Potential Output in Denmark

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1. INTRODUCTION AND SUMMARY

In Denmark as well as in all other countries, economic developments are characterised by fluctuations in the short term. Growth in the gross domestic product, GDP, is high in some years, while other years see low – sometimes even negative – growth.

Disentangling such short-term movements from the underlying long-term trend in the economy's productive capacity is an important issue in macroeconomic studies. The concepts of *potential output* and *output gap* play a key role in this context. Potential output is the output level that the economy can sustain without inflationary pressures arising in the longer term. The output gap is the deviation of actual output from this level.

These concepts are relevant in many areas of economic policy. The development in potential output is typically regarded as an indicator of long-term economic trends and is often used as a measure of the effects of structural policy steps. The output gap, on the other hand, is regarded as a summary indicator of the cyclical position of the economy and is probably the most frequently used measure of this. Consequently, output gap estimates play a decisive role in macroeconomic forecasts and in the conduct of macroeconomic stabilisation policies. Moreover, the output gap is often seen as an indicator of the balance between supply and demand in the economy and hence of the pressure on the economy's resources. A positive output gap provides an indication of macroeconomic imbalances which will generate inflationary pressures and which are not sustainable in the longer term. This indicator property is especially relevant for central banks that use the output gap as a tool for monitoring inflationary developments. Finally, the size of the output gap is central to assessments of the long-term perspectives of fiscal policy, since the output gap is a central element of the calculation of the underlying fiscal position.

1 In writing this article, we have benefited from comments and suggestions by Christian Møller Dahl. The views and conclusions expressed in this article are strictly those of the authors. Any errors or omissions remain the responsibility of the authors.

The key challenge when using the concepts of potential output and the output gap is that they are *theoretical* concepts and therefore not directly observable. This means that, as opposed to *actual* output, *potential* output cannot be calculated by systematic collection of statistical information. Instead, potential output and the output gap must be estimated. Over the years, economists and econometricians have developed a variety of estimation methods for this exact purpose.

In this article we estimate the development in potential output and the output gap in the Danish economy since 1985. The estimations are based on the production function method, in which the economy's total output is modelled as a function of capital, labour and productivity. The production function method is one of the most frequently used methods for measuring potential output, and it is used by e.g. the OECD, the European Commission and the Danish Ministry of Finance, cf. Beffy et al. (2006), D'Auria et al. (2010) and Ministry of Finance (2004).

The calculations in this article – which are based on the latest national accounts statistics – show that the Danish economy experienced a severe overheating in the years leading up to the financial crisis. A low unemployment rate and strong capacity pressures in Danish firms contributed to a GDP level that was considerably higher than potential GDP in these years. As a result of the intense labour market pressures, wage growth was markedly higher than warranted by labour productivity growth.

The economic crisis that followed the 2008 global financial crisis led to a dramatic acceleration of the slowdown that had hit the Danish economy already in late 2007, and resulted in large negative output gaps in 2009 and 2010. The distance to potential output has since narrowed considerably, however, and we estimate the output gap to be -1.6 per cent of potential GDP in the 2nd quarter of 2011.

The negative output gap is first and foremost expressed by a participation rate that is somewhat below its structural level, while net unemployment is close to the level that is consistent with stable medium-term development in wages and prices. So if labour demand should increase, a sustainable rise in employment would require a relatively quick expansion of the labour force. The large number of persons in active labour market programmes and jobseeking students indicates that such an expansion could be feasible, since it can reasonably be assumed that many of these persons would be both willing and able to return to the labour market, should employment opportunities improve.

Productivity in the Danish economy, measured in terms of total factor productivity, TFP, fell strongly in the wake of the financial crisis in 2008 and 2009. The TFP level has recovered somewhat since the trough in

2009, but remains below the 2006 peak. Since the spare capacity in Danish firms has now almost returned to its historical average, we find that productivity is again close to its structural level.

In sum, our overall assessment is that there is currently modest spare capacity in the Danish economy. The gap between actual and potential output has narrowed considerably since 2009, and despite the outlook for limited growth in the current half-year we expect this gradual closing of the negative output gap to continue in the coming years.

It is important to bear in mind, however, that the exact size of the output gap will always be subject to great uncertainty, and the analyses in this article show that the estimates may be considerably revised at a later date – due to revision of existing national accounts figures or new data releases. This applies especially to the estimates of the elements of greatest interest, i.e. the current size of the output gap and its near-term development. Hence, output gap estimates can never stand alone in assessments of the current cyclical position, but should be integrated as an important element of the overall assessment.

This article continues as follows: Section 2 presents various views of the concept of potential output and specifies how the concept is used in this article. Section 3 describes our general estimation approach in this article, i.e. the production function method. In sections 4-6 we describe estimation methods and results for the structural level of each sub-component in the production function. In section 7, these results are put together to construct estimates of potential output and the output gap in the Danish economy since 1985. Section 8 discusses the sensitivity of the results to alternative unemployment measures, while section 9 focuses on the uncertainty associated with estimates of the current size of the output gap when the future course of the economy is unknown. Section 10 discusses how the financial crisis in 2008 and 2009 may have impacted the potential output level in Denmark. In section 11, we proceed to illustrate one of the most important uses of output gap estimates, i.e. for calculations of the structural fiscal balance. The results and principal conclusions of this article are summarised in Part 1 of this Monetary Review.

2. WHAT IS POTENTIAL OUTPUT?

Potential output and the output gap are key concepts in discussions of macroeconomic policy, for academic economists as well as practitioners. But despite their central role, the precise meaning of these concepts is often only vaguely defined. The reason is that different economists apply the concepts in different ways. One contributory factor has been the evolution of macroeconomic theory, leading to several changes in the view of how potential output can and should be defined. The latest novelty is the increased use of DSGE models, which has enabled a number of model-based definitions of the concept, cf. Box 2.1.

The original definition of the concept of potential output, which can be traced back to Okun (1962), referred to the output level that can be achieved under full employment, the latter being interpreted as a situation with very low unemployment. This definition implies that actual output can exceed potential output only under rare circumstances.

However, this original definition of the concept has been superseded. The most popular view today is that potential output is the output level that is consistent with stable inflation. Contrary to Okun's original characterisation, this definition implies that actual output can and will deviate, both positively and negatively, from potential output. But a prolonged period of positive output gaps will result in uncontrollable inflation hikes. Ultimately, this will harm the future growth potential in the economy so that the actual output level is reduced. According to this view, potential output is equal to the economy's *sustainable* output level.[1]

Another popular view is that potential output can be interpreted as the *trend* of actual output. According to this view, the potential output level can be viewed as an indicator of the development in actual output in the longer term, e.g. over a 10-year period. The interpretation of potential output as closely related to the trend output level also entails that the *output gap*, i.e. the relative deviation between actual and potential output, may be regarded as an expression of the cyclical position. As a result, potential output is sometimes also referred to as the output level achieved in a "normal" state of the business cycle.

Although the latter two definitions of potential output are conceptually different, they often appear concurrently, cf. e.g. Arnold (2009). This reflects the implicit view that output may deviate from the sustainable, inflation-neutral level for a short time only. So if this level evolves smoothly over time, it will coincide with the trend of the actual output level.

In an economy characterised by frequent, temporary shocks to the supply side, the inflation-neutral output level will *not* evolve smoothly, however, so the two notions of potential output will not necessarily

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1 According to Okun's original representation, a high output level would, over time, lead to *high*, but not *rising* inflation. The theory that only one output level is consistent with stable inflation stems from two almost contemporaneous articles by Friedman (1968) and Phelps (1967). These articles focused on the relation between *unemployment* and inflation and led to the theory of the *natural unemployment level*. Hence, the modern view of potential output can also be characterised as the output level achieved when unemployment is at its natural level, cf. also Congdon (2008).

POTENTIAL OUTPUT IN NEW-KEYNESIAN DSGE MODELS Box 2.1

The development of new-Keynesian DSGE (*Dynamic Stochastic General Equilibrium*) models has entailed a number of new, model-consistent definitions of potential output, each related to the traditional definitions described in the main text. The DSGE literature distinguishes between three definitions of potential output, cf. Vetlov et al. (2011):

Trend output is the level towards which output would converge in the absence of all temporary shocks, corresponding to a long-term steady-state development. In DSGE models, trend output normally follows a stable course over time and is probably the model-consistent concept that is related most closely to traditional definitions of potential output.

Natural output is the level of output that would be achieved with full flexibility of all prices in the product and labour markets. Given the slow adaptation, due to nominal rigidities, of prices and wages to their long-term equilibrium values in these models, the level of actual output will generally deviate from this level. This concept is closely related to the traditional view of potential output as an inflation-stabilising level of output: In many new-Keynesian models, the deviation of output from its natural level is decisive for inflation dynamics, and stabilisation of inflation requires stabilisation of output at its natural level. As opposed to the traditional view of potential output, this natural level is also volatile in the short term, since it is impacted by a large number of potential shocks to the economy. Moreover, the changes in the natural output level will typically show a positive correlation with changes in actual output, even within the time horizons that are normally associated with the duration of the business cycle. Consequently, the deviation from the natural level is a suitable indicator of short-term inflationary pressures, but not suitable as a measure of the cyclical position.[1]

Efficient output is defined as the output level that would be achieved with full price flexibility and *perfect competition* in all markets. It is the optimum output level in a welfare context. Due to market imperfections, the actual output level will, however, always be lower than the efficient level, and the concept is thus related to Okun's original definition of the potential output level as the "best achievable". But where Okun saw deviations from the potential level as a consequence of insufficient demand, the DSGE literature offers another conclusion: Deviations between the efficient and the natural output levels are due to imperfect competition in the product, service and labour markets and should therefore be addressed via *structural policy*. Conversely, persistent attempts to press output upwards to the efficient level via demand-stimulating policy will only result in higher inflation. These policies should therefore not aim at *minimising* the gap between actual and efficient output, but at *stabilising* it.

[1] The business cycle is here envisaged as lasting 2-8 years, in accordance with the general view in the literature.

coincide. A sudden, short-lived oil price increase, for example, will exert upward pressure on prices, resulting in a reduction of the output level that can be sustained without inflationary pressures arising. Since the increase is short-lived and the effect on output therefore only temporary, *trend* output will, by definition, not change. Hence, in order to ensure consistency between the two concepts, potential output is often defined as *the output level that is consistent with stable inflation in the absence of temporary supply shocks.*

In the remainder of this article, the concept of potential output refers to this latter definition. Consequently, we focus on potential output as a phenomenon that usually evolves smoothly over time. This implies that the resulting output gap estimates can be seen as measures of the cyclical position, making the estimates particularly relevant from a fiscal point of view, cf., among others, Basu and Fernald (2009): In the longer term, fiscal developments are closely related to developments in the output *trend*, and when the output gap expresses a measure of the deviation from this trend, the gap can be used to calculate indicators of the underlying fiscal position, cf. also section 11.

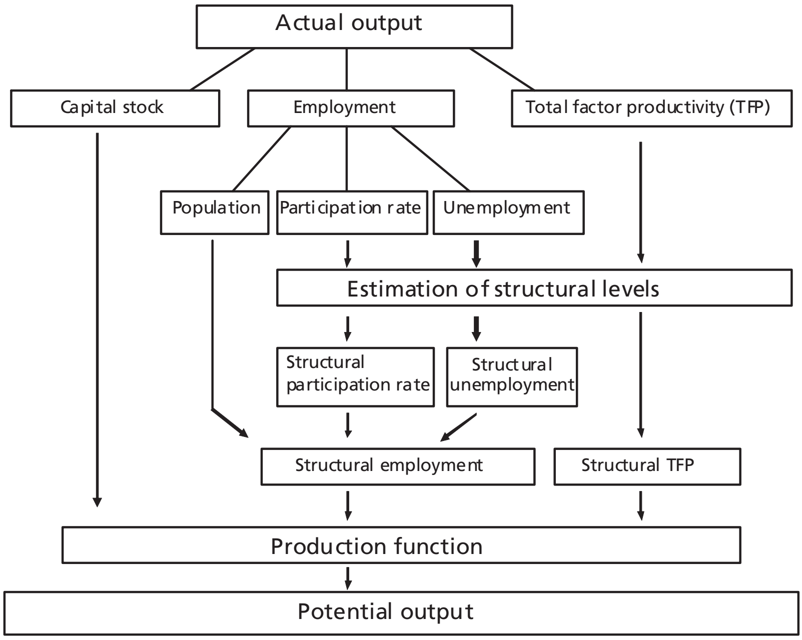
Moreover, the output gap estimates in this article may be interpreted as indicators of the inflationary pressures in the Danish economy in the *longer* term. Hence, persistently positive output gaps over several years will be a clear indication of mounting risk of overheating of the economy and rising inflationary pressures, while persistently negative output gaps will be an indication of falling inflationary pressures. However, it is important to bear in mind that the estimated output gaps are not necessarily accurate indicators of inflationary pressures in the *short* term, and that, accordingly, they do not represent a suitable benchmark for short-term inflation stabilisation. Such a benchmark would require estimation of another concept of potential output, which would allow a higher degree of volatility in the short term in the event of temporary supply shocks.

3. ESTIMATION OF POTENTIAL OUTPUT: THE PRODUCTION FUNCTION METHOD

In this article, the potential output level in Denmark is estimated using the production function method. Under this approach, total output in the economy, measured by GDP, is modelled as a specific function of measurable input factors (typically capital and labour) and total factor productivity, TFP. The latter is an overall measure of how efficienctly the production factors are used in the production process. Potential output is then calculated as the output level achieved when each input factor is at its structural level, cf. Chart 3.1.

The principal advantage of the production function method is that it allows decomposition of the output gap into the deviations of the underlying production factors from their structural levels. Similarly, the rate of growth in potential output can be decomposed into the growth

THE PRODUCTION FUNCTION METHOD FOR CALCULATION OF POTENTIAL OUTPUT Chart 3.1



rates for the structural levels of the input factors. Hence, this method provides for economic interpretation of developments in potential output and the output gap, thereby contributing to enhanced insight into the supply side of the economy. Such insight cannot be gained by methods estimating potential output solely by looking at the development in GDP, cf. also Box 3.1.

However, the production function method also has certain weaknesses: Firstly, the method requires specific assumptions regarding the form of the production function, and these assumptions influence the final estimate of potential output. Secondly, the method imposes substantial demands on the data, which can be difficult to meet. This applies especially to data for the size of the capital stock, which is typically subject to considerable uncertainty.

Finally, it is worth emphasising that the production function method as such is not an actual estimation method, but rather a framework that allows decomposition of potential output and the output gap into contributions from the underlying factors. The final estimate of potential output ultimately depends entirely on how the structural level of each

ALTERNATIVE METHODS FOR ESTIMATION OF POTENTIAL OUTPUT Box 3.1

The production function method is the most widespread method for estimation of potential output and the output gap, but it is far from being the only method. The most popular methods can be divided into the following main categories:

Univariate filters reflect statistical methods dividing GDP into a trend component and a cyclical component solely by means of mechanical transformations of actual GDP data. The Hodrick-Prescott filter (Hodrick and Prescott 1997) and the Baxter-King filter (Baxter and King, 1999) are popular examples. The principal advantage of these methods is that they are easy to use. But these filters are rarely anchored in economic theory, and the exact nature of what is estimated may be unclear. Moreover, the methods are subject to an *end point problem*, which makes it difficult to estimate the trend near the start and end points of the series. As regards the Hodrick-Prescott filter, the end point problem entails, for example, that the estimate of the trend in the last period of the time series will often be too close to the actual value in that period. This is problematic, given that the last period of the time series is typically the *present* period. In economic-policy planning, the estimate of the last period of the time series will therefore be of greatest interest.

Unobserved Components Models, UCM, reflect a more complex model approach where the trend in GDP is estimated by including information from other economic time series besides the GDP series itself. This approach is based on known macroeconomic relationships, often inspired by economic theory. The typical example is an expectations-augmented Phillips curve establishing a link between the output gap (or the unemployment gap) and inflation in the short term. The fundamental idea is that high and rising inflation indicates that output is above its potential level, while falling inflation indicates the opposite. In combination with a fully specified statistical model for the development in potential output, such relationships may be used for estimation of the level of potential output by means of recursive estimation methods, such as the frequently used Kalman filter. Kuttner (1994) and Apel and Jansson (1999) are examples of estimations of potential output based on the UCM approach. Theoretically, the UCM approach is considerably more satisfactory than simple, univariate filtering methods. But in practice, it can be difficult to apply this method in a satisfactory way, and the results will often be sensitive to detailed assumptions about the underlying statistical processes.

Estimates based on DSGE models constitute the newest type of estimates of potential output and the output gap and are not yet widely used. The European Central Bank, ECB, estimates potential output for the euro area within the framework of the *New Area Wide Model*, cf. Vetlov et al. (2011). DSGE-based estimates are fully model-consistent and therefore the theoretically most well-founded. In addition, this approach provides for a sophisticated distinction between various theoretical definitions of potential output, cf. also Box 2.1. The main drawback of this approach is that it requires development of a full DSGE model, which is highly time-consuming. Moreover, the estimates produced by such a model will be entirely dependent on the underlying model properties, and they will be more difficult to communicate to outsiders than estimates based on simpler and more well-known methods.

It is worth noting that several of these estimation methods can be used directly or in combination with the production function method. Previously, a widely used method was to use the framework of the production function method to decompose

CONTINUED Box 3.1

actual GDP into capital, labour and TFP, followed by application of the HP filter for cyclical adjustment of each of these components. However, simple HP filtering is increasingly found to be unsatisfactory, and more advanced methods are gaining ground, such as the UCM approach to estimation of the structural components within the framework of the production function approach. This type of combined approach is used in this article.

input factor is estimated. Sections 4-6 provide further descriptions of the methods used in this article to estimate these structural levels.

Application of the production function method in this article

The aggregate production function we use is a standard Cobb-Douglas function with constant returns to scale. Consequently, output in quarter *t* is modelled as:

$$Y_t = TFP_t \cdot L_t^{\alpha} K_t^{1-\alpha}, \tag{3.1}$$

where Y is GDP in volume terms, K is capital in volume terms, L is labour input and TFP is total factor productivity.

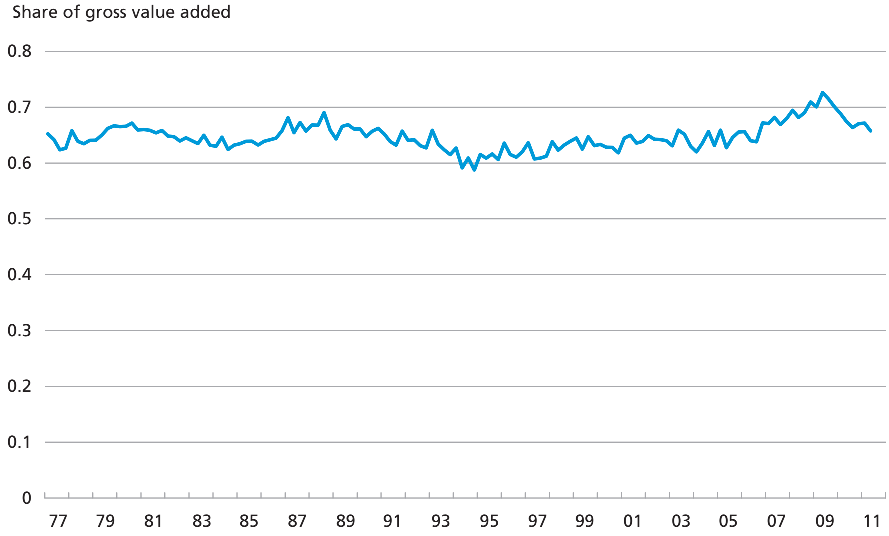
The main motivation for choosing the Cobb-Douglas function as our aggregate production function is simplicity. However, the specification also has a number of characteristics that are broadly consistent with empirical macroeconomic observations. For example, assuming perfect competition and profit maximisation, a production technology as in (3.1) will result in a constant wage share, equal to the elasticity of output with respect to labour, α. This is consistent with the fact that the wage share has been relatively constant over a large number of years, with limited fluctuations around an average of approximately 65 per cent, cf. Chart 3.2. In light of this observation, the parameter α is assumed to be equal to 0.65 in all periods.

Labour input is measured as the number of persons in employment, decomposed as:

$$L_t = B_t \cdot E_t \cdot (1 - u_t), \tag{3.2}$$

where B is the number of persons in the population in the 16-66 age group, E is the participation rate, and u is the unemployment rate. The unemployment rate is measured here as net unemployment, in full-time equivalents, divided by the number of persons in the labour force. The labour force is defined as the number of employed plus unemployed

WAGE SHARE IN THE PRIVATE NON-AGRICULTURAL SECTOR Chart 3.2



Source: Statistics Denmark and own calculations.

persons. The participation rate is defined as the number of persons in the labour force divided by the number of persons in the population in the 16-66 age group.[1]

The size of the capital stock is based on annual national accounts data. It is measured as the net stock of fixed real capital in 2000 prices, chained values, covering all industries and all types of capital[2]. The figures are converted to quarterly frequency by combining information about the size of gross fixed investment from the quarterly national accounts with an assumption of constant depreciation rates within the same year.

The TFP level is calculated residually on the basis of equations (3.1) and (3.2) as:

$$TFP_t = \frac{Y_t}{K_t^{1-\alpha}(B_t \cdot E_t \cdot (1-u_t))^{\alpha}} .$$

1 Ideally, labour input should be calculated as the number of hours worked rather than the number of employed. However, since the available time series for the number of hours worked in Denmark are burdened with considerable measurement problems, calculations based on them would generate noise in the estimates of structural employment and potential output. When labour input is measured as the number of employed, as is the case here, fluctuations in the average number of hours worked per employed person will be reflected in fluctuations in the TFP estimate.

2 The national accounts distinguish between gross and net stocks. In the gross stock, the value of a capital good is calculated as the replacement cost, disregarding the age dimension. Consequently, capital equipment of a given quality is assumed to have the same value, irrespective of its remaining lifetime. The net stock, on the other hand, takes into account the deterioration over time of the value of capital equipment due to wear and tear and a diminishing remaining lifetime. It remains an open question whether the gross stock would be a better measure of the productive capacity of the capital stock. However, we use the net stock, with a view to comparability with Danmarks Nationalbank's macroeconometric model, MONA.

This implies that all factors, other than capital and labour, that contribute to total output are included in TFP. Similarly, any measurement errors in the calculations of capital and labour will impact on the TFP measure. Moreover, fluctuations in the utilisation rate of the capital stock will be reflected in the TFP estimate, cf. also later.

Potential output, Y^*, is defined as the output level achieved when all components of the production function are at their structural levels. We then get:

$$Y_t^* = TFP_t^* \cdot (B_t^* \cdot E_t^* \cdot (1 - u_t^*))^{\alpha} (K_t^*)^{1-\alpha}, \quad (3.3)$$

where an asterisk indicates the structural level of the variable in question.

For the number of people in the population in the 16-66 age group and the size of the capital stock, the structural quantities are defined as being equal to the actual quantities. Hence, by definition, it applies that $B^* = B$ and $K^* = K$. As regards the population, this definition reflects that demographic changes are regarded as fully structural and therefore independent of the business cycle.[1] As regards the capital stock, the definition mainly reflects that the time series for the size of the capital stock is relatively smooth already at the point of departure, despite the strong volatility of investment in fixed real capital. The reason is that, in a given quarter, net investment is very small relative to the size of the accumulated capital stock. Conceptually, the definition $K^* = K$ implies that the estimate of potential output should be regarded as *conditional* on the current size of the capital stock. Consequently, the final result for potential output may be seen as an estimate of the sustainable output level of the economy, *given* the present volume of available capital.[2]

In contrast, the time series for TFP, the participation rate and unemployment are affected by both cyclical and structural movements, and it requires formal estimation to separate them. In sections 4-6 we describe how we define and estimate each of these structural components.

The output gap, Y^g, is defined as the relative deviation between the actual output level and potential output. Combining equations (3.1),

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1 In this way, the potential cyclical effects on the volume of immigration and emigration are ignored. Conversely, smoothing out the time series for the number of persons in the 16-66 age group would remove a considerable amount of the variation in the series that should be regarded as structurally driven, i.e. the variation attributable to the number of births and deaths.

2 This interpretation is parallelled in the new-Keynesian DSGE literature, in which a distinction is made between *conditional potential output* and *unconditional potential output*. The former is the output level that would be achieved if all nominal rigidities in the economy ceased to exist as from today, while the latter is the level that would be achieved in the absence of nominal rigidities in all past and future periods. The difference is that the size of the capital stock is taken as given in the first definition, while the definition of *unconditional potential output* implies that the potential level of the capital stock may deviate from the actual level.

(3.2) and (3.3) and the definitions $B^* = B$ and $K^* = K$, the ouput gap can then be approximated:

$$\begin{aligned} Y^g &= \ln Y_t - \ln Y_t^* \\ &\approx \ln TFP_t - \ln TFP_t^* + \alpha \cdot (\ln E_t - \ln E_t^* - (u_t - u_t^*)). \end{aligned} \tag{3.4}$$

This shows that the output gap is given by the sum of a TFP gap and a participation rate gap less an unemployment gap. The latter two are weighted using the parameter α, which is assumed to be equal to 0.65 as mentioned previously.

In this article, we use the expressions in (3.3) and (3.4) to calculate estimates of potential output and the output gap in the Danish economy for each quarter since 1985. The motivation for the choice of starting year is that the introduction of the fixed-exchange-rate policy in the autumn of 1982 represented a change of regime in the Danish economy, which resulted in permanent shifts in key economic mechanisms, including the formation of prices, wages and expectations. Since it is more than likely that the transition to the new regime happened gradually over several years, it makes sense to choose 1985 as the starting year for the estimation period.

The data are based on official statistics from Statistics Denmark for the period from the 1st quarter of 1985 to the 2nd quarter of 2011. These statistics have been supplemented with Danmarks Nationalbank's latest forecast for the Danish economy until end-2013. We have included the forecast-based data for two reasons: to calculate estimates of the development in the output potential and the output gap in the near future, and to improve the accuracy of the estimates of the current output gap, cf. section 9. The forecast itself is described in more detail in "Recent Economic and Monetary Trends" in Part 1 of this Monetary Review.

4. STRUCTURAL UNEMPLOYMENT

Structural unemployment is an important component in the determination of potential output. But it also attracts interest in its own right, since the theory of a structural or *natural* level of unemployment is a key element of understanding the labour market and the relationship between unemployment on the one hand and wage and price developments on the other.

Generally, structural unemployment can be defined as the level of unemployment that is *sustainable* in the long term. In this case, sustainability is typically interpreted as stable inflation. This reflects the predominant view that a trade-off exists between unemployment and inflation in the *short term*. Consequently, when unemployment is low, inflation is high and vice versa. In the *long* term, however, there is no such trade-off, meaning that a low level of unemployment will, over time, not only entail high but also *rising* inflation. But persistent increases in inflation are not sustainable in the long term and will ultimately lead to higher unemployment. Accordingly, structural unemployment is often defined as the level of unemployment that is consistent with stable inflation. As a result, structural unemployment is often referred to as *NAIRU,* which stands for *Non-Accelerating Inflation Rate of Unemployment.*

But in a small, open economy like Denmark, pursuing a fixed-exchange-rate policy against a major currency, stable inflation *per se* is not a sufficient criterion for sustainability. The level at which inflation is stabilised is also important. Hence, for the fixed-exchange-rate policy to be sustainable, inflation must not persistently exceed inflation in the currency anchor. Structural unemployment in Denmark can therefore be defined as the level of unemployment ensuring that price developments in Denmark mirror those in the euro area in the medium term.

Just like potential output, structural unemployment is a theoretical concept that is not directly observable. This entails uncertainty as to how much unemployment can fall without inflationary pressures arising. The structural level may change over time, further reinforcing the challenge. Possible reasons are e.g. changes in the unemployment benefit system (where the degree of compensation, in particular, is considered an important factor), the relative balance between the organisations in the labour market, or the degree of match between the qualifications of the unemployed and the qualifications demanded by the employers. The list of factors that may potentially affect structural unemployment is long, and it must be borne in mind that there is considerable uncertainty as to which factors determine its level.

Estimation method in this article

In this article, structural unemployment is determined by means of an *Unobserved Components Model,* UCM. The UCM approach was first applied to estimation of structural unemployment in the USA by Staiger, Stock and Watson (1996) and Gordon (1997, 1998). The method has been used increasingly since then and is currently applied by the OECD, the European Commission and the Danish Ministry of Finance, among others.

The UCM approach requires no explicit assumptions regarding the specific factors causing changes in the unobservable variables – in this case structural unemployment and the unemployment gap. Instead, the values of the unobservable variables are inferred from assumed relations between the unobservables and some observable variables. These observation relations are typically inspired by economic theory. At the same time, a full statistical model is constructed to account for developments in the unobservable variables, where changes over time are attributed to unspecified, stochastic shocks. On the basis of the full model, the values of the unobservable variables can then be estimated using the Kalman filter, a mathematical method for deriving information on the true values of variables that can only be observed subject to measurement error and other random influences.

In UCM estimations of structural unemployment, an expectations-augmented Phillips curve is often used as the key economic relation. This curve states a negative relation between the unemployment gap and the *change* in inflation.[1] The rationale is that rising inflation, everything else being equal, indicates that unemployment is below its structural level, implying a negative unemployment gap. Falling inflation, on the other hand, indicates a positive unemployment gap. This rationale is evident, given the general definition of structural unemployment as the level of unemployment at which inflation is constant.

However, this approach is problematic when applied to Danish quarterly data. The reason is that, as a result of the fixed-exchange-rate policy, price developments in Denmark are anchored in price developments in the euro area in the medium term. This means that price developments are less governed by internal factors, including influences from the labour market. Moreover, in the short term, price developments in a small, open economy like Denmark are driven by fluctuations in e.g. energy and food prices, which can be very volatile. The outcome of these factors together is that the observed correlation between unemployment and inflation is not strong and stable enough to be used for identification of the size of the unemployment gap in the Danish economy.

Developments in the Danish labour market, on the other hand, play a decisive role in wage developments. Consequently, this article's estimations of structural unemployment are, instead, based on a negative correlation between the unemployment gap and the development in the *wage share* in the private non-agricultural sector. If the unemployment rate is below its structural level, the resulting pressure on the

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[1] Such formulation of the expectations-augmented Phillips curve rests on the assumption that expected inflation can be approximated by lagged inflation.

labour market will lead to higher wage inflation. Given that domestic prices are, as mentioned, anchored to price developments in the euro area, higher wage inflation will entail a faster pace of real wage inflation compared with productivity. This will cause the wage share to rise and the profit ratio to fall.

Consequently, the development in the wage share, which is an observable variable, may be used to extract information on the size of the unemployment gap: A high and rising wage share indicates, everything else being equal, that unemployment is below its structural level, whereas a low and falling wage share indicates that unemployment is higher than its structural level.

The relation between unemployment and the wage share prompts the following equation:

$$\Delta ws_t = \sum_{i=1}^{4}\left(\delta_i (ws_{t-i} - ws^*)\right) - \gamma \cdot u_{t-1}^c + \mathbf{x}_t{}'\beta + \varepsilon_t \,, \qquad (4.1)$$

where ws is the logarithm of the wage share in the private non-agricultural sector, u^c is the unemployment gap, i.e. the deviation between actual and structural unemployment, and $\mathbf{x}$ is a vector of control variables, capturing the effect of temporary supply shocks. These comprise commodity price inflation, changes in the effective exchange rate and the growth in average labour productivity in the private non-agricultural sector. All control variables are expressed as deviations from their respective sample means and are included both contemporaneously and with one and two lags. The error term ε is assumed to be normally distributed with a mean of zero.

Equation (4.1) describes a dynamic process for the logarithm of the wage share. The change in the share is modelled as a function of the wage share level in the preceding quarters, unemployment in the preceding quarter and a number of temporary supply shocks. If unemployment is maintained at its structural level, whereby the unemployment gap is zero, the wage share, in the absence of temporary supply shocks, will, over time, converge towards its long-term level, ws^*, which is here assumed to be constant. The idea behind the equation is described in more detail in Box 4.1, while the Technical Appendix contains a more formal description of the theoretical basis.

The wage share equation in (4.1) is combined with a statistical model of the developments in structural unemployment and the unemployment gap. The full model is estimated using the Kalman filter and an iterative maximum likelihood procedure. The model and the estimation method are described in more detail in the Appendix.

THE RELATIONSHIP BETWEEN THE UNEMPLOYMENT GAP AND THE WAGE SHARE Box 4.1

The wage share is defined as the share of value added that is used for compensation of wage earners and can be calculated as total wage compensation divided by gross value added. The change in the logarithm of the wage share can be written as:

$$\Delta ws_t = \Delta w_t - \Delta p_t - \Delta y_t$$

where w is average nominal hourly wages, p is a measure of the price level (in producer prices), and y is average productivity per hour worked, all in logarithms. It follows that the relative change in the wage share is given by growth in real wages less growth in average hourly productivity.

In the long term, the wage share will typically be determined by structural factors on the supply side of the economy, including firms' production technology and competition climate. Assuming perfect competition and Cobb-Douglas production technology, the firms' profit maximisation will, for instance, entail a wage share equal to the elasticity of output with respect to labour input. The assumption of a constant, long-term equilibrium value for the wage share is supported by relatively constant wage shares in Denmark and a number of other countries over long periods, cf. also section 3.

In the short and medium term, however, the wage share may deviate from its long-term equilibrium value, possibly due to a certain lag in firms' adjustment to new conditions. For example, rigid nominal prices and wages may entail that real wages do not adjust immediately to cyclical fluctuations in productivity, so these fluctuations will cause the wage share to change. The wage share can also be changed as a result of temporary fluctuations in factors that can affect firms' pricing. These factors include commodity price inflation and the effective exchange rate.

Finally, fluctuations in unemployment may cause the wage share to deviate from its long-term level. A low rate of unemployment will typically generate an upward pressure on wages. In a small, open economy like Denmark, pursuing a fixed-exchange-rate policy against a major currency, price dynamics tend to be more closely linked to price developments in the currency anchor, and the pass-through of the higher wage inflation to price inflation will thus be less than complete. For a given rate of increase in productivity, this entails a higher wage share. Conversely, the wage share tends to fall when unemployment is high.

Structural unemployment is the level of unemployment that is consistent with convergence of the wage share, in the medium term, towards its equilibrium level. Actual unemployment may deviate from this level in the short and medium term. If unemployment is below its structural level, entailing a negative unemployment gap, the wage share will, all other things being equal, rise, while the profit ratio will fall. The result is deterioration of the competitiveness of Danish firms, which will eventually lead to lower demand for labour and higher unemployment, until the wage share returns to a normal level. Consequently, unemployment cannot deviate permanently from its structural level.

The relationship between the unemployment gap and the wage share is reflected by the inclusion of the unemployment gap on the right-hand side of equation (4.1). In addition, the formulation of the equation explicitly takes into account the developments

CONTINUED Box 4.1

in productivity, commodity prices and the effective krone rate, respectively, which may all influence the course of the wage share, cf. above. Finally, lagged values of the wage share level are included. This reflects an error correction mechanism prompted by the possible impact on hourly wages of the historical ratio between real wages and productivity, cf. e.g. Blanchard and Katz (1999).

Estimation results: Structural unemployment in Denmark since 1985

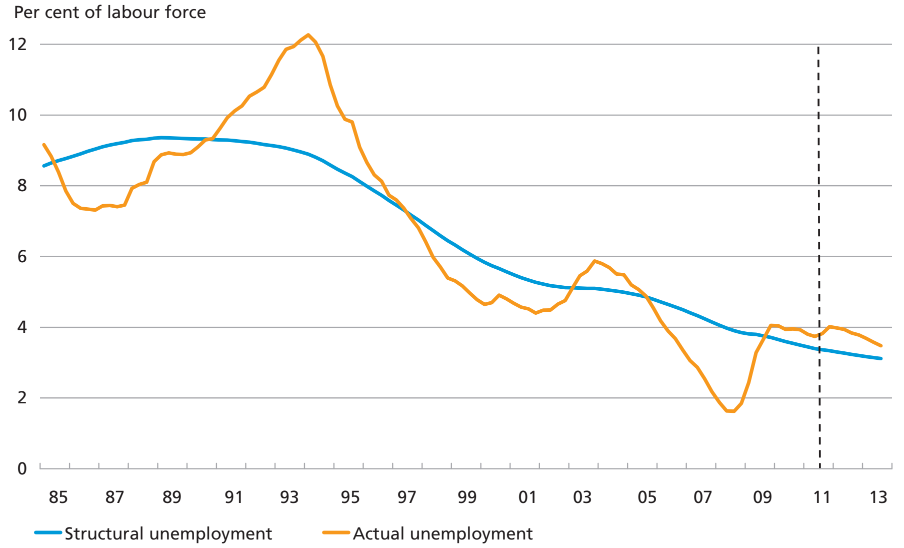
Table 4.1 shows estimates of the coefficient parameters in equation (4.1). The results show a clear and significant negative relation between the change in the wage share and the estimated unemployment gap. An increase in the unemployment gap by 1 percentage point is, all else equal, associated with a relative drop in the wage share of approximately 0.4 per cent in the subsequent quarter. Furthermore, for a given unemployment gap, a 1 percentage point increase in average labour productivity is associated with an average drop in the wage share of 0.6 per cent. This reflects a limited initial reaction in the growth rate of real wages, which increases less than one-for-one with the growth rate of productivity. Moreover, higher commodity price inflation causes the wage share to decline, but with a lag of a few quarters.

The coefficients on the lagged values of the wage share indicate a certain persistence. A short-lived increase in unemployment, for example, continues to impact the wage share for several quarters. However, in the absence of new shocks, the wage share will, over time, converge towards its long-term value, estimated here at 0.65. This is well

ESTIMATION OF EQUATION (4.1) Table 4.1

Variable	Coefficient	Standard error
Unemployment gap (t-1)	-0.40	(0.16)
Wage share gap (t-1)	-0.67	(0.09)
Wage share gap (t-2)	0.30	(0.09)
Wage share gap (t-3)	0.01	(0.09)
Wage share gap (t-4)	0.26	(0.07)
Commodity price inflation (t)	-0.03	(0.03)
Commodity price inflation (t-1)	0.08	(0.03)
Commodity price inflation (t-2)	-0.10	(0.03)
ΔEffective krone rate (t)	0.02	(0.12)
ΔEffective krone rate (t-1)	0.47	(0.12)
ΔEffective krone rate (t-2)	-0.14	(0.12)
Productivity growth (t)	-0.62	(0.08)
Productivity growth (t-1)	-0.33	(0.10)
Productivity growth (t-2)	-0.05	(0.10)
Long-run equilibrium wage share	0.65	

ACTUAL AND STRUCTURAL UNEMPLOYMENT — Chart 4.1



Note: Unemployment is calculated as registered net unemployment. The labour force is calculated as the number of employed persons plus net unemployed persons. The figures to the right of the dashed line are based on Danmarks Nationalbank's latest forecast of the Danish economy.

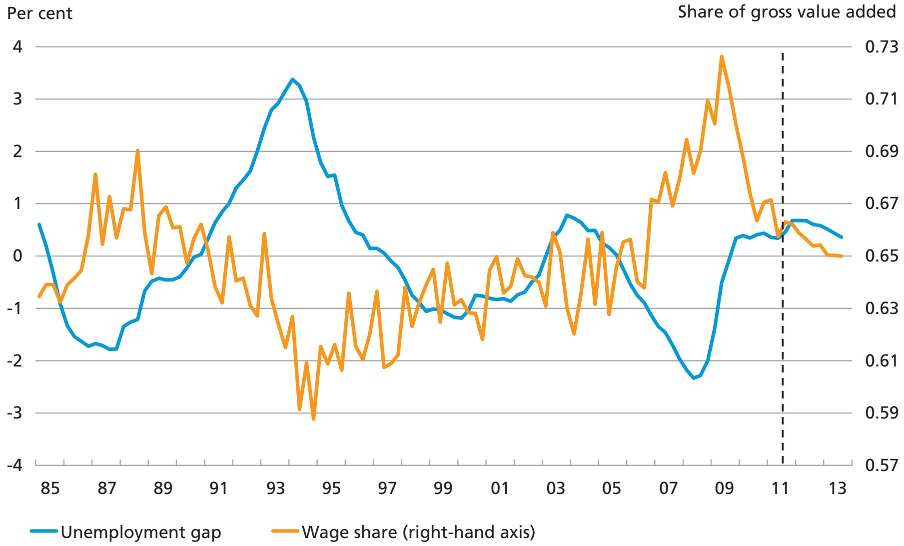
Source: Statistics Denmark and own calculations.

in accordance with the assumption that the parameter α in the aggregated Cobb-Douglas production function is equal to 0.65, cf. section 3.

Overall, structural unemployment has shown a remarkable decline over the period under review, cf. Chart 4.1. While structural net employment was more than 9 per cent of the labour force, corresponding to more than 260,000 persons, at the beginning of the 1990s, the current level is approximately 3.4 per cent of the labour force, or just under 100,000 full-time equivalents. The principal driver of this development is the labour market reforms – implemented particularly in the 1990s – which *inter alia* reduced the unemployment benefit entitlement period while also launching a far more active labour market policy than previously pursued. Before the reforms, the focus was on securing income support for the unemployed, but with the reforms, it shifted to ensuring their return to employment. The result of this reform drive was a notable and sustained drop in structural unemployment.

Compared with the development in actual unemployment, the development in structural unemployment prompts the assessment that the unemployment gap was positive over a prolonged period in the 1990s, but became negative just before the millennium rollover, cf. Chart 4.2. The recession in the early 2000s gave rise to a short-lived positive unemployment gap, while the most recent boom caused unemployment to plummet to far below its structural level. The recent economic crisis

UNEMPLOYMENT GAP AND WAGE SHARE Chart 4.2



Note: The figures to the right of the dashed line are based on Danmarks Nationalbank's latest forecast of the Danish economy.
Source: Statistics Denmark and own calculations.

saw a considerable rise in unemployment, closing the negative gap. The estimate for the 2nd quarter of 2011 indicates a positive unemployment gap of around 0.3 per cent of the labour force, or about 8,500 full-time equivalents.

The described development in the unemployment gap is closely related to the dynamics of the wage share, cf. Chart 4.2. During the recession in the early 1990s, for example, the wage share fell to a level far below its historical average. Conversely, the strong overheating of the labour market in 2006-07 brought the wage share to new historical peaks, before the financial crisis triggered the abrupt reversal in the labour market.

5. STRUCTURAL PARTICIPATION RATE

In this article, the participation rate is defined as the number of persons in the labour force divided by the number of persons in the population in the 16-66 age group. The labour force is defined as the sum of the number of employed persons and the number of registered full-time unemployed persons. Consequently, jobseekers who are not included in registered unemployment, including persons in active labour-market programmes, self-supporting unemployed and jobseeking students, are not counted in the labour force.

A fall in employment during an economic slowdown will, however, result in a rising number of not only registered unemployed persons, but also persons in each of the groups mentioned above. This means that more people will disappear from the registered labour force, causing the participation rate to fall. Conversely, the participation rate tends to increase during a boom.

The increased use of foreign labour in recent years also contributes to this pattern. The reason is that foreigners employed at Danish firms count in employment and hence in the labour force, but not in the population figure. Since the number of foreigners employed at Danish firms is strongly dependent on the business cycle, this will also contribute to a procyclical participation rate.

The structural participation rate is the level that would be obtained in the absence of such cyclical fluctuations. In this article, we estimate the structural participation rate using an Unobserved Components Model, as was also the case with structural unemployment. The central observation equation of the model is given as

$$shortage_t = \eta_0 + \eta_1 \cdot E_t^c + \varepsilon_t \ , \quad (5.1)$$

where $shortage$ is the share of industrial firms reporting a shortage of labour (seasonally adjusted), E^c is the deviation between the actual and structural participation rate, and ε is an error term that is assumed to be normally distributed with a mean of zero. The rationale behind the equation is that when the participation rate is higher than its structural level, the amount of spare labour in the economy will be limited, and more firms will experience a labour shortage. Conversely, a participation rate below the structural level will be associated with substantial reserves of potential labour, and fewer firms will experience a labour shortage.

The development in the structural participation rate, E^*, is described in terms of the following equation:

$$E_t^* = E_{t-1}^* - \Delta ogy_t + \mu \cdot \Delta leave_t - 0{,}5 \cdot \Delta sh6066_t + \zeta_t \, . \quad (5.2)$$

According to the equation, the structural participation rate is specified as a random walk with drift. The drift changes over time, depending on the development in the observable variables Δogy, $\Delta leave$ and $\Delta sh6066$. The variable Δogy denotes the change in the number of participants in an early retirement programme known as *transitional allowance*, while $\Delta leave$ denotes the change in the number of participants in leave schemes. These variables are included with a view to explicitly taking into account the impact of a number of labour market policy measures that were implemented in the 1990s and caused the participation rate to plummet. One of these measures was the introduction of transitional allowance. This retirement scheme was introduced in 1992 and later extended in 1994, bringing the number of participants in the scheme to just under 47,000. The scheme was closed for admissions in 1996, and the participation rate for the age group in question then rose in step with the phasing out of the scheme. From the 1st quarter of 1992 to the 1st quarter of 1996, the variable Δogy is equal to the quarterly change in the number of recipients of transitional allowance, divided by the number of persons in the 16-66 age group. Outside this period, the value of Δogy is zero. The coefficient on Δogy is set at -1. This restricttion represents the assumption that for each person admitted to the transitional allowance scheme, the structural labour force is also reduced by 1 person.

Another labour market policy measure in the 1990s was the implementation (and subsequent phasing out) of leave schemes for childminding, education and sabbaticals. At the beginning of 1995, the total number of participants in these schemes peaked at just over 87,000 people. The variable $\Delta leave$ takes the value of zero until 1992, when the number of participants in the schemes is registered for the first time. After this time, $\Delta leave$ equals the change in the number of participants in the leave schemes, divided by the number of persons in the 16-66 age group. The coefficient on $\Delta leave$ is not subject to restrictions. The reason is that part of the variation in the number of participants in the leave schemes must be assumed to be cyclical, whereby the relationship between the number of participants and the reduction of the structural labour force is less than 1:1.

The variable $\Delta sh6066$ denotes the change in the ratio between the number of persons in the 60-66 age group and the 16-66 age group. This variable is included in order to take into account that a larger share of 60-66 year-olds in the population will result in a lower structural participation rate, since this age group, irrespective of the cyclical position, has a lower participation rate than the rest of the population of working age. One of the reasons is access to the early retirement scheme. The coefficient on $\Delta sh6066$ is set at -0.5, reflecting that the average difference in the participation rate between the 16-59 age group and the 60-66 age group was approximately 50 percentage points in the period 1985-2005.

The development in the cyclical component of the participation rate, E^c, is described by the equation:

$$E_t^c = \phi_1 \cdot E_{t-1}^c + \phi_2 \cdot E_{t-2}^c + \chi \cdot u_{t-1}^c + \kappa_t \ , \qquad (5.3)$$

PARAMETER ESTIMATES IN THE MODEL FOR STRUCTURAL PARTICIPATION RATE — Table 5.1

Parameter	Estimate	Standard error
μ	-0.76	(0.36)
φ_1	0.84	(0.11)
φ_2	0.04	(0.11)
χ	-0.04	(0.05)
η_0	0.02	(0.00)
η_1	1.32	(0.55)

Source: Own calculations.

where u^c is the unemployment gap and κ is a noise term that is assumed to be normally distributed with a mean of zero. The lagged values of E^c on the right-hand side capture persistence in the cyclical component of the participation rate. One reason for such persistence is that persons who leave the labour force temporarily due to lack of job opportunities become engaged in other activities, e.g. studying. Once they have left the labour force, there is a certain probability that they will remain outside the labour force in the following quarters. The unemployment gap in the preceding quarter is included on the right-hand side of the equation to capture a "discouraged worker" effect, since higher unemployment can result in more people not finding employment and leaving the labour force. The unemployment gap is estimated as described in the preceding section. It is then treated as an exogenous, observable variable in the estimation of the structural participation rate.

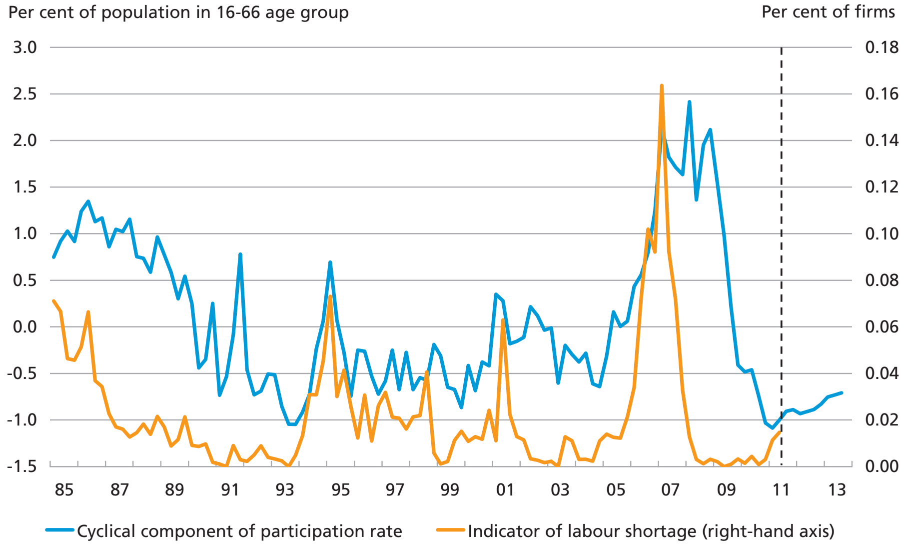
The model is estimated using the Kalman filter.[1] All parameter estimates have the expected signs, cf. Table 5.1. The coefficient on $\Delta leave$ in equation (5.2) is -0.76, i.e. 76 per cent of the inflow to and outflow from the leave schemes of the 1990s is counted as structural changes in the labour force. The coefficient on the unemployment gap in equation (5.3) has the expected negative sign, in accordance with a discouraged worker effect, but is not statistically significant. Finally, the coefficient on the cyclical component of the participation rate in equation (5.1) is positive and clearly significant. Thus, as expected, there is a positive relationship between the shortage of labour in manufacturing and the estimated cyclical component of the participation rate, cf. Chart 5.1.

The estimated series for the structural participation rate shows a strong drop in the period 1992-96, cf. Chart 5.2. This is directly attributable to the sudden growth in the number of persons in leave schemes and on transitional allowance in those years. Subsequently, the gradual phasing out of the leave schemes prompted small, abrupt increases in the

[1] The model and the estimation method and results are described in more detail in the Appendix.

CYCLICAL COMPONENT OF THE PARTICIPATION RATE AND LABOUR SHORTAGE IN MANUFACTURING INDUSTRY

Chart 5.1

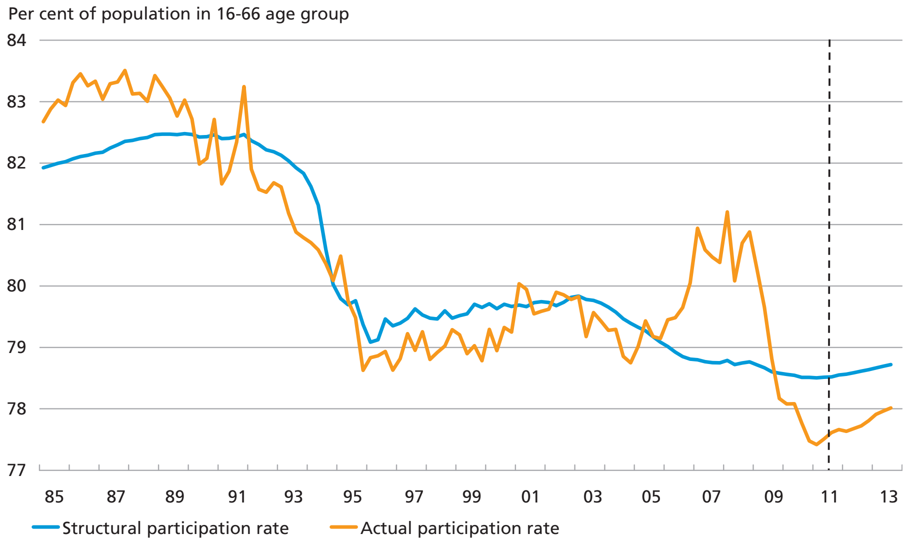


Note: The indicator of labour shortage indicates the share of firms in manufacturing industry, weighted by number of employees, reporting labour shortage as a production constraint. The figures to the right of the dashed line are based on Danmarks Nationalbank's latest forecast of the Danish economy.

Source: Statistics Denmark and own calculations.

ACTUAL AND STRUCTURAL PARTICIPATION RATE

Chart 5.2



Note: The participation rate is calculated as the number of persons in the labour force divided by the number of persons in the population in the 16-66 age group. The figures to the right of the dashed line are based on Danmarks Nationalbank's latest forecast of the Danish economy.

Source: Statistics Denmark and own calculations.

structural participation rate in the following years in step with the reduction of the number of participants in the schemes.

From 2003 until today, the structural participation rate has fallen gradually by just over 1 percentage point in total, whereby it is estimated to be 78.5 per cent in the 2nd quarter of 2011. This fall can be attributed to an increasing share of 60-66-year-olds in the total population of working age.

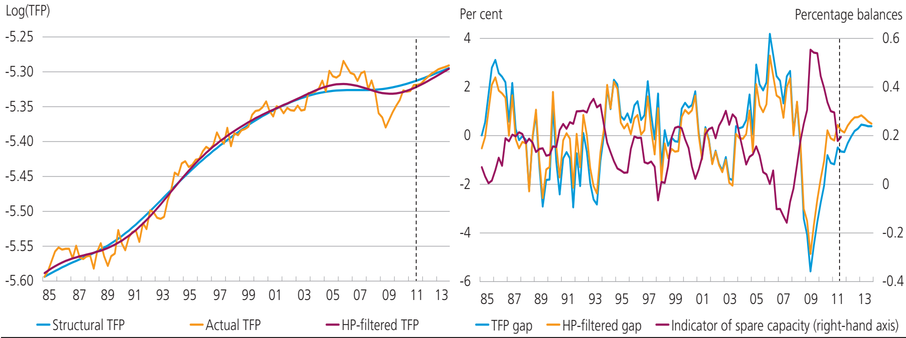
Given the actual development in the participation rate, the described development in the structural participation rate results in large, positive estimates of the cyclical component of the participation rate between the 1st quarter of 2006 and the 4th quarter of 2008, cf. also Chart 5.1. The participation rate then plummeted, as the effects of the economic crisis intensified, and the estimated cyclical component is strongly negative from the 4th quarter of 2009 onwards. In the 2nd quarter of 2011, the participation rate is approximately 1 percentage point below its structural level, corresponding to a negative labour force gap of just over 37,000 persons.

6. STRUCTURAL TOTAL FACTOR PRODUCTIVITY

The structural TFP level is estimated using an extended HP filter. The extended HP filter is based on the standard univariate HP filter that estimates the trend in a time series on the basis of a trade-off between smoothness in the estimated trend and minimisation of deviations between the actual values in the series and the trend. While the standard HP filter estimates are based solely on the development in the actual series, the extended HP filter also includes information from one or more external variables. Hence, this method can be regarded as a compromise between the simplicity of the HP filter and the higher degree of complexity in multivariate estimation methods, including the UCM approach. A formal description of the extended HP filter is given in the Appendix.

The estimation of structural TFP utilises information on the degree of spare capacity in manufacturing industries. The rationale behind this approach is that the firms' capacity utilisation is strongly dependent on cyclical factors. The reason is that after a cyclical turning point, it typically takes some time for firms to adapt their input of capital and labour. Consequently, during a demand-driven economic upturn, firms will respond to the increased demand by e.g. letting their employees work longer hours and more intensively and by utilising machinery and buildings more intensively. This results in higher capacity utilisation. On the other hand, an economic downturn will typically be accompanied by falling capacity utilisation.

STRUCTURAL TFP AND TFP GAP Chart 6.1



Note: The indicator of spare capacity is the share of firms (weighted by number of employees) reporting more than sufficient production capacity, less the share of firms (weighted by number of employees) reporting less than sufficient production capacity. The figures to the right of the dashed line are based on Danmarks Nationalbank's latest forecast of the Danish economy.
Source: Statistics Denmark and own calculations.

But such fluctuations in the degree of utilisation of the input factors are not captured sufficiently by the applied measures of capital and labour. Instead, they are reflected in the TFP estimate. For example, an increase in the amount of overtime will, other things being equal, result in a higher TFP estimate when the total labour input is measured by the number of persons, as is the case here. Hence, the TFP estimate can be seen as an imperfect measure of true total factor productivity where cyclical fluctuations in capacity utilisation generate measurement error. This necessitates cyclical adjustment of the estimated TFP series, and information on the degree of spare capacity can be exploited for this purpose.

The estimated series for structural TFP based on the extended HP filter follows the latest development in the actual TFP less closely than is the case for the series based on the standard HP filter, cf. Chart 6.1. The former series results in larger TFP gaps during the most recent boom and numerically larger negative gaps during the subsequent crisis. This reflects that the extended HP filter – as opposed to the standard HP filter – takes into account the unusually high capacity utilisation rates during the boom, as well as the extraordinarily high number of firms reporting spare capacity during the subsequent crisis, cf. Chart 6.1.[1]

Against this background it can be argued that the estimates of structural TFP based on the standard HP filter attach too much weight to

[1] We have also experimented with an UCM approach estimating structural TFP by means of the Kalman filter. The central observation equation in the model is a positive correlation between the TFP gap and capacity utilisation in manufacturing industries, while we have experimented with a number of different assumptions regarding the statistical model for structural TFP. In all of our experiments, the Kalman filter estimates resulted in *even* larger positive TFP gaps in the years 2005-07 than was the case with the extended HP filter.

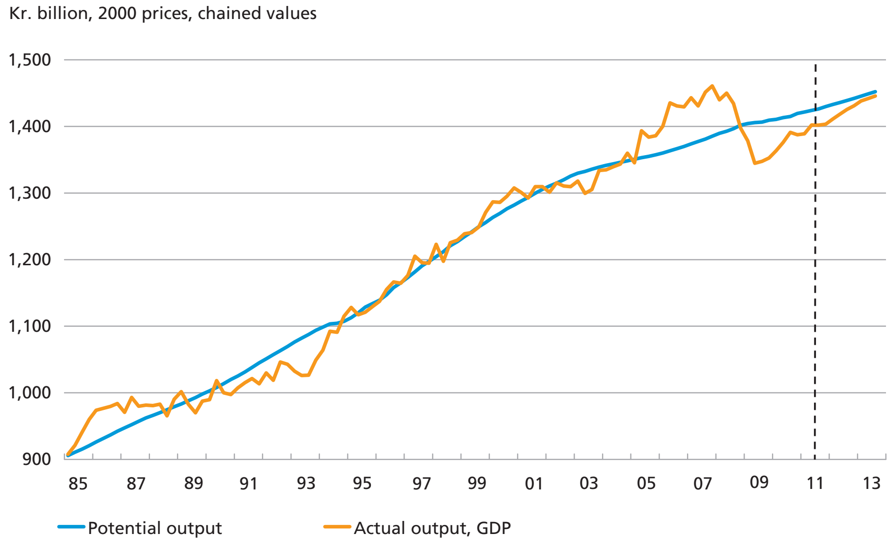
the actual TFP development, and that the development in the HP-filtered series is less credible than the corresponding development in the series based on estimates using the extended HP filter.

7. POTENTIAL OUTPUT AND THE OUTPUT GAP

On the basis of the estimates of the structural levels of unemployment, the participation rate and TFP, the level of potential output can be calculated as described in section 3. Chart 7.1 shows the resulting series. It is noteworthy that the rate of increase in potential output in recent years has been lower than in the preceding decades. In particular, the curve flattened in the early 2000s when growth in potential output seems to decline. Another "kink" can be observed in the autumn of 2008 in the wake of the global financial crisis, cf. also section 10.

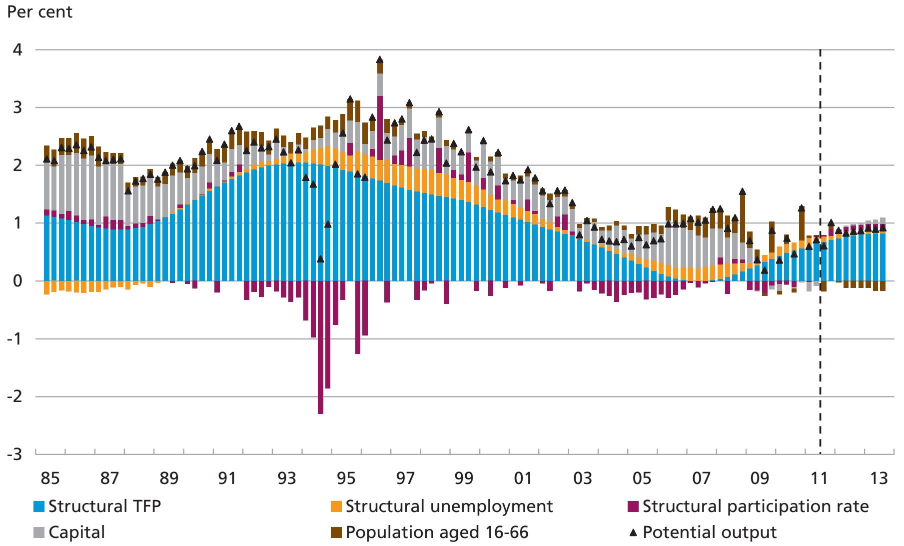
Chart 7.2 shows a decomposition of the growth rates in potential output. The declining growth since the mid-1990s can be primarily attributed to weaker development in TFP. A drop in the structural participation rate, combined with slower capital accumulation, contributed further to the fall in the growth rate in potential output seen around 2003. Capital accumulation contributed substantially to potential output growth during the most recent boom, however, but then declined to almost zero when the financial crisis caused a collapse in investment levels.

ACTUAL AND POTENTIAL OUTPUT — Chart 7.1



Note: The figures to the right of the dashed line are based on Danmarks Nationalbank's latest forecast of the Danish economy.
Source: Statistics Denmark and own calculations.

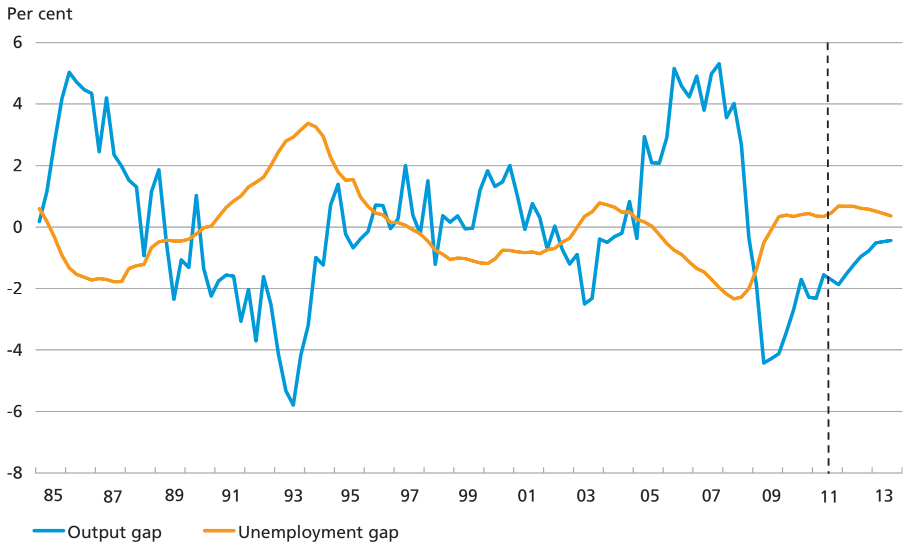
DECOMPOSITION OF GROWTH IN POTENTIAL OUTPUT — Chart 7.2



Note: The growth rates for each quarter have been converted to annual growth rates. The figures to the right of the dashed line are based on Danmarks Nationalbank's latest forecast of the Danish economy.
Source: Statistics Denmark and own calculations.

The estimated series for the output gap is shown in Chart 7.3 together with the estimated series for the unemployment gap. There is a strong negative correlation between the output gap and the unemployment gap. Moreover, the unemployment gap tends to lag the output gap by a

OUTPUT GAP AND UNEMPLOYMENT GAP — Chart 7.3



Note: The figures to the right of the dashed line are based on Danmarks Nationalbank's latest forecast of the Danish economy.
Source: Statistics Denmark and own calculations.

couple of quarters. This is a well-known cyclical phenomenon. The explanation is that, at the beginning of a demand-driven cyclical turning point, firms tend to respond to changed sales opportunities by adjusting the rate of capacity utilisation and the size of their inventories. It takes a couple of quarters for them to adjust the demand for labour, as reflected in unemployment.

According to the estimates, the output gap was large and positive at the beginning of the period, followed by a marked economic slowdown in the early 1990s. The decade ended with a substantial boom, which was then replaced by a short-lived downturn in the first years of the 2000s. In the years 2005-07, the Danish economy experienced a strong boom and overheating, which resulted in output gaps exceeding 4 per cent and considerable pressure on the labour market. But a cyclical turning point set in at the end of 2007, and the incipient slowdown escalated dramatically after the outbreak of the financial crisis in 2008 and the resulting sudden braking of the world economy. The recession bottomed out in the 2nd quarter of 2009, when GDP was more than 4 per cent below its potential level.

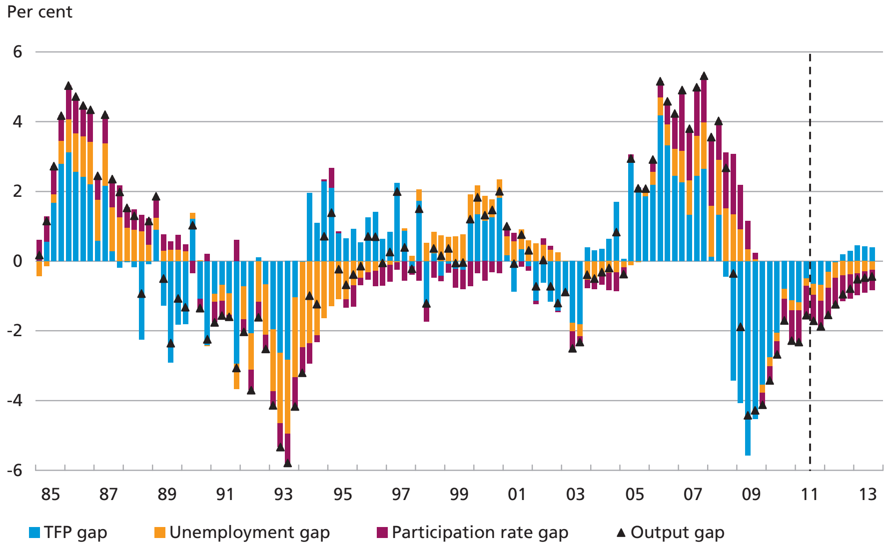
However, the GDP level has rallied since the abrupt falls in 2008 and 2009. Compared with modest growth rates in the level of potential output, this has led to considerable narrowing of the output gap, which is estimated to be -1.6 per cent of potential output in the 2nd quarter of 2011. Against the backdrop of Danmarks Nationalbank's forecast for the Danish economy, described in "Recent Economic and Monetary Trends" in Part 1 of this Monetary Review, our assessment is that the output gap will widen a little in the remainder of 2011, followed by a resumed narrowing in the coming years.

The output gap can be decomposed into contributions from TFP, the unemployment gap and the participation rate gap, cf. Chart 7.4. The large, positive output gaps in the years 2006-08 initially took the form of large, positive TFP gaps. Towards the end of the boom, immediately before the outbreak of the financial crisis, the composition of the output gap shifted, however, as the strong boom resulted in considerable pressure on the labour market.

Moreover, it should be noted that the large, negative output gaps in the early 1990s were primarily reflected in large unemployment gaps. In contrast, the large, negative output gaps in 2009 and 2010 primarily appeared as abrupt falls in TFP. One contributing factor can be the suddenness of the crisis' eruption, given that the labour market typically adjusts to cyclical turning points with a certain lag.

Indeed, the composition of the output gap has shifted in the expected direction since the onset of the crisis. The TFP gap has narrowed numer-

DECOMPOSITION OF OUTPUT GAP Chart 7.4



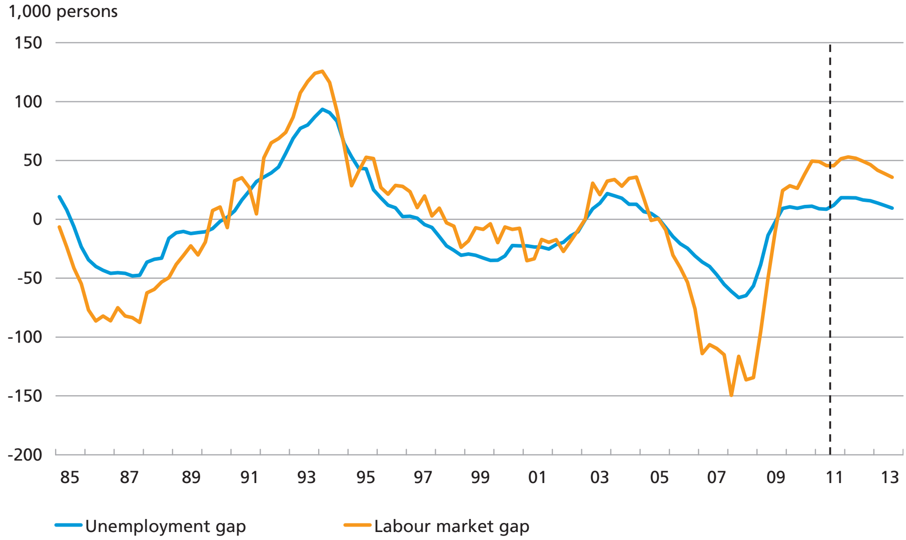
Note: The figures to the right of the dashed line are based on Danmarks Nationalbank's latest forecast of the Danish economy.
Source: Statistics Denmark and own calculations.

ically, constituting a smaller share of the total output gap in the 2nd quarter of 2011 than in 2009. It should be borne in mind, however, that the contribution of the unemployment gap to the output gap is still very modest. This means that the labour market's response to the cyclical turning point has first and foremost taken the form of a marked gap in the participation rate.

This development indicates a new pattern in the labour market's response to cyclical fluctuations. Previously, cyclical fluctuations were primarily reflected in fluctuations in the unemployment gap. In recent years, however, they have increasingly been reflected in cyclical fluctuations in the size of the labour force (and hence the participation rate). This is evident in that the fluctuations in the *labour market gap*, reflecting the unemployment gap and the labour force gap together, have been considerably more pronounced than the corresponding fluctuations in the unemployment gap, cf. Chart 7.5. One contributing factor could be the increased extent of active labour market programmes, since persons in these programmes are counted as being outside the labour force, the latter being calculated as the sum of employed and net unemployed persons, cf. also the next section.

The considerable fluctuations in the labour force gap in recent years should be seen as a reminder that it is not enough to focus solely on the development in the unemployment gap when assessing labour market pressures.

UNEMPLOYMENT AND LABOUR MARKET GAPS Chart 7.5



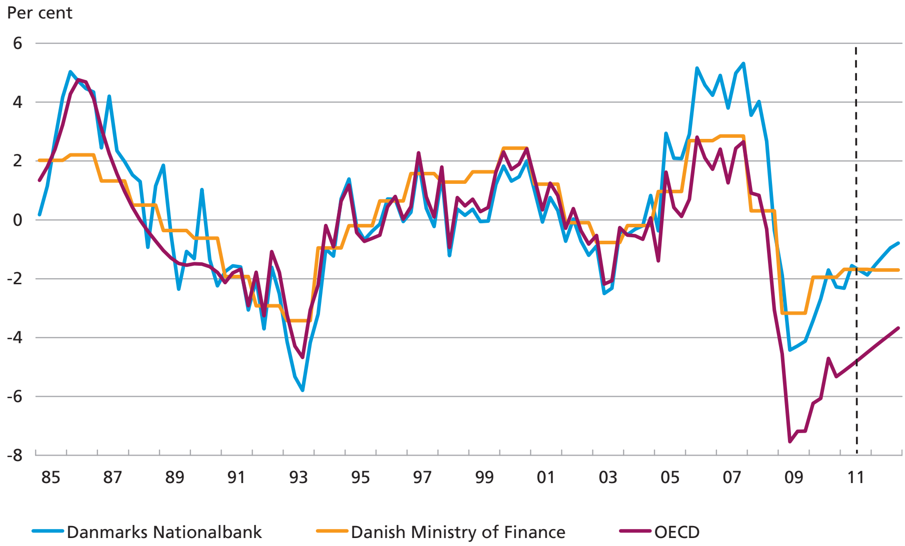
Note: The labour market gap is defined as the unemployment gap less the labour force gap; the latter is calculated as the participation rate gap multiplied by the number of persons in the population in the 16-66 age group. A positive labour market gap indicates that the total input of labour is below the structural level, e.g. because unemployment is higher than the structural unemployment rate or because the labour force is below its structural level. The figures to the right of the dashed line are based on Danmarks Nationalbank's latest forecast of the Danish economy.

Source: Statistics Denmark and own calculations.

Comparison with estimates from other institutions

In most of the estimation period, the described estimates of the output gap mirror corresponding estimates from the Danish Ministry of Finance and the OECD relatively closely, cf. Chart 7.6. In general, the three time series paint the same overall picture of the cyclical patterns in Denmark over the last 25 years. But there are also substantial differences between the three series, not least in recent years, when the estimates of the output gap calculated in this article deviate from the estimates of the other institutions. In particular, the calculations in the preceding sections indicate that the overheating of the Danish economy in the years leading up to the financial crisis was considerably stronger than assessed by the Ministry of Finance and the OECD. This reflects that our estimate of the level of potential output in these years is lower than the estimates of the other institutions. Compared with the OECD's estimate, our estimate of current potential output remains somewhat lower, which means that the OECD's estimate of the output gap is currently considerably more negative than the corresponding estimate in this article. The Ministry of Finance, on the other hand, finds that the level of potential output fell between 2008 and 2009, which brings their estimate of the current output gap to almost the same level as the

OUTPUT GAP ESTIMATED BY VARIOUS INSTITUTIONS Chart 7.6



Source: Danish Ministry of Finance, OECD *Economic Outlook* and own calculations.

estimate in this article. The differences between the three time series illustrate that estimates of the size of the output gap depend on the specific choice of estimation method.

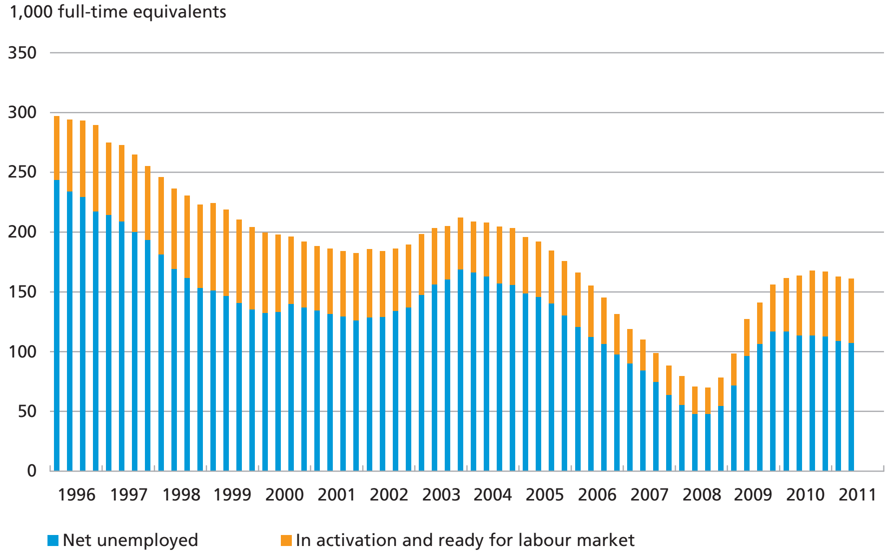
8. ALTERNATIVE UNEMPLOYMENT CONCEPTS

As mentioned, the estimates of potential output and the output gap are calculated using registered net unemployment as a measure of unemployment. Since 2010, Statistics Denmark has also published statistics for another unemployment concept, i.e. *gross unemployment*. The difference between the two unemployment concepts is that persons in active labour market programmes who are ready for the labour market (also called employment-ready persons in activation) are included in gross unemployment but not in net unemployment.

The number of such persons has risen markedly in recent years, cf. Chart 8.1, in 2010 constituting 31 per cent of the total number of gross unemployed. Some of the reasons are earlier activation of unemployed persons and amended refund rules for local governments, encouraging them to increase the extent of active labour market programmes.

Consequently, gross unemployment is considerably higher than net unemployment, and the estimates of structural unemployment and the unemployment gap therefore depend on the choice of unemployment concept. It is important to bear in mind that since gross unemployment is generally higher than net unemployment, the estimate of structural

COMPOSITION OF SEASONALLY ADJUSTED GROSS UNEMPLOYMENT Chart 8.1



Source: Statistics Denmark, Ministry of Employment and own calculations.

gross unemployment will also be higher than the estimate of structural net unemployment, However, it also means that it cannot be concluded in advance that the unemployment gap, i.e. the difference between actual and structural unemployment, will be correspondingly larger when the concept of gross unemployment is used.

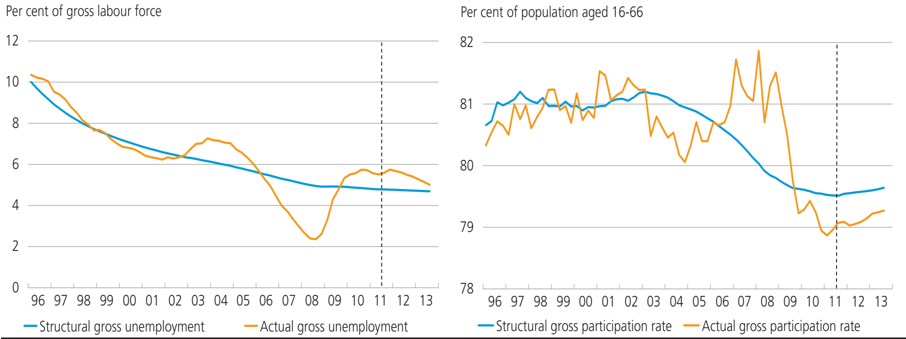
An alternative definition of unemployment also impacts on the calculation of the participation rate. For example, if the unemployment concept used is net unemployment, employment-ready persons in activation are not included in the labour force, while they are included if the concept of gross unemployment is used. The choice of unemployment concept thus influences the estimates of both structural unemployment and the structural participation rate, and the overall effect on the estimate of potential output will depend on the changes in both these estimates.

As an illustration of this point we have estimated time series for the structural levels of unemployment and the participation rate when calculating unemployment as the number of gross unemployed. The estimates are calculated using the same methods as described in sections 4 and 5, with the significant difference that the official statistics for gross unemployment are only available from 2007, while estimates are available back to 1996.[1] Principally, this means that it is not possible to

[1] Statistics Denmark's time series for gross unemployment goes back to 2007. Statistics from 1996 to 2006 are estimates based on statistics from the Ministry of Employment.

STRUCTURAL GROSS UNEMPLOYMENT AND STRUCTURAL GROSS PARTICIPATION RATE

Chart 8.2



Note: Unemployment is calculated as registered gross unemployment. The gross labour force is defined as the number of employed persons plus gross unemployed persons, while the gross participation rate is calculated as the gross labour force divided by the number of persons in the population in the 16-66 age group. The figures to the right of the dashed line are based on Danmarks Nationalbank's latest forecast of the Danish economy.

Source: Statistics Denmark, Ministry of Employment and own calculations.

provide a description of the development in structural unemployment and potential output over a relatively long period using this unemployment concept. In addition, given the shorter time series, the estimates of the structural levels are subject to greater uncertainty, which is one of the reasons why we have chosen to focus on net unemployment as the key unemployment concept in this article.

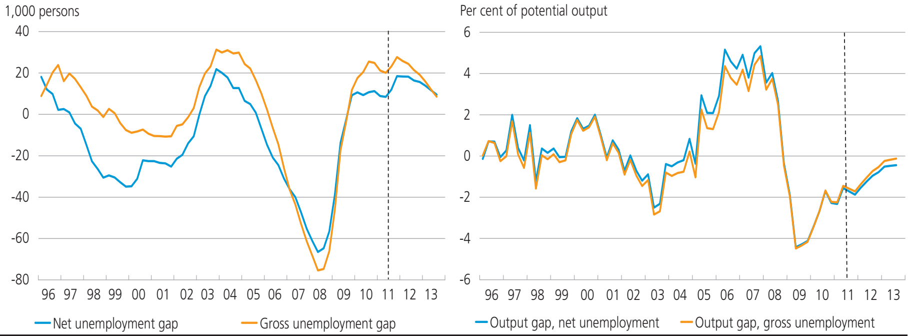
The estimated time series based on the concept of gross unemployment paint the same overall picture as in the preceding sections, cf. Chart 8.2. Structural unemployment has declined over the entire period under review, but the pace of the decline has slowed down in recent years. After a slight increase in the early 2000s, the structural participation rate has fallen considerably since then.

When measured by the number of persons, the size of the unemployment gap varies, as expected, according to whether gross or net unemployment is used, cf. Chart 8.3. For example, the unemployment gap in the 2nd quarter of 2011 is estimated to be just over 20,000 persons, using gross unemployment. For comparison, the net unemployment gap in the same quarter is estimated to be approximately 8,500 persons, cf. section 4.

But the higher gross unemployment gap is offset by a numerically smaller labour force gap. For example, the labour force gap in the 2nd quarter of 2011 is estimated to be only -20,600 persons using the concept of gross unemployment, against -37,000 persons in section 5. Consequently, the total labour market gap, given by the unemployment gap less the labour force gap, shows no great difference when the concept of gross unemployment is used, compared with net unemploy-

UNEMPLOYMENT GAP AND OUTPUT GAP WITH GROSS AND NET UNEMPLOYMENT

Chart 8.3



Note: The figures to the right of the dashed line are based on Danmarks Nationalbank's latest forecast of the Danish economy.
Source: Statistics Denmark, Ministry of Employment and own calculations.

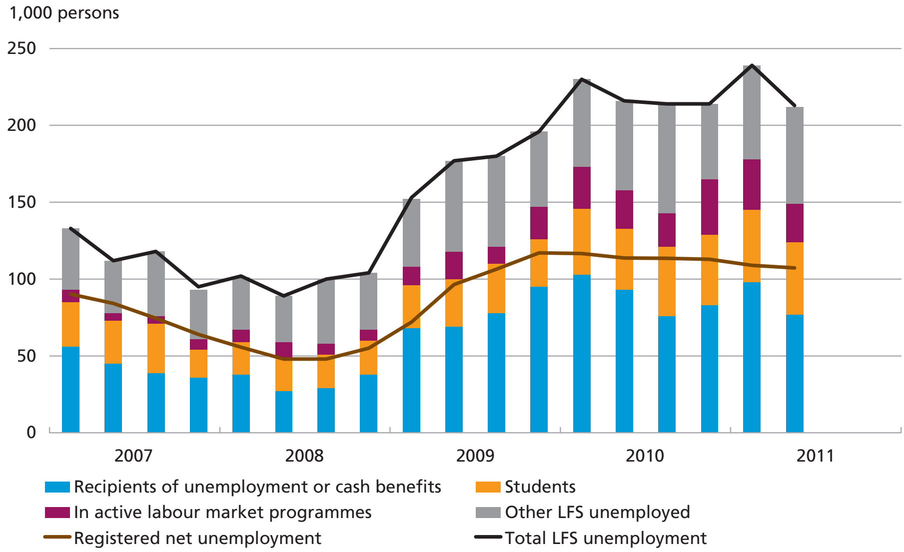
ment. As a result, the estimate of the output gap in the 2nd quarter of 2011 is roughly the same based on gross unemployment as based on net unemployment, cf. Chart 8.3. Again, this emphasises the importance of not limiting the focus to the unemployment gap when assessing capacity pressures in the economy.

A similar point applies to a third unemployment concept, LFS (labour force survey) unemployment, which is based on a continuous, interview-based survey of the population's attachment to the labour market. The calculation of LFS unemployment includes groups of jobseekers who are not included in the register-based unemployment figures. These include e.g. jobseeking students and self-supporting unemployed persons. Increases in the number of LFS unemployed persons in these groups have been a principal factor contributing to the considerably stronger growth in LFS unemployment since 2008, compared with registered net unemployment, cf. Chart 8.4.[1]

The estimation methods applied to net unemployment in section 4 have turned out not to produce satisfactory results for LFS unemployment. This is, among other factors, because statistics for the latter are only available back to 1995. Consequently, a formal estimation of structural unemployment based on the LFS unemployment concept is outside the scope of this article. However, given the stronger increase in LFS unemployment since 2008, a natural conjecture is that our estimate of the current unemployment gap would have been somewhat larger if we had used LFS unemployment as the measure of unemployment.

[1] Note that comparisons between LFS unemployment and registered unemployment are complicated by the fact that registered unemployment – as opposed to LFS unemployment – is expressed as the number of *full-time equivalents*. A half-time unemployed person, for example, is included as 0.5 full-time equivalents in registered unemployment statistics, but as an employed person in the LFS statistics.

LFS UNEMPLOYMENT AND REGISTERED NET UNEMPLOYMENT Chart 8.4



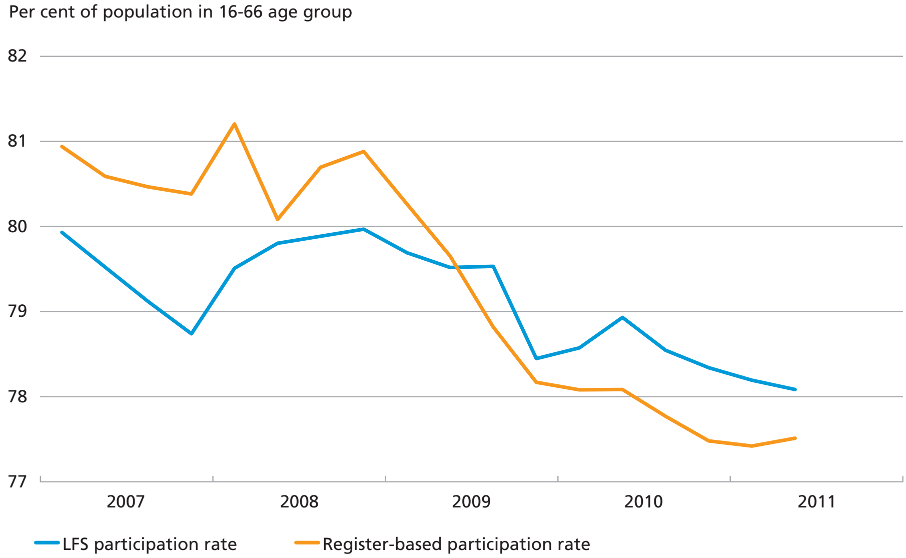
Note: Recipients of unemployment benefits and cash benefits should here be taken to mean recipients of unemployment benefits who are not in active labour market programmes and recipients of cash benefits who are not in such programmes and who are ready for the labour market, match categories 1-3. This means that these persons are also included in registered unemployment.
Source: Statistics Denmark.

On the other hand, it is also likely that our estimate of the participation rate gap would have been numerically lower if the estimations had been based on LFS unemployment. This is because the LFS participation rate – calculated as the number of LFS-unemployed persons plus LFS-employees divided by the number of persons in the 16-66 age group – has declined at a more moderate pace since 2008 than the participation rate based on the net unemployment concept, cf. Chart 8.5. A possible reason is that a large number of those who lost their jobs in the wake of the financial crisis were reclassified as self-supporting jobseekers or job-seeking students. In the LFS statistics, they were counted as unemployed, while they were outside the labour force in the statistics for registered unemployment.

In conclusion, we emphasise that, in general, the choice of unemployment concept has only limited impact on the estimates of potential output and the size of the output gap. But the *composition* of the output gap will depend on the choice of unemployment concept: Different unemployment concepts will therefore result in different estimates of structural unemployment and the unemployment gap, while also producing varying estimates of the structural participation rate and the labour force gap – and the differences in the labour force gap will typically tend to neutralise the differences in the unemployment gap.

LFS-BASED PARTICIPATION RATE AND REGISTER-BASED PARTICIPATION RATE

Chart 8.5



Note: The LFS participation rate is calculated here as the number of LFS-employed and LFS-unemployed relative to the number of persons in the population in the 16-66 age group. The register-based participation rate is calculated as the number of persons in the labour force, divided by the number of persons in the population in the 16-66 age group, where the labour force is calculated as the number of employed plus the number of net unemployed.
Source: Statistics Denmark and own calculations.

Consequently, using an alternative concept of unemployment will typically shift the decomposition of the output gap, but not its overall size.

9. UNCERTAINTY AND RELIABILITY IN REAL TIME

As described in the introduction, the output gap is one of the most frequently used indicators of the cyclical position. In macroeconomic policy planning, a natural focus point is the *current* output gap (and any forecasts of the outlook), while assessments of the output gap in previous periods are often of secondary interest.

Ideally, estimates of the output gap should therefore be available with a minimum lag in order to provide policy-makers with the optimum information basis about current capacity pressures in the economy. In practice, however, it is well known that early estimates of potential output and the output gap are associated with considerable uncertainty, and that they are often subject to substantial revisions as new information becomes available, cf. e.g. Orphanides and Van Norden (2002) and Koske and Pain (2008).

Revisions of estimates of potential output and the output gap are attributable to revisions of *previously published* statistics for actual output, unemployment, wage share, etc., as well as the release of *new* statistics for these measures.

Especially GDP in the national accounts is subject to revisions of existing figures, while e.g. the unemployment figures are usually revised to a lower degree. Revisions of the actual GDP figures will result in revisions of the output gap estimate, both directly and via revisions of the estimate of potential output.

The release of figures for later quarters may lead to revisions of the estimated output gap via two channels: Firstly, the new data points will contribute new information on the current cyclical position, which may lead to reassessment of the cyclical position in previous quarters. This is known as the end point problem, since it is particularly the estimates at the end of the sample that are reassessed when new data points are added. To mitigate this problem, the sample is often expanded with forecast-based figures for future quarters.[1]

Secondly, the addition of new figures may lead to changes in the model used in the estimations of the unobservable variables. Such changes will typically be updates of key parameter estimates, which will lead to reassessment of the estimates of the unobservable variables in all preceding periods. This problem is particularly relevant for Unobserved Components Models, which normally involve estimation of a large number of parameters.

The UCM approach described in section 4 provides for calculation of confidence intervals for the unobservable variables, and in principle, they can illustrate the degree of uncertainty. However, in the calculation of the confidence intervals, it is assumed that both the "true" form of the model and the values of the model parameters are known, and possible revisions of the existing data are not taken into account. Hence, the confidence intervals will tend to underestimate the real uncertainty, cf. Orphanides and Van Norden (2002). Finally, the UCM approach provides only confidence intervals for the individual sub-components to which it is applied, and the individual confidence intervals cannot be readily combined into an aggregate confidence interval for potential output or the output gap.

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[1] The end point problem is particularly pronounced for simple, statistical filtering methods, such as the HP filter. The reason is that the HP filter calculates the structural component in a time series as the weighted average of the actual values of the variable in both previous and subsequent periods. At the end of the sample, the values in the subsequent periods are not available, however, so the algorithm behind the HP filter will attach extra weight to the latest observation in the time series. As a result, the estimated structural component will often be close to the actual value in this period, often bringing the estimated gap close to zero. A frequently used solution, which is also used in this article, is to apply forecast-based figures for the future periods to avoid excessive weight on the last observation of the time series.

Real-time estimates of the output gap

In this section, we calculate *real-time estimates* of the output gap as an alternative illustration of the degree of uncertainty. The calculations result in a time series of estimates of the output gap in the period from the 1st quarter of 2000 to the 2nd quarter of 2011. Each element of the time series represents a *contemporaneous* estimate of the output gap in the relevant quarter – or more specifically the estimate that would have been achieved on the basis of the amount of information available immediately after the release of the first preliminary national accounts for the quarter in question. The real-time estimate for the 1st quarter of 2011, for example, reflects the estimate of the output gap in the 1st quarter of 2011 that we would have achieved on the basis of the data available immediately after 31 May 2011, which marked Statistics Denmark's release of the first preliminary national accounts for the 1st quarter. The method behind the calculations of the real-time estimates is described in more detail in Box 9.1.

The *final* estimates of the output gap are the estimates we can obtain with the full amount of information presently available, including revi-

CALCULATION OF REAL-TIME ESTIMATES — Box 9.1

The construction of real-time estimates of the output gap is in three steps: The first step consists of constructing a *real-time data set* for each quarter in the period from the 1st quarter of 2000 to the 2nd quarter of 2011. The real-time data set for a given quarter *t* reflects the information available at the time of release of the first preliminary national accounts for quarter *t*. But subsequent revisions of the figures are not reflected in the real-time data sets. In addition to the actual figures available at the time, each real-time data set also includes forecast-based figures for developments in the subsequent quarters. The forecast figures are from Danmarks Nationalbank's own forecasts from the time in question, whereby the full real-time data set is a close reflection of the total amount of information that was available to Danmarks Nationalbank at the time.

In the second step, the real-time data sets are used to estimate time series for the output gap, here called *vintages*. The estimation method is as described in sections 3-6, resulting in one vintage for each real-time data set. The vintage associated with real-time data set *t* describes the development in the output gap from 1985 until quarter *t*, as we would have assessed it to be at the time, if we had applied the same estimation method as in this article. The estimate of the output gap for quarter *t* from vintage *t* is called the *real-time estimate* of the output gap in quarter *t*.

The third and last step consists of production of a *real-time series* based on the vintages, comprising all real-time estimates from the 1st quarter of 2000 to the 2nd quarter of 2011. Hence, the real-time series provides a picture of how the original estimates of the output gap would have been if we had used the estimation method in this article on an ongoing basis for each quarter since 2000.

sions of previously published national accounts statistics. Consequently, the final estimates correspond to the estimates presented in section 7, which are the main findings of this article.

Comparison of the real-time estimates of the output gap and the final estimates gives us an impression of the amount of future revisions that can be expected when a new estimate of the output gap is first published.[1]

The real-time estimates of the output gap show a strong correlation with the final estimates (a correlation coefficient of 0.91), and in 89 per cent of the cases the two series produce output gaps with the same sign. In addition, the real-time estimates capture cyclical turning points (expressed as a change in the sign of the output gap) with a reasonable timeliness, cf. Chart 9.1.

But when it comes to the *size* of the output gap, the real-time estimates are subject to considerable uncertainty, and pronounced revision is sometimes necessary. The analyses in this article indicate that this occurs particularly when the fluctuations in the output gap are numerically large: For example, the real-time estimates of the output gap in the first half of the previous decade, when cyclical developments were rather stable, are relatively close to the final estimates. But the real-time estimates in the boom years 2006-07 are markedly lower than the final estimates and would thus have led to strong upward revision. In the crisis years 2009 and 2010, the real-time estimates are, on the other hand, considerably higher than the final estimates and would therefore have triggered a substantial downward adjustment of the output gap.

The revisions of the output gap can be decomposed into revisions of the real-time estimates of the TFP gap, unemployment gap and participation rate gap. The revisions of the TFP gap are the primary cause of revisions of the overall output gap, cf. Chart 9.2. Particularly for the boom years 2006-07, the real-time estimates of the TFP gap are notably lower than the final estimates, which may explain almost the entire difference between the corresponding estimates of the output gap in this period. Compared with the TFP gap, revisions of real-time estimates

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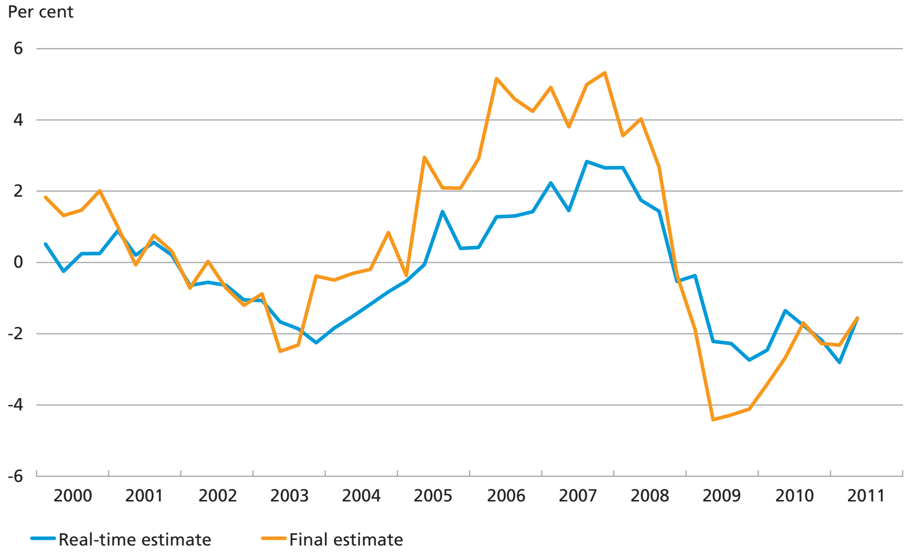
[1] It can be argued, however, that these results exaggerate the extent of expected future revisions. In the calculation of the real-time estimates, the contents of the forecasts at the time are thus taken as given. In reality, a forecast will always be based on a number of judgements, and – formal or informal – assessments of the output gap are often included as an important element of the basis for these judgements. Consequently, output gap estimates and economic forecasts have a mutual impact, and they are typically the result of an iterative process including many other factors as well. If Danmarks Nationalbank had actually produced formal estimates of the output gap on a current basis, these estimates would most likely have been included in the preparation of the current forecasts, which would then have come out differently. In turn, this would have resulted in other real-time estimates of the output gap, which would probably have been closer to the final estimates.

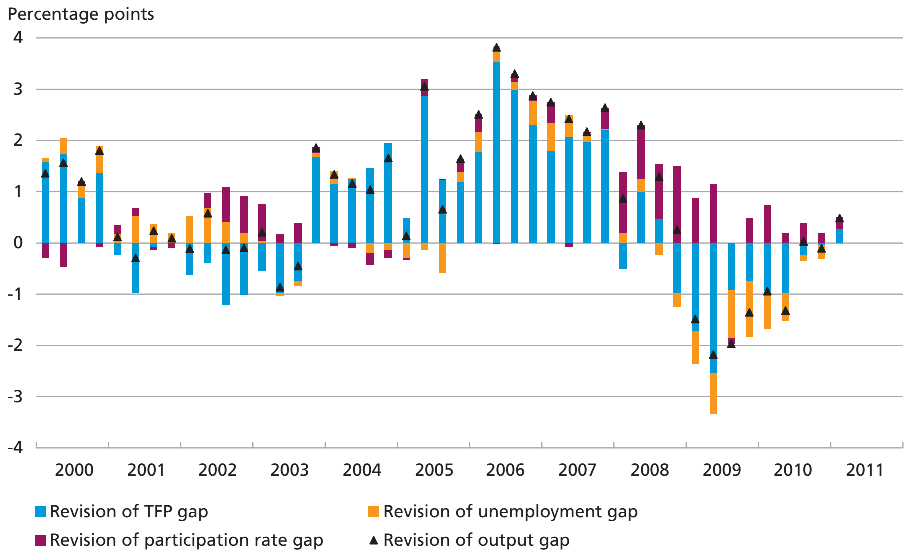
REAL-TIME ESTIMATES AND FINAL ESTIMATES FOR THE OUTPUT GAP Chart 9.1



Source: Statistics Denmark and own calculations.

of the unemployment gap and the participation rate generally play a minor role, although revisions of the unemployment gap contributed substantially to the marked deviations between the real-time estimates and the final estimates of the output gap in 2009 and 2010.

DECOMPOSITION OF REVISIONS OF REAL-TIME ESTIMATES OF THE OUPUT GAP Chart 9.2



Source: Own calculations.

Quasi real-time estimates

One of the reasons for the marked revisions of the real-time estimates of the TFP gap is that revisions of existing GDP data are directly reflected in the TFP series, which is calculated as a residual, cf. section 3. We have performed a so-called quasi real-time analysis in order to throw light on the role of revisions of the existing data. This analysis follows the same method as the real-time analysis mentioned above, but with the difference that we apply only final, revised data for the actual values of GDP, unemployment, wage share, etc.[1] Relative to the final estimates, this difference implies that the quasi real-time estimates use only data up to and including the quarter in question, while the final estimates are based on all information from the current data set. Consequently, the quasi real-time estimate of the output gap in a given quarter is an expression of what our output gap estimate would have been at the time, if we had known the final national accounts data for the quarter in question and the preceding quarters. The difference between the quasi real-time estimates and the real-time estimates can be seen as an indication of the extent of the contribution of revisions of the existing data basis to the uncertainty in real-time estimates of the output gap.

Compared with the real-time estimates, the quasi real-time estimates are generally considerably closer to the final estimates, cf. Chart 9.3. This indicates that the uncertainty of the real-time estimates of the output gap is to a large extent attributable to uncertainty concerning the actual data, and revisions of the existing figures are a considerable source of revision of output gap estimates. As expected, the TFP gap, in particular, is affected by revisions of the actual data. This is seen by the fact that the gap to the final estimates is considerably smaller when the real-time estimates are based on revised data than when they are based on the original, unrevised data. But the TFP gap is still the main cause of revision of the output gap. Even in the absence of revision of the national accounts statistics, the real-time estimates of TFP would be subject to substantially greater uncertainty, compared with the corresponding estimates of the unemployment gap and the structural participation rate, cf. Chart 9.3.

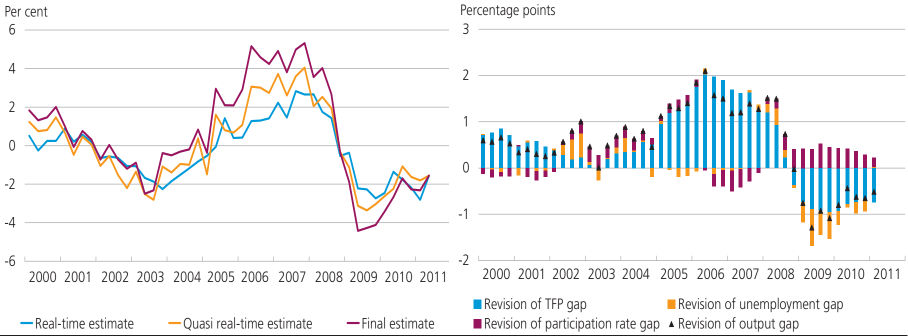
Economic policy implications

The great uncertainty associated with real-time estimates of the output gap raises the question of how policy-makers should address such esti-

[1] However, the data used in the calculation of the quasi real-time estimates do not include forecast-based figures for the following quarters. The reason is that in a given quarter, the forecast would have been different if we, at that time, had known the revised national accounts data. Hence, the quasi real-time estimate of the output gap in quarter *t* is based on the final, revised national accounts data up to and including quarter *t*.

REAL-TIME ESTIMATES, QUASI REAL-TIME ESTIMATES AND FINAL ESTIMATES OF THE OUTPUT GAP

Chart 9.3



Note: The right-hand Chart breaks down the difference between the quasi real-time estimate and the final estimate of the output gap into the corresponding differences for the TFP gap, unemployment gap and participation rate gap.

Source: Own calculations.

mates. In this connection, it should primarily be emphasised that real-time estimates of the output gap are far from uninformative. After all, the correlation between the real-time estimates and the final estimates is strong, as mentioned above.[1] But it should also be underlined that assessments of the cyclical position and capacity pressures in the economy should not rest on real-time estimates of the output gap alone. It is important to include other indicators too.[2] The real-time estimates from the years running up to the financial crisis can be taken as an illustration of this point: Given the amount of information available at that time, the method applied in this article would have resulted in positive estimates of the output gap in these years, which would have been an early indicator of overheating. On the other hand, it is also clear that the subsequent development has led to strong upward adjustment of the output gap in the period in question. This suggests that the method applied in this article would hardly have been sufficient to clarify the serious degree of overheating at the time when political countermeasures could have been implemented.[3]

The consequences of the uncertainty surrounding the output gap to optimum monetary policy have been analysed extensively in the theoretical literature. The studies in this literature typically apply simple macroeconomic models, assuming that a central bank's behaviour can be

1 This point of view is shared by Koske and Pain (2008), among others, while e.g. Orphanides and Williams (2002) are somewhat more critical. The latter authors recommend that economic policy decisions should be based on observable changes in actual output and unemployment rather than on estimates of the unobservable output and unemployment gaps.

2 Other indicators include growth in actual output, questionnaire surveys of capacity pressures and inflation expectations, the development in nominal wages and prices, etc.

3 In May 2006, the Danish Ministry of Finance estimated the output gap for that year to be 1.5 per cent (*Economic Survey*, May 2006), while the most recent estimate for 2006 is 2.7 per cent (*Economic Survey*, August 2011), supporting this interpretation.

described by a simple, monetary-policy rule (often a Taylor rule). A frequently occurring result in the literature is that the greater the uncertainty about the size of the output gap, the less weight should the central bank attach to the output gap in the construction of the monetary-policy rule.[1]

In Denmark, however, monetary policy is committed to maintaining the fixed-exchange-rate policy, so macroeconomic stabilisation is left to fiscal policy. In view of this, it is unfortunate that the implications of uncertainty regarding the output gap to fiscal stabilisation policy have not received very much attention in the literature so far. But one aspect of fiscal policy has attracted attention, namely the consequences for calculations of the structural balance. Since the output gap is included as a decisive component in these calculations, cf. section 11, uncertainty about its size will necessarily spread to the estimate of the structural balance, cf. Koske and Pain (2008). Consequently, it should be endeavoured to take qualified estimates of the extent of the uncertainty into account when calculating the structural balance. Moreover, in order to avoid overly optimistic assessments of the underlying fiscal position, it could also be argued that special weight should be attached to the calculations of the structural balance that are based on "prudent" estimates of the output gap, cf. also D'Auria et al. (2010).

10. POTENTIAL OUTPUT BEFORE AND AFTER THE FINANCIAL CRISIS

The turmoil in the financial markets, which began in 2007 and escalated dramatically in autumn 2008, led to a strong decline in economic activity in Denmark. The real economic consequences of the crisis are analysed in Abildgren et al. (2011). Their macroeconomic analyses focus on the consequences for *actual* output, however, while the underlying trend in GDP is generally taken as given. In this section our point of view is the opposite, since our focus is on the consequences of the crisis for *potential* output. In that sense, this section complements the analyses in Abildgren et al. (2011).

Throughout this article, we have interpreted the concept of potential output as a stable, slowly changing underlying level of actual output, cf. section 2. This interpretation is reasonable, in so far as the development in actual output follows a normal course, where temporary shocks to the economy result in varying, yet dampened, fluctuations in the annual growth rates for GDP. It is more doubtful, however, whether the characterisation of potential output as a relatively stable underlying level

[1] An overview of this literature is contained in Koske and Pain (2008).

also holds water in a situation like the one in 2008 and 2009, when GDP fell by approximately 8 per cent over a few quarters, relative to the peak in 2007. Since there is every probability that shocks of this magnitude will have persistent consequences also on the supply side of the economy, it can be envisaged that the development in the level of potential output will be more abrupt in this period, and that this will not necessarily be captured using the estimation method mentioned previously.[1]

In the following we discuss the likely impacts of the financial crisis on the level of potential output. In line with the overall approach of this article, our point of departure is to decompose potential output, using the production function method, into capital, population, participation rate, unemployment and TFP.

Capital

The most obvious impact channel from the crisis to the productive capacity of the Danish economy is via the negative effects on the level of investment. Investment levels are strongly cyclical and tend to fall in the short term when the economy is hit by strong, negative shocks. This short-term effect is reinforced by the credit squeeze typically resulting from a financial crisis. The crisis is also likely to affect the volume of investment in the longer term due to increased uncertainty about the economic outlook. This greater uncertainty may lead to higher risk premiums, thereby dampening the investment appetite. The level of investment has been particularly hard hit during the current crisis, which has brought the accumulation of capital in the Danish economy to a complete halt, cf. Chart 10.1. Hence, the capital stock is smaller today than before the outbreak of the crisis, which has contributed to lower growth in the level of potential output.

As a rough estimate of the magnitude of this effect, we can compare the actual development in the capital stock with the development in the years 2000-08, which resulted in an average quarterly growth rate of 0.32 per cent. If this development had continued after 2008, the capital stock today would have been 3.3 per cent larger than its actual size. Under the usual assumption that the elasticity of output with respect to capital is 0.35, this missed capital accumulation can be translated into a loss of potential output of just over 1 per cent.

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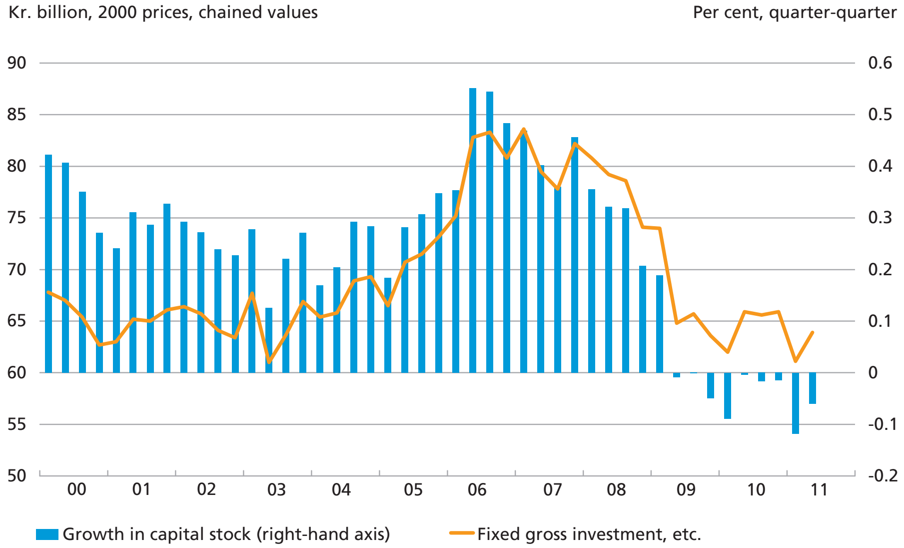
1 This interpretation is supported by the empirical literature on the consequences of financial crises to *actual* output: The historical experience of a large number of countries shows that financial crises often entail permanent, or at least prolonged, output losses, cf. *inter alia* Cerra and Saxena (2008), IMF (2009), Cecchetti et al. (2009) and Reinhart and Rogoff (2009). See also the analyses of the real economic consequences of Danish banking crises in Abildgren et al. (2011) in this Monetary Review.

GROSS FIXED INVESTMENT AND GROWTH IN CAPITAL STOCK | Chart 10.1



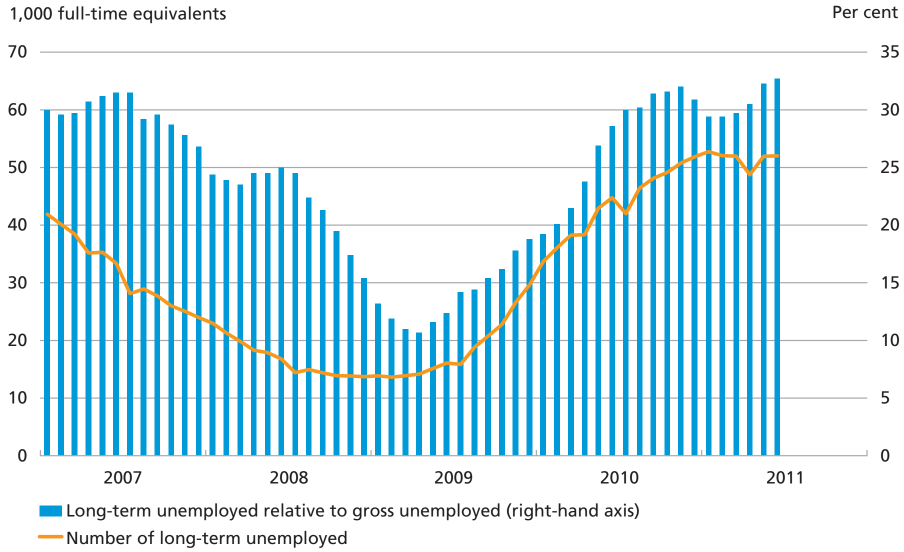
Note: Quarterly figures for the capital stock are calculated by combining annual national accounts figures with quarterly national accounts figures for gross fixed investment, given the assumption of a constant depreciation rate within the same year.
Source: Statistics Denmark and own calculations.

Besides the investment channel, the crisis may also have had a negative impact on the size of the capital stock via extraordinary depreciation of existing capital equipment. Many firms have failed, and, moreover, the remaining firms are likely to have a large need of restructuring. The possible implication is that part of the existing capital stock has become redundant, or at least that the value has deteriorated strongly. However, the extent of such extraordinary deterioration of value is difficult to measure, and it is doubtful whether the effect of the deterioration is captured in the official figures for the size of the capital stock. Consequently, there is a real risk of measurement error in the size of the capital stock, and such errors will be registered in the TFP residual. A non-measurable fall in the capital stock will thus result in a lower TFP measure, cf. below.

Structural unemployment and structural participation rate

The financial crisis triggered a sharp increase in actual unemployment in 2009 following the historically low levels in 2007 and 2008. Such increase in actual unemployment may possibly impact the development in structural unemployment via *hysteresis* effects: Persons who are jobless for a relatively long time and unable to keep their qualifications updated will be harder to re-employ, which entails an increase in structural unemployment, cf. Blanchard and Summers (1986) and Blanchard (1991).

LONG-TERM UNEMPLOYMENT SINCE 2007 | Chart 10.2



Note: The number of long-term unemployed persons is calculated as the number of gross unemployed recipients of unemployment benefits or cash or start-up benefits in match group 1, converted to full-time equivalents who have been unemployed/in activation for at least 80 per cent of the time within the last 52 weeks.
Source: Jobindsats.dk

The risk of hysteresis is particularly great if the increase in actual unemployment results in a higher number of long-term unemployed, cf. Ball (2009). Long-term unemployment rose by almost 40,000 in the period from March 2009 to January 2011, cf. Chart 10.2. Viewed in isolation, this causes some concern about the consequences for structural unemployment in the longer term. It should be noted, however, that the rise in long-term unemployment can to a reasonable extent be characterised as a return to normal, and the share of long-term unemployment out of total gross unemployment is now roughly the same as at the beginning of 2007.

Indeed, the results in section 4 do not indicate any increase in structural employment since the outbreak of the crisis. Unemployment has risen, but has so far stabilised at a level close to the level which – also before the crisis – is generally considered to be consistent with stable wage and price developments. Despite the higher unemployment, the wage share is still somewhat higher than its historical average, cf. Chart 4.2, although it has declined substantially from the peak in 2009.

All in all, developments so far seem to indicate that the increase in unemployment in the wake of the financial crisis can be characterised as closing a large, negative unemployment gap, while there are no indications of higher structural unemployment. One contributing factor may be the reduction of the maximum unemployment benefit entitlement period from four to two years, which was adopted as part of the unemployment benefit reform in May 2010 and should be expected to lead to lower structural unemployment.

Theoretically, the effect of the financial crisis on the structural participation rate may go either way: In the short term, poorer employment opportunities may induce more people to withdraw from the labour market as a result of a "discouraged worker effect", cf. section 5. Conversely, the falls in equity and house prices that characterised the early stages of the financial crisis may have made it more difficult to withdraw from the labour market, which may have contributed to a higher participation rate, especially for the elderly part of the population. Both of these short-term effects may influence the structural participation rate in the medium term, as it may be difficult to return to the labour market after withdrawal.

The actual development in the participation rate since autumn 2008 indicates that the first of the above effects has dominated. In this connection it is worth noting that the decline in the participation rate is, to a higher degree than in previous recessions, attributable to an increase in the number of students, cf. also section 8. It is likely that part of this group will be willing to return to the labour market when cyclical conditions normalise, pointing to a limited negative impact on the structural participation rate.

It should be emphasised, however, that this does not mean that the participation rate will return to its 2007 level: As a result of the overheating of the Danish economy, the participation rate in that year was considerably *higher* than its structural level, cf. section 5, so a return to this situation would not be sustainable.

Structural TFP

Theoretically, the effect of the financial crisis on TFP is the most inconclusive. This reflects general uncertainty as to what exactly the TFP residual measures and which factors determine its size. On the negative side, it can be argued that the cyclical downturn, the tighter credit terms and reduced risk appetite will reduce the volume of investment in research and development in the private sector, which will result in lower TFP growth in the long term. Moreover, credit squeezes may delay necessary, but costly restructuring measures in the economy, locking resources in less productive sectors. On the other hand, a severe recession may have a "purging effect", weeding out the least productive firms in the economy and freeing resources to more productive firms.[1]

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[1] This effect is often called "creative destruction", cf. Schumpeter (1942).

The results in sections 6 and 7 indicate that the growth in structural TFP has been unusually low after the financial crisis, but according to the estimates here, this effect began already *before* the outbreak of the crisis. The lowest growth rates for structural TFP are thus observed in 2006 and 2007. Compared with the development in actual TFP, this results in large, positive TFP gaps immediately before the crisis and correspondingly large, negative gaps in 2009 and 2010. This is interpreted as significant overheating of the Danish economy in the years leading up to the financial crisis, and, as a result, the eruption of the financial crisis triggered a "boom-bust" pattern around an otherwise stable potential level. The unusually high capacity utilisation in manufacturing industry before the crisis also supports this interpretation to a certain extent, cf. also section 6.

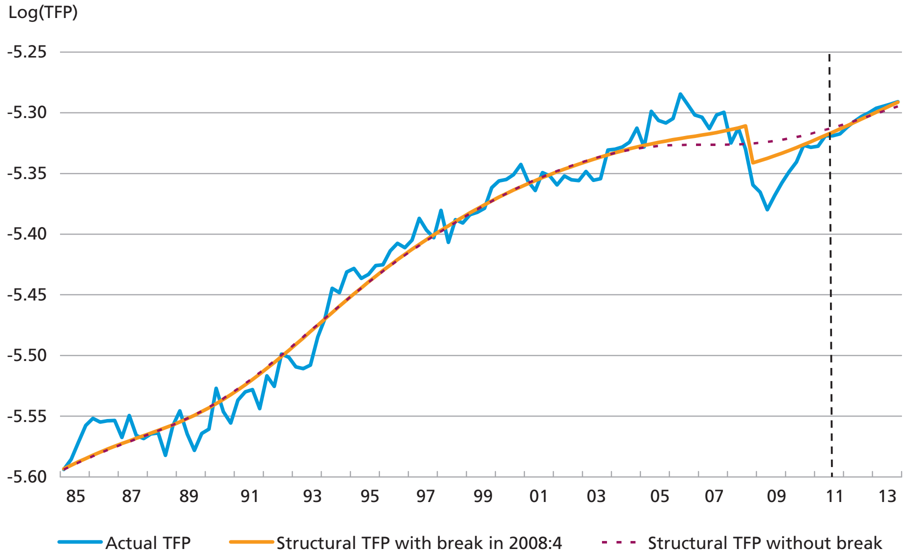
It should be borne in mind, however, that the results for the TFP gap depend to a high degree on our assumptions and especially on our view of potential output as a level that may change over time, but does not fluctuate sharply from quarter to quarter. Using the extended HP filter, the abrupt fall in actual TFP in the 4th quarter of 2008 will not only lead to reduction of the estimates of structural TFP in the subsequent quarters, but will also pull down the estimates for the *preceding* quarters.

As mentioned in the introduction to this section, it is not evident that the level of potential output and its components, including structural TFP, followed a stable, smooth course at the end of 2008 and the beginning of 2009. Extraordinary depreciation of capital equipment that was suddenly made redundant may, for example, have led to an abrupt one-time drop in the level of potential output. As explained above, this will most likely appear as a drop in the *measured* TFP level.

Consequently, as an alternative to the estimates in section 6, we have performed an estimation of structural TFP allowing a one-time structural break in the 4th quarter of 2008. The size of the break has been estimated by including a simple dummy variable in the extended HP filter. This approach causes the estimated series for structural TFP to dive by 3 per cent in the 4th quarter of 2008, cf. Chart 10.3. Compared with the estimates without a structural break, and in accordance with expectations, this means that the estimates of structural TFP are higher during the boom leading up to the break, but lower in the crisis years after the break.

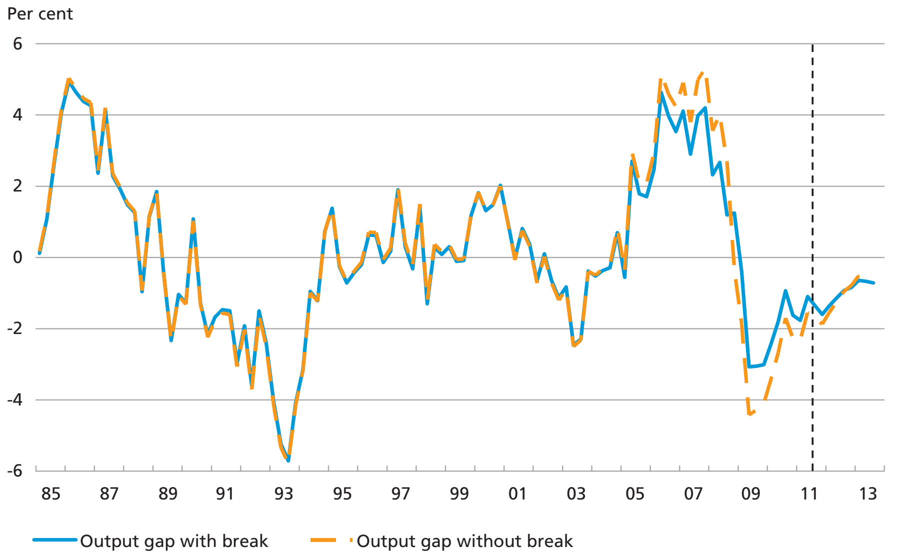
In combination with the development in capital, population, structural unemployment and structural participation rate, the sudden drop in structural TFP causes the level of potential output to fall by 2.7 per cent in the 4th quarter of 2008. Relative to the estimations without a structural break, this implies somewhat more dampened fluctuations in the output gap around of the turn of the year 2008/09, cf. Chart 10.4. This

STRUCTURAL TFP WITH BREAK IN Q4 2008 Chart 10.3



Note: The figures to the right of the dashed line are based on Danmarks Nationalbank's latest forecast of the Danish economy.
Source: Statistics Denmark and own calculations.

OUTPUT GAP WITH AND WITHOUT STRUCTURAL BREAK IN Q4 2008 Chart 10.4



Note: In the estimation with a break in structural TFP, the potential output level drops 2.7 per cent in Q4 2008. In the estimation without a break, it rises 0.4 per cent in the same quarter. The figures to the right of the dashed line are based on Danmarks Nationalbank's latest forecast of the Danish economy.
Source: Statistics Denmark and own calculations.

reflects that the estimated level of potential output tends to mirror the development in actual GDP to a higher degree. But it is notable that the output gaps in 2005-07 remain unusually large.

The estimation with the structural break provides for a slightly different interpretation of recent years' events: According to this interpretation, the growth in potential output in the boom years 2005-07 was indeed dampened, but not unusually low, until a sudden collapse in the world economy in the autumn of 2008 led to a unique drop in the productive capacity of the Danish economy. However, even with this interpretation, it remains beyond doubt that the Danish economy was exposed to severe capacity pressures in the years leading up to the financial crisis.

11. FROM OUTPUT GAP TO STRUCTURAL BALANCE

Public finances in Denmark are subject to a high degree of cyclicality. In a boom, the public finances automatically improve since higher output, incomes, consumption and employment generate higher revenue from direct and indirect taxes and lower expenditure for transfer incomes. Conversely, revenue tends to fall and expenditure tends to increase in an economic slowdown. As a result of this cyclicality, the actual balance is not an accurate measure of the underlying fiscal position.

In this section, we illustrate how estimates of the output gap can be used to calculate a *structural* budget balance. The structural balance is the actual balance adjusted for cyclical and temporary fluctuations, thus providing a more accurate picture of the fiscal position in the medium term. Several economic institutions calculate estimates of the structural budget balance in Denmark, and the calculations are included in their assessments of the public finances and current fiscal policy. The structural balance plays a particularly large role in an EU context, since it is included as a key element in the European Commission's assessment of a member state's compliance with the Stability and Growth Pact.

The structural balance is calculated on the basis of the actual balance in the following way:

Structural balance = Actual balance – Cyclical component – Special items

The *cyclical component* reflects the effect of the cyclical position on the actual balance. Estimates of the size of the output gap are a key element of the calculation of this component. Consequently, the first step in identifying the cyclical component is to find out how the cyclical position deviates from normal, and the output gap is indeed a measure of the sign and size of this deviation.

Another necessary element of the calculation of the cyclical component is to determine the sensitivity of the balance to fluctuations in the output gap. The sensitivity can be expressed by a budget factor indicating the change in the government balance as a percentage of GDP on a 1 percentage point increase in the output gap. In this article, the budget factor is calculated by means of simulations using Danmarks Nationalbank's macroeconometric model of the Danish economy, MONA. The simulations describe the change in a number of government revenue and expenditure items on an increase in GDP as a result of higher aggregate demand, summarising the results as a total budget factor, cf. Box 11.1. The cyclical contribution to the actual budget balance in a given year is then found by multiplying the budget factor by the output gap for the year in question.

The last step in the calculation of the structural balance is to adjust for a number of *special* items. They are budget items that may fluctuate substantially from year to year due to temporary factors, so they may exert strong influence on the actual budget balance in a particular year, but their fluctuations do not systematically follow the business cycle.
These items consist firstly of revenue from pension yield tax, which is strongly influenced by the development in equity and bond prices. Furthermore, we adjust for corporate tax proceeds, excluding revenue from North Sea production, since the fluctuations of the corporate tax of the financial sector do not necessarily match the business cycle. North Sea revenue is subject to separate adjustment, since it is strongly affected by oil price fluctuations. Car sales follow the business cycle only to a limited extent, so separate adjustment is made for revenue from registration fees. In addition, we make adjustment for the possible variation in government interest costs from year to year due to temporary interest-rate fluctuations. Similarly other special budget items, e.g. transfers to the EU, Greenland and the Faroe Islands, early retirement benefits and state pensions to Danes abroad, etc. are subject to separate adjustment as they do not follow the business cycle either. In this article, we apply the adjustments of these special items performed by the Danish Ministry of Finance.[1]

[1] The adjustment of the respective items is described in more detail in Skaarup (2005) and Economic Survey, August 2011.

CALCULATION OF BUDGET FACTOR IN MONA — Box 11.1

The cyclical component in a given year is calculated on the basis of the formula:

$$\mathit{Cyclical\ component}_t = \varepsilon_t \cdot \mathit{output\ gap}_t \,.$$

The output gap has been converted to annual levels, based on the quarterly figures from the preceding sections. The parameter ε is the budget factor defined as:

$$\varepsilon_t \equiv \frac{d(B_t / (P_t \cdot Y_t))}{dY_t / Y_t^*} \,,$$

where *B* is the government budget balance, *P* is the GDP deflator, and *Y* is real GDP. The budget factor indicates how much the government balance, in per cent of nominal GDP, changes when real GDP changes by 1 per cent of potential GDP. The expression of the budget factor can be rewritten as:

$$\varepsilon_t = \left[\sum_i \left(e_{B^i,Y} \cdot \frac{B_t^i}{P_t \cdot Y_t} \right) - \frac{B_t}{P_t \cdot Y_t} \right] \frac{Y_t^*}{Y_t} \,, \qquad (11.1)$$

where $e_{B^i,Y}$ denotes the elasticity of budget item *i* with respect to real GDP.[1] The expression in square brackets in equation (11.1) captures the effect on the balance, as a percentage of GDP, of a 1 per cent increase in GDP. It consists firstly of the sum of changes in the individual budget items.[2] For a given budget item, this can be found by multiplying the elasticity of the budget item with respect to real GDP by the item's share of nominal GDP in the year in question. The elasticities are here assumed to be constant, while the budget items' shares of nominal GDP may vary year-on-year. This can result in variation in the budget factor over time, cf. Chart 11.1. The second term in the square brackets represents the effect of a numerical decline, all other things being equal, of the balance as a percentage of nominal GDP when GDP increases. Finally, the expression in the square brackets is multiplied by the ratio of potential GDP to actual GDP in the year in question, thus taking into account that the output gap is expressed as a percentage of *potential* GDP.

The elasticities of the individual budget items are estimated using simulations in MONA. The simulations investigate the fiscal effects of a shock to the model's private demand components, increasing real GDP by 1 per cent relative to a similar scenario without shocks.

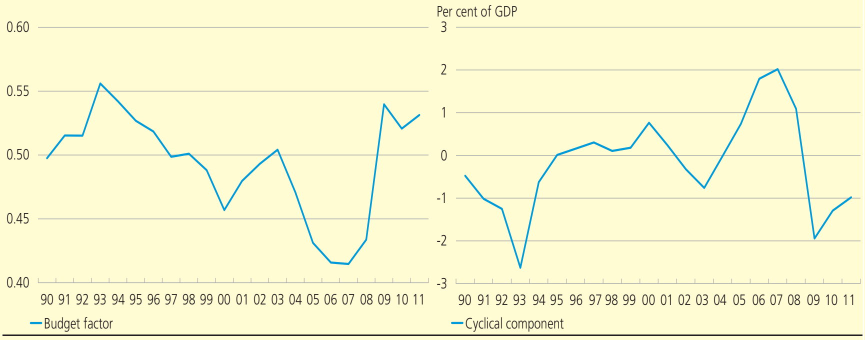
When modelling the demand shock, it is necessary to determine the demand component driving the increase in GDP. This choice is of great significance to the effect on the individual budget items and hence to the size of the total budget elasticity. This reflects varying taxation of different demand components, among other factors. For example, private consumption has a high content of indirect taxes and duties, whereas this content is very low in export goods. Ideally, this should lead to varying budget factors depending on the type of shock that should be regarded as the cause of the size of the output gap in the year in question. The estimates of the output gap as calculated in this article contain no information about the factors determining the size of the output gap, however.[3]

CONTINUED Box 11.1

Consequently, the increase in GDP is modelled by means of a proportional shock to private consumption, private investment and net exports. The calculated budget factor can thus be taken as an expression of the sensitivity of public finances to "average", demand-driven cyclical fluctuations.

The budget sensitivity to changes in GDP also depends on the choice of time horizon. The reason is a certain lag in the adjustment of employment, prices and wages. For example, the rise in employment in MONA will not occur until a few quarters after the increase in demand, resulting in a certain lag in the effect on the public finances. In order to take this into account, the final elasticity of each budget item is calculated as an average of elasticities in the first three years after the demand shock.

BUDGET FACTOR CALCULATED IN MONA AND THE CYCLICAL COMPONENT Chart 11.1



Note: The cyclical component is calculated by multiplying the budget factor for the year in question by the output gap in the same year.
Source: MONA data bank and own calculations.

The calculated budget factor fluctuates between 0.42 and 0.56, averaging 0.49. This budget sensitivity is somewhat lower than the findings of other institutions for Denmark. The Danish Ministry of Finance finds that the government budget changes by 0.79 per cent of GDP on a change by 1 percentage point in the output gap (Skaarup, 2005), while the OECD's corresponding figure is 0.59 per cent of GDP (André and Girouard, 2005). The differences can be attributed to different estimation approaches, among other factors. The OECD calculates the budget factor on the basis of econometric estimations of elasticities for a limited number of budget items, cf. André and Girouard (2005), while the Ministry of Finance applies a different macroeconometric model, ADAM, than we do. Despite strong similarities, the two models are not identical, so the calculations produce different budget factors. For example, ADAM takes into account the progressivity of income taxation, while MONA applies an average income tax rate to all income levels. This can contribute to a lower budget factor calculated in MONA.

[1] In the derivation of the expression in equation (11.1) it is assumed that the GDP deflator *P* is exogenous, disregarding that a demand-driven change in GDP will also lead to a change in the price level. In the short term, the change is limited, however, so we exclude this effect from our calculation of the budget factor.

[2] In this connection, expenditure items are counted as negative budget items, while revenue items are counted as positive budget items.

[3] Note that the output gap can, of course, be decomposed into the TFP gap, unemployment gap and participation rate gap, cf. previous sections. Such decomposition shows *how* the output gap is reflected on the supply side of the economy, but not *why* it has that particular size.

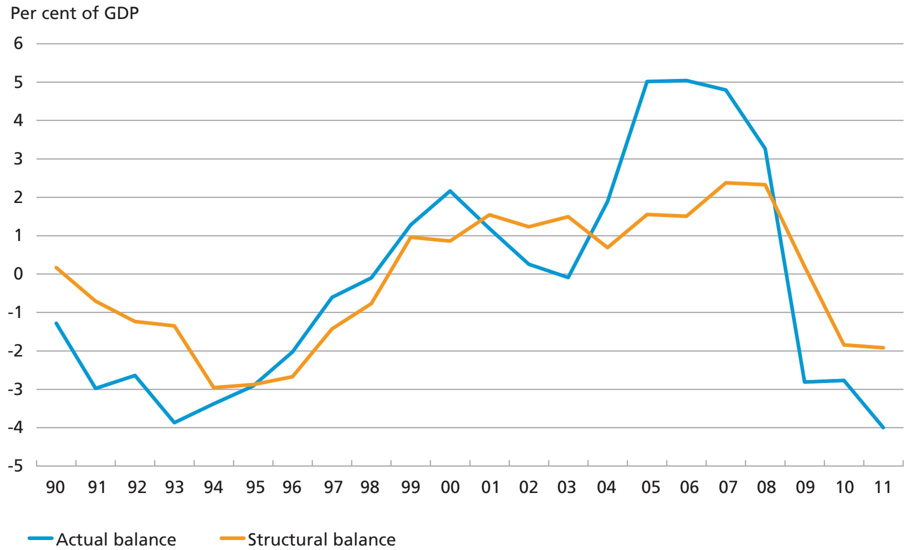
Denmark's structural budget balance 1990-2011

The structural balance has fluctuated strongly since 1990, cf. Chart 11.2, mainly due to the effects of structural policy, but also to fiscal policy activism. The structural balance deteriorated in the early 1990s. The weakening was particularly pronounced in 1994, which can be attributed to the easing of fiscal policy in connection with the "kick-start". From 1996 until the millennium rollover, the structural balance improved from a significant deficit to a surplus. This should be viewed in the light of an increase in structural employment in the same period due to the marked fall in structural unemployment, which was partly offset by a lower participation rate, however. Moreover, the expansionary measures in connection with the kick-start were gradually rolled back as the economy recovered, which also contributed to strengthening the structural balance.

From 2000 until the outbreak of the financial crisis in late 2008, the structural balance showed a stable surplus. The expansionary measures, such as the tax cuts in 2004 and increased public consumption, were offset by lower structural unemployment, lower interest costs and, especially in 2005-08, an extraordinarily high level of investment, which boosted potential output.

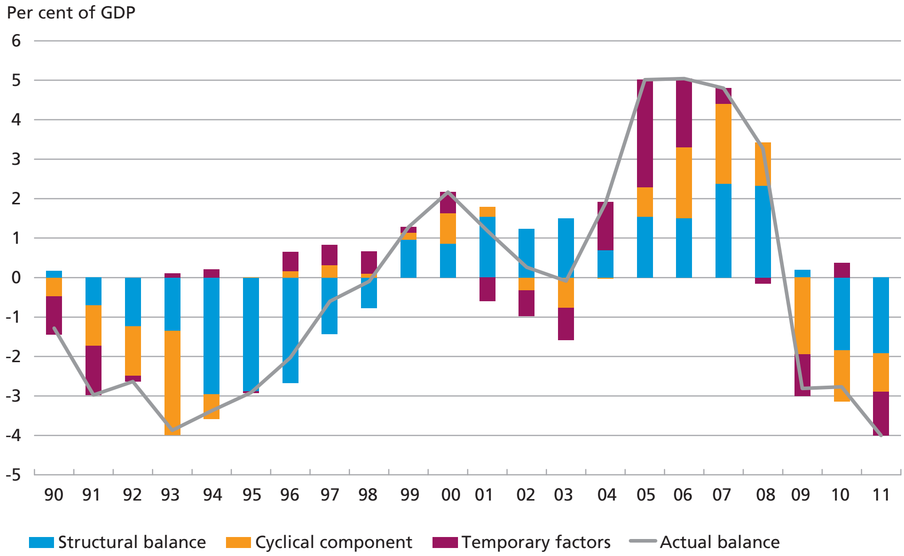
2009-10 saw a considerable deterioration of the structural balance due to the pronounced easing of fiscal policy as a reaction to the severe economic slowdown in the wake of the financial crisis. This deterioration

STRUCTURAL BALANCE IN DENMARK 1990-2011 Chart 11.2



Note: Figures for 2011 are based on Danmarks Nationalbank's latest forecast for the Danish economy.
Source: Statistics Denmark, Ministry of Finance and own calculations.

DECOMPOSITION OF ACTUAL BUDGET BALANCE 1990-2011 Chart 11.3



Note: Figures for 2011 are based on Danmarks Nationalbank's latest forecast for the Danish economy.
Source: Statistics Denmark, Ministry of Finance and own calculations.

has now ceased as a result of, *inter alia*, the subsequent consolidation effort in the form of e.g. the Fiscal Consolidation Agreement from May 2010. The structural deficit for 2011 has been estimated to be 1.9 per cent of GDP.

The substantial discretionary easing in 2009 and 2010 was supplemented by the effect of the automatic stabilisers that play a key role for the government budget in Denmark. The cyclical component is a decisive factor for the development in the actual balance, cf. Chart 11.3. For example, in the period 2005-08, the cyclical component contributed significantly to the surplus on the actual balance, whereas it strongly reduced the actual balance in 2009-11.

TECHNICAL APPENDIX

Theoretical basis for equation (4.1)

The theoretical background for the specification in equation (4.1) is based on the following simple model of wage and price developments in a small, open economy:

$$\Delta w_t = \Delta p_t + \eta \cdot \Delta y_t - \theta(w - p - y)_{t-1} - \gamma \cdot u_{t-1} + a_{t-1} + \varepsilon_t^w \qquad (A.1)$$

$$\Delta p_t = \lambda(\Delta w_t - \Delta y_t - \omega(1 - (w - p - y)_{t-1} - q^*)) + (1 - \lambda)\pi_t^* + \varepsilon_t^\pi . \qquad (A.2)$$

Equation (A.1) states that the relative change in nominal wages depends on price inflation (Δp), productivity growth (Δy), unemployment in the preceding quarter (u) and a temporary noise term (ε^w). The variable a captures time-varying structural factors influencing wage formation. One example is the degree of compensation of the unemployment benefit system, but a may also include other factors that may be difficult to observe or measure. If the parameter θ is positive, wage inflation is furthermore influenced by an error correction mechanism via the ratio of real wages (w-p) to productivity (y) in the last quarter. If productivity is approximated by average labour productivity, this ratio corresponds to the wage share. There is empirical evidence that this error correction mechanism is a widespread phenomenon in wage formation in European labour markets, cf. Blanchard and Katz (1999).

Equation (A.2) states that domestic inflation depends on domestic factors, inflation abroad (π^*) and temporary price shocks (ε^π). The domestic factors firstly depend on unit labour costs, expressed as growth in real wages less productivity growth. In addition, a positive relation is assumed between inflation and the wage share in the last period. The reason is that firms will aim at a certain profit ratio, q^*. Consequently, if the wage share becomes too high and the profit ratio too low, they will try to counter this by raising prices more than warranted by inflation abroad and the development in unit labour costs.

Long-term equilibrium is defined as a situation without temporary shocks where productivity growth equals its long-term trend value, Δy^*, and where the growth rate for real wages equals that for productivity. Furthermore, in a small, open economy pursuing a fixed-exchange-rate policy, inflation in the long-term equilibrium must equal inflation abroad. By combining equations (A.1) and (A.2) and inserting $\varepsilon^w = \varepsilon^\pi = 0$, $\Delta y = \Delta y^*$, $\Delta w - \Delta p = \Delta y$ and $\Delta p = \pi^*$, structural unemployment in period t-1 is given as:

$$u_{t-1}^* = \frac{1}{\gamma}\left(a_{t-1} - \theta(1 - q^*) - (1 - \eta)\Delta y^*\right).$$

Hence, equation (A.1) can be rewritten as

$$\Delta ws_t = -\theta\left(ws_{t-1} - ws^*\right) - \gamma \cdot (u_{t-1} - u_{t-1}^*) - (1 - \eta)(\Delta y - \Delta y^*) + \varepsilon_t^w , \qquad (A.3)$$

where $ws \equiv w - p - y$ can be approximated by the wage share, and $ws^* \equiv 1 - q^*$ is the equilibrium wage share. It should be noted that equations (4.1) and A.3) are almost identical. The only difference is that (4.1) includes more lags of the wage share and productivity growth on the right-hand side, and that equation (4.1) includes more control variables that are likely to exert temporary influence on the change in the wage share.

Technical description of the model for structural unemployment

The model consists of the following equations:

$$u_t = u_t^* + u_t^c \tag{A.4}$$

$$\Delta u_t^* = \rho \cdot \Delta u_{t-1}^* + \zeta_t \;,\; \zeta \sim N(0,\sigma_\zeta^2) \tag{A.5}$$

$$u_t^c = \alpha_1 u_{t-1}^c + \alpha_2 u_{t-2}^c + \kappa_t \;,\; \kappa \sim N(0,\sigma_\kappa^2) \tag{A.6}$$

$$\Delta ws_t = \sum_{i=1}^{4}\left(\alpha_i (ws_{t-i} - ws^*)\right) - \gamma \cdot u_{t-1}^c + \mathbf{x}_t{}'\beta + \varepsilon_t \;,\; \varepsilon \sim N(0,\sigma_\varepsilon^2)\,, \tag{A.7}$$

where u denotes registered net unemployment as a percentage of the labour force. In equation (A.4), this is decomposed into a structural component, u^*, and a cyclical component, u^c.

Equation (A.5) models the change in structural unemployment as an autoregressive process without specifying the drivers of the change. The parameter ρ indicates the persistence of the process. In the special case $\rho = 1$, the change in u^* will follow a random walk, whereby u^* is I(2). In the special case $\rho = 0$, u^* will itself follow a random walk and thus be I(1).

In equation (A.6), it is assumed that the cyclical component can be described by an AR(2) process. The autoregressive parameters α_1 and α_2 should meet the conditions for stationarity $|\alpha_2| < 1$, $\alpha_1 + \alpha_2 < 1$ and $\alpha_2 - \alpha_1 < 1$. This formulation takes into account that cyclical fluctuations in unemployment may be persistent, but they are still temporary. Hence, the equation implies that actual unemployment, viewed over longer periods, tends to fluctuate around structural unemployment.

Equation (A.7) describes a relation between the cyclical component of unemployment and the change in the wage share, Δws, conditioned on the latter's previous deviations from its equilibrium value, ws^*, and a number of control variables. The composition of the control variables is given by:

$$\begin{aligned}\mathbf{x}_t{}'\beta = {} & \beta_1(\Delta raa_t - \overline{\Delta raa}) + \beta_2(\Delta raa_{t-1} - \overline{\Delta raa}) + \beta_3(\Delta raa_{t-2} - \overline{\Delta raa}) \\ & + \beta_4(\Delta efkrks_t - \overline{\Delta efkrks}) + \beta_5(\Delta efkrks_{t-1} - \overline{\Delta efkrks}) + \beta_6(\Delta efkrks_{t-2} - \overline{\Delta efkrks}) \\ & + \beta_7(\Delta prod_t - \overline{\Delta prod}) + \beta_8(\Delta prod_{t-1} - \overline{\Delta prod}) + \beta_9(\Delta prod_{t-2} - \overline{\Delta prod})\end{aligned}$$

where raa is the commodity price index, $efkrks$ is the effective krone rate, and $prod$ is hourly productivity in the private non-agricultural sector, all in logarithms. A bar above a variable denotes the variable's sample average. Equation (A.7) corresponds to equation (4.1) and is described in more detail in section 4.

The noise terms ζ, κ and ε are assumed to be normally distributed, independently and identically distributed over time and mutually uncorrelated.

PARAMETER ESTIMATES IN THE ESTIMATION OF STRUCTURAL UNEMPLOYMENT

Table A.1

Parameter	Estimate	Standard error
Coefficients on variables in equation (A.7):		
Unemployment gap (t-1)	-0.40	(0.16)
Wage share (t-1)	-0.67	(0.09)
Wage share (t-2)	0.30	(0.09)
Wage share (t-3)	0.01	(0.09)
Wage share (t-4)	0.26	(0.07)
Commodity price inflation (t)	-0.03	(0.03)
Commodity price inflation (t-1)	0.08	(0.03)
Commodity price inflation (t-2)	-0.10	(0.03)
ΔEffective krone rate (t)	0.02	(0.12)
ΔEffective krone rate (t-1)	0.47	(0.12)
ΔEffective krone rate (t-2)	-0.14	(0.12)
Productivity growth (t)	-0.62	(0.08)
Productivity growth (t-1)	-0.33	(0.10)
Productivity growth (t-2)	-0.05	(0.10)
Long-run equilibrium wage share, ws^*	0.65	
Other parameters:		
α_1	1.719	(0.073)
α_2	-0.752	(0.071)
ρ	0.898	(0.051)
σ_ε	0.052	(0.185)
σ_κ	0.007	(0.024)
σ_ζ	0.002	

Source: Own calculations.

The unobservable variables and model parameters are estimated by maximum likelihood and the double-sided "smoothing" Kalman filter. For an elaboration on the estimation method, reference is made to e.g. Hamilton (1994, Chapter 13). It is well-known that the maximum likelihood method suffers from a potential stability problem that may lead to the variances of the unobservable variables being estimated to be zero. In order to circumvent this problem, the model is estimated subject to the restriction that the variances σ_ζ^2 and σ_κ^2 are locked in the ratio 1/15. The ratio has been chosen so that the smoothness of the resulting series for structural unemployment corresponds to our priors, as suggested by Gordon (1997). All other parameters, including the equilibrium value of the wage share, are estimated without restrictions. The full number of parameter estimates is reported in Table A.1.

Technical description of the model for structural participation rate

The model consists of the following equations:

$$E_t = E_t^* + E_t^c \tag{A.8}$$

$$E_t^* = E_{t-1}^* - \Delta ogy_t + \mu_1 \cdot \Delta leave_t + \mu_2 \cdot \Delta sh6066_t + \zeta_t , \quad \zeta \sim N(0,\sigma_\zeta^2) \tag{A.9}$$

$$E_t^c = \phi_1 \cdot E_{t-1}^c + \phi_2 \cdot E_{t-2}^c + \chi \cdot u_{t-1}^c + \kappa_t \;, \quad \kappa \sim N(0,\sigma_\kappa^2) \tag{A.10}$$

$$shortage_t = \eta_0 + \eta_1 \cdot E_t^c + \varepsilon_t \;, \quad \varepsilon \sim N(0,\sigma_\varepsilon^2) \tag{A.11}$$

PARAMETER ESTIMATES IN THE MODEL FOR STRUCTURAL PARTICIPATION RATE — Table A.2

Parameter	Estimate	Standard error
Coefficient parameters:		
μ	-0.76	(0.36)
φ_1	0.84	(0.11)
φ_2	0.04	(0.11)
χ	-0.04	(0.05)
η_0	0.02	(0.00)
η_1	1.32	(0.55)
Variance parameters:		
σ_κ	0.002	(0.003)
σ_ε	0.015	(0.020)
σ_ζ	0.001	

Source: Own calculations.

Equation (A.8) denotes that the participation rate, E, can be decomposed into a structural component, E^*, and a cyclical component, E^c. The remaining equations are described in detail in section 5. As in the model for structural unemployment, it is assumed that all noise terms are normally distributed, independently and identically distributed over time and mutually uncorrelated.

The model is estimated using the Kalman filter and maximum likelihood under a restriction locking the variances σ_ζ^2 and σ_κ^2 in the ratio 1/15. All other parameters are estimated freely. All parameter estimates are reported in Table A.2.

The extended HP filter and structural TFP

In the extended HP filtering of TFP, the logarithm of actual TFP is specified as the sum of a structural component, a cyclical component and a noise term representing non-cyclical stochastic shocks. At the same time, an unknown, linear relationship is postulated between the cyclical component and an external indicator, here the indicator of spare capacity. The estimated series for structural TFP and the TFP gap are then found by solving the following minimisation problem:

$$\min_{\{tfp_t^*, tfp_t^c\}_{t=1}^T, \beta_0, \beta_1} \left(\begin{array}{l} \lambda \sum_{t=3}^{T} \left(\Delta tfp_t^* - \Delta tfp_{t-1}^* \right)^2 + \sum_{t=1}^{T} \left(tfp_t - tfp_t^* - tfp_t^c \right)^2 \\ + \sum_{t=1}^{T} \left(tfp_t^c - \beta_0 - \beta_1 \cdot sparecap_t \right)^2 \end{array} \right), \tag{A.12}$$

where tfp_t denotes the logarithm of observed TFP in quarter t, tfp_t^* is the estimated level of the logarithm of structural TFP, tfp_t^c is the estimate of the logarithm of the cyclical component and $sparecap_t$ is the indicator of spare capacity. The latter is scaled so that the value zero corresponds to the average value in the sample. The parameter λ is the well-known weighting parameter from the HP filter, which is set here at the standard value 1600 for quarterly data.

OLS REGRESSION OF CYCLICAL TFP COMPONENT ON INDICATOR OF SPARE CAPACITY

Table A.3

Variable	Coefficient	Standard error
Spare capacity	-0.10	(0.00)
Constant	-0.00	(0.00)
R^2	0.83	
Observations	119	

Source: Own calculations.

The parameters β_0 and β_1 describe the relationship between the indicator of spare capacity and the estimated cyclical component of TFP. They are both estimated simultaneously with the series for structural and cyclical TFP, and the estimates correspond to the parameter estimates obtained by OLS regression of the estimated series for the cyclical component on the indicator. In relation to the well-known HP filter, the extended HP filter includes an extra criterion in the objective function in (A.12), i.e. the sum of the squared residuals from such regression.

Table A.3 shows the regression output from the estimated relationship between the cyclical TFP component and the indicator of spare capacity. The coefficient on the indicator of spare capacity is negative and clearly significant. This means that higher capacity utilisation is associated with a larger cyclical TFP component, so minimisation of the sum in (A.12) will result in a larger estimate of the cyclical component of TFP in the quarters with high capacity utilisation.

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