The natural real interest rate in Denmark has declined

The natural real interest rate has declined since the 1990s

The decline is attributable to both structural and cyclical factors and was particularly pronounced during the financial crisis. The natural real interest rate also remains currently low.

Demographic trends pull down interest rates

The natural real interest rate in Denmark is affected by both Danish and foreign conditions. Demographics provide a particularly negative contribution to interest rates, and this is expected to continue in the coming years.

Higher emphasis on other monetary policy tools

Lower natural interest rates have prompted central banks to apply unconventional monetary policies. Spillover effects from similar measures could affect the Danish economy in the future.
The natural real interest rate in Denmark has declined since the 1990s

The trend in the natural real interest rate, referred to as \( r^* \), has implications for the transmission of Danmarks Nationalbank’s interest rates to the Danish economy. The level of \( r^* \) is essential to the monetary policy position, i.e. whether Danmarks Nationalbank’s monetary policy rates stimulate or dampen growth, cf. Box 1.

Hence, a low monetary policy rate will not necessarily stimulate growth in activity. That depends on how low \( r^* \) is. If the real interest rate is lower than \( r^* \), it will stimulate consumption and investment. This will cause economic growth to exceed the structural growth rate and gradually close the output gap, i.e. the difference between actual and potential output, in a recession. Higher monetary policy rates may also still have a stimulating effect as long as the level of the real interest rate remains below \( r^* \).

In Denmark, \( r^* \) has declined since the early 1990s, as shown by an estimation based on Pedersen (2015), cf. Chart 1. The level of \( r^* \) implies a significant degree of estimation uncertainty. The decline in \( r^* \) since the 1990s is a robust result, however, and is commonly found in studies across countries and different estimation techniques, cf. Brand et al. (2018). \( r^* \) dropped and became negative during the financial crisis in the second half of the 2000s. Most recently, \( r^* \) has increased slightly, but it remains substantially below the pre-crisis level. A similar trend is seen in other advanced economies. To some extent, the drop in \( r^* \) can be explained by population ageing and thus a smaller share of the population of working age. This demographic trend can be expected to continue in the near future, so indications are that \( r^* \) will remain low in the coming years.

A low natural real interest rate challenges central banks

While Danmarks Nationalbank meets the objective of stable prices in Denmark via the fixed exchange rate policy, many other central banks use the interest rate to target inflation directly. This applies to the European Central Bank, ECB, among others. In such a
monetary policy regime, the monetary policy rate is adjusted in order to influence economic activity and hence inflation. For these central banks, a combination of three elements may pose challenges to the implementation of monetary policy:

1. Monetary policy rates are subject to a lower bound, reflecting that the interest rate on cash always equals zero. The lower bound is not known, but the negative interest rates in recent years have shown that it is below zero.

2. In response to an economic downturn with low inflation, monetary policy rates must be set so as to ensure that the real interest rate is lower than $r^*$. 

3. There are clear indications that $r^*$ has declined significantly since the mid-1990s, both internationally and in Denmark.

A lower bound for monetary policy rates and a low $r^*$ challenge the ability of monetary policy to stimulate growth in a recession. This is because the lower the $r^*$ is, the more likely the monetary policy rate is to hit a lower bound. The consequence may be more periods during which conventional monetary policy is less able to stimulate the economy. This does not mean that the monetary policy options are exhausted, however. But it does mean that central banks must, to a higher degree, use unconventional monetary policies in their response to new downturns. For example, the ECB has purchased financial assets during the period after the financial crisis.

Changes to monetary policy rates have also contributed to stabilising fluctuations in the Danish economy, cf. Ravn (2012). This is because monetary policy rates in Denmark follow those of the ECB, and Danish cyclical conditions to a large extent mirror those of the euro area. The low level of interest rates has thus supported the upswing after the financial crisis, cf. Jensen and Pedersen (2019) and Danmarks Nationalbank (2019).

A low level for $r^*$ in Denmark could imply that interest rates will have a less stimulating effect on activity in a recession. However, a spillover effect on the Danish economy may still be expected from unconventional measures applied by the ECB. For example, the ECB’s asset purchase programme for government bonds in the euro area has contributed to a decline in long-term interest rates in Denmark, cf. Jensen et al. (2017).

Overall, Danish growth has pulled down the natural real interest rate since the 1990s

The natural real interest rate cannot be observed directly

The major challenges of $r^*$ are that it cannot be observed and that it varies with developments in the economy. This analysis therefore applies the method in Pedersen (2015) to estimate $r^*$ for Denmark. The method uses statistical methods and economic theory to derive $r^*$. The result can be seen in Chart 1.

In the model from Pedersen (2015), $r^*$ can be written as the total effect of the structural growth rate of the Danish economy and other factors that are not captured directly by the model. Because Denmark is a small open economy that is heavily influenced by international factors, it is difficult to distinguish between national and international factors as drivers of the development in Danish $r^*$. Below, we first review the development of the structural growth rate in Denmark, as structural growth is an important factor for the estimated $r^*$ in the model in Pedersen (2015). We then analyse international conditions that could affect Danish $r^*$, but which are not included in the model.

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1 The probability of the monetary policy rate being constrained by a lower bound has not been calculated in the analysis. Other studies based on macroeconomic models indicate that the lower the $r^*$ is, the more frequently the lower bound will bind, see Kiley and Roberts (2017). If conventional monetary policy is unable to stimulate activity due to the combination of a lower effective bound for monetary policy rates and low $r^*$, this represents secular stagnation, cf. Summers (2014).
Decline in structural growth pulls down the natural real interest rate

Structural growth reflects the development in productivity, labour and capital stock. High expected future growth means a stronger incentive to bring forward consumption from the future until today. Therefore, interest rates need to rise to increase the incentive to save today, and thus to ensure equilibrium between saving and consumption. Conversely, lower productivity growth will exert downward pressure on interest rates. At the same time, a smaller labour force reduces the productivity of the capital stock. As interest rates reflect return on capital, a smaller labour force will also reduce the level of interest rates.

Structural growth in Denmark showed a downward trend from the mid-1990s until 2010, cf. Chart 2. This had a negative impact on $r^*$. Since 2010, structural growth in Denmark has risen again. Viewed in isolation, this has contributed to a slight increase in $r^*$.

Most of the decline in the structural growth rate in Denmark since the mid-1990s can be attributed to lower productivity growth, cf. Danielsen et al. (2017). This trend is also seen in other advanced economies, and it is difficult to distinguish the effects from productivity trends in Denmark and abroad. To this should be added a small contribution from the demographic development that has reduced the share of the population of working age.

During the financial crisis, $r^*$ declined substantially relative to structural growth in Denmark. The large deviation from the structural growth development may be attributed to both cyclical and international factors.

The low natural real interest rate reflects international factors to a large extent

The natural real interest rate in Denmark mirrors interest rate developments abroad

Natural real interest rates are currently low in many countries. A recurring result in analyses of advanced economies such as the USA, the euro area, the UK, Canada and Sweden is that $r^*$ has followed a downward trend since the late 1980s. Like Denmark, these countries experienced a strong decline during the financial crisis from 2008.\(^2\)

$r^*$ developments in Denmark and abroad show a close link between $r^*$ in Denmark and interest rate developments in the euro area and the USA, cf. Chart 3 and Box 2. This link reflects similar developments in factors such as structural growth and demographics across advanced economies.

The close link between $r^*$ in Denmark and abroad should also be viewed in the light of spillover effects via free international capital flows whereby capital and savings seek the highest returns. International capital flows imply that variations in return are equalised across economies. Hence, in the longer term, $r^*$ in Denmark can be expected to adapt if it has deviated from developments in $r^*$ in the euro area and the USA for some time. So

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\(^2\) See e.g. Holston et al. (2017), Brand et al. (2018) and Armelius et al. (2018).
in order to analyse further why \( r^* \) in Denmark has declined in recent years, it is necessary to review both national and especially international factors.

**Both national and international factors contribute negatively**

In Denmark, \( r^* \) has declined by approximately 4 percentage points since 1996. Below, we review how much of the decline can be explained by the effect of structural growth in Denmark based on the estimation of \( r^* \) and by a decomposition of other factors, cf. Box 3. The other factors also influence interest rates in Denmark, but are not included in the model in Pedersen (2015) used in this analysis. Due to the close link between \( r^* \) in Denmark and international interest rates, mainly international factors that have affected \( r^* \) developments in Denmark are reviewed below.³

The largest contributions come from three main factors: changes in structural growth in Denmark (0.8 percentage point), international demographics (1.5 percentage points) and saving behaviour in emerging market economies (1.2 percentage points), cf. Chart 4.⁴ These factors can explain more than three fourths of the overall decline in \( r^* \) since 1996. In addition, there has been a small contribution from temporary changes in saving behaviour caused by financial uncertainty.

**Financial unrest may lead to temporary drops in the natural real interest rate**

During the financial crisis and the subsequent debt crisis in some euro area member states, \( r^* \) in Denmark was pulled further down by uncertainty in the international financial markets. Reduced risk appetite boosted demand for safe assets such as government bonds rather than equities. The effect was lower yields on safe assets. Although

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³ Developments in demographics and productivity growth in Denmark coincide with developments in the euro area. As a result, contributions from changes in Denmark and the euro area may be identical. Because Denmark is a small open economy in which interest rates are highly dependent on interest rates abroad, the analysis includes demographics in the euro area. At the same time, the demographic effect from the size of the labour force in Denmark is captured by the contribution from structural growth.

⁴ The size of contributions from changes in structural growth, demographics and saving behaviour in emerging market economies to \( r^* \) since the 1990s is close to estimates from studies of other countries, cf. the literature review in Brand et al. (2018) and Pescatori and Turunen (2015). For a discussion of factors that may give rise to changes in \( r^* \), see also Woodford (2003).
the effects on $r^*$ of the financial crisis and the debt crisis in a number of euro area member states are temporary and likely to disappear in the longer term, the decline in risk appetite may persist for some time. For example, households in Denmark have continued to consolidate for many years after the crisis, cf. Mortensen et al. (2017) and Hviid and Kuchler (2017).

**Higher savings in emerging market economies push down the natural real interest rate**

Other changes in international saving behaviour affecting $r^*$ in Denmark are of a more sustained nature than financial uncertainty. In some of the emerging market economies, savings are typically placed in bonds from advanced economies, including Denmark. The sustained increase in demand for bonds in those countries has pushed down interest rates. Growing demand for savings in foreign currency is reflected in e.g. the accumulation of substantial foreign exchange reserves in emerging market economies, cf. Chart 5.

Higher demand in emerging market economies – especially China – for savings in foreign bonds should be viewed in the context of the economic and demographic developments of those countries. Demand for private pension and insurance savings has grown due to a combination of three factors: a higher income level, increased life expectancy and a low level of public social security.

**International demographics contribute negatively to the natural real interest rate**

Interest rates in Denmark are strongly affected by demographic trends abroad, particularly in the euro area, cf. Chart 6.

$r^*$ in Denmark is influenced by three effects of demographic trends in the euro area. (1) A smaller labour force in the euro area means lower returns on the capital stock. The level of interest rates reflects the return on capital, and the level of interest rates in the euro area is reduced, which transmits to $r^*$ in Denmark. (2) At the same time, longer life expectancy will increase the incentive for pension saving, which has a negative effect on interest rates as it boosts demand for savings. (3) Conversely, a higher percentage of pensioners means lower savings. All else equal, this will increase $r^*$. When the effects are added up, demographic changes in the euro area have influenced $r^*$ in Denmark negatively.
Studies of other economies indicate that demographic trends in recent years have had a negative effect on \( r^* \) of the same magnitude as in Denmark.\(^5\)

Projections show that the trend with a lower share of the population of working age is likely to continue, both in Denmark and abroad. Accordingly, \( r^* \) in Denmark can be expected to remain at a low level in the coming years and perhaps decline even further. Studies based on other economies indicate a further expected decline in \( r^* \) of between 0.25 and 0.50 percentage points towards 2030, cf. e.g. Papetti (2018) and Bielecki et al. (2018).

**A new normal for monetary policy?**

The situation with a low \( r^* \) may be regarded as a new normal for monetary policy in central banks with an independent monetary policy, including the ECB and the Fed, cf. e.g. Bernanke (2017) and Brainard (2017). All else equal, the implication of a low \( r^* \) is that monetary policy rates must be lower than previously to stimulate the real economy. At the same time, a lower bound for monetary policy rates could present a challenge to central banks attempting to stimulate the economy through conventional monetary policy. An increased probability of hitting the lower bound for monetary policy rates implies that unconventional monetary policy instruments such as asset purchases could become conventional. In other words, purchases of financial assets and large central bank balance sheets may be here to stay. In the future, unconventional monetary policy may also have an impact on economic developments in Denmark through the spillover effect from the ECB’s monetary policy.

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\(^5\) Carvalho et al. (2016) find that a decline in \( r^* \) of approximately 1.5 percentage points from 1990 to 2014 can be attributed to the demographics of developed economies. Similar results are found in Eggertsson et al. (2017) and Gagnon et al. (2016) for the USA.
Adjustment of the Danish natural real interest rate to the natural real interest rates in the euro area and the USA

The analysis uses a cointegration analysis to illustrate that \( r^* \) developments in Denmark mirror changes in \( r^* \) in the euro area and the USA. In other words, that changes in Danish \( r^* \) to a great extent reflect international conditions or similar conditions across countries. The point of departure is an Engle-Granger two-step procedure inspired by Armelius et al. (2018), who analyse \( r^* \) in Sweden. If the natural real interest rates across countries cointegrate, this means that they follow a common stochastic trend. In an ADF test, it cannot be ruled out that \( r^*_{DK}, r^*_{EA} \) and \( r^*_{US} \) each contain a unit root and follow a random walk, cf. the table below:

<table>
<thead>
<tr>
<th>Variable</th>
<th>0 lags</th>
<th>1 lag</th>
<th>2 lags</th>
</tr>
</thead>
<tbody>
<tr>
<td>( r^*_{DK} )</td>
<td>-0.80</td>
<td>-0.83</td>
<td>-0.86</td>
</tr>
<tr>
<td>( r^*_{EA} )</td>
<td>-0.78</td>
<td>-1.14</td>
<td>-1.26</td>
</tr>
<tr>
<td>( r^*_{US} )</td>
<td>-0.40</td>
<td>-0.48</td>
<td>-0.43</td>
</tr>
</tbody>
</table>

Critical value: -2.89

The first step of the Engle-Granger procedure is to set up an equilibrium relation between the Danish \( r^* \) and \( r^* \) for the euro area and the USA, respectively, which is estimated by OLS:

\[
r^*_{DK} = \beta_0 + \beta_{EA} r^*_{EA} + \beta_{US} r^*_{US} + \epsilon_t
\]

If the residual of the equilibrium relation, \( \epsilon_t \), is stationary, this is an indication that interest rates follow a common stochastic trend that disappears in the equilibrium relation. In an ADF test, it is rejected that \( \epsilon_t \) is non-stationary, cf. the table below:

<table>
<thead>
<tr>
<th>Variable</th>
<th>ADF t value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \epsilon_t )</td>
<td>-3.66*</td>
</tr>
</tbody>
</table>

Critical value: -3.74

A stationary residual indicates that a cointegration relation – in other words, an equilibrium relation – exists between \( r^*_{DK}, r^*_{EA} \) and \( r^*_{US} \). The next step of the Engle-Granger procedure is to set up an error correction model for the Danish \( r^* \). As \( \epsilon_t \) seems to follow an AR(1) process, \( \epsilon_t \) can be rewritten to the following error correction model:

\[
\Delta r^*_{DK} = \mu + \theta_{EA} \Delta r^*_{EA} + \theta_{US} \Delta r^*_{US} + \alpha \epsilon_{t-1} + u_t
\]

The model estimates the adjustment of \( \Delta r^*_{DK} \) towards equilibrium when it deviates from the equilibrium relation to \( r^*_{EA} \) and \( r^*_{US} \), i.e. an error correction term, \( \alpha \), which is significantly negative, cf. the table below. The significantly negative error correction term indicates that the Danish \( r^* \) will revert towards the equilibrium relation with \( r^* \) in the euro area and the USA in case of deviations. The half-life period for the fluctuations is approximately 1.5 years. In the short term, \( r^*_{DK} \) is affected only by changes in \( r^*_{EA} \) within the same quarter. 32 per cent of a change in \( r^* \) in the euro area will transmit to Denmark within the same quarter.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Robust standard errors</th>
<th>P value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \mu )</td>
<td>-0.04*</td>
<td>0.02</td>
<td>0.07</td>
</tr>
<tr>
<td>( \Delta r^*_{EA} )</td>
<td>0.52**</td>
<td>0.13</td>
<td>0.02</td>
</tr>
<tr>
<td>( \Delta r^*_{US} )</td>
<td>0.18</td>
<td>0.11</td>
<td>0.11</td>
</tr>
<tr>
<td>( \epsilon_{t-1} )</td>
<td>-0.10**</td>
<td>0.05</td>
<td>0.03</td>
</tr>
</tbody>
</table>

Observations: 94

\[ R^2 \] 0.19

1 Quarterly data from 1995 until the 3rd quarter of 2018 is applied. ** denotes significance at the 5 per cent level and * at the 10 per cent level. Natural real interest rates are from Holston et al. (2017) and own estimation. The results from the estimation of the error correction model are shown with standard errors that are robust to autocorrelation and heteroscedasticity. The results are robust to the use of the maximum likelihood estimator instead of the OLS estimator.
Decomposition of changes in the natural real interest rate

\( r^* \) is estimated using the method of Pedersen (2015), where \( r^* \) is a function of two terms: \( g_e \), which is the structural growth rate of the economy, and \( Z_e \), which captures information that is not contained in the model:

\[
r_i^* = g_e + Z_e
\]

This estimation provides information about contributions from the structural growth rate in Denmark to \( r^* \), based e.g. on developments in monetary policy rates, real activity and inflation in a simple macroeconomic model. The model does not include information on the development in \( Z_e \), as that is only a residual in Pedersen (2015).

In this analysis, we seek an explanation for the development in \( Z_e \) in the above estimation to better understand developments in \( r^* \). We find drivers behind the estimated development in \( Z_e \) by including information from outside the model, inspired by the study of \( r^* \) in the USA by Pescatori and Turunen (2015), and by the review of the literature on \( r^* \) by Brand et al. (2018).

In the estimation of \( r^*, Z_e \) is assumed to follow a stochastic process with a unit root, \( Z_e = Z_{e,-1} + \varepsilon_e \). As \( Z_{e,-1} \) is I(1), contributions to changes in \( Z_e \), \( \Delta Z_e \), are estimated. There are also indications that the explanatory variables of the model are non-stationary. Therefore, the explanatory variables are also transformed into first differences. When \( \Delta Z_e \) is included as a dependent variable, this is equivalent to including \( \varepsilon_e \) as a dependent variable. This means that the model estimates shocks to \( Z_e \), which are designed to have a mean value of 0. Accordingly, a constant is excluded from the regression model (statistical tests have been performed, showing that \( \varepsilon_e \) has a mean value of 0, and a constant is also included insignificantly in the model):

\[
\Delta Z_e = \beta_1 \Delta Z_{e,-1} + \beta_2 \Delta E_A.LABFORCE_i + \beta_3 \Delta E_A.OLD_i + \beta_4 \Delta E_ME.RES_i + \beta_5 \Delta U_S.VOLA_i + \nu_i
\]

\( \Delta Z_{e,-1} \) is included to avoid autocorrelation in the residuals and to achieve a model that meets model specification tests. \( E_A.LABFORCE \), is the share of the population of working age (15-64 years) in the euro area. \( E_A.OLD_i \) is the share of the population of retirement age (65+ years) in the euro area. The demographic structure is included as it affects return on capital and the total share of savers in the economy. \( E_ME.RES_i \) is foreign reserves as a percentage of US GDP in emerging market economies (here approximated as Brazil, China, Hong Kong, India, Russia, Singapore, South Korea and Taiwan) and is included as a measure of increased demand for savings in emerging market economies. \( U_S.VOLA_i \) is the deviation from the average daily volatility in the S&P 500 index over the last two years and represents international stock market uncertainty. Uncertainty is included as the general level of financial uncertainty affects demand for safe assets. It is difficult to distinguish contributions from demographic changes in the euro area and international financial uncertainty from demographic changes and financial uncertainty in Denmark, since these variables show almost identical trends. However, because Denmark is a small open economy where interest rates are heavily influenced by international interest rates, cf. the results in Box 2, the analysis focuses exclusively on international variables. Standard errors that are robust to autocorrelation and heteroscedasticity are applied. The results of the estimation are shown in the table below:

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Robust standard errors</th>
<th>P value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Delta Z_{e,-1} )</td>
<td>0.26***</td>
<td>0.07</td>
<td>0.001</td>
</tr>
<tr>
<td>( \Delta E_A.LABFORCE )</td>
<td>2.33**</td>
<td>0.78</td>
<td>0.004</td>
</tr>
<tr>
<td>( \Delta E_A.OLD )</td>
<td>1.11**</td>
<td>0.54</td>
<td>0.04</td>
</tr>
<tr>
<td>( \Delta E_ME.RES )</td>
<td>-0.03**</td>
<td>0.01</td>
<td>0.02</td>
</tr>
<tr>
<td>( \Delta U_S.VOLA )</td>
<td>-0.03***</td>
<td>0.01</td>
<td>0.001</td>
</tr>
<tr>
<td>Observations</td>
<td>87</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Log likelihood</td>
<td>55.6</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

1. Quarterly data from 1996 to 2017 is applied. *** denotes significance at the 1 per cent level, ** at the 5 per cent level and * at the 10 per cent level. Demographic variables that are available on an annual basis only have been linearly interpolated. The differences have then been calculated as 5-quarter moving averages to prevent the estimation from being affected by variation caused by the data interpolation. Data is from the European Commission, Eurostat, Thomson Reuters Datastream and own estimation. The results are robust to the use of the maximum likelihood estimator instead of the OLS estimator.
Decomposition of changes in the natural real interest rate

Box 3
Continued

\( Z_t \) is positively affected by the size of the labour force in the euro area. A smaller labour force reduces the marginal product of capital and thus interest rates. Moreover, longer life expectancy, leading to a smaller share of people of working age relative to pensioners, also increases the incentive for pension saving. On the other hand, a smaller share of savers will exert upward pressure on interest rates. So all else equal, the effect of the share of pensioners is positive.

Higher savings in foreign currency in emerging market economies put downward pressure on \( Z_t \) by increasing demand for bonds in the advanced economies. To this should be added temporary changes in saving behaviour due to international financial uncertainty.

Literature


