DANMARKS NATIONALBANK

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Expansions do not end due to old age

Large variations in length of expansions

Viewed across 19 OECD countries over the period 1970-2018, the length of expansions varies a lot. The majority last between three and nine years, and the most recent Danish expansion lasted for a little more than ten years. Only about 20 per cent of all expansions last longer.

Read more

A global downturn increases the risk of a long recession

The corona pandemic means that a very large proportion of the world's countries are in recession at the same time. This increases the risk of a prolonged and deep recession in Denmark, even though the Danish economy, seen in isolation, is robust and healthy.

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The pandemic hits an economy with imbalances harder

The absence of significant imbalances prior to the corona pandemic may mitigate the downturn. A balanced economy means that Danish households and businesses are better equipped to withstand the crisis.

Read more

> Historically, most economic expansions fade after this long. How confident are you that our economy won't slip back into recession in the near term?
>
> "Expansions do not die of old age.
>
> *Janet Yellen, Chair of the Federal Reserve, in 2015. The question was asked to Janet Yellen at a 2015 press conference.*

At the time the world was hit by the corona pandemic, the expansion had been extraordinarily long in many countries. The US economy, for example, had been growing continuously for more than a decade, making it the longest expansion in the US since the 1850s when the recording of cyclical fluctuations began.[1] In Denmark too, the expansion had lasted for more than ten years.

Economic growth is rarely this smooth and stable over long periods of time. History shows that periods of growth and good job opportunities are sometimes replaced by shorter periods characterised by falls in production and employment. Such fluctuations are referred to as business cycles where expansions give way to recession. A full business cycle is generally estimated to last between three and nine years.

Large fluctuations in economic activity are costly and harmful to welfare. One example of this is that unemployment rises during recessions, which may have a great impact on the households concerned. In addition, economic uncertainty impedes the investment appetite, resulting in reduced innovation and growth. An important objective of economic policy, including monetary policy, is thus to stabilise the economic development to reduce cyclical fluctuations.

In this analysis, we first look at how different economic conditions and events affect the risk of an expansion ending and a recession beginning. Among other things, we take a closer look at the importance of excess lending and housing bubbles and whether expansions die of old age or whether they are like Peter Pan, meaning that they never grow old, and die only if something unexpectedly hits them – what economists refer to as 'exogenous shocks' or *black swans*.[2] A current example of an exogenous shock is the outbreak of the corona pandemic. Secondly, we examine how quickly economies recover, i.e. what factors impact the length of recessions.

First of all, however, we need to clarify what is meant by 'expansion' and 'recession' in this analysis.

Expansions and recessions are defined on the basis of turning points in economic activity

There is no clear definition of business cycles. We use the 'classic' definition, which is based on turning points in a country's total production, i.e. GDP.[3] When analysing the length of business cycles, it is not expedient to apply, for example, fluctuations in output gaps, which otherwise constitute important input to economic policy, see Box 1.

Periods from a trough to a peak are referred to as an expansion, while periods from a peak to a trough are defined as a recession. In order to avoid small arbitrary falls in GDP being defined as a recession, we add the criteria of rising unemployment during a recession. The method of using turning points to

1 The longest expansion up until then lasted ten years, from March 1991 to March 2001.

2 The theory of black swans is a metaphor describing an event that comes as a surprise, has a major impact and which is *subsequently* most often explained by information available before the event occurred. Nassim Nicholas Taleb is the person most often credited with having introduced the concept in economics, see, for example, Taleb (2010).

3 See, for example, OECD (*link*).

determine expansions and recessions is applied, for example, by the National Bureau of Economic Research, NBER, which, however, applies an even wider set of indicators than just GDP and unemployment.

Turning points are identified across several OECD countries using an algorithm often described in literature.[4] A recession is characterised by at least two consecutive quarters of negative growth and rising unemployment. When applying this definition, both expansions and recessions may, in principle, be arbitrarily long. This is important when we want to examine whether expansions turn into recessions as the expansion ages.

Large variations in length of expansions

Looking at the period 1970-2019 across a number of OECD countries reveals both short and very long expansions, see Chart 1. Approximately 20 per cent of the expansions last longer than ten years. Recessions are typically quite short, and the vast majority do not last more than two years. The length of a normal business cycle, i.e. an expansion followed by a recession, is just below eight years.

Economic imbalances increase the risk of expansions turning into recessions

To analyse the length of expansions and recessions, we estimate the risk of a recession occurring based on the length of the expansion, i.e. what is the risk of recession if the expansion has lasted for, say, 20 quarters? More specifically, we estimate so-called 'hazard rates', i.e. conditional survival probabilities, which is a key concept in survival analysis, see Appendix 2 for a description of the statistical method.

4 The algorithm specifies minimum lengths for expansion, recession and overall cycle. The algorithm has, for instance, been applied in studies prepared by Bank for International Settlements, BIS, see Drehmann (2012), other studies prepared by Danmarks Nationalbank, Grinderslev et al (2017), IMF, see WEO (2010) and Claessens et al (2008), and the Federal Reserve, see Berge and Pfajfar (2019).

Growth cycles and output gaps

Box 1

Business cycles can be defined as fluctuations in economic activity around a potential level, i.e. fluctuations in the output gap. Such cycles are called growth cycles. Output gap is a key concept in the macroeconomic debate. It indicates whether the economic activity is greater or smaller than the underlying structural level, i.e. a level of economic activity with normal capacity utilisation.[1] This constitutes important input to the implementation of economic policy and forms the basis for many policy recommendations, for example regarding whether the economic policy should be tightened or eased.

At first glance, it would seem natural to examine the length of business cycles on the basis of output gaps, i.e. to estimate the risk of economic activity moving below the structural level. However, this is not appropriate for this analysis. To make a very rough approximation, the structural production is estimated as a moving average. If the growth level is high for a long period of time, the estimate for the structural production will increase gradually, and vice versa. This gives you, per construction, cycles that cannot be arbitrarily long. And, by definition, this entails that positive output gaps – activity above the potential – are followed by an economic downturn and production below the potential. This implies, per construction, that the risk of recession increases, the older the expansion is. In this analysis, we therefore apply definitions of expansion and recession that do not result in business cycles of a length which is, to a large extent, determined mechanically by the statistical techniques used.

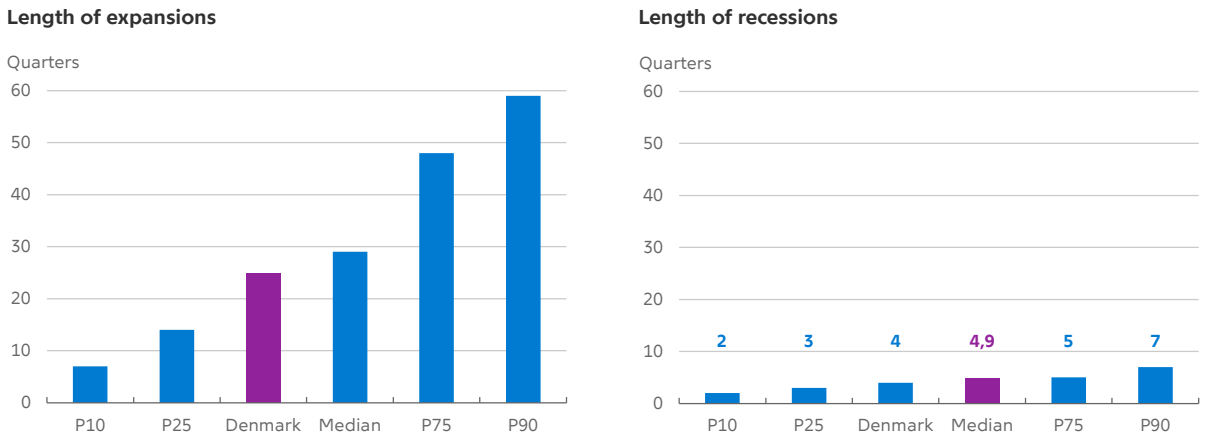
1. See Danielsen and Jørgensen (2015) for a more detailed description of Danmarks Nationalbank's estimation of output gaps.

What is a *hazard rate*?

The hazard rate measures the risk of an expansion dying, i.e. the risk that the economy goes into recession during the period from time t to t+1, provided that the expansion has lasted up until time t. A hazard rate is thus a conditional probability. A flat hazard rate means that the risk of going into recession does not increase in step with the length of the expansion. Conversely, an increasing hazard rate means that the longer an expansion has lasted, the higher the risk is of a recession occurring. See Appendix 2 for a more detailed description of hazard rates and the model applied in the analysis.

The length of business cycles is within a wide range

Chart 1



Note: Expansions are defined as periods during which the economy is not in recession. See the appendix for a detailed description of the dating method and underlying data.
Source: Macrobond and own calculations.

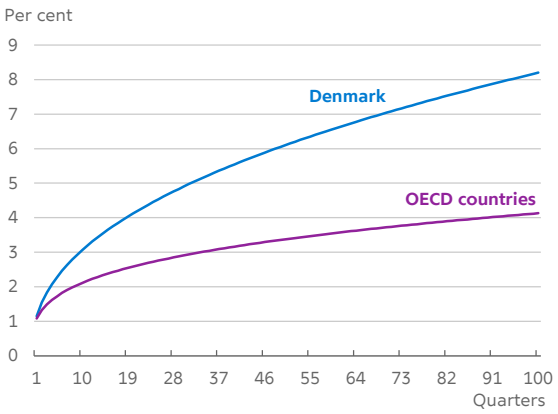
A simple hazard rate is conditional on time only. It is, however, also possible to estimate the risk of recession conditional on the build-up of economic imbalances, e.g. housing bubbles, as well as external events such as strong increases in energy prices or large declines in share prices. This will be addressed later in this analysis. First, we look at the scenario where the hazard rate is only conditional on time.

The risk of an expansion turning into a recession increases slightly in step with the length of the expansion

Fortunately, recessions are relatively rare. This means that solid results cannot be obtained by analysing one country only. We therefore estimate the hazard rate on a wider set of OECD countries and find that it increases slightly over time, see Chart 2.[5] According to the estimate, the expected median residual lifetime of a five-year old expansion is four and a quarter years, while it is three and a half years for a ten-year old expansion.[6]

The risk of recession increases slightly in step with the length of the expansion

Chart 2



Note: See Appendix 2 for a description of the calculation of hazard rates.
Source: Own calculations.

5 The estimate for the parameter determining the slope is approximately 1.4. Similar estimates have been found in several other studies, e.g. Beaudry and Portier (2019). Conversely, Rudebusch (2016) estimates that the hazard rate in the US is flat. However, this conclusion is based solely on US data after World War II, meaning that the confidence interval for the estimate is quite wide.

6 A median residual lifetime is specified. This applies to the rest of the analysis.

The hazard rate is slightly higher for Denmark. This is partly due to the fact that we have had a few more recessions than other OECD countries since 1970. In addition, the slope is also steeper. However, the uncertainty associated with the Danish hazard rate is significantly greater as the estimate is only based on seven recessions. Statistically speaking, the two curves are not identical, but the differences are small: The slope of the hazard rate for Denmark is approximately 1.4, while it is approximately 1.3 for the other countries for which data is available.

Several factors may increase the risk of an expansion turning into a recession

In order to examine why the risk of recession increases slightly over time, we take a closer look at a number of factors which are considered to affect the length of expansions. They can be divided into three groups:

- *Imbalances*, e.g. credit and/or housing bubbles. Imbalances are expected to shorten expansions and extend recessions.

- *The starting point*, e.g. the capacity available when the expansion sets in. For example, if there is much capacity available, the expansion is expected to last longer.

- *Shocks*, e.g. major changes in oil prices, recessions in other countries and more or less random events such as the outbreak of a pandemic, including coronavirus. Such shocks are also expected to shorten expansions.

Imbalances may arise if households, businesses and perhaps also public authorities become over-optimistic. Households may, for example, buy expensive homes and raise extra loans in anticipation of an increase in their wages or the value of their home. The risk of such over-optimism occurring is likely to increase the longer an expansion has lasted, thus increasing the risk of recession in step with the length of the expansion. This supports the fact that the build-up of imbalances is key to understanding why the risk of recession increases slightly over time. Some exogenous shocks such as strong changes in oil prices or the outbreak of pandemics like the coronavirus are, on the other hand, probably more or less independent of time. However, the risk of recessions in other countries – which are also defined as exogenous shocks – may increase over time if, for example, such recessions are the result of imbalances.

To assess the importance of the various factors, we estimate hazard rates where the risk of recession at any given time is conditional on the individual factors, see Appendix 3 for a precise description of the model.[7]

Significant imbalances increase the risk of recession

An important objective of economic policy is to avoid major fluctuations in the economy, such as a strong fall in employment. Most central banks have set up this objective, either as an explicit objective or indirectly via an

Imbalance indicator based on four variables Box 2

The imbalance indicator is based on data regarding unemployment, housing prices, total lending to households and non-financial corporations as well as the balance of payments.

For the first three variables, an imbalance is defined as the difference between a trend estimated with an HP filter and the actual development. The HP filter is run with a parameter set such that if the filter had been run on GDP, it would produce the same business cycles as those found using the algorithm described previously in the text. The balance of payments is stated in per cent of GDP, and an imbalance reflects a deficit. The measures of imbalances are relatively simple, but easy to estimate. Other measures, such as financial cycles, have not been used, as they are not readily available across countries.

All variables correlate greatly. Consequently, if all variables were introduced simultaneously in the econometric model, the model would suffer from multicollinearity, resulting in unstable coefficients. Therefore, an overall measure of imbalances is prepared as the first component of a principal component analysis. The remaining principal components do not significantly affect the length of expansions.

7 It should be noted that pandemic outbreaks such as the corona pandemic cannot be used as a condition, since the last major pandemic was the Spanish flu in the 1920s, where we do not have data for other variables such as GDP.

objective of stable prices and a stable financial system. To assess how imbalances affect the risk of recession, we estimate hazard rates based on imbalances. More specifically, we set up an imbalance indicator based on house prices, lending, unemployment and the balance of payments, see Box 2.

To interpret the results, we compare the indicator in Denmark prior to the financial crisis with a balanced economy, i.e. where the imbalance indicator is zero, see Chart 3.

Significant imbalances in the economy shift the entire hazard function upwards. This means that the risk of the expansion turning into a recession increases, regardless of how long the expansion has lasted. Furthermore, the hazard function becomes steeper. This shows that the risk of recession increases faster over time for an unbalanced economy compared to a balanced economy.

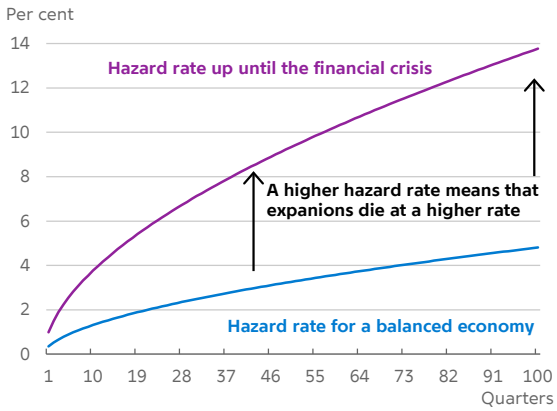
For an expansion that has lasted five years, the risk of recession increases in the next quarter from around 2 per cent if no imbalances exist to around 6 per cent in case of imbalances of a size similar to those of the pre-financial crisis period. This corresponds to the expected median residual lifetime of a five-year old expansion falling from eight years where no imbalances exist to just three years in case of significant imbalances.

Another way of illustrating the importance of imbalances is by way of survival probabilities. If no imbalances exist, about half of all expansions will last longer than 40 quarters. If, on the other hand, there are imbalances as large as those seen in the period leading up to the financial crisis, they will only survive for about 20 quarters, see Chart 4.

A balanced Danish economy before the outbreak of the corona pandemic shortens the downturn

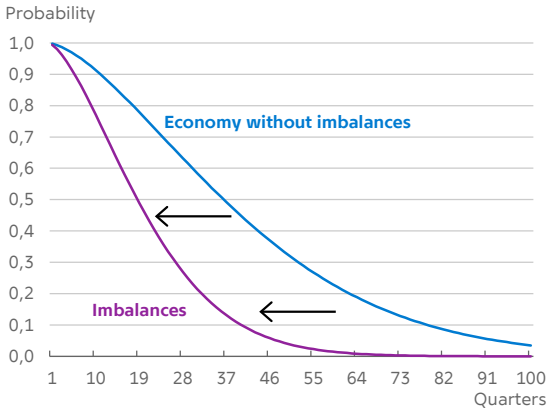
Imbalances thus significantly increase the risk of recession. At the same time, the model indicates that imbalances render economies less resilient to negative shocks to the economy. Such shocks could be a sharp fall in share prices or a global recession as a result of the corona pandemic. Let us look at an example: A 15 per cent drop in share prices hits an economy during an expansion that has lasted five years. In a balanced economy, this shock would cause the hazard rate to increase by 2 percentage points less than if it had hit an unbalanced economy.

Imbalances reduce length of expansions Chart 3



Note: The chart cannot be directly compared to Chart 1, partly because this chart takes country-specific conditions into account, i.e. fixed effects have been included in the country-level estimation, and partly because imbalances are evaluated at different levels. See Appendix 2 for a more detailed description of the estimation.
Source: Own calculations.

Expansion less likely to continue in an unbalanced economy Chart 4



Note: The chart shows the probability of an expansion continuing over time. The blue line reflects a situation with no imbalances in the economy. This is compared to a situation with imbalances similar to those seen in Denmark prior to the financial crisis (purple line).
Source: Own calculations.

The results show that central banks' strong focus on identifying and avoiding the build-up of imbalances is justified. If imbalances are avoided, expansions will typically last significantly longer. The results also indicate that if the Danish economy had been unbalanced, as was the case before the financial crisis, the current recession in the Danish economy triggered by the corona pandemic would be even deeper.

Stable prices extend expansions

A key objective for central banks is to ensure a low, stable inflation as high inflation is typically an indication of the economy overheating, for instance where unemployment is very low and wages are rising rapidly. In Denmark, an accelerated wage growth may reduce competitiveness, resulting in a slowdown in overall growth. Inflation can thus be seen as an indicator of an imbalance in, for example, the labour market.

We therefore estimate how inflation affects the risk of recession. The results indicate that higher inflation shortens expansions, see Chart 5. If, for example, inflation increases by 1 percentage point above the central banks' inflation target of typically 2 to 3 per cent, the risk of recession in the next quarter increases by approximately 30 per cent for an expansion that has lasted for five years, and the median residual lifetime decreases by about two years.

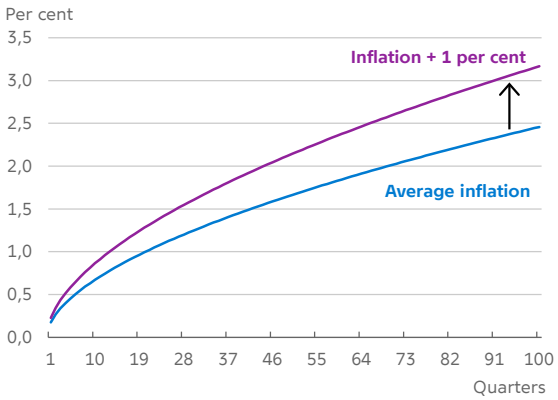
High inflation, as was seen in the 1970s, thus leads to a less stable economic development. It illustrates the importance of central banks ensuring low and stable inflation.

It should be noted that the model is not suitable for assessing the impact of inflation below the inflation target, as is currently the case. Inflation below the target will probably not reduce the risk of recession, as implied by the model at first glance.[8]

Slow expansion after severe recession

Other analyses have pointed out that restoring an economy which has fallen far below its potential

Higher inflation is an indication of an increased risk of recession Chart 5



Note: See Appendix 2 for a more detailed description of the estimation.
Source: Own calculations.

may take a long time.[9] For example, the current long expansions in the US and Denmark before the corona pandemic might have been caused by the fact that the preceding recession, i.e. the financial crisis, was one of the worst in history, with the economy therefore being far below its potential.

A hazard rate estimation that is conditional on the amount of available resources at the beginning of the expansion shows that expansions are typically longer if the preceding recession was severe. Consequently, an expansion following a crisis similar to the financial crisis in Denmark can be expected to be about two years longer than in the case of a mild recession.[10]

Global recessions imply national recessions

A solid economic policy and a healthy economy reduce the risk of expansions turning into recessions, but do not completely eliminate the risk of recessions. Economies risk being hit by external events that may impact economic activity. Such events or external shocks can-

8 In order to assess the impact of inflation below the target, we would have to introduce the concept of nonlinearity. We have, however, chosen to disregard this to avoid complicating the model further. Inflation was generally higher in the 1970s than today. Therefore, the correlation may perhaps to a higher extent capture the relationship between the higher inflation at the beginning of the data period and shorter expansions. Since the 1970s, the central banks' increased focus on fighting high and volatile inflation has contributed to a more stable development in inflation and in the economy.

9 This theory is called the *Plucking Theory*, cf. Tasci and Zevanove (2019).

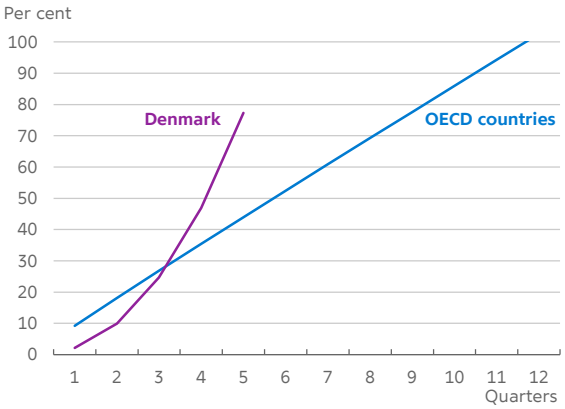
10 The impact is, however, small. One of the reasons is that random events can cause a recession even though the economy is still recovering from the preceding recession. The results are not driven by the financial crisis. Disregarding the financial crisis, the slope of the hazard rate falls from about 1.56 to about 1.46. The available resources coefficient increases slightly from -0.147 to -0.17, while the statistical significance decreases.

not be avoided and are difficult to predict. An example of such a shock is the corona pandemic. However, as mentioned earlier, the negative impact of such shocks on the economy is smaller in a balanced economy.

In the model, we include, among other things, share and oil prices as well as a proportion of other countries in expansion. Only share prices and the proportion of other countries in expansion are significant. Both coefficients are negative/positve as expected. For example, substantial falls in share prices cause the hazard rate to increase, while a higher proportion of countries in expansion reduces the risk of recession, see Table 1 of Appendix 3.

The most important factor for the duration of an expansion is macroeconomic developments in trading partner countries. If the majority of other countries are in recession, the risk of the Danish expansion turning into a recession thus increases considerably.

Recessions are short, and the probability of an expansion setting in increases significantly over time Chart 6



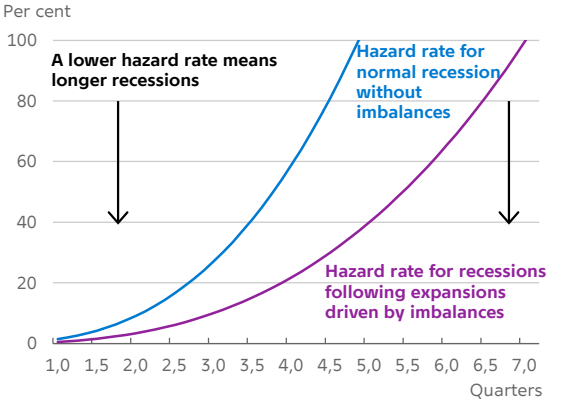
Note: See Appendix 2 for a more detailed description of the estimation.
Source: Own calculations.

Longer recessions following expansions driven by imbalances

Fortunately, recessions are relatively short, typically lasting about one year. The probability of a recession ending and an expansion setting in increases significantly over time, see Chart 6. In fact, the rate at which a recession turns into an expansion increases progressively over time – recessions die of old age. This indicates that the economies included in the data used are typically relatively flexible and thus able to quickly adapt to changing economic conditions. It may also reflect the fact that when negative events have pushed economies into recession, central banks have eased their monetary policy stances and the fiscal policy has stimulated demand.

Recessions occurring in the context of significant imbalances are, however, significantly longer and typically also deeper.[11] This can be illustrated by making the hazard rate for recessions conditional on the starting point of the imbalances at the end of the preceding expansion.

Recessions are longer if they follow expansions driven by imbalances Chart 7



Note: See Appendix 2 for a more detailed description of the estimation.
Source: Own calculations.

For recessions following expansions driven by significant imbalances, the hazard function is shifted downwards and becomes flatter. This means that the

11 The correlation between the length and depth of the recessions is about one half. The correlation between the length of and accumulated loss caused by the recession, as measured in terms of GDP, is approximately three quarters, see also Appendix 3.

(conditional) probability of a recession turning into an expansion decreases, resulting in a longer median residual lifetime of the recession, see Chart 7.

Recessions become longer if other economies are in recession at the same time

However, factors other than imbalances may lead to severe recession. This is particularly true of global recessions where several countries are experiencing economic downturns simultaneously. If the world economy is included as a condition, the hazard rate shifts significantly downwards, see Chart 8.

More specifically, the hazard rate is made conditional on the proportion of countries in recession at the same time. In case of a global crisis, where the majority of the world's economies enter into recession simultaneously, the expected recession length increases by 50 per cent from two to three years compared to a situation where only the Danish economy is in recession.

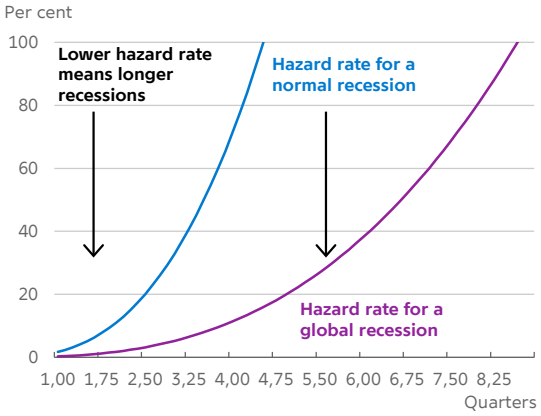
Great uncertainty about the length of the current recession

There is currently extraordinary uncertainty about the growth outlook for the coming years. On the one hand, before the corona pandemic the Danish economy was healthy and in balance and the expansion had been historically long. The same was true for many other developed economies. According to the model, the absence of imbalances implies, seen in isolation, that once the pandemic has been contained, the activity level will return to its previous level relatively quickly, which means that the current recession is likely to be short-lived.

On the other hand, the model indicates a significant risk of a long recession. Many countries are currently in a deep recession due to the corona pandemic, and global recessions are typically considerably longer and deeper as shown in this analysis. At the same time, limited room for manoeuvre in several countries when it comes to fiscal and monetary policy may leave less scope for stimulating the global economy than, for example, during the financial crisis. Consequently, although there are signs that the spread of the virus has been contained and economies are reopening, the recovery of the Danish economy depends largely on the development in the global economy. The longer the global recession lasts, the longer the Danish recession will be.

The more economies in recession at the same time, the longer the recession

Chart 8



Note: See Appendix 2 for a more detailed description of the estimation. The hazard rate of a normal recession (blue line) cannot be compared directly to the normal recession in Chart 7 as the hazard rate needs to be evaluated in its covariates.
Source: Own calculations.

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Appendix 1: Data

The analysis is based on data covering 19 developed countries

Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Italy, Japan, the Netherlands, New Zealand, Norway, Portugal, Spain, Sweden, Switzerland, the UK and the US.

Using a set of countries allows us to extract information about considerably more expansions and recessions, making the estimations more robust. The number of economies has been determined partly based on data availability and partly based on the desire to analyse countries that are roughly identical.

In addition to GDP, the analysis also applies data on house prices, lending to households and non-financial corporations, balance of payments as a percentage of GDP, unemployment, inflation measured on the basis of annual changes in consumer prices, monetary policy interest rates and oil prices. Nominal variables are deflated by consumer prices. Oil prices are included in the model as the net change in oil prices over the past 12 months, see Hamilton (1996). Data from Macrobond.

Appendix 2: The econometric model

Hazard rates is a key concept in survival analysis, i.e. analysis of the expected time until an event occurs. Other methods applied more frequently in macroeconomics, such as regression analysis or logistic regression, are not useful in this context. These methods do not take into account events that affect the length of business cycles and vary over time. Neither do they consider expansions or recessions which have not yet ended, including, for example, the current expansion in Denmark that has not yet turned into a recession.

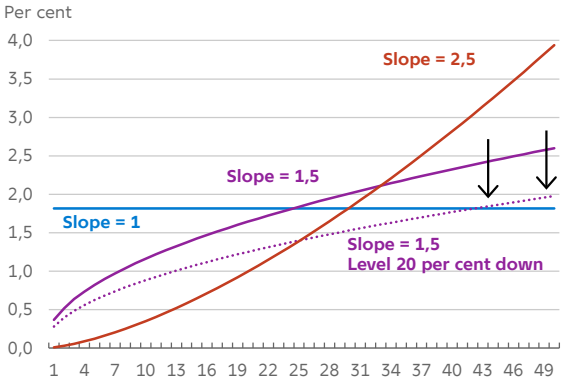
In this context, the time until an expansion turns into a recession and vice versa is analysed. The probability of an expansion turning into a recession is conditional on the length of the expansion, i.e. what is the probability of a recession if the expansion has lasted, say, ten quarters?

In order to calculate how often an event occurs, where the time elapsed without an event having occurred is included as a condition, the survival probability, *S(t)*, must be known. It describes the probability of an event occurring only after time *t*, i.e. *S(t)=P(T>t)*. Given that *P(T>t)=1-P(T<t)*, the survival probability can be calculated as 1 minus the distribution function *F(t)*, i.e. *S(t) = 1-F(t)*.

The hazard rate is calculated as the density function, *f(t)*, (the unconditional probability) divided by the survival probability, i.e. *h(t) = f(t)/S(t)*.

Example 1
If, for example, a coin is flipped, the probability of heads will increase over time (i.e. the more times you flip the coin). However, the probability of heads, conditional on x number of heads having occurred already, remains unchanged over time. Therefore, the fact that you only rarely see heads occurring 20 times in a row does not imply that the probability of tails increases over time. In this case, the hazard rate is flat.

Example of the impact on the hazard rate of different values for slope and level parameters Chart 9



Note: Stylised example of different hazard rates for different parameterisations of the Weibull distribution.
Source: Own calculations.

Example 2
What is the risk of dying? Looking at the density function, a higher proportion of the population dies at the age of 40 than at the age of 100. This is, of course, due to the fact that not many people live to the age of 100. In order to assess the mortality risk, you must base the assessment on the number of people in each age group, i.e. the survival probability. This is exactly how hazard rates are calculated. The hazard rate for the mortality risk is an increasing function of age.

Weibull
The analysis applies a so-called Weibull distribution, which is often used in survival analysis. The hazard rate $h(\tau)$ as a function of time τ in the Weibull model can be parameterised as follows:[12]

$$h(\tau) = \alpha\lambda\tau^{\alpha-1}.$$

The α parameter determines the time dependency and thus the slope of the hazard rate. If it equals 1, the survival probability is not dependent on time.

12 This is the model in continuous time. The estimation performed in the analysis applies discrete time as quarterly data is used. The intuition can, however, be more easily explained by applying the functional form of the model stated in continuous time.

If α is greater than 1 but less than 2, the probability of a given condition changing increases over time. If is greater than 2, the probability of an event occurring increases explosively over time.

The parameter λ changes the hazard rate level, meaning that for a given slope value, a higher λ value will move the entire hazard rate upwards.

The three types of covariates, i.e. data that is likely to increase or decrease the length of business cycles, are introduced into the model by being included in the hazard rate level, λ:

$$\lambda = \exp(\beta' X_t)$$

where $\beta' X_t$ is data for the variables and coefficients, respectively. The variables are described in the following paragraph. These are the coefficients that we seek to estimate in the model. The coefficients may be interpreted as the percentage change in the hazard rate if the covariate in question changes by 1 unit.

For example, if the coefficient is negative for an expansion, the covariate in question will reduce the probability of the expansion turning into a recession. The expansion will thus, all things being equal, become longer, and the hazard rate will move downwards. Similarly, if the coefficient is negative for a recession, the recession will become longer. This is illustrated in Chart 9.

Appendix 3:
Results of the estimation

Expansion model
The analysis uses two statistical estimation models – an expansion model and a recession model. The expansion model includes the short monetary policy interest rate, share and oil prices, the share of other countries in expansion, inflation, the development in unemployment during the preceding recession and an imbalance target, see Box 2.

The estimation results for the expansion model are shown in Table 1.

Recession model
The recession model differs to some extent from the expansion model, as the length of the recessions is significantly shorter. This means that many of the variables included in the expansion model do not contribute significantly to determining the probability of a recession ending, i.e. the variables that kill expansions are not necessarily the same as the ones that kill recessions.

The recession model includes oil prices, the monetary policy interest rate, the share of countries in recession, public debt as a percentage of GDP, the development in unemployment during the preceding expansion and the extent of the imbalance at the beginning of the recession.

The estimation results for the expansion model are shown in Table 2.

Estimation results for expansion Table 1

Expansion	
Short interest rate	-0,08
	(-1,04)
Share price	-0,03***
	(-4,99)
Oil price	0,06
	(0,04)
Proportion of countries in expansion	-2,06***
	(-4,76)
Inflation	0,25***
	(3,61)
Imbalances	0,42***
	(3,40)
Development in unemployment during the preceding recession	-0,15*
	(-1,96)
Constant	-6,03***
	(-5,16)
alpha	1,56***
	(3,60)
Observations	1,990
*** p<0,01, ** p<0,05, * p<0,1	

Note: The standard errors are cluster robust, and country-specific effects have been taken into account by including dummy variables.
Source: Own calculations.

Estimation results for recessions Table 2

Recessions	
Short interest rate	-0,08
	(-1,30)
Public debt	-0,03*
	(-1,92)
Oil price	-2,87
	(-0,61)
Proportion of countries in expansion	-2,04**
	(-2,23)
Development in unemployment during the preceding expansion	-0,04
	(-0,35)
Imbalances in preceding expansion	-0,28*
	(-1,85)
Constant	-0,03
	(-0,02)
	3,73***
	(11,00)
Observations	225
*** p<0,01, ** p<0,05, * p<0,1	

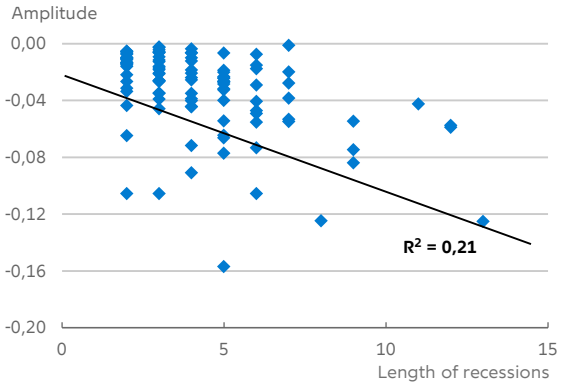
Note: The standard errors are cluster robust, and country-specific effects have been taken into account by including dummy variables.
Source: Own calculations.

Appendix 4: Correlation between the length and depth of recessions

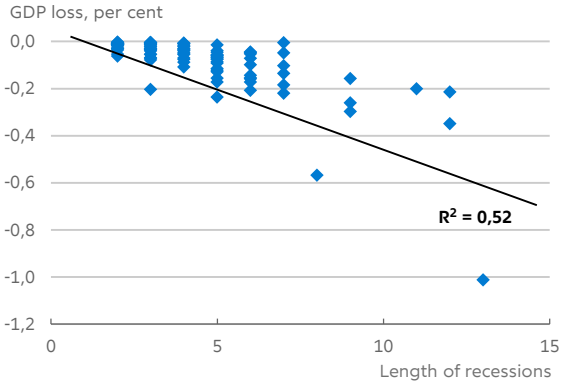
We analyse the length of expansions and recessions, but not, for example, the depth of the recession. However, the data used reflects a close correlation between the length and depth of recessions, see Chart 10. A long recession is typically also deep and involves a large overall loss of economic activity.

A longer recession is typically also deeper and leads to greater production losses Chart 10

Length of recessions and size of fluctuations



Length of recessions and size of GDP loss



Note: Expansions are defined as periods during which the economy is not in recession. Some recessions see a growth in GDP, as recessions in some economies are based on a broader set of indicators. See above for a detailed description of the dating method.
Source: Macrobond and own calculations.

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