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Detecting turning points in the Danish economy in real time

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Abstract
This working paper documents an econometric model for detecting turning points in the Danish economy in real time. The model is a mixed-frequency model using both monthly and quarterly data, which can be estimated on an unbalanced panel of data and be updated immediately as data comes through. The model allows the user to provide both a statement about the probability of the Danish economy being in a recession or an expansion as well as in terms of a quantitative estimate of the growth rate on a monthly basis. Further, the model provides a measure of the uncertainty surrounding such a nowcast. The properties of the model are shown and the real-time performance of the model is analysed. It is shown that the model gives correct signals about the activity in the Danish economy, also in real time. The model has been in use in Danmarks Nationalbank since the beginning of 2020, and nowcasts and the uncertainty surrounding this nowcast produced by the model have been published since summer 2020.

Resume
I papiret dokumenteres en økonometrisk model for at afdække vendepunkter i dansk økonomi i realtid. Modellen anvender både månedlige og kvartalsvise data og kan estimeres på et ubalanceret panel af data, der kan opdateres øjeblikkeligt, når nye data offentliggøres. Modellen kan bruges både til at give en vurdering af sandsynligheden for, at dansk økonomi befinder sig i henholdsvis en recession og en ekspansion, og til at give et kvantitativt estimat over realvæksten i dansk økonomi med månedlig frekvens. Samtidig giver modellen et bud på usikkerheden om dette estimat. I papiret analyseres modellens egenskaber. Det vises, at modellen giver korrekte signaler om aktivitetsniveauet i dansk økonomi i realtid. Modellen har været anvendt i Nationalbanken siden begyndelsen af 2020. Modellens nowcasts og vurdering af usikkerheden er siden sommeren 2020 blevet publiceret på Nationalbankens hjemmeside.

Key words
Economic activity and employment; Models; Forecasting.

JEL classification
E32; C22; E27.

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The author alone is responsible for any remaining errors.

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This working paper documents an econometric model for detecting turning points in the Danish economy in real time. The model is a mixed-frequency model using both monthly and quarterly data, which can be estimated on an unbalanced panel of data and be updated immediately as data comes through. The model allows the user to provide both a statement about the probability of the Danish economy being in a recession or an expansion as well as in terms of a quantitative estimate of the growth rate on a monthly basis. Further, the model provides a measure of the uncertainty surrounding such a nowcast. The properties of the model are shown and the real time performance of the model is analysed. It is shown that the model gives correct signals about the activity in the Danish economy also in real time. The model has been in use in Danmarks Nationalbank since the beginning of 2020, and nowcasts produced by the model have been published since summer 2020.

*Address: Nationalbanken, Langelinie Alle 47, 2100 Copenhagen Ø. The author wishes to thank colleagues at Danmarks Nationalbank for their valuable comments and suggestions, especially Adrian Michael Bay Schmith, Jakob Guldbæk Mikkelsen and seminar participants at Danmarks Nationalbank for useful discussions. The viewpoints and conclusions stated are the responsibility of the author, and do not necessarily reflect the views of Danmarks Nationalbank. Part of the model was developed in collaboration between the author and various central banks under the Working Group of Econometric Modeling of the European System of Central Banks. The author especially wishes to thank Danilo Leiva-Leon and Gabriel Perez-Quiroz for leading the expert group under the ECB, where a large part of this model was developed.

1. Introduction

To be able to give timely and precise policy recommendations to policy makers, it is important to detect turning points in the business cycle in real time as information hits the economy. To detect turning points, one way forward is to rely on the quarterly GDP figures published by national statistical bureaus. In Denmark, Statistics Denmark produces flash estimates of GDP with a lag of only around 45 days from the end of the quarter, thus providing timely information about the economy. However, information from other indicators and sources continuously becomes available, and it can be of interest to update the belief of the current state of the economy as soon as new information hits the economy. This has especially been the case during the COVID-19 crisis, where uncertainty about the impact of the pandemic has been elevated. And having a model to do so comes with extra advantages, including the ability to conduct scenarios and probability statements.

Another advantage of having a model is that it can tell economists what a given indicator means for growth, all else being equal. It might be that the economist knows more about the economy than the model, and that this information might lead to deviations from what the model points towards, e.g. that a lock-down of the economy due to a pandemic has been announced, which so far has not been reflected in data. But such a deviation from the relationship between data and real activity estimated using the models must then be justified by the user through a ”story”, e.g. the announcement of a lock down due to a pandemic, and hence explain why a judgment-based forecast of GDP might deviate from this historical relationship.

While expansions are more homogeneous, recessions are all different. Some have been due to oil price shocks like in the 70s, others have been caused by excess leverage and financial excesses like during the build-up to the great financial crisis. Lately, a recession has been caused by a global pandemic; the COVID-19 crisis. Two main defining properties of business cycles are comovements between various real indicators and nonlinear dynamics. As pointed out in Jensen et al. (2020), advanced economies have been characterised by an increasingly negative business-cycle asymmetry over the last three decades: periods of growth seem to be long-lasting, while downturns seem to be short but very abrupt, see also Kramp and Pedersen (2020) for the case of Denmark.

The aim of the model is to capture all these points, i.e., first, to use timely available information and deal with the characteristics of data production, publication delays and ragged ends. Ragged ends means that not all data becomes available at the same time during the month or quarter, but the user wishes to explore information as it becomes available thus needing a model, which can be estimated when part of the data set is not available. Second, it should be multivariate and thus include more information than simply one indicator like GDP. Third, it must be non-linear. Fourth, it should capture all types of recessions, and not just a particular type of recessions, like for example financial crises. Fifth, it should be easy to use and as automatic as possible such that the model can be updated weekly or even daily without large costs.

To this end, a dynamic non-linear Markov-switching factor model is set up based on the model in Leiva-Leon et al. (2020) which was developed in a working group under the ECB with the participation of Danmarks Nationalbank. The output of the model is both a statement about the stance of the Danish economy in terms of probabilities and a density forecast for GDP growth. Given the non-linearities inherent in the model, these densities can be non-Gaussian, while the nowcasts, taken to be, e.g., the median of the densities, can provide a quantitative statement about the growth rate of the Danish economy. The aim of this paper is to document this model for Denmark. The model builds on fairly large literature, which began with the model in Hamilton (1989). This model was only based on one factor, GDP. Later, Chauvet (1998) included a set of real activity indicators with the aim of summarising more information into a regime-switching factor. This framework was extended by, e.g., Camacho et al. (2014) to include both information at the monthly frequency and real GDP growth.

This work began before the COVID-19 crisis. Up until that crisis, it was apparent that all other crises besides the great financial crisis were difficult to detect basically because all other crisis are small blips in the data compared to the financial crisis. An extension to the framework in, e.g., Camacho et al. (2014) was therefore needed for the model to be able to detect all recessions. The idea is to let the average growth rate in a recession be stochastic: All recessions are different, while all expansions are the same.

The model includes six variables, quarterly GDP, and five monthly variables, imports, exports, industrial expectations, unemployment and a business sentiment indicator for services. The model cannot handle much larger data sets due to the non-linearities inherent in the model for capturing the skewness in the business cycle. The main idea is to mimic the way quarterly GDP is constructed by including data from the production side and the demand side. In doing so, the model is able to capture all types of recessions and not only, say, financial crises.

There are other approaches to detecting turning points or growth in real time. One approach is the approach taken in another real-time nowcasting model for the Danish economy developed during the COVID-19 crisis, see Schmith and Grenestam (2021). This approach builds on a much richer data set, the idea being to include as much information as possible on different frequencies. This approach has its pros and cons relative to the framework set up in this study. The richer data set allows the model to be reestimated more frequently, and the model can utilise the large information set available, including crossings of the large bridges in Denmark, credit card transactions and so forth. However, this also makes the model difficult to extend to a non-linear setup, and thus to model non-Gaussian shocks to the economy. Further, the model is neither able to provide probability statements about growth nor can the model framework provide density forecasts and thus estimates of uncertainty of the current stance of the economy. These relative strengths of the two approaches to estimate nowcasts for the Danish economy have led Danmarks Nationalbank to apply both methods in the process of monitoring the Danish economy.

The model is set up in section (2). Focus will be on explaining the intuition behind the various features of the model and why they have been added. The econometric techniques used to estimate the model can be found in the literature. The selection of data is explained in section (3), followed in section (4) by a thorough explanation of the output from the model and how it should be interpreted. Section (5) forms one of the most important parts of this paper, namely an analysis of the real-time properties of the model. The conclusion in section (6) includes a description of how this model and the model in Schmith and Grenestam (2021) have been used in Danmarks Nationalbank.

2. The model: A non-linear factor model for detecting turning points in real time

The model summarises the information contained in a set of indicators into a factor which approximates the business-cycle dynamics of the Danish economy. In this section, the model is set up, the subsequent section discusses data, while the econometric techniques to extract the factor from data are explained in detail in Leiva-Leon et al. (2020). The model builds on extensive literature within forecasting using factor models, e.g. Camacho et al. (2014), Camacho et al. (2015), Camacho and Quiros (2011), Camacho and Perez-Quiros (2010) and Doz et al. (2020).

2.1. Setting up the model

The proposed model summarises the information contained in a set of real activity indicators into a common factor. This factor is an index that proxies the business-cycle dynamics of a given economy and can thus be interpreted as measuring real activity like GDP. The factor is subject to regime switches in the mean: the mean of the factor is relatively high outside a recessionary period and relatively low in a downturn. The mean of the factor is thus state-dependent. Further, the model allows for heterogeneity of the recessions: On average, expansions evolve at the same underlying rate, while recessions evolve at a different underlying rate for each recession.

Specifically, the model is a dynamic factor model, in which the common factor, f_t, follows Markov-switching dynamics subject to time-varying means. The process for the factor evolves according to:

$$f_t = \mu_0 (1 - s_t) + \mu_1 s_t + s_t x_t + e_{f,t}, e_{f,t} \sim i.i.d.N\left(0, \sigma_e^2\right).$$

The idea is that this factor approximates real activity in the economy based on the set of data. The variable, s_t, can take values within the interval $[0; 1]$. It is 0 when the economy is in a normal state and 1 when the economy is in an abnormal state. I will come back to what is meant by these terms. The mean in the factor is a weighted average of s_t and the mean growth rate of the factor in these two states. Hence, if the economy is in a normal state, with $s_t = 0$, the mean of the factor is μ_0. When the state variable s_t resides outside the extreme of the bounds 0 and 1, the mean of the factor is an average of the two mean factors weighted by the probabilities $(1 - s_t)\mu_o + s_t\mu_1$. The variable s_t is in turn assumed to follow a two-state Markov chain defined by transition probabilities:

$$Pr(s_t = j | s_{t-1} = i, s_{t-2} = h, ...) = Pr(s_t = j | s_{t-1} = i) = p_{ij}.$$

However, the factor is also affected by the unobserved stochastic term denoted by x_t. It evolves over time according to the following process:

$$x_t = s_t x_{t-1} + (1 - s_t) v_t, v_t \sim N\left(0, \sigma_v^2\right) i.i.d.$$

If the economy resides in a normal state with a probability equal to 1 at time t, $s_t = 0$, the unobserved term is equal to white noise with no impact on the factor, $f_t = \mu_0$. However, in abnormal periods, when $s_t = 1$, the factor x_t remains fixed at the drawn value of the stochastic process, $v_t = v_t^0$, such that $x_t = x_{t-1} + v_t^0$. Consequently, the factor in such a state evolves according to $f_t = \mu_1 + x_t$.

These states and mean parameters can be given the following interpretations. With $\mu_0 > \mu_1$, if $s_t = 0$, the normal state, the economy can be said to reside in a relatively high growth state and if $s_t = 1$, the economy is in a relatively low growth stage. The mean in the factor is a weighted average of s_t and the mean growth rate of the factor in these two states. Thus, μ_o and μ_1 can be interpreted as the growth rate of the factor in an *expansion* and *recession*, respectively. Or in states of the world where the economy is growing above its long-run average or below. As can be seen from these terms, in the case of recession the mean growth rate is equal to $\mu_1 + x_t$. Therefore, it is in a recession that the mean of the factor is augmented with x_t and kept fixed during the recession.

The expectation is that regime switching enhances the ability of the model to provide a reliable estimate of the state of the economy. Intuitively, this can be explained as follows in a simple linear setting using an AR(1) model as the forecast model with a constant mean

$$y_t = c + \rho y_{t-1} + v_t, v_t \sim i.i.d.N\left(0, \sigma_v^2\right).$$

Here y_t is the variable of interest, i.e. GDP. The forecasts from such an AR(1) model would imply mean reversion to $\mu = \frac{c}{(1-\rho)}$ according to $E_t[y_t] = \mu + \rho^s \left(y_t - \mu\right)$. Here, s denotes the horizon for the forecast. When s goes to infinity, the forecasts goes towards μ assuming $\left|\rho < 1\right|$. The parameter would be constant estimated as a mean across the sample. Hence, a forecast of GDP would always converge geometrically to the mean growth rate using a model with a constant mean. However, in a non-linear setting as the one used in the current work, the point where the forecast would converge would depend on the state of the economy. As an example, if the economy has entered a low growth period, the forecast would revert not to the constant mean value, μ, but to μ_1.

This expected feature of the model is confirmed in Figure (1). The figure shows nowcasts and real-time forecasts for each month from 1 June 2005 to 1 December 2020 24 quarters ahead.[1] The forecasts are divided into four sub-groups. The first group (light grey) includes the forecasts produced by the model before the outbreak of the financial crisis. The second group (red) includes forecasts during the financial crisis. The third group (dark grey) includes forecasts from the end of the financial crisis up until the outbreak of the COVID-19 crisis. The final group (blue) is forecasts produced during the COVID-19 crisis. These periods capture high-growth periods and crisis periods. In contrast to a non-linear model, the forecasts converge to different end points depending on the state of the economy and the mean growth rates in the different periods. The COVID-19 crisis period stands out. During this period, the forecasts converge to a growth rate in GDP of around -0.25. It can also be seen that the model downgrades the mean growth rate in the economy; the point

[1] Details of the real time analysis are provided in section (5).

of convergence for the forecasts is higher for the forecasts conducted before the financial crisis compared to the period between the end of the financial crisis and the outbreak of the COVID-19 crisis.

The idea and motivation behind adding the term x_t to the factor is that if the model had just two states, it probably would attribute the financial crisis only to be a recession; the rest of the recessions in the Danish economy would be small blips compared to the financial crisis, and mild future recessions in comparison with the financial crisis would not be captured by the model.[2] Against this background, something needs to inform the model, that the financial crisis was large and unusual. Hence, all normal episodes share the same mean of factor, while each abnormal episode involves a different mean. The interpretation is that all expansions are the same, while all recessions are different.[3]

Each indicator is assumed to be influenced by a common component contemporaneously, $\gamma_i f_t$, and an idiosyncratic component, ζ. The latter component is assumed to follow the auto-regressive dynamics of order 2.

$$\zeta_{i,t} = \rho_{i,1}\zeta_{i,t-1} + \rho_{i,2}\zeta_{i,t-2} + e_t, e_t \sim N\left(0, \sigma_e^2\right) i.i.d.$$

Data at monthly frequency is then expressed as

$$y_{i,t} = \gamma_i f_t + \zeta_{i,t}.$$

The model also includes one variable with a quarterly frequency, GDP. To handle mixed frequencies in the model, the idea in Mariano and Murasawa (2003) is applied expressing quarter-on-quarter growth rates into month-on-month *unobserved* growth rates.[4]

[2]The model was developed before the outbreak of the COVID-19 pandemia and the consequent economic crisis.

[3]The idea behind the introduction of the factor is inspired by the opening remarks to Leo Tolstoys' Anna Karenina: "Happy families are all alike; every unhappy family is unhappy in its own way.", Tolstoj (1873). Here in terms of movements in the business-cycle: Expansions are all alike; every recession is different in its own way.

[4]Let Y_t^Q denote an indicator observed quarterly observable every third period. Then $log\left(Y_t^Q\right) = \frac{1}{3}\left(log\left(Y_{1,t}^m\right) + log\left(Y_{1,t-1}^m\right) + log\left(Y_{1,t-2}^m\right)\right)$, such that Y_t^Q is the geometric mean of the three parts in the brackets. Then taking the three-period differences we arrive at $log\left(Y_t^Q\right) - log\left(Y_{t-3}^Q\right) = \frac{1}{3}\left(log\left(Y_{1,t}^m\right) - log\left(Y_{1,t-3}^m\right)\right) - \frac{1}{3}\left(log\left(Y_{1,t-1}^m\right) - log\left(Y_{1,t-4}^m\right)\right) - \frac{1}{3}\left(log\left(Y_{1,t-2}^m\right) - log\left(Y_{1,t-5}^m\right)\right)$. Collecting terms and letting small case letters denote logs of the variables, the expression can be written as $y_t^Q = \frac{1}{3}y_{1,t} + \frac{2}{3}y_{t-1} + y_{1,t-2} + \frac{2}{3}y_{t-3} + \frac{1}{3}y_{t-4}$ Hence, the model observes y_t^Q every third period but it never observes $y_{1,t}$.

2.2. Estimation

The next step is to extract the factor from the set of observed variables. To this end, the model builds both on previous work and work done in the above-mentioned working group. This paper refers to the work in Leiva-Leon et al. (2020), Chauvet (1998) and Camacho et al. (2014) for details. Only the overall picture and intuition are provided in the current work. The model is estimated using Bayesian methods producing inference on parameters and latent variables, including the factor. While in principle the model could be estimated using classical techniques, the Bayesian method applied is relatively fast and efficient.

3. Selecting the variables

First, the next steps are to choose the number of state variables and, second, to determine which data provides the best signal about the business-cycle. When doing so, the main criteria for the model must be recalled. The model must use timely available information and deal with the characteristics of data production, publication delays, and ragged ends. This involves both choosing the number of monthly and quarterly variables as well as thinking about data delay. Ragged ends means that data is published at different times during the month or quarter. An example is that in Denmark unemployment numbers are published each month, while GDP is only available at a quarterly frequency. Likewise, even monthly indicators are published at different times during the months. However, economists want to get updated estimates when new information is available and not wait until a full panel of data is available. The model setup can handle these ragged ends, in which some data points are not available, while others are and hence, the model can be estimated, including parameters using a non-balanced panel of data.

Further, the model needs to be updated on a regularly basis and it must therefore be as easy to use as possible and as automatic as possible. And it should capture all types of recessions, and not only some recessions related to specific events. Further, due to the non-linearities in the model, this set of variables must be rather small or the estimation becomes too time-consuming.

To this end, I follow the literature and choose variables which mimic national accounts procedures, see, e.g., Stock and Watson (1991). The idea is to use variables for the production side of the economy, usually industrial production, variables capturing demand, usually sales, and variables capturing income, usually real personal income and employment.

For the Danish economy this is not sufficient as it is a very open economy. Hence, to the demand side of the model is added exports and imports. Further, the model is expanded by including a measure for the demand for services, which makes up a rather large and increasing part of the Danish economy. To these monthly variables is added GDP, leaving the model with five monthly variables and one quarterly variable:

1. GDP
2. Imports
3. Exports
4. Industrial expectations
5. Unemployment
6. Business sentiment indicator for services

Industrial expectations and the business sentiment indicator are in logs. The rest of the variables are expressed in growth rates with respect to the previous period. All data is seasonally adjusted and standardised prior to entering the corresponding model. The model is estimated from Q1-2000 up until today.[5] While some variables cover a longer time span, the limited sample size is due to the rather short sample for the business indicators.

This set of variables is rather small, but as shown in, e.g., Stock and Watson (2016), Leiva-Leon et al. (2020), Chauvet (1998) and Camacho et al. (2014) a small set of variables is sufficient to capture the current stance of the economy in real time. One further advantage of using a small set of variables mimicking national accounts is that they can capture all types of recessions; the performance of the model does not depend on whether the recession in question is due to a financial crisis, whether it is caused by oil price shocks or lock-downs due to the eruption of a pandemic. If the shock in question has an effect on the real economy, it will be reflected in variables in the national accounts.

How are the variables chosen? While the intuition behind the chosen variables is clear, the model also leaves a lot of possible candidates out and these could perhaps help to improve the properties of the model. The chosen variables were determined by evaluating the correlation between the estimated factor based on that set of variables and GDP growth. This correlation approaches approximately 0.85 for the baseline model.

[5]This documentation was written in January-February 2021.

4. Estimation results

In what follows, the output of the model is shown with a focus on the following output from the model: The probabilities of negative growth in the current quarter, the probability of low growth, the probability of a technical recession, the factor capturing the heterogeneity of recessions, distributions around GDP nowcasts and the nowcasts.

4.1. Estimated probabilities

One advantage of the model is its ability to give an assessment of the economy in terms of probabilities. In what follows, I will analyse the performance of the model through three types of probabilities. Figure (2(a)) shows the estimated probability of expansions/recessions, i.e. the variable s_t in equation (2.1). The model detects the turning points in historical data proving a probability of 1 of the Danish economy entering recession during the financial crisis and the COVID-19 crisis. The model also points to an elevated probability of the Danish economy entering in recession during 2003. In the end, this period is not characterised as being a recession; only a low growth period in the Danish economy. The same pattern can be seen during the outbreak of the sovereign debt crisis of 2011-2012, although the model points to a markedly lower probability. This is in sample using information from the full sample. However, it is assuring to see that the model captures events in the Danish economy which are expected to be found in the sample. In section (5) a real time analysis is conducted, which is a stricter test of the performance of the model.

The probability of ending up in a low growth state is not the same as the economy being in a recession. A technical recession is usually defined as two consecutive quarters of negative growth. In the model, the probability of growth being below zero in the current month and three months ahead is calculated. This probability can be calculated through the density forecast for GDP growth and multiply it by the probability of GDP growth being negative times the probability of GDP being negative three months ahead. This probability is shown in Figure (2(b)). Compared to the probability of low growth, the probability of a recession is more volatile. It captures the same historical episodes in the Danish economy as the probability of low growth, but reacts more to economic events. As an example, the sovereign debt crisis is more visible.

4.2. Capturing heterogeneity of recessions

As discussed in the introduction, the model was developed before the outbreak of the COVID-19 crisis. It has been common in the literature to estimate two-state models with one regime capturing recessions and the other state capturing expansions reflecting how macroeconomists usually think about business-cycles, see, e.g., Camacho et al. (2014) or Hamilton (1989). When estimating, say, a two-state model on this data sample, the only recession the model could pick up was the financial crisis being the only large recession in the data; the rest of the recessions in the data are merely small blips.[6] To avoid the model only capturing this crisis as being a recession, the two-state model needs to be expanded with either three states, stochastic volatility or as being found the best solution, heterogeneous mean recessions. Figure (3) shows this factor which helps to capture the heterogeneity of recessions. The factor is relatively stable in normal times, i.e., outside the two large recessions in the sample, the financial crisis and the COVID-19 crisis. It is also stable during the very mild recession preceding the financial crisis, and the low growth period during the sovereign debt crisis. However, during those larger recessions, highlighted with a grey shaded area, the factor decreases thus helping to capture the depth of the crisis.[7]

4.3. The monthly nowcasts

Besides providing a statement about the state of the economy in terms of probabilities, the model also provides a quantitative estimate of the growth rate. These are shown in Figure (4) together with actual GDP. The nowcasts in this full-sample analysis coincide with GDP by construction in quarters where the model has observed GDP. Outside these quarters, the model provides an estimate of the growth rate in the economy each month recalling that the frequency of the model is monthly. However, it is difficult to validate the accuracy of these estimates as there are no official estimates of the activity in the Danish economy at a monthly frequency. The two major crises in the sample can give a hint of the validity of these monthly estimates of the real activity in the Danish economy. Although the model is estimated using information from these crises, the model does not know the actual GDP growth rate at a monthly frequency.[8] Looking at the current COVID-19 crisis, the depth of

[6]This was especially the case across the countries participating in the ECB working group developing part of the model.

[7]Interestingly, the factor falls by almost the same amount during the COVID-19 crisis and during the financial crisis. This is, however, a coincidence, as the size of the fall in the factor is assumed to be normally distributed.

[8]In section (5) the real-time performance of the model is analysed.

the crisis coincides with publications of GDP numbers. GDP decreased from around -1.75 to the depth of the crisis of close to - 8 per cent, and the monthly GDP estimates from the model follow these patterns. During the financial crisis, the monthly GDP estimates are harder to validate. They point to slowdowns in monthly activity of around -7 to -8 per cent, while GDP only decreased by around -2. While the crisis was severe and long-lived, it is not known whether during these months it was as severe, as pointed out by the model.

4.4. Providing uncertainty bounds around nowcasts

The model estimates a density forecast of GDP growth each month and thus a density. Consequently the model can provide a measure of uncertainty around the growth rate of GDP. The density forecast is calculated using state space representation evaluated at the estimated parameters from the Bayesian estimation.

The full sample distributions can be found in Figure (5). From the figure can be seen that the uncertainty surrounding the nowcasts varies through time. On average, the uncertainty around the GDP nowcasts is smaller in months in which the model observes GDP, reflecting its importance for the developments in the real activity factor.

The density is a mixture of two normal distributions weighted by the estimated probabilities of the growth rate being in a ”low” growth regime or ”high” growth regime. The model can thus provide skewed distributions capturing non-Gaussian distributed GDP growth. In the sample and outside periods of crisis, the estimated distributions are close to following a normal distribution. However, during periods of recessions, kurtosis and skewness increase markedly. Movements in the shape of the distributions are thus clearly related to large crises.[9]

As an example, during the current COVID-19 crisis, the model at first points to great uncertainty about GDP growth, while one month later, the distribution is significantly tighter reflecting the greater certainty about low activity of around 7 per cent. This, however, cannot be observed during the financial crisis. Here, the uncertainty surrounding the depth of the crisis was larger although the mean of the distribution is comparable to the mean of the distribution during the COVID-19 crisis (highlighted in blue in the figure). As a result, the model pointed to relatively low uncertainty about the stance of the Danish economy during some months of the COVID-19 crisis – according to the model, real activity in the economy was certain to fall considerably and then rebound. During the financial crisis, the

[9] As shown in Adrian et al. (2019), it is the financial cycle which can give rise to highly skewed distributions around GDP growth. The financial cycle was included in the model as a test, but it did not markedly change the results shown in Figure (5) and did not improve the model’s ability to nowcast real activity.

model found that activity would be low which was reflected in a sharp downward shift in the distribution of GDP growth, but compared to the COVID-19 crisis the model pointed to relatively higher uncertainty about exactly how low activity would be.

5. Real-time performance of the model

The main focus for nowcasting is to be able to capture recessions in *real time*. A test of the real-time performance of the model needs to be conducted without the benefit of hindsight; 10 years after the financial crisis a modeller knows the origins of the crisis and would include a combination of credit and house price growth. However, using that information in a test of the model's ability to capture the financial crisis would not be a real-time analysis. And using those variables might not improve the performance of the model, and might in fact give wrong signals in other recessions, see Gadea Rivas and Perez-Quiros (2015).

In what follows, the model is estimated up until June 2005. From there and onwards, the model is recursively evaluated out-of-sample adding one month of data at a time. In each month, the probability of a recession is compared to GDP growth and the final sample probability of a recession at that point is compared.[10] Through the probabilities, not only is the model's ability to detect the low and high growth episodes in the Danish economy analysed when they actually occurred, but the end-point problems of the model are also evaluated.

5.1. Evaluating recession probabilities in real time

Figures (6(a)) and (6(b)), respectively, show the probability of the economy being in a low growth state and the economy is residing in a technical recession. These figures are the real-time equivalent of Figure (2(a)) and Figure (2(b)). These probabilities are shown together with the probabilities estimated using the full sample. These probabilities can be regarded as analysing the ability of the model to detect turning points in real time.

The real-time probabilities of the economy being in a low growth state are shown in Figure (6(a)). First, the real-time probabilities are more volatile reflecting an end-points problem and, second, the model seems to estimate a higher probability of a low state regime on average at the end of the sample. However, when the economy was in recession, like during the great financial crisis, the model does provide a clear signal in real time that something bad is going to happen. The real-time probability spikes before the full-sample

[10] The data used in the model is only to a lesser degree subject to revisions except for GDP, and these revisions are not taken into account in the exercise. In the exercise, the currently available publications of GDP are used.

probability, but the signal is clear. A lesson from this figure is that real-time probabilities of below 30-40 per cent do not necessarily imply that a low growth regime is in the making. As described in the introduction, the output from the model must necessarily be complemented with what users of the model also know about the state of the economy.

That being said, the probabilities of a technical recession in real time are much closer to their full-sample counterpart. These probabilities are calculated as follows. The model is estimated up until period t, and a nowcast and a forecast two quarters ahead is made. If the model forecasts a technical recession, a dummy is set to 1; otherwise it is 0. This is done across all the simulations, 10.000, for that point in time, and the probability of a technical recession is thus estimated. As can be seen, these probabilities are not more volatile than their full-sample counterparts and overall are quite close to them. Hence, real-time probabilities of technical recessions are robust in real time and give a clear signal about the state of the economy, also in real time.

Looking at the nowcasts in Figure (7), a similar story naturally emerges. As can be seen, the activity factor produced in real time resembles quite closely the full-sample equivalents. As a result, the activity factor, or nowcasts, is robust in real time and gives a clear signal about the state of the economy.

6. Conclusion

Motivated by a wish to follow developments in the Danish economy in real time and to obtain a data-driven estimate of the effects of releases of indicators on future GDP growth, this paper proposes an econometric framework for detecting turning points in real time, i.e. to nowcast real activity in the Danish economy. A mixed-frequency dynamic factor model which can take account of the rather steep declines in activity seen recently was introduced and estimated on data for Denmark. The model can handle mixed frequencies and ragged ends and is easy to update and thus use in policy work on a daily basis. The estimates show that the model is able to detect the recessionary episodes in the Danish economy from 2000 until today in real time.

The model has been in use at Danmarks Nationalbank since January 2020, and in June 2020 the distribution of future GDP growth was published on Danmarks Nationalbank's website.[11] This was complemented by the application of a different nowcasting model built around the approach in Bańbura et al. (2010), see also the discussion in the introduction. The forecasts and densities were updated weekly through week 26 to week 53 in 2020.

[11] At the time of writing, the current nowcasts and the historical nowcasts can be found at https://www.nationalbanken.dk/da/vidensbank/tema/Sider/Nowcastmodeller.aspx.

These real-time nowcasts using data available during each week of this six month period can be seen in Figure (8). This episode was clearly unique with unprecedented fluctuations in Danish GDP. The models do provide nowcasts which capture these movements, and the actual GDP numbers fall just within the error bounds provided by this model, see also Figure (5). While the two nowcasting models differ greatly, they do not differ greatly in their forecasting performance during this unusual period, the model analysed in this paper providing slightly better nowcasts. The current plan is to continue to publish output from the models while the economists at Danmarks Nationalbank continuously learn about the strengths and weaknesses of the model and continue to update and improve them.

7. Tables and figures

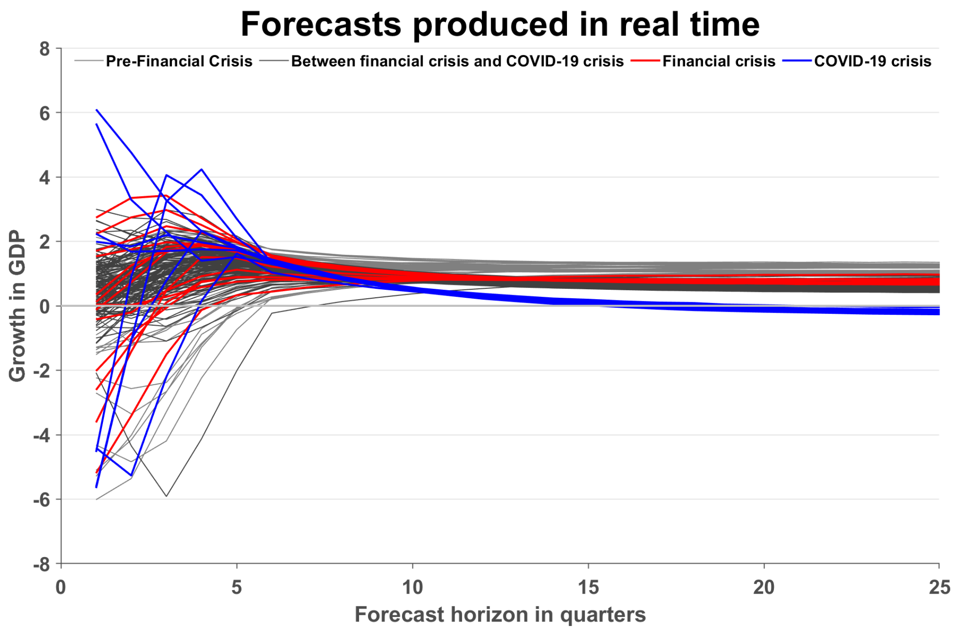

Figure 1: **Forecasting using the model**
This figure shows the real-time nowcast and forecasts for each month from 1 June 2005 to 1 December 2020 24 quarters ahead. The forecasts are divided into four sub-groups. The first group (light grey) includes the forecasts produced by the model before the outbreak of the financial crisis. The second group (red) includes forecasts during the financial crisis. The third group (dark grey) includes forecasts from the end of the financial crisis up until the outbreak of the COVID-19 crisis. The final group (blue) is forecasts produced during the COVID-19 crises.

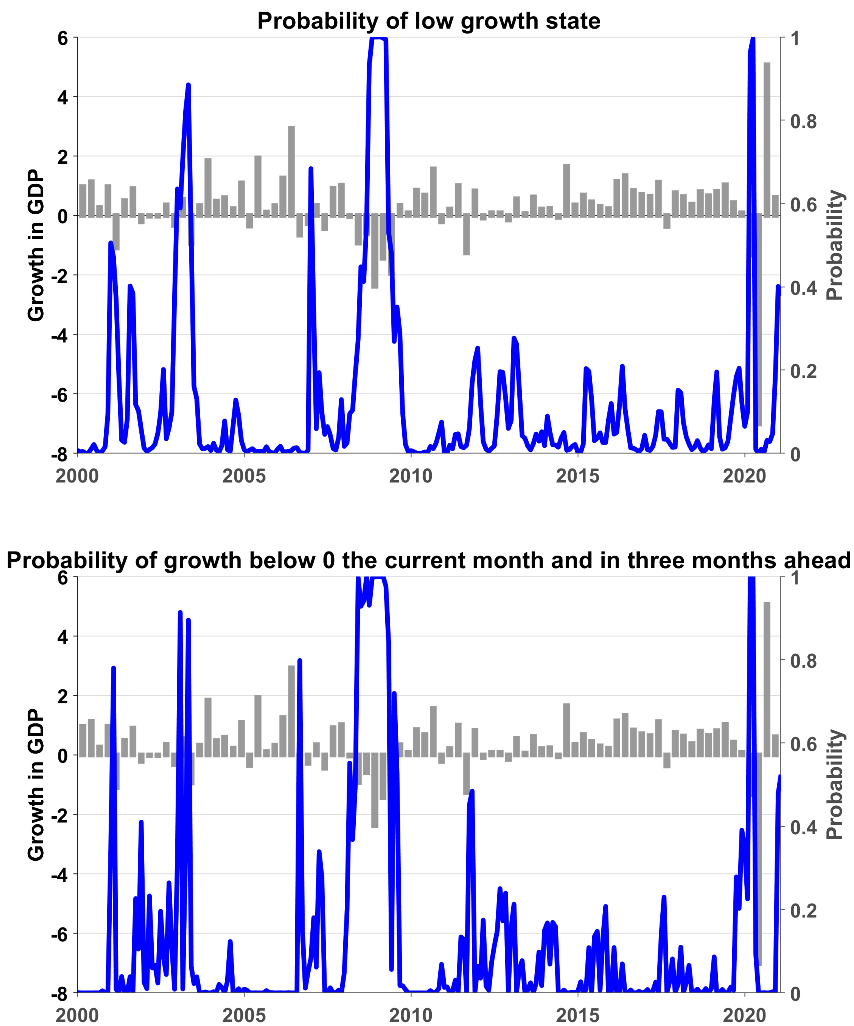

Figure 2: **Probability statements about the state of the economy**
The top figure shows the estimated probabilities of low growth from the model.
The bottom figure shows the estimated probabilities from the model of the economy ending up in a technical recession which is defined as two consecutive quarters of negative growth. Both probabilities are estimated on Danish data from the beginning of 2000 to mid-January 2021. The figure also shows the actual GDP growth rates. Both probabilities are estimated on Danish data from the beginning of 2000 to mid-January 2021.
In the figure is also shown actual GDP growth rates.

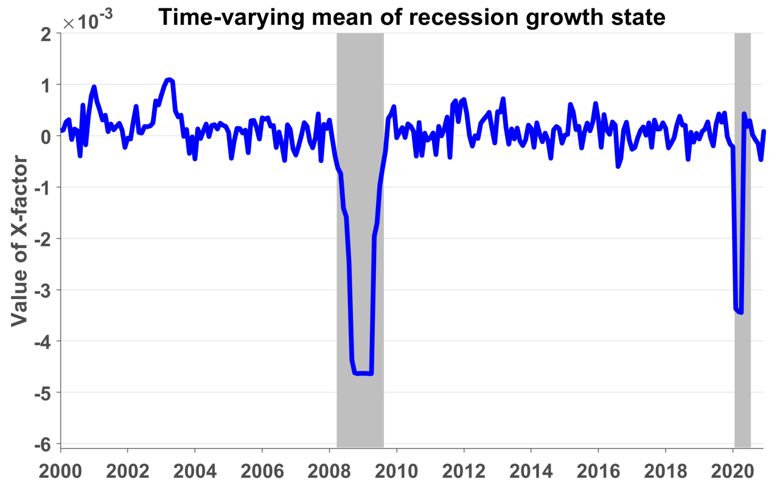

Figure 3: **Factor capturing the depth of the recession**
This figure shows the x-factor in equation (2.1) from the model estimated on Danish data from the beginning of 2000 to mid-January 2021. The shaded areas represent the financial crisis and the COVID-19 crisis, respectively.

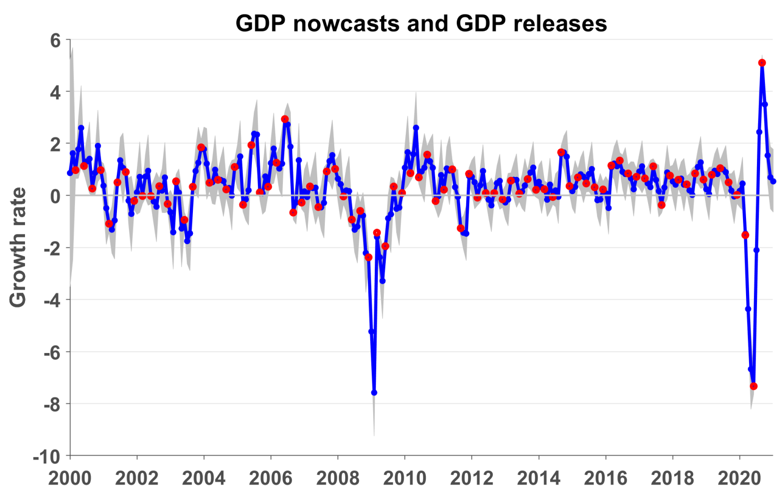

Figure 4: **Nowcasts estimated using the model**
This figure shows the nowcasts produced by the model using the full data sample. Grey-shaded areas denote the 95 per cent confidence bounds. The red dots denote GDP releases every quarter. These are the revised and final quarterly growth rates available at the time of writing this paper. The blue dots represent the model's nowcast of the activity in the Danish economy, which is close to coinciding with actual growth in GDP during months with official GDP estimates. For the later part of the sample, the most recent growth rates are used. The blue line represents the nowcasts produced by the model each month.

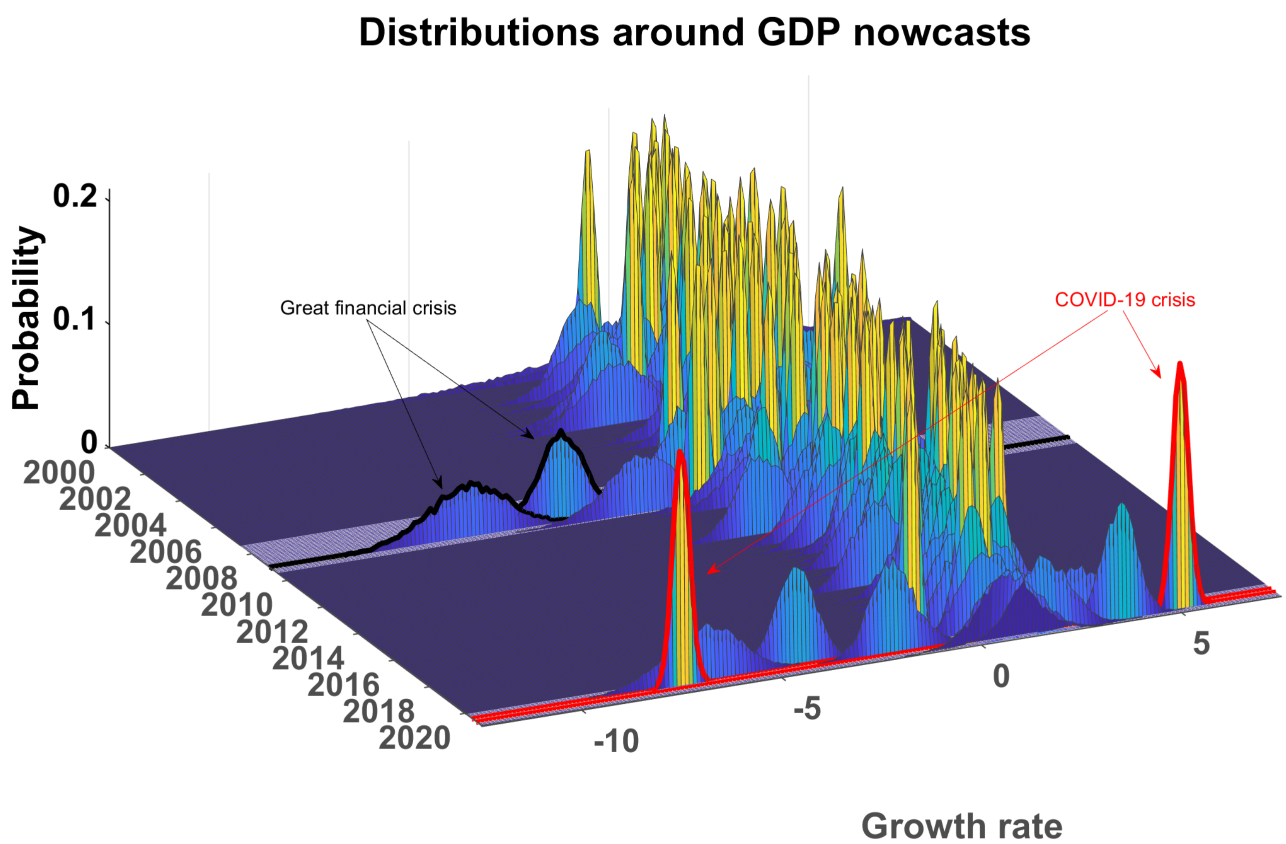

Figure 5: **Estimated distributions around the model's nowcasts**
This figure shows the distributions of GDP growth estimated using the model on Danish data from the beginning of 2000 to mid-January 2021. The shaded areas represent the financial crisis and the COVID-19 crisis.

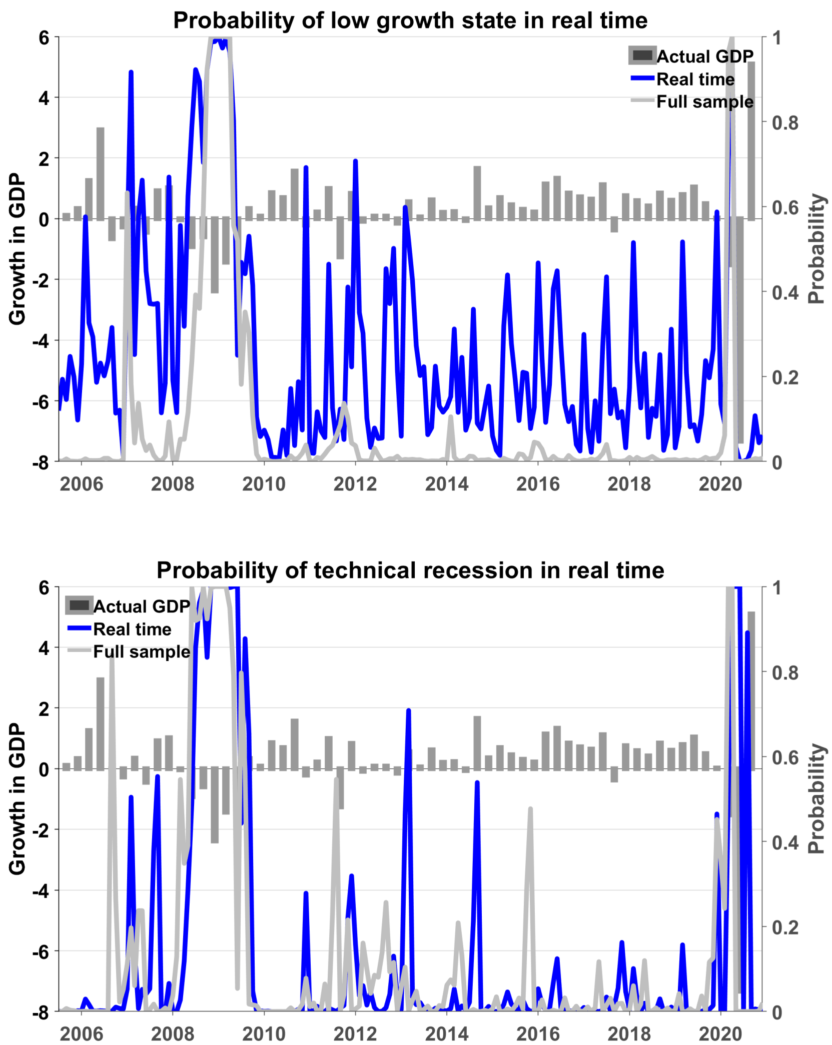

Figure 6: **Real-time evaluation of the model: Estimated probabilities**
The top figure shows the probability of a low growth regime estimated in real time.
The bottom figure shows the probability of a technical recession in real time, see the main text for details.
The grey lines reproduce the respective probabilities estimated using information from the full sample shown in Figures (2(a))-(2(b)).

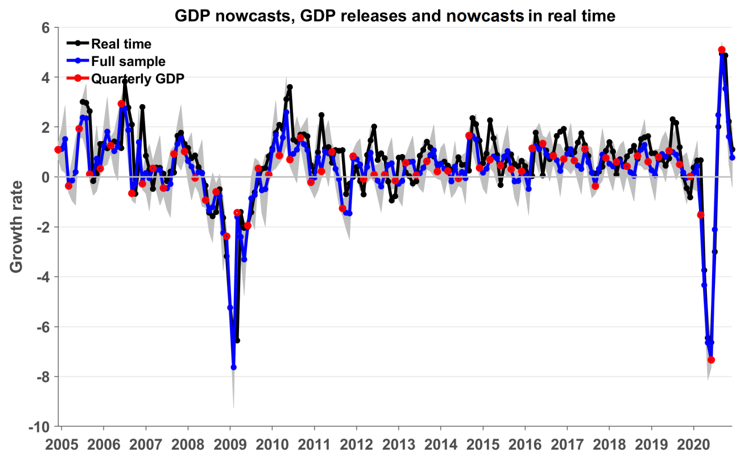

Figure 7: **Real-time evaluation of the model: Nowcast**
This figure shows the nowcasts produced by the model in real time. Grey-shaded areas denote the 95 per cent confidence bounds estimated on the full sample. The red dots denote GDP releases every quarter. These are revised and final quarterly growth rates. For the later part of the sample, the most recent growth rates are used. The blue line represents the nowcasts produced by the model each month, while the black line represents the nowcasts produced in real time.

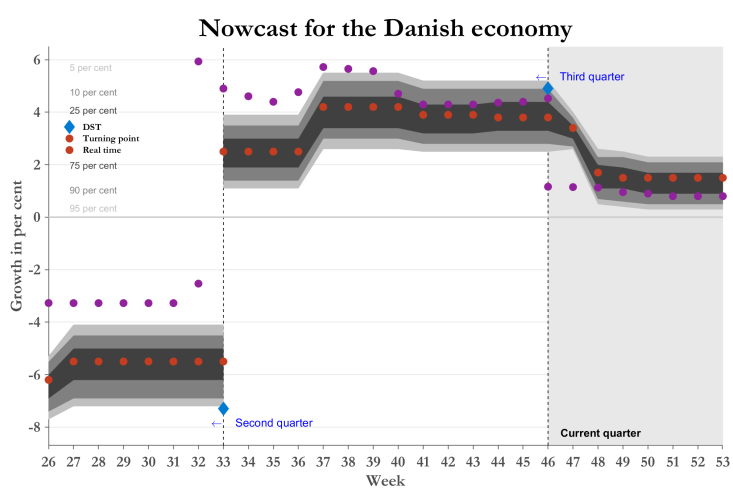

Figure 8: **The published nowcasts with distributions from the model and the model in Schmith and Grenestam (2021)**
This figure shows the published nowcasts from the model and the model from mid-June 2020 until the end of 2020. The figure also shows GDP growth rates published by Statistics Denmark and nowcasts from the model in Schmith and Grenestam (2021).

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