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Danmarks Nationalbank

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A comparison of DEA and SFA

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The efficiency of Danish banks before and during the crisis

A comparison of DEA and SFA[1]

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December 2013

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Abstract

This paper analyses the development in relative efficiency of Danish banks in the period 2001-2012. Using Data Envelopment Analysis (DEA) and Stochastic Frontier Analysis (SFA) techniques, we find that mean relative efficiency increased during the expansive period 2003-2007, while it decreased during the crisis years 2008-2010. During the recent years, mean relative efficiency increased again, possibly as a result of adjustment of inputs to the reduced output growth as well as the general consolidation in the banking sector. Furthermore, we find a considerable but not perfect correlation between efficiency rankings of banks as estimated by DEA and SFA. Finally, an international benchmarking exercise using data from 203 European banks and banking groups in 2012 shows that larger Danish banks and banking groups, when compared to their peers in other European countries, are distributed largely along most of the spectrum of efficiency.

Key words: Bank efficiency; DEA; SFA

JEL Classification: C44; G21.

Resumé (Danish summary)

Denne artikel analyserer udviklen i danske pengeinstitutters relative efficiens i perioden 2001-2012. Ved brug af analyseteknikkerne Dataindhyldningsanalyse (Data Envelopment Analysis, DEA) og Stochastic Frontier Analysis (SFA) finder vi at danske pengeinstitutters gennemsnitlige relative efficiens steg i den ekspansive periode 2003-2007, mens den faldt under krisen i 2008-2010. I de seneste år er den gennemsnitlige relative efficiens steget igen, sandsynligvis som konsekvens af bankernes tilpasning af inputs til den reducerede outputvækst samt den generelle konsolidering i sektoren. Desuden finder vi en betydelig, men ikke fuldstændig, korrelation mellem rangordningerne af pengeinstitutter baseret på DEA og SFA. Endelig viser en international sammenligning af europæiske bankkoncerner baseret på data fra 203 pengeinstitutter og bankkoncerner i 2012 at større danske pengeinstitutter og bankkoncerner, sammenlignet med øvrige europæiske, er fordelt bredt over det meste af efficiensspektret.

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1. Introduction

The banking sector plays an important role for the transmission of capital from economic agents with savings surpluses to agents in need of financing. In a macroeconomic perspective, efficiency of the banking sector in its provision of financial intermediation services is important for the overall allocation of resources. In addition, efficiency of the banking sector plays a role for the monetary transmission mechanism and for the stability of the financial system. Using accounting data for all Danish banks from the period 2001-2012, this study analyses the relative efficiency of Danish banks over time, including an investigation of factors related to efficiency. In addition, we perform an international benchmarking analysis of the larger Danish banks using a cross-sectional dataset covering listed banks in a range of European countries in 2012.

We find that the mean relative efficiency of the Danish banks increased in the years 2003-2007. During the financial crisis in the period 2008-2010, mean relative efficiency decreased, a finding which can be ascribed to a smaller growth in output while inputs were not adjusted as quickly. In the recent years 2010-2012, mean relative efficiency increased again. Before the crisis, larger and better capitalized banks were found to be more efficient, while no clear relations between bank characteristics and relative efficiency are found in the most recent period. These findings suggest that, following the crisis, many banks have adjusted their inputs, for example personnel and administration expenses, to reflect the lower level of outputs compared to the pre-crisis level. In addition, banks which failed or were taken over by other banks during the crisis were on average less efficient than the other banks, which suggests that the consolidation of the Danish banking sector following the crisis has contributed to an increase in the efficiency of the banking system. This interpretation is supported by the international benchmarking analysis, which shows that Danish banks did not differ markedly from their peers in other European countries in terms of efficiency in 2012; in particular, there is no evidence that Danish banks in general are poorer performers in terms of efficiency than other European banks. Some of the larger Danish banks are clearly efficient compared to peers in other European countries, while a number of banks have potential for further efficiency improvements.

Efficiency analysis has gained in popularity over the past decades, mainly based on the methodological frameworks of Data Envelopment Analysis (DEA) and Stochastic Frontier Analysis (SFA). Relatively few studies, however, use both methodologies, but those which do, often find that efficiency scores vary considerably across methods and specifications. Therefore, this paper provides a detailed comparison of results obtained with DEA and SFA.

The study contributes to the literature in several respects. By relying on a dataset covering the period 2001-2012, we are able to address the development in aggregate and bank specific efficiency both in expansive times in the middle of the 00's and during the recent financial crisis. A special focus is on the extent to which the efficiency rankings of banks differ when different models and specifications are used. We take the consequence of the considerable debate in the literature concerning the treatment of deposits in models of bank efficiency by reporting all results throughout the paper both based on models in which deposits are treated as inputs and on models in which deposits are treated as outputs. Furthermore, in addition to the analysis of bank specific relative efficiency estimated by DEA and SFA, we provide a discussion of the development in mean relative efficiency over time and an econometric analysis of the factors associated with efficiency. And finally, the international benchmarking exercise provides additional results useful for the interpretation of the results based on the national sample.

Though a significant, positive correlation between efficiency scores obtained by DEA and SFA is present in all cases, the correlation is not perfect. DEA tend to provide somewhat more stable rankings of banks over time than SFA, and also to be less influenced by the specification of deposits as an input or an output; however with considerable variation over time. One part of the explanation may be that a relatively large fraction of banks are considered as fully efficient in the DEA estimation, meaning that the probability of keeping that status over time or in a different specification is fairly high. The correlation between estimated efficiency scores using DEA and SFA and more traditional accounting-based measures is found to be quite low. This is in line with previous findings in the literature. Finally, the importance of using multiple methods to cross check findings is underlined by the international comparison, in which the assessment of Danish banks vary across methods. DEA assesses the Danish banks to be more efficient in general than SFA.

The paper proceeds as follows: Section two briefly reviews related literature and section three proceeds by presenting the two efficiency methods DEA and SFA and the specifications used here. Section four presents the data, while section five presents and discusses the results and section six concludes.

2. A brief review of related literature

Studies of bank efficiency have a long history, and have been based on increasingly advanced methods along with the methodological advances in DEA and SFA. Berger and Humphrey (1997) provide an overview of 130 studies of bank efficiency published until 1997. They find that estimated average efficiency levels vary substantially across methods, contexts and model specifications.

Only few studies have considered the efficiency of Danish banks. Bukh (1995) is the most comprehensive DEA study of the efficiency of Danish banks. Overall, he finds relatively large efficiency differences across banks. The main reason for inefficiency was found to be production at an inefficient scale. Bukh furthermore focuses on the optimal bank size, and he does not find support for the hypothesis that larger banks have lower costs.

The part of our analysis, which is concerned with the distribution and stability of efficiency rankings over time and across methods is closely related to the research by Fiorentino et al. (2006), who compare the performance of DEA and SFA for German banks. Our results complement the findings of Fiorentino et al. by using a comparable approach in a different country setting and with data covering a different time period spanning both the boom in the middle of the 00's as well as the economic slowdown from 2008 onwards. The results are not directly comparable, however, due to differences in the modelling approaches. DEA and SFA are compared largely along the lines of the framework of Bauer et al. (1998), as we compare the ranking of banks across methods and stability over time within methods, as well as the relations of estimated efficiency scores to other, more traditional performance measures. In addition to these comparisons based on Bauer et al., we assess the importance of the specification of inputs and outputs by treating deposits as an input and as an output, respectively, throughout the paper.

Another relevant strand of literature is concerned with the development in efficiency over time and the causes thereof. Martín-Oliver et al. (2013) find that two thirds of the estimated productivity growth of Spanish banks in the years 2000-2007 could be ascribed to changed business practices, such as the large increase in credit to the housing market, a shift towards short-term funding and the leveraging process of banks' balance sheets. More generally, this points to the importance of investigating the root causes of productivity development, and in particular of being cautious when interpreting an observed increase in efficiency as current models are not fully able to take into account the risks associated with e.g. such a change in business models.

3. Efficiency: Concepts and methodology

This study has three main purposes; namely (1) an assessment of the relative efficiency of Danish banks, including identification of factors related to efficiency, (2) a methodological comparison of DEA and SFA methods, and (3) an international benchmarking exercise of the larger Danish banks.

Most studies aiming at establishing an efficiency measure for individual production units rely on the empirical deviation from a perfect input-output allocation. Efficiency is for each unit calculated as the distance to the optimal (frontier) production function used as a benchmark. Obviously, different specifications of the frontier may lead to different results. The remainder of this section introduces the two most widely used techniques for estimating the frontier, DEA and SFA, along with a presentation of the empirical specification used in this study. The aim is not a full presentation of DEA and SFA, for an introduction to the methods, see e.g. Bogetoft and Otto (2011).

3.1 Data Envelopment Analysis (DEA)

Data Envelopment Analysis (DEA) is a linear programming technique originating in operations research and mathematical programming. DEA estimates a production frontier using information on inputs and outputs by 'enveloping' the observed combinations of inputs and outputs. The envelopment technique implies that all 'best' performers along the different dimensions are used to form the production frontier through local linear interpolation. This means that DEA is a non-parametric method. The advantage is that no assumptions regarding functional form of the production frontier is needed; on the other hand, this comes at the cost of not being able to distinguish between true efficiency differences and noise in the data. The estimated efficiency frontier can be used as a basis for determining the efficiency of individual banks. By construction, a number of banks are used to form the frontier, meaning that they will be ranked as fully efficient (an efficiency score of 1). For banks which are not used to form the frontier, a measure of the distance from the realised input / output combination to the frontier can be used to determine the efficiency of the bank relative to the fully efficient banks. Only the empirically observed input / output combinations are used to construct the frontier. This means that our DEA results can provide no indication regarding the efficiency of the banks on the frontier relative to banks not included in the dataset or relative to a theoretical frontier of what is technologically feasible. The distance measure used here (and in many other applications) is the Farrell distance function (Farrell, 1957), which gives rise to a Farrell efficiency estimate (the *efficiency score*) for each bank.

DEA was introduced by Charnes et al. (1978) in a model with constant returns to scale and input orientation. Since then, several contributions have developed versions of the DEA model, most important was the introduction of variable returns to scale and the complementary approach of output orientation. It is clear that the original approach of constant returns to scale is not appropriate for the description of banking technology. The model used in the present analysis is, therefore, a variable returns to scale model[3]. In technical terms, the model with variable returns to scale constructs the frontier as a convex hull of intersecting planes instead of the conical hull estimated by the constant returns to scale model

(Fiorentino et al., 2006). Hence, the variable returns to scale model envelops the data closer than the constant returns to scale model, a fact which implies that efficiency estimates are equal to, or greater than, those of the constant returns to scale model (Banker et al., 1984).

3.2 Stochastic Frontier Analysis (SFA)

A popular alternative to the deterministic, non-parametric DEA approach is the Stochastic Frontier Analysis (SFA). The main differences between DEA and SFA lies in exactly the two characteristics of DEA stated above; while DEA is deterministic and non-parametric, SFA assumes a stochastic relationship between input and output and is it is a parametric approach. The main advantage of SFA over DEA is that SFA does not ascribe the deviation from the estimated frontier completely to inefficiency; the distance between the observed input / output combination and the frontier may be ascribed partly to inefficiency and partly to noise in the data, originating from e.g. imprecise measurement of inputs and/or outputs. This feature comes at the cost of the need to impose a functional form to enable estimation of the parameters of the model. DEA originates in mathematical programming, while SFA is rooted in econometric theory. In this paper, the two approaches are seen as complementary rather than competing.

As banking is inherently a multi-input, multi-output production process, a standard production function with one output and multiple inputs is not flexible enough to provide a sufficiently comprehensive benchmarking of banks. One popular solution is to estimate a cost function instead. However, estimation of cost functions requires information on costs and prices. An alternative solution, which we will use here, is to use a distance function to describe the combination of inputs and outputs observed for a given bank. In practice, we use a translog distance function, a functional form which have showed to be useful in previous applications[4]. To be more specific, we follow Bogetoft and Otto (2011, p. 243) in specifying a translog distance function as

$$\log\left(\frac{1}{x_m}\right) = a_0 + \sum_{i=1}^{m-1} a_i \log\frac{x_i}{x_m} + \sum_{j=1}^{n} b_j \log y_j + \frac{1}{2}\sum_{i=1}^{m-1}\sum_{j=1}^{m-1} A_{ij} \log\frac{x_i}{x_m}\log\frac{x_j}{x_m}$$
$$+\frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} B_{ij} \log y_i \log y_j + \frac{1}{2}\sum_{i=1}^{m-1}\sum_{j=1}^{n} C_{ij} \log\frac{x_i}{x_m}\log y_j + v + u$$

[3] In practice, we utilize the R package Benchmarking (Bogetoft and Otto, 2013) to estimate an input oriented DEA model with variable returns to scale for each year in the study period. Robustness checks (results not reported in this paper) have been performed using the pooled sample (all observations over time) for estimation of the frontier.
[4] One particularly nice feature of the translog function is that estimation of the second-order derivative is done simultaneously with the parameter estimation. This means that no assumptions regarding the relation between inputs and outputs are needed; whether inputs and outputs are substitutes or complements is treated as an empirical question (Bogetoft and Otto, 2011, p. 243).

Where $x_1 \cdots x_m$ are inputs and $y_1 \cdots y_j$ are outputs, $v \sim N(0, \sigma_v^2)$ is an error term and $u \sim N_+(0, \sigma_u^2)$ represents the possible inefficiency of the firm (u follows the half-normal distribution to ensure that $u \geq 0$). As the SFA model is estimated with maximum likelihood, the results do not depend on which variable is estimated as a residual (the x_m variable).

The efficiency score (the technical efficiency) for each bank is estimated by minimizing the mean square error, the method used by most applications (for details, see section 7.10 in Bogetoft and Otto, 2011).

3.3 Input and output measures in the banking sector

The main data issue for the assessment of bank efficiency and, more generally, of bank production, is the specification of output measures. This issue is not limited to efficiency studies, also e.g. national accounts frameworks treat the production in the financial sector differently from production in non-financial corporations. Traditionally, studies of bank efficiency have been based on either a production approach or an intermediation approach. In the production approach, firms in the financial sector are thought of as units using capital and labor to produce loans and deposit services. Relevant output measures therefore include the number of accounts or transactions, and input could be measured as the total costs. The intermediation approach views banks as intermediaries, which produce financial services. Hence, output measures include loans and investments, while inputs include, in addition to capital and labor, the financing costs.

It is widely discussed in the literature whether deposits are to be treated as inputs or outputs. Based on the ideas in the production approach, deposits may be considered inputs (funding) used in the production of loans and other assets. On the other hand, deposits may also be viewed as outputs as they may be rather complex products, often accompanied with e.g. credit cards and other payment services. Following the ideas of the intermediation approach, deposits may therefore be argued to be outputs; which is also more in line with the national accounts perspective of production of financial services. Due to the substantial disagreement in the literature, throughout the paper, we estimate separate models treating deposits as an input and an output, respectively.

The frontier models estimated in this study may not be very informative about the efficiency of the largest banks in the population due to the few comparable banks. For the DEA model with variable returns to scale, it is even the case that the largest bank by definition will be ranked as fully efficient. This problem is partly overcome in the international benchmarking exercise, in which the largest Danish banks are compared to peers from other European countries.

As noted, the efficiency analysis results in an efficiency score per bank for each year. A graphical and a descriptive approach is used to assess the development in the mean and distribution of efficiency scores over time. In order to investigate which characteristics of a bank that is related to efficiency (see section 5.5), a Tobit model is used to relate bank characteristics to the efficiency scores estimated by DEA. The merits of a Tobit model in this context is, that it takes into account that the efficiency scores are censored in the interval [0 , 1], and in particular that the banks used to construct the efficiency frontier all have an efficiency score which is exactly 1. The relation between bank characteristics and the SFA efficiency scores is estimated by OLS as the SFA efficiency scores are not censored (while still limited to the interval [0 , 1]). While useful information can be extracted from the second stage regressions, one caveat is that the dependent variable, the efficiency score, is itself an estimate. Currently, no methods are convincing in taking this fact into account (that is, using both the point estimate and an estimate of its precision), so the second stage results should be interpreted with this in mind.

4. Data

The data used for the national benchmarking analysis is accounting data collected by the Danish Financial Supervisory Authority for all banks in Denmark[5]. Section 3 discussed the most important choices regarding variables. Table 1 presents descriptive statistics for each year[6]. Two points are immediately clear from an inspection of the table. First, a time trend is clear for most variables. This has implications for our modelling strategy, so that the efficiency models in this study, on which our main results are based, are based on observations from each year and not on pooled observations over time. And second, data is highly right-skewed. A relatively small number of banks are substantially larger than the others. All banks, which exist in Denmark at the end of a given year, are included in the sample. Mortgage banks are not included.

For the second part of the analysis, the international benchmarking exercise, we use a cross sectional dataset from 2012 from the SNL database[7], covering the EU15 plus Norway and Switzerland. Included in the dataset are mainly banks listed at the stock exchange, a fact which implies that the number of banks included varies substantially among countries as the listing practice vary. Naturally, banks included in the dataset are heterogeneous.

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[5] Data is downloadable from the website of the Danish FSA, http://www.ftnet.dk.

[6] For presentational reasons, only means and medians are shown in the table.

[7] See www.snl.com for more information.

Table 1: Descriptive statistics by year

Variable	Stat	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011	2012
Inputs													
Personnel and administration expenses	Median	16 141	16 974	20 607	23 709	29 225	40 036	48 308	52 110	56 385	59 103	60 268	68 462
	Mean	168 631	174 182	181 003	194 623	212 883	246 620	295 391	336 113	360 689	378 375	425 869	520 912
Interest expenses	Median	9 554	8 120	7 617	7 282	9 341	18 922	40 763	56 307	33 504	25 488	27 319	19 568
	Mean	328 016	271 859	225 566	246 854	369 026	563 141	873 270	1 097 448	575 669	358 966	439 635	422 960
Depreciation of immaterial and material assets	Median	731	683	830	918	756	1 227	1 416	1 232	1 442	1 570	1 541	1 116
	Mean	9 709	8 047	7 347	7 735	14 336	17 086	20 225	48 212	45 996	28 138	33 084	43 676
Outputs													
Total loans	Median	197 979	194 241	206 021	285 856	393 795	582 607	978 995	1 083 081	1 099 028	1 091 726	1 181 100	1 060 028
	Mean	4 550 267	4 988 608	5 339 378	6 222 938	7 858 340	10 609 553	14 151 945	15 519 355	13 867 069	14 843 937	15 450 459	18 349 631
Interest income	Median	27 326	26 192	26 291	29 823	37 197	62 054	88 556	100 835	99 246	89 370	92 138	99 110
	Mean	514 790	472 652	438 201	453 428	602 736	804 556	1 167 089	1 482 534	1 103 995	837 824	895 702	960 760
Non-interest income (fees, provisions, etc.)	Median	3 532	4 172	5 415	6 637	11 044	17 914	23 023	22 956	20 462	20 098	20 282	23 797
	Mean	81 006	82 804	90 702	103 953	132 396	144 556	170 827	174 363	176 447	203 145	217 829	275 147
Securities	Median	67 140	65 394	76 746	100 020	131 261	136 401	161 813	144 452	181 311	278 870	222 113	346 805
	Mean	2 505 692	2 845 916	3 483 477	3 489 834	3 643 093	4 469 644	5 201 542	6 258 017	7 308 730	7 349 031	8 095 160	10 428 680
Shares, etc.	Median	8 225	8 804	15 696	24 744	30 784	40 912	47 640	41 496	49 227	55 035	55 697	51 976
	Mean	206 493	173 907	222 469	252 975	159 416	209 466	225 533	159 132	184 535	225 707	227 416	309 469
Specially treated variable													
Deposits and other debt	Median	275 942	285 275	335 264	372 253	556 842	663 448	882 672	1 064 582	1 128 551	1 305 984	1 315 219	1 494 384
	Mean	4 382 740	4 715 940	5 328 053	6 041 188	6 771 965	7 832 394	10 401 308	11 575 888	11 835 395	12 301 887	13 444 807	17 175 030
Number of banks		185	180	176	172	161	152	146	138	132	123	113	93

Note: All values measured in DKK (DKK 1 ≈ EUR 0.13).

As our interest in the international benchmarking exercise is mainly to consider the larger banks, we restrict the dataset to banks whose total assets exceed 1 billion Euro. In contrast to the national level data, which are at the level of the individual banks, the international sample consists of consolidated banking groups and individual banks which are not part of a banking group. That is, banking groups present in different countries, and banking groups composed by two or more units within the same country, are consolidated and represented by one observation in the dataset for the country of residence of the group head office. Table 2 provides an overview of the data in terms of geographical distribution and descriptive statistics.

Table 2: Characteristics of the sample used for the international benchmarking exercise

Panel A: Geographical distribution of the sample

Country	No. of banks
Denmark	**15**
Other Nordic countries	
Finland	2
Norway	18
Sweden	5
West and Central Europe	
France	5
Austria	15
Belgium	2
Switzerland	30
Germany	20
Great Britain	18
Ireland	5
Luxembourg	4
Netherlands	10
Southern Europe	
Spain	18
Greece	4
Italy	27
Portugal	5
Total	203

Panel B: Descriptive statistics

Variable	Mean	Median
Inputs		
Number of employees	14 422	2 090
Interest expenses	19 969 729	3 620 132
Outputs - basic model		
Interest income	33 066 170	6 262 837
Operational income	22 316 025	4 226 874
Total loans	553 947 892	125 316 091
Securities and shares	419 853 186	36 983 871
Specially treated variable		
Deposits and other debt	461 677 428	89 009 275

Note: All values except number of employees are denoted in DKK

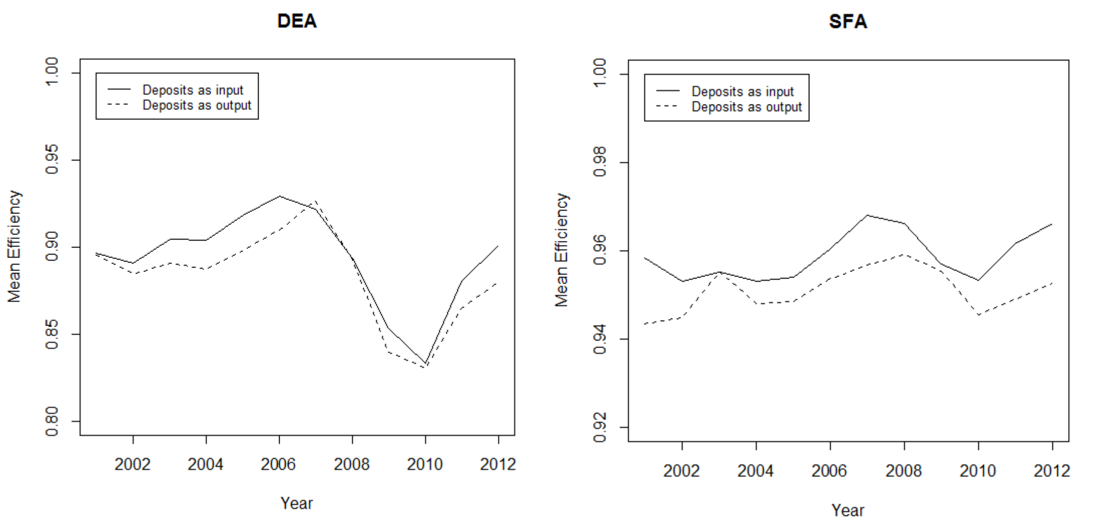
5. Results

5.1 The development of mean efficiency over time

Before turning to the analysis of individual efficiency scores, we in this subsection consider the development of the mean efficiency score over the period 2001-2012. It is important to keep in mind that mean efficiency is a relative measure, which may be influenced by many factors over time. Most importantly, mean efficiency is not informative about the level of efficiency of the banking system, but only a measure of the central tendency of the relative efficiencies. Even if all banks in a country are inefficient relative to similar banks in other countries, the mean relative efficiency may be high if many banks are close to the 'best practice' in the country. Nonetheless, mean efficiency score is the preferred indicator used in comparisons across efficiency studies in different contexts and time periods (e.g. Berger and Humphrey, 1997); and it is informative as an input to policymakers concerned with market structures, deregulation, mergers, etc. as well as it may be useful in assessing the need for further analysis of the underlying individual efficiency estimates in search of e.g. best practices. As Denmark is a small, open economy with a relatively deregulated financial system and substantial presence of foreign banks, the international competition increases the likelihood that relative efficiency measures can be informative of absolute efficiency levels as well. This idea is supported by the results of the international benchmarking exercise presented in section 5.6, in which there is no evidence that larger Danish banks in 2012 differed from their European peers in terms of efficiency (though, naturally, it is not clear to which extent this was the case in earlier periods as well).

Differences in estimated efficiency levels and the magnitude of variation between the DEA and SFA estimates are inherent in the methods and discussed further below. However, the two methods largely agree on their assessment of the development in relative efficiency over time, although the exact timing of the shifts in some cases vary by 1-2 years, cf. figure 1[8]. In the first years 2001-2002, there was a decrease in mean relative efficiency. Internationally, this period was characterized by the dot-com bubble, but in terms of the mean relative efficiency of Danish banks, the decrease was only temporary. In the years 2003-2007, mean relative efficiency of Danish banks increased substantially. This should be seen in light of the rapid expansion of the banks' credit supply which preceded the crisis. During the crisis years 2007-2010, mean relative efficiency decreased along with the decreased growth in outputs. This indicates that for many banks, the adjustment on the input side was markedly slower than the reduction in output growth during the crisis. In the last years of the sample, 2010-2012, mean

MEAN EFFICIENCY SCORE OF DANISH BANKS OVER TIME USING DEA AND SFA | Figure 1



Source: Own calculations based on data from the Danish Financial Supervisory Authority.

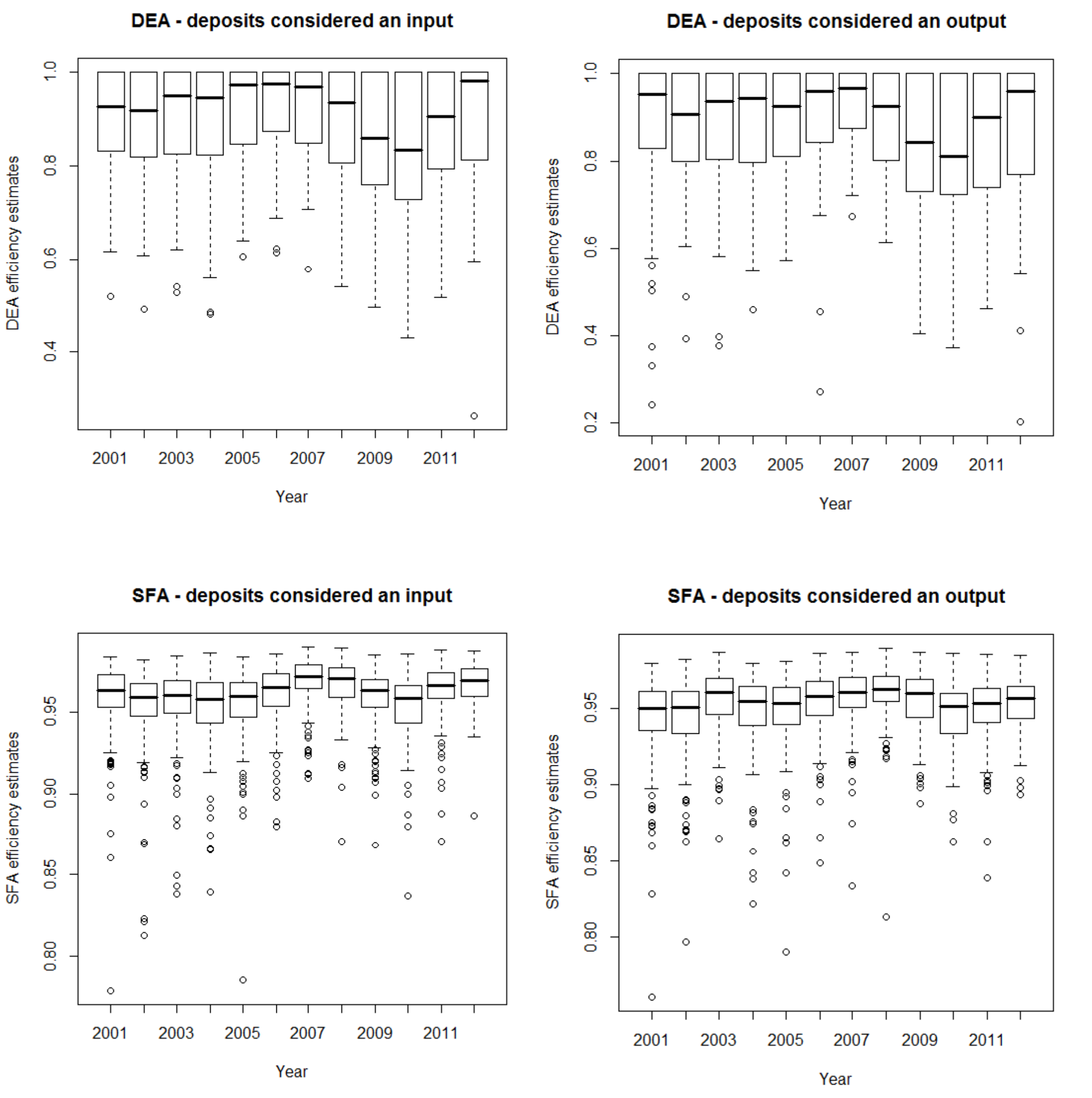
relative efficiency increased again. This can be interpreted as a structural adjustment on the input side to the new output levels. In addition, the increase in mean relative efficiency in those recent years indicates that the general consolidation in the banking sector, in the form of mergers and acquisitions as well as the failure of a number of banks, may be contributing to an increase in the efficiency of the Danish banking sector. We return to this in section 5.5.

Overall, based on the results from 2001-2012, mean bank efficiency decreases in crisis times, while it increases in more expansive periods. This is in line with the findings of Martín-Oliver et al. (2013). Furthermore, they find that part of the explanation is the high growth in lending often associated with periods of economic expansion. The large increase in credit is often argued to constitute a part of the reason for the financial crisis, but in efficiency studies loan growth is normally contributing to an increase in efficiency. To fully reflect efficiency, the increased volume of loans should ideally be countered in the model by a measure of the quality of the loan portfolio. In practice, however, it is difficult to find a timely indicator of the quality of the loan portfolio. In supplementary models (results not reported here) we include provisions and loan impairments as undesired outputs, but results are not comparable to our main results for the crisis period, in which the banks realise the losses incurred by loans supplied in the pre-crisis years[9]. Hence, an observed increase in mean

[8] As the data basis for the figure is the whole population, no confidence intervals are displayed in the figure. However, there may still be uncertainty originating in measurement error (probably quite small) and, of course, originating in the appropriateness of the model specifications and dimensions chosen to represent efficiency.

[9] A possible ex post modelling strategy could utilize the observed provisions during the crisis as an estimate of the riskiness of the loan portfolio before the crisis. To investigate the merits of this strategy, for each bank we plugged in the average of the observed provisions in 2008-2012 and reestimated the models for the pre-crisis years 2005-07, in which the loan growth was on its highest. In this robustness check, we treated provisions as an undesirable outcome. The distribution of DEA and SFA efficiency scores for 2005-07 using the ex post information on provisions did not differ markedly from the distribution using the actual provisions. However, such calculations

DISTRIBUTION OF ESTIMATED EFFICIENCY SCORES, DEA AND SFA MODELS Figure 2



Source: Own calculations based on data from the Danish Financial Supervisory Authority.

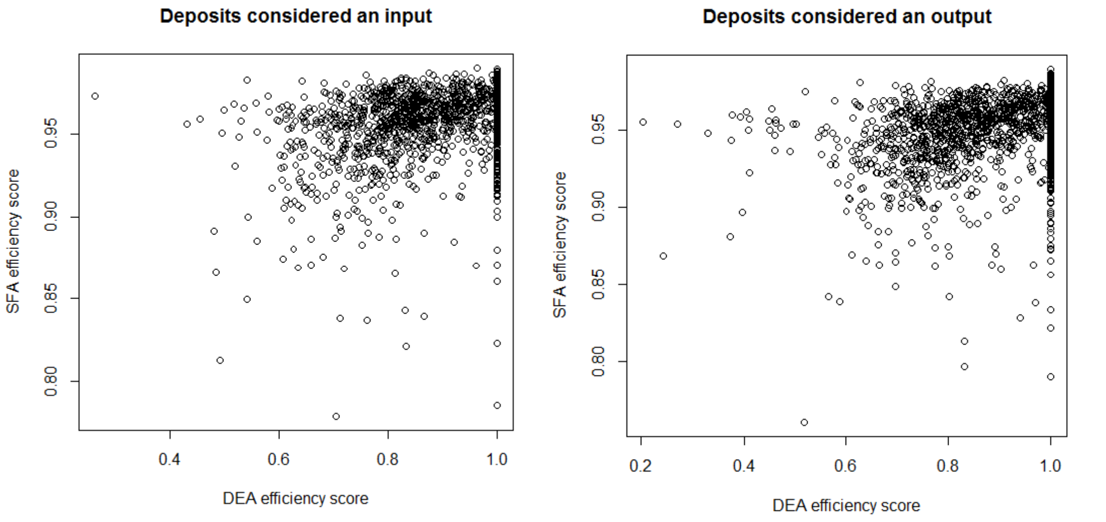
efficiency may be come with externalities, and should be subject to further investigation before drawing conclusions regarding financial stability.

5.2 Distribution and ranking of efficiency

The distributions of efficiency scores based on the estimation of the basic DEA and SFA models for each year in the period 2001-2012 are graphed in figure 2. In the DEA models, a

can only be based on the banks, which existed at least one of the years in the period 2008-12. It was also found that the average relative efficiency was higher for this sample of 'surviving' banks than for all banks in the population in 2005-07. Hence, part of the explanation for the absense of an effect of taking into account later provisions may be that some banks with poor quality loan portfolios failed during the crisis and hence, are excluded from the population. This means that their true losses / provisions are not reflected in the estimations. Even if a modelling strategy along these lines were succesful, it would be of little help for regular monitoring of the efficiency of the banking sector, as it requires information on the quality of the loan portfolio which is only available with a substantial time lag, and only if a stress scenario has actually occured.

COMPARISON OF EFFICIENCY ESTIMATES WITH DEA AND SFA | Figure 3



Source: Own calculations based on data from the Danish Financial Supervisory Authority.

relatively large fraction of the banks in each year is considered as fully efficient. That is, the efficiency frontier is constructed by a relatively large number of banks. The development in median efficiency shows the same pattern as the development in mean efficiency discussed above, albeit in some cases more pronounced. Based on the DEA estimates, median efficiency was in 2012 equal to or higher than median efficiency before the crisis, while it is not as clear in the case of SFA.

One of the central issues for this study is to compare the empirical performance of the DEA and SFA methods. Figure 3 plots the efficiency scores obtained by SFA and DEA for all bank-observations over time. For the banks with a DEA efficiency score less than 1, a somewhat reasonable relation between the efficiency scores obtained by the two approaches can be identified, although there is still a number of outliers. The banks which have received an efficiency score of 1 in the DEA model are characterized by a large variation in SFA scores, although they have generally been ranked at a high efficiency level by SFA as well. This is as expected, as the efficiency frontier estimated by SFA is not a linear combination of the 'best performing' units, but allows for measurement error.

Figure 3 furthermore documents that the SFA scores are more concentrated than the DEA scores; a pattern which can be ascribed to the fact that not all variation is interpreted as efficiency differences in the SFA model. This pattern is also found previously, see for example Berger and Humphrey (1997). Hence, the level of the efficiency scores may be difficult to interpret. As a first step towards comparison of the efficiency scores, it is instructive to consider the number of banks, which are consistently ranked as, e.g., poor performers across methods. For example, in 2012, 23 per cent of the banks have efficiency

Table 3: Rank order correlations among different approaches

	All observations				DEA efficiency < 1			
	DEA (I) SFA (I)	DEA (O) SFA (O)	DEA (I) DEA (O)	SFA (I) SFA (O)	DEA (I) SFA (I)	DEA (O) SFA (O)	DEA (I) DEA (O)	SFA (I) SFA (O)
Year	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
2001	0.34	0.30	0.85	0.71	0.52	0.46	0.70	0.73
2002	0.30	0.35	0.88	0.62	0.53	0.50	0.76	0.66
2003	0.35	0.32	0.87	0.60	0.36	0.41	0.77	0.53
2004	0.37	0.39	0.85	0.61	0.41	0.45	0.65	0.61
2005	0.47	0.42	0.82	0.72	0.49	0.51	0.56	0.75
2006	0.45	0.39	0.74	0.77	0.52	0.36	0.50	0.77
2007	0.24	0.39	0.93	0.71	0.30	0.41	0.70	0.79
2008	0.33	0.43	0.89	0.67	0.43	0.42	0.83	0.55
2009	0.34	0.40	0.96	0.65	0.35	0.41	0.93	0.62
2010	0.32	0.44	0.91	0.72	0.41	0.46	0.84	0.74
2011	0.38	0.35	0.87	0.61	0.43	0.45	0.84	0.65
2012	0.29	0.31	0.74	0.70	0.01	0.24	0.60	0.78

Note: The table displays the Spearman Rank Order Correlation Coefficient between the efficiency rankings obtained by the two approaches displayed in the first two rows. The parantheses refer to the treatment of deposits as inputs (I) and outputs (O), respectively. The right part of the table is based only on the observations, which have a DEA efficiency score less than one (when deposits are considered an input).

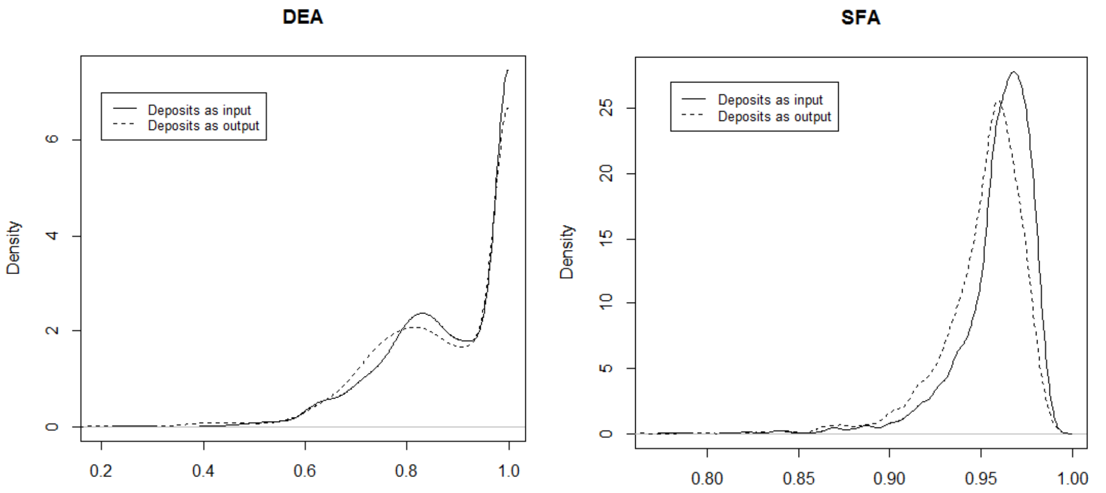
scores in the lowest half (i.e. less than the median) in all four models and specifications. 35 per cent have efficiency scores in the lowest half in at least three of four models.

A more formal comparison of results produced by DEA and SFA may be based on the degree to which the two approaches rank the banks in terms of efficiency in a similar way. Table 3 shows the Spearman Rank Order Correlation Coefficient (henceforth Spearman correlation) between the estimated rankings of banks using SFA and DEA[10]. The left part of the table is based on all bank efficiency scores, while the right part is based only on the observations, which have a DEA score less than one. The reason for this latter split is the presence of a relatively large number of banks, which in the DEA models are deemed fully efficient, implies that a full ranking of banks cannot be constructed. It is therefore difficult to compare the SFA and DEA scores (and the scores obtained by two DEA-approaches) in these cases, where an additional assumption of a 'shared rank' is needed to calculate the Spearman correlation.

The Spearman correlation between the efficiency rankings obtained by DEA and SFA varies considerably across years and output specifications (columns 1, 2, 5 and 6). In general, the correlations are considerable although not perfect. In all cases, they are positive and

[10] Spearmans Rank Order Correlation Coefficient (or Spearman's Rho) is a measure of the degree to which the relationship between two variables can be characterized by a monotonic function. A Spearman correlation of 1 signals that the ranking of observations according to one variable is equal to the ranking according to the other variable, while a Spearman correlation of -1 means that the ranking according to one variable is the opposite as the ranking according to the other variable.

DISTRIBUTION OF EFFICIENCY SCORES WITH ALTERNATIVE TREATMENTS OF DEPOSITS Figure 4



Note: Kernel density estimates (Gaussian kernel). Efficiency scores are based on annual frontier estimates. All efficiency scores over time are pooled in the figure.
Source: Own calculations based on data from the Danish Financial Supervisory Authority.

significant. This finding is in line with the results reported by Fiorentino et al. (2006) for German banks, although the correlations are slightly higher in this study.

Somewhat higher correlations are found between efficiency scores obtained with different input / output specifications of the DEA and SFA models. That is, the ranking of banks in terms of efficiency is less influenced by the specification of inputs and outputs (specifically the treatment of deposits) than the choice of method (DEA or SFA). SFA ranks banks more consistently irrespective of whether they are in the upper or lower part of the ranking (columns 4 and 8 are relatively similar). It is not surprising that the correlations for DEA in column 3 are large, as many banks are considered fully efficient in both approaches. To compare DEA rankings, it is preferable to consider the correlations in column 7, which are quite variable over time.

As noted in section 3.3, the treatment of deposits is widely debated in the literature. Figure 4 depicts the distribution of estimated efficiency scores when deposits are considered an input and an output, respectively. All observations over time are included in the figures. The difference between the distributions arising from a different treatment of deposits is evidently quite small.

5.3 Stability of rankings over time

Naturally, the efficiency ranking of banks changes over time. However, variables related to banks' balance sheets are relatively persistent in the short run, which should give rise to a certain persistency in efficiency scores over time. In addition, if efficiency scores should form

Table 4: Stability of efficiency rankings over time

Years between rankings	DEA (I)	DEA (O)	SFA (I)	SFA (O)
	(1)	(2)	(3)	(4)
All years				
1	0.76	0.78	0.65	0.67
2	0.66	0.68	0.53	0.57
3	0.61	0.61	0.49	0.52
4	0.53	0.53	0.43	0.45
Pre-crisis (2001 - 2006)				
1	0.76	0.83	0.67	0.67
2	0.70	0.75	0.52	0.55
3	0.65	0.71	0.43	0.46
4	0.61	0.67	0.38	0.42
Crisis (2008-2012)				
1	0.77	0.76	0.63	0.63
2	0.69	0.65	0.55	0.56
3	0.60	0.58	0.59	0.53
4	0.51	0.45	0.47	0.41

Note: The table shows the average of the Spearmans rank order correlation coefficients calculated for all year-pairs in the sample period separated by the number of years given in the first column. The parantheses refer to the treatment of deposits as inputs (I) and outputs (O), respectively.

the basis for regulatory policy measures, some persistency over time is needed to consistently identify the inefficient banks.

To test the stability of efficiency rankings produced by SFA and DEA over time, we calculate the pairwise Spearman's rank order correlation coefficient for the rankings produced by the DEA and SFA estimations for all pairs of years in the period 2001-2012, separated by 1, 2, 3 and 4 years, respectively[11]. Results are presented in table 4 as overall averages, and separately for the pre-crisis period (defined here as 2001-2006) and the crisis period (2008-2012).

In general, correlations between rankings over time are positive and significant, cf. table 4. Stability of rankings decreases over time, i.e. as the number of years between rankings increases. Stability of rankings is more pronounced for DEA than for SFA. Both of these results are in line with those found by Fiorentino et al. (2006) for German banks. The

[11] To illustrate, the Spearman's rank order correlation coefficient is calculated for the rankings produced by DEA in 2001 and 2002, in 2002 and 2003, and so on. Afterwards, the same calculation is performed for the rankings in 2001 and 2003, 2002 and 2004 and so on. The same principle is used for rankings separated by 3 and 4 years, and the whole procedure is repeated using SFA.

Table 5: Correlations between efficiency scores and other performance measures

		All observations				DEA efficiency < 1			
Indicator	Year	DEA (I)	DEA (O)	SFA (I)	SFA (O)	DEA (I)	DEA (O)	SFA (I)	SFA (O)
ROE	2011	0.29	0.29	-0.02	0.12	0.01	0.02	0.08	0.21
	2008	0.10	0.17	0.24	0.24	0.08	0.19	0.33	0.32
	2005	-0.15	-0.21	-0.07	-0.07	0.07	-0.06	-0.03	-0.05
ROA	2011	0.19	0.15	0.12	0.16	-0.13	-0.22	0.13	0.15
	2008	0.19	0.26	0.22	0.21	0.11	0.31	0.36	0.40
	2005	0.15	0.09	0.06	0.06	0.20	-0.01	0.23	0.20
Income / cost	2011	0.26	0.26	0.11	0.15	0.46	0.37	0.38	0.41
	2008	0.11	0.13	0.12	0.09	0.03	0.09	0.17	0.11
	2005	0.24	0.31	0.13	0.11	0.01	0.12	0.02	0.05

magnitude of the correlations is also comparable to their results for annual samples[12]. One part of the explanation for the apparent better performance of DEA may, however, be that a relative large fraction of banks are considered as fully efficient in the DEA estimation, meaning that the probability of keeping that status over time or in a different specification is fairly high. Stability of rankings was higher in the pre-crisis period than during the crisis; a result which calls for higher frequency in the monitoring of efficiency in turbulent periods than in periods with less stress on the financial markets.

5.4 Comparison of efficiency scores and other performance measures

DEA and SFA incorporates more information than traditional performance measures, such as Return On Assets (ROA) and Return On Equity (ROE). The merits of DEA and SFA as performance evaluation tools can therefore not solely be evaluated on the basis of the degree to which efficiency scores are correlated with more traditional measures. On the other hand, a certain degree of correlation is expected and is certainly desired if DEA or SFA should be used for performance evaluation. In addition, when conclusions regarding policy measures are based on performance evaluation by DEA and SFA, results are more consistent if they can be confirmed, at least to some extent, by the use of other performance measures.

Table 5 shows the pairwise correlations between the efficiency scores obtained by DEA and SFA and three accounting based indicators, namely ROE, ROA and the income/cost ratio. The correlations are calculated for 2005, 2008 and 2011 to cover different parts of the business cycle. Results show a quite low correlation between estimated efficiency scores and traditional accounting-based indicators. This finding is in line with previous studies (e.g. Bauer et al., 1998; Fiorentino et al., 2006), and it confirms that efficiency measures contain additional information compared with accounting-based indicators. Also in line with previous studies, we generally find that the correlation between efficiency scores and the cost-based indicator (the income/cost-ratio) is higher than that between efficiency scores and the return-based indicators (ROA and ROE). An exception is the correlations based on all observations for 2011, which indicate that ROE and the income/cost ratio are equally much related to the efficiency scores. Furthermore, the efficiency scores generated by the SFA specification treating deposits as an input exhibit a quite low and in some cases negative correlation with the accounting-based measures, in particular for 2011. The SFA specification treating deposits as an output is more stable regarding its correlation with traditional performance measures. Furthermore, the correlations between the DEA scores and traditional performance measures are in some cases larger than for SFA, but also more varying over time.

Summing up, we have, in line with previous studies, found a quite low correlation between DEA and SFA efficiency scores and more traditional accounting-based performance indicators. One explanation may be the modelling approach and the use of more information than the traditional ratios. However, the efficiency scores generated by the SFA model treating deposits as an input seems to be so unrelated to traditional measures that the results based on this model should be interpreted with care.

5.5 Determinants of the development in efficiency

In order to assess the importance of different bank characteristics for efficiency, we in this section carry out a second stage regression analysis using the estimated efficiency scores as dependent variables[13]. The results are useful in order to understand the development in efficiency over time as presented in section 5.1. As the DEA approach generates a number of efficiency scores with a value of one (i.e. the observations used to construct the efficiency frontier), the second stage analysis of the DEA results makes use of a standard Tobit model[14]. For SFA, we use OLS in the second stage analysis. The second stage analysis is carried out using data from 2005, 2008 and 2011 to cover different parts of the business cycle. The regression results cannot necessarily be interpreted to represent a causal relation between the bank characteristics and efficiency; however, it is still informative to present the results in a regression framework compared to raw correlations.

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[12] Fiorentino et al. (2006) find that stability over time is larger when the basis for the frontier estimation is annual samples, like in this application. Especially, this points to a potential of further developing SFA estimators capable of exploiting the panel structure of micro-data, along the lines of Battese and Coelli (1988).

[13] As noted in section 3.3, the practice of using the estimated efficiency scores in a second stage regression is not entirely satisfactorily as the uncertainty on the efficiency scores is not taken into account in the second stage. Hence, the significance tests in the second stage should be interpreted with this additional uncertainty in mind.

[14] A Tobit model is a regression model suitable for censored dependent variables, i.e. dependent variables which are limited to a certain interval. See, e.g., Wooldridge (2002), p. 519f.

Table 6: Second stage regression results

Dependent variable: Efficiency score estimated by	DEA (I)		DEA (O)		SFA (I)		SFA (O)	
Estimation method (second stage regression)	Tobit		Tobit		OLS		OLS	
2011								
Log of total assets	0.0260	*	0.0258	*	-0.0018		-0.0011	
Business loans as a percentage of total loans	-0.0019		-0.0014		0.0001		0.0003	
Core capital	0.0005		0.0006		0.0000		0.0001	
Interest risk	0.0074		0.0046		-0.0004		0.0004	
Foreign exchange risk	0.2097		0.0739		0.0015		0.0011	
Loans / deposits	0.0001		-0.0005		0.0000		0.0000	
2008								
Log of total assets	0.0350	***	0.0300	***	0.0002		0.0005	
Business loans as a percentage of total loans	0.0041	***	0.0024	*	-0.0001		-0.0002	
Core capital	0.0104	***	0.0095	***	0.0001		0.0000	
Interest risk	0.0076		0.0027		-0.0009		-0.0008	
Foreign exchange risk	0.0854		0.0624		-0.0001		-0.0004	
Loans / deposits	0.0000		-0.0001	**	0.0000		0.0000	
2005								
Log of total assets	0.0465	***	0.0555	***	0.0015	**	0.0021	**
Business loans as a percentage of total loans	0.0003		-0.0002		-0.0005	***	-0.0006	***
Core capital	0.0087	***	0.0083	***	0.0002		0.0001	
Interest risk	0.0052		0.0049		-0.0001		0.0000	
Foreign exchange risk	-0.0129	*	-0.0212	**	0.0005		-0.0001	
Loans / deposits	0.0003		-0.0001		0.0000	**	0.0000	*

Note: Values are coefficient estimates from Tobit and OLS estimations. The dependent variable is the efficiency score obtained by DEA or SFA, treating deposits as inputs (I) or outputs (O). Coefficient estimates are not directly comparable across Tobit and OLS models and for the case of Tobit, coefficients are not to be interpreted as marginal effects. ***, ** and * denotes significance at the 1%, 5% and 10% level, respectively.

Different dimensions of bank characteristics are covered by the analysis[15]. First, the size of the bank is represented by the logarithm of total assets. Second, we represent the bank's business model by loans to non-financial enterprises as a share of total loans. This variable is based on the banks' reporting to Danmarks Nationalbank for the balance sheet statistics. Third, we cover the capitalization of the bank by the core capital ratio as calculated by the Danish Financial Supervisory Authority. Fourth, risk characteristics are included in order to assess whether riskiness of the bank's operations have implications for efficiency. In particular, we include interest risk and foreign exchange risk as calculated by the Danish FSA. And finally, the funding structure of the bank is represented by the share of loans to deposits.

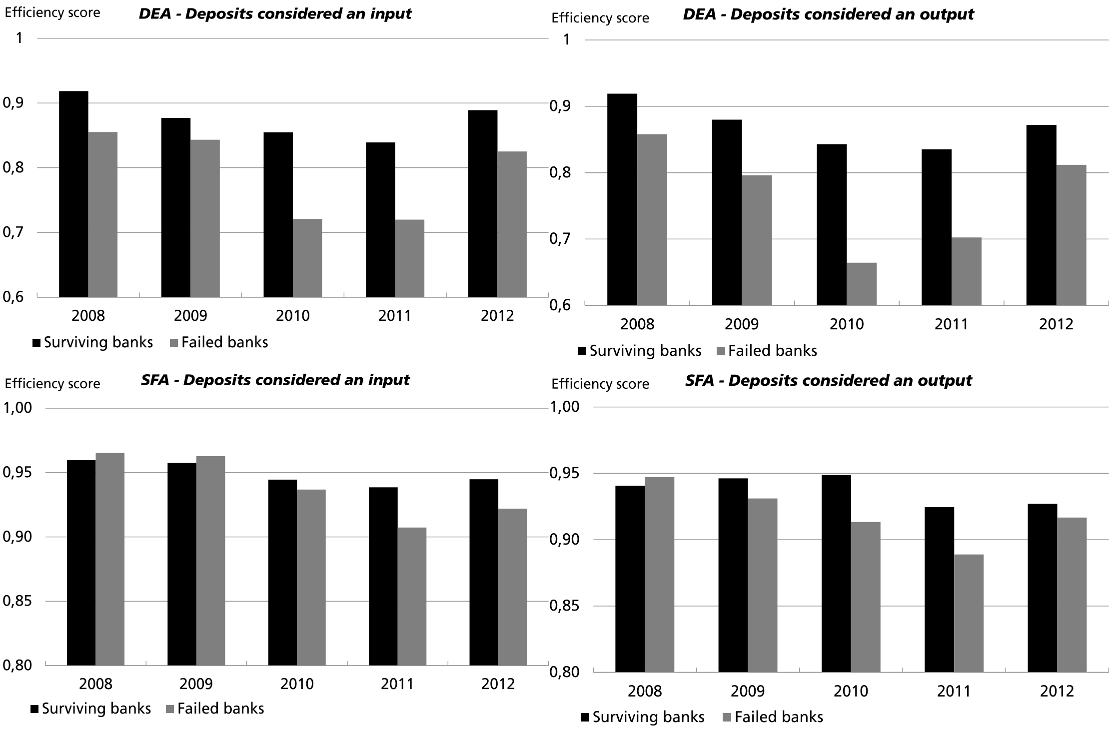
[15] The choice of independent variables is largely directed by the availability of data, as the variables need to be available for all banks.

Results of the second stage analysis are displayed in table 6. Before the crisis, in 2005, the size of the bank is clearly related to efficiency, meaning that the larger banks are on average more efficient. This is true irrespective of whether one considers the DEA or SFA models. In the SFA models, there is also evidence that banks focused less on enterprises are more efficient. According to the DEA models, more capitalized banks are more efficient, and there is also evidence that banks with less risky business models (less foreign exchange risk) are more efficient.

During the crisis in 2008, the results regarding bank size, enterprise focus and capitalization still hold in the case of DEA. In 2011, however, most bank characteristics, with the exception of bank size in the DEA models, are not significantly related to efficiency. One interpretation of this is that the increase in relative efficiency after the crisis has limited the efficiency differences between different types of banks. The general consolidation in the sector, both within and between banks, may have contributed to this development. This would, for example, be the case if the least efficient banks have failed or have been taken over by other banks during the crisis.

To investigate the efficiency characteristics of the banks, which failed during the crisis, figure 5 displays, for each year during and after the crisis, the mean efficiency of banks (in the previous year), which have failed, closed down their business or been taken over by other banks during the year (i.e. banks which do not exist as separate entities in the population at

THE EFFICIENCY OF FAILED BANKS AND SURVIVING BANKS DURING AND AFTER THE CRISIS — Figure 5



Note: The term "failed banks" refers to banks which have defaulted, closed or been taken over by other banks. The year in the figures refer to the year in which the bank leaves the population, and the efficiency scores refers to the end of the previous year.
Source: Own calculations based on data from the Danish Financial Supervisory Authority.

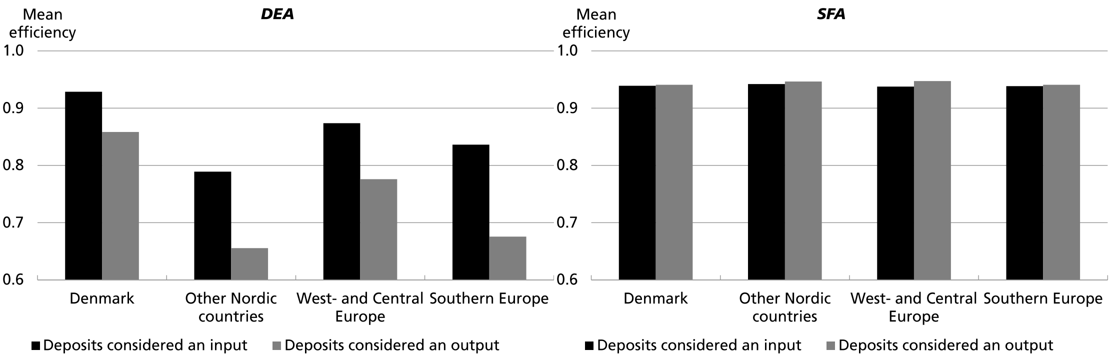
the end of the year, but which existed at the end of the previous year) and the mean efficiency of banks which have not failed or been taken over during the year (survivers). Assessed by all four methods and specifications used throughout the paper, the banks which failed during and after the crisis are on average less efficient than the banks which survived over the same horizon[16]. Overall, the results supports a conclusion that the increase in mean efficiency observed over the past few years (2010-2012) has in fact been supported by the consolidation in the banking sector, as the banks which have failed or been taken over by other banks in general were less efficient than the remaining banks, and as efficiency is not related to specific bank characteristics to the same extent as before the crisis.

5.6 An international benchmarking exercise

The results presented thus far can, strictly speaking, only be informative regarding the relative efficiency of the banks within the Danish banking system. That is, we do not use other information to identify the frontier (i.e. the best practice) than precisely the Danish banks included in the sample. To complete the picture, we in this section carry out a benchmarking exercise of the Danish banks against a range of European banks. Naturally, a wide variety of factors may impact such an analysis. Most importantly, different production possibilities caused by countries being at different parts of the business cycles may play a role. On the other hand, differences in business models and differences in the framework conditions across countries reflect differences in efficiency and should be counted as such, whether or not the banks themselves can influence them.

As noted in section 4, due to data availability, the variables included in the international benchmarking exercise differ somewhat from those included in the analysis based on Danish data. Due to the variation in the number of banks from each country included in the sample,

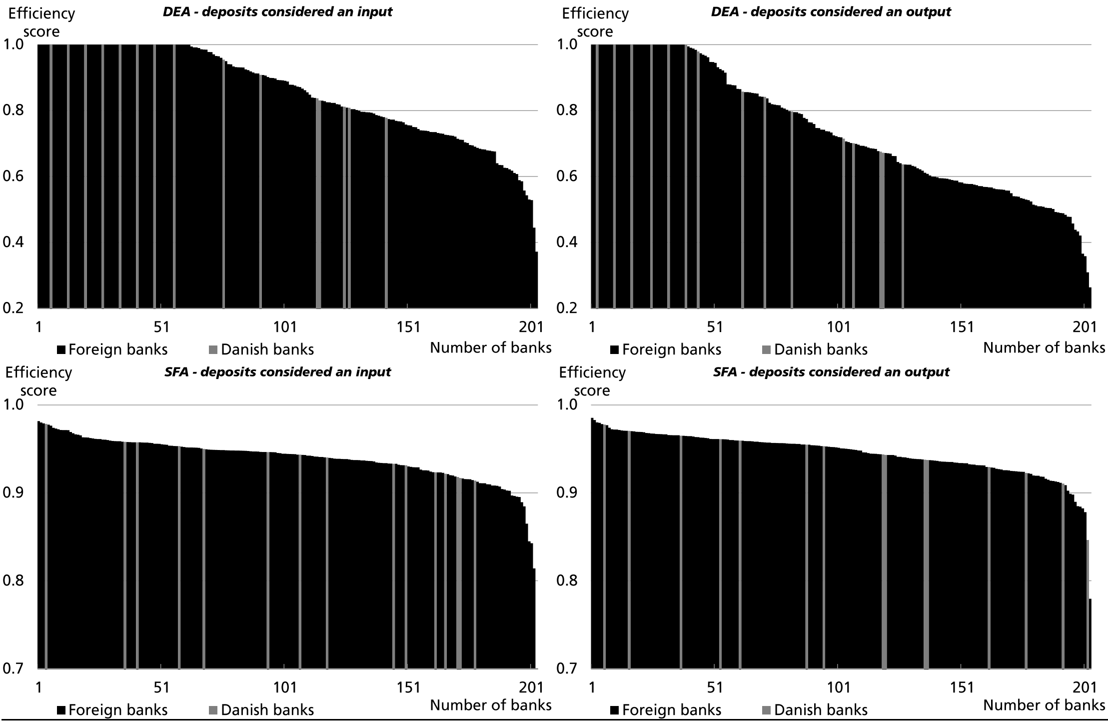
ESTIMATED MEAN EFFICIENCY OF LARGER DANISH BANKS AND BANKS IN DIFFERENT EUROPEAN REGIONS Figure 6



Note: Other Nordic countries refers to Finland, Norway and Sweden. West- and Central Europe refers to Austria, Belgium, France, Germany, Great Britain, Ireland, Luxembourg, Netherlands and Switzerland. Southern Europe refers to Greece, Italy, Portugal and Spain.
Source: Own calculations based on data from the SNL database.

[16] Note that the figure for banks failing in 2008 refers to the efficiency score in 2007 (i.e. before the crisis started in Denmark), which may explain the slightly opposing results for 2008 in the SFA models.

DISTRIBUTION OF EFFICIENCY SCORES FOR DANISH AND EUROPEAN BANKS Figure 7



Note: The figures show the distribution of efficiency scores estimated with DEA and SFA, treating deposits as inputs and outputs, respectively.
Source: Own calculations based on data from the SNL database.

for the purposes of reporting of results, benchmark banks are aggregated in regions and no individual country results are reported. Figure 6 displays the mean efficiency scores for the larger Danish banks included in the analysis as well as for the other banks grouped in European regions. As mean efficiency is not informative on the distribution of efficiency scores, figure 7 depicts the distribution of estimated efficiency scores for the 203 European banks in the sample – with the Danish banks highlighted.

Because of the differences between the efficiency scores and rankings produced by DEA and SFA in this international benchmarking exercise, and because of the relatively low number of Danish observations, we abstain from stronger conclusions regarding the efficiency of Danish banks relative to other European banks, and simply conclude that the larger Danish banks do not seem to be particularly inefficient compared to their European peers. This is comforting also for a possible generalization of the results presented in the previous sections of the paper. Those results were only strictly interpreted as informative of the relative efficiency within the Danish banking sector, but since we in this section has shown that, at least in 2012, the larger Danish banks were internationally comparable in terms of efficiency, the development found using Danish data alone may likely also be interpreted as being informative of the development in 'absolute' efficiency, if such a concept exists.

6. Concluding remarks

This paper has considered the efficiency of the Danish banking sector from different perspectives. Using two state-of-the-art methods for benchmarking analysis, DEA and SFA, we analyse the development in relative efficiency over time. Following this, we proceed with an analysis of the distribution of efficiency with a special focus on the extent to which DEA and SFA produces comparable results. Furthermore, we analyse the stability of rankings over time, the correlation between efficiency scores and traditional accounting based performance measures and the extent to which a range of bank characteristics are related to efficiency. We conclude by an international benchmarking exercise to overcome the challenge that the results based only on Danish banks may not necessarily be interpreted as informative on more than the relative efficiency of Danish banks compared to the best performing Danish banks.

The main findings can be summarized as follows. First, mean relative efficiency increased in the period leading up to the financial crisis beginning in 2008. During the crisis years mean relative efficiency decreased, but recent years have again seen an increase in mean relative efficiency. The increase in efficiency is likely to be the result of a general consolidation in the sector, as banks which failed or were taken over by other banks during the recent years were on average less efficient than the remaining banks, and as efficiency is not to the same extent as before the crisis correlated with specific bank characteristics. Second, efficiency rankings produced by DEA and SFA differ somewhat. This demonstrates the importance of checking the robustness of findings related to efficiency by using different methods. Third, the international benchmarking exercise shows that the larger Danish banks are similar in terms of efficiency to their peers from other European countries, suggesting that the development in mean relative efficiency as found using Danish data may also be informative on the development in absolute efficiency.

A number of issues which deserve attention for future research have emerged from this study. First, a comprehensive international benchmarking exercise of European banks over time could be informative to identify characteristics of efficient banking systems. An analysis of the development of efficiency of banks in different countries over time poses a lot of challenges to be accounted for, such as differences in accounting standards, exchange rates and countries being at different stages of the business cycle. Second, such an analysis, as well as a study like the present, may benefit from the development of dynamic DEA and SFA methods based on panel data. And finally, the importance of taking into account risk characteristics, such as the quality of the loan portfolio, is a very important topic for further research, in order to gain insight into the nature of an observed development in efficiency.

7. References

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