The Rising Cost of Climate Change: Evidence from the Bond Market[*]

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Abstract

The level of the social discount rate (SDR) is a crucial factor for evaluating the costs of climate change. We demonstrate that the equilibrium or steady-state real interest rate is the fundamental anchor for market-based SDRs. Much recent research has pointed to a decrease in the equilibrium real interest rate since the 1990s. Using new estimates of this decline, we document a pronounced downward shift in the entire term structure of SDRs in recent decades. This lower new normal for interest rates and SDRs has substantially boosted the estimated economic loss from climate change and the social cost of carbon.

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1 Introduction

When the costs and benefits of an action are distributed over time, their assessment requires discounting future values to produce present values that can be appropriately compared. This discounting reflects the fact that a given amount of resources is generally more valuable today than in the future, and it is especially important for a cost-benefit analysis addressing climate change. The anthropogenic increase in greenhouse gases—the cause of climate change—will dissipate extremely slowly: undisturbed, a significant fraction of today's emissions of greenhouse gases, notably carbon dioxide, will remain in the atmosphere for centuries warming the earth. Therefore, current carbon pollution is creating climate hazards for many generations to come. The precise value of the discount rate is critically important for making such intergenerational assessments given that the costs are distributed over a very long time span and the effect of discounting increases exponentially with time.

For assessing *public policy* actions, a comparison of inflation-adjusted costs and benefits at different horizons employs the social discount rate (SDR). For a cost-benefit analysis of climate change that spans decades and centuries, determining the appropriate value of the SDR is a key consideration, and there is a very large literature analyzing and debating the choice of the SDR in this context. One major divide in determining the appropriate level of the SDR has been between the prescriptive and descriptive approaches (Arrow et al., 1996). The former stresses a normative view that derives an SDR based on ethical fundamentals, while the descriptive approach relies on observed financial market valuations, which implicitly embed public preferences about intertemporal trade-offs. The SDRs based on the prescriptive approach have generally been lower than those based on the descriptive approach. Indeed, an analysis of a survey of academic experts on social discounting by Drupp et al. (2018) indicates that estimates of a pure descriptive SDR tend to be about 2 percentage points higher than estimates of a pure prescriptive SDR. This disparity has been one important impediment to reaching a consensus on the SDR to use for valuing future damages from climate change.

However, we argue that the situation has changed in recent decades as a shift in economic and financial fundamentals has resulted in a lower new normal for interest rates, which in turn implies lower discount rates at all horizons. Specifically, there is extensive evidence that the equilibrium or steady-state real interest rate, commonly denoted as r_t^*, has fallen since the 1990s.[1] One key driver of this steady decline in real interest rates is the dramatic demographic transition going on around the world—especially the trend toward an aging population. Over long samples and across countries, the working age fraction of the total population and the

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[1] Discussions and estimates of the decline in r_t^* include Laubach and Williams (2016), Del Negro et al. (2017), Christensen and Rudebusch (2019), and Bauer and Rudebusch (2020).

average life expectancy of individuals appear to be linked to the real interest rate.[2] Such co-movements are relevant for many countries in which a longer life expectancy and a steady retirement age appear to have resulted in more aggregate saving for post-employment years and lower real interest rates. Other underlying economic drivers of the decline in r_t^* include lower productivity growth, a fall in the price of capital goods, and strong precautionary saving flows from emerging market economies (Summers, 2014; Hamilton et al., 2016). These forces have tended to increase global savings or reduce desired investment, and as a result have exerted downward pressure on the equilibrium real rate. Moreover, these various economic trends show no signs of reversing and are likely to persist for some time with important effects for evaluating long-term issues such as climate change.

Indeed, the gradual reduction in r_t^* has profound implications for the economics of climate change that have generally gone unrecognized. Therefore, in this paper, we use an asset pricing framework to show theoretically and empirically the implications of this secular decline on the term structure of SDRs. We find that the current market-based SDR—calculated using prevailing data on risk-free real interest rates within a modern asset pricing structure—is much lower than than just two or three decades ago and much closer to the low discount rates often obtained from a prescriptive approach. Hence, a descriptive approach to the SDRs that incorporates current estimates of r_t^* results in a substantially higher discounted present value of future damages from marginally more carbon emissions—the social cost of carbon—than in the past. In essence, we demonstrate that the prevailing lower new normal for interest rates in turn implies a higher new normal for the market-based present value of climate change damages.

Based on the insight that r_t^* is a key element underlying descriptive social discounting, our paper makes three contributions to the debate about the appropriate SDR and the associated cost-benefit analysis of climate change. First, from a macro-finance perspective, as in Bauer and Rudebusch (2020), we show that conceptually r_t^* is the crucial anchor of the term structure of discount rates. The central importance of r_t^* reflects the fact that it acts as a level factor for this term structure. While short-term SDRs depend on current levels of interest rates and longer-horizon SDRs are reduced by the effect of uncertainty about the future (Weitzman, 1998, 2001; Gollier, 2002), the entire term structure of SDRs moves one-for-one with the equilibrium real interest rate. Intuitively, the reason for this anchoring is that r_t^* corresponds to the long-run trend, that is, the *permanent component* of real interest rates. Therefore,

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[2] Carvalho et al. (2016) and Lunsford and West (2019) investigate the empirical link between demographic trends and the equilibrium real interest rate. Gerlagh et al. (2017) use a calibrated, analytical model with overlapping generations to show theoretically that changes in population dynamics can have important effects on capital returns and optimal climate policies.

while various factors affect real interest rates at different maturities and horizons, shifts in this long-run trend move real rates at all maturities, and hence affect the entire term structure of SDRs.

Next, we quantify the fall in r_t^* in recent decades and its effect on the term structure of SDRs. Over the past quarter century, market interest rates have declined substantially. The empirical evidence indicates that the equilibrium real interest rate has been the main driver pushing bond yields lower, while falling inflation expectations and risk premia had only an ancillary role.[3] We provide new r_t^* estimates from a range of different time series models of the real interest rate using government bond and inflation expectations. Using our baseline model—an unobserved-components model for the one-year real interest rate, estimated using Bayesian methods—we find that r_t^* declined a bit more than 1 percentage point since the 1990s, which is consistent with existing macro-finance estimates from the literature. We then use our time series model estimates to calculate the implied term structure of SDRs at two different points in time: in 1990, before the equilibrium rate started its substantial downward drift, and in 2019, at the end of our sample. Consistent with our theoretical macro-finance framework, our empirical results show that the decline in r_t^* in recent decades has markedly shifted down the entire term structure of discount rates. We also find this result to be robust to the use of various alternative empirical specifications that allow for a possible shift in the long-run trend component of interest rates. In contrast, existing models in this literature have generally imposed a constant r^* by assuming stationary, mean-reverting dynamic specifications.[4] Our key finding is that market-based estimates of the SDR that account for the shift in the equilibrium rate are now much lower than in the past and notably below estimates from a couple of decades ago.

Finally, we illustrate the important policy implications of a lower r_t^* in the context of the economics of climate change. The pronounced downward shift in social discount rates has broad policy relevance for a range of issues that require consideration of intergenerational discounting, including public projects on infrastructure, health care, and so forth. We focus on what is widely viewed as the most pressing policy challenge of this century: assessing the economic impacts of climate change and incorporating such assessments into public policies on climate change mitigation and adaption. Specifically, we quantify how much the downward shift in the term structure of SDRs alters the social cost of carbon (SCC)—the present value

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[3] This attribution is supported by several recent studies including Laubach and Williams (2016), Del Negro et al. (2017), Christensen and Rudebusch (2019), and Bauer and Rudebusch (2020).

[4] Previous empirical work using stationary time series models for the SDR include Newell and Pizer (2003), Groom et al. (2007), Gollier et al. (2008), Hepburn et al. (2009), and Freeman et al. (2015). Newell and Pizer (2003) also include a random walk model but anchor their SDR estimates at the historical average real interest rate of four percent.

of all future marginal damages from an increase in carbon emissions—using damage estimates from the DICE model of William Nordhaus.[5] Given the long time lag of many of these future damages, the term structure of discount rates plays a central role in determining this present value. Our empirical models showing a gradual fall in r_t^* in recent decades imply a substantial increase in the SCC. Indeed, we estimate that the decline in r_t^* since the 1990s has caused the SCC to at least double in size. That is, the r_t^* decline has quantitatively important implications for valuing the economic consequences of climate change. Unlike past descriptive analyses, our estimates based on the current financial environment yield lower discount rates that are more closely aligned with previous low prescriptive ones, such as Stern (2007). Similarly, the associated higher SCC values are more consistent with strong efforts toward climate change mitigation.

Climate change is a global issue, while our empirical analysis uses U.S. data, namely, Treasury yields and price inflation, to quantify the decline in r_t^*. However, the structural factors underlying the measured drop in r_t^* in the United States—for example, the shifts in demographics, saving behavior, and productivity—are observed in many other countries as well. Furthermore, given generally low impediments to the cross-border mobility of capital, equilibrium real rates in different countries are equalized to a substantial degree by international financial flows.[6] Indeed, several papers have documented a global secular decline in r_t^* as interest rates have remained persistently low in many countries (Holston et al., 2017; Del Negro et al., 2019). Consequently, the SDR term structure has likely shifted lower worldwide, and our evidence from the United States is relevant for a calculation of the global SCC.

The paper is structured as follows. In Section 2, we provide a brief background on the role of discounting in the economics of climate change. Section 3 describes theoretically how r_t^* anchors and acts as a level shifter for the entire term structure of SDRs. Section 4 documents the empirical decline of r_t^* in the United States. In Section 5, we present new estimates of the term structure of SDRs, which show a substantial downward shift since the 1990s. Section 6 presents calculations of the SCC and demonstrates the substantial economic implications of lower discount rates for valuing future losses from climate change. Section 7 concludes.

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[5] Our approach to calculating the SCC is similar to that in Newell and Pizer (2003) and subsequent studies following their example (e.g., Freeman et al., 2015).

[6] To the extent that U.S. interest rates are affected, say, by foreign demand for safe assets, then U.S. yields also incorporate information about the social rate of time preference in other countries.

2 Discount rates in the economics of climate change

To set the stage for our analysis, we first outline key concepts and earlier approaches in the use of SDRs in the economics of climate change. Extensive surveys of the role of discounting in public policy and climate change include Arrow et al. (2014) and Gollier and Hammitt (2014).

In the cost-benefit analysis of public projects, some or most of the benefits of a given investment may accrue in the future. Consider a representative example in climate change economics: a mitigation project that reduces the amount of carbon in the atmosphere (either by reduced emissions or removal and storage). The up-front costs of this project must be weighed against the future benefits of reducing the environmental and economic damages that result from global warming, specifically, debilitating temperature increases and extremes, rising sea levels, species extinction, and destructive weather events, among others. The expected timing and magnitude of the potential benefits (reduced damages) of a carbon mitigation project are hard to estimate. However, researchers have used Integrated Assessment Models (IAMs), which link changes in greenhouse gas concentrations to climate and economic outcomes, to obtain complete time profiles of the benefits of carbon mitigation for cost-benefit analyses (Greenstone et al., 2013). Importantly, a substantial share of the damage from carbon emissions occurs in the far future. A significant fraction of greenhouse gas emissions remain in the atmosphere for a century and more, and the benefits from reducing them accrue over a similar period. In order to value these future benefits (which are adjusted for price inflation and expressed in real terms), it is necessary to discount them back to the present, so that the stream of future benefits over time is collapsed into a single number, their *present value*. The real interest rate used for discounting is a crucial determinant of the present value of these future benefits, with higher discount rates naturally lowering the present value. When evaluating public projects (as opposed to investment projects undertaken by a private corporation) the appropriate rate is the SDR.

To specify an SDR, the literature has generally taken one of two approaches (Arrow et al., 1996): a normative/prescriptive approach that recommends an SDR that conforms to certain standards of optimality and fairness, or a positive/descriptive approach that recommends an SDR based on observed market valuations. The former approach, detailed in Gollier (2013), focuses on economic models, behavioral parameters, and normative judgments, and, depending on specific assumptions, can obtain quite small discount rates. A prominent example is the famous Stern (2007) Review, which assessed climate change damages using a normative SDR of approximately 1.4 percent. By contrast, the descriptive approach, which is employed in this paper, is empirically based and infers SDRs from observed rates of return on capital and financial investments, which range from essentially risk-free Treasury bills to stock market indices.[7] The empirical descriptive approach has typically resulted in larger discount rates than the prescriptive method. Indeed, the fact that the normative SDR used in the Stern Review was lower than what a descriptive approach would recommend was viewed as a critical failing by Nordhaus (2007): "The Review's unambiguous conclusions about the need for extreme immediate action will not survive the substitution of assumptions that are more consistent with today's marketplace real interest rates and savings rates" (p. 686).[8] Similarly, Drupp et al. (2018) found that the SDRs recommended by experts depended on the relative weight given to normative versus positive concerns and that this dependence was so strong that a purely positive SDR would be 2 percentage points *higher* than a completely normative one.

Public regulatory agencies have used both normative and market-based SDRs (Arrow et al., 2014; Greenstone et al., 2013). Even with a purely descriptive approach, there are differing views about which market rates of return should inform social discounting and how to account for the risk associated with future damages. For example, the U.S. Office of Management and Budget has recommended that regulatory agencies consider both 7 and 3 percent real discount rates based on historical data (U.S. Office of Managment and Budget, 2003). The higher rate reflects an estimate of the average before-tax rate of return to private capital in the U.S. economy—a broad measure including returns to real estate and small business capital as well as corporate capital. The lower rate reflects "the rate that the average saver uses to discount future consumption," for which "the real rate of return on long-term government debt may provide a fair approximation," and from 1973 to 2003, the real rate of return on 10-year U.S. Treasury bonds averaged around 3 percent.[9]

We pursue a similar descriptive approach and obtain SDRs based on observed rates of return in financial markets. However, we focus on real (inflation-adjusted) government bond yields in order to estimate risk-free social discount rates, which reflect the social rate of time preference. Risk-free SDRs have long been an important benchmark for social discounting.[10] They are appropriate to discount expected future payoffs that are (i) certain, or (ii) riskless in the sense of being uncorrelated with future growth and marginal utility, or (iii) risk-adjusted

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[7] Prominent examples of a descriptive approach in climate change cost-benefit analysis include Newell and Pizer (2003), the U.S. interagency working group summarized in Greenstone et al. (2013), and Nordhaus (2014). Moreover, Nordhaus (2007, p. 692) argues that normative discount rates are "irrelevant" for public policy and that determining the appropriate benchmark for climate investments requires a careful look at "the *real* real interest rate." We argue that the "real" real interest rate has fallen to a lower new normal.

[8] Also, see Weitzman (2001, 2007).

[9] A U.S. interagency working group on the social cost of carbon calculated the impact of emissions using three different discount rates: 2.5, 3, and 5 percent (Greenstone et al., 2013).

[10] See, for example, Newell and Pizer (2003), Groom et al. (2007), and Drupp et al. (2018). Li and Pizer (2018) advocate using a risk-free interest rate, a "consumption rate," for discounting long-run damages and strongly caution against using the return on capital for long-run discounting.

and thus expressed as certainty-equivalent expected payoffs. By contrast, SDRs based on estimates of the economy-wide return on capital are conceptually problematic and fraught with serious measurement problems. Conceptually, the return on capital is the appropriate discount rate only for projects with risk characteristics exactly like those of the aggregate productive capital stock. Empirically, the measurement of an economy-wide return is very uncertain, with difficulties, for example, in accounting for separate returns to capital and labor in unincorporated businesses, determining economic depreciation rates, and imputing the intangible capital stock, say, of intellectual property.[11] Risk premia and economic distortions, including market power and asymmetric information, create a divergence between the rate of return that savers earn and the private rate of return to capital. The sizable wedge between these two returns has also widened considerably in recent decades, and there is much agreement that growing risk premia play an important role in this widening gap.[12]

Risk-free SDRs constitute a benchmark that exclusively captures time discounting, but in many cases, uncertain future payoffs will make it necessary to incorporate risk discounting as well. That is, the risk characteristics of the future benefits or damages a project help determine the appropriate SDR. Standard asset pricing logic implies that a project is risky if its payoffs are positively correlated with aggregate consumption growth—that is, if the so-called consumption β is positive—and discounting should use a rate above the risk-free SDR. Conversely, for a negative β project, the discount rate should be below the risk-free rate, as the project provides hedging benefits against future aggregate risks. However, there is no consensus about the risk characteristics of climate change mitigation projects. For example, Dietz et al. (2018) estimate an overall positive "climate beta," while Giglio et al. (2018) argue for a negative one.[13] Estimation of the climate beta is an important but difficult issue for the economics of climate change. In this paper, we concentrate on time discounting, based on the premise that more accurate estimation of risk-free SDRs is an important stepping stone in the literature on social discounting. Conceptually, the risk characteristics of any projects under consideration could be accounted for by adjusting either our risk-free discount rates or the expected future payoffs. In the latter case, "certainty-equivalent" payoffs are adjusted

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[11] Economy-wide estimates are usually motivated by historical stock market returns as a proxy estimate for the overall return on capital (Weitzman, 2007), although the equity market covers only a limited portion of the aggregate capital stock.

[12] Caballero et al. (2017) and Farhi and Gourio (2019) examine the growing divergence between the risk-free rate and the aggregate return on capital and consider a variety of possible factors including increases in risk premia, market power, unmeasured intangibles, and savings supply together with a slowdown in the growth of technology.

[13] An additional layer of complexity is that each mitigation or adaption project will have different risk characteristics and a specific consumption β and so will require a tailored SDR to appropriately discount its future payoffs.

for their systematic risk—for example, certainty-equivalent payoffs for a risky project with a positive consumption β are lower than the expected payoffs—and can thus be discounted using risk-free rates.[14]

In addition, a single value for the SDR is unlikely to be suitable for discounting all future costs and benefits. Instead, the appropriate discount rate for various costs and benefits will vary with the horizon of their realization. That is, there should be a *term structure* of discount rates for discounting future costs and benefits that takes into account their timing. In contrast to the deep disagreement about the overall level of the SDR, there is more of a consensus that the SDR term structure should decline with the length of the discounting horizon.[15] In an empirical analysis, it is therefore crucial to model the entire term structure of SDRs, as we do in this paper. Our results provide further support for a downward-sloping SDR term structure from a finance perspective.

In brief, we take a descriptive approach to social discounting and consider what observed and projected risk-free interest rates imply for the term structure of SDRs based on real government bond yields. Since government bonds generally have maturities of only 30 years or less—too short to derive SDRs at the very long horizons needed to discount climate change damages—we will combine the available market data with empirical interest rate models to derive the requisite long-maturity SDRs. Our methodology, based on econometric models for real interest rates combined with asset pricing methodology, was labeled the "econometric expected net present value (ENPV)" approach by Freeman and Groom (2016). This approach is based on the notion that the stochastic process generating future interest rates can be estimated from historic data, and uses the assumption of risk-neutrality, i.e., the expectations hypothesis, to derive long-term SDRs from expected future short-term SDRs. Other studies following the ENPV approach include Newell and Pizer (2003), Groom et al. (2007), Gollier et al. (2008), Hepburn et al. (2009), and Freeman et al. (2015). Our key difference with these previous studies is that we allow for the existence of a long-run trend in real interest rates, instead of imposing mean reversion to a fixed long-run mean. Allowing for this nonstationarity turns out to be vitally important for estimating term structures of SDRs and the social cost of carbon.

3 The role of the equilibrium real rate in discounting

We now introduce a macro-finance framework for discounting and show theoretically why the long-run trend in the real rate, r^*_t, is the crucial determinant of SDRs. The fundamental

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[14] For a useful treatment of the role of risk for discounting, see Gollier (2013, ch. 12).

[15] See, for example, Weitzman (2001), Newell and Pizer (2003), Cropper et al. (2014) and Arrow et al. (2014).

concept is a real zero-coupon bond, that is, a claim at time t to one unit of consumption at future time $t+n$. The price of an n-period bond, which we denote by $P_t^{(n)}$, is also the appropriate discount factor for valuing future payoffs at time $t+n$, provided that these payoffs are either certain/riskless or, in case they are risky, first translated into certainty-equivalent payoffs.[16] The present value of future payoffs, X_{t+n}, is then

$$PV_t = P_t^{(n)} E_t \left(X_{t+n} \right). \tag{1}$$

The short-term real interest rate r_t corresponds to the log-return on a one-period real bond, that is $\exp(r_t) = 1/P_t^{(1)}$. Under the assumption of *risk-neutrality* (the expectations hypothsis), the n-period discount factor is related to expected future short-term interest rates by

$$P_t^{(n)} = E_t \left[\exp \left(- \sum_{j=0}^{n-1} r_{t+j} \right) \right]. \tag{2}$$

The "certainty-equivalent discount rate" in the language of Weitzman (1998), known in the fixed income literature as a (continuously compounded) real zero-coupon bond yield, is

$$y_t^{(n)} = -\frac{1}{n} \log P_t^{(n)}. \tag{3}$$

Finally, the forward discount rate that is used to discount payoffs in $t+n+1$ back to period $t+n$, is $f_t^{(n)} = \log P_t^{(n)} - \log P_t^{(n+1)}$.

To derive long-term rates from our dynamic model of short-term rates we follow the existing literature and impose the expectations hypothesis. That is, we assume that there is no term premium in long-term discount rates. While there is ample evidence that long-term bond yields for maturities of, say, five or ten years include a term premium and thus differ from the expected return of rolling over short-term bonds, no such evidence exists for long-run SDRs with maturities of hundreds of years, simply because these are not observable. Instead of undertaking a heroic attempt of simultaneously modeling the dynamics of discount factors, fundamentals, and risk premia over the very long run, we follow the tradition of obtaining long-run SDRs from a model of short-run SDRs under the expectations hypothesis, as in, for example, Weitzman (2001), Newell and Pizer (2003), and Freeman et al. (2015).

The crucial determinant of the term structure of SDRs is the stochastic process of the short-term real rate, r_t. Much of the literature on fixed income modeling and social discounting

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[16] Even stochastic payoffs can be riskless, namely if they are uncorrelated with the future real stochastic discount factor, that is, with future real interest rates and marginal utility. Risky payoffs can be discounted with risk-free rates if they are first translated into certainty-equivalent payoffs; see the discussion in Section 2.

has assumed that this process is stationary (or $I(0)$ in time series terminology). Under this assumption, r_t exhibits reversion to a fixed long-run mean, which is the constant equilibrium real rate, $r^* = E(r_t)$. This assumption of stationarity is, however, at odds with evidence that real interest rates contain a slow-moving trend component (e.g., Rose, 1988) and with a growing literature that documents a pronounced decline in the equilibrium real rate—as surveyed in Section 4. Indeed, Bauer and Rudebusch (2020) show that it is crucial to include a stochastic trend in the real rate process in order to fully capture the highly persistent component of interest rates and explain the dynamic behavior of bond risk premia. Given the compelling evidence for nonstationarity in real interest rates, we relax the assumption of a constant mean and allow the long-run mean of the real interest rate to be potentially time-varying—a so-called shifting endpoint. In time series terminology, we allow for the real short rate to contain a stochastic trend (i.e., to be $I(1)$).[17] The quantitative importance of such a stochastic trend for SDRs and discounting climate damages is an empirical question that we investigate below.

Formally, we define the equilibrium real rate as the long-run stochastic trend component of r_t, that is,

$$r_t^* = \lim_{h\to\infty} E_t r_{t+h}, \tag{4}$$

which represents the real rate that prevails in the economy after all shocks have died out. Therefore, the stochastic trend in the real short rate is precisely r_t^*.[18] As a matter of definition, we can decompose the real short rate into trend and non-trend components,

$$r_t = r_t^* + \tilde{r}_t, \tag{5}$$

where the non-trend "cyclical" component $\tilde{r}_t$ is stationary and has a long-run mean of zero. Long-term discount rates—whether expressed as yields or forward rates—are determined by expectations of future short-term discount rates,

$$E_t(r_{t+h}) = r_t^* + E_t(\tilde{r}_{t+h}). \tag{6}$$

How far these expectations deviate from r_t^* depends on the size of the current cyclical deviation $\tilde{r}_t$ and its speed of mean reversion to zero. Over the very long horizons relevant for discounting future climate change, expectations of future short-term discount rates will generally be close

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[17] The process is $I(1)$ if the first difference, $r_t - r_{t-1}$, is covariance stationary. This includes the special case that r_t itself is already stationary, or $I(0)$.

[18] This is the trend concept of Beveridge and Nelson (1981) assuming that r_t contains no deterministic time trend. In general, r_t^* is a martingale and specifically a random walk (without drift) when innovations $r_t^* - r_{t-1}^*$ are *iid*, which we assume below.

to r_t^*. But an important insight is that r_t^* affects the full future path of expected rates equally and thus acts as a *level factor* that can shift the entire term structure of SDRs.

Of course, long-term discount rates are *not* equal to average expected future short-term rates due to the nonlinear relationship (3) between bond prices and yields. The resulting Jensen inequality or "convexity" effect underlies the insight of Weitzman (1998, 2001) that discount rates are decreasing with maturity in an environment of uncertainty (Cropper et al., 2014). We can use equations (2) and (3) to obtain this expression for discount rates:

$$y_t^{(n)} = -\frac{1}{n} \log E_t \left[\exp \left(-\sum_{j=0}^{n-1} r_{t+j} \right) \right] = \frac{1}{n} \sum_{j=0}^{n-1} E_t r_{t+j} + z_t^{(n)} = r_t^* + \frac{1}{n} \sum_{j=0}^{n-1} E_t \tilde{r}_{t+j} + z_t^{(n)}, \quad (7)$$

where $z_t^{(n)}$ is a convexity term that is negative and declining with maturity n.[19] Short-term discount rates are sensitive to near-term cyclical fluctuations in real rates, $\tilde{r}_t$, while long-term discount rates are pushed down by negative convexity effects. However, discount rates of *all* maturities vary with the equilibrium real rate r_t^*. Indeed, discount rates are all affected equally by r_t^*, which acts as a level factor for the term structure of SDRs. Appendix A formally derives this result in the context of a simple parametric model for the real rate.

The well-known Ramsey equation for the social discount rate, which arises in a simple intertemporal utility optimization setting, provides a common frame of reference in the literature on social discounting. We can extend the standard Ramsey formulation to show how accounting for structural economic change naturally gives rise to a social discount rate that includes a time-varying trend component r_t^*. With time-separable power utility, the usual intertemporal optimality condition can be rewritten to yield the following expression for the term structure of SDRs:

$$y_t^{(n)} = \delta + \frac{\gamma}{n} E_t \log(c_{t+n}/c_t) - \frac{\gamma^2}{2n} Var_t \log c_{t+n}, \quad (8)$$

where δ is the pure rate of time preference, γ is the curvature parameter of the period-utility function, and c_t is consumption (e.g., Gollier, 2013, ch. 4). For constant consumption growth, we obtain the classic Ramsey equation, and for *iid* Gaussian consumption growth, equation (8) yields the extended Ramsey equation.[20] As a more general formulation, we specify consumption growth as

$$\Delta \log c_{t+1} = g_t^* + \tilde{g}_t + \varepsilon_{t+1},$$

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[19] How quickly and to what limit $z_t^{(n)}$ declines as n increases depends on the stochastic properties of r_t. If r_t is conditionally Gaussian (and bond prices thus log-normal), then $z_t^{(n)} = -\frac{1}{2n} Var_t \sum_{j=0}^{n-1} r_{t+j}$.

[20] In both cases the term structure of SDRs is flat because there is no uncertainty about future values of r_t.

with a randow-walk trend component, $g^*_t = g^*_{t-1} + \eta_t$, a stationary, mean-zero cyclical component, $\tilde{g}_t = \phi \tilde{g}_{t-1} + u_t$, and an innovation ε_{t+1}. The shocks η_t, u_t and ε_t are assumed to be *iid* and Gaussian. The crucial feature is that consumption growth has a shifting endpoint, g^*_t, instead of a constant long-run mean.

With this consumption growth process, the term structure of SDRs is

$$y^{(n)}_t = \delta + \gamma g^*_t + \frac{\gamma(1-\phi^n)}{n(1-\phi)}\tilde{g}_t - \frac{\gamma^2}{2n} Var_t \log c_{t+n}, \tag{9}$$

where g^*_t is a level factor and $\tilde{g}_t$ is a slope factor (a high value of $\tilde{g}_t$ implies a downward slope). The final term reflects the convexity of the discount factor and of course declines in n, but its exact formulation and behavior are of second-order interest here.[21] The short-term rate and its trend are

$$r_t = y^{(1)}_t = \delta + \gamma(g^*_t + \tilde{g}_t) - \frac{\gamma^2}{2}\sigma^2_\varepsilon \quad \text{and} \quad r^*_t = \delta + \gamma g^*_t - \frac{\gamma^2}{2}\sigma^2_\varepsilon.$$

That is, the real-rate trend r^*_t and the trend in consumption growth g^*_t are connected linearly, and either one can be viewed as the level factor in the term structure of SDRs. Therefore, any long-run structural economic change that shifts the trend component in consumption growth also gives rise to a trend component and a level factor in real interest rates. Whether this theoretical result is of empirical importance—that is, whether r^*_t exhibits meaningful variation over time that matters quantitatively for the term structure of SDRs—is the question we turn to next.

4 The recent decline in the equilibrium real rate

Much research has examined the structural economic factors pushing down the equilibrium real rate in recent decades.[22] Important candidate explanations of a lower new normal for real interest rates include slower trend productivity growth, an aging population, and the reduced capital intensity of production. Such underlying fundamental economic shifts appear to have boosted global saving, reduced investment demand, and, as a result, lowered the steady-state real interest rate. Examinations of the longer-run historical record generally find that demographic variables have the most reliable connection with real interest rates (Lunsford and West, 2019). Not surprisingly then, the link between the changing demographic structure of

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[21] Specifically, the final term decreases rapidly with maturity n and is unbounded. It does not converge to a finite limit because consumption growth contains a unit root and thus $Var_t \log c_{t+n}$ is $O(n^2)$.

[22] Discussions of the decline in r^*_t include Summers (2014), Hamilton et al. (2016), Del Negro et al. (2017), Holston et al. (2017), Christensen and Rudebusch (2019), Bauer and Rudebusch (2020), and many others.

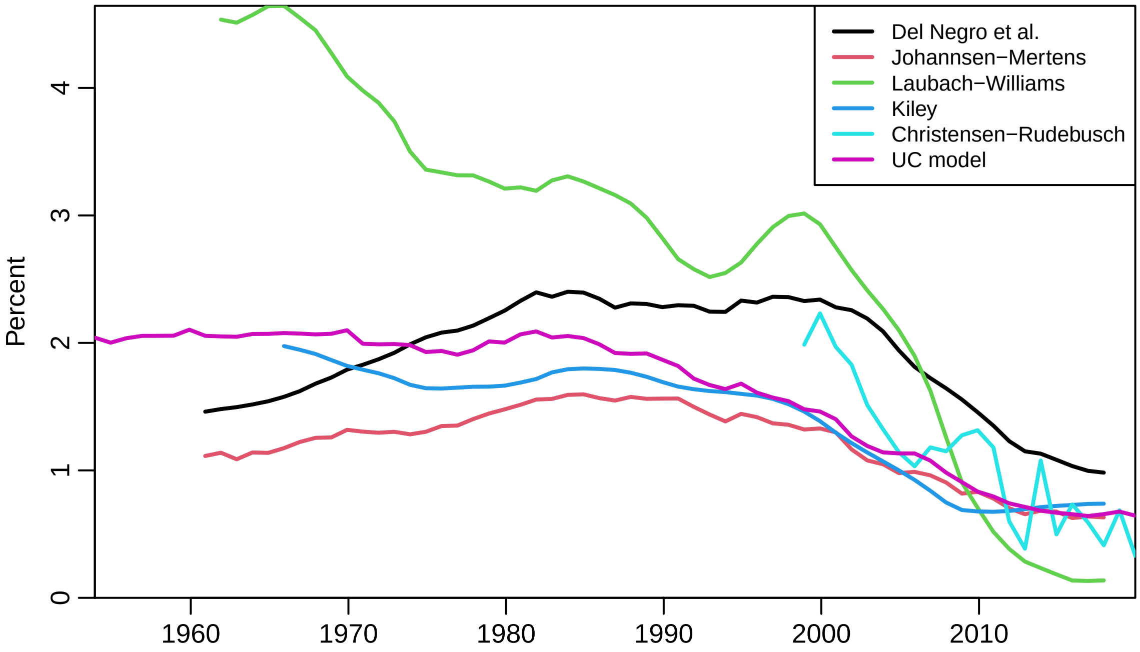
global economies and real interest rates has been the focus of recent theoretical and empirical work (e.g., Carvalho et al., 2016). Moreover, the various economic forces that have lowered the equilibrium real rate appear likely to persist. Demographic forces, in particular, are expected to continue to exert downward pressure on real interest rates going forward (Gagnon et al., 2016). Consequently, the structural shifts in the balance of supply and demand for aggregate savings and the downward trend in the equilibrium real rate are unlikely to be reversed in the foreseeable future.

The secular decline in the equilibrium real rate has garnered much interest because it has important implications for a wide range of issues. For various macroeconomic debates during the past decade, such as worries about secular stagnation (Summers, 2014), the drop in r_t^* has been a central concern. For monetary policy discussions, r_t^* is a key neutral policy benchmark and a signal of possible policy constraints (Yellen, 2015). Financial investors also track r_t^* as an anchor for projections of future private discount rates used in valuing financial assets (Clarida, 2014). This paper argues that the special role r_t^* has for the public economics of climate change should be added to this list.

Given the importance of the equilibrium real interest rate, researchers have used various empirical methods from macroeconomics and finance to try to pin it down. For example, Laubach and Williams (2003, 2016) infer r_t^* by using the Kalman filter to distinguish the real interest rate trend and cycle with a simple macroecomic model and data on a nominal short-term interest rate, consumer price inflation, and the output gap. Johanssen and Mertens (2016) and Lubik and Matthes (2015) provide closely related r_t^* estimates from a similar filtering of the macroeconomic data. By contrast, Christensen and Rudebusch (2019) use a finance model to estimate r_t^* using only real bond yields measured from inflation-indexed debt—namely, U.S. Treasury Inflation-Protected Securities (TIPS). These securities can provide a fairly direct reading on real yields since 1997 when the TIPS program was launched, and an arbitrage-free dynamic term structure model is used to help identify the underlying r_t^* in spite of the presence of potentially sizable liquidity and risk premia. Such a finance-based measure of r_t^* has several potential advantages relative to macro-based estimates as it does not depend on a correct specification of the output and inflation dynamics. Our new estimates of r_t^* in the next section are broadly similar in spirit to the finance-based estimates.

Along with this variety of methods, there are several somewhat different conceptual definitions of the equilibrium real rate in the literature. Some researchers focus on a short-run equilibrium rate, which represents the current value of the real rate that would be consistent with the economy at full employment and stable inflation. Others consider a very long-run empirical equilibrium rate defined as the real rate that would prevail in the infinite future, as calculated, for example, from a statistical trend-cycle decomposition of real rates. In practice, these different definitions appear to be closely related, and in many models they coincide, for example, as in Laubach and Williams (2003). For our purpose, the long-run trend is the relevant concept, since that is what matters for the term structure of social discount rates.

Figure 1: Macro-finance estimates of the equilibrium real interest rate



Alternative published estimates of the equilibrium real interest rate, r_t^*. The estimates are smoothed/two-sided estimates of the state-space models with macroeconomic and financial variables in Del Negro et al. (2017), Johanssen and Mertens (2016), Laubach and Williams (2016), Kiley (2020) and Christensen and Rudebusch (2019). The series are quarterly from 1971:Q4 to 2018:Q1.

In Figure 1, we show existing macro-finance estimates of r_t^* from a range of empirical studies. All of these estimates are consistent with our definition of r_t^* as the stochastic trend in the real short rate in equation (4).

- Del Negro, Giannone, Giannoni and Tambalotti (2017)(DGGT) use a Bayesian framework to estimate a linear state-space model with common trends r_t^* and π_t^*. Their estimation uses five data series: observed price inflation, long-run inflation expectations, 3-month Treasury bill rate, 20-year Treasury yield, and long-run survey expectations of the 3-month Treasury yield.

- Johanssen and Mertens (2016) (JM) similarly estimate the long-run real rate trend from a time series model, but explicitly account for the zero lower bound on nominal rates and account for stochastic volatility.

- Laubach and Williams (2003, 2016) (LW) use a simple macroeconomic model and the Kalman filter to infer the *neutral* real interest rate, that is, the level of the real rate consistent with real output at potential and inflation at target.

- Kiley (2020) augments the Laubach-Williams model to account for changes in financial conditions. Both specifications assume that the neutral rate follows a martingale, so these r_t^* estimates are consistent with the long-run concept we employ.

- Christensen and Rudebusch (2019) estimate a dynamic term structure model for real (TIPS) yields, as discussed above. The estimation uses a Kalman filter, and r_t^* is estimated as the five-year-ahead five-year average of the expected future real short rate.

In addition, Figure 1 also shows our own estimate of r_t^* described in the next section, which is obtained from an unobserved-components (UC) model for the real one-year U.S. Treasury bond yield (the baseline model of our empirical analysis of SDRs below).

Table 1: Estimates of the equilibrium real interest rate (r_t^*)

Source of estimate	1990s	2010s	Change
Del Negro et al. (2017)	2.3	1.1	-1.2
Johannsen and Mertens (2016)	1.4	0.7	-0.7
Laubach and Williams (2016)	2.8	0.3	-2.5
Kiley (2015)	1.6	0.7	-0.9
Christensen and Rudebusch (2019)	2.1	0.6	-1.5
UC model, 1y rate	1.6	0.7	-0.9
Mean	1.96	0.68	-1.28

Model estimates of r_t^* (in percent) during recent decades and the changes between these decadal averages (in percentage points). The "1990s" value is the average r_t^* from 1990 through 1999, except for the estimate of Christensen and Rudebusch (2019), where we report the average from 1998 (the first available observation) through 1999. The "2010s" value is the average r_t^* from 2010 through 2018 or 2019, depending on data availability. The "UC model, 1y rate" estimates are based on our own empirical unobserved-components model for the inflation-adjusted one-year U.S. Treasury bond yield that is described in Section 5. Mean values—averaged across all six models—are shown at the bottom.

It is evident from Figure 1 that the various estimates of r_t^* have all declined substantially from the 1990s to recent years. The exact magnitude and pattern of the decline differs across models: These differences reflect the estimation and specification uncertainty that is a feature of statistical inference about long-run trends, and in particular about the equilibrium real rate (Laubach and Williams, 2016). But the overall pattern of a pronounced decline in the estimated path of r_t^* since the 1990s is quite consistent across all of the various specifications. Table 1 summarizes the time profiles of these estimates. For each model, the table provides the average r_t^* during the 1990s and the 2010s and the difference between these two decadal averages. The bottom line in the table also provides the averages across the six estimates. All estimates show a decline across the decades, with a mean decline of 1-1/4 percentage points.

5 New estimates of the term structure of discount rates

In this section, we estimate new models for the real short-term interest rate r_t that (i) provide complete representations of a time-varying trend component r_t^* as an equilibrium real rate, (ii) quantify the decline in r_t^* since the 1990s, and (iii) demonstrate the implications of the decline in r_t^* for the entire term structure of SDRs.

Our baseline model is a univariate unobserved-components (UC) model, similar in econometric structure to various empirical macroeconomic models ranging from Watson (1986) to Bauer and Rudebusch (2020), among many others. This UC model incorporates the trend-cycle decomposition in equation (5) together with these specifications for each component:

$$r_t^* = r_{t-1}^* + u_t, \quad u_t \sim N(0, \sigma_u^2) \tag{10}$$

$$\tilde{r}_t = \phi \tilde{r}_{t-1} + v_t, \quad u_t \sim N(0, \sigma_v^2). \tag{11}$$

We use annual data on the short-term real interest rate, r_t, measured as the difference between the nominal one-year yield and inflation expectations over the next year. That is, we use the ex ante real short rate, $r_t = i_t - E_t\pi_{t+1}$, which is the difference between the nominal short rate, i_t, and inflation expectations, $E_t\pi_{t+1}$. The nominal yield is the one-year constant-maturity Treasury yield. Inflation expectations are the median of the Livingston survey expectations for inflation in the consumer price index (CPI). Our annual sample, consisting of December observations of yields and survey expectations, starts in 1953 and ends in 2019.

To estimate the model, we cast it into a state-space form with r_t^* and $\tilde{r}_t$ as state variables, equations (10) and (11) as transition equations, and (5) as the measurement equation. We estimate this model using Bayesian methods and employ uninformative prior distributions for the parameters, with the exception of σ_u, which is the innovation variance of r_t^*. To encourage a smooth trend r_t^*, we use a tight prior around a low value for this variance, similar to Del Negro et al. (2017) and Bauer and Rudebusch (2020).[23] For estimation, we use a Markov chain Monte Carlo (MCMC) sampler, where we draw the unobserved state variables using

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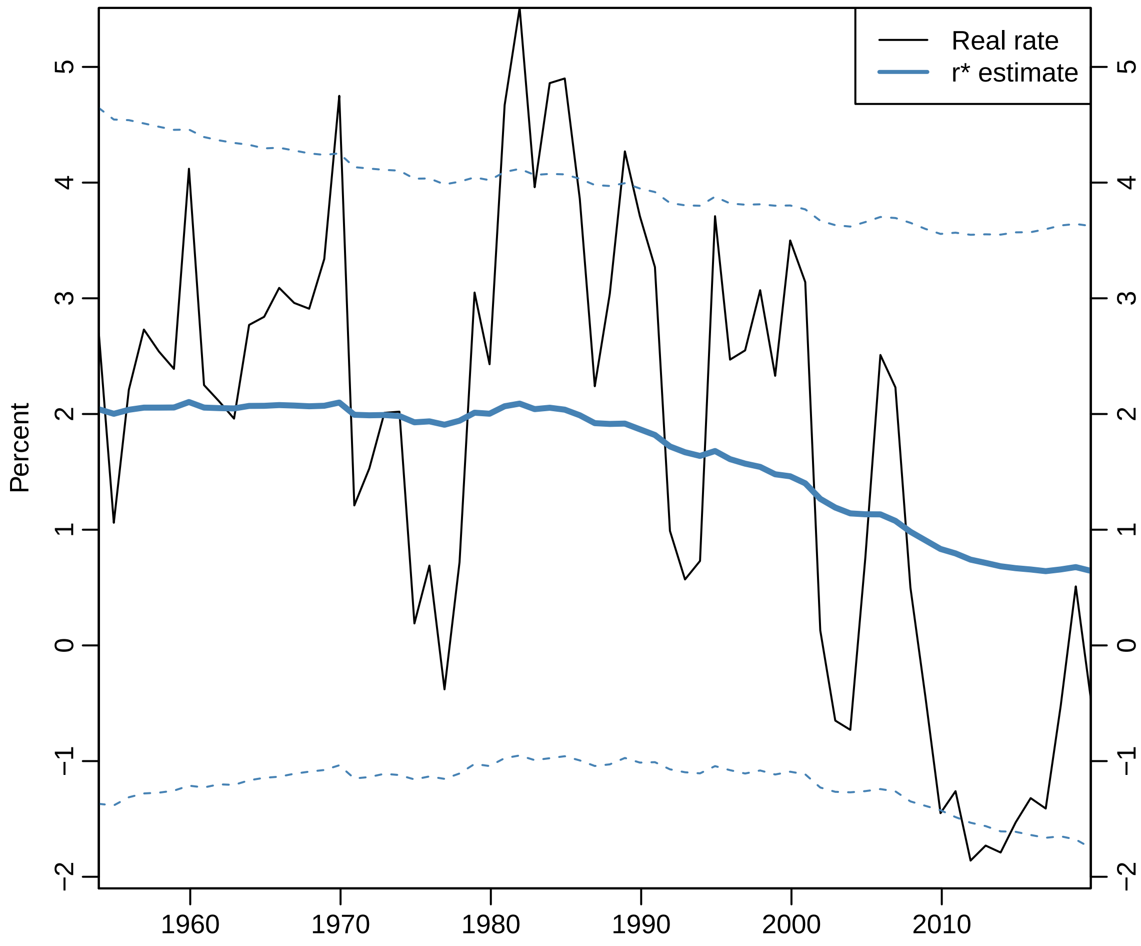
[23] Specifically, the prior distribution for this variance is inverse-gamma, $IG(\alpha/2, \delta/2)$, with $\alpha = 100$ and $\delta = 0.04(\alpha + 2)$. This implies that the mode is 0.01, and the variance of the change in r_t^* over 100 years is 4, i.e., the standard deviation is 2 percent.

the simulation smoother of Durbin and Koopman (2002) and the parameters using standard Gibbs steps.[24]

Figure 2: Estimates of equilibrium real interest rate from baseline UC model



The real rate shows annual data for the one-year U.S. Treasury yield adjusted for inflation expectations over the sample from 1953 to 2019. Based on this series, the resulting estimate of the equilibrium real rate, r_t^*, is a Bayesian posterior mean from the univariate UC model. Dashed lines show the 95% Bayesian credibility intervals for r_t^*.

Figure 2 displays the one-year real interest rate and the resulting equilibrium real rate inferred from our baseline UC model with the MCMC algorithm. This estimated r_t^* declines about 1.2 percentage points over the last three decades of our sample, from 1990 to 2019. Dashed lines indicate 95 percent posterior credibility intervals, which show the sizable estimation uncertainty that is a typical feature of statistical inference about latent trends (Laubach

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[24] We sample five separate MCMC chains from random starting values, each with 20,000 iterations, of which we discard the first half as a burn-in sample. Standard diagnostics indicate that the MCMC chains have converged to their stationary distribution, which is the joint posterior distribution of the model.

and Williams, 2003).

We now turn to the term structure of discount rates that is implied by the UC model. As in Newell and Pizer (2003) and a subsequent literature described in Section 3, we obtain long-term discount rates from the time series model of the short-term real rate by using simulations and and the expectations hypothsis. The crucial difference with previous work, however, is that the long-run mean of the short rate is allowed to change over time. We compare two different years, 1990 and 2019, to illustrate how the term structure of discount rates has shifted in recent decades. To calculate a model-implied term structure for each year, we use the following simulation approach:

1. For each year, $t = 1990$ or 2019, use the posterior mean estimates of the state variables r_t^* and $\tilde{r}_t$ as well as the posterior means for the parameters σ_u, σ_v, and ϕ.[25]
2. Simulate 100,000 paths of the state variables from the year-t starting values out to a horizon of 400 years.
3. Calculate the effective real short rate using a *shadow rate* specification to ensure non-negativity: $r_t = \max(0, r_t^* + \tilde{r}_t)$.
4. Calculate a term structure of long-run discount rates from the simulated short rate paths using equations (2) and (3).

Before turning to the results, we briefly discuss the non-negativity constraint. In financial economics, a shadow-rate specification is commonly used to ensure the non-negativity of nominal interest rates, which reflects the possible arbitrage with zero-interest cash (Christensen and Rudebusch, 2015; Bauer and Rudebusch, 2016).[26] For real interest rates, no such arbitrage argument can be made, and negative real rates of short and medium maturities have been observed. Therefore, imposing a lower bound on real rates is rare in term structure models. However, a non-negativity constraint is generally viewed as desirable in empirical modeling of social discount rates, and model-based estimates of long-term SDRs are generally constrained to remain positive (Newell and Pizer, 2003; Groom et al., 2007; Freeman et al., 2015). The imposition of this constraint reflects the power of discounting, as even slightly negative long-term SDRs can produce exploding present values of future damages and imply an implausibly large willingness of the current generation to pay for reduced carbon emissions. In addition, it

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[25] Using an alternative simulation procedure that accounted for estimation and filtering uncertainty by drawing from the posterior distributions of the parameters and the state variables did not materially change our results.

[26] While recent years have shown that nominal rates can in practice dip into negative territory due to limits to this arbitrage, an effective lower bound slightly below zero is still likely to exist.

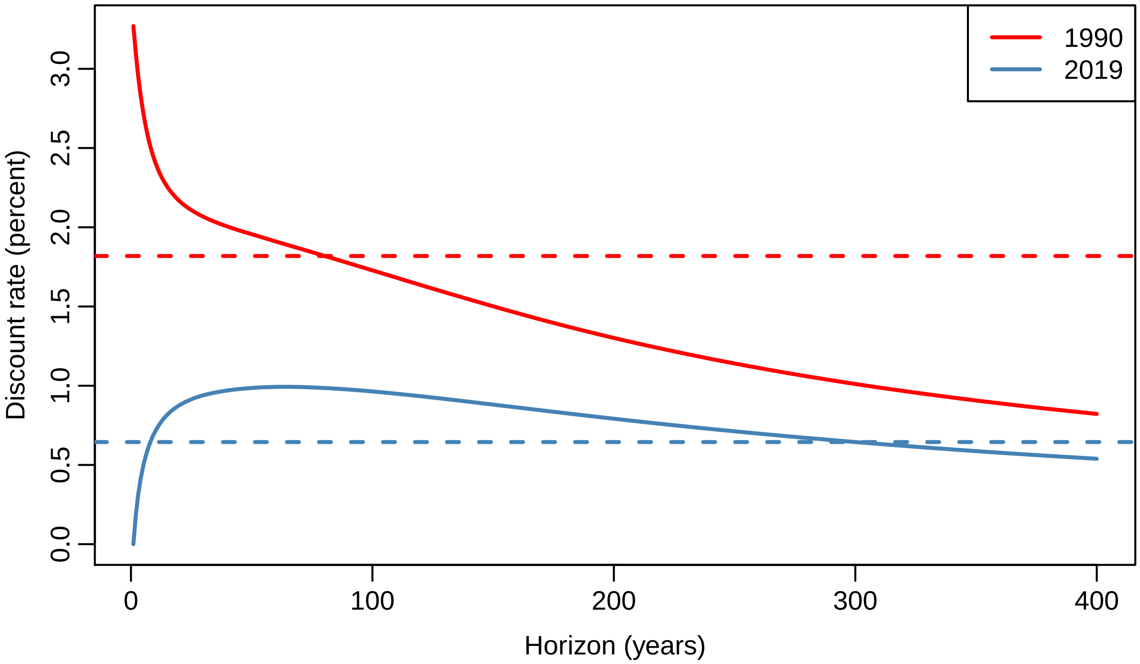
is generally viewed that "negative [real] rates are unlikely to persist for long periods" (Gollier et al., 2008, p. 771), which is consistent with the empirical observation that long-term real rates, such as TIPS yields with ten or more years of maturity, typically remain positive as well. As a result, we use the shadow-rate approach to ensure non-negative term structures of SDRs, which in our view is preferable over alternatives such as discarding simulated paths with real rates outside certain bounds (e.g., Freeman et al., 2015) or simulating the logarithm of the real rate (e.g., Newell and Pizer, 2003).

A related issue is that most of our empirical models have a random walk component to capture the persistence of real interest rates. Such a time series specification is widely used to represent slowly moving trends (Campbell and Perron, 1991). However, a random walk component in interest rates implies that uncertainty about the level of future yields is unbounded, so the bond convexity/Weitzman effect eventually dominates and pushes model-implied rates into negative territory. Indeed, the unbounded long-run variance is especially problematic as interest rates eventually diverge to minus infinity (Bauer and Rudebusch, 2020). Thus, although we employ the parsimonious and popular random walk specification, it is useful to temper its more extreme implications in the limit and impose non-negativity.

Figure 3 shows the implied term structures of discount rates in 1990 (red solid line) and 2019 (blue solid line) from our baseline UC model. Aside from an initial hump for short maturities in the latest term structure, both term structures ultimately decline because of uncertainty about future short-term real rates and the convexity/Weitzman effect. For the same reason, long-term SDRs ultimately fall below the levels of r_t^* that anchor them—the dashed lines—and gradually approach their lower bound of zero, though at a 400-year horizon they are still just below 1 percent. Most interesting is the downward shift over time in these discount rate term structures. Due to the decline in r_t^* over the past 30 years, the entire SDR term structure has shifted lower. At the short end, the downward shift is larger than the decline in r_t^*, due to the larger fall in the observed real short rate. At the long end, the downward shift is smaller than the decline in r_t^* due to the presence of the zero lower bound. All SDRs, however, shift down due to the one-for-one decline in long-run expectations; see equation (7). That is, the pronounced and widely supported decline in the equilibrium real interest rate translates into a significant downward shift in discount rates at all horizons.

To explore the robustness of our results, we estimate four additional models that have a time-varying trend component r_t^* and complement our benchmark specification. Here we describe and summarize the results of the alternative models, while Appendix B shows the detailed results, including all estimated r_t^* series and model-implied SDRs. The first alternative is identical to our UC model but uses the ten-year yield instead of a one-year yield. Using

Figure 3: Term structure of discount rates from baseline UC model



Term structures of discount rates (real yields) calculated using simulations from UC model in equations (5), (10) and (11), the posterior mean estimates for parameters and state variables, and a shadow-rate constraint for the real short rate that ensures non-negative discount rates. The red term structure is based on the real rate and estimated r_t^* in 1990, and the blue term structure uses the values for 2019. The dashed lines show the model-based estimates of r_t^* in those two years.

a long-term instead of a short-term yield more closely follows the existing literature such as Newell and Pizer (2003). To proxy for the 10-year real yield, we use the 10-year U.S. Treasury yield and subtract a popular measure of long-run inflation expectations, the time-varying perceived inflation target rate (PTR) from the Federal Reserve's main macroeconomic model.[27] Due to the data availability for PTR this annual sample starts in 1963. Our UC model estimate for this ten-year real rate series shows a greater decline in the trend component r_t^* of 1.9 percentage points. However, a portion of this larger decline may be explained by a modest reduction in the term premium in long-term Treasury yields during this sample (Bauer and Rudebusch, 2020).

A second alternative, which is very similar to the specification in Del Negro et al. (2017), explicitly models long-run trends in the real interest rate and inflation. It is a state-space model that includes as state variables the trend and cycle components of the real short rate (r_t^* and $\tilde{r}_t$) and the inflation rate ($\pi_t = \pi_t^* + \tilde{\pi}_t$, which are defined analogously to equations (4) and (5)). The observed data series are the nominal short rate (one-year Treasury yield),

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[27] See `https://www.federalreserve.gov/econres/us-models-about.htm` for more information.

inflation (one-year price inflation for personal consumption expenditures excluding food and energy), and PTR as a proxy for π_t^*. This specification explicitly models the inflation process, so inflation expectations are model-consistent. The results from this model are broadly similar to our baseline UC model with an estimated decline r_t^* of 1.0 percentage point between 1990 and 2019.

For the last two alternatives, we modify the form of the r_t^* nonstationarity more substantially. While models with stochastic trend components of the sort described above are commonly used in empirical macroeconomics due to their good forecast performance (Campbell and Perron, 1991), they also have some drawbacks. If taken literally, the assumption of a random walk component in interest rates may seem unappealing, and, as noted above, the resulting unbounded forecast error variance can be inconvenient for asset pricing. Accordingly, we also consider a simple alternative specification for the short-term real rate that nevertheless captures our central idea of a shifting long-run mean. Namely, we allow for a deterministic shift in the unconditional mean using the model

$$r_t = \phi r_{t-1} + \alpha + \beta D(\tau)_t + u_t, \tag{12}$$

where $D(\tau)_t$ is a dummy variable equal to zero before period τ and equal to one from τ until the end of the sample. That is, we estimate a first-order autoregressive model for the real rate with a change in the intercept in period τ. The unconditional mean of r_t is $\alpha/(1-\phi)$ in the first part of the sample and $(\alpha+\beta)/(1-\phi)$ in the later sample period. For simplicity, we choose as the breakpoint τ the year 1995, based on the large body of empirical evidence (some of it visualized in Figure 1) that the equilibrium real interest rate began a downward shift in the 1990s.[28] We estimate this mean-shift model for both the one-year and ten-year real Treasury yields. In both cases, we find strong statistical evidence for $\beta < 0$, further supporting the view that the long-run interest rate mean has shifted lower. For the one-year real rate, the long-run mean declines from 2.6 percent to about zero, and for the ten-year rate it declines from 3.8 percent to 1.3 percent. We examine simulations that start from the real rate in 1990 or 2019 using the long-run mean of either the early or the later part of the sample, respectively. The resulting term structures of discount rates show a somewhat more pronounced downward shift compared with our UC and state-space models.

It is worth highlighting that our estimates of long-run SDRs are based on historical data for interest rates that cover a similarly long sample period as in previous work (e.g., Newell

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[28] We have also considered a data-based choice of τ, searching for the year that optimized the least-squares fit of the model. The optimal break date τ depends on the interest rate series but is generally in the late 1990s. This data-based procedure complicates statistical inference about β and produced qualitatively very similar results, so we focus on the case described in the text.

and Pizer, 2003, and follow-up studies). That is, previous analyses also estimated SDRs for maturities of hundreds of years based on a relatively shorter estimation sample of postwar interest rates. Therefore, our analysis differs not in the length of the estimation sample nor in the inference about long-run maturities from such a sample. Instead, the crucial difference is that previous work has estimated a constant r^* as the postwar sample mean of real interest rates, and assumed that this r^* was the relevant anchor appropriate to infer long-run discount rates. By contrast, our analysis takes into account that r_t^* may vary due to structural economic changes over the estimation sample. Our long-run mean and the anchor for long-run SDRs is not the simple postwar sample average—as in earlier work—but incorporates a more nuanced and flexible reading of interest rate dynamics during that sample. Evidence from longer historical data samples of real rates, as in Schmelzing (2020), shows the importance of allowing for time variation in the underlying trend r_t^* and thus supports our analysis.

6 Implications for the social cost of carbon

One of the most pressing applications of social discounting is the valuation of climate change damages due to human activities such as burning fossil fuels. The associated greenhouse gas emissions will remain in the atmosphere for many generations, and the level of the discount rate is crucial for assessing the resulting medium- and long-term future damages. We have already documented that the downward shift in the equilibrium real interest rate in recent decades has pushed down the level of the entire term structure of discount rates. Here, we provide a quantitative example of the implications of this reduction for assessing the economic costs of climate change.

This example builds on a large literature that calculates the social cost of carbon (SCC), that is, the discounted present value of the future damages caused by one additional ton of carbon dioxide (CO_2) emissions. This calculation requires (i) a time profile of the marginal future consumption damages caused by an increase in current emissions, and (ii) a term structure of discount rates (see Appendix C for a detailed derivation). To understand the effect of discounting on the SCC, we take the path of marginal damages as given and discount it using past and current term structures of discount rates. An IAM can estimate the future consumption damages from an increase in current CO_2 emissions, and we follow earlier studies (e.g., Newell and Pizer, 2003; Groom et al., 2007; Freeman et al., 2015) by using the well-known DICE model developed by William Nordhaus.[29]

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[29] This approach is similar to van der Ploeg and Rezai (2019) who assume that private agents discount the future with the standard DICE parameters but the social planner sets carbon prices using a lower discount rate. Therefore, economy-wide discount rates within the DICE model generally differ from the discount rates

For comparability, we report results for the marginal damage profile used in Newell and Pizer (2003) and subsequent work, which is based on the DICE-94 model of Nordhaus (1994). These are marginal consumption damages over the next 400 years from an additional ton of CO_2 emissions in the (arbitrary) baseline year 2000.[30] Differences in the SCC that we calculate relative to this earlier literature can be attributed to the term structure of discount rates.

The DICE model serves as a useful frame of reference that has been widely used in previous research to assess the economic implications of changes in SDRs. However, the assumed future damages resulting from higher carbon emissions are quite uncertain, as stressed by Pindyck (2013). Furthermore, Dietz et al. (2020) argue that the DICE model and other IAMs in the economics literature are severely misspecified—relative to physical climate models—and exhibit an excessively long lag between CO_2 emissions and subsequent global warming. Hänsel et al. (2020) provide a similar critique. To investigate the robustness of our results, we have also carried out our SCC calculations using two other versions of the DICE model, including not only the most revent vintage DICE-2016, discussed in Nordhaus (2017), but also the DICE-FAIR-Geoffroy model proposed in Dietz et al. (2020) that trys to make the DICE climate module more consistent with the current physical science representations. These more recent model versions generally imply larger and more back-loaded marginal damage profiles. But the results of our calculations using these alternative model profiles yielded qualitatively similar conclusions as in the main text below, as shown in Appendix D.

To calculate the SCC we add up the stream of discounted damages over the next 400 years to get a present value of future damages, that is,

$$SCC_t = \sum_{n=0}^{400} P_t^{(n)} E_t(MD_{t+n}) \tag{13}$$

where $E_t(MD_{t+n})$ is the model-based estimate for future marginal consumption damages in year $t+n$ estimated from the DICE model (see Appendix C). For the DICE-94 model, the future consumption damages are given in constant 1989 dollars, and the model yields an SCC estimate of \$5.29 (see Nordhaus, 1994). If we assume a constant 4 percent discount rate, then the SCC is \$5.7. This value is in line with conventional analyses that use constant discount

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used to calculate the SCC from the model's climate damages. To some extent, the DICE model profile of marginal damages is endogenous to assumptions about interest rates, which, for example, affect the time paths of capital and output. However, changes in interest rates arguably have only second-order effects on future economic damages from climate change but first-order effects via discounting on the present value of these damages.

[30] This damage sequence is shown in Figure 6 of the working paper version of Newell and Pizer (2003). The time path of future economic damages increases steadily after the additional carbon release and reaches a maximum after about a century and then slowly declines over the remainder of the horizon.

rates and produce low estimates of climate change damages and support modest mitigation efforts (Newell and Pizer, 2003). Instead, if we assume a constant 2 percent discount rate, which is closer to the 1990s estimates of the equilibrium real interest rate, the social cost of carbon jumps to \$21.7, highlighting the importance of the level of the discount rate.

Table 2: Estimates of the SCC (\$ per metric ton of CO_2)

Model	Change in r_t^* (p.p.)	1990	2019	Change
UC model, 1y rate	-1.2	31.8	68.7	116%
UC model, 10y rate	-1.9	14.3	58.9	313%
State-space model, 1y rate	-1.0	29.8	58.5	96%
Mean shift model, 1y rate	-2.6	13.3	95.3	618%
Mean shift model, 10y rate	-2.4	6.3	37.9	503%

For our five empirical models, the estimated mean change in each model-based r_t^* from 1990 to 2019 is shown in percentage points. Estimates of the social cost of carbon (SCC) are based on the same path of marginal damages as in Newell and Pizer (2003): the marginal changes in future consumption over the next 400 years (in constant 1989 U.S. dollars) from one extra ton of CO_2 emitted in the arbitrary baseline year 2000, as implied by the DICE-94 model. The columns "1990" and "2019" show SCC calculations using model-implied term structures of social discount rates for 1990 and 2019.

However, a constant discount rate makes little sense in an uncertain world with the bond convexity/Weitzman effect ensuring that the term structure of SDRs is declining. Estimates from Newell and Pizer (2003) that account for this decline, starting from a 4 percent real rate at the zero horizon, range from \$6.5 to \$10.4, depending on the model. But these estimates do not incorporate the secular decline in the equilibrium real rate and the resulting downward shift in the term structure of SDRs that we documented above.

We demonstrate the effects of these secular shifts using the five empirical models described in Section 5, and report the resulting changes in the SCC in Table 2. We focus on our benchmark UC model and first consider the SCC in 1990 by fixing the estimate of r_t^* at the model's 1990 level. The resulting term structure of social discount rates corresponds to the red line in Figure 3. Although the equilibrium real rate in 1990 for this benchmark case is estimated to be very close to 2 percent, the declining term structure of discount rates in the presence of uncertainty and a persistent short rate raises the SCC (relative to the value obtained for a constant 2 percent SDR) to \$31.8. By comparison, the next column uses the equilibrium real rate in 2019, which is more than 1 percentage point below the value from 1990. The decline in r_t^* pushes down the entire term structure of SDRs—corresponding to the blue line in Figure 3. Using these lower discount rates increases the SCC to \$68.7. That is, even with the same marginal damage function, the latest term structure estimates increase the SCC by about 37 dollars, which amounts to an 116 percent increase.

This substantial increase in the SCC is robust to the use of the four other models for real rates that we use to obtain term structures of SDRs. With the UC model applied to the 10-year yield, discount rates decline even more, resulting in a dramatic increase in the SCC, which quadruples from \$14.3 to \$58.9. With a state-space model that explicitly includes inflation expectations, nominal rates and real rates, the SCC almost exactly doubles from \$29.8 to \$58.5. For the simple mean shift model shown in equation (12), the increase in the SCC is the largest, due to the even more pronounced decline in the estimated long-run mean and the level of the term structure of SDRs: the SCC increases sevenfold and sixfold, for the one- and ten-year rate models, respectively.

To sum up, the fall in the equilibrium real rate over the past few decades has sizable economic implications for assessing the cost of climate damages. The lower discount rates that result from the decline in r_t^*—the anchor of the SDR term structure—necessarily lead to higher climate damage estimates. Our example quantifying this change suggests that the decline in r_t^* over the past few decades substantially raises the SCC, generally at least doubling it according to our estimates.[31]

7 Conclusion

This paper has shown that the recent secular decline in the equilibrium real interest rate has important consequences for the economics of climate change. Namely, the lower new normal in interest rates—that is, a drop in r_t^*—lowers the term structure of discount rates and boosts the social cost of carbon. Quantitatively, our baseline model estimate of about a 1 percentage point decline in r_t^* since the 1990s results in approximate doubling of the SCC in the United States using a standard measure of the future damages from climate change.

Our results have notable policy implications. Previous analyses of social discounting revealed a sizable gap between a prescriptive approach that yielded low discount rates (e.g., Stern, 2007) and a descriptive approach that gave substantially higher ones (e.g., Nordhaus, 2007). This gap in discount rates led to very different recommendations about the requisite degree of climate policy activism. However, after accounting for the recent decline in r_t^*, a descriptive analysis leads to SDRs that are in close alignment with the low rates from a prescriptive approach. An empirically based discount rate analysis that takes into account recent evidence of a lower term structure of discount rates and a higher estimated SCC provides support for a more active approach to combating climate change—along the lines advocated in the Stern Review.

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[31] The increases in our SCC estimates are broadly consistent with the theoretical results of Gerlagh et al. (2017) who provide a model in which the discount factor reacts to ongoing demographic trends.

Our paper is a first step toward incorporating new evidence on the equilibrium real interest rate into the economics of climate change, and more remains to be done. For example, our analysis uses estimates of r_t^* that do not explicitly take into account the wide-ranging structural changes caused by climate change that are in train for the economy. Instead, as climate change continues, it will likely have effects on interest rates and r_t^*. Global warming may well lower the level of interest rates due to adverse effects on trend productivity and output growth or increased precautionary savings to protect against climate disasters. If this is the case, then the feedback from climate change to interest rates would tend to strengthen our result that the appropriate interest rate benchmark to use for discounting climate change is lower than what has been observed in the past. More generally, modeling the joint determination of the equilibrium real interest rate, economic growth, and global warming may reveal feedback effects with substantial consequences for valuing the consequences of climate change. Despite useful first steps in this direction, as in Hambel et al. (2020), accounting for the general equilibrium and feedback effects of the climate system and interest rates in IAMs remains an important and challenging area for future climate change research.

Another promising direction for future work is to leverage modern tools of financial economics to make better use of the information in the term structures of market-observed real bond yields. While our analysis, like previous work using market rates, has generally focused on the time series dimension of the data, there is important cross-sectional information in, for example, U.S. TIPS yields or U.K. inflation-linked bond yields, that can provide important cross-sectional information to improve inference about the term structure of discount rates. In addition, government yield curves in other countries should be informative for estimation of long-term SDRs, especially because some countries have issued government bonds with very long maturities.

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Appendix

A Dynamic term structure model with r_t^* anchor

As described in Section 3, the equilibrium real interest rate, r_t^*, anchors the term structure of SDRs. In this appendix, we derive this result using a simple parametric model for the real short rate and the assumption of risk-neutrality. This specification provides a tractable affine dynamic term structure model for real interest rates.

We assume that r_t^* is a random walk without drift, that the cyclical component, $\tilde{r}_t$, follows a first-order autoregressive process, and that innovations are Gaussian:

$$r_t^* = r_{t-1}^* + u_t, \quad u_t \sim N(0, \sigma_u^2) \tag{A.1}$$

$$\tilde{r}_t = \phi \tilde{r}_{t-1} + v_t, \quad u_t \sim N(0, \sigma_v^2). \tag{A.2}$$

The assumption of risk-neutrality implies that the stochastic discount factor is $M_t = \exp(-r_t)$.

This model implies that the discount factor/bond price is exponentially affine in the two risk factors:

$$P_t^{(n)} = \exp(A_n + B_n r_t^* + C_n \tilde{r}_t), \tag{A.3}$$

where the affine loadings follow the recursions

$$A_{n+1} = A_n + \frac{1}{2} B_n^2 \sigma_u^2 + \frac{1}{2} C_n^2 \sigma_v^2 \tag{A.4}$$

$$B_{n+1} = -1 + B_n \tag{A.5}$$

$$C_{n+1} = -1 + \phi C_n, \tag{A.6}$$

with initial conditions $A_0 = B_0 = C_0 = 0$, and solutions $B_n = -n$ and $C_n = -(1 - \phi^n)/(1 - \phi)$. These results are easily derived by positing the structure in (A.3), using the bond price recursion $P_t^{(n+1)} = \exp(-r_t) E_t P_{t+1}^{(n)}$ and matching coefficients, similar to Ang and Piazzesi (2003). Yields are then given as $y_t^{(n)} = -A_n/n + r_t^* + (1 - \phi^n)/[n(1 - \phi)]\tilde{r}_t$. Forward rates are

$$f_t^{(n)} = -\frac{1}{2} B_n^2 \sigma_u^2 - \frac{1}{2} C_n^2 \sigma_v^2 + r_t^* + \phi^n \tilde{r}_t, \tag{A.7}$$

where the first two terms capture the convexity effects due to Jensen's inequality, and the last two terms reflect expectations. Note that in the special case where r_t is stationary, $\sigma_u^2 = 0$, the limiting forward rate (for $n \to \infty$) is a constant equal to $-\frac{1}{2(1-\phi)^2}\sigma_v^2 + r^*$, whereas if $\sigma_u^2 > 0$ the forward rate diverges to minus infinity. Equation (A.7) illustrates that short-term discount rates are affected by cyclical deviations from trend, $\tilde{r}_t$, that long-term rates are pushed down by convexity, and, crucially, that all discount rates are equally affected by r_t^*.

B Estimates of r_t^* and SDRs from alternative models

This appendix compares our baseline estimates for r_t^* and the associated term structures of SDRs to estimates from four alternative empirical time series models:

(1) UC model for 1y real rate (our baseline model)

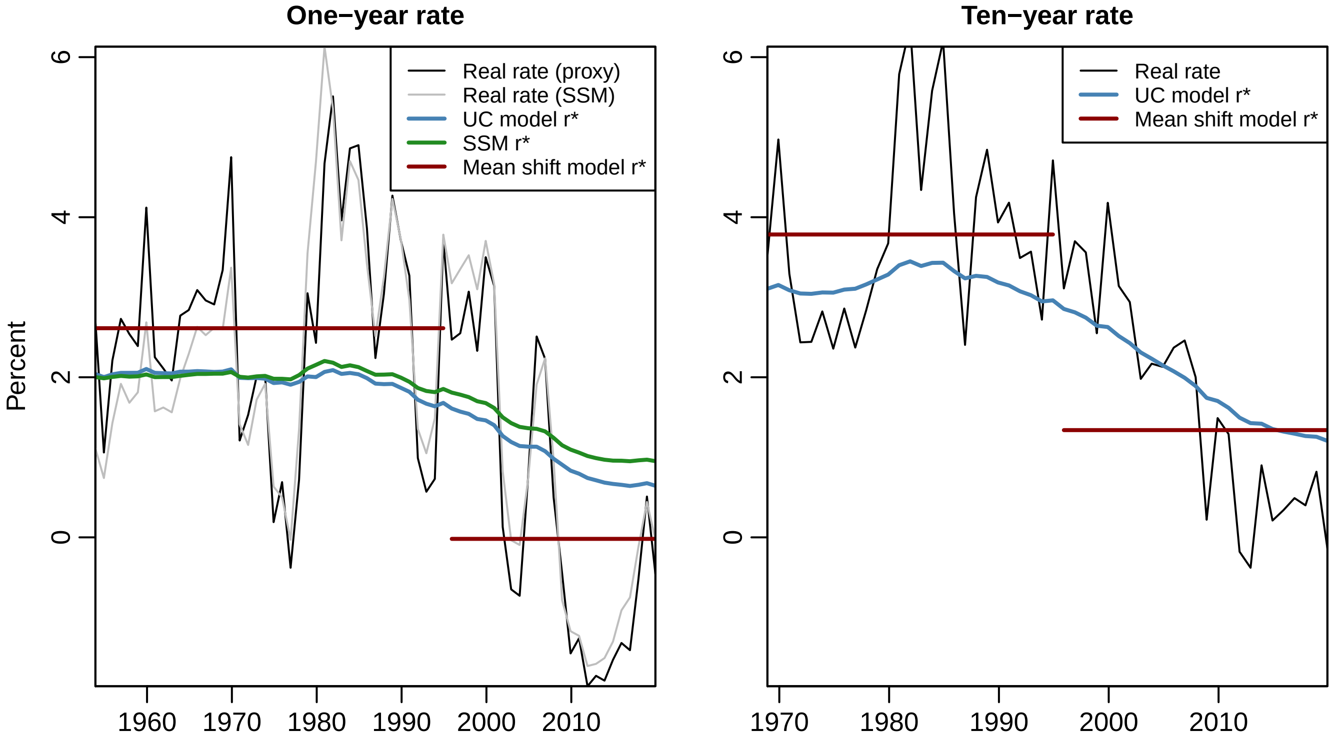
(2) UC model for 10y real rate

(3) State-space model for 1y real rate

(4) Mean shift model for 1y real rate

(5) Mean shift model for 10y real rate

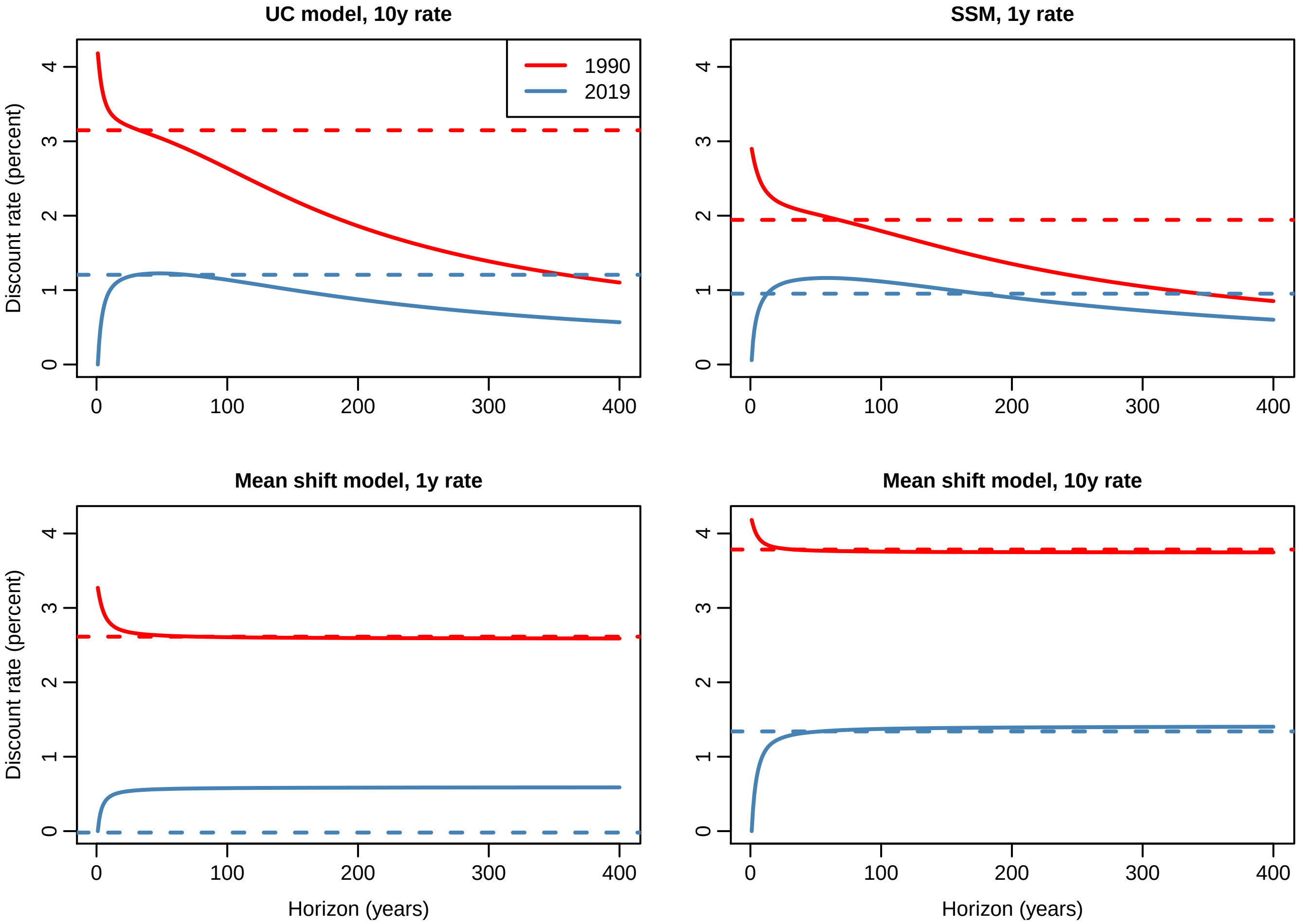
Figure B.1: Alternative model-based estimates of r_t^*



Short- and long-term real interest rates and alternative model-based estimates of the equilibrium real interest rate. The left panel shows the one-year real interest rate, the filtered ex-ante real rate from the state-space model (SSM), and three alternative model estimates of r_t^* (sample period 1953 to 2019). The right panel shows the ten-year real interest rate and two alternative model estimates of r_t^* based on this long rate (sample period 1968 to 2019).

Figure B.1 plots real interest rates along with the model estimates of r_t^*. The left panel shows two measures of the one-year real interest rate: the version employed in Section 5 (the difference between the one-year nominal U.S. Treasury yield and one-year median CPI inflation expectations from the Livingston survey) and the ex-ante real rate filtered from our state-space model. The left panel also shows the r_t^* estimates from models (1), (3), and (4). The right panel shows our ten-year real interest rate (the difference between the ten-year nominal U.S. Treasury yield and PTR, a measure of long-term inflation expectations) together with the r_t^* estimates from models (2) and (5).

Figure B.2: Alternative model-based term structures of SDRs



Alternative model-based estimates of the term structure of social discount rates (SDRs). The term structures are calculated using simulations from the alternative models, under a shadow-rate constraint that ensures non-negative rates. The red and blue lines are the term structures based on the real rate and estimated r_t^* in 1990 and 2019, respectively. The dashed lines show the model-based estimates of r_t^* in those two years.

In the main text, Figure 3 shows the term structures of SDRs for our baseline model (1). For the four alternative models, Figure B.2 plots the implied term structures. The term structures are calculated using simulations from the models similar to what is described in Section 5, including a shadow-rate constraint on the real interest rate that ensures non-negativity. As before, the term structures are shown for values of r_t^* in 1990 and 2019.

C Method for calculating the social cost of carbon

The social cost of carbon at time t is defined as

$$SCC(t) = -\frac{\partial W}{\partial E(t)} \Big/ \frac{\partial W}{\partial C(t)} \ , \tag{C.1}$$

where $C(t)$ is aggregate consumption, $E(t)$ represents emissions, and W is welfare; see for example Nordhaus (2017). Welfare in the DICE model is

$$W = \sum_{t=0}^{T} R^t U(c(t)) L(t), \tag{C.2}$$

where the discount factor is $R = \frac{1}{1+\rho}$ with rate of time preference ρ, period utility is $U(c) = \frac{c^{1-\alpha}}{1-\alpha}$, a function of per-capita consumption c, and population is $L(t)$. With these definitions, we can rewrite the SCC as follows

$$\begin{aligned} SCC(t) &= -\sum_{\tau=t}^{T} R^{\tau} \frac{\partial U(c(\tau))}{\partial E(t)} L(\tau) \Big/ R^t \frac{\partial U(c(t))}{\partial C(t)} L(t) \\ &= -\sum_{\tau=t}^{T} R^{\tau-t} \frac{\partial U(c(\tau))}{\partial C(\tau)} \frac{\partial C(\tau)}{\partial E(t)} \frac{L(\tau)}{L(t)} \Big/ \frac{\partial U(c(t))}{\partial C(t)} \\ &= -\sum_{\tau=t}^{T} R^{\tau-t} \left(\frac{c(\tau)}{c(t)}\right)^{-\alpha} \frac{\partial C(\tau)}{\partial E(t)} \end{aligned}$$

The first equality holds because (i) only period utility in t is affected by a marginal change in consumption $C(t)$ and (ii) only utility from period t onward is affected by a marginal change in emissions $E(t)$. The second equality uses the fact that emissions affect utility by changing current and future consumption. The third equality substitutes the partial derivative $\partial U(c)/\partial C = c^{-\alpha}/L$. The end result is that, in the DICE model, the SCC is the present value of future marginal consumption damages $MD_{t+n} = -\frac{\partial C(t+n)}{\partial E(t)}$, which are known at t since there is no uncertainty in the model. The future damages are discounted using the consumption discount factor $R^n \left(\frac{c(t+n)}{c(t)}\right)^{-\alpha}$, which depends on pure time discounting and consumption growth between t and $t+n$, just like the discount rate in the classic Ramsey equation. We replace this discount factor by $P_t^{(n)}$ to compute changes in the value of the SCC.

D Social cost of carbon from alternative DICE models

Here we report detailed results for SCC calculations from two different versions of the DICE-2016 model. For each model, we follow the approach of Newell and Pizer (2003) to calculate an SCC using our own term structures of SDRs instead of the model's own internal discount rates: We run the model with a one-period shock to CO_2 emissions in 2015, and calculate the consumption damages by comparing this model run to the results without a shock. We double-check that we can reproduce the model's internal SCC estimate using the discount rates within the DICE model and the marginal consumption damages we obtained in this manner.[32]

Table D.1: Estimates of the SCC from alternative DICE models

Model	Change in r_t^* (p.p.)	1990	2019	Change
DICE-2016 model				
UC model, 1y rate	-1.2	809.4	2207.9	173%
UC model, 10y rate	-1.9	287.5	1905.8	563%
State-space model, 1y rate	-1.0	733.7	1777.7	142%
Mean shift model, 1y rate	-2.6	110.0	2789.1	2436%
Mean shift model, 10y rate	-2.4	40.5	547.3	1251%
DICE-FAIR-Geoffroy model				
UC model, 1y rate	-1.2	323.3	867.2	168%
UC model, 10y rate	-1.9	119.2	749.1	528%
State-space model, 1y rate	-1.0	294.0	700.5	138%
Mean shift model, 1y rate	-2.6	51.3	1106.5	2057%
Mean shift model, 10y rate	-2.4	22.7	224.5	889%

For our five empirical models, the estimated mean change in each the model-based r_t^* from 1990 to 2019 is shown in percentage points. Estimates of the social cost of carbon (SCC) are in (constant 2010) dollars per metric ton of CO_2, using two different versions of the DICE-2016 model. In both cases, we take 2015 as the base year and use model-implied marginal damages over the next 400 years from one extra ton of CO_2 emissions. The top panel shows results for the standard DICE-2016 model of Nordhaus (2017). The bottom panel shows results for the DICE-FAIR-Geoffroy model proposed by Dietz et al. (2020). The colums "1990" and "2019" show SCC estimates using our model-based term structures of social discount rates for 1990 and 2019.

Our first model is the baseline version of the DICE-2016R2 model, used in Nordhaus (2017), among others.[33] It implies an SCC for 2015 of \$31.2 (in constant 2010 dollars) under the model's own internal discount rates. For a constant 4% discount rate the SCC is \$36.1. Moving to our own discount rates, the SCC increases in most cases dramatically. This is due to two factors: First, our risk-free discount rates are substantially lower than the model-internal discount rates, which Nordhaus calibrated to historical stock returns. Second, the temperature changes and climate damages from an increase in current emissions occur mostly in the (far) future, so that lower discount rates imply larger changes than for more front-loaded damages as in older versions of the DICE model (such as DICE-94 which was used by Newell and Pizer (2003) and for which we report results in the main text). Despite the much higher absolute

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[32] As in Newell and Pizer (2003), we use the baseline or "no-controls" case in all of our model runs, instead of the optimized path of emissions.

[33] The GAMS code for this model is available at `https://sites.google.com/site/williamdnordhaus/dice-rice`.

magnitudes of the SCC estimates, the implications of a decline in r_t^* are qualitatlvey simlar as for the results reported in Section 6: The SCC increases substantially as a result of the downward shift in the term structures of discount rates.

Our second model is from Dietz et al. (2020). These authors propose several adjustments to the DICE-2016 model that bring the model's implications closer in line with current climate science. For their DICE-FAIR-Geoffroy model version, they use an improved carbon cycle including saturation effects in carbon sinks (FAIR) and an updated warming model (Geoffroy). In addition, they use the exogenous CO_2 emissions path from the IPCC's RCP2.6 scenario. An appendix to Dietz et al. (2020) provides GAMS code for this model, which we we use for our analysis. This model implies a 2015 SCC of \$17.4 under the model's own discount rates, while for a constant 4% discount rate the SCC is \$20.9. Using our own discount rates, the SCC estimates are again substantially higher than the model-internal SCC, but the increase is less dramatic than for the standard DICE-2016 model because damages from higher current emissions are somewhat more front-loaded. Thus, the present value of climate change damages is somewhat less sensitive to discount rates, as emphasized by Dietz et al. (2020). Importantly, even in this case a downward shift in SDRs due to a lower r_t^* has very similar economic implications: The SCC increases by roughly similar multiples as in the case of the DICE-2016 model, underlining the important consequences of accounting for a lower new normal for interest rates in assessments of climate change damages.