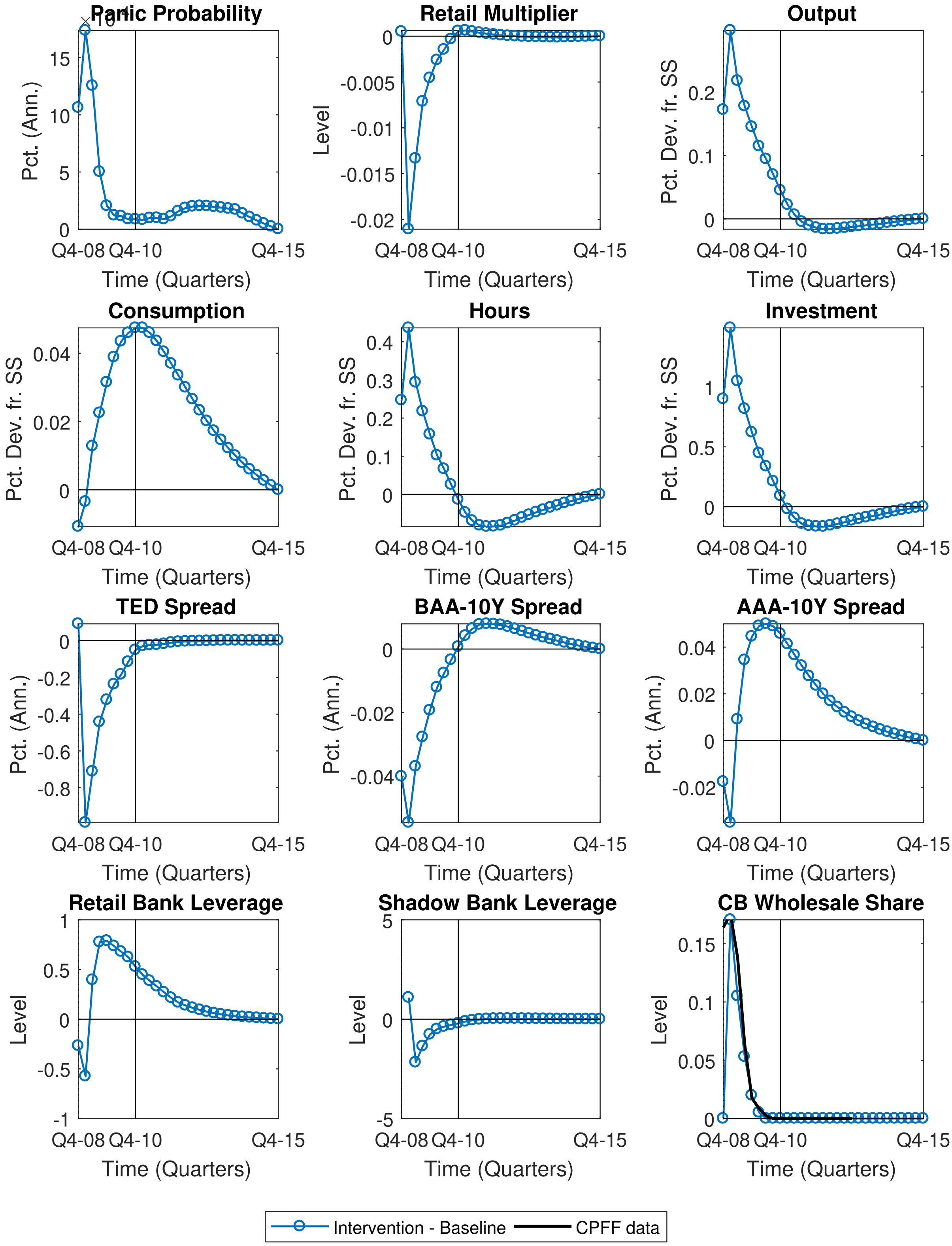

Figure 9: The Effects of a Central Bank Intervention on the Wholesale Funding Market.

frequency of banking panics decreases by about 10 percent or 0.5 percentage points per year. Note that this reduction of volatility occurs despite an increased leverage in both the retail banking sector and the shadow banking sector. Finally, the lower probability of banking panics leads to a small increase in the level of consumption, output and investment.

8 Conclusion

We study the macroeconomic effects of a banking panic in a quantitative macroeconomic model with retail banks and shadow banks, connected through a wholesale funding market. In our model, banking panics take the form of rollover crises on the wholesale funding market. The model is quantitatively consistent with the dynamics of macroeconomic variables and asset prices during the US financial crisis. In particular, due to occasionally binding financial constraints of retail banks becoming binding, the model produces a slow run period with elevated credit spreads on the wholesale funding market, followed by a fast run. We show that the slow run makes the fast run more likely by deteriorating the balance sheets of shadow banks.

We discuss the policy implications. A government intervention as lender of last resort on the wholesale funding market, similar to the Commercial Paper Funding Facility instituted by the Federal Reserve System in October 2008, can reduce credit spreads and relax the financial constraints of both retail and shadow banks. This in turn reduces the likelihood and severity of banking panics.

An interesting extension of our model would be to include nominal debt. A bank run could then result in a Fisher (1933)-style debt deflation spiral: The initial effects of the run depresses goods prices, which worsens the real debt burden of banks, which in turn depresses investment, and so on. Bank runs can then lead to episodes that cause the economy to be at the lower bound of the nominal policy interest rate. In this case, the possibility of bank runs will also affect how monetary policy should be conducted.

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Appendix

For Online Publication

A Data

A.1 Data Sources

Name	Data	Source
Macroeconomic Data		
Output	Real Gross Domestic Product	BEA
Investment	Real Gross Private Domestic Investment	BEA
Consumption	Real Personal Consumption Expenditures	BEA
Hours	Non-farm Business Sector: Hours of All Persons	BLS
BAA-10Y Spread	Moody's Seasoned Baa Corporate Bond Yield Relative to Yield on 10-Year Treasury Constant Maturity	Federal Reserve Bank of St. Louis and Moody's
AAA-10Y Spread	Moody's Seasoned Aaa Corporate Bond Yield Relative to Yield on 10-Year Treasury Constant Maturity	Federal Reserve Bank of St. Louis and Moody's
TED Spread	TED Spread	Federal Reserve Bank of St. Louis
Data from Compustat		
Market Value of Equity	Common Shares Outstanding $\times$ Price (Close) – Quarter	Compustat
Short-Term Debt	Debt in Current Liabilities	Compustat
Long-Term Debt	Long-Term Debt – Total	Compustat
Accounts Payable	Accounts Payable	Compustat

For the figures, output, investment, consumption, and hours are detrended using CBO potential estimates for output and hours, normalizing the deviations from trend to zero at the start of the figures in 2004Q2. For the tables, we detrend them using a log-quadratic trend estimated using data from 1986Q1 to 2018Q4. We also detrend the market value of equity using a log-quadratic trend for both figures and tables, normalizing it to zero in 2004Q2.

A.2 Definition of Retail and Shadow Banks in Compustat

We define retail and shadow banks in Compustat as described in Table 5.

B Full Statement of the Equilibrium Conditions

Denote variables in the no-run equilibrium by X_t and variables in the run equilibrium by X_t^*. We introduce the following notation to denote state-contingent variables:

$$\mathbf{X}_{t+1} = \begin{cases} X_{t+1}^* & \text{if } x_{t+1}^* \leq 1 \text{ and sunspot observed} \\ X_{t+1} & \text{if } x_{t+1}^* \leq 1 \text{ and no sunspot observed or } x_{t+1}^* > 1 \end{cases}$$

B.1 No Run Equilibrium

- Household:
  - Capital:

$$\left(Q_t + f_t^H\right) = \mathbb{E}_t\left(\mathbf{\Lambda}_{t,t+1}^H \mathbf{R}_{t+1}^A\right) \tag{B.1}$$

Type	SIC Code	Description
Retail Banks		
	602	Commercial Banks
	603	Savings Institutions
	671	Holding Offices
Shadow Banks		
	614	Personal Credit Institutions
	615	Business Credit Institutions
	616	Mortgage Bankers & Brokers
	617	Finance Lessors
	620	Brokers & Dealers
	621	Security Brokers & Dealers

Table 5: Definition of Retail and Shadow Banks.

- Deposits:

$$1 = \mathbb{E}_t\left(\mathbf{\Lambda}^H_{t,t+1} R^D_{t+1}\right) \tag{B.2}$$

- Labor

$$\mu L_t^{\phi} = (C_t^H)^{-\sigma} W_t \tag{B.3}$$

- Stochastic Discount Factor

$$\mathbf{\Lambda}^H_{t,t+1} = \beta\left(\frac{\mathbf{C}^H_{t+1}}{C^H_t}\right)^{-\sigma} \tag{B.4}$$

• Shadow Bank:

- Value Function

$$\Omega^S_t n^S_t = \mathbb{E}_t \tilde{\mathbf{\Omega}}^S_{t+1}\mathbf{n}^S_{t+1} \tag{B.5}$$

- Shadow Bank Stochastic Discount Factor

$$\tilde{\mathbf{\Omega}}^S_{t+1} = \mathbf{\Lambda}^H_{t,t+1}\left[\sigma^S + (1-\sigma^S)\mathbf{\Omega}^S_{t+1}\right] \tag{B.6}$$

- Balance Sheet Constraint

$$Q_t a^S_{t+1} = n^S_t + b^S_{t+1} \tag{B.7}$$

- Incentive Constraint

$$\psi Q_t\left[a^S_{t+1} + b^S_{t+1} - \omega b^S_{t+1}\right] = \Omega^S_t n^S_t \tag{B.8}$$

- Net Worth Law of Motion

$$\mathbf{n}^S_{t+1} = \max\left\{\mathbf{R}^A_{t+1}(a^S_{t+1} + R^B_{t+1} b^S_{t+1}, 0\right\} \tag{B.9}$$

• Retail Bank:

– Value Function

$$\Omega_t^R n_t^R = \mathbb{E}_t \tilde{\boldsymbol{\Omega}}_{t+1}^R \mathbf{n}_{t+1}^R \tag{B.10}$$

– Retail Bank Stochastic Discount Factor

$$\tilde{\boldsymbol{\Omega}}_{t+1}^R = \boldsymbol{\Lambda}_{t,t+1}^H \left[\sigma^R + (1-\sigma^R)\boldsymbol{\Omega}_{t+1}^R\right] \tag{B.11}$$

– Balance Sheet Constraint

$$Q_t a_{t+1}^R + b_{t+1}^R = n_t^R + d_{t+1}^R \tag{B.12}$$

– Net Worth Law of Motion

$$\mathbf{n}_{t+1}^R = \max\left\{\mathbf{R}_{t+1}^A a_{t+1}^R + \min\{\mathbf{x}_{t+1}, 1\} R_{t+1}^B b_{t+1}^R - R_{t+1}^D d_{t+1}^R, 0\right\} \tag{B.13}$$

– For the remaining two equations, there are two cases:

  * Case 1: Binding Incentive Constraint

    · Incentive Constraint

$$\psi\left[\left(Q_t + f_t^R\right) a_{t+1}^R + \gamma b_{t+1}^S\right] = \Omega_t^S n_t^S \tag{B.14}$$

    · Wholesale Lending FOC

$$\gamma \mathbb{E}_t \tilde{\boldsymbol{\Omega}}_{t+1}^R \left[\frac{\mathbf{R}_{t+1}^A}{Q_t + f_t^R} - R_{t+1}^D\right] = \mathbb{E}_t \tilde{\boldsymbol{\Omega}}_{t+1}^R \left[\min\{\mathbf{x}_{t+1}, 1\} R_{t+1}^B - R_{t+1}^D\right] \tag{B.15}$$

  * Case 2: Non-binding Incentive Constraint

    · Retail Lending FOC

$$\mathbb{E}_t \tilde{\boldsymbol{\Omega}}_{t+1}^R \left[\frac{\mathbf{R}_{t+1}^A}{Q_t + f_t^R} - R_{t+1}^D\right] = 0 \tag{B.16}$$

    · Wholesale Lending FOC

$$\mathbb{E}_t \tilde{\boldsymbol{\Omega}}_{t+1}^R \left[\min\{\mathbf{x}_{t+1}, 1\} R_{t+1}^B - R_{t+1}^D\right] = 0 \tag{B.17}$$

• Capital Goods Producers:

– Production Function

$$X_t = \left[\frac{\theta_1}{1-\theta_0}\left(\frac{I_t}{K_t}\right)^{1-\theta_0} + \theta_2\right] K_t \tag{B.18}$$

– FOC

$$Q_t = \frac{1}{\theta_1}\left(\frac{I_t}{K_t}\right)^{\theta_0} \tag{B.19}$$

• Intermediate Goods Producers:

– Production Function

$$Y_t = K_t^{\alpha} L_t^{1-\alpha} \tag{B.20}$$

– Phillips Curve

$$(\Pi_t - 1)\Pi_t - \frac{\varepsilon}{\rho^R}\left(\frac{1}{\mathcal{M}_t} - \frac{\varepsilon - 1}{\varepsilon}\right) = \mathbb{E}_t \boldsymbol{\Lambda}^H_{t,t+1} \frac{\mathbf{Y_{t+1}}}{Y_t}(\boldsymbol{\Pi}_{t+1} - 1)\boldsymbol{\Pi}_{t+1} \tag{B.21}$$

– Marginal Cost

$$\frac{1}{\mathcal{M}_t} = \left(\frac{W_t}{1-\alpha}\right)^{1-\alpha}\left(\frac{r_t^K}{\alpha}\right)^{\alpha} \tag{B.22}$$

– Factor Prices

$$r_t^K = \frac{1}{\mathcal{M}_t}\alpha K_t^{\alpha-1} L_t^{1-\alpha} \tag{B.23}$$

$$W_t = \frac{1}{\mathcal{M}_t}(1-\alpha) K_t^{\alpha} L_t^{-\alpha} \tag{B.24}$$

- Loan Servicing Firms:

  – Household fee:

$$f_t^H = \frac{\eta^H}{(C_t^H)^{-\sigma}} \max\left\{\frac{A_{t+1}^H}{A_{t+1}} - \zeta^H, 0\right\} \tag{B.25}$$

  – Retail bank fee:

$$f_t^R = \frac{\eta^R}{(C_t^H)^{-\sigma}} \max\left\{\frac{A_{t+1}^R}{A_{t+1}} - \zeta^R, 0\right\} \tag{B.26}$$

- Monetary Policy

$$R_{t+1}^N = \frac{1}{\beta}\pi_t^{\kappa^\pi}\left(\frac{\mathcal{M}_t}{\mathcal{M}_t^n}\right)^{\kappa^y}. \tag{B.27}$$

- Fisher Equation

$$R_{t+1}^D = \frac{R_{t+1}^N}{\Pi_t}. \tag{B.28}$$

- Aggregate Laws of Motion:

  – Aggregate Retail Bank Net Worth

$$\mathbf{N}_{t+1}^R = \max\left\{\left(\mathbf{R}_{t+1}^A \frac{a_{t+1}^R}{n_t^R} + \tilde{\mathbf{R}}_{t+1}^B \frac{b_{t+1}^R}{n_t^R} - R_{t+1}^D \frac{d_{t+1}^R}{n_t^R}\right) N_t^R, 0\right\} + \nu^R \mathbf{K}_{t+1} \tag{B.29}$$

  – Aggregate Shadow Bank Net Worth

$$\mathbf{N}_{t+1}^S = \begin{cases} \max\left\{\left(\mathbf{R}_{t+1}^A \frac{a_{t+1}^S}{n_t^S} - R_{t+1}^B \frac{b_{t+1}^S}{n_t^S}\right) N_t^S, 0\right\} + \nu^S \mathbf{K}_{t+1} & \text{if no run in } t+1 \\ 0 & \text{if run in } t+1 \end{cases} \tag{B.30}$$

– Shadow Bank Recovery Value

$$\mathbf{x}_{t+1} = \frac{\mathbf{R}^A_{t+1} a^S_{t+1}}{R^B_{t+1} b^S_{t+1}} \tag{B.31}$$

– End-of-period Capital

$$S_{t+1} = (1-\delta)K_t + X_t \tag{B.32}$$

– Beginning-of-period Capital

$$K_t = Z_t S_t \tag{B.33}$$

– Capital Quality

$$\ln(Z_t) = \rho_Z \ln(Z_{t-1}) + \epsilon_t \tag{B.34}$$

– Sunspot

$$\Xi_t = \begin{cases} 1 & \text{with prob. } \pi \\ 0 & \text{with prob. } 1-\pi \end{cases} \tag{B.35}$$

- Aggregate Resource Constraint

$$C^H_t = Y_t(1 - \frac{\rho^R}{2}(\Pi_t - 1)^2) - I_t - G \tag{B.36}$$

B.2 Run Equilibrium

We do not repeat the equilibrium conditions which do not change relative to the no-run equilibrium.

- Retail Bank:

– Value Function

$$\Omega^R_t n^R_t = \mathbb{E}_t \tilde{\boldsymbol{\Omega}}^R_{t+1} \mathbf{n}^R_{t+1} \tag{B.37}$$

– Retail Bank Stochastic Discount Factor

$$\tilde{\boldsymbol{\Omega}}^R_{t+1} = \boldsymbol{\Lambda}^H_{t,t+1}\left[\sigma^R + (1-\sigma^R)\boldsymbol{\Omega}^R_{t+1}\right] \tag{B.38}$$

– Balance Sheet Constraint

$$Q_t a^R_{t+1} = n^R_t + d^R_{t+1} \tag{B.39}$$

– Net Worth Law of Motion

$$\mathbf{n}^R_{t+1} = \max\left\{\mathbf{R}^A_{t+1} a^R_{t+1} - R^D_{t+1} d^R_{t+1}, 0\right\} \tag{B.40}$$

– For the remaining equation, there are two cases:

∗ Case 1: Binding Incentive Constraint

· Incentive Constraint

$$\psi\left[\left(Q_t + f^R_t\right) a^R_{t+1}\right] = \Omega^S_t n^S_t \tag{B.41}$$

* Case 2: Non-binding Incentive Constraint
  · Retail Lending FOC

$$\mathbb{E}_t \tilde{\boldsymbol{\Omega}}^R_{t+1} \left[\frac{\mathbf{R}^A_{t+1}}{Q_t + f^R_t} - R^D_{t+1} \right] = 0 \tag{B.42}$$

- Aggregate Laws of Motion:
  - Aggregate Retail Bank Net Worth

$$\mathbf{N}^R_{t+1} = \max \left\{ \left(\mathbf{R}^A_{t+1} \frac{a^R_{t+1}}{n^R_t} - R^D_{t+1} \frac{d^R_{t+1}}{n^R_t} \right) N^R_t, 0 \right\} + \nu^R \mathbf{K}_{t+1} \tag{B.43}$$

  - Aggregate Shadow Bank Net Worth

$$\mathbf{N}^S_{t+1} = \nu^S \left[(1 - \sigma^S) K_t + \mathbf{K}_{t+1} \right] \tag{B.44}$$

- Aggregate Resource Constraint

$$C^H_t = Y_t (1 - \frac{\rho^R}{2} (\Pi_t - 1)^2) - I_t - G - \nu^S K_t \tag{B.45}$$

C Computation

C.1 Solution

We solve the model non-linearly using projection methods. Solving the model non-linearly is important, because bank runs can lead to large deviations from steady state, where perturbation algorithms are inaccurate.

The state space of the model is $\mathcal{S} = (n^R, n^S, K, Z)$ in the no bank run equilibrium and $\mathcal{S}^* = (n^{R,*}, K, Z)$ in the bank run equilibrium. n^R is the net worth of incumbent retail bankers, n^S the net worth of incumbent shadow bankers. We use Smolyak grids $\mathcal{S}_i, i = 1, \ldots, N$ and $\mathcal{S}^*_i, i = 1, \ldots, N^*$ of order $\mu = 5$ for the endogenous and exogenous states. We compute the Smolyak grid and polynomials using the toolbox by Judd, Maliar, Maliar, and Valero (2014).

We need to find the following policy functions for the no-run equilibrium: $\mathbf{C^H}(\mathcal{S})$, $\mathbf{V^R}(\mathcal{S})$, $\mathbf{V^S}(\mathcal{S})$, $\mathbf{Q}(\mathcal{S})$, $\mathbf{L}(\mathcal{S})$ and $\boldsymbol{\Pi}(\mathcal{S})$. Denote those functions as $\mathbf{V}(\mathcal{S})$. For the run equilibrium, we need to find policy functions $\mathbf{C}^{\mathbf{H},*}(\mathcal{S}^*)$, $\mathbf{V}^{\mathbf{R},*}(\mathcal{S}^*)$, $\mathbf{Q}^*(\mathcal{S}^*)$ and $\mathbf{L}^*(\mathcal{S}^*)$. Denote those functions as $\mathbf{V}^*(\mathcal{S})$. We compute one set of functions for both the case of the binding and the non-binding incentive constraints of retail banks. Between grid points, we approximate these functions using polynomial basis functions $\mathcal{P}(\mathcal{S})$. We compute the polynomial coefficients by imposing that the polynomial approximations must be equal to the original functions on the grid. Specifically, denoting the polynomial coefficients by α, we impose

$$\mathcal{P}(\mathcal{S}_i)\alpha_V \equiv \mathbf{V}(\mathcal{S}_i) = V(\mathcal{S}_i) \quad i = 1, \ldots, N. \tag{C.1}$$

for all N grid points. We use an anisotropic grid with 6th-order Smolyak polynomials for the net worth of retail and shadow banks and 5th-order polynomials for capital and the capital quality shock.

We also need to find laws of motion for the future endogenous state variables, $\mathbf{n^{R}}'(\mathcal{S}, \varepsilon^{Z'}, \Xi')$, $\mathbf{n^{S}}'(\mathcal{S}, \varepsilon^{Z'}, \Xi')$, $\mathbf{K}'(\mathcal{S}, \varepsilon^{Z'}, \Xi')$, the probability of a banking panic $\mathbf{p}'(\mathcal{S}, \varepsilon^{Z'}, \Xi')$, and the recovery rates $\mathbf{x}'(\mathcal{S}, \varepsilon^{Z'}, \Xi')$ and $\mathbf{x}^{*'}(\mathcal{S}, \varepsilon^{Z'}, \Xi')$. Collect those laws of motion as $\mathbf{T}(\mathcal{S}, \varepsilon^{Z'}, \Xi')$, and the corresponding laws of motion for the bank run equilibrium as $\mathbf{T}^*(\mathcal{S}^*, \varepsilon^{Z'}, \Xi')$ The laws of motion depend on both the realization of the next period capital quality shock $\varepsilon^{Z'}$ and the sunspot Ξ'.

With this in mind, we will now outline our solution algorithm. Suppose we are in iteration k and have initial guesses for the policy functions $\mathbf{V_{(k)}}(\mathcal{S})$ and $\mathbf{V^*_{(k)}}(\mathcal{S})$, as well as laws of motion $\mathbf{T_{(k)}}(\mathcal{S}, \varepsilon^{Z'}, \Xi')$ and $\mathbf{T^*_{(k)}}(\mathcal{S}, \varepsilon^{Z'}, \Xi')$.

1. Given the value functions and the future net worth, compute the future value functions and capital prices as

$$V'_{(k)} = \mathbf{V}_{(k)}(\mathbf{T}_{(k)}(\mathcal{S}, \varepsilon^{Z'}, \Xi'))$$

2. Compute the expected value functions for the forward-looking equations B.1, B.2, B.5, B.10, B.15, B.16, B.17, and B.21.

3. Find the new policy functions and equilibrium prices.

4. Using the new policy functions and equilibrium prics, find the new value functions and laws of motion $\mathbf{\tilde{V}_{(k+1)}}(\mathcal{S})$ and $\mathbf{\tilde{T}_{(k+1)}}(\mathcal{S}, \varepsilon^{Z'}, \Xi')$.

5. Repeat steps 1 to 4 for the run equilibrium to find $\mathbf{\tilde{V}^*_{(k+1)}}(\mathcal{S})$ and $\mathbf{\tilde{T}^*_{(k+1)}}(\mathcal{S}, \varepsilon^{Z'}, \Xi')$.

6. Compute errors as

$$\begin{aligned}\varepsilon^V &= \max |\mathbf{\tilde{V}_{(k+1)}}(\mathcal{S}) - \mathbf{V_{(k)}}(\mathcal{S})| \\ \varepsilon^T &= \max \mathbb{E}|\mathbf{\tilde{T}_{(k+1)}}(\mathcal{S}, \varepsilon^{Z'}, \Xi') - \mathbf{T_{(k)}}(\mathcal{S}, \varepsilon^{Z'}, \Xi')|\end{aligned}$$

7. Update the value functions and laws of motion with some attenuation:

$$\begin{aligned}\mathbf{V_{(k+1)}}(\mathcal{S}) &= \iota \mathbf{V_{(k)}}(\mathcal{S}) + (1-\iota)\mathbf{\tilde{V}_{(k+1)}}(\mathcal{S}) \\ \mathbf{T_{(k+1)}}(\mathcal{S}, \varepsilon^{Z'}, \Xi') &= \iota \mathbf{T_{(k)}}(\mathcal{S}, \varepsilon^{Z'}, \Xi') + (1-\iota)\mathbf{\tilde{T}_{(k+1)}}(\mathcal{S}, \varepsilon^{Z'}, \Xi')\end{aligned}$$

8. Repeat until the errors ε^V and ε^T are less than 1e-3. The errors in the consumption policy function of the household and the capital price function are much smaller, around 1e-5.

C.2 Precision of the Solution

To gauge the precision of the solution, we compute Euler errors as proposed in Judd (1992) for equations B.1 and B.2. We can see that the Errors are typically small, with means between -4 and -5 in log10-scale.

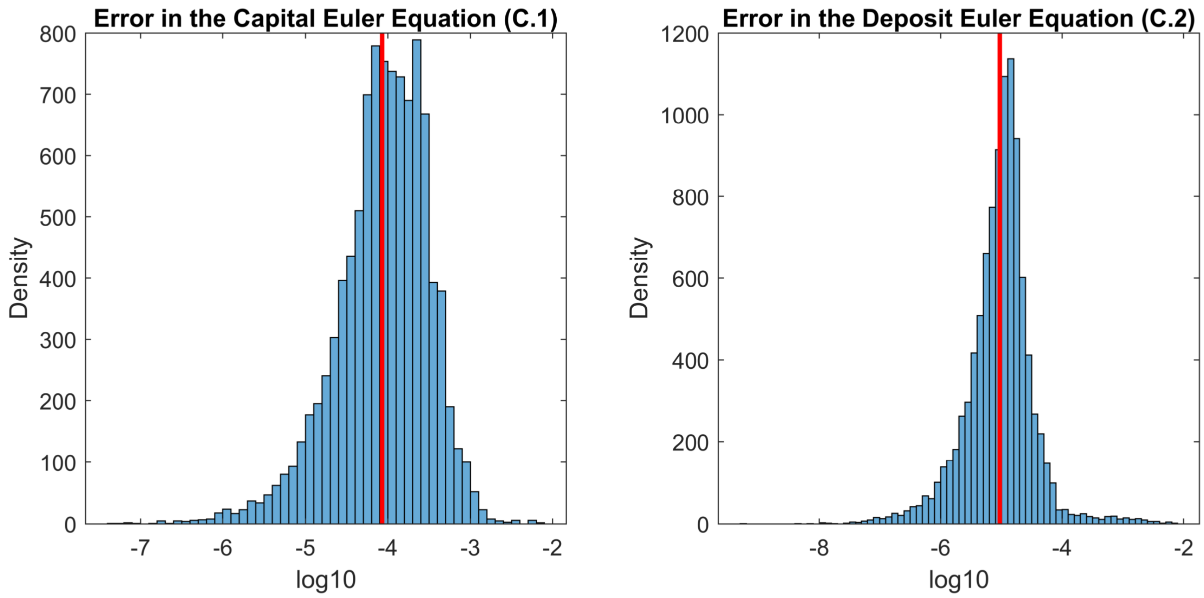

Figure 10: Errors in the Euler equations B.1 and B.2.

Note: Based on a simulation of 1000 economies for 2000 periods, discarding the first 1000 periods as burn-in. The red line denotes the average Euler error.

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