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News Uncertainty in Brexit UK

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Abstract

After the Brexit referendum the behavior of the UK economy defied widespread expectations, as it did not exhibit a V-shaped recession, but a slow decline in production. We show that this pattern of propagation arises when uncertainty is about future, rather than current fundamentals, and if the expected duration of uncertainty is sufficiently long. We reach this conclusion within the confines of a heterogeneous firms model featuring news uncertainty, instead of conventional uncertainty shocks. In the quantitative analysis, uncertainty is informed by firm-level probability distributions on the expected effect of Brexit on sales.

Resumé

Modsat manges forventninger oplevede den britiske økonomi ikke en V-formet recession efter afstemningen om Brexit men et langsomt fald i produktionen. Vi viser, at dette mønster opstår, når der knytter sig usikkerhed til fremtiden snarere end til nuværende fundamentale forhold, og når den forventede varighed af usikkerheden er tilstrækkelig lang. Vi når frem til denne konklusion inden for rammerne af en model med heterogene virksomheder og usikkerhed omkring nyheder i stedet for traditionelle usikkerhedsstød. I den kvantitative analyse er usikkerhed baseret på virksomhedsspecifikke sandsynlighedsfordelinger over den forventede effekt af Brexit på omsætningen.

Key words

Recent economic and monetary trends; Productivity and competitiveness.

JEL classification

E22; E32; E65; O40

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News Uncertainty in Brexit UK*

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Abstract

After the Brexit referendum, the behavior of the UK economy defied widespread expectations, as it did not exhibit a V-shaped recession, but a slow decline in production. We show that this pattern of propagation arises when uncertainty is about future, rather than current fundamentals, and if the expected duration of uncertainty is sufficiently long. We reach this conclusion within the confines of a heterogeneous firms model featuring news uncertainty rather than conventional uncertainty shocks. In the quantitative analysis, uncertainty is informed by firm-level probability distributions on the expected effect of Brexit on sales.

Keywords: news shocks, uncertainty, firms heterogeneity, long-run productivity, survey data.
JEL codes: E22, E32, E65, O40

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1 Introduction

Does uncertainty matter for the business cycle? After the vote to leave the EU in the Brexit referendum of June 2016, the behavior of the UK economy has instilled the doubt that maybe uncertainty is not as important as previously thought. Indeed, the academic literature has shown that uncertainty shocks propagate in a V-shaped fashion, inducing a sharp and short-lived contraction in aggregate production (Bloom 2009). Consistent with this view, prior to the referendum, a number of policy institutions predicted a sharp recession conditional on a victory of the leave campaign (HM Treasury 2016). But this contraction failed to materialize, leading to dramatic upward revisions in the short-term forecast of economic activity. The Brexit referendum is an ideal case study to gauge the importance of uncertainty, for the sheer magnitude of uncertainty sparked, which involves not only the future legislation of trade agreements with the EU and the rest of the world, but also the future regulation of migration flows, domestic industrial policy, banking regulation, taxation policy etc.

Figure 1 shows that soon after the Brexit referendum, the employment rate, as well as capital and output per capita slowly started to fall below trend growth. So the figure suggests that the Brexit referendum has likely produced a material impact over the next three years. But if this is the case, these damaging effects did not come through a sudden contraction in economic activity, the manifestation of conventional uncertainty shocks, but via a gradual decline in output and in the factors of production.[1]

The starting point of our analysis is the observation that Brexit uncertainty hardly fits the notion of a canonical uncertainty shock, and therefore it should not be expected to propagate accordingly. An uncertainty shock is defined as a second-moment shock to aggregate and idiosyncratic fundamentals over the short run. But in the case of Brexit, uncertainty is primarily about fundamentals in the long run. Moreover, Brexit will affect fundamentals directly only after it takes place; the outcome of the referendum itself does not affect fundamentals directly, but only indirectly, through expectations. We therefore model the referendum as a news shock, with the time horizon and the content of the news being uncertain. We make use of survey data on firm-level distributions of beliefs about both the expected Brexit date and expected impact of Brexit on sales, to inform uncertainty in the model. The aim of the analysis is twofold. First, we aim to understand the impact of the Brexit referendum on UK macroeconomic dynamics. Second, and more broadly, we take Brexit as a laboratory to learn about the role of uncertainty in news shocks, which we believe characterizes most episodes of economic policy uncertainty.

[1] Born et al. (2019) provide evidence for the gradual slowdown of GDP using a more sophisticated methodology to compute the trend.

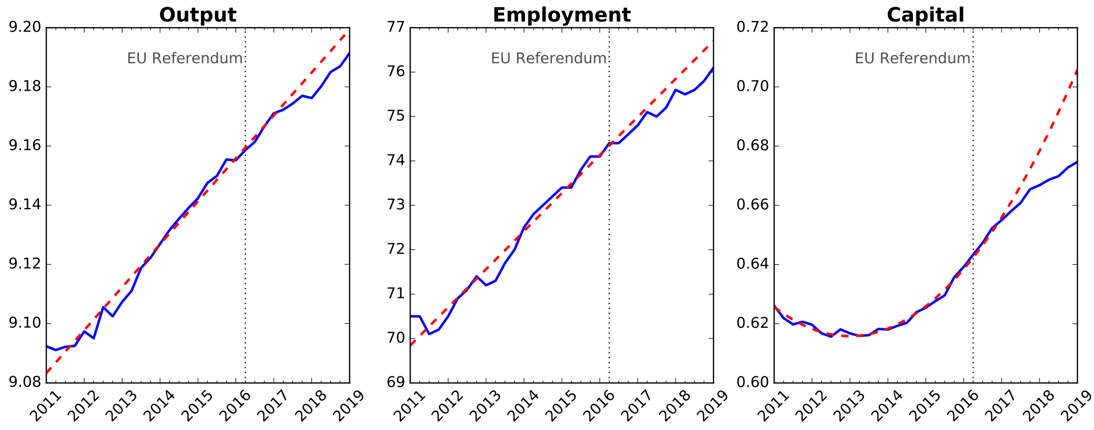


Figure 1: UK Macroeconomic Aggregates and Their Pre-Referendum Trend

Notes: *The figure shows real GDP per capita, the employment rate and capital per capita in deviations from their trend. Output and capital per capita are expressed in logs, the rate of employment is in level. Trend growth is linear for output and employment and quadratic for capital, and is calculated from 2011Q1 to 2016Q2.*

We assume that the stationary equilibrium of a heterogeneous-firms economy *à la* Khan and Thomas (2008) is hit by the unexpected news that a package of policy reforms may be implemented in the future, with firms being uncertain about which policy will be implemented. A specific Brexit policy, say Hard or Soft Brexit, is defined as a combination of aggregate and firm-level TFP shocks that are permanent. Because the model is real, these shocks should not be interpreted literally as technology shocks, but more broadly as permanent sales shocks, or the outcome of idiosyncratic policy wedges in the spirit of Restuccia and Rogerson (2008) and Buera and Shin (2013, 2017).

We show that the model can quantitatively account for the slowdown of the UK economy illustrated in Figure 1. The main lesson, beyond the specific case of Brexit, is that the effects of uncertainty depend greatly on whether uncertainty is associated with current or future fundamentals, and on its expected duration.

2 Model

We present a simple, discrete-time economy with a unit mass of heterogeneous firms facing non-convex costs of adjusting capital. We deliberately take a partial equilibrium approach and abstract from the explicit modeling of households. Indeed, we do not want to impose equality between domestic savings and investment for the UK economy.[2]

[2]Introducing a consumption and savings decision has implications for the transition, but not for the long-run effects of Brexit, which are pinned down by the survey data. This extension implies that the permanent income shock induced by

2.0.1 Environment

Each period, firms produce by renting labor at the wage rate w and using their capital, which evolves according to $k' = (1-\delta)k + i$, where $\delta \in (0,1)$ is a depreciation rate, i denotes investment and $k \in \mathbb{R}_+$. Firms are characterized by four idiosyncratic state variables: idiosyncratic productivity, an idiosyncratic policy state, capital and a fixed cost associated with investment.

The evolution of individual productivity, $a \in \mathcal{A} = \{a_1, ..., a_{N_a}\}$, is governed by an exogenous stochastic AR(1) process: $log(a') = \psi log(a) + \epsilon$, where $\epsilon \sim \mathcal{N}(0, \sigma)$ and $\psi \in [0,1)$. The idiosyncratic policy state $z \in \mathcal{Z} = \{z_+, z_-, z_0\}$ captures whether the implementation of the Brexit policy contributes a positive, negative or neutral permanent component to firm-level productivity, respectively. This state becomes relevant only after the policy is implemented, at which time firms take an independent draw of z. Specifically, firms will draw z_+ and z_- with associated probability q_+ and q_-, respectively, and z_0 with probability $1 - q_+ - q_-$. These assumptions generate idiosyncratic uncertainty over the outcome of the policy. Finally, we assume that actively changing capital is subject to a fixed, stochastic cost of adjustment ϕ.[3]

In addition to these idiosyncratic states, we introduce a fifth state variable, $\zeta \in \{\zeta^P, \zeta^N, \zeta^S, \zeta^H\}$, which captures the aggregate state of Brexit. ζ^P denotes the pre-referendum aggregate state of the economy. Because the outcome of the referendum was largely unexpected, we assume that this steady state does not contemplate the possibility that the Brexit referendum may happen. ζ^N denotes instead the aggregate state of the negotiations, which starts when the news is unexpectedly announced and ends either if Brexit happens, or if it is ruled out once and for all. Specifically, we assume that while the economy is in the state ζ^N, Brexit takes place at Poisson rate θ. In this case, the economy transitions either to the absorbing state ζ^H, with probability γ^H, or to the other absorbing state ζ^S, with probability $\gamma^S = 1 - \gamma^H$, depending on which policy is implemented, Hard or Soft. In a given period, there is also the possibility that a remain shock strikes with probability γ^R, in which case the economy reverts to the initial absorbing state ζ^P. We interpret this shock as the outcome of a second referendum, which overrules the first one with the decision to remain in the EU once and for all. If Brexit is neither implemented nor scrapped, the negotiations drag on to the next period with period probability $1 - \theta - \gamma^R$. The state ζ^N can be interpreted as an aggregate state of uncertainty, since the presence of aggregate, idiosyncratic and timing

the Brexit referendum generates a massive increase in precautionary savings. If savings equals investment, the news of the referendum induces an investment boom, which is at odds with the data.

[3] We assume that adjustment costs are stochastic for computational reasons. By making the policy function continuous, this assumption allows for better accuracy and faster execution without affecting the results.

uncertainty accompanies the negotiations.

2.0.2 Production Function

The production function is:

$$f = \tilde{\tau} a (k^{\alpha} l^{1-\alpha})^{(1-\nu)},$$

where $1-\nu$ governs the degree of decreasing returns to scale. $\tilde{\tau}$ captures instead the effects of Brexit on firm-level productivity. We assume that unless Brexit happens, that is, in either of the states ζ^P and ζ^N, all firms face $\tilde{\tau} = 1$. Conditional on Brexit happening instead, firms will be subject to a distribution of values for $\tilde{\tau}$. Specifically:

$$\tilde{\tau} \equiv \tau(z_i, \zeta^j, a) = \begin{cases} 1 & \text{for} \quad j = \{P, N\}, \\ (1+X^j)(1+x_i^j)a^{\omega} & \text{for} \quad j = \{S, H\}, \quad i \in \{z_+, z_-, z_0\}, \end{cases} \tag{1}$$

where $X^j \in \mathbb{R}$ is the parameter capturing the permanent aggregate productivity shock associated with the aggregate policy state $j \in \{\zeta^S, \zeta^H\}$ and $x_i^j \in \mathbb{R}$ is the parameter associated with the permanent idiosyncratic TFP shock, which is contingent on both the aggregate policy state j and the idiosyncratic policy state i. We assume that $x_i^j = 0$ for $i = z_0$. In this case, Brexit affects the firms with idiosyncratic state z_0 only through the aggregate shocks X^j. Moreover, it is assumed that the effect of Brexit on sales depends on productivity via the elasticity parameter $\omega \in \mathbb{R}$. A negative value of ω implies that, everything else equal, firms with higher productivity lose more or gain less from Brexit.[4] This specification is the most parsimonious one that allows us to match the empirical moments discussed in Section 3. We emphasize that because the model is real, a change in $\tilde{\tau}$ should not be interpreted literally as a technology shock, but more broadly as a permanent sales shock, or as the outcome of idiosyncratic policy wedges in the spirit of Restuccia and Rogerson (2008).

2.0.3 Firms

Every period, a firm can invest in any future level of capital k' only upon payment of the fixed adjustment cost ϕ, drawn independently every period from the same uniform distribution $G(\phi) \sim \mathcal{U}\left[0, \overline{\phi}\right]$. We summarize the joint distribution of firms over (a, z, k) by a probability measure $\mu(a, z, k)$.

[4] $\omega \neq 0$ also allows for heterogeneity in the response of policy uncertainty across firms, a robust finding documented by Hassan et al. (2019).

At the beginning of the period, after observing the aggregate state of the economy ζ, and inheriting its capital stock from the previous period, the firm takes a new draw of productivity and, conditional on Brexit taking place, also a new draw of the idiosyncratic policy state z. It then chooses its current level of employment, produces and pays its workers. Next, it draws its fixed cost ϕ. The current period value of a firm before drawing the adjustment cost can be represented as the expected end-of-period value over all possible realizations of ϕ:

$$v(a,z,k;\zeta) = \int_0^{\overline{\phi}} \tilde{v}(a,z,k,\phi;\zeta)\; G(d\phi). \tag{2}$$

Once the firm observes the fixed adjustment cost ϕ, it decides whether to adjust its capital or not. The optimal choice solves:

$$\tilde{v}(a,z,k,\phi;\zeta) = \quad \max\, \{-\phi + v^A(a,z,k;\zeta)\;,\; v^{NA}(a,z,k;\zeta)\}, \tag{3}$$

where v^A and v^{NA} represent the value functions of adjusting and non-adjusting capital, respectively. In turn, v^A solves:

$$\begin{aligned} v^A(a,z,k;\zeta) = \max_{k',l}\; \Big[& f(a,z,k,l;\zeta) - wl - k' + (1-\delta)k - \mathbb{I}(i<0)\,|i|\,\chi \\ & + \mathbb{E}_{\Gamma_a,\Gamma_z,\Gamma_\zeta} \beta v(a',z',k';\zeta'|a,z,\zeta)\Big], \end{aligned} \tag{4}$$

subject to the laws of motion for the aggregate policy state Γ_ζ, idiosyncratic productivity Γ_a, and the individual policy state Γ_z. In the above problem, β is the discount factor, and $\mathbb{I}$ an indicator function taking the value of one if the firm disinvests and zero otherwise. The parameter χ governs the partial irreversibility of investment—the resale price of capital is only a share $(1-\chi)$ of its purchase price. The expression in the first line represents the period flow of profits, after substituting investment using the law of motion for capital.

The value function $v^{NA}(a,z,k;\zeta,\mu)$ solves:

$$\begin{aligned} v^{NA}(a,z,k;\zeta) = \max_{l}\; \Big[& f(a,z,k,l;\zeta) - wl \\ & + \mathbb{E}_{\Gamma_a,\Gamma_z,\Gamma_\zeta} \beta v(a',z',(1-\delta)k;\zeta'|a,z,\zeta)\Big], \end{aligned} \tag{5}$$

subject to the laws of motion for the states a, z, and ζ, which implies that the firm brings into the next period its current capital, after depreciation.

We define a threshold adjustment cost $\phi^T(a,z,k;\zeta)$ that makes each firm indifferent between adjusting and not adjusting:

$$v^A(a,z,k;\zeta) - \phi^T(a,z,k;\zeta) = v^{NA}(a,z,k;\zeta). \tag{6}$$

Firms optimally adjust their capital only if $\phi \leqslant \phi^T(a,z,k;\zeta)$. As a result, the optimal investment decision exhibits an inaction region for any value of $\phi > \phi^T$. Aggregate and idiosyncratic uncertainty about the future policy states widens this region, with firms freezing investment, or disinvestment, as they wait for uncertainty to resolve.

3 Calibration

The model comprises three sets of parameters which affect the pre-referendum steady state, the stochastic process for Brexit and the policy shocks that characterize the post-Brexit steady states, respectively. We calibrate the pre-referendum steady state using a standard parametrization from the literature, and then impose a calibrated process for Brexit.

3.0.1 Pre-Referendum Steady State

In the calibration one period equals one quarter. The steady state is meant to represent the UK economy over the period 1992-2015. The wage w is set to 1.651, implying average working hours equal to 33.7% of the time endowment Office for National Statistics (2018). We set the discount factor β to 0.995 to match an average annual real interest rate of 1.8% World Bank Open Data (2018). The parameters of the production function, α and ν, are set to the standard values of 0.333 and 0.250, respectively, in accordance with Bloom et al. (2018).

For the parameters governing the persistence and the variance of the stochastic idiosyncratic productivity process, we rely on the values set by Bloom et al. (2018) for the US economy, which are 0.95 and 0.051, respectively. The capital depreciation rate δ is fixed at 0.020 implying an annual depreciation of 8.2% Office for National Statistics (2016). The upper support of the uniform fixed-cost distribution, $\overline{\phi}$, is set to $1.5e^{-6}$ in order to match the fact that 8% of US firms every year remain inactive with respect to investment (Kahn and Thomas 2008).[5] Finally, χ is set to 0.339 to match a resale loss of capital of

[5] Assuming non-convex costs of adjustment is necessary to hit this target.

33.9% (Bloom 2009).

3.0.2 Brexit Stochastic Process

The stochastic transition set in motion by the referendum is governed by the following parameters: the period probability that Brexit takes place, θ; the probability that, conditional on Brexit occurring, Brexit is Hard, γ^H; the period probability that the decision of the Brexit referendum is reversed, γ^R.

For their calibration we rely on the Decision Maker Panel (2017-2018) (DMP). This is a new, large survey that is representative of UK businesses with ten employees or more. It was launched in August 2016 by the Bank of England, together with Stanford and Nottingham University, and was designed to investigate how the uncertainty sparked by the Brexit referendum would affect UK businesses. Every question is answered by the CFO or the CEO of a firm and has, on average, 2,500 respondents. For further details see Bloom et al. (2019a).

We first calibrate θ and γ^R using question S.18 of Nov.2017-Jan.2018, which asks: *"What do you think is the probability of the UK leaving the EU (after the end of any transitional arrangements) in each of the following years?:* i*) 2019,* ii) *2020,* iii) *2021,* iv) *2022,* v) *2023 or later,* vi) *Never"*. The answers to this question indicate that the average probability of the UK not having left the EU by the end of 2022 is 18%. In turn, this probability comprises a 9% chance of Brexit happening any time after 2022 and a 9% chance of Brexit never taking place.

In the model, the probability of the UK not having left the EU by the end of 2022 can be expressed as the probability of remaining in the negotiation state for 26 consecutive quarters (from 2016Q3 up to and including 2022Q4):

$$Pr(\text{UK remaining until end 2022}) = (1 - \theta - \gamma^R)^{26}. \tag{7}$$

The probability that the UK never leaves the EU can be represented in the model by the probability that the decision of the Brexit referendum is eventually reversed:

$$\begin{aligned} &Pr(\text{UK never leaving}) = \\ &\sum_{i=1}^{\infty}(1 - \theta - \gamma^R)^{i-1}\gamma^R = \frac{\gamma^R}{1 - (1 - \theta - \gamma^R)}. \end{aligned} \tag{8}$$

Assigning the values of 0.18 and 0.09 to the L.H.S. of equations (7) and (8), we can solve for $\theta =$

0.0581 and $\gamma^R = 0.0057$.

To calibrate the conditional probability of Hard Brexit, γ^H, we make use of Question S.25 of Feb.-Apr.2018, which is informative about the unconditional probability of Hard Brexit. The question asks: *"What probability do you attach to a disorderly Brexit, whereby no-deal is reached by the end of March 2019?"*. The average answer to this question is 44%.

In the model, Hard Brexit is not restricted to taking place before any particular date but follows a Poisson process. The unconditional probability of Hard Brexit happening at any future point in time is:

$$Pr(\text{Hard Brexit}) = \sum_{i=1}^{\infty}(1-\theta-\gamma^R)^{i-1}\theta\gamma^H = \frac{\theta\gamma^H}{1-(1-\theta-\gamma^R)}. \tag{9}$$

Equipped with the values for γ^R and θ, and making use of $Pr(\text{Hard Brexit}) = 0.44$, we solve equation (9) and get $\gamma^H = 0.485$.

3.0.3 Brexit Policy Parameters

For the calibration of the following nine Brexit policy parameters, X^j, x^j_+, x^j_-, q_+, q_- and ω for $j = S, H$, we make use of two questions from the DMP, whose results, averaged across respondent firms, are reported in Table 1. The first question asks: *"How likely do you think it is that the eventual agreement will have the following effects, compared to what would have been the case had the UK remained a member of the EU, with five scenarios provided about the effect on sales at home and abroad: i) a large positive effect adding* 10% *or more, ii) modest positive effect adding less than* 10%*, iii) make little difference, iv) modest negative effect subtracting less than* 10%*, v) large negative effect subtracting* 10% *or more."* [6]

The second question asks about the expected effects of Brexit on sales, conditional on the case where the UK exits the EU without a deal (Hard Brexit). The question asks: *"How likely do you think it is that [a no-deal] outcome will have the following effect on the sales of your business, compared to what would have been the case had the UK remained a member of the EU: i) a large positive effect adding* 10% *or more, ii) modest positive effect adding less than* 10%*, iii) make little difference, iv) modest negative effect subtracting less than* 10%*, v) large negative effect subtracting* 10% *or more."* In both questions each respondent is therefore asked to attach a probability to each of these five outcomes, thereby providing an individual probability distribution of the expected effect of Brexit on their sales. The first question elicits

[6]The question asks about the expected effects of "the eventual agreement" and not about the effects of a possible Free Trade Agreement, and is therefore intended to capture average individual expectations over all possible Brexit outcomes, including the case where no deal is reached. In this case the eventual agreement is the fall-back position dictated by WTO terms (cf. Bloom et al. 2019b).

Table 1: Moments and Parameters of the Calibration

Panel A: *Moments*

'Expected impact of Brexit on sales, average probability"			*"Expected impact of disorderly Brexit on sales, average probability"*		
	Data	*Model*		*Data*	*Model*
$< -10\%$	17.5	17.7	$< -10\%$	19.7	19.4
$> -10\%$	27.6	28.1	$> -10\%$	26.9	26.4
$\simeq 0\%$	37.2	35.5	$\simeq 0\%$	43.5	44.0
$< 10\%$	12.3	12.1	$< 10\%$	6.9	7.3
$> 10\%$	5.4	6.6	$> 10\%$	2.9	2.8
	Data	*Model*		*Data*	*Model*
Average standard deviation of sales expectations	5.9	6.9	*Elasticity of expected sales to productivity*	-0.37	-0.36

Panel B: *Parameters affecting Hard and Soft Brexit steady states*

Aggregate policy parameters		*Idiosyncratic policy parameters*				*Elasticity to productivity*	*Idiosyncratic probabilities*	
X^S	X^H	x^S_+	x^S_-	x^H_+	x^H_-	ω	q_+	q_-
0.010	-0.006	0.046	-0.060	0.055	-0.046	-0.002	0.101	0.459

Panel C: *Parameters affecting Brexit stochastic transition*

θ	γ^S	γ^H	γ^R
0.058	0.515	0.485	0.006

Notes: *Panel A compares the moments of the survey data used for the calibration of the Brexit policy with the corresponding moments in the model. The top left corner of Panel A reports the unconditional probability distribution of the expected impact of Brexit on sales, averaged across respondent firms weighted by the employment shares; the top right corner of Panel A reports a similar probability distribution for the case of a no-deal outcome, i.e. Hard Brexit. The unconditional distribution reflects sales expectations taken over all possible Brexit scenarios, including the case of no deal. Panel B of this table reports the values of the policy parameters calibrated using the moments from Panel A. Parameters with the superscripts S and H relate to the aggregate states of Soft and Hard Brexit, respectively. The parameters with subscripts* $-$ *and* $+$ *refer to the idiosyncratic policy states* z_- *and* z_+*, respectively. Panel C reports the values of the parameters governing the Brexit stochastic process.*
Source: *The question in the top left corner of Panel A is S.8 of Nov.2017-Jan.2018 from the Decision Maker Panel (2017-2018). The question on the top right corner has not been published online and we thank the Bank of England for sharing the data. The average standard deviation of sales expectations and the elasticity of expected sales to productivity have been calculated by Bloom et al. (2019b, 2019c). None of these two statistics have been published, and we thank the authors for sharing their results. The parameters governing the Brexit stochastic process have been calibrated using Question S.18 of Nov.2017-Jan.2018 and S.25 of Feb.-Apr.2018 from the Decision Maker Panel (2017-2018).*

information on the unconditional distribution, while the second is conditional on Hard Brexit.

Each question provides us with four independent moments which can be used as calibration targets. The four moments in the aggregate distribution about the sales impact of Hard Brexit elicit direct information on the Hard Brexit policy parameters X^H, x^H_-, x^H_+. Given this information, the probability bins of the unconditional distribution, together with our assumption that there can be only two types of Brexit, directly identify the policy parameters of the Soft Brexit policy, X^S, x^S_-, x^S_+. Together, these two distributions also pin down the probability parameters q_- and q_+.

To elicit information that identifies how post-Brexit sales correlate with firm-level productivity, as captured by the parameter ω, we use the estimate by Bloom et al. (2019c) of the elasticity of expected sales to firm-level productivity in the DMP data. Their finding of a negative correlation of -0.37 implies that more productive firms expect heavier losses from Brexit.[7] Finally, we also target the average dispersion of the individual histograms characterizing the unconditional post-Brexit sales expectations. Because we have nine parameters for ten calibration targets, our model is over-identified. How well the model captures this measure of average dispersion in firms' expectations is, therefore, a useful validation test for the formulation of the policy in eq.(1).

To compare model outcomes with the data, we compute artificial probability distributions for the firms in our model in the same way as they are computed in the DMP: we produce five-bin histograms for each firm, both unconditional and conditional on Hard Brexit, and then average across all firms. We calibrate the parameters of the policy using the method of simulated moments (MSM), that is, we minimize the squared deviation of model moments from the targets.

Table 1 reports the moments targeted in our calibration and their corresponding values in the model. Looking at the data on the expected effect of Brexit on sales, we see that firms placed more weight on Brexit reducing sales than on it increasing them. This is even more so in the case of Hard Brexit. We also note that a good chunk of the mass in the unconditional probability distribution assigned to positive effects on sales shifts to the left in the case of Hard Brexit, raising the probability of receiving no material impact. When mapping the probability distribution of the data to those constructed in the model, we assume that expected changes in sales that are lower than 2% in absolute value correspond to firms reporting in the survey that Brexit would make "little difference".

Table 1 shows that the model matches all targeted moments remarkably well.

[7] We thank the authors for sharing their unpublished results.

4 Quantitative Analysis

4.1 The Effects of Brexit in the Long Run

The Hard and Soft Brexit steady states implied by our calibration imply large economic losses, with GDP falling by 7.6% in the case of Hard Brexit and 4.5% in the case of Soft. Aggregate TFP also falls by 2.0% and 1.1% in the cases of Hard and Soft Brexit, respectively. The result that Brexit implies large economic losses is implied by two features of the data: the unconditional distribution of sales expectations being heavily skewed to the left, and expected post-Brexit sales being negatively correlated with productivity.

The DMP data only allows the model to identify two Brexit scenarios: Hard and Soft. The Hard Brexit scenario neatly corresponds to the no-deal Brexit case in the DMP. In this scenario, the model delivers a fall in GDP that coincides with the government's central estimate of -7.6% (HM Government, 2018). However, Soft Brexit is not identified as neatly. Indeed, Soft Brexit represents the average case in which the UK and the EU strike a deal, which includes scenarios like the Single Market or a Free Trade Agreement (FTA). The long-run estimate of the impact of Soft Brexit on GDP in our model (-4.5%) comes remarkably close to the government's estimates of the average impact of a FTA agreement, -4.9%, revealing that business expectations reflect the declared political intentions.

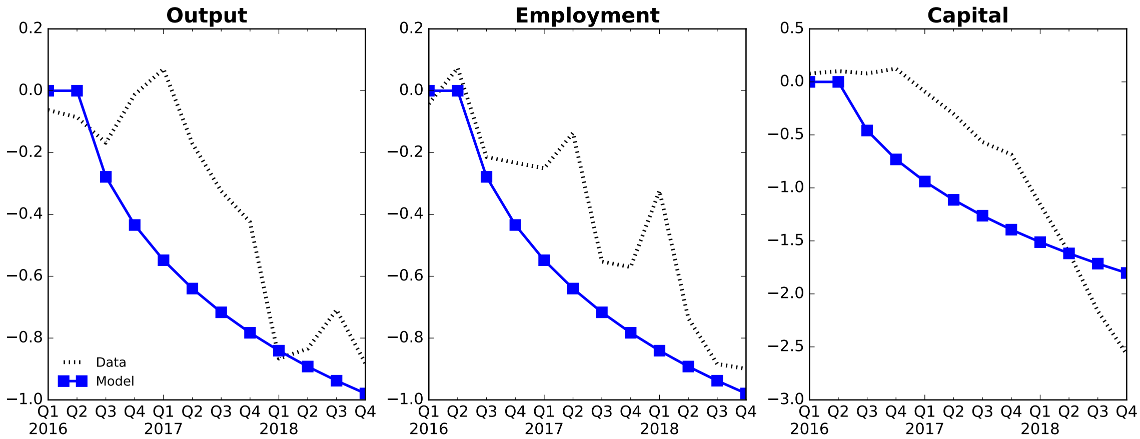
We note that these steady-state calculations are based on the beliefs of current firms, some of which may be forced out of business by Brexit, as the new economic environment makes room for new entrepreneurial activities to emerge in a process of creative destruction. Because these future potential entrepreneurs are not part of the survey, the above results may overestimate the true long-run effects of Brexit. Moreover, the individual expectations of UK businesses may be affected by individual ideological biases, and hence may not correctly reflect the true effects of Brexit on sales. However, because we are ultimately interested in assessing the effects of Brexit uncertainty in the short run, it is the expectations of the *current* UK businesses that matter for their investment decisions, however biased they may be.

4.2 The Effects of the Referendum

4.2.1 Model vs. Data

To assess the short-term impact of the Brexit referendum, we trace the model's responses under policy uncertainty. Figure 2 compares the behavior of output, employment and capital in the model and in the

Figure 2: Economy's Response to the Brexit Referendum



Notes: *Model response to the news of the Brexit referendum (blue line with square markers) vs. the behavior of output per capita, capital per capita and the employment rate in the data (gray dotted line). The responses of the model are in percentage deviations from steady state; the behavior of the data represents percentage deviations from their pre-referendum trend, as reported in Figure 1.*

data, over the period 2016Q1-2018Q4, where 2016Q2 is the quarter of the referendum. The response of the model is shown by the blue solid line with square markers, and all variables are expressed in percentage deviations from steady state. The dotted gray line shows the behavior of the data reported in Figure 1, in percentage deviations from trend.

The figure reveals that the news of the referendum alone accounts remarkably well for the deviation of output and employment from the counterfactual pre-referendum trend. Three years on, output is 1% below trend and the cumulative loss has been around 1.8% of annual GDP. These results are quantitatively similar to the empirical findings in Born et al. (2019) and NIESR (2019). The model also explains much of the behavior of capital, though it front-loads part of the disinvestment. This is due to the assumption of a constant Brexit Poisson rate, θ. This assumption is motivated purely by computational reasons. A more realistic approach would be to assume a Poisson rate that increases over time, starting close to zero and then rising when approaching the first cliff edge of the negotiations in March 2019. We show below that in our model the effects of the news are smaller, the longer the expected duration of the negotiations. An increasing Poisson rate would therefore imply that the impact of uncertainty on investment intensifies as the expected date of the resolution approaches. The response of investment would therefore be backloaded, thereby producing the same slope in the behavior of capital in the data.

Overall, the model seems to do a rather good job at capturing the behavior of GDP and the production inputs over the post-referendum period.

4.2.2 Disentangling the Role of First and Second Moments, Duration and Heterogeneous Incidence

We now investigate the transmission mechanism, disentangling the separate effects of first and second moments, duration and heterogeneous incidence of the sales shocks by productivity.

In a first experiment, we compute the response to the referendum in the counterfactual scenario where we zero the first moment of the news about aggregate steady-state TFP, while keeping the second moment fixed. We do so by changing the values of the aggregate shocks in Soft and Hard Brexit, X^S and X^H, while leaving all other parameters in Table 1 unchanged.[8] The behavior of the economy, traced by the orange line with triangle markers in Figure 3, reveals a strong attenuation in the fall of output and the inputs of production relative to the baseline case, i.e., the model as calibrated in Table 1 (thick gray line). Yet, investment still falls, dragging down production, due to the second-moment effect. Faced with uncertainty about whether Brexit will result in sales gains or losses, firms respond by freezing investment to save on the cost of adjustment, as they wait for uncertainty to resolve. This "wait-and-see" response produces a cumulative fall in output of -0.7% by 2018Q4, compared to a fall of -1.8% in the baseline case, where the first moment is active. When zeroing the first moment, the stock-market index is nearly unchanged right after the referendum, whereas it falls by -4.3% in the baseline.

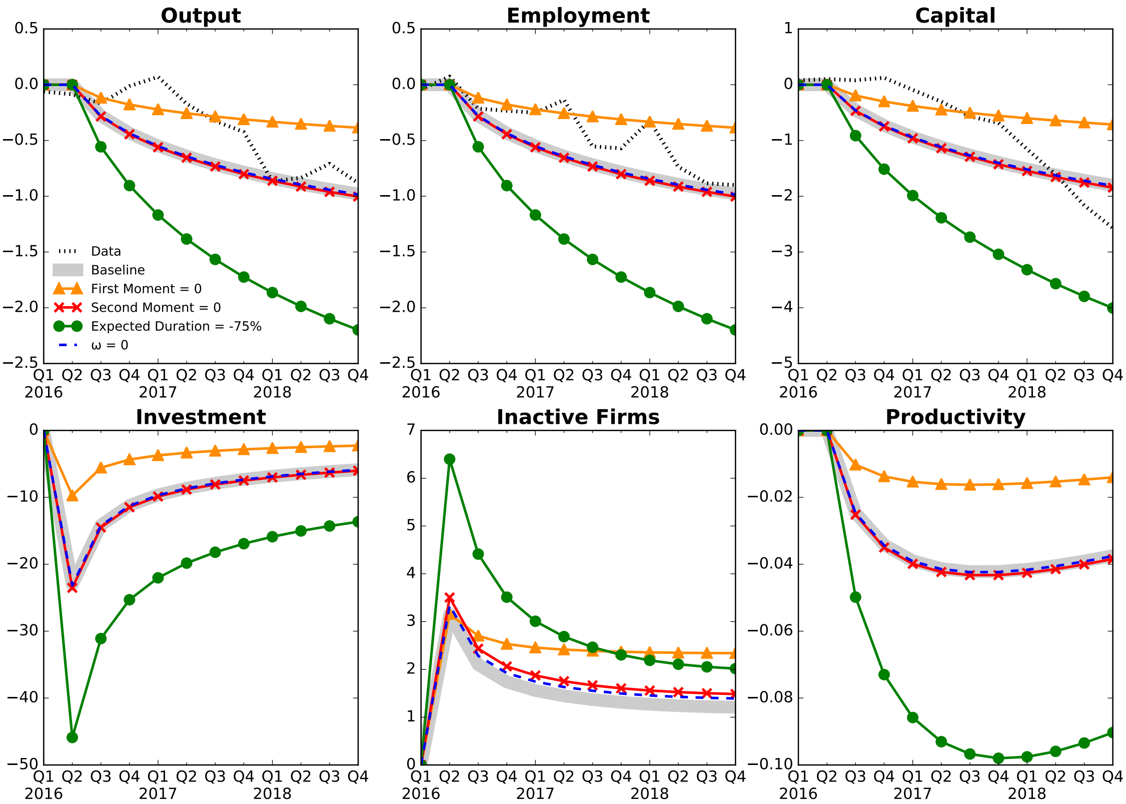
In a second experiment, we trace the response of the model when we zero the second moment of the news about aggregate steady-state TFP, while leaving the first moment as in the baseline. This is achieved by changing the parameters X^S and X^H, setting the probability of a second referendum γ^R to zero, rescaling the Brexit Poisson rate θ to preserve the same duration as in the baseline, and leaving all other parameters unchanged.[9] In this case, the firms anticipate that Brexit will eventually take place, with the aggregate impact on steady-state TFP being the same in Soft and Hard Brexit. The response of the economy, traced by the red line with x-markers, is virtually identical to the baseline. This exercise reveals that the importance of the second moment in this class of models is tied to the first moment of the news. When aggregate uncertainty is symmetric around the zero mean of the news (the orange line), the effect of the second moment is substantial; when instead the first moment is strongly negative, and uncertainty is effectively about the occurrence of a scenario that can be either bad (Soft Brexit) or very bad (Hard Brexit), the effect of the second moment is negligible.

Intuitively, if firms are uncertain whether they will invest or disinvest after the shock takes place,

[8] We set $X^S = 0.028$ and $X^H = 0.008$.

[9] We set $X^S = 0.007$, $X^H = -0.0001$ and $\theta = 0.0638$.

Figure 3: The Role of First and Second Moments, Duration and Heterogeneous Incidence



Notes: *The figure shows the propagation of the Brexit referendum shock in the baseline case (the thick gray line), against the counterfactual scenarios where we zero the first moment of the news about aggregate steady-state TFP keeping the second moment unchanged relative to the baseline (the orange line with triangle markers), we zero the second moment keeping the first moment unchanged (red line with x-markers), we remove the effects of the heterogeneous impact of the Brexit shock keeping the first and second moments unchanged (broken blue line), and we reduce the duration of the negotiations, keeping first and second moments unchanged (the green line with circle markers). The dotted gray line reports the behavior of the data in percentage deviations from their pre-Referendum trend shown in Figure 1. All variables are expressed in percentage deviations from the initial steady state, apart from the share of inactive firms, which is in percentage points deviations.*

then they wait and see, to save on future adjustment costs. The value of waiting, in this case, increases with the second moment, just like in the case of a standard uncertainty shock (Bloom, 2009). But if firms expect to eventually disinvest in any case, being uncertain only about the amount of disinvestment, then they start to decumulate capital straightaway, either actively or by attrition, and mean-preserving spreads between future steady-state outcomes matter less. Indeed, Figure 3 shows that the increase in inaction and the fall in investments in the baseline are unrelated to the second moment. Rather, inaction increases during the negotiation period because the Brexit stochastic process makes firms more reluctant to top up their capital stock, as it naturally depreciates over time; the investment freeze is motivated by the fear of ending up with too much capital should Brexit take place, which entails future adjustment costs. So the uncertainty that matters here, is about the *timing* of the Brexit “first-moment shock”.

To make the point sharper, we investigate the role of the expected duration of the negotiations by simultaneously changing the Poisson rates θ and γ^R, while keeping the unconditional probability of Brexit unchanged, which ensures that the first and second moments of aggregate TFP are also unchanged.[10] These two parameters govern the probability that the negotiations end in a given period, either because of Brexit, or because of a second referendum. The green line with circles in Figure 3 reports transitional dynamics for the case in which the expected duration of the negotiations is reduced to one year, i.e., about 75% less than in the baseline.

The results indicate that the impact of the referendum is stronger when the expected duration of the negotiations is shorter: the cumulative loss of GDP from 2016Q2 to 2018Q4 is -3.8% in the low duration scenario, compared to -1.8% in the baseline. Reducing the expected duration leads to an increase in inaction, a sharp contraction in investment and a faster decumulation of capital. Moreover, the stock-market index falls by -4.5% right after the referendum, compared to a fall of -4.3% in the baseline.

Why does inaction increase as duration falls? Faced with the likely prospect of suffering sales losses if Brexit takes place, firms have an incentive to freeze investments. But freezing investments is costly, as it implies operating at a sub-optimal scale of production. So firms trade off the inefficiency cost of inaction, against the future costs of disinvestment if Brexit takes place. If chances are higher that aggregate-state uncertainty resolves in a given period, more firms find it worthwhile to ”wait and see”.

The policy implications substantiate the arguments made by the Deputy Governor of Monetary Policy at the Bank of England, who reflects on the consequences of having postponed the Brexit deadline repeatedly since March 2019. In his words:

“A repeated series of cliff-edges, each of which is expected to be decisive but in reality just gives way to the next cliff, is more damaging for investment than if it had been clear at the outset that the process will take time(...) For a given duration of uncertainty, it’s better that people are aware of it from the start, rather than being repeatedly surprised at how long the process takes”.[11]

So a possible reason why the Brexit referendum has not produced as dramatic effects as anticipated is that the projections failed to account for the role of the expected duration of the negotiations on

[10] Setting $\theta = 0.232$ and $\gamma^R = 0.023$ satisfies eq.(8), which implies that despite the rise in θ relative to the baseline calibration, there is still a 9% unconditional probability that Brexit would never happen.

[11] Broadbent (2019), p.4.

investment decisions. We also note that the role of the expected duration of uncertainty in the propagation of the news shock in our model is different from the role of persistence in the propagation of a typical uncertainty shock, which is defined as a second-moment shock to current fundamentals. In the latter case, the share of inactive firms increases with the persistence of the shock (Bloom et al. 2007); in our model, instead, it falls with the average duration of uncertainty. The difference is due to the assumption that in our model uncertainty is sparked by a news shock, which by definition has a direct impact only on future fundamentals. The longer the expected duration of uncertainty, the further away in time is the expected moment in which fundamentals will be affected. As a result, the more subdued the response to the shock, because of the mechanism discussed above. An uncertainty shock instead has a direct impact on the variance of current fundamentals. The longer its persistence, the stronger the response. So it matters a lot whether uncertainty is about current or future fundamentals, just as it matters how long uncertainty is expected to last.

We believe that many episodes of economic policy uncertainty, like Brexit, the threat of trade wars, the possible dissolution of the euro area, or more generally proposals of institutional or fiscal reforms, can be interpreted as news about a future that may be several years away. In these cases, economic policy uncertainty associated with the news may contribute to a gradual slowdown of the economy, depending on the expected duration of uncertainty, with very different propagation relative to the canonical uncertainty shocks studied in the literature. These considerations align well with the empirical evidence in Baker, Bloom and Davis (2016), who find that shocks to economic policy uncertainty produce persistent, rather than V-shaped, impulse responses.

Finally, we investigate the role of heterogeneity in the impact of the Brexit shock, by setting $\omega = 0$, and rescaling X^S and X^H to keep the first and second moments unchanged.[12] In this case, depicted by the broken blue line, aggregate and idiosyncratic productivity shocks affect all firms identically, irrespective of their productivity. The responses are virtually unchanged relative to the baseline. So while the parameter ω is necessary to match the moments in Table 1, it does not affect transitional dynamics beyond its effect on the first moment.

[12] We set $X^S = 0.010$ and $X^H = -0.006$.

5 Conclusions

We have investigated the effects of the Brexit referendum using a heterogeneous firms model where the uncertainty about the beliefs associated with different Brexit outcomes is informed by expectation data. The quantitative analysis reveals that the recent slowdown of the UK economy can be explained by the referendum. So this "puzzling" behavior is no longer a puzzle if one models Brexit uncertainty carefully. Our results also indicate that the expected duration of the negotiations is key for the propagation of the news, and that its effects are larger, the sooner uncertainty is expected to resolve. The policy implication is that to keep postponing the Brexit deadline generates a succession of cliff edges in the negotiations that, by setting up expectations of a quick resolution of uncertainty, maximizes its damage.

Moving beyond the case of Brexit, our results indicate that uncertainty may propagate differently, depending on whether it concerns current or future fundamentals and on how long it is expected to last. While typical episodes of uncertainty, like those observed during recessions, are about the state of fundamentals over the short run and are expected to resolve relatively soon, economic policy uncertainty often stems from news about fundamentals in the long run, and may be expected to last for longer. More work remains to be done to sharpen our understanding of how these propagation mechanisms differ.

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Appendix A Method of Simulated Moments

The Method of Simulated Moments (MSM) retrieves the parameters that minimize the sum of squared residuals between the moments of the data and those of the model. The minimization problem reads as follows:

$$\Theta = \arg\min_{\Theta} \mathbf{d}(\Theta)' W \mathbf{d}(\Theta), \tag{10}$$

where Θ is a $N \times 1$ vector of parameters, $\mathbf{d}(\Theta)$ is an $M \times 1$ vector of residuals, and W is an $M \times M$ weighting matrix. It is required that there should be at least as many parameters (N) as moments (M), that is $N \geq M$. In the case of $N = M$ the model is just-identified, whereas if $N > M$ the model is over-identified. Note that setting W as a matrix with the reciprocal of the squared data moments on the diagonal and zero elsewhere implies that solving eq. 10 is equivalent to minimizing the sum of squared residuals between the moments of the data and those of the model.

In order to solve the MSM we rely on the root-finding method of Nelder and Mead (1965). Since we use a local root-finding method, we conduct robustness checks by altering both the initial starting values and the step factor. We find that the results are not sensitive to such modifications.

Appendix B Computational Strategy

This section describes the numerical methods used to compute the model's steady state, as well as the transitional dynamics under policy uncertainty. We first describe how the state space is discretized.

B.1 State Space Discretization

The model contains four states: idiosyncratic productivity (a), idiosyncratic policy state (z), idiosyncratic capital (k), aggregate state (ζ). The discretization of the four states is as follows:

- The idiosyncratic productivity (a) is discretized into a grid $\mathbf{a} \in \{\bar{a}_1, ..., \bar{a}_{N_a}\}$ comprising of $N_a = 15$ log-linearly spaced points.
- The idiosyncratic policy state (z) is discretized into a grid containing $N_z = 3$ idiosyncratic states represented by $\mathbf{z} \in \{\bar{z}_+, \bar{z}_-, \bar{z}_0\}$.
- The idiosyncratic capital (k) is discretized into a grid $\mathbf{k} \in \{\bar{k}_1, ..., \bar{k}_{N_k}\}$ containing $N_k = 100$ points spaced log-linearly between 1×10^0 and 1×10^2.
- The aggregate states (ζ) are four ($N_\zeta = 4$): the pre-referendum state (ζ^P), the negotiations state (ζ^N), the Soft Brexit state (ζ^S) and the Hard Brexit state (ζ^H). These aggregate states can be represented by the following grid $\mathbf{Z} \in \{\overline{\zeta^P}, \overline{\zeta^N}, \overline{\zeta^S}, \overline{\zeta^H}\}$.

The state space is $N_a \times N_k \times N_z \times N_\zeta$, or more specifically $15 \times 100 \times 3 \times 4$.

We also discretize the following three exogenous stochastic processes:

- The stochastic process of the aggregate states can be represented by the transition matrix Γ_ζ of size $N_\zeta \times N_\zeta$, where $\sum_{l=1}^{N_\zeta} \pi_{j,l}^{\zeta} = 1$ for all $j \in \{1, ..., N_\zeta\}$. The transition matrix probabilities is given by: The transition matrix for the aggregate policy state of the economy is given by:

$$\Gamma_\zeta(\zeta_{t+1} = \zeta^i | \zeta_t = \zeta^q) =$$

$$\begin{array}{lc} \downarrow \zeta_t^q, \zeta_{t+1}^i \rightarrow & \begin{array}{cccc} \zeta^N & \zeta^S & \zeta^H & \zeta^P \end{array} \\ \begin{array}{l} \zeta^N \\ \zeta^S \\ \zeta^H \\ \zeta^P \end{array} & \begin{pmatrix} 1-\gamma^R-\theta & \theta(1-\gamma^H) & \theta\gamma^H & \gamma^R \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \end{array}. \tag{11}$$

- The stochastic process of idiosyncratic productivity can be represented by the transition matrix Γ_a of size $N_a \times N_a$ discretized using the Tauchen's method.
- The stochastic process of the idiosyncratic policy states can be represented by the transition matrix Γ_z of size $N_a \times N_a \times N_z \times N_z$, where $\Gamma_z(\zeta' = \zeta^j, z' | \zeta = \zeta^i, z) = \mathbb{I}$ if $i = \{P, S, H\}$ and $j = \{P, N, S, H\}$, as these states do not entail the draw of z. Moreover, if $i = \{N\}$, $j = \{N, P\}$, and $z_n = \{z_+, z_0, z_-\}$: $\Gamma_z(\zeta' = \zeta^j, z' | \zeta = \zeta^i, z) = \mathbb{I}$. However, if $i = \{N\}$, $j = \{S, H\}$, and $z_n = \{z_+, z_0, z_-\}$:

$$\Gamma_z(\zeta' = \zeta^j, z' | \zeta = \zeta^i, z) = \begin{array}{lc} \downarrow z, z' \rightarrow & \begin{array}{ccc} z_+ & z_0 & z_- \end{array} \\ \begin{array}{l} z_+ \\ z_0 \\ z_- \end{array} & \begin{pmatrix} q_+ & (1-q_+-q_-) & q_- \\ q_+ & (1-q_+-q_-) & q_- \\ q_+ & (1-q_+-q_-) & q_- \end{pmatrix} \end{array}. \tag{12}$$

B.2 Steady State

We compute the stochastic steady states of the pre-referendum economy abstracting from the possibility that Brexit may happen, i.e. we do not calculate the *"risky"* steady state. In what follows, we describe the solution algorithm based on value function iteration.

B.3 Steady State Solution Algorithm:

1. Solve the problem of the firms using value function iteration, given the prices β and w:

   (a) guess an initial value function $V_g(a, z, k)$, for instance $V_g(e, z, k) = 0$;

(b) solve for $V^{NA}(a,z,k)$ and $V^{A}(a,z,k)$ by taking expectations over the exogenous processes of a and z and using $V'(a,z,k) = V_g(a,z,k)$, and obtain the policy functions $K(a,z,k)$ and $L(a,z,k)$;

(c) using $V^{NA}(a,z,k)$ and $V^{A}(a,z,k)$, find $\tilde{V}(a,z,k,\phi)$;

(d) then find the policy function for the fixed capital adjustment cost threshold $\phi^T(a,z,k)$;

(e) calculate $V(a,z,k)$ by taking expectations of $\tilde{V}(a,z,k,\phi)$ over ϕ using the threshold $\phi^T(a,z,k)$;

(f) check whether the absolute percentage deviation between the guessed value function $V_g(a,z,k)$ and the obtained value function $V(a,z,k)$ is within a pre-set tolerance. If the absolute deviation is smaller than the tolerance then exit the algorithm and save the optimal policy functions ($K(a,z,k)$,$L(a,z,k)$,$\phi^T(a,z,k)$), otherwise update the guess $V_g(a,z,k) = V(a,z,k)$ and repeat steps (a)-(e) until convergence.

2. Using the policy functions $K(a,z,k)$ and $\phi^T(a,z,k)$, solve for the stationary distribution as a fixed point, defined as $\mu'(a',z',k') = \mu(a,z,k)$, by iterating on the distribution of firms over idiosyncratic productivity, idiosyncratic policy, and idiosyncratic capital holdings. In doing so, the transitional probability matrices, Γ_a and Γ_z, for the exogenous processes for a and z, respectively, are used for the evolution of the distribution:

$$\begin{aligned} &\mu'(a',z',k') \\ &= \sum_{a\in\mathbf{a}}^{N_a}\sum_{z\in\mathbf{z}}^{N_z}\mu(a,z,k)\Gamma_a(a'=a_l|a=a_q)\Gamma_z(z'=z_i|z=z_j)\mathbb{I}(k',a,z,k), \end{aligned} \tag{13}$$

where $\mathbb{I}(k',a,z,k) = 1$ if $k' = K(a,z,k)$ and 0 otherwise.

3. Once the stationary distribution is obtained, it is possible to multiply it by the relevant policy decision to obtain the aggregates K, L, Y, I.

B.4 Transitional Dynamics under Policy Uncertainty

We solve for the transitional dynamics under policy uncertainty. The model features policy uncertainty, coming through the stochastic processes represented by the transition matrices Γ_ζ and Γ_z.

We set $T = 100$ and $N^* = 16$, where T is the total number of periods in the simulation and N^* denotes the first period in which uncertainty is resolved. For the transition from ζ^P to ζ^j where $j \in \{S,H,P\}$:

1. Solve the model for the initial steady state (ζ^P) using value function iteration and obtain the initial distribution $\mu_0(a,z,k)$ by solving the fixed point of the stationary distribution.

2. Solve the model for all the aggregate states with the aggregate policy stochastic process (Γ_ζ) and the idiosyncratic policy stochastic process (Γ_z) and obtain the optimal policy functions $K_t(a,z,k;\zeta^i)$, $L_t(a,z,k;\zeta^i)$, and $\phi_t^T(a,z,k;\zeta^i)$ where $i \in \{P,N,S,H\}$ using value function iteration.

3. Using the optimal policy functions and $\mu_{t-1}(a,z,k)$, obtain aggregates and solve for the next period distribution $\mu_t(a,z,k)$ for $t = 1,...,N^*$ under the aggregate state ζ^N.

4. Again, using the optimal policy functions and $\mu_{t-1}(a,z,k)$, obtain aggregates and solve for the next period distribution $\mu_t(a,z,k)$ for $t = N^*+1,...,T$ under the aggregate state ζ^j.

We have used alternative maximum time periods for the algorithm, namely, $T = 200, 300$ and checked that the results do not change.

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