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DANMARKS NATIONALBANK

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The role of refinancing in the interest rate pass-through to fixed-rate mortgage contracts

Jonas Ladegaard Hensch
jlh@nationalbanken.dk
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Abstract

In this paper, I study how mortgage refinancing influences the interest rate pass-through to household budgets via fixed-rate mortgage contracts in Denmark. I develop a model based on a state-dependent process of household actions that endogenously determines household refinancing decisions as a function of their incentives. I show that the model leads to a dynamic equilibrium, in which all households with positive incentives respond (over a period of adjustment) to monetary policy. The dynamic equilibrium formalizes the interest rate pass-through into an analytical expression, which I use to assess the empirical relevance of the model and the contribution of the refinancing channel in that context. I estimate the aggregate response via a cointegrated VAR model in the period 2008-2020, and complement the results with detailed mortgage data at individual household level. I document that the long-run interest rate pass-through is significantly below unity in the years after the financial crisis and subsequently converges towards a level close to unity. I argue that the result is driven by a structural shift in refinancing incentives in the years after the financial crisis. The result can be used to understand the importance of asymmetric effects in the refinancing channel of monetary policy transmission.

Resume

I dette papir studerer jeg, hvordan omlægninger af fastforrentede realkreditlån påvirker rentegennemslaget til husholdningernes budgetter i Danmark. Jeg udvikler en model baseret på en proces af tilstand-afhængighed, der endogent bestemmer husholdningernes konverteringsbeslutninger som en funktion af deres incitamenter. Jeg viser, at modellen leder til en dynamisk ligevægt, hvori alle husholdningerne med positive incitamenter, konverterer (over en tilpasningsperiode) til et pengepolitisk stød. Den dynamiske ligevægt formaliserer rentegennemslaget til et analytisk udtryk, der kan benyttes til at vurdere den empiriske relevans af modellen og bidraget fra konverteringskanelen i den sammenhæng. Jeg estimerer det samlede gennemslag via en kointegreret VAR model i perioden 2008-2020, og komplementerer resultaterne med deltaljeret mikrodata på husholdningernes fastforrentede realkreditlån. Jeg dokumenterer, at rentegennemslaget er signifikant under én i årene efter finanskrisen og derefter konvergerer mod et niveau tæt på én. Jeg argumenter, at resultatet er drevet af et strukturelt skift i incitamenter til at omlægge lån i årene efter finanskrisen. Resultatet kan anvendes til at forstå vigtigheden af asymmetriske effekter i konverteringskanelen i den pengepolitiske transmission.

Key words

Fixed-rate mortgages; Inaction; Interest rate pass-through; Monetary transmission; Refinancing

JEL classification

E43; E44; E52; G40; G50

Acknowledgements

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The authors alone are responsible for any remaining errors.

DANMARKS NATIONALBANK
ANALYSES

LOW
FOR
LONG

Low for long

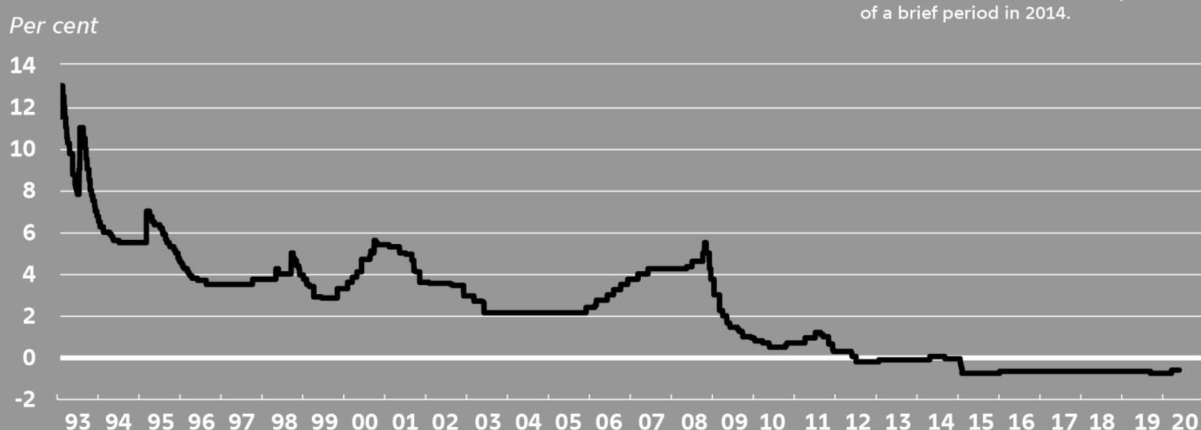
Denmark was the first country to introduce negative monetary policy rates in 2012. Since then, Switzerland, Sweden, Japan and the euro area have followed suit.

Very low and in some cases negative interest rates have characterised the past decade across the advanced economies. There are several reasons why interest rates have fallen to the current low levels. Low interest rates reflect the fact that inflation has been subdued in many countries, but structural changes in household and corporate savings and investment behaviour are also part of the explanation.

These developments have brought monetary policy and the economy into uncharted waters, which is why Danmarks Nationalbank will be issuing a series of publications on the topic of which this Working Paper is one.

Danmarks Nationalbank's interest rate

Danmarks Nationalbank's key interest rate has been negative since the summer of 2012, with the exception of a brief period in 2014.



DANMARKS
NATIONALBANK

The role of refinancing in the interest rate pass-through to fixed-rate mortgage contracts

Jonas Ladegaard Hensch[‡]
Danmarks Nationalbank

April 2021

Abstract

In this paper, I study how mortgage refinancing influences the interest rate pass-through to household budgets via fixed-rate mortgage contracts in Denmark. I develop a model based on a state-dependent process of household actions that endogenously determines household refinancing decisions as a function of their incentives. I show that the model leads to a dynamic equilibrium, in which all households with positive incentives respond (over a period of adjustment) to monetary policy. The dynamic equilibrium formalizes the interest rate pass-through into an analytical expression, which I use to assess the empirical relevance of the model and the contribution of the refinancing channel in that context. I estimate the aggregate response via a cointegrated VAR model in the period 2008-2020, and complement the results with detailed mortgage data at individual household level. I document that the long-run interest rate pass-through is significantly below unity in the years after the financial crisis and subsequently converges towards a level close to unity. I argue that the result is driven by a shift in refinancing incentives in the years after the financial crisis. The result can be used to understand the importance of asymmetric effects in the refinancing channel of monetary policy transmission.

Keywords: *Fixed-rate mortgages; Inaction; Interest rate pass-through; Monetary transmission; Refinancing*

JEL classification: *E43, E44, E52, G40, G50*

[‡]Danmarks Nationalbank. E-mail: jlh@nationalbanken.dk.

The views expressed in this paper are those of the author, and do not necessarily correspond to those of Danmarks Nationalbank. I would like to thank Kim Abildgren, Søren Lejsgaard Autrup, Denis Gorea and Jakob Guldbæk Mikkelsen and Carsten Paysen Rosenskjold for useful comments and suggestions; and Henrik Yde Andersen and Jacob Marott Sundram for outstanding data assistance. The author alone is responsible for any remaining errors.

1 Introduction

Housing finance is an important channel in the transmission of monetary policy to the real economy (Bernanke and Gertler (1995)). Expansionary monetary policy stimulates the economy by lowering rates, which in turn increases household consumption. However, in a fixed-rate mortgage system, lower mortgage rates relieve the budgets only of households that refinance their mortgages. Such budget relief is related to household decision-making, which primarily depends on the potential savings from refinancing relative to the refinancing costs.

A growing body of literature documents that mortgage refinancing plays an important role in the monetary transmission to household consumption (Agarwal et al. (2015), Auclert (2019), Beraja et al. (2019), Di Maggio et al. (2017) and Wong (2019)). However, what remains less documented is a better understanding of the interplay between refinancing and the interest rate pass-through to fixed-rate mortgage contracts. The interest-rate pass-through is important to monetary transmission, and identifying the role of refinancing in that context would contribute to shedding light on how household consumption responds to monetary policy.

In this paper, I study how mortgage refinancing influences the interest-rate pass-through to homeowners' fixed-rate mortgages. First, I develop a theoretical framework to clarify how refinancing and the pass-through to homeowners' fixed-rate mortgage rates are inherently related. The main building block in the model is a state-dependent process of household actions that endogenously determines household refinancing decisions as a function of their incentives. Refinancing incentives depend on both homogeneous characteristics like market rates, and more household-specific characteristics, such as existing interest rates and refinancing cost. Inspired by the task-based models developed in both economic growth and labor literatures, I model refinancing incentives on a continuum interval by exploiting the heterogeneity of refinancing incentives at the household level. The task-inspired setup ensures the existence of a range of borrowers who represent the borrowers for which interest rate savings if refinancing are not sufficiently strong. Beyond this range of borrowers, interest rate savings are sufficiently strong, which from a purely monetary point of view triggers the decision to refinance.

A new strand of empirical literature emphasizes the role of financial frictions such as, inaction in households' refinancing decisions. Inaction in mortgage refinancing is common, and delays refinancing because the actions to refinance often occur long after the incentive to take them has been present (Berger et al. (2019), Eichenbaum (2019) and Andersen et al. (2020)). I introduce inaction in my model by allowing psychological costs of refinancing to shift the range of borrowers with sufficiently strong refinancing incentives. Psychological costs can reflect the value of time spent executing a refinancing, possibly augmented by behavioral present bias that makes households reluctant to incur current time costs for the sake of future benefits (Laibson (1997), O'Donoghue and Rabin (1999)). In my model, the inherent nature of inaction slows down the response to refinancing incentives, implying that only borrowers with sufficiently large refinancing incentives react instantaneously to changes in monetary policy.

I show that there exist a dynamic equilibrium in which all borrowers who initially have positive refinaning incentives respond to monetary policy. Within the dynamic equilibrium, the magnitude of the interest-rate pass-through depends on the size of the change in the market rate: If the expansionary shock to the market rate is sufficiently large, all households will (over a period of adjustment) respond to the shock by refinancing, leading to a one-to-one interest rate pass-through in equilibrium. If the shock is small, not all households will initially face positive incentives, and the pass-through will be less than one-to-one in equilibrium. The model thus demonstrates that the size of the interest rate shock is decisive for the share of borrowers who consider refinancing, explaining why refinancing typically comes in surges during periods of substantial declines in interest rates. As a result, the model formalises that the endogeneity of refinancing decisions leads to a non-linearity in the interest rate pass-through to fixed-rate mortgages.

To assess the empirical relevance of the theoretical model, I use evidence from Danish mortgage data. Denmark is highly suited for such analysis due to the high prevalence of long-term fixed-rate mortgages. Mortgage debt accounts for around 60-65 percent of household financing in Denmark, where around half of the mortgage debt is financed by long-term fixed-rate mortgages. The Danish mortgage system is broadly similar to the US mortgage system, where homeowners also can refinance their fixed-rate mortgages without penalties.

First, I use a cointegrated VAR model to estimate the macroeconomic response of a monetary policy shock to homeowners' fixed-rate mortgage rates in the period 2008-2020. I show that a monetary policy shock, defined as a shock to the expected monetary policy rates, is transmitted to household budgets at a speed of 3.7 per cent monthly. Economically, it means that it takes around 5-6 years before a shock to expected monetary policy rates has been fully transmitted in households' average interest rates. The empirical result matches the magnitude of the option-adjusted duration on fixed-rate mortgage bonds, which during the sample period is also around 5-6 years. Option-adjusted duration measures the probability that the fixed-rate mortgage bonds will be prepaid, and can thus be interpreted as the ex-ante expected lifetime of fixed-rate mortgages. The large coincidence between the estimates and the option-adjusted duration supports the plausibility of my empirical results.

I also show that the long-run pass-through is significantly below but close to unity during the sample period. However, the long-run pass-through varies slightly over time and depends specifically on the occurrence of periods of elevated refinancing activities. In that context, I document that interest rate pass-through is lower in the beginning of the sample and afterwards jumps permanently around 2014 to a level close to unity. The lower pass-through in the beginning of the sample reflects the fact that the refinancing incentives were lower in the beginning of the sample due to the interest rate environment in the years up to the financial crisis was characterized by increasing rates, which had affected homeowners incentives to refinance in the years after the financial crisis. As a result, the share of borrowers with positive incentives might have been lower in the beginning of the sample, thus reducing the magnitude of the interest rate pass-through. My results contribute to shedding light on the asymmetric effects of the refinancing channel on monetary transmission.

A weakness of the cointegrated VAR model is that it cannot decompose the aggregate estimates into underlying determinants. To more comprehensively examine the role of refinancing in the interest-rate pass-through to homeowners' fixed-rate mortgages, I complement the empirical analysis with administrative mortgage data on Danish households in the period 2010-2018. The data allows me to disentangle the contributions from the individual channels to the aggregate response by using the identities derived from the theoretical model. I show that speed of adjustment is close but slightly below the estimate found in the empirical model, taking differences in data sources and sample periods into account. I also document that the refinancing channel has boosted the speed at which a monetary shock spreads to homeowners' mortgage rates by around 30 per cent compared to a situation without refinancing. The mortgage data at household level also reveals that the pass-through is somewhat lower in the beginning of the sample period, and the level is more or less in line with the estimate from the cointegrated VAR model. This confirms that the interest rate pass-through was significantly lower at the beginning of the sample period, primarily driven by a larger fraction of borrowers with no incentives to refinance. The result highlights that the magnitude of the refinancing channel is endogenously determined by the development in the market rate.

The paper relates to at least two existing strands of literature. The first strand examines the importance of mortgage markets and the refinancing channel in the transmission of monetary policy to the real economy (Agarwal et al. 2015, Auclert 2019, Beraja et al. 2019, Di Maggio et al. 2017, Greenwald 2018 and Wong (2019)). I contribute to that literature in at least one important way by going a step deeper in the monetary transmission and only focus on the role of refinancing in the response from a monetary policy shock to homeowernes' fixed-rate mortgage rates. In this context, I theoretically documents that the size of the interest rate shock is decisive for the share of borrowers who ultimately end by refinancing, thus explaining why refinancing typically comes in surges during periods of substantial declines in interest rates. Empirically, I show that the magnitude of the interest-rate pass-through depends on the developments in the refinancing channel. This insight adds to the understanding of the role of refinancing in monetary transmission. The second strand of literature studies the distribution of mortgage rates across borrowers and emphasizes the role of transaction costs and inaction in explaining refinancing decisions. Examples include Bhutta and Keys (2016), Berger et al. (2019), Eichenbaum (2019) and Andersen et al. (2020). I add to that literature by theorectically quantifying the role of inaction in the pass-through to home owners' mortgage rates, and show that a higher degree of inaction delays the interest-rate pass-through.

2 The danish market for fixed-rate mortgages

The Danish mortgage market offers three different types of loans: variable-rate loans, adjustable-rate loans and loans with a fixed rate throughout the loan term. All mortgage types make use of the balance principle and the match-funding principle. This means that the loan is i) matched by the issuance of a bond, and ii) a borrower can prepay the loan by rebuying the bond. Danish mortgage bonds are covered bonds that are collateralized by pools of mortgages. In Denmark, mortgage banks act as intermediaries between investors and borrowers. Investors purchase mortgage bonds which are issued by the mortgage bank, while borrowers take out mortgages from the banks. All lending is secured, and once banks have initially screened borrowers, they have no further influence on mortgage rates, which are determined by the market.

The issuances of mortgage bonds are currently handled by seven mortgage banks (the so-called market makers) offering very similar mortgage rates and administration fees. When borrowers raise new fixed-rate mortgages, they pay the coupons on the bonds and fees to the mortgage bank. Fees include issuance costs to the mortgage credit institution and administrative costs to the mortgage bank to cover, for example compensation for the incurred credit risk of a default. Mortgage banks are exposed to credit risks as they have to cover any losses from borrowers defaulting, while investors are unaffected by defaults as long as the bank remains solvent. In effect, bond investors bear interest rate and prepayment risks, while the mortgage banks retain the credit risk. The size of the administration fee depends on the loan-to-value (LTV) ratio on the mortgage and is slightly 70 basis points on average.

Homeowners in Denmark can refinance their fixed-rate mortgages without incurring penalties in relation to the level of their interest rates. The reason is that fixed-rate mortgages are based on callable bonds, which gives the borrower the right (but not the obligation) to prepay the loan four times a year at face value (i.e. at a price of 100). This protects homeowners from having to pay a very high price if they wish to buy back the debt before the loan matures. When a borrower refinances, the mortgage bank repurchases the mortgage bond (corresponds to the size of the mortgage debt). The value of the received repurchase depends on whether the bonds are bought back at market value or at face value. In an environment of continously declining interest rates, repurchase of fixed-rate mortgage bonds always occurs at face value, and refinancing thus requires repurchase of the full face value of the mortgage bond. In Denmark, fixed-rate mortgages are issued with discrete coupon rates, historically at integer levels, but more recently at 50-basis point intervals. The issuances never take place at a premium to face value, and instead bonds are under normal circumstances issued at a discount to face value. This means that a homeowner who refinances to a lower interest rate can prepay at face value and raise a new mortgages at a price below face value, say e.g. 95-99, entailing a capital loss of 1-5 per cent without the one-off refinancing costs. In other words, it means that in Denmark the interest saved over the lifetime of the loan when refinancing fixed-rate mortgage contracts within an environment of falling interest rate is determined by the spread between the coupon rate on the old mortgage and the effective yield at issue on the new mortgage.

In an environment of increasing interest rates, the repurchase of fixed-rate mortgage bonds will instead take place at market value. Consequently, refinancing to a higher coupon rate in the Danish mortgage system entails a debt reduction for the homeowner. Again, whether the homeowner ultimately ends up with a debt reduction depends on the one-off refinancing cost. In sum, this implies that the incentives to refinance differ in the two cases: In an environment of falling interest rates, homeowners obtain a cut in interest rates as the mortgage bond is prepaid at face value, while in an environment of increasing interest rates, refinancing will not give rise to an interest rate cut, but instead a debt reduction, as the bond is prepaid at market value.

The Danish mortgage system is broadly similar to the US mortgage system, where homeowners also can refinance their fixed-rate mortgages without penalties. An important difference between the two mortgage systems is that refinancing always occurs at face value in the US mortgage system. The advantage of repurchasing the bond at market value when interest rates are increasing can thus not be exploited by US homeowners. This might make refinancing more common in Denmark during environments of increasing interest rates. For more details about the mortgage systems, see Campbell (2013) and Gyntelberg et al. (2012).

3 THEORETICAL FRAMEWORK

I start with a version of my model without entry and exit of borrowers and absence of debt amortization. It allows me to introduce the main setup in the simplest fashion and quantify the main effects of mortgage refinancing in the interest rate pass-through to homeowners' fixed-rate mortgage contracts.

3.1 Environment

Consider a continuum of borrowers $j_t \in [0, m]$, each financing their home debt $d_t(j_t)$ via fixed-rate mortgages. In every period of time t, a borrower j_t can either choose to (i) refinance to a new fixed-rate mortgage; or (ii) keep its existing fixed-rate mortgage. Borrower j_t's action depends on its decision in the previous period of time, which is inherently a function of refinancing incentives.

Each borrower considers the following refinancing incentives at time t:

$$I_t(j_t) = i_t^h(j_t) - i^m, \quad \text{for} \quad j_t \in [0, m], \tag{1}$$

where $i_t^h(j_t)$ defines borrower j_t's interest rate on its existing fixed-rate mortgage and i^m defines the interest on the fixed-rate mortgage bond available to borrowers if refinancing (henceforth the market rate). Without loss of generality, I disregard household-specific refinancing costs to simplify the structure of the equilibrium.[3] Equation (1) states that refinancing incentives for borrower j_t equals the interest savings from refinancing, corresponding to the spread between the interest rate on the old mortgage and the market rate on the new mortgage. Borrower j_t has positive refinancing incentives if $I_t(j_t) > 0$, and non-positive incentives if $I_t(j_t) \leq 0$.

Throughout, I impose the following assumption:

Assumption 3.1. *$i_t^h(j_t)$ is weakly increasing in j_t for all $j_t \in [0, m]$, and has the boundary condition $i_t^h(0) \leq i^m \leq i_t^h(m)$.*

Assumption 3.1 ensures that borrowers' interest rates are ordered, such that higher-index borrowers at time t have higher interest rates, and vice versa. It will guarantee that higher-index borrowers have equal or higher refinancing incentives relative to low-index borrowers. The imposed properties of $i_t^h(j_t)$ leads to the following lemma:

Lemma 3.1. *Given the function of refinancing incentives (equation (1)), there must exist a range of borrowers, denoted $j_t \in [0, j_t^*]$, which satisfies: $i_t^h(j_t) \leq i^m$, where $0 \leq j_t^* \leq m$. In the interior case $0 < j_t^* < m$, there exists a range of borrowers who face positive incentives, i.e. $i_t^h(j_t) > i^m$ for $j_t \in [j_t^*, m]$, and a range of borrowers who face non-positive incentives, i.e. $i_t^h(j_t) \leq i^m$ for $j_t \in [0, j_t^*]$.*
Proof: *It follows directly from equation (1) and assumption 3.1.*

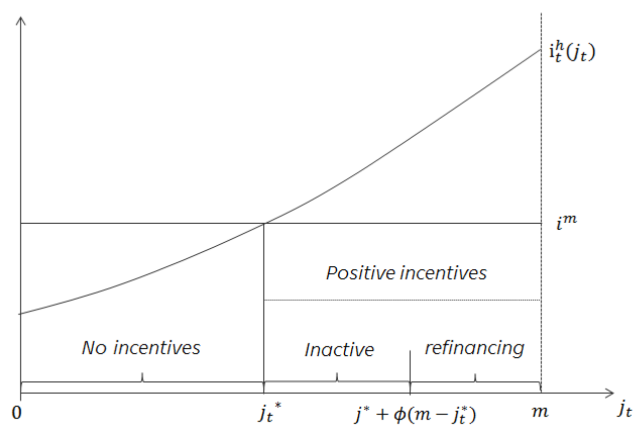


Figure 1: *The figure illustrates that a borrower $j_t \leq j_t^*$ has non-positive refinancing incentives, while a borrower $j_t > j_t^*$ has positive refinancing incentives. Among borrowers with positive incentives, the figures illustrates that only the share $1 - \phi$ refinances when $j_t^* < m$ due to inaction.*

[3]The introduction of refinancing cost does not changes the qualitative results if refinancing costs are more or less identical across borrowers. With refinancing costs, refinancing incentives would instead depend on the interest rate savings from refinancing less the one-off refinancing cost. As mentioned above, incentives in Denmark might also be affected by the benefits from obtaining a debt reduction when refinancing to a higher interest rate. I disregard this scenario.

Lemma 3.1 implies that a borrower $j_t \leq j_t^*$ has non-positive refinancing incentives, while a borrower $j_t > j_t^*$ has positive refinancing incentives, as illustrated in figure 1. In equilibrium, borrowers with non-positive incentives will not end by refinancing due to lack of financial motives, while borrowers with positive incentives will in constrast end up refinancing. This highlights that there does not necessarily exist one unique threshold borrower who represents the borrower for whom the potential savings from refinancing equal zero. Instead, the properties of $i_t^h(j_t)$ allow for the existence of a subcontinuum of borrowers with zero refinancing incentives, which for example arises in the wake of refinancing booms where several borrowers have refinanced to the same interest rate.

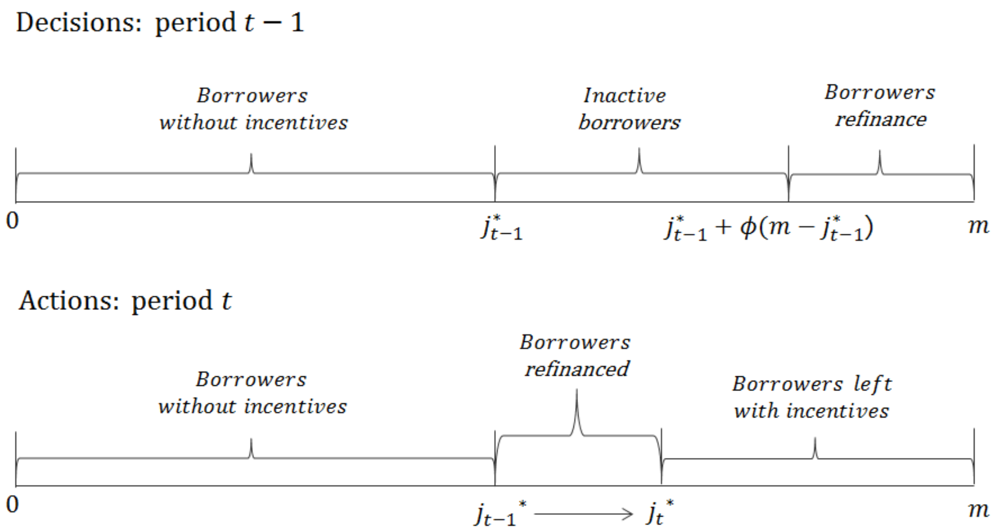


Figure 2: *The space of borrowers. Upper panel depicts the space of allocation of borrowers' decisions at $t-1$. Bottom panel depicts the borrowers' resulting actions at time t given their decisions.*

Borrowers with positive incentives are divided into two types. The first type is called "active", while the second type is called "inactive". Inaction is triggered by psychological costs of refinancing that shift j_t^* and thus widen the range of borrowers who do not refinance. I assume that an exogenous share $\phi \in (0, 1)$ of the borrowers with positive refinancing incentives are inactive. Without loss of generality, I let ϕ be the share of borrowers with the least strong positive incentives to refinance be inactive. In effect, the share of inactive borrowers is exogenously determined by the continuum of borrowers with positive incentives, whereas the share of borrowers with positive incentives, and hence the probability of considering a refinancing, are endogenously determined. Inactive borrowers belong to the range $j_t \in [j_t^*, j_t^* + \phi(m - j_t^*)]$, while active borrowers belong to the range $j_t \in [j_t^* + \phi(m_t - j_t^*), m]$.

Borrowers who are active at time $t-1$ refinance, and thus obtain a new fixed-rate mortgage at time t with an interest rate equating to $i_t^h(j_t) = i^m$. Lemma 3.1 implies that the market rate of newly issued mortgage bonds is no greater than the interest rate of new mortgage bonds. As a result, refinancing will never be attractive when a borrower have already refinanced to the market rate.

Borrowers who are inactive at time $t-1$ keep their existing fixed-rate mortgages going into period t, consistent with the interest-rate path $i_t^h(j_t) = i_{t-1}^h(j_{t-1})$ for all $j_{t-1} \in [j_{t-1}^*, j_{t-1}^* + \phi(m - j_{t-1}^*)]$. Borrowers with no refinancing incentives at time $t-1$, i.e. borrowers who belong to the interval $j_t \in [0, j_t^*]$, naturally keep their existing fixed-rate mortgages going into period t. Consequently, at time t, they face the interest rate path $i_t^h(j_t) = i_{t-1}^h(j_{t-1})$.

Figure 2 depicts the allocated space of borrowers' decisions at $t-1$ (upper panel) and their resulting actions at time t (bottom panel) based on their decisions at time $t-1$.

In sum, a borrower j's fixed-rate mortgage rate at time t conditional on its decision at time $t-1$ can formally be written as:[4]

$$i_t^h(j_t) = \begin{cases} i^m & \text{if } j_{t-1} \in [j_{t-1}^* + \phi(m - j_{t-1}^*), m] \\ i_{t-1}^h(j_{t-1}) & \text{if } j_{t-1} \in [j_{t-1}^*, j_{t-1}^* + \phi(m - j_{t-1}^*)] \\ i_{t-1}^h(j_{t-1}) & \text{if } j_{t-1} \in [0, j_{t-1}^*] \end{cases} \tag{2}$$

Equation (2) is a state-dependent process that endogenously determines a borrower's interest rate action as a function of its decision in the previous period. The process takes the stickiness of refinancing decisions into account by allowing for inaction. Inaction implies that homeowners respond slowly to changes in the market rate, which delays mortgage refinancing. As a result, depending on the degree of inaction, the average interest rate will evolve smoothly over time.

Debt accumulation by borrower j_t depends also on its financing decision. If refinancing, debt changes can only result from additional/less borrowing in relation to the refinancing. If borrower j_t does not refinance (either due to inaction or lack of incentives), its debt remains unchanged.[5] Let κ be an exogenous parameter that measures the relative change in debt when refinancing. Based on borrower j_t's decision at time $t-1$, its debt at time t must satisfy:

$$d_t(j_t) = \begin{cases} (1+\kappa) d_{t-1}(j_{t-1}) & \text{if } j_{t-1} \in [j_{t-1}^* + \phi(m - j_{t-1}^*), m] \\ d_{t-1}(j_{t-1}) & \text{if } j_{t-1} \in [j_{t-1}^*, j_{t-1}^* + \phi(m - j_{t-1}^*)] \\ d_{t-1}(j_{t-1}) & \text{if } j_{t-1} \in [0, j_{t-1}^*] \end{cases} \tag{3}$$

[4] The process of state dependency assumes that a homeowner only changes interest rates if refinancing occurs before the existing mortgage matures. The assumption is reasonable as fixed-rate mortgages are mainly long-term bonds with a term to maturity up to 30 years.

[5] I allow for instaments of debt in the full model described in section 3.4.

3.2 Aggregation

Let $D_t = \int_0^m d_t(j_t)dj$ be the aggregate fixed-rate mortgage debt at time t. According to equation (3), aggregate debt accumulation must reflect the debt raised by borrowers who refinance and the existing debt from borrowers who not refinance:

$$D_t = (1+\kappa)\int_{j^*_{t-1}+\phi(m-j^*_{t-1})}^{m} d_{t-1}(j_{t-1})dj + \int_0^{j^*_{t-1}} d_{t-1}(j_{t-1})dj + \int_{j^*_{t-1}}^{j^*_{t-1}+\phi(m-j^*_{t-1})} d_{t-1}(j_{t-1})dj \quad (4)$$

Subsequently, I let $\varphi_t(j_t) = d_t(j_t)/D_t$ be the individual debt share of borrower j_t. Given the debt accumulation equation in (4), individual debt shares must satisfy:

$$\begin{aligned} 1 =& \left(\int_0^{j^*_{t-1}} \varphi_{t-1}(j_{t-1})dj + \int_{j^*_{t-1}}^{j^*_{t-1}+\phi(m-j^*_{t-1})} \varphi_{t-1}(j_{t-1})dj\right)\Lambda^d_{(t-1,t)} \\ &+ (1+\kappa)\Lambda^d_{(t-1,t)}\int_{j^*_{t-1}+\phi(m-j^*_{t-1})}^{m} \varphi_{t-1}(j_{t-1})dj \\ =& \underbrace{(1+\kappa)\Omega^r_{t-1}\Lambda^d_{(t-1,t)}}_{\substack{\text{aggregate debt share}\\ \text{refinancing}}} + \underbrace{\Omega^s_{t-1}\Lambda^d_{(t-1,t)}}_{\substack{\text{aggregate debt share}\\ \text{no incentives}}} + \underbrace{\Omega^i_{t-1}\Lambda^d_{(t-1,t)}}_{\substack{\text{aggregate debt share}\\ \text{inactive}}}, \end{aligned} \quad (5)$$

where $\Omega^r_{t-1} = \int_{j^*_{t-1}+\phi(m-j^*_{t-1})}^{m} \varphi_{t-1}(j_{t-1})dj$ is the aggregate debt share of borrowers who refinance in period $t-1$, $\Omega^s_{t-1} = \int_0^{j^*_{t-1}} \varphi_{t-1}(j_{t-1})dj$ is the aggregate debt share of borrowers without incentives in perod $t-1$, $\Omega^i_{t-1} = \int_{j^*_{t-1}}^{j^*_{t-1}+\phi(m-j^*_{t-1})} \varphi_{t-1}(j_{t-1})dj$ is the aggregate debt share of inactive borrowers in period $t-1$, and $\Lambda^d_{(t-1,t)} = D_{t-1}/D_t$.

Hence, I can determine the average fixed-rate mortgage rate, which is by construction defined as $i^h = \int_0^m i^h_t(j_t)dj$ for all $j_t \in [0,m]$. The law of motion of borrowers' average interest rate follows the proposition:

Proposition 3.1. *The change in the average interest rate on fixed-rate mortgages is:*

$$\begin{aligned} \Delta i^h_t =& \underbrace{\int_{j^*_{t-1}+\phi(m-j^*_{t-1})}^{m} (1+\kappa)\left(i^m - i^h_{t-1}\right)\Lambda^d_{(t-1,t)}\varphi_{t-1}(j_{t-1})dj}_{\text{refinancing channel}} \\ &+ \underbrace{\int_0^{j^*_{t-1}} \left(i^h_{t-1}(j_{t-1}) - i^h_{t-1}\right)\Lambda^d_{(t-1,t)}\varphi_{t-1}(j_{t-1})dj}_{\text{no incentive channel}} \\ &+ \underbrace{\int_{j^*_{t-1}}^{j^*_{t-1}+\phi(m-j^*_{t-1})} \left(i^h_{t-1}(j_{t-1}) - i^h_{t-1}\right)\Lambda^d_{(t-1,t)}\varphi_{t-1}(j_{t-1})dj}_{\text{inaction channel}} \end{aligned} \quad (6)$$

where $\Delta i^h_t = i^h_t - i^h_{t-1}$.

Proof: *See appendix A.1.*

Proposition 3.1 states that homeowners' average interest rate can be decomposed into three channels that solely exist due to the refinancing option on fixed-rate mortgages.

1. *The refinancing channel* reflects how refinancing by active households is materialised into Δi_t^h. On the household-specific level, refinancing dynamics was highlighted in equation (2), where the change in borrower j's interest rate was decisively determined by each household's incentives to refinance. On the aggregate scale, the magnitude of the refinancing channel is determined by borrowers' tendency to change debt when refinancing, captured by κ, and the aggregate share of refinanced debt, Ω_{t-1}^r, which is positively affected by the number of households who refinance, that is $(1-\phi)(m-j_{t-1}^*)$. The sign of the channel depends intuitively on whether the active borrowers refinance to a lower or higher rate than the average. The importance of refinancing can be illustrated by considering a case where no borrowers have incentives to refinance, corresponding to $j_{t-1}^* = m$. Here, it is straightforward that Δi_t^h would only work through the no incentive channel and the inaction channel. The refinancing channel thus boosts the speed at which i^m is transmitted into i_t^h.

2. *The no incentive channel* covers how borrowers without the incentives to refinance contributes to the development in average interest rate. The channel has ambiguous effects on Δi_t^h, depending on whether the interest rates on the borrowers without refinancing incentives are above or below the average. If rates are above, the channel contributes in isolation to a decline in the average interest rate, and vice versa.

3. *The inaction channel* works, by and large, like the no incentive channel. One key difference is that the magnitude of the channel is affected by the degree of inaction. If the share of inactive borrowers increases (i.e. a higher ϕ), it leads to more refinancing failures. In every point in time, this limits the interest rate response from the market rate, which thus slowdown the transmission of monetary policy. In the boundary case $\phi = 1$, the inaction channel perfectly dominates the refinancing channel, and homeowners do not respond on monetary policy changes.

3.3 Equilibrium

Given the set of borrowers $j_t \in [0, m]$, I can characterize the dynamic equilibrium in terms of j_t^*, individual debt shares $\varphi_t(j_t)$, individual interest rates $i_t^h(j_t)$, the average interest rate i_t^h, and total debt D_t. A *balanced equilibrium path* is a dynamic equilibrium in which $\{j_t^*, i_t^h(j_t), \varphi_t(j_t), i_t^h, D_t\}$ are constant over time.

Inaction introduces transitional dynamics into j_t^* that is endogenously determined by the refinancing incentives condition (lemma 3.1). As active borrowers refinance, j_t^* shifts with a speed $(1-\phi)(m_t - j_t^*)$. Consequently, j_t^* is gradually pushed towards m as long as $j_t^* < m$. The transition occurs until no borrowers are left with positive refinancing incentives.

Specifically, the dynamics of j_t^* must satisfy:

$$j_t^* = j_{t-1}^* + (1-\phi)(m - j_{t-1}^*) = \phi j_{t-1}^* + (1-\phi)m, \tag{7}$$

for $m \geq j_0^* \geq 0$. The value j_0^* represents the initial value of j_t^*. If $j_0^* = 0$, all borrowers initially face positive incentives, and will thus refinance in the transition towards equilibrium. If instead $m > j_0^* > 0$, only a fraction of borrowers initially face positive incentives, and full refinancing will not occur in equilibrium.

The dynamic system comprising lemma 3.1 and the equation (7) determines the process of state dependency in household actions. According to equation (7), the locus $\Delta j_t^* = 0$ satisfies

$$\Delta j_t^* = 0 \quad \text{for} \quad j^* = m, \tag{8}$$

The equilibrium condition $j^* = m$ is sustained for a given value of i^m. The existence of j_0^* implies that the law of motion of homeowners' average interest rate can be rewritten as follows:

$$\begin{aligned}
\Delta i_t^h =& \underbrace{(1+\kappa)\Lambda^d_{(t-1,t)} \int_{j_{t-1}^* + \phi(m - j_{t-1}^*)}^{m} \varphi_{t-1}(j_{t-1})}_{\substack{\text{speed of adjustment} \\ \text{refinancing}}} \underbrace{\left(i^m - i_{t-1}^h\right) dj}_{\substack{\text{long-run pass-through} \\ \text{refinancing}}} \\
&+ \underbrace{\int_{j_{t-1}^*}^{j_{t-1}^* + \phi(m - j_{t-1}^*)} \varphi_{t-1}(j_{t-1}) \Lambda^d_{(t-1,t)}}_{\substack{\text{speed of adjustment} \\ \text{inaction}}} \underbrace{\left(i_{t-1}^h(j_{t-1}) - i_{t-1}^h\right) dj}_{\substack{\text{long-run pass-through} \\ \text{inaction}}} \\
&+ \underbrace{\int_{j_0^*}^{j_{t-1}^*} \varphi_{t-1}(j_{t-1}) \Lambda^d_{(t-1,t)}}_{\substack{\text{speed of adjustment} \\ \text{have refinanced}}} \underbrace{\left(i_{t-1}^h(j_{t-1}) - i_{t-1}^h\right) dj}_{\substack{\text{long-run pass-through} \\ \text{have refinanced}}} \\
&+ \underbrace{\int_{0}^{j_0^*} \varphi_{t-1}(j_{t-1}) \Lambda^d_{(t-1,t)}}_{\substack{\text{speed of adjustment} \\ \text{no response}}} \underbrace{\left(i_{t-1}^h(j_{t-1}) - i_{t-1}^h\right) dj}_{\substack{\text{long-run pass-through} \\ \text{no response}}},
\end{aligned} \tag{9}$$

In contrast to propostion 3.1, equation (9) takes the initial value of j_t^* into account, and dynamically expresses how the market rate is transmitted into homeowners' fixed-rate mortgage rates. According to lemma 3.1, the value of j_0^* is determined by the magnitude of the market rate compared to the borrowers' existing fixed-rate mortgage rates. If i^m is relatively large, j_0^* is relatively small as the majority of borrowers initially face positive incentives, and vice versa. In each point in time, the market rate is only transmitted into the borrowers who refinance. As a result, adjustment towards the long-run equilibrium takes time and occurs temporarily with a speed that depends on the fraction of refinanced debt in each period. The speed of adjustment slows down as $j_t^* \to m$ (i.e. $\Omega_{t-1}^r \to 0$) due to a relatively smaller share of borrowers being left with positive incentives in every period during transition towards steady state.

A balanced equilibrium requires $\Delta i_t^h = 0$. The equilibrium path of i_t^h can be determined by the following proposition:

Proposition 3.2 (Equilibrium path of average interest rate). *When $\Delta i_t^h = 0$, a balanced equilibrium path of i_t^h exists. Along this path $i_{t-1}^{h^*}$ satisfies*

$$i_{t-1}^{h^*} = \left(1 - \Lambda_{(t-1,t)}^d \int_0^{j_0^*} \varphi_{t-1}(j_{t-1})dj\right) i^m + \Lambda_{(t-1,t)}^d \int_0^{j_0^*} i_{t-1}^h(j_{t-1})\varphi_{t-1}(j_{t-1})dj$$
$$+ \Lambda_{(t-1,t)}^d \left(\int_{j_{t-1}^*}^{j_{t-1}^*+\phi(m-j_{t-1}^*)} \left(i_{t-1}^h(j_{t-1}) - i^m\right) \varphi_{t-1}(j_{t-1})dj\right), \tag{10}$$

for a given value of j_0^*. The interest rate path $i_{t-1}^{h^*}$ can take the following two forms depending on the value of j_0^*:

- ***Case (i):*** *If $j_0^* = 0$, the average interest rate follows the path:*
  $i_{t-1}^{h^*} = i^m + \Lambda_{(t-1,t)}^d \mu_{t-1}^i$.
- ***Case (ii):*** *If $0 < j_0^* \leq m$, the average interest rate follows the path:*
  $i_{t-1}^{h^*} = \left(1 - \Lambda_{(t-1,t)}^d \int_0^{j_0^*} \varphi_{t-1}(j_{t-1})dj\right) i^m + \Lambda_{(t-1,t)}^d \mu_{t-1}^h + \Lambda_{(t-1,t)}^d \mu_{t-1}^i$.

$\mu_{t-1}^h = \int_0^{j_0^*} i^h(j) i_{t-1}^h(j_{t-1})\varphi_{t-1}(j_{t-1})dj < i^m$ is the average interest rate of borrowers who initially have non-positive incentives, and $\mu_{t-1}^s = \left(\int_{j_{t-1}^*}^{j_{t-1}^*+\phi(m-j_{t-1}^*)} \left(i_{t-1}^h(j_{t-1}) - i^m\right) \varphi_{t-1}(j_{t-1})dj\right)$
Proof: *See appendix A.1.*

Figure 3 presents the phase diagram for the system of difference equations comprising the dynamic threshold equation (equation (7)) and the interest rate pass-through equation (equation (9)). This system of difference equations determines the structure of the dynamic equilibrium in the (j_t^*, i_t^h) space. The locus $\Delta j_t^* = 0$ is according to (8) $j_t^* = m$, i.e. a pure vertical line in the (j_t^*, i_t^h) space.

The slope of the locus $\Delta i_t^h = 0$ is always negative in j_t^* and declines towards zero as $j_t^* \to m$. This is a result of the fact that the fraction of inactive borrowers gradually gets smaller in the transition towards equilibrium and the fact that inactive borrowers always have interest rates that are above the market rate. Hence, refinancing gradually pulls down the average interest rate until it reaches its long-run target.

The locus $\Delta i_t^h = 0$ has different long-run properties depending on j_0^*. In the upper panel of figure 3, I have shown case (i) in proposition 3.2, and in the bottom panel of figure 3, I have shown case (ii) in proposition 3.2. In both panels, I have illustrated the cases where $i_0^{h^*} > i^m$, which occurs when $\Lambda_{(0,1)}^d \int_0^{j_0^*+\phi(m-j_0^*)} i_0^h(j_0)\varphi_0(j_0)dj > \left(\Lambda_{(0,1)}^d \int_0^{j_0^*+\phi(m-j_0^*)} \varphi_0(j_0)dj\right) i^m$.

The loci $\Delta i_t^h = 0$ and $\Delta j_t^* = 0$ intersect at the point E that characterizes the obtainable steady state in the dynamic system given the set of initial values $\{j_0^*, i_0^h\}$. The steady state is a global

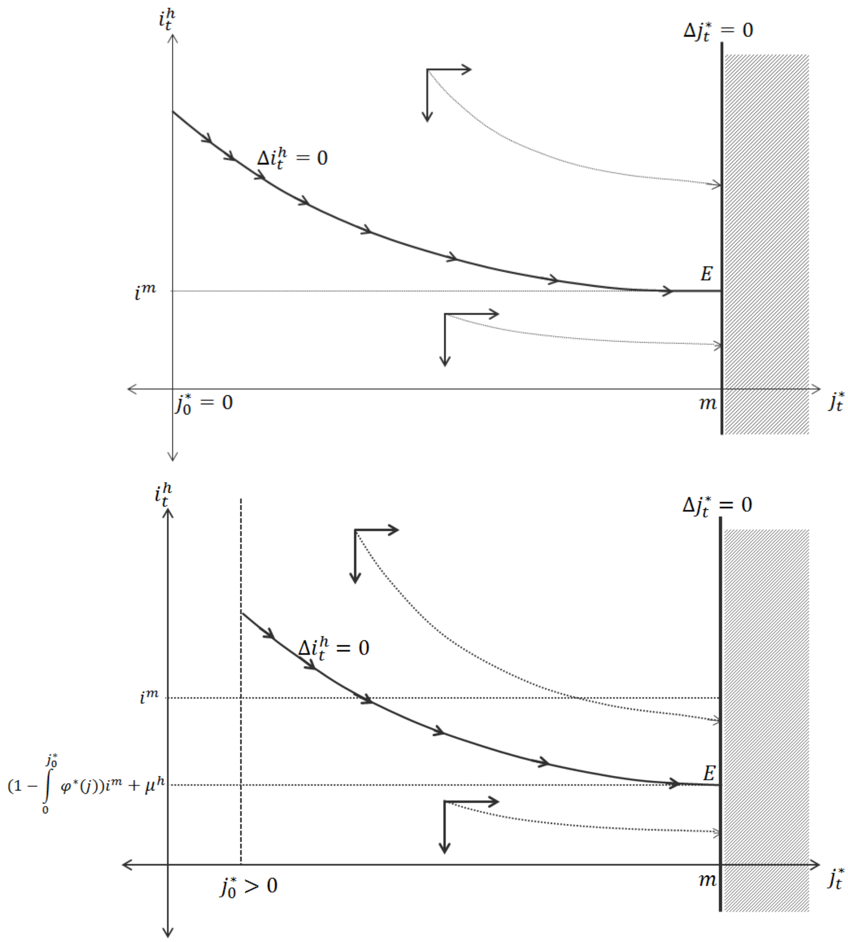


Figure 3: *Dynamic equilibrium. Upper panel illustrates case (i) in proposition 3.2, and bottom panel illustrates case (ii) in proposition 3.2.*

saddle point, meaning that all paths for $j_t^* \in [0, m]$ asymptotically converge towards a well-defined equilibrium located on the locus $\Delta j_t^* = 0$. As a consequence, all well-defined paths make up the stable arm in the system.

The initial value of i_0^h is predetermined by homeowners' previous decisions and adjustment thus takes time. Hence, at time $t = 0$, the system must be somewhere on the vertical line $j_0^* = 0$ in case (i) or $j_0^* > 0$ in case (ii). Here, convergence occurs as a positive number of borrowers $m - j_0^* > 0$ face positive incentives and thus refinance to the market rate. Due to inaction, refinancing occurs slowly and gradually pulls the average interest rate downwards. When all borrowers who initially faced positive incentives have refinanced, the system has reached its balanced equilibrium.

Proposition 3.3 (Balanced equilibrium). *When $\Delta j_t^* = 0$ and $\Delta i_t^h = 0$, a balanced equilibrium $\{j^*, \varphi^*(j), i^{h^*}(j), i^{h^*}, D^*\}$ exists and is unique. In equilibrium, the average interest rate is:*

$$i^{h^*} = \underbrace{\left(1 - \int_0^{j_0^*} \varphi^*(j)dj\right)}_{\text{magnitude of interest rate pass-through}} i^m + \underbrace{\int_0^{j_0^*} i^{h^*}(j)\varphi^*(j)dj}_{\text{no response mark-up}}, \tag{11}$$

for a given value of j_0^*. The long-run interest rate pass-through of i^{h^*} can take the following two forms:

- ***(One-to-one pass-through)*** *For $j_0^* = 0$, the long-run pass through is $i^{h^*} = i^m$ and occurs under case (i) in proposition 3.2. It requires that all borrowers initially face positive incentives, so that all borrowers refinance when $j_t^* \to m$.*
- ***(Less than one-to-one pass-through)*** *For $m \geq j_0^* > 0$, the long-run pass through is $i^{h^*} = \left(1 - \int_0^{j_0^*} \varphi^*(j)dj\right) i^m + \mu^h < i^m$ and occurs under case (ii) in proposition 3.2. It requires that not all borrowers initially face non-positive incentives.*

Proof: *See appendix A.1.*

Proposition 3.3 reveals that the long-run interest rate pass-through is a weighted average of interest rates on those borrowers who have responded to i^m during transition (i.e. borrowers who have refinanced) and those who have not responded to i^m during transition (i.e. borrowers with no incentives). The first term captures the contribution in interest rate terms from refinancing, while the second term captures the contribution in interest rate terms from no refinancing. In equilibrium, the average interest rate is thus proportional to the market rate with a multiplier equal to the share of refinanced debt plus a mark-up comprising the average interest rate on non-refinanced debt. The balanced equilibrium is solely consistent with a one-to-one interest rate pass-through if all borrowers initially face positive incentives and thus refinance in the transition towards steady state. If a fraction of borrowers initially start out with positive incentivess, the dynamic equilibrium is instead consistent with less than a one-to-one pass-through. Here, the magnitude of the pass-through depends intuitively on the number of borrowers who do not initially face incentives and their respective debt shares.

The dynamic equilibrium should be qualified as a medium-run equilibrium due to the relatively slow convergence in j_t^*. In reality, the market rate is changing instanstaneously, and the system will never reach equilibrium before new shocks occur. The equilibrium should thus be interpreted as a resting point towards which the average interest rate is drawn after it has been pushed away.

3.3.1 The effect of a shock to the market rate

Consider an expansionary monetary policy shock corresponding to a decline in i^m. A lower i^m leads to a permanent downward shift in the locus $\Delta i_t^h = 0$ according to proposition 3.2, while the locus

$\Delta j_t^* = 0$ remains unchanged, as shown in figure 4. The immediate effect of a decline in i^m is a downward jump in j_t^*, where the size of the jump depends on the change in borrowers' refinancing incentives (i.e. lemma 3.1), which are inherently determined by the magnitude of the shock to i^m. The illustration in figure 4 relies on case (ii) under proposition 3.2, implying that j_t^* jumps to a level $j_0^* > 0$ when the shock hits. As a consequence, the system jumps instantanously from the old steady state E_0 towards the point E_0' where the level of i_t^h still remains unchanged.

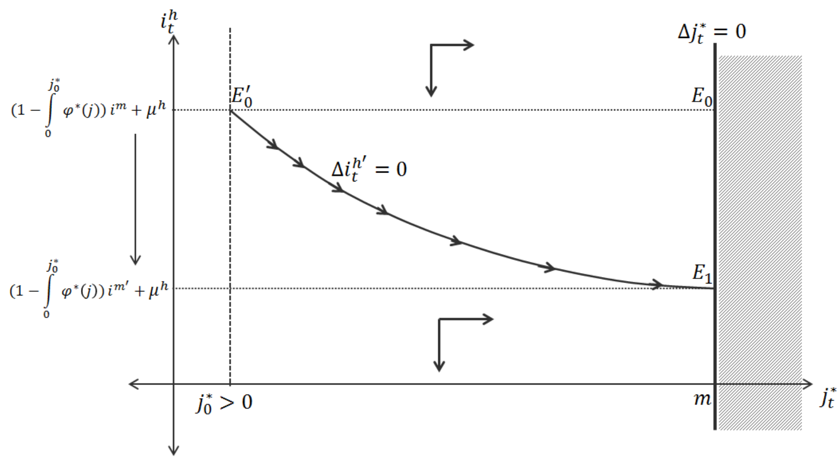


Figure 4: *Illustation of a permanent decline in* i^m

In the point E_0' refinancing takes off. In the transition towards the new equilibrium refinancing occurs slowly and gradually pulls the average interest rate downwards along the saddle path until the borrowers who initially faced positive incentives have refinanced. This defines the new equilibrium, E_1, where the average interest rate is below the market rate. The new equilibrium is consistent with a less than one-to-one interest rate pass-through, reflecting that only some borrowers initially face positive refinancing incentives.

The large dependency between the size of the shock (i.e. the starting point of j_t^*) and the fraction of borrowers who refinance formalizes why refinancing takes place in surges during periods of large declines in the market rate. As a result, the model explains why the endogenous nature of refinancing leads to a non-linearity in the interest rate pass-through to household's fixed-rate mortgages.

The system of difference equations reveals that a tightening of monetary policy (corresponding to an increase in i^m) does not lead to a change in average interest rates. This is due to the fact that j_t^* remains unchanged as refinancing incentives for all homeowners deteriorate when the shock hits. As a result, the system will not converge towards a new equilibrium since no borrowers have incentive to refinance. The differences in the response on the average interest rate depending on whether the monetary policy shock is contractionary or expansionary summarize the asymmetric effects of monetary policy in the refinancing channel.

3.4 Full model: introducing entry, exit and amortization

So far, I have entirely focused on existing borrowers and their respective decisions. Two channels that have been left out are how inflows and outflows of debt affect the interest rate pass-through by changing the composition of interest rates across borrowers and debt levels. Entry of new borrowers can arise from housing relocation, homeowners who are shifting from rented accommodation to owner-occupied dwellings, or households who have previously financed housing debt through for example adjustable-rate mortgages. Exit of borrowers matches events like housing relocation and substitution to other loan types.

Let $j_t \in [j_t^* - \omega_t^n, j_t^*]$ define a new borrower that enters at time t, where ω_t^n is the number of new borrowers. I assume that a constant share $\gamma^n \in (0,1)$ of the existing number of borrowers enters in each period, such that $\omega_t^n = \gamma^n m_t$. Subsequently, I assume $j_t \in [0, \omega_t^p]$ denote a borrower that repays at time t, where $\omega_t^p < m_t$ defines the number of borrowers who repays. I assume that a constant share $\gamma^p \in (0,1)$ of all existing borrowers repay at time t, so that $\omega_{t-1}^p = \gamma^p m_{t-1}$. Henceforth, I will refer to γ^p as *the exit effect*.

The repayment assumption implies that it is the range of borrowers with least strongest refinancing incentives who repay in each period, implying that the decision to exit is independent of refinancing incentives and other potentiel outside options. As a result, the assumption matches events like housing relocation that typically occur independently of the development in refinancing incentives. Figure 5 depicts the allocated space of borrowers' decisions at time $t-1$ (upper panel) and their resulting actions at time t based on their decisions at time $t-1$ (bottom panel).

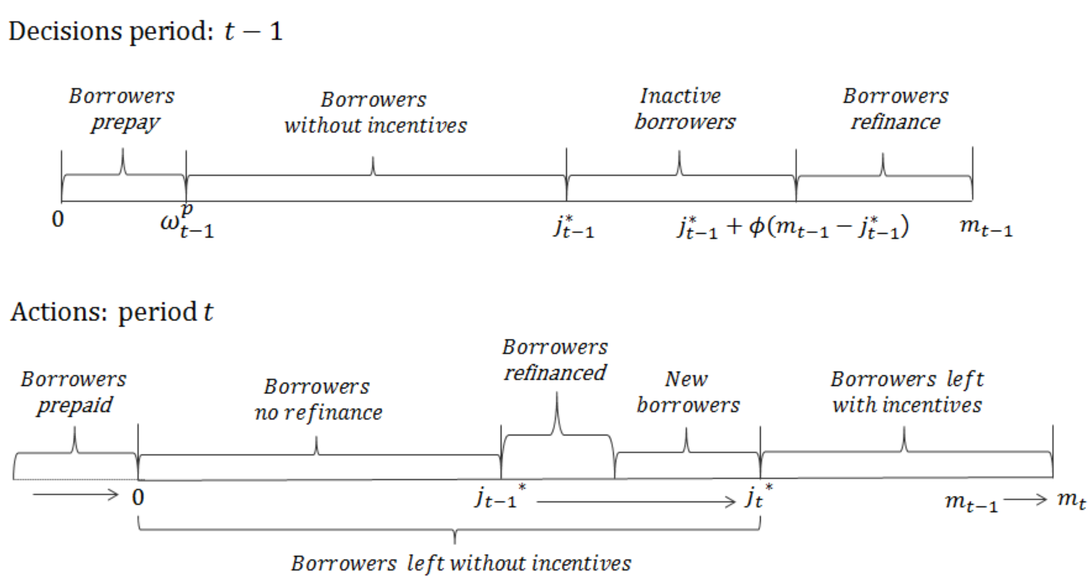


Figure 5: *The space of borrowers. Upper panel depicts the space of allocation of borrowers decisions at $t-1$. Bottom panel depicts the borrowers resulting actions at time t given their decisions and entry and exit of borrowers.*

New borrowers who enter obtain the market rate, and thus face the interest rate $i_t^h(j_t) = i^m$ at time t. The introduction of repayments implies that a exisiting borrower's interest rate at time t conditional on its decision at time $t-1$ is given by the state-dependent process:

$$i_t^h(j_t) = \begin{cases} i^m & \text{if } j_{t-1} \in [j_t^* - \gamma^n m_t, j_t^*] \\ i^m & \text{if } j_{t-1} \in [\max(j_{t-1}^* + \phi\chi_{t-1}, \gamma^p m_{t-1}), m_{t-1}] \\ i_{t-1}^h(j_{t-1}) & \text{if } j_{t-1} \in [\max(j_{t-1}^*, \gamma^p m_{t-1}), \max(j_{t-1}^* + \phi\chi_{t-1}, \gamma^p m_{t-1})] \\ i_{t-1}^h(j_{t-1}) & \text{if } j_{t-1} \in [\gamma^p m_{t-1}, \max(j_{t-1}^*, \gamma^p m_{t-1})] \\ 0 & \text{if } j_{t-1} \in [0, \gamma^p m_{t-1}] \end{cases} \tag{12}$$

where $\chi_{t-1} = m_{t-1} - j_{t-1}^*$.

I also introduce debt amortization. As a result, borrowers who do not refinance or repay, do no longer have constant debt levels. Instead, instalments of debt ensure that individual debt levels are declining over time. Let $\lambda \in (0,1)$ be the rate of amortization. Hence, debt by borrower j_t now follows the state-dependent process of debt accumulation:

$$d_t(j_t) = \begin{cases} d_t & \text{if } j_{t-1} \in [j_t^* - \gamma^m m_t, j_t^*] \\ (1+\kappa)d_{t-1}(j_{t-1}) & \text{if } j_{t-1} \in [\max(j_{t-1}^* + \phi\chi_{t-1}, \gamma^p m_{t-1}), m_{t-1}] \\ (1-\lambda)d_{t-1}(j_{t-1}) & \text{if } j_{t-1} \in [\max(j_{t-1}^*, \gamma^p m_{t-1}), \max(j_{t-1}^* + \phi\chi_{t-1}, \gamma^p m_{t-1})] \\ (1-\lambda)d_{t-1}(j_{t-1}) & \text{if } j_{t-1} \in [\gamma^p m_{t-1}, \max(j_{t-1}^*, \gamma^p m_{t-1})] \\ 0 & \text{if } j_{t-1} \in [0, \gamma^p m_{t-1}] \end{cases} \tag{13}$$

Inflows and outflows of borrowers ensure that the number of borrowers on the continuum interval is not necessarily constant over time. The law of motion for the number of borrowers is:

$$m_t = \omega_t^n + m_{t-1} - \omega_{t-1}^p = \frac{1-\gamma^p}{1-\gamma^n} m_{t-1} = \Theta_m m_{t-1}, \quad \text{for} \quad m_0 > 0, \tag{14}$$

where $\Theta_m = (1-\gamma^p)/(1-\gamma^n)$ measures the fraction of existing borrowers who are replaced by new borrowers with interest rates equal to the market rate. I will henceforth refer to Θ_m as *the replacement effect*. To avoid $m_t \to 0$, I assume that $\Delta m_t \geq 0$, causing that the share of borrowers who enter exceeds or equals the share who exit ($\gamma^n \geq \gamma^p$).Consequently, the replacement effect will always be above (or equal) unity $\Theta_m \geq 1$.

3.4.1 Aggregation

I only go through the main extentions here, while the rest of the extended model is presented in appendix A.1. The law of motion of the average interest is determined by the proposition:

Proposition 3.4. *With replacement of debt, the law of motion of homeowners' average interest rate is:*

$$
\begin{aligned}
\Delta i_t^h = & \underbrace{\int_{\max(j_{t-1}^*+\phi\chi_{t-1},\gamma^p m_{t-1})}^{m_{t-1}} \Big((1+\kappa)\Lambda_{(t-1,t)}^d (i^m - i_{t-1}^h) - (i_{t-1}^h(j_{t-1}) - i_{t-1}^h)\Big)\varphi_{t-1}(j_{t-1})dj}_{\text{refinancing channel}} \quad (15)\\
& \underbrace{- \lambda\Lambda_{(t-1,t)}^d \int_{\gamma^p m_{t-1}}^{\max(j_{t-1}^*,\gamma^p m_{t-1})} \left(i_{t-1}^h(j_{t-1}) - i_{t-1}^h\right)\varphi_{t-1}(j_{t-1})dj + \Xi_{(t-1,t)}^s}_{\text{no incentive channel}}\\
& \underbrace{- \lambda\Lambda_{(t-1,t)}^d \int_{\max(j_{t-1}^*,\gamma^p m_{t-1})}^{\max(j_{t-1}^*+\phi\chi_{t-1},\gamma^p m_{t-1})} \left(i_{t-1}^h(j_{t-1}) - i_{t-1}^h\right)\varphi_{t-1}(j_{t-1})dj + \Xi_{(t-1,t)}^i}_{\text{inaction channel}}\\
& + \underbrace{\int_{j_t^*-\gamma^n m_t}^{j_t^*} \left(i^m - i_{t-1}^h\right)\varphi_t(j_t)dj}_{\text{entry channel}} - \underbrace{\int_0^{\gamma^p m_{t-1}} \left(i_{t-1}^h(j_{t-1}) - i_{t-1}^h\right)\varphi_{t-1}(j_{t-1})dj}_{\text{exit channel}}
\end{aligned}
$$

where $\Xi_{(t-1,t)}^s = (\Lambda_{(t-1,t)}^d - 1)\int_{\gamma^p m_{t-1}}^{\max(j_{t-1}^*,\gamma^p m_{t-1})} \left(i_{t-1}^h(j_{t-1}) - i_{t-1}^h\right)\varphi_{t-1}(j_{t-1})dj$ *and* $\Xi_{(t-1,t)}^i = (\Lambda_{(t-1,t)}^d - 1) = \int_{\max(j_{t-1}^*,\gamma^p m_{t-1})}^{\max(j_{t-1}^*+\phi\chi_{t-1},\gamma^p m_{t-1})} \left(i_{t-1}^h(j_{t-1}) - i_{t-1}^h\right)\varphi_{t-1}(j_{t-1})dj$.
Proof: *See appendix A.1.*

Proposition 3.4 shows that homeowners' average interest rate can be decomposed into additionally two channels. While *the refinancing channel*, *the inaction channel* and *the no incentive channel* exist solely due to the refinancing option on fixed-rate mortgages, *the entry channel* and *the exit channel* are classical channels when assessing pass-throughs to average interest rates.

In contrast to the baseline model, the no incentive channel and the inaction channel additionally cover how outflow of debt from instalments contributes to the average interest rate. Amortization affects Δi_t^h via two effects. First, the magnitudes of the channels are positively affected by λ. This reflects the fact that a higher rate of amortization reduces the share of (non-refinanced) debt, which mechanically raises the importance of refinanced debt in the determination of the average interest rate in the subsequent period. As a result, a larger λ thus influences the pass-through positively by reallocating debt towards mortgages with interest rates equal to the market rate. Second, amortization has also ambiguous effects on Δi_t^h, depending on whether the interest rates on the amortized debt are above or below the average. If rates are above, the channel contributes in isolation to a decline in the average interest rate, and vice versa.

1. *The entry channel* works, by and large, like the refinancing channel because new borrowers also finance their mortgages directly at the market rate. In isolation, the entry channel contributes to pulling the average interest rate towards the market rate, where the magnitude

of the contribution is logically increasing in the share of debt raised by new borrowers. The sign of the effect depends thus on whether new borrowers enter to a lower or higher rate than the average interest rate. A dissimilarity between the entry channel and the refinancing channel is that the decision to enter are not endogenously determined by refinancing incentives. As a result, the entry channel contributes to pulling the average interest rate towards the market rate even through interest rates are increasing. Consequently, the entry channel help mitigate the asymmetries in the interest rate pass-through to fixed-rate mortgages.

2. *The exit channel* affects the interest rate pass-through via the same two effects as amortization. First, repayments make survived debt and new debt more influential in the determination of the average interest rate.[6] Second, the exit channel boosts the speed of adjustment if repayments pull the average interest rate in the same direction as the market rate, meaning that if the interest rates on repaid debt exceed the average interest rate concurrently with the market rate being below the average rate, and vice versa, repayments boost speed of adjustment.

3.4.2 Equilibrium

The law of motion of j_t^* states:

$$j_t^* = j_{t-1}^* + (1-\phi)(m_{t-1} - j_{t-1}^*) + \frac{\gamma^n - \gamma^p}{1-\gamma^n} m_{t-1} \tag{16}$$

Equation (16) shows that j_t^* will not be constant in equilibrium if $\Delta m_t > 0$. However, the refinancing incentive condition (lemma 3.1) still implies that active borrowers gradually refinance until no borrowers are left with positive refinancing incentives (i.e. $j_t^* = m_t$) given j_t^* is initially below m_t. In equilibrium, j_t^* will thus equal m_t. Consequently, I consider the function:

$$\Lambda(j_t^*, m_t) = m_t - j_t^* = -j_{t-1}^* - (1-\phi)(m_{t-1} - j_{t-1}^*) + m_{t-1} \tag{17}$$

In equilibrium, I have $\Lambda(j_t^*, m_t) = 0$, which leads to the following equilibrium condition:

$$\Lambda(j_t^*, m_t) = 0 \quad \text{for} \quad j_t^* = m_t \tag{18}$$

Inflows and outflows of borrowers induce that individual debt shares $\varphi_t(j_t)$, individual interest rates $i_t^h(j_t)$, the average interest rate i_t^h, and total debt do not necessarily converge towards constant levels as $j_t^* \to m_t$. The intuition is that the continuous flows of repayments gradually replace all existing debt despite no borrowers have incentive to refinance. In the long-run, all initial debt will hence end up being ultimately replaced by new debt. In practice, new debt and repayments

[6] However, the scale of this the two effects might differ. Amortization eliminates debt by the rate λ, while the exit channel eliminates debt by the rate of unity. On the other hand, the share of prepaid debt is typically small compared to the debt share from instalments.

are slow-moving variables compared to changes in refinancing incentives. Refinancing has thus relatively fast-moving effects on the interest rate pass-through compared to the time it takes before debt raised by new borrowers has replaced all existing debt. As a result, I will distinguish between two types of equilibria. First, I characterize a dynamic equilibrium only in terms j_t^* growing with the same rate as m_t. I define this scenario as a medium-run equilibrium. Second, I characterize a dynamic equilibrium in terms of j_t^*, individual interest rates $i_t^h(j_t)$, and the average interest rate i_t^h. A *balanced equilibrium path* is a dynamic equilibrium in which the variables $\{i_t^h(j_t), i_t^h\}$ are constant over time and j_t^* grows with the same rate as m_t. I define this scenario as a long-run equilibrium.

Proposition 3.5 (Medium-run equilibrium with replacement of debt). *When $\Delta i_t^h = 0$, $\Lambda(j_t^*, m_t) = 0$, an equilibrium path where $j_t^* = m_t$ exists for $t = \bar{t}^m \geq t_0$ is unique. Along this equilibrium path, the average interest rate is:*

$$i_{t-1}^{h^*} = \underbrace{\left(1 - (1-\lambda)^t \Lambda_{(t-1,t)}^d \int_{\gamma^p m_{t-1}}^{j_0^* \times \max\left(\max\left(1-\gamma^p \tilde{m}_0 \frac{1-\Theta_m^{t-1}}{1-\Theta_m}, 0\right), \gamma^p \tilde{m}_0 \Theta_m^{t-1}\right)} \varphi_0(j_0) dj\right)}_{\text{magnitude of interest rate pass-through}} i^m$$
$$+ \underbrace{(1-\lambda)^t \Lambda_{(t-1,t)}^d \int_{\gamma^p m_{t-1}}^{j_0^* \times \max\left(\max\left(1-\gamma^p \tilde{m}_0 \frac{1-\Theta_m^{t-1}}{1-\Theta_m}, 0\right), \gamma^p \tilde{m}_0 \Theta_m^{t-1}\right)} i_{t-1}^h(j_{t-1}) \varphi_0(j_0) dj}_{\text{no response mark-up}}, \tag{19}$$

for a given values of j_0^* and $\tilde{m}_0 = m_0/j_0^*$. The long-run pass-through of $i_{t-1}^{h^*}$ can take the following two forms:

- ***(One-to-one pass-through)*** *For $j_0^* = 0$ or $1 - \gamma^p \tilde{m}_0 \frac{1-\Theta_m^{t-1}}{1-\Theta_m} \leq \gamma^p \tilde{m}_0 \Theta_m^{t-1}$, the long-run pass through is $i^{h^*} = i^m$ and occurs if (i) no borrowers initially face non-positive incentives repay as $j_t^* \to m_t$; or (ii) if the borrowers who initially face non-positive incentives repay as $j_t^* \to m_t$.*
- ***(Less than one-to-one pass-through)*** *For $m_0 \geq j_0^* > 0$ or $1 - \gamma^p \tilde{m}_0 \frac{1-\Theta_m^{t-1}}{1-\Theta_m} > \gamma^p \tilde{m}_0 \Theta_m^{t-1}$, the long-run pass through is*

$$i_{t-1}^{h^*} = \left(1 - (1-\lambda)^t \Lambda_{(t-1,t)}^d \int_{\gamma^p m_{t-1}}^{j_0^* \times \max\left(\max\left(1-\gamma^p \tilde{m}_0 \frac{1-\Theta_m^{t-1}}{1-\Theta_m}, 0\right), \gamma^p \tilde{m}_0 \Theta_m^{t-1}\right)} \varphi_0(j_0) dj\right) i^m$$
$$+ \mu_{(t-1,t)}^h < i^m$$

and occurs if not all borrowers initially face non-positive incentives; or (ii) if not all borrowers who initially face non-positive incentives repay as $j_t^* \to m_t$, where $\mu_{(t-1,t)}^h = (1-\lambda)^t \Lambda_{(t-1,t)}^d \int_{\gamma^p m_{t-1}}^{j_0^* \times \max\left(\max\left(1-\gamma^p \tilde{m}_0 \frac{1-\Theta_m^{t-1}}{1-\Theta_m}, 0\right), \gamma^p \tilde{m}_0 \Theta_m^{t-1}\right)} i_{t-1}^h(j_{t-1}) \varphi_0(j_0) dj$.

Proof: *See appendix A.1.*

Proposition 3.6 (Long-run equilibrium with replacement of debt). *When* $\Delta i_t^h = 0$ *and* $\Lambda(j_t^*, m_t) = 0$*, a balanced equilibrium path* $\{i^{h^*}(j_t), i^{h^*}, j_t^*\}$ *exists and is unique for* $t = \bar{t}^l \geq \bar{t}^m$*, where* $\{i^{h^*}(j_t), i^{h^*}\}$ *are constant over time, and* j_t^* *grows with the rate of* m_t*. Along this balanced equilibrium path, the average interest rate is* $i^{h^*} = i^m$*.*
Proof: *See appendix A.1.*

Proposition 3.5 states that the medium-run interest rate pass-through also depends on entry and exit of borrowers (i.e. the parameters γ^p and Θ_m) and debt amortization (i.e. parameter λ). The inflows and outflows of debt imply that the magnitude of the interest rate pass-through is increasing in (i) the exit effect (i.e. the parameter γ^p); (ii) the replacement effect (i.e. the paramterer Θ_m); and (iii) the rate of debt amortization (i.e. the parameter λ^p). The intuition is that repayments and amortization reduce the share of debt with interest rates different from the market rate, automatically making the contribution (measured in interest rate terms) from refinanced debt and new debt more influential in the determination of the average interest rate.[7] As a result, the interest rate pass-through will always be larger compared to the baseline model in the case where some borrowers do not initially face positive refinancing incentives. Another major difference is that a one-to-one interest rate pass-through can occur in the medium run despite all existing borrowers do not initially face positive refinancing. This will occur if the exit effect is sufficiently large to eliminates all exisiting borrowers who initially faced non-positive refinancing incentives.

Proposition 3.6 highlights that a one-to-one interest rate pass-through will always take place in the long run. The intuition is that borrowers who initially had non-positive refinancing incentives will gradually be replaced by new borrowers, implying that no borrowers will be left with interest rates different from the market rate when $t = t^l < \infty$. The speed of adjustment to the long-run equilibrium is logically increasing in the magnitudes of the exit effect, the replacement effect and the amortization rate. (i.e. the parameters γ^p,Θ_m and λ) for $j_0^* > 0$. For reasonable values of γ^n, γ^p and λ, the convergence towards the long-run equilibrium is relatively slow, especially compared to the speed of adjustment of the threshold borrower j_t^*. Consequently, the balanced long-run equilibrium should be qualified as a very long-run equilibrium that will never be relevant in practice due to the instantaneous changes in the market rate.

Figure 6 presents the phase diagram for the system of difference equations comprising the modified threshold equation (equation (18)) and the dynamic interest rate pass-through equation (equation (A.11) in appendix A.1). The locus $\Delta\Lambda(j_t^*, m_t) = 0$ is still a pure vertical line in the (j_t^*, i_t^h) space. When $t_0 \leq t < \bar{t}^m$, the slope of the locus $\Delta i_t^h = 0$ is negative for $i_{t-1}^{h^*} \geq i^m$, while the sign of the slope can be ambiguous for $i_{t-1}^{h^*} < i^m$. In the latter, the sign of the slope depends on the magnitudes between the inaction channel, the no incentive channel and the entry the exit channels.

[7] In the boundary case $\lambda = 1$ or $\gamma^p = 1$, all debt which is not refinanced is completely repaid. It leads to a one-to-one response, as all survived debt is refinanced immediately.

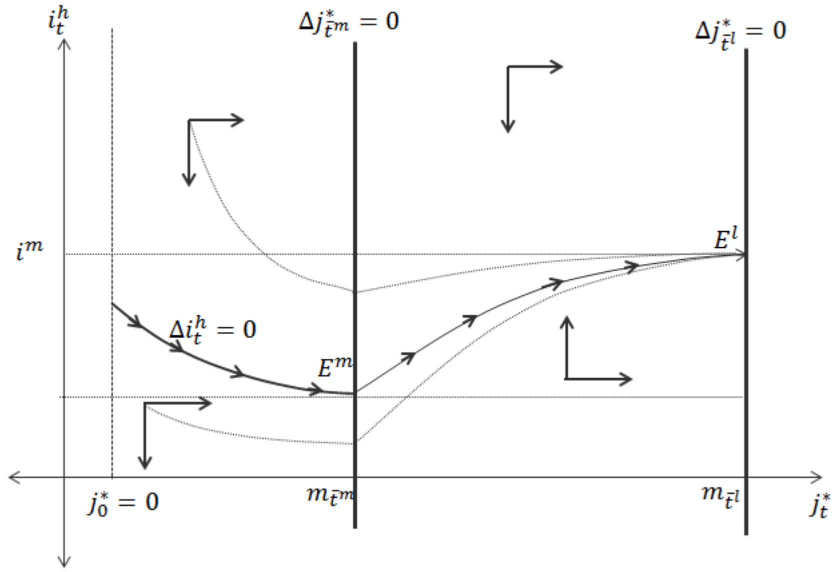


Figure 6: *Dynamic equilibrium with entry and exit based on case (ii) in proposition 3.5.*

If the negative contribution on the slope from inactive borrowers who are refinancing outweighs the positive contribution from repayments and replacement of debt, the slope is negative, and vice versa. Details are presented in appendix A.1.

For all $t \geq \bar{t}^m$, the slope of the locus $\Delta i_t^h = 0$ is zero in case (i) and positive in case (ii). The intuition is that the replacement of borrowers and debt amortization do not play a role in the interest rate pass-through if all borrowers have refinanced when $t = \bar{t}^m$. If not, replacement of debt gradually pulls the average interest rate towards the market rate.

In figure 6, I have shown case (ii) in proposition 3.5 based on the assumption that the slope is negative for all $t < \bar{t}^m$. Given the set of initial values $\{j_0^*, i_0^h\}$, the loci $\Delta i_t^h = 0$ and $\Delta\Lambda(j^*_{\bar{t}^m}, m_{\bar{t}^m}) = 0$ intersect at the point E^m, which characterizes the medium-run equilibrium. Furthermore, the loci $\Delta i_t^h = 0$ and $\Delta\Lambda(j^*_{\bar{t}^l}, m_{\bar{t}^l}) = 0$ intersect at the point E^l, which denotes the long-run steady-state path in the dynamic system that exists for all $t \in [\bar{t}^l, \infty[$. The steady-state path constitutes the global saddle path in the dynamic system, meaning that all paths for $j_t^* \in [0, m_t]$ asymptotically converge towards this well-defined path.

The transition towards the long-run equilibrium clarifies that the entry channel has a symmetrical response on the average interest rate. As a result, an increase in the market rate now leads to a one-to-one interest rate pass-thorugh because the replacement of debt mechanically pulls the average interest rate towards the market rate. The speed of adjustment is, however, still much lower when the system is hit by a contractionary monetary policy shock compared to an expansionary one. Intuitively, this covers that the refinancing channel still responds asymmetrically to changes in the market rate. In effect, the interest rate pass-through is thus weaker in periods of contractionary monetary policy despite the entry channel contributes to mitigating the asymmetric response on the interest-rate pass-through.

4 Empirical framework

In this section, I examine the empirical relevance of the theoretical model using evidence from Denmark. First, I use a cointegrated VAR model to estimate the macroeconomic response from monetary policy shocks to homeowners' fixed-rate mortgage rates. Subesequently, I complement the empirical analysis with evidence from administrative data on Danish households. The household data allows me to construct measures of the individual parameters in the theoretical model, and thus makes it possible to decompose the contributions from the individual channels to the aggregate response.

4.1 The empirical specification

Before taking the theoretical model to the data, there are some issues that need to be considered. The first issue relates to how amortization and repayments affect speed of adjustment. According to the theoretical model, amortization and repayments boost speed of adjustment if both factors pull the average interest in the same direction as the market rate. I empirically allow for this by imposing the definition:

Definition 4.1. *Let $\Omega_t^p = \int_0^{\omega_t^p} \varphi_t(j_t)dj$ be the aggregate debt share that is repaid in period t and $\Omega_t^a = \int_{\omega_t^p}^{j_t^*+\phi(m_t-j_t^*)} \varphi_t(j_t)dj$ the aggregate debt share that is amortized in period t. Subsequently, I define*

$$\tilde{\Omega}_t^p = \begin{cases} \Omega_t^p & \text{if } i_t^h(j_t) \gtrless i_t^h \quad \text{and} \quad i^m \lesseqgtr i_t^h \quad \text{for} \quad j_t \in (0, \omega_t^p) \\ -\Omega_t^p & \text{if } i_t^h(j_t) \lesseqgtr i_t^h \quad \text{and} \quad i^m \gtrless i_t^h \quad \text{for} \quad j_t \in (0, \omega_t^p) \\ 0 & \text{Otherwise} \end{cases}$$

$$\tilde{\Omega}_t^a = \begin{cases} \Omega_t^a & \text{if } i_t^h(j_t) \gtrless i_t^h \quad \text{and} \quad i^m \lesseqgtr i_t^h \quad \text{for} \quad j_t \in (\omega_t^p, j_t^* + \phi(m_t - j_t^*)) \\ -\Omega_t^a & \text{if } i_t^h(j_t) \lesseqgtr i_t^h \quad \text{and} \quad i^m \gtrless i_t^h \quad \text{for} \quad j_t \in (\omega_t^p, j_t^* + \phi(m_t - j_t^*)) \\ 0 & \text{Otherwise} \end{cases}$$

The second issue relates to interest rates on homeowners who decide to repay. In the theoretical model, repayments exclusively occured for borrowers who faced the least strong refinancing incentives. In reality, repayments can occur on the whole continuum. To empirically account for this heterogeneity of interest rates for borrowers who repay, I Impose the assumption:

Assumption 4.1. *For all* $j_t \in [0, \omega_t^p]$, *I define* $i_t(j_t) = \rho_t^p i^h$, *for* $\rho_t^p > 0$.

Assumption 4.1 states that the interest rates on borrowers who prepaid at time t are proportional to the average interest rate with the homogenous multiplier ρ_t^p. If $\rho_t^p > 1$ the average interest rate on borrowers who exit is above the average interest rate, and vice versa.

The third issue relates to heterogeneity of interest rates for borrowers who initially face non-positive incentives. In the theoretical model, borrowers who initially start out with non-positive incentives only had interest rates which were below the market rate. This is not necessarily the case in practice, which might have implications for the magnitude of the interest rate pass-through. Hence, I impose the following assumption:

Assumption 4.2. *For all* $j_t \in [\omega_t^p, j_0^*]$, *I define* $i_t(j_t) = \rho_t^s i_t^m$, *for* $\rho_t^s > 0$.

Assumption 4.2 states that the interest rates on borrowers who initially face non-positive incentives are proportional to the market rate with the homogenous multiplier ρ_t^s. Both the assumption 4.1 and 4.2 disregard household-specific variation in the data, and ρ_t^p and ρ_t^s can be viewed as exogenously given. As a result, ρ_t^p and ρ_t^s are common across households and can hence be seen as parameters to be estimated.

The fourth issue relates to parameter constancy. As a starting point, I assume parameter constancy throughout the empirical analysis. Parameter constancy corresponds to a special case, where new debt shares, refinanced debt shares, and repaid debt shares are constant over time. In the sections 4.3.4 and 4.3, I examine how the empirical results change when allowing for time-dependent coefficients, which correspond to the theoretical case of endogenous refinancing incentives used in my model.

The fifth issue relates to the endogeneity of the market rate. In general, market rates on fixed-rate mortgages are endogenously determined by the term structure of the underlying bonds. Due to the prepayment option on callable bonds the market rate is comprised by expected future risk-free short-term rates, market risk premiums and the prepayment spread.[8] Let r_t^f be the expectation structure of short-term risk-free rates and ψ_t market risk premiums on fixed-rate mortgage bonds. In a dynamic setting, the term structure on a fixed-rate mortgage can be written as follows:[9]

$$\Delta i_t^m = \Upsilon_t + \Omega^m (i_{t-1}^m - r_{t-1}^f - \psi_{t-1} - \mu^m) + \varepsilon_t^m, \tag{20}$$

[8] Expected risk-free short-term rates and market risk premium are well-known determinants of traditional long-term bonds, while the prepayment spread is only a factor due to the prepayment option. Hence, the prepayment spread captures the additional return, measured in yield terms, an investor requires to be compensated for cash flows being highly uncertain, as borrowers have the option of redeeming whenever they find it appropriate. The value of the prepayment spread cannot be negative, as the borrower is not under any obligation to exercise the embedded option.

[9] See appendix A.2 for details. Equation (20) relaxes the instantaneous response of a shock to monetary policy rates and instead introduces adjustment dynamics, which from an empirical perspective makes sense when aggregating individual bond data on daily observations to compounded data on a monthly basis. This reflects the fact that the aggregation may blur the very precise information about current effects between the variables.

where Ω^m is the speed at which a monetary policy shock is transmitted into the market rate after accounting for short-run effects that are expressed by Υ_t. μ^m reflects the magnitude of the prepayment spread, and ε_t^m is Gaussian innovations with zero mean and variance σ^2.

Based on these considerations, the interest rate pass-through on fixed-rate mortgages to be estimated is:[10]

$$\Delta i_t^h = \underbrace{\Omega^n \Delta i_t^m}_{\text{current effect}} + \underbrace{(\Omega^n + \tilde{\Omega}^r + \lambda\tilde{\Omega}^a + \tilde{\Omega}^p)}_{\text{speed of adjustment}} \times \underbrace{\left[i_{t-1}^h - \left(\frac{\Omega^n - \lambda\bar{\Omega}^s \rho^s}{\Omega^n + \Omega^p(\rho^p - 1) - \lambda\bar{\Omega}^s} \right) i_{t-1}^m \right]}_{\text{long-run pass-through}}, \tag{21}$$

where $\bar{\Omega}^s$ measures the aggregate debt share from borrowers who initially have non-positive refinancing incentives. Current effects enter the regression equation because changes in the market rate can have immediate effects on the average interest rates via debt raised by new borrowers.[11] The dynamic system comprised by equations (20) and (21) delivers the testable empirical framework.

The empirical model holds the testable predictions:

- One-to-one long-run interest rate pass-through if $\rho^p = 1$ and $\bar{\Omega}^s = 0$.
- One-to-one long-run interest rate pass-through if $\rho^p = 1$ and $\rho^s = 1$
- Otherwise, a less than one-to-one long-run interest rate pass-through.

The intuition of the testable predictions follows the arguments in the previous section. A less than one-to-one pass-through is empirically relevant in cases where incentives are structurally lower over a longer period. This includes an environment of increasing interest rates or in periods in the wake of increasing interest rates, for example, reflecting that a larger fraction of homeowners face market rates that are above or close to their existing interest rates during these periods.

4.2 The empirical model

Consider a data vector x_t and the identity $x_t = x_{t-1} + \Delta x_t$. Hence, the system comprised by equations (21) and (20) can be written as a special case of the cointegrated VAR model with k lags conditional on time-independent coefficients:

$$\Delta x_t = \Gamma_1 \Delta x_{t-1} + \Gamma_2 \Delta x_{t-2} + \cdots + \Gamma_{k-1} \Delta x_{t-(k-1)} + \alpha\beta' \tilde{x}_{t-1} + \mu_0 + \varepsilon_t \tag{22}$$

where $\tilde{x}_{t-1} = (x_{t-1}, 1)'$ is of dimension 5×1 , α is of dimension $4 \times r$ and β is of dimension $5 \times r$, the short-run parameters $\Gamma_1, ..., \Gamma_{k-1}$ are 4×4 matrices, and ε_t is a 4×1 sequence of independent Gaussian innovations with zero mean and the covariance matrix $\Sigma > 0$. The constant in the cointegrated space controls for both the level of the prepayment spread and the potential

[10] For details see appendix A.1.

[11] Recall that $i_t^m = \Delta i_t^m + i_{t-1}^m$.

no response mark-up. If the levels of x_t are cointegrated with r long-run relations, $\Pi = \alpha\beta'$ must have reduced rank (Johansen (1996)). The cointegration rank, denoted by r, divides data into r relations, in which the adjustment to equilibrium takes place, and $4 \times r$ common stochastic trends. Hence, the choice of rank will be very influential on whether the subsequent econometric analysis coincides with the theoretical model. According to the theoretical specifications in (21) and (20), the cointegration rank should be $r = 2$ with the following long-run restrictions:

$$\alpha = \begin{pmatrix} \Omega^n\Omega^m & \Omega^n + \tilde{\Omega}^r + \tilde{\Omega}^p + \lambda\tilde{\Omega}^a \\ \Omega^m & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}, \quad \beta' = \begin{pmatrix} 0 & 1 & -1 & -1 & -\mu^m \\ 1 & -\frac{\Omega^n - \lambda\bar{\Omega}^s\rho^s}{\Omega^n + \Omega^p(\rho^p - 1) - \lambda\bar{\Omega}^s} & 0 & 0 & 0 \end{pmatrix} \tag{23}$$

where $\tilde{\Omega}^r = (1+\kappa)\Omega^r$. The elements of the α-matrix are known as the adjustment coefficients (sometimes referred to the speed of adjustment) and capture how the variables will react to a deviation from the equilibrium relationships $\beta'\tilde{x}_t$. For instance, the compound coefficient $\Omega^n + \tilde{\Omega}^r + \tilde{\Omega}^p + \lambda\tilde{\Omega}^a$ measures the proportion of the deviation from the equilibrium $\beta_2'\tilde{x}_t$ the variable i_t^h is eliminating each month after short-run effects. The two zero rows in α mean that both expected monetary policy rates and market risk premiums are not, according to theory, expected to respond to disequilibria. It means that the variables are expected to be weakly exogenous variables that do not respond to other variables in the system.

The elements of the β matrix are the coefficients that determine the long-run relationships of the variables. In this empirical application, the β coefficients can be interpreted as the magnitudes at which the different variables affect the pass-throughs to i_t^h and i_t^m, respectively. Both the α and β coefficients are thus very informative in the assessment of the theoretical model.

To clarify how the theory can formally be tested within the cointegrated VAR framework, consider an expansionary exogenous shock to expected monetary policy rates, corresponding to $\varepsilon_t^{r^f} < 0$ in (22). This will immediately lower the market rate through short-run effects and potentially the average rate, depending on the magnitude of current effects, which in turn will gradually transmit into a lower average rate through the entry and refinancing transmission channels (and perhaps the amortization and the exit channels depending on the direction they push the average interest rate). The adjustment will continue to occur as long as new borrowers and active borrowers can take advantage of entering and refinancing, respectively, to lower market rates. In isolation, this will feed into a stationary relationship, where borrowers cannot finance at rates different from the average. The stationary relationship spanned by i_t^h and i_t^m thus expresses the long-run pass-through.

4.3 Empirical analysis using aggregated data

The first part of the empirical analysis which deals with estimating the macroeconomic response on homeowners' fixed-rate mortgages rates is divided into two steps. The first stage focuses on (i) the formulation of a well-specified unrestricted cointegrated VAR model that includes misspecification analysis and determination of the cointegration rank; and (ii) tests the joint restrictions imposed by the theoretical model and identifies the full short-run and long-run structure of the model. The objective of identifying the short-run structure is to obtain shocks that are causally meaningful and can be economically interpreted, while the objective of identifying the long-run structure is to determine how the responses to the variables of those shocks are adjusted towards their long-run equilibria. The second step deals with parameter stability, which is highly relevant for assessing (i) the validity of the empirical model and (ii) how the pass-through reacts to different events, including refinancing booms and different interest rate regimes. Two types of recursive estimations are performed: forward-recursive estimations and rolling-windows estimations.

4.3.1 Data

The dataset covers four main variables, which can be summarized in the following data vector $x_t = (i_t^h : i_t^m : r_t^f : OAS_t)$, where i_t^h is the average interest rate on fixed-rate mortgages, i_t^m is the market rate on fixed-rate mortgages, r_t^f measures expected monetary policy rates, and OAS_t is the option-adjusted spread on i_t^m. Data is based on monthly observations, and the sample covers the period $2008(1) - 2020(7)$.[12]

The average interest rate is based on 30-year fixed-rate mortgages which constitute the largest share of fixed-rate mortgages in Denmark, accounting for approximately 80 per cent of total outstanding. i_t^h is constructed by a weighted average on each household interest rate. The development in the average interest rate is shown in the upper panel of figure 7 and depends on the composition of debt across coupon rates shown in the bottom panel of figure 7. Fees are not included in the data measurement since I am interested in measuring the "clean effect" of the individuals borrowers' response to monetary policy. The inclusion of fees would blur the empirical results, as it would not be clear whether the identified effect on i_t^h was caused by a monetary policy shock, for example, or simply by changed fees.

The market rate is based on a series of 30-year fixed-rate benchmark bonds. A benchmark bond is a newly issued bond with a price below par value, allowing borrowers, as per agreement with their mortgage bank, to raise or refinance mortgage debt to the interest rate on the bond. When the price of the bond exceeds par value (or declines below a price of 95-96), it can no longer be characterized as a benchmark bond. Consequently, the benchmark series will be replaced by a new benchmark bond with a price below but sufficiently close to par value. Therefore, the market rate is by construction a compound series of rates on 30-year fixed-rate mortgage bonds, where each individual bond was open to financing at the time it was characterized as a benchmark bond.[13] The market rate on 30-year fixed-rate mortgages is depicted in figure 8.

In general, constructing a measure that exclusively reflects expectations of future monetary policy rates is complicated. A well-known method is to use forward rates on Overnight Index Swaps

[12] Data on i_t^h is based on monthly observations, while the data on i_t^m, r_t^f and OAS_t is based on daily observations and thus aggregated to monthly observations by simple averages.

[13] I have used Finance Denmark's definition of benchmark bonds.

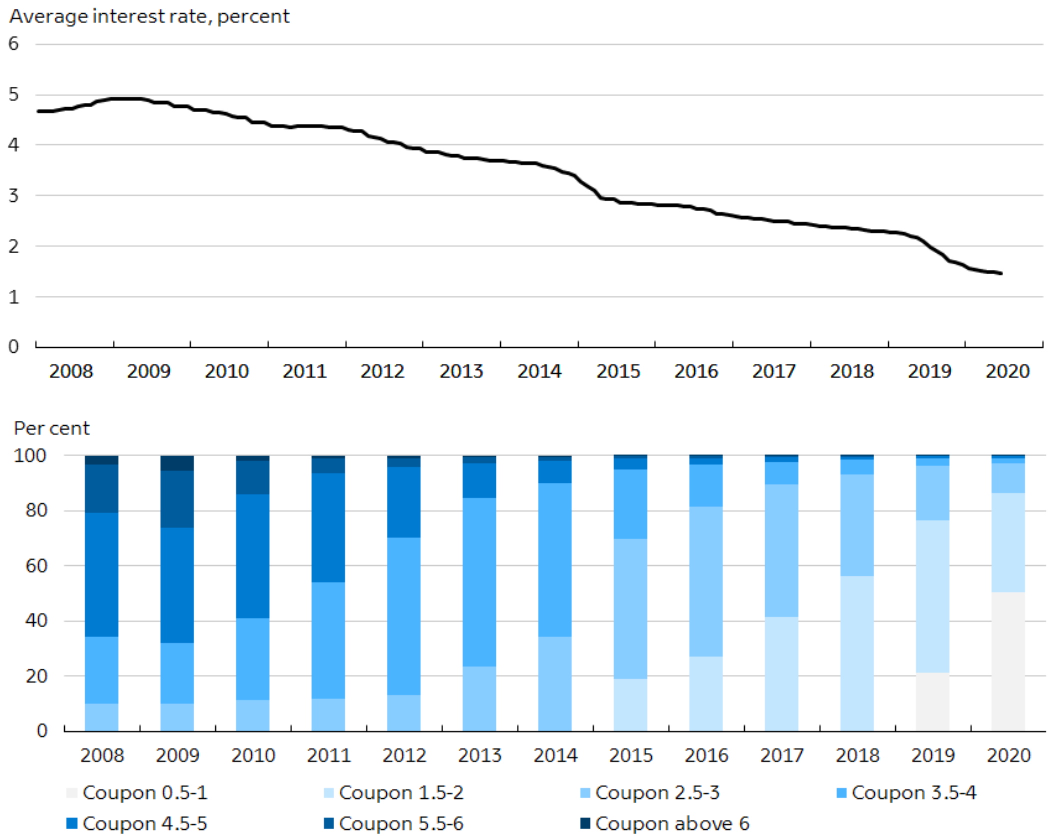


Figure 7: *Development in the average interest rate of 30-year fixed-rate mortgages (upper panel). Composition of coupon rates on 30-year fixed-rate mortgages (bottom panel).*
Source: Danmarks Nationalbank, Refinitiv Eikon and own calculations.

(OIS) that can be interpreted as the market participants' implied expectations of future policy rates.[14] One caveat is, however, that forward rates on OIS swaps also contain term premiums that may blur the actual expectation structure for money market rates. Nevertheless, term premiums on OIS forward rates in euro area have been low and have fluctuated very little since the financial crisis ((McCoy (2019)).[15]

In Denmark, the OIS rate is known as the Cita-swap rate and is based on daily unsecured tomorrow/next money market rates. To construct a single variable that captures the entire forward structure, I calculate based on 1-month Cita-swap forward rates a swap rate that matches the duration of the benchmark bond at every date. One limitation is, however, that data on Cita-swap

[14]Overnight Index Swaps are financial contracts to exchange a payment corresponding to the difference between a fixed rate (which is the OIS rate) and a daily money market rate at the end of the contract.

[15]The sole objective of monetary policy in Denmark is to maintain the fixed exchange rate between the Danish krone and the euro. Hence, Danish monetary policy rates closely track those of the euro area in absence of foreign exchange market pressures, and monetary policy shocks in the euro area is typically transmitted by an one-to-one-factor to Danish policy rates. As a result, interest rates on Danish securities respond similarly to monetary policy actions by the ECB as European bonds (Autrup and Jensen 2021)).

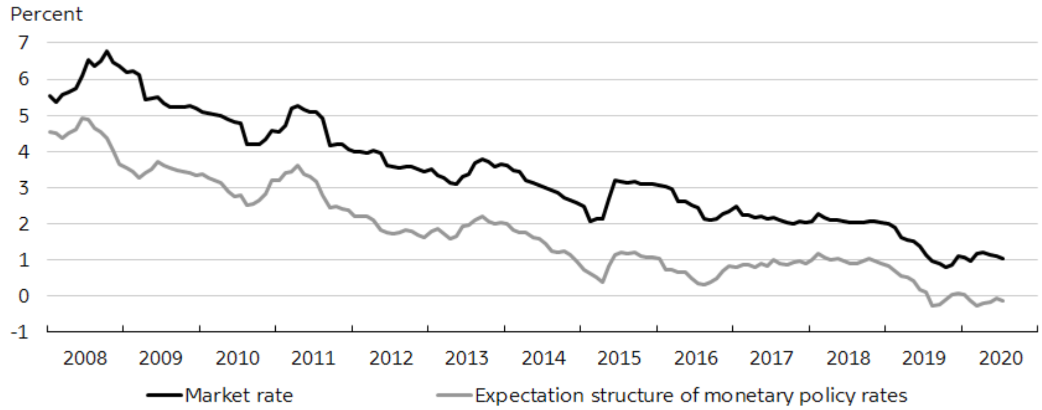


Figure 8: *Development in the market rate on 30-year fixed-rate mortgages and the expectation structure of monetary policy rates.*
Source: Refinitiv Eikon, Rio Scanrate and own calculations.

rates is only available from the end of 2013. To deal with that, I expand the sample by applying the same methodology on forward rates on Eonia-swap before the end of 2013 and add it to the remaining sub-sample after adjusting for differences in the monetary policy spread to the euro area. That delivers a proxy of expected monetary policy rates in Denmark due to the fixed exchange rate regime. The development in r_t^f is shown in figure 8.

The option-adjusted spread (OAS) measures the additional yield investors require for buying the cash flows of a callable bond compared to holding a portfolio of all short-term rates adjusted for the value of the option. OAS can thus be interpreted as credit and liquidity premiums or omitted prepayment risk factors. Theoretically, OAS equals the difference between the theoretical price and the observed market price, transformed into an interest rate differential. Details are presented in appendix A.2.

Developments in OAS are shown in figure 9. The level of OAS has generally been limited over time, reflecting the fact that credit and liquidity risks on Danish mortgages are low in normal times, mainly due to high credit quality (AAA rating) of Danish mortgages and a highly liquid market. Despite low OAS in normal times, substantial OAS expansions have occurred during short periods of financial distress.[16]

[16]Unconventional monetary policy actions have also affected OAS developments. In general, asset purchase programs (APP) have become important monetary policy tools, which has led to spillovers to the global bond market, including the Danish bond markets through ECB APP. This channel is known as the portfolio rebalancing channel and can affect credit, liquidity, term and prepayment premiums through cross-country spillovers (Krishnamurthy and Vissing-Jørgensen (2011)). The gradual narrowing of OAS since 2015-2016 can reflect spillovers from ECB implementation of quantitative easing in March 2015 (Autrup and Jensen (2021)), where ECB started to purchase euro-denominated investment-grade securities issued by euro area governments, banks, agencies and European institutions in the secondary market.

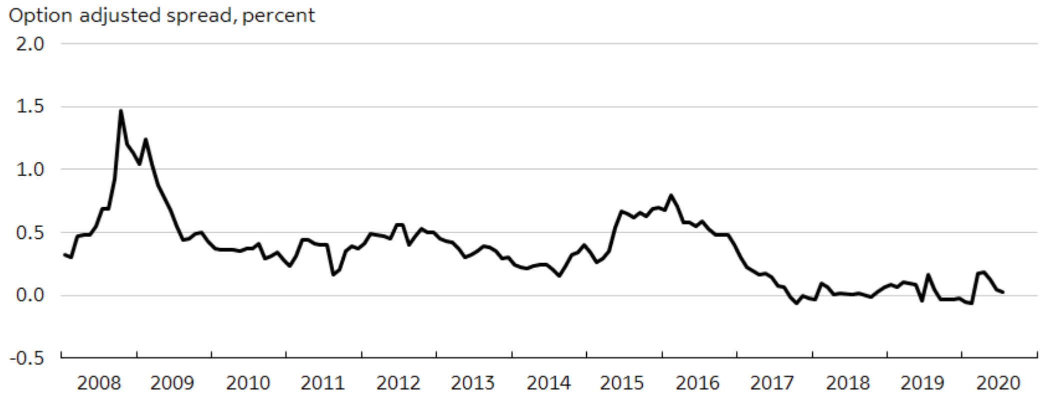


Figure 9: *Developments in the option-adjusted spread on the benchmark series of 30-year mortgage bonds.*
Source: Rio Scanrate and own calculations.

4.3.2 Lag length determination and misspecification analysis

The stochastic variation in the data is assessed by using a combination of the general-to-specific procedure and the information criteria (SC, HQ and AIC). The test statistics indicate that two lags for r_t^f and one lags for the remaining three variables are satisfactory in terms of maximizing the influence of the data, while simultaneously minimizing the complexity of the autoregressive structure.

In general, the choice of lag length is only valid under the assumption of a correctly specified model. Obtaining a well-specified model requires that special events (typically extraordinary shocks) that the model is not intended to explain are taken into account, as such events obscure and bias the estimated coefficients. By following the procedure in Juselius (2006), I observe several innovational outliers that lead to extraordinary, large non-normal shocks in the dynamic autoregressive modelling, initially due to large residuals.[17] To deal with these extraordinary, large non-normal shocks, I introduce the following unrestricted dummies $\mathbb{D}_t = (D^{tr}_{08:10_t}, D^{p}_{10:8_t}, D^{p}_{15:5_t}, D^{tr}_{20:3_t})$, where D^p_t and D^{tr}_t denote permanent and transitory dummies, respectively.[18]

A well-specified unrestricted VAR model is a statistical model for which it seems reasonable,

[17] I observe two extraordinary intervention shocks with permanent effects. These shocks reflect the fact that shifts to the benchmark serie of 30-year mortgage bonds sometimes lead to extraordinary shocks primarily in OAS_t and i_t^m. Such shocks can effectively be taken into account by including permanent blip dummies as they are affected by the VAR dynamics. Furthermore, I observe two transitory innovational outliers in OAS_t in relation to the financial crisis in the fall 2008 and the COVID-19-crisis in March 2020, respectively. Such delayed dynamic effects in the data can effectively be taken into account by adding transitory dummies in periods where the innovational outliers are observed.

[18] Permanent blip dummies have the form $(0, ..., 0, 0, 1, 0, 0, ..., 0)$ and take changes in level-shift in the data into account, while transitory dummies have for example the form $(0, ..., 0, 0, 0.5, -0.5, 0, ...)$. There are still some moderate permanent intervention outliers left in model. However, the remaining outliers lead to few misspecified residuals that do not disturb the autoregressive modelling. Including too many dummy variables would potentially be more costly than beneficial, as dummies in general may absorb explanatory power from the variables.

based on the residual analysis, to assume that the errors do not exhibit autocorrelation, non-normality, or autoregressive conditional heteroscedasticity (ARCH). The misspecification tests of the single equations and the multivariate system suggest that the unrestricted VAR model is well-specified. Details are presented in appendix A.3.

Taking lag length and unrestricted dummy variables into account, the cointegrated VAR model to consider can thus be written as:

$$\Delta x_t = \Gamma_1 \Delta x_{t-1} + \Gamma_2 \Delta x_{t-2} + \alpha\beta' \tilde{x}_{t-1} + \phi \mathbb{D}_t + \mu_0 + \varepsilon_t, \tag{24}$$

The long-run restrictions in equation (23) suggests two long-run stationary relationships. To test whether the data is consistent with a cointegration rank of $r = 2$, I perform several test statistics. Details are shown in appendix A.3. In summary, the statistical model suggests a cointegration rank of $r = 2$, which is in line with the theoretical setup. As a result, I continue with the choice of $r = 2$ and let the identification process determine whether it is consistent with the theoretical specifications. It turns out that the choice of $r = 2$ ensures economically meaningful long-run properties that are consistent with the theory.

4.3.3 Identification

Motivated by the choice of cointegration rank, I will in this subsection go a step deeper in terms of imposing identifying restrictions on the long-run and short-run structures, and assessing whether they are consistent with the theoretical restrictions.

Before imposing identifying restrictions, it is appropriate to examine whether some of the variables are weakly exogenous. The theoretical long-run restrictions state that both r_t^f and OAS_t should be weakly exogenous, as the variables should solely influence the long-run stochastic path of other variables, while at the same time not being affected by them. Both r_t^f and OAS_t can individually and jointly be characterized as weakly exogenous, while the remaining variables, which are i_t^h and i_t^m, can as expected be characterized as endogenous variables.[19]

Long-run identification requires that I impose linear restrictions on β, so that each cointegration vector cannot be composed by linear combinations in the remaining cointegration space.[20] As the ultimate purpose is to identify stable long-run relationships that potentially match the empirical specification, the identification approach is based on a combination of statistics and the presumed theoretical restrictions. The results are reported in table 1, where the t-values shown in the parentheses are based on the asymptotic standard errors.

First, I impose an adequate set of over-identifying restrictions on β that correspond to the

[19]The test statistics are reported in appendix A.3.

[20]Formally, identification of the long-run structure specifies s_i free parameters in the cointegration vector, such that the concentrated model can then be restricted to: $R_{0t} = \sum_{i=1}^{2} \alpha_i \zeta_i' W_i' R_{1t} + \varepsilon_t$, where ζ_i are $s_i \times 1$ vector of unrestricted coefficients and W_i is a known design matrix of dimension $5 \times s_i$, reflecting testable linear hypotheses. The principle of identification is to choose W_i so that $\tilde{\beta}_i$ cannot be composed by linear combinations of the remaining cointegration vector.

	$\mathbb{H}_1$				$\mathbb{H}_2$			
	α_1	α_2	β_1	β_2	α_1	α_2	β_1	β_2
i_t^h	$-\ 0$ $(\cdots)$	-0.0379 (-12.6)	0 $(\cdots)$	1 $(\cdots)$	$-\ 0$ $(\cdots)$	$-\ 0.037$ (-12.5)	0 $(\cdots)$	1 $(\cdots)$
i_t^m	-0.284 $-(5.5)$	0 $(\cdots)$	1 $(\cdots)$	$-\ 0.871$ $-(53.9)$	-0.254 $(-5.9$	0 $(\cdots)$	1 $(\cdots)$	$-\ 0.867$ (-52.4)
OAS_t	$\cdots$	$\cdots$	$-\ 1.28$ $-(12.7)$	0 $(\cdots)$	$\cdots$	$\cdots$	$-\ 1$ $(\cdots)$	0 $(\cdots)$
r_t^f	$\cdots$	$\cdots$	$-\ 1$ $(\cdots)$	0 $(\cdots)$	$\cdots$	$\cdots$	$-\ 1$ $(\cdots)$	0 $\cdots$
1	$\cdots$	$\cdots$	$-\ 1.10$ $-(23.3)$	0 $(\cdots)$	$\cdots$	$\cdots$	$-\ 1.2$ (-34.9)	0 $(\cdots)$
LR statistic	5.06				11.49			
P-value	0.65				0.18			
Distribution	$\chi^2(7)$				$\chi^2(8)$			

Table 1: *Identification of the long-run structure.* t-values are based on asymptotic standard errors and shown in parentheses.

theoretical restrictions in (23) except for a unit restriction on OAS_t. The results are reported under $\mathbb{H}_1$, and the null hypothesis of joint non-stationarity can easily be rejected jointly with a p-value of 0.65, suggesting that the two cointegration vectors can individually and jointly be characterized as long-run equilibria.

From $\mathbb{H}_1$ I also note that the α coefficient to i_t^h in the first cointegration relationship is effectively restricted to zero due to low asymptotic standard errors.[21] The intuition is that current effects on i_t^h are captured by short-run effects rather than error-correction dynamics.

Subsequently, I test whether the β coefficient to OAS_t can be restricted to unity in line with the theoretical restrictions. The result is reported under $\mathbb{H}_2$, and the system can be statistically accepted with a p-value of 0.18. Despite the explanatory power in terms of stationary declines when moving from $\mathbb{H}_1$ to $\mathbb{H}_2$, the hypothesis can still be accepted, and the system is thus robust to the additional over-identifying restriction. However, to maintain an empirical model that has very convincing statistical properties and is consistent with the economic theory, I will continue with the empirically identified model under $\mathbb{H}_1$, while having in mind that the model under $\mathbb{H}_2$ is still statistically and economically relevant. As a robustness check I have identified the long-run structure under the hypotheses $\mathbb{H}_1$-$\mathbb{H}_2$ without the presence of dummies variables. The results are presented in table A.4. in appendix A.3, and both hypotheses are robust to these changes.

So far, the only testable theoretical restrictions that has not been empirically scrutinized is the β coefficient to the long-run interest rate pass-through on fixed-rate mortgages. To test whether the statistical model is consistent with one-to-one pass-through, I impose a unity identifying restriction on the coefficient to i_t^m under $\mathbb{H}_1$. The additional over-identifying restriction is clearly rejected with a p-value of 0.00. This suggests that the long-run interest rate pass-through is close to but

[21]The null hypothesis of identifying restrictions on α is given by $\alpha = (A_1\iota_1 : A_2\iota_2)$, where A_i is a $2 \times s_i$ vector, ι_i is of dimension $s_i \times 1$, and s_i captures the numbers of non-zero α-coefficient in column i. Conditional on the identified structure of $\tilde{\beta}$, the concentrated model under the null hypothesis is thus: $R_{0t} = \sum_{i=1}^{2} A_i\iota_i\zeta_i' W_i R_{1t} + \varepsilon_t$, where the LR test procedure can then be derived by partitioning the system and subsequently solving the standard eigenvalue problem, see Johansen (1996).

significantly below unity.

Identification of the short-run structure tries to deal with over-parametrization and residual correlation, and hence allows me to economically determine whether a monetary policy shock causally affects the average interest rate. As it is standard in the literature, I use the Cholesky decomposition to identify structural shocks. Consequently, I impose short-run identifying restrictions via the contemporaneous matrix $A_0 = \hat{\Sigma}^{(-1/2)}$, which is an upper triangular matrix. Conditional on the identified long-run structure, I can thus rewrite (24) as follows:

$$A_0 \Delta x_t = A_0 \Gamma_1 \Delta x_{t-1} + A_0 \Gamma_2 \Delta x_{t-2} + A_0 \alpha (\hat{\beta})' \tilde{x}_{t-1} + A_0 \phi \mathbb{D}_t + A_0 \mu + A_0 \varepsilon_t, \tag{25}$$

where $\hat{\beta}$ is the identified long-run structure. The causal ordering follows the identification scheme

$$\Delta r_t^f \rightarrow \Delta OAS_t \rightarrow \Delta i_t^m \rightarrow \Delta i_t^h, \tag{26}$$

The ordering is rooted in the theoretical specifications that delivered a representation of the causal ordering: r_t^f, OAS_t and i_t^m are expected to have current effects on i_t^h, and r_t^f and OAS_t are expected to have current effects on i_t^m and so forth. Empirically, the identified long-run structure also supports the ordering, as both r_t^f and OAS_t are weakly exogenous and error correction towards the long-run pass-through was is taking place through i_t^h. The only subset of the ordering that is not straightforward is the order between r_t^f and OAS_t. Nevertheless, monetary policy can, according to the signaling channel, at least from a theoretical perspective affect risk premiums, thus favoring the economic relevance of the ordering in (26).[22]

The identified short-run structure is presented in appendix A.3. The estimated coefficients are also highly intuitive and correspond to the theoretical restrictions. Neither Δr_t^f and ΔOAS_t depend upon any other endogenous variables, reconfirming the validity of the causal ordering. As expected, contemporaneous effects on risk premiums and expected monetary policy rates have a major impact on Δi_t^m and minor impact on Δi_t^h. The magnitude of the current effects on i_t^m differs slightly from the term structure representation, which again covers that data is aggregated from daily to monthly observations. Finally, the over-identified short-run structure also reveals that only i_t^m and i_t^h error-correct to their respective long-run equilibria with almost identical magnitudes and significances as under the identification of the long-run structure.

To evaluate the overall effect of a shock to expected monetary policy rates, I calculate the impulse response on i_t^h from one unit shock of r_t^f based on the identified model. The results are shown in the left panel of figure 10. The impulse response on i_t^h reveals that a shock to expected monetary policy rates is very persistent and has significant impact on homeowners' fixed-rate mortgage rates several years after the realization of the shock. This is due to the fact that the shock is transmitted at a speed of 3.7 per cent monthly after current effects to the long-run equilibrium. Economically,

[22] In appendix A.3, I document that the result is robust to changing the variables r_t^f and OAS_t in the causal ordering.

it takes around 5 years before 90 per cent of a 1 percentage point shock has been transmitted to homeowners' interest rates.

The speed of adjustment matches the magnitude of the option-adjusted duration of fixed-rate mortgage bonds. Option-adjusted duration measures the probability that the underlying bond will be prepaid and can hence be interpreted as the expected remaining maturity of the bond. During the sample period the option-adjusted duration has been 5-6 years on average, as shown in the right panel of figure 10. Intuitively, this means that the ex-ante expected lifetime on a benchmark bond of a 30-year fixed-rate mortgage has been around 5-6 years between 2008 and 2020. The coherence between the estimates and the option-adjusted duration supports the plausibility of my empirical results.

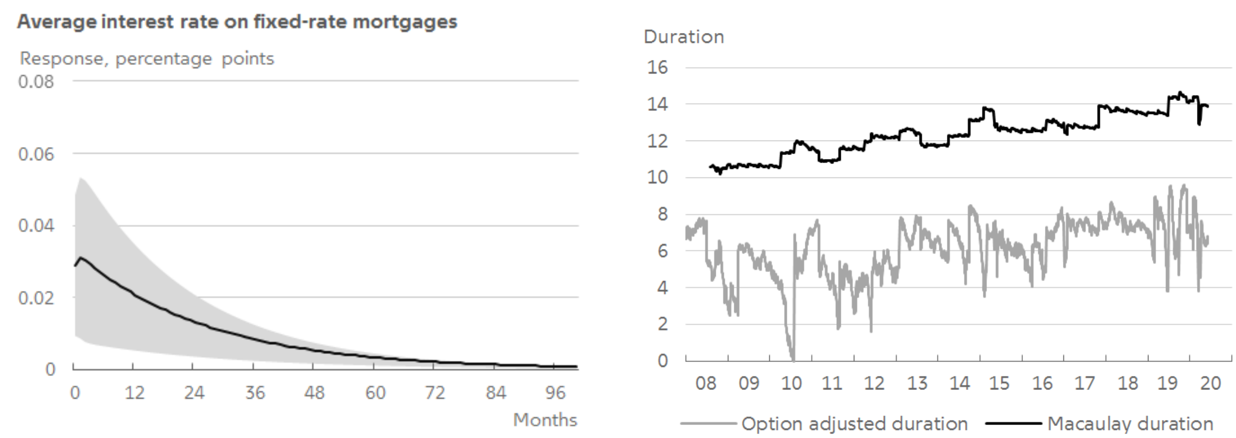


Figure 10: *Impulse response of average interest rate on 30-year fixed-rate mortgages (left panel) and duration on 30-year fixed mortgage bonds (right panel).*
Note: Left panel: Individual response on average fixed-rate mortgage rates from a 1 percentage point shock to expected monetary policy rates. Light grey area marks 95 percent confidence intervals. Right panel: Macaulay duration and option-adjusted duration of 30-year fixed-rate benchmark bonds.
Source: Rio Scanrate and own calculations.

4.3.4 Long-run stability and implications of refinancing booms

The endogenous nature of refinancing implies that the estimated coefficients may suffer from instability during the sample period. In this subsection, I will present forward-recursive tests and rolling-windows estimations of the full model and the individual coefficients of the identified long-run structure. In contrast to the full-sample estimation that provides estimates based on the maximum number of observations, the idea of recursive analysis is to spot potential changes and structural breaks in the estimated coefficients.

As a starting point, I consider a recursive test of the full long-run model, here clarified by the recursively calculated log likelihood test (Hansen og Johansen (1999)). Intuitively, the test compared the influence of data of the sub-samples and the baseline sample, adjusted for the relative length of the baseline sample and the number of parameters. The test statistics are shown under $\mathbb{H}_1$

in appendix A.4, where the 95 percent quantile is characterized by the horizontal dashed line. The graph clearly indicates that constancy of the long-run structure can jointly be accepted, recalling that variability in the beginning of the sub-sample is quite prevalent when the baseline sample is relatively small compared to the number of parameters.

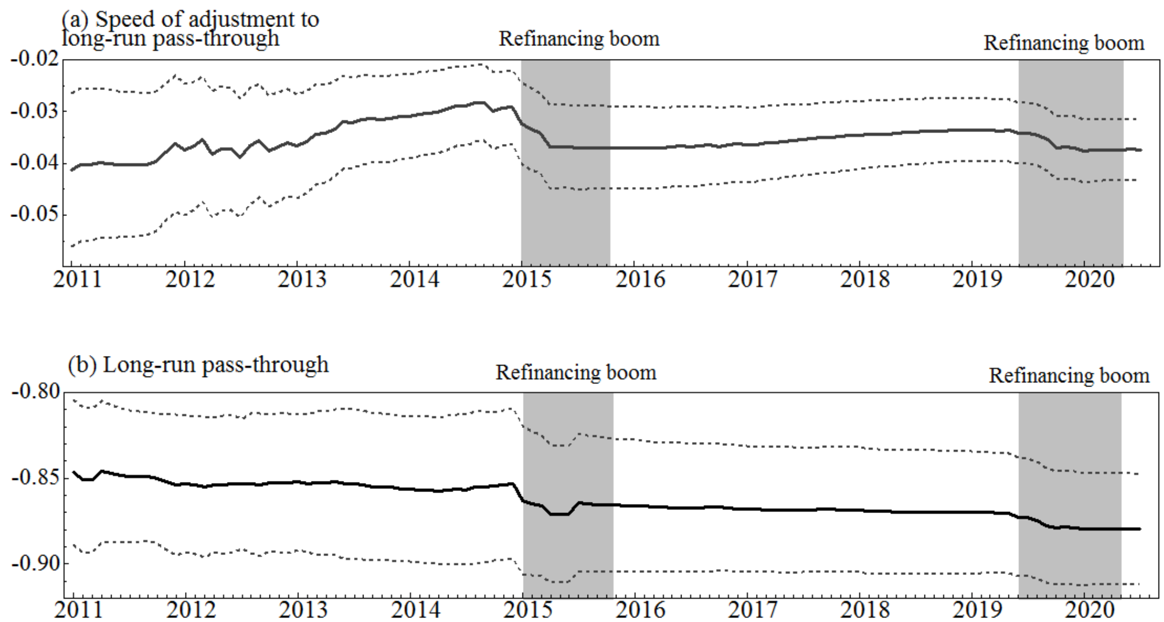


Figure 11: *Forward-recursive tests*
Note: The figure depicts the results of the model for various sample lengths, holding the initial year fixed. The dashed lines indicate the 95 percent confidence bands. The recursively calculated coefficients of $\alpha(t_1)$ and $\beta(t_1)$ to the pass-through relation are based on the sub-samples $t_1 = 2010(12), ..., 2020(7)$. In each baseline sample, all short-run parameters are fixed at their full-sample estimates. The recursive graphs are produced by comparing the space of the individual full-sample estimate with the accompanying spanned estimates of the respective sub-samples, see Hansen and Johansen (1999) for details.

The forward-recursive estimates of the speed-of-adjustment coefficient and the coefficient to the long-run pass-through are shown in figure 11.[23] Generally, the graphs illustrate that the estimates have been remarkably stable over the considered sample, and the narrowing of the confidence bands reflects increasing information on the long-run parameters. It is clear that both estimates increase slightly and temporarily during the period 2014-2015 and in 2019, which were periods characterized by extraordinary refinancing activity.

Forward-recursive estimation has an inherent tendency to misjudge potential changes in the parameters towards the end of the full sample, as it adheres to the starting-point observations. In order to more accurately assess the timing of changes in the coefficients, specifically related to refinancing booms and more structural changes in interest rate developments, I also perform rolling-window estimations. The estimations are showed in figure 12. Despite the modest variation in the estimated coefficient parameter constancy still cannot be rejected.

[23]The individual α and β estimates to the remaining cointegration relationship are shown in appendix A.4.

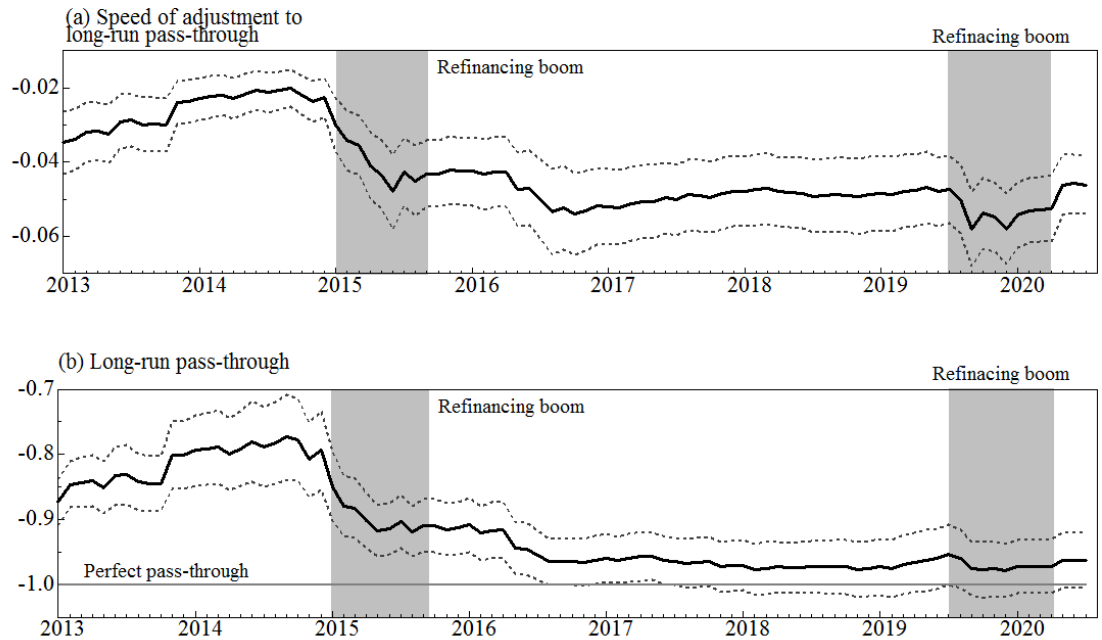


Figure 12: *Rolling-window estimation*
Note: The figure illustrates the results of the 5-year rolling-window estimation based on the concentrated model. The 5-year samples are: $(2008(1) - 2013(1) - (2005(7) - 2020(7))$, and the dashed lines indicate the 95% confidence bands. In each sample, all short-run parameters are fixed at their full-sample estimates. The start and end years of the recursive sample are not decisive for the results and the size of the rolling window is chosen adequately large in order to prevent an excessive drop in the power of the estimates.

In contrast to forward-recursive tests, the coefficient to the interest rate pass-through seems to jump more permanently around the refinancing boom in 2014-2015. According to the theoretical model, the long-run pass-through was affected by the fraction of borrowers who did not initially have incentives to refinance in the presence of an expansionary shock to the market rate. In the period between 2008 and 2012, the market rate was above or close to the average interest rate, and refinancing incentives might thus have been structurally lower. The reason is that the interest rate environment in the years up to the financial crisis were characterized by increasing rates, which affected homeowners' incentives to refinance in the years after the financial crisis. As a result, the share of borrowers with positive incentives might have been lower at the beginning of the sample. Since 2012, the actual market rate has been substantially below the average rate, which has contributed to spurring refinancing incentives and thus increasing the interest rate pass-through close to unity.

The lower pass-through in the beginning of the sample contributes to shedding light on the asymmetric effects in the interest-rate pass-through to fixed-rate mortgages. The findings are relevant for the literature on the refinancing channel of monetary policy transmission (Agarwal et al. 2015, Auclert 2019, Beraja et al. 2019, Di Maggio et al. 2017). In general, expansionary monetary policy stimulates the economy in part by lowering rates, which in turn increases household consumption. However, in a fixed-rate mortgage system, lower mortgage rates relieve the budgets only of households that refinance their mortgages. Such budget relief depends on refinancing incentives that rely on interest rate developments. When market rates increase, incentives decline, which leads to a larger share of refinancing failures, even long after market rates have stopped increasing. This slows down the interest rate pass-through and thus delays the transmission of monetary policy to household consumption. As discussed above, refinancing incentives are potentially also influenced by the possibility of a debt reduction when refinancing to higher interest rates. This effect might contribute to mitigating the slowdown in the interest rate pass-through during environments with increasing interest rates.

The estimation of the speed-of-adjustment coefficient was also permanently enhanced around the refinancing boom in 2014-2015 and temporarily during the refinancing boom in 2019. According to the theoretical model, sufficient declines in the market rate spur refinancing and thus increase $\tilde{\Omega}_t^r$ substantially compared to a situation without a monetary policy shock. The market rate was hit by extraordinary declines both at the beginning of 2015 and during 2019, for example, and may explain the increase in the speed-of-adjustment coefficient. In the next section, I document that the increase were indeed driven by extraordinary refinancing activity.

4.4 Evidence from administrative data on danish households

A weakness of the cointegrated VAR model is that it cannot disentangle the aggregate estimates into their underlying determinants. As a result, the model cannot shed light on fundamental questions, such as the role of the refinancing in the monetary transmission to homeowners' fixed-rate mortgage rates.

To answer such questions, I complement the empirical analysis with mortgage data at household level. The data allows me to construct measures of the single parameters in the theoretical model, and thus makes it possible to decompose the contributions from the individual channels to the aggregate response. Simultaneously, the mortgage data allows me to compare the predictions in the theoretical model with the estimates from the empirical model, and through that evaluate the accuracy of the model.

I obtain the household-specific mortgage data from Danmarks Nationalbank, which in turn obtains the data from mortgage banks through the Association of Danish Mortgage Banks and the Danish Mortgage Banks' Federation. The data is annual for the period between 2010 and 2018, and covers all mortgage banks and all mortgages in Denmark. I have identification numbers for mortgages and information on mortgage terms (principal, outstanding principal, coupon rates, annual fees, maturity, issue date, etc.).

The data is shown in table 2 and covers descriptive statistics for all single parameters in the theoretical model. For example, the debt share from refinancing, i.e. the parameter Ω_t^r, means that 13 per cent on average of all fixed-rate mortgage debt in a given year is refinanced by existing borrowers.

	Parameter	mean	min	max
Debt share new lending	Ω_t^n	0.133	0.070	0.201
Debt share refinancing	Ω_t^r	0.130	0.038	0.229
Debt share remain	Ω_t^a	0.736	0.570	0.893
Debt share remain contribution speed of adjustment (monthly)	$\tilde{\Omega}_t^a$	0.978	0.964	0.991
Debt share repayments	Ω_t^p	0.109	0.064	0.203
Debt share repayiments contribution speed of adjustment (monthly)	$\tilde{\Omega}_t^p$	0.009	0.006	0.017
Amortisation rate	λ_t	0.038	0.029	0.049
Changed borrowing refinancing	κ_t	1.080	0.996	1.161
Debt shares no incentives	$\bar{\Omega}_t^s$	0.584	0.414	0.849
Interest rates relative to market rate no incentives	ρ_t^s	1.066	0.996	1.122
Interest rates relative to average interest rate repayments	ρ_t^p	1.113	1.042	1.197
Speed of adjustment (monthly)	$\Omega_t^n + \tilde{\Omega}_t^r + \tilde{\Omega}_t^p + \lambda_t \tilde{\Omega}_t^a$	0.034	0.021	0.046
Interest rate pass-through	$\frac{\Omega_t^n - \lambda_t \bar{\Omega}_t^s \rho_t^s}{\Omega_t^n + \Omega_t^p(\rho_t^p - 1) - \lambda_t \bar{\Omega}_t^s}$	0.888	0.737	0.973

Table 2: *Evidence from household-specific mortgage data in the period between 2010 and 2018.*
Note: The parameter are calculated using all mortgages taken by households in Denmark with a single fixed rate mortgage in the period 2010-2018. The fraction of all fixed-rate mortgage debt with negative refinancing incentives in a given year is calculated using the method in Agarwal, Driscoll, and Laibson (2013).
Source: Own calculations based on register data from Statistics Denmark.

According to the empirical specification (i.e. equation (20)), the speed of adjustment was made up of $\Omega_t^n + \tilde{\Omega}_t^r + \tilde{\Omega}_t^p + \lambda_t \tilde{\Omega}_t^a$. When calculating the speed-of-adjustment coefficients, I transform the contribution from the underlying channels into a monthly scale, so the household data is comparable to the predictions from the empirical model. The household data suggests that the speed of adjustment is 3.4 percent monthly on average in the period between 2010 and 2018. The estimated value is slightly below the estimate from the empirical model that is 3.7 percent on a monthly basis. The slight difference in the estimates can be due to several reasons. First, the sample periods are not identical, which for example implies that the large refinancing boom in 2019 is not included in the household data. It has tended to reduce the estimate of the speed of adjustment for the household data. Second, the VAR model estimate is based on monthly observations, whereas the household data is based on annual observations transformed into a monthly scale. Third, macroeconomic and microeconomic data is by construction not always directly comparable. Despite these dissimilarities between the data sources, the values are, however, relatively identical, supporting the economic relevance of the empirical results and the applicability of the theoretical model.

Developments in the speed-of-adjustment estimate and its underlying determinants based on household data are reported in panel (a) of figure 13. In general, the estimates are broadly stable over the period, which coheres with the results from the recursive analysis in the previous section. The household data also reveals that speed of adjustment was temporarily higher during the refinancing booms in 2012 and 2014-2015. In addition, the household data documents that the larger speed-of-adjustment estimates during the refinancing booms were driven by the refinancing channel. During the refinancing booms, the household data shows that refinancing approximately doubled the speed at which monetary policy shocks spreads to home owners' mortgages. Economically, this means that the transmission of monetary policy to homeowners' budgets would have been around 40 percent slower without refinancing during these booms.

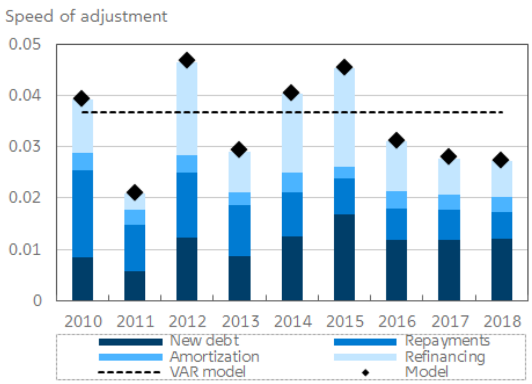
Table 2 also documents that the interest rate pass-through is 0.89 on average and varies between 0.74 and 0.97 during the period 2010-2018. According to panel (b) of figure 13, the long-run pass-through was somewhat lower in the beginning of period, which coincides with the evidence from the recursive analysis in the previous section. The lower pass-through at the beginning of the sample reflects that the debt share of borrowers without refinancing incentives (denoted by the parameter $\bar{\Omega}_t^s$) was higher in that period.[24] The developments in $\bar{\Omega}_t^s$ strengthens the view that refinancing is more common when the incentives to refinance, measured as the interest rate savings relative to the one-off refinancing cost, are sufficiently strong. The household data thus reconfirms that asymmetries in refinancing responses may play a role in the monetary transmission for household financing home debt through fixed-rate mortgage contracts.

4.4.1 Applying the theoretical model

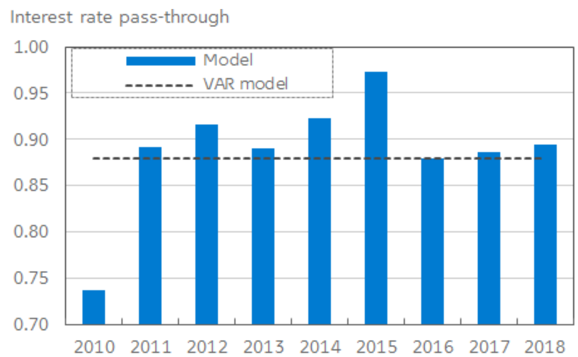
To assess the overall applicability of the theoretical model, I perform model simulation for an initial value of i_0^h given the parameters in table 2 and the observed developments in i_t^m. The simulation of i_t^h is presented in panel (c) of figure 13 for the period between 2010 and 2018 with the contributions from the underlying channels depicted in panel (d). It is clear that the simulated development in i_t^h evolves remarkably close to the observed value of i_t^h. The simulated values of i_t^h are 0.15 percentage points on average above the observed values, which only corresponds to a 4 percent deviation from the data. As a result, the model is quite accurate in timing the observed development in i_t^h.

I can also use the model to explore the importance of refinancing in a hypothetical simulation. As a first exercise, I consider a hypothetical simulation of i_t^h without refinancing of fixed-rate mortgages. Panel (e) of figure 13 depicts the results. Not surprisingly, the absence of refinancing limits the pass-through substantially compared to the baseline case with refinancing. The speed of adjustment is hypothetically reduced by 1.1 percentage points according to the model predictions, corresponding to a 30 percent decline in monetary transmission compared to a situation with refinancing. Another relevant exercise is a hypothetical simulation of i_t^h without refinancing from fixed-rate mortgages to all kind of mortgages, i.e. both fixed-rate, variable-rate, and adjustable-rate mortgages. Panel (f) of figure 13 presents the results of the simulation. In contrast to the case with refinancing of fixed-rate mortgages only, the interest rate pass-through has further slowed down due to a total decline in the speed of adjustment of 1.5 percentage points compared to the baseline case with full refinancing.

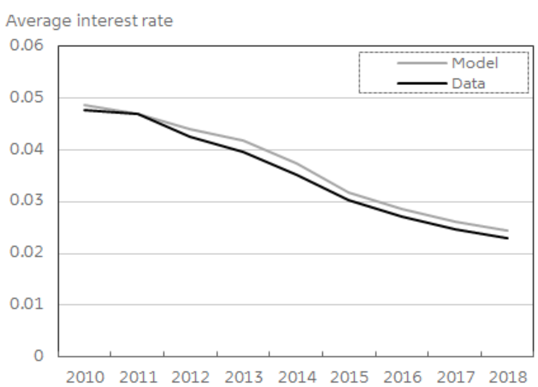
[24] The value of $\bar{\Omega}_t^s$ is constructed by calculating each borrower's interest rate savings if refinancing and the cost of refinancing. The calculation of the optimal refinancing cost follows the methodology in Agarwal, Driscoll, and Laibson (2013), and takes both the fixed cost of refinancing into account and the option value of waiting for further interest-rate declines.



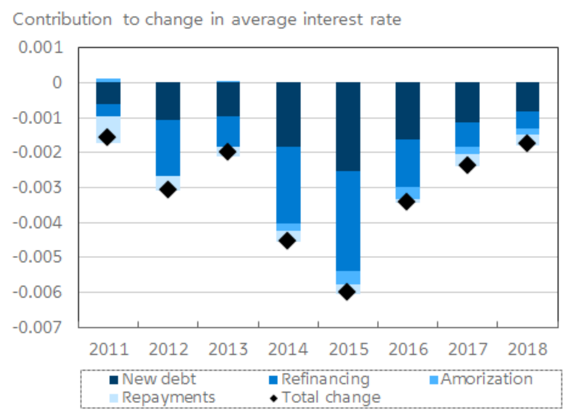
(a) *Speed of adjustment*



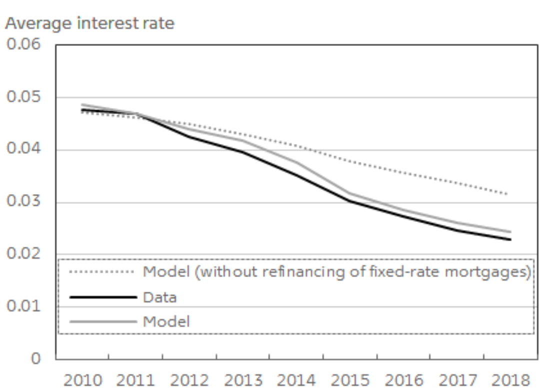
(b) *Interest rate pass-through*



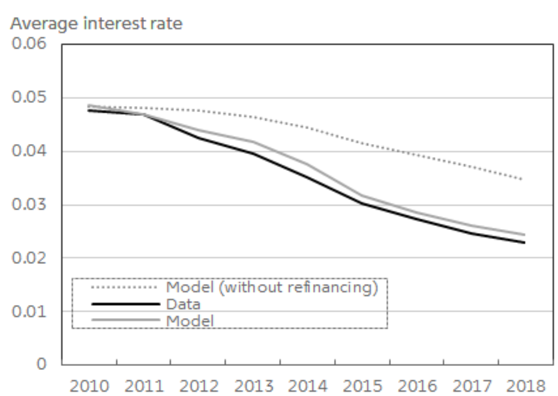
(c) *Simulation baseline model*



(d) *Simulation baseline model*



(e) *Simulation model without refinancing of fixed-rate mortgages*



(f) *Simulation model without refinancing*

Figure 13: *Evidence from household data and simulations of the theorectical model*

Depending on household behavior, the hypothetical simulation indicates that, the slowdown in the interest rate pass-through to fixed-rate mortgages rates can turn out to be sizable if the economy is hit by a prolonged period of increasing interest rates. Depending on the importance of fixed-rate mortgages in total household financing, this can potentially delay the monetary transmission to household consumption.

5 Concluding Remarks

In this paper, I document and theoretically model the interest rate pass-through on fixed-rate mortgages and the role of refinancing behavior in that context. I develop a model based on a state-dependent process that endogenously determines household refinancing decisions as a function of their incentives. The model adds fixed psychological costs to the direct financial costs of refinancing, and thereby decreases the range of borrowers that triggers refinancing. This ensures that households with positive refinancing incentives respond slowly to an interest rate shock. I show that the model is consistent with a dynamic equilibrium in which all households with positive incentives end up refinancing, while households with non-positive incentives refrain from refinancing. The model thus demonstrates that the size of the interest rate shock is decisive for the share of borrowers who consider refinancing, explaining why refinancing typically comes in surges during periods of substantial declines in interest rates. As a result, the model formalises that the endogeneity of refinancing decisions leads to a non-linearity in the interest rate pass-through to fixed-rate mortgages.

I document that the dynamic equilibrium of the model leads to an analytical expression for the long-run interest rate pass-through, which I use to assess the empirical relevance of the model and the contribution of the refinancing channel in that context. I estimate the model using data from Denmark for the period between 2008 and 2020, an environment that is advantageous for studying such types of dynamics due to high prevalence of fixed-rate mortgages in the Danish mortgage system. I estimate the interest rate pass-through via a cointegrated VAR model and complement the results with detailed mortgage data at individual household level. My results suggest that the long-run interest rate pass-through is significantly below unity in the years after the financial crisis and subsequently converges towards a level close to unity. I document that the result is driven by a considerable share of homeowners who initially faced non-positive refinancing incentives in the wake of the financial crisis. This might be a result of the fact that the interest rate environment in the years up to the financial crisis were characterized by increasing rates, which meant that refinancing incentives, measured as the interest rate savings relative to refinancing costs, were not sufficiently strong to trigger refinancing.

I interpret the shift in the pass-through as evidence of asymmetries in the pass-through to homeowners' fixed-rate mortgage rates. My findings are relevant for the literature on the refinancing channel of monetary policy transmission (Agarwal et al. 2015, Auclert 2019, Beraja et al. 2019, Di Maggio et al. 2017). In general, an expansionary monetary policy stimulates the economy in part by lowering rates, which in turn increases household consumption. However, in a fixed-rate mortgage system, lower mortgage rates relieve the budgets only of households that refinance their mortgages. Such budget relief depends on refinancing incentives that rely on interest rate developments. When market rates increase, incentives decline, which leads to more refinancing failures. As a result, refinancing will not take place to the same extent in an environment of increasing interest rates. This slows down the interest rate pass-through and thus delays the transmission of monetary policy to household consumption. My results imply that the effect of a tightening monetary policy on household consumption might be weakened in economies predominated by fixed-rate mortgages.

Most of the observations in this study cover a period with falling or unchanged interest rates. It would be interesting to extend my analysis by using data covering a period with increasing interest rates. Such investigation would allow me to more comprehensively examine the asymmetric effects in the interest rate pass-through to fixed-rate mortgages. Other useful extensions could be to examine how refinancing from fixed-rate mortgages to variable-rate mortgages influences the interest rate pass-through of monetary policy to mortgage rates.

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A Appendix

A.1 Theoretical framework

Proof of propostion 3.1:
The average interest rate is:

$$
\begin{aligned}
i_t^h = &\int_0^m i_t^h(j_t)\varphi_t(j_t)dj = \int_0^{j_{t-1}^*} i_t^h(j_t)\varphi_t(j_t)dj + \\
&\int_{j_{t-1}^*}^{j_{t-1}^*+\phi(m-j_{t-1}^*)} i_t^h(j_t)\varphi_t(j_t)dj + \int_{j_{t-1}^*+\phi(m-j_{t-1}^*)}^m i_t^h(j_t)\varphi_t(j_t)dj \\
= &\int_0^{j_{t-1}^*} i_{t-1}^h(j_{t-1})\Lambda_{(t-1,t)}^d\varphi_{t-1}(j_{t-1})dj + \\
&\int_{j_{t-1}^*}^{j_{t-1}^*+\phi(m-j_{t-1}^*)} i_{t-1}^h(j_{t-1})\Lambda_{(t-1,t)}^d\varphi_{t-1}(j_{t-1})dj + (1+\kappa)\int_{j_{t-1}^*+\phi(m-j_{t-1}^*)}^m i^m\Lambda_{(t-1,t)}^d\varphi_{t-1}(j_{t-1})dj
\end{aligned}
$$

Subsequently, the change in the average interest rate is:

$$
\begin{aligned}
\Delta i_t^h = &i_t^h - i_{t-1}^h \\
= &i_t^h - \left((1+\kappa)\Lambda_{(t-1,t)}^d\Omega_{t-1}^r + \Omega_{t-1}^s\Lambda_{(t-1,t)}^d + \Omega_{t-1}^i\Lambda_{(t-1,t)}^d\right) i_{t-1}^h \\
= &(1+\kappa)\int_{j_{t-1}^*+\phi(m-j_{t-1}^*)}^m i^m\Lambda_{(t-1,t)}^d\varphi_{t-1}(j_{t-1})dj - (1+\kappa)\int_{j_{t-1}^*+\phi(m-j_{t-1}^*)}^m i_{t-1}^h\Lambda_{(t-1,t)}^d\varphi_{t-1}(j_{t-1})dj \\
&+ \int_{j_{t-1}^*}^{j_{t-1}^*+\phi(m-j_{t-1}^*)} i_{t-1}^h(j_{t-1})\Lambda_{(t-1,t)}^d\varphi_{t-1}(j_{t-1})dj - \int_{j_{t-1}^*}^{j_{t-1}^*+\phi(m-j_{t-1}^*)} i_{t-1}^h\Lambda_{(t-1,t)}^d\varphi_{t-1}(j_{t-1})dj \\
&+ \int_0^{j_{t-1}^*} i_{t-1}^h(j_{t-1})\Lambda_{(t-1,t)}^d\varphi_{t-1}(j_{t-1})dj - \int_0^{j_{t-1}^*} i_{t-1}^h\Lambda_{(t-1,t)}^d\varphi_{t-1}(j_{t-1})dj \\
= &(1+\kappa)\int_{j_{t-1}^*+\phi(m-j_{t-1}^*)}^m \left(i^m - i_{t-1}^h\right)\Lambda_{(t-1,t)}^d\varphi_{t-1}(j_{t-1})dj \\
&+ \int_{j_{t-1}^*}^{j_{t-1}^*+\phi(m-j_{t-1}^*)} \left(i_{t-1}^h(j_{t-1}) - i_{t-1}^h\right)\Lambda_{(t-1,t)}^d\varphi_{t-1}(j_{t-1})dj \\
&+ \int_0^{j_{t-1}^*} \left(i_{t-1}^h(j_{t-1}) - i_{t-1}^h\right)\Lambda_{(t-1,t)}^d\varphi_{t-1}(j_{t-1})dj
\end{aligned}
$$

Proof of propostion 3.2:

From equation (9), one gets:

$$\begin{aligned}
\Delta i_t^h =& \int_0^{j_0^*} \left(i_{t-1}^h(j_{t-1}) - i_{t-1}^h\right) \Lambda_{(t-1,t)}^d \varphi_{t-1}(j_{t-1})dj + \int_{j_0^*}^{j_{t-1}^*} \left(i_{t-1}^h(j_{t-1}) - i_{t-1}^h\right) \Lambda_{(t-1,t)}^d \varphi_{t-1}(j_{t-1})dj + \\
& \int_{j_{t-1}^*}^{j_{t-1}^*+\phi(m-j_{t-1}^*)} \left(i_{t-1}^h(j_{t-1}) - i_{t-1}^h\right) \Lambda_{(t-1,t)}^d \varphi_{t-1}(j_{t-1})dj + \\
& (1+\kappa) \int_{j_{t-1}^*+\phi(m-j_{t-1}^*)}^{m} \left(i^m - i_{t-1}^h\right) \Lambda_{(t-1,t)}^d \varphi_{t-1}(j_{t-1})dj, \\
=& \int_0^{j_0^*} i_{t-1}^h(j_{t-1})\Lambda_{(t-1,t)}^d \varphi_{t-1}(j_{t-1})dj + \int_{j_0^*}^{j_{t-1}^*} i_{t-1}^h(j_{t-1})\Lambda_{(t-1,t)}^d \varphi_{t-1}(j_{t-1})dj \\
& + \int_{j_{t-1}^*}^{j_{t-1}^*+\phi(m-j_{t-1}^*)} i_{t-1}^h(j_{t-1})\Lambda_{(t-1,t)}^d \varphi_{t-1}(j_{t-1})dj + (1+\kappa)\int_{j_{t-1}^*+\phi(m-j_{t-1}^*)}^{m} i^m \Lambda_{(t-1,t)}^d \varphi_{t-1}(j_{t-1})dj - i_{t-1}^h,
\end{aligned}$$

Balanced equilibrium requires $\Delta i_t^h = 0$. Hence, I get:

$$\begin{aligned}
i_{t-1}^{h^*} =& \int_0^{j_0^*} i_{t-1}^h(j_{t-1})\Lambda_{(t-1,t)}^d \varphi_{t-1}(j_{t-1})dj + \int_{j_0^*}^{j_{t-1}^*} i_{t-1}^h(j_{t-1})\Lambda_{(t-1,t)}^d \varphi_{t-1}(j_{t-1})dj \\
& + \int_{j_{t-1}^*}^{j_{t-1}^*+\phi(m-j_{t-1}^*)} i_{t-1}^h(j_{t-1})\Lambda_{(t-1,t)}^d \varphi_{t-1}(j_{t-1})dj + (1+\kappa)\int_{j_{t-1}^*+\phi(m-j_{t-1}^*)}^{m} i^m \Lambda_{(t-1,t)}^d \varphi_{t-1}(j_{t-1})dj \\
=& i^m - \Lambda_{(t-1,t)}^d \left(\int_0^{j_0^*} \varphi_{t-1}(j_{t-1})dj\right) i^m - \\
& \Lambda_{(t-1,t)}^d \left(\int_{j_0^*}^{j_{t-1}^*} \varphi_{t-1}(j_{t-1})dj\right) i^m - \Lambda_{(t-1,t)}^d \left(\int_{j_{t-1}^*}^{j_{t-1}^*+\phi(m-j_{t-1}^*)} \varphi_{t-1}(j_{t-1})dj\right) i^m + \\
& \Lambda_{(t-1,t)}^d \int_0^{j_0^*} i_{t-1}^h(j_{t-1})\varphi_{t-1}(j_{t-1})dj + \int_{j_0^*}^{j_{t-1}^*} i_{t-1}^h(j_{t-1})\Lambda_{(t-1,t)}^d \varphi_{t-1}(j_{t-1})dj + \\
& \int_{j_{t-1}^*}^{j_{t-1}^*+\phi(m-j_{t-1}^*)} i_{t-1}^h(j_{t-1})\Lambda_{(t-1,t)}^d \varphi_{t-1}(j_{t-1})dj \\
=& \left(1 - \Lambda_{(t-1,t)}^d \int_0^{j_0^*} \varphi_{t-1}(j_{t-1})dj\right) i^m + \Lambda_{(t-1,t)}^d \int_0^{j_0^*} i_{t-1}^h(j_{t-1})\varphi_{t-1}(j_{t-1})dj + \\
& + \Lambda_{(t-1,t)}^d \int_{j_{t-1}^*}^{j_{t-1}^*+\phi(m-j_{t-1}^*)} \left(i_{t-1}^h(j_{t-1}) - i^m\right) \varphi_{t-1}(j_{t-1})dj,
\end{aligned}$$

where $\Lambda_{(t-1,t)}^d \int_{j_0^*}^{j_{t-1}^*} \left(i_{t-1}^h(j_{t-1}) - i^m\right) \varphi_{t-1}(j_{t-1})dj = 0$ as $i_{t-1}^h(j_{t-1}) = i^m$ for $j_{t-1} \in [j_0^*, j_{t-1}^*]$.

Proof of propostion 3.3:

Equation (7) implies that $j_t^* = j^* = m$. According to the equations (2) and (3), the interest rate and the debt share of borrower j must also be constant in equilibrium due to the constancy of j^*.

Consequently, I have $i^h_{t-1}(j_{t-1}) = i^{h^*}(j)$ and $d_{t-1}(j_{t-1}) = d^*(j)$. The constancy of individual debt levels causes that aggregate debt is also constant in equilibrium:

$$D_t = \int_0^{j_0^*} d^*(j)dj + \int_{j_0^*}^{m} d^*(j)dj = D^* \tag{A.1}$$

From (A.1), it follows that $\Lambda^d_{(t-1,t)} = \Lambda^{d^*} = 1$. Finally, proposition 3.2 can rewritten as follows when $j^* = m$

$$i^{h^*} = \left(1 - \int_0^{j_0^*} \varphi^*(j)dj\right) i^m + \int_0^{j_0^*} i^{h^*}(j)\varphi^*(j)dj$$

Full model: Introducing entry, exit and amortization

Aggregate debt at time t is:

$$\begin{aligned}
D_t &= \int_0^{m_t} d_t(j_t)dj = \int_{j_t^*-\gamma^n m_t}^{j_t^*} d_t(j_t)dj + \int_0^{m_{t-1}} d_t(j_t)dj - \int_0^{\gamma^p m_{t-1}} d_t(j_t)dj \\
&= \int_{j_t^*-\gamma^n m_t}^{j_t^*} d_t(j_t)dj + \int_{\gamma^p m_{t-1}}^{m_{t-1}} d_t(j_t)dj \\
&= \underbrace{(1-\lambda)\int_{\gamma^p m_{t-1}}^{\max(j^*_{t-1},\gamma^p m_{t-1})} d_{t-1}(j_{t-1})dj + (1-\lambda)\int_{\max(j^*_{t-1},\gamma^p m_{t-1})}^{\max(j^*_{t-1}+\phi\chi_{t-1},\gamma^p m_{t-1})} d_{t-1}(j_{t-1})dj}_{\text{debt service}} \\
&\quad + \underbrace{\int_{j_t^*}^{j_t^*-\gamma^n m_t} d_t(j_t)dj}_{\text{new debt}} + \underbrace{(1+\kappa)\int_{\max(j^*_{t-1}+\phi\chi_{t-1},\gamma^p m_{t-1})}^{m_{t-1}} d_{t-1}(j_{t-1})dj}_{\text{refinanced debt}}
\end{aligned} \tag{A.2}$$

The law of motion of aggregate debt is given by:

$$\begin{aligned}
\Delta D_t =& D_t - D_{t-1} = \int_{j_t^*-\gamma^n m_t}^{j_t^*} d_t(j_t)dj + \kappa \int_{\max(j^*_{t-1}+\phi\chi_{t-1},\gamma^p m_{t-1})}^{m_{t-1}} d_{t-1}(j_{t-1})dj \\
&-\lambda \int_{\gamma^p m_{t-1}}^{\max(j^*_{t-1},\gamma^p m_{t-1})} d_{t-1}(j_{t-1})dj - \lambda \int_{\max(j^*_{t-1},\gamma^p m_{t-1})}^{\max(j^*_{t-1}+\phi\chi_{t-1},\gamma^p m_{t-1})} d_{t-1}(j_{t-1})dj - \int_0^{\gamma^p m_{t-1}} d_{t-1}(j_{t-1})dj
\end{aligned}$$

Subsequently, individual debt shares at t satisfy the equation:

$$
\begin{aligned}
1 = & \int_{j_t^*-\gamma^n m_t}^{j_t^*} \varphi_t(j_t)dj + (1+\kappa)\Lambda^d_{(t-1,t)} \int_{max(j^*_{t-1}+\phi\chi_{t-1},\gamma^p m_{t-1})}^{m_{t-1}} \varphi_{t-1}(j_{t-1})dj \\
& +(1-\lambda)\Lambda^d_{(t-1,t)} \int_{\gamma^p m_{t-1}}^{max(j^*_{t-1},\gamma^p m_{t-1})} \varphi_{t-1}(j_{t-1})dj + (1-\lambda)\Lambda^d_{(t-1,t)} \int_{max(j^*_{t-1},\gamma^p m_{t-1})}^{max(j^*_{t-1}+\phi\chi_{t-1},\gamma^p m_{t-1})} \varphi_{t-1}(j_{t-1})dj + \\
= & \underbrace{\Omega^n_t}_{\substack{\text{aggregate debt share}\\ \text{new debt}}} + \underbrace{(1+\kappa)\Lambda^d_{(t-1,t)}\Omega^r_{t-1}}_{\substack{\text{aggregate debt share}\\ \text{refinanced}}} + \underbrace{(1-\lambda)\Lambda^d_{(t-1,t)}\Omega^s_{t-1},}_{\substack{\text{aggregate debt share}\\ \text{no incentives}}} + \underbrace{(1-\lambda)\Lambda^d_{(t-1,t)}\Omega^i_{t-1},}_{\substack{\text{aggregate debt share}\\ \text{inactive}}}
\end{aligned}
\tag{A.3}
$$

while individual debt shares at $t-1$ satisfy the equation:

$$
\begin{aligned}
1 = & \int_{\max(j^*_{t-1}+\phi\chi_{t-1},\gamma^p m_{t-1})}^{m_{t-1}} \varphi_{t-1}(j_{t-1})dj + \int_{\gamma^p m_{t-1}}^{\max(j^*_{t-1},\gamma^p m_{t-1})} \varphi_{t-1}(j_{t-1})dj \\
& + \int_{\max(j^*_{t-1},\gamma^p m_{t-1})}^{\max(j^*_{t-1}+\phi\chi_{t-1},\gamma^p m_{t-1})} \varphi_{t-1}(j_{t-1})dj + \int_0^{\gamma^p m_{t-1}} \varphi_{t-1}(j_{t-1})dj \\
= & \underbrace{\Omega^r_{t-1}}_{\substack{\text{aggregate debt share}\\ \text{refinancing}}} + \underbrace{\Omega^s_{t-1}}_{\substack{\text{aggregate debt share}\\ \text{no incentives}}} + \underbrace{\Omega^i_{t-1},}_{\substack{\text{aggregate debt share}\\ \text{inactive}}} + \underbrace{\Omega^p_{t-1}}_{\substack{\text{aggregate debt share}\\ \text{repayments}}}
\end{aligned}
\tag{A.4}
$$

where $\Omega^n_t = \int_{j_t^*-\gamma^n m_t}^{j_t^*} \varphi_t(j_t)dj$ is the aggregate debt share of new borrowers who raise fixed-rate mortgages at time t, and $\Omega^p_{t-1} = \int_0^{\gamma^p m_{t-1}} \varphi_{t-1}(j_{t-1})dj$ is the aggregate debt share of existing borrowers who repay at time $t-1$.

Proof of propostion 3.4:

The average interest rate is:

$$
\begin{aligned}
i^h_t = & \int_0^{m_t} i^h_t(j_t)\varphi_t(j_t)dj = \int_{j_t^*-\gamma^n}^{j_t^*} i^h_t(j_t)\varphi_t(j_t)dj + \int_{\gamma^p m_{t-1}}^{m_{t-1}} i^h_t(j_t)\varphi_t(j_t)dj \\
= & \int_{j_t^*-\gamma^n m_t}^{j_t^*} i^m \varphi_t(j_t)dj + (1+\kappa)\Lambda^d_{(t-1,t)} \int_{\max(j^*_{t-1}+\phi\chi_{t-1},\gamma^p m_{t-1})}^{m_{t-1}} i^m \varphi_{t-1}(j_{t-1})dj \\
& +(1-\lambda)\Lambda^d_{(t-1,t)} \int_{\gamma^p m_{t-1}}^{\max(j^*_{t-1},\gamma^p m_{t-1})} i^h_{t-1}(j_{t-1})\varphi_{t-1}(j_{t-1})dj \\
& +(1-\lambda)\Lambda^d_{(t-1,t)} \int_{\max(j^*_{t-1},\gamma^p m_{t-1})}^{\max(j^*_{t-1}+\phi\chi_{t-1},\gamma^p m_{t-1})} i^h_{t-1}(j_{t-1})\varphi_{t-1}(j_{t-1})dj
\end{aligned}
\tag{A.5}
$$

Subseqeuently, I can determine the change in average interest rate:

$$
\begin{aligned}
\Delta i^h_t =& i^h_t - i^h_{t-1} \\
=& \int_{j^*_t-\gamma^n m_t}^{j^*_t} i^m \varphi_t(j_t)dj + (1+\kappa)\Lambda^d_{(t-1,t)} \int_{\max(j^*_{t-1}+\phi\chi_{t-1},\gamma^p m_{t-1})}^{m_{t-1}} i^m \varphi_{t-1}(j_{t-1})dj \\
&+(1-\lambda)\Lambda^d_{(t-1,t)} \int_{\gamma^p m_{t-1}}^{\max(j^*_{t-1},\gamma^p m_{t-1})} i^h_{t-1}(j_{t-1})\varphi_{t-1}(j_{t-1})dj \\
&+(1-\lambda)\Lambda^d_{(t-1,t)} \int_{\max(j^*_{t-1},\gamma^p m_{t-1})}^{\max(j^*_{t-1}+\phi\chi_{t-1},\gamma^p m_{t-1})} i^h_{t-1}(j_{t-1})\varphi_{t-1}(j_{t-1})dj - i^h_{t-1} \\
=& \int_{j^*_t-\gamma^n m_t}^{j^*_t} i^h_t(j_t)\varphi_t(j_t)dj + (1+\kappa)\Lambda^d_{(t-1,t)} \int_{\max(j^*_{t-1}+\phi\chi_{t-1},\gamma^p m_{t-1})}^{m_{t-1}} i^m \varphi_{t-1}(j_{t-1})dj \\
&- \int_{\max(j^*_{t-1}+\phi\chi_{t-1},\gamma^p m_{t-1})}^{m_{t-1}} i^h_{t-1}(j_{t-1})\varphi_{t-1}(j_{t-1})dj - \lambda\Lambda^d_{(t-1,t)} \int_{\gamma^p m_{t-1}}^{\max(j^*_{t-1},\gamma^p m_{t-1})} i^h_{t-1}(j_{t-1})\varphi_{t-1}(j_{t-1})dj \\
&-\lambda\Lambda^d_{(t-1,t)} \int_{\max(j^*_{t-1},\gamma^p m_{t-1})}^{\max(j^*_{t-1}+\phi\chi_{t-1},,\gamma^p m_{t-1})} i^h_{t-1}(j_{t-1})\varphi_{t-1}(j_{t-1})dj \\
&+(\Lambda^d_{(t-1,t)}-1) \int_{\gamma^p m_{t-1}}^{\max(j^*_{t-1}+\phi\chi_{t-1},\gamma^p m_{t-1})} i^h_{t-1}(j_{t-1})\varphi_{t-1}(j_{t-1})d - \int_0^{\gamma^p m_{t-1}} i^h_{t-1}(j_{t-1})\varphi_t(j_{t-1})dj
\end{aligned} \tag{A.6}
$$

By exploiting

$$0 = \Omega^n_t + (1+\kappa)\Lambda^d_{(t-1,t)}\Omega^r_{t-1} + (1-\lambda)\Lambda^d_{(t-1,t)}\Omega^s_{t-1} + (1-\lambda)\Lambda^d_{(t-1,t)}\Omega^i_{t-1} - (\Omega^r_{t-1}+\Omega^i_{t-1}+\Omega^s_{t-1}+\Omega^p_{t-1}),$$

I can rewrite (A.6) in following way :

$$
\begin{aligned}
\Delta i^h_t =& \int_{j^*_t-\gamma^n m_t}^{j^*_t} \left(i^m - i^h_{t-1}\right)\varphi_t(j_t)dj - \int_0^{\gamma^p m_{t-1}} \left(i^h_{t-1}(j_{t-1}) - i^h_{t-1}\right)\varphi_t(j_t)dj \\
&+ \int_{\max(j^*_{t-1}+\phi\chi_{t-1,\gamma^p m_{t-1}})}^{m_{t-1}} \left((1+\kappa)\Lambda^d_{(t-1,t)}(i^m - i^h_{t-1}) - (i_{t-1}(j_{t-1}) - i^h_{t-1})\right)\varphi_{t-1}(j_{t-1})dj \\
&-\lambda\Lambda^d_{(t-1,t)} \int_{\gamma^p m_{t-1}}^{\max(j^*_{t-1},\gamma^p m_{t-1})} \left(i^h_{t-1}(j_{t-1}) - i^h_{t-1}\right)\varphi_{t-1}(j_{t-1})dj \\
&-\lambda\Lambda^d_{(t-1,t)} \int_{\max(j^*_{t-1},\gamma^p m_{t-1})}^{\max(j^*_{t-1}+\phi\chi_{t-1},\gamma^p m_{t-1})} \left(i^h_{t-1}(j_{t-1}) - i^h_{t-1}\right)\varphi_{t-1}(j_{t-1})dj \\
&+(\Lambda^d_{(t-1,t)}-1) \int_{\gamma^p m_{t-1}}^{\max(j^*_{t-1}+\phi\chi_{t-1},\gamma^p m_{t-1})} \left(i^h_{t-1}(j_{t-1}) - i^h_{t-1}\right)\varphi_{t-1}(j_{t-1})dj
\end{aligned} \tag{A.7}
$$

Proof of propostion 3.5:

Let E_t denotes the accumulated number of borrowers who have prepaid at t:

$$E_t = E_{t-1} + \omega^p_{t-1} = E_{t-2} + \omega^p_{t-1} + \omega^p_{t-2} = E_{t-n} + \omega^p_{t-1} + \omega^p_{t-2} + ... + \omega^p_{t-n}, \tag{A.8}$$

where $E_0 = \omega^p_0$. Solving A.8, I get:

$$\begin{aligned} E_t =& \sum_{k=0}^{t-1} \omega^p_k dk = \sum_{k=0}^{t-1} \gamma^p m_k dk = \sum_{k=0}^{t-1} \gamma^p m_0 \Theta^k_m dk = \gamma^p m_0 \sum_{k=0}^{t-1} \Theta^k_m dk \\ =& \gamma^p m_0 \frac{1-\Theta^t_m}{1-\Theta_m} = \gamma^p j^*_0 \tilde{m}_0 \frac{1-\Theta^t_m}{1-\Theta_m}, \end{aligned}$$

where $\tilde{m}_0 = m_0/j^*_0$. Subsequently, I let S_t be the remaining number of borrowers at time t who initially faced non-positive incentives:

$$S_t = \max(0, j^*_0 - E_t) = \max\left(j^*_0 \left(1 - \gamma^p \tilde{m}_0 \frac{1-\Theta^t_m}{1-\Theta_m}\right), 0\right) \tag{A.9}$$

To impose the medium-run equilibrium, I consider the equilibrium path of the average interest rate. From the equations (A.4), (A.5), and (A.9) I obtain:

$$\begin{aligned} \Delta i^h_t =& \left(1 - (1-\lambda)\Lambda^d_{(t-1,t)} \left(\int_{\gamma^p m_{t-1}}^{\max(j^*_{t-1}, \gamma^p m_{t-1})} \varphi_{t-1}(j_{t-1}) dj + \int_{\max(j^*_{t-1}, \gamma^p m_{t-1})}^{\max(j^*_{t-1} + \phi\chi_{t-1}, \gamma^p m_{t-1})} \varphi_{t-1}(j_{t-1}) dj \right)\right) i^m \\ &+ (1-\lambda)\Lambda^d_{(t-1,t)} \int_{\gamma^p m_{t-1}}^{\max(j^*_{t-1}, \gamma^p m_{t-1})} i^h_{t-1}(j_{t-1}) \varphi_{t-1}(j_{t-1}) dj \\ &+ (1-\lambda)\Lambda^d_{(t-1,t)} \int_{\max(j^*_{t-1}, \gamma^p m_{t-1})}^{\max(j^*_{t-1} + \phi\chi_{t-1}, \gamma^p m_{t-1})} i^h_{t-1}(j_{t-1}) \varphi_{t-1}(j_{t-1}) dj \\ =& \left(1 - (1-\lambda)\Lambda^d_{(t-1,t)} \int_{\gamma^p m_{t-1}}^{\max(S_{t-1}, \gamma^p m_{t-1})} \varphi_{t-1}(j_{t-1}) dj \right) i^m \\ &+ (1-\lambda)\Lambda^d_{(t-1,t)} \int_{\gamma^p m_{t-1}}^{\max(S_{t-1}, \gamma^p m_{t-1})} i^h_{t-1}(j_{t-1}) \varphi_{t-1}(j_{t-1}) dj - i^h_{t-1} \\ &+ (1-\lambda)\Lambda^d_{(t-1,t)} \int_{\max(S_{t-1}, \gamma^p m_{t-1})}^{\max(j^*_{t-1}, \gamma^p m_{t-1})} \left(i^h_{t-1}(j_{t-1}) - i^m\right) \varphi_{t-1}(j_{t-1}) dj \\ &+ (1-\lambda)\Lambda^d_{(t-1,t)} \int_{\max(j^*_{t-1}, \gamma^p m_{t-1})}^{\max(j^*_{t-1} + \phi\chi_{t-1}, \gamma^p m_{t-1})} \left(i^h_{t-1}(j_{t-1}) - i^m\right) \varphi_{t-1}(j_{t-1}) dj \\ =& \left(1 - (1-\lambda)\Lambda^d_{(t-1,t)} \int_{\gamma^p m_{t-1}}^{\max(S_{t-1}, \gamma^p m_{t-1})} \varphi_{t-1}(j_{t-1}) dj \right) i^m \\ &+ (1-\lambda)\Lambda^d_{(t-1,t)} \int_{\gamma^p m_{t-1}}^{\max(S_{t-1}, \gamma^p m_{t-1})} i^h_{t-1}(j_{t-1}) \varphi_{t-1}(j_{t-1}) dj - i^h_{t-1} \\ &+ (1-\lambda)\Lambda^d_{(t-1,t)} \int_{\max(j^*_{t-1}, \gamma^p m_{t-1})}^{\max(j^*_{t-1} + \phi\chi_{t-1}, \gamma^p m_{t-1})} \left(i^h_{t-1}(j_{t-1}) - i^m\right) \varphi_{t-1}(j_{t-1}) dj, \end{aligned} \tag{A.10}$$

where $(1-\lambda)\Lambda^d_{(t-1,t)}\int_{\max(S_{t-1},\gamma^p m_{t-1})}^{\max(j^*_{t-1},\gamma^p m_{t-1})}\left(i^h_{t-1}(j_{t-1})-i^m\right)\varphi_{t-1}(j_{t-1})dj=0$ because $i^h_{t-1}(j_{t-1})=i^m$ for $j_t\in[max(S_{t-1},\gamma^p m_{t-1}),\max(j^*_{t-1},\gamma^p m_{t-1})]$. Moreover, I have exploited the fact:

$$\int_{\gamma^p m_{t-1}}^{\max(j^*_{t-1},\gamma^p m_{t-1})}\varphi_{t-1}(j_{t-1})dj=\int_{\gamma^p m_{t-1}}^{\max(S_{t-1},\gamma^p m_{t-1})}\varphi_{t-1}(j_{t-1})dj+\int_{\max(S_{t-1},\gamma^p m_{t-1})}^{\max(j^*_{t-1},\gamma^p m_{t-1})}\varphi_{t-1}(j_{t-1})dj$$

By combining equations (A.9)-(A.10), I obtain the following proposition:

Proposition A.1 (Equilibrium path of average interest rate with replacement of debt). *When $\Delta i^h_t=0$, a balanced equilibrium path of i^h_t exists. Along this path $i^{h^*}_{t-1}$ satisfies*

$$\begin{aligned}
i^{h^*}_{t-1}=&\left(1-(1-\lambda)\Lambda^d_{(t-1,t)}\int_{\gamma^p m_{t-1}}^{j^*_0\times\max\left(\max\left(1-\gamma^p\tilde{m}_0\frac{1-\Theta_m^{t-1}}{1-\Theta_m},0\right),\gamma^p\tilde{m}_0\Theta_m^{t-1}\right)}\varphi_{t-1}(j_{t-1})dj\right)i^m\\
&+(1-\lambda)^t\Lambda^d_{(t-1,t)}\int_{\gamma^p m_{t-1}}^{j^*_0\times\max\left(\max\left(1-\gamma^p\tilde{m}_0\frac{1-\Theta_m^{t-1}}{1-\Theta_m},0\right),\gamma^p\tilde{m}_0\Theta_m^{t-1}\right)}i^h_{t-1}(j_{t-1})\varphi_0(j_0)dj\\
&+(1-\lambda)^t\Lambda^d_{(t-1,t)}\int_{\max(j^*_{t-1},\gamma^p m_{t-1})}^{\max(j^*_{t-1}+\phi\chi_{t-1},\gamma^p m_{t-1})}\left(i^h_{t-1}(j_{t-1})-i^m\right)\varphi_0(j_0)dj
\end{aligned}$$

for a given values of j^*_0 and $\tilde{m}_0$. The interest rate path $i^{h^*}_{t-1}$ can take the following two forms depending on the value of j^*_0:

- ***Case (i):*** *If $j^*_0=0$ or $1-\gamma^p\tilde{m}_0\frac{1-\Theta_m^{t-1}}{1-\Theta_m}\leq\gamma^p\tilde{m}_0\Theta_m^{t-1}$, the average interest rate follows the path: $i^{h^*}_{t-1}=i^m+(1-\lambda)\mu^i_{(t-1,t)}$.*
- ***Case (ii):*** *If $j^*_0>0$ or $1-\gamma^p\tilde{m}_0\frac{1-\Theta_m^{t-1}}{1-\Theta_m}>\gamma^p\tilde{m}_0\Theta_m^{t-1}$, the average interest rate follows the path:*

$$\begin{aligned}
i^{h^*}_{t-1}=&\left(1-(1-\lambda)\Lambda^d_{(t-1,t)}\int_{\gamma^p m_{t-1}}^{j^*_0\times\max\left(\max\left(1-\gamma^p\tilde{m}_0\frac{1-\Theta_m^{t-1}}{1-\Theta_m},0\right),\gamma^p\tilde{m}_0\Theta_m^{t-1}\right)}\varphi_{t-1}(j_{t-1})dj\right)i^m\\
&+(1-\lambda)(\mu^h_{(t-1,t)}+\mu^i_{(t-1,t)})
\end{aligned}$$

where $\mu^i_{(t-1,t)}=\Lambda^d_{(t-1,t)}\int_{\max(j^*_{t-1},\gamma^p m_{t-1})}^{\max(j^*_{t-1}+\phi\chi_{t-1},\gamma^p m_{t-1})}\left(i^h_{t-1}(j_{t-1})-i^m\right)\varphi_{t-1}(j_{t-1})dj$ is the average fixed interest rate of inactive borrowers.

The law of motion of $\Lambda(j^*_t,m_t)$ implies that $j^*_t=m_t$ (see equation (18)). Setting $j^*_t=m_t$ in proposition A.1 for $t = t^m < \infty$, I get:

$$i_{t-1}^{h^*} = \left(1 - (1-\lambda)^t \Lambda^d_{(t-1,t)} \int_{\gamma^p m_{t-1}}^{j_0^* \times \max\left(\max\left(1-\gamma^p \tilde{m}_0 \frac{1-\Theta_m^{t-1}}{1-\Theta_m},0\right),\gamma^p \tilde{m}_0 \Theta_m^{t-1}\right)} \varphi_0(j_0) dj\right) i^m$$
$$+(1-\lambda)^t \Lambda^d_{(t-1,t)} \int_{\gamma^p m_{t-1}}^{j_0^* \times \max\left(\max\left(1-\gamma^p \tilde{m}_0 \frac{1-\Theta_m^{t-1}}{1-\Theta_m},0\right),\gamma^p \tilde{m}_0 \Theta_m^{t-1}\right)} i_{t-1}^h(j_{t-1}) \varphi_0(j_0) dj \tag{A.11}$$

which proves proposition 3.5.

Proof of propostion 3.6:
From equation (A.11), I realize:

$$\lim_{t\to\infty} \max\left(1 - \gamma^p \tilde{m}_0 \frac{1-\Theta_m^{t-1}}{1-\Theta_m}, 0\right) = 0,$$

and

$$\lim_{t\to\infty} \gamma^p \tilde{m}_0 \Theta_m^{t-1} = \infty$$

As a result, there must exist a point in time $t = t^l \geq t^m \geq t_0$, where:

$$\max\left(1 - \gamma^p \tilde{m}_0 \frac{1-\Theta_m^{t-1}}{1-\Theta_m}, 0\right) < \gamma^p \tilde{m}_0 \Theta_m^{t-1},$$

such that

$$\lim_{t\to\bar{t}^l} \left[j_0^* \times \max\left(\max\left(1 - \gamma^p \tilde{m}_0 \frac{1-\Theta_m^{t-1}}{1-\Theta_m}, 0\right), \gamma^p \tilde{m}_0 \Theta_m^{t-1}\right)\right] = j_0^* \times \gamma^p \tilde{m}_0 \Theta_m^{t-1} = \gamma^p m_{t-1}$$

Finally, I must have:

$$i^{h^*} = \lim_{t\to\bar{t}^l} i_{t-1}^{h^*} = i^m \tag{A.12}$$

Empirical specification

Let $i_t(j_t) = \rho_t^r i_t^h$ for $j_t \in [\max(j_t^* + \phi\chi_t, \gamma^p m_t), m_t]$, and $i_t(j_t) = \rho_t^a i_t^m$ for $j_t \in [\gamma^p m_t, \max((j_t^* + \phi\chi_t, \gamma^p m_t)]$ By using assumption 4.1, I can rewrite proposition 3.4 as follows:

$$\begin{aligned} \Delta i_t^h =& (1+\kappa)\Lambda^d_{(t-1,t)} \Omega^r_{t-1}(i^m - \rho^r_{t-1} i^h_{t-1}) - \Omega^r_{t-1}(1-\rho^r_{t-1}) i^h_{t-1} \\ & -\lambda \Lambda^d_{(t-1,t)} \Omega^a_{t-1}(i^m \rho^a_{t-1} - i^h_{t-1}) + (\Lambda^d_{(t-1,t)} - 1)\Omega^a_{t-1}(\rho^a_{t-1} i^m - i^h_{t-1}) + \Omega^n_t (i^m - i^h_{t-1}) - \Omega^p_{t-1}(\rho^p_{t-1} - 1) i^h_{t-1} \end{aligned} \tag{A.13}$$

In equilibrium ($j_t^* = m_t$), I have

$$\Delta i_t^h = -\left(1 - \Lambda^d_{(t-1,t)}(1-\lambda)\right) \bar{\Omega}^s_{t-1}(i^m \rho^s_{t-1} - i^h_{t-1}) + \Omega^n_t (i^m - i^h_{t-1}) - \Omega^p_{t-1}(\rho^p_{t-1} - 1) i^h_{t-1},$$

where I have used assumption 4.2. For $\Delta i^h_{t-1} = 0$, I get:

$$i^h_{t-1} = \frac{\Omega^n_t - \left(1 - \Lambda^d_{(t-1,t)}(1-\lambda)\right)\bar{\Omega}^s_{t-1}\rho^s_{t-1}}{\Omega^n_t - \left(1 - \Lambda^d_{(t-1,t)}(1-\lambda)\right)\bar{\Omega}^s_{t-1} + \Omega^p_{t-1}(\rho^p_{t-1} - 1)} i^m_{t-1} \tag{A.14}$$

Assume parameter constancy, I get:

$$i^h_{t-1} = \frac{\Omega^n - \lambda\bar{\Omega}^s\rho^s}{\Omega^n - \lambda\bar{\Omega}^s + \Omega^p(\rho^p - 1)} i^m_{t-1}, \tag{A.15}$$

which corresponds to the long-run interest pass-through stated in empirical specification (equation (21)). Given that equation (A.15) constitutes the long-run equilibrium, I can rewrite (A.13) conditional on parameter constancy as follows:

$$\begin{aligned}
\Delta i^h_t =& (1+\kappa)\Lambda^d_{(t-1,t)}\Omega^r_{t-1}(i^m - \rho^r_{t-1} i^h_{t-1}) - \Omega^r_{t-1}(1-\rho^r_{t-1})i^h_{t-1} \\
&-\lambda\Lambda^d_{(t-1,t)}\Omega^a_{t-1}(i^m\rho^a_{t-1} - i^h_{t-1}) + (\Lambda^d_{(t-1,t)} - 1)\Omega^a_{t-1}(\rho^a_{t-1}i^m - i^h_{t-1}) + \Omega^n_t(i^m - i^h_{t-1}) - \Omega^p_{t-1}(\rho^p_{t-1} - 1)i^h_{t-1} \\
&(\tilde{\Omega}^r + \Omega^n - \lambda\Omega^a - \Omega^p)\left(i^h_{t-1} - \frac{\Omega^n - \lambda\bar{\Omega}^s\rho^s}{\Omega^n - \lambda\bar{\Omega}^s + \Omega^p(\rho^p - 1)} i^m_{t-1}\right) \\
&(\tilde{\Omega}^r + \Omega^n + \lambda\tilde{\Omega}^a + \tilde{\Omega}^p)\left(i^h_{t-1} - \frac{\Omega^n - \lambda\bar{\Omega}^s\rho^s}{\Omega^n - \lambda\bar{\Omega}^s + \Omega^p(\rho^p - 1)} i^m_{t-1}\right)
\end{aligned} \tag{A.16}$$

A.2 Determinants of market rates on fixed-rate mortgages

The market rate on fixed-rate mortgages is determined by the yield to maturity of the underlying mortgage bonds. Due to the prepayment option, homeowners normally raise their mortgages slightly below face value. When price of a mortgage bond increases above face value or falls substantially below face value, the bond is replaced by a new mortgage bond with a price below and close to face value.

Interest rates on callable bonds can be determined by using the well-known expectations theory of term structure. According to the linearized version of the expectation theory, the interest rate on callable bonds is the sum of the expected future risk-free short-term rates, market risk premiums and the prepayment spread. Hence, the market rate at time t is:

$$i^m_{t,n} = \frac{1}{n}\sum_{i=0}^{n-1} \mathbb{E}_t\left(r^f_{t+i} + \theta_{t+i} + \psi_{t+i}\right), \tag{A.17}$$

where $\mathbb{E}_t$ is expectations to short-term rates evaluated at time t, r^f_{t+i} denotes risk-free rates which naturally reflect expectations to monetary policy rates, $\psi_{b,t+i}$ is market risk premium, and $\theta_{b,t+i}$ is the prepayment spread.

Both r^f_{t+i} and ψ_{t+i} are well-known determinants of traditional long-term bonds, while $\theta_{b,t+i}$ is only a factor due to the prepayment option. Consequently, the price of a callable bond corresponds to the price of a traditional, non-convertible, bond less the price of the option. Hence, the price of callable bonds will never be higher than similar non-callable bonds when the term structure is upward-sloping, as is usually the case. As a consequence, the yield to maturity is by construction larger compared to a standard, non-convertible, bond with identical properties. The notation used in the empirical analysis is $r_t^f = \sum_{i=0}^{n-1} \mathbb{E}_t r_{t+i}^f$, $\theta_t = \frac{1}{n}\sum_{i=0}^{n-1} \mathbb{E}_t \theta_{t+i}$, and $\psi_t = \frac{1}{n}\sum_{i=0}^{n-1} \mathbb{E}_t \psi_{t+i}$.

Subsequently, I transform the term structure equation (i.e. equation (A.17)) into a dynamic equation by assuming that the prepayment spread follows a stationary process. The transformation is needed to make (A.17) empirically testable, as it would otherwise suffer from over-identification. Specifically, I assume

Assumption A.1. *$\theta_t = \mu + \eta_t$, where $\mu > 0$ and η_t satisfy the following stationary process:*

$$\eta_t = \sum_{s=1}^{v} \zeta_s \xi_{t-s} + \varepsilon_t^m,$$

where $\zeta(1) = 1 - \zeta_1 - \zeta_2 - ... - \zeta_m > 0$ and ε_t^m is Gaussian innovations with zero mean and variance σ^2.

Intuitively, stationarity of the prepayment spread is expected as the options on the benchmark bonds are always at-the-money.[25] Given the autoregressive dynamic of θ_t, I can rewrite (A.17) as equation (20) in the main text.

Modeling the spread value of prepayment risk

In practice, the spread value of prepayment risk is typically estimated by advanced prepayment models. I use an existing prepayment model developed by Scanrate Financial Systems to calculate the spread value. The estimation procedure of prepayment models is quite complex, and a detailed description is hence beyond the scope of this paper. Below, I will go briefly through how the prepayment spread is calculated.

For a standard bond, future cash flows are known in advance. This is not the case for callable bonds. If I let s_{t+i} be spot zero-coupon rates, then the theoretical price of a callable bond with maturity n is:

$$\tilde{p}_{n,t}^c = \sum_{i=1}^{n} \frac{\mathbb{E}_t z_{t+i}}{(1 + s_{t+i})^i}, \tag{A.18}$$

where z_{t+1} is the cash flow at time $t + i$. Due to prepayment the risk, the price of a callable bond will always be lower than or equal to that of a similar standard bond with identical discount rates as $\mathbb{E}_t z_{t+i} \leq z_{t+i}$. The estimated prepayment spread can therefore be found by adding the value θ_t

[25] An ATM option is an option that would lead to zero cash flow if exercised immediately. This implies that the strike price of the option (equivalent to the face value of the bond (i.e. price 100) is equal/close to the market price of the bond.

to all spot zero-coupon rates, such that:

$$\tilde{p}_{n,t}^{c} = \sum_{i=1}^{n} \frac{\mathbb{E}_t z_{t+i}}{(1+s_{t+i})^i} = \sum_{i=1}^{n} \frac{z_{t+i}}{(1+s_{t+i}+\theta_{t+i})^i}, \tag{A.19}$$

As the theoretical price cannot be observed, one needs to estimate $\mathbb{E}_t z_{t+i}$ in order to calculate θ_t. $\mathbb{E}_t z_{t+i}$ is determined by estimating the expected prepayment rates for the fixed-rate mortgage bond.

Calculating the option-adjusted spread

Let $p_{n,t}^{c}$ be the observed market price of a callable bond. The option-adjusted spread (i.e. ψ_t according to the (A.17) equation) can be calculated residually by setting $p_{n,b,t}^{c} = \tilde{p}_{n,b,t}^{c}$, so that:

$$p_{n,b,t}^{c} = \sum_{i=1}^{n} \frac{\mathbb{E}_t z_{t+i}}{(1+s_{t+i}+\psi_t)^i}, \tag{A.20}$$

The option-adjusted spread defined in section 4.2 is calculated for s_{t+i} equals to forward rates on 6-month IBOR swaps in Danish krone (also called 6-month Cibor swaps). The use of plain-vanilla IBOR swap rates as discount rates slightly affects the interpretation of OAS. This is due to the fact that Cibor-swap rates might contain minor interbank risks, meaning that OAS cannot necessarily be interpreted as the actual size of risk premiums. Instead, it should more be interpreted as the additional market risk premium on the callable bond compared to the IBOR swap curve.

A.3 Test statistics

Misspecification tests

Table A.1 reports the results of the misspecification tests of the single equations and the multivariate system for the unrestricted VAR model. The null hypotheses of no autocorrelation in all the single equations and for the multivariate system are clearly accepted. The null hypotheses of no ARCH-effects in the residuals are also accepted in all the single equations. Turning to the normality tests, the null hypotheses of normally distributed errors are rejected in the single equations for i_t^h, i_t^m, OAS_t and in the multivariate system, while residuals in the r_t^f equation are accepted to behave Gaussian. According to the reports, the rejections of the normality assumption might essentially be due to excess kurtosis caused by remaining moderate outliers. Even though non-Gaussian behaving residuals may lead to inefficient estimates, simulation studies have shown that statistical inference is robust to excess kurtosis, see Juselius (2006). Consequently, the misspecification due to rejected normality may not be a serious problem for the remaining analysis.

	AR(1-1)		AR(1-2)		ARCH(1-2)		Normality		Excess kurtosis	Skewness
Δi_t^h	0.47	[0.50]	0.23	[0.79]	0.18	[0.84]	47.8	[0.00]	4.56	-1.43
Δi_t^m	0.15	[0.70]	0.69	[0.50]	0.05	[0.94]	15.8	[0.01]	5.43	-0.57
Δr_t^f	0.39	[0.53]	0.35	[0.70]	1.5	[0.23]	0.27	[0.87]	3.57	0.09
ΔOAS_t	0.56	[0.46]	1.88	[0.16]	1.50	[0.23]	13.4	[0.01]	5.10	0.24
Multivariate tests:	1.48	[0.10]	1.25	[0.22]	...		34.6	[0.01]		

Table A.1: *Test statistics for misspecification of the unrestricted VAR(*$1,2$*) system.* AR (1-1) and AR (1-2) are F-tests for autocorrelated residuals up to second order. The single equations and the multivariate tests are distributed as F(1,138), F(2,137) and F(16,403) F(32,473), respectively. ARCH (1-2) test for ARCH effects up to second order. The single equations are distributed as F(2,145). Finally, the tests for normality are distributed as $\chi^2(2)$ and $\chi^2(8)$ in the single equations and the multivariate tests, respectively.

Determining the cointegration rank

First, I consider the Johansen trace test. The test is based on (24) expressed in terms of its concentrated model. Formally, the concentrated model can be derived by transforming the CVAR(2) into compact form:

$$C_{0t} = \alpha\beta' \underbrace{\tilde{x}_{t-1}}_{C_{1t}} + \tilde{\Gamma} \underbrace{\begin{pmatrix} \Delta x_{t-1} \\ \mu_0 \\ \mathbb{D}_t \end{pmatrix}}_{C_{2t}} + \varepsilon_t,$$

where $\tilde{\Gamma}$ is a vector capturing the coefficients to the short-run parameters, the unrestricted mean and the unrestricted dummy variables, respectively. In order to estimate $\alpha\beta'$, I concentrate out the effect of C_{2t} on C_{0t} and C_{1t}, respectively, and then regress the cleaned C_{0t} (i.e. the residual called R_{0t}) on the cleaned C_{1t} (i.e. the residual called R_{1t}).[26] Hence, I obtain $R_{0t} = \alpha\beta' R_{1t} + \varepsilon_t$, where R_{0t} is the short-run adjusted vector of endogenous variables, and R_{1t} is the lagged short-run adjusted vector of the endogenous variables and the restricted mean, respectively. The transformation of the cointegrated VAR model ensures that the long-run part of the model can be economically interpreted, as all short-run dynamics, unrestricted dummies and deterministic components have been concentrated out. As a result, I am left with a statistical model that solely captures the adjustment that takes place towards the long-run equilibrium relations. The ML estimator can then be derived by using the general two-step approach (Johansen (1996)). Thus, the LR (trace) test statistic for two nested models, say $H(p)$ and $H(r)$, is:

$$\tau_{p-r} = LR(H(r)|H(p)) = -T \sum_{i=r+1}^{p} log(1 - E_i),$$

where the models meet the nested sequence $H(0) \subset \cdots \subset H(r) \subset \cdots \subset H(p)$, and E_i captures the eigenvalues, explicitly linked to the cointegration vector i. The asymptotic distribution of the rank tests converges in probability to some kind of a Dickey-Fuller distribution containing functionals of

[26]For more details, see Johansen (1996).

Brownian motions. As the distribution generally depends on deterministic specifications, such as an unrestricted constant and a constant restricted to the cointegration space, the distribution of the asymptotic trace test needs to be simulated (Nielsen (2004)).[27]

The LR test statistics based on the top-bottom procedure are reported in table A.2. The null hypothesis of no cointegration is rejected for $r \leq 2$, while $r = 3$ is clearly accepted with a p-value of 0.182. In the right panel, I have performed bootstrap likelihood ratio tests that try to deal with the poor approximation of asymptotic inference in finite samples (especially in small samples).[28] Based on Cavaliere et al. (2012), the bootstrap version of the LR test rejects that $r \leq 1$, while the null hypothesis on $r = 2$ cannot be rejected. In sum, the LR test statistics suggest that a cointegration rank of either $r = 2$ or $r = 3$ maximizes the explanatory power of the model in terms of stationarity.[29]

		Simulated LR test					Bootstrap test	
p-r	r	EigValue	Trace	Trace*	P-value	P-value*	P-value	P-value*
4	0	0.52	193.4	190.9	[0.000]	[0.000]	[0.000]	[0.000]
3	1	0.33	83.60	82.8	[0.000]	[0.000]	[0.000]	[0.000]
2	2	0.11	22.95	22.82	[0.019]	[0.020]	[0.019]	[0.236]
1	3	0.04	6.28	6.27	[0.181]	[0.182]	[0.176]	[0.306]

Table A.2: *Rank determination based on a simulated asymptotic distribution of Johansen trace test and bootstrap testing.* Asymptotic tables have been simulated based on the program developed by Nielsen (2004*b*). P-values are based on 5% critical values. The test statistics marked with an asterisk are for the Bartlett-corrected trace test. In contrast to the general trace test, the Bartlett-corrected trace test corrects for small sample bias which in general leads to over-sized tests, see Johansen (2000, 2002*a* and 2002*b*).

A general way of interpreting the LR test is that the magnitude of the eigenvalues reflects the degree of stationarity one is capable of obtaining by a linear combination of the variables when all short-run fluctuations are ignored. This link between the LR test and the eigenvalues implies that one can alternatively focus on the characteristic roots (the inverse of the eigenvalues) in the

[27] In contrast, dummies for additive and innovational outliers do not influence the shape of the asymptotic distribution, as they correct for single observation shocks, see Johansen (1996, chapter 11).

[28] The substantial asymptotic distortion is a result of the complexity of the VAR model's dynamics, and the construction of the trace test that does not allow asymptotically for short-run effects, which turn out to be influential for the trace test in small samples, see Johansen (2002*b*). Since bootstrap testing, which is based on a well-defined data-generating process, has the advantage that it converges much faster to the true distribution, inference will automatically be much more reliable in small samples.

[29] The bootstrap Johansen trace test is implemented by estimating the restricted model $H(r)$, so that one obtains the estimates in (24) and the estimated residuals, $\{\hat{\varepsilon}_t\}$. Hence, the bootstrap test is then consistent with the generation of artificial samples:

$$\Delta x_t^* = \hat{\alpha}\hat{\tilde{\beta}}' \begin{pmatrix} x_{t-1}^* \\ 1 \end{pmatrix} + \hat{\Gamma}_1 \Delta x_{t-1}^* + \hat{\mu}_0 + \hat{\phi} d_t + \varepsilon_t^*,$$

where $x_{-1}^* = x_{-1}^a$, $x_0^* = x_0$, and ε_t^* is drawn with replacement from the estimated residuals $\{\hat{\varepsilon}_t\}$. On each sample, one can calculate the test statistic for $H(r)$ and $H(p)$, respectively. Based on this sampling scheme, one can schematically re-estimate the restricted model and then simulate the distribution under the null hypothesis by using the generated bootstrap data. The bootstrapped p-values are generated from 400 replications, and the sampling scheme is based on wild bootstrap.

determination of $H(r)$; if the $r+1$'th cointegration vector does not constitute a stationary relations, then the characteristic root (which is not part of the $p-r$ unit roots) will be close to unity. Table 2 also documents that the largest unrestricted root is indisputably smallest for $r = 2$, while the choice of $r \geq 3$ could potentially lead to models containing I(2) trends. Another implication of the eigenvalues is that they depend quadratically on the estimated error-correction coefficients. As a higher eigenvalue tends to improve the stationarity of a potential cointegration vector, then it has to be consistent with larger error-correction dynamics. Taking into account the fact that t-statistics of the error-correction coefficients are not normally distributed, the estimated coefficients (not depicted here) suggest clearly a cointegration rank of $r = 2$.

In summary, most tests pointed to a cointegration rank of $r = 2$, while few a others preferred $r = 3$ cointegration relationships. As the cointegration rank divides data into r relations, in which the adjustment to equilibrium takes place, and $p-r$ common stochastic trends, the choice of r will be very influential on whether the subsequent statistical analysis coincides with the expected economic hypothesis. A wrong choice of r can lead to wrong economic interpretations, as it leaves out additional information about the long-run equilibrium properties.

Identification

The test of long-run weak exogeneity is inspired by Johansen (1996), having the following null hypothesis of weak exogeneity: $\alpha = \tilde{H}\alpha_1$, where $\tilde{H}$ is a $4 \times s$ matrix and α_1 is of dimension $s \times r$ of non-zero α-coefficients. As the model contains two common stochastic trends, I must have that $s \geq 2$, so that the number of variables which are adjusting to the long-run relation i has to be larger than or equal to the number of long-run relations. According to table A.3, both the individual and joint tests suggest that r_t^f and OAS_t can be characterized as weakly exogenous, while the remaining variables, which are i_t^h and i_t^m, can as expected be characterized as endogenous variables.

v	$\chi(v)^2$	p-value	i_t^h	i_t^m	r_t^f	OAS_t
2	85.6	0.00	$\otimes$			
2	20.1	0.00		$\otimes$		
2	2.90	0.24			$\otimes$	
2	1.81	0.41				$\otimes$
4	4.06	0.40			$\otimes$	$\otimes$

Table A.3: *Tests of long-run weak exogeneity*

The estimates of the just-identified short-run structure in (25), based on the ordering in (26) are reported in table A.4 to the left. The triangular form of the VAR is exactly identified by $p(p-1)/2$ zero restrictions on A_0 and the transformed covariance matrix. Because residuals are uncorrelated in the VAR system, the OLS estimator is equivalent to FIML, and the coefficients can be estimated efficiently by OLS equation by equation. The lagged variables Δi_{t-1}^h, Δi_{t-1}^m and OAS_{t-1} were found to be insignificant in the whole system, and are logically removed prior to estimation. Empirically, the insignificance of Δi_{t-1}^h and Δi_{t-1}^m is also expected and consistent with the economic theory, as

	$\mathbb{H}_1$				$\mathbb{H}_2$			
	α_1	α_2	β_1	β_2	α_1	α_2	β_1	β_2
i_t^h	$-\ 0$ $(\cdots)$	-0.0383 (-12.6)	0 $(\cdots)$	1 $(\cdots)$	$-\ 0$ $(\cdots)$	-0.0374 (-12.4)	0 $(\cdots)$	1 $(\cdots)$
i_t^m	-0.311 $-(6.8)$	0 $(\cdots)$	1 $(\cdots)$	$-\ 0.877$ $-(55.1)$	-0.28 $(-6.1$	0 $(\cdots)$	1 $(\cdots)$	$-\ 0.873$ (-53.5)
OAS_t	$\cdots$	$\cdots$	$-\ 1.28$ $-(12.5)$	0 $(\cdots)$	$\cdots$	$\cdots$	$-\ 1$ $(\cdots)$	0 $(\cdots)$
r_t^f	$\cdots$	$\cdots$	$-\ 1$ $(\cdots)$	0 $(\cdots)$	$\cdots$	$\cdots$	$-\ 1$ $(\cdots)$	0 $\cdots$
1	$\cdots$	$\cdots$	$-\ 1.11$ $-(23.9)$	0 $(\cdots)$	$\cdots$	$\cdots$	$-\ 1.21$ (-36.2)	0 $(\cdots)$
LR statistic	4.75				11.4			
P-value	0.69				0.18			
Distribution	$\chi^2(7)$				$\chi^2(8)$			

Table A.4: *Identification of the long-run structure without dummy variables.* t-values are based on asymptotic standard errors and shown in parentheses.

all explanatory power of anticipated effects is captured in the adjustment towards the two long-run equilibria $(\hat{\beta})'\tilde{x}_{t-1}$.

The just-identified short-run system is still over-parameterized due to several insignificant estimates. In order to obtain a more parsimonious system that still allows for structural shocks, I subsequently impose over-identifying restrictions without creating any significant correlation in the residuals. Relaxing the $p-1$ zero restrictions on the residual covariance matrix implies that OLS estimation equation by equation is no longer equivalent to maximum likelihood estimation. Hence, the system should be estimated simultaneously subject to the chosen generically over-identifying restrictions via FIML.

In order to minimize the contemporaneous effects, I expand the identification scheme to include directed graph analysis of the covariance structure (Hoover 2005), which aims at uncovering causal links between the current effects that deliver uncorrelated residuals, still keeping the individual equations conditionally independent, similarly to the just-identified triangular system. The parsimonious over-identified system is reported in table A.4 to the right, and all 22 zero restrictions imposed cannot be rejected with a p-value of 0.60.

The correlation of the residuals is reported at the bottom of table A.4 and is negligible and not significantly different from zero. It suggests that the statistical representation of the data is generically identified, where shocks can be interpreted as structural and the response to the variables adjust to well-behaved long-run relationships.

A.4 Additional figures

	Just-identified short-run structure				Over-identified short-run structure			
	Δi_t^h	Δi_t^m	ΔOAS_t	Δr_t^f	Δi_t^h	Δi_t^m	ΔOAS_t	Δr_t^f
Δi_t^h	−1 (···)	− 0 (···)	0 (···)	0 (···)	−1 (···)	− 0 (···)	0 (···)	0 (···)
Δi_t^m	0.016 (0.605)	−1 (···)	0 (···)	0 (···)	0.019 (1.52)	−1 (···)	0 (···)	0 (···)
ΔOAS_t	0.042 (1.03)	0.685*** (5.65)	−1 (···)	0 (···)	0.063*** (2.66)	0.647*** (9.44)	−1 (···)	0 (···)
Δr_t^f	0.031 (1.35)	0.487*** (7.96)	0.022 (0.497)	−1 (···)	0.026 (1.05)	0.56 *** (3.63)	0 (···)	−1 (···)
Δr_{t-1}^f	0.002 (0.121	−0.007 (−0.107)	−0.022 (0.460)	0.297*** (3.32)	0 (···)	0 (···)	0 (···)	0.348*** (5.31)
Δr_{t-2}^f	0.017 (0.940)	0.098* (1.71)	−0.014 −(0.335)	0.061 (0.754)	0 (···)	0 (···)	0 (···)	0 (···)
$(\hat{\beta}_1)\tilde{x}_{t-1}$	0.000 (0.0004)	−0.268*** (−5.91)	0.023 (0.716)	−0.001 (−0.128)	0 (···)	−0.27 *** (−5.07)	0 (···)	0 (···)
$(\hat{\beta}_2)\tilde{x}_{t-1}$	−0.0366*** (−6.44)	0.008 (0.488)	0.004 (0.300)	0.052** (2.16)	−0.0366*** (−8.58)	0 (···)	0 (···)	0 (···)
μ	0.001 (0.086)	−0.004 (−0.322)	−0.011 (−1.29)	−0.05 *** (−2.98)	0 (···)	0 (···)	0 (···)	−0.034 (−2.15)
Test of over-identifying restrictions:								
LR statistic		···				19.721		
P-value		···				0.60		
Distribution		···				$\chi^2(22)$		
Σ (standard errors on the diagonal, off-diagonal elements are standardized correlation of structural residuals)								
Δi_t^h	0.0257				0.0255			
Δi_t^m	0	0.0853			−0.002	0.084		
ΔOAS_t	0	0	0.0615		−0.052	0.019	0.0607	
Δr_t^f	0	0	0	0.1219	0.005	−0.0801	0.041	0.122

Table A.5: *Identification of the short-run structure.* t-values are based on Newey-West standard errors and shown in parentheses. The unrestricted dummy variables are included in the identification and insignificant estimates are in line with the other variables also restricted to zero. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

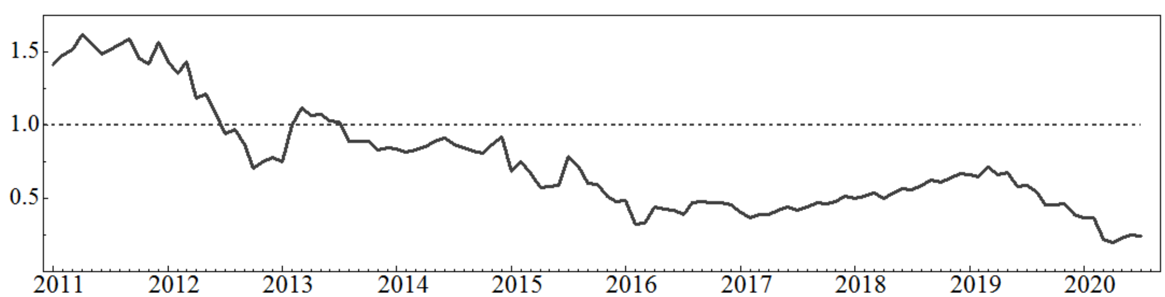
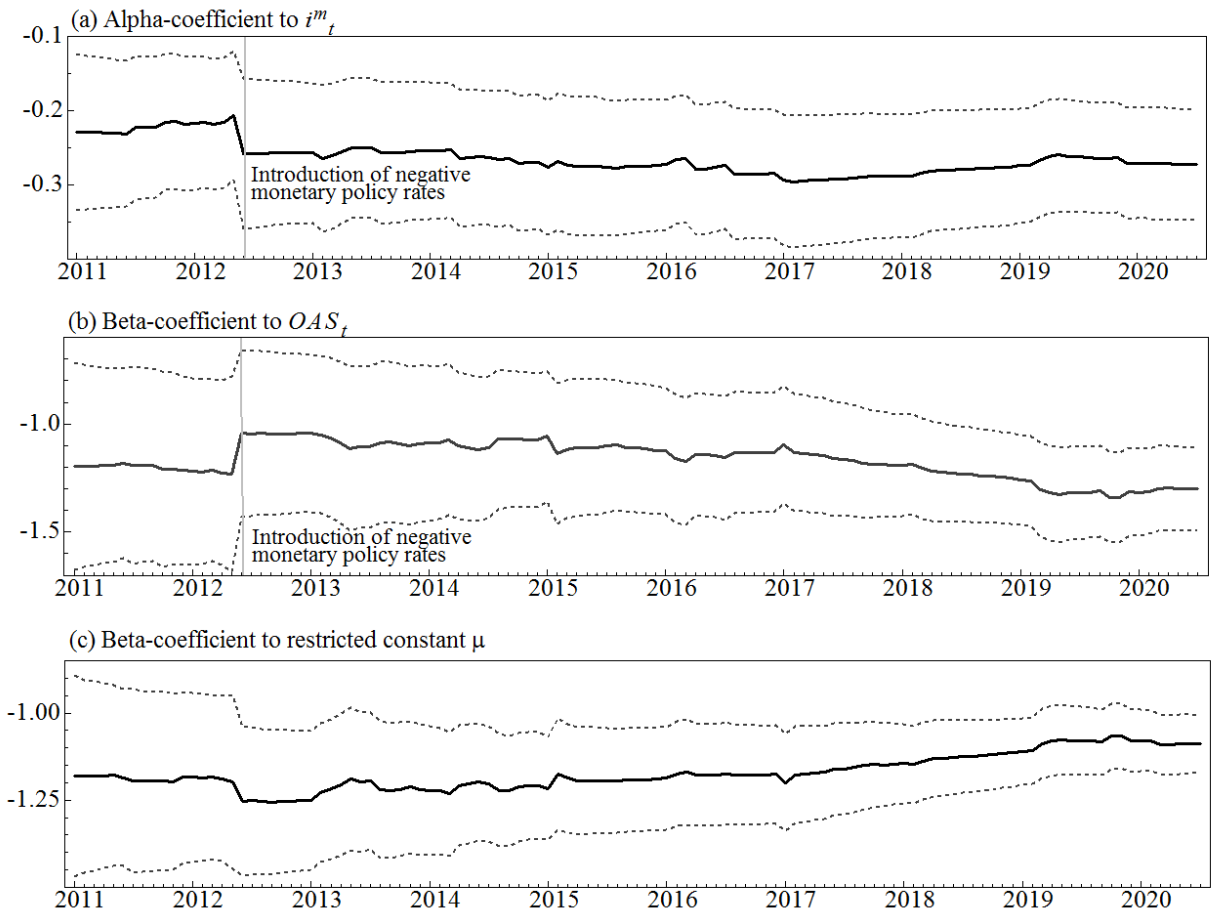


Figure A.1: *Forward recursive LR test*
Note: The recursively calculated LR test is based on the concentrated model and the sub-samples: $t_1 = 2010(12), ..., 2020(7)$. The dashed lines indicate the 95% confidence bands.



A.2: *Forward-recursive tests*

Note: The figure depicts the results of the model for various sample lengths, holding the initial year fixed. The estimation is based on the concentrated model and the sub-samples: $(2008(1) - 2010(12)) - (2008(1) - 2020(7))$. The dashed lines indicate the 95% confidence bands. Panel (a) shows the recursively calculated $\alpha(t_1)$-coefficient to i_t^m in the first long-run cointegration relationships, panel (b) shows the recursively calculated $\beta(t_1)$-coefficient to OAS_t in the first long-run cointegration relationships and, panel (c) shows the recursively calculated $\beta(t_1)$-coefficients to the restricted mean in the first long-run cointegration relationships. In each baseline sample, all short-run parameters are fixed at their full-sample estimates.

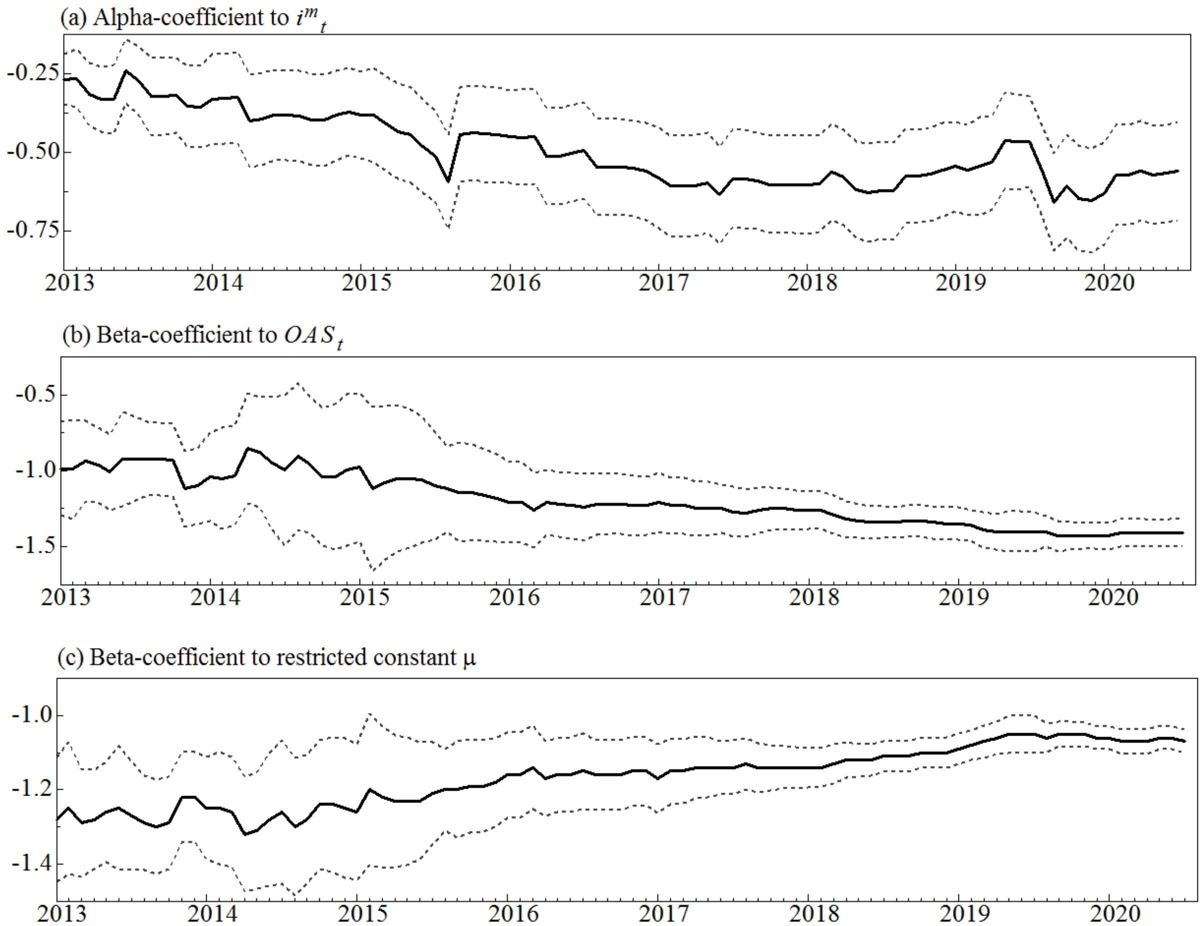


Figure A.3: *Rolling-window estimation*
Note: The figure illustrates the results of the 5-year rolling-window estimation based on the concentrated model. The 5-year samples are: $(2008(1) - 2013(1) - (2005(7) - 2020(7))$ and the dashed lines indicate the 95% confidence bands. Panel (a) shows the rolling-window calculated $\alpha(t_1)$-coefficient to i_t^m in the first long-run cointegration relationships, panel (b) shows the rolling-window calculated $\beta(t_1)$-coefficient to OAS_t in the first long-run cointegration relationships, and panel (c) shows the rolling-window calculated $\beta(t_1)$-coefficients to the restricted mean in the first long-run cointegration relationships. In each sample, all short-run parameters are fixed at their full-sample estimates.

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