Natural rates across the Atlantic

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August 2017

Abstract

The paper estimates a closed-economy medium-scale DSGE model for the United States and the euro-area to assess the current level of the natural rate of interest and shed light on its drivers. The dynamics of the model are driven by permanent and transitory shocks that bear some connection with the explanations put forward in the literature to explain the secular downward trend in real interest rates. The results show that the natural rate has trended downward, contributing to lowering nominal and real interest rates. Risk premium shocks, a short-cut for changes in households' preference for safe assets, have been the main driver in the euro area; in the United States, shocks to the risk premium and to the efficiency of investment, which proxy the functioning of the financial sector, have played a major role. These differences in the importance of the various shocks underscore the importance of adopting a structural approach with a detailed stochastic structure, featuring both permanent and transitory shocks.

JEL classification codes: C51; E32; E43; E52.

Keywords: natural rate of interest; monetary policy; DSGE model; Bayesian methods.

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The views expressed in the articles are those of the authors and do not involve the responsibility of the Banca d'Italia.

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"*As a consequence,* the natural rate of interest […] has fallen over time, to very low or even negative levels*. And whatever the drivers behind this, central banks have to take it into account and cut their policy rates to commensurately lower levels*."
Mario Draghi, President of the ECB, 2nd DIW Europe Lecture, German Institute for Economic Research, Berlin, 25 October 2016.

"[…] *What about interest rates? Like job growth and GDP growth,* the new normal for interest rates is likely much lower than we are used to*. There is growing evidence of a significant decline in the natural rate of interest, or r* (r-star), over the past quarter-century to historically low levels."*
John C. Williams, President and CEO of the Federal Reserve Bank of San Francisco, presentation to the Federal Home Loan Bank of San Francisco's 2016 Member Conference, San Francisco California, 21 October 2016.

1. Introduction and motivation

In recent years, several papers have estimated "equilibrium" interest rates using a variety of methods. Such heightened interest was sparked by the observation that nominal rates have been on a declining trend in advanced economies since the eighties. Albeit falling inflation expectations and risk premia have contributed to these developments – in an environment characterized by increased attention of central banks to price stability all around the world – the fall in nominal rates has been associated primarily with a decline in the real component.

Within the debate on the causes of the decline, the concept of the "natural rate of interest" – defined by Knutt Wicksell (1898) as the real interest rate that balances desired saving and planned investment – has regained importance after the speech by Summers (2014) on secular stagnation in the US. According to this view, the natural rate has been pushed downward by secular factors acting on both the demand and supply of savings. The resulting downward pressure on nominal rates can limit the effectiveness of conventional monetary policy by increasing the likelihood of hitting the effective lower bound to the policy rates. According to an alternative, and partly competing view, the current low interest rate environment can be rationalized by appealing to financial factors and to the legacy of the global financial crisis (Borio 2014and Lo and Rogoff 2015).

As the policy implications from these two views can be different, it is essential for the academia and policymakers to identify the factors behind the world-wide decline in real rates (17th Geneva Report on the World Economy, 2015). This is particularly true for those central banks that have embarked in unconventional measures and that, in some cases, have brought policy rates into negative territory. These measures will be unwound in the next few years, but the pace and timing will crucially hinge on where the natural rate is estimated to be and how it is expected to evolve in the medium-term.

In this paper, we estimate a medium-scale DSGE model to answer two set of questions: what is the current level of the natural rate in the US and in the euro area and what are the main factors underlying its developments over the last decades, and in particular since the global financial crisis. Within the DSGE literature, the natural rate of interest is defined as the real rate which arises in an economy in which output is at its potential level and inflation is at the central bank's target; in this paper we use this definition and compute the natural rate by shutting down nominal rigidities (Woodford, 2003). The model is endowed with a rich set of shocks which allow us to gauge which of the explanations put forward in the literature for the decline in interest rates gets more support from the data.

There are several papers providing empirical estimates of the natural rate to which we connect. This fast-growing literature can be broadly divided in three strands (Giammarioli and Valla, 2004). The first one uses pure time-series methods – based on either univariate filtering methods (Weber, Lemke and Worms, 2008) or multivariate models (Lubik and Matthes, 2015; Hamilton et al., 2016) – to extract the low-frequency component from market data on yields on index-linked bonds (Christensen and Rudebusch, 2017). A second approach uses semi-structural econometric models with the natural rate as a latent variable: in these models, the natural rate is typically assumed to depend on the (trend) growth rate of potential output and a transitory component. The Kalman filter is used to jointly estimate the natural rate and potential output growth (Laubach and Williams 2003, Holston, Laubach and Williams, 2017, Pescatori and Turunen, 2016 for the US; Mésonnier and Renne, 2007 and Fries et al., 2016 for the euro area). The third approach relies on structural models: they can be either overlapping generation (OLG) models (Gagnon, Johannsen and López-Salido, 2016; Kara and von Thadden, 2016; Eggertsson, Mehrotra and Robbins, 2017) or DSGE models of the New Keynesian tradition (Edge et al., 2008, Justiniano and Primiceri, 2010, Barsky, Justiniano and Melosi, 2014, Curdia, Ferrero, Ng and Tambalotti, 2015, Del Negro et al., 2015, Del Negro et al., 2017 all for the US; Hristov, 2016 for the euro area).

We use a structural model since in our view this approach has the decisive advantage of providing economic intuition for the underling drivers of the natural rate (Neiss and Nelson, 2003). However, we do not go to the extreme of developing an OLG (or heterogeneous agents) model whose results will be less easy to digest for policymakers. Our approach, instead, is somewhat reminiscent of a growth accounting exercise: we take a standard workhorse model routinely used in central banks (e.g. Smets and Wouters, 2007) and enrich it with two set of shocks: permanent shocks that affect the *trend growth rate* of the economy and transitory shocks that drive its *cyclical* component. In this way, we keep a close link with the Laubach and Williams (2003) and Holston, Laubach and Williams (2017) approach; we differ, however, from this method in that we provide more structure to the transitory drivers of the natural rate and the other macroeconomic variables. The shocks play the role of "wedges" the model requires to explain the data. As these wedges are chosen to "speak" to the theories for the post-financial crisis low interest rate environment, the idea is to let the estimation tell us which theories are more consistent with the data. We consider this to be our contribution to the literature.

Our results point to qualitatively similar developments in the natural rates in the two economies. The natural rate reached its maximum level in early eighties in the US and early nineties in the euro area. Since then, it has declined persistently until the first years of this century, when it reached essentially zero per cent. After a surge in 2007 and 2008, it started declining again and reached a minimum of approximately -1 per cent in the euro area and -3 per cent in the US, on average in the period 2014-16. When compared with the literature, our results confirm a common finding in recent studies on the US – using either DSGE models or the Laubach and Williams (2003) approach – i.e. the fact that the natural rate has been on a declining trend at least since the onset of the 21st century. Our results are broadly in line with these estimates, in particular as far as long-run developments are concerned.

With regards to the drivers, the natural rate in the US and the euro area is driven primarily by shocks to the cyclical component, with those affecting the growth rate of output in the long-run playing a more muted role. In the euro area, risk premium shocks, possibly capturing changes in the preference for safe assets, explain the bulk of the decline after the global financial crisis; in the US, an important role is also played by the shock to the marginal efficiency of investment, which in a model without a financial sector captures inefficiencies in financial intermediation that diminishes the efficiency with which resources are channeled from savers to productive borrowers (Justiniano et al., 2011). The importance of the risk premium for the natural rate in the US has been documented by Del Negro et al. (2017). The differences in the contribution of the shocks in the two economies underscore the importance of adopting a structural approach with a rich stochastic structure, featuring both permanent and transitory shocks, to the estimation of the natural rate. Our results show that the downward trend in natural rates is due to unfavorable transitory but persistent shocks and suggest the need for investing in models in which financial factors, including the presence of safe assets, play a key role.

The remainder of the paper is as follows. Section 2 presents the model and Section 3 describes estimation; Section 4 presents the estimates and the historical decomposition of the natural rate. Section 5 discusses the robustness of the findings. Section 6 concludes, drawing policy implications.

2. The model

This Section provides a bird's eye view of the model. Appendix A provides more details.

A representative household maximises her utility function by choosing consumption, labour supply, capital and investment, subject to a budget constraint and a capital accumulation law. Households rent the capital stock and supply labour to firms. Changes in households' preference for safe assets shift the demand for savings away from capital. Households have preferences for smooth changes in consumption and hours worked. The presence of habit formation in labour is equivalent to assuming learning-by-doing (Chang et al., 2002 and Bouakez and Kano, 2006) by households. Households' preferences are subject to shocks that affect their degree of patience and their disutility from working.

A representative firm produces a homogeneous intermediate good with a production technology that uses labour and capital. The production function is shifted by permanent (labour augmenting) and transitory shocks to technology. The efficiency of newly installed capital (i.e. the investment specific technology) is subject to permanent shocks. A continuum of monopolistically competitive firms differentiates the homogeneous good and choose the prices charged to households taking into account the demand and the costs for changing these prices. This (menu) cost is a function of the deviation of newly set prices from a combination of past inflation and the inflation target set by the central bank. Firms' mark-up is subject to transitory shocks.

The central bank sets the policy rate according to a Taylor rule. A monetary policy shock randomly changes the policy rate, together with a persistent shock to the inflation target. We do not attempt to model unconventional monetary policy. The various measures adopted by central banks after the outbreak of the global financial crisis constitute a major break in monetary policy making, which would be difficult to deal with in a medium-scale DSGE model. We also do not take into account the presence of a lower bound to the policy rates, but we do carry out some robustness along this dimension (Section 5).

We use the same model for the US and the euro area, which are assumed to be closed economies, in line with the literature; each model is estimated separately. This choice is motivated by the results in Smets and Wouters (2005), who show that over the period 1974-2002 the two economies are remarkably similar in terms of drivers and propagation mechanism.

The model does not feature neither financial frictions (e.g. Bernanke et al., 1999 and Iacoviello, 2005) nor a banking sector. The reason is that, ideally, fitting the model for both the US and the euro area would require including both features, although with varying degrees, in order for the model to be consistent with the functioning of the financial sector in both economies. Moreover, data on credit spread, firms' leverage or lending rates are not available for the euro area in the seventies and eighties. Some of the shocks included in the model may be capturing, in a reduced-form, the functioning of financial markets (Justiniano et al. 2011; see section 2.2).

2.1 Permanent shocks and stochastic trends

The model features a balanced-growth path which is determined by the growth rate of three unit root processes with stochastic drift: labour augmenting technology, investment-specific technology and labour supply. These drifts, which are modelled as stochastic AR(1) processes, affect the growth rate of the economy along the balanced-growth path. This gives the model a chance to account for the very persistent decline of real and natural interest rates documented in the literature. In this section, we discuss each of these shocks in turn, connecting them to various potential explanations put forward in the literature for the decline in real interest rates.

The process for the labour-augmenting technology shock (King, Plosser and Rebelo, 1985) is:

$$\log A_t = \log A_{t-1} + \gamma_t^A$$

where the (gross) growth rate γ_t^A follows the AR(1) process with γ_A being the steady state growth:

$$\gamma_t^A = \gamma_A + \rho_A \gamma_{t-1}^A + \varepsilon_t^A$$

with ε_t^A being a zero-mean random variable with normal distribution and standard deviation σ_A. These shocks can potentially capture the trend decline in the rate of innovation which, as argued by Gordon (2012 and 2014), can explain the decline in real rates. The process for the investment specific technology (IST) shock (Justiniano et al., 2011) is:

$$\log A_t^K = \log A_{t-1}^k + \gamma_t^{AK}$$

where the (gross) growth rate γ_t^{AK} follows the AR(1) process with γ_{AK} as steady state growth:

$$\gamma_t^{AK} = \gamma_{AK} + \rho_{AK} \gamma_{t-1}^{AK} + \varepsilon_t^{AK}$$

with ε_t^{AK} being a zero-mean random variable with normal distribution and standard deviation σ_{AK}. This component of the growth rate of the economy captures the effects of the persistent decline observed in the relative price of investment goods, in particular in the US (Thwaites, 2015). Finally, the process for the labour supply shock (Chang et al., 2007) is:

$$\log B_t = \log B_{t-1} + \gamma_t^B$$

where the growth rate γ_t^B follows the AR(1) process with γ_B being the steady state growth:

$$\gamma_t^B = \gamma_B + \rho_B \gamma_{t-1}^B + \varepsilon_t^B$$

with ε_t^B being a zero-mean random variable with normal distribution and standard deviation σ_B. Labour supply shocks may capture permanent shifts in the labour input due to structural changes in demography, household production technology or preferences (Chang et al., 2007).

The specification of the utility function and of the technology process yield a balance-growth path for the real variables. Along this path, real consumption and real output grow at the rate equal to:

$$\gamma_t^C = \gamma_t^A \, \gamma_t^B \, (\gamma_t^{AK})^{\frac{\alpha}{1-\alpha}}$$

where α is the capital share in the production function. Real investment and capital grow faster than real consumption, at the rate:

$$\gamma_t^I = \gamma_t^A \, \gamma_t^B \, (\gamma_t^{AK})^{\frac{1}{1-\alpha}}$$

while the price of investment relative to consumption declines at the rate $\gamma_t^q = \frac{1}{\gamma_t^{AK}}$ .The trend in hours is only determined by the drift in the labour supply process, γ_t^B, while real wages grow at the rate:

$$\gamma_t^w = \gamma_t^A \, (\gamma_t^{AK})^{\frac{1}{1-\alpha}}$$

which does not depend on the labour supply process.

2.2 Transitory shocks

Fluctuations around the balance growth path are driven by transitory, but potentially persistent, shocks to monetary policy, firms' mark-up, total factor productivity (TFP), marginal efficiency of investment (MEI), risk premium for investing in safe assets and households' discount factor.

The monetary policy and mark-up shocks are independent and identically distributed (i.i.d.) random variables with normal distributions and are meant to capture the high frequency movements in the short-term nominal interest rate and inflation. The stationary technology shock (Kydland and Prescott, 1982) is included in line with the empirical DSGE literature.

The shock to the marginal efficiency of investment (MEI) affects the production of installed capital from investment goods or, more broadly, the transformation of savings into future productive capital input. Justiniano et al. (2010) show that this shock explains most of the variability of output and hours at business cycle frequencies. Given that this shock can augment or diminish the efficiency with which savings are turned into investment goods, one possible interpretation of it is as a proxy for more fundamental disturbances to the functioning of the financial sector (Justiniano et al. 2011).

As far as the risk premium shock (Smets and Wouters, 2003 and 2007) is concerned, Fisher (2015) argues that this shock, although originally appended to the representative household's linearized Euler equation without any structural interpretation, can be given one. The author shows that this shock acts as a shifter of the demand for safe and liquid assets (such as short-term Treasury securities) and thus has an interpretation as a "liquidity" or a "flight to safety" shock. When this shock hits, consumers have an incentive to save more through a safe asset (as opposed to save in physical capital). This means that the shock can generate positive business cycle co-movements among investment, consumption, output and hours worked (Fisher 2015), and thus its effects are isomorphic to those arising from an increase precautionary savings. Caballero and Farhi (2017) develop a model to assess the macroeconomic implications of safe asset shortages.

The risk premium should not be confused with a credit spread shock arising from the presence of financial frictions (Christiano et al., 2014), which the model does not feature, as this shock capture exogenous changes in borrowers' riskiness.

The shock to the households' discount factor induces compensating changes in consumption and investment. To the extent that it influences households' propensity to save, this shock might best be thought of as capturing the effects of a saving glut (Bernanke, 2005) or deleveraging (Eggertsson and Krugman, 2011). While a shock to the discount factor affect consumption and savings, the shock to the risk premium influences, for a given amount of savings, the choice between capital and the safe asset.

2.3 The natural rate

In line with the existing literature we define the natural rate of interest (r_t^n) as the real short-term interest rate that is consistent with output being at its flexible price level ($Y_t = Y_t^f\ \forall\ t$; the output gap is closed) and inflation being at the central bank's target ($\pi_t = \bar{\pi}_t\ \forall\ t$). In practice, the natural rate is the real rate that arises in an economy in which there are no nominal (price) rigidities. When computing the natural rate, we do not undo the steady state distortion arising from the presence of mark-ups in the good-producing sector. This implies that real disturbances may affect the natural rate and the "efficient" rate of interest differently.

The natural rate r_t^n is determined by the Euler equation for the choice of the one-period nominal safe bond in the flexible price model:

$$r_t^n = \frac{\tilde{\lambda}_t}{\beta E_t \left(\frac{\tilde{\lambda}_{t+1}}{\gamma_{t+1}^c}\right) \vartheta_t}$$

where $\tilde{\lambda}_t$ is the Lagrange multiplier of the household's budget constraint (in the stationary version of the model) and ϑ_t is the risk premium shock. The natural rate of interest declines, *ceteris paribus* and disregarding the effects of the other shocks that indirectly affect it, when households increase the demand for safe assets and when the growth rate of real consumption is expected to increase. In computing the natural rate, we do not set to zero any of the shocks, including the risk premium, that do have an impact on real variables in the flexible price model. By construction monetary policy and mark-up shocks do not affect the natural rate. It is important to acknowledge that the natural rate is driven by stationary AR(1) shocks, as the equations above show. This is important in order to assess the role of the different shocks in driving the natural rate, in particular in the frequency domain (section 4.4).

We close this section with a discussion of the potential implications of the lower bound to policy rates for our analysis. To this end, it is important to distinguish between the US and the euro area. The Federal Open Market Committee of the Federal Reserve lowered the target for the Federal funds rate to the effective lower bound in December 2008. Then, the Federal Reserve adopted various large-scale asset purchase programmes and resorted on forward guidance to further loosen the monetary stance. The European Central Bank (ECB) reached the effective lower bound to policy rates in March 2016. In light of these developments, the lower bound could be more of concern for the analysis for the US, given that the model is linearized around steady state and therefore does not take into account the presence of the lower bound. The lower bound could potentially affect the estimation of the monetary policy shocks: as the policy rate does not respond to shocks affecting inflation and output, these shocks may also show up as monetary policy shocks. In this case, the estimate of the natural rate may not be affected. In order to assess whether the estimate of the natural rate could be influenced by the inclusion of the lower bound period in the estimation sample, we have estimated the model for the US over the period 1980-2008 and also used a measure of shadow rate as the short-term rate. The results are discussed in Section 5.

3. Bayesian estimation

In this Section, we present the methodology for estimating the model and inferring about the natural rate. The model is estimated with Bayesian methods, as commonly done in the literature (An and Schorfheide, 2007). The linearized model is cast into a state-space representation and the

Kalman filter is used to compute the likelihood function $L(\theta|Y^T)$ of the observed variables Y^T. Given a prior distribution for the parameters of the model θ, $P(\theta)$, the posterior distribution $P(\theta|Y^T)$ is:

$$p(\theta|Y^T) \propto L(\theta|Y^T)\, p(\theta)\ .$$

Inference on $P(\theta|Y^T)$ and all the computations are carried out with Dynare (http://www.dynare.org/).

3.1 Data

The model is fit to seven macroeconomic variables at annual frequency for the US and the euro area for the period 1971-2016. We use annual data rather than quarterly since we are interested in capturing the low frequency movements in the natural rate of interest rather than the short-run (below two year) developments. In Section 5 we test the robustness of the results to alternative sample periods.

The following time series are used in the estimation: per-capita real consumption growth, per-capita real investment growth, changes in the relative (to consumption) price of investment, inflation, a short-term interest rate, the growth of labour input (relative to population) and a measure of inflation target. The choice for the latter variable deserves some discussion. As for the US, we use the Philadelphia Fed long-term expectations merged with Blue Chip and Livingston Survey data. As for the euro area, we use the implicit target for the ECB as estimated by Lippi and Neri (2007) who follow Gerlach and Svensson (2003) in modelling an implicit time-varying inflation objective for the euro area as a whole based on a partial adjustment mechanism towards the Bundesbank's target. Data on these measures of inflation targets are available for the US since 1979 and for the euro area since 1982. The observations for the previous years are estimated with the Kalman filter used in the computation of the likelihood. Regarding the labour input, we used total employment over population for the euro area, given that hours worked are not available for the full sample, and total per-capita hours worked for the US.[1]

The time series with clear trends are made stationary before estimation by taking first differences. We do not subtract the mean from the data since we are interested in estimating the parameters characterizing the steady state of model.

[1] We could have used an additional equation linking hours worked to employment, as in Smets and Wouters (2003). However, we in our view this choice should not alter the results.

The measurement equations describe the mapping between data and model counterparts:

$$\Delta C_t = \bar{\gamma}^C + \tilde{C}_t - \tilde{C}_{t-1} + \tilde{\gamma}_t^C$$

$$\Delta I_t = \bar{\gamma}^I + \tilde{I}_t - \tilde{I}_{t-1} + \tilde{\gamma}_t^I$$

$$\Delta q_t = \bar{\gamma}^q + \tilde{q}_t - \tilde{q}_{t-1} + \tilde{\gamma}_t^q$$

$$\Delta h_t = \bar{\gamma}^B + \tilde{h}_t - \tilde{h}_{t-1} + \tilde{\gamma}_t^B$$

$$\pi_t = \bar{\pi} + \tilde{\pi}_t$$

$$\pi_t^* = \bar{\pi}^* + \tilde{\tilde{\pi}}_t^*$$

$$\mathrm{R}_t = \frac{\bar{\pi}\bar{\gamma}^C}{\beta} + \tilde{R}_t$$

where a ˜ denotes deviation from steady state and a ¯ the steady state value of the variable; π_t^* is the inflation target. The steady state growth rates of real consumption and investment, the relative price of investment and hours worked are described in Section 2.1.

3.2 Prior distributions

Priors are added to take external information into account and to add curvature to the likelihood function that may be relatively flat along some dimensions (Canova and Sala, 2009).

The prior distributions are assumed to be relatively flat, so as to let the data speak. In Section 5 we assess the robustness of the results to the choice of the prior. The priors on the parameters of the stochastic processes are harmonized. The standard errors of the innovations are assumed to follow an inverse-gamma distribution with a mean of 0.01 and standard deviation of 0.05, leading to a very loose prior. The persistence of the autoregressive parameter has a beta distribution with mean and standard deviation equal to, respectively, 0.5 and 0.2. The steady state gross growth rate of the labour-augmenting technology shock has a normal distribution with mean and standard deviation equal to, respectively, 1.01 and 0.005; the growth rate of the investment-specific technology shock has a normal distribution with mean and standard deviation equal to, respectively, 1.02 and 0.005, considering that Justiniano et al. (2011) find that the latter technology grows (almost twice) faster than the labour-augmenting one between 1982 and 2009. The growth rate of the labour supply process has also a normal distribution with mean and standard deviation equal to, respectively, 1.00 and 0.005. The steady-state inflation rate and the discount rate have also normal distribution, with means equal to, respectively, 0.02 and 1.0, and standard deviations equal to 0.005 and 0.02.

Four parameters are calibrated. The depreciation rate δ is set at 0.025, the capital share in the production function, α, at 0.36, the steady state mark-up of monopolistic competitive firms at 1.2 (implying a demand elasticity in the Dixit-Stiglitz aggregator, ϱ, of 6; see Appendix A) and the (Frisch) labour supply elasticity is fixed at 1.5. These values are in line with the existing literature.

3.3 Inference

Inference on the parameters is based on the random walk version of the Metropolis Hastings algorithm. Tables 1a and 1b show, respectively for the US and the euro area, the mean and standard deviation of the prior, as well as the functional form, and the median, and the 0.90 highest posterior density (HPD) intervals obtained with 20 parallel chains each of length 25,000 draws in order to have a large number of initial values for the algorithm. The initial 20 per cent of the draws in each chain is dropped as burn-in sample. The inference is based on a random sample of 20,000 draws. In order to check for the convergence of the simulation, we have also generated 500,000 draws from the posterior distribution of the parameters using a single chain and also five chains each of length 200,000 draws. The standard convergence diagnostics reported by Dynare suggest that convergence to the posterior distribution has been achieved to a satisfactory extent in all the cases.

All in all, the mean values of the marginal posterior distributions of the structural parameters are similar across countries. The parameter measuring the costs for adjusting investment is lower than the values found in the literature (Smets and Wouters, 2007 and Justiniano et al., 2011). The costs for adjusting the price of the final goods are similar in the US and the euro area. We do not compare the value of the parameter with other estimates (e.g. Ireland, 2003 for the US) as the frequency of our model is annual rather con quarterly. However, the low posterior mean suggests a rather low degree of price rigidity. Indexation of prices is almost equally split between past inflation and the inflation target; comparison of the marginal prior and posterior distributions, however, suggests that this parameter may not be well identified. The degree of habit formation in consumption and labour supply are relatively high and broadly similar to the values found in the literature. As far as the parameters of the monetary policy rule is concerned, the mean values of the marginal posterior distributions show that central banks adjust the policy rate more in response to the deviations of inflation from the target than to output. The degree of interest rate inertia is similar in the two economies.

Interestingly, the mean of the marginal posterior distribution of the steady state growth of labour-augmenting technology is larger in the euro area than in the US whereas the opposite is true for the steady state growth rate of the investment technological progress, consistently with a faster decline of the relative price of investment in the US. The steady state gross growth rate of the labour supply shock is equal to one in both economies, suggesting no trend in per-capita aggregates due to the labour supply process. These mean values imply that the steady state growth rates of real per-capita consumption and investment are larger in the US than in the euro area: the former is equal to, respectively, 1.8 and 1.4 per cent in the US and in the euro area, the latter is equal to 4.2 and 1.8.

The mean of the posterior distribution of the steady state of the inflation target is equal to 2.3 per cent in the euro area and 2.5 in the US. While these values are larger than the official targets of the Federal Reserve and the ECB, it is important to note that the estimated inflation target since 1999 has averaged, respectively, 2.0 and 1.9 per cent in the two economies.

The persistence of the shock processes is, in general, low and similar in the two economies. The only exception is the process for the inflation target, which is, as expectedly, very persistent in both economies. The mean of the marginal posterior distributions of the standard deviation of the innovations to the shocks are low in general.

3.4 Transmission of shocks

The dynamics of the model in response to the shocks are important to assess the persistence of the natural rate. To this end, this section briefly discusses the transmission of the shocks that matter in the flexible price version of the model, in which the natural rate of interest is defined and plays a role.

Figures 1 and 2 report the mean of the posterior distribution of the impulse responses of the natural and real rates together with the corresponding gap (first row), and of real consumption and investment growth (second row) for the US (Figure 1) and the euro area (Figure 2). The size of the shocks is set to the posterior mean of the standard deviation of the innovation to the shock processes. The responses are qualitatively similar in the two economies, both in terms of magnitude and persistence.

The interest rate gap increases after positive labour-augmenting and risk premium shocks, as the real rate falls by less than its natural counterpart; consumption and investment increase in response to the former shock and decline after the latter. An increase in the risk premium (i.e. an increase in the preference for the safe asset) reduces the amount of resources available for investment and consumption, lowering the natural and real interest rates as households are willing to accept a lower but safer return on their savings. The gap declines in response to the shock to the discount factor and to the marginal efficiency of investment, as the real rate increase by less than the natural rate. In response to these shocks, consumption and investment move in opposite directions, as households divert resources from one purpose to the other: in the case of the shock to the discount factor, as households become more or less patient (demand side effect), in the other case because the technology converting investment goods into newly installed capital either improves or worsen (supply side effect).

Under all shocks, the response of the interest rate gap is short-lived: from the second or third year after the shocks, the real rate tracks very closely its natural counterpart.

4. The natural rate of interest and its drivers

This Section presents the estimates of the natural rate and the ex-ante real interest rate in the US and in the euro area and shows their decomposition in terms of the contribution of the shocks. Figures 3 and 4 present the mean of the posterior distribution of the smoothed natural rate. Concerning the euro area (Figure 3), the following results emerge.

First, the natural rate was low and negative in the early seventies. Second, the upward trend that started at the end of the seventies continued until 1990, reaching a maximum of about 8 per cent. Since then, the natural rate has persistently declined until 2003, when it reached 0.0 per cent. After a surge in 2007 and 2008 (to 2.7 per cent), the natural rate started declining again and reached a minimum of -1.6 in 2015. Third, the natural rate was lower during the sovereign debt crisis than after the outbreak of the global financial crisis. This result is consistent with the view that flight to quality concerns were more widespread in the sovereign crisis, when markets were considering the possibility of euro-area countries defaulting on their public debt or even the break-up of the monetary union. García and Gimeno (2014) provide support to this interpretation and show that developments in euro area bond markets in the period 2009-13 were significantly affected by flight to safety and flight to liquidity. Fourth, the estimated real rate tracks very well its natural counterpart, possibly reflecting the low degree of price rigidity. However, there are significant differences between the two rates: the difference between the natural rate and the real rate, the so-called real interest rate gap, that can be used as an indicator of the stance of monetary policy (Neiss and Nelson, 2003), was large and positive in mid-eighties, late nineties and 2009, right after the outbreak of the global financial crisis. Finally, the interest rate gap was positive between 2013 and 2015 (0.9 per cent on average in the three years), suggesting a relatively tight stance of monetary policy: only in 2016, after the launch of the Asset Purchase Programme by the ECB, the gap turned negative.

In the US, the natural rate of interest shares some similarities to the euro area counterpart (Figure 4), in particular as far as the low frequency developments are concerned (Table 3). The natural rate peaked (7.5 per cent) in 1980 and since then it has declined persistently with cycles of varying length, reaching a minimum of -4.6 per cent in 2015, well below the euro area counterpart.

The main difference between the natural rate in the US and the euro area is the period in which they peaked. While in the US it reached its maximum level in the early eighties, the peak in the euro area was reached in the early nineties. Table 3 confirms that the natural rates have declined in both economies since the eighties and have reached negative values with the outbreak of the global financial crisis (-2.9 and -1.6 between 2009 and 2016 in, respectively, the US and the euro area).

Despite that natural rates differ among the two economies (Figure 5), the natural rate gaps (Figure 6) are quite similar and correlated (the correlation over the period 1980-2016 is 0.7). This result suggests that the monetary policy cycles in the two economies have been relatively synchronized: policies were restrictive in the first half of the eighties, in mid-nineties and immediately after the outbreak of the global financial crisis, and expansionary after the outbreak of the global financial and sovereign crises, albeit with varying timing and degree.

Our estimates for the natural rates are broadly similar to those available in the recent literature, in particular when based on DSGE models and for the US. Figures 7 and 8 show the comparison of our estimates with those available in the literature. Among the DSGE-based estimates, the one in Barsky et al. (2014) display the largest fluctuations; the estimate by Holston, Laubach and Williams (2017), the smoothest among the various estimates, gradually reached historical minima in 2016, having declined sharply after the outbreak of the global financial crisis. Estimates for the euro area are more heterogeneous. In most of the cases, the natural rate was close to zero between 2003 and 2005, positive and high in 2008 and negative in the following years in the case of the DSGE-based estimates, or very close to zero in the case of the semi-structural approach (Holston, Laubach and Williams, 2017). Our estimate is close to the ones in Hristov (2016), who relies on the basic Smets and Wouters (2007) model and on an augmented version that features the financial accelerator à la Bernanke, Gertler and Gilchrist (1999). The estimate by Fries et al. (2016) is different from all the others, in particular in 2008, when the natural rate is very negative. The authors employ the approach by Laubach and Williams (2003) to estimate country-specific natural rates for each of the largest four economies of the euro area.

The next sub-sections decompose the historical evolution of the natural (and real) rates over in the euro area and the US in terms of the contribution of the shocks; this historical decomposition allows for a story-telling of the decline in these rates and provides an explanation for their current low levels. Section 4.4 analyses the contribution of the various shocks to the dynamics of the natural rate (as well as the real interest rate, consumption and investment growth) at different frequencies to disentangle its medium-term (i.e. business cycle frequencies) and long-term drivers.

The decomposition of the deviations of the natural and ex-ante real rates from their steady state are computing setting the parameters at their marginal posterior mean.

4.1. Shock decomposition: the euro area

The shock decomposition of the natural rate in the euro area is shown in Figure 9. The largest contribution comes from the risk premium shock, which explains almost all the fluctuations around the steady state. The contributions of the shock to the discount rate and to the labour-augmenting technology, although small on average, have been positive since the beginning of the sample, but turned negative after the outbreak of the global financial crisis. Risk premium shocks do not explain much of real consumption and investment growth, which are driven primarily by the various technology shocks, with the largest contribution coming from the labour-augmenting one.

Monetary policy shocks have contributed to raising the real interest rate (Figure 10) in the eighties and to lowering it between 2002 and 2005 and after the outbreak of the global financial crisis, with the maximum negative contribution reached in 2012. In 2014 and 2015, the contribution of monetary policy shocks to the real rate has been nil.

4.2. Shock decomposition: the United States

The shock decomposition of the natural rate in the US is shown in Figure 11. Shocks to the labour-augmenting technology and to the marginal efficiency of investment (MEI) play the largest role over the whole period. The risk premium, instead, has become an important source of fluctuations in the natural rate after the outbreak of the global financial crisis; since then, the risk premium and the MEI shocks account for almost all the deviations from steady state. As argued by Justiniano et al. (2011), the MEI shock may be a proxy for more fundamental disturbances to the functioning of the financial sector. The risk premium shock may be capturing households' preference for safe assets (Fisher, 2015), either because of increased riskiness in the economy or because of aging of the population (Jones, 2016).[2] Del Negro et al. (2017) show that the natural rate in the US has been pushed downward since the early nineties by the premia on safe and liquid assets, which bear some similarities with our risk premium or preference for safe assets shock. According to their historical decomposition, the risk premium shock is the most important driver of the (short-term)

[2] Jones (2016) shows that the aggregate consequences of demographic changes in lifecycle models are well approximated by a representative agent RBC model with trends in productivity, marginal utility of consumption and labour wedge.

natural rate in the whole sample period.[3] The labour-augmenting technology shock has played an important role in raising the natural rate in the late nineties and before the outbreak of the global financial crisis. Turning to the real interest rate (Figure 12), the shock decomposition shows that just after the outbreak of the financial crisis, monetary policy shocks contributed to substantially lowering the real rate, in particular in 2008.

4.3. Shock decomposition: a discussion

Comparison of the results for the euro area and the US highlights important differences: the most striking is the role of the risk premium shock and of the shock to the marginal efficiency of investment. The former is the dominant source of fluctuations of the natural rate in the euro area in the whole sample; in the US, this shock plays an important role only after the outbreak of the global financial crisis, which led to a sharp increase in the preference for safe assets.

These findings suggest that the risk premium in the euro area may be capturing the effects of other shocks before the outbreak of the global financial crisis, and in particular between the early eighties and the early nineties. One possibility is that this shock captures the heterogeneity in monetary policies or in the yields on government securities. In order to provide some support to this interpretation, we have computed the correlations between, on the one hand, the posterior mean of the risk premium process and, on the other, the difference between the average short-term rates in Italy and Spain and those of France and Germany and the same difference for the long-term yields. In the first case, the correlation is equal to 0.33; in the second it increases to 0.45. This, admittedly very simple, analysis suggests that the risk premium may be capturing the heterogeneity in interest rates within the euro area, at least in the period before the beginning of the monetary union in 1999. We recognize that imposing that the euro area is a single economy before 1999 may be a restrictive assumption for the purpose of our analysis. In Section 5, we assess the robustness of the findings to the choice of the estimation period for the euro area. We leave to future research the possibility to set up and estimate a monetary union version of our DSGE model that takes into account the country differences in the natural rates and their impact on the aggregate euro-area natural rate.

4.4. Short and long-run drivers of the natural rate

After having provided a description of the shocks that have driven the natural and the real interest rates, we compute the contribution of the various shocks to their variance and to that of real

[3] It is not possible to compare with more details our estimates with those of Del Negro et al. (2017) since in their analysis the contributions of the shocks are grouped into four categories.

consumption and investment growth in the frequency domain. In this way, we are able to identify the shocks that matter at business cycle frequencies and in the long-run (Justiniano et al., 2010).

In the US, in the long-run (frequency equal to zero in the spectrum) shocks to the efficiency of investment and to the risk premium explain all the variance of the natural rate (Figure 13); as we move along the spectrum towards higher frequencies, the fraction of these shocks declines and the contribution of the shocks to the total factor productivity (the stationary technology) and to the labour-augmenting technology gradually increases. At business cycle frequencies, 30 per cent of the variance of the natural rate is due to shocks to the marginal efficiency of investment, 15 per cent to the risk premium shock, and about 50 per cent to the technology shocks. Investment-specific technology shock, that directly affect the relative price of investment, do not play a significant role in driving the natural rate both in the medium and long-run. Turning to the real interest rate, risk premium shocks and shocks to the marginal efficiency of investment explain almost all the variance at low frequencies. As the frequency increases, monetary policy shocks become the dominant source of fluctuations in the real interest rate.

Real consumption growth is driven by shocks to the labour-augmenting technology (50 per cent in the long-run) and labour supply (40 per cent in the long-run) almost at all frequencies. Investment growth at business cycle frequencies is driven primarily by shocks to the marginal efficiency of investment (80 per cent), in line with Justiniano et al. (2010), followed by shocks to the labour-augmenting technology and to the supply of labour. At low frequencies, investment-specific technology shocks account for around 35 per cent of the variance of real investment growth.

Turning to the euro area (Figure 14), the natural rate is driven only by shocks to the risk premium and to stationary technology (TFP) shock; the relative weight of the two shocks varies with the frequency, with the former becoming less important as the frequency increases. The importance of monetary policy shocks for the real rate increases at higher frequencies, as in the US, whereas the role of the risk premium declines. In the long-run, these shocks explain almost all the variance of the real rate. At business cycle frequencies, shocks to the inflation target explain around 15 per cent of the variance of the real rate.

Real consumption growth is driven at all frequencies by shocks to the labour-augmenting technology (70 per cent in the long-run) and to the supply of labour (around 20 per cent). Regarding real investment growth, shocks to the labour-augmenting technology play the largest role at low frequencies (around 55 per cent), together with shocks to the investment-specific technology, which accounts for around 25 per cent. Along the spectrum, the importance of shocks to the risk premium and to the marginal efficiency of investment increase, reaching, respectively, 35 and 15 per cent.

Summing up. All in all, the results of the historical decomposition and the variance decomposition in the frequency domain suggest that the drivers of the natural rate in the US and the euro area are different: in the euro area, risk premium shocks are the main source of fluctuations, while in the US the picture is more mixed, with risk premium shocks playing a smaller role compared with the euro area and shocks to technology and investment being the main drivers of the natural rate. The relative importance of these shocks varies along the spectrum.

5. Robustness

In this Section, we test the robustness of the results along several dimensions, including the choice of the sample period and the parameterization of the prior distributions.

Sample period. The seventies were a period of large fluctuations in nominal and real interest rates. The Federal Reserve's monetary policy underwent a significant change after the arrival of Volcker as chairman of the FOMC in October 1979 (Goodfriend and King, 2005). In early eighties' the FOMC began focusing more on inflation developments with the objective of lowering inflation expectations by keeping monetary conditions tight for a prolonged period. Due to this important change in US monetary policy-making, we estimated the model over the period 1980-2016 both for the US and the euro area. Figures C1 and C2 in Appendix C compare the estimates of the natural rate obtained with the two sample periods: 1971-2016 and 1980-2016. The comparison shows that there are limited differences between the two estimates for both economies. Tables C1 and C2 in Appendix C report the summary statistics of the marginal posterior distributions of the parameters. Comparison with Tables 1a and 1b show that the summary statistics are similar, with larger differences in the case of the parameter measuring price rigidity. In the post-1980 sample, the posterior mean of this parameter is larger than in the full sample.

Looser priors. In the Bayesian framework, the posterior distribution of the parameters can be influenced by the tightness of the prior when the likelihood function is computed on a small number of observations. To test for the robustness of our results, we have made the prior distributions of all the parameters except the autoregressive coefficient of the shocks looser by doubling the standard deviations. Tables C3 and C4 in Appendix C report the summary statistics of the marginal posterior distributions of the parameters. Also in this case, comparison with Tables 1a and 1b show that the summary statistics are very similar, with somewhat larger differences in the case of the parameter measuring the degree of price rigidity. The posterior mean of this parameter when the prior is looser is slightly lower than in the baseline calibration of the prior. Figures C3 and C4 in Appendix C show that the estimates of the natural rates are very similar to those obtained in the baseline specification of the prior distribution.

As a further robustness check, we have increased the mean of the prior for the parameter measuring the cost for adjusting good prices to 50 (from 20) and the standard deviation to 20 (from 5). The posterior mean for the euro area with this specification of the prior is equal to 9.34, compared with 11.3 in the baseline specification. The smoothed natural rate is almost identical to the estimate obtained in the baseline case. As for the US, the posterior mean of the parameter measuring price rigidity under the alternative specification of the prior is almost identical, 13.8, to the baseline specification.

Fixed steady state. As a further robustness check we have estimated the model fixing the steady state growth rates of the permanent shocks, the steady state inflation target and the discount factor as to match the average growth rates of consumption and investment, inflation and the real interest rate. The other parameters were estimated. The resulting natural rate are broadly similar to those obtained in Section 4 in terms of fluctuations and long-term developments. In the euro area, the estimate of the natural rate with fixed steady state reaches a lower level after the outbreak of the global financial crisis (-1.8 per cent on average, compared with -0.5 in the baseline specification of the model). The estimated natural rate in 2016 in the euro area is around -0.8 per cent in both cases. Also for the US, the estimate of the natural rate is lower compared with the baseline case in the period after the global financial crisis; the natural rate in 2016 is equal to -2.6 in the case with fixed steady states and -1.9 in the baseline case.

The lower bound period in the US. We have estimated the model over the period 1980-2008, thus ending the sample when the global financial crisis broke out and the Federal Reserve brought the target for the Federal funds rate at its effective lower bound (0.00-0.25 per cent). We also estimated the model using a measure of shadow rate (Krippner, 2016) after 2008 to account for the asset purchases and forward guidance implemented by the Federal Reserve. These two exercises are meant to shed light on the potential impact of failing to account for the lower bound on the estimates of the natural rate. In both cases, the estimates up to and including 2008 are very similar; some differences arise for the years 2011-2013 when using the shadow rate. However, the estimates for the 2014-2016 period are very similar in all the cases (figure C5 in Appendix C). We leave to future research the possibility to incorporate the lower bound constraint in estimating the model, following the analysis by Guerrieri and Iacoviello (2017).

The euro area post-1993 sample. Finally, we have estimated the model over the period 1994-2016, thus starting after the period in which shocks to the risk premium brought the natural rate persistently above the steady state (figure 9). The historical decomposition yields similar results to those obtained with the full sample (figure C6 in Appendix C); shocks to the risk premium are still the dominant source of decline in the natural rate after the global financial crisis.[4]

6. Concluding remarks

Understanding why real interest rates have fallen in the last decades and why they have remained at very low levels after the outbreak of the global financial crisis is of utmost importance for monetary policy-making. Assessing the current level of the natural rate of interest and how it may evolve in the next few years is even more important, as central banks in advanced economies are will sooner or later normalize their monetary policies.

A theoretical framework is required to infer the level of the natural rate from macroeconomic data, due to its nature of unobservable variable. This paper has presented closed-economy DSGE model of the US and euro-area with a rich stochastic structure consisting of permanent and transitory shocks to capture the impact of some of the factors that have been put forward in the literature to explain the decline of interest rates since the eighties.

The analysis shows that the natural rate has been trending downward over the last decades. Shocks to the risk premium, capturing changes in the preference for safe assets, to the efficiency of investment and to technology have been the main drivers, with varying degree depending on the economy and the period. Our results underscore the importance of adopting a structural approach, with a rich stochastic structure, to the estimation of natural rates.

The pace at which natural rates may recover from current historically low levels depend on households' preference for (scarce) safe assets and the capability of the financial sector to finance investment and support economic growth. Central banks should take into account how natural rates may evolve over time when designing the normalization path of monetary policies in order to avoid hitting the effective lower bound to the policy rates. At the same time, structural policies aiming at financing investment in R&D and raising potential output growth should be undertaken, along with growth-friendly fiscal policies and measures to enhance the functioning of the financial sector.

[4] The model estimated for Germany between 1980 and 2016 confirms the importance of risk premium shocks.

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Table 1a. Structural parameters of the model: United States

	Prior distribution			Posterior distribution		
	Prior type	Mean	Std. dev.	Mean	90% HPD	
ω	gamma	2.0	0.50	1.195	0.652	1.729
κ_P	gamma	20.0	5.00	13.860	9.284	18.423
γ_P	beta	0.50	0.10	0.491	0.337	0.648
γ_c	beta	0.50	0.025	0.505	0.464	0.545
γ_h	beta	0.60	0.050	0.638	0.575	0.702
φ_R	beta	0.70	0.10	0.594	0.508	0.681
φ_π	normal	2.50	0.20	2.678	2.377	2.962
φ_y	normal	0.50	0.10	0.226	0.115	0.335
γ_A	normal	1.01	0.005	1.005	1.000	1.011
γ_{AK}	normal	1.02	0.005	1.024	1.020	1.028
γ_B	normal	1.00	0.005	0.999	0.994	1.005
π	normal	0.02	0.005	0.026	0.019	0.032
β	normal	1.00	0.020	0.998	0.988	1.008
ρ_A	beta	0.500	0.200	0.247	0.129	0.363
ρ_B	beta	0.500	0.200	0.072	0.012	0.126
ρ_{AK}	beta	0.500	0.200	0.541	0.395	0.693
ρ_C	beta	0.500	0.200	0.554	0.238	0.882
ρ_π	beta	0.500	0.200	0.929	0.886	0.972
ρ_i	beta	0.500	0.200	0.575	0.428	0.733
ρ_z	beta	0.500	0.200	0.673	0.405	0.958
ρ_σ	beta	0.500	0.200	0.645	0.412	0.902
σ_A	inverse gamma	0.010	0.050	0.031	0.024	0.040
σ_B	inverse gamma	0.010	0.050	0.033	0.025	0.041
σ_{AK}	inverse gamma	0.010	0.050	0.012	0.010	0.014
σ_C	inverse gamma	0.010	0.050	0.006	0.002	0.009
σ_R	inverse gamma	0.010	0.050	0.016	0.012	0.019
σ_p	inverse gamma	0.010	0.050	0.011	0.002	0.022
σ_π	inverse gamma	0.010	0.050	0.005	0.004	0.006
σ_i	inverse gamma	0.010	0.050	0.092	0.052	0.131
σ_σ	inverse gamma	0.010	0.050	0.005	0.003	0.006
σ_z	inverse gamma	0.010	0.050	0.006	0.002	0.010

Table 1b. Structural parameters of the model: euro area

	Prior distribution			Posterior distribution		
	Prior type	Mean	Std. dev.	Mean	90% HPD	
ω	gamma	2.0	0.50	1.098	0.694	1.493
κ_P	gamma	20.0	5.00	11.303	7.389	15.142
γ_P	beta	0.50	0.10	0.434	0.273	0.587
γ_c	beta	0.50	0.025	0.524	0.482	0.566
γ_h	beta	0.60	0.050	0.614	0.553	0.674
φ_R	beta	0.70	0.10	0.616	0.543	0.690
φ_π	normal	2.50	0.20	2.731	2.449	2.996
φ_y	normal	0.50	0.10	0.152	-0.022	0.329
γ_A	normal	1.01	0.005	1.012	1.007	1.017
γ_{AK}	normal	1.02	0.005	1.004	1.000	1.008
γ_B	normal	1.00	0.005	1.000	0.996	1.004
π	normal	0.02	0.005	0.023	0.016	0.030
β	normal	1.00	0.020	0.997	0.988	1.006
ρ_A	beta	0.500	0.200	0.505	0.302	0.661
ρ_B	beta	0.500	0.200	0.117	0.022	0.209
ρ_{AK}	beta	0.500	0.200	0.518	0.345	0.700
ρ_C	beta	0.500	0.200	0.711	0.382	0.965
ρ_π	beta	0.500	0.200	0.961	0.933	0.992
ρ_i	beta	0.500	0.200	0.605	0.273	0.949
ρ_z	beta	0.500	0.200	0.743	0.504	0.987
ρ_σ	beta	0.500	0.200	0.747	0.610	0.885
σ_A	Inverse gamma	0.010	0.050	0.016	0.010	0.022
σ_B	Inverse gamma	0.010	0.050	0.016	0.012	0.020
σ_{AK}	Inverse gamma	0.010	0.050	0.007	0.005	0.008
σ_C	Inverse gamma	0.010	0.050	0.010	0.003	0.016
σ_R	Inverse gamma	0.010	0.050	0.011	0.009	0.014
σ_p	Inverse gamma	0.010	0.050	0.009	0.002	0.016
σ_π	Inverse gamma	0.010	0.050	0.004	0.003	0.005
σ_i	Inverse gamma	0.010	0.050	0.014	0.003	0.024
σ_σ	Inverse gamma	0.010	0.050	0.005	0.003	0.007
σ_z	Inverse gamma	0.010	0.050	0.005	0.002	0.009

Table 2. Structural parameters: a comparison

	United States			Euro area		
	Mean	90% HPD		Mean	90% HPD	
ω	1.195	0.652	1.729	1.098	0.694	1.493
κ_P	13.860	9.284	18.423	11.303	7.389	15.142
γ_P	0.491	0.337	0.648	0.434	0.273	0.587
γ_c	0.505	0.464	0.545	0.524	0.482	0.566
γ_h	0.638	0.575	0.702	0.614	0.553	0.674
φ_R	0.594	0.508	0.681	0.616	0.543	0.690
φ_π	2.678	2.377	2.962	2.731	2.449	2.996
φ_y	0.226	0.115	0.335	0.152	-0.022	0.329
γ_A	1.005	1.000	1.011	1.012	1.007	1.017
γ_{AK}	1.024	1.020	1.028	1.004	1.000	1.008
γ_B	0.999	0.994	1.005	1.000	0.996	1.004
π	0.026	0.019	0.032	0.023	0.016	0.030
β	0.998	0.988	1.008	0.997	0.988	1.006
ρ_A	0.247	0.129	0.363	0.505	0.302	0.661
ρ_B	0.072	0.012	0.126	0.117	0.022	0.209
ρ_{AK}	0.541	0.395	0.693	0.518	0.345	0.700
ρ_C	0.554	0.238	0.882	0.711	0.382	0.965
ρ_π	0.929	0.886	0.972	0.961	0.933	0.992
ρ_i	0.575	0.428	0.733	0.605	0.273	0.949
ρ_z	0.673	0.405	0.958	0.743	0.504	0.987
ρ_σ	0.645	0.412	0.902	0.747	0.610	0.885
σ_A	0.031	0.024	0.040	0.016	0.010	0.022
σ_B	0.033	0.025	0.041	0.016	0.012	0.020
σ_{AK}	0.012	0.010	0.014	0.007	0.005	0.008
σ_C	0.006	0.002	0.009	0.010	0.003	0.016
σ_R	0.016	0.012	0.019	0.011	0.009	0.014
σ_p	0.011	0.002	0.022	0.009	0.002	0.016
σ_π	0.005	0.004	0.006	0.004	0.003	0.005
σ_i	0.092	0.052	0.131	0.014	0.003	0.024
σ_σ	0.005	0.003	0.006	0.005	0.003	0.007
σ_z	0.006	0.002	0.010	0.005	0.002	0.009

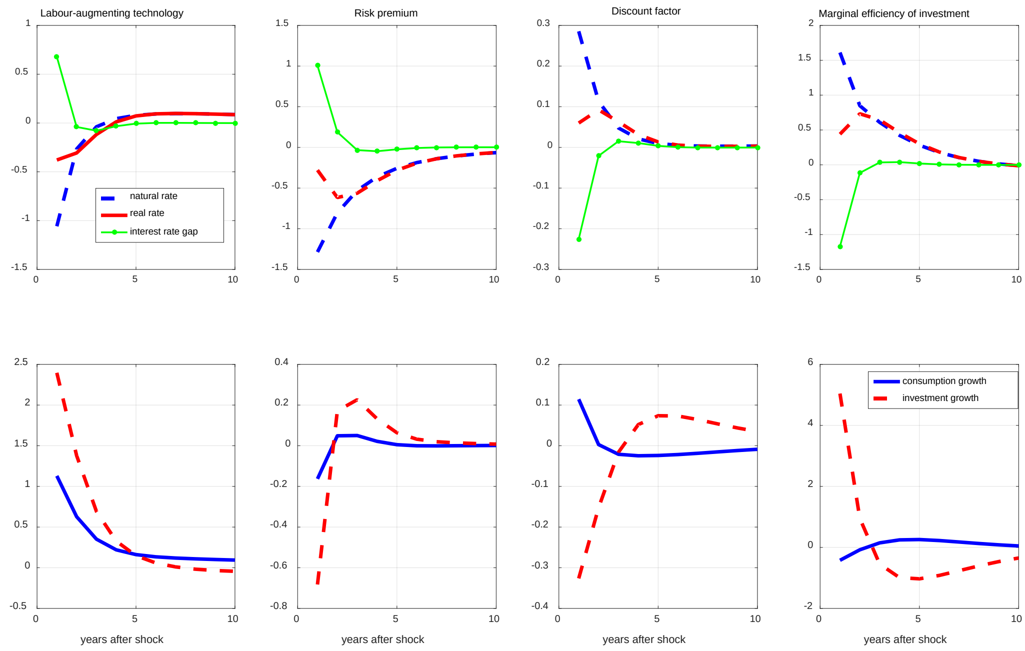
Table 3. Natural rate of interest: period averages

	United States	Euro area
1971-1980	1.1	0.6
1981-1990	3.7	4.0
1991-2000	2.1	3.5
2001-2007	0.4	0.9
2009-2016	-2.9	-1.6

Notes: the averages are computed using the mean of the marginal posterior distribution of the smoothed estimates of the natural rate for each year.

Figure 1. Selected impulse responses: United States

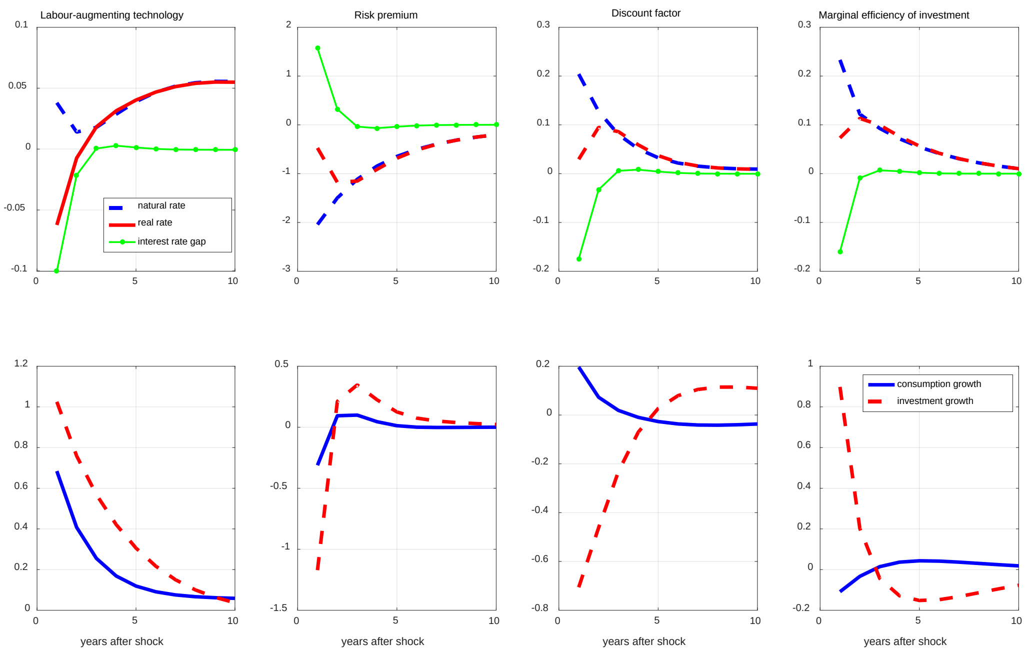
(*percentage deviation from steady state*)



Notes: mean of the marginal posterior distributions for each year of impulse horizon.

Figure 2. Selected impulse responses: euro area

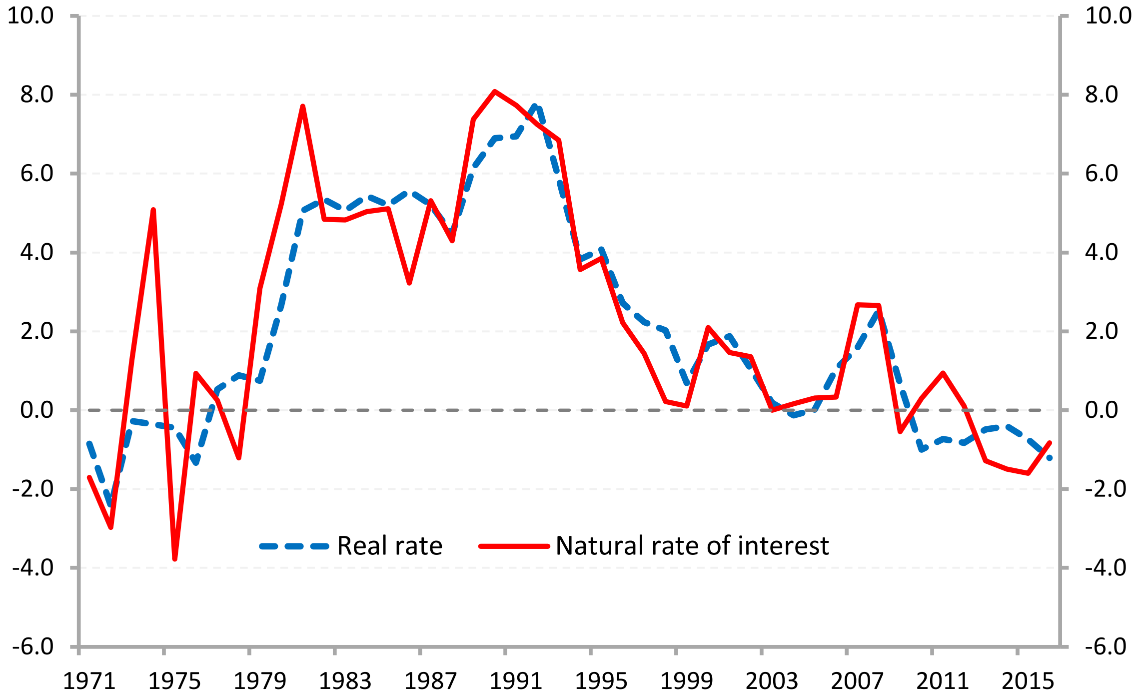
(*percentage deviation from steady state*)



Notes: mean of the marginal posterior distributions for each year of impulse horizon.

Figure 3. The natural rate of interest in the euro area

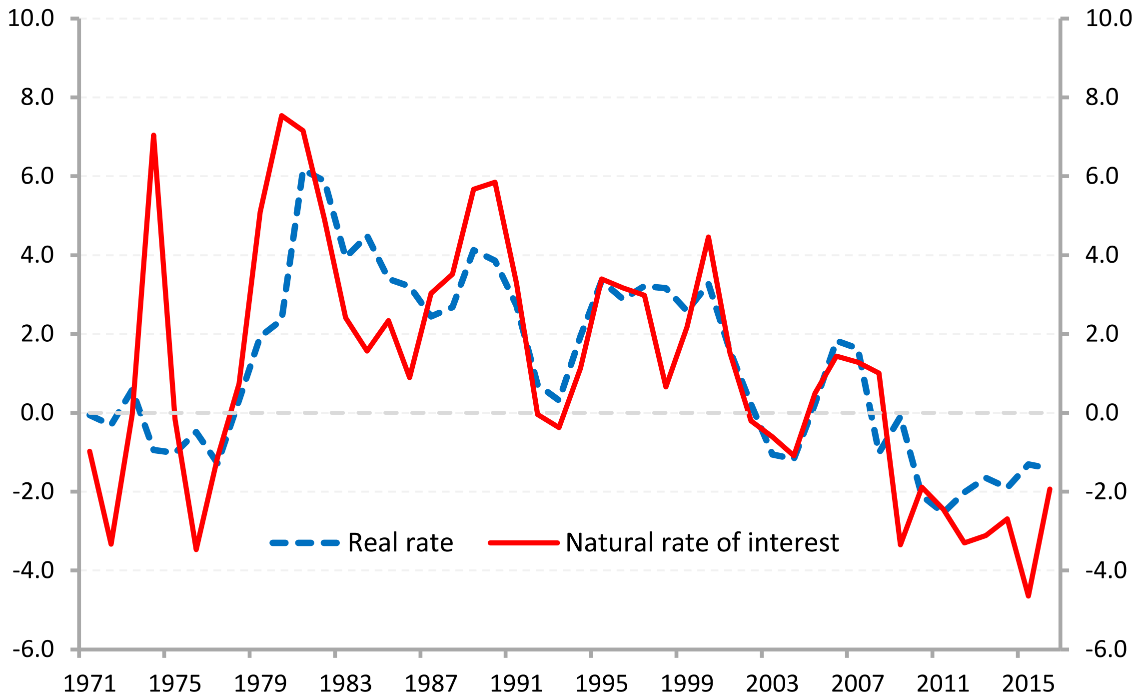
(*per cent*)



Notes: mean of the marginal posterior distribution of the smoothed estimates for each year.

Figure 4. The natural rate of interest in the United States

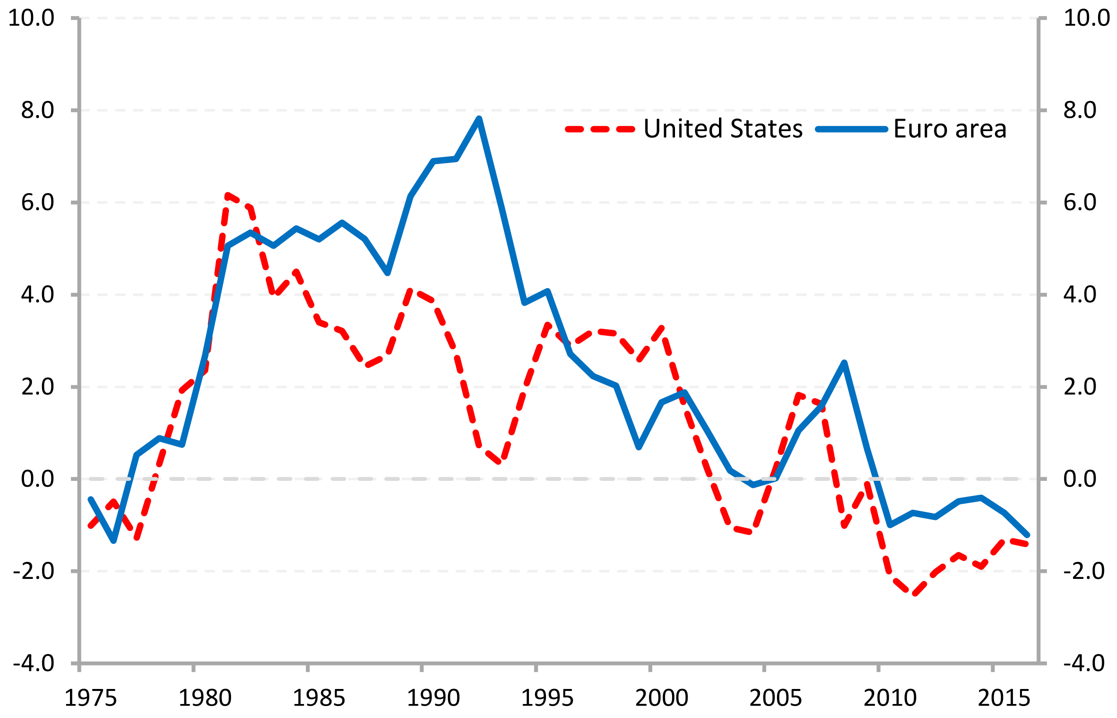
(*per cent*)



Notes: mean of the marginal posterior distribution of the smoothed estimates for each year.

Figure 5. The natural rate of interest: a comparison

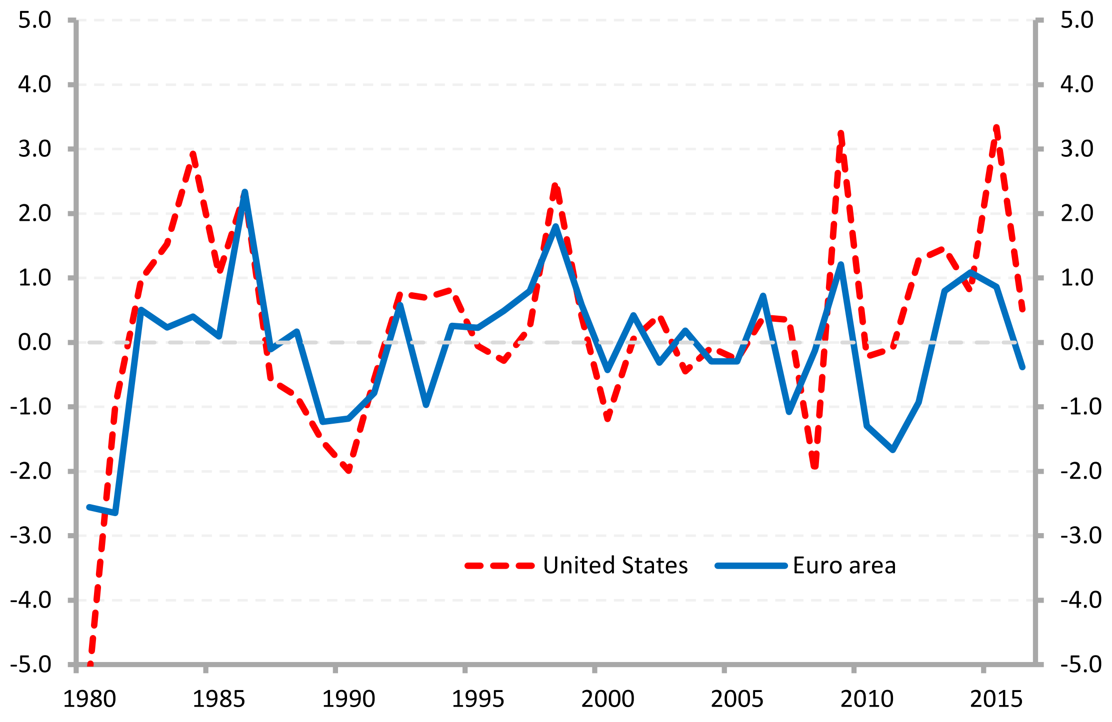
(*per cent*)



Notes: mean of the marginal posterior distribution of the smoothed estimates for each year.

Figure 6. The natural rate gap in the euro area and the US

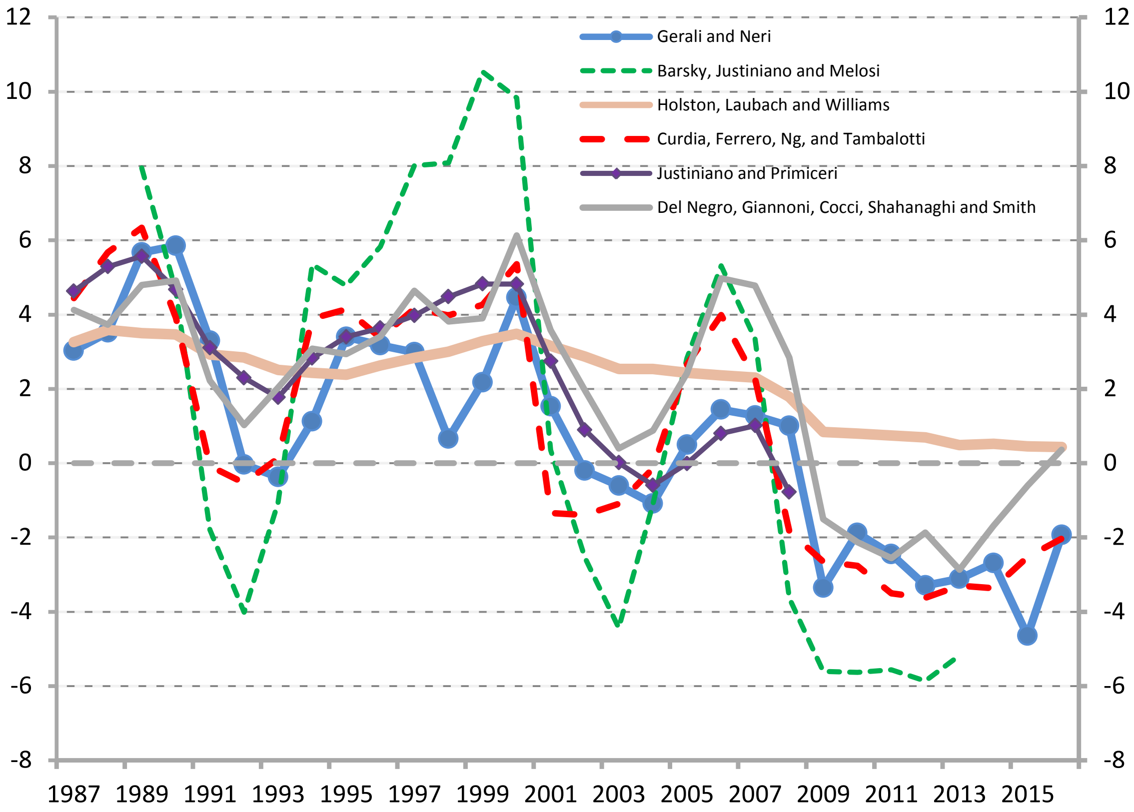
(*percentage points*)



Notes: mean of the marginal posterior distribution of the smoothed estimates for each year.

Figure 7. The natural rate of interest in the United States: comparison of estimates

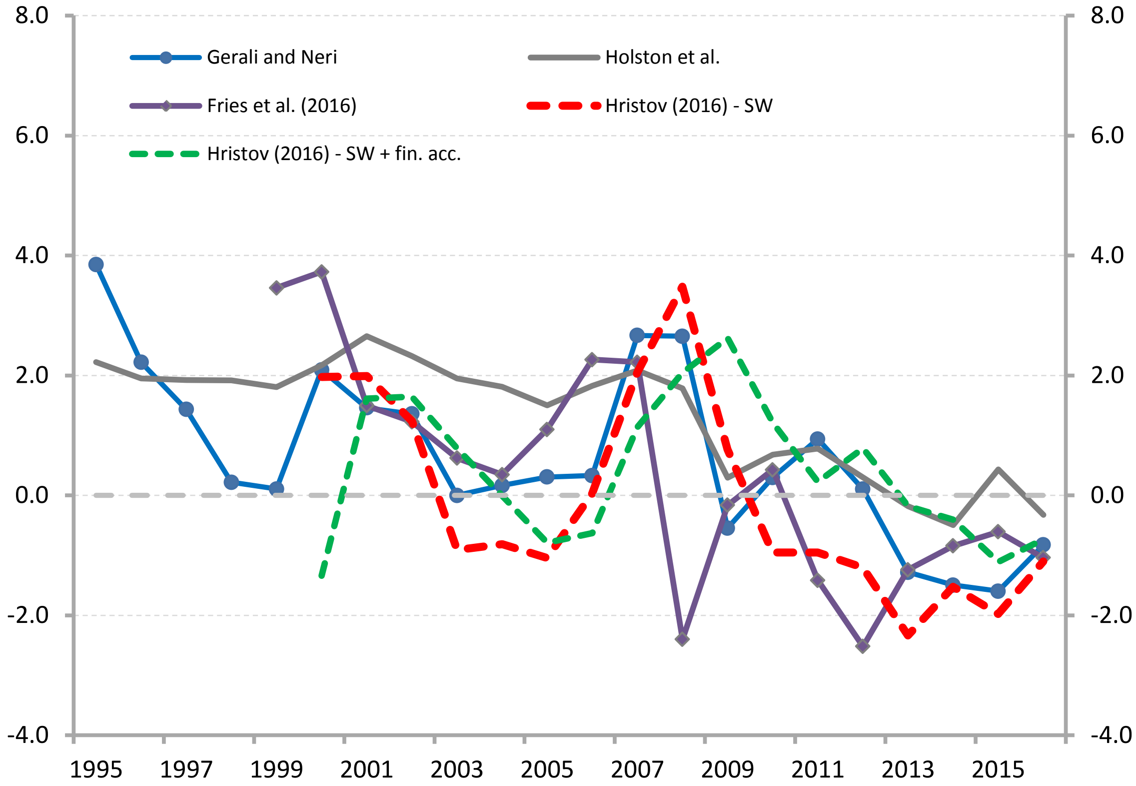
(*per cent*)



Note: Barsky, Justiniano and Melosi: American Economic Review Papers and Proceedings, 2014 – Holston, Laubach and Williams: Journal of International Economics, 2017 – Curdia, Ferrero. Ng and Tambalotti, Journal of Monetary Economics, 2015 – Justiniano and Primiceri, Chicago FED “Economic Perspectives”, 2010 – Del Negro, Giannoni, Cocci, Shahanaghi and Smith, Liberty Street Economics, 2015 – Gerali and Neri: mean of the marginal posterior distribution of the smoothed estimates for each year.

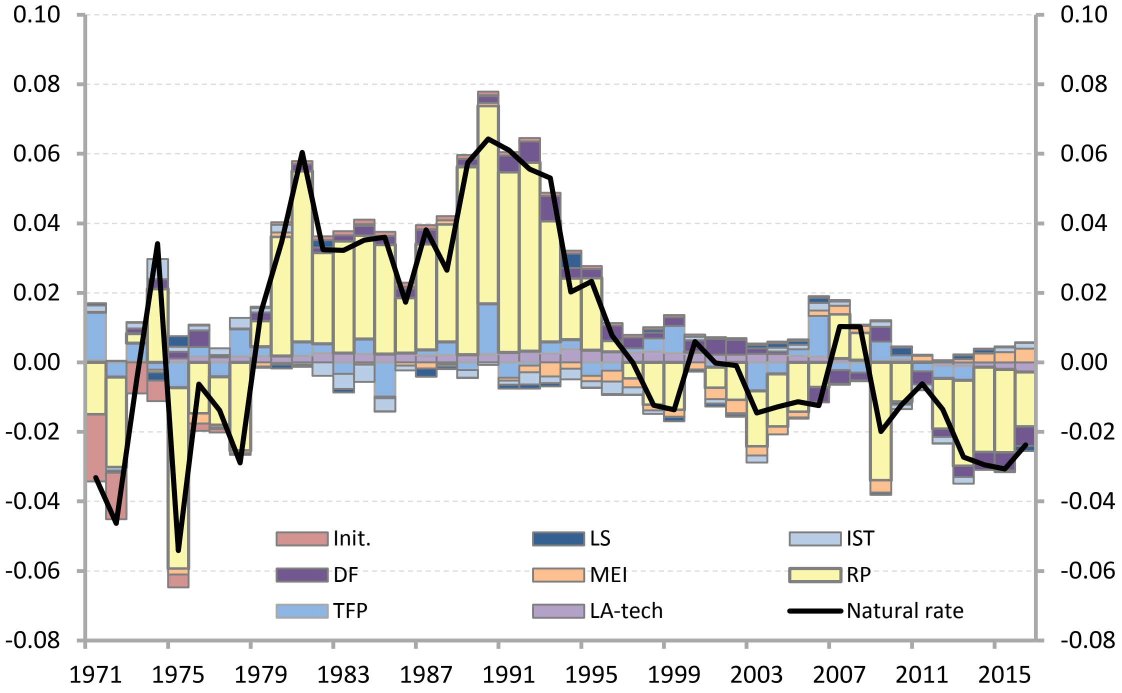
Figure 8. The natural rate of interest in the euro area: comparison of estimates

(*per cent*)



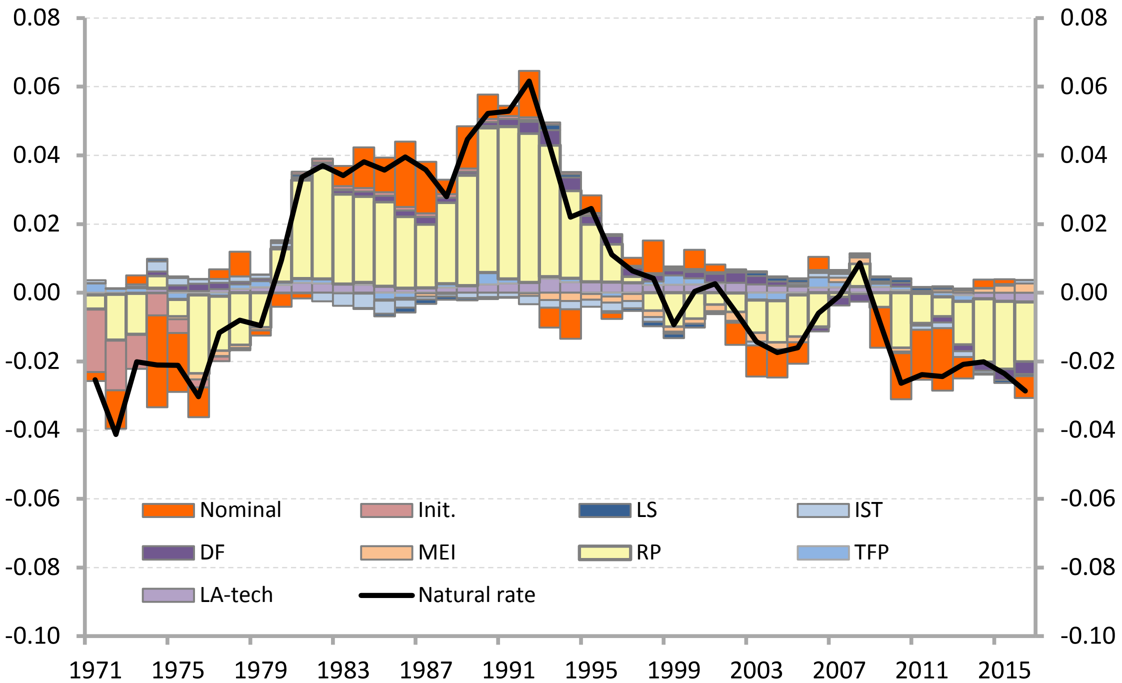
Note: Fries, Mésonnier, Mouabbi and Renne, Banque de France Working Paper, 2016 – Holston, Laubach and Williams: Journal of International Economics, 2017 – Hristov, CESifo Forum, 2016 – Gerali and Neri: mean of the marginal posterior distribution of the smoothed estimates for each year.

Figure 9. The natural rate of interest in the euro area: shock decomposition



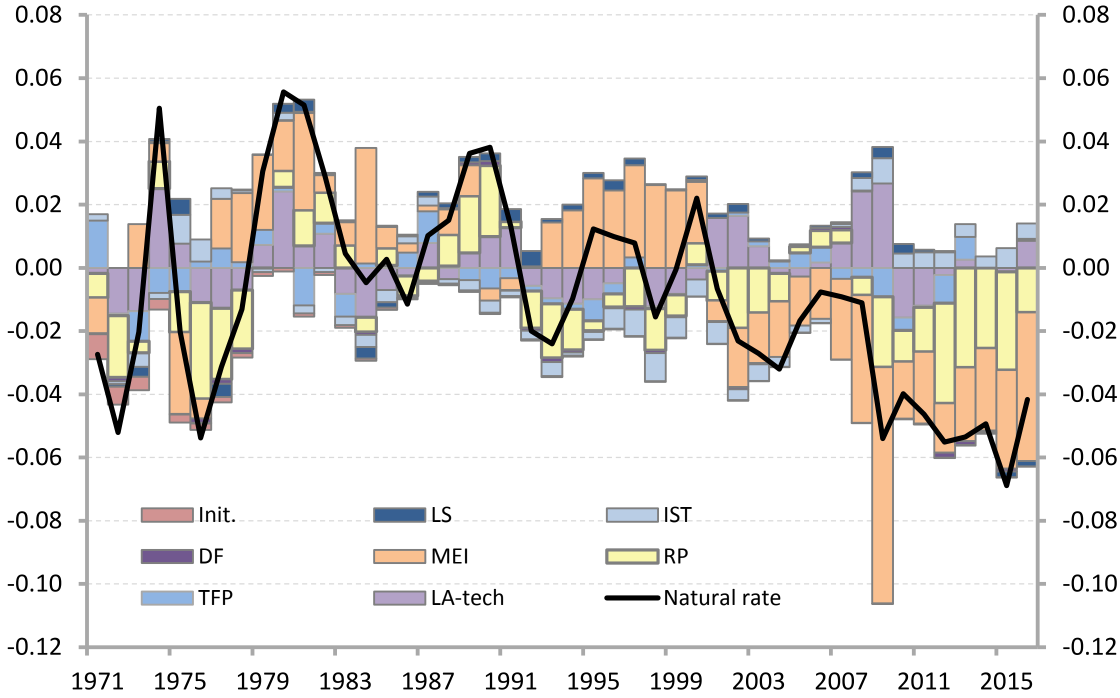
Notes: mean of the marginal posterior distribution of the smoothed estimates and shock contribution for each year. The chart plots the deviation of the natural rate of interest from the steady state. Init. = initial conditions; LS = labour supply; IST = investment specific technology; DF = discount factor; MEI = marginal efficiency of investment; RP = risk premium; TFP = total factor productivity; LA-tech = labour-augmenting technology.

Figure 10. The real interest rate in the euro area: shock decomposition



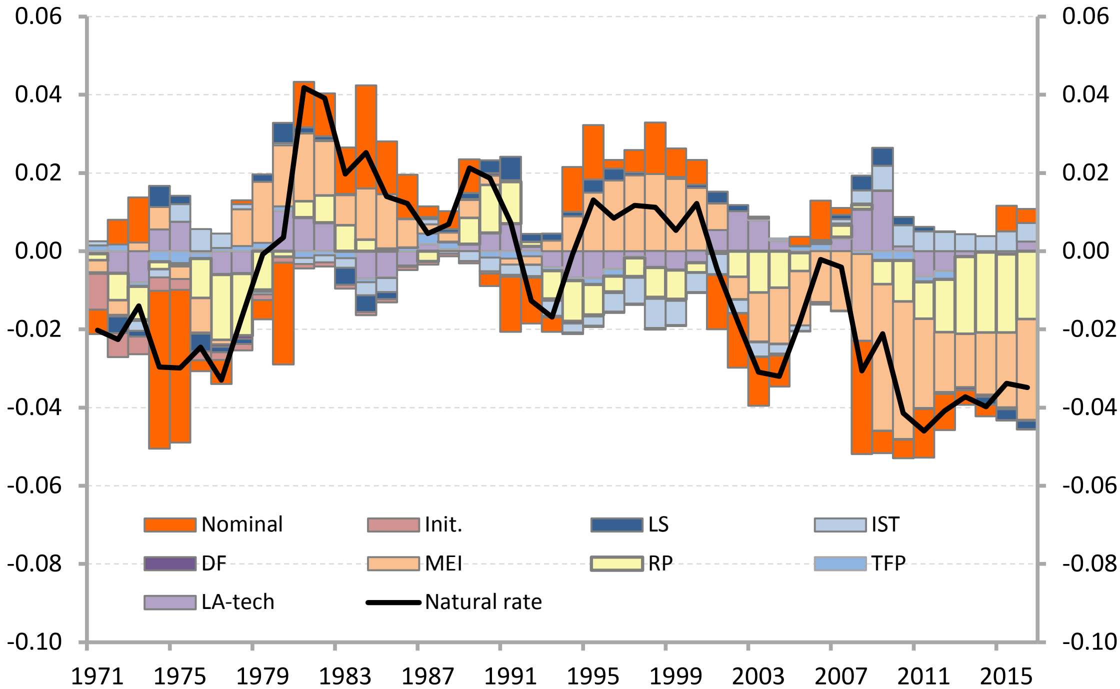
Notes: see note to figure 9.

Figure 11. The natural rate of interest in the US: shock decomposition



Notes: mean of the marginal posterior distribution of the smoothed estimates and shock contribution for each year. The chart plots the deviation of the natural rate of interest from the steady state. Init. = initial conditions; LS = labour supply; IST = investment specific technology; DF = discount factor; MEI = marginal efficiency of investment; RP = risk premium; TFP = total factor productivity; LA-tech = labour-augmenting technology.

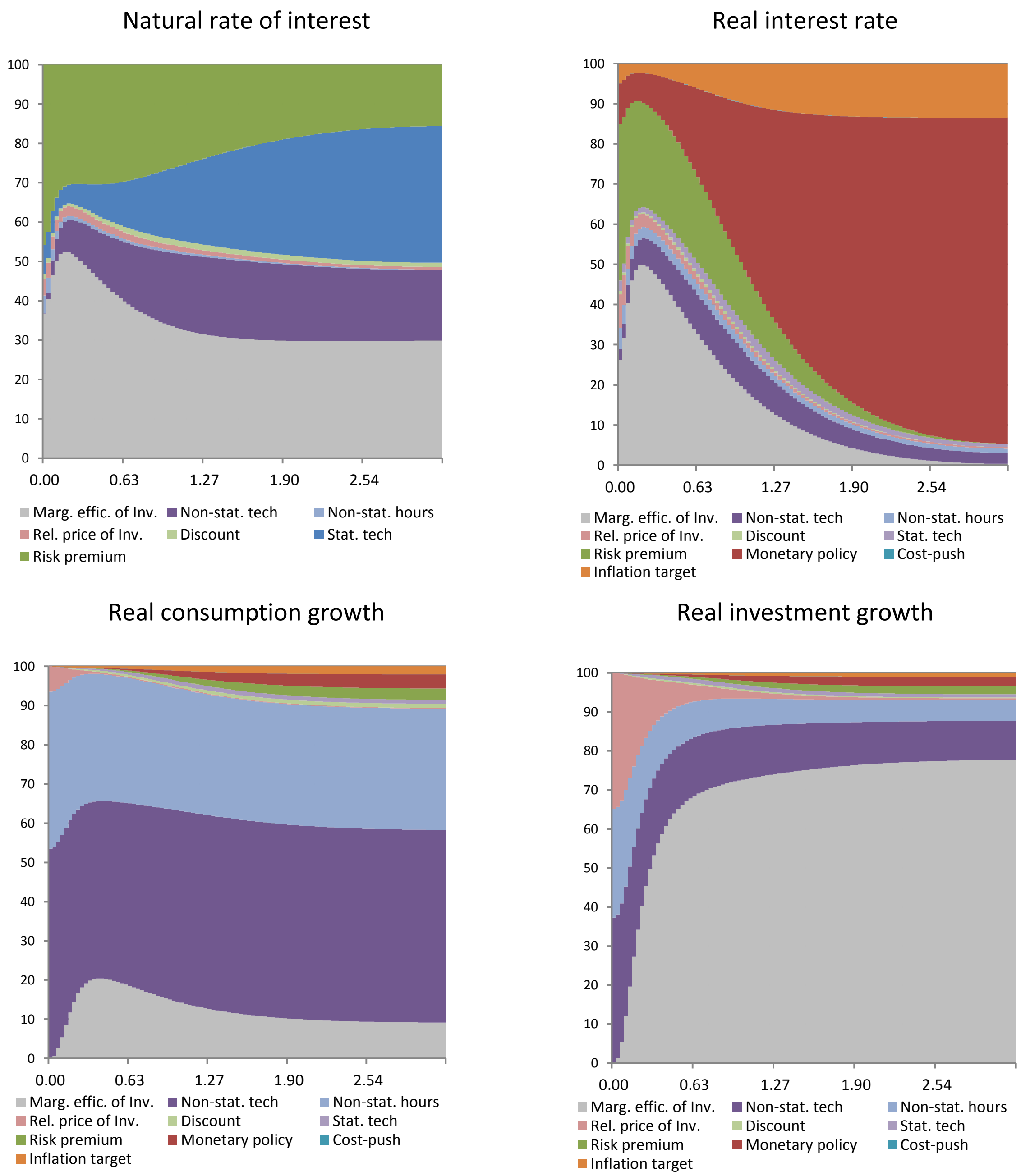
Figure 12. The real interest rate in the US: shock decomposition



Notes: see note to figure 11.

Figure 13: The natural rate of interest, the real rate, consumption and investment growth in the United States: variance decomposition in the frequency domain

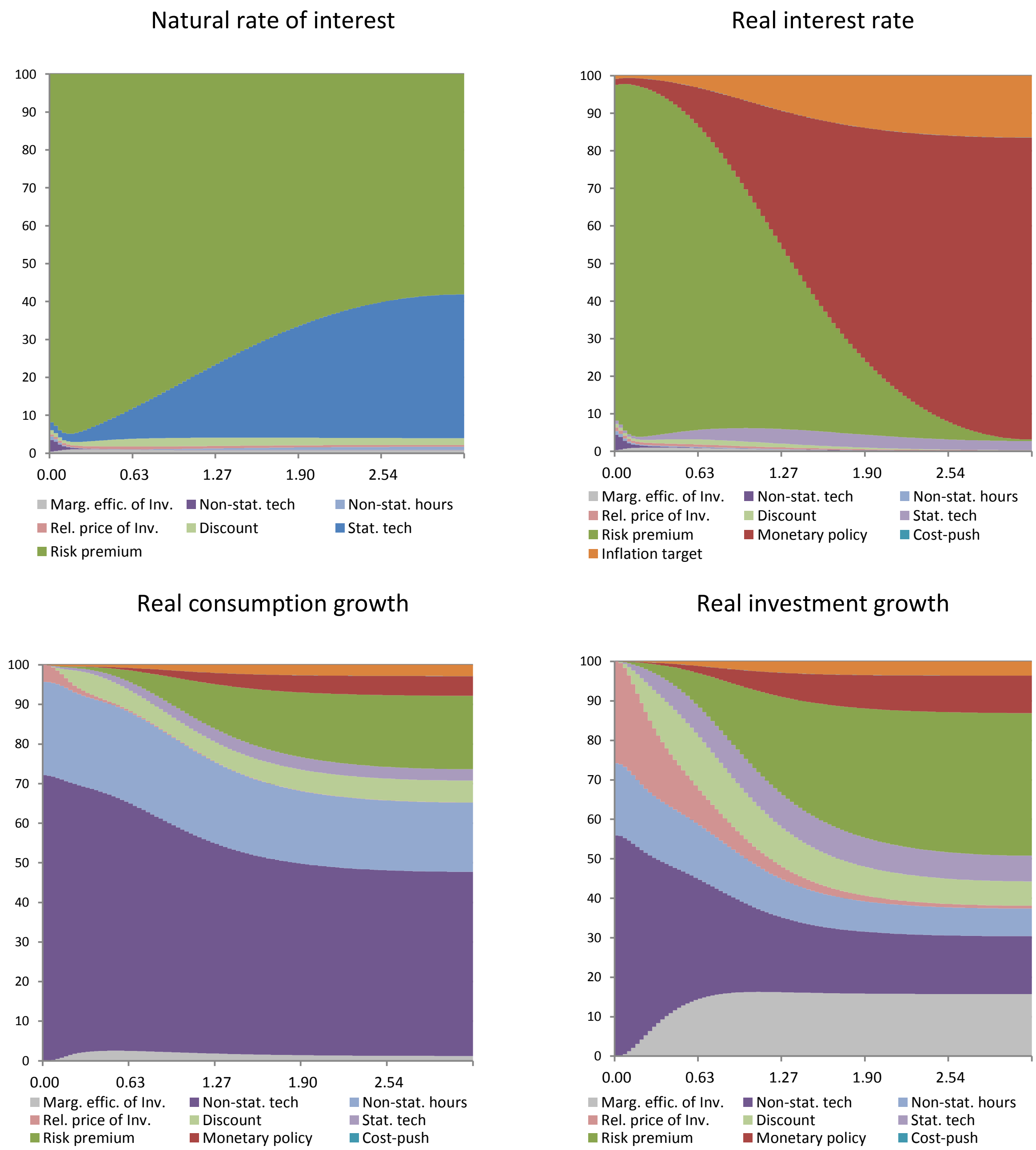
(*percentage point*)



Notes: variance share of the shocks as a function of the spectrum frequencies. Results based on the mean of the marginal posterior distribution of the parameters and of the smoothed estimates of the natural rate. The horizontal axis measures the frequency of the fluctuations: the range between $\frac{2\pi}{32} = 0.19$ and $\frac{2\pi}{6} = 1.05$ identifies business cycle fluctuations. Values close to zero refer to long run fluctuations.

Figure 14: The natural rate of interest, the real rate, consumption and investment growth in the euro area: variance decomposition in the frequency domain

(*percentage point*)



Notes: variance share of the shocks as a function of the spectrum frequencies. Results based on the mean of the marginal posterior distribution of the parameters and of the smoothed estimates of the natural rate. The horizontal axis measures the frequency of the fluctuations: the range between $\frac{2\pi}{32} = 0.19$ and $\frac{2\pi}{6} = 1.05$ identifies business cycle fluctuations. Values close to zero refer to long run fluctuations.

Appendix A: the model equations

The representative household problem

The household maximized her period utility function:

$$U(c_t, h_t) = \log c_t - \frac{\left(\frac{h_t}{B_t}\right)^{1+\frac{1}{\nu}}}{1+\frac{1}{\nu}}$$

subject to the budget constraint, in terms of the price of consumption goods,

$$c_t + \frac{i_t}{A_t^k} + \frac{b_t}{R_t} = w_t h_t + r_t^k k_t + \frac{b_{t-1}}{\pi_t}$$

and the law of motion of the capital stock

$$k_{t+1} = (1-\delta)k_t + \left[1 - S\left(\frac{i_t}{i_{t-1}}\right)\right] i_t$$

Where A_t^k is a permanent shock to the investment specific technology, B_t is a permanent shock to the disutility of working, c_t is consumption, h_t is the labour supply, w_t is the real wage, r_t^k is the rental rate of the aggregate capital stock k_t, which is rented to firms, and i_t is investment. The price of investment relative to consumption goods is $\frac{1}{A_t^k}$. The function S measures the costs for adjusting investment relative to the previous level. The parameter ν measures the elasticity of the labour supply while δ is the depreciation rate of the stock of capital.

The maximization problem is:

$$Max_{c_t,h_t} E_t \sum_{t=0}^{\infty} \beta^t \chi_t \left[\log c_t - \frac{\left(\frac{h_t}{B_t}\right)^{1+\frac{1}{\nu}}}{1+\frac{1}{\nu}} + \lambda_t \left(w_t h_t + r_t^k k_t - c_t - \frac{i_t}{A_t^k} \right) \right.$$
$$\left. + \lambda_t \mu_t \left((1-\delta)k_t + \left[1 - S\left(\frac{i_t}{i_{t-1}}\right)\right] i_t - k_{t+1} \right) \right]$$

where λ_t is the Lagrange multiplier of the budget constraint, μ_t the multiplier of the capital accumulation constraint and χ_t the shock to the households' discount factor.

The representative finished goods-producing firm

The representative finished goods-producing firm uses $\mathrm{Y_t(i)}$ units of each intermediate good $i \in [0, 1]$ to produce $\mathrm{Y_t}$ units of the finished good according to the constant returns to scale technology described by

$$\left[\int_0^1 \mathrm{Y_t(i)}^{\frac{\varrho-1}{\varrho}}\,\mathrm{di}\right]^{\frac{\varrho}{\varrho-1}} \geq Y_t$$

in which ϱ is the elasticity of demand.

Intermediate good i sells at the nominal price $\mathrm{P_t(i)}$, while the finished good sells at the nominal price $\mathrm{P_t}$; given these prices, the finished goods-producing firm chooses $\mathrm{Y_t}$ and $\mathrm{Y_t(i)}$ for all $i \in [0, 1]$ to maximize its profits

$$P_t Y_t - \int_0^1 P_t(i) Y_t(i)\, di$$

subject to the constraint imposed by the production function above.

The representative intermediate goods-producing firm

The representative intermediate goods-producing firm hires $\mathrm{h_t(i)}$ units of labor and rents $\mathrm{k_t(i)}$ units of capital from the representative household during period t in order to produce $\mathrm{Y_t(i)}$ units of intermediate good i according to the constant returns to scale technology

$$Y_t(i) = Z_t k_t(i)^{\alpha}[A_t h_t(i)]^{1-\alpha}$$

where $\mathrm{A_t}$ is the labor-augmenting technological progress, $\mathrm{Z_t}$ is a stationary shock to total factor productivity, and α is the capital share in the production function. Technology follows a unit root process with stochastic drift γ_t^{A} which has an AR(1) representation.

Since intermediate goods substitute imperfectly for one another as inputs to producing the finished good, the representative intermediate goods-producing firm sells its output in a monopolistically competitive market; during each period t, it sets a nominal price $P_t(i)$, subject to the requirement that it satisfy the representative finished goods-producing firm's demand, taking P_t and Y_t as given. In addition, the representative intermediate goods-producing firm faces a cost of adjusting its nominal price, measured in units of the finished good, when maximising profits. The firm optimization problem is the following:

$$Max_{P_{t(i)}} E_t \sum_{t=0}^{\infty} \beta^t \lambda_t \left\{\left[\frac{P_{t(i)}}{P_t}\right] Y_t(i) - \frac{\mathrm{w_t h_t + r_t^k k_t}}{P_t} - \frac{\kappa_P}{2}\left[\frac{P_t(i)}{(\bar{\pi}_t)^{\gamma_P}(\pi_{t-1})^{1-\gamma_P} P_{t-1}(i)} - 1\right]^2 Y_t\right\} +$$
$$+\sum_{t=0}^{\infty} \beta^t\, \Xi_t (Y_t(i) - Z_t k_t(i)^{\alpha}[A_t h_t(i)]^{1-\alpha})$$

where κ_P measures the cost for adjusting the price (scaled by total output Y_t) and Ξ_t the multiplier on the production function constraint.

The firm chooses the price $P_t(i)$ taking into account the trade-off between paying the costs for adjusting prices and maximising profits.

Monetary policy

The central bank conducts monetary policy by adjusting the short-term rate in response to deviations of inflation from the target and to output from its steady state:

$$R_t = \left(\frac{\bar{\gamma}_c\bar{\pi}}{\beta}\right)^{1-\phi_R}\left[\left(\frac{\pi_t}{\pi_t^*}\right)^{\phi_\pi}\left(\frac{y_t}{\bar{y}\Upsilon_t^y}\right)^{\phi_y}\right]^{1-\phi_R} R_{t-1}^{\phi_R}\,\varepsilon_t^R$$

where $\left(\frac{\bar{\gamma}_c\bar{\pi}}{\beta}\right)$ is the steady state nominal interest rate, Υ_t^y is the trend of output, $\bar{y}$ is the de-trended steady state, ε_t^R is monetary policy shock, and ϕ_R, ϕ_π and ϕ_y are parameters measuring, respectively, the degree of inertia in interest rate setting, the response to deviations of inflation from the central bank's target, π_t^*, and to deviations of output from its steady state.

Finally, the resource constraint of the economy is given by:

$$Y_t = C_t + I_t - \frac{\phi_P}{2}\left[\frac{\pi_t}{(\bar{\pi}_t)^{\gamma_P}(\pi_{t-1})^{1-\gamma_P}} - 1\right]^2 Y_t$$

where $C_t = \int_0^1 c_t(i)di$, $I_t = \int_0^1 i_t(i)di$ and $Y_t = \int_0^1 y_t(i)di$.

Appendix B: data and sources

United States

Real consumption growth: annual changes in real personal consumption expenditures, billions of chained 2009 dollars, not seasonally adjusted, annual. (FRED Economic data code: DPCERX1A020NBEA).
Source: US Bureau of Economic Analysis.

Real investment growth: annual changes in real gross private domestic investment, fixed investment, equipment; billions of chained 2009 dollars, not seasonally adjusted, annual. (FRED Economic data code: Y033RX1A020NBEA).
Source: US Bureau of Economic Analysis.

Inflation: annual changes in personal consumption expenditures chain-type price index; index 2009=100, annual, not seasonally adjusted. (FRED Economic data code: DPCERG3A086NBEA). *Source*: US Bureau of Economic Analysis.

Short-term interest rate: 3-month Treasury Bill rate, secondary market rate, percent, annual, not seasonally adjusted (FRED Economic data code: TB3MS).
Source: Federal Reserve Board.

Total hours (per-capita) worked growth: civilian employment level (FRED Economic data code: CE16OV; thousands of persons, annual, seasonally adjusted) * nonfarm business sector (FRED Economic data code: PRS85006023; average weekly hours) / total population (FRED Economic data code: POP).
Source: US Bureau of Labour Supply and US Bureau of the Census.

Inflation target: long-term inflation expectations from the Federal Reserve Bank of Philadelphia FED (Survey of Professional Forecasters, SPF) minus 0.5 percentage points per year. Missing data are taken from the file available at the following link:
https://www.philadelphiafed.org/-/media/research-and-data/real-time-center/survey-of-professional-forecasters/historical-data/additional-cpie10.xls?la=en
Source: Federal Reserve Bank of Philadelphia*.*

Growth rate of relative price of investment: growth of the inverse of the relative price of consumption to price of equipment (code: relativePrice in Excel file by John Fernald, Federal Reserve Bank of San Francisco).
Source: Federal Reserve Bank of San Francisco.

Population growth: annual changes in total population: all ages including armed forces overseas, annual percentage change, annual, not seasonally adjusted (FRED Economic data code: POP_PC1).
Source: US Bureau of the Census.

Euro area

Real consumption growth: annual changes in private final consumption expenditure at 2010 prices, billions of euro/ECU, not seasonally adjusted, annual (AMECO code: OCPH). Euro area (12 countries) merged with EA12 (including D_W West-Germany).
Source: AMECO, European Commission.

Real investment growth: annual changes in gross fixed capital formation at 2010 prices: total economy, billions of euro/ECU, not seasonally adjusted, annual. (AMECO code: OIGT). Euro area (12 countries) merged with EA12 (including D_W West-Germany).
Source: AMECO, European Commission.

Inflation: annual changes in the Harmonized Index of Consumer Prices. Past data are backdated in growth rates using the AWM series. Neither seasonally nor working day adjusted data, Index base year 1996 = 100 (AWM code: HICP). Data for 2016 are taken from the ECB
Source: Area-wide model database, Euro Area Business Cycle Network.
Link: http://eabcn.org/page/area-wide-model

Short-term interest rate: 3-month money market rate, backdated with the corresponding series contained in the original database (BIS and AMECO). (AWM code: STN).
Source: Area-wide model database, Euro Area Business Cycle Network.
Link: http://eabcn.org/page/area-wide-model

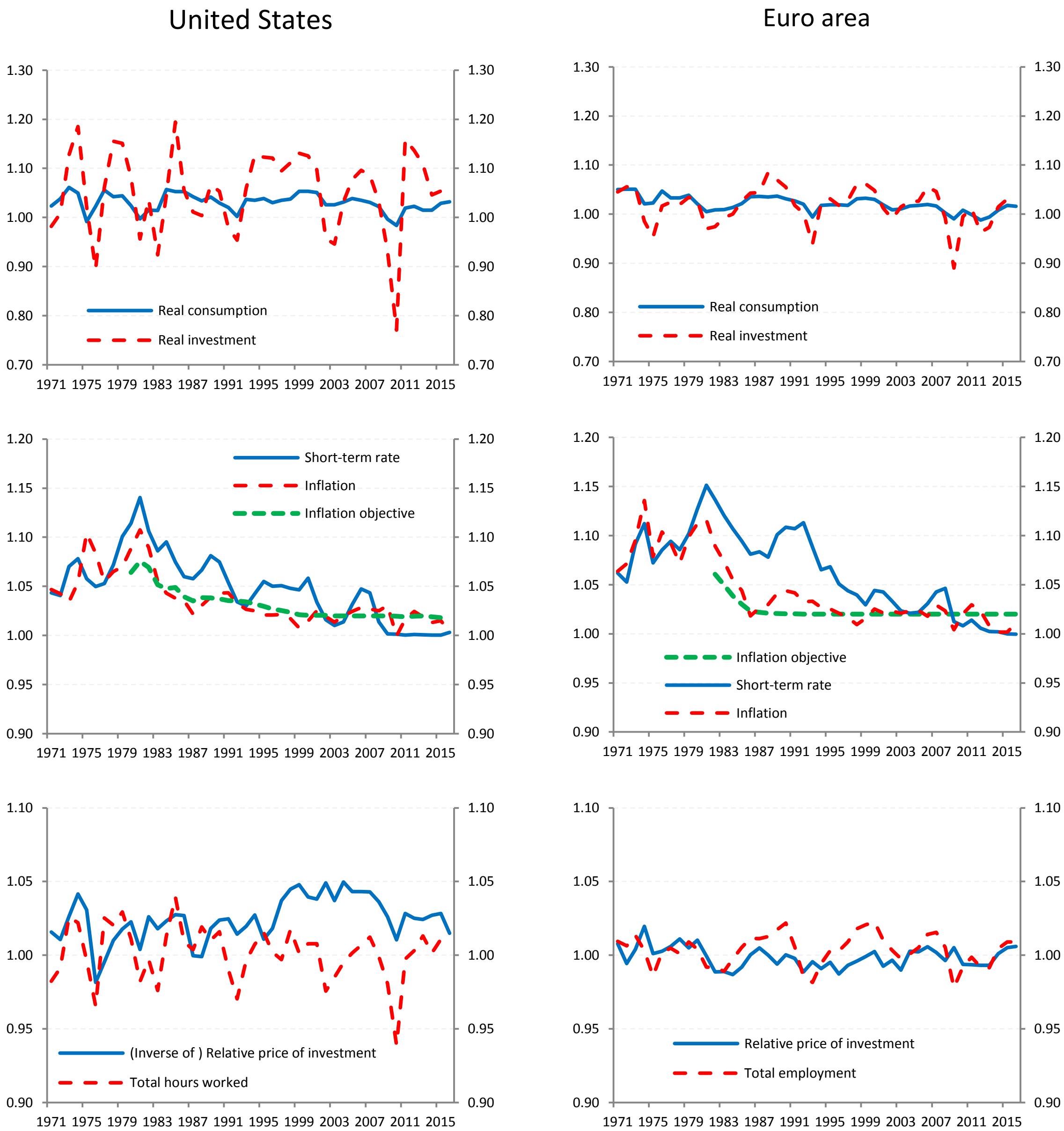
Total employment (over population) growth: growth of ratio between total employees, all activities, thousands of persons, calendar and seasonally adjusted data (AWM code: LEN) and total population (national accounts, euro area 12 countries AMECO code: NPTD), thousands of persons. Total employees are taken from the Statistical Data Warehouse and backdated in rates of growth
Source: Area-wide model database, Euro Area Business Cycle Network and AMECO, European Commission.

Inflation target: implicit target for the European Central Bank as estimated in Lippi and Neri (2007) on the basis of the Bundesbank unavoidable inflation rate.
Source: Lippi and Neri (*Journal of Monetary Economics*, 2007).

Growth rate of relative price of investment: growth of the ratio of price deflator of gross fixed capital formation for the total economy (AMECO code: PIGT) to the price deflator of private final consumption expenditure (AMECO code: PCPH).
Source: AMECO, European Commission.

Population growth: annual changes in total population, thousands of persons (national accounts; AMECO code: NPTD).
Source: AMECO, European Commission.

Figure B1. Data



Note: for the estimation, real consumption and investment, total hours and total employment are normalized by population. Real consumption, real investment, the relative price of investment, total hours worked and total employment are expressed as gross annual growth rate. Inflation, inflation expectations and the inflation target, and the short-term rate are expressed as gross annual percentage changes.

Appendix C: robustness

Table C1. Structural parameters of the model: United States, 1980-2016

	Prior distribution			Posterior distribution		
	Prior type	Mean	Std. dev.	Mean	90% HPD	
ω	gamma	2.0	0.50	1.391	0.734	2.021
κ_P	gamma	20.0	5.00	22.930	15.948	29.778
γ_P	beta	0.50	0.10	0.587	0.441	0.725
γ_c	beta	0.50	0.025	0.499	0.459	0.539
γ_h	beta	0.60	0.050	0.619	0.557	0.681
φ_R	beta	0.70	0.10	0.640	0.543	0.732
φ_π	normal	2.50	0.20	2.679	2.383	2.975
φ_y	normal	0.50	0.10	0.284	0.150	0.419
γ_A	normal	1.01	0.005	1.004	0.999	1.010
γ_{AK}	normal	1.02	0.005	1.024	1.019	1.029
γ_B	normal	1.00	0.005	0.999	0.994	1.004
π	normal	0.02	0.005	0.026	0.020	0.033
β	normal	1.00	0.020	0.995	0.982	1.007
ρ_A	beta	0.500	0.200	0.354	0.211	0.493
ρ_B	beta	0.500	0.200	0.134	0.029	0.233
ρ_{AK}	beta	0.500	0.200	0.614	0.461	0.769
ρ_C	beta	0.500	0.200	0.531	0.209	0.867
ρ_π	beta	0.500	0.200	0.953	0.920	0.987
ρ_i	beta	0.500	0.200	0.598	0.424	0.771
ρ_z	beta	0.500	0.200	0.648	0.358	0.945
ρ_σ	beta	0.500	0.200	0.688	0.465	0.928
σ_A	inverse gamma	0.050	0.050	0.026	0.019	0.033
σ_B	inverse gamma	0.050	0.050	0.022	0.016	0.027
σ_{AK}	inverse gamma	0.050	0.050	0.010	0.008	0.012
σ_C	inverse gamma	0.050	0.050	0.006	0.002	0.011
σ_R	inverse gamma	0.050	0.050	0.014	0.011	0.017
σ_p	inverse gamma	0.050	0.050	0.009	0.002	0.016
σ_π	inverse gamma	0.050	0.050	0.004	0.003	0.005
σ_i	inverse gamma	0.050	0.050	0.103	0.052	0.152
σ_σ	inverse gamma	0.050	0.050	0.006	0.003	0.008
σ_z	inverse gamma	0.050	0.050	0.005	0.003	0.008

Table C2. Structural parameters of the model: euro area, 1980-2016

	Prior distribution			Posterior distribution		
	Prior type	Mean	Std. dev.	Mean	90% HPD	
ω	gamma	2.0	0.50	1.131	0.734	1.523
κ_P	gamma	20.0	5.00	18.334	12.746	23.806
γ_P	beta	0.50	0.10	0.490	0.333	0.648
γ_c	beta	0.50	0.025	0.510	0.468	0.551
γ_h	beta	0.60	0.050	0.652	0.595	0.713
φ_R	beta	0.70	0.10	0.638	0.560	0.717
φ_π	normal	2.50	0.20	2.745	2.465	3.019
φ_y	normal	0.50	0.10	0.195	0.039	0.357
γ_A	normal	1.01	0.005	1.009	1.004	1.013
γ_{AK}	normal	1.02	0.005	1.004	1.001	1.007
γ_B	normal	1.00	0.005	0.999	0.995	1.003
π	normal	0.02	0.005	0.023	0.016	0.029
β	normal	1.00	0.020	0.991	0.980	1.001
ρ_A	beta	0.500	0.200	0.543	0.322	0.760
ρ_B	beta	0.500	0.200	0.114	0.020	0.199
ρ_{AK}	beta	0.500	0.200	0.460	0.263	0.665
ρ_C	beta	0.500	0.200	0.717	0.389	0.965
ρ_π	beta	0.500	0.200	0.946	0.906	0.989
ρ_i	beta	0.500	0.200	0.643	0.299	0.961
ρ_z	beta	0.500	0.200	0.746	0.490	0.985
ρ_σ	beta	0.500	0.200	0.784	0.667	0.904
σ_A	inverse gamma	0.050	0.050	0.013	0.006	0.019
σ_B	inverse gamma	0.050	0.050	0.018	0.013	0.022
σ_{AK}	inverse gamma	0.050	0.050	0.006	0.005	0.007
σ_C	inverse gamma	0.050	0.050	0.009	0.003	0.014
σ_R	inverse gamma	0.050	0.050	0.011	0.008	0.013
σ_p	inverse gamma	0.050	0.050	0.008	0.002	0.016
σ_π	inverse gamma	0.050	0.050	0.004	0.003	0.005
σ_i	inverse gamma	0.050	0.050	0.012	0.003	0.021
σ_σ	inverse gamma	0.050	0.050	0.005	0.003	0.006
σ_z	inverse gamma	0.050	0.050	0.005	0.002	0.009

Table C3. Structural parameters of the model: United States, looser priors

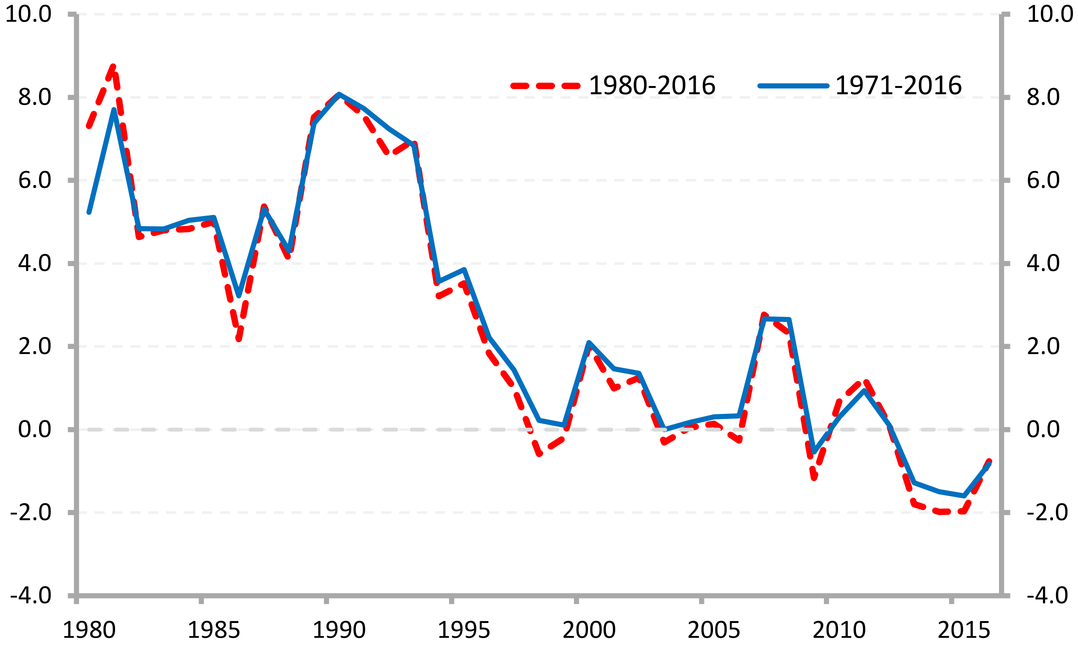
	Prior distribution			Posterior distribution		
	Prior type	Mean	Std. dev.	Mean	90% HPD	
ω	gamma	2.0	1.000	0.464	0.091	0.840
κ_P	gamma	20.0	10.000	10.992	5.226	16.619
γ_P	beta	0.50	0.200	0.556	0.267	0.859
γ_c	beta	0.50	0.050	0.518	0.437	0.597
γ_h	beta	0.60	0.100	0.625	0.535	0.717
φ_R	beta	0.70	0.200	0.564	0.438	0.695
φ_π	normal	2.50	0.400	3.013	2.500	3.536
φ_y	normal	0.50	0.200	0.164	0.015	0.312
γ_A	normal	1.01	0.010	1.003	0.995	1.010
γ_{AK}	normal	1.02	0.010	1.025	1.020	1.030
γ_B	normal	1.00	0.010	0.999	0.993	1.006
π	normal	0.02	0.010	0.030	0.022	0.039
β	normal	1.00	0.040	0.999	0.990	1.008
ρ_A	beta	0.500	0.200	0.277	0.156	0.397
ρ_B	beta	0.500	0.200	0.095	0.019	0.167
ρ_{AK}	beta	0.500	0.200	0.463	0.294	0.632
ρ_C	beta	0.500	0.200	0.566	0.249	0.896
ρ_π	beta	0.500	0.200	0.921	0.875	0.968
ρ_i	beta	0.500	0.200	0.705	0.541	0.873
ρ_z	beta	0.500	0.200	0.656	0.375	0.947
ρ_σ	beta	0.500	0.200	0.616	0.385	0.852
σ_A	inverse gamma	0.050	0.100	0.032	0.025	0.039
σ_B	inverse gamma	0.050	0.100	0.032	0.023	0.041
σ_{AK}	inverse gamma	0.050	0.100	0.012	0.010	0.014
σ_C	inverse gamma	0.050	0.100	0.006	0.002	0.009
σ_R	inverse gamma	0.050	0.100	0.017	0.013	0.021
σ_p	inverse gamma	0.050	0.100	0.010	0.002	0.018
σ_π	inverse gamma	0.050	0.100	0.005	0.004	0.006
σ_i	inverse gamma	0.050	0.100	0.047	0.022	0.071
σ_σ	inverse gamma	0.050	0.100	0.004	0.003	0.006
σ_z	inverse gamma	0.050	0.100	0.006	0.002	0.009

Table C4. Structural parameters of the model: euro area, looser priors

	Prior distribution			Posterior distribution		
	Prior type	Mean	Std. dev.	Mean	90% HPD	
ω	gamma	2.0	1.000	0.455	0.155	0.739
κ_P	gamma	20.0	10.000	6.973	2.841	10.787
γ_P	beta	0.50	0.200	0.392	0.094	0.675
γ_c	beta	0.50	0.050	0.561	0.484	0.638
γ_h	beta	0.60	0.100	0.577	0.486	0.667
φ_R	beta	0.70	0.200	0.550	0.464	0.636
φ_π	normal	2.50	0.400	2.924	2.532	3.304
φ_y	normal	0.50	0.200	-0.125	-0.273	0.022
γ_A	normal	1.01	0.010	1.017	1.011	1.023
γ_{AK}	normal	1.02	0.010	1.003	1.000	1.007
γ_B	normal	1.00	0.010	1.002	0.998	1.006
π	normal	0.02	0.010	0.024	0.015	0.034
β	normal	1.00	0.040	1.006	0.997	1.016
ρ_A	beta	0.500	0.200	0.419	0.289	0.546
ρ_B	beta	0.500	0.200	0.162	0.046	0.270
ρ_{AK}	beta	0.500	0.200	0.489	0.334	0.645
ρ_C	beta	0.500	0.200	0.759	0.519	0.967
ρ_π	beta	0.500	0.200	0.963	0.935	0.992
ρ_i	beta	0.500	0.200	0.557	0.234	0.922
ρ_z	beta	0.500	0.200	0.713	0.461	0.951
ρ_σ	beta	0.500	0.200	0.829	0.730	0.931
σ_A	inverse gamma	0.050	0.100	0.019	0.014	0.024
σ_B	inverse gamma	0.050	0.100	0.014	0.010	0.017
σ_{AK}	inverse gamma	0.050	0.100	0.007	0.005	0.008
σ_C	inverse gamma	0.050	0.100	0.008	0.003	0.013
σ_R	inverse gamma	0.050	0.100	0.011	0.008	0.013
σ_p	inverse gamma	0.050	0.100	0.009	0.002	0.017
σ_π	inverse gamma	0.050	0.100	0.004	0.003	0.005
σ_i	inverse gamma	0.050	0.100	0.009	0.003	0.014
σ_σ	inverse gamma	0.050	0.100	0.003	0.002	0.005
σ_z	inverse gamma	0.050	0.100	0.005	0.002	0.007

Figure C1. The natural rate of interest in the euro area: 1971-2016 and 1980-2016

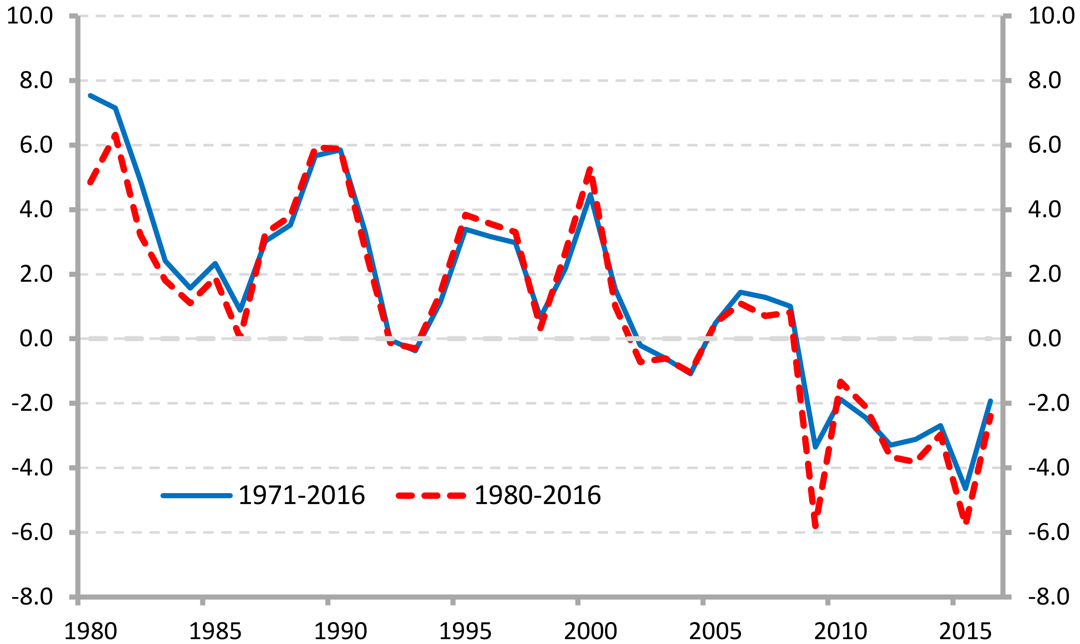
(*per cent*)



Notes: mean of the marginal posterior distribution of the smoothed estimates for each year.

Figure C2. The natural rate of interest in the United States: 1970-2016 and 1980-2016

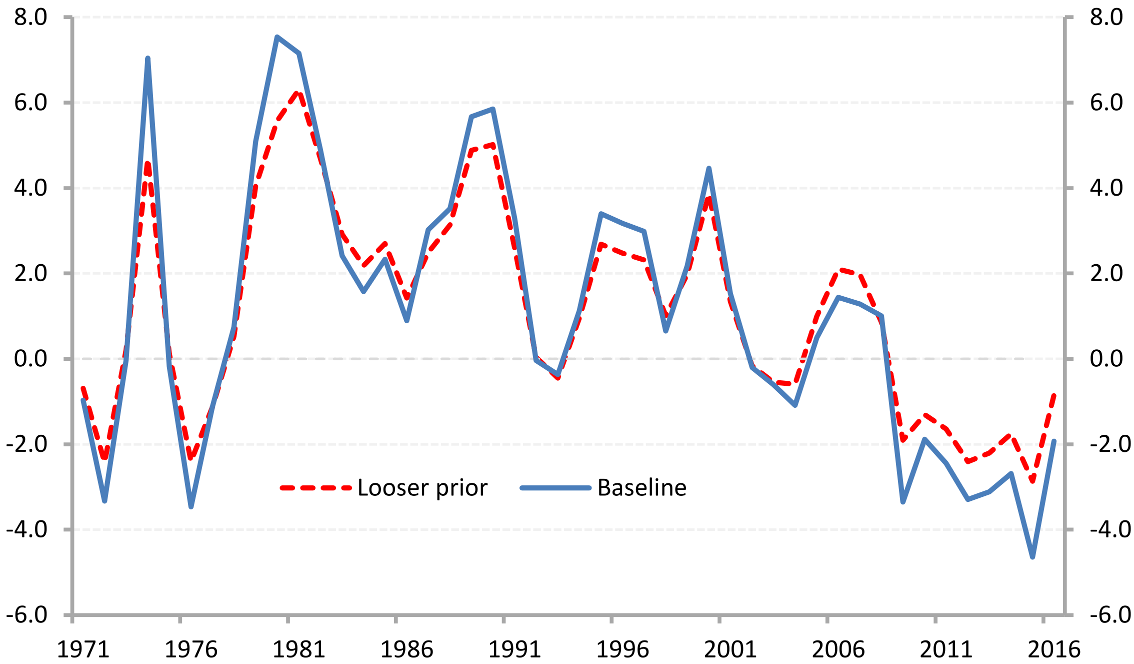
(*per cent*)



Notes: mean of the marginal posterior distribution of the smoothed estimates for each year.

Figure C3. The natural rate of interest in the United States: looser priors

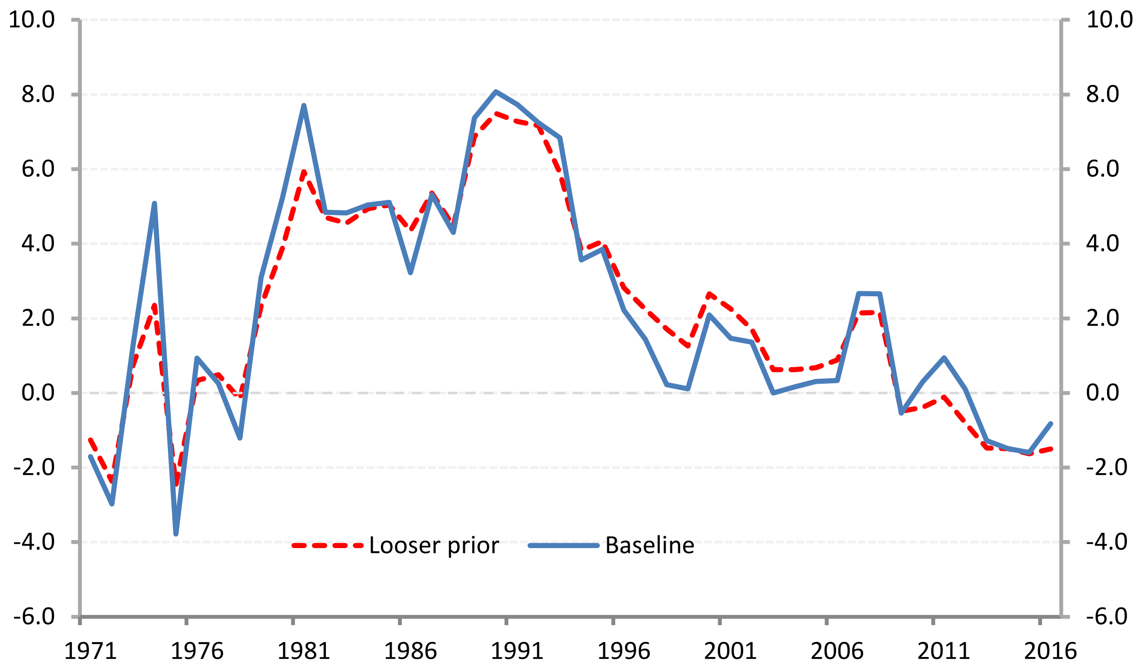
(*per cent*)



Notes: median values of the marginal posterior distributions for each year of the sample. The model is estimated with looser prior on all the parameters, with the only exception of the autoregressive coefficients of the shocks.

Figure C4. The natural rate of interest in the euro area: looser priors

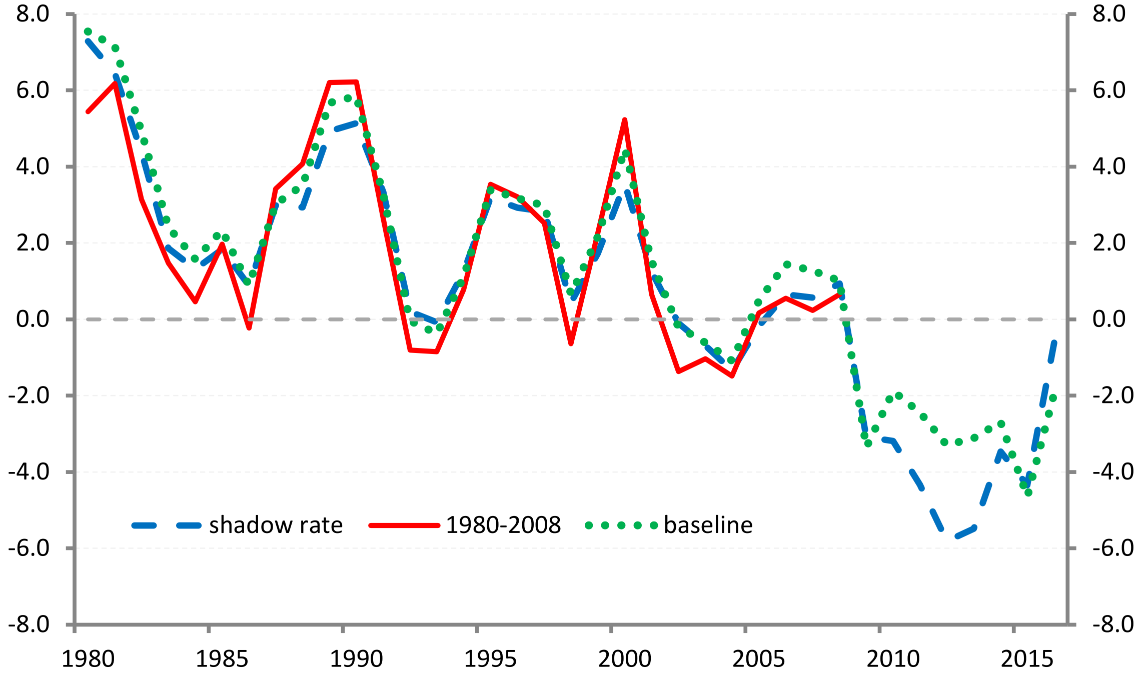
(*per cent*)



Notes: median values of the marginal posterior distributions for each year of the sample. The model is estimated with looser prior on all the parameters, with the only exception of the autoregressive coefficients of the shocks.

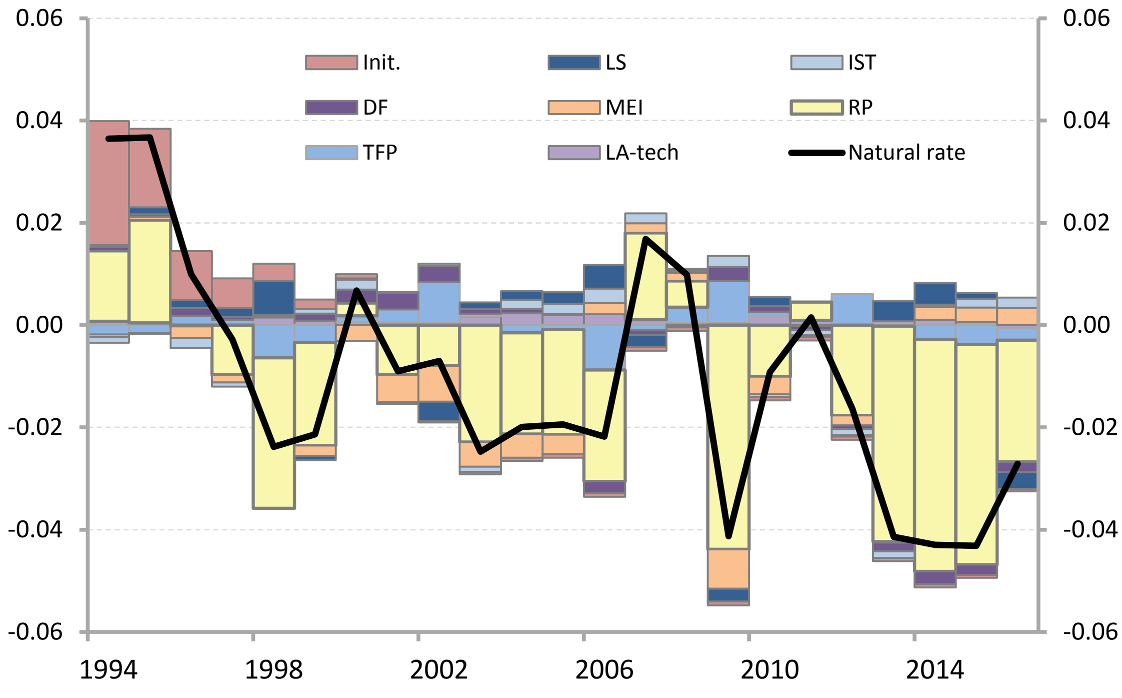
Figure C5. The role of the lower bound to policy rates in the United

(*per cent*)



Notes: mean of the marginal posterior distribution of the smoothed estimates for each year.

Figure C6. The natural rate in the euro area: post-1994 sample, shock decomposition



Notes: mean of the marginal posterior distribution of the smoothed estimates and shock contribution for each year. The chart plots the deviation of the natural rate of interest from the steady state. Init. = initial conditions; LS = labour supply; IST = investment specific technology; DF = discount factor; MEI = marginal efficiency of investment; RP = risk premium; TFP = total factor productivity; LA-tech = labour-augmenting technology.